Review Article

EEG Signal Analysis Methods Based on Steady State Visual Evoked Potential Stimuli for the Development of Brain Computer Interfaces: A Review

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ABSTRACT

Recently, brain computer interface (BCI) research has increased because of its application value in neural engineering and neuroscience, BCI Systems can provide online communication between a human or animal brain and external devices without depending on the normal output pathways of peripheral nerves and muscles. BCI applications include communication devices for disabled people, neuroprotheses and games. The most popular BCIs is based on steady state visual evoked potential (SSVEP) that can be recognized through detecting the dominant frequency components in the recorded electroencephalography (EEG) signals. BCI performance depends on correctly and fast decoding the user intentions and is critical to employ a reliable signal processing methods to detect and extract the components of de EEG signals recording. In this paper, mathematical tools used to design brain computer interface (BCI) systems based on electroencephalogram (EEG) signals obtain by visual stimulus are reviewed.

Keywords: Brain Computer Interface, BCI; Steady State Visual Evoked Potential, SSVEP; EEG Signal Analysis.

INTRODUCTION

A brain computer interface (BCI) is a system that translates the electrophysiological activity of a nervous system signals that can be measurable by an electromechanical device, generally an electroencephalogram (EEG). The BCI systems that are aimed to provide a channel of non-muscle communication for sending commands to the outside world using the electrical activity of the brain allowing interaction between the human brain and a computer[19].

The electrical activity of the brain generates brain states as a result of different patterns of neural interaction. These patterns result in waves, which are characterized by different amplitudes and frequencies. The human brain electrical activity present due to two causes. The first is internal, that is, due to inadvertent operation, such as control of respiration, digestion etc. and will of the individual, to move your body, speak or think, etc. The second cause of brain activity is the occurrence of external stimuli, through a bodily sense.

The BCI systems can be classified according to the acquisition of signals:

- **Endogenous:** brain rhythms based systems depend on the user's ability to control their electrophysiological activity, such as the EEG amplitude in a specific frequency band over a particular area of the cerebral cortex. We can classify the endogenous systems:

  a) *Motor Imagery* (MI) which are based on a paradigm of two or more kinds of MI, stroke of the right or left hand, feet, tongue, etc., or other brain tasks like rotating a cube, performing arithmetic, etc. These rhythms have variations for both the execution of a real movement to the imagination of a move or preparing to it.

  b) *Slow Cortical Potentials* (SCP), which involves slow changes in voltage, generated on the cerebral cortex, with a variable duration between 0.5 and 10 seconds. They are typically associated with movement and other functions involving cortical activation. It has been shown that people can learn to control these potential[4].

- **Exogenous:** These are based on event-related potentials (ERP) systems depend on the electrophysiological activity evoked by external stimuli and do not require intensive training stage. We can classify exogenous in BCI systems:

  (a) Event-based potentials P300, which refer to a peak amplitude on the EEG approximately 300 ms after a rare auditory or visual stimulus occurred, hence the name P300. Usually the person presents a set of stimuli that only a few are related to the intention of the person. Thus, the stimulus of interest, to be infrequent and be mixed with other more common stimuli cause the appearance of a potential P300 in brain activity of the person. This potential is seen mainly in the central and parietal areas of the cerebral cortex.

  (b) Visual Events Potential (VEP) or *Potential for Visual Events Steady State* (SSVEP), which are detected on the EEG in the visual area of the cerebral cortex after the user a visual stimulus, has been applied. When a person focuses his gaze on a flickering image at a certain frequency, that frequency can be detected by analyzing the spectrum of the EEG signal.

  (c) Auditory Events Potentials (AEP), which are detected on the EEG auditory cortex area presenting the user with sound, sources at different frequencies, the user to focus on one of them and generates potential Systems the same frequency as the stimulus.

This paper focus on the analysis methodologies signals on exogenous BCI systems based on VEP and SSVEP.

EEG SIGNALS

The set of signals derived from an electroencephalogram (EEG) is composed of a series of electrical potentials that fluctuate over time on different channels. Each channel represents an electrode placed on the scalp. Each EEG equipment has a certain sampling rate to quantify, which indicates the number of updates per second that a signal can be output (Figure 1).

To obtain brain signals an electrode grid based on the ***International System Companies EEG 10-20*** is used; system defined for placing electrodes on the scalp, so called because the electrodes are spaced between 10% and 20% of the total distance between points recognizable brain (Figure 2). Reference marks are A1 (nasion) A2 (inion) the rest of the electrodes are identified by a letter indicating the area on which the electrical activity is obtained: "F" (frontal lobe), "P" (parietal lobe), "T" (temporal lobe) "O" (occipital lobe), "C" (frontal lobe), "FP" (frontal pole). The number is used to identify the cerebral hemisphere, odd numbers denotes the electrodes located on the left side and even numbers on the right hand side. The suffix "Z" indicates the centerline of the brain[15], as shown on Figure 3.

EEG interpretation commonly based on the calculation of the amplitude values and measures peak values in registers in event-related potentials (ERP), obtained under conditions of absence of external excitation and of the person, thus, cortical sources ERP assume values different from the spontaneous EEG activity nature. The relationship between the placements of the electrodes on the scalp with different physiological activities of the user[18]. This document shows a review of different techniques focused on visual events (VEP) for the development of Brain Computer Interface systems (BCI)[9] Figure 3.

STEADY STATE VISUAL EVOKED POTENTIAL (SSVEP)

Visual Evoked Potentials (VEP) which appeared in EEG records are the response of human brain for visual stimuli such as strobe, video display and others image screens. VEP is generally used for analyzing the brain function on a visual system[2]. The amplitude of a VEP signal is rather small in comparison with the activity of EEG background signals. Therefore, the stimulus-locked averaging is usually adopted for extracting the VEP components. Conventional averaging method requires enough number of data for an accurate estimation of VEP, though the quality of recorded raw data is not often acceptable[7]. Other studies uses **Steady State Visual Evoked Potentials** (SSVEP) for BCI systems (Friman 2007; Wolpaw, 2000, 2002; Kelly, 2005; Xia, 2013; among others). SSVEPs are oscillatory brain responses produced in the visual cortex by repetitive visual stimulus. SSVEP generally occur in the occipital and parietal lobes[23]. SSVEPs approach allows to collect a large number of trial within a short amount of time[1]. Many studies reported successful integration using VEP or SSVEP stimuli for **brain computer interface** (BCI's) systems. In an SSVEP-BCI system, The targets are encoded by a single frequency or various combinations of frequencies. Different commands can be transmitted by shifting the subject's attention to the coded targets[23]. This document aims to show different methodologies used in several studies of EEG signals for the development of BCIs using different kind of SSVEP stimuli on different experimental design and applications.

SSVEP METHODS

SSVEP had been increasingly used for the development of BCI human-computer communication[2]. The signal is often extracted non-invasively from EEG. Base on the literature, several methodologies had been proposed for classification of features[2,21,6], Table 1.

Assisted Closed Loop (ACL)

The use of closed loop interaction with biological nervous systems for observation and control purposes goes back to the beginnings of electrophysiology in the 1940s when the voltage clamp technique was developed. Later on, the dynamic clamp technology to implement artificial membrane or synaptic conductance has produced many examples of successful closed loop interactions with neural systems at the cellular and circuit levels[5]. ACL approach to optimize the efficiency of SSVEP based BCI which might have a large impact for applied uses, such as computer control and biomedical or prosthetic uses.

ACL is used to select the set of the four top stimulation frequencies by compatibility for each subject in a some experimental context. Stimulation frequencies are defined as valid if their S_f exceeds a prefixed threshold (set to 10) any time during the ongoing visual stimulation. For N valid frequencies, the frequency corresponding to the largest S_f gets an initial score of $s_1(0) = N$, the second to best $s_2(0) = N - 1$, etc. The frequency corresponding to the lowest S_f gets a score of $s_N(0) = 1$. Finally, the four best scores define the selection of the four top stimulation frequencies.

As the next step, we calculate the following compatibility measure between all possible pairs of frequencies x and y taking into account a measure of their distance and their scores:

$$c_{xy}(t) = \alpha\left(s_x(t) + s_y(t)\right) + \beta d_{xy} \qquad (1)$$

t represents the iteration number, α and β weights to the distance and the scores respectively (e.g. $\alpha = 1.5$ and $\beta = 1$), the values for α and β were set empirically based on several trials and d_{xy} is a measure of the distance between two specific frequencies f_x and f_y and is calculated as:

$$d_{xy} = \left|f_x - f_y\right| \qquad (2)$$

The first step is to identify pairs of frequencies with optimal compatibility. This search consists of $3\,N/4$ iterations, each of them divided in to 16 steps with a resting period at its end. The ACL departs from the scores calculated in the scanning procedure $s_1(0), s_2(0), \dots, s_N(0)$: they are modified in the successive iterations to search for the best compatibility. In each iteration, the subject has to follow a sequence of visual stimulus focusing upon. The stimulus frequencies are chosen by selecting $max_{xy}\left(c_{xy}\right)$ at the end of the iteration. To update the scores, take into account both the success rate and the time as:

$$s_x(t) = s_x(t-1)(\delta \cdot SR - \gamma T) \qquad (3)$$

Where SR is the success rate (correct SSVEP the number of possible detections), δ and γ are parameters of the ACL algorithm (e.g. $\delta = 1.2$ and $\gamma = 0.02$). T is the duration of the detection in seconds. The values for δ and γ are chosen based upon the range of SR and T several simulations.

Each $c_{XY}(t)$ is updated by the new scores after each iteration. Once this procedure has run $p = \lfloor 3\,N/4 \rfloor$ times, the highest $c_{xy}(p)$ is selected and a new set is created with the union of both frequencies. The next highest $c_{x'y'}(p)$ disjoint from the previous set is chosen and a new set is constructed. This is repeated $\lfloor N/2 \rfloor$ times because this is the total number of possible disjoint pairs. It is ensured that each set is disjoint from all others. $p = \lfloor 3\,N/4 \rfloor$ is chosen to test $\lfloor 3\,N/2 \rfloor$ frequencies, so that the best frequencies are tested more than once. It is important to note that the duration of the frequency tests has to be restricted.

The second step, the selection of the four frequencies. The same procedure as in the first part is employed, but instead of single frequencies, sets of two frequencies are used. The values of $s_{x'}(p+1)$of each set are adjusted according to the values $c_{xy}(p)$, where $x' = x \cup y$. In this way, the set with the highest value gets $s_{1'}(p+1) = \lfloor N/2 \rfloor$, the second best $s_{2'}(p+1) = \lfloor N/2 \rfloor - 1$and soon. The last one gets $s_{\lfloor N/2 \rfloor'}(p+1) = 1$. From this point of the algorithm on, these sets are indivisible.

Using the same procedure performed with two frequencies, the process is repeated with four of them. The compatibility and the score actualization rules are still the same. The only difference is the distance measure for Equation (1) calculated as:

$$d_{xy} = \frac{\sum_{i=1}^{2k} \sum_{j=1}^{2k} |f_i - f_j|}{2k(2k-1)} \quad (4)$$

Where k is the number of frequencies of each set (in this case 2), and f_i and f_j are the individual frequencies taken from the union of the sets x and y. x and y refer to sets of two frequencies while in Equation (2) x and y referred to individual frequencies. This distance expresses the arithmetic mean of all possible pairs in the set resulting from the union of the initial sets$x$and$y$. Note that for$k = 1$, this distance measure is exactly the same distance (2) as used in the first part of the algorithm. In this second part $\lceil 3N/8 \rceil$ iterations are performed, which is $N/2$ (the number of disjoint sets) times $3/4$ (see above)[5].

Canonical Correlation Analysis (CCA)

CCA method uses channel covariance information, which tends to increase the signal to noise ratio (SNR) and reduce the computational cost for online systems. CCA reflected the correlation relationship between EEG response signals and classical Fourier series at the stimulus frequency and its harmonics. Bin (2009) used CCA algorithm to develop an online BCI system for detecting SSVEP signals without complicated training procedures[2].

CCA is a way of making sense of cross-covariance matrices. If we have two vectors $X = (X_1, \ldots, X_n)$ and $Y = (Y_1, \ldots, Y_m)$ of random variables with finite second moments and there are correlations among the variables. Then canonical-correlation analysis will find linear combinations of$X_i$and$Y_j$which have maximum correlation with each other.
Consider the source signal for SSVEP,X, is the output of linear system with stimulus signal, Y, as the input.Yat certain frequency$f$can be discomposed into Fourier series of its harmonics

$(sin(2\pi ft), cos(2\pi ft), sin(4\pi ft), \ldots)$:

$$Y = \begin{cases} sin\,(2\pi ft) \\ cos\,(2\pi ft) \\ sin\,(4\pi ft) \\ cos\,(4\pi ft) \\ sin\,(6\pi ft) \\ cos\,(6\pi ft) \end{cases} t = \frac{1}{s}, \frac{2}{s}, \ldots \frac{T}{s} \quad (5)$$

Where f is the fundamental frequency T is the number of sampling points and S is the sample rate. The algorithm can find a pair of linear combinations, $x = X^T W_x$ and $y = Y^T W_x$ for X and Y, to maximize the correlation between two canonical variables, $x$and$y$, by solving the following optimization problem:

$$max_{W_x W_y} \rho(x, y) = \frac{E[x^T y]}{\sqrt{E[x^T x] E[y^T y]}} = \frac{E[W_X^T X Y^T W_Y]}{\sqrt{E[W_X^T X X^T W_X] E[W_Y^T Y Y^T W_Y]}} \quad (6)$$

The canonical correlation$\rho$is utilized as the CCA coefficient obtained with the frequency of reference signals[2,8].

Common Spatial Pattern (CSP)

Common Spatial Patterns (CSP) is a

powerful signal-processing technique used for feature extraction in EEG-based BCIs. The CSP algorithm computes spatial filters whose purpose is achieving optimal discrimination when using band power features; thus, it increases the signal-to-noise ratio and reduces adverse effects of volume conduction[1].

CSP maximizes the variance on the spatially filtered signals under one condition while minimizing it for the other condition. CSP was developed as a two-class spatial filtering technique that aims to maximize feature variations for one class and simultaneously minimize feature variations for the other class[16]. CSP analysis is applied to band-pass filtered signals in order to obtain an effective discrimination of mental states between the two conditions. CSP projects the signal $x(t) \in R^C$in the original sensor space to$x_{CSP} \in R^C$, which lives in the surrogate sensor space, as follows:

$$x_{CSP}(t) = W^T x_{(t)} \tag{7}$$

Each column vector $w_j \in R^C, j = 1,2, \ldots, C$of a matrix $A = (W^{-1})^T \in R^{C \times C}$ is a spatialpattern.While for classifications only the special filters are used, only the patterns allow or a physiological interpretation of the CSP components[1].

Distinctive Sensitive Learning Vector Quantization (DSLVQ)

Distinction Sensitive Learning Vector Quantization (DSLVQ) is a modified version of Kohonen's Learning Vector Quantization (LVQ), classifier (1990) it incorporates an additional feature scaling to compensate for relevance differences among the input features[12]. LVQ can be applied to many pattern recognition domains like, e.g., speech recognition Kohonen (1990) or classification of EEG patterns, Flotzinger (1991), (Pregenzer, 1994). Pregenzer (1999) have introduced DSLVQ algorithm, it uses a weighted distance function, which affects the characteristics that often contribute to misclassification, reducing influence of those characteristics that are important for correct classifications.

For the weighted distance function of DSLVQ a global weights vector $w$is used. DS distance show following:

$$DSdist(x, y, w) = \sqrt{\sum_{i=1}^{n} (w_i [x_. - y_i])^2} \tag{8}$$

This vector stores the distinctiveness, the relevance, of every single feature. The weights vector$w$can be seen as a scaling transformation from the original feature space into a DS-feature space. This transformation increases distances for very distinctive features and decreases distances for common features[11].$w$must be updated with every learning iteration. It may be assumed that, for a learning iteration$t$, $m_i(t)$and$m_j(t)$are the two closest vectors to the training sample$x(t)$, that$m_i(t)$and $m_j(t)$belong to the same and to a different class as$x(t)$, respectively, and that$x(t)$falls into the "window" learning. The weights learning are described with the following equation:

$$w(t+1) = norm\left[threshold\left(w(t) + \alpha(t)\left(w_n(t) - w(t)\right)\right)\right] \tag{9}$$

With

$$norm(y) = \frac{y}{\sum_{i=1}^{n} |y_i|}$$
$$w_n = norm\left[d_i(t) - d_j(t)\right] \tag{10}$$
$$d_i(t) = |x(t) - m_i(t)|$$
$$d_j(t) = |x(t) - m_j(t)|$$

The thresholding in the weights update, which cuts all weights values below 0.0001 and over 1 to 0.0001 and 1, respectively, ensures that the weights values cannot become negative. If a large number m vectors is employed, it should be considered that, when the same learning factor is taken and weights learning, the weights vector is updated. m times more often than the average vector. Therefore, in the case of many vectors, a smaller learning factor might be used for weights learning[11].

Empirical Mode Decomposition (EMD)

The EMD approach attempts to sequentially decompose a signal into a finite number of intrinsic mode functions (IMFs). Each IMF represents a simple oscillation signal, is an analytical, self-constructed, well-defined, data-driven, function whose amplitudes and frequencies vary with time[24].The sifting process decomposes each EEG epoch into a set of IMFs by the sifting process through the following steps:

a) Set $\vec{h}(1) = \vec{x}$
b) identify all the local extrema in $\vec{h}^{(1)}$, including local maximum and local minimum;
c) connect all the local maxima/minima using a cubic spline to generate the upper/lower envelopes;
d) generate a local mean curve, $\vec{m}$, by averaging the upper and lower envelopes;
e) calculate the pre-IMF, $\vec{h}^{(2)}$, by subtracting the local mean, $\vec{m}$, from $\vec{h}^{(1)}$, i.e., $\vec{h}^{(2)} = \vec{h}^{(1)} - \vec{m}$
f) Repeat steps (b)–(e) for k iterations until the difference between two continuing pre-IMFs, SD^k, reaches a userdefined stoppage criterion, ε, i.e.

$$SD^k = \frac{\left\|\vec{h}^{(k+1)} - \vec{h}^{(k)}\right\|^2}{\left\|\vec{h}^{(k)}\right\|^2} < \varepsilon \qquad (11)$$

Where $\|\cdot\|$ denotes the Euclidean distance;
g) Set $\vec{c}_1 = \vec{h}^{(k)}$ as the first IMF;
h) Calculate $\vec{r} = \vec{x} - \vec{c}_1$;
i) Replace $\vec{x}$ in step (a) by $\vec{r}$ and repeating steps from (b) to (h)(sifting process), to find other IMFs, $\vec{c}_1, \vec{c}_2, \dots, \vec{c}_j$;
j) Stop the sifting process when the residue function $\vec{r} = \vec{x} - \sum_{j=1}^{J} \vec{c}_j$ becomes a monotonic function that cannot extract any more IMFs.

After applying the EMD process to an EEG epoch, $\vec{x}$ can be represented by a monotonic residue function, $\vec{r}$, plus a set of posteriori-defined IMF basis, $\vec{c}_1, \vec{c}_2, \dots$ and , $\vec{c}_j$, where J is the number of IMFs extracted from $\vec{x}$ and each $\vec{c}_k$, $1 \le k \le J$, is a $1 \times N$ vector. The IMFs can be arranged in a $J \times N$ matrix, C, where each row $\vec{c}_k$ represents the kth IMF:

$$C = \begin{bmatrix} \vec{c}_1 \\ \vec{c}_2 \\ \vdots \\ \vec{c}_j \end{bmatrix}_{J \times N} \qquad (12)$$

Least Absolute shrinkage and selection operator (LASSO)

Proposed by Tibshirani (1996), this method can provide an analytical solution and a low-variance estimate with high interpretability for a linear regression due to its sparsity constraint. LASSO method was applied to recognize SSVEP signals to archive the better effect than of CCA in a short time window. Can provide an analytical solution and a low-variance estimate with high interpretability for a linear regression due to its sparsity constraint[2].

Consider a standard linear regression model for the observations of the responsey $\in R^n$

$$y = X\beta + \varepsilon \qquad (13)$$

Where y is a $n \times 1$ vector, $X = (x_1, x_2, \dots, x_p)$ denotes a $n \times p$ design matrix, and ε represents a noise vector with the zero mean and constant variance. The LASSO estimate is then given by:

$$\hat{\beta} = arg_{\beta} min \left(\|y - X\beta\|_2^2 + \lambda\|B\|_1\right) \qquad (14)$$

Where $\|\cdot\|_1, \|\cdot\|_2$ denote the $l_1 - norm$ and $l_2 - norm$ respectively. λ is a penalty parameter which encourages a sparse solution $\hat{\beta}$.Quadratic programming can solve the optimization problem depicted by Eq. (6)[21].

Minimum Energy Combination (MEC)

Minimum Energy Combination is derived from the principal component analysis (PCA)[10]. Nan (2011), Volosyak (2010) and Friman (2007) used MEC method in his research's. Assuming N_y electrodes, N_t is data length is and N_h the number of harmonics. The formula $Y = XA + B$ represent the linear EEG signal modeling. A contains all the amplitudes for all electrode signals, X is SSVEP information matrix of size $N_t \times 2N_h$, B is the noise, artifacts and all the information that are not relevant to the SSVEP response.

The different electrodes signals must be combined into a channel in order to extract discriminate features. Define

$$s = \sum_{i=1}^{N_y} w_i y_i = Yw \qquad (15)$$

s is a channel signal defined as a linear combination of each electrode signal yi_i and several sets of weights, $w$can be used to create several channels.

More generally, we can create several channels by making different combinations of the original electrode signals

$$S = YW \qquad (16)$$

Where W is a $N_y \times N_s$ matrix containing the weights for each combination in its columns. The optimal choice of weight matrix Wdepends on the nature of the SSVEP signal. The noise signals can be canceled as much as possible by combining of the electrode signals. Firstly, remove any potential SSVEP components from all the electrode signals by using the orthogonal projection

$$Y_1 = Y - X(X^TX)^{-1}X^TY \qquad (17)$$

Y_1 contains approximately only noise, artifacts and background activity. The weight vector W which minimizes the variance of Y_1 can be found by optimizing:

$$min\|Y_1W\|^2 = min(W^TY_1^TY_1W) \qquad (18)$$

Which has the solution in the eigenvector that corresponds with the smallest eigenvalue of the covariance of Y_1. In order to increase the robustness, not only the eigenvector of the smallest eigenvalue but also those eigenvectors of the next largest eigenvalues are utilized here. About 10% of the variance of the date is included to construct the spatial filter[10].

The SSVEP signal power estimation is defined as

$$\hat{P} = \frac{1}{N_sN_h}\sum_{i=1}^{N_s}\sum_{k=1}^{N_h}\|X_k^Ts_i\|^2 \qquad (19)$$

EEG signals from multiple channels are calculated by the above steps and then the stimulus frequency corresponding to the maximum signal power is obtained[6,10,17].

Multivariate synchronization index (MSI)

Zhang(2014) proposed a Multivariate synchronization index (MSI) for frequency recognition. This measure characterized the synchronization between multichannel EEGs and the reference signals, the latter of which were defined according to the stimulus frequency. The MSI must also create a reference signal from the stimulus frequencies used in an SSVEP-based BCI system, similarly to CCA and MEC[23].

Consider a EEG data set $X \in R^{M\times P}$(M channels $\times$ P temporal points). To implement the MSI for SSVEP recognition, construct a reference signal set $Y \in R^{2H\times P}$at a certain stimulus frequency f as

$$R = \begin{bmatrix} sin\,(2\pi f t) \\ cos\,(2\pi f t) \\ \vdots \\ sin\,(2\pi H f t) \\ cos\,(2\pi H f t) \end{bmatrix}, t = \frac{1}{f_8}, \frac{2}{f_8}, \cdots \frac{P}{f_8} \qquad (20)$$

Where H denotes the number of used harmonics and f_s is the sampling rate. Assuming both the X and Y have been normalized to have zero means and unit variances, the auto-covariance and cross-covariance matrices can be respectively estimated as

$$\begin{aligned} C_{xx} &= \frac{1}{P} X X^T \\ C_{yy} &= \frac{1}{P} Y Y^T \\ C_{xy} &= C_{yx} = \frac{1}{P} X Y^T \end{aligned} \qquad (21)$$

The cross-correlation matrix without autocorrelation is then computed as

$$G = \begin{bmatrix} I_{M\times M} & C_{xx}^{-\left(\frac{1}{2}\right)} C_{xy} C_{yy}^{-\left(\frac{1}{2}\right)} \\ C_{yy}^{-\left(\frac{1}{2}\right)} C_{yx} C_{xx}^{-\left(\frac{1}{2}\right)} & I_{2H\times 2H} \end{bmatrix} \qquad (22)$$

Where $I_{M\times M}$ denotes the identity matrix of dimension M. The synchronization index between X and Y can be estimated as

$$s = 1 + \frac{\sum_{i=1}^{M+2H} \lambda_i log\,(\lambda_i)}{log\,(N+2H)} \qquad (23)$$

Where λ_i is the $i-th$ normalized eigenvalue of matrix G. Assume there are K stimulus frequencies to be recognize. Through constructing the reference signal set at each of the stimulus frequency, we estimate the synchronization indices $s_1, s_2, \dots, s_k$ for all of the K stimulus frequencies[23]. The SSVEP target frequency is then recognized by

$$f_t = argmax_{f_k} s_k, k = 1,2, \dots, K \qquad (24)$$

Power Spectral Density Analysis (PSDA)

The PSDA method is often used as a method of SSVEP detection, which is related to signal processing in frequency domain. The implementation of the power spectral density analysis is performed by looking at the power densities around the stimulus frequencies and obtaining a signal/noise ratio as

$$S_k = 10 log_{10}\left(\frac{nP(f_k)}{\sum_{m=1}^{\frac{n}{2}} P(f_k + m f_{res}) + P(f_k - m f_{res})}\right) \qquad (25)$$

Where n is the number of points near at the frequency stimulus, $P(f_k)$ is the power density of the stimulus frequencies and f_{res} is the resolution frequency, which depends on the number of samples used in the Fourier transform. $P(f_k + m f_{res})$ and $P(f_k - m f_{res})$ are power densities around the target frequency[3].

Stability Coefficient (SC)

The SSVEP signals can be divided into two parts, one part is caused by the repetitive stimulus, which can be considered approximately as a constant, for this part, the amplitude difference between near time points is approximately zero. The other part comes from the spontaneous EEG, and the amplitude difference between the near temporal points may be not zero, it may vary with time. For the existence of the variable part, the amplitude difference between near time points in SSVEP may be not zero, and it may vary with time but not so violently as that in spontaneous EEG. Wu (2008) define a parameter called the ***stability coefficient*** (SC), which is the absolute ratio between the amplitude difference and the amplitude sum of two near temporal points. From this definition, it is clear that the average SC of SSVEP is smaller than that of the corresponding frequency in the spontaneous EEG within a given period; the smaller the SC, the more stationary the SSVEP.

The SC of two near time points is defined as

$$SC = \frac{|N_a - N_b|}{(N_a + N_b)} \quad (26)$$

Where N_a or N_b is the SSVEP amplitude obtained by wavelet analysis at two near sample points. For a period SC is defined as

$$SC = \frac{\sum_{n=2}^{m}\left|\frac{|N_n - N_{(n-1)}|}{N_n + N_{(n-1)}}\right|}{(m-1)} \quad (27)$$

Where m is the total of sample point in this period, and N_1 to N_n mean the SSVEP amplitude calculated by wavelet analysis at each sample point. The average SC of the fundamental frequency in two minutes SSVEP was computed independently for each electrode, and the electrode, which had the smallest SC under each stimulus, was selected as the signal electrode[24].

Sequence Detection (SD)

Consider the EEG signal is divided into several subsequences. The subsequence consists of EEG data within a time window (TW) and slides on a sliding window (MW) between consecutive subsequences (Fig. 4). In a subsequence, the CCA coefficients ρ are compute first. Then the instantaneous probability ratio of $Pr_i$of the stimulus frequency i is calculated as

$$Pr_i = \frac{\rho_i}{M} \quad (28)$$

where M is defined as

$$M = \frac{\sum_n \rho_j}{n} j = 1,2,\ldots,n \quad (29)$$

and n is the number of stimulus frequencies. After m subsequences, the SD coefficient S_j^m, which denotes the probability ratio of thestimulus frequency f, can be formulated as

$$S_j^m = Pr_f^1 \times Pr_f^2 \times \ldots \times Pr_f^{m-1} \quad (30)$$

A defined threshold of SD, T, is used for making a decision. If $S_f^m \geq T$, the stimulus frequency f is determined as the target frequency. If $S_f^m < T$, the SD is continued to compute the S_f^{m+1} in the next subsequence[2].

COMPARISON BETWEEN METHODS

Table 2 shows a comparative of methodologies and algorithms presented above based on its main features.

Other important features to determine the performance of BCI systems based SSVEP are the Signal Noise Radio (SNR) and the Information Transfer Rate (ITR), which are defined as

$$SNR = 10log\frac{P_{signal}}{P_{noise}} = 10log\frac{\left(\frac{A}{\sqrt{2}}\right)}{\sigma^2} \quad (31)$$

Where P_{signal} and $P_{noise}$are de power of the signal and the power of the noise, respectively. A is the amplitude of de signals and σ^2 is the variance of the noise[10,13,21,22,23].

$$ITR = \left[log_2 N + Plog_2 P + (1 - P)log_2\left(\frac{1-P}{N-1}\right)\right] N_T \quad (32)$$

Where N is the possible targets in a SSVEP-BCI system, P is t he probability that the desired choice will be selected by the users; and $N_T$is the number of correct commands per minute[2,3,5,8,10,21-23].

DISCUSSION

Develop techniques to improve the performance of BCI systems is priority for researchers in this area. Future work in the area of BCI systems should be based on improving their performance as well the SNR and ITR. Knowledge of the techniques used currently open possibility of such techniques wet or creating new paradigms. These challenges have led to develop hybrid techniques (BCI-Hybrids) which aims to use the combination of neuro-mecanism based SSVEP and some others, as motor imagery (MI) or P300, it is considered that the combination of these elements opens up the possibility of greater BCI accuracy in systems claiming control devices. Hybrid Systems BCI techniques may be revised in the future.

CONCLUSION

In this paper a review of the main methods for analyzing EEG signals for the development of BCI systems based on SSVEP performed. Techniques for single-channel and multichannel signals analysis are presented, and the use of different experiments as blinking and performing tasks for the evaluation of the techniques review. The technical explained, extracted main signal characteristics and have different ways of performing wet BCI systems.

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Table 1. List of the most used methodologies for the analysis of signals SSVEP-based BCI

Method	Author(s)	Year
Assisted Closed Loop (ACL)	Fernandez-Vargas *et al*	2013
Canonical Correlation Analysis (CCA)	Lin *et al* Bin *et al* Wenya *et al* Zhang *et al* Castillo-Garcia *et al* Jian *et al* Cao *et al*	2006 2009 2011 2012 2014 2014 2015
Common spatial patern (CSP)	Song *et al* Acqualagna *et al*	2013 2015
Distinctive Sensitive Learning Vector Quantization (DSLVQ)	Müller-Putz *et al*	2005, 2008
Empirical Mode Decomposition (EMD)	Chi-Hsun *et al*	2011
Least absolute shrinkage and selection operator (LASSO)	Zhang *et al* Cao *et al*	2012 2015
Minimum Energy Combination (MEC)	Friman *et al* Volosyak *et al* Wenya *et al*	2007 2010 2011
Multivariate synchronization index (MSI)	Zhang *et al*	2014
Power Spectral Density-Based Analysis (PSDA)	Liavas *et al* Middendorf *et al* Gao *et al* Lalor *et al* Mukesh *et al* Castillo-Garcia *et al*	1998 2000 2003 2005 2006 2014
Stability Coefficient (SC)	Nan *et al*	2008
Sequence detection (SD)	Wald Cao *et al*	1943 2015

Table 2. Describe SSVEP Methods, their critical properties and features

Method	Analysis Type	Feature	Advantage	Disadvantage
Assisted Closed Loop (ACL)	Single Channel	• Optimize parameters of the stimuli • Improve real time BCI systems	• Adaptive to user differences • Improve ITR[1] performance	
Canonical Correlation Analysis (CCA)	Multichannel	• method that considers a periodic pattern with the same frequency as the stimulus frequency • measure the correlations between the brain signals and the given stimuli frequencies • obtains the maximum similarity between two data sets		• has lower deviation, • higher detection accuracy and higher SNR[2] • Require fixed TWL[3] for estimation of the dominant frequency components
Common Spatial Patern (CSP)	Multichannel	• spatial filtering technique that maximizes the variance of band-passed EEG signals • maximizing the variance on the spatially filtered • band-pass filtered signals in order to obtain an effective discrimination of mental states between the two conditions	• efficient tool to identify discriminative spatial brain activity • BCI classification tasks	
Distinctive Sensitive Learning Vector Quantization	Single Channel	• feature extraction based on spectral information	• Uses strategy to divide a classification problem into	

[1] Information Transfer Rate
[2] Signal to Noise Ratio
[3] Time Window Length

(DSLVQ)			subproblems and to find an optimal linear approximation for each subproblem.	
Empirical Mode Decomposition (EMD)	Single Channel	• used to extract time-frequency information from a nonlinear and nonstationary signal • efficient method of analyzing nonlinear and non-stationary data.	• Noise reduction	
Least Absolute Shrinkage and Selection Operator (LASSO)	Single Channel	• analytical solution • low-variance estimate • high interpretability for a linear regression		
Minimum Energy Combination (MEC)	Multichannel	• Combines multiple electrode signals to less number of channels in order to cancel noise as much as possible	• high detection accuracy • high SNR • no calibration data for noise estimation • remove noises in multichannel data	• Require fixed TWL for estimation of the dominant frequency components
Multivariate synchronization index (MSI)	Multichannel	• estimate the synchronization between the actual mixed signals and the reference signals as a potential index for recognizing the stimulus frequency		• Require fixed TWL for estimation of the dominant frequency components
Power Spectral Density-Based Analysis (PSDA)	Multichannel	• looking the power densities around the stimulus frequencies and obtaining a SNR	Improve de average classification accuracy	• Require fixed TWL for estimation of the dominant frequency components • Sensitive external noise • Require calibration methods • High computational cost

Stability Coefficient (SC)	Single Channel		• Frequency recognition within a short time period.	
Sequence detection (SD)	Multichannel	• Improve the performance of SSVEP recognition • Improve occurrence selection	• High SNR	• High ITR

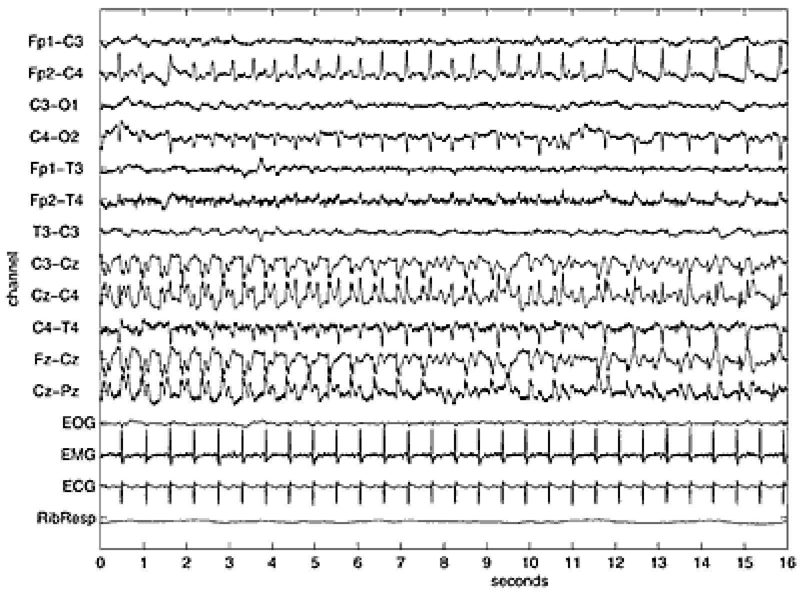

Figure 1. Set of EEG signals, 16 channels for 16 seconds sampling[25]

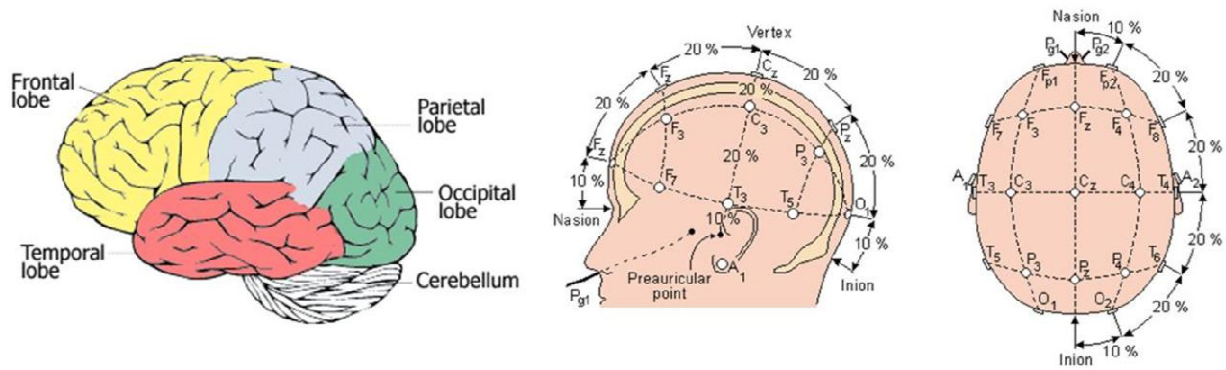

Figure 2. Identifying parts of the brain and location of electrodes in the International System 10-20 EEG Companies[14].

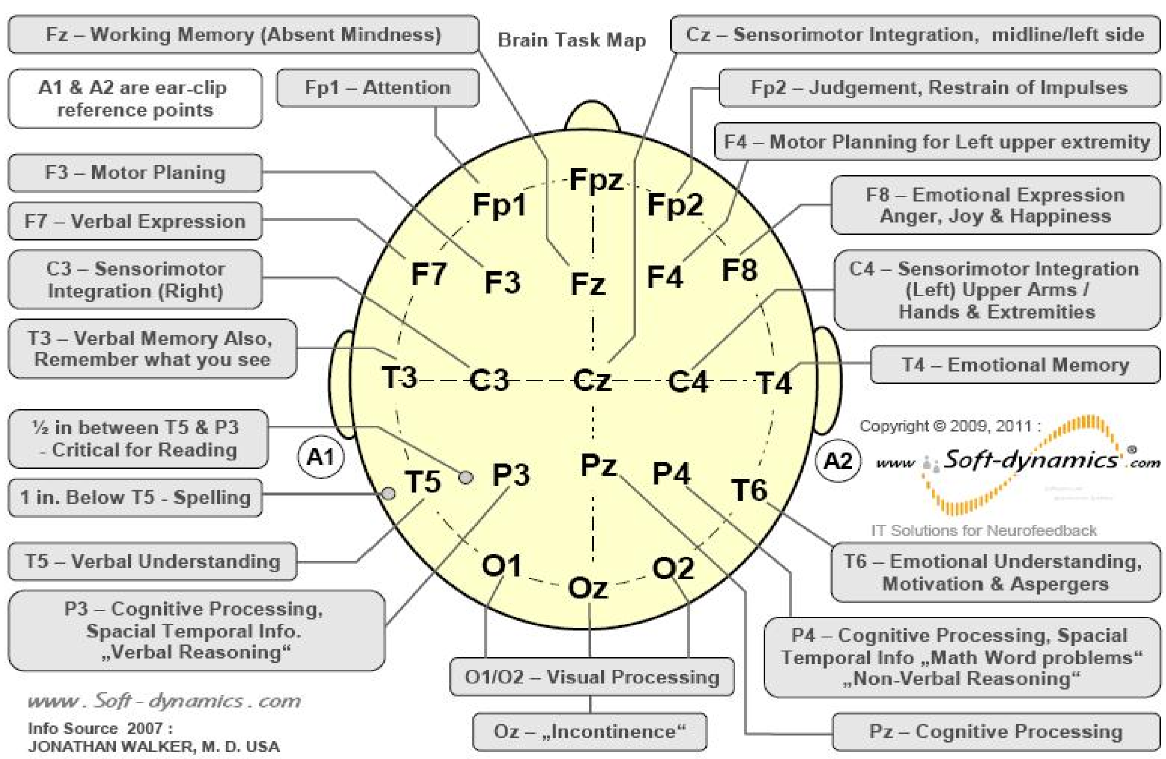

Figure 3. Identification of electrodes international 10-20 system and Map of brain activities with the location of the electrodes on the scalp[18].

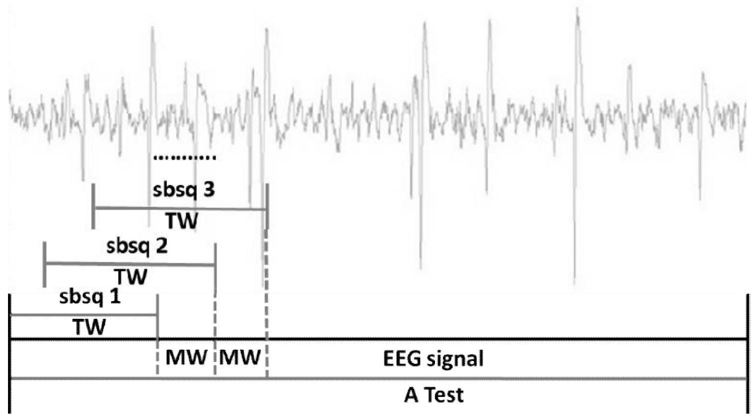

Figure 4. EEG signal with SSVEP stimuli. The test is form by subsequences ($sbsq_i$) within a time window (TW) and it is a slid with a moving window (MW)[2].