The FMB Algorithm

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Abstract

This paper introduces how to perform intersection detection of pair of static/dynamic cuboid/tetrahedron in 2D/3D by using the Fourier-Motzkin elimination method.

Contents

Introduction

This paper introduces the FMB (Fourier-Motzkin-Baillehache) algorithm which can be used to perform intersection detection of moving and resting parallelepipeds and triangles in 2D, and cuboids and tetrahedrons in 3D.

The detection result is returned has a boolean (intersection / no intersection), and if there is intersection a bounding box of the intersection.

The two first sections introduce how the problem can be expressed as a system of linear inequation, and its resolution using the Fourier-Motzkin method.

The algorithm of the solution and its implementation in the C programming language are detailed in the four following sections.

The last two sections introduce the validation and qualification in term of relative performance of the FMB algorithm against the SAT algorithm.

1 The problem as a system of linear inequations

1.1 Notations and definitions

- $[M]_{r,c}$ is the component at column c and row r of the matrix M
- $[V]_r$ is the r-th component of the vector $\overrightarrow{V}$
- the term "frame" is used indifferently for parallelepiped, triangle, cuboid and tetrahedron.

1.2 Static case

The two Frames are represented as a vector origin and a number of component vectors equal to the dimension D of the space where live the Frames. Each vector is of dimension equal to D.

Lets call $\mathbb{A}$ and $\mathbb{B}$ the two Frames tested for intersection. If A and B are two cuboids:

$$\mathbb{A} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{A}}.\overrightarrow{X} \end{array} \right\} \tag{1}$$

$$\mathbb{B} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \overrightarrow{O_{\mathbb{B}}} + C_{\mathbb{B}}.\overrightarrow{X} \end{array} \right\} \tag{2}$$

where $\overrightarrow{O_{\mathbb{A}}}$ is the origin of $\mathbb{A}$ and $C_{\mathbb{A}}$ is the matrix of the components of A (one component per column). Idem for $\overrightarrow{O_{\mathbb{B}}}$ and $C_{\mathbb{B}}$.

If $\mathbb{A}$ and $\mathbb{B}$ are two tetrahedrons:

$$\mathbb{A} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{A}}.\overrightarrow{X} \end{array} \right\} \tag{3}$$

$$\mathbb{B} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ \overrightarrow{O_{\mathbb{B}}} + C_{\mathbb{B}}.\overrightarrow{X} \end{array} \right\} \tag{4}$$

I'll assume the Frames are well formed, i.e. their components matrix is invertible. It is then possible to express $\mathbb{B}$ in $\mathbb{A}$'s coordinates system, noted as $\mathbb{B}_{\mathbb{A}}$. If $\mathbb{B}$ is a cuboid:

$$\mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ C_{\mathbb{A}}^{-1}.(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}) \end{array} \right\} \tag{5}$$

If $\mathbb{B}$ is a tetrahedron:

$$\mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ C_{\mathbb{A}}^{-1}.(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}) \end{array} \right\} \tag{6}$$

$\mathbb{A}$ in its own coordinates system becomes, for a cuboid:

$$\mathbb{A}_{\mathbb{A}} = \left\{ \overrightarrow{X} \in [0.0, 1.0]^D \right\} \tag{7}$$

and for a tetrahedron:

$$\mathbb{A}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \end{array} \right\} \tag{8}$$

The intersection of $\mathbb{A}$ and $\mathbb{B}$ in $\mathbb{A}$'s coordinates sytem, can then be expressed as follow.

If $\mathbb{A}$ and $\mathbb{B}$ are two cuboids:

$$\left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}\right) \cap [0.0, 1.0]^D \end{array} \right\} \tag{9}$$

If $\mathbb{A}$ is a cuboid and $\mathbb{B}$ is a tetrahedron:

$$\left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}\right) \cap [0.0, 1.0]^D \end{array} \right\} \tag{10}$$

If $\mathbb{A}$ is a tetrahedron and $\mathbb{B}$ is a cuboid:

$$\left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}\right) \cap [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} \left[C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}\right)\right]_i \leq 1.0 \end{array} \right\} \tag{11}$$

If $\mathbb{A}$ and $\mathbb{B}$ are two tetrahedrons:

$$\left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ C_{\mathbb{A}}^{-1}.(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}) \cap [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} \left[C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}\right)\right]_i \leq 1.0 \end{array} \right\} \tag{12}$$

These can in turn be expressed as systems of linear inequations as follows, given the two shortcuts $\overrightarrow{O_{\mathbb{B}_{\mathbb{A}}}} = C_{\mathbb{A}}^{-1}.(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}})$ and $C_{\mathbb{B}_{\mathbb{A}}} = C_{\mathbb{A}}^{-1}.C_{\mathbb{B}}$.

If $\mathbb{A}$ and $\mathbb{B}$ are two cuboids:

$$\left\{ \begin{array}{rcl} [X]_0 & \leq & 1.0 \\ & \ldots & \\ [X]_{D-1} & \leq & 1.0 \\ -[X]_0 & \leq & 0.0 \\ & \ldots & \\ -[X]_{D-1} & \leq & 0.0 \\ \sum_{i=0}^{D-1} [C_{\mathbb{B}_{\mathbb{A}}}]_{0,i} \cdot [X]_i & \leq & 1.0 - [O_{\mathbb{B}_{\mathbb{A}}}]_0 \\ & \ldots & \\ \sum_{i=0}^{D-1} [C_{\mathbb{B}_{\mathbb{A}}}]_{D-1,i} \cdot [X]_i & \leq & 1.0 - [O_{\mathbb{B}_{\mathbb{A}}}]_{D-1} \\ -\sum_{i=0}^{D-1} [C_{\mathbb{B}_{\mathbb{A}}}]_{0,i} \cdot [X]_i & \leq & [O_{\mathbb{B}_{\mathbb{A}}}]_0 \\ & \ldots & \\ -\sum_{i=0}^{D-1} [C_{\mathbb{B}_{\mathbb{A}}}]_{D-1,i} \cdot [X]_i & \leq & [O_{\mathbb{B}_{\mathbb{A}}}]_{D-1} \end{array} \right. \tag{13}$$

If $\mathbb{A}$ is a cuboid and $\mathbb{B}$ is a tetrahedron:

$$\left\{\begin{array}{rcl} -[X]_0 & \leq & 0.0 \\ \ldots & & \\ -[X]_{D-1} & \leq & 0.0 \\ \sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{0,i}.[X]_i & \leq & 1.0-[O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ \sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{D-1,i}.[X]_i & \leq & 1.0-[O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ -\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{0,i}.[X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ -\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{D-1,i}.[X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ \sum_{i=0}^{D-1}[X]_i & \leq & 1.0 \end{array}\right. \tag{14}$$

If $\mathbb{A}$ is a tetrahedron and $\mathbb{B}$ is a cuboid:

$$\left\{\begin{array}{rcl} [X]_0 & \leq & 1.0 \\ \ldots & & \\ [X]_{D-1} & \leq & 1.0 \\ -[X]_0 & \leq & 0.0 \\ \ldots & & \\ -[X]_{D-1} & \leq & 0.0 \\ -\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{0,i}.[X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ -\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{D-1,i}.[X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ \sum_{j=0}^{D-1}\left(\left(\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{j,i}\right).[X]_i\right) & \leq & 1.0-\sum_{i=0}^{D-1}[O_{\mathbb{B}_\mathbb{A}}]_i \end{array}\right. \tag{15}$$

If $\mathbb{A}$ and $\mathbb{B}$ are two tetrahedrons:

$$\left\{\begin{array}{rcl} -[X]_0 & \leq & 0.0 \\ \ldots & & \\ -[X]_{D-1} & \leq & 0.0 \\ -\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{0,i}.[X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ -\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{D-1,i}.[X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ \sum_{i=0}^{D-1}[X]_i & \leq & 1.0 \\ \sum_{j=0}^{D-1}\left(\left(\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{j,i}\right).[X]_i\right) & \leq & 1.0-\sum_{i=0}^{D-1}[O_{\mathbb{B}_\mathbb{A}}]_i \end{array}\right. \tag{16}$$

1.3 Dynamic case

If the frames $\mathbb{A}$ and $\mathbb{B}$ are moving linearly along the vectors $\overrightarrow{V_\mathbb{A}}$ and $\overrightarrow{V_\mathbb{B}}$ respectively during the interval of time $t \in [0.0, 1.0]$, the above definition of

the problem is modified as follow.

If A and B are two cuboids:

$$\mathbb{A} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{A}}.\overrightarrow{X} + \overrightarrow{V_{\mathbb{A}}}.t \end{array} \right\} \tag{17}$$

$$\mathbb{B} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ \overrightarrow{O_{\mathbb{B}}} + C_{\mathbb{B}}.\overrightarrow{X} + \overrightarrow{V_{\mathbb{B}}}.t \end{array} \right\} \tag{18}$$

where $\overrightarrow{O_{\mathbb{A}}}$ is the origin of $\mathbb{A}$ and $C_{\mathbb{A}}$ is the matrix of the components of A (one component per column). Idem for $\overrightarrow{O_{\mathbb{B}}}$ and $C_{\mathbb{B}}$.

If $\mathbb{A}$ and $\mathbb{B}$ are two tetrahedrons:

$$\mathbb{A} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{A}}.\overrightarrow{X} + \overrightarrow{V_{\mathbb{A}}}.t \end{array} \right\} \tag{19}$$

$$\mathbb{B} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ \overrightarrow{O_{\mathbb{B}}} + C_{\mathbb{B}}.\overrightarrow{X} + \overrightarrow{V_{\mathbb{B}}}.t \end{array} \right\} \tag{20}$$

If $\mathbb{B}$ is a cuboid, $\mathbb{B}_{\mathbb{A}}$ becomes:

$$\mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right) \end{array} \right\} \tag{21}$$

If $\mathbb{B}$ is a tetrahedron:

$$\mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right) \end{array} \right\} \tag{22}$$

$\mathbb{A}$ in its own coordinates system has the same definition as in the static case. For a cuboid:

$$\mathbb{A}_{\mathbb{A}} = \left\{ \overrightarrow{X} \in [0.0, 1.0]^D \right\} \tag{23}$$

and for a tetrahedron:

$$\mathbb{A}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \end{array} \right\} \tag{24}$$

The intersection of $\mathbb{A}$ and $\mathbb{B}$ in $\mathbb{A}$'s coordinates sytem, can then be expressed as follow.

If $\mathbb{A}$ and $\mathbb{B}$ are two cuboids:

$$\left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right) \cap [0.0, 1.0]^D \end{array} \right\} \tag{25}$$

If $\mathbb{A}$ is a cuboid and $\mathbb{B}$ is a tetrahedron:

$$\left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right) \cap [0.0, 1.0]^D \end{array} \right\} \tag{26}$$

If $\mathbb{A}$ is a tetrahedron and $\mathbb{B}$ is a cuboid:

$$\left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right) \cap [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} \left[C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}\right)\right]_i \leq 1.0 \end{array} \right\} \tag{27}$$

If $\mathbb{A}$ and $\mathbb{B}$ are two tetrahedrons:

$$\left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right) \cap [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} \left[C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}\right)\right]_i \leq 1.0 \end{array} \right\} \tag{28}$$

These lead to the following systems of linear inequations, given the three shortcuts $\overrightarrow{O_{\mathbb{B}_{\mathbb{A}}}} = C_{\mathbb{A}}^{-1}.(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}})$, $\overrightarrow{V_{\mathbb{B}_{\mathbb{A}}}} = C_{\mathbb{A}}^{-1}.(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}})$ and $C_{\mathbb{B}_{\mathbb{A}}} = C_{\mathbb{A}}^{-1}.C_{\mathbb{B}}$.

If $\mathbb{A}$ and $\mathbb{B}$ are two cuboids:

$$\left\{\begin{array}{rcl} t & \leq & 1.0 \\ -t & \leq & 0.0 \\ [X]_0 & \leq & 1.0 \\ \ldots & & \\ [X]_{D-1} & \leq & 1.0 \\ -[X]_0 & \leq & 0.0 \\ \ldots & & \\ -[X]_{D-1} & \leq & 0.0 \\ [V_{\mathbb{B}_\mathbb{A}}]_0 .t + \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{0,i} [X]_i & \leq & 1.0 - [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ [V_{\mathbb{B}_\mathbb{A}}]_{D-1} .t + \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{D-1,i} [X]_i & \leq & 1.0 - [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ -[V_{\mathbb{B}_\mathbb{A}}]_0 .t - \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{0,i} [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ -[V_{\mathbb{B}_\mathbb{A}}]_{D-1} .t - \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{D-1,i} [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \end{array}\right. \tag{29}$$

If $\mathbb{A}$ is a cuboid and $\mathbb{B}$ is a tetrahedron:

$$\left\{\begin{array}{rcl} t & \leq & 1.0 \\ -t & \leq & 0.0 \\ -[X]_0 & \leq & 0.0 \\ \ldots & & \\ -[X]_{D-1} & \leq & 0.0 \\ [V_{\mathbb{B}_\mathbb{A}}]_0 .t + \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{0,i} [X]_i & \leq & 1.0 - [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ [V_{\mathbb{B}_\mathbb{A}}]_{D-1} .t + \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{D-1,i} [X]_i & \leq & 1.0 - [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ -[V_{\mathbb{B}_\mathbb{A}}]_0 .t - \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{0,i} [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ -[V_{\mathbb{B}_\mathbb{A}}]_{D-1} .t - \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{D-1,i} [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ \sum_{i=0}^{D-1} [X]_i & \leq & 1.0 \end{array}\right. \tag{30}$$

If $\mathbb{A}$ is a tetrahedron and $\mathbb{B}$ is a cuboid:

$$\left\{\begin{array}{rcl} t & \leq & 1.0 \\ -t & \leq & 0.0 \\ [X]_0 & \leq & 1.0 \\ \ldots & & \\ [X]_{D-1} & \leq & 1.0 \\ -[X]_0 & \leq & 0.0 \\ \ldots & & \\ -[X]_{D-1} & \leq & 0.0 \\ -\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{0,i}[X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ -[V_{\mathbb{B}_\mathbb{A}}]_{D-1}.t - \sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{D-1,i}[X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ \sum_{j=0}^{D-1}\left([V_{\mathbb{B}_\mathbb{A}}]_j.t + \sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{j,i}[X]_i\right) & \leq & 1.0 - \sum_{i=0}^{D-1}[O_{\mathbb{B}_\mathbb{A}}]_i \end{array}\right. \quad (31)$$

If $\mathbb{A}$ and $\mathbb{B}$ are two tetrahedrons:

$$\left\{\begin{array}{rcl} t & \leq & 1.0 \\ -t & \leq & 0.0 \\ -[X]_0 & \leq & 0.0 \\ \ldots & & \\ -[X]_{D-1} & \leq & 0.0 \\ -\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{0,i}[X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ -[V_{\mathbb{B}_\mathbb{A}}]_{D-1}.t - \sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{D-1,i}[X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ \sum_{i=0}^{D-1}[X]_i & \leq & 1.0 \\ \sum_{j=0}^{D-1}\left([V_{\mathbb{B}_\mathbb{A}}]_j.t + \sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{j,i}[X]_i\right) & \leq & 1.0 - \sum_{i=0}^{D-1}[O_{\mathbb{B}_\mathbb{A}}]_i \end{array}\right. \quad (32)$$

2 Resolution of the problem by Fourier-Motzkin method

2.1 The Fourier-Motzkin elimination method

The Fourier-Motzkin elimination method has been introduced by J.J.-B. Fourier in 1827 [1], and described in the Ph.D. thesis of T.S. Motzkin in 1936 [2]. This is a generalization of the Gaussian elimination method to linear systems of inequalities. This method consists of eliminating one variable of the system and rewrite a new system accordingly. Then the elimination operation is repeated on another variable in the new system, and so on until we obtain a trivial system with only one variable. From there, a solution

for each variable can be obtained if it exists. The variable elimination is performed as follow.
Lets write the linear system $\mathcal{I}$ of m inequalities and n variables as

$$\left\{\begin{array}{llll} a_{11}.x_1 & +a_{12}.x_2 & +\cdots & +a_{1n}.x_n \quad \leq b_1 \\ a_{21}.x_1 & +a_{22}.x_2 & +\cdots & +a_{2n}.x_n \quad \leq b_2 \\ & & \vdots & \\ a_{m1}.x_1 & +a_{m2}.x_2 & +\cdots & +a_{mn}.x_n \quad \leq b_m \end{array}\right. \tag{33}$$

with

$$\begin{array}{c} i \in 1, 2, ..., m \\ j \in 1, 2, ..., n \\ x_i \in \mathbb{R} \\ a_{ij} \in \mathbb{R} \\ b_j \in \mathbb{R} \end{array} \tag{34}$$

To eliminate the first variable x_1, lets multiply each inequality by $1.0/|a_{i1}|$ where $a_{i1} \neq 0.0$. The system becomes

$$\left\{\begin{array}{llll} x_1 & +a'_{i2}.x_2 & +\cdots & +a'_{in}.x_n \quad \leq b'_i \qquad (i \in \mathcal{I}_+) \\ & a_{i2}.x_2 & +\cdots & +a_{in}.x_n \quad \leq b_i \qquad (i \in \mathcal{I}_0) \\ -x_1 & +a'_{i2}.x_2 & +\cdots & +a'_{in}.x_n \quad \leq b'_i \qquad (i \in \mathcal{I}_-) \end{array}\right. \tag{35}$$

where

$$\begin{array}{c} \mathcal{I}_+ = \{i : a_{i1} > 0.0\} \\ \mathcal{I}_0 = \{i : a_{i1} = 0.0\} \\ \mathcal{I}_- = \{i : a_{i1} < 0.0\} \\ a'_{ij} = a_{ij}/|a_{i1}| \\ b'_i = b_i/|a_{i1}| \end{array}$$

Then $x_1, x_2, \cdots, x_n \in \mathbb{R}^n$ is a solution of $\mathcal{I}$ if and only if

$$\left\{\begin{array}{ll} \sum_{j=2}^{n}((a'_{kj} + a'_{lj}).x_j) \leq b'_k + b'_l & (k \in \mathcal{I}_+, l \in \mathcal{I}_-) \\ \sum_{j=2}^{n}(a_{ij}.x_j) \leq b_i & i \in \mathcal{I}_0 \end{array}\right. \tag{36}$$

and

$$\max_{l \in \mathcal{I}_-}(\sum_{j=2}^{n}(a'_{lj}.x_j) - b'_l) \leq x_1 \leq \min_{k \in \mathcal{I}_+}(b'_k - \sum_{j=2}^{n}(a'_{kj}.x_j)) \tag{37}$$

The same method is then applied on this new system to eliminate the second variable x_2, and so on until we reach the inequality

$$\max_{l \in \mathcal{I}''^{\cdots'}_-}(-b''^{\cdots'}_l) \leq x_n \leq \min_{k \in \mathcal{I}''^{\cdots'}_+}(b''^{\cdots'}_k) \tag{38}$$

If this inequality has no solution, then neither the system $\mathcal{I}$. If it has a solution, the minimum and maximum are the bounding values for the variable x_n. One can get a particular solution to the system $\mathcal{I}$ by choosing a value for x_n between these bounding values, which allow us to set a particular value for the variable x_{n-1}, and so on back up to x_1.

2.2 Application of the Fourier-Motzkin method to the intersection problem

The Fourier-Motzkin method can be directly applied to obtain the bounds of each variable, if the system has a solution. If the system has no solution, the method will eventually reach an inconsistent inequality.

One solution $\overrightarrow{S}$ within the bounds obtained by the resolution of the system is expressed in the Frame $\mathbb{B}$'s coordinates system. One can get the equivalent coordinates $\overrightarrow{S'}$ in the real world's coordinates system as follow:

$$\overrightarrow{S'} = \overrightarrow{O_{\mathbb{B}}} + C_{\mathbb{B}}.\overrightarrow{S} \tag{39}$$

Only one inconsistent inequality is sufficient to prove the absence of solution, and then the non intersection of the Frames. One shall check the inconsistence of each inequality as soon as possible during the resolution of the system to optimize the speed of the algorithm.

A suficient condition for one inequality $\sum_i a_i X_i \leq Y$ to be inconsistent is, given that $\forall i, X_i \in [0.0, 1.0]$:

$$Y < \sum_{i \in I^-} a_i \tag{40}$$

where $I^- = \{i, a_i < 0.0\}$.

2.3 About the size of system of linear inequation

During implementation in languages where the developper needs to manage memory itself the size of the systems (35) resulting from variable elimination is necessary but cannot be forecasted. Instead, a maximum size can be calculated as follow.

Lets call n_-, n_+ and n_0 the size of, respectively, $\mathcal{I}_-$, $\mathcal{I}_+$ and $\mathcal{I}_0$, and N the number of inequalities in the original system and N' the number inequalities

in the resulting system. We have:

$$n_- + n_+ + n_0 = N \tag{41}$$

and

$$n_-.n_+ + n_0 = N' \tag{42}$$

Now lets define $K = N - n_0$, then we have:

$$n_- + n_+ = K \tag{43}$$

then,

$$n_-.n_+ = n_-(K - n_-) \tag{44}$$

then,

$$n_-.n_+ = K.n_- n_-^2 \tag{45}$$

The right part is polynomial whose maximum is reached for $n_- = K/2$. Then,

$$n_-.n_+ \leq K^2/2 - K^2/4 \tag{46}$$

or,

$$n_-.n_+ \leq K^2/4 \tag{47}$$

and putting back the definition of K

$$n_-.n_+ \leq (N - n_0)^2/4 \tag{48}$$

which is also

$$n_-.n_+ \leq N^2/4 \tag{49}$$

From (42) we get,

$$N' \leq N^2/4 - n_0 \tag{50}$$

and getting rid of the n_0 knowing that $n_0 \geq 0$,

$$N' \leq N^2/4 \tag{51}$$

The maximum number of inequation in the initial system is defined for each case (2D/3D, static/dynamic) in the previous section. This leads to the following maximum number of inequations:

	N	N'	N''	N'''
$2Dstatic$	8	16		
$2Ddynamic$	10	25	157	
$3Dstatic$	12	36	324	
$3Ddynamic$	14	49	601	90301

3 Algorithms of the solution

In this section I introduce the algorithms of the solution of the previous section for each case (static/dynamic and 2D/3D), and the algorithms to manipulate the structure used to represent the cuboid and tetrahedron.

Algorithms are given in pseudo code, and consequently without any optimization based on properties of one given language. One can refer to the C implementation in the following sections for possible optimization in this language.

Algorithms are also given independantly from each other. Code commonalization may be possible if one plans to gather several cases together, but this is dependant of the implementation and thus left to the developper responsibility.

3.1 2D static

```
ENUM FrameType
  FrameCuboid,
  FrameTetrahedron
END ENUM

STRUCT AABB2D
  // x,y
  real min[2]
  real max[2]
END STRUCT

STRUCT Frame2D
  FrameType type
  real orig[2]
  // comp[iComp][iAxis]
  real comp[2][2]
  AABB2D bdgBox
  real invComp[2][2]
END STRUCT

FUNCTION powi(base, exp)
    res = 1
    FOR i=0..(exp - 1)
      res = res * base
    END FOR
    RETURN res
END FUNCTION

FUNCTION Frame2DPrint(that)
  IF that.type == FrameTetrahedron
    PRINT "T"
  ELSE IF that.type == FrameCuboid
    PRINT "C"
```

```
  END IF
  PRINT "o("
  FOR i = 0..1
    PRINT that.orig[i]
    IF i < 1
      PRINT ","
    END IF
  END FOR
  comp = ['x','y']
  FOR j = 0..1
    PRINT ") " comp[j] "("
    FOR i = 0..1
      PRINT that.comp[j][i]
      IF i < 1
        PRINT ","
      END IF
    END FOR
  END FOR
  PRINT ")"
END FUNCTION

FUNCTION AABB2DPrint(that)
  PRINT "minXY("
  FOR i = 0..1
    PRINT that.min[i]
    IF i < 1
      PRINT ","
    END IF
  END FOR
  PRINT ")-maxXY("
  FOR i = 0..1
    PRINT that.max[i]
    IF i < 1
      PRINT ","
    END IF
  END FOR
  PRINT ")"
END FUNCTION

FUNCTION Frame2DExportBdgBox(that, bdgBox, bdgBoxProj)
  FOR i = 0..1
    bdgBoxProj.max[i] = that.orig[i]
    FOR j = 0..1
      bdgBoxProj.max[i] =
        bdgBoxProj.max[i] + that.comp[j][i] * bdgBox.min[j]
    END FOR
    bdgBoxProj.min[i] = bdgBoxProj.max[i]
  END FOR
  nbVertices = powi(2, 2)
  FOR iVertex = 1..(nbVertices - 1)
    FOR i = 0..1
      IF (iVertex & (1 << i)) == TRUE
        v[i] = bdgBox.max[i]
      ELSE
        v[i] = bdgBox.min[i]
      END IF
    END FOR
    FOR i = 0..1
      w[i] = that.orig[i]
      FOR j = 0..1
        w[i] = w[i] + that.comp[j][i] * v[j]
      END FOR
```

```
    END FOR
    FOR i = 0..1
      IF bdgBoxProj.min[i] > w[i]
        bdgBoxProj.min[i] = w[i]
      END IF
      IF bdgBoxProj.max[i] < w[i]
        bdgBoxProj.max[i] = w[i]
      END IF
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame2DImportFrame(P, Q, Qp)
  FOR i = 0..1
    v[i] = Q.orig[i] - P.orig[i]
  END FOR
  FOR i = 0..1
    Qp.orig[i] = 0.0
    FOR j = 0..1
      Qp.orig[i] = Qp.orig[i] + P.invComp[j][i] * v[j]
      Qp.comp[j][i] = 0.0
      FOR k = 0..1
        Qp.comp[j][i] = Qp.comp[j][i] + P.invComp[k][i] * Q.comp[j][k]
      END FOR
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame2DUpdateInv(that)
  det = that.comp[0][0] * that.comp[1][1] -
    that.comp[1][0] * that.comp[0][1]
  that.invComp[0][0] = that.comp[1][1] / det
  that.invComp[0][1] = -that.comp[0][1] / det
  that.invComp[1][0] = -that.comp[1][0] / det
  that.invComp[1][1] = that.comp[0][0] / det
END FUNCTION

FUNCTION Frame2DCreateStatic(type, orig, comp)
  that.type = type
  FOR iAxis = 0..1
    that.orig[iAxis] = orig[iAxis]
    FOR iComp = 0..1
      that.comp[iComp][iAxis] = comp[iComp][iAxis]
    END FOR
  END FOR
  FOR iAxis = 0..1
    min = orig[iAxis]
    max = orig[iAxis]
    FOR iComp = 0..1
      IF that.type == FrameCuboid
        IF that.comp[iComp][iAxis] < 0.0
          min += that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0
          max += that.comp[iComp][iAxis]
        END IF
      ELSE IF that.type == FrameTetrahedron
        IF that.comp[iComp][iAxis] < 0.0 AND
          min > orig[iAxis] + that.comp[iComp][iAxis]
          min = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0 AND
```

```
          max < orig[iAxis] + that.comp[iComp][iAxis]
          max = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
      END IF
    END FOR
    that.bdgBox.min[iAxis] = min
    that.bdgBox.max[iAxis] = max
  END FOR
  Frame2DUpdateInv(that)
  RETURN that
END FUNCTION

FUNCTION Sgn(v)
  IF 0.0 < v
    a = 1
  ELSE
    a = 0
  END IF
  IF v < 0.0
    b = 1
  ELSE
    b = 0
  END IF
  RETURN A - B
END FUNCTION

FUNCTION Neg(x)
  IF x < 0.0
    RETURN x
  ELSE
    RETURN 0.0
  END IF
END FUNCTION

FST_VAR = 0
SND_VAR = 1

FUNCTION ElimVar2D(iVar, M, Y, nbRows, nbCols, Mp, Yp, nbRemainRows)
  nbRemainRows = 0
  FOR iRow = 0..(nbRows - 2)
    FOR jRow = (iRow + 1)..(nbRows - 1)
      IF sgn(M[iRow][iVar]) <> sgn(M[jRow][iVar]) AND
          M[iRow][iVar] <> 0.0 AND
          M[jRow][iVar] <> 0.0
        sumNegCoeff = 0.0
        jCol = 0
        FOR iCol = 0..(nbCols - 1)
          IF iCol <> iVar
            Mp[nbRemainRows][jCol] =
              M[iRow][iCol] / fabs(M[iRow][iVar]) +
              M[jRow][iCol] / fabs(M[jRow][iVar])
            sumNegCoeff += neg(Mp[nbRemainRows][jCol])
            jCol = jcol + 1
          END IF
        END FOR
        Yp[nbRemainRows] =
          Y[iRow] / fabs(M[iRow][iVar]) +
          Y[jRow] / fabs(M[jRow][iVar])
        IF Yp[nbRemainRows] < sumNegCoeff
          RETURN TRUE
        END IF
        nbRemainRows = nbRemainRows + 1
```

```
      END IF
    END FOR
  END FOR
  FOR iRow = 0..(nbRows - 1)
    IF M[iRow][iVar] == 0.0
      jCol = 0
      FOR iCol = 0..(nbCols - 1)
        IF iCol <> iVar
          Mp[nbRemainRows][jCol] = M[iRow][iCol]
          jCol = jCol + 1
        END IF
      END FOR
      Yp[nbRemainRows] = Y[iRow]
      nbRemainRows = nbRemainRows + 1
    END IF
  END FOR
  RETURN FALSE
END FUNCTION

FUNCTION GetBound2D(iVar, M, Y, nbRows, bdgBox)
  bdgBox.min[iVar] = 0.0
  bdgBox.max[iVar] = 1.0
  FOR jRow = 0..(nbRows - 1)
    IF M[jRow][0] > 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.max[iVar] > y
        bdgBox.max[iVar] = y
      END IF
    ELSE IF M[jRow][0] < 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.min[iVar] < y
        bdgBox.min[iVar] = y
      END IF
    END IF
  END FOR
END FUNCTION

FUNCTION FMBTestIntersection2D(that, tho, bdgBox)
  Frame2DImportFrame(that, tho, &thoProj)
  M[0][0] = -thoProj.comp[0][0]
  M[0][1] = -thoProj.comp[1][0]
  Y[0] = thoProj.orig[0]
  IF Y[0] < neg(M[0][0]) + neg(M[0][1])
    RETURN FALSE
  END IF
  M[1][0] = -thoProj.comp[0][1]
  M[1][1] = -thoProj.comp[1][1]
  Y[1] = thoProj.orig[1]
  IF Y[1] < neg(M[1][0]) + neg(M[1][1])
    RETURN FALSE
  END IF
  M[2][0] = -1.0
  M[2][1] = 0.0
  Y[2] = 0.0
  M[3][0] = 0.0
  M[3][1] = -1.0
  Y[3] = 0.0
  nbRows = 4
  IF that.type == FrameCuboid
    M[nbRows][0] = thoProj.comp[0][0]
    M[nbRows][1] = thoProj.comp[1][0]
    Y[nbRows] = 1.0 - thoProj.orig[0]
```

```
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1])
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
    M[nbRows][0] = thoProj.comp[0][1]
    M[nbRows][1] = thoProj.comp[1][1]
    Y[nbRows] = 1.0 - thoProj.orig[1]
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1])
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
  ELSE
    M[nbRows][0] = thoProj.comp[0][0] + thoProj.comp[0][1]
    M[nbRows][1] = thoProj.comp[1][0] + thoProj.comp[1][1]
    Y[nbRows] = 1.0 - thoProj.orig[0] - thoProj.orig[1]
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1])
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
  END
  IF tho.type == FrameCuboid
    M[nbRows][0] = 1.0
    M[nbRows][1] = 0.0
    Y[nbRows] = 1.0
    nbRows = nbRows + 1
    M[nbRows][0] = 0.0
    M[nbRows][1] = 1.0
    Y[nbRows] = 1.0
    nbRows = nbRows + 1
  ELSE
    M[nbRows][0] = 1.0
    M[nbRows][1] = 1.0
    Y[nbRows] = 1.0
    nbRows = nbRows + 1
  END
  inconsistency = ElimVar2D(FST_VAR, M, Y, nbRows, 2, Mp, Yp, nbRowsP)
  IF inconsistency == TRUE
    RETURN FALSE
  END
  GetBound2D(SND_VAR, Mp, Yp, nbRowsP, bdgBoxLocal)
  IF bdgBoxLocal.min[SND_VAR] >= bdgBoxLocal.max[SND_VAR]
    RETURN FALSE
  END
  ElimVar2D(SND_VAR, M, Y, nbRows, 2, Mp, Yp, nbRowsP)
  GetBound2D(FST_VAR, Mp, Yp, nbRowsP, bdgBoxLocal)
  bdgBox = bdgBoxLocal
  RETURN TRUE
END

origP2D = [0.0, 0.0]
compP2D = [
  [1.0, 0.0],
  [0.0, 1.0]]
P2D = Frame2DCreateStatic(FrameCuboid, origP2D, compP2D)
origQ2D = [0.0, 0.0]
compQ2D = [
  [1.0, 0.0],
  [0.0, 1.0]]
Q2D = Frame2DCreateStatic(FrameCuboid, origQ2D, compQ2D)
isIntersecting2D = FMBTestIntersection2D(P2D, Q2D, bdgBox2DLocal)
if isIntersecting2D == TRUE
  PRINT "Intersection detected."
```

```
  Frame2DExportBdgBox(Q2D, bdgBox2DLocal, bdgBox2D);
  AABB2DPrint(bdgBox2D)
ELSE
  PRINT "No intersection."
END IF
```

3.2 3D static

```
ENUM FrameType
  FrameCuboid,
  FrameTetrahedron
END ENUM

STRUCT AABB3D
  // x,y,z
  real min[3]
  real max[3]
END STRUCT

STRUCT Frame3D
  FrameType type
  real orig[3]
  // comp[iComp][iAxis]
  real comp[3][3]
  AABB3D bdgBox
  real invComp[3][3]
END STRUCT

FUNCTION powi(base, exp)
    res = 1
    FOR i=0..(exp - 1)
      res = res * base
    END FOR
    RETURN res
END FUNCTION

FUNCTION Frame3DPrint(that)
  IF that.type == FrameTetrahedron
    PRINT "T"
  ELSE IF that.type == FrameCuboid
    PRINT "C"
  END IF
  PRINT "o("
  FOR i = 0..2
    PRINT that.orig[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  comp = ['x','y','z']
  FOR j = 0..2
    PRINT ") " comp[j] "("
    FOR i = 0..2
      PRINT that.comp[j][i]
      IF i < 2
        PRINT ","
      END IF
    END FOR
  END FOR
  PRINT ")"
END FUNCTION
```

```
FUNCTION AABB3DPrint(that)
  PRINT "minXYZ("
  FOR i = 0..2
    PRINT that.min[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  PRINT ")-maxXYZ("
  FOR i = 0..2
    PRINT that.max[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  PRINT ")"
END FUNCTION

FUNCTION Frame3DExportBdgBox(that, bdgBox, bdgBoxProj)
  FOR i = 0..2
    bdgBoxProj.max[i] = that.orig[i]
    FOR j = 0..2
      bdgBoxProj.max[i] =
        bdgBoxProj.max[i] + that.comp[j][i] * bdgBox.min[j]
    END FOR
    bdgBoxProj.min[i] = bdgBoxProj.max[i]
  END FOR
  nbVertices = powi(2, 3)
  FOR iVertex = 1..(nbVertices - 1)
    FOR i = 0..2
      IF (iVertex & (1 << i)) == TRUE
        v[i] = bdgBox.max[i]
      ELSE
        v[i] = bdgBox.min[i]
      END IF
    END FOR
    FOR i = 0..2
      w[i] = that.orig[i]
      FOR j = 0..2
        w[i] = w[i] + that.comp[j][i] * v[j]
      END FOR
    END FOR
    FOR i = 0..2
      IF bdgBoxProj.min[i] > w[i]
        bdgBoxProj.min[i] = w[i]
      END IF
      IF bdgBoxProj.max[i] < w[i]
        bdgBoxProj.max[i] = w[i]
      END IF
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame3DImPortFrame(P, Q, Qp)
  FOR i = 0..2
    v[i] = Q.orig[i] - P.orig[i]
  END FOR
  FOR i = 0..2
    Qp.orig[i] = 0.0
    FOR j = 0..2
      Qp.orig[i] = Qp.orig[i] + P.invComp[j][i] * v[j]
```

```
      Qp.comp[j][i] = 0.0
      FOR k = 0..2
        Qp.comp[j][i] = Qp.comp[j][i] + P.invComp[k][i] * Q.comp[j][k]
      END FOR
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame3DUpdateInv(that)
  det =
    that.comp[0][0] * (that.comp[1][1] * that.comp[2][2] -
    that.comp[1][2] * that.comp[2][1]) -
    that.comp[1][0] * (that.comp[0][1] * that.comp[2][2] -
    that.comp[0][2] * that.comp[2][1]) +
    that.comp[2][0] * (that.comp[0][1] * that.comp[1][2] -
    that.comp[0][2] * that.comp[1][1])
  that.invComp[0][0] = (that.comp[1][1] * that.comp[2][2] -
    that.comp[2][1] * that.comp[1][2]) / det
  that.invComp[0][1] = (that.comp[2][1] * that.comp[0][2] -
    that.comp[2][2] * that.comp[0][1]) / det
  that.invComp[0][2] = (that.comp[0][1] * that.comp[1][2] -
    that.comp[0][2] * that.comp[1][1]) / det
  that.invComp[1][0] = (that.comp[2][0] * that.comp[1][2] -
    that.comp[2][2] * that.comp[1][0]) / det
  that.invComp[1][1] = (that.comp[0][0] * that.comp[2][2] -
    that.comp[2][0] * that.comp[0][2]) / det
  that.invComp[1][2] = (that.comp[0][2] * that.comp[1][0] -
    that.comp[1][2] * that.comp[0][0]) / det
  that.invComp[2][0] = (that.comp[1][0] * that.comp[2][1] -
    that.comp[2][0] * that.comp[1][1]) / det
  that.invComp[2][1] = (that.comp[0][1] * that.comp[2][0] -
    that.comp[2][1] * that.comp[0][0]) / det
  that.invComp[2][2] = (that.comp[0][0] * that.comp[1][1] -
    that.comp[1][0] * that.comp[0][1]) / det
END FUNCTION

FUNCTION Frame3DCreateStatic(type, orig, comp)
  that.type = type
  FOR iAxis = 0..2
    that.orig[iAxis] = orig[iAxis]
    FOR iComp = 0..2
      that.comp[iComp][iAxis] = comp[iComp][iAxis]
    END FOR
  END FOR
  FOR iAxis = 0..2
    min = orig[iAxis]
    max = orig[iAxis]
    FOR iComp = 0..2
      IF that.type == FrameCuboid) {
        IF that.comp[iComp][iAxis] < 0.0
          min += that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0
          max += that.comp[iComp][iAxis]
        END IF
      ELSE IF that.type == FrameTetrahedron
        IF that.comp[iComp][iAxis] < 0.0 AND
          min > orig[iAxis] + that.comp[iComp][iAxis]
          min = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0 AND
          max < orig[iAxis] + that.comp[iComp][iAxis]
```

```
          max = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
      END IF
    END FOR
    that.bdgBox.min[iAxis] = min
    that.bdgBox.max[iAxis] = max
  END FOR
  Frame3DUpdateInv(that)
  RETURN that
END FUNCTION

FUNCTION Sgn(v)
  IF 0.0 < v
    a = 1
  ELSE
    a = 0
  END IF
  IF v < 0.0
    b = 1
  ELSE
    b = 0
  END IF
  RETURN A - B
END FUNCTION

FUNCTION Neg(x)
  IF x < 0.0
    RETURN x
  ELSE
    RETURN 0.0
  END IF
END FUNCTION

FST_VAR = 0
SND_VAR = 1
THD_VAR = 2

FUNCTION ElimVar3D(iVar, M, Y, nbRows, nbCols, Mp, Yp, nbRemainRows)
  nbRemainRows = 0
  FOR iRow = 0..(nbRows - 2)
    FOR jRow = (iRow + 1)..(nbRows - 1)
      IF sgn(M[iRow][iVar]) <> sgn(M[jRow][iVar]) AND
          M[iRow][iVar] <> 0.0 AND
          M[jRow][iVar] <> 0.0
        sumNegCoeff = 0.0
        jCol = 0
        FOR iCol = 0..(nbCols - 1)
          IF iCol <> iVar
            Mp[nbRemainRows][jCol] =
              M[iRow][iCol] / fabs(M[iRow][iVar]) +
              M[jRow][iCol] / fabs(M[jRow][iVar])
            sumNegCoeff += neg(Mp[nbRemainRows][jCol])
            jCol = jCol + 1
          END IF
        END FOR
        Yp[nbRemainRows] =
          Y[iRow] / fabs(M[iRow][iVar]) +
          Y[jRow] / fabs(M[jRow][iVar])
        IF Yp[nbRemainRows] < sumNegCoeff
          RETURN TRUE
        END IF
        nbRemainRows = nbRemainRows
```

```
      END IF
    END FOR
  END FOR
  FOR iRow = 0..(nbRows - 1)
    IF M[iRow][iVar] == 0.0
      jCol = 0
      FOR iCol = 0..(nbCols  -1)
        IF iCol <> iVar
          Mp[nbRemainRows][jCol] = M[iRow][iCol]
          jCol = jCol + 1
        END IF
      END FOR
      Yp[nbRemainRows] = Y[iRow]
      nbRemainRows = nbRemainRows + 1
    END IF
  END FOR
  RETURN FALSE
END FUNCTION

FUNCTION GetBound3D(iVar, M, Y, nbRows, bdgBox)
  bdgBox.min[iVar] = 0.0
  bdgBox.max[iVar] = 1.0
  FOR jRow = 0..(nbRows - 1)
    IF M[jRow][0] > 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.max[iVar] > y
        bdgBox.max[iVar] = y
      END IF
    ELSE IF M[jRow][0] < 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.min[iVar] < y
        bdgBox.min[iVar] = y
      END IF
    END IF
  END FOR
END FUNCTION

FUNCTION FMBTestIntersection3D(that, tho, bdgBox)
  Frame3DImportFrame(that, tho, thoProj)
  M[0][0] = -thoProj.comp[0][0]
  M[0][1] = -thoProj.comp[1][0]
  M[0][2] = -thoProj.comp[2][0]
  Y[0] = thoProj.orig[0]
  IF Y[0] < neg(M[0][0]) + neg(M[0][1]) + neg(M[0][2])
    RETURN FALSE
  END IF
  M[1][0] = -thoProj.comp[0][1]
  M[1][1] = -thoProj.comp[1][1]
  M[1][2] = -thoProj.comp[2][1]
  Y[1] = thoProj.orig[1]
  IF Y[1] < neg(M[1][0]) + neg(M[1][1]) + neg(M[1][2])
    RETURN FALSE
  END IF
  M[2][0] = -thoProj.comp[0][2]
  M[2][1] = -thoProj.comp[1][2]
  M[2][2] = -thoProj.comp[2][2]
  Y[2] = thoProj.orig[2]
  IF Y[2] < neg(M[2][0]) + neg(M[2][1]) + neg(M[2][2])
    RETURN FALSE
  END IF
  M[3][0] = -1.0
  M[3][1] = 0.0
```

```
M[3][2] = 0.0
Y[3] = 0.0
M[4][0] = 0.0
M[4][1] = -1.0
M[4][2] = 0.0
Y[4] = 0.0
M[5][0] = 0.0
M[5][1] = 0.0
M[5][2] = -1.0
Y[5] = 0.0
nbRows = 6
IF that.type == FrameCuboid
  M[nbRows][0] = thoProj.comp[0][0]
  M[nbRows][1] = thoProj.comp[1][0]
  M[nbRows][2] = thoProj.comp[2][0]
  Y[nbRows] = 1.0 - thoProj.orig[0]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
  M[nbRows][0] = thoProj.comp[0][1]
  M[nbRows][1] = thoProj.comp[1][1]
  M[nbRows][2] = thoProj.comp[2][1]
  Y[nbRows] = 1.0 - thoProj.orig[1]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
  M[nbRows][0] = thoProj.comp[0][2]
  M[nbRows][1] = thoProj.comp[1][2]
  M[nbRows][2] = thoProj.comp[2][2]
  Y[nbRows] = 1.0 - thoProj.orig[2]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
ELSE
  M[nbRows][0] =
    thoProj.comp[0][0] + thoProj.comp[0][1] + thoProj.comp[0][2]
  M[nbRows][1] =
    thoProj.comp[1][0] + thoProj.comp[1][1] + thoProj.comp[1][2]
  M[nbRows][2] =
    thoProj.comp[2][0] + thoProj.comp[2][1] + thoProj.comp[2][2]
  Y[nbRows] =
    1.0 - thoProj.orig[0] - thoProj.orig[1] - thoProj.orig[2]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
END
IF tho.type == FrameCuboid {
  M[nbRows][0] = 1.0
  M[nbRows][1] = 0.0
  M[nbRows][2] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
  M[nbRows][0] = 0.0
  M[nbRows][1] = 1.0
```

```
    M[nbRows][2] = 0.0
    Y[nbRows] = 1.0
    nbRows = nbRows + 1
    M[nbRows][0] = 0.0
    M[nbRows][1] = 0.0
    M[nbRows][2] = 1.0
    Y[nbRows] = 1.0
    nbRows = nbRows + 1
  ELSE
    M[nbRows][0] = 1.0
    M[nbRows][1] = 1.0
    M[nbRows][2] = 1.0
    Y[nbRows] = 1.0
    nbRows = nbRows + 1
  END
  inconsistency =
    ElimVar3D(FST_VAR, M, Y, nbRows, 3, Mp, Yp, nbRowsP)
  IF inconsistency == TRUE
    RETURN FALSE
  END
  inconsistency =
    ElimVar3D(FST_VAR, Mp, Yp, nbRowsP, 2, Mpp, Ypp, nbRowsPP)
  IF inconsistency == TRUE
    RETURN FALSE
  END
  GetBound3D(THD_VAR, Mpp, Ypp, nbRowsPP, bdgBoxLocal)
  IF bdgBoxLocal.min[THD_VAR] >= bdgBoxLocal.max[THD_VAR]
    RETURN FALSE
  END
  ElimVar3D(SND_VAR, Mp, Yp, nbRowsP, 2, Mpp, Ypp, nbRowsPP)
  GetBound3D(SND_VAR, Mpp, Ypp, nbRowsPP, bdgBoxLocal)
  ElimVar3D(THD_VAR, M, Y, nbRows, 3, Mp, Yp, nbRowsP)
  ElimVar3D( SND_VAR, Mp, Yp, nbRowsP, 2, Mpp, Ypp, nbRowsPP)
  GetBound3D(FST_VAR, Mpp, Ypp, nbRowsPP, bdgBoxLocal)
  bdgBox = bdgBoxLocal
  RETURN TRUE
END

origP3D = [0.0, 0.0, 0.0]
compP3D = [
  [1.0, 0.0, 0.0],
  [0.0, 1.0, 0.0],
  [0.0, 0.0, 1.0]]
P3D = Frame3DCreateStatic(FrameTetrahedron, origP3D, compP3D)
origQ3D = [0.5, 0.5, 0.5]
compQ3D = [
  [2.0, 0.0, 0.0],
  [0.0, 2.0, 0.0],
  [0.0, 0.0, 2.0]]
Q3D = Frame3DCreateStatic(FrameTetrahedron, origQ3D, compQ3D)
isIntersecting3D = FMBTestIntersection3D(P3D, Q3D, bdgBox3DLocal)
IF isIntersecting3D == TRUE
  PRINT "Intersection detected."
  Frame3DExportBdgBox(Q3D, bdgBox3DLocal, bdgBox3D)
  AABB3DPrint(bdgBox3D)
ELSE
  PRINT "No intersection."
END IF
```

3.3 2D dynamic

```
ENUM FrameType
  FrameCuboid,
  FrameTetrahedron
END ENUM

STRUCT AABB2DTime
  // x,y,t
  real min[3]
  real max[3]
END STRUCT

STRUCT Frame2DTime
  FrameType type
  real orig[2]
  // comp[iComp][iAxis]
  real comp[2][2]
  AABB2DTime bdgBox
  real invComp[2][2]
  real speed[2]
END STRUCT

FUNCTION powi(base, exp)
    res = 1
    FOR i=0..(exp - 1)
      res = res * base
    END FOR
    RETURN res
END FUNCTION

FUNCTION Frame2DTimePrint(that)
  IF that.type == FrameTetrahedron
    PRINT "T"
  ELSE IF that.type == FrameCuboid
    PRINT "C"
  END IF
  PRINT "o("
  FOR i = 0..1
    PRINT that.orig[i]
    IF i < 1
      PRINT ","
    END IF
  END FOR
  PRINT ") s("
  FOR i = 0..1
    PRINT that.speed[i]
    IF i < 1
      PRINT ","
    END IF
  END FOR
  comp = ['x', 'y']
  FOR j = 0..1
    PRINT ") " comp[j] "("
    FOR i = 0..1
      PRINT that.comp[j][i]
      IF i < 1
        PRINT ","
      END IF
    END FOR
  END FOR
  PRINT ")"
END FUNCTION
```

```
FUNCTION AABB2DTimePrint(that)
  PRINT "minXYT("
  FOR i = 0..2
    PRINT that.min[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  PRINT ")-maxXYT("
  FOR i = 0..2
    PRINT that.max[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  PRINT ")"
END FUNCTION

FUNCTION Frame2DTimeExportBdgBox(that, bdgBox, bdgBoxProj)
  bdgBoxProj.min[2] = bdgBox.min[2]
  bdgBoxProj.max[2] = bdgBox.max[2]
  FOR i = 0..1
    bdgBoxProj.max[i] = that.orig[i] + that.speed[i] * bdgBox.min[2]
    FOR j = 0..1
      bdgBoxProj.max[i] =
        bdgBoxProj.max[i] + that.comp[j][i] * bdgBox.min[j]
    END FOR
    bdgBoxProj.min[i] = bdgBoxProj.max[i]
  END FOR
  nbVertices = powi(2, 2)
  FOR iVertex = 1..(nbVertices - 1)
    FOR i = 0..1
      IF (iVertex & (1 << i)) == TRUE
        v[i] = bdgBox.max[i]
      ELSE
        v[i] = bdgBox.min[i]
      END IF
    END FOR
    FOR i = 0..1
      w[i] = that.orig[i]
      FOR j = 0..1
        w[i] = w[i] + that.comp[j][i] * v[j]
      END FOR
    END FOR
    FOR i = 0..1
      IF bdgBoxProj.min[i] > w[i] + that.speed[i] * bdgBox.min[2]
        bdgBoxProj.min[i] = w[i] + that.speed[i] * bdgBox.min[2]
      END IF
      IF bdgBoxProj.min[i] > w[i] + that.speed[i] * bdgBox.max[2]
        bdgBoxProj.min[i] = w[i] + that.speed[i] * bdgBox.max[2]
      END IF
      IF bdgBoxProj.max[i] < w[i] + that.speed[i] * bdgBox.min[2]
        bdgBoxProj.max[i] = w[i] + that.speed[i] * bdgBox.min[2]
      END IF
      IF bdgBoxProj.max[i] < w[i] + that.speed[i] * bdgBox.max[2]
        bdgBoxProj.max[i] = w[i] + that.speed[i] * bdgBox.max[2]
      END IF
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame2DTimeImPortFrame(P, Q, Qp)
```

```
  FOR i = 0..1
    v[i] = Q.orig[i] - P.orig[i]
    s[i] = Q.speed[i] - P.speed[i]
  END FOR
  FOR i = 0..1
    Qp.orig[i] = 0.0
    Qp.speed[i] = 0.0
    FOR j = 0..1
      Qp.orig[i] = Qp.orig[i] + P.invComp[j][i] * v[j]
      Qp.speed[i] = Qp.speed[i] + P.invComp[j][i] * s[j]
      Qp.comp[j][i] = 0.0
      FOR k = 0..1
        Qp.comp[j][i] = Qp.comp[j][i] + P.invComp[k][i] * Q.comp[j][k]
      END FOR
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame2DTimeUpdateInv(that)
  det = that.comp[0][0] * that.comp[1][1] -
    that.comp[1][0] * that.comp[0][1]
  that.invComp[0][0] = that.comp[1][1] / det
  that.invComp[0][1] = -that.comp[0][1] / det
  that.invComp[1][0] = -that.comp[1][0] / det
  that.invComp[1][1] = that.comp[0][0] / det
END FUNCTION

FUNCTION Frame2DTimeCreateStatic(type, orig, comp)
  that.type = type
  FOR iAxis = 0..1
    that.orig[iAxis] = orig[iAxis]
    that.speed[iAxis] = speed[iAxis]
    FOR iComp = 0..1
      that.comp[iComp][iAxis] = comp[iComp][iAxis]
    END FOR
  END FOR
  FOR iAxis = 0..1
    min = orig[iAxis]
    max = orig[iAxis]
    FOR iComp = 0..1
      IF that.type == FrameCuboid
        IF that.comp[iComp][iAxis] < 0.0
          min += that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0
          max += that.comp[iComp][iAxis]
        END IF
      ELSE IF that.type == FrameTetrahedron
        IF that.comp[iComp][iAxis] < 0.0 AND
          min > orig[iAxis] + that.comp[iComp][iAxis]
          min = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0 AND
          max < orig[iAxis] + that.comp[iComp][iAxis]
          max = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
      END IF
    END FOR
    IF that.speed[iAxis] < 0.0
      min = min + that.speed[iAxis]
    END IF
    IF that.speed[iAxis] > 0.0
```

```
      max = max + that.speed[iAxis]
    END IF
    that.bdgBox.min[iAxis] = min
    that.bdgBox.max[iAxis] = max
  END FOR
  that.bdgBox.min[2] = 0.0
  that.bdgBox.max[2] = 1.0
  Frame2DTimeUpdateInv(that)
  RETURN that
END FUNCTION

FUNCTION Sgn(v)
  IF 0.0 < v
    a = 1
  ELSE
    a = 0
  END IF
  IF v < 0.0
    b = 1
  ELSE
    b = 0
  END IF
  RETURN A - B
END FUNCTION

FUNCTION Neg(x)
  IF x < 0.0
    RETURN x
  ELSE
    RETURN 0.0
  END IF
END FUNCTION

FST_VAR = 0
SND_VAR = 1
THD_VAR = 2

FUNCTION ElimVar2DTime(iVar, M, Y, nbRows, nbCols, Mp, Yp, nbRemainRows)
  nbRemainRows = 0
  FOR iRow = 0..(nbRows - 2)
    FOR jRow = (iRow + 1)..(nbRows - 1)
      IF sgn(M[iRow][iVar])  <> sgn(M[jRow][iVar]) AND
          M[iRow][iVar] <> 0.0 AND
          M[jRow][iVar] <> 0.0
        sumNegCoeff = 0.0
        jCol = 0
        FOR iCol = 0..(nbCols - 1)
          IF iCol <> iVar
            Mp[nbRemainRows][jCol] =
              M[iRow][iCol] / fabs(M[iRow][iVar]) +
              M[jRow][iCol] / fabs(M[jRow][iVar])
            sumNegCoeff += neg(Mp[nbRemainRows][jCol])
            jCol = jCol + 1
          END IF
        END FOR
        Yp[nbRemainRows] =
          Y[iRow] / fabs(M[iRow][iVar]) +
          Y[jRow] / fabs(M[jRow][iVar])
        IF Yp[nbRemainRows] < sumNegCoeff
          RETURN TRUE
        END IF
      nbRemainRows = nbRemainRows + 1
```

```
      END IF
    END FOR
  END FOR
  FOR (int iRow = 0
       iRow < nbRows
       ++iRow) {
    IF M[iRow][iVar] == 0.0
      jCol = 0
      FOR iCol = 0..(nbCols - 1)
        IF iCol <> iVar
          Mp[nbRemainRows][jCol] = M[iRow][iCol]
          jCol = jCol + 1
        END IF
      END FOR
      Yp[nbRemainRows] = Y[iRow]
      nbRemainRows = nbRemainRows + 1
    END IF
  END FOR
  RETURN FALSE
END FUNCTION

FUNCTION GetBound2DTime(iVar, M, Y, nbRows, bdgBox)
  bdgBox.min[iVar] = 0.0
  bdgBox.max[iVar] = 1.0
  FOR jRow = 0..(nbRows - 1)
    IF M[jRow][0] > 0.0
      double y = Y[jRow] / M[jRow][0]
      IF bdgBox.max[iVar] > y
        bdgBox.max[iVar] = y
      END IF
    ELSE IF M[jRow][0] < 0.0
      double y = Y[jRow] / M[jRow][0]
      IF bdgBox.min[iVar] < y
        bdgBox.min[iVar] = y
      END IF
    END IF
  END FOR
END FUNCTION

FUNCTION FMBTestIntersection2DTime(that, tho, bdgBox)
  Frame2DTimeImportFrame(that, tho, &thoProj)
  M[0][0] = -thoProj.comp[0][0]
  M[0][1] = -thoProj.comp[1][0]
  M[0][2] = -thoProj.speed[0]
  Y[0] = thoProj.orig[0]
  IF (Y[0] < neg(M[0][0]) + neg(M[0][1]) + neg(M[0][2]))
    RETURN FALSE
  END IF
  M[1][0] = -thoProj.comp[0][1]
  M[1][1] = -thoProj.comp[1][1]
  M[1][2] = -thoProj.speed[1]
  Y[1] = thoProj.orig[1]
  IF (Y[1] < neg(M[1][0]) + neg(M[1][1]) + neg(M[1][2]))
    RETURN FALSE
  END IF
  M[2][0] = -1.0
  M[2][1] = 0.0
  M[2][2] = 0.0
  Y[2] = 0.0
  M[3][0] = 0.0
  M[3][1] = -1.0
  M[3][2] = 0.0
```

```
Y[3] = 0.0
M[4][0] = 0.0
M[4][1] = 0.0
M[4][2] = 1.0
Y[4] = 1.0
M[5][0] = 0.0
M[5][1] = 0.0
M[5][2] = -1.0
Y[5] = 0.0
nbRows = 6
IF that.type == FrameCuboid
  M[nbRows][0] = thoProj.comp[0][0]
  M[nbRows][1] = thoProj.comp[1][0]
  M[nbRows][2] = thoProj.speed[0]
  Y[nbRows] = 1.0 - thoProj.orig[0]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
  M[nbRows][0] = thoProj.comp[0][1]
  M[nbRows][1] = thoProj.comp[1][1]
  M[nbRows][2] = thoProj.speed[1]
  Y[nbRows] = 1.0 - thoProj.orig[1]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2])
    RETURN FALSE
  nbRows = nbRows + 1
ELSE
  M[nbRows][0] = thoProj.comp[0][0] + thoProj.comp[0][1]
  M[nbRows][1] = thoProj.comp[1][0] + thoProj.comp[1][1]
  M[nbRows][2] = thoProj.speed[0] + thoProj.speed[1]
  Y[nbRows] = 1.0 - thoProj.orig[0] - thoProj.orig[1]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2])
    RETURN FALSE
  nbRows = nbRows + 1
END
IF tho.type == FrameCuboid
  M[nbRows][0] = 1.0
  M[nbRows][1] = 0.0
  M[nbRows][2] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
  M[nbRows][0] = 0.0
  M[nbRows][1] = 1.0
  M[nbRows][2] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
ELSE
  M[nbRows][0] = 1.0
  M[nbRows][1] = 1.0
  M[nbRows][2] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
END IF
inconsistency =
  ElimVar2DTime(FST_VAR, M, Y, nbRows, 3, Mp, Yp, nbRowsP)
IF inconsistency == TRUE
  RETURN FALSE
END IF
inconsistency =
```

```
    ElimVar2DTime(FST_VAR, Mp, Yp, nbRowsP, 2, Mpp, Ypp, nbRowsPP)
  IF inconsistency == TRUE
    RETURN FALSE
  END IF
  GetBound2DTime(THD_VAR, Mpp, Ypp, nbRowsPP, bdgBoxLocal)
  IF bdgBoxLocal.min[THD_VAR] >= bdgBoxLocal.max[THD_VAR]
    RETURN FALSE
  END IF
  ElimVar2DTime(SND_VAR, Mp, Yp, nbRowsP, 2, Mpp, Ypp, nbRowsPP)
  GetBound2DTime(SND_VAR, Mpp, Ypp, nbRowsPP, bdgBoxLocal)
  ElimVar2DTime(THD_VAR, M, Y, nbRows, 3, Mp, Yp, nbRowsP)
  ElimVar2DTime(SND_VAR, Mp, Yp, nbRowsP, 2, Mpp, Ypp, nbRowsPP)
  GetBound2DTime(FST_VAR, Mpp, Ypp, nbRowsPP, bdgBoxLocal)
  bdgBox = bdgBoxLocal
  RETURN TRUE
END FUNCTION

origP2DTime = [0.0, 0.0]
speedP2DTime = [0.0, 0.0]
compP2DTime = [
  [1.0, 0.0],
  [0.0, 1.0]]
P2DTime =
  Frame2DTimeCreateStatic(
    FrameCuboid, origP2DTime, speedP2DTime, compP2DTime)
origQ2DTime = [0.0,0.0]
speedQ2DTime = [0.0,0.0]
compQ2DTime = [
  [1.0, 0.0],
  [0.0, 1.0]]
Q2DTime =
  Frame2DTimeCreateStatic(
    FrameCuboid, origQ2DTime, speedQ2DTime, compQ2DTime)
isIntersecting2DTime =
  FMBTestIntersection2DTime(P2DTime, Q2DTime, bdgBox2DTimeLocal)
IF isIntersecting2DTime == TRUE
  PRINT "Intersection detected."
  Frame2DTimeExportBdgBox(Q2DTime, bdgBox2DTimeLocal, bdgBox2DTime)
  AABB2DTimePrint(bdgBox2DTime)
ELSE
  PRINT "No intersection."
END IF
```

3.4 3D dynamic

```
ENUM FrameType
  FrameCuboid,
  FrameTetrahedron
END ENUM

STRUCT AABB3DTime
  // x,y,z,t
  real min[4]
  real max[4]
END STRUCT

STRUCT Frame3DTime
  FrameType type
  real orig[3]
  // comp[iComp][iAxis]
  real comp[3][3]
```

```
  AABB3DTime bdgBox
  real invComp[3][3]
  real speed[3]
END STRUCT

FUNCTION powi(base, exp)
    res = 1
    FOR i=0..(exp - 1)
      res = res * base
    END FOR
    RETURN res
END FUNCTION

FUNCTION Frame3DTimePrint(that)
  IF that.type == FrameTetrahedron
    PRINT "T"
  ELSE IF that.type == FrameCuboid
    PRINT "C"
  END IF
  PRINT "o("
  FOR (i = 0..2
    PRINT that.orig[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  PRINT " s("
  FOR i = 0..2
    PRINT that.speed[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  comp = ['x', 'y', 'z']
  FOR j = 0..2
    PRINT " " comp[j] "("
    FOR i = 0..2
      PRINT that.comp[j][i]
      IF i < 2
        PRINT ","
      END IF
    END FOR
  END FOR
  PRINT ""
END FUNCTION

FUNCTION AABB3DTimePrint(that)
  PRINT "minXYZT("
  FOR i = 0..3
    PRINT that.min[i]
    IF i < 3
      PRINT ","
    END IF
  END FOR
  PRINT ")-maxXYZT("
  FOR i = 0..3
    PRINT that.max[i]
    IF i < 3
      PRINT ","
    END IF
  END FOR
  PRINT ")"
```

```
END FUNCTION

FUNCTION Frame3DTimeExportBdgBox(that, bdgBox, bdgBoxProj)
  bdgBoxProj.min[3] = bdgBox.min[3]
  bdgBoxProj.max[3] = bdgBox.max[3]
  FOR i = 0..2
    bdgBoxProj.max[i] = that.orig[i] + that.speed[i] * bdgBox.min[3]
    FOR j = 0..2
      bdgBoxProj.max[i] =
        bdgBoxProj.max[i] + that.comp[j][i] * bdgBox.min[j]
    END FOR
    bdgBoxProj.min[i] = bdgBoxProj.max[i]
  END FOR
  nbVertices = powi(2, 3)
  FOR iVertex = 1..(nbVertices - 1)
    FOR i = 0..2
      IF (iVertex & (1 << i)) == TRUE
        v[i] = bdgBox.max[i]
      ELSE
        v[i] = bdgBox.min[i]
      END IF
    END FOR
    FOR i = 0..2
      w[i] = that.orig[i]
      FOR j = 0..2
        w[i] = w[i] + that.comp[j][i] * v[j]
      END FOR
    END FOR
    FOR i = 0..2
      IF bdgBoxProj.min[i] > w[i] + that.speed[i] * bdgBox.min[3]
        bdgBoxProj.min[i] = w[i] + that.speed[i] * bdgBox.min[3]
      END IF
      IF bdgBoxProj.min[i] > w[i] + that.speed[i] * bdgBox.max[3]
        bdgBoxProj.min[i] = w[i] + that.speed[i] * bdgBox.max[3]
      END IF
      IF bdgBoxProj.max[i] < w[i] + that.speed[i] * bdgBox.min[3]
        bdgBoxProj.max[i] = w[i] + that.speed[i] * bdgBox.min[3]
      END IF
      IF bdgBoxProj.max[i] < w[i] + that.speed[i] * bdgBox.max[3]
        bdgBoxProj.max[i] = w[i] + that.speed[i] * bdgBox.max[3]
      END IF
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame3DTimeImPortFrame(P, Q, Qp)
  FOR i = 0..2
    v[i] = Q.orig[i] - P.orig[i]
    s[i] = Q.speed[i] - P.speed[i]
  END FOR
  FOR i = 0..2
    Qp.orig[i] = 0.0
    Qp.speed[i] = 0.0
    FOR j = 0..2
      Qp.orig[i] = Qp.orig[i] + P.invComp[j][i] * v[j]
      Qp.speed[i] = Qp.speed[i] + P.invComp[j][i] * s[j]
      Qp.comp[j][i] = 0.0
      FOR k = 0..2
        Qp.comp[j][i] = Qp.comp[j][i] + P.invComp[k][i] * Q.comp[j][k]
      END FOR
    END FOR
  END FOR
```

```
END FUNCTION

FUNCTION Frame3DTimeUpdateInv(that)
  det =
    that.comp[0][0] *
    (that.comp[1][1] * that.comp[2][2] - that.comp[1][2] * that.comp[2][1])
        -
    that.comp[1][0] *
    (that.comp[0][1] * that.comp[2][2] - that.comp[0][2] * that.comp[2][1])
        +
    that.comp[2][0] *
    (that.comp[0][1] * that.comp[1][2] - that.comp[0][2] * that.comp[1][1])
  that.invComp[0][0] = (that.comp[1][1] * that.comp[2][2] -
    that.comp[2][1] * that.comp[1][2]) / det
  that.invComp[0][1] = (that.comp[2][1] * that.comp[0][2] -
    that.comp[2][2] * that.comp[0][1]) / det
  that.invComp[0][2] = (that.comp[0][1] * that.comp[1][2] -
    that.comp[0][2] * that.comp[1][1]) / det
  that.invComp[1][0] = (that.comp[2][0] * that.comp[1][2] -
    that.comp[2][2] * that.comp[1][0]) / det
  that.invComp[1][1] = (that.comp[0][0] * that.comp[2][2] -
    that.comp[2][0] * that.comp[0][2]) / det
  that.invComp[1][2] = (that.comp[0][2] * that.comp[1][0] -
    that.comp[1][2] * that.comp[0][0]) / det
  that.invComp[2][0] = (that.comp[1][0] * that.comp[2][1] -
    that.comp[2][0] * that.comp[1][1]) / det
  that.invComp[2][1] = (that.comp[0][1] * that.comp[2][0] -
    that.comp[2][1] * that.comp[0][0]) / det
  that.invComp[2][2] = (that.comp[0][0] * that.comp[1][1] -
    that.comp[1][0] * that.comp[0][1]) / det
END FUNCTION

FUNCTION Frame3DTimeCreateStatic(type, orig, comp)
  that.type = type
  FOR iAxis = 0..2
    that.orig[iAxis] = orig[iAxis]
    that.speed[iAxis] = speed[iAxis]
    FOR iComp = 0..2
      that.comp[iComp][iAxis] = comp[iComp][iAxis]
    END FOR
  END FOR
  FOR iAxis = 0..2
    min = orig[iAxis]
    max = orig[iAxis]
    FOR iComp = 0..2
      IF that.type == FrameCuboid
        IF that.comp[iComp][iAxis] < 0.0
          min += that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0
          max += that.comp[iComp][iAxis]
        END IF
      ELSE IF that.type == FrameTetrahedron
        IF that.comp[iComp][iAxis] < 0.0 AND
          min > orig[iAxis] + that.comp[iComp][iAxis]
          min = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0 AND
          max < orig[iAxis] + that.comp[iComp][iAxis]
          max = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
      END IF
```

```
    END FOR
    IF that.speed[iAxis] < 0.0
      min = min + that.speed[iAxis]
    END IF
    IF that.speed[iAxis] > 0.0
      max = max + that.speed[iAxis]
    END IF
    that.bdgBox.min[iAxis] = min
    that.bdgBox.max[iAxis] = max
  END FOR
  that.bdgBox.min[3] = 0.0
  that.bdgBox.max[3] = 1.0
  Frame3DTimeUpdateInv(that)
  RETURN that
END FUNCTION

FUNCTION Sgn(v)
  IF 0.0 < v
    a = 1
  ELSE
    a = 0
  END IF
  IF v < 0.0
    b = 1
  ELSE
    b = 0
  END IF
  RETURN A - B
END FUNCTION

FUNCTION Neg(x)
  IF x < 0.0
    RETURN x
  ELSE
    RETURN 0.0
  END IF
END FUNCTION

FST_VAR = 0
SND_VAR = 1
THD_VAR = 2
FOR_VAR = 3

FUNCTION ElimVar3DTime(iVar, M, Y, nbRows, nbCols, Mp, Yp, nbRemainRows)
  nbRemainRows = 0
  FOR iRow = 0..(nbRows - 2)
    FOR jRow = (iRow + 1)..(nbRows - 1)
      IF Sgn(M[iRow][iVar]) <> sgn(M[jRow][iVar] AND
         M[iRow][iVar] <> 0.0 AND
         M[jRow][iVar] <> 0.0:
        sumNegCoeff = 0.0
        jCol = 0
        FOR iCol = 0..(nbCols - 1)
          IF iCol <> iVar
            Mp*nbRemainRows][jCol] =
              M[iRow][iCol] / fabs(M[iRow][iVar]) +
              M[jRow][iCol] / fabs(M[jRow][iVar])
            sumNegCoeff += neg(Mp[nbRemainRows][jCol])
            jCol = jCol + 1
          END IF
        END FOR
        Yp[nbRemainRows] =
```

```
          Y[iRow] / fabs(M[iRow][iVar]) +
          Y[jRow] / fabs(M[jRow][iVar])
        IF Yp[nbRemainRows] < sumNegCoeff
          RETURN TRUE
        END IF
        nbRemainRows = nbRemainRows + 1
      END IF
    END FOR
  END FOR
  FOR iRow = 0..(nbRows - 1)
    IF M[iRow][iVar] == 0.0
      jCol = 0
      FOR iCol = 0..(nbCols - 1)
        IF iCol <> iVar
          Mp[nbRemainRows][jCol] = M[iRow][iCol]
          jCol = jCol + 1
        END IF
      END FOR
      Yp[nbRemainRows] = Y[iRow]
      nbRemainRows = nbRemainRows + 1
    END IF
  END FOR
  RETURN FALSE
END FUNCTION

FUNCTION GetBound3DTime(iVar, M, Y, nbRows, bdgBox)
  bdgBox.min[iVar] = 0.0
  bdgBox.max[iVar] = 1.0
  FOR jRow = 0..(nbRows - 1)
    IF M[jRow][0] > 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.max[iVar] > y
        bdgBox.max[iVar] = y
      END IF
    ELSE IF M[jRow][0] < 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.min[iVar] < y
        bdgBox.min[iVar] = y
      END IF
    END IF
  END FOR
END FUNCTION

FUNCTION FMBTestIntersection3DTime(that, tho, bdgBox)
  Frame3DTimeImportFrame(that, tho, thoProj)
  M[0][0] = -thoProj.comp[0][0]
  M[0][1] = -thoProj.comp[1][0]
  M[0][2] = -thoProj.comp[2][0]
  M[0][3] = -thoProj.speed[0]
  Y[0] = thoProj.orig[0]
  IF (Y[0] < neg(M[0][0]) + neg(M[0][1]) + neg(M[0][2]) + neg(M[0][3]))
    RETURN FALSE
  END IF
  M[1][0] = -thoProj.comp[0][1]
  M[1][1] = -thoProj.comp[1][1]
  M[1][2] = -thoProj.comp[2][1]
  M[1][3] = -thoProj.speed[1]
  Y[1] = thoProj.orig[1]
  IF Y[1] < neg(M[1][0]) + neg(M[1][1]) + neg(M[1][2]) + neg(M[1][3])
    RETURN FALSE
  END IF
  M[2][0] = -thoProj.comp[0][2]
```

```
M[2][1] = -thoProj.comp[1][2]
M[2][2] = -thoProj.comp[2][2]
M[2][3] = -thoProj.speed[2]
Y[2] = thoProj.orig[2]
IF (Y[2] < neg(M[2][0]) + neg(M[2][1]) + neg(M[2][2]) + neg(M[2][3]))
  RETURN FALSE
nbRows = 3
IF that.type == FrameCuboid
  M[nbRows][0] = thoProj.comp[0][0]
  M[nbRows][1] = thoProj.comp[1][0]
  M[nbRows][2] = thoProj.comp[2][0]
  M[nbRows][3] = thoProj.speed[0]
  Y[nbRows] = 1.0 - thoProj.orig[0]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2]) + neg(M[nbRows][3])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
  M[nbRows][0] = thoProj.comp[0][1]
  M[nbRows][1] = thoProj.comp[1][1]
  M[nbRows][2] = thoProj.comp[2][1]
  M[nbRows][3] = thoProj.speed[1]
  Y[nbRows] = 1.0 - thoProj.orig[1]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2]) + neg(M[nbRows][3])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
  M[nbRows][0] = thoProj.comp[0][2]
  M[nbRows][1] = thoProj.comp[1][2]
  M[nbRows][2] = thoProj.comp[2][2]
  M[nbRows][3] = thoProj.speed[2]
  Y[nbRows] = 1.0 - thoProj.orig[2]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2]) + neg(M[nbRows][3])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
ELSE
  M[nbRows][0] =
    thoProj.comp[0][0] + thoProj.comp[0][1] + thoProj.comp[0][2]
  M[nbRows][1] =
    thoProj.comp[1][0] + thoProj.comp[1][1] + thoProj.comp[1][2]
  M[nbRows][2] =
    thoProj.comp[2][0] + thoProj.comp[2][1] + thoProj.comp[2][2]
  M[nbRows][3] = thoProj.speed[0] + thoProj.speed[1] + thoProj.speed[2]
  Y[nbRows] = 1.0 - thoProj.orig[0] - thoProj.orig[1] - thoProj.orig[2]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2]) + neg(M[nbRows][3])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
END IF
IF tho.type == FrameCuboid
  M[nbRows][0] = 1.0
  M[nbRows][1] = 0.0
  M[nbRows][2] = 0.0
  M[nbRows][3] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
  M[nbRows][0] = 0.0
  M[nbRows][1] = 1.0
```

```
  M[nbRows][2] = 0.0
  M[nbRows][3] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
  M[nbRows][0] = 0.0
  M[nbRows][1] = 0.0
  M[nbRows][2] = 1.0
  M[nbRows][3] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
ELSE
  M[nbRows][0] = 1.0
  M[nbRows][1] = 1.0
  M[nbRows][2] = 1.0
  M[nbRows][3] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
END IF
M[nbRows][0] = -1.0
M[nbRows][1] = 0.0
M[nbRows][2] = 0.0
M[nbRows][3] = 0.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
M[nbRows][0] = 0.0
M[nbRows][1] = -1.0
M[nbRows][2] = 0.0
M[nbRows][3] = 0.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
M[nbRows][0] = 0.0
M[nbRows][1] = 0.0
M[nbRows][2] = -1.0
M[nbRows][3] = 0.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
M[nbRows][0] = 0.0
M[nbRows][1] = 0.0
M[nbRows][2] = 0.0
M[nbRows][3] = 1.0
Y[nbRows] = 1.0
nbRows = nbRows + 1
M[nbRows][0] = 0.0
M[nbRows][1] = 0.0
M[nbRows][2] = 0.0
M[nbRows][3] = -1.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
inconsistency =
  ElimVar3DTime(FST_VAR, M, Y, nbRows, 4, Mp, Yp, nbRowsP)
IF inconsistency == TRUE
  RETURN FALSE
END IF
inconsistency =
  ElimVar3DTime(FST_VAR, Mp, Yp, nbRowsP, 3, Mpp, Ypp, nbRowsPP)
IF inconsistency == TRUE
  RETURN FALSE
END IF
inconsistency =
  ElimVar3DTime(FST_VAR, Mpp, Ypp, nbRowsPP, 2, Mppp, Yppp, nbRowsPPP)
IF inconsistency == TRUE
  RETURN FALSE
```

```
  END IF
  GetBound3DTime(FOR_VAR, Mppp, Yppp, nbRowsPPP, bdgBoxLocal)
  IF bdgBoxLocal.min[FOR_VAR] >= bdgBoxLocal.max[FOR_VAR]
    RETURN FALSE
  END IF
  ElimVar3DTime(SND_VAR, Mpp, Ypp, nbRowsPP, 2, Mppp, Yppp, nbRowsPPP)
  GetBound3DTime(THD_VAR, Mppp, Yppp, nbRowsPPP, bdgBoxLocal)
  ElimVar3DTime(FOR_VAR, M, Y, nbRows, 4, Mp, Yp, nbRowsP)
  ElimVar3DTime(THD_VAR, Mp, Yp, nbRowsP, 3, Mpp, Ypp, nbRowsPP)
  ElimVar3DTime(SND_VAR, Mpp, Ypp, nbRowsPP, 2, Mppp, Yppp, nbRowsPPP)
  GetBound3DTime(FST_VAR, Mppp, Yppp, nbRowsPPP, bdgBoxLocal)
  ElimVar3DTime(FST_VAR, Mpp, Ypp, nbRowsPP, 2, Mppp, Yppp, nbRowsPPP)
  GetBound3DTime(SND_VAR, Mppp, Yppp, nbRowsPPP, bdgBoxLocal)
  bdgBox = bdgBoxLocal
  RETURN TRUE
END FUNCTION

origP3DTime = [0.0, 0.0, 0.0]
speedP3DTime = [0.0, 0.0, 0.0]
compP3DTime = [
  [1.0, 0.0, 0.0],
  [0.0, 1.0, 0.0],
  [0.0, 0.0, 1.0]]
P3DTime =
  Frame3DTimeCreateStatic(
    FrameCuboid, origP3DTime, speedP3DTime, compP3DTime)
origQ3DTime = [0.0, 0.0, 0.0]
speedQ3DTime = [0.0, 0.0, 0.0]
compQ3DTime = [
  [1.0, 0.0, 0.0],
  [0.0, 1.0, 0.0],
  [0.0, 0.0, 1.0]]
Q3DTime =
  Frame3DTimeCreateStatic(
    FrameCuboid, origQ3DTime, speedQ3DTime, compQ3DTime)
isIntersecting3DTime =
  FMBTestIntersection3DTime(P3DTime, Q3DTime, bdgBox3DTimeLocal)
IF isIntersecting3DTime
  PRINT "Intersection detected."
  Frame3DTimeExportBdgBox(Q3DTime, bdgBox3DTimeLocal, bdgBox3DTime)
  AABB3DTimePrint(bdgBox3DTime)
ELSE
  PRINT "No intersection."
END IF
```

4 Implementation of the algorithms in C

In this section I introduce an implementation of the algorithms of the previous section in the C language.

4.1 Frames

4.1.1 Header

```
#ifndef __FRAME_H_
#define __FRAME_H_
```

```
// ------------- Includes -------------

#include <math.h>
#include <stdio.h>
#include <stdlib.h>

// ------------- Macros -------------

// ------------- Enumerations -------------

typedef enum {
  FrameCuboid,
  FrameTetrahedron
} FrameType;

// ------------- Data structures -------------

// Axis aligned bounding box structure
typedef struct {
  // x,y
  double min[2];
  double max[2];
} AABB2D;

typedef struct {
  // x,y,z
  double min[3];
  double max[3];
} AABB3D;

typedef struct {
  // x,y,t
  double min[3];
  double max[3];
} AABB2DTime;

typedef struct {
  // x,y,z,t
  double min[4];
  double max[4];
} AABB3DTime;

// Axis unaligned cuboid and tetrahedron structure
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
  // AABB of the frame
  AABB2D bdgBox;
  // Inverted components used during computation
  double invComp[2][2];
} Frame2D;

typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
  // AABB of the frame
  AABB3D bdgBox;
  // Inverted components used during computation
  double invComp[3][3];
```

```
} Frame3D;

typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
  // AABB of the frame
  AABB2DTime bdgBox;
  // Inverted components used during computation
  double invComp[2][2];
  double speed[2];
} Frame2DTime;

typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
  // AABB of the frame
  AABB3DTime bdgBox;
  // Inverted components used during computation
  double invComp[3][3];
  double speed[3];
} Frame3DTime;

// ------------- Functions declaration -------------

// Print the AABB 'that' on stdout
// Output format is
// (min[0], min[1], min[2], min[3])-(max[0], max[1], max[2], max[3])
void AABB2DPrint(const AABB2D* const that);
void AABB3DPrint(const AABB3D* const that);
void AABB2DTimePrint(const AABB2DTime* const that);
void AABB3DTimePrint(const AABB3DTime* const that);

// Print the Frame 'that' on stdout
// Output format is
// (orig[0], orig[1], orig[2])
// (comp[0][0], comp[0][1], comp[0][2])
// (comp[1][0], comp[1][1], comp[1][2])
// (comp[2][0], comp[2][1], comp[2][2])
// (speed[0], speed[1], speed[2])
void Frame2DPrint(const Frame2D* const that);
void Frame3DPrint(const Frame3D* const that);
void Frame2DTimePrint(const Frame2DTime* const that);
void Frame3DTimePrint(const Frame3DTime* const that);

// Create a static Frame structure of FrameType 'type',
// at position 'orig' with components 'comp' ([iComp][iAxis])
Frame2D Frame2DCreateStatic(
  const FrameType type,
     const double orig[2],
     const double comp[2][2]);
Frame3D Frame3DCreateStatic(
  const FrameType type,
     const double orig[3],
     const double comp[3][3]);
Frame2DTime Frame2DTimeCreateStatic(
  const FrameType type,
     const double orig[2],
     const double speed[2],
     const double comp[2][2]);
Frame3DTime Frame3DTimeCreateStatic(
```

```
  const FrameType type,
     const double orig[3],
     const double speed[3],
     const double comp[3][3]);

// Project the Frame 'Q' in the Frame 'P' 's coordinates system and
// memorize the result in the Frame 'Qp'
void Frame2DImportFrame(
  const Frame2D* const P,
  const Frame2D* const Q,
        Frame2D* const Qp);
void Frame3DImportFrame(
  const Frame3D* const P,
  const Frame3D* const Q,
        Frame3D* const Qp);
void Frame2DTimeImportFrame(
  const Frame2DTime* const P,
  const Frame2DTime* const Q,
        Frame2DTime* const Qp);
void Frame3DTimeImportFrame(
  const Frame3DTime* const P,
  const Frame3DTime* const Q,
        Frame3DTime* const Qp);

// Export the AABB 'bdgBox' from 'that' 's coordinates system to
// the real coordinates system and update 'bdgBox' with the resulting
// AABB
void Frame2DExportBdgBox(
  const Frame2D* const that,
   const AABB2D* const bdgBox,
         AABB2D* const bdgBoxProj);
void Frame3DExportBdgBox(
  const Frame3D* const that,
   const AABB3D* const bdgBox,
         AABB3D* const bdgBoxProj);
void Frame2DTimeExportBdgBox(
  const Frame2DTime* const that,
   const AABB2DTime* const bdgBox,
         AABB2DTime* const bdgBoxProj);
void Frame3DTimeExportBdgBox(
  const Frame3DTime* const that,
   const AABB3DTime* const bdgBox,
         AABB3DTime* const bdgBoxProj);

// Power function for integer base and exponent
// Return 'base' ^ 'exp'
int powi(
           int base,
  unsigned int exp);

#endif
```

4.1.2 Body

```
#include "frame.h"

// ------------- Macros -------------

#define EPSILON 0.0000001

// ------------- Functions declaration -------------
```

```
// Update the inverse components of the Frame 'that'
void Frame2DUpdateInv(Frame2D* const that);
void Frame3DUpdateInv(Frame3D* const that);
void Frame2DTimeUpdateInv(Frame2DTime* const that);
void Frame3DTimeUpdateInv(Frame3DTime* const that);

// ------------- Functions implementation -------------

// Create a static Frame structure of FrameType 'type',
// at position 'orig' with components 'comp'
// arrangement is comp[iComp][iAxis]
Frame2D Frame2DCreateStatic(
  const FrameType type,
     const double orig[2],
     const double comp[2][2]) {

  // Create the new Frame
  Frame2D that;
  that.type = type;
  for (int iAxis = 2;
       iAxis--;) {

    that.orig[iAxis] = orig[iAxis];

    for (int iComp = 2;
         iComp--;) {

      that.comp[iComp][iAxis] = comp[iComp][iAxis];

    }

  }

  // Create the bounding box
  for (int iAxis = 2;
       iAxis--;) {

    double min = orig[iAxis];
    double max = orig[iAxis];

    for (int iComp = 2;
         iComp--;) {

      if (that.type == FrameCuboid) {

        if (that.comp[iComp][iAxis] < 0.0) {

          min += that.comp[iComp][iAxis];

        }

        if (that.comp[iComp][iAxis] > 0.0) {

          max += that.comp[iComp][iAxis];

        }

      } else if (that.type == FrameTetrahedron) {

        if (that.comp[iComp][iAxis] < 0.0 &&
          min > orig[iAxis] + that.comp[iComp][iAxis]) {
```

```
          min = orig[iAxis] + that.comp[iComp][iAxis];

        }

        if (that.comp[iComp][iAxis] > 0.0 &&
          max < orig[iAxis] + that.comp[iComp][iAxis]) {

          max = orig[iAxis] + that.comp[iComp][iAxis];

        }

      }

    }

    that.bdgBox.min[iAxis] = min;
    that.bdgBox.max[iAxis] = max;

  }


  // Calculate the inverse matrix
  Frame2DUpdateInv(&that);

  // Return the new Frame
  return that;

}

Frame3D Frame3DCreateStatic(
  const FrameType type,
     const double orig[3],
     const double comp[3][3]) {

  // Create the new Frame
  Frame3D that;
  that.type = type;
  for (int iAxis = 3;
       iAxis--;) {

    that.orig[iAxis] = orig[iAxis];

    for (int iComp = 3;
         iComp--;) {

      that.comp[iComp][iAxis] = comp[iComp][iAxis];

    }

  }

  // Create the bounding box
  for (int iAxis = 3;
       iAxis--;) {

    double min = orig[iAxis];
    double max = orig[iAxis];

    for (int iComp = 3;
         iComp--;) {
```

```
      if (that.type == FrameCuboid) {

        if (that.comp[iComp][iAxis] < 0.0) {

          min += that.comp[iComp][iAxis];

        }

        if (that.comp[iComp][iAxis] > 0.0) {

          max += that.comp[iComp][iAxis];

        }

      } else if (that.type == FrameTetrahedron) {

        if (that.comp[iComp][iAxis] < 0.0 &&
          min > orig[iAxis] + that.comp[iComp][iAxis]) {

          min = orig[iAxis] + that.comp[iComp][iAxis];

        }

        if (that.comp[iComp][iAxis] > 0.0 &&
          max < orig[iAxis] + that.comp[iComp][iAxis]) {

          max = orig[iAxis] + that.comp[iComp][iAxis];

        }

      }

    }

    that.bdgBox.min[iAxis] = min;
    that.bdgBox.max[iAxis] = max;

  }


  // Calculate the inverse matrix
  Frame3DUpdateInv(&that);

  // Return the new Frame
  return that;

}

Frame2DTime Frame2DTimeCreateStatic(
  const FrameType type,
     const double orig[2],
     const double speed[2],
     const double comp[2][2]) {

  // Create the new Frame
  Frame2DTime that;
  that.type = type;
  for (int iAxis = 2;
       iAxis--;) {

    that.orig[iAxis] = orig[iAxis];
    that.speed[iAxis] = speed[iAxis];
```

```
  for (int iComp = 2;
       iComp--;) {

    that.comp[iComp][iAxis] = comp[iComp][iAxis];

  }

}

// Create the bounding box
for (int iAxis = 2;
     iAxis--;) {

  double min = orig[iAxis];
  double max = orig[iAxis];

  for (int iComp = 2;
       iComp--;) {

    if (that.type == FrameCuboid) {

      if (that.comp[iComp][iAxis] < 0.0) {

        min += that.comp[iComp][iAxis];

      }

      if (that.comp[iComp][iAxis] > 0.0) {

        max += that.comp[iComp][iAxis];

      }

    } else if (that.type == FrameTetrahedron) {

      if (that.comp[iComp][iAxis] < 0.0 &&
        min > orig[iAxis] + that.comp[iComp][iAxis]) {

        min = orig[iAxis] + that.comp[iComp][iAxis];

      }

      if (that.comp[iComp][iAxis] > 0.0 &&
        max < orig[iAxis] + that.comp[iComp][iAxis]) {

        max = orig[iAxis] + that.comp[iComp][iAxis];

      }

    }

  }

  if (that.speed[iAxis] < 0.0) {

    min += that.speed[iAxis];

  }

  if (that.speed[iAxis] > 0.0) {
```

```
      max += that.speed[iAxis];

    }

    that.bdgBox.min[iAxis] = min;
    that.bdgBox.max[iAxis] = max;

  }

  that.bdgBox.min[2] = 0.0;
  that.bdgBox.max[2] = 1.0;


  // Calculate the inverse matrix
  Frame2DTimeUpdateInv(&that);

  // Return the new Frame
  return that;

}

Frame3DTime Frame3DTimeCreateStatic(
  const FrameType type,
     const double orig[3],
     const double speed[3],
     const double comp[3][3]) {

  // Create the new Frame
  Frame3DTime that;
  that.type = type;
  for (int iAxis = 3;
       iAxis--;) {

    that.orig[iAxis] = orig[iAxis];
    that.speed[iAxis] = speed[iAxis];

    for (int iComp = 3;
         iComp--;) {

      that.comp[iComp][iAxis] = comp[iComp][iAxis];

    }

  }

  // Create the bounding box
  for (int iAxis = 3;
       iAxis--;) {

    double min = orig[iAxis];
    double max = orig[iAxis];

    for (int iComp = 3;
         iComp--;) {

      if (that.type == FrameCuboid) {

        if (that.comp[iComp][iAxis] < 0.0) {

          min += that.comp[iComp][iAxis];

        }
```

```
        if (that.comp[iComp][iAxis] > 0.0) {

          max += that.comp[iComp][iAxis];

        }

      } else if (that.type == FrameTetrahedron) {

        if (that.comp[iComp][iAxis] < 0.0 &&
          min > orig[iAxis] + that.comp[iComp][iAxis]) {

          min = orig[iAxis] + that.comp[iComp][iAxis];

        }

        if (that.comp[iComp][iAxis] > 0.0 &&
          max < orig[iAxis] + that.comp[iComp][iAxis]) {

          max = orig[iAxis] + that.comp[iComp][iAxis];

        }

      }

    }

    if (that.speed[iAxis] < 0.0) {

      min += that.speed[iAxis];

    }

    if (that.speed[iAxis] > 0.0) {

      max += that.speed[iAxis];

    }

    that.bdgBox.min[iAxis] = min;
    that.bdgBox.max[iAxis] = max;

  }

  that.bdgBox.min[3] = 0.0;
  that.bdgBox.max[3] = 1.0;

  // Calculate the inverse matrix
  Frame3DTimeUpdateInv(&that);

  // Return the new Frame
  return that;

}

// Update the inverse components of the Frame 'that'
void Frame2DUpdateInv(Frame2D* const that) {

  // Shortcuts
  double (*tc)[2] = that->comp;
  double (*tic)[2] = that->invComp;
```

```
  double det = tc[0][0] * tc[1][1] - tc[1][0] * tc[0][1];
  if (fabs(det) < EPSILON) {
    fprintf(stderr,
      "FrameUpdateInv: det == 0.0\n");
    exit(1);
  }

  tic[0][0] = tc[1][1] / det;
  tic[0][1] = -tc[0][1] / det;
  tic[1][0] = -tc[1][0] / det;
  tic[1][1] = tc[0][0] / det;

}

void Frame3DUpdateInv(Frame3D* const that) {

  // Shortcuts
  double (*tc)[3] = that->comp;
  double (*tic)[3] = that->invComp;

  // Update the inverse components
  double det =
    tc[0][0] * (tc[1][1] * tc[2][2] - tc[1][2] * tc[2][1]) -
    tc[1][0] * (tc[0][1] * tc[2][2] - tc[0][2] * tc[2][1]) +
    tc[2][0] * (tc[0][1] * tc[1][2] - tc[0][2] * tc[1][1]);
  if (fabs(det) < EPSILON) {
    fprintf(stderr,
      "FrameUpdateInv: det == 0.0\n");
    exit(1);
  }

  tic[0][0] = (tc[1][1] * tc[2][2] - tc[2][1] * tc[1][2]) / det;
  tic[0][1] = (tc[2][1] * tc[0][2] - tc[2][2] * tc[0][1]) / det;
  tic[0][2] = (tc[0][1] * tc[1][2] - tc[0][2] * tc[1][1]) / det;
  tic[1][0] = (tc[2][0] * tc[1][2] - tc[2][2] * tc[1][0]) / det;
  tic[1][1] = (tc[0][0] * tc[2][2] - tc[2][0] * tc[0][2]) / det;
  tic[1][2] = (tc[0][2] * tc[1][0] - tc[1][2] * tc[0][0]) / det;
  tic[2][0] = (tc[1][0] * tc[2][1] - tc[2][0] * tc[1][1]) / det;
  tic[2][1] = (tc[0][1] * tc[2][0] - tc[2][1] * tc[0][0]) / det;
  tic[2][2] = (tc[0][0] * tc[1][1] - tc[1][0] * tc[0][1]) / det;

}

// Update the inverse components of the Frame 'that'
void Frame2DTimeUpdateInv(Frame2DTime* const that) {

  // Shortcuts
  double (*tc)[2] = that->comp;
  double (*tic)[2] = that->invComp;

  double det = tc[0][0] * tc[1][1] - tc[1][0] * tc[0][1];
  if (fabs(det) < EPSILON) {
    fprintf(stderr,
      "FrameUpdateInv: det == 0.0\n");
    exit(1);
  }

  tic[0][0] = tc[1][1] / det;
  tic[0][1] = -tc[0][1] / det;
  tic[1][0] = -tc[1][0] / det;
  tic[1][1] = tc[0][0] / det;
```

```
}

void Frame3DTimeUpdateInv(Frame3DTime* const that) {

  // Shortcuts
  double (*tc)[3] = that->comp;
  double (*tic)[3] = that->invComp;

  // Update the inverse components
  double det =
    tc[0][0] * (tc[1][1] * tc[2][2] - tc[1][2] * tc[2][1]) -
    tc[1][0] * (tc[0][1] * tc[2][2] - tc[0][2] * tc[2][1]) +
    tc[2][0] * (tc[0][1] * tc[1][2] - tc[0][2] * tc[1][1]);
  if (fabs(det) < EPSILON) {
    fprintf(stderr,
      "FrameUpdateInv: det == 0.0\n");
    exit(1);
  }

  tic[0][0] = (tc[1][1] * tc[2][2] - tc[2][1] * tc[1][2]) / det;
  tic[0][1] = (tc[2][1] * tc[0][2] - tc[2][2] * tc[0][1]) / det;
  tic[0][2] = (tc[0][1] * tc[1][2] - tc[0][2] * tc[1][1]) / det;
  tic[1][0] = (tc[2][0] * tc[1][2] - tc[2][2] * tc[1][0]) / det;
  tic[1][1] = (tc[0][0] * tc[2][2] - tc[2][0] * tc[0][2]) / det;
  tic[1][2] = (tc[0][2] * tc[1][0] - tc[1][2] * tc[0][0]) / det;
  tic[2][0] = (tc[1][0] * tc[2][1] - tc[2][0] * tc[1][1]) / det;
  tic[2][1] = (tc[0][1] * tc[2][0] - tc[2][1] * tc[0][0]) / det;
  tic[2][2] = (tc[0][0] * tc[1][1] - tc[1][0] * tc[0][1]) / det;

}

// Project the Frame 'Q' in the Frame 'P' 's coordinates system and
// memorize the result in the Frame 'Qp'
void Frame2DImportFrame(
  const Frame2D* const P,
  const Frame2D* const Q,
        Frame2D* const Qp) {

  // Shortcuts
  const double*  qo  = Q->orig;
        double*  qpo = Qp->orig;
  const double*  po  = P->orig;

  const double  (*pi)[2] = P->invComp;
        double (*qpc)[2] = Qp->comp;
  const double  (*qc)[2] = Q->comp;

  // Calculate the projection
  double v[2];
  for (int i = 2;
       i--;) {

    v[i] = qo[i] - po[i];

  }

  for (int i = 2;
       i--;) {

    qpo[i] = 0.0;

    for (int j = 2;
```

```
         j--;) {

      qpo[i] += pi[j][i] * v[j];
      qpc[j][i] = 0.0;

      for (int k = 2;
           k--;) {

        qpc[j][i] += pi[k][i] * qc[j][k];

      }
    }
  }
}

void Frame3DImportFrame(
  const Frame3D* const P,
  const Frame3D* const Q,
        Frame3D* const Qp) {

  // Shortcuts
  const double*  qo  = Q->orig;
        double*  qpo = Qp->orig;
  const double*  po  = P->orig;

  const double  (*pi)[3] = P->invComp;
        double (*qpc)[3] = Qp->comp;
  const double  (*qc)[3] = Q->comp;

  // Calculate the projection
  double v[3];
  for (int i = 3;
       i--;) {

    v[i] = qo[i] - po[i];

  }

  for (int i = 3;
       i--;) {

    qpo[i] = 0.0;

    for (int j = 3;
         j--;) {

      qpo[i] += pi[j][i] * v[j];
      qpc[j][i] = 0.0;

      for (int k = 3;
           k--;) {

        qpc[j][i] += pi[k][i] * qc[j][k];

      }
    }
  }
}

void Frame2DTimeImportFrame(
  const Frame2DTime* const P,
  const Frame2DTime* const Q,
```

```
        Frame2DTime* const Qp) {

  // Shortcuts
  const double*  qo  = Q->orig;
        double*  qpo = Qp->orig;
  const double*  po  = P->orig;

  const double*  qs  = Q->speed;
        double*  qps = Qp->speed;
  const double*  ps  = P->speed;

  const double  (*pi)[2] = P->invComp;
        double (*qpc)[2] = Qp->comp;
  const double  (*qc)[2] = Q->comp;

  // Calculate the projection
  double v[2];
  double s[2];
  for (int i = 2;
       i--;) {

    v[i] = qo[i] - po[i];
    s[i] = qs[i] - ps[i];

  }

  for (int i = 2;
       i--;) {

    qpo[i] = 0.0;
    qps[i] = 0.0;

    for (int j = 2;
         j--;) {

      qpo[i] += pi[j][i] * v[j];
      qps[i] += pi[j][i] * s[j];
      qpc[j][i] = 0.0;

      for (int k = 2;
           k--;) {

        qpc[j][i] += pi[k][i] * qc[j][k];

      }
    }
  }
}

void Frame3DTimeImportFrame(
  const Frame3DTime* const P,
  const Frame3DTime* const Q,
        Frame3DTime* const Qp) {

  // Shortcuts
  const double*  qo  = Q->orig;
        double*  qpo = Qp->orig;
  const double*  po  = P->orig;

  const double*  qs  = Q->speed;
        double*  qps = Qp->speed;
  const double*  ps  = P->speed;
```

```
  const double  (*pi)[3] = P->invComp;
        double (*qpc)[3] = Qp->comp;
  const double  (*qc)[3] = Q->comp;

  // Calculate the projection
  double v[3];
  double s[3];
  for (int i = 3;
       i--;) {

    v[i] = qo[i] - po[i];
    s[i] = qs[i] - ps[i];

  }

  for (int i = 3;
       i--;) {

    qpo[i] = 0.0;
    qps[i] = 0.0;

    for (int j = 3;
         j--;) {

      qpo[i] += pi[j][i] * v[j];
      qps[i] += pi[j][i] * s[j];
      qpc[j][i] = 0.0;

      for (int k = 3;
           k--;) {

        qpc[j][i] += pi[k][i] * qc[j][k];

      }
    }
  }
}

// Export the AABB 'bdgBox' from 'that' 's coordinates system to
// the real coordinates system and update 'bdgBox' with the resulting
// AABB
void Frame2DExportBdgBox(
  const Frame2D* const that,
   const AABB2D* const bdgBox,
         AABB2D* const bdgBoxProj) {

  // Shortcuts
  const double* to    = that->orig;
  const double* bbmi  = bdgBox->min;
  const double* bbma  = bdgBox->max;
        double* bbpmi = bdgBoxProj->min;
        double* bbpma = bdgBoxProj->max;

  const double (*tc)[2] = that->comp;

  // Initialise the coordinates of the result AABB with the projection
  // of the first corner of the AABB in argument
  for (int i = 2;
       i--;) {

    bbpma[i] = to[i];
```

```
  for (int j = 2;
       j--;) {

    bbpma[i] += tc[j][i] * bbmi[j];

  }

  bbpmi[i] = bbpma[i];

}

// Loop on vertices of the AABB
// skip the first vertex which is the origin already computed above
int nbVertices = powi(2, 2);
for (int iVertex = nbVertices;
     iVertex-- && iVertex;) {

  // Declare a variable to memorize the coordinates of the vertex in
  // 'that' 's coordinates system
  double v[2];

  // Calculate the coordinates of the vertex in
  // 'that' 's coordinates system
  for (int i = 2;
       i--;) {

    v[i] = ((iVertex & (1 << i)) ? bbma[i] : bbmi[i]);

  }

  // Declare a variable to memorize the projected coordinates
  // in real coordinates system
  double w[2];

  // Project the vertex to real coordinates system
  for (int i = 2;
       i--;) {

    w[i] = to[i];

    for (int j = 2;
         j--;) {

      w[i] += tc[j][i] * v[j];

    }
  }

  // Update the coordinates of the result AABB
  for (int i = 2;
       i--;) {

    if (bbpmi[i] > w[i]) {

      bbpmi[i] = w[i];

    }
    if (bbpma[i] < w[i]) {

      bbpma[i] = w[i];
```

```
      }
    }
  }

}

void Frame3DExportBdgBox(
  const Frame3D* const that,
   const AABB3D* const bdgBox,
         AABB3D* const bdgBoxProj) {

  // Shortcuts
  const double* to    = that->orig;
  const double* bbmi  = bdgBox->min;
  const double* bbma  = bdgBox->max;
        double* bbpmi = bdgBoxProj->min;
        double* bbpma = bdgBoxProj->max;

  const double (*tc)[3] = that->comp;

  // Initialise the coordinates of the result AABB with the projection
  // of the first corner of the AABB in argument
  for (int i = 3;
       i--;) {

    bbpma[i] = to[i];

    for (int j = 3;
         j--;) {

      bbpma[i] += tc[j][i] * bbmi[j];

    }

    bbpmi[i] = bbpma[i];

  }

  // Loop on vertices of the AABB
  // skip the first vertex which is the origin already computed above
  int nbVertices = powi(2, 3);
  for (int iVertex = nbVertices;
       iVertex-- && iVertex;) {

    // Declare a variable to memorize the coordinates of the vertex in
    // 'that' 's coordinates system
    double v[3];

    // Calculate the coordinates of the vertex in
    // 'that' 's coordinates system
    for (int i = 3;
         i--;) {

      v[i] = ((iVertex & (1 << i)) ? bbma[i] : bbmi[i]);

    }

    // Declare a variable to memorize the projected coordinates
    // in real coordinates system
    double w[3];

    // Project the vertex to real coordinates system
```

```
    for (int i = 3;
         i--;) {

      w[i] = to[i];

      for (int j = 3;
           j--;) {

        w[i] += tc[j][i] * v[j];

      }
    }

    // Update the coordinates of the result AABB
    for (int i = 3;
         i--;) {

      if (bbpmi[i] > w[i]) {

        bbpmi[i] = w[i];

      }
      if (bbpma[i] < w[i]) {

        bbpma[i] = w[i];

      }
    }
  }

}

void Frame2DTimeExportBdgBox(
  const Frame2DTime* const that,
   const AABB2DTime* const bdgBox,
         AABB2DTime* const bdgBoxProj) {

  // Shortcuts
  const double* to    = that->orig;
  const double* ts    = that->speed;
  const double* bbmi  = bdgBox->min;
  const double* bbma  = bdgBox->max;
        double* bbpmi = bdgBoxProj->min;
        double* bbpma = bdgBoxProj->max;
  const double (*tc)[2] = that->comp;

  // The time component is not affected
  bbpmi[2] = bbmi[2];
  bbpma[2] = bbma[2];

  // Initialise the coordinates of the result AABB with the projection
  // of the first corner of the AABB in argument
  for (int i = 2;
       i--;) {

    bbpma[i] = to[i] + ts[i] * bbmi[2];

    for (int j = 2;
         j--;) {

      bbpma[i] += tc[j][i] * bbmi[j];
```

```
  }

  bbpmi[i] = bbpma[i];

}

// Loop on vertices of the AABB
// skip the first vertex which is the origin already computed above
int nbVertices = powi(2, 2);
for (int iVertex = nbVertices;
     iVertex-- && iVertex;) {

  // Declare a variable to memorize the coordinates of the vertex in
  // 'that' 's coordinates system
  double v[2];

  // Calculate the coordinates of the vertex in
  // 'that' 's coordinates system
  for (int i = 2;
       i--;) {

    v[i] = ((iVertex & (1 << i)) ? bbma[i] : bbmi[i]);

  }

  // Declare a variable to memorize the projected coordinates
  // in real coordinates system
  double w[2];

  // Project the vertex to real coordinates system
  for (int i = 2;
       i--;) {

    w[i] = to[i];

    for (int j = 2;
         j--;) {

      w[i] += tc[j][i] * v[j];

    }
  }

  // Update the coordinates of the result AABB
  for (int i = 2;
       i--;) {

    if (bbpmi[i] > w[i] + ts[i] * bbmi[2]) {

      bbpmi[i] = w[i] + ts[i] * bbmi[2];

    }
    if (bbpmi[i] > w[i] + ts[i] * bbma[2]) {

      bbpmi[i] = w[i] + ts[i] * bbma[2];

    }
    if (bbpma[i] < w[i] + ts[i] * bbmi[2]) {

      bbpma[i] = w[i] + ts[i] * bbmi[2];

    }
```

```
      if (bbpma[i] < w[i] + ts[i] * bbma[2]) {

        bbpma[i] = w[i] + ts[i] * bbma[2];

      }
    }
  }

}

void Frame3DTimeExportBdgBox(
  const Frame3DTime* const that,
   const AABB3DTime* const bdgBox,
         AABB3DTime* const bdgBoxProj) {

  // Shortcuts
  const double* to    = that->orig;
  const double* ts    = that->speed;
  const double* bbmi  = bdgBox->min;
  const double* bbma  = bdgBox->max;
        double* bbpmi = bdgBoxProj->min;
        double* bbpma = bdgBoxProj->max;
  const double (*tc)[3] = that->comp;

  // The time component is not affected
  bbpmi[3] = bbmi[3];
  bbpma[3] = bbma[3];

  // Initialise the coordinates of the result AABB with the projection
  // of the first corner of the AABB in argument
  for (int i = 3;
       i--;) {

    bbpma[i] = to[i] + ts[i] * bbmi[3];

    for (int j = 3;
         j--;) {

      bbpma[i] += tc[j][i] * bbmi[j];

    }

    bbpmi[i] = bbpma[i];

  }

  // Loop on vertices of the AABB
  // skip the first vertex which is the origin already computed above
  int nbVertices = powi(2, 3);
  for (int iVertex = nbVertices;
       iVertex-- && iVertex;) {

    // Declare a variable to memorize the coordinates of the vertex in
    // 'that' 's coordinates system
    double v[3];

    // Calculate the coordinates of the vertex in
    // 'that' 's coordinates system
    for (int i = 3;
         i--;) {

      v[i] = ((iVertex & (1 << i)) ? bbma[i] : bbmi[i]);
```

```
    }

    // Declare a variable to memorize the projected coordinates
    // in real coordinates system
    double w[3];

    // Project the vertex to real coordinates system
    for (int i = 3;
         i--;) {

      w[i] = to[i];

      for (int j = 3;
           j--;) {

        w[i] += tc[j][i] * v[j];

      }
    }

    // Update the coordinates of the result AABB
    for (int i = 3;
         i--;) {

      if (bbpmi[i] > w[i] + ts[i] * bbmi[3]) {

        bbpmi[i] = w[i] + ts[i] * bbmi[3];

      }
      if (bbpmi[i] > w[i] + ts[i] * bbma[3]) {

        bbpmi[i] = w[i] + ts[i] * bbma[3];

      }
      if (bbpma[i] < w[i] + ts[i] * bbmi[3]) {

        bbpma[i] = w[i] + ts[i] * bbmi[3];

      }
      if (bbpma[i] < w[i] + ts[i] * bbma[3]) {

        bbpma[i] = w[i] + ts[i] * bbma[3];

      }
    }
  }

}

// Print the AABB 'that' on stdout
// Output format is (min[0], min[1], ...)-(max[0], max[1], ...)
void AABB2DPrint(const AABB2D* const that) {

  printf("minXY(");
  for (int i = 0;
       i < 2;
       ++i) {

    printf("%f", that->min[i]);
    if (i < 1)
      printf(",");
```

```
  }
  printf(")-maxXY(");
  for (int i = 0;
       i < 2;
       ++i) {

    printf("%f", that->max[i]);
    if (i < 1)
      printf(",");

  }
  printf(")");

}

void AABB3DPrint(const AABB3D* const that) {

  printf("minXYZ(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->min[i]);
    if (i < 2)
      printf(",");

  }
  printf(")-maxXYZ(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->max[i]);
    if (i < 2)
      printf(",");

  }
  printf(")");

}

void AABB2DTimePrint(const AABB2DTime* const that) {

  printf("minXYT(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->min[i]);
    if (i < 2)
      printf(",");

  }
  printf(")-maxXYT(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->max[i]);
    if (i < 2)
      printf(",");
```

```
  }
  printf(")");

}

void AABB3DTimePrint(const AABB3DTime* const that) {

  printf("minXYZT(");
  for (int i = 0;
       i < 4;
       ++i) {

    printf("%f", that->min[i]);
    if (i < 3)
      printf(",");

  }
  printf(")-maxXYZT(");
  for (int i = 0;
       i < 4;
       ++i) {

    printf("%f", that->max[i]);
    if (i < 3)
      printf(",");

  }
  printf(")");

}

// Print the Frame 'that' on stdout
// Output format is (orig[0], orig[1], orig[2])
// (comp[0][0], comp[0][1], comp[0][2])
// (comp[1][0], comp[1][1], comp[1][2])
// (comp[2][0], comp[2][1], comp[2][2])
void Frame2DPrint(const Frame2D* const that) {
  if (that->type == FrameTetrahedron) {
    printf("T");
  } else if (that->type == FrameCuboid) {
    printf("C");
  }
  printf("o(");
  for (int i = 0;
       i < 2;
       ++i) {

    printf("%f", that->orig[i]);
    if (i < 1)
      printf(",");

  }
  char comp[2] = {'x', 'y'};
  for (int j = 0;
       j < 2;
       ++j) {
    printf(") %c(", comp[j]);
    for (int i = 0;
         i < 2;
         ++i) {
```

```
      printf("%f", that->comp[j][i]);
      if (i < 1)
        printf(",");

    }
  }
  printf(")");

}

void Frame3DPrint(const Frame3D* const that) {
  if (that->type == FrameTetrahedron) {
    printf("T");
  } else if (that->type == FrameCuboid) {
    printf("C");
  }
  printf("o(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->orig[i]);
    if (i < 2)
      printf(",");

  }
  char comp[3] = {'x', 'y', 'z'};
  for (int j = 0;
       j < 3;
       ++j) {
    printf(") %c(", comp[j]);
    for (int i = 0;
         i < 3;
         ++i) {

      printf("%f", that->comp[j][i]);
      if (i < 2)
        printf(",");

    }
  }
  printf(")");

}

void Frame2DTimePrint(const Frame2DTime* const that) {
  if (that->type == FrameTetrahedron) {
    printf("T");
  } else if (that->type == FrameCuboid) {
    printf("C");
  }
  printf("o(");
  for (int i = 0;
       i < 2;
       ++i) {

    printf("%f", that->orig[i]);
    if (i < 1)
      printf(",");

  }
  printf(") s(");
```

```
  for (int i = 0;
       i < 2;
       ++i) {

    printf("%f", that->speed[i]);
    if (i < 1)
      printf(",");

  }
  char comp[2] = {'x', 'y'};
  for (int j = 0;
       j < 2;
       ++j) {
    printf(") %c(", comp[j]);
    for (int i = 0;
         i < 2;
         ++i) {

      printf("%f", that->comp[j][i]);
      if (i < 1)
        printf(",");

    }
  }
  printf(")");

}

void Frame3DTimePrint(const Frame3DTime* const that) {
  if (that->type == FrameTetrahedron) {
    printf("T");
  } else if (that->type == FrameCuboid) {
    printf("C");
  }
  printf("o(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->orig[i]);
    if (i < 2)
      printf(",");

  }
  printf(") s(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->speed[i]);
    if (i < 2)
      printf(",");

  }
  char comp[3] = {'x', 'y', 'z'};
  for (int j = 0;
       j < 3;
       ++j) {
    printf(") %c(", comp[j]);
    for (int i = 0;
         i < 3;
         ++i) {
```

```
      printf("%f", that->comp[j][i]);
      if (i < 2)
        printf(",");

    }
  }
  printf(")");

}

// Power function for integer base and exponent
// Return 'base' ^ 'exp'
int powi(
           int base,
  unsigned int exp) {

    int res = 1;
    for (;
         exp;
         --exp) {

      res *= base;

    }
    return res;
}
```

4.2 FMB

4.2.1 2D static

Header

```
#ifndef __FMB2D_H_
#define __FMB2D_H_

#include <stdbool.h>
#include "frame.h"

// ------------- Functions declaration -------------

// Test for intersection between Frame 'that' and Frame 'tho'
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into 'bdgBox', else 'bdgBox' is not modified
// If 'bdgBox' is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in 'tho' 's local coordinates system
bool FMBTestIntersection2D(
  const Frame2D* const that,
  const Frame2D* const tho,
         AABB2D* const bdgBox);

#endif
```

Body

```
#include "fmb2d.h"

// ------------- Macros -------------

// Return 1.0 if v is positive, -1.0 if v is negative, 0.0 else
#define sgn(v) (((0.0 < (v)) ? 1 : 0) - (((v) < 0.0) ? 1 : 0))

// Return x if x is negative, 0.0 else
#define neg(x) (x < 0.0 ? x : 0.0)

#define FST_VAR 0
#define SND_VAR 1

#define EPSILON 0.0000001

// ------------- Functions declaration -------------

// Eliminate the 'iVar'-th variable in the system 'M'.X<='Y'
// using the Fourier-Motzkin method and return
// the resulting system in 'Mp' and 'Yp', and the number of rows of
// the resulting system in 'nbRemainRows'
// Return false if the system becomes inconsistent during elimination,
// else return true
bool ElimVar2D(
     const int iVar,
  const double (*M)[2],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[2],
        double* Yp,
    int* const nbRemainRows);

// Get the bounds of the 'iVar'-th variable in the 'nbRows' rows
// system 'M'.X<='Y' and store them in the 'iVar'-th axis of the
// AABB 'bdgBox'
// ('M' arrangement is [iRow][iCol])
// The system is supposed to have been reduced to only one variable
// per row, the one in argument, which can be located in a different
// column than 'iVar'
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBound2D(
     const int iVar,
  const double (*M)[2],
  const double* Y,
     const int nbRows,
   AABB2D* const bdgBox);

// ------------- Functions implementation -------------

// Eliminate the 'iVar'-th variable in the system 'M'.X<='Y'
// using the Fourier-Motzkin method and return
// the resulting system in 'Mp' and 'Yp', and the number of rows of
// the resulting system in 'nbRemainRows'
// ('M' arrangement is [iRow][iCol])
// Return true if the system becomes inconsistent during elimination,
// else return false
bool ElimVar2D(
     const int iVar,
  const double (*M)[2],
  const double* Y,
```

```
   const int nbRows,
   const int nbCols,
      double (*Mp)[2],
      double* Yp,
  int* const nbRemainRows) {

// Initialize the number of rows in the result system
*nbRemainRows = 0;

// First we process the rows where the eliminated variable is not null

// For each row except the last one
for (int iRow = 0;
     iRow < nbRows - 1;
     ++iRow) {

  // Shortcuts
  int sgnMIRowIVar = sgn(M[iRow][iVar]);
  double fabsMIRowIVar = fabs(M[iRow][iVar]);
  double YIRowDivideByFabsMIRowIVar = Y[iRow] / fabsMIRowIVar;

  // For each following rows
  for (int jRow = iRow + 1;
       jRow < nbRows;
       ++jRow) {

    // If coefficients of the eliminated variable in the two rows have
    // different signs and are not null
    if (sgnMIRowIVar != sgn(M[jRow][iVar]) &&
        fabsMIRowIVar > EPSILON &&
        fabs(M[jRow][iVar]) > EPSILON) {

      // Declare a variable to memorize the sum of the negative
      // coefficients in the row
      double sumNegCoeff = 0.0;

      // Add the sum of the two normed (relative to the eliminated
      // variable) rows into the result system. This actually
      // eliminate the variable while keeping the constraints on
      // others variables
      for (int iCol = 0, jCol = 0;
           iCol < nbCols;
           ++iCol ) {

        if (iCol != iVar) {

          Mp[*nbRemainRows][jCol] =
            M[iRow][iCol] / fabsMIRowIVar +
            M[jRow][iCol] / fabs(M[jRow][iVar]);

          // Update the sum of the negative coefficient
          sumNegCoeff += neg(Mp[*nbRemainRows][jCol]);

          // Increment the number of columns in the new inequality
          ++jCol;

        }

      }

      // Update the right side of the inequality
      Yp[*nbRemainRows] =
```

```
        YIRowDivideByFabsMIRowIVar +
        Y[jRow] / fabs(M[jRow][iVar]);

      // If the right side of the inequality if lower than the sum of
      // negative coefficients in the row
      // (Add epsilon for numerical imprecision)
      if (Yp[*nbRemainRows] < sumNegCoeff - EPSILON) {

        // Given that X is in [0,1], the system is inconsistent
        return true;

      }

      // Increment the nb of rows into the result system
      ++(*nbRemainRows);

    }

  }

}

// Then we copy and compress the rows where the eliminated
// variable is null

// Loop on rows of the input system
for (int iRow = 0;
     iRow < nbRows;
     ++iRow) {

  // Shortcut
  const double* MiRow = M[iRow];

  // If the coefficient of the eliminated variable is null on
  // this row
  if (fabs(MiRow[iVar]) < EPSILON) {

    // Shortcut
    double* MpnbRemainRows = Mp[*nbRemainRows];

    // Copy this row into the result system excluding the eliminated
    // variable
    for (int iCol = 0, jCol = 0;
         iCol < nbCols;
         ++iCol) {

      if (iCol != iVar) {

        MpnbRemainRows[jCol] = MiRow[iCol];

        ++jCol;

      }

    }

    Yp[*nbRemainRows] = Y[iRow];

    // Increment the nb of rows into the result system
    ++(*nbRemainRows);

  }
```

```
  }

  // If we reach here the system is not inconsistent
  return false;

}

// Get the bounds of the 'iVar'-th variable in the 'nbRows' rows
// system 'M'.X<='Y' and store them in the 'iVar'-th axis of the
// AABB 'bdgBox'
// ('M' arrangement is [iRow][iCol])
// The system is supposed to have been reduced to only one variable
// per row, the one in argument
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBound2D(
     const int iVar,
  const double (*M)[2],
  const double* Y,
     const int nbRows,
   AABB2D* const bdgBox) {

  // Shortcuts
  double* min = bdgBox->min + iVar;
  double* max = bdgBox->max + iVar;

  // Initialize the bounds to there maximum maximum and minimum minimum
  *min = 0.0;
  *max = 1.0;

  // Loop on rows
  for (int jRow = 0;
       jRow < nbRows;
       ++jRow) {

    // Shortcut
    double MjRowiVar = M[jRow][0];

    // If this row has been reduced to the variable in argument
    // and it has a strictly positive coefficient
    if (MjRowiVar > EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is lower than the current maximum bound
      if (*max > y) {

        // Update the maximum bound
        *max = y;

      }

    // Else, if this row has been reduced to the variable in argument
    // and it has a strictly negative coefficient
    } else if (MjRowiVar < -EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is greater than the current minimum bound
```

```
      if (*min < y) {

        // Update the minimum bound
        *min = y;

      }

    }

  }

}

// Test for intersection between Frame 'that' and Frame 'tho'
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into 'bdgBox', else 'bdgBox' is not modified
// If 'bdgBox' is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in 'tho' 's local coordinates system
bool FMBTestIntersection2D(
  const Frame2D* const that,
  const Frame2D* const tho,
        AABB2D* const bdgBox) {
//Frame2DPrint(that);printf("\n");
//Frame2DPrint(tho);printf("\n");
  // Get the projection of the Frame 'tho' in Frame 'that' coordinates
  // system
  Frame2D thoProj;
  Frame2DImportFrame(that, tho, &thoProj);

  // Declare two variables to memorize the system to be solved M.X <= Y
  // (M arrangement is [iRow][iCol])
  double M[8][2];
  double Y[8];

  // Create the inequality system

  // -sum_iC_j,iX_i<=O_j
  M[0][0] = -thoProj.comp[0][0];
  M[0][1] = -thoProj.comp[1][0];
  Y[0] = thoProj.orig[0];
  if (Y[0] < neg(M[0][0]) + neg(M[0][1]))
    return false;

  M[1][0] = -thoProj.comp[0][1];
  M[1][1] = -thoProj.comp[1][1];
  Y[1] = thoProj.orig[1];
  if (Y[1] < neg(M[1][0]) + neg(M[1][1]))
    return false;

  // -X_i <= 0.0
  M[2][0] = -1.0;
  M[2][1] = 0.0;
  Y[2] = 0.0;

  M[3][0] = 0.0;
  M[3][1] = -1.0;
  Y[3] = 0.0;
```

```
// Variable to memorise the nb of rows in the system
int nbRows = 4;

if (that->type == FrameCuboid) {

  // sum_iC_j,iX_i<=1.0-O_j
  M[nbRows][0] = thoProj.comp[0][0];
  M[nbRows][1] = thoProj.comp[1][0];
  Y[nbRows] = 1.0 - thoProj.orig[0];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]))
    return false;
  ++nbRows;

  M[nbRows][0] = thoProj.comp[0][1];
  M[nbRows][1] = thoProj.comp[1][1];
  Y[nbRows] = 1.0 - thoProj.orig[1];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]))
    return false;
  ++nbRows;

} else {

  // sum_j(sum_iC_j,iX_i)<=1.0-sum_iO_i
  M[nbRows][0] = thoProj.comp[0][0] + thoProj.comp[0][1];
  M[nbRows][1] = thoProj.comp[1][0] + thoProj.comp[1][1];
  Y[nbRows] = 1.0 - thoProj.orig[0] - thoProj.orig[1];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]))
    return false;
  ++nbRows;

}

if (tho->type == FrameCuboid) {

  // X_i <= 1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

  M[nbRows][0] = 0.0;
  M[nbRows][1] = 1.0;
  Y[nbRows] = 1.0;
  ++nbRows;

} else {

  // sum_iX_i<=1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 1.0;
  Y[nbRows] = 1.0;
  ++nbRows;

}

// Solve the system

// Declare a AABB to memorize the bounding box of the intersection
// in the coordinates system of that
AABB2D bdgBoxLocal;
```

```
// Declare variables to eliminate the first variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mp[24][2];
//double Yp[24];
double Mp[11][2];
double Yp[11];
int nbRowsP;

// Eliminate the first variable
bool inconsistency =
  ElimVar2D(
    FST_VAR,
    M,
    Y,
    nbRows,
    2,
    Mp,
    Yp,
    &nbRowsP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;

}

// Get the bounds for the remaining second variable
GetBound2D(
  SND_VAR,
  Mp,
  Yp,
  nbRowsP,
  &bdgBoxLocal);

// If the bounds are inconsistent
if (bdgBoxLocal.min[SND_VAR] >= bdgBoxLocal.max[SND_VAR]) {

  // The two Frames are not in intersection
  return false;

// Else, if the bounds are consistent here it means
// the two Frames are in intersection.
// If the user hasn't requested for the resulting bounding box
} else if (bdgBox == NULL) {

  // Immediately return true
  return true;

}

// Now starts again from the initial systems and eliminate the
// second variable to get the bounds of the first variable
// No need to check for consistency because we already know here
// that the Frames are intersecting and the system is consistent
inconsistency =
  ElimVar2D(
    SND_VAR,
```

```
      M,
      Y,
      nbRows,
      2,
      Mp,
      Yp,
      &nbRowsP);

  // Get the bounds for the remaining first variable
  GetBound2D(
    FST_VAR,
    Mp,
    Yp,
    nbRowsP,
    &bdgBoxLocal);

  // If the user requested the resulting bounding box
  if (bdgBox != NULL) {

    // Memorize the result
    *bdgBox = bdgBoxLocal;

  }

  // If we've reached here the two Frames are intersecting
  return true;

}
```

4.2.2 3D static

Header

```
#ifndef __FMB3D_H_
#define __FMB3D_H_

#include <stdbool.h>
#include "frame.h"

// ------------- Functions declaration -------------

// Test for intersection between Frame 'that' and Frame 'tho'
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into 'bdgBox', else 'bdgBox' is not modified
// If 'bdgBox' is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in 'tho' 's local coordinates system
bool FMBTestIntersection3D(
  const Frame3D* const that,
  const Frame3D* const tho,
        AABB3D* const bdgBox);

#endif
```

Body

```
#include "fmb3d.h"

// ------------- Macros -------------

// Return 1.0 if v is positive, -1.0 if v is negative, 0.0 else
#define sgn(v) (((0.0 < (v)) ? 1 : 0) - (((v) < 0.0) ? 1 : 0))

// Return x if x is negative, 0.0 else
#define neg(x) (x < 0.0 ? x : 0.0)

#define FST_VAR 0
#define SND_VAR 1
#define THD_VAR 2

#define EPSILON 0.0000001

// ------------- Functions declaration -------------

// Eliminate the 'iVar'-th variable in the system 'M'.X<='Y'
// using the Fourier-Motzkin method and return
// the resulting system in 'Mp' and 'Yp', and the number of rows of
// the resulting system in 'nbRemainRows'
// Return false if the system becomes inconsistent during elimination,
// else return true
bool ElimVar3D(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[3],
        double* Yp,
    int* const nbRemainRows);

// Get the bounds of the 'iVar'-th variable in the 'nbRows' rows
// system 'M'.X<='Y' and store them in the 'iVar'-th axis of the
// AABB 'bdgBox'
// ('M' arrangement is [iRow][iCol])
// The system is supposed to have been reduced to only one variable
// per row, the one in argument, which can be located in a different
// column than 'iVar'
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBound3D(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
   AABB3D* const bdgBox);

// ------------- Functions implementation -------------

// Eliminate the 'iVar'-th variable in the system 'M'.X<='Y'
// using the Fourier-Motzkin method and return
// the resulting system in 'Mp' and 'Yp', and the number of rows of
// the resulting system in 'nbRemainRows'
// ('M' arrangement is [iRow][iCol])
// Return true if the system becomes inconsistent during elimination,
// else return false
bool ElimVar3D(
     const int iVar,
  const double (*M)[3],
```

```
const double* Y,
   const int nbRows,
   const int nbCols,
      double (*Mp)[3],
      double* Yp,
  int* const nbRemainRows) {

// Initialize the number of rows in the result system
*nbRemainRows = 0;

// First we process the rows where the eliminated variable is not null

// For each row except the last one
for (int iRow = 0;
     iRow < nbRows - 1;
     ++iRow) {

  // Shortcuts
  int sgnMIRowIVar = sgn(M[iRow][iVar]);
  double fabsMIRowIVar = fabs(M[iRow][iVar]);
  double YIRowDivideByFabsMIRowIVar = Y[iRow] / fabsMIRowIVar;

  // For each following rows
  for (int jRow = iRow + 1;
       jRow < nbRows;
       ++jRow) {

    // If coefficients of the eliminated variable in the two rows have
    // different signs and are not null
    if (sgnMIRowIVar != sgn(M[jRow][iVar]) &&
        fabsMIRowIVar > EPSILON &&
        fabs(M[jRow][iVar]) > EPSILON) {

      // Declare a variable to memorize the sum of the negative
      // coefficients in the row
      double sumNegCoeff = 0.0;

      // Add the sum of the two normed (relative to the eliminated
      // variable) rows into the result system. This actually
      // eliminate the variable while keeping the constraints on
      // others variables
      for (int iCol = 0, jCol = 0;
           iCol < nbCols;
           ++iCol ) {

        if (iCol != iVar) {

          Mp[*nbRemainRows][jCol] =
            M[iRow][iCol] / fabsMIRowIVar +
            M[jRow][iCol] / fabs(M[jRow][iVar]);

          // Update the sum of the negative coefficient
          sumNegCoeff += neg(Mp[*nbRemainRows][jCol]);

          // Increment the number of columns in the new inequality
          ++jCol;

        }

      }

      // Update the right side of the inequality
```

```
      Yp[*nbRemainRows] =
        YIRowDivideByFabsMIRowIVar +
        Y[jRow] / fabs(M[jRow][iVar]);

      // If the right side of the inequality if lower than the sum of
      // negative coefficients in the row
      // (Add epsilon for numerical imprecision)
      if (Yp[*nbRemainRows] < sumNegCoeff - EPSILON) {

        // Given that X is in [0,1], the system is inconsistent
        return true;

      }

      // Increment the nb of rows into the result system
      ++(*nbRemainRows);

    }

  }

}

// Then we copy and compress the rows where the eliminated
// variable is null

// Loop on rows of the input system
for (int iRow = 0;
     iRow < nbRows;
     ++iRow) {

  // Shortcut
  const double* MiRow = M[iRow];

  // If the coefficient of the eliminated variable is null on
  // this row
  if (fabs(MiRow[iVar]) < EPSILON) {

    // Shortcut
    double* MpnbRemainRows = Mp[*nbRemainRows];

    // Copy this row into the result system excluding the eliminated
    // variable
    for (int iCol = 0, jCol = 0;
         iCol < nbCols;
         ++iCol) {

      if (iCol != iVar) {

        MpnbRemainRows[jCol] = MiRow[iCol];

        ++jCol;

      }

    }

    Yp[*nbRemainRows] = Y[iRow];

    // Increment the nb of rows into the result system
    ++(*nbRemainRows);
```

```
    }

  }

  // If we reach here the system is not inconsistent
  return false;

}

// Get the bounds of the 'iVar'-th variable in the 'nbRows' rows
// system 'M'.X<='Y' and store them in the 'iVar'-th axis of the
// AABB 'bdgBox'
// ('M' arrangement is [iRow][iCol])
// The system is supposed to have been reduced to only one variable
// per row, the one in argument
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBound3D(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
   AABB3D* const bdgBox) {

  // Shortcuts
  double* min = bdgBox->min + iVar;
  double* max = bdgBox->max + iVar;

  // Initialize the bounds to there maximum maximum and minimum minimum
  *min = 0.0;
  *max = 1.0;

  // Loop on rows
  for (int jRow = 0;
       jRow < nbRows;
       ++jRow) {

    // Shortcut
    double MjRowiVar = M[jRow][0];

    // If this row has been reduced to the variable in argument
    // and it has a strictly positive coefficient
    if (MjRowiVar > EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is lower than the current maximum bound
      if (*max > y) {

        // Update the maximum bound
        *max = y;

      }

    // Else, if this row has been reduced to the variable in argument
    // and it has a strictly negative coefficient
    } else if (MjRowiVar < -EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;
```

```
      // If the value is greater than the current minimum bound
      if (*min < y) {

        // Update the minimum bound
        *min = y;

      }

    }

  }

}

// Test for intersection between Frame 'that' and Frame 'tho'
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into 'bdgBox', else 'bdgBox' is not modified
// If 'bdgBox' is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in 'tho' 's local coordinates system
bool FMBTestIntersection3D(
  const Frame3D* const that,
  const Frame3D* const tho,
        AABB3D* const bdgBox) {

  // Get the projection of the Frame 'tho' in Frame 'that' coordinates
  // system
  Frame3D thoProj;
  Frame3DImportFrame(that, tho, &thoProj);

  // Declare two variables to memorize the system to be solved M.X <= Y
  // (M arrangement is [iRow][iCol])
  double M[12][3];
  double Y[12];

  // Create the inequality system

  // -sum_iC_j,iX_i<=0_j
  M[0][0] = -thoProj.comp[0][0];
  M[0][1] = -thoProj.comp[1][0];
  M[0][2] = -thoProj.comp[2][0];
  Y[0] = thoProj.orig[0];
  if (Y[0] < neg(M[0][0]) + neg(M[0][1]) + neg(M[0][2]))
    return false;

  M[1][0] = -thoProj.comp[0][1];
  M[1][1] = -thoProj.comp[1][1];
  M[1][2] = -thoProj.comp[2][1];
  Y[1] = thoProj.orig[1];
  if (Y[1] < neg(M[1][0]) + neg(M[1][1]) + neg(M[1][2]))
    return false;

  M[2][0] = -thoProj.comp[0][2];
  M[2][1] = -thoProj.comp[1][2];
  M[2][2] = -thoProj.comp[2][2];
  Y[2] = thoProj.orig[2];
  if (Y[2] < neg(M[2][0]) + neg(M[2][1]) + neg(M[2][2]))
    return false;
```

```
// -X_i <= 0.0
M[3][0] = -1.0;
M[3][1] = 0.0;
M[3][2] = 0.0;
Y[3] = 0.0;

M[4][0] = 0.0;
M[4][1] = -1.0;
M[4][2] = 0.0;
Y[4] = 0.0;

M[5][0] = 0.0;
M[5][1] = 0.0;
M[5][2] = -1.0;
Y[5] = 0.0;

// Variable to memorise the nb of rows in the system
int nbRows = 6;

if (that->type == FrameCuboid) {

  // sum_iC_j,iX_i<=1.0-O_j
  M[nbRows][0] = thoProj.comp[0][0];
  M[nbRows][1] = thoProj.comp[1][0];
  M[nbRows][2] = thoProj.comp[2][0];
  Y[nbRows] = 1.0 - thoProj.orig[0];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

  M[nbRows][0] = thoProj.comp[0][1];
  M[nbRows][1] = thoProj.comp[1][1];
  M[nbRows][2] = thoProj.comp[2][1];
  Y[nbRows] = 1.0 - thoProj.orig[1];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

  M[nbRows][0] = thoProj.comp[0][2];
  M[nbRows][1] = thoProj.comp[1][2];
  M[nbRows][2] = thoProj.comp[2][2];
  Y[nbRows] = 1.0 - thoProj.orig[2];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

} else {

  // sum_j(sum_iC_j,iX_i)<=1.0-sum_iO_i
  M[nbRows][0] =
    thoProj.comp[0][0] + thoProj.comp[0][1] + thoProj.comp[0][2];
  M[nbRows][1] =
    thoProj.comp[1][0] + thoProj.comp[1][1] + thoProj.comp[1][2];
  M[nbRows][2] =
    thoProj.comp[2][0] + thoProj.comp[2][1] + thoProj.comp[2][2];
  Y[nbRows] =
    1.0 - thoProj.orig[0] - thoProj.orig[1] - thoProj.orig[2];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
```

```
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

}

if (tho->type == FrameCuboid) {

  // X_i <= 1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 0.0;
  M[nbRows][2] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

  M[nbRows][0] = 0.0;
  M[nbRows][1] = 1.0;
  M[nbRows][2] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

  M[nbRows][0] = 0.0;
  M[nbRows][1] = 0.0;
  M[nbRows][2] = 1.0;
  Y[nbRows] = 1.0;
  ++nbRows;

} else {

  // sum_iX_i<=1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 1.0;
  M[nbRows][2] = 1.0;
  Y[nbRows] = 1.0;
  ++nbRows;

}

// Solve the system

// Declare a AABB to memorize the bounding box of the intersection
// in the coordinates system of that
AABB3D bdgBoxLocal;

// Declare variables to eliminate the first variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mp[48][3];
//double Yp[48];
double Mp[20][3];
double Yp[20];
int nbRowsP;

// Eliminate the first variable in the original system
bool inconsistency =
  ElimVar3D(
    FST_VAR,
    M,
    Y,
    nbRows,
```

```
    3,
    Mp,
    Yp,
    &nbRowsP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;

}

// Declare variables to eliminate the second variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mpp[624][3];
//double Ypp[624];
double Mpp[55][3];
double Ypp[55];
int nbRowsPP;

// Eliminate the second variable (which is the first in the new system)
inconsistency =
  ElimVar3D(
    FST_VAR,
    Mp,
    Yp,
    nbRowsP,
    2,
    Mpp,
    Ypp,
    &nbRowsPP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;

}

// Get the bounds for the remaining third variable
GetBound3D(
  THD_VAR,
  Mpp,
  Ypp,
  nbRowsPP,
  &bdgBoxLocal);

// If the bounds are inconstent
if (bdgBoxLocal.min[THD_VAR] >= bdgBoxLocal.max[THD_VAR]) {

  // The two Frames are not in intersection
  return false;

// Else, if the bounds are consistent here it means
// the two Frames are in intersection.
// If the user hasn't requested for the resulting bounding box
} else if (bdgBox == NULL) {
```

```
  // Immediately return true
  return true;

}

// Eliminate the third variable (which is the first in the new
// system)
inconsistency =
  ElimVar3D(
    SND_VAR,
    Mp,
    Yp,
    nbRowsP,
    2,
    Mpp,
    Ypp,
    &nbRowsPP);

// Get the bounds for the remaining second variable
GetBound3D(
  SND_VAR,
  Mpp,
  Ypp,
  nbRowsPP,
  &bdgBoxLocal);

// Now starts again from the initial systems and eliminate the
// second and third variables to get the bounds of the first variable
// No need to check for consistency because we already know here
// that the Frames are intersecting and the system is consistent
inconsistency =
  ElimVar3D(
    THD_VAR,
    M,
    Y,
    nbRows,
    3,
    Mp,
    Yp,
    &nbRowsP);

inconsistency =
  ElimVar3D(
    SND_VAR,
    Mp,
    Yp,
    nbRowsP,
    2,
    Mpp,
    Ypp,
    &nbRowsPP);

GetBound3D(
  FST_VAR,
  Mpp,
  Ypp,
  nbRowsPP,
  &bdgBoxLocal);

// If the user requested the resulting bounding box
if (bdgBox != NULL) {
```

```
    // Memorize the result
    *bdgBox = bdgBoxLocal;

  }

  // If we've reached here the two Frames are intersecting
  return true;

}
```

4.2.3 2D dynamic

Header

```
#ifndef __FMB2DT_H_
#define __FMB2DT_H_

#include <stdbool.h>
#include "frame.h"

// ------------- Functions declaration -------------

// Test for intersection between Frame 'that' and Frame 'tho'
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into 'bdgBox', else 'bdgBox' is not modified
// If 'bdgBox' is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in 'tho' 's local coordinates system
bool FMBTestIntersection2DTime(
  const Frame2DTime* const that,
  const Frame2DTime* const tho,
         AABB2DTime* const bdgBox);

#endif
```

Body

```
#include "fmb2dt.h"

// ------------- Macros -------------

// Return 1.0 if v is positive, -1.0 if v is negative, 0.0 else
#define sgn(v) (((0.0 < (v)) ? 1 : 0) - (((v) < 0.0) ? 1 : 0))

// Return x if x is negative, 0.0 else
#define neg(x) (x < 0.0 ? x : 0.0)

#define FST_VAR 0
#define SND_VAR 1
#define THD_VAR 2

#define EPSILON 0.0000001

// ------------- Functions declaration -------------
```

```
// Eliminate the 'iVar'-th variable in the system 'M'.X<='Y'
// using the Fourier-Motzkin method and return
// the resulting system in 'Mp' and 'Yp', and the number of rows of
// the resulting system in 'nbRemainRows'
// Return false if the system becomes inconsistent during elimination,
// else return true
bool ElimVar2DTime(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[3],
        double* Yp,
    int* const nbRemainRows);

// Get the bounds of the 'iVar'-th variable in the 'nbRows' rows
// system 'M'.X<='Y' and store them in the 'iVar'-th axis of the
// AABB 'bdgBox'
// ('M' arrangement is [iRow][iCol])
// The system is supposed to have been reduced to only one variable
// per row, the one in argument, which can be located in a different
// column than 'iVar'
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBound2DTime(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
   AABB2DTime* const bdgBox);

// ------------- Functions implementation -------------

// Eliminate the 'iVar'-th variable in the system 'M'.X<='Y'
// using the Fourier-Motzkin method and return
// the resulting system in 'Mp' and 'Yp', and the number of rows of
// the resulting system in 'nbRemainRows'
// ('M' arrangement is [iRow][iCol])
// Return true if the system becomes inconsistent during elimination,
// else return false
bool ElimVar2DTime(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[3],
        double* Yp,
    int* const nbRemainRows) {

  // Initialize the number of rows in the result system
  *nbRemainRows = 0;

  // First we process the rows where the eliminated variable is not null

  // For each row except the last one
  for (int iRow = 0;
       iRow < nbRows - 1;
       ++iRow) {

    // Shortcuts
```

```
int sgnMIRowIVar = sgn(M[iRow][iVar]);
double fabsMIRowIVar = fabs(M[iRow][iVar]);
double YIRowDivideByFabsMIRowIVar = Y[iRow] / fabsMIRowIVar;

// For each following rows
for (int jRow = iRow + 1;
     jRow < nbRows;
     ++jRow) {

  // If coefficients of the eliminated variable in the two rows have
  // different signs and are not null
  if (sgnMIRowIVar != sgn(M[jRow][iVar]) &&
      fabsMIRowIVar > EPSILON &&
      fabs(M[jRow][iVar]) > EPSILON) {

    // Declare a variable to memorize the sum of the negative
    // coefficients in the row
    double sumNegCoeff = 0.0;

    // Add the sum of the two normed (relative to the eliminated
    // variable) rows into the result system. This actually
    // eliminate the variable while keeping the constraints on
    // others variables
    for (int iCol = 0, jCol = 0;
         iCol < nbCols;
         ++iCol ) {

      if (iCol != iVar) {

        Mp[*nbRemainRows][jCol] =
          M[iRow][iCol] / fabsMIRowIVar +
          M[jRow][iCol] / fabs(M[jRow][iVar]);

        // Update the sum of the negative coefficient
        sumNegCoeff += neg(Mp[*nbRemainRows][jCol]);

        // Increment the number of columns in the new inequality
        ++jCol;

      }

    }

    // Update the right side of the inequality
    Yp[*nbRemainRows] =
      YIRowDivideByFabsMIRowIVar +
      Y[jRow] / fabs(M[jRow][iVar]);

    // If the right side of the inequality if lower than the sum of
    // negative coefficients in the row
    // (Add epsilon for numerical imprecision)
    if (Yp[*nbRemainRows] < sumNegCoeff - EPSILON) {

      // Given that X is in [0,1], the system is inconsistent
      return true;

    }

    // Increment the nb of rows into the result system
    ++(*nbRemainRows);

  }
```

```
    }

  }

  // Then we copy and compress the rows where the eliminated
  // variable is null

  // Loop on rows of the input system
  for (int iRow = 0;
       iRow < nbRows;
       ++iRow) {

    // Shortcut
    const double* MiRow = M[iRow];

    // If the coefficient of the eliminated variable is null on
    // this row
    if (fabs(MiRow[iVar]) < EPSILON) {

      // Shortcut
      double* MpnbRemainRows = Mp[*nbRemainRows];

      // Copy this row into the result system excluding the eliminated
      // variable
      for (int iCol = 0, jCol = 0;
           iCol < nbCols;
           ++iCol) {

        if (iCol != iVar) {

          MpnbRemainRows[jCol] = MiRow[iCol];

          ++jCol;

        }

      }

      Yp[*nbRemainRows] = Y[iRow];

      // Increment the nb of rows into the result system
      ++(*nbRemainRows);

    }

  }

  // If we reach here the system is not inconsistent
  return false;

}

// Get the bounds of the 'iVar'-th variable in the 'nbRows' rows
// system 'M'.X<='Y' and store them in the 'iVar'-th axis of the
// AABB 'bdgBox'
// ('M' arrangement is [iRow][iCol])
// The system is supposed to have been reduced to only one variable
// per row, the one in argument
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBound2DTime(
```

```
    const int iVar,
  const double (*M)[3],
  const double* Y,
    const int nbRows,
   AABB2DTime* const bdgBox) {

  // Shortcuts
  double* min = bdgBox->min + iVar;
  double* max = bdgBox->max + iVar;

  // Initialize the bounds to there maximum maximum and minimum minimum
  *min = 0.0;
  *max = 1.0;

  // Loop on rows
  for (int jRow = 0;
       jRow < nbRows;
       ++jRow) {

    // Shortcut
    double MjRowiVar = M[jRow][0];

    // If this row has been reduced to the variable in argument
    // and it has a strictly positive coefficient
    if (MjRowiVar > EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is lower than the current maximum bound
      if (*max > y) {

        // Update the maximum bound
        *max = y;

      }

    // Else, if this row has been reduced to the variable in argument
    // and it has a strictly negative coefficient
    } else if (MjRowiVar < -EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is greater than the current minimum bound
      if (*min < y) {

        // Update the minimum bound
        *min = y;

      }

    }

  }

}

// Test for intersection between Frame 'that' and Frame 'tho'
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into 'bdgBox', else 'bdgBox' is not modified
```

```
// If 'bdgBox' is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in 'tho' 's local coordinates system
bool FMBTestIntersection2DTime(
  const Frame2DTime* const that,
  const Frame2DTime* const tho,
        AABB2DTime* const bdgBox) {

  // Get the projection of the Frame 'tho' in Frame 'that' coordinates
  // system
  Frame2DTime thoProj;
  Frame2DTimeImportFrame(that, tho, &thoProj);

  // Declare two variables to memorize the system to be solved M.X <= Y
  // (M arrangement is [iRow][iCol])
  double M[10][3];
  double Y[10];

  // Create the inequality system

  // -V_jT-sum_iC_j,iX_i<=O_j
  M[0][0] = -thoProj.comp[0][0];
  M[0][1] = -thoProj.comp[1][0];
  M[0][2] = -thoProj.speed[0];
  Y[0] = thoProj.orig[0];
  if (Y[0] < neg(M[0][0]) + neg(M[0][1]) + neg(M[0][2]))
    return false;

  M[1][0] = -thoProj.comp[0][1];
  M[1][1] = -thoProj.comp[1][1];
  M[1][2] = -thoProj.speed[1];
  Y[1] = thoProj.orig[1];
  if (Y[1] < neg(M[1][0]) + neg(M[1][1]) + neg(M[1][2]))
    return false;

  // -X_i <= 0.0
  M[2][0] = -1.0;
  M[2][1] = 0.0;
  M[2][2] = 0.0;
  Y[2] = 0.0;

  M[3][0] = 0.0;
  M[3][1] = -1.0;
  M[3][2] = 0.0;
  Y[3] = 0.0;

  // 0.0 <= t <= 1.0
  M[4][0] = 0.0;
  M[4][1] = 0.0;
  M[4][2] = 1.0;
  Y[4] = 1.0;

  M[5][0] = 0.0;
  M[5][1] = 0.0;
  M[5][2] = -1.0;
  Y[5] = 0.0;

  // Variable to memorise the nb of rows in the system
  int nbRows = 6;
```

```
if (that->type == FrameCuboid) {

  // V_jT+sum_iC_j,iX_i<=1.0-O_j
  M[nbRows][0] = thoProj.comp[0][0];
  M[nbRows][1] = thoProj.comp[1][0];
  M[nbRows][2] = thoProj.speed[0];
  Y[nbRows] = 1.0 - thoProj.orig[0];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

  M[nbRows][0] = thoProj.comp[0][1];
  M[nbRows][1] = thoProj.comp[1][1];
  M[nbRows][2] = thoProj.speed[1];
  Y[nbRows] = 1.0 - thoProj.orig[1];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

} else {

  // sum_j(V_jT+sum_iC_j,iX_i)<=1.0-sum_iO_i
  M[nbRows][0] = thoProj.comp[0][0] + thoProj.comp[0][1];
  M[nbRows][1] = thoProj.comp[1][0] + thoProj.comp[1][1];
  M[nbRows][2] = thoProj.speed[0] + thoProj.speed[1];
  Y[nbRows] = 1.0 - thoProj.orig[0] - thoProj.orig[1];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

}

if (tho->type == FrameCuboid) {

  // X_i <= 1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 0.0;
  M[nbRows][2] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

  M[nbRows][0] = 0.0;
  M[nbRows][1] = 1.0;
  M[nbRows][2] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

} else {

  // sum_iX_i<=1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 1.0;
  M[nbRows][2] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

}
```

```
// Solve the system

// Declare a AABB to memorize the bounding box of the intersection
// in the coordinates system of that
AABB2DTime bdgBoxLocal;

// Declare variables to eliminate the first variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mp[35][3];
//double Yp[35];
double Mp[13][3];
double Yp[13];
int nbRowsP;

// Eliminate the first variable in the original system
bool inconsistency =
  ElimVar2DTime(
    FST_VAR,
    M,
    Y,
    nbRows,
    3,
    Mp,
    Yp,
    &nbRowsP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;

}

// Declare variables to eliminate the second variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mpp[342][3];
//double Ypp[342];
double Mpp[21][3];
double Ypp[21];
int nbRowsPP;

// Eliminate the second variable (which is the first in the new system)
inconsistency =
  ElimVar2DTime(
    FST_VAR,
    Mp,
    Yp,
    nbRowsP,
    2,
    Mpp,
    Ypp,
    &nbRowsPP);

// If the system is inconsistent
if (inconsistency == true) {
```

```
  // The two Frames are not in intersection
  return false;

}

// Get the bounds for the remaining third variable
GetBound2DTime(
  THD_VAR,
  Mpp,
  Ypp,
  nbRowsPP,
  &bdgBoxLocal);

// If the bounds are inconstent
if (bdgBoxLocal.min[THD_VAR] >= bdgBoxLocal.max[THD_VAR]) {

  // The two Frames are not in intersection
  return false;

// Else, if the bounds are consistent here it means
// the two Frames are in intersection.
// If the user hasn't requested for the resulting bounding box
} else if (bdgBox == NULL) {

  // Immediately return true
  return true;

}

// Eliminate the third variable (which is the second in the new
// system)
inconsistency =
  ElimVar2DTime(
    SND_VAR,
    Mp,
    Yp,
    nbRowsP,
    2,
    Mpp,
    Ypp,
    &nbRowsPP);

// Get the bounds for the remaining second variable
GetBound2DTime(
  SND_VAR,
  Mpp,
  Ypp,
  nbRowsPP,
  &bdgBoxLocal);

// Now starts again from the initial systems and eliminate the
// second and third variables to get the bounds of the first variable
// No need to check for consistency because we already know here
// that the Frames are intersecting and the system is consistent
inconsistency =
  ElimVar2DTime(
    THD_VAR,
    M,
    Y,
    nbRows,
    3,
```

```
      Mp,
      Yp,
      &nbRowsP);

  inconsistency =
    ElimVar2DTime(
      SND_VAR,
      Mp,
      Yp,
      nbRowsP,
      2,
      Mpp,
      Ypp,
      &nbRowsPP);

  GetBound2DTime(
    FST_VAR,
    Mpp,
    Ypp,
    nbRowsPP,
    &bdgBoxLocal);

  // If the user requested the resulting bounding box
  if (bdgBox != NULL) {

    // Memorize the result
    *bdgBox = bdgBoxLocal;

  }

  // If we've reached here the two Frames are intersecting
  return true;

}
```

4.2.4 3D dynamic

Header

```
#ifndef __FMB3DT_H_
#define __FMB3DT_H_

#include <stdbool.h>
#include "frame.h"

// ------------- Functions declaration -------------

// Test for intersection between Frame 'that' and Frame 'tho'
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into 'bdgBox', else 'bdgBox' is not modified
// If 'bdgBox' is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in 'tho' 's local coordinates system
bool FMBTestIntersection3DTime(
  const Frame3DTime* const that,
  const Frame3DTime* const tho,
```

```
         AABB3DTime* const bdgBox);

#endif
```

Body

```
#include "fmb3dt.h"

// ------------- Macros -------------

// Return 1.0 if v is positive, -1.0 if v is negative, 0.0 else
#define sgn(v) (((0.0 < (v)) ? 1 : 0) - (((v) < 0.0) ? 1 : 0))

// Return x if x is negative, 0.0 else
#define neg(x) (x < 0.0 ? x : 0.0)

#define FST_VAR 0
#define SND_VAR 1
#define THD_VAR 2
#define FOR_VAR 3

#define EPSILON 0.0000001

// ------------- Functions declaration -------------

// Eliminate the 'iVar'-th variable in the system 'M'.X<='Y'
// using the Fourier-Motzkin method and return
// the resulting system in 'Mp' and 'Yp', and the number of rows of
// the resulting system in 'nbRemainRows'
// Return false if the system becomes inconsistent during elimination,
// else return true
bool ElimVar3DTime(
     const int iVar,
  const double (*M)[4],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[4],
        double* Yp,
    int* const nbRemainRows);

// Get the bounds of the 'iVar'-th variable in the 'nbRows' rows
// system 'M'.X<='Y' and store them in the 'iVar'-th axis of the
// AABB 'bdgBox'
// ('M' arrangement is [iRow][iCol])
// The system is supposed to have been reduced to only one variable
// per row, the one in argument, which can be located in a different
// column than 'iVar'
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBound3DTime(
     const int iVar,
  const double (*M)[4],
  const double* Y,
     const int nbRows,
   AABB3DTime* const bdgBox);

// ------------- Functions implementation -------------

// Eliminate the 'iVar'-th variable in the system 'M'.X<='Y'
// using the Fourier-Motzkin method and return
// the resulting system in 'Mp' and 'Yp', and the number of rows of
```

```
// the resulting system in 'nbRemainRows'
// ('M' arrangement is [iRow][iCol])
// Return true if the system becomes inconsistent during elimination,
// else return false
bool ElimVar3DTime(
     const int iVar,
  const double (*M)[4],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[4],
        double* Yp,
    int* const nbRemainRows) {

  // Initialize the number of rows in the result system
  *nbRemainRows = 0;

  // First we process the rows where the eliminated variable is not null

  // For each row except the last one
  for (int iRow = 0;
       iRow < nbRows - 1;
       ++iRow) {

    // Shortcuts
    int sgnMIRowIVar = sgn(M[iRow][iVar]);
    double fabsMIRowIVar = fabs(M[iRow][iVar]);
    double YIRowDivideByFabsMIRowIVar = Y[iRow] / fabsMIRowIVar;

    // For each following rows
    for (int jRow = iRow + 1;
         jRow < nbRows;
         ++jRow) {

      // If coefficients of the eliminated variable in the two rows have
      // different signs and are not null
      if (sgnMIRowIVar != sgn(M[jRow][iVar]) &&
          fabsMIRowIVar > EPSILON &&
          fabs(M[jRow][iVar]) > EPSILON) {

        // Declare a variable to memorize the sum of the negative
        // coefficients in the row
        double sumNegCoeff = 0.0;

        // Add the sum of the two normed (relative to the eliminated
        // variable) rows into the result system. This actually
        // eliminate the variable while keeping the constraints on
        // others variables
        for (int iCol = 0, jCol = 0;
             iCol < nbCols;
             ++iCol ) {

          if (iCol != iVar) {

            Mp[*nbRemainRows][jCol] =
              M[iRow][iCol] / fabsMIRowIVar +
              M[jRow][iCol] / fabs(M[jRow][iVar]);

            // Update the sum of the negative coefficient
            sumNegCoeff += neg(Mp[*nbRemainRows][jCol]);

            // Increment the number of columns in the new inequality
```

```
          ++jCol;

        }

      }

      // Update the right side of the inequality
      Yp[*nbRemainRows] =
        YIRowDivideByFabsMIRowIVar +
        Y[jRow] / fabs(M[jRow][iVar]);

      // If the right side of the inequality if lower than the sum of
      // negative coefficients in the row
      // (Add epsilon for numerical imprecision)
      if (Yp[*nbRemainRows] < sumNegCoeff - EPSILON) {

        // Given that X is in [0,1], the system is inconsistent
        return true;

      }

      // Increment the nb of rows into the result system
      ++(*nbRemainRows);

    }

  }

}

// Then we copy and compress the rows where the eliminated
// variable is null

// Loop on rows of the input system
for (int iRow = 0;
     iRow < nbRows;
     ++iRow) {

  // Shortcut
  const double* MiRow = M[iRow];

  // If the coefficient of the eliminated variable is null on
  // this row
  if (fabs(MiRow[iVar]) < EPSILON) {

    // Shortcut
    double* MpnbRemainRows = Mp[*nbRemainRows];

    // Copy this row into the result system excluding the eliminated
    // variable
    for (int iCol = 0, jCol = 0;
         iCol < nbCols;
         ++iCol) {

      if (iCol != iVar) {

        MpnbRemainRows[jCol] = MiRow[iCol];

        ++jCol;

      }
```

```
      }

      Yp[*nbRemainRows] = Y[iRow];

      // Increment the nb of rows into the result system
      ++(*nbRemainRows);

    }

  }

  // If we reach here the system is not inconsistent
  return false;

}

// Get the bounds of the 'iVar'-th variable in the 'nbRows' rows
// system 'M'.X<='Y' and store them in the 'iVar'-th axis of the
// AABB 'bdgBox'
// ('M' arrangement is [iRow][iCol])
// The system is supposed to have been reduced to only one variable
// per row, the one in argument
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBound3DTime(
     const int iVar,
  const double (*M)[4],
  const double* Y,
     const int nbRows,
   AABB3DTime* const bdgBox) {

  // Shortcuts
  double* min = bdgBox->min + iVar;
  double* max = bdgBox->max + iVar;

  // Initialize the bounds to there maximum maximum and minimum minimum
  *min = 0.0;
  *max = 1.0;

  // Loop on rows
  for (int jRow = 0;
       jRow < nbRows;
       ++jRow) {

    // Shortcut
    double MjRowiVar = M[jRow][0];

    // If this row has been reduced to the variable in argument
    // and it has a strictly positive coefficient
    if (MjRowiVar > EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is lower than the current maximum bound
      if (*max > y) {

        // Update the maximum bound
        *max = y;

      }
```

```
    // Else, if this row has been reduced to the variable in argument
    // and it has a strictly negative coefficient
    } else if (MjRowiVar < -EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is greater than the current minimum bound
      if (*min < y) {

        // Update the minimum bound
        *min = y;

      }

    }

  }

}

// Test for intersection between Frame 'that' and Frame 'tho'
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into 'bdgBox', else 'bdgBox' is not modified
// If 'bdgBox' is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in 'tho' 's local coordinates system
bool FMBTestIntersection3DTime(
  const Frame3DTime* const that,
  const Frame3DTime* const tho,
        AABB3DTime* const bdgBox) {

  // Get the projection of the Frame 'tho' in Frame 'that' coordinates
  // system
  Frame3DTime thoProj;
  Frame3DTimeImportFrame(that, tho, &thoProj);

  // Declare two variables to memorize the system to be solved M.X <= Y
  // (M arrangement is [iRow][iCol])
  double M[14][4];
  double Y[14];

  // Create the inequality system

  // -V_jT-sum_iC_j,iX_i<=O_j
  M[0][0] = -thoProj.comp[0][0];
  M[0][1] = -thoProj.comp[1][0];
  M[0][2] = -thoProj.comp[2][0];
  M[0][3] = -thoProj.speed[0];
  Y[0] = thoProj.orig[0];
  if (Y[0] < neg(M[0][0]) + neg(M[0][1]) + neg(M[0][2]) + neg(M[0][3]))
    return false;

  M[1][0] = -thoProj.comp[0][1];
  M[1][1] = -thoProj.comp[1][1];
  M[1][2] = -thoProj.comp[2][1];
  M[1][3] = -thoProj.speed[1];
  Y[1] = thoProj.orig[1];
```

```
if (Y[1] < neg(M[1][0]) + neg(M[1][1]) + neg(M[1][2]) + neg(M[1][3]))
  return false;

M[2][0] = -thoProj.comp[0][2];
M[2][1] = -thoProj.comp[1][2];
M[2][2] = -thoProj.comp[2][2];
M[2][3] = -thoProj.speed[2];
Y[2] = thoProj.orig[2];
if (Y[2] < neg(M[2][0]) + neg(M[2][1]) + neg(M[2][2]) + neg(M[2][3]))
  return false;

// Variable to memorise the nb of rows in the system
int nbRows = 3;

if (that->type == FrameCuboid) {

  // V_jT+sum_iC_j,iX_i<=1.0-O_j
  M[nbRows][0] = thoProj.comp[0][0];
  M[nbRows][1] = thoProj.comp[1][0];
  M[nbRows][2] = thoProj.comp[2][0];
  M[nbRows][3] = thoProj.speed[0];
  Y[nbRows] = 1.0 - thoProj.orig[0];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]) + neg(M[nbRows][3]))
    return false;
  ++nbRows;

  M[nbRows][0] = thoProj.comp[0][1];
  M[nbRows][1] = thoProj.comp[1][1];
  M[nbRows][2] = thoProj.comp[2][1];
  M[nbRows][3] = thoProj.speed[1];
  Y[nbRows] = 1.0 - thoProj.orig[1];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]) + neg(M[nbRows][3]))
    return false;
  ++nbRows;

  M[nbRows][0] = thoProj.comp[0][2];
  M[nbRows][1] = thoProj.comp[1][2];
  M[nbRows][2] = thoProj.comp[2][2];
  M[nbRows][3] = thoProj.speed[2];
  Y[nbRows] = 1.0 - thoProj.orig[2];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]) + neg(M[nbRows][3]))
    return false;
  ++nbRows;

} else {

  // sum_j(V_jT+sum_iC_j,iX_i)<=1.0-sum_iO_i
  M[nbRows][0] =
    thoProj.comp[0][0] + thoProj.comp[0][1] + thoProj.comp[0][2];
  M[nbRows][1] =
    thoProj.comp[1][0] + thoProj.comp[1][1] + thoProj.comp[1][2];
  M[nbRows][2] =
    thoProj.comp[2][0] + thoProj.comp[2][1] + thoProj.comp[2][2];
  M[nbRows][3] = thoProj.speed[0] + thoProj.speed[1] + thoProj.speed[2];
  Y[nbRows] = 1.0 - thoProj.orig[0] - thoProj.orig[1] - thoProj.orig[2];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]) + neg(M[nbRows][3]))
    return false;
  ++nbRows;
```

```
}

if (tho->type == FrameCuboid) {

  // X_i <= 1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 0.0;
  M[nbRows][2] = 0.0;
  M[nbRows][3] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

  M[nbRows][0] = 0.0;
  M[nbRows][1] = 1.0;
  M[nbRows][2] = 0.0;
  M[nbRows][3] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

  M[nbRows][0] = 0.0;
  M[nbRows][1] = 0.0;
  M[nbRows][2] = 1.0;
  M[nbRows][3] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

} else {

  // sum_iX_i<=1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 1.0;
  M[nbRows][2] = 1.0;
  M[nbRows][3] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

}

// -X_i <= 0.0
M[nbRows][0] = -1.0;
M[nbRows][1] = 0.0;
M[nbRows][2] = 0.0;
M[nbRows][3] = 0.0;
Y[nbRows] = 0.0;
++nbRows;

M[nbRows][0] = 0.0;
M[nbRows][1] = -1.0;
M[nbRows][2] = 0.0;
M[nbRows][3] = 0.0;
Y[nbRows] = 0.0;
++nbRows;

M[nbRows][0] = 0.0;
M[nbRows][1] = 0.0;
M[nbRows][2] = -1.0;
M[nbRows][3] = 0.0;
Y[nbRows] = 0.0;
++nbRows;

// 0.0 <= t <= 1.0
```

```
M[nbRows][0] = 0.0;
M[nbRows][1] = 0.0;
M[nbRows][2] = 0.0;
M[nbRows][3] = 1.0;
Y[nbRows] = 1.0;
++nbRows;

M[nbRows][0] = 0.0;
M[nbRows][1] = 0.0;
M[nbRows][2] = 0.0;
M[nbRows][3] = -1.0;
Y[nbRows] = 0.0;
++nbRows;

// Solve the system

// Declare a AABB to memorize the bounding box of the intersection
// in the coordinates system of that
AABB3DTime bdgBoxLocal;

// Declare variables to eliminate the first variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mp[63][4];
//double Yp[63];
double Mp[22][4];
double Yp[22];
int nbRowsP;

// Eliminate the first variable in the original system
bool inconsistency =
  ElimVar3DTime(
    FST_VAR,
    M,
    Y,
    nbRows,
    4,
    Mp,
    Yp,
    &nbRowsP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;

}

// Declare variables to eliminate the second variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mpp[1056][4];
//double Ypp[1056];
double Mpp[57][4];
double Ypp[57];
int nbRowsPP;
```

```
// Eliminate the second variable (which is the first in the new system)
inconsistency =
  ElimVar3DTime(
    FST_VAR,
    Mp,
    Yp,
    nbRowsP,
    3,
    Mpp,
    Ypp,
    &nbRowsPP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;

}

// Declare variables to eliminate the third variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mppp[279840][4];
//double Yppp[279840];
double Mppp[560][4];
double Yppp[560];
int nbRowsPPP;

// Eliminate the third variable (which is the first in the new system)
inconsistency =
  ElimVar3DTime(
    FST_VAR,
    Mpp,
    Ypp,
    nbRowsPP,
    2,
    Mppp,
    Yppp,
    &nbRowsPPP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;

}

// Get the bounds for the remaining fourth variable
GetBound3DTime(
  FOR_VAR,
  Mppp,
  Yppp,
  nbRowsPPP,
  &bdgBoxLocal);

// If the bounds are inconstent
if (bdgBoxLocal.min[FOR_VAR] >= bdgBoxLocal.max[FOR_VAR]) {
```

```
  // The two Frames are not in intersection
  return false;

// Else, if the bounds are consistent here it means
// the two Frames are in intersection.
// If the user hasn't requested for the resulting bounding box
} else if (bdgBox == NULL) {

  // Immediately return true
  return true;

}

// Eliminate the fourth variable (which is the second in the new
// system)
inconsistency =
  ElimVar3DTime(
    SND_VAR,
    Mpp,
    Ypp,
    nbRowsPP,
    2,
    Mppp,
    Yppp,
    &nbRowsPPP);

// Get the bounds for the remaining third variable
GetBound3DTime(
  THD_VAR,
  Mppp,
  Yppp,
  nbRowsPPP,
  &bdgBoxLocal);

// Now starts again from the initial systems and eliminate the
// third and fourth variables to get the bounds of the first and
// second variables.
// No need to check for consistency because we already know here
// that the Frames are intersecting and the system is consistent
inconsistency =
  ElimVar3DTime(
    FOR_VAR,
    M,
    Y,
    nbRows,
    4,
    Mp,
    Yp,
    &nbRowsP);

inconsistency =
  ElimVar3DTime(
    THD_VAR,
    Mp,
    Yp,
    nbRowsP,
    3,
    Mpp,
    Ypp,
    &nbRowsPP);

inconsistency =
```

```
    ElimVar3DTime(
      SND_VAR,
      Mpp,
      Ypp,
      nbRowsPP,
      2,
      Mppp,
      Yppp,
      &nbRowsPPP);

  GetBound3DTime(
    FST_VAR,
    Mppp,
    Yppp,
    nbRowsPPP,
    &bdgBoxLocal);

  inconsistency =
    ElimVar3DTime(
      FST_VAR,
      Mpp,
      Ypp,
      nbRowsPP,
      2,
      Mppp,
      Yppp,
      &nbRowsPPP);

  GetBound3DTime(
    SND_VAR,
    Mppp,
    Yppp,
    nbRowsPPP,
    &bdgBoxLocal);

  // If the user requested the resulting bounding box
  if (bdgBox != NULL) {

    // Memorize the result
    *bdgBox = bdgBoxLocal;

  }

  // If we've reached here the two Frames are intersecting
  return true;

}
```

5 Minimal example of use

In this section I give a minimal example of how to use the code given in the previous section.

5.1 2D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>

// Include FMB algorithm library
#include "fmb2d.h"

// Main function
int main(int argc, char** argv) {

  // Create the two objects to be tested for intersection
  double origP2D[2] = {0.0, 0.0};
  double compP2D[2][2] = {
    {1.0, 0.0},  // First component
    {0.0, 1.0}}; // Second component
  Frame2D P2D =
    Frame2DCreateStatic(
      FrameCuboid,
      origP2D,
      compP2D);

  double origQ2D[2] = {0.0, 0.0};
  double compQ2D[2][2] = {
    {1.0, 0.0},
    {0.0, 1.0}};
  Frame2D Q2D =
    Frame2DCreateStatic(
      FrameCuboid,
      origQ2D,
      compQ2D);

  // Declare a variable to memorize the result of the intersection
  // detection
  AABB2D bdgBox2DLocal;

  // Test for intersection between P and Q
  bool isIntersecting2D =
    FMBTestIntersection2D(
      &P2D,
      &Q2D,
      &bdgBox2DLocal);

  // If the two objects are intersecting
  if (isIntersecting2D) {

    printf("Intersection detected in AABB ");

    // Export the local bounding box toward the real coordinates
    // system
    AABB2D bdgBox2D;
    Frame2DExportBdgBox(
      &Q2D,
      &bdgBox2DLocal,
      &bdgBox2D);

    // Clip with the AABB of 'Q2D' and 'P2D' to improve results
    for (int iAxis = 2;
         iAxis--;) {

      if (bdgBox2D.min[iAxis] < P2D.bdgBox.min[iAxis]) {
```

```
        bdgBox2D.min[iAxis] = P2D.bdgBox.min[iAxis];

      }
      if (bdgBox2D.max[iAxis] > P2D.bdgBox.max[iAxis]) {

        bdgBox2D.max[iAxis] = P2D.bdgBox.max[iAxis];

      }

      if (bdgBox2D.min[iAxis] < Q2D.bdgBox.min[iAxis]) {

        bdgBox2D.min[iAxis] = Q2D.bdgBox.min[iAxis];

      }
      if (bdgBox2D.max[iAxis] > Q2D.bdgBox.max[iAxis]) {

        bdgBox2D.max[iAxis] = Q2D.bdgBox.max[iAxis];

      }

    }

    AABB2DPrint(&bdgBox2D);
    printf("\n");

  // Else, the two objects are not intersecting
  } else {

    printf("No intersection.\n");

  }

  return 0;

}
```

5.2 3D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>

// Include FMB algorithm library
#include "fmb3d.h"

// Main function
int main(int argc, char** argv) {

  // Create the two objects to be tested for intersection
  double origP3D[3] = {0.0, 0.0, 0.0};
  double compP3D[3][3] = {
    {1.0, 0.0, 0.0},  // First component
    {0.0, 1.0, 0.0},  // Second component
    {0.0, 0.0, 1.0}}; // Third component
  Frame3D P3D =
    Frame3DCreateStatic(
      FrameTetrahedron,
      origP3D,
      compP3D);
```

```
double origQ3D[3] = {0.0, 0.0, 0.0};
double compQ3D[3][3] = {
  {1.0, 0.0, 0.0},
  {0.0, 1.0, 0.0},
  {0.0, 0.0, 1.0}};
Frame3D Q3D =
  Frame3DCreateStatic(
    FrameTetrahedron,
    origQ3D,
    compQ3D);

// Declare a variable to memorize the result of the intersection
// detection
AABB3D bdgBox3DLocal;

// Test for intersection between P and Q
bool isIntersecting3D =
  FMBTestIntersection3D(
    &P3D,
    &Q3D,
    &bdgBox3DLocal);

// If the two objects are intersecting
if (isIntersecting3D) {

  printf("Intersection detected in AABB ");

  // Export the local bounding box toward the real coordinates
  // system
  AABB3D bdgBox3D;
  Frame3DExportBdgBox(
    &Q3D,
    &bdgBox3DLocal,
    &bdgBox3D);

  // Clip with the AABB of 'Q3D' and 'P3D' to improve results
  for (int iAxis = 2;
       iAxis--;) {

    if (bdgBox3D.min[iAxis] < P3D.bdgBox.min[iAxis]) {

      bdgBox3D.min[iAxis] = P3D.bdgBox.min[iAxis];

    }
    if (bdgBox3D.max[iAxis] > P3D.bdgBox.max[iAxis]) {

      bdgBox3D.max[iAxis] = P3D.bdgBox.max[iAxis];

    }

    if (bdgBox3D.min[iAxis] < Q3D.bdgBox.min[iAxis]) {

      bdgBox3D.min[iAxis] = Q3D.bdgBox.min[iAxis];

    }
    if (bdgBox3D.max[iAxis] > Q3D.bdgBox.max[iAxis]) {

      bdgBox3D.max[iAxis] = Q3D.bdgBox.max[iAxis];

    }

  }
```

```
    AABB3DPrint(&bdgBox3D);
    printf("\n");

  // Else, the two objects are not intersecting
  } else {

    printf("No intersection.\n");

  }

  return 0;
}
```

5.3 2D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>

// Include FMB algorithm library
#include "fmb2dt.h"

// Main function
int main(int argc, char** argv) {

  // Create the two objects to be tested for intersection
  double origP2DTime[2] = {0.0, 0.0};
  double speedP2DTime[2] = {0.0, 0.0};
  double compP2DTime[2][2] = {
    {1.0, 0.0},  // First component
    {0.0, 1.0}}; // Second component
  Frame2DTime P2DTime =
    Frame2DTimeCreateStatic(
      FrameCuboid,
      origP2DTime,
      speedP2DTime,
      compP2DTime);

  double origQ2DTime[2] = {0.0,0.0};
  double speedQ2DTime[2] = {0.0,0.0};
  double compQ2DTime[2][2] = {
    {1.0, 0.0},
    {0.0, 1.0}};
  Frame2DTime Q2DTime =
    Frame2DTimeCreateStatic(
      FrameCuboid,
      origQ2DTime,
      speedQ2DTime,
      compQ2DTime);

  // Declare a variable to memorize the result of the intersection
  // detection
  AABB2DTime bdgBox2DTimeLocal;

  // Test for intersection between P and Q
  bool isIntersecting2DTime =
    FMBTestIntersection2DTime(
      &P2DTime,
      &Q2DTime,
```

```
      &bdgBox2DTimeLocal);

  // If the two objects are intersecting
  if (isIntersecting2DTime) {

    printf("Intersection detected in AABB ");

    // Export the local bounding box toward the real coordinates
    // system
    AABB2DTime bdgBox2DTime;
    Frame2DTimeExportBdgBox(
      &Q2DTime,
      &bdgBox2DTimeLocal,
      &bdgBox2DTime);

    AABB2DTimePrint(&bdgBox2DTime);
    printf("\n");

  // Else, the two objects are not intersecting
  } else {

    printf("No intersection.\n");

  }

  return 0;
}
```

5.4 3D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>

// Include FMB algorithm library
#include "fmb3dt.h"

// Main function
int main(int argc, char** argv) {

  // Create the two objects to be tested for intersection
  double origP3DTime[3] = {0.0, 0.0, 0.0};
  double speedP3DTime[3] = {0.0, 0.0, 0.0};
  double compP3DTime[3][3] = {
    {1.0, 0.0, 0.0},  // First component
    {0.0, 1.0, 0.0},  // Second component
    {0.0, 0.0, 1.0}}; // Third component
  Frame3DTime P3DTime =
    Frame3DTimeCreateStatic(
      FrameCuboid,
      origP3DTime,
      speedP3DTime,
      compP3DTime);

  double origQ3DTime[3] = {0.0, 0.0, 0.0};
  double speedQ3DTime[3] = {0.0, 0.0, 0.0};
  double compQ3DTime[3][3] = {
    {1.0, 0.0, 0.0},
    {0.0, 1.0, 0.0},
    {0.0, 0.0, 1.0}};
```

```
  Frame3DTime Q3DTime =
    Frame3DTimeCreateStatic(
      FrameCuboid,
      origQ3DTime,
      speedQ3DTime,
      compQ3DTime);

  // Declare a variable to memorize the result of the intersection
  // detection
  AABB3DTime bdgBox3DTimeLocal;

  // Test for intersection between P and Q
  bool isIntersecting3DTime =
    FMBTestIntersection3DTime(
      &P3DTime,
      &Q3DTime,
      &bdgBox3DTimeLocal);

  // If the two objects are intersecting
  if (isIntersecting3DTime) {

    printf("Intersection detected in AABB ");

    // Export the local bounding box toward the real coordinates
    // system
    AABB3DTime bdgBox3DTime;
    Frame3DTimeExportBdgBox(
      &Q3DTime,
      &bdgBox3DTimeLocal,
      &bdgBox3DTime);

    AABB3DTimePrint(&bdgBox3DTime);
    printf("\n");

  // Else, the two objects are not intersecting
  } else {

    printf("No intersection.\n");

  }

  return 0;
}
```

6 Unit tests

In this section I introduce the code I've used to test the algorithm and its implementation. The test consists of running the algorithm on a set of cases for which the solution has been computed by hand. The code of the implementation of the SAT algorithm is given in annex (p.211)

6.1 Code

6.1.1 2D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>

// Include FMB and SAT algorithm library
#include "fmb2d.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of tests of the validation
#define NB_TESTS 1000000

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Global variables to count nb of tests resulting in intersection
// and no intersection
unsigned long int nbInter;
unsigned long int nbNoInter;

// Helper structure to pass arguments to the Validation function
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
} Param2D;

// Validation function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and check the results are identical
void Validation2D(
  const Param2D paramP,
  const Param2D paramQ) {

  // Create the two Frames
  Frame2D P =
    Frame2DCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.comp);

  Frame2D Q =
    Frame2DCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame2D* that = &P;
  Frame2D* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Test intersection with FMB
    bool isIntersectingFMB =
```

```
      FMBTestIntersection2D(
        that,
        tho,
        NULL);

    // Test intersection with SAT
    bool isIntersectingSAT =
      SATTestIntersection2D(
        that,
        tho);

    // If the results are different
    if (isIntersectingFMB != isIntersectingSAT) {

      // Print the disagreement
      printf("Validation2D has failed\n");
      Frame2DPrint(that);
      printf(" against ");
      Frame2DPrint(tho);
      printf("\n");
      printf("FMB : ");
      if (isIntersectingFMB == false)
        printf("no ");
      printf("intersection\n");
      printf("SAT : ");
      if (isIntersectingSAT == false)
        printf("no ");
      printf("intersection\n");

      // Stop the validation
      exit(0);

    }

    // If the Frames are in intersection
    if (isIntersectingFMB == true) {

      // Update the number of intersection
      nbInter++;

    // If the Frames are not in intersection
    } else {

      // Update the number of no intersection
      nbNoInter++;

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

// Main function
void Validate2D(void) {

  // Initialise the random generator
  srandom(time(NULL));
```

```
// Declare two variables to memorize the arguments to the
// Validation function
Param2D paramP;
Param2D paramQ;

// Initialize the number of intersection and no intersection
nbInter = 0;
nbNoInter = 0;

// Loop on the tests
for (unsigned long iTest = NB_TESTS;
     iTest--;) {

  // Create two random Frame definitions
  Param2D* param = &paramP;
  for (int iParam = 2;
       iParam--;) {

    // 50% chance of being a Cuboid or a Tetrahedron
    if (rnd() < 0.5)
      param->type = FrameCuboid;
    else
      param->type = FrameTetrahedron;

    for (int iAxis = 2;
         iAxis--;) {

      param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      for (int iComp = 2;
           iComp--;) {

        param->comp[iComp][iAxis] =
          -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      }

    }

    param = &paramQ;

  }

  // Calculate the determinant of the Frames' components matrix
  double detP =
    paramP.comp[0][0] * paramP.comp[1][1] -
    paramP.comp[1][0] * paramP.comp[0][1];

  double detQ =
    paramQ.comp[0][0] * paramQ.comp[1][1] -
    paramQ.comp[1][0] * paramQ.comp[0][1];

  // If the determinants are not null, ie the Frame are not degenerate
  if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

    // Run the validation on the two Frames
    Validation2D(
      paramP,
      paramQ);

  }
```

```
  }

  // If we reached it means the validation was successfull
  // Print results
  printf("Validation2D has succeed.\n");
  printf("Tested %lu intersections ", nbInter);
  printf("and %lu no intersections\n", nbNoInter);

}

int main(int argc, char** argv) {

  printf("===== 2D static ======\n");
  Validate2D();

  return 0;
}
```

6.1.2 3D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>

// Include FMB and SAT algorithm library
#include "fmb3d.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of tests of the validation
#define NB_TESTS 1000000

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Global variables to count nb of tests resulting in intersection
// and no intersection
unsigned long int nbInter;
unsigned long int nbNoInter;

// Helper structure to pass arguments to the Validation function
typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
} Param3D;

// Validation function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and check the results are identical
void Validation3D(
  const Param3D paramP,
  const Param3D paramQ) {

  // Create the two Frames
  Frame3D P =
```

```
  Frame3DCreateStatic(
    paramP.type,
    paramP.orig,
    paramP.comp);

Frame3D Q =
  Frame3DCreateStatic(
    paramQ.type,
    paramQ.orig,
    paramQ.comp);

// Helper variables to loop on the pair (that, tho) and (tho, that)
Frame3D* that = &P;
Frame3D* tho = &Q;

// Loop on pairs of Frames
for (int iPair = 2;
     iPair--;) {

  // Test intersection with FMB
  bool isIntersectingFMB =
    FMBTestIntersection3D(
      that,
      tho,
      NULL);

  // Test intersection with SAT
  bool isIntersectingSAT =
    SATTestIntersection3D(
      that,
      tho);

  // If the results are different
  if (isIntersectingFMB != isIntersectingSAT) {

    // Print the disagreement
    printf("Validation3D has failed\n");
    Frame3DPrint(that);
    printf(" against ");
    Frame3DPrint(tho);
    printf("\n");
    printf("FMB : ");
    if (isIntersectingFMB == false)
      printf("no ");
    printf("intersection\n");
    printf("SAT : ");
    if (isIntersectingSAT == false)
      printf("no ");
    printf("intersection\n");

    // Stop the validation
    exit(0);

  }

  // If the Frames are in intersection
  if (isIntersectingFMB == true) {

    // Update the number of intersection
    nbInter++;

  // If the Frames are not in intersection
```

```
    } else {

      // Update the number of no intersection
      nbNoInter++;

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Validate3D(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Declare two variables to memorize the arguments to the
  // Validation function
  Param3D paramP;
  Param3D paramQ;

  // Initialize the number of intersection and no intersection
  nbInter = 0;
  nbNoInter = 0;

  // Loop on the tests
  for (unsigned long iTest = NB_TESTS;
       iTest--;) {

    // Create two random Frame definitions
    Param3D* param = &paramP;
    for (int iParam = 2;
         iParam--;) {

      // 50% chance of being a Cuboid or a Tetrahedron
      if (rnd() < 0.5)
        param->type = FrameCuboid;
      else
        param->type = FrameTetrahedron;

      for (int iAxis = 3;
           iAxis--;) {

        param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        for (int iComp = 3;
             iComp--;) {

          param->comp[iComp][iAxis] =
            -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        }

      }

      param = &paramQ;

    }
```

```
    // Calculate the determinant of the Frames' components matrix
    double detP =
      paramP.comp[0][0] * (paramP.comp[1][1] * paramP.comp[2][2]-
      paramP.comp[1][2] * paramP.comp[2][1]) -
      paramP.comp[1][0] * (paramP.comp[0][1] * paramP.comp[2][2]-
      paramP.comp[0][2] * paramP.comp[2][1]) +
      paramP.comp[2][0] * (paramP.comp[0][1] * paramP.comp[1][2]-
      paramP.comp[0][2] * paramP.comp[1][1]);

    double detQ =
      paramQ.comp[0][0] * (paramQ.comp[1][1] * paramQ.comp[2][2]-
      paramQ.comp[1][2] * paramQ.comp[2][1]) -
      paramQ.comp[1][0] * (paramQ.comp[0][1] * paramQ.comp[2][2]-
      paramQ.comp[0][2] * paramQ.comp[2][1]) +
      paramQ.comp[2][0] * (paramQ.comp[0][1] * paramQ.comp[1][2]-
      paramQ.comp[0][2] * paramQ.comp[1][1]);

    // If the determinants are not null, ie the Frame are not degenerate
    if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

      // Run the validation on the two Frames
      Validation3D(
        paramP,
        paramQ);

    }

  }

  // If we reached it means the validation was successfull
  // Print results
  printf("Validation3D has succeed.\n");
  printf("Tested %lu intersections ", nbInter);
  printf("and %lu no intersections\n", nbNoInter);

}

int main(int argc, char** argv) {

  printf("===== 3D static ======\n");
  Validate3D();

  return 0;
}
```

6.1.3 2D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>

// Include FMB and SAT algorithm library
#include "fmb2dt.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
```

```
#define RANGE_AXIS 100.0
// Nb of tests of the validation
#define NB_TESTS 1000000

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Global variables to count nb of tests resulting in intersection
// and no intersection
unsigned long int nbInter;
unsigned long int nbNoInter;

// Helper structure to pass arguments to the Validation function
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
  double speed[2];
} Param2DTime;

// Validation function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and check the results are identical
void Validation2DTime(
  const Param2DTime paramP,
  const Param2DTime paramQ) {

  // Create the two Frames
  Frame2DTime P =
    Frame2DTimeCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.speed,
      paramP.comp);

  Frame2DTime Q =
    Frame2DTimeCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.speed,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame2DTime* that = &P;
  Frame2DTime* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Test intersection with FMB
    bool isIntersectingFMB =
      FMBTestIntersection2DTime(
        that,
        tho,
        NULL);

    // Test intersection with SAT
    bool isIntersectingSAT =
      SATTestIntersection2DTime(
        that,
        tho);
```

```
    // If the results are different
    if (isIntersectingFMB != isIntersectingSAT) {

      // Print the disagreement
      printf("Validation2D has failed\n");
      Frame2DTimePrint(that);
      printf(" against ");
      Frame2DTimePrint(tho);
      printf("\n");
      printf("FMB : ");
      if (isIntersectingFMB == false)
        printf("no ");
      printf("intersection\n");
      printf("SAT : ");
      if (isIntersectingSAT == false)
        printf("no ");
      printf("intersection\n");

      // Stop the validation
      exit(0);

    }

    // If the Frames are in intersection
    if (isIntersectingFMB == true) {

      // Update the number of intersection
      nbInter++;

    // If the Frames are not in intersection
    } else {

      // Update the number of no intersection
      nbNoInter++;

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

// Main function
void Validate2DTime(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Declare two variables to memorize the arguments to the
  // Validation function
  Param2DTime paramP;
  Param2DTime paramQ;

  // Initialize the number of intersection and no intersection
  nbInter = 0;
  nbNoInter = 0;

  // Loop on the tests
```

```
  for (unsigned long iTest = NB_TESTS;
       iTest--;) {

    // Create two random Frame definitions
    Param2DTime* param = &paramP;
    for (int iParam = 2;
         iParam--;) {

      // 50% chance of being a Cuboid or a Tetrahedron
      if (rnd() < 0.5)
        param->type = FrameCuboid;
      else
        param->type = FrameTetrahedron;

      for (int iAxis = 2;
           iAxis--;) {

        param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;
        param->speed[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        for (int iComp = 2;
             iComp--;) {

          param->comp[iComp][iAxis] =
            -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        }

      }

      param = &paramQ;

    }

    // Calculate the determinant of the Frames' components matrix
    double detP =
      paramP.comp[0][0] * paramP.comp[1][1] -
      paramP.comp[1][0] * paramP.comp[0][1];

    double detQ =
      paramQ.comp[0][0] * paramQ.comp[1][1] -
      paramQ.comp[1][0] * paramQ.comp[0][1];

    // If the determinants are not null, ie the Frame are not degenerate
    if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

      // Run the validation on the two Frames
      Validation2DTime(
        paramP,
        paramQ);

    }

  }

  // If we reached it means the validation was successfull
  // Print results
  printf("Validation2DTime has succeed.\n");
  printf("Tested %lu intersections ", nbInter);
  printf("and %lu no intersections\n", nbNoInter);

}
```

```
int main(int argc, char** argv) {

  printf("===== 2D dynamic ======\n");
  Validate2DTime();

  return 0;
}
```

6.1.4 3D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>

// Include FMB and SAT algorithm library
#include "fmb3dt.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of tests of the validation
#define NB_TESTS 1000000

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Global variables to count nb of tests resulting in intersection
// and no intersection
unsigned long int nbInter;
unsigned long int nbNoInter;

// Helper structure to pass arguments to the Validation function
typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
  double speed[3];
} Param3DTime;

// Validation function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and check the results are identical
void Validation3DTime(
  const Param3DTime paramP,
  const Param3DTime paramQ) {

  // Create the two Frames
  Frame3DTime P =
    Frame3DTimeCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.speed,
      paramP.comp);

  Frame3DTime Q =
    Frame3DTimeCreateStatic(
```

```
    paramQ.type,
    paramQ.orig,
    paramQ.speed,
    paramQ.comp);

// Helper variables to loop on the pair (that, tho) and (tho, that)
Frame3DTime* that = &P;
Frame3DTime* tho = &Q;

// Loop on pairs of Frames
for (int iPair = 2;
     iPair--;) {

  // Test intersection with FMB
  bool isIntersectingFMB =
    FMBTestIntersection3DTime(
      that,
      tho,
      NULL);

  // Test intersection with SAT
  bool isIntersectingSAT =
    SATTestIntersection3DTime(
      that,
      tho);

  // If the results are different
  if (isIntersectingFMB != isIntersectingSAT) {

    // Print the disagreement
    printf("Validation3D has failed\n");
    Frame3DTimePrint(that);
    printf(" against ");
    Frame3DTimePrint(tho);
    printf("\n");
    printf("FMB : ");
    if (isIntersectingFMB == false)
      printf("no ");
    printf("intersection\n");
    printf("SAT : ");
    if (isIntersectingSAT == false)
      printf("no ");
    printf("intersection\n");

    // Stop the validation
    exit(0);

  }

  // If the Frames are in intersection
  if (isIntersectingFMB == true) {

    // Update the number of intersection
    nbInter++;

  // If the Frames are not in intersection
  } else {

    // Update the number of no intersection
    nbNoInter++;

  }
```

```
    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

// Main function
void Validate3DTime(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Declare two variables to memorize the arguments to the
  // Validation function
  Param3DTime paramP;
  Param3DTime paramQ;

  // Initialize the number of intersection and no intersection
  nbInter = 0;
  nbNoInter = 0;

  // Loop on the tests
  for (unsigned long iTest = NB_TESTS;
       iTest--;) {

    // Create two random Frame definitions
    Param3DTime* param = &paramP;
    for (int iParam = 2;
         iParam--;) {

      // 50% chance of being a Cuboid or a Tetrahedron
      if (rnd() < 0.5)
        param->type = FrameCuboid;
      else
        param->type = FrameTetrahedron;

      for (int iAxis = 3;
           iAxis--;) {

        param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;
        param->speed[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        for (int iComp = 3;
             iComp--;) {

          param->comp[iComp][iAxis] =
            -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        }

      }

      param = &paramQ;

    }

    // Calculate the determinant of the Frames' components matrix
    double detP =
      paramP.comp[0][0] * (paramP.comp[1][1] * paramP.comp[2][2]-
```

```
      paramP.comp[1][2] * paramP.comp[2][1]) -
      paramP.comp[1][0] * (paramP.comp[0][1] * paramP.comp[2][2] -
      paramP.comp[0][2] * paramP.comp[2][1]) +
      paramP.comp[2][0] * (paramP.comp[0][1] * paramP.comp[1][2] -
      paramP.comp[0][2] * paramP.comp[1][1]);

    double detQ =
      paramQ.comp[0][0] * (paramQ.comp[1][1] * paramQ.comp[2][2] -
      paramQ.comp[1][2] * paramQ.comp[2][1]) -
      paramQ.comp[1][0] * (paramQ.comp[0][1] * paramQ.comp[2][2] -
      paramQ.comp[0][2] * paramQ.comp[2][1]) +
      paramQ.comp[2][0] * (paramQ.comp[0][1] * paramQ.comp[1][2] -
      paramQ.comp[0][2] * paramQ.comp[1][1]);

    // If the determinants are not null, ie the Frame are not degenerate
    if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

      // Run the validation on the two Frames
      Validation3DTime(
        paramP,
        paramQ);

    }

  }

  // If we reached it means the validation was successfull
  // Print results
  printf("Validation3DTime has succeed.\n");
  printf("Tested %lu intersections ", nbInter);
  printf("and %lu no intersections\n", nbNoInter);

}

int main(int argc, char** argv) {

  printf("===== 3D dynamic ======\n");
  Validate3DTime();

  return 0;
}
```

6.2 Results

6.2.1 2D static

```
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
```

```
Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

Co(-0.500000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed

Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(-0.500000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.250000,-0.250000) x(0.500000,0.000000) y(0.000000,2.000000)
Succeed

Co(0.250000,-0.250000) x(0.500000,0.000000) y(0.000000,2.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(-0.250000,0.250000) x(2.000000,0.000000) y(0.000000,0.500000)
Succeed

Co(-0.250000,0.250000) x(2.000000,0.000000) y(0.000000,0.500000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

Co(0.000000,0.000000) x(1.000000,1.000000) y(-1.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,1.000000) y(-1.000000,1.000000)
Succeed

Co(-0.500000,-0.500000) x(1.000000,1.000000) y(-1.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(-0.500000,-0.500000) x(1.000000,1.000000) y(-1.000000,1.000000)
Succeed

Co(1.500000,1.500000) x(1.000000,-1.000000) y(-1.000000,-1.000000)
against
Co(1.000000,0.000000) x(-1.000000,0.000000) y(0.000000,1.000000)
Succeed

Co(1.000000,0.000000) x(-1.000000,0.000000) y(0.000000,1.000000)
against
```

```
Co(1.500000,1.500000) x(1.000000,-1.000000) y(-1.000000,-1.000000)
Succeed

Co(1.000000,0.500000) x(-0.500000,0.500000) y(-0.500000,-0.500000)
against
Co(0.000000,1.000000) x(1.000000,0.000000) y(0.000000,-1.000000)
Succeed

Co(0.000000,1.000000) x(1.000000,0.000000) y(0.000000,-1.000000)
against
Co(1.000000,0.500000) x(-0.500000,0.500000) y(-0.500000,-0.500000)
Succeed

Co(0.000000,0.000000) x(1.000000,0.000000) y(1.000000,1.000000)
against
Co(2.000000,-1.000000) x(0.000000,1.000000) y(-0.500000,1.000000)
Succeed

Co(2.000000,-1.000000) x(0.000000,1.000000) y(-0.500000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(1.000000,1.000000)
Succeed

Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
Co(1.000000,1.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed

Co(1.000000,1.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed

Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
Co(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed

Co(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed

To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
Co(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed

Co(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed

Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
To(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed

To(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed
```

```
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(0.000000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

To(0.000000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

Co(0.500000,0.500000) x(-0.500000,0.000000) y(0.000000,-0.500000)
against
To(0.000000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

To(0.000000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.500000,0.500000) x(-0.500000,0.000000) y(0.000000,-0.500000)
Succeed

Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed

To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(1.500000,1.500000) x(-1.500000,0.000000) y(0.000000,-1.500000)
Succeed

To(1.500000,1.500000) x(-1.500000,0.000000) y(0.000000,-1.500000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed

To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(1.010000,1.010000) x(-1.000000,0.000000) y(0.000000,-1.000000)
 Succeed

To(1.010000,1.010000) x(-1.000000,0.000000) y(0.000000,-1.000000)
against
To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed

To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
```

```
against
To(1.000000,1.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed

To(1.000000,1.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed

To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
To(1.010000,1.500000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
 Succeed

To(1.010000,1.500000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
 Succeed

All unit tests 2D have succeed.
```

6.2.2 3D static

```
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.500000,0.500000,0.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed

Co(0.500000,0.500000,0.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(-0.500000,-0.500000,-0.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed

Co(-0.500000,-0.500000,-0.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
```

```
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(1.500000,1.500000,1.500000) x(-1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,-1.000000)
Succeed

Co(1.500000,1.500000,1.500000) x(-1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,-1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.500000,1.500000,-1.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed

Co(0.500000,1.500000,-1.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,-1.000000)
against
Co(0.500000,1.500000,-1.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed

Co(0.500000,1.500000,-1.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,-1.000000)
Succeed

Co(-1.010000,-1.010000,-1.010000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(-1.010000,-1.010000,-1.010000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
 Succeed

Co(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
```

```
against
Co(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed

Co(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
Succeed

To(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed

Co(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
To(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
 Succeed

Co(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed

To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
Succeed

To(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed

To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
To(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
 Succeed

To(-0.500000,-1.000000,-0.500000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed

To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
```

```
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
To(-0.500000,-1.000000,-0.500000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
Succeed

All unit tests 3D have succeed.
```

6.2.3 2D dynamic

```
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(-1.000000,0.000000) s(-1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
 Succeed

Co(-1.000000,0.000000) s(-1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
 Succeed

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(-1.010000,-1.010000) s(1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
 Succeed

Co(-1.010000,-1.010000) s(1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
 Succeed

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(-1.000000,0.000000) s(1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed

Co(-1.000000,0.000000) s(1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(-1.000000,0.250000) s(4.000000,0.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
Succeed

Co(-1.000000,0.250000) s(4.000000,0.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
```

```
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.250000,-1.000000) s(0.000000,4.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
Succeed

Co(0.250000,-1.000000) s(0.000000,4.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.900000,-1.000000) s(0.000000,4.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
Succeed

Co(0.900000,-1.000000) s(0.000000,4.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed

All unit tests 2DTime have succeed.
```

6.2.4 3D dynamic

```
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(-1.000000,0.000000,0.000000) s(-1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
 Succeed

Co(-1.000000,0.000000,0.000000) s(-1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
 Succeed

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(-1.010000,-1.010000,0.000000) s(1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
```

```
    (0.000000,0.000000,1.000000)
 Succeed

Co(-1.010000,-1.010000,0.000000) s(1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
 Succeed

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(-1.000000,0.000000,0.000000) s(1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed

Co(-1.000000,0.000000,0.000000) s(1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(-1.000000,0.250000,0.000000) s(4.000000,0.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed

Co(-1.000000,0.250000,0.000000) s(4.000000,0.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.250000,-1.000000,0.000000) s(0.000000,4.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed

Co(0.250000,-1.000000,0.000000) s(0.000000,4.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
```

```
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.900000,-1.000000,0.000000) s(0.000000,4.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed

Co(0.900000,-1.000000,0.000000) s(0.000000,4.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed

All unit tests 3DTime have succeed.
```

7 Validation against SAT

In this section I introduce the code I've used to validate the algorithm and its implementation. The validation consists of running the FMB algorithm on randomly generated pairs of Frame and check that its result is equal to the one of running the SAT algorithm on the same pair of Frames. The code of the implementation of the SAT algorithm is given in annex (p.211)

7.1 Code

7.1.1 2D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>

// Include FMB and SAT algorithm library
#include "fmb2d.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of tests of the validation
#define NB_TESTS 1000000

// Helper macro to generate random number in [0.0, 1.0]
```

```
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Global variables to count nb of tests resulting in intersection
// and no intersection
unsigned long int nbInter;
unsigned long int nbNoInter;

// Helper structure to pass arguments to the Validation function
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
} Param2D;

// Validation function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and check the results are identical
void Validation2D(
  const Param2D paramP,
  const Param2D paramQ) {

  // Create the two Frames
  Frame2D P =
    Frame2DCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.comp);

  Frame2D Q =
    Frame2DCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame2D* that = &P;
  Frame2D* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Test intersection with FMB
    bool isIntersectingFMB =
      FMBTestIntersection2D(
        that,
        tho,
        NULL);

    // Test intersection with SAT
    bool isIntersectingSAT =
      SATTestIntersection2D(
        that,
        tho);

    // If the results are different
    if (isIntersectingFMB != isIntersectingSAT) {

      // Print the disagreement
      printf("Validation2D has failed\n");
      Frame2DPrint(that);
      printf(" against ");
```

```
      Frame2DPrint(tho);
      printf("\n");
      printf("FMB : ");
      if (isIntersectingFMB == false)
        printf("no ");
      printf("intersection\n");
      printf("SAT : ");
      if (isIntersectingSAT == false)
        printf("no ");
      printf("intersection\n");

      // Stop the validation
      exit(0);

    }

    // If the Frames are in intersection
    if (isIntersectingFMB == true) {

      // Update the number of intersection
      nbInter++;

    // If the Frames are not in intersection
    } else {

      // Update the number of no intersection
      nbNoInter++;

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

// Main function
void Validate2D(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Declare two variables to memorize the arguments to the
  // Validation function
  Param2D paramP;
  Param2D paramQ;

  // Initialize the number of intersection and no intersection
  nbInter = 0;
  nbNoInter = 0;

  // Loop on the tests
  for (unsigned long iTest = NB_TESTS;
       iTest--;) {

    // Create two random Frame definitions
    Param2D* param = &paramP;
    for (int iParam = 2;
         iParam--;) {
```

```
      // 50% chance of being a Cuboid or a Tetrahedron
      if (rnd() < 0.5)
        param->type = FrameCuboid;
      else
        param->type = FrameTetrahedron;

      for (int iAxis = 2;
           iAxis--;) {

        param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        for (int iComp = 2;
             iComp--;) {

          param->comp[iComp][iAxis] =
            -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        }

      }

      param = &paramQ;

    }

    // Calculate the determinant of the Frames' components matrix
    double detP =
      paramP.comp[0][0] * paramP.comp[1][1] -
      paramP.comp[1][0] * paramP.comp[0][1];

    double detQ =
      paramQ.comp[0][0] * paramQ.comp[1][1] -
      paramQ.comp[1][0] * paramQ.comp[0][1];

    // If the determinants are not null, ie the Frame are not degenerate
    if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

      // Run the validation on the two Frames
      Validation2D(
        paramP,
        paramQ);

    }

  }

  // If we reached it means the validation was successfull
  // Print results
  printf("Validation2D has succeed.\n");
  printf("Tested %lu intersections ", nbInter);
  printf("and %lu no intersections\n", nbNoInter);

}

int main(int argc, char** argv) {

  printf("===== 2D static ======\n");
  Validate2D();

  return 0;
}
```

7.1.2 3D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>

// Include FMB and SAT algorithm library
#include "fmb3d.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of tests of the validation
#define NB_TESTS 1000000

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Global variables to count nb of tests resulting in intersection
// and no intersection
unsigned long int nbInter;
unsigned long int nbNoInter;

// Helper structure to pass arguments to the Validation function
typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
} Param3D;

// Validation function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and check the results are identical
void Validation3D(
  const Param3D paramP,
  const Param3D paramQ) {

  // Create the two Frames
  Frame3D P =
    Frame3DCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.comp);

  Frame3D Q =
    Frame3DCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame3D* that = &P;
  Frame3D* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {
```

```
    // Test intersection with FMB
    bool isIntersectingFMB =
      FMBTestIntersection3D(
        that,
        tho,
        NULL);

    // Test intersection with SAT
    bool isIntersectingSAT =
      SATTestIntersection3D(
        that,
        tho);

    // If the results are different
    if (isIntersectingFMB != isIntersectingSAT) {

      // Print the disagreement
      printf("Validation3D has failed\n");
      Frame3DPrint(that);
      printf(" against ");
      Frame3DPrint(tho);
      printf("\n");
      printf("FMB : ");
      if (isIntersectingFMB == false)
        printf("no ");
      printf("intersection\n");
      printf("SAT : ");
      if (isIntersectingSAT == false)
        printf("no ");
      printf("intersection\n");

      // Stop the validation
      exit(0);

    }

    // If the Frames are in intersection
    if (isIntersectingFMB == true) {

      // Update the number of intersection
      nbInter++;

    // If the Frames are not in intersection
    } else {

      // Update the number of no intersection
      nbNoInter++;

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Validate3D(void) {

  // Initialise the random generator
```

```
srandom(time(NULL));

// Declare two variables to memorize the arguments to the
// Validation function
Param3D paramP;
Param3D paramQ;

// Initialize the number of intersection and no intersection
nbInter = 0;
nbNoInter = 0;

// Loop on the tests
for (unsigned long iTest = NB_TESTS;
     iTest--;) {

  // Create two random Frame definitions
  Param3D* param = &paramP;
  for (int iParam = 2;
       iParam--;) {

    // 50% chance of being a Cuboid or a Tetrahedron
    if (rnd() < 0.5)
      param->type = FrameCuboid;
    else
      param->type = FrameTetrahedron;

    for (int iAxis = 3;
         iAxis--;) {

      param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      for (int iComp = 3;
           iComp--;) {

        param->comp[iComp][iAxis] =
          -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      }

    }

    param = &paramQ;

  }

  // Calculate the determinant of the Frames' components matrix
  double detP =
    paramP.comp[0][0] * (paramP.comp[1][1] * paramP.comp[2][2]-
    paramP.comp[1][2] * paramP.comp[2][1]) -
    paramP.comp[1][0] * (paramP.comp[0][1] * paramP.comp[2][2]-
    paramP.comp[0][2] * paramP.comp[2][1]) +
    paramP.comp[2][0] * (paramP.comp[0][1] * paramP.comp[1][2]-
    paramP.comp[0][2] * paramP.comp[1][1]);

  double detQ =
    paramQ.comp[0][0] * (paramQ.comp[1][1] * paramQ.comp[2][2]-
    paramQ.comp[1][2] * paramQ.comp[2][1]) -
    paramQ.comp[1][0] * (paramQ.comp[0][1] * paramQ.comp[2][2]-
    paramQ.comp[0][2] * paramQ.comp[2][1]) +
    paramQ.comp[2][0] * (paramQ.comp[0][1] * paramQ.comp[1][2]-
    paramQ.comp[0][2] * paramQ.comp[1][1]);
```

```
    // If the determinants are not null, ie the Frame are not degenerate
    if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

      // Run the validation on the two Frames
      Validation3D(
        paramP,
        paramQ);

    }

  }

  // If we reached it means the validation was successfull
  // Print results
  printf("Validation3D has succeed.\n");
  printf("Tested %lu intersections ", nbInter);
  printf("and %lu no intersections\n", nbNoInter);

}

int main(int argc, char** argv) {

  printf("===== 3D static ======\n");
  Validate3D();

  return 0;
}
```

7.1.3 2D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>

// Include FMB and SAT algorithm library
#include "fmb2dt.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of tests of the validation
#define NB_TESTS 1000000

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Global variables to count nb of tests resulting in intersection
// and no intersection
unsigned long int nbInter;
unsigned long int nbNoInter;

// Helper structure to pass arguments to the Validation function
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
  double speed[2];
```

```
} Param2DTime;

// Validation function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and check the results are identical
void Validation2DTime(
  const Param2DTime paramP,
  const Param2DTime paramQ) {

  // Create the two Frames
  Frame2DTime P =
    Frame2DTimeCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.speed,
      paramP.comp);

  Frame2DTime Q =
    Frame2DTimeCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.speed,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame2DTime* that = &P;
  Frame2DTime* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Test intersection with FMB
    bool isIntersectingFMB =
      FMBTestIntersection2DTime(
        that,
        tho,
        NULL);

    // Test intersection with SAT
    bool isIntersectingSAT =
      SATTestIntersection2DTime(
        that,
        tho);

    // If the results are different
    if (isIntersectingFMB != isIntersectingSAT) {

      // Print the disagreement
      printf("Validation2D has failed\n");
      Frame2DTimePrint(that);
      printf(" against ");
      Frame2DTimePrint(tho);
      printf("\n");
      printf("FMB : ");
      if (isIntersectingFMB == false)
        printf("no ");
      printf("intersection\n");
      printf("SAT : ");
      if (isIntersectingSAT == false)
        printf("no ");
      printf("intersection\n");
```

```
      // Stop the validation
      exit(0);

    }

    // If the Frames are in intersection
    if (isIntersectingFMB == true) {

      // Update the number of intersection
      nbInter++;

    // If the Frames are not in intersection
    } else {

      // Update the number of no intersection
      nbNoInter++;

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

// Main function
void Validate2DTime(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Declare two variables to memorize the arguments to the
  // Validation function
  Param2DTime paramP;
  Param2DTime paramQ;

  // Initialize the number of intersection and no intersection
  nbInter = 0;
  nbNoInter = 0;

  // Loop on the tests
  for (unsigned long iTest = NB_TESTS;
       iTest--;) {

    // Create two random Frame definitions
    Param2DTime* param = &paramP;
    for (int iParam = 2;
         iParam--;) {

      // 50% chance of being a Cuboid or a Tetrahedron
      if (rnd() < 0.5)
        param->type = FrameCuboid;
      else
        param->type = FrameTetrahedron;

      for (int iAxis = 2;
           iAxis--;) {

        param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;
```

```
        param->speed[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        for (int iComp = 2;
             iComp--;) {

          param->comp[iComp][iAxis] =
            -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        }

      }

      param = &paramQ;

    }

    // Calculate the determinant of the Frames' components matrix
    double detP =
      paramP.comp[0][0] * paramP.comp[1][1] -
      paramP.comp[1][0] * paramP.comp[0][1];

    double detQ =
      paramQ.comp[0][0] * paramQ.comp[1][1] -
      paramQ.comp[1][0] * paramQ.comp[0][1];

    // If the determinants are not null, ie the Frame are not degenerate
    if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

      // Run the validation on the two Frames
      Validation2DTime(
        paramP,
        paramQ);

    }

  }

  // If we reached it means the validation was successfull
  // Print results
  printf("Validation2DTime has succeed.\n");
  printf("Tested %lu intersections ", nbInter);
  printf("and %lu no intersections\n", nbNoInter);

}

int main(int argc, char** argv) {

  printf("===== 2D dynamic ======\n");
  Validate2DTime();

  return 0;
}
```

7.1.4 3D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>
```

```
// Include FMB and SAT algorithm library
#include "fmb3dt.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of tests of the validation
#define NB_TESTS 1000000

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Global variables to count nb of tests resulting in intersection
// and no intersection
unsigned long int nbInter;
unsigned long int nbNoInter;

// Helper structure to pass arguments to the Validation function
typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
  double speed[3];
} Param3DTime;

// Validation function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and check the results are identical
void Validation3DTime(
  const Param3DTime paramP,
  const Param3DTime paramQ) {

  // Create the two Frames
  Frame3DTime P =
    Frame3DTimeCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.speed,
      paramP.comp);

  Frame3DTime Q =
    Frame3DTimeCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.speed,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame3DTime* that = &P;
  Frame3DTime* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Test intersection with FMB
    bool isIntersectingFMB =
      FMBTestIntersection3DTime(
        that,
        tho,
```

```
        NULL);

    // Test intersection with SAT
    bool isIntersectingSAT =
      SATTestIntersection3DTime(
        that,
        tho);

    // If the results are different
    if (isIntersectingFMB != isIntersectingSAT) {

      // Print the disagreement
      printf("Validation3D has failed\n");
      Frame3DTimePrint(that);
      printf(" against ");
      Frame3DTimePrint(tho);
      printf("\n");
      printf("FMB : ");
      if (isIntersectingFMB == false)
        printf("no ");
      printf("intersection\n");
      printf("SAT : ");
      if (isIntersectingSAT == false)
        printf("no ");
      printf("intersection\n");

      // Stop the validation
      exit(0);

    }

    // If the Frames are in intersection
    if (isIntersectingFMB == true) {

      // Update the number of intersection
      nbInter++;

    // If the Frames are not in intersection
    } else {

      // Update the number of no intersection
      nbNoInter++;

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

// Main function
void Validate3DTime(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Declare two variables to memorize the arguments to the
  // Validation function
  Param3DTime paramP;
```

```
Param3DTime paramQ;

// Initialize the number of intersection and no intersection
nbInter = 0;
nbNoInter = 0;

// Loop on the tests
for (unsigned long iTest = NB_TESTS;
     iTest--;) {

  // Create two random Frame definitions
  Param3DTime* param = &paramP;
  for (int iParam = 2;
       iParam--;) {

    // 50% chance of being a Cuboid or a Tetrahedron
    if (rnd() < 0.5)
      param->type = FrameCuboid;
    else
      param->type = FrameTetrahedron;

    for (int iAxis = 3;
         iAxis--;) {

      param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;
      param->speed[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      for (int iComp = 3;
           iComp--;) {

        param->comp[iComp][iAxis] =
          -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      }

    }

    param = &paramQ;

  }

  // Calculate the determinant of the Frames' components matrix
  double detP =
    paramP.comp[0][0] * (paramP.comp[1][1] * paramP.comp[2][2]-
    paramP.comp[1][2] * paramP.comp[2][1]) -
    paramP.comp[1][0] * (paramP.comp[0][1] * paramP.comp[2][2]-
    paramP.comp[0][2] * paramP.comp[2][1]) +
    paramP.comp[2][0] * (paramP.comp[0][1] * paramP.comp[1][2]-
    paramP.comp[0][2] * paramP.comp[1][1]);

  double detQ =
    paramQ.comp[0][0] * (paramQ.comp[1][1] * paramQ.comp[2][2]-
    paramQ.comp[1][2] * paramQ.comp[2][1]) -
    paramQ.comp[1][0] * (paramQ.comp[0][1] * paramQ.comp[2][2]-
    paramQ.comp[0][2] * paramQ.comp[2][1]) +
    paramQ.comp[2][0] * (paramQ.comp[0][1] * paramQ.comp[1][2]-
    paramQ.comp[0][2] * paramQ.comp[1][1]);

  // If the determinants are not null, ie the Frame are not degenerate
  if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

    // Run the validation on the two Frames
```

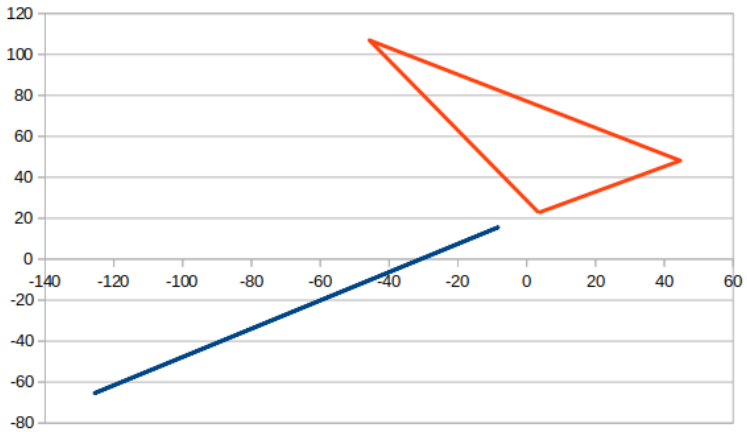
```
      Validation3DTime(
        paramP,
        paramQ);

    }

  }

  // If we reached it means the validation was successfull
  // Print results
  printf("Validation3DTime has succeed.\n");
  printf("Tested %lu intersections ", nbInter);
  printf("and %lu no intersections\n", nbNoInter);

}

int main(int argc, char** argv) {

  printf("===== 3D dynamic ======\n");
  Validate3DTime();

  return 0;
}
```

7.2 Results

7.2.1 Failures

Validation has failed in one case: when one or both of the frame are degenerated (at least two of there components ae colinear). An example is given below for reference:

```
    ===== 2D static ======
Validation2D has failed
Co(-63.571705,-22.581119) x(55.239119,38.152177) y(-62.031537,-42.843548) against To(3.474294,22.751011)
x(-49.195251,84.166201) y(41.179031,-95.350316)
FMB : intersection
SAT : no intersection
```



This case can be detected and avoided prior to the intersection test by checking the determinant of the frame: degenerated frames have a null determinant. In the example above the determinant of the first frame is equal to -0.001667.

7.2.2 2D static

```
===== 2D static ======
Validation2D has succeed.
Tested 466398 intersections and 1533536 no intersections
```

7.2.3 2D dynamic

```
===== 2D dynamic ======
Validation2DTime has succeed.
Tested 745264 intersections and 1254682 no intersections
```

7.2.4 3D static

```
===== 3D static ======
Validation3D has succeed.
Tested 314452 intersections and 1685546 no intersections
```

7.2.5 3D dynamic

```
===== 3D dynamic ======
Validation3DTime has succeed.
Tested 523938 intersections and 1476062 no intersections
```

8 Qualification against SAT

In this section I introduce the code I've used to qualify the algorithm and its implementation. The qualification consists of running the FMB algorithm

on randomly generated pairs of Frame, and check its execution time against the one of running the SAT algorithm on the same pair of Frames.

8.1 Code

8.1.1 2D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>
#include <sys/time.h>

// Include FMB and SAT algorithm library
#include "fmb2d.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of run
#define NB_RUNS 9
// Nb of tests per run
#define NB_TESTS 100000
// Nb of times the test is run on one pair of frame, used to
// slow down the processus and be able to measure time
#define NB_REPEAT_2D 1500

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Helper structure to pass arguments to the Qualification function
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
} Param2D;

// Global variables to count nb of tests resulting in intersection
// and no intersection, and min/max/total time of execution for each
double minInter;
double maxInter;
double sumInter;
unsigned long countInter;
double minNoInter;
double maxNoInter;
double sumNoInter;
unsigned long countNoInter;

double minInterCC;
double maxInterCC;
double sumInterCC;
unsigned long countInterCC;
double minNoInterCC;
double maxNoInterCC;
```

```
double sumNoInterCC;
unsigned long countNoInterCC;

double minInterCT;
double maxInterCT;
double sumInterCT;
unsigned long countInterCT;
double minNoInterCT;
double maxNoInterCT;
double sumNoInterCT;
unsigned long countNoInterCT;

double minInterTC;
double maxInterTC;
double sumInterTC;
unsigned long countInterTC;
double minNoInterTC;
double maxNoInterTC;
double sumNoInterTC;
unsigned long countNoInterTC;

double minInterTT;
double maxInterTT;
double sumInterTT;
unsigned long countInterTT;
double minNoInterTT;
double maxNoInterTT;
double sumNoInterTT;
unsigned long countNoInterTT;

// Qualification function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and measure the time of execution of each
void Qualification2DStatic(
        const Param2D paramP,
        const Param2D paramQ) {

  // Create the two Frames
  Frame2D P =
    Frame2DCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.comp);

  Frame2D Q =
    Frame2DCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame2D* that = &P;
  Frame2D* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Declare an array to memorize the results of the repeated
    // test on the same pair,
    // to prevent optimization from the compiler to remove the for loop
    bool isIntersectingFMB[NB_REPEAT_2D] = {false};
```

```
// Start measuring time
struct timeval start;
gettimeofday(&start, NULL);

// Run the FMB intersection test
for (int i = NB_REPEAT_2D;
     i--;) {

  isIntersectingFMB[i] =
    FMBTestIntersection2D(
      that,
      tho,
      NULL);
}

// Stop measuring time
struct timeval stop;
gettimeofday(&stop, NULL);

// Calculate the delay of execution
unsigned long deltausFMB = 0;
if (stop.tv_sec < start.tv_sec) {
  printf("time warps, try again\n");
  exit(0);
}
if (stop.tv_sec > start.tv_sec + 1) {
  printf("deltausFMB >> 1s, decrease NB_REPEAT\n");
  exit(0);
}
if (stop.tv_usec < start.tv_usec) {
  deltausFMB = stop.tv_sec - start.tv_sec;
  deltausFMB += stop.tv_usec + 1000000 - start.tv_usec;
} else {
  deltausFMB = stop.tv_usec - start.tv_usec;
}

// Declare an array to memorize the results of the repeated
// test on the same pair,
// to prevent optimization from the compiler to remove the for loop
bool isIntersectingSAT[NB_REPEAT_2D] = {false};

// Start measuring time
gettimeofday(&start, NULL);

// Run the FMB intersection test
for (int i = NB_REPEAT_2D;
     i--;) {

  isIntersectingSAT[i] =
    SATTestIntersection2D(
      that,
      tho);

}

// Stop measuring time
gettimeofday(&stop, NULL);

// Calculate the delay of execution
unsigned long deltausSAT = 0;
if (stop.tv_sec < start.tv_sec) {
```

```
  printf("time warps, try again\n");
  exit(0);
}
if (stop.tv_sec > start.tv_sec + 1) {
  printf("deltausSAT >> 1s, decrease NB_REPEAT\n");
  exit(0);
}
if (stop.tv_usec < start.tv_usec) {
  deltausSAT = stop.tv_sec - start.tv_sec;
  deltausSAT += stop.tv_usec + 1000000 - start.tv_usec;
} else {
  deltausSAT = stop.tv_usec - start.tv_usec;
}

// If the delays are greater than 10ms
if (deltausFMB >= 10 && deltausSAT >= 10) {

  // If FMB and SAT disagrees
  if (isIntersectingFMB[0] != isIntersectingSAT[0]) {

    printf("Qualification has failed\n");
    Frame2DPrint(that);
    printf(" against ");
    Frame2DPrint(tho);
    printf("\n");
    printf("FMB : ");
    if (isIntersectingFMB[0] == false)
      printf("no ");
    printf("intersection\n");
    printf("SAT : ");
    if (isIntersectingSAT[0] == false)
      printf("no ");
    printf("intersection\n");

    // Stop the qualification test
    exit(0);

  }

  // Get the ratio of execution time
  double ratio = ((double)deltausFMB) / ((double)deltausSAT);

  // If the Frames intersect
  if (isIntersectingSAT[0] == true) {

    // Update the counters
    if (countInter == 0) {

      minInter = ratio;
      maxInter = ratio;

    } else {

      if (minInter > ratio)
        minInter = ratio;
      if (maxInter < ratio)
        maxInter = ratio;

    }
    sumInter += ratio;
    ++countInter;
```

```
if (paramP.type == FrameCuboid &&
    paramQ.type == FrameCuboid) {

  if (countInterCC == 0) {

    minInterCC = ratio;
    maxInterCC = ratio;

  } else {

    if (minInterCC > ratio)
      minInterCC = ratio;
    if (maxInterCC < ratio)
      maxInterCC = ratio;

  }
  sumInterCC += ratio;
  ++countInterCC;

} else if (paramP.type == FrameCuboid &&
           paramQ.type == FrameTetrahedron) {

  if (countInterCT == 0) {

    minInterCT = ratio;
    maxInterCT = ratio;

  } else {

    if (minInterCT > ratio)
      minInterCT = ratio;
    if (maxInterCT < ratio)
      maxInterCT = ratio;

  }
  sumInterCT += ratio;
  ++countInterCT;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameCuboid) {

  if (countInterTC == 0) {

    minInterTC = ratio;
    maxInterTC = ratio;

  } else {

    if (minInterTC > ratio)
      minInterTC = ratio;
    if (maxInterTC < ratio)
      maxInterTC = ratio;

  }
  sumInterTC += ratio;
  ++countInterTC;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameTetrahedron) {

  if (countInterTT == 0) {
```

```
      minInterTT = ratio;
      maxInterTT = ratio;

    } else {

      if (minInterTT > ratio)
        minInterTT = ratio;
      if (maxInterTT < ratio)
        maxInterTT = ratio;

    }
    sumInterTT += ratio;
    ++countInterTT;

  }

// Else, the Frames do not intersect
} else {

  // Update the counters
  if (countNoInter == 0) {

    minNoInter = ratio;
    maxNoInter = ratio;

  } else {

    if (minNoInter > ratio)
      minNoInter = ratio;
    if (maxNoInter < ratio)
      maxNoInter = ratio;

  }
  sumNoInter += ratio;
  ++countNoInter;

  if (paramP.type == FrameCuboid &&
      paramQ.type == FrameCuboid) {

    if (countNoInterCC == 0) {

      minNoInterCC = ratio;
      maxNoInterCC = ratio;

    } else {

      if (minNoInterCC > ratio)
        minNoInterCC = ratio;
      if (maxNoInterCC < ratio)
        maxNoInterCC = ratio;

    }
    sumNoInterCC += ratio;
    ++countNoInterCC;

  } else if (paramP.type == FrameCuboid &&
             paramQ.type == FrameTetrahedron) {

    if (countNoInterCT == 0) {

      minNoInterCT = ratio;
      maxNoInterCT = ratio;
```

```
      } else {

        if (minNoInterCT > ratio)
          minNoInterCT = ratio;
        if (maxNoInterCT < ratio)
          maxNoInterCT = ratio;

      }
      sumNoInterCT += ratio;
      ++countNoInterCT;

    } else if (paramP.type == FrameTetrahedron &&
               paramQ.type == FrameCuboid) {

      if (countNoInterTC == 0) {

        minNoInterTC = ratio;
        maxNoInterTC = ratio;

      } else {

        if (minNoInterTC > ratio)
          minNoInterTC = ratio;
        if (maxNoInterTC < ratio)
          maxNoInterTC = ratio;

      }
      sumNoInterTC += ratio;
      ++countNoInterTC;

    } else if (paramP.type == FrameTetrahedron &&
               paramQ.type == FrameTetrahedron) {

      if (countNoInterTT == 0) {

        minNoInterTT = ratio;
        maxNoInterTT = ratio;

      } else {

        if (minNoInterTT > ratio)
          minNoInterTT = ratio;
        if (maxNoInterTT < ratio)
          maxNoInterTT = ratio;

      }
      sumNoInterTT += ratio;
      ++countNoInterTT;

    }
  }

// Else, if time of execution for FMB was less than a 10ms
} else if (deltausFMB < 10) {

  printf("deltausFMB < 10ms, increase NB_REPEAT\n");
  exit(0);

// Else, if time of execution for SAT was less than a 10ms
} else if (deltausSAT < 10) {
```

```
      printf("deltausSAT < 10ms, increase NB_REPEAT\n");
      exit(0);

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Qualify2DStatic(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Loop on runs
  for (int iRun = 0;
       iRun < NB_RUNS;
       ++iRun) {

    // Ratio intersection/no intersection for the displayed results
    double ratioInter = 0.1 + 0.8 * (double)iRun / (double)(NB_RUNS - 1);

    // Initialize counters
    minInter = 0.0;
    maxInter = 0.0;
    sumInter = 0.0;
    countInter = 0;
    minNoInter = 0.0;
    maxNoInter = 0.0;
    sumNoInter = 0.0;
    countNoInter = 0;

    minInterCC = 0.0;
    maxInterCC = 0.0;
    sumInterCC = 0.0;
    countInterCC = 0;
    minNoInterCC = 0.0;
    maxNoInterCC = 0.0;
    sumNoInterCC = 0.0;
    countNoInterCC = 0;

    minInterCT = 0.0;
    maxInterCT = 0.0;
    sumInterCT = 0.0;
    countInterCT = 0;
    minNoInterCT = 0.0;
    maxNoInterCT = 0.0;
    sumNoInterCT = 0.0;
    countNoInterCT = 0;

    minInterTC = 0.0;
    maxInterTC = 0.0;
    sumInterTC = 0.0;
    countInterTC = 0;
    minNoInterTC = 0.0;
    maxNoInterTC = 0.0;
    sumNoInterTC = 0.0;
    countNoInterTC = 0;
```

```
minInterTT = 0.0;
maxInterTT = 0.0;
sumInterTT = 0.0;
countInterTT = 0;
minNoInterTT = 0.0;
maxNoInterTT = 0.0;
sumNoInterTT = 0.0;
countNoInterTT = 0;

// Declare two variables to memozie the arguments to the
// Qualification function
Param2D paramP;
Param2D paramQ;

// Loop on the number of tests
for (unsigned long iTest = NB_TESTS;
     iTest--;) {

  // Create two random Frame definitions
  Param2D* param = &paramP;
  for (int iParam = 2;
       iParam--;) {

    // 50% chance of being a Cuboid or a Tetrahedron
    if (rnd() < 0.5)
      param->type = FrameCuboid;
    else
      param->type = FrameTetrahedron;

    for (int iAxis = 2;
         iAxis--;) {

      param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      for (int iComp = 2;
           iComp--;) {

        param->comp[iComp][iAxis] =
          -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      }

    }

    param = &paramQ;

  }

  // Calculate the determinant of the Frames' components matrix

  double detP =
    paramP.comp[0][0] * paramP.comp[1][1] -
    paramP.comp[1][0] * paramP.comp[0][1];

  double detQ =
    paramQ.comp[0][0] * paramQ.comp[1][1] -
    paramQ.comp[1][0] * paramQ.comp[0][1];

  // If the determinants are not null, ie the Frame are not degenerate
  if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {
```

```
    // Run the validation on the two Frames
    Qualification2DStatic(
      paramP,
      paramQ);

  }

}

// Display the results
if (iRun == 0) {

  printf("percPairInter\t");
  printf("countInter\tcountNoInter\t");
  printf("minInter\tavgInter\tmaxInter\t");
  printf("minNoInter\tavgNoInter\tmaxNoInter\t");
  printf("minTotal\tavgTotal\tmaxTotal\t");

  printf("countInterCC\tcountNoInterCC\t");
  printf("minInterCC\tavgInterCC\tmaxInterCC\t");
  printf("minNoInterCC\tavgNoInterCC\tmaxNoInterCC\t");
  printf("minTotalCC\tavgTotalCC\tmaxTotalCC\t");

  printf("countInterCT\tcountNoInterCT\t");
  printf("minInterCT\tavgInterCT\tmaxInterCT\t");
  printf("minNoInterCT\tavgNoInterCT\tmaxNoInterCT\t");
  printf("minTotalCT\tavgTotalCT\tmaxTotalCT\t");

  printf("countInterTC\tcountNoInterTC\t");
  printf("minInterTC\tavgInterTC\tmaxInterTC\t");
  printf("minNoInterTC\tavgNoInterTC\tmaxNoInterTC\t");
  printf("minTotalTC\tavgTotalTC\tmaxTotalTC\t");

  printf("countInterTT\tcountNoInterTT\t");
  printf("minInterTT\tavgInterTT\tmaxInterTT\t");
  printf("minNoInterTT\tavgNoInterTT\tmaxNoInterTT\t");
  printf("minTotalTT\tavgTotalTT\tmaxTotalTT\n");

}

printf("%.1f\t", ratioInter);

printf("%lu\t%lu\t", countInter, countNoInter);
double avgInter = sumInter / (double)countInter;
printf("%f\t%f\t%f\t", minInter, avgInter, maxInter);
double avgNoInter = sumNoInter / (double)countNoInter;
printf("%f\t%f\t%f\t", minNoInter, avgNoInter, maxNoInter);
double avg =
  ratioInter * avgInter + (1.0 - ratioInter) * avgNoInter;
printf("%f\t%f\t%f\t",
  (minNoInter < minInter ? minNoInter : minInter),
  avg,
  (maxNoInter > maxInter ? maxNoInter : maxInter));

printf("%lu\t%lu\t", countInterCC, countNoInterCC);
double avgInterCC = sumInterCC / (double)countInterCC;
printf("%f\t%f\t%f\t", minInterCC, avgInterCC, maxInterCC);
double avgNoInterCC = sumNoInterCC / (double)countNoInterCC;
printf("%f\t%f\t%f\t", minNoInterCC, avgNoInterCC, maxNoInterCC);
double avgCC =
  ratioInter * avgInterCC + (1.0 - ratioInter) * avgNoInterCC;
printf("%f\t%f\t%f\t",
```

```
      (minNoInterCC < minInterCC ? minNoInterCC : minInterCC),
      avgCC,
      (maxNoInterCC > maxInterCC ? maxNoInterCC : maxInterCC));

    printf("%lu\t%lu\t", countInterCT, countNoInterCT);
    double avgInterCT = sumInterCT / (double)countInterCT;
    printf("%f\t%f\t%f\t", minInterCT, avgInterCT, maxInterCT);
    double avgNoInterCT = sumNoInterCT / (double)countNoInterCT;
    printf("%f\t%f\t%f\t", minNoInterCT, avgNoInterCT, maxNoInterCT);
    double avgCT =
      ratioInter * avgInterCT + (1.0 - ratioInter) * avgNoInterCT;
    printf("%f\t%f\t%f\t",
      (minNoInterCT < minInterCT ? minNoInterCT : minInterCT),
      avgCT,
      (maxNoInterCT > maxInterCT ? maxNoInterCT : maxInterCT));

    printf("%lu\t%lu\t", countInterTC, countNoInterTC);
    double avgInterTC = sumInterTC / (double)countInterTC;
    printf("%f\t%f\t%f\t", minInterTC, avgInterTC, maxInterTC);
    double avgNoInterTC = sumNoInterTC / (double)countNoInterTC;
    printf("%f\t%f\t%f\t", minNoInterTC, avgNoInterTC, maxNoInterTC);
    double avgTC =
      ratioInter * avgInterTC + (1.0 - ratioInter) * avgNoInterTC;
    printf("%f\t%f\t%f\t",
      (minNoInterTC < minInterTC ? minNoInterTC : minInterTC),
      avgTC,
      (maxNoInterTC > maxInterTC ? maxNoInterTC : maxInterTC));

    printf("%lu\t%lu\t", countInterTT, countNoInterTT);
    double avgInterTT = sumInterTT / (double)countInterTT;
    printf("%f\t%f\t%f\t", minInterTT, avgInterTT, maxInterTT);
    double avgNoInterTT = sumNoInterTT / (double)countNoInterTT;
    printf("%f\t%f\t%f\t", minNoInterTT, avgNoInterTT, maxNoInterTT);
    double avgTT =
      ratioInter * avgInterTT + (1.0 - ratioInter) * avgNoInterTT;
    printf("%f\t%f\t%f\n",
      (minNoInterTT < minInterTT ? minNoInterTT : minInterTT),
      avgTT,
      (maxNoInterTT > maxInterTT ? maxNoInterTT : maxInterTT));

  }

}

int main(int argc, char** argv) {

  Qualify2DStatic();

  return 0;
}
```

8.1.2 3D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>
#include <sys/time.h>

// Include FMB and SAT algorithm library
```

```
#include "fmb3d.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of run
#define NB_RUNS 9
// Nb of tests per run
#define NB_TESTS 100000
// Nb of times the test is run on one pair of frame, used to
// slow down the processus and be able to measure time
#define NB_REPEAT_3D 800

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Helper structure to pass arguments to the Qualification function
typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
} Param3D;

// Global variables to count nb of tests resulting in intersection
// and no intersection, and min/max/total time of execution for each
double minInter;
double maxInter;
double sumInter;
unsigned long countInter;
double minNoInter;
double maxNoInter;
double sumNoInter;
unsigned long countNoInter;

double minInterCC;
double maxInterCC;
double sumInterCC;
unsigned long countInterCC;
double minNoInterCC;
double maxNoInterCC;
double sumNoInterCC;
unsigned long countNoInterCC;

double minInterCT;
double maxInterCT;
double sumInterCT;
unsigned long countInterCT;
double minNoInterCT;
double maxNoInterCT;
double sumNoInterCT;
unsigned long countNoInterCT;

double minInterTC;
double maxInterTC;
double sumInterTC;
unsigned long countInterTC;
double minNoInterTC;
double maxNoInterTC;
double sumNoInterTC;
unsigned long countNoInterTC;
```

```
double minInterTT;
double maxInterTT;
double sumInterTT;
unsigned long countInterTT;
double minNoInterTT;
double maxNoInterTT;
double sumNoInterTT;
unsigned long countNoInterTT;

// Qualification function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and measure the time of execution of each
void Qualification3DStatic(
        const Param3D paramP,
        const Param3D paramQ) {

  // Create the two Frames
  Frame3D P =
    Frame3DCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.comp);

  Frame3D Q =
    Frame3DCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame3D* that = &P;
  Frame3D* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Declare an array to memorize the results of the repeated
    // test on the same pair,
    // to prevent optimization from the compiler to remove the for loop
    bool isIntersectingFMB[NB_REPEAT_3D] = {false};

    // Start measuring time
    struct timeval start;
    gettimeofday(&start, NULL);

    // Run the FMB intersection test
    for (int i = NB_REPEAT_3D;
         i--;) {

      isIntersectingFMB[i] =
        FMBTestIntersection3D(
          that,
          tho,
          NULL);
    }

    // Stop measuring time
    struct timeval stop;
    gettimeofday(&stop, NULL);
```

```
// Calculate the delay of execution
unsigned long deltausFMB = 0;
if (stop.tv_sec < start.tv_sec) {
  printf("time warps, try again\n");
  exit(0);
}
if (stop.tv_sec > start.tv_sec + 1) {
  printf("deltausFMB >> 1s, decrease NB_REPEAT\n");
  exit(0);
}
if (stop.tv_usec < start.tv_usec) {
  deltausFMB = stop.tv_sec - start.tv_sec;
  deltausFMB += stop.tv_usec + 1000000 - start.tv_usec;
} else {
  deltausFMB = stop.tv_usec - start.tv_usec;
}

// Declare an array to memorize the results of the repeated
// test on the same pair,
// to prevent optimization from the compiler to remove the for loop
bool isIntersectingSAT[NB_REPEAT_3D] = {false};

// Start measuring time
gettimeofday(&start, NULL);

// Run the FMB intersection test
for (int i = NB_REPEAT_3D;
     i--;) {

  isIntersectingSAT[i] =
    SATTestIntersection3D(
      that,
      tho);

}

// Stop measuring time
gettimeofday(&stop, NULL);

// Calculate the delay of execution
unsigned long deltausSAT = 0;
if (stop.tv_sec < start.tv_sec) {
  printf("time warps, try again\n");
  exit(0);
}
if (stop.tv_sec > start.tv_sec + 1) {
  printf("deltausSAT >> 1s, decrease NB_REPEAT\n");
  exit(0);
}
if (stop.tv_usec < start.tv_usec) {
  deltausSAT = stop.tv_sec - start.tv_sec;
  deltausSAT += stop.tv_usec + 1000000 - start.tv_usec;
} else {
  deltausSAT = stop.tv_usec - start.tv_usec;
}

// If the delays are greater than 10ms
if (deltausFMB >= 10 && deltausSAT >= 10) {

  // If FMB and SAT disagrees
  if (isIntersectingFMB[0] != isIntersectingSAT[0]) {
```

```
  printf("Qualification has failed\n");
  Frame3DPrint(that);
  printf(" against ");
  Frame3DPrint(tho);
  printf("\n");
  printf("FMB : ");
  if (isIntersectingFMB[0] == false)
    printf("no ");
  printf("intersection\n");
  printf("SAT : ");
  if (isIntersectingSAT[0] == false)
    printf("no ");
  printf("intersection\n");

  // Stop the qualification test
  exit(0);

}

// Get the ratio of execution time
double ratio = ((double)deltausFMB) / ((double)deltausSAT);

// If the Frames intersect
if (isIntersectingSAT[0] == true) {

  // Update the counters
  if (countInter == 0) {

    minInter = ratio;
    maxInter = ratio;

  } else {

    if (minInter > ratio)
      minInter = ratio;
    if (maxInter < ratio)
      maxInter = ratio;

  }
  sumInter += ratio;
  ++countInter;

  if (paramP.type == FrameCuboid &&
      paramQ.type == FrameCuboid) {

    if (countInterCC == 0) {

      minInterCC = ratio;
      maxInterCC = ratio;

    } else {

      if (minInterCC > ratio)
        minInterCC = ratio;
      if (maxInterCC < ratio)
        maxInterCC = ratio;

    }
    sumInterCC += ratio;
    ++countInterCC;

  } else if (paramP.type == FrameCuboid &&
```

```
            paramQ.type == FrameTetrahedron) {

    if (countInterCT == 0) {

      minInterCT = ratio;
      maxInterCT = ratio;

    } else {

      if (minInterCT > ratio)
        minInterCT = ratio;
      if (maxInterCT < ratio)
        maxInterCT = ratio;

    }
    sumInterCT += ratio;
    ++countInterCT;

  } else if (paramP.type == FrameTetrahedron &&
            paramQ.type == FrameCuboid) {

    if (countInterTC == 0) {

      minInterTC = ratio;
      maxInterTC = ratio;

    } else {

      if (minInterTC > ratio)
        minInterTC = ratio;
      if (maxInterTC < ratio)
        maxInterTC = ratio;

    }
    sumInterTC += ratio;
    ++countInterTC;

  } else if (paramP.type == FrameTetrahedron &&
            paramQ.type == FrameTetrahedron) {

    if (countInterTT == 0) {

      minInterTT = ratio;
      maxInterTT = ratio;

    } else {

      if (minInterTT > ratio)
        minInterTT = ratio;
      if (maxInterTT < ratio)
        maxInterTT = ratio;

    }
    sumInterTT += ratio;
    ++countInterTT;

  }

// Else, the Frames do not intersect
} else {

  // Update the counters
```

```
if (countNoInter == 0) {

  minNoInter = ratio;
  maxNoInter = ratio;

} else {

  if (minNoInter > ratio)
    minNoInter = ratio;
  if (maxNoInter < ratio)
    maxNoInter = ratio;

}
sumNoInter += ratio;
++countNoInter;

if (paramP.type == FrameCuboid &&
    paramQ.type == FrameCuboid) {

  if (countNoInterCC == 0) {

    minNoInterCC = ratio;
    maxNoInterCC = ratio;

  } else {

    if (minNoInterCC > ratio)
      minNoInterCC = ratio;
    if (maxNoInterCC < ratio)
      maxNoInterCC = ratio;

  }
  sumNoInterCC += ratio;
  ++countNoInterCC;

} else if (paramP.type == FrameCuboid &&
           paramQ.type == FrameTetrahedron) {

  if (countNoInterCT == 0) {

    minNoInterCT = ratio;
    maxNoInterCT = ratio;

  } else {

    if (minNoInterCT > ratio)
      minNoInterCT = ratio;
    if (maxNoInterCT < ratio)
      maxNoInterCT = ratio;

  }
  sumNoInterCT += ratio;
  ++countNoInterCT;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameCuboid) {

  if (countNoInterTC == 0) {

    minNoInterTC = ratio;
    maxNoInterTC = ratio;
```

```
          } else {

            if (minNoInterTC > ratio)
              minNoInterTC = ratio;
            if (maxNoInterTC < ratio)
              maxNoInterTC = ratio;

          }
          sumNoInterTC += ratio;
          ++countNoInterTC;

        } else if (paramP.type == FrameTetrahedron &&
                   paramQ.type == FrameTetrahedron) {

          if (countNoInterTT == 0) {

            minNoInterTT = ratio;
            maxNoInterTT = ratio;

          } else {

            if (minNoInterTT > ratio)
              minNoInterTT = ratio;
            if (maxNoInterTT < ratio)
              maxNoInterTT = ratio;

          }
          sumNoInterTT += ratio;
          ++countNoInterTT;

        }
      }

    // Else, if time of execution for FMB was less than a 10ms
    } else if (deltausFMB < 10) {

      printf("deltausFMB < 10ms, increase NB_REPEAT\n");
      exit(0);

    // Else, if time of execution for SAT was less than a 10ms
    } else if (deltausSAT < 10) {

      printf("deltausSAT < 10ms, increase NB_REPEAT\n");
      exit(0);

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Qualify3DStatic(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Loop on runs
  for (int iRun = 0;
```

```
   iRun < NB_RUNS;
   ++iRun) {

// Ratio intersection/no intersection for the displayed results
double ratioInter = 0.1 + 0.8 * (double)iRun / (double)(NB_RUNS - 1);

// Initialize counters
minInter = 0.0;
maxInter = 0.0;
sumInter = 0.0;
countInter = 0;
minNoInter = 0.0;
maxNoInter = 0.0;
sumNoInter = 0.0;
countNoInter = 0;

minInterCC = 0.0;
maxInterCC = 0.0;
sumInterCC = 0.0;
countInterCC = 0;
minNoInterCC = 0.0;
maxNoInterCC = 0.0;
sumNoInterCC = 0.0;
countNoInterCC = 0;

minInterCT = 0.0;
maxInterCT = 0.0;
sumInterCT = 0.0;
countInterCT = 0;
minNoInterCT = 0.0;
maxNoInterCT = 0.0;
sumNoInterCT = 0.0;
countNoInterCT = 0;

minInterTC = 0.0;
maxInterTC = 0.0;
sumInterTC = 0.0;
countInterTC = 0;
minNoInterTC = 0.0;
maxNoInterTC = 0.0;
sumNoInterTC = 0.0;
countNoInterTC = 0;

minInterTT = 0.0;
maxInterTT = 0.0;
sumInterTT = 0.0;
countInterTT = 0;
minNoInterTT = 0.0;
maxNoInterTT = 0.0;
sumNoInterTT = 0.0;
countNoInterTT = 0;

// Declare two variables to memozie the arguments to the
// Qualification function
Param3D paramP;
Param3D paramQ;

// Loop on the number of tests
for (unsigned long iTest = NB_TESTS;
     iTest--;) {

  // Create two random Frame definitions
```

```
  Param3D* param = &paramP;
  for (int iParam = 2;
       iParam--;) {

    // 50% chance of being a Cuboid or a Tetrahedron
    if (rnd() < 0.5)
      param->type = FrameCuboid;
    else
      param->type = FrameTetrahedron;

    for (int iAxis = 3;
         iAxis--;) {

      param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      for (int iComp = 3;
           iComp--;) {

        param->comp[iComp][iAxis] =
          -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      }

    }

    param = &paramQ;

  }

  // Calculate the determinant of the Frames' components matrix
  double detP =
    paramP.comp[0][0] * (paramP.comp[1][1] * paramP.comp[2][2]-
    paramP.comp[1][2] * paramP.comp[2][1]) -
    paramP.comp[1][0] * (paramP.comp[0][1] * paramP.comp[2][2]-
    paramP.comp[0][2] * paramP.comp[2][1]) +
    paramP.comp[2][0] * (paramP.comp[0][1] * paramP.comp[1][2]-
    paramP.comp[0][2] * paramP.comp[1][1]);

  double detQ =
    paramQ.comp[0][0] * (paramQ.comp[1][1] * paramQ.comp[2][2]-
    paramQ.comp[1][2] * paramQ.comp[2][1]) -
    paramQ.comp[1][0] * (paramQ.comp[0][1] * paramQ.comp[2][2]-
    paramQ.comp[0][2] * paramQ.comp[2][1]) +
    paramQ.comp[2][0] * (paramQ.comp[0][1] * paramQ.comp[1][2]-
    paramQ.comp[0][2] * paramQ.comp[1][1]);

  // If the determinants are not null, ie the Frame are not degenerate
  if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

    // Run the validation on the two Frames
    Qualification3DStatic(
      paramP,
      paramQ);

  }

}

// Display the results
if (iRun == 0) {

  printf("percPairInter\t");
```

```
  printf("countInter\tcountNoInter\t");
  printf("minInter\tavgInter\tmaxInter\t");
  printf("minNoInter\tavgNoInter\tmaxNoInter\t");
  printf("minTotal\tavgTotal\tmaxTotal\t");

  printf("countInterCC\tcountNoInterCC\t");
  printf("minInterCC\tavgInterCC\tmaxInterCC\t");
  printf("minNoInterCC\tavgNoInterCC\tmaxNoInterCC\t");
  printf("minTotalCC\tavgTotalCC\tmaxTotalCC\t");

  printf("countInterCT\tcountNoInterCT\t");
  printf("minInterCT\tavgInterCT\tmaxInterCT\t");
  printf("minNoInterCT\tavgNoInterCT\tmaxNoInterCT\t");
  printf("minTotalCT\tavgTotalCT\tmaxTotalCT\t");

  printf("countInterTC\tcountNoInterTC\t");
  printf("minInterTC\tavgInterTC\tmaxInterTC\t");
  printf("minNoInterTC\tavgNoInterTC\tmaxNoInterTC\t");
  printf("minTotalTC\tavgTotalTC\tmaxTotalTC\t");

  printf("countInterTT\tcountNoInterTT\t");
  printf("minInterTT\tavgInterTT\tmaxInterTT\t");
  printf("minNoInterTT\tavgNoInterTT\tmaxNoInterTT\t");
  printf("minTotalTT\tavgTotalTT\tmaxTotalTT\n");

}

printf("%.1f\t", ratioInter);

printf("%lu\t%lu\t", countInter, countNoInter);
double avgInter = sumInter / (double)countInter;
printf("%f\t%f\t%f\t", minInter, avgInter, maxInter);
double avgNoInter = sumNoInter / (double)countNoInter;
printf("%f\t%f\t%f\t", minNoInter, avgNoInter, maxNoInter);
double avg =
  ratioInter * avgInter + (1.0 - ratioInter) * avgNoInter;
printf("%f\t%f\t%f\t",
  (minNoInter < minInter ? minNoInter : minInter),
  avg,
  (maxNoInter > maxInter ? maxNoInter : maxInter));

printf("%lu\t%lu\t", countInterCC, countNoInterCC);
double avgInterCC = sumInterCC / (double)countInterCC;
printf("%f\t%f\t%f\t", minInterCC, avgInterCC, maxInterCC);
double avgNoInterCC = sumNoInterCC / (double)countNoInterCC;
printf("%f\t%f\t%f\t", minNoInterCC, avgNoInterCC, maxNoInterCC);
double avgCC =
  ratioInter * avgInterCC + (1.0 - ratioInter) * avgNoInterCC;
printf("%f\t%f\t%f\t",
  (minNoInterCC < minInterCC ? minNoInterCC : minInterCC),
  avgCC,
  (maxNoInterCC > maxInterCC ? maxNoInterCC : maxInterCC));

printf("%lu\t%lu\t", countInterCT, countNoInterCT);
double avgInterCT = sumInterCT / (double)countInterCT;
printf("%f\t%f\t%f\t", minInterCT, avgInterCT, maxInterCT);
double avgNoInterCT = sumNoInterCT / (double)countNoInterCT;
printf("%f\t%f\t%f\t", minNoInterCT, avgNoInterCT, maxNoInterCT);
double avgCT =
  ratioInter * avgInterCT + (1.0 - ratioInter) * avgNoInterCT;
printf("%f\t%f\t%f\t",
  (minNoInterCT < minInterCT ? minNoInterCT : minInterCT),
```

```
      avgCT,
      (maxNoInterCT > maxInterCT ? maxNoInterCT : maxInterCT));

    printf("%lu\t%lu\t", countInterTC, countNoInterTC);
    double avgInterTC = sumInterTC / (double)countInterTC;
    printf("%f\t%f\t%f\t", minInterTC, avgInterTC, maxInterTC);
    double avgNoInterTC = sumNoInterTC / (double)countNoInterTC;
    printf("%f\t%f\t%f\t", minNoInterTC, avgNoInterTC, maxNoInterTC);
    double avgTC =
      ratioInter * avgInterTC + (1.0 - ratioInter) * avgNoInterTC;
    printf("%f\t%f\t%f\t",
      (minNoInterTC < minInterTC ? minNoInterTC : minInterTC),
      avgTC,
      (maxNoInterTC > maxInterTC ? maxNoInterTC : maxInterTC));

    printf("%lu\t%lu\t", countInterTT, countNoInterTT);
    double avgInterTT = sumInterTT / (double)countInterTT;
    printf("%f\t%f\t%f\t", minInterTT, avgInterTT, maxInterTT);
    double avgNoInterTT = sumNoInterTT / (double)countNoInterTT;
    printf("%f\t%f\t%f\t", minNoInterTT, avgNoInterTT, maxNoInterTT);
    double avgTT =
      ratioInter * avgInterTT + (1.0 - ratioInter) * avgNoInterTT;
    printf("%f\t%f\t%f\n",
      (minNoInterTT < minInterTT ? minNoInterTT : minInterTT),
      avgTT,
      (maxNoInterTT > maxInterTT ? maxNoInterTT : maxInterTT));

  }

}

int main(int argc, char** argv) {

  Qualify3DStatic();

  return 0;
}
```

8.1.3 2D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>
#include <sys/time.h>

// Include FMB and SAT algorithm library
#include "fmb2dt.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of run
#define NB_RUNS 9
// Nb of tests per run
#define NB_TESTS 100000
// Nb of times the test is run on one pair of frame, used to
// slow down the processus and be able to measure time
```

```
#define NB_REPEAT_2D 1500

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Helper structure to pass arguments to the Qualification function
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
  double speed[2];
} Param2DTime;

// Global variables to count nb of tests resulting in intersection
// and no intersection, and min/max/total time of execution for each
double minInter;
double maxInter;
double sumInter;
unsigned long countInter;
double minNoInter;
double maxNoInter;
double sumNoInter;
unsigned long countNoInter;

double minInterCC;
double maxInterCC;
double sumInterCC;
unsigned long countInterCC;
double minNoInterCC;
double maxNoInterCC;
double sumNoInterCC;
unsigned long countNoInterCC;

double minInterCT;
double maxInterCT;
double sumInterCT;
unsigned long countInterCT;
double minNoInterCT;
double maxNoInterCT;
double sumNoInterCT;
unsigned long countNoInterCT;

double minInterTC;
double maxInterTC;
double sumInterTC;
unsigned long countInterTC;
double minNoInterTC;
double maxNoInterTC;
double sumNoInterTC;
unsigned long countNoInterTC;

double minInterTT;
double maxInterTT;
double sumInterTT;
unsigned long countInterTT;
double minNoInterTT;
double maxNoInterTT;
double sumNoInterTT;
unsigned long countNoInterTT;

// Qualification function
// Takes two Frame definition as input, run the intersection test on
```

```
// them with FMB and SAT, and measure the time of execution of each
void Qualification2DDynamic(
        const Param2DTime paramP,
        const Param2DTime paramQ) {

  // Create the two Frames
  Frame2DTime P =
    Frame2DTimeCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.speed,
      paramP.comp);

  Frame2DTime Q =
    Frame2DTimeCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.speed,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame2DTime* that = &P;
  Frame2DTime* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Declare an array to memorize the results of the repeated
    // test on the same pair,
    // to prevent optimization from the compiler to remove the for loop
    bool isIntersectingFMB[NB_REPEAT_2D] = {false};

    // Start measuring time
    struct timeval start;
    gettimeofday(&start, NULL);

    // Run the FMB intersection test
    for (int i = NB_REPEAT_2D;
         i--;) {

      isIntersectingFMB[i] =
        FMBTestIntersection2DTime(
          that,
          tho,
          NULL);
    }

    // Stop measuring time
    struct timeval stop;
    gettimeofday(&stop, NULL);

    // Calculate the delay of execution
    unsigned long deltausFMB = 0;
    if (stop.tv_sec < start.tv_sec) {
      printf("time warps, try again\n");
      exit(0);
    }
    if (stop.tv_sec > start.tv_sec + 1) {
      printf("deltausFMB >> 1s, decrease NB_REPEAT\n");
      exit(0);
    }
```

```
if (stop.tv_usec < start.tv_usec) {
  deltausFMB = stop.tv_sec - start.tv_sec;
  deltausFMB += stop.tv_usec + 1000000 - start.tv_usec;
} else {
  deltausFMB = stop.tv_usec - start.tv_usec;
}

// Declare an array to memorize the results of the repeated
// test on the same pair,
// to prevent optimization from the compiler to remove the for loop
bool isIntersectingSAT[NB_REPEAT_2D] = {false};

// Start measuring time
gettimeofday(&start, NULL);

// Run the FMB intersection test
for (int i = NB_REPEAT_2D;
     i--;) {

  isIntersectingSAT[i] =
    SATTestIntersection2DTime(
      that,
      tho);

}

// Stop measuring time
gettimeofday(&stop, NULL);

// Calculate the delay of execution
unsigned long deltausSAT = 0;
if (stop.tv_sec < start.tv_sec) {
  printf("time warps, try again\n");
  exit(0);
}
if (stop.tv_sec > start.tv_sec + 1) {
  printf("deltausSAT >> 1s, decrease NB_REPEAT\n");
  exit(0);
}
if (stop.tv_usec < start.tv_usec) {
  deltausSAT = stop.tv_sec - start.tv_sec;
  deltausSAT += stop.tv_usec + 1000000 - start.tv_usec;
} else {
  deltausSAT = stop.tv_usec - start.tv_usec;
}

// If the delays are greater than 10ms
if (deltausFMB >= 10 && deltausSAT >= 10) {

  // If FMB and SAT disagrees
  if (isIntersectingFMB[0] != isIntersectingSAT[0]) {

    printf("Qualification has failed\n");
    Frame2DTimePrint(that);
    printf(" against ");
    Frame2DTimePrint(tho);
    printf("\n");
    printf("FMB : ");
    if (isIntersectingFMB[0] == false)
      printf("no ");
    printf("intersection\n");
    printf("SAT : ");
```

```
  if (isIntersectingSAT[0] == false)
    printf("no ");
  printf("intersection\n");

  // Stop the qualification test
  exit(0);

}

// Get the ratio of execution time
double ratio = ((double)deltausFMB) / ((double)deltausSAT);

// If the Frames intersect
if (isIntersectingSAT[0] == true) {

  // Update the counters
  if (countInter == 0) {

    minInter = ratio;
    maxInter = ratio;

  } else {

    if (minInter > ratio)
      minInter = ratio;
    if (maxInter < ratio)
      maxInter = ratio;

  }
  sumInter += ratio;
  ++countInter;

  if (paramP.type == FrameCuboid &&
      paramQ.type == FrameCuboid) {

    if (countInterCC == 0) {

      minInterCC = ratio;
      maxInterCC = ratio;

    } else {

      if (minInterCC > ratio)
        minInterCC = ratio;
      if (maxInterCC < ratio)
        maxInterCC = ratio;

    }
    sumInterCC += ratio;
    ++countInterCC;

  } else if (paramP.type == FrameCuboid &&
             paramQ.type == FrameTetrahedron) {

    if (countInterCT == 0) {

      minInterCT = ratio;
      maxInterCT = ratio;

    } else {

      if (minInterCT > ratio)
```

```
        minInterCT = ratio;
      if (maxInterCT < ratio)
        maxInterCT = ratio;

    }
    sumInterCT += ratio;
    ++countInterCT;

  } else if (paramP.type == FrameTetrahedron &&
             paramQ.type == FrameCuboid) {

    if (countInterTC == 0) {

      minInterTC = ratio;
      maxInterTC = ratio;

    } else {

      if (minInterTC > ratio)
        minInterTC = ratio;
      if (maxInterTC < ratio)
        maxInterTC = ratio;

    }
    sumInterTC += ratio;
    ++countInterTC;

  } else if (paramP.type == FrameTetrahedron &&
             paramQ.type == FrameTetrahedron) {

    if (countInterTT == 0) {

      minInterTT = ratio;
      maxInterTT = ratio;

    } else {

      if (minInterTT > ratio)
        minInterTT = ratio;
      if (maxInterTT < ratio)
        maxInterTT = ratio;

    }
    sumInterTT += ratio;
    ++countInterTT;

  }

// Else, the Frames do not intersect
} else {

  // Update the counters
  if (countNoInter == 0) {

    minNoInter = ratio;
    maxNoInter = ratio;

  } else {

    if (minNoInter > ratio)
      minNoInter = ratio;
    if (maxNoInter < ratio)
```

```
    maxNoInter = ratio;

}
sumNoInter += ratio;
++countNoInter;

if (paramP.type == FrameCuboid &&
    paramQ.type == FrameCuboid) {

  if (countNoInterCC == 0) {

    minNoInterCC = ratio;
    maxNoInterCC = ratio;

  } else {

    if (minNoInterCC > ratio)
      minNoInterCC = ratio;
    if (maxNoInterCC < ratio)
      maxNoInterCC = ratio;

  }
  sumNoInterCC += ratio;
  ++countNoInterCC;

} else if (paramP.type == FrameCuboid &&
           paramQ.type == FrameTetrahedron) {

  if (countNoInterCT == 0) {

    minNoInterCT = ratio;
    maxNoInterCT = ratio;

  } else {

    if (minNoInterCT > ratio)
      minNoInterCT = ratio;
    if (maxNoInterCT < ratio)
      maxNoInterCT = ratio;

  }
  sumNoInterCT += ratio;
  ++countNoInterCT;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameCuboid) {

  if (countNoInterTC == 0) {

    minNoInterTC = ratio;
    maxNoInterTC = ratio;

  } else {

    if (minNoInterTC > ratio)
      minNoInterTC = ratio;
    if (maxNoInterTC < ratio)
      maxNoInterTC = ratio;

  }
  sumNoInterTC += ratio;
  ++countNoInterTC;
```

```
        } else if (paramP.type == FrameTetrahedron &&
                   paramQ.type == FrameTetrahedron) {

          if (countNoInterTT == 0) {

            minNoInterTT = ratio;
            maxNoInterTT = ratio;

          } else {

            if (minNoInterTT > ratio)
              minNoInterTT = ratio;
            if (maxNoInterTT < ratio)
              maxNoInterTT = ratio;

          }
          sumNoInterTT += ratio;
          ++countNoInterTT;

        }
      }

    // Else, if time of execution for FMB was less than a 10ms
    } else if (deltausFMB < 10) {

      printf("deltausFMB < 10ms, increase NB_REPEAT\n");
      exit(0);

    // Else, if time of execution for SAT was less than a 10ms
    } else if (deltausSAT < 10) {

      printf("deltausSAT < 10ms, increase NB_REPEAT\n");
      exit(0);

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Qualify2DDynamic(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Loop on runs
  for (int iRun = 0;
       iRun < NB_RUNS;
       ++iRun) {

    // Ratio intersection/no intersection for the displayed results
    double ratioInter = 0.1 + 0.8 * (double)iRun / (double)(NB_RUNS - 1);

    // Initialize counters
    minInter = 0.0;
    maxInter = 0.0;
    sumInter = 0.0;
```

```
countInter = 0;
minNoInter = 0.0;
maxNoInter = 0.0;
sumNoInter = 0.0;
countNoInter = 0;

minInterCC = 0.0;
maxInterCC = 0.0;
sumInterCC = 0.0;
countInterCC = 0;
minNoInterCC = 0.0;
maxNoInterCC = 0.0;
sumNoInterCC = 0.0;
countNoInterCC = 0;

minInterCT = 0.0;
maxInterCT = 0.0;
sumInterCT = 0.0;
countInterCT = 0;
minNoInterCT = 0.0;
maxNoInterCT = 0.0;
sumNoInterCT = 0.0;
countNoInterCT = 0;

minInterTC = 0.0;
maxInterTC = 0.0;
sumInterTC = 0.0;
countInterTC = 0;
minNoInterTC = 0.0;
maxNoInterTC = 0.0;
sumNoInterTC = 0.0;
countNoInterTC = 0;

minInterTT = 0.0;
maxInterTT = 0.0;
sumInterTT = 0.0;
countInterTT = 0;
minNoInterTT = 0.0;
maxNoInterTT = 0.0;
sumNoInterTT = 0.0;
countNoInterTT = 0;

// Declare two variables to memozie the arguments to the
// Qualification function
Param2DTime paramP;
Param2DTime paramQ;

// Loop on the number of tests
for (unsigned long iTest = NB_TESTS;
     iTest--;) {

  // Create two random Frame definitions
  Param2DTime* param = &paramP;
  for (int iParam = 2;
       iParam--;) {

    // 50% chance of being a Cuboid or a Tetrahedron
    if (rnd() < 0.5)
      param->type = FrameCuboid;
    else
      param->type = FrameTetrahedron;
```

```
    for (int iAxis = 2;
         iAxis--;) {

      param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;
      param->speed[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      for (int iComp = 2;
           iComp--;) {

        param->comp[iComp][iAxis] =
          -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      }

    }

    param = &paramQ;

  }

  // Calculate the determinant of the Frames' components matrix

  double detP =
    paramP.comp[0][0] * paramP.comp[1][1] -
    paramP.comp[1][0] * paramP.comp[0][1];

  double detQ =
    paramQ.comp[0][0] * paramQ.comp[1][1] -
    paramQ.comp[1][0] * paramQ.comp[0][1];

  // If the determinants are not null, ie the Frame are not degenerate
  if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

    // Run the validation on the two Frames
    Qualification2DDynamic(
      paramP,
      paramQ);

  }

}

// Display the results
if (iRun == 0) {

  printf("percPairInter\t");
  printf("countInter\tcountNoInter\t");
  printf("minInter\tavgInter\tmaxInter\t");
  printf("minNoInter\tavgNoInter\tmaxNoInter\t");
  printf("minTotal\tavgTotal\tmaxTotal\t");

  printf("countInterCC\tcountNoInterCC\t");
  printf("minInterCC\tavgInterCC\tmaxInterCC\t");
  printf("minNoInterCC\tavgNoInterCC\tmaxNoInterCC\t");
  printf("minTotalCC\tavgTotalCC\tmaxTotalCC\t");

  printf("countInterCT\tcountNoInterCT\t");
  printf("minInterCT\tavgInterCT\tmaxInterCT\t");
  printf("minNoInterCT\tavgNoInterCT\tmaxNoInterCT\t");
  printf("minTotalCT\tavgTotalCT\tmaxTotalCT\t");

  printf("countInterTC\tcountNoInterTC\t");
```

```
  printf("minInterTC\tavgInterTC\tmaxInterTC\t");
  printf("minNoInterTC\tavgNoInterTC\tmaxNoInterTC\t");
  printf("minTotalTC\tavgTotalTC\tmaxTotalTC\t");

  printf("countInterTT\tcountNoInterTT\t");
  printf("minInterTT\tavgInterTT\tmaxInterTT\t");
  printf("minNoInterTT\tavgNoInterTT\tmaxNoInterTT\t");
  printf("minTotalTT\tavgTotalTT\tmaxTotalTT\n");

}

printf("%.1f\t", ratioInter);

printf("%lu\t%lu\t", countInter, countNoInter);
double avgInter = sumInter / (double)countInter;
printf("%f\t%f\t%f\t", minInter, avgInter, maxInter);
double avgNoInter = sumNoInter / (double)countNoInter;
printf("%f\t%f\t%f\t", minNoInter, avgNoInter, maxNoInter);
double avg =
  ratioInter * avgInter + (1.0 - ratioInter) * avgNoInter;
printf("%f\t%f\t%f\t",
  (minNoInter < minInter ? minNoInter : minInter),
  avg,
  (maxNoInter > maxInter ? maxNoInter : maxInter));

printf("%lu\t%lu\t", countInterCC, countNoInterCC);
double avgInterCC = sumInterCC / (double)countInterCC;
printf("%f\t%f\t%f\t", minInterCC, avgInterCC, maxInterCC);
double avgNoInterCC = sumNoInterCC / (double)countNoInterCC;
printf("%f\t%f\t%f\t", minNoInterCC, avgNoInterCC, maxNoInterCC);
double avgCC =
  ratioInter * avgInterCC + (1.0 - ratioInter) * avgNoInterCC;
printf("%f\t%f\t%f\t",
  (minNoInterCC < minInterCC ? minNoInterCC : minInterCC),
  avgCC,
  (maxNoInterCC > maxInterCC ? maxNoInterCC : maxInterCC));

printf("%lu\t%lu\t", countInterCT, countNoInterCT);
double avgInterCT = sumInterCT / (double)countInterCT;
printf("%f\t%f\t%f\t", minInterCT, avgInterCT, maxInterCT);
double avgNoInterCT = sumNoInterCT / (double)countNoInterCT;
printf("%f\t%f\t%f\t", minNoInterCT, avgNoInterCT, maxNoInterCT);
double avgCT =
  ratioInter * avgInterCT + (1.0 - ratioInter) * avgNoInterCT;
printf("%f\t%f\t%f\t",
  (minNoInterCT < minInterCT ? minNoInterCT : minInterCT),
  avgCT,
  (maxNoInterCT > maxInterCT ? maxNoInterCT : maxInterCT));

printf("%lu\t%lu\t", countInterTC, countNoInterTC);
double avgInterTC = sumInterTC / (double)countInterTC;
printf("%f\t%f\t%f\t", minInterTC, avgInterTC, maxInterTC);
double avgNoInterTC = sumNoInterTC / (double)countNoInterTC;
printf("%f\t%f\t%f\t", minNoInterTC, avgNoInterTC, maxNoInterTC);
double avgTC =
  ratioInter * avgInterTC + (1.0 - ratioInter) * avgNoInterTC;
printf("%f\t%f\t%f\t",
  (minNoInterTC < minInterTC ? minNoInterTC : minInterTC),
  avgTC,
  (maxNoInterTC > maxInterTC ? maxNoInterTC : maxInterTC));

printf("%lu\t%lu\t", countInterTT, countNoInterTT);
```

```
    double avgInterTT = sumInterTT / (double)countInterTT;
    printf("%f\t%f\t%f\t", minInterTT, avgInterTT, maxInterTT);
    double avgNoInterTT = sumNoInterTT / (double)countNoInterTT;
    printf("%f\t%f\t%f\t", minNoInterTT, avgNoInterTT, maxNoInterTT);
    double avgTT =
      ratioInter * avgInterTT + (1.0 - ratioInter) * avgNoInterTT;
    printf("%f\t%f\t%f\n",
      (minNoInterTT < minInterTT ? minNoInterTT : minInterTT),
      avgTT,
      (maxNoInterTT > maxInterTT ? maxNoInterTT : maxInterTT));

  }

}

int main(int argc, char** argv) {

  Qualify2DDynamic();

  return 0;
}
```

8.1.4 3D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>
#include <sys/time.h>

// Include FMB and SAT algorithm library
#include "fmb3dt.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of run
#define NB_RUNS 9
// Nb of tests per run
#define NB_TESTS 100000
// Nb of times the test is run on one pair of frame, used to
// slow down the processus and be able to measure time
#define NB_REPEAT_3D 800

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Helper structure to pass arguments to the Qualification function
typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
  double speed[3];
} Param3DTime;

// Global variables to count nb of tests resulting in intersection
// and no intersection, and min/max/total time of execution for each
double minInter;
```

```
double maxInter;
double sumInter;
unsigned long countInter;
double minNoInter;
double maxNoInter;
double sumNoInter;
unsigned long countNoInter;

double minInterCC;
double maxInterCC;
double sumInterCC;
unsigned long countInterCC;
double minNoInterCC;
double maxNoInterCC;
double sumNoInterCC;
unsigned long countNoInterCC;

double minInterCT;
double maxInterCT;
double sumInterCT;
unsigned long countInterCT;
double minNoInterCT;
double maxNoInterCT;
double sumNoInterCT;
unsigned long countNoInterCT;

double minInterTC;
double maxInterTC;
double sumInterTC;
unsigned long countInterTC;
double minNoInterTC;
double maxNoInterTC;
double sumNoInterTC;
unsigned long countNoInterTC;

double minInterTT;
double maxInterTT;
double sumInterTT;
unsigned long countInterTT;
double minNoInterTT;
double maxNoInterTT;
double sumNoInterTT;
unsigned long countNoInterTT;

// Qualification function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and measure the time of execution of each
void Qualification3DDynamic(
        const Param3DTime paramP,
        const Param3DTime paramQ) {

  // Create the two Frames
  Frame3DTime P =
    Frame3DTimeCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.speed,
      paramP.comp);

  Frame3DTime Q =
    Frame3DTimeCreateStatic(
      paramQ.type,
```

```
    paramQ.orig,
    paramQ.speed,
    paramQ.comp);

// Helper variables to loop on the pair (that, tho) and (tho, that)
Frame3DTime* that = &P;
Frame3DTime* tho = &Q;

// Loop on pairs of Frames
for (int iPair = 2;
     iPair--;) {

  // Declare an array to memorize the results of the repeated
  // test on the same pair,
  // to prevent optimization from the compiler to remove the for loop
  bool isIntersectingFMB[NB_REPEAT_3D] = {false};

  // Start measuring time
  struct timeval start;
  gettimeofday(&start, NULL);

  // Run the FMB intersection test
  for (int i = NB_REPEAT_3D;
       i--;) {

    isIntersectingFMB[i] =
      FMBTestIntersection3DTime(
        that,
        tho,
        NULL);
  }

  // Stop measuring time
  struct timeval stop;
  gettimeofday(&stop, NULL);

  // Calculate the delay of execution
  unsigned long deltausFMB = 0;
  if (stop.tv_sec < start.tv_sec) {
    printf("time warps, try again\n");
    exit(0);
  }
  if (stop.tv_sec > start.tv_sec + 1) {
    printf("deltausFMB >> 1s, decrease NB_REPEAT\n");
    exit(0);
  }
  if (stop.tv_usec < start.tv_usec) {
    deltausFMB = stop.tv_sec - start.tv_sec;
    deltausFMB += stop.tv_usec + 1000000 - start.tv_usec;
  } else {
    deltausFMB = stop.tv_usec - start.tv_usec;
  }

  // Declare an array to memorize the results of the repeated
  // test on the same pair,
  // to prevent optimization from the compiler to remove the for loop
  bool isIntersectingSAT[NB_REPEAT_3D] = {false};

  // Start measuring time
  gettimeofday(&start, NULL);

  // Run the FMB intersection test
```

```
for (int i = NB_REPEAT_3D;
     i--;) {

  isIntersectingSAT[i] =
    SATTestIntersection3DTime(
      that,
      tho);

}

// Stop measuring time
gettimeofday(&stop, NULL);

// Calculate the delay of execution
unsigned long deltausSAT = 0;
if (stop.tv_sec < start.tv_sec) {
  printf("time warps, try again\n");
  exit(0);
}
if (stop.tv_sec > start.tv_sec + 1) {
  printf("deltausSAT >> 1s, decrease NB_REPEAT\n");
  exit(0);
}
if (stop.tv_usec < start.tv_usec) {
  deltausSAT = stop.tv_sec - start.tv_sec;
  deltausSAT += stop.tv_usec + 1000000 - start.tv_usec;
} else {
  deltausSAT = stop.tv_usec - start.tv_usec;
}

// If the delays are greater than 10ms
if (deltausFMB >= 10 && deltausSAT >= 10) {

  // If FMB and SAT disagrees
  if (isIntersectingFMB[0] != isIntersectingSAT[0]) {

    printf("Qualification has failed\n");
    Frame3DTimePrint(that);
    printf(" against ");
    Frame3DTimePrint(tho);
    printf("\n");
    printf("FMB : ");
    if (isIntersectingFMB[0] == false)
      printf("no ");
    printf("intersection\n");
    printf("SAT : ");
    if (isIntersectingSAT[0] == false)
      printf("no ");
    printf("intersection\n");

    // Stop the qualification test
    exit(0);

  }

  // Get the ratio of execution time
  double ratio = ((double)deltausFMB) / ((double)deltausSAT);

  // If the Frames intersect
  if (isIntersectingSAT[0] == true) {

    // Update the counters
```

```
if (countInter == 0) {

  minInter = ratio;
  maxInter = ratio;

} else {

  if (minInter > ratio)
    minInter = ratio;
  if (maxInter < ratio)
    maxInter = ratio;

}
sumInter += ratio;
++countInter;

if (paramP.type == FrameCuboid &&
    paramQ.type == FrameCuboid) {

  if (countInterCC == 0) {

    minInterCC = ratio;
    maxInterCC = ratio;

  } else {

    if (minInterCC > ratio)
      minInterCC = ratio;
    if (maxInterCC < ratio)
      maxInterCC = ratio;

  }
  sumInterCC += ratio;
  ++countInterCC;

} else if (paramP.type == FrameCuboid &&
           paramQ.type == FrameTetrahedron) {

  if (countInterCT == 0) {

    minInterCT = ratio;
    maxInterCT = ratio;

  } else {

    if (minInterCT > ratio)
      minInterCT = ratio;
    if (maxInterCT < ratio)
      maxInterCT = ratio;

  }
  sumInterCT += ratio;
  ++countInterCT;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameCuboid) {

  if (countInterTC == 0) {

    minInterTC = ratio;
    maxInterTC = ratio;
```

```
    } else {

      if (minInterTC > ratio)
        minInterTC = ratio;
      if (maxInterTC < ratio)
        maxInterTC = ratio;

    }
    sumInterTC += ratio;
    ++countInterTC;

  } else if (paramP.type == FrameTetrahedron &&
             paramQ.type == FrameTetrahedron) {

    if (countInterTT == 0) {

      minInterTT = ratio;
      maxInterTT = ratio;

    } else {

      if (minInterTT > ratio)
        minInterTT = ratio;
      if (maxInterTT < ratio)
        maxInterTT = ratio;

    }
    sumInterTT += ratio;
    ++countInterTT;

  }

// Else, the Frames do not intersect
} else {

  // Update the counters
  if (countNoInter == 0) {

    minNoInter = ratio;
    maxNoInter = ratio;

  } else {

    if (minNoInter > ratio)
      minNoInter = ratio;
    if (maxNoInter < ratio)
      maxNoInter = ratio;

  }
  sumNoInter += ratio;
  ++countNoInter;

  if (paramP.type == FrameCuboid &&
      paramQ.type == FrameCuboid) {

    if (countNoInterCC == 0) {

      minNoInterCC = ratio;
      maxNoInterCC = ratio;

    } else {
```

```
    if (minNoInterCC > ratio)
      minNoInterCC = ratio;
    if (maxNoInterCC < ratio)
      maxNoInterCC = ratio;

  }
  sumNoInterCC += ratio;
  ++countNoInterCC;

} else if (paramP.type == FrameCuboid &&
           paramQ.type == FrameTetrahedron) {

  if (countNoInterCT == 0) {

    minNoInterCT = ratio;
    maxNoInterCT = ratio;

  } else {

    if (minNoInterCT > ratio)
      minNoInterCT = ratio;
    if (maxNoInterCT < ratio)
      maxNoInterCT = ratio;

  }
  sumNoInterCT += ratio;
  ++countNoInterCT;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameCuboid) {

  if (countNoInterTC == 0) {

    minNoInterTC = ratio;
    maxNoInterTC = ratio;

  } else {

    if (minNoInterTC > ratio)
      minNoInterTC = ratio;
    if (maxNoInterTC < ratio)
      maxNoInterTC = ratio;

  }
  sumNoInterTC += ratio;
  ++countNoInterTC;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameTetrahedron) {

  if (countNoInterTT == 0) {

    minNoInterTT = ratio;
    maxNoInterTT = ratio;

  } else {

    if (minNoInterTT > ratio)
      minNoInterTT = ratio;
    if (maxNoInterTT < ratio)
      maxNoInterTT = ratio;
```

```
          }
          sumNoInterTT += ratio;
          ++countNoInterTT;

        }
      }

    // Else, if time of execution for FMB was less than a 10ms
    } else if (deltausFMB < 10) {

      printf("deltausFMB < 10ms, increase NB_REPEAT\n");
      exit(0);

    // Else, if time of execution for SAT was less than a 10ms
    } else if (deltausSAT < 10) {

      printf("deltausSAT < 10ms, increase NB_REPEAT\n");
      exit(0);

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Qualify3DDynamic(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Loop on runs
  for (int iRun = 0;
       iRun < NB_RUNS;
       ++iRun) {

    // Ratio intersection/no intersection for the displayed results
    double ratioInter = 0.1 + 0.8 * (double)iRun / (double)(NB_RUNS - 1);

    // Initialize counters
    minInter = 0.0;
    maxInter = 0.0;
    sumInter = 0.0;
    countInter = 0;
    minNoInter = 0.0;
    maxNoInter = 0.0;
    sumNoInter = 0.0;
    countNoInter = 0;

    minInterCC = 0.0;
    maxInterCC = 0.0;
    sumInterCC = 0.0;
    countInterCC = 0;
    minNoInterCC = 0.0;
    maxNoInterCC = 0.0;
    sumNoInterCC = 0.0;
    countNoInterCC = 0;

    minInterCT = 0.0;
```

```
maxInterCT = 0.0;
sumInterCT = 0.0;
countInterCT = 0;
minNoInterCT = 0.0;
maxNoInterCT = 0.0;
sumNoInterCT = 0.0;
countNoInterCT = 0;

minInterTC = 0.0;
maxInterTC = 0.0;
sumInterTC = 0.0;
countInterTC = 0;
minNoInterTC = 0.0;
maxNoInterTC = 0.0;
sumNoInterTC = 0.0;
countNoInterTC = 0;

minInterTT = 0.0;
maxInterTT = 0.0;
sumInterTT = 0.0;
countInterTT = 0;
minNoInterTT = 0.0;
maxNoInterTT = 0.0;
sumNoInterTT = 0.0;
countNoInterTT = 0;

// Declare two variables to memozie the arguments to the
// Qualification function
Param3DTime paramP;
Param3DTime paramQ;

// Loop on the number of tests
for (unsigned long iTest = NB_TESTS;
     iTest--;) {

  // Create two random Frame definitions
  Param3DTime* param = &paramP;
  for (int iParam = 2;
       iParam--;) {

    // 50% chance of being a Cuboid or a Tetrahedron
    if (rnd() < 0.5)
      param->type = FrameCuboid;
    else
      param->type = FrameTetrahedron;

    for (int iAxis = 3;
         iAxis--;) {

      param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;
      param->speed[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      for (int iComp = 3;
           iComp--;) {

        param->comp[iComp][iAxis] =
          -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      }

    }
```

```
    param = &paramQ;

  }

  // Calculate the determinant of the Frames' components matrix

double detP =
  paramP.comp[0][0] * (paramP.comp[1][1] * paramP.comp[2][2]-
  paramP.comp[1][2] * paramP.comp[2][1]) -
  paramP.comp[1][0] * (paramP.comp[0][1] * paramP.comp[2][2]-
  paramP.comp[0][2] * paramP.comp[2][1]) +
  paramP.comp[2][0] * (paramP.comp[0][1] * paramP.comp[1][2]-
  paramP.comp[0][2] * paramP.comp[1][1]);

double detQ =
  paramQ.comp[0][0] * (paramQ.comp[1][1] * paramQ.comp[2][2]-
  paramQ.comp[1][2] * paramQ.comp[2][1]) -
  paramQ.comp[1][0] * (paramQ.comp[0][1] * paramQ.comp[2][2]-
  paramQ.comp[0][2] * paramQ.comp[2][1]) +
  paramQ.comp[2][0] * (paramQ.comp[0][1] * paramQ.comp[1][2]-
  paramQ.comp[0][2] * paramQ.comp[1][1]);

  // If the determinants are not null, ie the Frame are not degenerate
  if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

    // Run the validation on the two Frames
    Qualification3DDynamic(
      paramP,
      paramQ);

  }

}

// Display the results
if (iRun == 0) {

  printf("percPairInter\t");
  printf("countInter\tcountNoInter\t");
  printf("minInter\tavgInter\tmaxInter\t");
  printf("minNoInter\tavgNoInter\tmaxNoInter\t");
  printf("minTotal\tavgTotal\tmaxTotal\t");

  printf("countInterCC\tcountNoInterCC\t");
  printf("minInterCC\tavgInterCC\tmaxInterCC\t");
  printf("minNoInterCC\tavgNoInterCC\tmaxNoInterCC\t");
  printf("minTotalCC\tavgTotalCC\tmaxTotalCC\t");

  printf("countInterCT\tcountNoInterCT\t");
  printf("minInterCT\tavgInterCT\tmaxInterCT\t");
  printf("minNoInterCT\tavgNoInterCT\tmaxNoInterCT\t");
  printf("minTotalCT\tavgTotalCT\tmaxTotalCT\t");

  printf("countInterTC\tcountNoInterTC\t");
  printf("minInterTC\tavgInterTC\tmaxInterTC\t");
  printf("minNoInterTC\tavgNoInterTC\tmaxNoInterTC\t");
  printf("minTotalTC\tavgTotalTC\tmaxTotalTC\t");

  printf("countInterTT\tcountNoInterTT\t");
  printf("minInterTT\tavgInterTT\tmaxInterTT\t");
  printf("minNoInterTT\tavgNoInterTT\tmaxNoInterTT\t");
  printf("minTotalTT\tavgTotalTT\tmaxTotalTT\n");
```

```
}

printf("%.1f\t", ratioInter);

printf("%lu\t%lu\t", countInter, countNoInter);
double avgInter = sumInter / (double)countInter;
printf("%f\t%f\t%f\t", minInter, avgInter, maxInter);
double avgNoInter = sumNoInter / (double)countNoInter;
printf("%f\t%f\t%f\t", minNoInter, avgNoInter, maxNoInter);
double avg =
  ratioInter * avgInter + (1.0 - ratioInter) * avgNoInter;
printf("%f\t%f\t%f\t",
  (minNoInter < minInter ? minNoInter : minInter),
  avg,
  (maxNoInter > maxInter ? maxNoInter : maxInter));

printf("%lu\t%lu\t", countInterCC, countNoInterCC);
double avgInterCC = sumInterCC / (double)countInterCC;
printf("%f\t%f\t%f\t", minInterCC, avgInterCC, maxInterCC);
double avgNoInterCC = sumNoInterCC / (double)countNoInterCC;
printf("%f\t%f\t%f\t", minNoInterCC, avgNoInterCC, maxNoInterCC);
double avgCC =
  ratioInter * avgInterCC + (1.0 - ratioInter) * avgNoInterCC;
printf("%f\t%f\t%f\t",
  (minNoInterCC < minInterCC ? minNoInterCC : minInterCC),
  avgCC,
  (maxNoInterCC > maxInterCC ? maxNoInterCC : maxInterCC));

printf("%lu\t%lu\t", countInterCT, countNoInterCT);
double avgInterCT = sumInterCT / (double)countInterCT;
printf("%f\t%f\t%f\t", minInterCT, avgInterCT, maxInterCT);
double avgNoInterCT = sumNoInterCT / (double)countNoInterCT;
printf("%f\t%f\t%f\t", minNoInterCT, avgNoInterCT, maxNoInterCT);
double avgCT =
  ratioInter * avgInterCT + (1.0 - ratioInter) * avgNoInterCT;
printf("%f\t%f\t%f\t",
  (minNoInterCT < minInterCT ? minNoInterCT : minInterCT),
  avgCT,
  (maxNoInterCT > maxInterCT ? maxNoInterCT : maxInterCT));

printf("%lu\t%lu\t", countInterTC, countNoInterTC);
double avgInterTC = sumInterTC / (double)countInterTC;
printf("%f\t%f\t%f\t", minInterTC, avgInterTC, maxInterTC);
double avgNoInterTC = sumNoInterTC / (double)countNoInterTC;
printf("%f\t%f\t%f\t", minNoInterTC, avgNoInterTC, maxNoInterTC);
double avgTC =
  ratioInter * avgInterTC + (1.0 - ratioInter) * avgNoInterTC;
printf("%f\t%f\t%f\t",
  (minNoInterTC < minInterTC ? minNoInterTC : minInterTC),
  avgTC,
  (maxNoInterTC > maxInterTC ? maxNoInterTC : maxInterTC));

printf("%lu\t%lu\t", countInterTT, countNoInterTT);
double avgInterTT = sumInterTT / (double)countInterTT;
printf("%f\t%f\t%f\t", minInterTT, avgInterTT, maxInterTT);
double avgNoInterTT = sumNoInterTT / (double)countNoInterTT;
printf("%f\t%f\t%f\t", minNoInterTT, avgNoInterTT, maxNoInterTT);
double avgTT =
  ratioInter * avgInterTT + (1.0 - ratioInter) * avgNoInterTT;
printf("%f\t%f\t%f\n",
  (minNoInterTT < minInterTT ? minNoInterTT : minInterTT),
```

```
      avgTT,
      (maxNoInterTT > maxInterTT ? maxNoInterTT : maxInterTT));

  }

}

int main(int argc, char** argv) {

  Qualify3DDynamic();

  return 0;
}
```

8.2 Results

8.2.1 2D static

```
percPairInter   countInter      countNoInter    minInter        avgInter
            maxInter        minNoInter      avgNoInter      maxNoInter
    minTotal        avgTotal        maxTotal        countInterCC
    countNoInterCC  minInterCC      avgInterCC      maxInterCC
    minNoInterCC    avgNoInterCC    maxNoInterCC    minTotalCC
    avgTotalCC      maxTotalCC      countInterCT    countNoInterCT
    minInterCT      avgInterCT      maxInterCT      minNoInterCT
    avgNoInterCT    maxNoInterCT    minTotalCT      avgTotalCT
    maxTotalCT      countInterTC    countNoInterTC  minInterTC
    avgInterTC      maxInterTC      minNoInterTC    avgNoInterTC
    maxNoInterTC    minTotalTC      avgTotalTC      maxTotalTC
    countInterTT    countNoInterTT  minInterTT      avgInterTT
    maxInterTT      minNoInterTT    avgNoInterTT    maxNoInterTT
    minTotalTT      avgTotalTT      maxTotalTT
0.1     46412   153580  0.616883        1.657549        3.596774
    0.166667        0.837315        6.388889        0.166667        0.919338
            6.388889        12818   36632   1.234043        2.125033
    3.181818        0.222222        0.744752        4.048780        0.222222
            0.882780        4.048780        11682   38364   0.866667
    1.615855        3.596774        0.175676        0.844797        6.333333
            0.175676        0.921903        6.333333        11728   38496
    0.671141        1.610385        2.816667        0.183099        0.848145
            6.388889        0.183099        0.924369        6.388889
    10184   40088   0.616883        1.171295        2.265060        0.166667
            0.904338        5.785714        0.166667        0.931033
    5.785714
0.2     46928   153066  0.719178        1.657465        2.981481
    0.279070        0.837966        6.400000        0.279070        1.001866
            6.400000        12868   37134   1.500000        2.125174
    2.981481        0.305556        0.744184        3.128205        0.305556
            1.020382        3.128205        11840   38248   1.083333
    1.615471        2.250000        0.279070        0.844854        6.400000
            0.279070        0.998977        6.400000        12034   37996
    1.062500        1.610490        2.344262        0.317073        0.847222
            6.266667        0.317073        0.999876        6.266667
    10186   39688   0.719178        1.170918        1.787879        0.323944
            0.910212        5.928571        0.323944        0.962353
    5.928571
0.3     46670   153318  0.566502        1.654960        4.323077
    0.160000        0.835920        6.266667        0.160000        1.081632
            6.266667        12720   37350   0.797203        2.124592
    4.323077        0.297297        0.744088        3.275000        0.297297
            1.158239        4.323077        11940   37896   0.606061
```

```
    1.615371        2.278689        0.279070        0.849089        6.266667
            0.279070        1.078973        6.266667        11736   38274
    0.844037        1.610313        3.055556        0.160000        0.843415
            6.266667        0.160000        1.073484        6.266667
    10274   39798   0.566502        1.170530        2.276923        0.230769
            0.902358        6.266667        0.230769        0.982809
    6.266667
0.4     47332   152666  0.877778        1.660218        2.763636
    0.162500        0.837145        6.266667        0.162500        1.166375
            6.266667        13204   36732   1.469136        2.125862
    2.763636        0.162500        0.745593        3.051282        0.162500
            1.297700        3.051282        11852   38294   1.086022
    1.615809        2.344828        0.295455        0.848284        6.266667
            0.295455        1.155294        6.266667        12002   38058
    1.009901        1.610637        2.049180        0.320513        0.846972
            6.200000        0.320513        1.152438        6.200000
    10274   39582   0.877778        1.170931        1.414286        0.205882
            0.901881        5.928571        0.205882        1.009501
    5.928571
0.5     46252   153738  0.420091        1.657266        2.800000
    0.177419        0.837769        6.266667        0.177419        1.247518
            6.266667        12790   36844   1.290323        2.125768
    2.800000        0.288889        0.746203        3.051282        0.288889
            1.435986        3.051282        11702   38544   0.420091
    1.615246        2.639344        0.200000        0.849703        6.266667
            0.200000        1.232475        6.266667        11568   38288
    0.724409        1.610654        2.383333        0.226415        0.848720
            6.266667        0.226415        1.229687        6.266667
    10192   40062   0.887640        1.170491        1.549296        0.177419
            0.900031        5.928571        0.177419        1.035261
    5.928571
0.6     46726   153270  0.784314        1.660095        2.907407
    0.180328        0.838758        6.266667        0.180328        1.331560
            6.266667        13016   36726   1.500000        2.125790
    2.907407        0.260000        0.749614        3.615385        0.260000
            1.575320        3.615385        11946   38082   1.096774
    1.615989        2.114754        0.288889        0.848005        6.266667
            0.288889        1.308796        6.266667        11606   38426
    1.000000        1.610769        2.467742        0.180328        0.847312
            6.266667        0.180328        1.305387        6.266667
    10158   40036   0.784314        1.171602        1.863636        0.272727
            0.903525        5.785714        0.272727        1.064371
    5.785714
0.7     46528   153472  0.800000        1.659042        3.018868
    0.213115        0.838176        6.266667        0.213115        1.412782
            6.266667        13016   37092   0.811688        2.123809
    3.018868        0.254902        0.749233        4.675000        0.254902
            1.711436        4.675000        11584   38102   0.979167
    1.616240        2.372881        0.295455        0.844277        6.266667
            0.295455        1.384651        6.266667        11736   38348
    0.853211        1.609627        2.866667        0.213115        0.850611
            6.266667        0.213115        1.381922        6.266667
    10192   39930   0.800000        1.171043        2.448718        0.222222
            0.903032        5.857143        0.222222        1.090640
    5.857143
0.8     46534   153462  0.886364        1.657115        4.557692
    0.220339        0.836960        6.266667        0.220339        1.493084
            6.266667        12910   36950   1.318966        2.125076
    4.557692        0.315789        0.746742        3.390244        0.315789
            1.849409        4.557692        11652   38442   1.074468
    1.616396        2.322034        0.220339        0.847616        6.200000
            0.220339        1.462640        6.200000        11636   38192
```

```
    1.068966        1.610109        2.135593        0.265306        0.845112
            6.266667        0.265306        1.457110        6.266667
    10336   39878   0.886364        1.171437        1.803030        0.323944
            0.902475        5.785714        0.323944        1.117644
    5.785714
0.9     46332   153660  0.815534        1.661000        4.400000
    0.203390        0.838067        6.466667        0.203390        1.578707
            6.466667        13112   36486   1.326531        2.124772
    4.400000        0.288889        0.744488        3.051282        0.288889
            1.986743        4.400000        11538   38816   1.000000
    1.615996        2.048387        0.203390        0.845152        6.266667
            0.203390        1.538911        6.266667        11520   38442
    1.108434        1.610439        2.620690        0.252632        0.843114
            6.466667        0.252632        1.533707        6.466667
    10162   39916   0.815534        1.171013        1.935065        0.305556
            0.911854        5.857143        0.305556        1.145097
    5.857143
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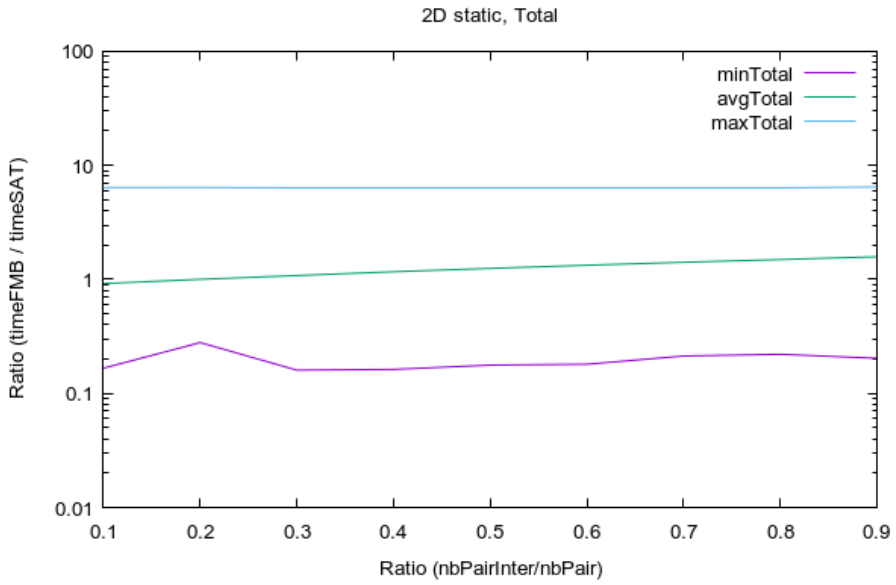
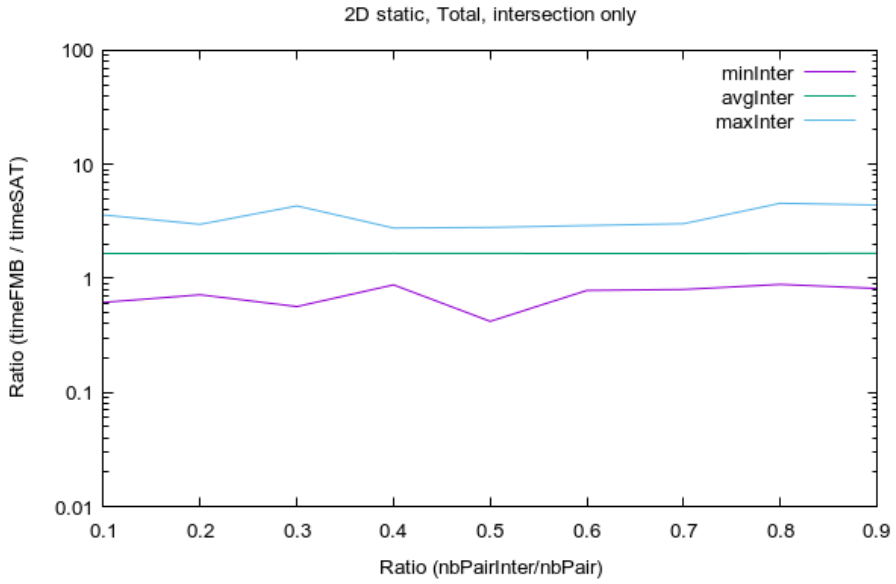
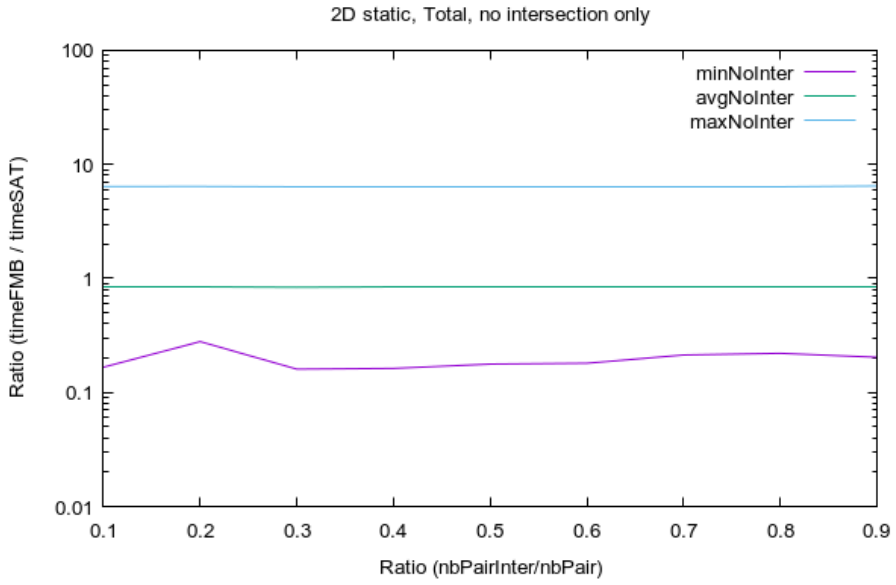
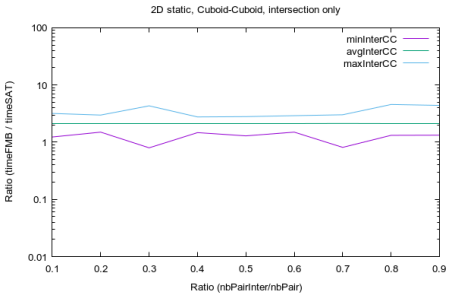
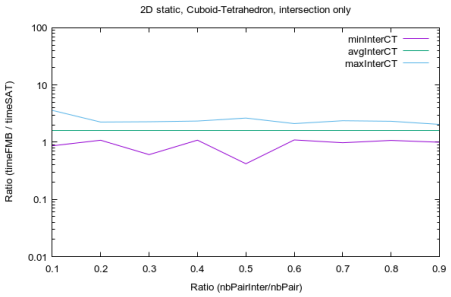
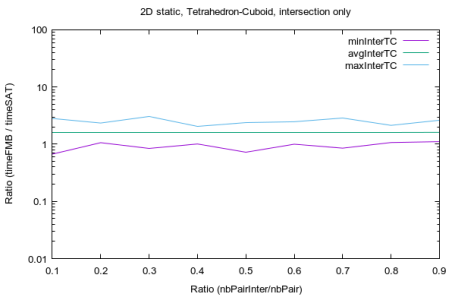


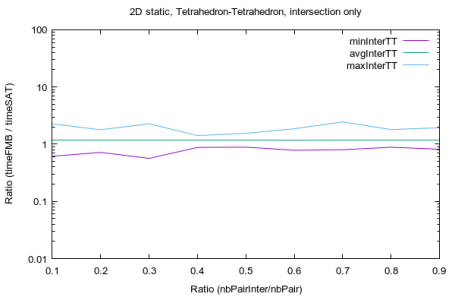

2D static, Total, intersection only
minInter
avgInter
maxInter
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

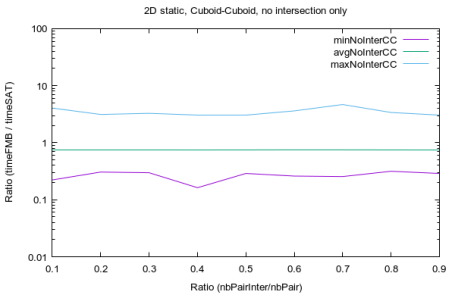

2D static, Total, no intersection only
minNoInter
avgNoInter
maxNoInter
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

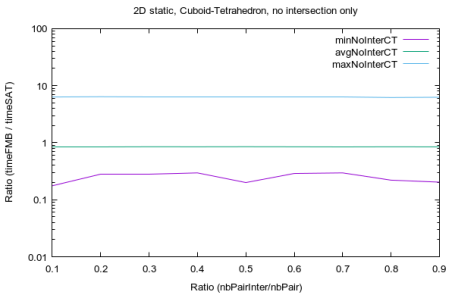


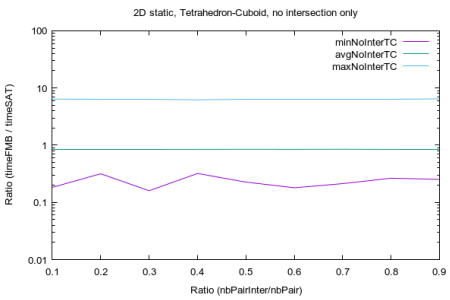


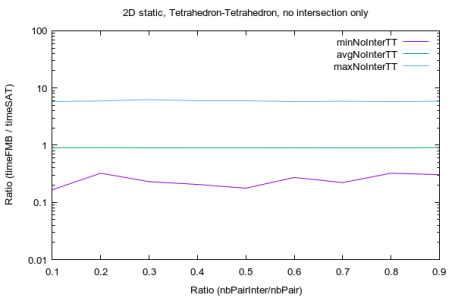












8.2.2 3D static

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percPairInter   countInter      countNoInter    minInter        avgInter
            maxInter        minNoInter      avgNoInter      maxNoInter
    minTotal        avgTotal        maxTotal        countInterCC
    countNoInterCC  minInterCC      avgInterCC      maxInterCC
    minNoInterCC    avgNoInterCC    maxNoInterCC    minTotalCC
    avgTotalCC      maxTotalCC      countInterCT    countNoInterCT
    minInterCT      avgInterCT      maxInterCT      minNoInterCT
    avgNoInterCT    maxNoInterCT    minTotalCT      avgTotalCT
    maxTotalCT      countInterTC    countNoInterTC  minInterTC
    avgInterTC      maxInterTC      minNoInterTC    avgNoInterTC
    maxNoInterTC    minTotalTC      avgTotalTC      maxTotalTC
    countInterTT    countNoInterTT  minInterTT      avgInterTT
    maxInterTT      minNoInterTT    avgNoInterTT    maxNoInterTT
    minTotalTT      avgTotalTT      maxTotalTT
0.1     31476   168524  0.170931        0.494268        1.161458
    0.036827        0.508954        9.161290        0.036827        0.507485
            9.161290        10692   39084   0.546685        0.740880
    1.161458        0.065463        0.365958        3.013889        0.065463
            0.403450        3.013889        7806    42344   0.206122
    0.414209        0.878477        0.045608        0.494690        9.000000
            0.045608        0.486642        9.000000        7826    42110
    0.228484        0.414822        0.645624        0.044304        0.497443
            9.161290        0.044304        0.489181        9.161290
    5152    44986   0.170931        0.224448        0.330037        0.036827
            0.657390        8.260870        0.036827        0.614096
    8.260870
0.2     31272   168728  0.147396        0.495383        0.983402
    0.036723        0.511060        9.483871        0.036723        0.507925
            9.483871        10756   39306   0.623457        0.740706
    0.983240        0.065463        0.370161        3.965986        0.065463
            0.444270        3.965986        7588    42420   0.271233
    0.414222        0.710267        0.043339        0.496994        8.843750
            0.043339        0.480439        8.843750        7790    42138
    0.200695        0.414515        0.983402        0.044586        0.496832
            9.483871        0.044586        0.480369        9.483871
    5138    44864   0.147396        0.224292        0.270960        0.036723
            0.661167        8.250000        0.036723        0.573792
    8.250000
0.3     31438   168562  0.183603        0.494386        0.900369
    0.036212        0.508618        9.129032        0.036212        0.504349
            9.129032        10622   39316   0.486352        0.740696
    0.900369        0.065022        0.366612        2.909091        0.065022
            0.478837        2.909091        7954    42388   0.269283
    0.414584        0.588840        0.044262        0.494289        8.875000
            0.044262        0.470378        8.875000        7824    41994
    0.269179        0.414899        0.571843        0.044335        0.493158
            9.129032        0.044335        0.469680        9.129032
    5038    44864   0.183603        0.224510        0.272040        0.036212
            0.661073        8.208333        0.036212        0.530104
    8.208333
0.4     31522   168478  0.181193        0.489008        0.990842
    0.036620        0.511956        8.625000        0.036620        0.502777
            8.625000        10394   39340   0.636656        0.741145
    0.990842        0.064302        0.370763        2.950355        0.064302
            0.518916        2.950355        7792    42080   0.273826
    0.414141        0.557522        0.044094        0.495137        8.625000
            0.044094        0.462738        8.625000        7840    41792
    0.270862        0.414862        0.573529        0.044728        0.499492
            8.375000        0.044728        0.465640        8.375000
    5496    45266   0.181193        0.224080        0.284326        0.036620
            0.661806        8.250000        0.036620        0.486716
    8.250000
```

```
0.5     31346   168654  0.181499        0.493568        0.877586
    0.036517        0.510896        8.935484        0.036517        0.502232
            8.935484        10622   39620   0.644951        0.740633
    0.877586        0.065611        0.367311        3.083916        0.065611
            0.553972        3.083916        7790    42516   0.259119
    0.414673        0.596154        0.044850        0.494958        8.593750
            0.044850        0.454816        8.593750        7734    41772
    0.270525        0.414724        0.574780        0.042254        0.496662
            8.935484        0.042254        0.455693        8.935484
    5200    44746   0.181499        0.224347        0.276904        0.036517
            0.666462        8.125000        0.036517        0.445404
    8.125000
0.6     31646   168354  0.175000        0.492506        1.241509
    0.035912        0.512378        8.967742        0.035912        0.500454
            8.967742        10648   39044   0.648734        0.740433
    1.241509        0.064877        0.368633        3.048951        0.064877
            0.591713        3.048951        7984    42024   0.271003
    0.414160        0.570162        0.044335        0.496820        8.687500
            0.044335        0.447224        8.687500        7758    42236
    0.270380        0.414504        0.573295        0.044586        0.497677
            8.967742        0.044586        0.447773        8.967742
    5256    45050   0.175000        0.224377        0.484222        0.035912
            0.665253        7.920000        0.035912        0.400728
    7.920000
0.7     31480   168520  0.181435        0.493408        0.869811
    0.036466        0.512449        9.387097        0.036466        0.499120
            9.387097        10576   39314   0.573913        0.740847
    0.869811        0.062635        0.368561        3.277027        0.062635
            0.629161        3.277027        7922    42110   0.269333
    0.414724        0.638235        0.043760        0.500615        8.843750
            0.043760        0.440491        8.843750        7878    42606
    0.273333        0.414704        0.683824        0.045677        0.498194
            9.387097        0.045677        0.439751        9.387097
    5104    44490   0.181435        0.224295        0.283272        0.036466
            0.664448        8.416667        0.036466        0.356341
    8.416667
0.8     30898   169102  0.182030        0.491303        0.921811
    0.035665        0.510769        9.281250        0.035665        0.495196
            9.281250        10284   39208   0.574230        0.740270
    0.858456        0.065169        0.366940        2.969388        0.065169
            0.665604        2.969388        7622    42314   0.267568
    0.414434        0.921811        0.043956        0.501325        8.741935
            0.043956        0.431812        8.741935        7856    42528
    0.263091        0.414506        0.831004        0.043548        0.490800
            9.281250        0.043548        0.429765        9.281250
    5136    45052   0.182030        0.224334        0.266350        0.035665
            0.663662        8.250000        0.035665        0.312199
    8.250000
0.9     31818   168182  0.184272        0.493482        1.134441
    0.035714        0.510785        9.064516        0.035714        0.495213
            9.064516        10804   39136   0.651540        0.740489
    1.134441        0.064877        0.369992        2.979167        0.064877
            0.703439        2.979167        7768    41786   0.267473
    0.414426        0.558174        0.044335        0.497496        9.064516
            0.044335        0.422733        9.064516        7922    42308
    0.271871        0.414667        0.612069        0.042857        0.494389
            9.064516        0.042857        0.422640        9.064516
    5324    44952   0.184272        0.224853        0.286571        0.035714
            0.661145        8.291667        0.035714        0.268482
    8.291667
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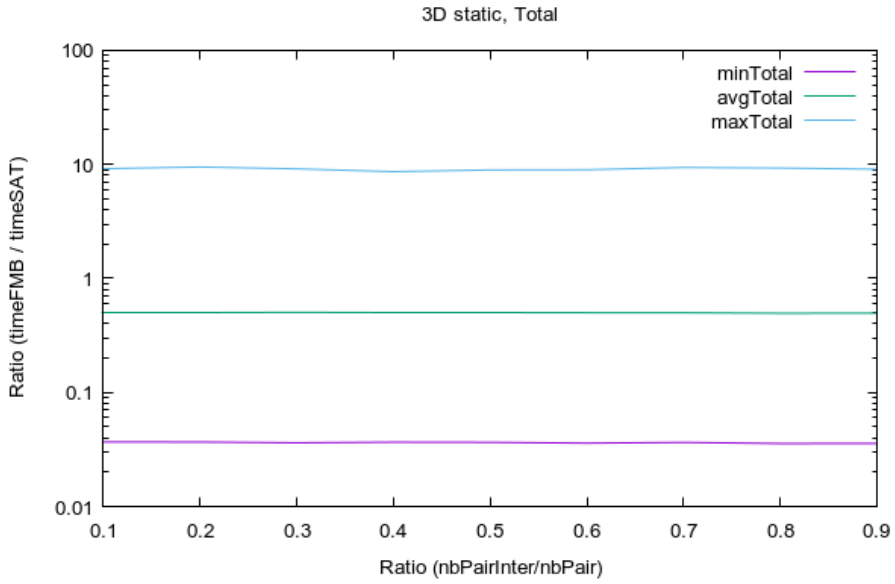
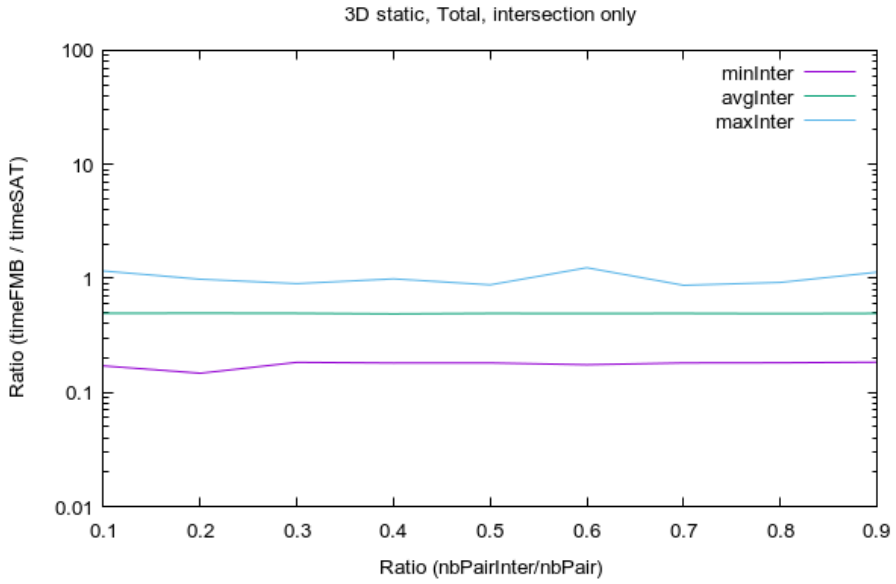
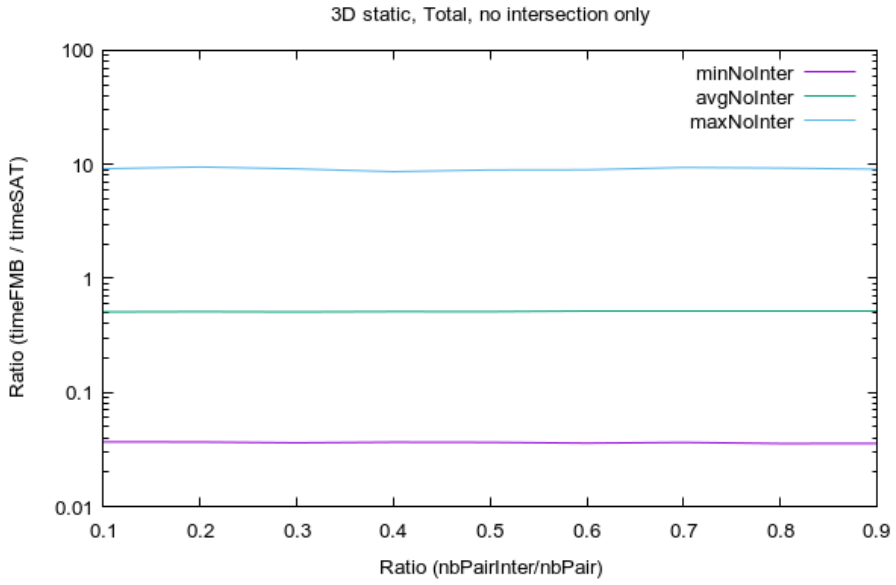
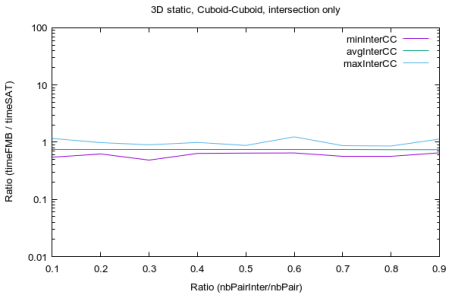
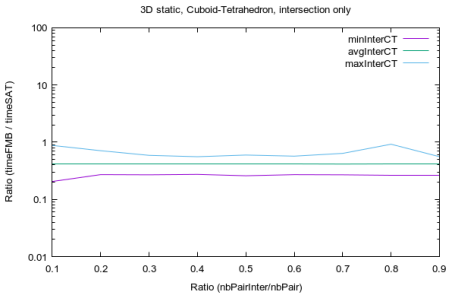
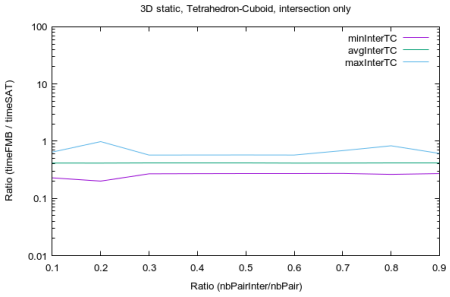

3D static, Total
minTotal
avgTotal
maxTotal
Ratio (timeFMB / timeSAT)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Ratio (nbPairInter/nbPair)

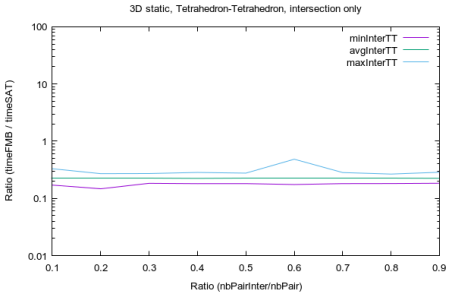

3D static, Total, intersection only
minInter
avgInter
maxInter
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

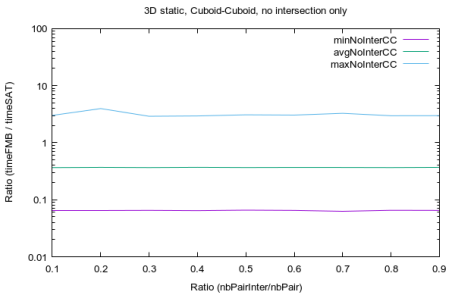

3D static, Total, no intersection only
minNoInter
avgNoInter
maxNoInter
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

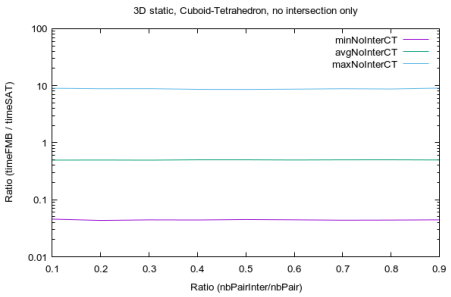


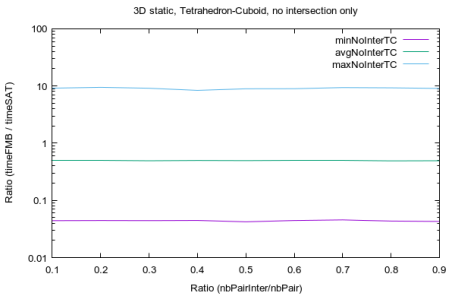


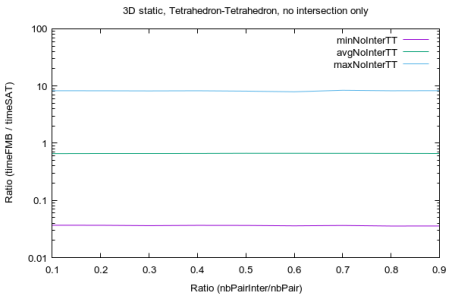












8.2.3 2D dynamic

```
percPairInter   countInter      countNoInter    minInter        avgInter
            maxInter        minNoInter      avgNoInter      maxNoInter
    minTotal        avgTotal        maxTotal        countInterCC
    countNoInterCC  minInterCC      avgInterCC      maxInterCC
    minNoInterCC    avgNoInterCC    maxNoInterCC    minTotalCC
    avgTotalCC      maxTotalCC      countInterCT    countNoInterCT
    minInterCT      avgInterCT      maxInterCT      minNoInterCT
    avgNoInterCT    maxNoInterCT    minTotalCT      avgTotalCT
    maxTotalCT      countInterTC    countNoInterTC  minInterTC
    avgInterTC      maxInterTC      minNoInterTC    avgNoInterTC
    maxNoInterTC    minTotalTC      avgTotalTC      maxTotalTC
    countInterTT    countNoInterTT  minInterTT      avgInterTT
    maxInterTT      minNoInterTT    avgNoInterTT    maxNoInterTT
    minTotalTT      avgTotalTT      maxTotalTT
0.1     74540   125456  1.072581        2.016891        4.849624
    0.140187        1.113797        17.115385       0.140187        1.204107
            17.115385       19906   29830   1.454237        2.539910
    4.849624        0.166667        1.072354        14.172414       0.166667
            1.219109        14.172414       18888   31332   1.250883
    1.974600        3.676259        0.140187        1.114282
    13.583333       0.140187        1.200314        13.583333       18610
    31518   1.072581        1.973950        3.461538        0.146789
    1.120753        17.115385       0.146789        1.206073
    17.115385       17136   32776   1.111111        1.502579        2.439024
            0.161290        1.144364        10.521739       0.161290
    1.180185        10.521739
0.2     74066   125930  1.194656        2.017957        2.954198
    0.125000        1.126331        13.652174       0.125000        1.304657
            13.652174       19986   30416   1.723077        2.539556
    2.954198        0.170455        1.077619        12.740741       0.170455
            1.370006        12.740741       18430   31182   1.360825
    1.974826        2.571429        0.125000        1.137328
    13.652174       0.125000        1.304827        13.652174       18590
    31314   1.353535        1.974034        2.699301        0.127119
    1.134871        13.500000       0.127119        1.302704
    13.500000       17060   33018   1.194656        1.501356        1.833333
            0.161616        1.152721        10.291667       0.161616
    1.222448        10.291667
0.3     74530   125464  1.115207        2.014464        4.582090
    0.142857        1.114772        13.625000       0.142857        1.384680
            13.625000       19818   30264   1.616505        2.539830
    4.582090        0.170213        1.058833        13.137931       0.170213
            1.503132        13.137931       18506   31020   1.215116
    1.975186        3.128571        0.144144        1.143663
    13.625000       0.144144        1.393120        13.625000       18780
    31510   1.223256        1.974334        3.394161        0.146789
    1.103604        13.615385       0.146789        1.364823
    13.615385       17426   32670   1.115207        1.501943        1.913043
            0.142857        1.149931        10.521739       0.142857
    1.255535        10.521739
0.4     74532   125462  1.078704        2.013686        4.640845
    0.119048        1.121286        14.560000       0.119048        1.478246
            14.560000       19744   30016   1.484848        2.540887
    3.740458        0.119048        1.090234        12.750000       0.119048
            1.670495        12.750000       18362   31348   1.166667
    1.975340        4.640845        0.146789        1.129813
    13.416667       0.146789        1.468024        13.416667       18910
    31358   1.141631        1.974553        2.652174        0.124031
    1.115450        14.560000       0.124031        1.459091
    14.560000       17516   32740   1.078704        1.501870        2.297468
            0.157895        1.147179        10.291667       0.157895
    1.289055        10.291667
```

```
0.5     74572   125416  0.959698        2.014620        3.803030
    0.145455        1.122265        15.806452       0.145455        1.568442
            15.806452       19986   30064   1.860335        2.539512
    3.803030        0.170000        1.050379        15.806452       0.170000
            1.794945        15.806452       18488   31280   1.104603
    1.974474        2.950355        0.145455        1.142531
    13.333333       0.145455        1.558502        13.333333       18624
    31754   1.247619        1.973859        3.333333        0.145455
    1.134469        14.200000       0.145455        1.554164
    14.200000       17474   32318   0.959698        1.500189        2.125786
            0.153846        1.157532        10.346154       0.153846
    1.328861        10.346154
0.6     74242   125748  1.227488        2.015532        2.931973
    0.130435        1.126601        14.200000       0.130435        1.659959
            14.200000       19898   30308   1.964912        2.539975
    2.931973        0.183908        1.077178        12.821429       0.183908
            1.954857        12.821429       18712   31802   1.371134
    1.974901        2.671533        0.130435        1.141280
    14.200000       0.130435        1.641453        14.200000       18334
    30836   1.316832        1.973841        2.649635        0.140496
    1.144666        13.250000       0.140496        1.642171
    13.250000       17298   32802   1.227488        1.500401        1.778481
            0.142857        1.141052        11.200000       0.142857
    1.356662        11.200000
0.7     74212   125778  1.114679        2.016154        3.326087
    0.121739        1.118372        13.375000       0.121739        1.746820
            13.375000       19812   30132   1.515748        2.540043
    3.137405        0.133803        1.053011        12.892857       0.133803
            2.093934        12.892857       18430   31462   1.231660
    1.975607        3.326087        0.125000        1.128547
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            0.160000        1.163462        10.521739       0.160000
    1.400048        10.521739
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    0.121429        1.113242        13.416667       0.121429        1.837631
            13.416667       20182   30158   1.883333        2.540169
    4.259542        0.152174        1.059715        13.259259       0.152174
            2.244078        13.259259       18494   31582   1.274882
    1.975098        3.635036        0.121429        1.112056
    13.375000       0.121429        1.802490        13.375000       18580
    31308   0.900000        1.973406        2.716312        0.146789
    1.127207        13.416667       0.146789        1.804167
    13.416667       17208   32484   1.154589        1.502994        2.527363
            0.159574        1.150630        12.578947       0.159574
    1.432521        12.578947
0.9     74412   125578  0.977860        2.014325        4.353383
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            14.500000       19812   29902   1.283019        2.539859
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    1.974208        3.905109        0.144144        1.137957
    14.500000       0.144144        1.890583        14.500000       18620
    31198   1.275304        1.973773        2.627737        0.135135
    1.131695        13.640000       0.135135        1.889565
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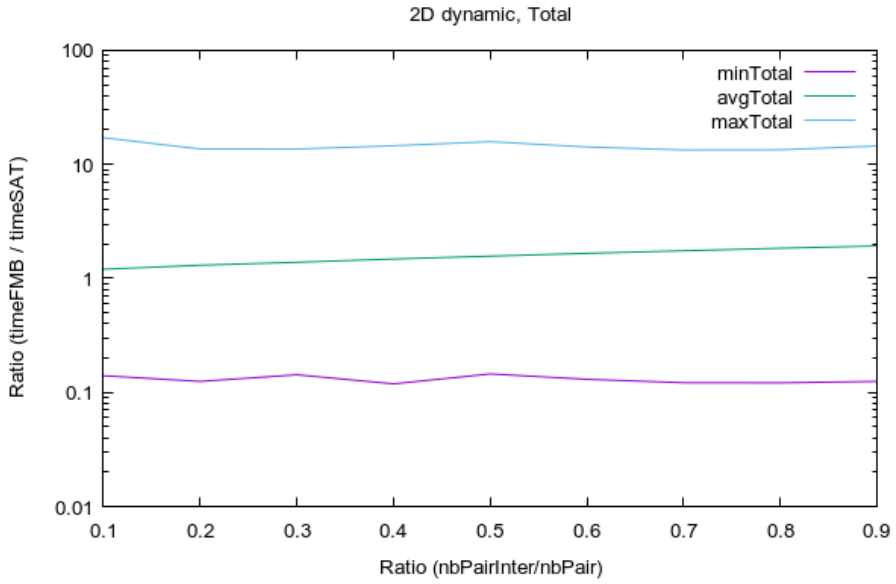
2D dynamic, Total
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avgTotal
maxTotal
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
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0.2
0.3
0.4
0.5
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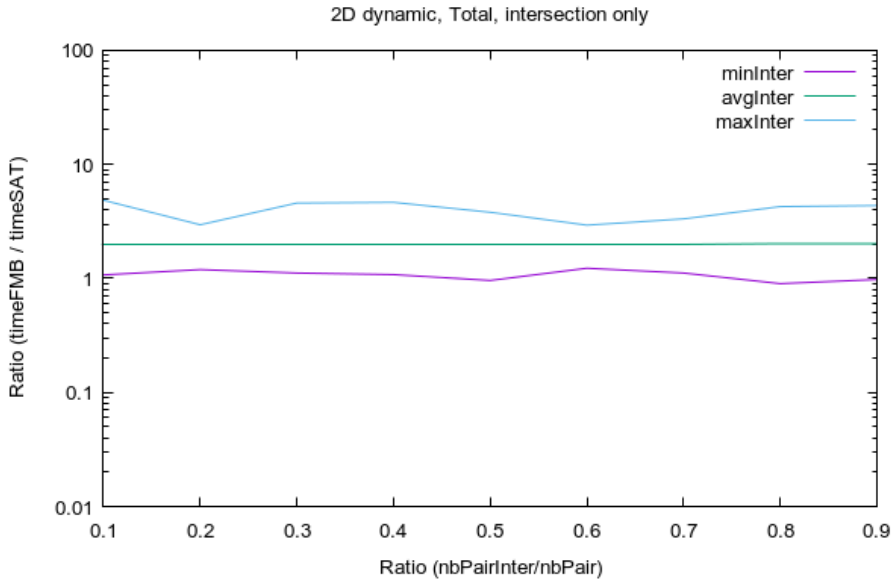
0.7
0.8
0.9


2D dynamic, Total, intersection only
minInter
avgInter
maxInter
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
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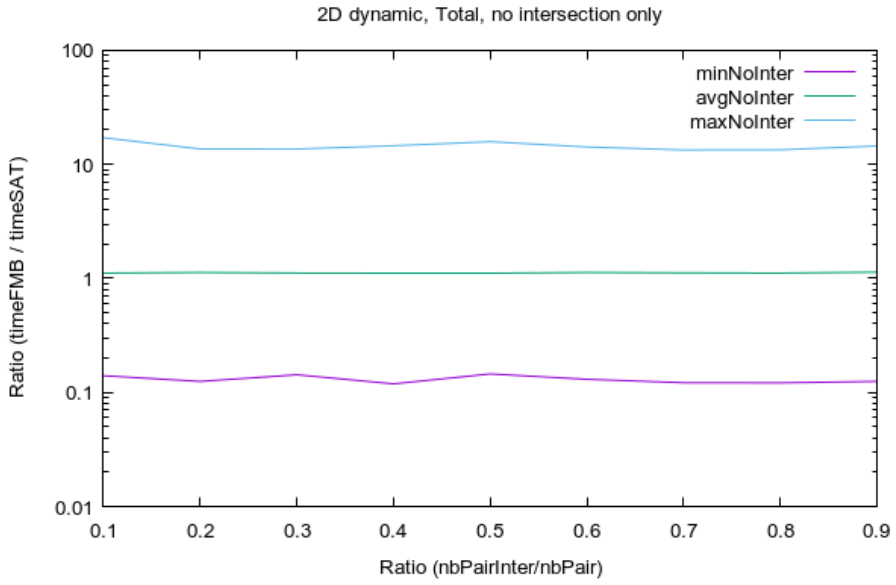
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minNoInter
avgNoInter
maxNoInter
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
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0.8
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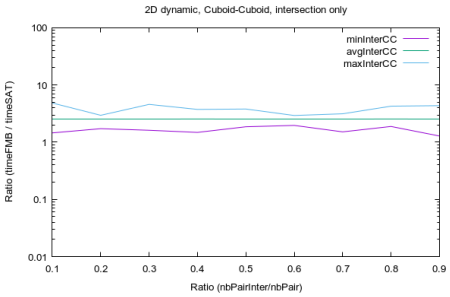

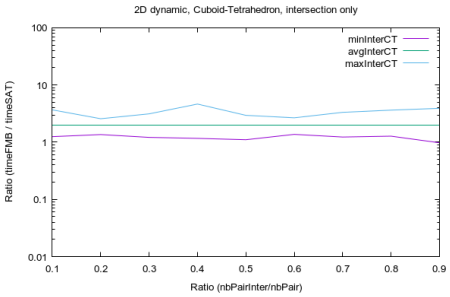

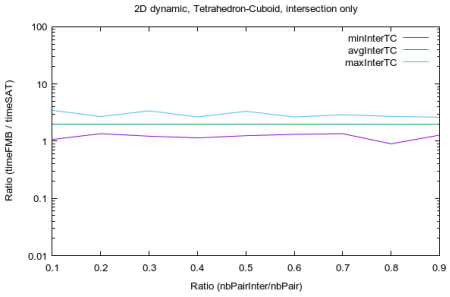

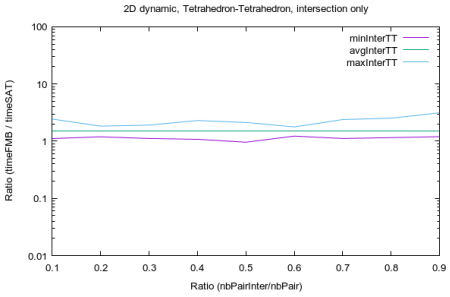

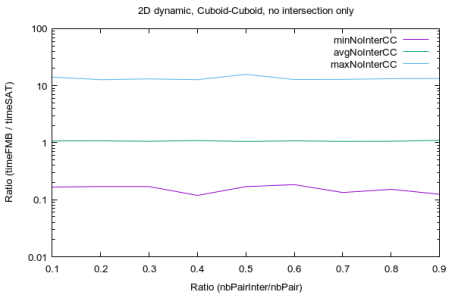

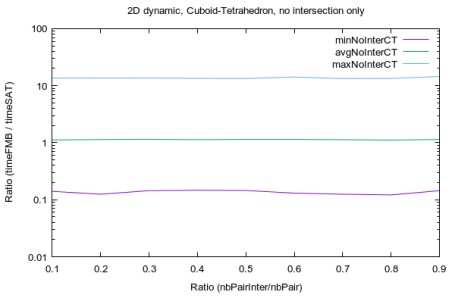

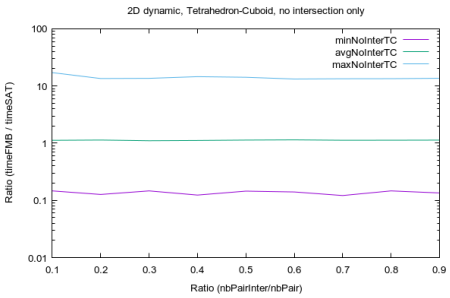

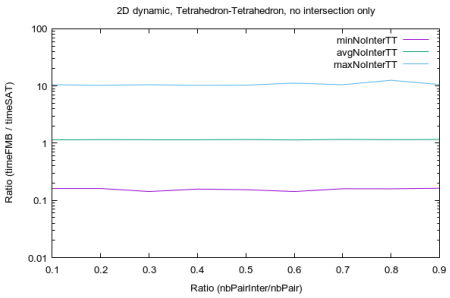

8.2.4 3D dynamic

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    countNoInterCC  minInterCC      avgInterCC      maxInterCC
    minNoInterCC    avgNoInterCC    maxNoInterCC    minTotalCC
    avgTotalCC      maxTotalCC      countInterCT    countNoInterCT
    minInterCT      avgInterCT      maxInterCT      minNoInterCT
    avgNoInterCT    maxNoInterCT    minTotalCT      avgTotalCT
    maxTotalCT      countInterTC    countNoInterTC  minInterTC
    avgInterTC      maxInterTC      minNoInterTC    avgNoInterTC
    maxNoInterTC    minTotalTC      avgTotalTC      maxTotalTC
    countInterTT    countNoInterTT  minInterTT      avgInterTT
    maxInterTT      minNoInterTT    avgNoInterTT    maxNoInterTT
    minTotalTT      avgTotalTT      maxTotalTT
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    0.024929        0.729255        28.428571       0.024929        0.816375
            28.428571       15776   34120   1.538105        2.615324
    3.265092        0.039615        0.630705        13.700893       0.039615
            0.829167        13.700893       13168   36682   0.466667
    1.393738        2.559970        0.025566        0.728236
    22.836735       0.025566        0.794786        22.836735       12996
    37414   0.466285        1.391845        2.476120        0.027888
    0.728085        22.836735       0.027888        0.794461
    22.836735       10008   39836   0.288525        0.543547        0.948420
            0.024929        0.815702        28.428571       0.024929
    0.788487        28.428571
0.2     52660   147340  0.292249        1.602534        4.039062
    0.025754        0.726991        29.342857       0.025754        0.902099
            29.342857       16088   34022   1.617647        2.615509
    4.039062        0.039958        0.647699        13.535398       0.039958
            1.041261        13.535398       13294   36748   0.457932
    1.395088        2.474041        0.026237        0.723097
    23.625000       0.026237        0.857495        23.625000       13094
    36700   0.470976        1.391214        3.104478        0.028998
    0.721707        22.720000       0.028998        0.855608
    22.720000       10184   39870   0.292249        0.544803        0.742556
            0.025754        0.803105        29.342857       0.025754
    0.751444        29.342857
0.3     52318   147682  0.292898        1.601704        3.961538
    0.025679        0.730448        30.176471       0.025679        0.991825
            30.176471       15946   33640   0.948944        2.614666
    3.961538        0.039054        0.645531        13.513158       0.039054
            1.236272        13.513158       13232   37072   0.466318
    1.391362        2.467814        0.028235        0.719220
    20.840000       0.028235        0.920863        20.840000       13050
    37026   0.456954        1.394344        2.605748        0.026616
    0.722152        22.800000       0.026616        0.923809
    22.800000       10090   39944   0.292898        0.544875        0.741851
            0.025679        0.820075        30.176471       0.025679
    0.737515        30.176471
0.4     52344   147656  0.288499        1.597924        3.397775
    0.025436        0.725079        30.828571       0.025436        1.074217
            30.828571       15988   33996   2.109453        2.614919
    3.397775        0.039236        0.622718        14.696970       0.039236
            1.419598        14.696970       13054   37214   0.469599
    1.391394        2.555973        0.026616        0.715950
    22.795918       0.026616        0.986128        22.795918       12960
    36512   0.464771        1.391468        2.563286        0.026718
    0.723789        22.019608       0.026718        0.990861
    22.019608       10342   39934   0.288499        0.545130        0.746852
            0.025436        0.821905        30.828571       0.025436
    0.711195        30.828571
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0.5     52456   147544  0.285071        1.602763        3.194542
    0.025660        0.732794        30.514286       0.025660        1.167778
            30.514286       15942   34190   1.858919        2.615660
    3.194542        0.040000        0.634864        13.819820       0.040000
            1.625262        13.819820       13284   36524   0.467855
    1.395456        2.505319        0.026257        0.740025
    23.833333       0.026257        1.067740        23.833333       13146
    36496   0.469677        1.395234        2.443858        0.026296
    0.722588        23.489796       0.026296        1.058911
    23.489796       10084   40334   0.285071        0.545090        0.762179
            0.025660        0.818494        30.514286       0.025660
    0.681792        30.514286
0.6     52558   147442  0.284326        1.606739        3.268930
    0.025907        0.721223        28.888889       0.025907        1.252533
            28.888889       16210   34030   1.690750        2.615304
    3.268930        0.039236        0.644433        13.619469       0.039236
            1.826956        13.619469       13136   36914   0.467658
    1.391212        2.713663        0.026100        0.695584
    24.021277       0.026100        1.112961        24.021277       13142
    37184   0.468952        1.392790        2.519490        0.026355
    0.723739        23.204082       0.026355        1.125170
    23.204082       10070   39314   0.284326        0.543587        0.801034
            0.025907        0.809386        28.888889       0.025907
    0.649906        28.888889
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    0.026012        0.717967        29.416667       0.026012        1.334245
            29.416667       16050   34004   1.686330        2.614795
    3.779299        0.037376        0.611985        13.684685       0.037376
            2.013952        13.684685       13344   36730   0.468731
    1.392435        2.832593        0.026100        0.722128
    23.708333       0.026100        1.191343        23.708333       12912
    36750   0.421024        1.391122        2.541791        0.026012
    0.704014        21.137255       0.026012        1.184989
    21.137255       10366   39844   0.288889        0.547828        0.957672
            0.026178        0.817449        29.416667       0.026178
    0.628714        29.416667
0.8     52274   147726  0.282595        1.599488        3.557336
    0.023985        0.723269        29.028571       0.023985        1.424244
            29.028571       15900   33794   1.881828        2.615188
    3.557336        0.039749        0.627566        13.558036       0.039749
            2.217663        13.558036       12964   37014   0.471279
    1.394053        2.649926        0.026799        0.723188
    23.125000       0.026799        1.259880        23.125000       13178
    37040   0.467126        1.393887        2.482890        0.027301
    0.714758        23.770833       0.027301        1.258061
    23.770833       10232   39878   0.282595        0.546228        0.933824
            0.023985        0.812353        29.028571       0.023985
    0.599453        29.028571
0.9     52830   147170  0.285622        1.600873        3.447076
    0.025473        0.730697        30.529412       0.025473        1.513855
            30.529412       16092   33588   2.127107        2.614492
    2.766014        0.038895        0.661782        13.606195       0.038895
            2.419221        13.606195       13448   36772   0.463859
    1.391804        3.447076        0.025622        0.704062
    22.840000       0.025622        1.323030        22.840000       13080
    36930   0.466881        1.391964        2.458709        0.026336
    0.715223        23.163265       0.026336        1.324290
    23.163265       10210   39880   0.285622        0.546310        0.735691
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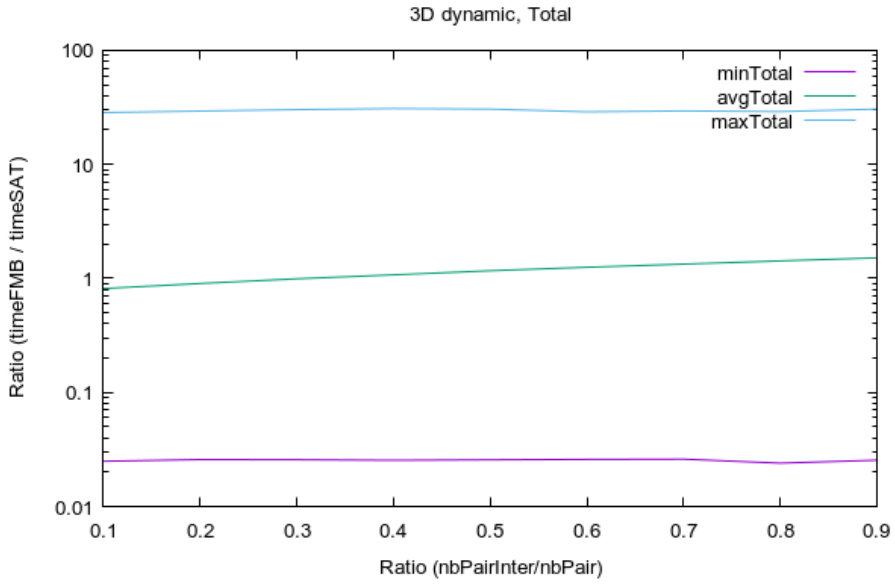
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avgTotal
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Ratio (nbPairInter/nbPair)
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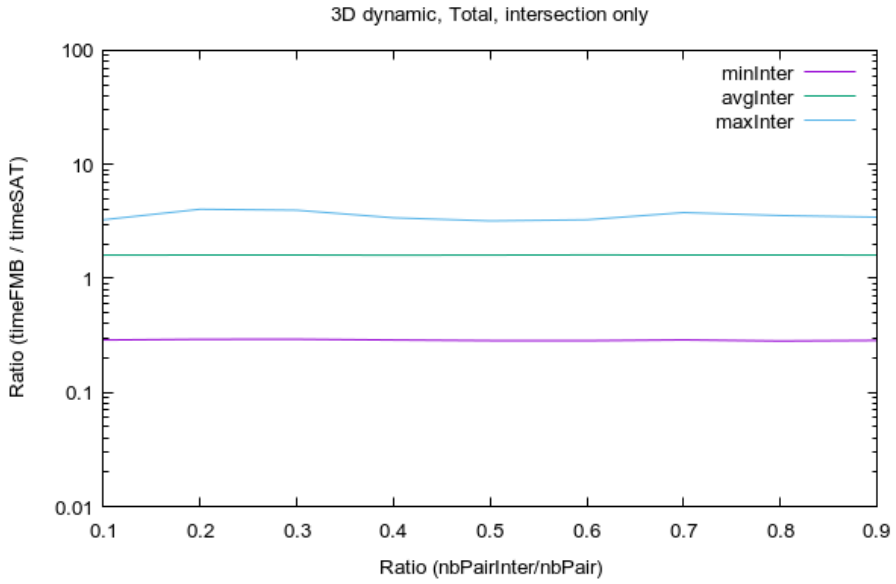
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Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
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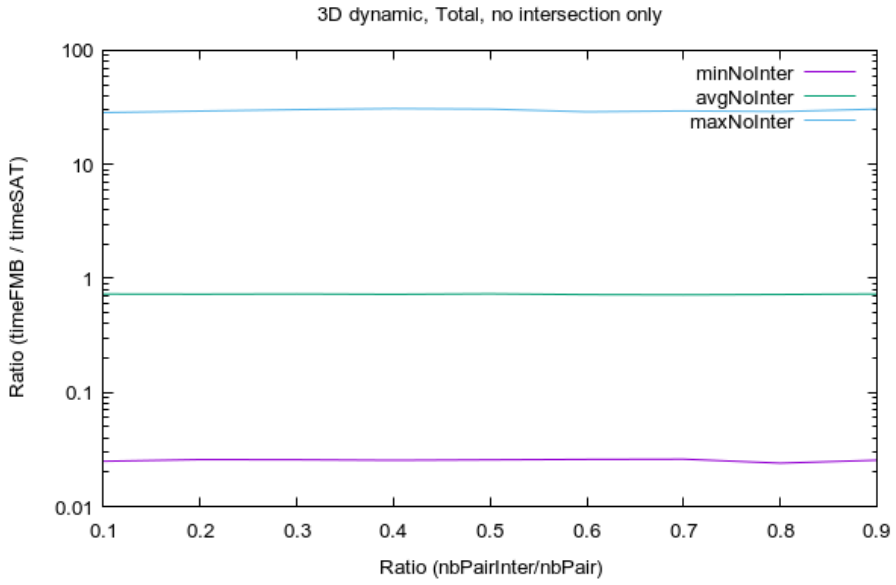
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avgNoInter
maxNoInter
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Ratio (nbPairInter/nbPair)
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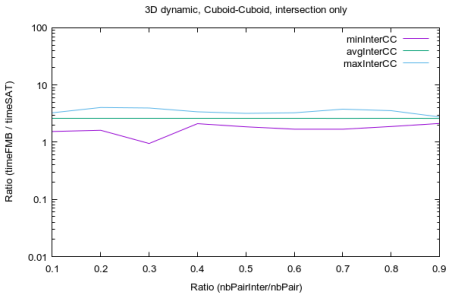
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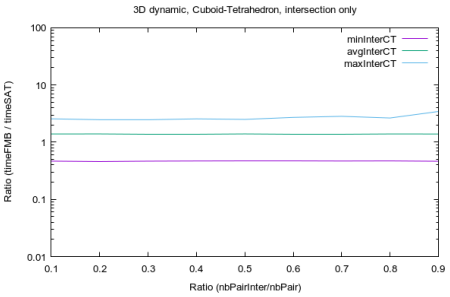
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)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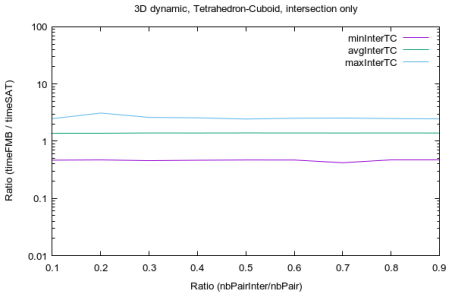
Ratio (nbPairInter/nbPair)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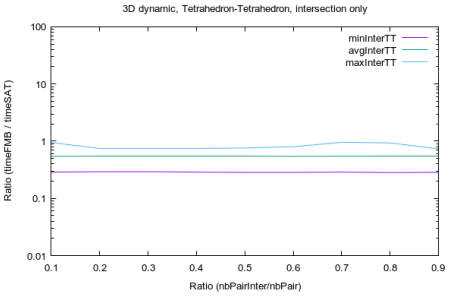
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)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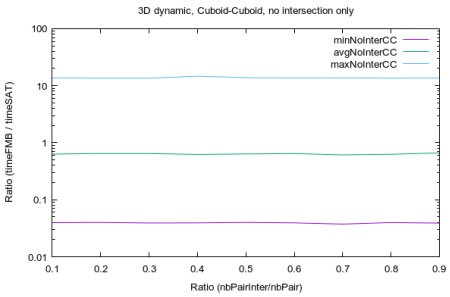
maxInterTT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)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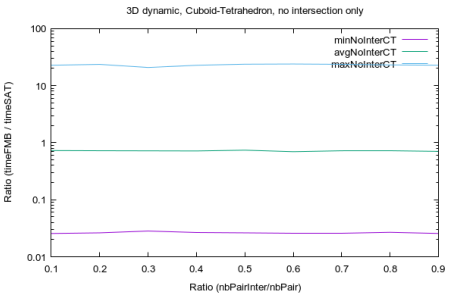
avgNoInterCC
maxNoInterCC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)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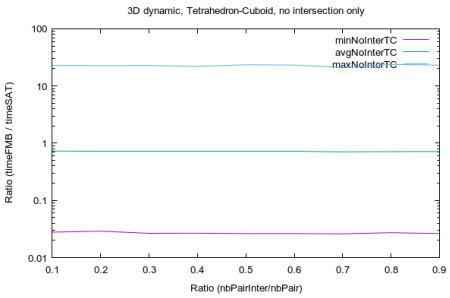
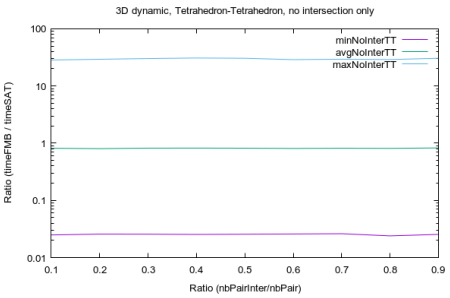
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)


3D dynamic, Tetrahedron-Cuboid, no intersection only
minNoInterTC
avgNoInterTC
maxNoInterTC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)


3D dynamic, Tetrahedron-Tetrahedron, no intersection only
minNoInterTT
avgNoInterTT
maxNoInterTT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)

9 Conclusion

The validation proves that the FMB algorithm correctly identifies intersection of pairs of Frames in accordance with the results of the SAT algorithm.

The qualification shows that the FMB is 1.2 to 1.8 times slower than the SAT algorithm in the 2D dynamic case. However it is around 2 times faster in the 3D static case, and up to 1.25 times faster in 3D dynamic and up to 1.1 times faster in the 2D static case if the percentage of tested pairs in intersection is less than, respectively, around 40% and 25%.

On one given pair of Frame, the relative speed of the FMB algorithm varies widely, from around 20 times slower to 50 times faster. This is explained by the way the 2 algorithms works: they both make the asumption that the Frames are intersecting and run through a series of tests to try to prove it wrong. This leads to best cases and worst cases for both algorithm: a non interesecting detected right from the first test, or one detected by the last test. These best and worst cases are different for the two algorithm as the tests they performed are completely different. But globally, the FMB algorithm has the advantage.

10 Annex

10.1 Runtime environment

Results introduce in this paper have been produced by compiling and running the corresponding algorithms in the following environment:

```
    > uname -v 40 18.04.1-Ubuntu SMP Thu Nov 14 12:06:39 UTC 2019
    > lshw -short H/W path Device Class Description ====================================================
system VC65-C1 /0 bus VC65-C1 /0/0 memory 64KiB BIOS /0/2f memory 16GiB System Memory /0/2f/0
memory [empty] /0/2f/1 memory 16GiB SODIMM DDR4 Synchronous 2400 MHz (0.4 ns) /0/39 memory 384KiB
L1 cache /0/3a memory 1536KiB L2 cache /0/3b memory 12MiB L3 cache /0/3c processor Intel(R) Core(TM)
i7-8700T CPU @ 2.40GHz /0/100 bridge 8th Gen Core Processor Host Bridge/DRAM Registers /0/100/2
display Intel Corporation /0/100/12 generic Cannon Lake PCH Thermal Controller /0/100/14 bus
Cannon Lake PCH USB 3.1 xHCI Host Controller /0/100/14/0 usb1 bus xHCI Host Controller /0/100/14/0/5
input ELECOM Wired Keyboard /0/100/14/0/6 input PTZ-630 /0/100/14/0/7 generic USB2.0-CRW /0/100/14/0/e
communication Bluetooth wireless interface /0/100/14/1 usb2 bus xHCI Host Controller /0/100/14.2
memory RAM memory /0/100/14.3 wlo1 network Wireless-AC 9560 [Jefferson Peak] /0/100/16 communication
Cannon Lake PCH HECI Controller /0/100/17 storage Cannon Lake PCH SATA AHCI Controller /0/100/1f
bridge Intel Corporation /0/100/1f.3 multimedia Cannon Lake PCH cAVS /0/100/1f.4 bus Cannon Lake
PCH SMBus Controller /0/100/1f.5 bus Cannon Lake PCH SPI Controller /0/100/1f.6 eno2 network
Ethernet Connection (7) I219-V /0/1 scsi0 storage /0/1/0.0.0 /dev/sda disk 128GB HFS128G39TND-N21
/0/1/0.0.0/1 volume 99MiB Windows FAT volume /0/1/0.0.0/2 /dev/sda2 volume 15MiB reserved partition
/0/1/0.0.0/3 /dev/sda3 volume 83GiB Windows NTFS volume /0/1/0.0.0/4 /dev/sda4 volume 499MiB
Windows NTFS volume /0/1/0.0.0/5 /dev/sda5 volume 35GiB EXT4 volume /0/2 scsi2 storage /0/2/0.0.0
```

/dev/sdb disk 500GB ST500LM034-2GH17 /0/2/0.0.0/1 /dev/sdb1 volume 463GiB EXT4 volume /0/2/0.0.0/2 /dev/sdb2 volume 499MiB Windows FAT volume /0/3 scsi5 storage /0/3/0.0.0 /dev/cdrom disk BD-RE BU50N /1 power To Be Filled By O.E.M.

> lscpu Architecture: $x86_64 CPU op-mode(s) : 32-bit, 64-bit Byte Order : Little Endian CPU(s) : 12 On-line CPU(s) list : 0-11 Thread(s) per core : 2 Core(s) per socket : 6 Socket(s) : 1 NUMA node(s) : 1 Vendor ID : Genuine Intel CPU family : 6 Model : 158 Model name : Intel(R) Core(TM) i7-8700 CPU @ 2.40 GHz Stepping : 10 CPU MHz : 1380.998 CPU max MHz : 4000.0000 CPU min MHz : 800.0000 Bogo MIPS : 4800.00 Virtualization : VT-x L1d cache : 32K L1i cache : 32K L2 cache : 256K L3 cache : 12288K NUMA node0 CPU(s) : 0-11 Flags : fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge mca cmov pat pse36 clflush dts acpi mmx fxsr sse sse2 ss ht tm pbe syscall nx pdpe1$

> gcc -v Using built-in specs. $COLLECT_GCC = gcc COLLECT_LTO_WRAPPER = /usr/lib/gcc/x86_64-linux-gnu/7/lto-wrapper OFFLOAD_TARGET_NAMES = nvptx-none OFFLOAD_TARGET_DEFAULT = 1 Target : x86_64-linux-gnu Configured with : ../src/configure -v --with-pkgversion='Ubuntu 7.4.0-1ubuntu1 18.04.1' --with-bugurl=file:///usr/share/doc/gcc-7/README.Bugs --enable-languages=c,ada,c++,go,brig,d,fortran,objc,obj-c++ --prefix=/usr --with-gcc-major-version-only --program-suffix=-7 --program-prefix=x86_64-linux-gnu- --enable-shared --enable-linker-build-id --libexecdir=/usr/lib --without-included-gettext --enable-threads=posix --libdir=/usr/lib --enable-nls --with-sysroot=/ --enable-clocale=gnu --enable-libstdcxx-debug --enable-libstdcxx-time=yes --with-default-libstdcxx-abi=new --enable-gnu-unique-object --disable-vtable-verify --enable-libmpx --enable-plugin --enable-default-pie --with-system-zlib --with-target-system-zlib --enable-objc-gc=auto --enable-multiarch --disable-werror --with-arch-32=i686 --with-abi=m64 --with-multilib-list=m32,m64,mx32 --enable-multilib --with-tune=generic --enable-offload-targets=nvptx-none --without-cuda-driver --enable-checking=release --build=x86_64-linux-gnu --host=x86_64-linux-gnu --target=x86_64-linux-gnu Thread model : posix gcc version 7.4.0 (Ubuntu 7.4.0-1ubuntu1 18.04.1)$

10.2 SAT implementation

In this section I introduce the code of the implementation of the SAT algorithm, used to validate and qualify the FMB algorithm.

10.2.1 Header

```
#ifndef __SAT_H_
#define __SAT_H_

#include <stdbool.h>
#include <string.h>
#include "frame.h"

// ------------- Functions declaration -------------

// Test for intersection between 2D Frame 'that' and 2D Frame 'tho'
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection2D(
  const Frame2D* const that,
  const Frame2D* const tho);
```

```
// Test for intersection between moving 2D Frame 'that' and 2D
// Frame 'tho'
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection2DTime(
  const Frame2DTime* const that,
  const Frame2DTime* const tho);

// Test for intersection between 3D Frame 'that' and 3D Frame 'tho'
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection3D(
  const Frame3D* const that,
  const Frame3D* const tho);

// Test for intersection between moving 3D Frame 'that' and 3D
// Frame 'tho'
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection3DTime(
  const Frame3DTime* const that,
  const Frame3DTime* const tho);

#endif
```

10.2.2 Body

```
#include "sat.h"

// ------------- Macros -------------

#define EPSILON 0.0000001

// ------------- Functions declaration -------------

// Check the intersection constraint along one axis
bool CheckAxis3D(
  const Frame3D* const that,
  const Frame3D* const tho,
  const double* const axis);

// Check the intersection constraint along one axis
bool CheckAxis3DTime(
  const Frame3DTime* const that,
  const Frame3DTime* const tho,
  const double* const axis,
  const double* const relSpeed);

// ------------- Functions implementation -------------

// Test for intersection between 2D Frame 'that' and 2D Frame 'tho'
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection2D(
  const Frame2D* const that,
  const Frame2D* const tho) {

  // Declare a variable to loop on Frames and commonalize code
  const Frame2D* frameEdge = that;

  // Loop to commonalize code when checking SAT based on that's edges
  // and then tho's edges
  for (int iFrame = 2;
       iFrame--;) {
```

```
// Shortcuts
FrameType frameEdgeType = frameEdge->type;
const double* frameEdgeCompA = frameEdge->comp[0];
const double* frameEdgeCompB = frameEdge->comp[1];

// Declare a variable to memorize the number of edges, by default 2
int nbEdges = 2;

// Declare a variable to memorize the third edge in case of
// tetrahedron
double thirdEdge[2];

// If the frame is a tetrahedron
if (frameEdgeType == FrameTetrahedron) {

  // Initialise the third edge
  thirdEdge[0] = frameEdgeCompB[0] - frameEdgeCompA[0];
  thirdEdge[1] = frameEdgeCompB[1] - frameEdgeCompA[1];

  // Correct the number of edges
  nbEdges = 3;

}

// Loop on the frame's edges
for (int iEdge = nbEdges;
     iEdge--;) {

  // Get the current edge
  const double* edge =
    (iEdge == 2 ? thirdEdge : frameEdge->comp[iEdge]);

  // Declare variables to memorize the boundaries of projection
  // of the two frames on the current edge
  double bdgBoxA[2];
  double bdgBoxB[2];

  // Declare two variables to loop on Frames and commonalize code
  const Frame2D* frame = that;
  double* bdgBox = bdgBoxA;

  // Loop on Frames
  for (int iFrame = 2;
       iFrame--;) {

    // Shortcuts
    const double* frameOrig = frame->orig;
    const double* frameCompA = frame->comp[0];
    const double* frameCompB = frame->comp[1];
    FrameType frameType = frame->type;

    // Get the number of vertices of frame
    int nbVertices = (frameType == FrameTetrahedron ? 3 : 4);

    // Declare a variable to memorize if the current vertex is
    // the first in the loop, used to initialize the boundaries
    bool firstVertex = true;

    // Loop on vertices of the frame
    for (int iVertex = nbVertices;
         iVertex--;) {
```

```
    // Get the vertex
    double vertex[2];
    vertex[0] = frameOrig[0];
    vertex[1] = frameOrig[1];
    switch (iVertex) {
      case 3:
        vertex[0] += frameCompA[0] + frameCompB[0];
        vertex[1] += frameCompA[1] + frameCompB[1];
        break;
      case 2:
        vertex[0] += frameCompA[0];
        vertex[1] += frameCompA[1];
        break;
      case 1:
        vertex[0] += frameCompB[0];
        vertex[1] += frameCompB[1];
        break;
      default:
        break;
    }

    // Get the projection of the vertex on the normal of the edge
    // Orientation of the normal doesn't matter, so we
    // use arbitrarily the normal (edge[1], -edge[0])
    double proj = vertex[0] * edge[1] - vertex[1] * edge[0];

    // If it's the first vertex
    if (firstVertex == true) {

        // Initialize the boundaries of the projection of the
        // Frame on the edge
        bdgBox[0] = proj;
        bdgBox[1] = proj;

        // Update the flag to memorize we did the first vertex
        firstVertex = false;

    // Else, it's not the first vertex
    } else {

      // Update the boundaries of the projection of the Frame on
      // the edge
      if (bdgBox[0] > proj)
        bdgBox[0] = proj;

      if (bdgBox[1] < proj)
        bdgBox[1] = proj;

    }

  }

  // Switch the frame to check the vertices of the second Frame
  frame = tho;
  bdgBox = bdgBoxB;

}

// If the projections of the two frames on the edge are
// not intersecting
if (bdgBoxB[1] < bdgBoxA[0] ||
    bdgBoxA[1] < bdgBoxB[0]) {
```

```
        // There exists an axis which separates the Frames,
        // thus they are not in intersection
        return false;

      }

    }

    // Switch the frames to test against the second Frame's edges
    frameEdge = tho;

  }

  // If we reaches here, it means the two Frames are intersecting
  return true;

}

// Test for intersection between moving 2D Frame 'that' and 2D
// Frame 'tho'
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection2DTime(
  const Frame2DTime* const that,
  const Frame2DTime* const tho) {

  // Declare a variable to loop on Frames and commonalize code
  const Frame2DTime* frameEdge = that;

  // Declare a variable to memorize the speed of tho relative to that
  double relSpeed[2];
  relSpeed[0] = tho->speed[0] - that->speed[0];
  relSpeed[1] = tho->speed[1] - that->speed[1];

  // Loop to commonalize code when checking SAT based on that's edges
  // and then tho's edges
  for (int iFrame = 2;
       iFrame--;) {

    // Shortcuts
    FrameType frameEdgeType = frameEdge->type;
    const double* frameEdgeCompA = frameEdge->comp[0];
    const double* frameEdgeCompB = frameEdge->comp[1];

    // Declare a variable to memorize the number of edges, by default 2
    int nbEdges = 2;

    // Declare a variable to memorize the third edge in case of
    // tetrahedron
    double thirdEdge[2];

    // If the frame is a tetrahedron
    if (frameEdgeType == FrameTetrahedron) {

      // Initialise the third edge
      thirdEdge[0] = frameEdgeCompB[0] - frameEdgeCompA[0];
      thirdEdge[1] = frameEdgeCompB[1] - frameEdgeCompA[1];

      // Correct the number of edges
      nbEdges = 3;

    }
```

```
// If the current frame is the second frame
if (iFrame == 1) {

  // Add one more edge to take into account the movement
  // of tho relative to that
  ++nbEdges;

}

// Loop on the frame's edges
for (int iEdge = nbEdges;
     iEdge--;) {

  // Get the current edge
  const double* edge =
    (iEdge == 3 ? relSpeed :
      (iEdge == 2 ?
        (frameEdgeType == FrameTetrahedron ? thirdEdge : relSpeed) :
        frameEdge->comp[iEdge]));

  // Declare variables to memorize the boundaries of projection
  // of the two frames on the current edge
  double bdgBoxA[2];
  double bdgBoxB[2];

  // Declare two variables to loop on Frames and commonalize code
  const Frame2DTime* frame = that;
  double* bdgBox = bdgBoxA;

  // Loop on Frames
  for (int iFrame = 2;
       iFrame--;) {

    // Shortcuts
    const double* frameOrig = frame->orig;
    const double* frameCompA = frame->comp[0];
    const double* frameCompB = frame->comp[1];
    FrameType frameType = frame->type;

    // Get the number of vertices of frame
    int nbVertices = (frameType == FrameTetrahedron ? 3 : 4);

    // Declare a variable to memorize if the current vertex is
    // the first in the loop, used to initialize the boundaries
    bool firstVertex = true;

    // Loop on vertices of the frame
    for (int iVertex = nbVertices;
         iVertex--;) {

      // Get the vertex
      double vertex[2];
      vertex[0] = frameOrig[0];
      vertex[1] = frameOrig[1];
      switch (iVertex) {
        case 3:
          vertex[0] += frameCompA[0] + frameCompB[0];
          vertex[1] += frameCompA[1] + frameCompB[1];
          break;
        case 2:
          vertex[0] += frameCompA[0];
```

```
      vertex[1] += frameCompA[1];
      break;
    case 1:
      vertex[0] += frameCompB[0];
      vertex[1] += frameCompB[1];
      break;
    default:
      break;
  }

  // Get the projection of the vertex on the normal of the edge
  // Orientation of the normal doesn't matter, so we
  // use arbitrarily the normal (edge[1], -edge[0])
  double proj = vertex[0] * edge[1] - vertex[1] * edge[0];

  // If it's the first vertex
  if (firstVertex == true) {

      // Initialize the boundaries of the projection of the
      // Frame on the edge
      bdgBox[0] = proj;
      bdgBox[1] = proj;

      // Update the flag to memorize we did the first vertex
      firstVertex = false;

  // Else, it's not the first vertex
  } else {

    // Update the boundaries of the projection of the Frame on
    // the edge
    if (bdgBox[0] > proj)
      bdgBox[0] = proj;

    if (bdgBox[1] < proj)
      bdgBox[1] = proj;

  }

  // If we are checking the second frame's vertices
  if (frame == tho) {

    // Check also the vertices moved by the relative speed
    vertex[0] += relSpeed[0];
    vertex[1] += relSpeed[1];

    proj = vertex[0] * edge[1] - vertex[1] * edge[0];

    if (bdgBox[0] > proj)
      bdgBox[0] = proj;

    if (bdgBox[1] < proj)
      bdgBox[1] = proj;

  }

}

// Switch the frame to check the vertices of the second Frame
frame = tho;
bdgBox = bdgBoxB;
```

```
      }

      // If the projections of the two frames on the edge are
      // not intersecting
      if (bdgBoxB[1] < bdgBoxA[0] ||
          bdgBoxA[1] < bdgBoxB[0]) {

         // There exists an axis which separates the Frames,
         // thus they are not in intersection
         return false;

      }

    }

    // Switch the frames to test against the second Frame's edges
    frameEdge = tho;

  }

  // If we reaches here, it means the two Frames are intersecting
  return true;

}

// Test for intersection between 3D Frame 'that' and 3D Frame 'tho'
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection3D(
  const Frame3D* const that,
  const Frame3D* const tho) {

  // Declare two variables to memorize the opposite edges in case
  // of tetrahedron
  double oppEdgesThat[3][3];
  double oppEdgesTho[3][3];

  // Declare two variables to memorize the number of edges, by default 3
  int nbEdgesThat = 3;
  int nbEdgesTho = 3;

  // If the first Frame is a tetrahedron
  if (that->type == FrameTetrahedron) {

    // Shortcuts
    const double* frameCompA = that->comp[0];
    const double* frameCompB = that->comp[1];
    const double* frameCompC = that->comp[2];

    // Initialise the opposite edges
    oppEdgesThat[0][0] = frameCompB[0] - frameCompA[0];
    oppEdgesThat[0][1] = frameCompB[1] - frameCompA[1];
    oppEdgesThat[0][2] = frameCompB[2] - frameCompA[2];

    oppEdgesThat[1][0] = frameCompB[0] - frameCompC[0];
    oppEdgesThat[1][1] = frameCompB[1] - frameCompC[1];
    oppEdgesThat[1][2] = frameCompB[2] - frameCompC[2];

    oppEdgesThat[2][0] = frameCompC[0] - frameCompA[0];
    oppEdgesThat[2][1] = frameCompC[1] - frameCompA[1];
    oppEdgesThat[2][2] = frameCompC[2] - frameCompA[2];

    // Correct the number of edges
```

```
  nbEdgesThat = 6;

}

// If the second Frame is a tetrahedron
if (tho->type == FrameTetrahedron) {

  // Shortcuts
  const double* frameCompA = tho->comp[0];
  const double* frameCompB = tho->comp[1];
  const double* frameCompC = tho->comp[2];

  // Initialise the opposite edges
  oppEdgesTho[0][0] = frameCompB[0] - frameCompA[0];
  oppEdgesTho[0][1] = frameCompB[1] - frameCompA[1];
  oppEdgesTho[0][2] = frameCompB[2] - frameCompA[2];

  oppEdgesTho[1][0] = frameCompB[0] - frameCompC[0];
  oppEdgesTho[1][1] = frameCompB[1] - frameCompC[1];
  oppEdgesTho[1][2] = frameCompB[2] - frameCompC[2];

  oppEdgesTho[2][0] = frameCompC[0] - frameCompA[0];
  oppEdgesTho[2][1] = frameCompC[1] - frameCompA[1];
  oppEdgesTho[2][2] = frameCompC[2] - frameCompA[2];

  // Correct the number of edges
  nbEdgesTho = 6;

}

// Declare variables to loop on Frames and commonalize code
const Frame3D* frame = that;
const double (*oppEdgesA)[3] = oppEdgesThat;

// Loop to commonalize code when checking SAT based on that's edges
// and then tho's edges
for (int iFrame = 2;
     iFrame--;) {

  // Shortcuts
  FrameType frameType = frame->type;
  const double* frameCompA = frame->comp[0];
  const double* frameCompB = frame->comp[1];
  const double* frameCompC = frame->comp[2];

  // Declare a variable to memorize the number of faces, by default 3
  int nbFaces = 3;

  // Declare a variable to memorize the normal to faces
  // Arrangement is normFaces[iFace][iAxis]
  double normFaces[4][3];

  // Initialise the normal to faces
  normFaces[0][0] =
    frameCompA[1] * frameCompB[2] -
    frameCompA[2] * frameCompB[1];
  normFaces[0][1] =
    frameCompA[2] * frameCompB[0] -
    frameCompA[0] * frameCompB[2];
  normFaces[0][2] =
    frameCompA[0] * frameCompB[1] -
    frameCompA[1] * frameCompB[0];
```

```
normFaces[1][0] =
  frameCompA[1] * frameCompC[2] -
  frameCompA[2] * frameCompC[1];
normFaces[1][1] =
  frameCompA[2] * frameCompC[0] -
  frameCompA[0] * frameCompC[2];
normFaces[1][2] =
  frameCompA[0] * frameCompC[1] -
  frameCompA[1] * frameCompC[0];

normFaces[2][0] =
  frameCompC[1] * frameCompB[2] -
  frameCompC[2] * frameCompB[1];
normFaces[2][1] =
  frameCompC[2] * frameCompB[0] -
  frameCompC[0] * frameCompB[2];
normFaces[2][2] =
  frameCompC[0] * frameCompB[1] -
  frameCompC[1] * frameCompB[0];

// If the frame is a tetrahedron
if (frameType == FrameTetrahedron) {

  // Shortcuts
  const double* oppEdgeA = oppEdgesA[0];
  const double* oppEdgeB = oppEdgesA[1];

  // Initialise the normal to the opposite face
  normFaces[3][0] =
    oppEdgeA[1] * oppEdgeB[2] -
    oppEdgeA[2] * oppEdgeB[1];
  normFaces[3][1] =
    oppEdgeA[2] * oppEdgeB[0] -
    oppEdgeA[0] * oppEdgeB[2];
  normFaces[3][2] =
    oppEdgeA[0] * oppEdgeB[1] -
    oppEdgeA[1] * oppEdgeB[0];

  // Correct the number of faces
  nbFaces = 4;

}

// Loop on the frame's faces
for (int iFace = nbFaces;
     iFace--;) {

  // Check against the current face's normal
  bool isIntersection =
    CheckAxis3D(
      that,
      tho,
      normFaces[iFace]);

  // If the axis is separating the Frames
  if (isIntersection == false) {

    // The Frames are not in intersection,
    // terminate the test
    return false;
```

```
      }

    }

    // Switch the frame to test against the second Frame
    frame = tho;
    oppEdgesA = oppEdgesTho;

  }

  // Loop on the pair of edges between the two frames
  for (int iEdgeThat = nbEdgesThat;
       iEdgeThat--;) {

    // Get the first edge
    const double* edgeThat =
      (iEdgeThat < 3 ?
        that->comp[iEdgeThat] :
        oppEdgesThat[iEdgeThat - 3]);

    for (int iEdgeTho = nbEdgesTho;
         iEdgeTho--;) {

      // Get the second edge
      const double* edgeTho =
        (iEdgeTho < 3 ?
          tho->comp[iEdgeTho] :
          oppEdgesTho[iEdgeTho - 3]);

      // Get the cross product of the two edges
      double axis[3];
      axis[0] = edgeThat[1] * edgeTho[2] - edgeThat[2] * edgeTho[1];
      axis[1] = edgeThat[2] * edgeTho[0] - edgeThat[0] * edgeTho[2];
      axis[2] = edgeThat[0] * edgeTho[1] - edgeThat[1] * edgeTho[0];

      // Check against the cross product of the two edges
      bool isIntersection =
        CheckAxis3D(
          that,
          tho,
          axis);

      // If the axis is separating the Frames
      if (isIntersection == false) {

        // The Frames are not in intersection,
        // terminate the test
        return false;

      }

    }

  }

  // If we reaches here, it means the two Frames are intersecting
  return true;

}

// Test for intersection between moving 3D Frame 'that' and 3D
// Frame 'tho'
```

```
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection3DTime(
  const Frame3DTime* const that,
  const Frame3DTime* const tho) {

  // Declare two variables to memorize the opposite edges in case
  // of tetrahedron
  double oppEdgesThat[3][3];
  double oppEdgesTho[3][3];

  // Declare a variable to memorize the speed of tho relative to that
  double relSpeed[3];
  relSpeed[0] = tho->speed[0] - that->speed[0];
  relSpeed[1] = tho->speed[1] - that->speed[1];
  relSpeed[2] = tho->speed[2] - that->speed[2];

  // Declare two variables to memorize the number of edges, by default 3
  int nbEdgesThat = 3;
  int nbEdgesTho = 3;

  // If the first Frame is a tetrahedron
  if (that->type == FrameTetrahedron) {

    // Shortcuts
    const double* frameCompA = that->comp[0];
    const double* frameCompB = that->comp[1];
    const double* frameCompC = that->comp[2];

    // Initialise the opposite edges
    oppEdgesThat[0][0] = frameCompB[0] - frameCompA[0];
    oppEdgesThat[0][1] = frameCompB[1] - frameCompA[1];
    oppEdgesThat[0][2] = frameCompB[2] - frameCompA[2];

    oppEdgesThat[1][0] = frameCompB[0] - frameCompC[0];
    oppEdgesThat[1][1] = frameCompB[1] - frameCompC[1];
    oppEdgesThat[1][2] = frameCompB[2] - frameCompC[2];

    oppEdgesThat[2][0] = frameCompC[0] - frameCompA[0];
    oppEdgesThat[2][1] = frameCompC[1] - frameCompA[1];
    oppEdgesThat[2][2] = frameCompC[2] - frameCompA[2];

    // Correct the number of edges
    nbEdgesThat = 6;

  }

  // If the second Frame is a tetrahedron
  if (tho->type == FrameTetrahedron) {

    // Shortcuts
    const double* frameCompA = tho->comp[0];
    const double* frameCompB = tho->comp[1];
    const double* frameCompC = tho->comp[2];

    // Initialise the opposite edges
    oppEdgesTho[0][0] = frameCompB[0] - frameCompA[0];
    oppEdgesTho[0][1] = frameCompB[1] - frameCompA[1];
    oppEdgesTho[0][2] = frameCompB[2] - frameCompA[2];

    oppEdgesTho[1][0] = frameCompB[0] - frameCompC[0];
    oppEdgesTho[1][1] = frameCompB[1] - frameCompC[1];
    oppEdgesTho[1][2] = frameCompB[2] - frameCompC[2];
```

```
  oppEdgesTho[2][0] = frameCompC[0] - frameCompA[0];
  oppEdgesTho[2][1] = frameCompC[1] - frameCompA[1];
  oppEdgesTho[2][2] = frameCompC[2] - frameCompA[2];

  // Correct the number of edges
  nbEdgesTho = 6;

}

// Declare variables to loop on Frames and commonalize code
const Frame3DTime* frame = that;
const double (*oppEdgesA)[3] = oppEdgesThat;

// Loop to commonalize code when checking SAT based on that's edges
// and then tho's edges
for (int iFrame = 2;
     iFrame--;) {

  // Shortcuts
  FrameType frameType = frame->type;
  const double* frameCompA = frame->comp[0];
  const double* frameCompB = frame->comp[1];
  const double* frameCompC = frame->comp[2];

  // Declare a variable to memorize the number of faces, by default 3
  int nbFaces = 3;

  // Declare a variable to memorize the normal to faces
  // Arrangement is normFaces[iFace][iAxis]
  double normFaces[10][3];

  // Initialise the normal to faces
  normFaces[0][0] =
    frameCompA[1] * frameCompB[2] -
    frameCompA[2] * frameCompB[1];
  normFaces[0][1] =
    frameCompA[2] * frameCompB[0] -
    frameCompA[0] * frameCompB[2];
  normFaces[0][2] =
    frameCompA[0] * frameCompB[1] -
    frameCompA[1] * frameCompB[0];

  normFaces[1][0] =
    frameCompA[1] * frameCompC[2] -
    frameCompA[2] * frameCompC[1];
  normFaces[1][1] =
    frameCompA[2] * frameCompC[0] -
    frameCompA[0] * frameCompC[2];
  normFaces[1][2] =
    frameCompA[0] * frameCompC[1] -
    frameCompA[1] * frameCompC[0];

  normFaces[2][0] =
    frameCompC[1] * frameCompB[2] -
    frameCompC[2] * frameCompB[1];
  normFaces[2][1] =
    frameCompC[2] * frameCompB[0] -
    frameCompC[0] * frameCompB[2];
  normFaces[2][2] =
    frameCompC[0] * frameCompB[1] -
    frameCompC[1] * frameCompB[0];
```

```
// If the frame is a tetrahedron
if (frameType == FrameTetrahedron) {

  // Shortcuts
  const double* oppEdgeA = oppEdgesA[0];
  const double* oppEdgeB = oppEdgesA[1];

  // Initialise the normal to the opposite face
  normFaces[3][0] =
    oppEdgeA[1] * oppEdgeB[2] -
    oppEdgeA[2] * oppEdgeB[1];
  normFaces[3][1] =
    oppEdgeA[2] * oppEdgeB[0] -
    oppEdgeA[0] * oppEdgeB[2];
  normFaces[3][2] =
    oppEdgeA[0] * oppEdgeB[1] -
    oppEdgeA[1] * oppEdgeB[0];

  // Correct the number of faces
  nbFaces = 4;

}

// If we are checking the frame 'tho'
if (frame == tho) {

  // Add the normal to the virtual faces created by the speed
  // of tho relative to that

  normFaces[nbFaces][0] =
    relSpeed[1] * frameCompA[2] -
    relSpeed[2] * frameCompA[1];
  normFaces[nbFaces][1] =
    relSpeed[2] * frameCompA[0] -
    relSpeed[0] * frameCompA[2];
  normFaces[nbFaces][2] =
    relSpeed[0] * frameCompA[1] -
    relSpeed[1] * frameCompA[0];
  if (fabs(normFaces[nbFaces][0]) > EPSILON ||
      fabs(normFaces[nbFaces][1]) > EPSILON ||
      fabs(normFaces[nbFaces][2]) > EPSILON)
    ++nbFaces;

  normFaces[nbFaces][0] =
    relSpeed[1] * frameCompB[2] -
    relSpeed[2] * frameCompB[1];
  normFaces[nbFaces][1] =
    relSpeed[2] * frameCompB[0] -
    relSpeed[0] * frameCompB[2];
  normFaces[nbFaces][2] =
    relSpeed[0] * frameCompB[1] -
    relSpeed[1] * frameCompB[0];
  if (fabs(normFaces[nbFaces][0]) > EPSILON ||
      fabs(normFaces[nbFaces][1]) > EPSILON ||
      fabs(normFaces[nbFaces][2]) > EPSILON)
    ++nbFaces;

  normFaces[nbFaces][0] =
    relSpeed[1] * frameCompC[2] -
    relSpeed[2] * frameCompC[1];
  normFaces[nbFaces][1] =
```

```
    relSpeed[2] * frameCompC[0] -
    relSpeed[0] * frameCompC[2];
  normFaces[nbFaces][2] =
    relSpeed[0] * frameCompC[1] -
    relSpeed[1] * frameCompC[0];
  if (fabs(normFaces[nbFaces][0]) > EPSILON ||
      fabs(normFaces[nbFaces][1]) > EPSILON ||
      fabs(normFaces[nbFaces][2]) > EPSILON)
    ++nbFaces;

  if (frameType == FrameTetrahedron) {

    const double* oppEdgeA = oppEdgesA[0];
    const double* oppEdgeB = oppEdgesA[1];
    const double* oppEdgeC = oppEdgesA[2];

    normFaces[nbFaces][0] =
      relSpeed[1] * oppEdgeA[2] -
      relSpeed[2] * oppEdgeA[1];
    normFaces[nbFaces][1] =
      relSpeed[2] * oppEdgeA[0] -
      relSpeed[0] * oppEdgeA[2];
    normFaces[nbFaces][2] =
      relSpeed[0] * oppEdgeA[1] -
      relSpeed[1] * oppEdgeA[0];
    if (fabs(normFaces[nbFaces][0]) > EPSILON ||
        fabs(normFaces[nbFaces][1]) > EPSILON ||
        fabs(normFaces[nbFaces][2]) > EPSILON)
      ++nbFaces;

    normFaces[nbFaces][0] =
      relSpeed[1] * oppEdgeB[2] -
      relSpeed[2] * oppEdgeB[1];
    normFaces[nbFaces][1] =
      relSpeed[2] * oppEdgeB[0] -
      relSpeed[0] * oppEdgeB[2];
    normFaces[nbFaces][2] =
      relSpeed[0] * oppEdgeB[1] -
      relSpeed[1] * oppEdgeB[0];
    if (fabs(normFaces[nbFaces][0]) > EPSILON ||
        fabs(normFaces[nbFaces][1]) > EPSILON ||
        fabs(normFaces[nbFaces][2]) > EPSILON)
      ++nbFaces;

    normFaces[nbFaces][0] =
      relSpeed[1] * oppEdgeC[2] -
      relSpeed[2] * oppEdgeC[1];
    normFaces[nbFaces][1] =
      relSpeed[2] * oppEdgeC[0] -
      relSpeed[0] * oppEdgeC[2];
    normFaces[nbFaces][2] =
      relSpeed[0] * oppEdgeC[1] -
      relSpeed[1] * oppEdgeC[0];
    if (fabs(normFaces[nbFaces][0]) > EPSILON ||
        fabs(normFaces[nbFaces][1]) > EPSILON ||
        fabs(normFaces[nbFaces][2]) > EPSILON)
      ++nbFaces;

  }
}

// Loop on the frame's faces
```

```
  for (int iFace = nbFaces;
       iFace--;) {

    // Check against the current face's normal
    bool isIntersection =
      CheckAxis3DTime(
        that,
        tho,
        normFaces[iFace],
        relSpeed);

    // If the axis is separating the Frames
    if (isIntersection == false) {

      // The Frames are not in intersection,
      // terminate the test
      return false;

    }

  }

  // Switch the frame to test against the second Frame
  frame = tho;
  oppEdgesA = oppEdgesTho;

}

// Loop on the pair of edges between the two frames
for (int iEdgeThat = nbEdgesThat;
     iEdgeThat--;) {

  // Get the first edge
  const double* edgeThat =
    (iEdgeThat < 3 ?
      that->comp[iEdgeThat] :
      oppEdgesThat[iEdgeThat - 3]);

  for (int iEdgeTho = nbEdgesTho + 1;
       iEdgeTho--;) {

    // Get the second edge
    const double* edgeTho =
      (iEdgeTho == nbEdgesTho ?
        relSpeed :
        (iEdgeTho < 3 ?
          tho->comp[iEdgeTho] :
          oppEdgesTho[iEdgeTho - 3]));

    // Get the cross product of the two edges
    double axis[3];
    axis[0] = edgeThat[1] * edgeTho[2] - edgeThat[2] * edgeTho[1];
    axis[1] = edgeThat[2] * edgeTho[0] - edgeThat[0] * edgeTho[2];
    axis[2] = edgeThat[0] * edgeTho[1] - edgeThat[1] * edgeTho[0];

    // Check against the cross product of the two edges
    bool isIntersection =
      CheckAxis3DTime(
        that,
        tho,
        axis,
        relSpeed);
```

```
      // If the axis is separating the Frames
      if (isIntersection == false) {

        // The Frames are not in intersection,
        // terminate the test
        return false;

      }

    }

  }

  // If we reaches here, it means the two Frames are intersecting
  return true;

}

// Check the intersection constraint for Frames 'that' and 'tho'
// relatively to 'axis'
bool CheckAxis3D(
  const Frame3D* const that,
  const Frame3D* const tho,
  const double* const axis) {

  // Declare variables to memorize the boundaries of projection
  // of the two frames on the current edge
  double bdgBoxA[2];
  double bdgBoxB[2];

  // Declare two variables to loop on Frames and commonalize code
  const Frame3D* frame = that;
  double* bdgBox = bdgBoxA;

  // Loop on Frames
  for (int iFrame = 2;
       iFrame--;) {

    // Shortcuts
    const double* frameOrig = frame->orig;
    const double* frameCompA = frame->comp[0];
    const double* frameCompB = frame->comp[1];
    const double* frameCompC = frame->comp[2];
    FrameType frameType = frame->type;

    // Get the number of vertices of frame
    int nbVertices = (frameType == FrameTetrahedron ? 4 : 8);

    // Declare a variable to memorize if the current vertex is
    // the first in the loop, used to initialize the boundaries
    bool firstVertex = true;

    // Loop on vertices of the frame
    for (int iVertex = nbVertices;
         iVertex--;) {

      // Get the vertex
      double vertex[3];
      vertex[0] = frameOrig[0];
      vertex[1] = frameOrig[1];
      vertex[2] = frameOrig[2];
```

```
switch (iVertex) {
  case 7:
    vertex[0] +=
      frameCompA[0] +  frameCompB[0] + frameCompC[0];
    vertex[1] +=
      frameCompA[1] +  frameCompB[1] + frameCompC[1];
    vertex[2] +=
      frameCompA[2] +  frameCompB[2] + frameCompC[2];
    break;
  case 6:
    vertex[0] += frameCompB[0] + frameCompC[0];
    vertex[1] += frameCompB[1] + frameCompC[1];
    vertex[2] += frameCompB[2] + frameCompC[2];
    break;
  case 5:
    vertex[0] += frameCompA[0] + frameCompC[0];
    vertex[1] += frameCompA[1] + frameCompC[1];
    vertex[2] += frameCompA[2] + frameCompC[2];
    break;
  case 4:
    vertex[0] += frameCompA[0] + frameCompB[0];
    vertex[1] += frameCompA[1] + frameCompB[1];
    vertex[2] += frameCompA[2] + frameCompB[2];
    break;
  case 3:
    vertex[0] += frameCompC[0];
    vertex[1] += frameCompC[1];
    vertex[2] += frameCompC[2];
    break;
  case 2:
    vertex[0] += frameCompB[0];
    vertex[1] += frameCompB[1];
    vertex[2] += frameCompB[2];
    break;
  case 1:
    vertex[0] += frameCompA[0];
    vertex[1] += frameCompA[1];
    vertex[2] += frameCompA[2];
    break;
  default:
    break;
}

// Get the projection of the vertex on the axis
double proj =
  vertex[0] * axis[0] +
  vertex[1] * axis[1] +
  vertex[2] * axis[2];

// If it's the first vertex
if (firstVertex == true) {

    // Initialize the boundaries of the projection of the
    // Frame on the edge
    bdgBox[0] = proj;
    bdgBox[1] = proj;

    // Update the flag to memorize we did the first vertex
    firstVertex = false;

// Else, it's not the first vertex
} else {
```

```
        // Update the boundaries of the projection of the Frame on
        // the edge
        if (bdgBox[0] > proj)
          bdgBox[0] = proj;

        if (bdgBox[1] < proj)
          bdgBox[1] = proj;

      }

    }

    // Switch the frame to check the vertices of the second Frame
    frame = tho;
    bdgBox = bdgBoxB;

  }

  // If the projections of the two frames on the edge are
  // not intersecting
  if (bdgBoxB[1] < bdgBoxA[0] ||
      bdgBoxA[1] < bdgBoxB[0]) {

     // There exists an axis which separates the Frames,
     // thus they are not in intersection
     return false;

  }

  // If we reaches here the two Frames are in intersection
  return true;

}

// Check the intersection constraint for Frames 'that' and 'tho'
// relatively to 'axis'
bool CheckAxis3DTime(
  const Frame3DTime* const that,
  const Frame3DTime* const tho,
  const double* const axis,
  const double* const relSpeed) {

  // Declare variables to memorize the boundaries of projection
  // of the two frames on the current edge
  double bdgBoxA[2];
  double bdgBoxB[2];

  // Declare two variables to loop on Frames and commonalize code
  const Frame3DTime* frame = that;
  double* bdgBox = bdgBoxA;

  // Loop on Frames
  for (int iFrame = 2;
       iFrame--;) {

    // Shortcuts
    const double* frameOrig = frame->orig;
    const double* frameCompA = frame->comp[0];
    const double* frameCompB = frame->comp[1];
    const double* frameCompC = frame->comp[2];
    FrameType frameType = frame->type;
```

```
// Get the number of vertices of frame
int nbVertices = (frameType == FrameTetrahedron ? 4 : 8);

// Declare a variable to memorize if the current vertex is
// the first in the loop, used to initialize the boundaries
bool firstVertex = true;

// Loop on vertices of the frame
for (int iVertex = nbVertices;
     iVertex--;) {

  // Get the vertex
  double vertex[3];
  vertex[0] = frameOrig[0];
  vertex[1] = frameOrig[1];
  vertex[2] = frameOrig[2];
  switch (iVertex) {
    case 7:
      vertex[0] +=
        frameCompA[0] +  frameCompB[0] + frameCompC[0];
      vertex[1] +=
        frameCompA[1] +  frameCompB[1] + frameCompC[1];
      vertex[2] +=
        frameCompA[2] +  frameCompB[2] + frameCompC[2];
      break;
    case 6:
      vertex[0] += frameCompB[0] + frameCompC[0];
      vertex[1] += frameCompB[1] + frameCompC[1];
      vertex[2] += frameCompB[2] + frameCompC[2];
      break;
    case 5:
      vertex[0] += frameCompA[0] + frameCompC[0];
      vertex[1] += frameCompA[1] + frameCompC[1];
      vertex[2] += frameCompA[2] + frameCompC[2];
      break;
    case 4:
      vertex[0] += frameCompA[0] + frameCompB[0];
      vertex[1] += frameCompA[1] + frameCompB[1];
      vertex[2] += frameCompA[2] + frameCompB[2];
      break;
    case 3:
      vertex[0] += frameCompC[0];
      vertex[1] += frameCompC[1];
      vertex[2] += frameCompC[2];
      break;
    case 2:
      vertex[0] += frameCompB[0];
      vertex[1] += frameCompB[1];
      vertex[2] += frameCompB[2];
      break;
    case 1:
      vertex[0] += frameCompA[0];
      vertex[1] += frameCompA[1];
      vertex[2] += frameCompA[2];
      break;
    default:
      break;
  }

  // Get the projection of the vertex on the axis
  double proj =
```

```
        vertex[0] * axis[0] +
        vertex[1] * axis[1] +
        vertex[2] * axis[2];

      // If it's the first vertex
      if (firstVertex == true) {

          // Initialize the boundaries of the projection of the
          // Frame on the edge
          bdgBox[0] = proj;
          bdgBox[1] = proj;

          // Update the flag to memorize we did the first vertex
          firstVertex = false;

      // Else, it's not the first vertex
      } else {

        // Update the boundaries of the projection of the Frame on
        // the edge
        if (bdgBox[0] > proj)
          bdgBox[0] = proj;

        if (bdgBox[1] < proj)
          bdgBox[1] = proj;

      }

      // If we are checking the second frame's vertices
      if (frame == tho) {

        // Check also the vertices moved by the relative speed
        vertex[0] += relSpeed[0];
        vertex[1] += relSpeed[1];
        vertex[2] += relSpeed[2];

      proj =
        vertex[0] * axis[0] +
        vertex[1] * axis[1] +
        vertex[2] * axis[2];

        if (bdgBox[0] > proj)
          bdgBox[0] = proj;

        if (bdgBox[1] < proj)
          bdgBox[1] = proj;

      }

    }

    // Switch the frame to check the vertices of the second Frame
    frame = tho;
    bdgBox = bdgBoxB;

  }

  // If the projections of the two frames on the edge are
  // not intersecting
  if (bdgBoxB[1] < bdgBoxA[0] ||
      bdgBoxA[1] < bdgBoxB[0]) {
```

```
      // There exists an axis which separates the Frames,
      // thus they are not in intersection
      return false;

    }

    // If we reaches here the two Frames are in intersection
    return true;

}
```

10.3 Makefile

In this section I introduce the Makefile used to compile the code given in the previous sections.

```
COMPILER=gcc
OPTIMIZATION=-O3

all : compile run plot doc

install :
        sudo apt-get install gnuplot

compile : main unitTests validation qualification

main : main2D main2DTime main3D main3DTime

main2D:
        cd 2D; make main; cd -

main2DTime:
        cd 2DTime; make main; cd -

main3D:
        cd 3D; make main; cd -

main3DTime:
        cd 3DTime; make main; cd -

unitTests : unitTests2D unitTests2DTime unitTests3D unitTests3DTime

unitTests2D:
        cd 2D; make unitTests; cd -

unitTests2DTime:
        cd 2DTime; make unitTests; cd -

unitTests3D:
        cd 3D; make unitTests; cd -

unitTests3DTime:
        cd 3DTime; make unitTests; cd -

validation : validation2D validation2DTime validation3D validation3DTime

validation2D:
        cd 2D; make validation; cd -
```

```
validation2DTime:
	cd 2DTime; make validation; cd -

validation3D:
	cd 3D; make validation; cd -

validation3DTime:
	cd 3DTime; make validation; cd -

qualification : qualification2D qualification2DTime qualification3D
    qualification3DTime

qualification2D:
	cd 2D; make qualification; cd -

qualification2DTime:
	cd 2DTime; make qualification; cd -

qualification3D:
	cd 3D; make qualification; cd -

qualification3DTime:
	cd 3DTime; make qualification; cd -

clean : clean2D clean2DTime clean3D clean3DTime

clean2D:
	cd 2D; make clean; cd -

clean2DTime:
	cd 2DTime; make clean; cd -

clean3D:
	cd 3D; make clean; cd -

clean3DTime:
	cd 3DTime; make clean; cd -

valgrind : valgrind2D valgrind2DTime valgrind3D valgrind3DTime

valgrind2D:
	cd 2D; make valgrind; cd -

valgrind2DTime:
	cd 2DTime; make valgrind; cd -

valgrind3D:
	cd 3D; make valgrind; cd -

valgrind3DTime:
	cd 3DTime; make valgrind; cd -

run : run2D run2DTime run3D run3DTime

run2D:
	cd 2D; ./main > ../Results/main2D.txt; ./unitTests > ../Results/
	    unitTests2D.txt; ./validation > ../Results/validation2D.txt;
	    grep failed ../Results/validation2D.txt; ./qualification > ../
	    Results/qualification2D.txt; grep failed ../Results/
	    qualification2D.txt; cd -

run3D:
```

```
	cd 3D; ./main > ../Results/main3D.txt; ./unitTests > ../Results/
	    unitTests3D.txt; ./validation > ../Results/validation3D.txt;
	    grep failed ../Results/validation3D.txt; ./qualification > ../
	    Results/qualification3D.txt; grep failed ../Results/
	    qualification3D.txt; cd -

run2DTime:
	cd 2DTime; ./main > ../Results/main2DTime.txt; ./unitTests > ../
	    Results/unitTests2DTime.txt; ./validation > ../Results/
	    validation2DTime.txt; grep failed ../Results/validation2DTime.
	    txt; ./qualification > ../Results/qualification2DTime.txt; grep
	    failed ../Results/qualification2DTime.txt; cd -

run3DTime:
	cd 3DTime; ./main > ../Results/main3DTime.txt; ./unitTests > ../
	    Results/unitTests3DTime.txt; ./validation > ../Results/
	    validation3DTime.txt; grep failed ../Results/validation3DTime.
	    txt; ./qualification > ../Results/qualification3DTime.txt; grep
	    failed ../Results/qualification3DTime.txt; cd -

plot: cleanPlot plot2D plot2DTime plot3D plot3DTime

cleanPlot:
	rm Results/*.png

plot2D:
	cd Results; gnuplot qualification2D.gnu < qualification2D.txt; cd -

plot2DTime:
	cd Results; gnuplot qualification2DTime.gnu < qualification2DTime.
	    txt; cd -

plot3D:
	cd Results; gnuplot qualification3D.gnu < qualification3D.txt; cd -

plot3DTime:
	cd Results; gnuplot qualification3DTime.gnu < qualification3DTime.
	    txt; cd -

doc:
	cd Doc; make latex; cd -
```

10.3.1 2D static

```
all : main unitTests validation qualification

COMPILER?=gcc
OPTIMIZATION?=-O3
BUILD_ARG=$(OPTIMIZATION) -I../SAT -I../Frame

main : main.o fmb2d.o frame.o Makefile
	$(COMPILER) -o main main.o fmb2d.o frame.o

main.o : main.c fmb2d.h ../Frame/frame.h Makefile
	$(COMPILER) -c main.c $(BUILD_ARG)

unitTests : unitTests.o fmb2d.o frame.o Makefile
	$(COMPILER) -o unitTests unitTests.o fmb2d.o frame.o $(LINK_ARG)
```

```
unitTests.o : unitTests.c fmb2d.h ../Frame/frame.h Makefile
	$(COMPILER) -c unitTests.c $(BUILD_ARG)

validation : validation.o fmb2d.o sat.o frame.o Makefile
	$(COMPILER) -o validation validation.o fmb2d.o sat.o frame.o

validation.o : validation.c fmb2d.h ../SAT/sat.h ../Frame/frame.h Makefile
	$(COMPILER) -c validation.c $(BUILD_ARG)

qualification : qualification.o fmb2d.o sat.o frame.o Makefile
	$(COMPILER) -o qualification qualification.o fmb2d.o sat.o frame.o $
	    (LINK_ARG)

qualification.o : qualification.c fmb2d.h ../SAT/sat.h ../Frame/frame.h
    Makefile
	$(COMPILER) -c qualification.c $(BUILD_ARG)

fmb2d.o : fmb2d.c fmb2d.h ../Frame/frame.h Makefile
	$(COMPILER) -c fmb2d.c $(BUILD_ARG)

sat.o : ../SAT/sat.c ../SAT/sat.h ../Frame/frame.h Makefile
	$(COMPILER) -c ../SAT/sat.c $(BUILD_ARG)

frame.o : ../Frame/frame.c ../Frame/frame.h Makefile
	$(COMPILER) -c ../Frame/frame.c $(BUILD_ARG)

clean :
	rm -f *.o main unitTests validation qualification

valgrind :
	valgrind -v --track-origins=yes --leak-check=full \
	--gen-suppressions=yes --show-leak-kinds=all ./main
```

10.3.2 3D static

```
all : main unitTests validation qualification

COMPILER?=gcc
OPTIMIZATION?=-O3
BUILD_ARG=$(OPTIMIZATION) -I../SAT -I../Frame

main : main.o fmb3d.o frame.o Makefile
	$(COMPILER) -o main main.o fmb3d.o frame.o

main.o : main.c fmb3d.h ../Frame/frame.h Makefile
	$(COMPILER) -c main.c $(BUILD_ARG)

unitTests : unitTests.o fmb3d.o frame.o Makefile
	$(COMPILER) -o unitTests unitTests.o fmb3d.o frame.o $(LINK_ARG)

unitTests.o : unitTests.c fmb3d.h ../Frame/frame.h Makefile
	$(COMPILER) -c unitTests.c $(BUILD_ARG)

validation : validation.o fmb3d.o sat.o frame.o Makefile
	$(COMPILER) -o validation validation.o fmb3d.o sat.o frame.o

validation.o : validation.c fmb3d.h ../SAT/sat.h ../Frame/frame.h Makefile
	$(COMPILER) -c validation.c $(BUILD_ARG)

qualification : qualification.o fmb3d.o sat.o frame.o Makefile
	$(COMPILER) -o qualification qualification.o fmb3d.o sat.o frame.o $
```

```
            (LINK_ARG)

qualification.o : qualification.c fmb3d.h ../SAT/sat.h ../Frame/frame.h
    Makefile
        $(COMPILER) -c qualification.c $(BUILD_ARG)

fmb3d.o : fmb3d.c fmb3d.h ../Frame/frame.h Makefile
        $(COMPILER) -c fmb3d.c $(BUILD_ARG)

sat.o : ../SAT/sat.c ../SAT/sat.h ../Frame/frame.h Makefile
        $(COMPILER) -c ../SAT/sat.c $(BUILD_ARG)

frame.o : ../Frame/frame.c ../Frame/frame.h Makefile
        $(COMPILER) -c ../Frame/frame.c $(BUILD_ARG)

clean :
        rm -f *.o main unitTests validation qualification

valgrind :
        valgrind -v --track-origins=yes --leak-check=full \
        --gen-suppressions=yes --show-leak-kinds=all ./main
```

10.3.3 2D dynamic

```
all : main unitTests validation qualification

COMPILER?=gcc
OPTIMIZATION?=-O3
BUILD_ARG=$(OPTIMIZATION) -I../SAT -I../Frame

main : main.o fmb2dt.o frame.o Makefile
        $(COMPILER) -o main main.o fmb2dt.o frame.o

main.o : main.c fmb2dt.h ../Frame/frame.h Makefile
        $(COMPILER) -c main.c $(BUILD_ARG)

unitTests : unitTests.o fmb2dt.o frame.o Makefile
        $(COMPILER) -o unitTests unitTests.o fmb2dt.o frame.o $(LINK_ARG)

unitTests.o : unitTests.c fmb2dt.h ../Frame/frame.h Makefile
        $(COMPILER) -c unitTests.c $(BUILD_ARG)

validation : validation.o fmb2dt.o sat.o frame.o Makefile
        $(COMPILER) -o validation validation.o fmb2dt.o sat.o frame.o

validation.o : validation.c fmb2dt.h ../SAT/sat.h ../Frame/frame.h Makefile
        $(COMPILER) -c validation.c $(BUILD_ARG)

qualification : qualification.o fmb2dt.o sat.o frame.o Makefile
        $(COMPILER) -o qualification qualification.o fmb2dt.o sat.o frame.o
            $(LINK_ARG)

qualification.o : qualification.c fmb2dt.h ../SAT/sat.h ../Frame/frame.h
    Makefile
        $(COMPILER) -c qualification.c $(BUILD_ARG)

fmb2dt.o : fmb2dt.c fmb2dt.h ../Frame/frame.h Makefile
        $(COMPILER) -c fmb2dt.c $(BUILD_ARG)

sat.o : ../SAT/sat.c ../SAT/sat.h ../Frame/frame.h Makefile
        $(COMPILER) -c ../SAT/sat.c $(BUILD_ARG)
```

```
frame.o : ../Frame/frame.c ../Frame/frame.h Makefile
        $(COMPILER) -c ../Frame/frame.c $(BUILD_ARG)

clean :
        rm -f *.o main unitTests validation qualification

valgrind :
        valgrind -v --track-origins=yes --leak-check=full \
        --gen-suppressions=yes --show-leak-kinds=all ./main
```

10.3.4 3D dynamic

```
all : main unitTests validation qualification

COMPILER?=gcc
OPTIMIZATION?=-O3
BUILD_ARG=$(OPTIMIZATION) -I../SAT -I../Frame

main : main.o fmb3dt.o frame.o Makefile
        $(COMPILER) -o main main.o fmb3dt.o frame.o

main.o : main.c fmb3dt.h ../Frame/frame.h Makefile
        $(COMPILER) -c main.c $(BUILD_ARG)

unitTests : unitTests.o fmb3dt.o frame.o Makefile
        $(COMPILER) -o unitTests unitTests.o fmb3dt.o frame.o $(LINK_ARG)

unitTests.o : unitTests.c fmb3dt.h ../Frame/frame.h Makefile
        $(COMPILER) -c unitTests.c $(BUILD_ARG)

validation : validation.o fmb3dt.o sat.o frame.o Makefile
        $(COMPILER) -o validation validation.o fmb3dt.o sat.o frame.o

validation.o : validation.c fmb3dt.h ../SAT/sat.h ../Frame/frame.h Makefile
        $(COMPILER) -c validation.c $(BUILD_ARG)

qualification : qualification.o fmb3dt.o sat.o frame.o Makefile
        $(COMPILER) -o qualification qualification.o fmb3dt.o sat.o frame.o
            $(LINK_ARG)

qualification.o : qualification.c fmb3dt.h ../SAT/sat.h ../Frame/frame.h
    Makefile
        $(COMPILER) -c qualification.c $(BUILD_ARG)

fmb3dt.o : fmb3dt.c fmb3dt.h ../Frame/frame.h Makefile
        $(COMPILER) -c fmb3dt.c $(BUILD_ARG)

sat.o : ../SAT/sat.c ../SAT/sat.h ../Frame/frame.h Makefile
        $(COMPILER) -c ../SAT/sat.c $(BUILD_ARG)

frame.o : ../Frame/frame.c ../Frame/frame.h Makefile
        $(COMPILER) -c ../Frame/frame.c $(BUILD_ARG)

clean :
        rm -f *.o main unitTests validation qualification

valgrind :
        valgrind -v --track-origins=yes --leak-check=full \
        --gen-suppressions=yes --show-leak-kinds=all ./main
```

References

[1] J.J.-B. Fourier. Oeuvres II. Paris, 1890

[2] T.S. Motzkin. *Beiträge zur Theorie der linearen Ungleichungen*. Thesis, 1936. Reprinted in: *Theodore S. Motzkin: selected papers* (D.Cantor et al., eds,), Birkhäuser, Boston, 1983.