Mean temperature of a planet using orbital settings and star properties

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Abstract

To achieve the goal of creating procedural solar systems with realistic properties, it is necessary to compute the temperature of each planet so that it can influence its geological generation, enabling the differenciation between frozen world from toasted ones from habitables ones. The approach I will use here is to use a planet's energy budget to compute its average temperature using only the physical properties of its star such as its temperature and radius and using the properties of the planet : its distance to the star, its albedo, and greenhouse effect.

Symbols

Here are the variables that we will be using :
- T_{star} is the temperature of the star in K
- R_{star} is the radius of the star in m
- T_{planet} is the temperature of the planet in K
- R_{planet} is the radius of the planet in m
- D is the distance between the planet and its star in m
- A is the albedo of the planet
- α is the greenhouse effect factor
- σ is the Stefan-Boltzmann constant

Hypothesis

The star will be assimilated to a perfect black body for the entirety of the demonstration.

1 The energy emitted by the star

The total energy emitted by the star can be obtained using Stefan-Boltzmann law to compute the energy per unit of surface ($\Phi_{star} = \sigma T_{star}^4$). Then we can multiply by the surface of the sun ($4\pi R_{star}^2$) :

$$P_{star} = \Phi_{star} \times S_{star}$$

$$P_{star} = 4\pi\sigma R_{star}^2 T_{star}^4$$

2 The energy received by the planet

The planet we are studying will only receive part of this energy. As the planet is at a distance D of the star, the energy of the star will be spread across the emission sphere centered on the star, whose radius is D.

Therefore we have to compute the per surface energy received by the planet at the distance D. We can then find the amount of energy received by the planet by multiplying by the surface of the apparent disk of the planet.

$$P_{received} = \frac{P_{star}}{S_{emission}} S_{planetdisk}$$

$$P_{received} = \frac{4\pi\sigma R_{star}^2 T_{star}^4}{4\pi D^2} \pi R_{planet}^2$$

$$P_{received} = \frac{\pi\sigma R_{star}^2 R_{planet}^2 T_{star}^4}{D^2}$$

3 Accounting for the albedo

Part of this energy is reflected into space because of the natural color of the planet. This phenomenon is called albedo, which represents the percentage of energy reflected into space. Knowing that we can compute the amount of energy absorbed by the planet :

$$P_{absorbed} = (1 - A) P_{received}$$

$$P_{absorbed} = (1 - A) \frac{\pi\sigma R_{star}^2 R_{planet}^2 T_{star}^4}{D^2}$$

4 Greenhouse effect

The energy absorbed is then reemitted by infrared radiation using the Stefan-Boltzmann law we used during part 1. We have already taken account of the albedo so we can consider all the remaining energy is absorbed, thus we do not need to specify the emissivity of the planet in the Stefan-Boltzmann formula :

$$P_{reemitted} = P_{absorbed} = \sigma T_{planet}^4 \times 4\pi R_{planet}^2$$

A fraction α of this reemitted energy will be sent back to the planet's surface because of the greenhouse effect.

We can then write the complete energy budget of the planet using this model :

$$P_{reemited} = P_{absorbed} + P_{greenhouse}$$

$$P_{greenhouse} = \alpha P_{reemited}$$

$$P_{absorbed} + \alpha P_{reemited} = P_{reemited}$$

$$\frac{1-A}{1-\alpha} P_{received} = P_{reemited}$$

5 Calculating the planet's average temperature

Using the relation we found, we can extract the temperature of the planet :

$$\frac{1-A}{1-\alpha} \times \frac{\pi\sigma R_{star}^2 R_{planet}^2 T_{star}^4}{D^2} = 4\pi\sigma R_{planet}^2 T_{planet}^4$$

$$\frac{1-A}{1-\alpha} \times \frac{R_{star}^2 T_{star}^4}{D^2} = 4T_{planet}^4$$

$$T_{planet}^4 = \frac{1-A}{1-\alpha} \times \frac{R_{star}^2}{4D^2} T_{star}^4$$

$$\boxed{T_{planet} = T_{star} \times \sqrt[4]{\frac{(1-A)R_{star}^2}{4(1-\alpha)D^2}}}$$

6 Testing the formula

Let's test this formula for the earth and the sun. We have $T_{star} = 5770K$, $R_{star} = 696 \times 10^6 m$, $D = 150 \times 10^9 m$, $A = 0.3$, $\alpha = 0.4$.

$$T_{planet} = 289K = 16°C$$

Close enough ! Moreover this simple model can also show how greenhouse effect can have a huge impact on the average temperature of earth : by changing α from 0.4 to 0.42, we already increase the temperature by more than two degrees on the surface of the planet !

7 Usability in procedural generation

The generation of procedural solar systems can be boiled down to the choice of the different physical parameters of the planets and the stars. As we have large amounts of data on stars like their radius and temperatures, generating these parameters in a realistic way is achievable. Therefore knowing T_{star} and R_{star} is not problematic.

The distance of the planet to its star is someting that can be chosen using a random range during the generation of the system, consequently we know the value of D.

The difficulties arise when talking about α and A. Indeed, the two parameters are interdependent, making the generation process difficult. An increase of temperature caused by an increase in α can for example reduce the amount of ice on a planet, thus reducing its albedo A, thus decreasing the temperature of the planet.

One way to avoid this complication could be choosing first the temperature of the planet in a reasonable range and then derivating the albedo and the the greenhouse effect, but it would defeat the purpose of the formula which was to compute the temperature.