The FMB Algorithm

P. Baillehache

January 19, 2020

Abstract

This paper introduces how to perform intersection detection of pair of static/dynamic cuboid/tetrahedron in 2D/3D by using the Fourier-Motzkin elimination method.

Contents

Introduction

This paper introduces the FMB (Fourier-Motzkin-Baillehache) algorithm which can be used to perform intersection detection of moving and resting parallelepipeds and triangles in 2D, and cuboids and tetrahedrons in 3D.

The detection result is returned has a boolean (intersection / no intersection), and if there is intersection, a bounding box of the intersection.

The two first sections introduce how the problem can be expressed as a system of linear inequation, and its resolution using the Fourier-Motzkin method.

The algorithm of the solution and its implementation in the C programming language are detailed in the three following sections.

The last three sections introduce the validation and qualification in term of relative performance of the FMB algorithm against the SAT algorithm.

1 The problem as a system of linear inequations

1.1 Notations and definitions

- $[M]_{r,c}$ is the component at column c and row r of the matrix M
- $[V]_r$ is the r-th component of the vector $\overrightarrow{V}$
- the term "Frame" is used indifferently for parallelepiped, triangle, cuboid and tetrahedron.

1.2 Static case

The two Frames are represented as a vector origin and a number of component vectors equal to the dimension D of the space where live the Frames. Each vector is of dimension equal to D.

Let's call $\mathbb{A}$ and $\mathbb{B}$ the two Frames tested for intersection. If $\mathbb{A}$ and $\mathbb{B}$ are two cuboids:

$$\mathbb{A} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{A}}.\overrightarrow{X} \end{array} \right\} \tag{1}$$

$$\mathbb{B} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \overrightarrow{O_{\mathbb{B}}} + C_{\mathbb{B}}.\overrightarrow{X} \end{array} \right\} \tag{2}$$

where $\overrightarrow{O_{\mathbb{A}}}$ is the origin of $\mathbb{A}$ and $C_{\mathbb{A}}$ is the matrix of the components of $\mathbb{A}$ (one component per column). Idem for $\overrightarrow{O_{\mathbb{B}}}$ and $C_{\mathbb{B}}$.

If $\mathbb{A}$ and $\mathbb{B}$ are two tetrahedrons:

$$\mathbb{A} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{A}}.\overrightarrow{X} \end{array} \right\} \tag{3}$$

$$\mathbb{B} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ \overrightarrow{O_{\mathbb{B}}} + C_{\mathbb{B}}.\overrightarrow{X} \end{array} \right\} \tag{4}$$

I'll assume the Frames are well formed, i.e. their components matrix is invertible. It is then possible to express $\mathbb{B}$ in $\mathbb{A}$'s coordinates system, noted as $\mathbb{B}_{\mathbb{A}}$. If $\mathbb{B}$ is a cuboid:

$$\mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ C_{\mathbb{A}}^{-1}.(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}) \end{array} \right\} \tag{5}$$

If $\mathbb{B}$ is a tetrahedron:

$$\mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ C_{\mathbb{A}}^{-1}.(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}) \end{array} \right\} \tag{6}$$

$\mathbb{A}$ in its own coordinates system becomes, for a cuboid:

$$\mathbb{A}_{\mathbb{A}} = \left\{ \overrightarrow{X} \in [0.0, 1.0]^D \right\} \tag{7}$$

and for a tetrahedron:

$$\mathbb{A}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \end{array} \right\} \tag{8}$$

The intersection of $\mathbb{A}$ and $\mathbb{B}$ in $\mathbb{A}$'s coordinates sytem, $\mathbb{A}_{\mathbb{A}} \cap \mathbb{B}_{\mathbb{A}}$,can then be expressed as follow.

If $\mathbb{A}$ and $\mathbb{B}$ are two cuboids:

$$\mathbb{A}_{\mathbb{A}} \cap \mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}\right) \cap [0.0, 1.0]^D \end{array} \right\} \tag{9}$$

If $\mathbb{A}$ is a cuboid and $\mathbb{B}$ is a tetrahedron:

$$\mathbb{A}_{\mathbb{A}} \cap \mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}\right) \cap [0.0, 1.0]^D \end{array} \right\} \tag{10}$$

If $\mathbb{A}$ is a tetrahedron and $\mathbb{B}$ is a cuboid:

$$\mathbb{A}_{\mathbb{A}} \cap \mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}\right) \cap [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} \left[C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}\right)\right]_i \leq 1.0 \end{array} \right\} \tag{11}$$

If $\mathbb{A}$ and $\mathbb{B}$ are two tetrahedrons:

$$\mathbb{A}_{\mathbb{A}} \cap \mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ C_{\mathbb{A}}^{-1}.(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}) \cap [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} \left[C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X}\right)\right]_i \leq 1.0 \end{array} \right\} \tag{12}$$

These can in turn be expressed as systems of linear inequations as follow, given the two shortcuts $\overrightarrow{O_{\mathbb{B}_{\mathbb{A}}}} = C_{\mathbb{A}}^{-1}.(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}})$ and $C_{\mathbb{B}_{\mathbb{A}}} = C_{\mathbb{A}}^{-1}.C_{\mathbb{B}}$.

If $\mathbb{A}$ and $\mathbb{B}$ are two cuboids:

$$\left\{\begin{array}{rcl} [X]_0 & \leq & 1.0 \\ \ldots & & \\ [X]_{D-1} & \leq & 1.0 \\ -[X]_0 & \leq & 0.0 \\ \ldots & & \\ -[X]_{D-1} & \leq & 0.0 \\ \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{0,i} . [X]_i & \leq & 1.0 - [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{D-1,i} . [X]_i & \leq & 1.0 - [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ -\sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{0,i} . [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ -\sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{D-1,i} . [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \end{array}\right. \tag{13}$$

If $\mathbb{A}$ is a cuboid and $\mathbb{B}$ is a tetrahedron:

$$\left\{\begin{array}{rcl} -[X]_0 & \leq & 0.0 \\ \ldots & & \\ -[X]_{D-1} & \leq & 0.0 \\ \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{0,i} . [X]_i & \leq & 1.0 - [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{D-1,i} . [X]_i & \leq & 1.0 - [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ -\sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{0,i} . [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ -\sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{D-1,i} . [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ \sum_{i=0}^{D-1} [X]_i & \leq & 1.0 \end{array}\right. \tag{14}$$

If $\mathbb{A}$ is a tetrahedron and $\mathbb{B}$ is a cuboid:

$$\left\{\begin{array}{rcl} [X]_0 & \leq & 1.0 \\ \ldots & & \\ [X]_{D-1} & \leq & 1.0 \\ -[X]_0 & \leq & 0.0 \\ \ldots & & \\ -[X]_{D-1} & \leq & 0.0 \\ -\sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{0,i} . [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \ldots & & \\ -\sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{D-1,i} . [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ \sum_{j=0}^{D-1} \left(\left(\sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{j,i} \right) . [X]_i \right) & \leq & 1.0 - \sum_{j=0}^{D-1} [O_{\mathbb{B}_\mathbb{A}}]_j \end{array}\right. \tag{15}$$

If $\mathbb{A}$ and $\mathbb{B}$ are two tetrahedrons:

$$\left\{\begin{array}{rcl} -[X]_0 & \leq & 0.0 \\ & \ldots & \\ -[X]_{D-1} & \leq & 0.0 \\ -\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{0,i}\cdot[X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ & \ldots & \\ -\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{D-1,i}\cdot[X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ \sum_{i=0}^{D-1}[X]_i & \leq & 1.0 \\ \sum_{j=0}^{D-1}\left(\left(\sum_{i=0}^{D-1}[C_{\mathbb{B}_\mathbb{A}}]_{j,i}\right).[X]_i\right) & \leq & 1.0-\sum_{j=0}^{D-1}[O_{\mathbb{B}_\mathbb{A}}]_j \end{array}\right. \tag{16}$$

1.3 Dynamic case

If the frames $\mathbb{A}$ and $\mathbb{B}$ are moving linearly along the vectors $\overrightarrow{V_\mathbb{A}}$ and $\overrightarrow{V_\mathbb{B}}$ respectively during the interval of time $t \in [0.0, 1.0]$, the above definition of the problem is modified as follow.

If $\mathbb{A}$ and $\mathbb{B}$ are two cuboids:

$$\mathbb{A} = \left\{\begin{array}{c} \overrightarrow{X} \in [0.0,1.0]^D \\ t \in [0.0,1.0] \\ \overrightarrow{O_\mathbb{A}} + C_\mathbb{A}.\overrightarrow{X} + \overrightarrow{V_\mathbb{A}}.t \end{array}\right\} \tag{17}$$

$$\mathbb{B} = \left\{\begin{array}{c} \overrightarrow{X} \in [0.0,1.0]^D \\ t \in [0.0,1.0] \\ \overrightarrow{O_\mathbb{B}} + C_\mathbb{B}.\overrightarrow{X} + \overrightarrow{V_\mathbb{B}}.t \end{array}\right\} \tag{18}$$

where $\overrightarrow{O_\mathbb{A}}$ is the origin of $\mathbb{A}$ and $C_\mathbb{A}$ is the matrix of the components of $\mathbb{A}$ (one component per column). Idem for $\overrightarrow{O_\mathbb{B}}$ and $C_\mathbb{B}$.

If $\mathbb{A}$ and $\mathbb{B}$ are two tetrahedrons:

$$\mathbb{A} = \left\{\begin{array}{c} \overrightarrow{X} \in [0.0,1.0]^D \\ t \in [0.0,1.0] \\ \sum_{i=0}^{D-1}[X]_i \leq 1.0 \\ \overrightarrow{O_\mathbb{A}} + C_\mathbb{A}.\overrightarrow{X} + \overrightarrow{V_\mathbb{A}}.t \end{array}\right\} \tag{19}$$

$$\mathbb{B} = \left\{\begin{array}{c} \overrightarrow{X} \in [0.0,1.0]^D \\ t \in [0.0,1.0] \\ \sum_{i=0}^{D-1}[X]_i \leq 1.0 \\ \overrightarrow{O_\mathbb{B}} + C_\mathbb{B}.\overrightarrow{X} + \overrightarrow{V_\mathbb{B}}.t \end{array}\right\} \tag{20}$$

If $\mathbb{B}$ is a cuboid, $\mathbb{B}_{\mathbb{A}}$ becomes:

$$\mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right) \end{array} \right\} \tag{21}$$

If $\mathbb{B}$ is a tetrahedron, $\mathbb{B}_{\mathbb{A}}$ becomes:

$$\mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right) \end{array} \right\} \tag{22}$$

$\mathbb{A}$ in its own coordinates system has the same definition as in the static case. For a cuboid:

$$\mathbb{A}_{\mathbb{A}} = \left\{ \overrightarrow{X} \in [0.0, 1.0]^D \right\} \tag{23}$$

and for a tetrahedron:

$$\mathbb{A}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \end{array} \right\} \tag{24}$$

The intersection of $\mathbb{A}$ and $\mathbb{B}$ in $\mathbb{A}$'s coordinates sytem, $\mathbb{A}_{\mathbb{A}} \cap \mathbb{B}_{\mathbb{A}}$, can then be expressed as follow.

If $\mathbb{A}$ and $\mathbb{B}$ are two cuboids:

$$\mathbb{A}_{\mathbb{A}} \cap \mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right) \cap [0.0, 1.0]^D \end{array} \right\} \tag{25}$$

If $\mathbb{A}$ is a cuboid and $\mathbb{B}$ is a tetrahedron:

$$\mathbb{A}_{\mathbb{A}} \cap \mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right) \cap [0.0, 1.0]^D \end{array} \right\} \tag{26}$$

If $\mathbb{A}$ is a tetrahedron and $\mathbb{B}$ is a cuboid:

$$\mathbb{A}_{\mathbb{A}} \cap \mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right) \cap [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} \left[C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right)\right]_i \leq 1.0 \end{array} \right\} \tag{27}$$

If $\mathbb{A}$ and $\mathbb{B}$ are two tetrahedrons:

$$\mathbb{A}_{\mathbb{A}} \cap \mathbb{B}_{\mathbb{A}} = \left\{ \begin{array}{c} \overrightarrow{X} \in [0.0, 1.0]^D \\ t \in [0.0, 1.0] \\ \sum_{i=0}^{D-1} [X]_i \leq 1.0 \\ C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right) \cap [0.0, 1.0]^D \\ \sum_{i=0}^{D-1} \left[C_{\mathbb{A}}^{-1}.\left(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}} + C_{\mathbb{B}}.\overrightarrow{X} + \left(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}}\right).t\right)\right]_i \leq 1.0 \end{array} \right\} \tag{28}$$

These lead to the following systems of linear inequations, given the three shortcuts $\overrightarrow{O_{\mathbb{B}_{\mathbb{A}}}} = C_{\mathbb{A}}^{-1}.(\overrightarrow{O_{\mathbb{B}}} - \overrightarrow{O_{\mathbb{A}}})$, $\overrightarrow{V_{\mathbb{B}_{\mathbb{A}}}} = C_{\mathbb{A}}^{-1}.(\overrightarrow{V_{\mathbb{B}}} - \overrightarrow{V_{\mathbb{A}}})$ and $C_{\mathbb{B}_{\mathbb{A}}} = C_{\mathbb{A}}^{-1}.C_{\mathbb{B}}$.

If $\mathbb{A}$ and $\mathbb{B}$ are two cuboids:

$$\left\{ \begin{array}{rcl} t & \leq & 1.0 \\ -t & \leq & 0.0 \\ [X]_0 & \leq & 1.0 \\ \ldots & & \\ [X]_{D-1} & \leq & 1.0 \\ -[X]_0 & \leq & 0.0 \\ \ldots & & \\ -[X]_{D-1} & \leq & 0.0 \\ [V_{\mathbb{B}_{\mathbb{A}}}]_0 .t + \sum_{i=0}^{D-1} [C_{\mathbb{B}_{\mathbb{A}}}]_{0,i} [X]_i & \leq & 1.0 - [O_{\mathbb{B}_{\mathbb{A}}}]_0 \\ \ldots & & \\ [V_{\mathbb{B}_{\mathbb{A}}}]_{D-1} .t + \sum_{i=0}^{D-1} [C_{\mathbb{B}_{\mathbb{A}}}]_{D-1,i} [X]_i & \leq & 1.0 - [O_{\mathbb{B}_{\mathbb{A}}}]_{D-1} \\ -[V_{\mathbb{B}_{\mathbb{A}}}]_0 .t - \sum_{i=0}^{D-1} [C_{\mathbb{B}_{\mathbb{A}}}]_{0,i} [X]_i & \leq & [O_{\mathbb{B}_{\mathbb{A}}}]_0 \\ \ldots & & \\ -[V_{\mathbb{B}_{\mathbb{A}}}]_{D-1} .t - \sum_{i=0}^{D-1} [C_{\mathbb{B}_{\mathbb{A}}}]_{D-1,i} [X]_i & \leq & [O_{\mathbb{B}_{\mathbb{A}}}]_{D-1} \end{array} \right. \tag{29}$$

If $\mathbb{A}$ is a cuboid and $\mathbb{B}$ is a tetrahedron:

$$\left\{ \begin{array}{rcl} t & \leq & 1.0 \\ -t & \leq & 0.0 \\ -[X]_0 & \leq & 0.0 \\ \ldots & & \\ -[X]_{D-1} & \leq & 0.0 \\ [V_{\mathbb{B}_{\mathbb{A}}}]_0 .t + \sum_{i=0}^{D-1} [C_{\mathbb{B}_{\mathbb{A}}}]_{0,i} [X]_i & \leq & 1.0 - [O_{\mathbb{B}_{\mathbb{A}}}]_0 \\ \ldots & & \\ [V_{\mathbb{B}_{\mathbb{A}}}]_{D-1} .t + \sum_{i=0}^{D-1} [C_{\mathbb{B}_{\mathbb{A}}}]_{D-1,i} [X]_i & \leq & 1.0 - [O_{\mathbb{B}_{\mathbb{A}}}]_{D-1} \\ -[V_{\mathbb{B}_{\mathbb{A}}}]_0 .t - \sum_{i=0}^{D-1} [C_{\mathbb{B}_{\mathbb{A}}}]_{0,i} [X]_i & \leq & [O_{\mathbb{B}_{\mathbb{A}}}]_0 \\ \ldots & & \\ -[V_{\mathbb{B}_{\mathbb{A}}}]_{D-1} .t - \sum_{i=0}^{D-1} [C_{\mathbb{B}_{\mathbb{A}}}]_{D-1,i} [X]_i & \leq & [O_{\mathbb{B}_{\mathbb{A}}}]_{D-1} \\ \sum_{i=0}^{D-1} [X]_i & \leq & 1.0 \end{array} \right. \tag{30}$$

If $\mathbb{A}$ is a tetrahedron and $\mathbb{B}$ is a cuboid:

$$\left\{\begin{array}{rcl} t & \leq & 1.0 \\ -t & \leq & 0.0 \\ [X]_0 & \leq & 1.0 \\ \dots & & \\ [X]_{D-1} & \leq & 1.0 \\ -[X]_0 & \leq & 0.0 \\ \dots & & \\ -[X]_{D-1} & \leq & 0.0 \\ -[V_{\mathbb{B}_\mathbb{A}}]_0 .t - \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{0,i} [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \dots & & \\ -[V_{\mathbb{B}_\mathbb{A}}]_{D-1} .t - \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{D-1,i} [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ \sum_{j=0}^{D-1} \left([V_{\mathbb{B}_\mathbb{A}}]_j .t + \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{j,i} [X]_i \right) & \leq & 1.0 - \sum_{j=0}^{D-1} [O_{\mathbb{B}_\mathbb{A}}]_j \end{array}\right. \tag{31}$$

If $\mathbb{A}$ and $\mathbb{B}$ are two tetrahedrons:

$$\left\{\begin{array}{rcl} t & \leq & 1.0 \\ -t & \leq & 0.0 \\ -[X]_0 & \leq & 0.0 \\ \dots & & \\ -[X]_{D-1} & \leq & 0.0 \\ -[V_{\mathbb{B}_\mathbb{A}}]_0 .t - \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{0,i} [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_0 \\ \dots & & \\ -[V_{\mathbb{B}_\mathbb{A}}]_{D-1} .t - \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{D-1,i} [X]_i & \leq & [O_{\mathbb{B}_\mathbb{A}}]_{D-1} \\ \sum_{i=0}^{D-1} [X]_i & \leq & 1.0 \\ \sum_{j=0}^{D-1} \left([V_{\mathbb{B}_\mathbb{A}}]_j .t + \sum_{i=0}^{D-1} [C_{\mathbb{B}_\mathbb{A}}]_{j,i} [X]_i \right) & \leq & 1.0 - \sum_{j=0}^{D-1} [O_{\mathbb{B}_\mathbb{A}}]_j \end{array}\right. \tag{32}$$

2 Resolution of the problem by Fourier-Motzkin method

2.1 The Fourier-Motzkin elimination method

The Fourier-Motzkin elimination method has been introduced by J.J.-B. Fourier in 1827 [1], and described in the Ph.D. thesis of T.S. Motzkin in 1936 [2]. This is a generalization of the Gaussian elimination method to linear systems of inequalities. This method consists of eliminating one variable of the system and rewrite a new system accordingly. Then the elimination operation is repeated on another variable in the new system, and so on until we obtain a trivial system with only one variable. From there, a solution

for each variable can be obtained if it exists. The variable elimination is performed as follow.
Lets write the linear system $\mathcal{I}$ of m inequalities and n variables as

$$\left\{\begin{array}{llll} a_{11}.x_1 & +a_{12}.x_2 & +\cdots & +a_{1n}.x_n \quad \leq b_1 \\ a_{21}.x_1 & +a_{22}.x_2 & +\cdots & +a_{2n}.x_n \quad \leq b_2 \\ & & \vdots & \\ a_{m1}.x_1 & +a_{m2}.x_2 & +\cdots & +a_{mn}.x_n \quad \leq b_m \end{array}\right. \tag{33}$$

with

$$\begin{array}{c} i \in 1, 2, ..., m \\ j \in 1, 2, ..., n \\ x_i \in \mathbb{R} \\ a_{ij} \in \mathbb{R} \\ b_j \in \mathbb{R} \end{array} \tag{34}$$

To eliminate the first variable x_1, lets multiply each inequality by $1.0/|a_{i1}|$ where $a_{i1} \neq 0.0$. The system becomes

$$\left\{\begin{array}{llll} x_1 & +a'_{i2}.x_2 & +\cdots & +a'_{in}.x_n \quad \leq b'_i \qquad (i \in \mathcal{I}_+) \\ & a_{i2}.x_2 & +\cdots & +a_{in}.x_n \quad \leq b_i \qquad (i \in \mathcal{I}_0) \\ -x_1 & +a'_{i2}.x_2 & +\cdots & +a'_{in}.x_n \quad \leq b'_i \qquad (i \in \mathcal{I}_-) \end{array}\right. \tag{35}$$

where

$$\begin{array}{c} \mathcal{I}_+ = \{i : a_{i1} > 0.0\} \\ \mathcal{I}_0 = \{i : a_{i1} = 0.0\} \\ \mathcal{I}_- = \{i : a_{i1} < 0.0\} \\ a'_{ij} = a_{ij}/|a_{i1}| \\ b'_i = b_i/|a_{i1}| \end{array}$$

Then $x_1, x_2, \cdots, x_n \in \mathbb{R}^n$ is a solution of $\mathcal{I}$ if and only if

$$\left\{\begin{array}{ll} \sum_{j=2}^{n}((a'_{kj} + a'_{lj}).x_j) \leq b'_k + b'_l & (k \in \mathcal{I}_+, l \in \mathcal{I}_-) \\ \sum_{j=2}^{n}(a_{ij}.x_j) \leq b_i & i \in \mathcal{I}_0 \end{array}\right. \tag{36}$$

and

$$\max_{l \in \mathcal{I}_-}(\sum_{j=2}^{n}(a'_{lj}.x_j) - b'_l) \leq x_1 \leq \min_{k \in \mathcal{I}_+}(b'_k - \sum_{j=2}^{n}(a'_{kj}.x_j)) \tag{37}$$

The same method is then applied on this new system to eliminate the second variable x_2, and so on until we reach the inequality

$$\max_{l \in \mathcal{I}''^{\cdots'}_-}(-b''^{\cdots'}_l) \leq x_n \leq \min_{k \in \mathcal{I}''^{\cdots'}_+}(b''^{\cdots'}_k) \tag{38}$$

If this inequality has no solution, then neither the system $\mathcal{I}$. If it has a solution, the minimum and maximum are the bounding values for the variable x_n. One can get a particular solution to the system $\mathcal{I}$ by choosing a value for x_n between these bounding values, which allows to set a particular value for the variable x_{n-1}, and so on back up to x_1.

2.2 Application of the Fourier-Motzkin method to the intersection problem

The Fourier-Motzkin method can be directly applied to the inequality systems of the previous section, to obtain the bounding box of the intersection, if the system has a solution. If the system has no solution, the method will eventually reach an inconsistent inequality, meaning there is no intersection between the two Frames.

One coordinate $\overrightarrow{S}$, or $(\overrightarrow{S}, t)$ in dynamic case, within the bounds obtained by the resolution of the system is expressed in the Frame $\mathbb{B}$'s coordinates system. One can get the equivalent coordinates $\overrightarrow{S'}$, or $(\overrightarrow{S}, t)$, in the real world's coordinates system as follow:

$$\overrightarrow{S'} = \overrightarrow{O_{\mathbb{B}}} + C_{\mathbb{B}}.\overrightarrow{S} \tag{39}$$

$$(\overrightarrow{S'}, t) = \left(\overrightarrow{O_{\mathbb{B}}} + C_{\mathbb{B}}.\overrightarrow{S} + \overrightarrow{V}_{\mathbb{B}}.t, t\right) \tag{40}$$

Only one inconsistent inequality is sufficient to prove the absence of solution, and then the non intersection of the Frames. Thus, one shall check the inconsistence of each inequality as soon as possible during the resolution of the system to optimize the speed of the algorithm.

A sufficient condition for one inequality $\sum_i a_i X_i \leq Y$ to be inconsistent is, given that $\forall i, X_i \in [0.0, 1.0]$:

$$Y < \sum_{i \in I^-} a_i \tag{41}$$

where $I^- = \{i, a_i < 0.0\}$.

2.3 About the size of the system of linear inequations

During implementation in languages where the developper needs to manage memory itself the size of the systems (35) resulting from variable elimination

is necessary but cannot be forecasted. Instead, a maximum size can be calculated as follow.

Let's call n_-, n_+ and n_0, each in $[0, \mathbb{N}]$, the size of, respectively, $\mathcal{I}_-$, $\mathcal{I}_+$ and $\mathcal{I}_0$, and N the number of inequalities in the original system and N' the number inequalities in the resulting system. We have:

$$n_- + n_+ + n_0 = N \tag{42}$$

and

$$n_-.n_+ + n_0 = N' \tag{43}$$

Now let's define $K = N - n_0$, then we have:

$$n_- + n_+ = K \tag{44}$$

then,

$$n_-.n_+ = n_-.(K - n_-) \tag{45}$$

then,

$$n_-.n_+ = K.n_- - n_-^2 \tag{46}$$

The right part is a polynomial whose maximum is reached for $n_- = K/2$. Then,

$$n_-.n_+ \leq K^2/2 - K^2/4 \tag{47}$$

or,

$$n_-.n_+ \leq K^2/4 \tag{48}$$

and putting back the definition of K

$$n_-.n_+ \leq (N - n_0)^2/4 \tag{49}$$

which is also

$$n_-.n_+ \leq N^2/4 \tag{50}$$

From (43) we get,

$$N' \leq N^2/4 + n_0 \tag{51}$$

and finally,

$$N' \leq N^2/4 + N \tag{52}$$

The maximum number of inequations in the initial system is defined for each case (2D/3D, static/dynamic) in the previous section. This leads to the following maximum number of inequations:

	N	N'	N''	N'''
$2Dstatic$	8	24		
$2Ddynamic$	10	35	342	
$3Dstatic$	12	48	624	
$3Ddynamic$	14	63	1056	279840

However, these values are much higher than the ones encountered in the case of the systems correpsonding to the intersection problem. It can be noticed that n_0 can be better estimated as the inequations corresponding to the constraints $0.0 \leq x \leq 1.0$ leads to, for N', $n_0 \in \{D-1, 2(D-1)\}$ in static case and $n_0 \in \{D+1, 2D+1\}$ in dynamic case. Thus we can reduce N' to:

	N	N'
$2Dstatic$	8	14
$2Ddynamic$	10	16
$3Dstatic$	12	27
$3Ddynamic$	14	29

and so on for N'' and N'''. In pratice, the maximum number of inequations encountered during validation were:

	N	N'	N''	N'''
$2Dstatic$	8	11		
$2Ddynamic$	10	13	21	
$3Dstatic$	12	20	55	
$3Ddynamic$	14	22	57	560

3 Algorithms of the solution

In this section I introduce the algorithms of the solution of the previous section for each case (static/dynamic and 2D/3D), and the algorithms to manipulate the structure used to represent the Frames.

Algorithms are given in pseudo code, and consequently without any optimization based on properties of one given language. One can refer to the C implementation in the following section for possible optimization in this language.

Algorithms are also given independantly from each other. Code commonalization may be possible if one plans to use several cases together, but this

is dependant of the implementation and thus left to the developper responsibility.

3.1 2D static

```
ENUM FrameType
  FrameCuboid,
  FrameTetrahedron
END ENUM

STRUCT AABB2D
  // x,y
  real min[2]
  real max[2]
END STRUCT

STRUCT Frame2D
  FrameType type
  real orig[2]
  // comp[iComp][iAxis]
  real comp[2][2]
  AABB2D bdgBox
  real invComp[2][2]
END STRUCT

FUNCTION powi(base, exp)
  res = 1
  FOR i=0 TO (exp - 1)
    res = res * base
  END FOR
  RETURN res
END FUNCTION

FUNCTION Frame2DPrint(that)
  IF that.type == FrameTetrahedron
    PRINT "T"
  ELSE IF that.type == FrameCuboid
    PRINT "C"
  END IF
  PRINT "o("
  FOR i = 0 TO 1
    PRINT that.orig[i]
    IF i < 1
      PRINT ","
    END IF
  END FOR
  comp = ["x","y"]
  FOR j = 0 TO 1
    PRINT ") ", comp[j], "("
    FOR i = 0 TO 1
      PRINT that.comp[j][i]
      IF i < 1
        PRINT ","
      END IF
    END FOR
  END FOR
  PRINT ")"
END FUNCTION
```

```
FUNCTION AABB2DPrint(that)
  PRINT "minXY("
  FOR i = 0 TO 1
    PRINT that.min[i]
    IF i < 1
      PRINT ","
    END IF
  END FOR
  PRINT ")-maxXY("
  FOR i = 0 TO 1
    PRINT that.max[i]
    IF i < 1
      PRINT ","
    END IF
  END FOR
  PRINT ")"
END FUNCTION

FUNCTION Frame2DExportBdgBox(that, bdgBox, bdgBoxProj)
  FOR i = 0 TO 1
    bdgBoxProj.max[i] = that.orig[i]
    FOR j = 0 TO 1
      bdgBoxProj.max[i] =
        bdgBoxProj.max[i] + that.comp[j][i] * bdgBox.min[j]
    END FOR
    bdgBoxProj.min[i] = bdgBoxProj.max[i]
  END FOR
  nbVertices = powi(2, 2)
  FOR iVertex = 1 TO (nbVertices - 1)
    FOR i = 0 TO 1
      IF BITWISEAND(iVertex, powi(2, i)) <> 0
        v[i] = bdgBox.max[i]
      ELSE
        v[i] = bdgBox.min[i]
      END IF
    END FOR
    FOR i = 0 TO 1
      w[i] = that.orig[i]
      FOR j = 0 TO 1
        w[i] = w[i] + that.comp[j][i] * v[j]
      END FOR
    END FOR
    FOR i = 0 TO 1
      IF bdgBoxProj.min[i] > w[i]
        bdgBoxProj.min[i] = w[i]
      END IF
      IF bdgBoxProj.max[i] < w[i]
        bdgBoxProj.max[i] = w[i]
      END IF
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame2DImportFrame(P, Q, Qp)
  FOR i = 0 TO 1
    v[i] = Q.orig[i] - P.orig[i]
  END FOR
  FOR i = 0 TO 1
    Qp.orig[i] = 0.0
    FOR j = 0 TO 1
      Qp.orig[i] = Qp.orig[i] + P.invComp[j][i] * v[j]
```

```
      Qp.comp[j][i] = 0.0
      FOR k = 0 TO 1
        Qp.comp[j][i] = Qp.comp[j][i] + P.invComp[k][i] * Q.comp[j][k]
      END FOR
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame2DUpdateInv(that)
  det = that.comp[0][0] * that.comp[1][1] -
    that.comp[1][0] * that.comp[0][1]
  that.invComp[0][0] = that.comp[1][1] / det
  that.invComp[0][1] = -that.comp[0][1] / det
  that.invComp[1][0] = -that.comp[1][0] / det
  that.invComp[1][1] = that.comp[0][0] / det
END FUNCTION

FUNCTION Frame2DCreateStatic(type, orig, comp)
  that.type = type
  FOR iAxis = 0 TO 1
    that.orig[iAxis] = orig[iAxis]
    FOR iComp = 0 TO 1
      that.comp[iComp][iAxis] = comp[iComp][iAxis]
    END FOR
  END FOR
  FOR iAxis = 0 TO 1
    min = orig[iAxis]
    max = orig[iAxis]
    FOR iComp = 0 TO 1
      IF that.type == FrameCuboid
        IF that.comp[iComp][iAxis] < 0.0
          min = min + that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0
          max = max + that.comp[iComp][iAxis]
        END IF
      ELSE IF that.type == FrameTetrahedron
        IF that.comp[iComp][iAxis] < 0.0 AND
          min > orig[iAxis] + that.comp[iComp][iAxis]
          min = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0 AND
          max < orig[iAxis] + that.comp[iComp][iAxis]
          max = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
      END IF
    END FOR
    that.bdgBox.min[iAxis] = min
    that.bdgBox.max[iAxis] = max
  END FOR
  Frame2DUpdateInv(that)
  RETURN that
END FUNCTION

FUNCTION Sgn(v)
  IF 0.0 < v
    a = 1
  ELSE
    a = 0
  END IF
  IF v < 0.0
    b = 1
```

```
  ELSE
    b = 0
  END IF
  RETURN A - B
END FUNCTION

FUNCTION Neg(x)
  IF x < 0.0
    RETURN x
  ELSE
    RETURN 0.0
  END IF
END FUNCTION

FST_VAR = 0
SND_VAR = 1

FUNCTION ElimVar2D(M, Y, nbRows, nbCols, Mp, Yp, nbRemainRows)
  nbRemainRows = 0
  FOR iRow = 0 TO (nbRows - 2)
    IF M[iRow][0] <> 0.0
      FOR jRow = (iRow + 1) TO (nbRows - 1)
        IF sgn(M[iRow][0]) <> sgn(M[jRow][0]) AND
            M[jRow][0] <> 0.0
          sumNegCoeff = 0.0
          jCol = 0
          FOR iCol = 1 TO (nbCols - 1)
            Mp[nbRemainRows][jCol] =
              M[iRow][iCol] / ABS(M[iRow][0]) +
              M[jRow][iCol] / ABS(M[jRow][0])
            sumNegCoeff = sumNegCoeff + neg(Mp[nbRemainRows][jCol])
            jCol = jCol + 1
          END FOR
          Yp[nbRemainRows] =
            Y[iRow] / ABS(M[iRow][0]) +
            Y[jRow] / ABS(M[jRow][0])
          IF Yp[nbRemainRows] < sumNegCoeff
            RETURN TRUE
          END IF
          nbRemainRows = nbRemainRows + 1
        END IF
      END FOR
    END IF
  END FOR
  FOR iRow = 0 TO (nbRows - 1)
    IF M[iRow][0] == 0.0
      jCol = 0
      FOR iCol = 1 TO (nbCols - 1)
        Mp[nbRemainRows][jCol] = M[iRow][iCol]
        jCol = jCol + 1
      END FOR
      Yp[nbRemainRows] = Y[iRow]
      nbRemainRows = nbRemainRows + 1
    END IF
  END FOR
  RETURN FALSE
END FUNCTION

FUNCTION GetBoundLastVar2D(iVar, M, Y, nbRows, bdgBox)
  bdgBox.min[iVar] = 0.0
  bdgBox.max[iVar] = 1.0
  FOR jRow = 0 TO (nbRows - 1)
```

```
    IF M[jRow][0] > 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.max[iVar] > y
        bdgBox.max[iVar] = y
      END IF
    ELSE IF M[jRow][0] < 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.min[iVar] < y
        bdgBox.min[iVar] = y
      END IF
    END IF
  END FOR
END FUNCTION

FUNCTION GetBoundVar2D(iVar, M, Y, nbRows, nbCols, bdgBox)
  bdgBox.min[iVar] = 0.0
  bdgBox.max[iVar] = 1.0
  FOR iRow = 0 .. TO (nbRows - 1)
    IF M[iRow][0] <> 0.0
      min = -1.0 * Y[iRow]
      max = Y[iRow]
      FOR iCol = 1 .. TO (nbCols - 1)
        IF M[iRow][iCol] > 0.0
          min = min + M[iRow][iCol] * bdgBox.min[iCol + iVar]
          max = max - M[iRow][iCol] * bdgBox.min[iCol + iVar]
        ELSE IF M[iRow][iCol] < 0.0
          min = min + M[iRow][iCol] * bdgBox.max[iCol + iVar]
          max = max - M[iRow][iCol] * bdgBox.max[iCol + iVar]
        END IF
      END FOR
      min = min / (-1.0 * M[iRow][0])
      max = max / M[iRow][0]
      IF bdgBox.min[iVar] > min
        bdgBox.min[iVar] = min
      END IF
      IF bdgBox.max[iVar] < max
        bdgBox.max[iVar] = max
      END IF
    END IF
  END FOR
END FUNCTION

FUNCTION FMBTestIntersection2D(that, tho, bdgBox)
  Frame2DImportFrame(that, tho, thoProj)
  M[0][0] = -thoProj.comp[0][0]
  M[0][1] = -thoProj.comp[1][0]
  Y[0] = thoProj.orig[0]
  IF Y[0] < neg(M[0][0]) + neg(M[0][1])
    RETURN FALSE
  END IF
  M[1][0] = -thoProj.comp[0][1]
  M[1][1] = -thoProj.comp[1][1]
  Y[1] = thoProj.orig[1]
  IF Y[1] < neg(M[1][0]) + neg(M[1][1])
    RETURN FALSE
  END IF
  nbRows = 2
  IF that.type == FrameCuboid
    M[nbRows][0] = thoProj.comp[0][0]
    M[nbRows][1] = thoProj.comp[1][0]
    Y[nbRows] = 1.0 - thoProj.orig[0]
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1])
```

```
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
    M[nbRows][0] = thoProj.comp[0][1]
    M[nbRows][1] = thoProj.comp[1][1]
    Y[nbRows] = 1.0 - thoProj.orig[1]
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1])
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
  ELSE
    M[nbRows][0] = thoProj.comp[0][0] + thoProj.comp[0][1]
    M[nbRows][1] = thoProj.comp[1][0] + thoProj.comp[1][1]
    Y[nbRows] = 1.0 - thoProj.orig[0] - thoProj.orig[1]
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1])
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
  END IF
  IF tho.type == FrameCuboid
    M[nbRows][0] = 1.0
    M[nbRows][1] = 0.0
    Y[nbRows] = 1.0
    nbRows = nbRows + 1
    M[nbRows][0] = 0.0
    M[nbRows][1] = 1.0
    Y[nbRows] = 1.0
    nbRows = nbRows + 1
  ELSE
    M[nbRows][0] = 1.0
    M[nbRows][1] = 1.0
    Y[nbRows] = 1.0
    nbRows = nbRows + 1
  END IF
  M[nbRows][0] = -1.0
  M[nbRows][1] = 0.0
  Y[nbRows] = 0.0
  nbRows = nbRows + 1
  M[nbRows][0] = 0.0
  M[nbRows][1] = -1.0
  Y[nbRows] = 0.0
  nbRows = nbRows + 1
  inconsistency = ElimVar2D(M, Y, nbRows, 2, Mp, Yp, nbRowsP)
  IF inconsistency == TRUE
    RETURN FALSE
  END IF
  GetBoundLastVar2D(SND_VAR, Mp, Yp, nbRowsP, bdgBoxLocal)
  IF bdgBoxLocal.min[SND_VAR] >= bdgBoxLocal.max[SND_VAR]
    RETURN FALSE
  ELSE
    GetBoundVar2D(FST_VAR, M, Y, nbRows, 2, bdgBoxLocal)
    bdgBox = bdgBoxLocal
  END IF
  RETURN TRUE
END FUNCTION

origP2D = [0.0, 0.0]
compP2D = [
  [1.0, 0.0],
  [0.0, 1.0]]
P2D = Frame2DCreateStatic(FrameCuboid, origP2D, compP2D)
origQ2D = [0.0, 0.0]
```

```
compQ2D = [
  [1.0, 0.0],
  [0.0, 1.0]]
Q2D = Frame2DCreateStatic(FrameCuboid, origQ2D, compQ2D)
isIntersecting2D = FMBTestIntersection2D(P2D, Q2D, bdgBox2DLocal)
IF isIntersecting2D == TRUE
  PRINT "Intersection detected."
  Frame2DExportBdgBox(Q2D, bdgBox2DLocal, bdgBox2D);
  AABB2DPrint(bdgBox2D)
ELSE
  PRINT "No intersection."
END IF
```

3.2 3D static

```
ENUM FrameType
  FrameCuboid,
  FrameTetrahedron
END ENUM

STRUCT AABB3D
  // x,y,z
  real min[3]
  real max[3]
END STRUCT

STRUCT Frame3D
  FrameType type
  real orig[3]
  // comp[iComp][iAxis]
  real comp[3][3]
  AABB3D bdgBox
  real invComp[3][3]
END STRUCT

FUNCTION powi(base, exp)
  res = 1
  FOR i=0 TO (exp - 1)
    res = res * base
  END FOR
  RETURN res
END FUNCTION

FUNCTION Frame3DPrint(that)
  IF that.type == FrameTetrahedron
    PRINT "T"
  ELSE IF that.type == FrameCuboid
    PRINT "C"
  END IF
  PRINT "o("
  FOR i = 0 TO 2
    PRINT that.orig[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  comp = ["x","y","z"]
  FOR j = 0 TO 2
    PRINT ") ", comp[j], "("
    FOR i = 0 TO 2
      PRINT that.comp[j][i]
```

```
      IF i < 2
        PRINT ","
      END IF
    END FOR
  END FOR
  PRINT ")"
END FUNCTION

FUNCTION AABB3DPrint(that)
  PRINT "minXYZ("
  FOR i = 0 TO 2
    PRINT that.min[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  PRINT ")-maxXYZ("
  FOR i = 0 TO 2
    PRINT that.max[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  PRINT ")"
END FUNCTION

FUNCTION Frame3DExportBdgBox(that, bdgBox, bdgBoxProj)
  FOR i = 0 TO 2
    bdgBoxProj.max[i] = that.orig[i]
    FOR j = 0 TO 2
      bdgBoxProj.max[i] =
        bdgBoxProj.max[i] + that.comp[j][i] * bdgBox.min[j]
    END FOR
    bdgBoxProj.min[i] = bdgBoxProj.max[i]
  END FOR
  nbVertices = powi(2, 3)
  FOR iVertex = 1 TO (nbVertices - 1)
    FOR i = 0 TO 2
      IF BITWISEAND(iVertex, powi(2, i)) <> 0
        v[i] = bdgBox.max[i]
      ELSE
        v[i] = bdgBox.min[i]
      END IF
    END FOR
    FOR i = 0 TO 2
      w[i] = that.orig[i]
      FOR j = 0 TO 2
        w[i] = w[i] + that.comp[j][i] * v[j]
      END FOR
    END FOR
    FOR i = 0 TO 2
      IF bdgBoxProj.min[i] > w[i]
        bdgBoxProj.min[i] = w[i]
      END IF
      IF bdgBoxProj.max[i] < w[i]
        bdgBoxProj.max[i] = w[i]
      END IF
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame3DImPortFrame(P, Q, Qp)
```

```
  FOR i = 0 TO 2
    v[i] = Q.orig[i] - P.orig[i]
  END FOR
  FOR i = 0 TO 2
    Qp.orig[i] = 0.0
    FOR j = 0 TO 2
      Qp.orig[i] = Qp.orig[i] + P.invComp[j][i] * v[j]
      Qp.comp[j][i] = 0.0
      FOR k = 0 TO 2
        Qp.comp[j][i] = Qp.comp[j][i] + P.invComp[k][i] * Q.comp[j][k]
      END FOR
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame3DUpdateInv(that)
  det =
    that.comp[0][0] * (that.comp[1][1] * that.comp[2][2] -
    that.comp[1][2] * that.comp[2][1]) -
    that.comp[1][0] * (that.comp[0][1] * that.comp[2][2] -
    that.comp[0][2] * that.comp[2][1]) +
    that.comp[2][0] * (that.comp[0][1] * that.comp[1][2] -
    that.comp[0][2] * that.comp[1][1])
  that.invComp[0][0] = (that.comp[1][1] * that.comp[2][2] -
    that.comp[2][1] * that.comp[1][2]) / det
  that.invComp[0][1] = (that.comp[2][1] * that.comp[0][2] -
    that.comp[2][2] * that.comp[0][1]) / det
  that.invComp[0][2] = (that.comp[0][1] * that.comp[1][2] -
    that.comp[0][2] * that.comp[1][1]) / det
  that.invComp[1][0] = (that.comp[2][0] * that.comp[1][2] -
    that.comp[2][2] * that.comp[1][0]) / det
  that.invComp[1][1] = (that.comp[0][0] * that.comp[2][2] -
    that.comp[2][0] * that.comp[0][2]) / det
  that.invComp[1][2] = (that.comp[0][2] * that.comp[1][0] -
    that.comp[1][2] * that.comp[0][0]) / det
  that.invComp[2][0] = (that.comp[1][0] * that.comp[2][1] -
    that.comp[2][0] * that.comp[1][1]) / det
  that.invComp[2][1] = (that.comp[0][1] * that.comp[2][0] -
    that.comp[2][1] * that.comp[0][0]) / det
  that.invComp[2][2] = (that.comp[0][0] * that.comp[1][1] -
    that.comp[1][0] * that.comp[0][1]) / det
END FUNCTION

FUNCTION Frame3DCreateStatic(type, orig, comp)
  that.type = type
  FOR iAxis = 0 TO 2
    that.orig[iAxis] = orig[iAxis]
    FOR iComp = 0 TO 2
      that.comp[iComp][iAxis] = comp[iComp][iAxis]
    END FOR
  END FOR
  FOR iAxis = 0 TO 2
    min = orig[iAxis]
    max = orig[iAxis]
    FOR iComp = 0 TO 2
      IF that.type == FrameCuboid
        IF that.comp[iComp][iAxis] < 0.0
          min = min + that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0
          max = max + that.comp[iComp][iAxis]
        END IF
```

```
      ELSE IF that.type == FrameTetrahedron
        IF that.comp[iComp][iAxis] < 0.0 AND
          min > orig[iAxis] + that.comp[iComp][iAxis]
          min = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0 AND
          max < orig[iAxis] + that.comp[iComp][iAxis]
          max = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
      END IF
    END FOR
    that.bdgBox.min[iAxis] = min
    that.bdgBox.max[iAxis] = max
  END FOR
  Frame3DUpdateInv(that)
  RETURN that
END FUNCTION

FUNCTION Sgn(v)
  IF 0.0 < v
    a = 1
  ELSE
    a = 0
  END IF
  IF v < 0.0
    b = 1
  ELSE
    b = 0
  END IF
  RETURN A - B
END FUNCTION

FUNCTION Neg(x)
  IF x < 0.0
    RETURN x
  ELSE
    RETURN 0.0
  END IF
END FUNCTION

FST_VAR = 0
SND_VAR = 1
THD_VAR = 2

FUNCTION ElimVar3D(M, Y, nbRows, nbCols, Mp, Yp, nbRemainRows)
  nbRemainRows = 0
  FOR iRow = 0 TO (nbRows - 2)
    IF M[iRow][0] <> 0.0
      FOR jRow = (iRow + 1) TO (nbRows - 1)
        IF sgn(M[iRow][0]) <> sgn(M[jRow][0]) AND
            M[jRow][0] <> 0.0
          sumNegCoeff = 0.0
          jCol = 0
          FOR iCol = 1 TO (nbCols - 1)
            Mp[nbRemainRows][jCol] =
              M[iRow][iCol] / ABS(M[iRow][0]) +
              M[jRow][iCol] / ABS(M[jRow][0])
            sumNegCoeff = sumNegCoeff + neg(Mp[nbRemainRows][jCol])
            jCol = jCol + 1
          END FOR
          Yp[nbRemainRows] =
            Y[iRow] / ABS(M[iRow][0]) +
```

```
            Y[jRow] / ABS(M[jRow][0])
          IF Yp[nbRemainRows] < sumNegCoeff
            RETURN TRUE
          END IF
          nbRemainRows = nbRemainRows + 1
        END IF
      END FOR
    END IF
  END FOR
  FOR iRow = 0 TO (nbRows - 1)
    IF M[iRow][0] == 0.0
      jCol = 0
      FOR iCol = 1 TO (nbCols - 1)
        Mp[nbRemainRows][jCol] = M[iRow][iCol]
        jCol = jCol + 1
      END FOR
      Yp[nbRemainRows] = Y[iRow]
      nbRemainRows = nbRemainRows + 1
    END IF
  END FOR
  RETURN FALSE
END FUNCTION

FUNCTION GetBoundLastVar3D(iVar, M, Y, nbRows, bdgBox)
  bdgBox.min[iVar] = 0.0
  bdgBox.max[iVar] = 1.0
  FOR jRow = 0 TO (nbRows - 1)
    IF M[jRow][0] > 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.max[iVar] > y
        bdgBox.max[iVar] = y
      END IF
    ELSE IF M[jRow][0] < 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.min[iVar] < y
        bdgBox.min[iVar] = y
      END IF
    END IF
  END FOR
END FUNCTION

FUNCTION GetBoundVar3D(iVar, M, Y, nbRows, nbCols, bdgBox)
  bdgBox.min[iVar] = 0.0
  bdgBox.max[iVar] = 1.0
  FOR iRow = 0 .. TO (nbRows - 1)
    IF M[iRow][0] <> 0.0
      min = -1.0 * Y[iRow]
      max = Y[iRow]
      FOR iCol = 1 .. TO (nbCols - 1)
        IF M[iRow][iCol] > 0.0
          min = min + M[iRow][iCol] * bdgBox.min[iCol + iVar]
          max = max - M[iRow][iCol] * bdgBox.min[iCol + iVar]
        ELSE IF M[iRow][iCol] < 0.0
          min = min + M[iRow][iCol] * bdgBox.max[iCol + iVar]
          max = max - M[iRow][iCol] * bdgBox.max[iCol + iVar]
        END IF
      END FOR
      min = min / (-1.0 * M[iRow][0])
      max = max / M[iRow][0]
      IF bdgBox.min[iVar] > min
        bdgBox.min[iVar] = min
      END IF
```

```
      IF bdgBox.max[iVar] < max
        bdgBox.max[iVar] = max
      END IF
    END IF
  END FOR
END FUNCTION

FUNCTION FMBTestIntersection3D(that, tho, bdgBox)
  Frame3DImportFrame(that, tho, thoProj)
  M[0][0] = -thoProj.comp[0][0]
  M[0][1] = -thoProj.comp[1][0]
  M[0][2] = -thoProj.comp[2][0]
  Y[0] = thoProj.orig[0]
  IF Y[0] < neg(M[0][0]) + neg(M[0][1]) + neg(M[0][2])
    RETURN FALSE
  END IF
  M[1][0] = -thoProj.comp[0][1]
  M[1][1] = -thoProj.comp[1][1]
  M[1][2] = -thoProj.comp[2][1]
  Y[1] = thoProj.orig[1]
  IF Y[1] < neg(M[1][0]) + neg(M[1][1]) + neg(M[1][2])
    RETURN FALSE
  END IF
  M[2][0] = -thoProj.comp[0][2]
  M[2][1] = -thoProj.comp[1][2]
  M[2][2] = -thoProj.comp[2][2]
  Y[2] = thoProj.orig[2]
  IF Y[2] < neg(M[2][0]) + neg(M[2][1]) + neg(M[2][2])
    RETURN FALSE
  END IF
  nbRows = 3
  IF that.type == FrameCuboid
    M[nbRows][0] = thoProj.comp[0][0]
    M[nbRows][1] = thoProj.comp[1][0]
    M[nbRows][2] = thoProj.comp[2][0]
    Y[nbRows] = 1.0 - thoProj.orig[0]
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                   neg(M[nbRows][2])
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
    M[nbRows][0] = thoProj.comp[0][1]
    M[nbRows][1] = thoProj.comp[1][1]
    M[nbRows][2] = thoProj.comp[2][1]
    Y[nbRows] = 1.0 - thoProj.orig[1]
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                   neg(M[nbRows][2])
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
    M[nbRows][0] = thoProj.comp[0][2]
    M[nbRows][1] = thoProj.comp[1][2]
    M[nbRows][2] = thoProj.comp[2][2]
    Y[nbRows] = 1.0 - thoProj.orig[2]
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                   neg(M[nbRows][2])
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
  ELSE
    M[nbRows][0] =
      thoProj.comp[0][0] + thoProj.comp[0][1] + thoProj.comp[0][2]
```

```
  M[nbRows][1] =
    thoProj.comp[1][0] + thoProj.comp[1][1] + thoProj.comp[1][2]
  M[nbRows][2] =
    thoProj.comp[2][0] + thoProj.comp[2][1] + thoProj.comp[2][2]
  Y[nbRows] =
    1.0 - thoProj.orig[0] - thoProj.orig[1] - thoProj.orig[2]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
END IF
IF tho.type == FrameCuboid
  M[nbRows][0] = 1.0
  M[nbRows][1] = 0.0
  M[nbRows][2] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
  M[nbRows][0] = 0.0
  M[nbRows][1] = 1.0
  M[nbRows][2] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
  M[nbRows][0] = 0.0
  M[nbRows][1] = 0.0
  M[nbRows][2] = 1.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
ELSE
  M[nbRows][0] = 1.0
  M[nbRows][1] = 1.0
  M[nbRows][2] = 1.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
END IF
M[nbRows][0] = -1.0
M[nbRows][1] = 0.0
M[nbRows][2] = 0.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
M[nbRows][0] = 0.0
M[nbRows][1] = -1.0
M[nbRows][2] = 0.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
M[nbRows][0] = 0.0
M[nbRows][1] = 0.0
M[nbRows][2] = -1.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
inconsistency =
  ElimVar3D(M, Y, nbRows, 3, Mp, Yp, nbRowsP)
IF inconsistency == TRUE
  RETURN FALSE
END IF
inconsistency =
  ElimVar3D(Mp, Yp, nbRowsP, 2, Mpp, Ypp, nbRowsPP)
IF inconsistency == TRUE
  RETURN FALSE
END IF
GetBoundLastVar3D(THD_VAR, Mpp, Ypp, nbRowsPP, bdgBoxLocal)
IF bdgBoxLocal.min[THD_VAR] >= bdgBoxLocal.max[THD_VAR]
```

```
    RETURN FALSE
  ELSE
    GetBoundVar3D(SND_VAR, Mp, Yp, nbRowsP, 2, bdgBoxLocal)
    GetBoundVar3D(FST_VAR, M, Y, nbRows, 3, bdgBoxLocal)
    bdgBox = bdgBoxLocal
  END IF
  RETURN TRUE
END FUNCTION

origP3D = [0.0, 0.0, 0.0]
compP3D = [
  [1.0, 0.0, 0.0],
  [0.0, 1.0, 0.0],
  [0.0, 0.0, 1.0]]
P3D = Frame3DCreateStatic(FrameTetrahedron, origP3D, compP3D)
origQ3D = [0.0, 0.0, 0.0]
compQ3D = [
  [1.0, 0.0, 0.0],
  [0.0, 1.0, 0.0],
  [0.0, 0.0, 1.0]]
Q3D = Frame3DCreateStatic(FrameTetrahedron, origQ3D, compQ3D)
isIntersecting3D = FMBTestIntersection3D(P3D, Q3D, bdgBox3DLocal)
IF isIntersecting3D == TRUE
  PRINT "Intersection detected."
  Frame3DExportBdgBox(Q3D, bdgBox3DLocal, bdgBox3D)
  AABB3DPrint(bdgBox3D)
ELSE
  PRINT "No intersection."
END IF
```

3.3 2D dynamic

```
ENUM FrameType
  FrameCuboid,
  FrameTetrahedron
END ENUM

STRUCT AABB2DTime
  // x,y,t
  real min[3]
  real max[3]
END STRUCT

STRUCT Frame2DTime
  FrameType type
  real orig[2]
  // comp[iComp][iAxis]
  real comp[2][2]
  AABB2DTime bdgBox
  real invComp[2][2]
  real speed[2]
END STRUCT

FUNCTION powi(base, exp)
  res = 1
  FOR i=0 TO (exp - 1)
    res = res * base
  END FOR
  RETURN res
END FUNCTION
```

```
FUNCTION Frame2DTimePrint(that)
  IF that.type == FrameTetrahedron
    PRINT "T"
  ELSE IF that.type == FrameCuboid
    PRINT "C"
  END IF
  PRINT "o("
  FOR i = 0 TO 1
    PRINT that.orig[i]
    IF i < 1
      PRINT ","
    END IF
  END FOR
  PRINT ") s("
  FOR i = 0 TO 1
    PRINT that.speed[i]
    IF i < 1
      PRINT ","
    END IF
  END FOR
  comp = ["x", "y"]
  FOR j = 0 TO 1
    PRINT ") ", comp[j], "("
    FOR i = 0 TO 1
      PRINT that.comp[j][i]
      IF i < 1
        PRINT ","
      END IF
    END FOR
  END FOR
  PRINT ")"
END FUNCTION

FUNCTION AABB2DTimePrint(that)
  PRINT "minXYT("
  FOR i = 0 TO 2
    PRINT that.min[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  PRINT ")-maxXYT("
  FOR i = 0 TO 2
    PRINT that.max[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  PRINT ")"
END FUNCTION

FUNCTION Frame2DTimeExportBdgBox(that, bdgBox, bdgBoxProj)
  bdgBoxProj.min[2] = bdgBox.min[2]
  bdgBoxProj.max[2] = bdgBox.max[2]
  FOR i = 0 TO 1
    bdgBoxProj.max[i] = that.orig[i] + that.speed[i] * bdgBox.min[2]
    FOR j = 0 TO 1
      bdgBoxProj.max[i] =
        bdgBoxProj.max[i] + that.comp[j][i] * bdgBox.min[j]
    END FOR
    bdgBoxProj.min[i] = bdgBoxProj.max[i]
  END FOR
```

```
  nbVertices = powi(2, 2)
  FOR iVertex = 1 TO (nbVertices - 1)
    FOR i = 0 TO 1
      IF BITWISEAND(iVertex, powi(2, i)) <> 0
        v[i] = bdgBox.max[i]
      ELSE
        v[i] = bdgBox.min[i]
      END IF
    END FOR
    FOR i = 0 TO 1
      w[i] = that.orig[i]
      FOR j = 0 TO 1
        w[i] = w[i] + that.comp[j][i] * v[j]
      END FOR
    END FOR
    FOR i = 0 TO 1
      IF bdgBoxProj.min[i] > w[i] + that.speed[i] * bdgBox.min[2]
        bdgBoxProj.min[i] = w[i] + that.speed[i] * bdgBox.min[2]
      END IF
      IF bdgBoxProj.min[i] > w[i] + that.speed[i] * bdgBox.max[2]
        bdgBoxProj.min[i] = w[i] + that.speed[i] * bdgBox.max[2]
      END IF
      IF bdgBoxProj.max[i] < w[i] + that.speed[i] * bdgBox.min[2]
        bdgBoxProj.max[i] = w[i] + that.speed[i] * bdgBox.min[2]
      END IF
      IF bdgBoxProj.max[i] < w[i] + that.speed[i] * bdgBox.max[2]
        bdgBoxProj.max[i] = w[i] + that.speed[i] * bdgBox.max[2]
      END IF
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame2DTimeImPortFrame(P, Q, Qp)
  FOR i = 0 TO 1
    v[i] = Q.orig[i] - P.orig[i]
    s[i] = Q.speed[i] - P.speed[i]
  END FOR
  FOR i = 0 TO 1
    Qp.orig[i] = 0.0
    Qp.speed[i] = 0.0
    FOR j = 0 TO 1
      Qp.orig[i] = Qp.orig[i] + P.invComp[j][i] * v[j]
      Qp.speed[i] = Qp.speed[i] + P.invComp[j][i] * s[j]
      Qp.comp[j][i] = 0.0
      FOR k = 0 TO 1
        Qp.comp[j][i] = Qp.comp[j][i] + P.invComp[k][i] * Q.comp[j][k]
      END FOR
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame2DTimeUpdateInv(that)
  det = that.comp[0][0] * that.comp[1][1] -
    that.comp[1][0] * that.comp[0][1]
  that.invComp[0][0] = that.comp[1][1] / det
  that.invComp[0][1] = -that.comp[0][1] / det
  that.invComp[1][0] = -that.comp[1][0] / det
  that.invComp[1][1] = that.comp[0][0] / det
END FUNCTION

FUNCTION Frame2DTimeCreateStatic(type, orig, comp)
  that.type = type
```

```
  FOR iAxis = 0 TO 1
    that.orig[iAxis] = orig[iAxis]
    that.speed[iAxis] = speed[iAxis]
    FOR iComp = 0 TO 1
      that.comp[iComp][iAxis] = comp[iComp][iAxis]
    END FOR
  END FOR
  FOR iAxis = 0 TO 1
    min = orig[iAxis]
    max = orig[iAxis]
    FOR iComp = 0 TO 1
      IF that.type == FrameCuboid
        IF that.comp[iComp][iAxis] < 0.0
          min = min + that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0
          max = max + that.comp[iComp][iAxis]
        END IF
      ELSE IF that.type == FrameTetrahedron
        IF that.comp[iComp][iAxis] < 0.0 AND
          min > orig[iAxis] + that.comp[iComp][iAxis]
          min = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0 AND
          max < orig[iAxis] + that.comp[iComp][iAxis]
          max = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
      END IF
    END FOR
    IF that.speed[iAxis] < 0.0
      min = min + that.speed[iAxis]
    END IF
    IF that.speed[iAxis] > 0.0
      max = max + that.speed[iAxis]
    END IF
    that.bdgBox.min[iAxis] = min
    that.bdgBox.max[iAxis] = max
  END FOR
  that.bdgBox.min[2] = 0.0
  that.bdgBox.max[2] = 1.0
  Frame2DTimeUpdateInv(that)
  RETURN that
END FUNCTION

FUNCTION Sgn(v)
  IF 0.0 < v
    a = 1
  ELSE
    a = 0
  END IF
  IF v < 0.0
    b = 1
  ELSE
    b = 0
  END IF
  RETURN A - B
END FUNCTION

FUNCTION Neg(x)
  IF x < 0.0
    RETURN x
  ELSE
```

```
    RETURN 0.0
  END IF
END FUNCTION

FST_VAR = 0
SND_VAR = 1
THD_VAR = 2

FUNCTION ElimVar2DTime(M, Y, nbRows, nbCols, Mp, Yp, nbRemainRows)
  nbRemainRows = 0
  FOR iRow = 0 TO (nbRows - 2)
    IF M[iRow][0] <> 0.0
      FOR jRow = (iRow + 1) TO (nbRows - 1)
        IF sgn(M[iRow][0]) <> sgn(M[jRow][0]) AND
            M[jRow][0] <> 0.0
          sumNegCoeff = 0.0
          jCol = 0
          FOR iCol = 1 TO (nbCols - 1)
            Mp[nbRemainRows][jCol] =
              M[iRow][iCol] / ABS(M[iRow][0]) +
              M[jRow][iCol] / ABS(M[jRow][0])
            sumNegCoeff = sumNegCoeff + neg(Mp[nbRemainRows][jCol])
            jCol = jCol + 1
          END FOR
          Yp[nbRemainRows] =
            Y[iRow] / ABS(M[iRow][0]) +
            Y[jRow] / ABS(M[jRow][0])
          IF Yp[nbRemainRows] < sumNegCoeff
            RETURN TRUE
          END IF
          nbRemainRows = nbRemainRows + 1
        END IF
      END FOR
    END IF
  END FOR
  FOR iRow = 0 TO (nbRows - 1)
    IF M[iRow][0] == 0.0
      jCol = 0
      FOR iCol = 1 TO (nbCols - 1)
        Mp[nbRemainRows][jCol] = M[iRow][iCol]
        jCol = jCol + 1
      END FOR
      Yp[nbRemainRows] = Y[iRow]
      nbRemainRows = nbRemainRows + 1
    END IF
  END FOR
  RETURN FALSE
END FUNCTION

FUNCTION GetBoundLastVar2DTime(iVar, M, Y, nbRows, bdgBox)
  bdgBox.min[iVar] = 0.0
  bdgBox.max[iVar] = 1.0
  FOR jRow = 0 TO (nbRows - 1)
    IF M[jRow][0] > 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.max[iVar] > y
        bdgBox.max[iVar] = y
      END IF
    ELSE IF M[jRow][0] < 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.min[iVar] < y
        bdgBox.min[iVar] = y
```

```
      END IF
    END IF
  END FOR
END FUNCTION

FUNCTION GetBoundVar2DTime(iVar, M, Y, nbRows, nbCols, bdgBox)
  bdgBox.min[iVar] = 0.0
  bdgBox.max[iVar] = 1.0
  FOR iRow = 0 .. TO (nbRows - 1)
    IF M[iRow][0] <> 0.0
      min = -1.0 * Y[iRow]
      max = Y[iRow]
      FOR iCol = 1 .. TO (nbCols - 1)
        IF M[iRow][iCol] > 0.0
          min = min + M[iRow][iCol] * bdgBox.min[iCol + iVar]
          max = max - M[iRow][iCol] * bdgBox.min[iCol + iVar]
        ELSE IF M[iRow][iCol] < 0.0
          min = min + M[iRow][iCol] * bdgBox.max[iCol + iVar]
          max = max - M[iRow][iCol] * bdgBox.max[iCol + iVar]
        END IF
      END FOR
      min = min / (-1.0 * M[iRow][0])
      max = max / M[iRow][0]
      IF bdgBox.min[iVar] > min
        bdgBox.min[iVar] = min
      END IF
      IF bdgBox.max[iVar] < max
        bdgBox.max[iVar] = max
      END IF
    END IF
  END FOR
END FUNCTION

FUNCTION FMBTestIntersection2DTime(that, tho, bdgBox)
  Frame2DTimeImportFrame(that, tho, thoProj)
  M[0][0] = -thoProj.comp[0][0]
  M[0][1] = -thoProj.comp[1][0]
  M[0][2] = -thoProj.speed[0]
  Y[0] = thoProj.orig[0]
  IF (Y[0] < neg(M[0][0]) + neg(M[0][1]) + neg(M[0][2]))
    RETURN FALSE
  END IF
  M[1][0] = -thoProj.comp[0][1]
  M[1][1] = -thoProj.comp[1][1]
  M[1][2] = -thoProj.speed[1]
  Y[1] = thoProj.orig[1]
  IF (Y[1] < neg(M[1][0]) + neg(M[1][1]) + neg(M[1][2]))
    RETURN FALSE
  END IF
  nbRows = 2
  IF that.type == FrameCuboid
    M[nbRows][0] = thoProj.comp[0][0]
    M[nbRows][1] = thoProj.comp[1][0]
    M[nbRows][2] = thoProj.speed[0]
    Y[nbRows] = 1.0 - thoProj.orig[0]
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                   neg(M[nbRows][2])
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
    M[nbRows][0] = thoProj.comp[0][1]
    M[nbRows][1] = thoProj.comp[1][1]
```

```
  M[nbRows][2] = thoProj.speed[1]
  Y[nbRows] = 1.0 - thoProj.orig[1]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
ELSE
  M[nbRows][0] = thoProj.comp[0][0] + thoProj.comp[0][1]
  M[nbRows][1] = thoProj.comp[1][0] + thoProj.comp[1][1]
  M[nbRows][2] = thoProj.speed[0] + thoProj.speed[1]
  Y[nbRows] = 1.0 - thoProj.orig[0] - thoProj.orig[1]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
END IF
IF tho.type == FrameCuboid
  M[nbRows][0] = 1.0
  M[nbRows][1] = 0.0
  M[nbRows][2] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
  M[nbRows][0] = 0.0
  M[nbRows][1] = 1.0
  M[nbRows][2] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
ELSE
  M[nbRows][0] = 1.0
  M[nbRows][1] = 1.0
  M[nbRows][2] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
END IF
M[nbRows][0] = -1.0
M[nbRows][1] = 0.0
M[nbRows][2] = 0.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
M[nbRows][0] = 0.0
M[nbRows][1] = -1.0
M[nbRows][2] = 0.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
M[nbRows][0] = 0.0
M[nbRows][1] = 0.0
M[nbRows][2] = 1.0
Y[nbRows] = 1.0
nbRows = nbRows + 1
M[nbRows][0] = 0.0
M[nbRows][1] = 0.0
M[nbRows][2] = -1.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
inconsistency =
  ElimVar2DTime(M, Y, nbRows, 3, Mp, Yp, nbRowsP)
IF inconsistency == TRUE
  RETURN FALSE
END IF
inconsistency =
```

```
    ElimVar2DTime(Mp, Yp, nbRowsP, 2, Mpp, Ypp, nbRowsPP)
  IF inconsistency == TRUE
    RETURN FALSE
  END IF
  GetBoundLastVar2DTime(THD_VAR, Mpp, Ypp, nbRowsPP, bdgBoxLocal)
  IF bdgBoxLocal.min[THD_VAR] >= bdgBoxLocal.max[THD_VAR]
    RETURN FALSE
  ELSE
    GetBoundVar2DTime(SND_VAR, Mp, Yp, nbRowsP, 2, bdgBoxLocal)
    GetBoundVar2DTime(FST_VAR, M, Y, nbRows, 3, bdgBoxLocal)
    bdgBox = bdgBoxLocal
  END IF
  RETURN TRUE
END FUNCTION

origP2DTime = [0.0, 0.0]
speedP2DTime = [0.0, 0.0]
compP2DTime = [
  [1.0, 0.0],
  [0.0, 1.0]]
P2DTime =
  Frame2DTimeCreateStatic(
    FrameCuboid, origP2DTime, speedP2DTime, compP2DTime)
origQ2DTime = [0.0,0.0]
speedQ2DTime = [0.0,0.0]
compQ2DTime = [
  [1.0, 0.0],
  [0.0, 1.0]]
Q2DTime =
  Frame2DTimeCreateStatic(
    FrameCuboid, origQ2DTime, speedQ2DTime, compQ2DTime)
isIntersecting2DTime =
  FMBTestIntersection2DTime(P2DTime, Q2DTime, bdgBox2DTimeLocal)
IF isIntersecting2DTime == TRUE
  PRINT "Intersection detected."
  Frame2DTimeExportBdgBox(Q2DTime, bdgBox2DTimeLocal, bdgBox2DTime)
  AABB2DTimePrint(bdgBox2DTime)
ELSE
  PRINT "No intersection."
END IF
```

3.4 3D dynamic

```
ENUM FrameType
  FrameCuboid,
  FrameTetrahedron
END ENUM

STRUCT AABB3DTime
  // x,y,z,t
  real min[4]
  real max[4]
END STRUCT

STRUCT Frame3DTime
  FrameType type
  real orig[3]
  // comp[iComp][iAxis]
  real comp[3][3]
  AABB3DTime bdgBox
  real invComp[3][3]
```

```
  real speed[3]
END STRUCT

FUNCTION powi(base, exp)
  res = 1
  FOR i=0 TO (exp - 1)
    res = res * base
  END FOR
  RETURN res
END FUNCTION

FUNCTION Frame3DTimePrint(that)
  IF that.type == FrameTetrahedron
    PRINT "T"
  ELSE IF that.type == FrameCuboid
    PRINT "C"
  END IF
  PRINT "o("
  FOR i = 0 TO 2
    PRINT that.orig[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  PRINT " s("
  FOR i = 0 TO 2
    PRINT that.speed[i]
    IF i < 2
      PRINT ","
    END IF
  END FOR
  comp = ["x", "y", "z"]
  FOR j = 0 TO 2
    PRINT " ", comp[j], "("
    FOR i = 0 TO 2
      PRINT that.comp[j][i]
      IF i < 2
        PRINT ","
      END IF
    END FOR
  END FOR
  PRINT ")"
END FUNCTION

FUNCTION AABB3DTimePrint(that)
  PRINT "minXYZT("
  FOR i = 0 TO 3
    PRINT that.min[i]
    IF i < 3
      PRINT ","
    END IF
  END FOR
  PRINT ")-maxXYZT("
  FOR i = 0 TO 3
    PRINT that.max[i]
    IF i < 3
      PRINT ","
    END IF
  END FOR
  PRINT ")"
END FUNCTION
```

```
FUNCTION Frame3DTimeExportBdgBox(that, bdgBox, bdgBoxProj)
  bdgBoxProj.min[3] = bdgBox.min[3]
  bdgBoxProj.max[3] = bdgBox.max[3]
  FOR i = 0 TO 2
    bdgBoxProj.max[i] = that.orig[i] + that.speed[i] * bdgBox.min[3]
    FOR j = 0 TO 2
      bdgBoxProj.max[i] =
        bdgBoxProj.max[i] + that.comp[j][i] * bdgBox.min[j]
    END FOR
    bdgBoxProj.min[i] = bdgBoxProj.max[i]
  END FOR
  nbVertices = powi(2, 3)
  FOR iVertex = 1 TO (nbVertices - 1)
    FOR i = 0 TO 2
      IF BITWISEAND(iVertex, powi(2, i)) <> 0
        v[i] = bdgBox.max[i]
      ELSE
        v[i] = bdgBox.min[i]
      END IF
    END FOR
    FOR i = 0 TO 2
      w[i] = that.orig[i]
      FOR j = 0 TO 2
        w[i] = w[i] + that.comp[j][i] * v[j]
      END FOR
    END FOR
    FOR i = 0 TO 2
      IF bdgBoxProj.min[i] > w[i] + that.speed[i] * bdgBox.min[3]
        bdgBoxProj.min[i] = w[i] + that.speed[i] * bdgBox.min[3]
      END IF
      IF bdgBoxProj.min[i] > w[i] + that.speed[i] * bdgBox.max[3]
        bdgBoxProj.min[i] = w[i] + that.speed[i] * bdgBox.max[3]
      END IF
      IF bdgBoxProj.max[i] < w[i] + that.speed[i] * bdgBox.min[3]
        bdgBoxProj.max[i] = w[i] + that.speed[i] * bdgBox.min[3]
      END IF
      IF bdgBoxProj.max[i] < w[i] + that.speed[i] * bdgBox.max[3]
        bdgBoxProj.max[i] = w[i] + that.speed[i] * bdgBox.max[3]
      END IF
    END FOR
  END FOR
END FUNCTION

FUNCTION Frame3DTimeImPortFrame(P, Q, Qp)
  FOR i = 0 TO 2
    v[i] = Q.orig[i] - P.orig[i]
    s[i] = Q.speed[i] - P.speed[i]
  END FOR
  FOR i = 0 TO 2
    Qp.orig[i] = 0.0
    Qp.speed[i] = 0.0
    FOR j = 0 TO 2
      Qp.orig[i] = Qp.orig[i] + P.invComp[j][i] * v[j]
      Qp.speed[i] = Qp.speed[i] + P.invComp[j][i] * s[j]
      Qp.comp[j][i] = 0.0
      FOR k = 0 TO 2
        Qp.comp[j][i] = Qp.comp[j][i] + P.invComp[k][i] * Q.comp[j][k]
      END FOR
    END FOR
  END FOR
END FUNCTION
```

```
FUNCTION Frame3DTimeUpdateInv(that)
  det =
    that.comp[0][0] *
    (that.comp[1][1] * that.comp[2][2] - that.comp[1][2] * that.comp[2][1])
        -
    that.comp[1][0] *
    (that.comp[0][1] * that.comp[2][2] - that.comp[0][2] * that.comp[2][1])
        +
    that.comp[2][0] *
    (that.comp[0][1] * that.comp[1][2] - that.comp[0][2] * that.comp[1][1])
  that.invComp[0][0] = (that.comp[1][1] * that.comp[2][2] -
    that.comp[2][1] * that.comp[1][2]) / det
  that.invComp[0][1] = (that.comp[2][1] * that.comp[0][2] -
    that.comp[2][2] * that.comp[0][1]) / det
  that.invComp[0][2] = (that.comp[0][1] * that.comp[1][2] -
    that.comp[0][2] * that.comp[1][1]) / det
  that.invComp[1][0] = (that.comp[2][0] * that.comp[1][2] -
    that.comp[2][2] * that.comp[1][0]) / det
  that.invComp[1][1] = (that.comp[0][0] * that.comp[2][2] -
    that.comp[2][0] * that.comp[0][2]) / det
  that.invComp[1][2] = (that.comp[0][2] * that.comp[1][0] -
    that.comp[1][2] * that.comp[0][0]) / det
  that.invComp[2][0] = (that.comp[1][0] * that.comp[2][1] -
    that.comp[2][0] * that.comp[1][1]) / det
  that.invComp[2][1] = (that.comp[0][1] * that.comp[2][0] -
    that.comp[2][1] * that.comp[0][0]) / det
  that.invComp[2][2] = (that.comp[0][0] * that.comp[1][1] -
    that.comp[1][0] * that.comp[0][1]) / det
END FUNCTION

FUNCTION Frame3DTimeCreateStatic(type, orig, comp)
  that.type = type
  FOR iAxis = 0 TO 2
    that.orig[iAxis] = orig[iAxis]
    that.speed[iAxis] = speed[iAxis]
    FOR iComp = 0 TO 2
      that.comp[iComp][iAxis] = comp[iComp][iAxis]
    END FOR
  END FOR
  FOR iAxis = 0 TO 2
    min = orig[iAxis]
    max = orig[iAxis]
    FOR iComp = 0 TO 2
      IF that.type == FrameCuboid
        IF that.comp[iComp][iAxis] < 0.0
          min = min + that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0
          max = max + that.comp[iComp][iAxis]
        END IF
      ELSE IF that.type == FrameTetrahedron
        IF that.comp[iComp][iAxis] < 0.0 AND
          min > orig[iAxis] + that.comp[iComp][iAxis]
          min = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
        IF that.comp[iComp][iAxis] > 0.0 AND
          max < orig[iAxis] + that.comp[iComp][iAxis]
          max = orig[iAxis] + that.comp[iComp][iAxis]
        END IF
      END IF
    END FOR
    IF that.speed[iAxis] < 0.0
```

```
      min = min + that.speed[iAxis]
    END IF
    IF that.speed[iAxis] > 0.0
      max = max + that.speed[iAxis]
    END IF
    that.bdgBox.min[iAxis] = min
    that.bdgBox.max[iAxis] = max
  END FOR
  that.bdgBox.min[3] = 0.0
  that.bdgBox.max[3] = 1.0
  Frame3DTimeUpdateInv(that)
  RETURN that
END FUNCTION

FUNCTION Sgn(v)
  IF 0.0 < v
    a = 1
  ELSE
    a = 0
  END IF
  IF v < 0.0
    b = 1
  ELSE
    b = 0
  END IF
  RETURN A - B
END FUNCTION

FUNCTION Neg(x)
  IF x < 0.0
    RETURN x
  ELSE
    RETURN 0.0
  END IF
END FUNCTION

FST_VAR = 0
SND_VAR = 1
THD_VAR = 2
FOR_VAR = 3

FUNCTION ElimVar3DTime(M, Y, nbRows, nbCols, Mp, Yp, nbRemainRows)
  nbRemainRows = 0
  FOR iRow = 0 TO (nbRows - 2)
    IF M[iRow][0] <> 0.0
      FOR jRow = (iRow + 1) TO (nbRows - 1)
        IF sgn(M[iRow][0]) <> sgn(M[jRow][0]) AND
            M[jRow][0] <> 0.0
          sumNegCoeff = 0.0
          jCol = 0
          FOR iCol = 1 TO (nbCols - 1)
            Mp[nbRemainRows][jCol] =
              M[iRow][iCol] / ABS(M[iRow][0]) +
              M[jRow][iCol] / ABS(M[jRow][0])
            sumNegCoeff = sumNegCoeff + neg(Mp[nbRemainRows][jCol])
            jCol = jCol + 1
          END FOR
          Yp[nbRemainRows] =
            Y[iRow] / ABS(M[iRow][0]) +
            Y[jRow] / ABS(M[jRow][0])
          IF Yp[nbRemainRows] < sumNegCoeff
            RETURN TRUE
```

```
          END IF
          nbRemainRows = nbRemainRows + 1
        END IF
      END FOR
    END IF
  END FOR
  FOR iRow = 0 TO (nbRows - 1)
    IF M[iRow][0] == 0.0
      jCol = 0
      FOR iCol = 1 TO (nbCols - 1)
        Mp[nbRemainRows][jCol] = M[iRow][iCol]
        jCol = jCol + 1
      END FOR
      Yp[nbRemainRows] = Y[iRow]
      nbRemainRows = nbRemainRows + 1
    END IF
  END FOR
  RETURN FALSE
END FUNCTION

FUNCTION GetBoundLastVar3DTime(iVar, M, Y, nbRows, bdgBox)
  bdgBox.min[iVar] = 0.0
  bdgBox.max[iVar] = 1.0
  FOR jRow = 0 TO (nbRows - 1)
    IF M[jRow][0] > 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.max[iVar] > y
        bdgBox.max[iVar] = y
      END IF
    ELSE IF M[jRow][0] < 0.0
      y = Y[jRow] / M[jRow][0]
      IF bdgBox.min[iVar] < y
        bdgBox.min[iVar] = y
      END IF
    END IF
  END FOR
END FUNCTION

FUNCTION GetBoundVar3DTime(iVar, M, Y, nbRows, nbCols, bdgBox)
  bdgBox.min[iVar] = 0.0
  bdgBox.max[iVar] = 1.0
  FOR iRow = 0 .. TO (nbRows - 1)
    IF M[iRow][0] <> 0.0
      min = -1.0 * Y[iRow]
      max = Y[iRow]
      FOR iCol = 1 .. TO (nbCols - 1)
        IF M[iRow][iCol] > 0.0
          min = min + M[iRow][iCol] * bdgBox.min[iCol + iVar]
          max = max - M[iRow][iCol] * bdgBox.min[iCol + iVar]
        ELSE IF M[iRow][iCol] < 0.0
          min = min + M[iRow][iCol] * bdgBox.max[iCol + iVar]
          max = max - M[iRow][iCol] * bdgBox.max[iCol + iVar]
        END IF
      END FOR
      min = min / (-1.0 * M[iRow][0])
      max = max / M[iRow][0]
      IF bdgBox.min[iVar] > min
        bdgBox.min[iVar] = min
      END IF
      IF bdgBox.max[iVar] < max
        bdgBox.max[iVar] = max
      END IF
```

```
    END IF
  END FOR
END FUNCTION

FUNCTION FMBTestIntersection3DTime(that, tho, bdgBox)
  Frame3DTimeImportFrame(that, tho, thoProj)
  M[0][0] = -thoProj.comp[0][0]
  M[0][1] = -thoProj.comp[1][0]
  M[0][2] = -thoProj.comp[2][0]
  M[0][3] = -thoProj.speed[0]
  Y[0] = thoProj.orig[0]
  IF Y[0] < neg(M[0][0]) + neg(M[0][1]) + neg(M[0][2]) + neg(M[0][3])
    RETURN FALSE
  END IF
  M[1][0] = -thoProj.comp[0][1]
  M[1][1] = -thoProj.comp[1][1]
  M[1][2] = -thoProj.comp[2][1]
  M[1][3] = -thoProj.speed[1]
  Y[1] = thoProj.orig[1]
  IF Y[1] < neg(M[1][0]) + neg(M[1][1]) + neg(M[1][2]) + neg(M[1][3])
    RETURN FALSE
  END IF
  M[2][0] = -thoProj.comp[0][2]
  M[2][1] = -thoProj.comp[1][2]
  M[2][2] = -thoProj.comp[2][2]
  M[2][3] = -thoProj.speed[2]
  Y[2] = thoProj.orig[2]
  IF Y[2] < neg(M[2][0]) + neg(M[2][1]) + neg(M[2][2]) + neg(M[2][3])
    RETURN FALSE
  END IF
  nbRows = 3
  IF that.type == FrameCuboid
    M[nbRows][0] = thoProj.comp[0][0]
    M[nbRows][1] = thoProj.comp[1][0]
    M[nbRows][2] = thoProj.comp[2][0]
    M[nbRows][3] = thoProj.speed[0]
    Y[nbRows] = 1.0 - thoProj.orig[0]
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                   neg(M[nbRows][2]) + neg(M[nbRows][3])
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
    M[nbRows][0] = thoProj.comp[0][1]
    M[nbRows][1] = thoProj.comp[1][1]
    M[nbRows][2] = thoProj.comp[2][1]
    M[nbRows][3] = thoProj.speed[1]
    Y[nbRows] = 1.0 - thoProj.orig[1]
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                   neg(M[nbRows][2]) + neg(M[nbRows][3])
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
    M[nbRows][0] = thoProj.comp[0][2]
    M[nbRows][1] = thoProj.comp[1][2]
    M[nbRows][2] = thoProj.comp[2][2]
    M[nbRows][3] = thoProj.speed[2]
    Y[nbRows] = 1.0 - thoProj.orig[2]
    IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                   neg(M[nbRows][2]) + neg(M[nbRows][3])
      RETURN FALSE
    END IF
    nbRows = nbRows + 1
```

```
ELSE
  M[nbRows][0] =
    thoProj.comp[0][0] + thoProj.comp[0][1] + thoProj.comp[0][2]
  M[nbRows][1] =
    thoProj.comp[1][0] + thoProj.comp[1][1] + thoProj.comp[1][2]
  M[nbRows][2] =
    thoProj.comp[2][0] + thoProj.comp[2][1] + thoProj.comp[2][2]
  M[nbRows][3] = thoProj.speed[0] + thoProj.speed[1] + thoProj.speed[2]
  Y[nbRows] = 1.0 - thoProj.orig[0] - thoProj.orig[1] - thoProj.orig[2]
  IF Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                 neg(M[nbRows][2]) + neg(M[nbRows][3])
    RETURN FALSE
  END IF
  nbRows = nbRows + 1
END IF
IF tho.type == FrameCuboid
  M[nbRows][0] = 1.0
  M[nbRows][1] = 0.0
  M[nbRows][2] = 0.0
  M[nbRows][3] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
  M[nbRows][0] = 0.0
  M[nbRows][1] = 1.0
  M[nbRows][2] = 0.0
  M[nbRows][3] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
  M[nbRows][0] = 0.0
  M[nbRows][1] = 0.0
  M[nbRows][2] = 1.0
  M[nbRows][3] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
ELSE
  M[nbRows][0] = 1.0
  M[nbRows][1] = 1.0
  M[nbRows][2] = 1.0
  M[nbRows][3] = 0.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
END IF
M[nbRows][0] = -1.0
M[nbRows][1] = 0.0
M[nbRows][2] = 0.0
M[nbRows][3] = 0.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
M[nbRows][0] = 0.0
M[nbRows][1] = -1.0
M[nbRows][2] = 0.0
M[nbRows][3] = 0.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
M[nbRows][0] = 0.0
M[nbRows][1] = 0.0
M[nbRows][2] = -1.0
M[nbRows][3] = 0.0
Y[nbRows] = 0.0
nbRows = nbRows + 1
M[nbRows][0] = 0.0
M[nbRows][1] = 0.0
```

```
  M[nbRows][2] = 0.0
  M[nbRows][3] = 1.0
  Y[nbRows] = 1.0
  nbRows = nbRows + 1
  M[nbRows][0] = 0.0
  M[nbRows][1] = 0.0
  M[nbRows][2] = 0.0
  M[nbRows][3] = -1.0
  Y[nbRows] = 0.0
  nbRows = nbRows + 1
  inconsistency =
    ElimVar3DTime(M, Y, nbRows, 4, Mp, Yp, nbRowsP)
  IF inconsistency == TRUE
    RETURN FALSE
  END IF
  inconsistency =
    ElimVar3DTime(Mp, Yp, nbRowsP, 3, Mpp, Ypp, nbRowsPP)
  IF inconsistency == TRUE
    RETURN FALSE
  END IF
  inconsistency =
    ElimVar3DTime(Mpp, Ypp, nbRowsPP, 2, Mppp, Yppp, nbRowsPPP)
  IF inconsistency == TRUE
    RETURN FALSE
  END IF
  GetBoundLastVar3DTime(FOR_VAR, Mppp, Yppp, nbRowsPPP, bdgBoxLocal)
  IF bdgBoxLocal.min[FOR_VAR] >= bdgBoxLocal.max[FOR_VAR]
    RETURN FALSE
  ELSE
    GetBoundVar3DTime(THD_VAR, Mpp, Ypp, nbRowsPP, 2, bdgBoxLocal)
    GetBoundVar3DTime(SND_VAR, Mp, Yp, nbRowsP, 3, bdgBoxLocal)
    GetBoundVar3DTime(FST_VAR, M, Y, nbRows, 4, bdgBoxLocal)
    bdgBox = bdgBoxLocal
  END IF
  RETURN TRUE
END FUNCTION

origP3DTime = [0.0, 0.0, 0.0]
speedP3DTime = [0.0, 0.0, 0.0]
compP3DTime = [
  [1.0, 0.0, 0.0],
  [0.0, 1.0, 0.0],
  [0.0, 0.0, 1.0]]
P3DTime =
  Frame3DTimeCreateStatic(
    FrameCuboid, origP3DTime, speedP3DTime, compP3DTime)
origQ3DTime = [0.0, 0.0, 0.0]
speedQ3DTime = [0.0, 0.0, 0.0]
compQ3DTime = [
  [1.0, 0.0, 0.0],
  [0.0, 1.0, 0.0],
  [0.0, 0.0, 1.0]]
Q3DTime =
  Frame3DTimeCreateStatic(
    FrameCuboid, origQ3DTime, speedQ3DTime, compQ3DTime)
isIntersecting3DTime =
  FMBTestIntersection3DTime(P3DTime, Q3DTime, bdgBox3DTimeLocal)
IF isIntersecting3DTime == TRUE
  PRINT "Intersection detected."
  Frame3DTimeExportBdgBox(Q3DTime, bdgBox3DTimeLocal, bdgBox3DTime)
  AABB3DTimePrint(bdgBox3DTime)
ELSE
```

```
  PRINT "No intersection."
END IF
```

4 Implementation of the algorithms in C

In this section I introduce an implementation of the algorithms of the previous section in the C language.

4.1 Frames

4.1.1 Header

```
#ifndef __FRAME_H_
#define __FRAME_H_

// ------------- Includes -------------

#include <math.h>
#include <stdio.h>
#include <stdlib.h>

// ------------- Macros -------------

// ------------- Enumerations -------------

typedef enum {
  FrameCuboid,
  FrameTetrahedron
} FrameType;

// ------------- Data structures -------------

// Axis aligned bounding box structure
typedef struct {
  // x,y
  double min[2];
  double max[2];
} AABB2D;

typedef struct {
  // x,y,z
  double min[3];
  double max[3];
} AABB3D;

typedef struct {
  // x,y,t
  double min[3];
  double max[3];
} AABB2DTime;

typedef struct {
  // x,y,z,t
  double min[4];
  double max[4];
} AABB3DTime;
```

```
// Axis unaligned cuboid and tetrahedron structure
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
  // AABB of the frame
  AABB2D bdgBox;
  // Inverted components used during computation
  double invComp[2][2];
} Frame2D;

typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
  // AABB of the frame
  AABB3D bdgBox;
  // Inverted components used during computation
  double invComp[3][3];
} Frame3D;

typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
  // AABB of the frame
  AABB2DTime bdgBox;
  // Inverted components used during computation
  double invComp[2][2];
  double speed[2];
} Frame2DTime;

typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
  // AABB of the frame
  AABB3DTime bdgBox;
  // Inverted components used during computation
  double invComp[3][3];
  double speed[3];
} Frame3DTime;

// ------------- Functions declaration -------------

// Print the AABB that on stdout
// Output format is
// (min[0], min[1], min[2], min[3])-(max[0], max[1], max[2], max[3])
void AABB2DPrint(const AABB2D* const that);
void AABB3DPrint(const AABB3D* const that);
void AABB2DTimePrint(const AABB2DTime* const that);
void AABB3DTimePrint(const AABB3DTime* const that);

// Print the Frame that on stdout
// Output format is
// T/C  <- type of Frame
// o(orig[0], orig[1], orig[2])
// s(speed[0], speed[1], speed[2])
// x(comp[0][0], comp[0][1], comp[0][2])
// y(comp[1][0], comp[1][1], comp[1][2])
// z(comp[2][0], comp[2][1], comp[2][2])
```

```
void Frame2DPrint(const Frame2D* const that);
void Frame3DPrint(const Frame3D* const that);
void Frame2DTimePrint(const Frame2DTime* const that);
void Frame3DTimePrint(const Frame3DTime* const that);

// Create a static Frame structure of FrameType type,
// at position orig with components comp ([iComp][iAxis])
// and speed
Frame2D Frame2DCreateStatic(
  const FrameType type,
     const double orig[2],
     const double comp[2][2]);
Frame3D Frame3DCreateStatic(
  const FrameType type,
     const double orig[3],
     const double comp[3][3]);
Frame2DTime Frame2DTimeCreateStatic(
  const FrameType type,
     const double orig[2],
     const double speed[2],
     const double comp[2][2]);
Frame3DTime Frame3DTimeCreateStatic(
  const FrameType type,
     const double orig[3],
     const double speed[3],
     const double comp[3][3]);

// Project the Frame Q in the Frame P's coordinates system and
// memorize the result in the Frame Qp
void Frame2DImportFrame(
  const Frame2D* const P,
  const Frame2D* const Q,
        Frame2D* const Qp);
void Frame3DImportFrame(
  const Frame3D* const P,
  const Frame3D* const Q,
        Frame3D* const Qp);
void Frame2DTimeImportFrame(
  const Frame2DTime* const P,
  const Frame2DTime* const Q,
        Frame2DTime* const Qp);
void Frame3DTimeImportFrame(
  const Frame3DTime* const P,
  const Frame3DTime* const Q,
        Frame3DTime* const Qp);

// Export the AABB bdgBox from that's coordinates system to
// the real coordinates system and update bdgBoxProj with the resulting
// AABB
void Frame2DExportBdgBox(
  const Frame2D* const that,
   const AABB2D* const bdgBox,
         AABB2D* const bdgBoxProj);
void Frame3DExportBdgBox(
  const Frame3D* const that,
   const AABB3D* const bdgBox,
         AABB3D* const bdgBoxProj);
void Frame2DTimeExportBdgBox(
  const Frame2DTime* const that,
   const AABB2DTime* const bdgBox,
         AABB2DTime* const bdgBoxProj);
void Frame3DTimeExportBdgBox(
```

```
  const Frame3DTime* const that,
   const AABB3DTime* const bdgBox,
         AABB3DTime* const bdgBoxProj);

// Power function for integer base and exponent
// Return base^exp
int powi(
           int base,
  unsigned int exp);

#endif
```

4.1.2 Body

```
#include "frame.h"

// ------------- Macros -------------

#define EPSILON 0.0000001

// ------------- Functions declaration -------------

// Update the inverse components of the Frame that
void Frame2DUpdateInv(Frame2D* const that);
void Frame3DUpdateInv(Frame3D* const that);
void Frame2DTimeUpdateInv(Frame2DTime* const that);
void Frame3DTimeUpdateInv(Frame3DTime* const that);

// ------------- Functions implementation -------------

// Create a static Frame structure of FrameType type,
// at position orig with components comp and speed
// arrangement is comp[iComp][iAxis]
Frame2D Frame2DCreateStatic(
  const FrameType type,
     const double orig[2],
     const double comp[2][2]) {

  // Create the new Frame
  Frame2D that;
  that.type = type;
  for (int iAxis = 2;
       iAxis--;) {

    that.orig[iAxis] = orig[iAxis];

    for (int iComp = 2;
         iComp--;) {

      that.comp[iComp][iAxis] = comp[iComp][iAxis];

    }

  }

  // Create the bounding box
  for (int iAxis = 2;
       iAxis--;) {

    double min = orig[iAxis];
    double max = orig[iAxis];
```

```
    for (int iComp = 2;
         iComp--;) {

      if (that.type == FrameCuboid) {

        if (that.comp[iComp][iAxis] < 0.0) {

          min += that.comp[iComp][iAxis];

        }

        if (that.comp[iComp][iAxis] > 0.0) {

          max += that.comp[iComp][iAxis];

        }

      } else if (that.type == FrameTetrahedron) {

        if (that.comp[iComp][iAxis] < 0.0 &&
          min > orig[iAxis] + that.comp[iComp][iAxis]) {

          min = orig[iAxis] + that.comp[iComp][iAxis];

        }

        if (that.comp[iComp][iAxis] > 0.0 &&
          max < orig[iAxis] + that.comp[iComp][iAxis]) {

          max = orig[iAxis] + that.comp[iComp][iAxis];

        }

      }

    }

    that.bdgBox.min[iAxis] = min;
    that.bdgBox.max[iAxis] = max;

  }


  // Calculate the inverse matrix
  Frame2DUpdateInv(&that);

  // Return the new Frame
  return that;

}

Frame3D Frame3DCreateStatic(
  const FrameType type,
     const double orig[3],
     const double comp[3][3]) {

  // Create the new Frame
  Frame3D that;
  that.type = type;
  for (int iAxis = 3;
       iAxis--;) {
```

```
  that.orig[iAxis] = orig[iAxis];

  for (int iComp = 3;
       iComp--;) {

    that.comp[iComp][iAxis] = comp[iComp][iAxis];

  }

}

// Create the bounding box
for (int iAxis = 3;
     iAxis--;) {

  double min = orig[iAxis];
  double max = orig[iAxis];

  for (int iComp = 3;
       iComp--;) {

    if (that.type == FrameCuboid) {

      if (that.comp[iComp][iAxis] < 0.0) {

        min += that.comp[iComp][iAxis];

      }

      if (that.comp[iComp][iAxis] > 0.0) {

        max += that.comp[iComp][iAxis];

      }

    } else if (that.type == FrameTetrahedron) {

      if (that.comp[iComp][iAxis] < 0.0 &&
        min > orig[iAxis] + that.comp[iComp][iAxis]) {

        min = orig[iAxis] + that.comp[iComp][iAxis];

      }

      if (that.comp[iComp][iAxis] > 0.0 &&
        max < orig[iAxis] + that.comp[iComp][iAxis]) {

        max = orig[iAxis] + that.comp[iComp][iAxis];

      }

    }

  }

  that.bdgBox.min[iAxis] = min;
  that.bdgBox.max[iAxis] = max;

}
```

```
  // Calculate the inverse matrix
  Frame3DUpdateInv(&that);

  // Return the new Frame
  return that;

}

Frame2DTime Frame2DTimeCreateStatic(
  const FrameType type,
     const double orig[2],
     const double speed[2],
     const double comp[2][2]) {

  // Create the new Frame
  Frame2DTime that;
  that.type = type;
  for (int iAxis = 2;
       iAxis--;) {

    that.orig[iAxis] = orig[iAxis];
    that.speed[iAxis] = speed[iAxis];

    for (int iComp = 2;
         iComp--;) {

      that.comp[iComp][iAxis] = comp[iComp][iAxis];

    }

  }

  // Create the bounding box
  for (int iAxis = 2;
       iAxis--;) {

    double min = orig[iAxis];
    double max = orig[iAxis];

    for (int iComp = 2;
         iComp--;) {

      if (that.type == FrameCuboid) {

        if (that.comp[iComp][iAxis] < 0.0) {

          min += that.comp[iComp][iAxis];

        }

        if (that.comp[iComp][iAxis] > 0.0) {

          max += that.comp[iComp][iAxis];

        }

      } else if (that.type == FrameTetrahedron) {

        if (that.comp[iComp][iAxis] < 0.0 &&
          min > orig[iAxis] + that.comp[iComp][iAxis]) {

          min = orig[iAxis] + that.comp[iComp][iAxis];
```

```
        }

        if (that.comp[iComp][iAxis] > 0.0 &&
          max < orig[iAxis] + that.comp[iComp][iAxis]) {

          max = orig[iAxis] + that.comp[iComp][iAxis];

        }

      }

    }

    if (that.speed[iAxis] < 0.0) {

      min += that.speed[iAxis];

    }

    if (that.speed[iAxis] > 0.0) {

      max += that.speed[iAxis];

    }

    that.bdgBox.min[iAxis] = min;
    that.bdgBox.max[iAxis] = max;

  }

  that.bdgBox.min[2] = 0.0;
  that.bdgBox.max[2] = 1.0;


  // Calculate the inverse matrix
  Frame2DTimeUpdateInv(&that);

  // Return the new Frame
  return that;

}

Frame3DTime Frame3DTimeCreateStatic(
  const FrameType type,
     const double orig[3],
     const double speed[3],
     const double comp[3][3]) {

  // Create the new Frame
  Frame3DTime that;
  that.type = type;
  for (int iAxis = 3;
       iAxis--;) {

    that.orig[iAxis] = orig[iAxis];
    that.speed[iAxis] = speed[iAxis];

    for (int iComp = 3;
         iComp--;) {

      that.comp[iComp][iAxis] = comp[iComp][iAxis];
```

```
  }

}

// Create the bounding box
for (int iAxis = 3;
     iAxis--;) {

  double min = orig[iAxis];
  double max = orig[iAxis];

  for (int iComp = 3;
       iComp--;) {

    if (that.type == FrameCuboid) {

      if (that.comp[iComp][iAxis] < 0.0) {

        min += that.comp[iComp][iAxis];

      }

      if (that.comp[iComp][iAxis] > 0.0) {

        max += that.comp[iComp][iAxis];

      }

    } else if (that.type == FrameTetrahedron) {

      if (that.comp[iComp][iAxis] < 0.0 &&
        min > orig[iAxis] + that.comp[iComp][iAxis]) {

        min = orig[iAxis] + that.comp[iComp][iAxis];

      }

      if (that.comp[iComp][iAxis] > 0.0 &&
        max < orig[iAxis] + that.comp[iComp][iAxis]) {

        max = orig[iAxis] + that.comp[iComp][iAxis];

      }

    }

  }

  if (that.speed[iAxis] < 0.0) {

    min += that.speed[iAxis];

  }

  if (that.speed[iAxis] > 0.0) {

    max += that.speed[iAxis];

  }

  that.bdgBox.min[iAxis] = min;
```

```
    that.bdgBox.max[iAxis] = max;

  }

  that.bdgBox.min[3] = 0.0;
  that.bdgBox.max[3] = 1.0;

  // Calculate the inverse matrix
  Frame3DTimeUpdateInv(&that);

  // Return the new Frame
  return that;

}

// Update the inverse components of the Frame that
void Frame2DUpdateInv(Frame2D* const that) {

  // Shortcuts
  double (*tc)[2] = that->comp;
  double (*tic)[2] = that->invComp;

  double det = tc[0][0] * tc[1][1] - tc[1][0] * tc[0][1];
  if (fabs(det) < EPSILON) {
    fprintf(stderr,
      "FrameUpdateInv: det == 0.0\n");
    exit(1);
  }

  tic[0][0] = tc[1][1] / det;
  tic[0][1] = -tc[0][1] / det;
  tic[1][0] = -tc[1][0] / det;
  tic[1][1] = tc[0][0] / det;

}

void Frame3DUpdateInv(Frame3D* const that) {

  // Shortcuts
  double (*tc)[3] = that->comp;
  double (*tic)[3] = that->invComp;

  // Update the inverse components
  double det =
    tc[0][0] * (tc[1][1] * tc[2][2] - tc[1][2] * tc[2][1]) -
    tc[1][0] * (tc[0][1] * tc[2][2] - tc[0][2] * tc[2][1]) +
    tc[2][0] * (tc[0][1] * tc[1][2] - tc[0][2] * tc[1][1]);
  if (fabs(det) < EPSILON) {
    fprintf(stderr,
      "FrameUpdateInv: det == 0.0\n");
    exit(1);
  }

  tic[0][0] = (tc[1][1] * tc[2][2] - tc[2][1] * tc[1][2]) / det;
  tic[0][1] = (tc[2][1] * tc[0][2] - tc[2][2] * tc[0][1]) / det;
  tic[0][2] = (tc[0][1] * tc[1][2] - tc[0][2] * tc[1][1]) / det;
  tic[1][0] = (tc[2][0] * tc[1][2] - tc[2][2] * tc[1][0]) / det;
  tic[1][1] = (tc[0][0] * tc[2][2] - tc[2][0] * tc[0][2]) / det;
  tic[1][2] = (tc[0][2] * tc[1][0] - tc[1][2] * tc[0][0]) / det;
  tic[2][0] = (tc[1][0] * tc[2][1] - tc[2][0] * tc[1][1]) / det;
  tic[2][1] = (tc[0][1] * tc[2][0] - tc[2][1] * tc[0][0]) / det;
  tic[2][2] = (tc[0][0] * tc[1][1] - tc[1][0] * tc[0][1]) / det;
```

```
}

// Update the inverse components of the Frame that
void Frame2DTimeUpdateInv(Frame2DTime* const that) {

  // Shortcuts
  double (*tc)[2] = that->comp;
  double (*tic)[2] = that->invComp;

  double det = tc[0][0] * tc[1][1] - tc[1][0] * tc[0][1];
  if (fabs(det) < EPSILON) {
    fprintf(stderr,
      "FrameUpdateInv: det == 0.0\n");
    exit(1);
  }

  tic[0][0] = tc[1][1] / det;
  tic[0][1] = -tc[0][1] / det;
  tic[1][0] = -tc[1][0] / det;
  tic[1][1] = tc[0][0] / det;

}

void Frame3DTimeUpdateInv(Frame3DTime* const that) {

  // Shortcuts
  double (*tc)[3] = that->comp;
  double (*tic)[3] = that->invComp;

  // Update the inverse components
  double det =
    tc[0][0] * (tc[1][1] * tc[2][2] - tc[1][2] * tc[2][1]) -
    tc[1][0] * (tc[0][1] * tc[2][2] - tc[0][2] * tc[2][1]) +
    tc[2][0] * (tc[0][1] * tc[1][2] - tc[0][2] * tc[1][1]);
  if (fabs(det) < EPSILON) {
    fprintf(stderr,
      "FrameUpdateInv: det == 0.0\n");
    exit(1);
  }

  tic[0][0] = (tc[1][1] * tc[2][2] - tc[2][1] * tc[1][2]) / det;
  tic[0][1] = (tc[2][1] * tc[0][2] - tc[2][2] * tc[0][1]) / det;
  tic[0][2] = (tc[0][1] * tc[1][2] - tc[0][2] * tc[1][1]) / det;
  tic[1][0] = (tc[2][0] * tc[1][2] - tc[2][2] * tc[1][0]) / det;
  tic[1][1] = (tc[0][0] * tc[2][2] - tc[2][0] * tc[0][2]) / det;
  tic[1][2] = (tc[0][2] * tc[1][0] - tc[1][2] * tc[0][0]) / det;
  tic[2][0] = (tc[1][0] * tc[2][1] - tc[2][0] * tc[1][1]) / det;
  tic[2][1] = (tc[0][1] * tc[2][0] - tc[2][1] * tc[0][0]) / det;
  tic[2][2] = (tc[0][0] * tc[1][1] - tc[1][0] * tc[0][1]) / det;

}

// Project the Frame Q in the Frame P's coordinates system and
// memorize the result in the Frame Qp
void Frame2DImportFrame(
  const Frame2D* const P,
  const Frame2D* const Q,
        Frame2D* const Qp) {

  // Shortcuts
  const double*  qo  = Q->orig;
```

```
        double*  qpo = Qp->orig;
  const double*  po  = P->orig;

  const double  (*pi)[2] = P->invComp;
        double (*qpc)[2] = Qp->comp;
  const double  (*qc)[2] = Q->comp;

  // Calculate the projection
  double v[2];
  for (int i = 2;
       i--;) {

    v[i] = qo[i] - po[i];

  }

  for (int i = 2;
       i--;) {

    qpo[i] = 0.0;

    for (int j = 2;
         j--;) {

      qpo[i] += pi[j][i] * v[j];
      qpc[j][i] = 0.0;

      for (int k = 2;
           k--;) {

        qpc[j][i] += pi[k][i] * qc[j][k];

      }
    }
  }
}

void Frame3DImportFrame(
  const Frame3D* const P,
  const Frame3D* const Q,
        Frame3D* const Qp) {

  // Shortcuts
  const double*  qo  = Q->orig;
        double*  qpo = Qp->orig;
  const double*  po  = P->orig;

  const double  (*pi)[3] = P->invComp;
        double (*qpc)[3] = Qp->comp;
  const double  (*qc)[3] = Q->comp;

  // Calculate the projection
  double v[3];
  for (int i = 3;
       i--;) {

    v[i] = qo[i] - po[i];

  }

  for (int i = 3;
       i--;) {
```

```
    qpo[i] = 0.0;

    for (int j = 3;
         j--;) {

      qpo[i] += pi[j][i] * v[j];
      qpc[j][i] = 0.0;

      for (int k = 3;
           k--;) {

        qpc[j][i] += pi[k][i] * qc[j][k];

      }
    }
  }
}

void Frame2DTimeImportFrame(
  const Frame2DTime* const P,
  const Frame2DTime* const Q,
        Frame2DTime* const Qp) {

  // Shortcuts
  const double*  qo  = Q->orig;
        double*  qpo = Qp->orig;
  const double*  po  = P->orig;

  const double*  qs  = Q->speed;
        double*  qps = Qp->speed;
  const double*  ps  = P->speed;

  const double  (*pi)[2] = P->invComp;
        double (*qpc)[2] = Qp->comp;
  const double  (*qc)[2] = Q->comp;

  // Calculate the projection
  double v[2];
  double s[2];
  for (int i = 2;
       i--;) {

    v[i] = qo[i] - po[i];
    s[i] = qs[i] - ps[i];

  }

  for (int i = 2;
       i--;) {

    qpo[i] = 0.0;
    qps[i] = 0.0;

    for (int j = 2;
         j--;) {

      qpo[i] += pi[j][i] * v[j];
      qps[i] += pi[j][i] * s[j];
      qpc[j][i] = 0.0;

      for (int k = 2;
```

```
          k--;) {

        qpc[j][i] += pi[k][i] * qc[j][k];

      }
    }
  }
}

void Frame3DTimeImportFrame(
  const Frame3DTime* const P,
  const Frame3DTime* const Q,
        Frame3DTime* const Qp) {

  // Shortcuts
  const double*  qo  = Q->orig;
        double*  qpo = Qp->orig;
  const double*  po  = P->orig;

  const double*  qs  = Q->speed;
        double*  qps = Qp->speed;
  const double*  ps  = P->speed;

  const double  (*pi)[3] = P->invComp;
        double (*qpc)[3] = Qp->comp;
  const double  (*qc)[3] = Q->comp;

  // Calculate the projection
  double v[3];
  double s[3];
  for (int i = 3;
       i--;) {

    v[i] = qo[i] - po[i];
    s[i] = qs[i] - ps[i];

  }

  for (int i = 3;
       i--;) {

    qpo[i] = 0.0;
    qps[i] = 0.0;

    for (int j = 3;
         j--;) {

      qpo[i] += pi[j][i] * v[j];
      qps[i] += pi[j][i] * s[j];
      qpc[j][i] = 0.0;

      for (int k = 3;
           k--;) {

        qpc[j][i] += pi[k][i] * qc[j][k];

      }
    }
  }
}

// Export the AABB bdgBox from that's coordinates system to
```

```
// the real coordinates system and update bdgBoxProj with the resulting
// AABB
void Frame2DExportBdgBox(
  const Frame2D* const that,
   const AABB2D* const bdgBox,
         AABB2D* const bdgBoxProj) {

  // Shortcuts
  const double* to    = that->orig;
  const double* bbmi  = bdgBox->min;
  const double* bbma  = bdgBox->max;
        double* bbpmi = bdgBoxProj->min;
        double* bbpma = bdgBoxProj->max;

  const double (*tc)[2] = that->comp;

  // Initialise the coordinates of the result AABB with the projection
  // of the first corner of the AABB in argument
  for (int i = 2;
       i--;) {

    bbpma[i] = to[i];

    for (int j = 2;
         j--;) {

      bbpma[i] += tc[j][i] * bbmi[j];

    }

    bbpmi[i] = bbpma[i];

  }

  // Loop on vertices of the AABB
  // skip the first vertex which is the origin already computed above
  int nbVertices = powi(2, 2);
  for (int iVertex = nbVertices;
       iVertex-- && iVertex;) {

    // Declare a variable to memorize the coordinates of the vertex in
    // 'that' 's coordinates system
    double v[2];

    // Calculate the coordinates of the vertex in
    // 'that' 's coordinates system
    for (int i = 2;
         i--;) {

      v[i] = ((iVertex & (1 << i)) ? bbma[i] : bbmi[i]);

    }

    // Declare a variable to memorize the projected coordinates
    // in real coordinates system
    double w[2];

    // Project the vertex to real coordinates system
    for (int i = 2;
         i--;) {

      w[i] = to[i];
```

```
      for (int j = 2;
           j--;) {

        w[i] += tc[j][i] * v[j];

      }
    }

    // Update the coordinates of the result AABB
    for (int i = 2;
         i--;) {

      if (bbpmi[i] > w[i]) {

        bbpmi[i] = w[i];

      }
      if (bbpma[i] < w[i]) {

        bbpma[i] = w[i];

      }
    }
  }

}

void Frame3DExportBdgBox(
  const Frame3D* const that,
   const AABB3D* const bdgBox,
         AABB3D* const bdgBoxProj) {

  // Shortcuts
  const double* to    = that->orig;
  const double* bbmi  = bdgBox->min;
  const double* bbma  = bdgBox->max;
        double* bbpmi = bdgBoxProj->min;
        double* bbpma = bdgBoxProj->max;

  const double (*tc)[3] = that->comp;

  // Initialise the coordinates of the result AABB with the projection
  // of the first corner of the AABB in argument
  for (int i = 3;
       i--;) {

    bbpma[i] = to[i];

    for (int j = 3;
         j--;) {

      bbpma[i] += tc[j][i] * bbmi[j];

    }

    bbpmi[i] = bbpma[i];

  }

  // Loop on vertices of the AABB
  // skip the first vertex which is the origin already computed above
```

```
  int nbVertices = powi(2, 3);
  for (int iVertex = nbVertices;
       iVertex-- && iVertex;) {

    // Declare a variable to memorize the coordinates of the vertex in
    // 'that' 's coordinates system
    double v[3];

    // Calculate the coordinates of the vertex in
    // 'that' 's coordinates system
    for (int i = 3;
         i--;) {

      v[i] = ((iVertex & (1 << i)) ? bbma[i] : bbmi[i]);

    }

    // Declare a variable to memorize the projected coordinates
    // in real coordinates system
    double w[3];

    // Project the vertex to real coordinates system
    for (int i = 3;
         i--;) {

      w[i] = to[i];

      for (int j = 3;
           j--;) {

        w[i] += tc[j][i] * v[j];

      }
    }

    // Update the coordinates of the result AABB
    for (int i = 3;
         i--;) {

      if (bbpmi[i] > w[i]) {

        bbpmi[i] = w[i];

      }
      if (bbpma[i] < w[i]) {

        bbpma[i] = w[i];

      }
    }
  }

}

void Frame2DTimeExportBdgBox(
  const Frame2DTime* const that,
   const AABB2DTime* const bdgBox,
         AABB2DTime* const bdgBoxProj) {

  // Shortcuts
  const double* to    = that->orig;
  const double* ts    = that->speed;
```

```
const double* bbmi  = bdgBox->min;
const double* bbma  = bdgBox->max;
      double* bbpmi = bdgBoxProj->min;
      double* bbpma = bdgBoxProj->max;
const double (*tc)[2] = that->comp;

// The time component is not affected
bbpmi[2] = bbmi[2];
bbpma[2] = bbma[2];

// Initialise the coordinates of the result AABB with the projection
// of the first corner of the AABB in argument
for (int i = 2;
     i--;) {

  bbpma[i] = to[i] + ts[i] * bbmi[2];

  for (int j = 2;
       j--;) {

    bbpma[i] += tc[j][i] * bbmi[j];

  }

  bbpmi[i] = bbpma[i];

}

// Loop on vertices of the AABB
// skip the first vertex which is the origin already computed above
int nbVertices = powi(2, 2);
for (int iVertex = nbVertices;
     iVertex-- && iVertex;) {

  // Declare a variable to memorize the coordinates of the vertex in
  // 'that' 's coordinates system
  double v[2];

  // Calculate the coordinates of the vertex in
  // 'that' 's coordinates system
  for (int i = 2;
       i--;) {

    v[i] = ((iVertex & (1 << i)) ? bbma[i] : bbmi[i]);

  }

  // Declare a variable to memorize the projected coordinates
  // in real coordinates system
  double w[2];

  // Project the vertex to real coordinates system
  for (int i = 2;
       i--;) {

    w[i] = to[i];

    for (int j = 2;
         j--;) {

      w[i] += tc[j][i] * v[j];
```

```
      }
    }

    // Update the coordinates of the result AABB
    for (int i = 2;
         i--;) {

      if (bbpmi[i] > w[i] + ts[i] * bbmi[2]) {

        bbpmi[i] = w[i] + ts[i] * bbmi[2];

      }
      if (bbpmi[i] > w[i] + ts[i] * bbma[2]) {

        bbpmi[i] = w[i] + ts[i] * bbma[2];

      }
      if (bbpma[i] < w[i] + ts[i] * bbmi[2]) {

        bbpma[i] = w[i] + ts[i] * bbmi[2];

      }
      if (bbpma[i] < w[i] + ts[i] * bbma[2]) {

        bbpma[i] = w[i] + ts[i] * bbma[2];

      }
    }
  }

}

void Frame3DTimeExportBdgBox(
  const Frame3DTime* const that,
   const AABB3DTime* const bdgBox,
         AABB3DTime* const bdgBoxProj) {

  // Shortcuts
  const double* to    = that->orig;
  const double* ts    = that->speed;
  const double* bbmi  = bdgBox->min;
  const double* bbma  = bdgBox->max;
        double* bbpmi = bdgBoxProj->min;
        double* bbpma = bdgBoxProj->max;
  const double (*tc)[3] = that->comp;

  // The time component is not affected
  bbpmi[3] = bbmi[3];
  bbpma[3] = bbma[3];

  // Initialise the coordinates of the result AABB with the projection
  // of the first corner of the AABB in argument
  for (int i = 3;
       i--;) {

    bbpma[i] = to[i] + ts[i] * bbmi[3];

    for (int j = 3;
         j--;) {

      bbpma[i] += tc[j][i] * bbmi[j];
```

```
  }

  bbpmi[i] = bbpma[i];

}

// Loop on vertices of the AABB
// skip the first vertex which is the origin already computed above
int nbVertices = powi(2, 3);
for (int iVertex = nbVertices;
     iVertex-- && iVertex;) {

  // Declare a variable to memorize the coordinates of the vertex in
  // 'that' 's coordinates system
  double v[3];

  // Calculate the coordinates of the vertex in
  // 'that' 's coordinates system
  for (int i = 3;
       i--;) {

    v[i] = ((iVertex & (1 << i)) ? bbma[i] : bbmi[i]);

  }

  // Declare a variable to memorize the projected coordinates
  // in real coordinates system
  double w[3];

  // Project the vertex to real coordinates system
  for (int i = 3;
       i--;) {

    w[i] = to[i];

    for (int j = 3;
         j--;) {

      w[i] += tc[j][i] * v[j];

    }
  }

  // Update the coordinates of the result AABB
  for (int i = 3;
       i--;) {

    if (bbpmi[i] > w[i] + ts[i] * bbmi[3]) {

      bbpmi[i] = w[i] + ts[i] * bbmi[3];

    }
    if (bbpmi[i] > w[i] + ts[i] * bbma[3]) {

      bbpmi[i] = w[i] + ts[i] * bbma[3];

    }
    if (bbpma[i] < w[i] + ts[i] * bbmi[3]) {

      bbpma[i] = w[i] + ts[i] * bbmi[3];

    }
```

```
      if (bbpma[i] < w[i] + ts[i] * bbma[3]) {

        bbpma[i] = w[i] + ts[i] * bbma[3];

      }
    }
  }

}

// Print the AABB that on stdout
// Output format is (min[0], min[1], ...)-(max[0], max[1], ...)
void AABB2DPrint(const AABB2D* const that) {

  printf("minXY(");
  for (int i = 0;
       i < 2;
       ++i) {

    printf("%f", that->min[i]);
    if (i < 1)
      printf(",");

  }
  printf(")-maxXY(");
  for (int i = 0;
       i < 2;
       ++i) {

    printf("%f", that->max[i]);
    if (i < 1)
      printf(",");

  }
  printf(")");

}

void AABB3DPrint(const AABB3D* const that) {

  printf("minXYZ(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->min[i]);
    if (i < 2)
      printf(",");

  }
  printf(")-maxXYZ(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->max[i]);
    if (i < 2)
      printf(",");

  }
  printf(")");
```

```
}

void AABB2DTimePrint(const AABB2DTime* const that) {

  printf("minXYT(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->min[i]);
    if (i < 2)
      printf(",");

  }
  printf(")-maxXYT(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->max[i]);
    if (i < 2)
      printf(",");

  }
  printf(")");

}

void AABB3DTimePrint(const AABB3DTime* const that) {

  printf("minXYZT(");
  for (int i = 0;
       i < 4;
       ++i) {

    printf("%f", that->min[i]);
    if (i < 3)
      printf(",");

  }
  printf(")-maxXYZT(");
  for (int i = 0;
       i < 4;
       ++i) {

    printf("%f", that->max[i]);
    if (i < 3)
      printf(",");

  }
  printf(")");

}

// Print the Frame that on stdout
// Output format is
// T/C  <- type of Frame
// o(orig[0], orig[1], orig[2])
// s(speed[0], speed[1], speed[2])
// x(comp[0][0], comp[0][1], comp[0][2])
// y(comp[1][0], comp[1][1], comp[1][2])
// z(comp[2][0], comp[2][1], comp[2][2])
```

```
void Frame2DPrint(const Frame2D* const that) {
  if (that->type == FrameTetrahedron) {
    printf("T");
  } else if (that->type == FrameCuboid) {
    printf("C");
  }
  printf("o(");
  for (int i = 0;
       i < 2;
       ++i) {

    printf("%f", that->orig[i]);
    if (i < 1)
      printf(",");

  }
  char comp[2] = {'x', 'y'};
  for (int j = 0;
       j < 2;
       ++j) {
    printf(") %c(", comp[j]);
    for (int i = 0;
         i < 2;
         ++i) {

      printf("%f", that->comp[j][i]);
      if (i < 1)
        printf(",");

    }
  }
  printf(")");

}

void Frame3DPrint(const Frame3D* const that) {
  if (that->type == FrameTetrahedron) {
    printf("T");
  } else if (that->type == FrameCuboid) {
    printf("C");
  }
  printf("o(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->orig[i]);
    if (i < 2)
      printf(",");

  }
  char comp[3] = {'x', 'y', 'z'};
  for (int j = 0;
       j < 3;
       ++j) {
    printf(") %c(", comp[j]);
    for (int i = 0;
         i < 3;
         ++i) {

      printf("%f", that->comp[j][i]);
      if (i < 2)
```

```
        printf(",");

    }
  }
  printf(")");

}

void Frame2DTimePrint(const Frame2DTime* const that) {
  if (that->type == FrameTetrahedron) {
    printf("T");
  } else if (that->type == FrameCuboid) {
    printf("C");
  }
  printf("o(");
  for (int i = 0;
       i < 2;
       ++i) {

    printf("%f", that->orig[i]);
    if (i < 1)
      printf(",");

  }
  printf(") s(");
  for (int i = 0;
       i < 2;
       ++i) {

    printf("%f", that->speed[i]);
    if (i < 1)
      printf(",");

  }
  char comp[2] = {'x', 'y'};
  for (int j = 0;
       j < 2;
       ++j) {
    printf(") %c(", comp[j]);
    for (int i = 0;
         i < 2;
         ++i) {

      printf("%f", that->comp[j][i]);
      if (i < 1)
        printf(",");

    }
  }
  printf(")");

}

void Frame3DTimePrint(const Frame3DTime* const that) {
  if (that->type == FrameTetrahedron) {
    printf("T");
  } else if (that->type == FrameCuboid) {
    printf("C");
  }
  printf("o(");
  for (int i = 0;
       i < 3;
```

```
       ++i) {

    printf("%f", that->orig[i]);
    if (i < 2)
      printf(",");

  }
  printf(") s(");
  for (int i = 0;
       i < 3;
       ++i) {

    printf("%f", that->speed[i]);
    if (i < 2)
      printf(",");

  }
  char comp[3] = {'x', 'y', 'z'};
  for (int j = 0;
       j < 3;
       ++j) {
    printf(") %c(", comp[j]);
    for (int i = 0;
         i < 3;
         ++i) {

      printf("%f", that->comp[j][i]);
      if (i < 2)
        printf(",");

    }
  }
  printf(")");

}

// Power function for integer base and exponent
// Return base^exp
int powi(
           int base,
  unsigned int exp) {

    int res = 1;
    for (;
         exp;
         --exp) {

      res *= base;

    }
    return res;
}
```

4.2 FMB

4.2.1 2D static

Header

```
#ifndef __FMB2D_H_
#define __FMB2D_H_
```

```
#include <stdbool.h>
#include "frame.h"

// ------------- Functions declaration -------------

// Test for intersection between Frame that and Frame tho
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into bdgBox, else bdgBox is not modified
// If bdgBox is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in tho's local coordinates system
bool FMBTestIntersection2D(
  const Frame2D* const that,
  const Frame2D* const tho,
         AABB2D* const bdgBox);

#endif
```

Body

```
#include "fmb2d.h"

// ------------- Macros -------------

// Return 1.0 if v is positive, -1.0 if v is negative, 0.0 else
#define sgn(v) (((0.0 < (v)) ? 1 : 0) - (((v) < 0.0) ? 1 : 0))

// Return x if x is negative, 0.0 else
#define neg(x) (x < 0.0 ? x : 0.0)

#define FST_VAR 0
#define SND_VAR 1

#define EPSILON 0.0000001

// ------------- Functions declaration -------------

// Eliminate the first variable in the system M.X<=Y
// using the Fourier-Motzkin method and return
// the resulting system in Mp and Yp, and the number of rows of
// the resulting system in nbRemainRows
// Return false if the system becomes inconsistent during elimination,
// else return true
bool ElimVar2D(
  const double (*M)[2],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[2],
        double* Yp,
    int* const nbRemainRows);

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y which has been reduced to only one variable
// and store them in the iVar-th axis of the
// AABB bdgBox
// (M arrangement is [iRow][iCol])
```

```
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBoundLastVar2D(
     const int iVar,
  const double (*M)[2],
  const double* Y,
     const int nbRows,
   AABB2D* const bdgBox);

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y where the iVar-th variable is on the first column
// and store them in the iVar-th axis of the AABB bdgBox
// (M arrangement is [iRow][iCol])
void GetBoundVar2D(
     const int iVar,
  const double (*M)[2],
  const double* Y,
     const int nbRows,
     const int nbCols,
   AABB2D* const bdgBox);

// ------------- Functions implementation -------------

// Eliminate the first variable in the system M.X<=Y
// using the Fourier-Motzkin method and return
// the resulting system in Mp and Yp, and the number of rows of
// the resulting system in nbRemainRows
// (M arrangement is [iRow][iCol])
// Return true if the system becomes inconsistent during elimination,
// else return false
bool ElimVar2D(
  const double (*M)[2],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[2],
        double* Yp,
    int* const nbRemainRows) {

  // Initialize the number of rows in the result system
  int nbResRows = 0;

  // First we process the rows where the eliminated variable is not null

  // For each row except the last one
  for (int iRow = 0;
       iRow < nbRows - 1;
       ++iRow) {

    // Shortcuts
    const double fabsMIRowIVar = fabs(M[iRow][0]);

    // If the coefficient for the eliminated vairable is not null
    // in this row
    if (fabsMIRowIVar > EPSILON) {

      // Shortcuts
      const double* MiRow = M[iRow];
      const int sgnMIRowIVar = sgn(MiRow[0]);
      const double YIRowDivideByFabsMIRowIVar = Y[iRow] / fabsMIRowIVar;

      // For each following rows
```

```
    for (int jRow = iRow + 1;
         jRow < nbRows;
         ++jRow) {

      // If coefficients of the eliminated variable in the two rows have
      // different signs and are not null
      if (sgnMIRowIVar != sgn(M[jRow][0]) &&
          fabs(M[jRow][0]) > EPSILON) {

        // Shortcuts
        const double* MjRow = M[jRow];
        const double fabsMjRow = fabs(MjRow[0]);

        // Declare a variable to memorize the sum of the negative
        // coefficients in the row
        double sumNegCoeff = 0.0;

        // Add the sum of the two normed (relative to the eliminated
        // variable) rows into the result system. This actually
        // eliminate the variable while keeping the constraints on
        // others variables
        for (int iCol = 1;
             iCol < nbCols;
             ++iCol ) {

          Mp[nbResRows][iCol - 1] =
            MiRow[iCol] / fabsMIRowIVar +
            MjRow[iCol] / fabsMjRow;

          // Update the sum of the negative coefficient
          sumNegCoeff += neg(Mp[nbResRows][iCol - 1]);

        }

        // Update the right side of the inequality
        Yp[nbResRows] =
          YIRowDivideByFabsMIRowIVar +
          Y[jRow] / fabsMjRow;

        // If the right side of the inequality if lower than the sum of
        // negative coefficients in the row
        // (Add epsilon for numerical imprecision)
        if (Yp[nbResRows] < sumNegCoeff - EPSILON) {

          // Given that X is in [0,1], the system is inconsistent
          return true;

        }

        // Increment the nb of rows into the result system
        ++nbResRows;

      }

    }

  }

}

// Then we copy and compress the rows where the eliminated
// variable is null
```

```
  // Loop on rows of the input system
  for (int iRow = 0;
       iRow < nbRows;
       ++iRow) {

    // Shortcut
    const double* MiRow = M[iRow];

    // If the coefficient of the eliminated variable is null on
    // this row
    if (fabs(MiRow[0]) < EPSILON) {

      // Shortcut
      double* MpnbResRows = Mp[nbResRows];

      // Copy this row into the result system excluding the eliminated
      // variable
      for (int iCol = 1;
           iCol < nbCols;
           ++iCol) {

        MpnbResRows[iCol - 1] = MiRow[iCol];

      }

      Yp[nbResRows] = Y[iRow];

      // Increment the nb of rows into the result system
      ++nbResRows;

    }

  }

  // Memorize the number of rows in the result system
  *nbRemainRows = nbResRows;

  // If we reach here the system is not inconsistent
  return false;

}

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y which has been reduced to only one variable
// and store them in the iVar-th axis of the
// AABB bdgBox
// (M arrangement is [iRow][iCol])
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBoundLastVar2D(
     const int iVar,
  const double (*M)[2],
  const double* Y,
     const int nbRows,
   AABB2D* const bdgBox) {

  // Shortcuts
  double* min = bdgBox->min + iVar;
  double* max = bdgBox->max + iVar;

  // Initialize the bounds to their maximum maximum and minimum minimum
```

```
  *min = 0.0;
  *max = 1.0;

  // Loop on rows
  for (int jRow = 0;
       jRow < nbRows;
       ++jRow) {

    // Shortcut
    double MjRowiVar = M[jRow][0];

    // If this row has been reduced to the variable in argument
    // and it has a strictly positive coefficient
    if (MjRowiVar > EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is lower than the current maximum bound
      if (*max > y) {

        // Update the maximum bound
        *max = y;

      }

    // Else, if this row has been reduced to the variable in argument
    // and it has a strictly negative coefficient
    } else if (MjRowiVar < -EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is greater than the current minimum bound
      if (*min < y) {

        // Update the minimum bound
        *min = y;

      }

    }

  }

}

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y where the iVar-th variable is on the first column
// and store them in the iVar-th axis of the AABB bdgBox
// (M arrangement is [iRow][iCol])
void GetBoundVar2D(
     const int iVar,
  const double (*M)[2],
  const double* Y,
     const int nbRows,
     const int nbCols,
   AABB2D* const bdgBox) {

  // Shortcuts
  double* bdgBoxMin = bdgBox->min;
  double* bdgBoxMax = bdgBox->max;
```

```
  // Initialize the bounds
  bdgBoxMin[iVar] = 0.0;
  bdgBoxMax[iVar] = 1.0;

  // Loop on the rows
  for (int iRow = 0;
       iRow < nbRows;
       ++iRow) {

    // Shortcuts
    const double* MIRow = M[iRow];
    double fabsMIRowIVar = fabs(MIRow[0]);

    // If the coefficient of the first variable on this row is not null
    if (fabsMIRowIVar > EPSILON) {

      // Declare two variables to memorize the min and max of the
      // requested variable in this row
      double min = -1.0 * Y[iRow];
      double max = Y[iRow];

      // Loop on columns except the first one which is the one of the
      // requested variable
      for (int iCol = 1;
           iCol < nbCols;
           ++iCol) {

        if (MIRow[iCol] > EPSILON) {
          min += MIRow[iCol] * bdgBoxMin[iCol + iVar];
          max -= MIRow[iCol] * bdgBoxMin[iCol + iVar];
        } else if (MIRow[iCol] < EPSILON) {
          min += MIRow[iCol] * bdgBoxMax[iCol + iVar];
          max -= MIRow[iCol] * bdgBoxMax[iCol + iVar];
        }

      }

      min /= -1.0 * MIRow[0];
      max /= MIRow[0];
      if (bdgBoxMin[iVar] > min) {

        bdgBoxMin[iVar] = min;

      }
      if (bdgBoxMax[iVar] < max) {

        bdgBoxMax[iVar] = max;

      }

    }

  }

}

// Test for intersection between Frame that and Frame tho
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into bdgBox, else bdgBox is not modified
// If bdgBox is null, the result AABB is not memorized (to use if
```

```
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in tho's local coordinates system
bool FMBTestIntersection2D(
  const Frame2D* const that,
  const Frame2D* const tho,
        AABB2D* const bdgBox) {

  // Get the projection of the Frame 'tho' in Frame 'that' coordinates
  // system
  Frame2D thoProj;
  Frame2DImportFrame(that, tho, &thoProj);

  // Declare two variables to memorize the system to be solved M.X <= Y
  // (M arrangement is [iRow][iCol])
  double M[8][2];
  double Y[8];

  // Create the inequality system

  // -sum_iC_j,iX_i<=0_j
  M[0][0] = -thoProj.comp[0][0];
  M[0][1] = -thoProj.comp[1][0];
  Y[0] = thoProj.orig[0];
  if (Y[0] < neg(M[0][0]) + neg(M[0][1]))
    return false;

  M[1][0] = -thoProj.comp[0][1];
  M[1][1] = -thoProj.comp[1][1];
  Y[1] = thoProj.orig[1];
  if (Y[1] < neg(M[1][0]) + neg(M[1][1]))
    return false;

  // Variable to memorise the nb of rows in the system
  int nbRows = 2;

  if (that->type == FrameCuboid) {

    // sum_iC_j,iX_i<=1.0-0_j
    M[nbRows][0] = thoProj.comp[0][0];
    M[nbRows][1] = thoProj.comp[1][0];
    Y[nbRows] = 1.0 - thoProj.orig[0];
    if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]))
      return false;
    ++nbRows;

    M[nbRows][0] = thoProj.comp[0][1];
    M[nbRows][1] = thoProj.comp[1][1];
    Y[nbRows] = 1.0 - thoProj.orig[1];
    if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]))
      return false;
    ++nbRows;

  } else {

    // sum_j(sum_iC_j,iX_i)<=1.0-sum_i0_i
    M[nbRows][0] = thoProj.comp[0][0] + thoProj.comp[0][1];
    M[nbRows][1] = thoProj.comp[1][0] + thoProj.comp[1][1];
    Y[nbRows] = 1.0 - thoProj.orig[0] - thoProj.orig[1];
    if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]))
```

```
    return false;
  ++nbRows;

}

if (tho->type == FrameCuboid) {

  // X_i <= 1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

  M[nbRows][0] = 0.0;
  M[nbRows][1] = 1.0;
  Y[nbRows] = 1.0;
  ++nbRows;

} else {

  // sum_iX_i<=1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 1.0;
  Y[nbRows] = 1.0;
  ++nbRows;

}

// -X_i <= 0.0
M[nbRows][0] = -1.0;
M[nbRows][1] = 0.0;
Y[nbRows] = 0.0;
++nbRows;

M[nbRows][0] = 0.0;
M[nbRows][1] = -1.0;
Y[nbRows] = 0.0;
++nbRows;

// Solve the system

// Declare a AABB to memorize the bounding box of the intersection
// in the coordinates system of tho
AABB2D bdgBoxLocal = {
  .min = {0.0, 0.0},
  .max = {0.0, 0.0}
};

// Declare variables to eliminate the first variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mp[24][2];
//double Yp[24];
double Mp[11][2];
double Yp[11];
int nbRowsP;

// Eliminate the first variable
bool inconsistency =
  ElimVar2D(
```

```
      M,
      Y,
      nbRows,
      2,
      Mp,
      Yp,
      &nbRowsP);

  // If the system is inconsistent
  if (inconsistency == true) {

    // The two Frames are not in intersection
    return false;

  }

  // Get the bounds for the remaining second variable
  GetBoundLastVar2D(
    SND_VAR,
    Mp,
    Yp,
    nbRowsP,
    &bdgBoxLocal);

  // If the bounds are inconsistent
  if (bdgBoxLocal.min[SND_VAR] >= bdgBoxLocal.max[SND_VAR]) {

    // The two Frames are not in intersection
    return false;

  // Else, if the bounds are consistent here it means
  // the two Frames are in intersection.
  // If the user has requested for the resulting bounding box
  } else if (bdgBox != NULL) {

    // Get the bounds of the first variable from the bounds of the
    // second one

    GetBoundVar2D(
       FST_VAR,
       M,
       Y,
       nbRows,
       2,
       &bdgBoxLocal);

    // Memorize the result
    *bdgBox = bdgBoxLocal;

  }

  // If we've reached here the two Frames are intersecting
  return true;

}
```

4.2.2 3D static

Header

```
#ifndef __FMB3D_H_
#define __FMB3D_H_

#include <stdbool.h>
#include "frame.h"

// ------------- Functions declaration -------------

// Test for intersection between Frame that and Frame tho
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into bdgBox, else bdgBox is not modified
// If bdgBox is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in tho's local coordinates system
bool FMBTestIntersection3D(
  const Frame3D* const that,
  const Frame3D* const tho,
        AABB3D* const bdgBox);

#endif
```

Body

```
#include "fmb3d.h"

// ------------- Macros -------------

// Return 1.0 if v is positive, -1.0 if v is negative, 0.0 else
#define sgn(v) (((0.0 < (v)) ? 1 : 0) - (((v) < 0.0) ? 1 : 0))

// Return x if x is negative, 0.0 else
#define neg(x) (x < 0.0 ? x : 0.0)

#define FST_VAR 0
#define SND_VAR 1
#define THD_VAR 2

#define EPSILON 0.0000001

// ------------- Functions declaration -------------

// Eliminate the first variable in the system M.X<=Y
// using the Fourier-Motzkin method and return
// the resulting system in Mp and Yp, and the number of rows of
// the resulting system in nbRemainRows
// Return false if the system becomes inconsistent during elimination,
// else return true
bool ElimVar3D(
  const double (*M)[3],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[3],
        double* Yp,
    int* const nbRemainRows);

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y which has been reduced to only one variable
```

```
// and store them in the iVar-th axis of the
// AABB bdgBox
// (M arrangement is [iRow][iCol])
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBoundLastVar3D(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
   AABB3D* const bdgBox);

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y where the iVar-th variable is on the first column
// and store them in the iVar-th axis of the AABB bdgBox
// (M arrangement is [iRow][iCol])
void GetBoundVar3D(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
     const int nbCols,
   AABB3D* const bdgBox);

// ------------- Functions implementation -------------

// Eliminate the first variable in the system M.X<=Y
// using the Fourier-Motzkin method and return
// the resulting system in Mp and Yp, and the number of rows of
// the resulting system in nbRemainRows
// (M arrangement is [iRow][iCol])
// Return true if the system becomes inconsistent during elimination,
// else return false
bool ElimVar3D(
  const double (*M)[3],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[3],
        double* Yp,
    int* const nbRemainRows) {

  // Initialize the number of rows in the result system
  int nbResRows = 0;

  // First we process the rows where the eliminated variable is not null

  // For each row except the last one
  for (int iRow = 0;
       iRow < nbRows - 1;
       ++iRow) {

    // Shortcuts
    const double fabsMIRowIVar = fabs(M[iRow][0]);

    // If the coefficient for the eliminated vairable is not null
    // in this row
    if (fabsMIRowIVar > EPSILON) {

      // Shortcuts
      const double* MiRow = M[iRow];
      const int sgnMIRowIVar = sgn(MiRow[0]);
```

```
    const double YIRowDivideByFabsMIRowIVar = Y[iRow] / fabsMIRowIVar;

    // For each following rows
    for (int jRow = iRow + 1;
         jRow < nbRows;
         ++jRow) {

      // If coefficients of the eliminated variable in the two rows have
      // different signs and are not null
      if (sgnMIRowIVar != sgn(M[jRow][0]) &&
          fabs(M[jRow][0]) > EPSILON) {

        // Shortcuts
        const double* MjRow = M[jRow];
        const double fabsMjRow = fabs(MjRow[0]);

        // Declare a variable to memorize the sum of the negative
        // coefficients in the row
        double sumNegCoeff = 0.0;

        // Add the sum of the two normed (relative to the eliminated
        // variable) rows into the result system. This actually
        // eliminate the variable while keeping the constraints on
        // others variables
        for (int iCol = 1;
             iCol < nbCols;
             ++iCol ) {

          Mp[nbResRows][iCol - 1] =
            MiRow[iCol] / fabsMIRowIVar +
            MjRow[iCol] / fabsMjRow;

          // Update the sum of the negative coefficient
          sumNegCoeff += neg(Mp[nbResRows][iCol - 1]);

        }

        // Update the right side of the inequality
        Yp[nbResRows] =
          YIRowDivideByFabsMIRowIVar +
          Y[jRow] / fabsMjRow;

        // If the right side of the inequality if lower than the sum of
        // negative coefficients in the row
        // (Add epsilon for numerical imprecision)
        if (Yp[nbResRows] < sumNegCoeff - EPSILON) {

          // Given that X is in [0,1], the system is inconsistent
          return true;

        }

        // Increment the nb of rows into the result system
        ++nbResRows;

      }

    }

  }

}
```

```
  // Then we copy and compress the rows where the eliminated
  // variable is null

  // Loop on rows of the input system
  for (int iRow = 0;
       iRow < nbRows;
       ++iRow) {

    // Shortcut
    const double* MiRow = M[iRow];

    // If the coefficient of the eliminated variable is null on
    // this row
    if (fabs(MiRow[0]) < EPSILON) {

      // Shortcut
      double* MpnbResRows = Mp[nbResRows];

      // Copy this row into the result system excluding the eliminated
      // variable
      for (int iCol = 1;
           iCol < nbCols;
           ++iCol) {

        MpnbResRows[iCol - 1] = MiRow[iCol];

      }

      Yp[nbResRows] = Y[iRow];

      // Increment the nb of rows into the result system
      ++nbResRows;

    }

  }

  // Memorize the number of rows in the result system
  *nbRemainRows = nbResRows;

  // If we reach here the system is not inconsistent
  return false;

}

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y which has been reduced to only one variable
// and store them in the iVar-th axis of the
// AABB bdgBox
// (M arrangement is [iRow][iCol])
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBoundLastVar3D(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
   AABB3D* const bdgBox) {

  // Shortcuts
  double* min = bdgBox->min + iVar;
```

```
  double* max = bdgBox->max + iVar;

  // Initialize the bounds to their maximum maximum and minimum minimum
  *min = 0.0;
  *max = 1.0;

  // Loop on rows
  for (int jRow = 0;
       jRow < nbRows;
       ++jRow) {

    // Shortcut
    double MjRowiVar = M[jRow][0];

    // If this row has been reduced to the variable in argument
    // and it has a strictly positive coefficient
    if (MjRowiVar > EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is lower than the current maximum bound
      if (*max > y) {

        // Update the maximum bound
        *max = y;

      }

    // Else, if this row has been reduced to the variable in argument
    // and it has a strictly negative coefficient
    } else if (MjRowiVar < -EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is greater than the current minimum bound
      if (*min < y) {

        // Update the minimum bound
        *min = y;

      }

    }

  }

}

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y where the iVar-th variable is on the first column
// and store them in the iVar-th axis of the AABB bdgBox
// (M arrangement is [iRow][iCol])
void GetBoundVar3D(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
     const int nbCols,
   AABB3D* const bdgBox) {
```

```
  // Shortcuts
  double* bdgBoxMin = bdgBox->min;
  double* bdgBoxMax = bdgBox->max;

  // Initialize the bounds
  bdgBoxMin[iVar] = 0.0;
  bdgBoxMax[iVar] = 1.0;

  // Loop on the rows
  for (int iRow = 0;
       iRow < nbRows;
       ++iRow) {

    // Shortcuts
    const double* MIRow = M[iRow];
    double fabsMIRowIVar = fabs(MIRow[0]);

    // If the coefficient of the first variable on this row is not null
    if (fabsMIRowIVar > EPSILON) {

      // Declare two variables to memorize the min and max of the
      // requested variable in this row
      double min = -1.0 * Y[iRow];
      double max = Y[iRow];

      // Loop on columns except the first one which is the one of the
      // requested variable
      for (int iCol = 1;
           iCol < nbCols;
           ++iCol) {

        if (MIRow[iCol] > EPSILON) {
          min += MIRow[iCol] * bdgBoxMin[iCol + iVar];
          max -= MIRow[iCol] * bdgBoxMin[iCol + iVar];
        } else if (MIRow[iCol] < EPSILON) {
          min += MIRow[iCol] * bdgBoxMax[iCol + iVar];
          max -= MIRow[iCol] * bdgBoxMax[iCol + iVar];
        }

      }

      min /= -1.0 * MIRow[0];
      max /= MIRow[0];
      if (bdgBoxMin[iVar] > min) {

        bdgBoxMin[iVar] = min;

      }
      if (bdgBoxMax[iVar] < max) {

        bdgBoxMax[iVar] = max;

      }

    }

  }

}

// Test for intersection between Frame that and Frame tho
// Return true if the two Frames are intersecting, else false
```

```
// If the Frame are intersecting the AABB of the intersection
// is stored into bdgBox, else bdgBox is not modified
// If bdgBox is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in tho's local coordinates system
bool FMBTestIntersection3D(
  const Frame3D* const that,
  const Frame3D* const tho,
        AABB3D* const bdgBox) {

  // Get the projection of the Frame 'tho' in Frame 'that' coordinates
  // system
  Frame3D thoProj;
  Frame3DImportFrame(that, tho, &thoProj);

  // Declare two variables to memorize the system to be solved M.X <= Y
  // (M arrangement is [iRow][iCol])
  double M[12][3];
  double Y[12];

  // Create the inequality system

  // -sum_iC_j,iX_i<=O_j
  M[0][0] = -thoProj.comp[0][0];
  M[0][1] = -thoProj.comp[1][0];
  M[0][2] = -thoProj.comp[2][0];
  Y[0] = thoProj.orig[0];
  if (Y[0] < neg(M[0][0]) + neg(M[0][1]) + neg(M[0][2]))
    return false;

  M[1][0] = -thoProj.comp[0][1];
  M[1][1] = -thoProj.comp[1][1];
  M[1][2] = -thoProj.comp[2][1];
  Y[1] = thoProj.orig[1];
  if (Y[1] < neg(M[1][0]) + neg(M[1][1]) + neg(M[1][2]))
    return false;

  M[2][0] = -thoProj.comp[0][2];
  M[2][1] = -thoProj.comp[1][2];
  M[2][2] = -thoProj.comp[2][2];
  Y[2] = thoProj.orig[2];
  if (Y[2] < neg(M[2][0]) + neg(M[2][1]) + neg(M[2][2]))
    return false;

  // Variable to memorise the nb of rows in the system
  int nbRows = 3;

  if (that->type == FrameCuboid) {

    // sum_iC_j,iX_i<=1.0-O_j
    M[nbRows][0] = thoProj.comp[0][0];
    M[nbRows][1] = thoProj.comp[1][0];
    M[nbRows][2] = thoProj.comp[2][0];
    Y[nbRows] = 1.0 - thoProj.orig[0];
    if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                    neg(M[nbRows][2]))
      return false;
    ++nbRows;
```

```
  M[nbRows][0] = thoProj.comp[0][1];
  M[nbRows][1] = thoProj.comp[1][1];
  M[nbRows][2] = thoProj.comp[2][1];
  Y[nbRows] = 1.0 - thoProj.orig[1];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

  M[nbRows][0] = thoProj.comp[0][2];
  M[nbRows][1] = thoProj.comp[1][2];
  M[nbRows][2] = thoProj.comp[2][2];
  Y[nbRows] = 1.0 - thoProj.orig[2];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

} else {

  // sum_j(sum_iC_j,iX_i)<=1.0-sum_iO_i
  M[nbRows][0] =
    thoProj.comp[0][0] + thoProj.comp[0][1] + thoProj.comp[0][2];
  M[nbRows][1] =
    thoProj.comp[1][0] + thoProj.comp[1][1] + thoProj.comp[1][2];
  M[nbRows][2] =
    thoProj.comp[2][0] + thoProj.comp[2][1] + thoProj.comp[2][2];
  Y[nbRows] =
    1.0 - thoProj.orig[0] - thoProj.orig[1] - thoProj.orig[2];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

}

if (tho->type == FrameCuboid) {

  // X_i <= 1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 0.0;
  M[nbRows][2] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

  M[nbRows][0] = 0.0;
  M[nbRows][1] = 1.0;
  M[nbRows][2] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

  M[nbRows][0] = 0.0;
  M[nbRows][1] = 0.0;
  M[nbRows][2] = 1.0;
  Y[nbRows] = 1.0;
  ++nbRows;

} else {

  // sum_iX_i<=1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 1.0;
```

```
  M[nbRows][2] = 1.0;
  Y[nbRows] = 1.0;
  ++nbRows;

}

// -X_i <= 0.0
M[nbRows][0] = -1.0;
M[nbRows][1] = 0.0;
M[nbRows][2] = 0.0;
Y[nbRows] = 0.0;
++nbRows;

M[nbRows][0] = 0.0;
M[nbRows][1] = -1.0;
M[nbRows][2] = 0.0;
Y[nbRows] = 0.0;
++nbRows;

M[nbRows][0] = 0.0;
M[nbRows][1] = 0.0;
M[nbRows][2] = -1.0;
Y[nbRows] = 0.0;
++nbRows;

// Solve the system

// Declare a AABB to memorize the bounding box of the intersection
// in the coordinates system of tho
AABB3D bdgBoxLocal = {
  .min = {0.0, 0.0, 0.0},
  .max = {0.0, 0.0, 0.0}
};

// Declare variables to eliminate the first variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mp[48][3];
//double Yp[48];
double Mp[20][3];
double Yp[20];
int nbRowsP;

// Eliminate the first variable in the original system
bool inconsistency =
  ElimVar3D(
    M,
    Y,
    nbRows,
    3,
    Mp,
    Yp,
    &nbRowsP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;
```

```
}

// Declare variables to eliminate the second variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mpp[624][3];
//double Ypp[624];
double Mpp[55][3];
double Ypp[55];
int nbRowsPP;

// Eliminate the second variable (which is the first in the new system)
inconsistency =
  ElimVar3D(
    Mp,
    Yp,
    nbRowsP,
    2,
    Mpp,
    Ypp,
    &nbRowsPP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;

}

// Get the bounds for the remaining third variable
GetBoundLastVar3D(
  THD_VAR,
  Mpp,
  Ypp,
  nbRowsPP,
  &bdgBoxLocal);

// If the bounds are inconsistent
if (bdgBoxLocal.min[THD_VAR] >= bdgBoxLocal.max[THD_VAR]) {

  // The two Frames are not in intersection
  return false;

// Else, if the bounds are consistent here it means
// the two Frames are in intersection.
// If the user has requested for the resulting bounding box
} else if (bdgBox != NULL) {

  // Get the bounds of the other variables

  GetBoundVar3D(
     SND_VAR,
     Mp,
     Yp,
     nbRowsP,
     2,
     &bdgBoxLocal);

  GetBoundVar3D(
```

```
        FST_VAR,
        M,
        Y,
        nbRows,
        3,
        &bdgBoxLocal);

    // Memorize the result
    *bdgBox = bdgBoxLocal;

  }

  // If we've reached here the two Frames are intersecting
  return true;

}
```

4.2.3 2D dynamic

Header

```
#ifndef __FMB2DT_H_
#define __FMB2DT_H_

#include <stdbool.h>
#include "frame.h"

// ------------- Functions declaration -------------

// Test for intersection between Frame that and Frame tho
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into bdgBox, else bdgBox is not modified
// If bdgBox is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in tho's local coordinates system
bool FMBTestIntersection2DTime(
  const Frame2DTime* const that,
  const Frame2DTime* const tho,
         AABB2DTime* const bdgBox);

#endif
```

Body

```
#include "fmb2dt.h"

// ------------- Macros -------------

// Return 1.0 if v is positive, -1.0 if v is negative, 0.0 else
#define sgn(v) (((0.0 < (v)) ? 1 : 0) - (((v) < 0.0) ? 1 : 0))

// Return x if x is negative, 0.0 else
#define neg(x) (x < 0.0 ? x : 0.0)

#define FST_VAR 0
#define SND_VAR 1
```

```
#define THD_VAR 2

#define EPSILON 0.0000001

// ------------- Functions declaration -------------

// Eliminate the first variable in the system M.X<=Y
// using the Fourier-Motzkin method and return
// the resulting system in Mp and Yp, and the number of rows of
// the resulting system in nbRemainRows
// Return false if the system becomes inconsistent during elimination,
// else return true
bool ElimVar2DTime(
  const double (*M)[3],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[3],
        double* Yp,
    int* const nbRemainRows);

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y which has been reduced to only one variable
// and store them in the iVar-th axis of the
// AABB bdgBox
// (M arrangement is [iRow][iCol])
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBoundLastVar2DTime(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
   AABB2DTime* const bdgBox);

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y where the iVar-th variable is on the first column
// and store them in the iVar-th axis of the AABB bdgBox
// (M arrangement is [iRow][iCol])
void GetBoundVar2DTime(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
     const int nbCols,
   AABB2DTime* const bdgBox);

// ------------- Functions implementation -------------

// Eliminate the first variable in the system M.X<=Y
// using the Fourier-Motzkin method and return
// the resulting system in Mp and Yp, and the number of rows of
// the resulting system in nbRemainRows
// (M arrangement is [iRow][iCol])
// Return true if the system becomes inconsistent during elimination,
// else return false
bool ElimVar2DTime(
  const double (*M)[3],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[3],
```

```
      double* Yp,
  int* const nbRemainRows) {

// Initialize the number of rows in the result system
int nbResRows = 0;

// First we process the rows where the eliminated variable is not null

// For each row except the last one
for (int iRow = 0;
     iRow < nbRows - 1;
     ++iRow) {

  // Shortcuts
  const double fabsMIRowIVar = fabs(M[iRow][0]);

  // If the coefficient for the eliminated vairable is not null
  // in this row
  if (fabsMIRowIVar > EPSILON) {

    // Shortcuts
    const double* MiRow = M[iRow];
    const int sgnMIRowIVar = sgn(MiRow[0]);
    const double YIRowDivideByFabsMIRowIVar = Y[iRow] / fabsMIRowIVar;

    // For each following rows
    for (int jRow = iRow + 1;
         jRow < nbRows;
         ++jRow) {

      // If coefficients of the eliminated variable in the two rows have
      // different signs and are not null
      if (sgnMIRowIVar != sgn(M[jRow][0]) &&
          fabs(M[jRow][0]) > EPSILON) {

        // Shortcuts
        const double* MjRow = M[jRow];
        const double fabsMjRow = fabs(MjRow[0]);

        // Declare a variable to memorize the sum of the negative
        // coefficients in the row
        double sumNegCoeff = 0.0;

        // Add the sum of the two normed (relative to the eliminated
        // variable) rows into the result system. This actually
        // eliminate the variable while keeping the constraints on
        // others variables
        for (int iCol = 1;
             iCol < nbCols;
             ++iCol ) {

          Mp[nbResRows][iCol - 1] =
            MiRow[iCol] / fabsMIRowIVar +
            MjRow[iCol] / fabsMjRow;

          // Update the sum of the negative coefficient
          sumNegCoeff += neg(Mp[nbResRows][iCol - 1]);

        }

        // Update the right side of the inequality
        Yp[nbResRows] =
```

```
          YIRowDivideByFabsMIRowIVar +
          Y[jRow] / fabsMjRow;

        // If the right side of the inequality if lower than the sum of
        // negative coefficients in the row
        // (Add epsilon for numerical imprecision)
        if (Yp[nbResRows] < sumNegCoeff - EPSILON) {

          // Given that X is in [0,1], the system is inconsistent
          return true;

        }

        // Increment the nb of rows into the result system
        ++nbResRows;

      }

    }

  }

}

// Then we copy and compress the rows where the eliminated
// variable is null

// Loop on rows of the input system
for (int iRow = 0;
     iRow < nbRows;
     ++iRow) {

  // Shortcut
  const double* MiRow = M[iRow];

  // If the coefficient of the eliminated variable is null on
  // this row
  if (fabs(MiRow[0]) < EPSILON) {

    // Shortcut
    double* MpnbResRows = Mp[nbResRows];

    // Copy this row into the result system excluding the eliminated
    // variable
    for (int iCol = 1;
         iCol < nbCols;
         ++iCol) {

      MpnbResRows[iCol - 1] = MiRow[iCol];

    }

    Yp[nbResRows] = Y[iRow];

    // Increment the nb of rows into the result system
    ++nbResRows;

  }

}

// Memorize the number of rows in the result system
```

```
  *nbRemainRows = nbResRows;

  // If we reach here the system is not inconsistent
  return false;

}

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y which has been reduced to only one variable
// and store them in the iVar-th axis of the
// AABB bdgBox
// (M arrangement is [iRow][iCol])
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBoundLastVar2DTime(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
   AABB2DTime* const bdgBox) {

  // Shortcuts
  double* min = bdgBox->min + iVar;
  double* max = bdgBox->max + iVar;

  // Initialize the bounds to there maximum maximum and minimum minimum
  *min = 0.0;
  *max = 1.0;

  // Loop on rows
  for (int jRow = 0;
       jRow < nbRows;
       ++jRow) {

    // Shortcut
    double MjRowiVar = M[jRow][0];

    // If this row has been reduced to the variable in argument
    // and it has a strictly positive coefficient
    if (MjRowiVar > EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is lower than the current maximum bound
      if (*max > y) {

        // Update the maximum bound
        *max = y;

      }

    // Else, if this row has been reduced to the variable in argument
    // and it has a strictly negative coefficient
    } else if (MjRowiVar < -EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is greater than the current minimum bound
      if (*min < y) {
```

```
        // Update the minimum bound
        *min = y;

      }

    }

  }

}

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y where the iVar-th variable is on the first column
// and store them in the iVar-th axis of the AABB bdgBox
// (M arrangement is [iRow][iCol])
void GetBoundVar2DTime(
     const int iVar,
  const double (*M)[3],
  const double* Y,
     const int nbRows,
     const int nbCols,
   AABB2DTime* const bdgBox) {

  // Shortcuts
  double* bdgBoxMin = bdgBox->min;
  double* bdgBoxMax = bdgBox->max;

  // Initialize the bounds
  bdgBoxMin[iVar] = 0.0;
  bdgBoxMax[iVar] = 1.0;

  // Loop on the rows
  for (int iRow = 0;
       iRow < nbRows;
       ++iRow) {

    // Shortcuts
    const double* MIRow = M[iRow];
    double fabsMIRowIVar = fabs(MIRow[0]);

    // If the coefficient of the first variable on this row is not null
    if (fabsMIRowIVar > EPSILON) {

      // Declare two variables to memorize the min and max of the
      // requested variable in this row
      double min = -1.0 * Y[iRow];
      double max = Y[iRow];

      // Loop on columns except the first one which is the one of the
      // requested variable
      for (int iCol = 1;
           iCol < nbCols;
           ++iCol) {

        if (MIRow[iCol] > EPSILON) {
          min += MIRow[iCol] * bdgBoxMin[iCol + iVar];
          max -= MIRow[iCol] * bdgBoxMin[iCol + iVar];
        } else if (MIRow[iCol] < EPSILON) {
          min += MIRow[iCol] * bdgBoxMax[iCol + iVar];
          max -= MIRow[iCol] * bdgBoxMax[iCol + iVar];
        }
```

```
      }

      min /= -1.0 * MIRow[0];
      max /= MIRow[0];
      if (bdgBoxMin[iVar] > min) {

        bdgBoxMin[iVar] = min;

      }
      if (bdgBoxMax[iVar] < max) {

        bdgBoxMax[iVar] = max;

      }

    }

  }

}

// Test for intersection between Frame that and Frame tho
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into bdgBox, else bdgBox is not modified
// If bdgBox is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in tho's local coordinates system
bool FMBTestIntersection2DTime(
  const Frame2DTime* const that,
  const Frame2DTime* const tho,
        AABB2DTime* const bdgBox) {

  // Get the projection of the Frame 'tho' in Frame 'that' coordinates
  // system
  Frame2DTime thoProj;
  Frame2DTimeImportFrame(that, tho, &thoProj);

  // Declare two variables to memorize the system to be solved M.X <= Y
  // (M arrangement is [iRow][iCol])
  double M[10][3];
  double Y[10];

  // Create the inequality system

  // -V_jT-sum_iC_j,iX_i<=O_j
  M[0][0] = -thoProj.comp[0][0];
  M[0][1] = -thoProj.comp[1][0];
  M[0][2] = -thoProj.speed[0];
  Y[0] = thoProj.orig[0];
  if (Y[0] < neg(M[0][0]) + neg(M[0][1]) + neg(M[0][2]))
    return false;

  M[1][0] = -thoProj.comp[0][1];
  M[1][1] = -thoProj.comp[1][1];
  M[1][2] = -thoProj.speed[1];
  Y[1] = thoProj.orig[1];
  if (Y[1] < neg(M[1][0]) + neg(M[1][1]) + neg(M[1][2]))
    return false;
```

```
// Variable to memorise the nb of rows in the system
int nbRows = 2;

if (that->type == FrameCuboid) {

  // V_jT+sum_iC_j,iX_i<=1.0-O_j
  M[nbRows][0] = thoProj.comp[0][0];
  M[nbRows][1] = thoProj.comp[1][0];
  M[nbRows][2] = thoProj.speed[0];
  Y[nbRows] = 1.0 - thoProj.orig[0];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

  M[nbRows][0] = thoProj.comp[0][1];
  M[nbRows][1] = thoProj.comp[1][1];
  M[nbRows][2] = thoProj.speed[1];
  Y[nbRows] = 1.0 - thoProj.orig[1];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

} else {

  // sum_j(V_jT+sum_iC_j,iX_i)<=1.0-sum_iO_i
  M[nbRows][0] = thoProj.comp[0][0] + thoProj.comp[0][1];
  M[nbRows][1] = thoProj.comp[1][0] + thoProj.comp[1][1];
  M[nbRows][2] = thoProj.speed[0] + thoProj.speed[1];
  Y[nbRows] = 1.0 - thoProj.orig[0] - thoProj.orig[1];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]))
    return false;
  ++nbRows;

}

if (tho->type == FrameCuboid) {

  // X_i <= 1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 0.0;
  M[nbRows][2] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

  M[nbRows][0] = 0.0;
  M[nbRows][1] = 1.0;
  M[nbRows][2] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

} else {

  // sum_iX_i<=1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 1.0;
  M[nbRows][2] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;
```

```
}

// -X_i <= 0.0
M[nbRows][0] = -1.0;
M[nbRows][1] = 0.0;
M[nbRows][2] = 0.0;
Y[nbRows] = 0.0;
++nbRows;

M[nbRows][0] = 0.0;
M[nbRows][1] = -1.0;
M[nbRows][2] = 0.0;
Y[nbRows] = 0.0;
++nbRows;

// 0.0 <= t <= 1.0
M[nbRows][0] = 0.0;
M[nbRows][1] = 0.0;
M[nbRows][2] = 1.0;
Y[nbRows] = 1.0;
++nbRows;

M[nbRows][0] = 0.0;
M[nbRows][1] = 0.0;
M[nbRows][2] = -1.0;
Y[nbRows] = 0.0;
++nbRows;

// Solve the system

// Declare a AABB to memorize the bounding box of the intersection
// in the coordinates system of tho
AABB2DTime bdgBoxLocal = {
  .min = {0.0, 0.0, 0.0},
  .max = {0.0, 0.0, 0.0}
};

// Declare variables to eliminate the first variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mp[35][3];
//double Yp[35];
double Mp[13][3];
double Yp[13];
int nbRowsP;

// Eliminate the first variable in the original system
bool inconsistency =
  ElimVar2DTime(
    M,
    Y,
    nbRows,
    3,
    Mp,
    Yp,
    &nbRowsP);

// If the system is inconsistent
if (inconsistency == true) {
```

```
  // The two Frames are not in intersection
  return false;

}

// Declare variables to eliminate the second variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mpp[342][3];
//double Ypp[342];
double Mpp[21][3];
double Ypp[21];
int nbRowsPP;

// Eliminate the second variable (which is the first in the new system)
inconsistency =
  ElimVar2DTime(
    Mp,
    Yp,
    nbRowsP,
    2,
    Mpp,
    Ypp,
    &nbRowsPP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;

}

// Get the bounds for the remaining third variable
GetBoundLastVar2DTime(
  THD_VAR,
  Mpp,
  Ypp,
  nbRowsPP,
  &bdgBoxLocal);

// If the bounds are inconsistent
if (bdgBoxLocal.min[THD_VAR] >= bdgBoxLocal.max[THD_VAR]) {

  // The two Frames are not in intersection
  return false;

// Else, if the bounds are consistent here it means
// the two Frames are in intersection.
// If the user has requested for the resulting bounding box
} else if (bdgBox != NULL) {

  // Get the bounds of the other variables

  GetBoundVar2DTime(
     SND_VAR,
     Mp,
     Yp,
     nbRowsP,
```

```
        2,
        &bdgBoxLocal);

    GetBoundVar2DTime(
        FST_VAR,
        M,
        Y,
        nbRows,
        3,
        &bdgBoxLocal);

    // Memorize the result
    *bdgBox = bdgBoxLocal;

  }

  // If we've reached here the two Frames are intersecting
  return true;

}
```

4.2.4 3D dynamic

Header

```
#ifndef __FMB3DT_H_
#define __FMB3DT_H_

#include <stdbool.h>
#include "frame.h"

// ------------- Functions declaration -------------

// Test for intersection between Frame that and Frame tho
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into bdgBox, else bdgBox is not modified
// If bdgBox is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in tho's local coordinates system
bool FMBTestIntersection3DTime(
  const Frame3DTime* const that,
  const Frame3DTime* const tho,
         AABB3DTime* const bdgBox);

#endif
```

Body

```
#include "fmb3dt.h"

// ------------- Macros -------------

// Return 1.0 if v is positive, -1.0 if v is negative, 0.0 else
#define sgn(v) (((0.0 < (v)) ? 1 : 0) - (((v) < 0.0) ? 1 : 0))

// Return x if x is negative, 0.0 else
```

```
#define neg(x) (x < 0.0 ? x : 0.0)

#define FST_VAR 0
#define SND_VAR 1
#define THD_VAR 2
#define FOR_VAR 3

#define EPSILON 0.0000001

// ------------- Functions declaration -------------

// Eliminate the first variable in the system M.X<=Y
// using the Fourier-Motzkin method and return
// the resulting system in Mp and Yp, and the number of rows of
// the resulting system in nbRemainRows
// Return false if the system becomes inconsistent during elimination,
// else return true
bool ElimVar3DTime(
  const double (*M)[4],
  const double* Y,
     const int nbRows,
     const int nbCols,
        double (*Mp)[4],
        double* Yp,
    int* const nbRemainRows);

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y which has been reduced to only one variable
// and store them in the iVar-th axis of the
// AABB bdgBox
// (M arrangement is [iRow][iCol])
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBoundLastVar3DTime(
     const int iVar,
  const double (*M)[4],
  const double* Y,
     const int nbRows,
   AABB3DTime* const bdgBox);

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y where the iVar-th variable is on the first column
// and store them in the iVar-th axis of the AABB bdgBox
// (M arrangement is [iRow][iCol])
void GetBoundVar3DTime(
     const int iVar,
  const double (*M)[4],
  const double* Y,
     const int nbRows,
     const int nbCols,
   AABB3DTime* const bdgBox);

// ------------- Functions implementation -------------

// Eliminate the first variable in the system M.X<=Y
// using the Fourier-Motzkin method and return
// the resulting system in Mp and Yp, and the number of rows of
// the resulting system in nbRemainRows
// (M arrangement is [iRow][iCol])
// Return true if the system becomes inconsistent during elimination,
// else return false
bool ElimVar3DTime(
```

```
const double (*M)[4],
const double* Y,
   const int nbRows,
   const int nbCols,
      double (*Mp)[4],
      double* Yp,
  int* const nbRemainRows) {

// Initialize the number of rows in the result system
int nbResRows = 0;

// First we process the rows where the eliminated variable is not null

// For each row except the last one
for (int iRow = 0;
     iRow < nbRows - 1;
     ++iRow) {

  // Shortcuts
  const double fabsMIRowIVar = fabs(M[iRow][0]);

  // If the coefficient for the eliminated vairable is not null
  // in this row
  if (fabsMIRowIVar > EPSILON) {

    // Shortcuts
    const double* MiRow = M[iRow];
    const int sgnMIRowIVar = sgn(MiRow[0]);
    const double YIRowDivideByFabsMIRowIVar = Y[iRow] / fabsMIRowIVar;

    // For each following rows
    for (int jRow = iRow + 1;
         jRow < nbRows;
         ++jRow) {

      // If coefficients of the eliminated variable in the two rows have
      // different signs and are not null
      if (sgnMIRowIVar != sgn(M[jRow][0]) &&
          fabs(M[jRow][0]) > EPSILON) {

        // Shortcuts
        const double* MjRow = M[jRow];
        const double fabsMjRow = fabs(MjRow[0]);

        // Declare a variable to memorize the sum of the negative
        // coefficients in the row
        double sumNegCoeff = 0.0;

        // Add the sum of the two normed (relative to the eliminated
        // variable) rows into the result system. This actually
        // eliminate the variable while keeping the constraints on
        // others variables
        for (int iCol = 1;
             iCol < nbCols;
             ++iCol ) {

          Mp[nbResRows][iCol - 1] =
            MiRow[iCol] / fabsMIRowIVar +
            MjRow[iCol] / fabsMjRow;

          // Update the sum of the negative coefficient
          sumNegCoeff += neg(Mp[nbResRows][iCol - 1]);
```

```
        }

        // Update the right side of the inequality
        Yp[nbResRows] =
          YIRowDivideByFabsMIRowIVar +
          Y[jRow] / fabsMjRow;

        // If the right side of the inequality if lower than the sum of
        // negative coefficients in the row
        // (Add epsilon for numerical imprecision)
        if (Yp[nbResRows] < sumNegCoeff - EPSILON) {

          // Given that X is in [0,1], the system is inconsistent
          return true;

        }

        // Increment the nb of rows into the result system
        ++nbResRows;

      }

    }

  }

}

// Then we copy and compress the rows where the eliminated
// variable is null

// Loop on rows of the input system
for (int iRow = 0;
     iRow < nbRows;
     ++iRow) {

  // Shortcut
  const double* MiRow = M[iRow];

  // If the coefficient of the eliminated variable is null on
  // this row
  if (fabs(MiRow[0]) < EPSILON) {

    // Shortcut
    double* MpnbResRows = Mp[nbResRows];

    // Copy this row into the result system excluding the eliminated
    // variable
    for (int iCol = 1;
         iCol < nbCols;
         ++iCol) {

      MpnbResRows[iCol - 1] = MiRow[iCol];

    }

    Yp[nbResRows] = Y[iRow];

    // Increment the nb of rows into the result system
    ++nbResRows;
```

```
    }

  }

  // Memorize the number of rows in the result system
  *nbRemainRows = nbResRows;

  // If we reach here the system is not inconsistent
  return false;

}

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y and store them in the iVar-th axis of the
// AABB bdgBox
// (M arrangement is [iRow][iCol])
// The system is supposed to have been reduced to only one variable
// per row, the one in argument
// May return inconsistent values (max < min), which would
// mean the system has no solution
void GetBoundLastVar3DTime(
     const int iVar,
  const double (*M)[4],
  const double* Y,
     const int nbRows,
   AABB3DTime* const bdgBox) {

  // Shortcuts
  double* min = bdgBox->min + iVar;
  double* max = bdgBox->max + iVar;

  // Initialize the bounds to there maximum maximum and minimum minimum
  *min = 0.0;
  *max = 1.0;

  // Loop on rows
  for (int jRow = 0;
       jRow < nbRows;
       ++jRow) {

    // Shortcut
    double MjRowiVar = M[jRow][0];

    // If this row has been reduced to the variable in argument
    // and it has a strictly positive coefficient
    if (MjRowiVar > EPSILON) {

      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is lower than the current maximum bound
      if (*max > y) {

        // Update the maximum bound
        *max = y;

      }

    // Else, if this row has been reduced to the variable in argument
    // and it has a strictly negative coefficient
    } else if (MjRowiVar < -EPSILON) {
```

```
      // Get the scaled value of Y for this row
      double y = Y[jRow] / MjRowiVar;

      // If the value is greater than the current minimum bound
      if (*min < y) {

        // Update the minimum bound
        *min = y;

      }

    }

  }

}

// Get the bounds of the iVar-th variable in the nbRows rows
// system M.X<=Y where the iVar-th variable is on the first column
// and store them in the iVar-th axis of the AABB bdgBox
// (M arrangement is [iRow][iCol])
void GetBoundVar3DTime(
     const int iVar,
  const double (*M)[4],
  const double* Y,
     const int nbRows,
     const int nbCols,
   AABB3DTime* const bdgBox) {

  // Shortcuts
  double* bdgBoxMin = bdgBox->min;
  double* bdgBoxMax = bdgBox->max;

  // Initialize the bounds
  bdgBoxMin[iVar] = 0.0;
  bdgBoxMax[iVar] = 1.0;

  // Loop on the rows
  for (int iRow = 0;
       iRow < nbRows;
       ++iRow) {

    // Shortcuts
    const double* MIRow = M[iRow];
    double fabsMIRowIVar = fabs(MIRow[0]);

    // If the coefficient of the first variable on this row is not null
    if (fabsMIRowIVar > EPSILON) {

      // Declare two variables to memorize the min and max of the
      // requested variable in this row
      double min = -1.0 * Y[iRow];
      double max = Y[iRow];

      // Loop on columns except the first one which is the one of the
      // requested variable
      for (int iCol = 1;
           iCol < nbCols;
           ++iCol) {

        if (MIRow[iCol] > EPSILON) {
          min += MIRow[iCol] * bdgBoxMin[iCol + iVar];
```

```
          max -= MIRow[iCol] * bdgBoxMin[iCol + iVar];
        } else if (MIRow[iCol] < EPSILON) {
          min += MIRow[iCol] * bdgBoxMax[iCol + iVar];
          max -= MIRow[iCol] * bdgBoxMax[iCol + iVar];
        }

      }

      min /= -1.0 * MIRow[0];
      max /= MIRow[0];
      if (bdgBoxMin[iVar] > min) {

        bdgBoxMin[iVar] = min;

      }
      if (bdgBoxMax[iVar] < max) {

        bdgBoxMax[iVar] = max;

      }

    }

  }

}

// Test for intersection between Frame that and Frame tho
// Return true if the two Frames are intersecting, else false
// If the Frame are intersecting the AABB of the intersection
// is stored into bdgBox, else bdgBox is not modified
// If bdgBox is null, the result AABB is not memorized (to use if
// unnecessary and want to speed up the algorithm)
// The resulting AABB may be larger than the smallest possible AABB
// The resulting AABB of FMBTestIntersection(A,B) may be different
// of the resulting AABB of FMBTestIntersection(B,A)
// The resulting AABB is given in tho's local coordinates system
bool FMBTestIntersection3DTime(
  const Frame3DTime* const that,
  const Frame3DTime* const tho,
        AABB3DTime* const bdgBox) {

  // Get the projection of the Frame 'tho' in Frame 'that' coordinates
  // system
  Frame3DTime thoProj;
  Frame3DTimeImportFrame(that, tho, &thoProj);

  // Declare two variables to memorize the system to be solved M.X <= Y
  // (M arrangement is [iRow][iCol])
  double M[14][4];
  double Y[14];

  // Create the inequality system

  // -V_jT-sum_iC_j,iX_i<=O_j
  M[0][0] = -thoProj.comp[0][0];
  M[0][1] = -thoProj.comp[1][0];
  M[0][2] = -thoProj.comp[2][0];
  M[0][3] = -thoProj.speed[0];
  Y[0] = thoProj.orig[0];
  if (Y[0] < neg(M[0][0]) + neg(M[0][1]) + neg(M[0][2]) + neg(M[0][3]))
    return false;
```

```
M[1][0] = -thoProj.comp[0][1];
M[1][1] = -thoProj.comp[1][1];
M[1][2] = -thoProj.comp[2][1];
M[1][3] = -thoProj.speed[1];
Y[1] = thoProj.orig[1];
if (Y[1] < neg(M[1][0]) + neg(M[1][1]) + neg(M[1][2]) + neg(M[1][3]))
  return false;

M[2][0] = -thoProj.comp[0][2];
M[2][1] = -thoProj.comp[1][2];
M[2][2] = -thoProj.comp[2][2];
M[2][3] = -thoProj.speed[2];
Y[2] = thoProj.orig[2];
if (Y[2] < neg(M[2][0]) + neg(M[2][1]) + neg(M[2][2]) + neg(M[2][3]))
  return false;

// Variable to memorize the nb of rows in the system
int nbRows = 3;

if (that->type == FrameCuboid) {

  // V_jT+sum_iC_j,iX_i<=1.0-O_j
  M[nbRows][0] = thoProj.comp[0][0];
  M[nbRows][1] = thoProj.comp[1][0];
  M[nbRows][2] = thoProj.comp[2][0];
  M[nbRows][3] = thoProj.speed[0];
  Y[nbRows] = 1.0 - thoProj.orig[0];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]) + neg(M[nbRows][3]))
    return false;
  ++nbRows;

  M[nbRows][0] = thoProj.comp[0][1];
  M[nbRows][1] = thoProj.comp[1][1];
  M[nbRows][2] = thoProj.comp[2][1];
  M[nbRows][3] = thoProj.speed[1];
  Y[nbRows] = 1.0 - thoProj.orig[1];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]) + neg(M[nbRows][3]))
    return false;
  ++nbRows;

  M[nbRows][0] = thoProj.comp[0][2];
  M[nbRows][1] = thoProj.comp[1][2];
  M[nbRows][2] = thoProj.comp[2][2];
  M[nbRows][3] = thoProj.speed[2];
  Y[nbRows] = 1.0 - thoProj.orig[2];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]) + neg(M[nbRows][3]))
    return false;
  ++nbRows;

} else {

  // sum_j(V_jT+sum_iC_j,iX_i)<=1.0-sum_iO_i
  M[nbRows][0] =
    thoProj.comp[0][0] + thoProj.comp[0][1] + thoProj.comp[0][2];
  M[nbRows][1] =
    thoProj.comp[1][0] + thoProj.comp[1][1] + thoProj.comp[1][2];
  M[nbRows][2] =
    thoProj.comp[2][0] + thoProj.comp[2][1] + thoProj.comp[2][2];
```

```
  M[nbRows][3] = thoProj.speed[0] + thoProj.speed[1] + thoProj.speed[2];
  Y[nbRows] = 1.0 - thoProj.orig[0] - thoProj.orig[1] - thoProj.orig[2];
  if (Y[nbRows] < neg(M[nbRows][0]) + neg(M[nbRows][1]) +
                  neg(M[nbRows][2]) + neg(M[nbRows][3]))
    return false;
  ++nbRows;

}

if (tho->type == FrameCuboid) {

  // X_i <= 1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 0.0;
  M[nbRows][2] = 0.0;
  M[nbRows][3] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

  M[nbRows][0] = 0.0;
  M[nbRows][1] = 1.0;
  M[nbRows][2] = 0.0;
  M[nbRows][3] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

  M[nbRows][0] = 0.0;
  M[nbRows][1] = 0.0;
  M[nbRows][2] = 1.0;
  M[nbRows][3] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

} else {

  // sum_iX_i<=1.0
  M[nbRows][0] = 1.0;
  M[nbRows][1] = 1.0;
  M[nbRows][2] = 1.0;
  M[nbRows][3] = 0.0;
  Y[nbRows] = 1.0;
  ++nbRows;

}

// -X_i <= 0.0
M[nbRows][0] = -1.0;
M[nbRows][1] = 0.0;
M[nbRows][2] = 0.0;
M[nbRows][3] = 0.0;
Y[nbRows] = 0.0;
++nbRows;

M[nbRows][0] = 0.0;
M[nbRows][1] = -1.0;
M[nbRows][2] = 0.0;
M[nbRows][3] = 0.0;
Y[nbRows] = 0.0;
++nbRows;

M[nbRows][0] = 0.0;
M[nbRows][1] = 0.0;
```

```
M[nbRows][2] = -1.0;
M[nbRows][3] = 0.0;
Y[nbRows] = 0.0;
++nbRows;

// 0.0 <= t <= 1.0
M[nbRows][0] = 0.0;
M[nbRows][1] = 0.0;
M[nbRows][2] = 0.0;
M[nbRows][3] = 1.0;
Y[nbRows] = 1.0;
++nbRows;

M[nbRows][0] = 0.0;
M[nbRows][1] = 0.0;
M[nbRows][2] = 0.0;
M[nbRows][3] = -1.0;
Y[nbRows] = 0.0;
++nbRows;

// Solve the system

// Declare a AABB to memorize the bounding box of the intersection
// in the coordinates system of that
AABB3DTime bdgBoxLocal = {
  .min = {0.0, 0.0, 0.0, 0.0},
  .max = {0.0, 0.0, 0.0, 0.0}
};

// Declare variables to eliminate the first variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mp[63][4];
//double Yp[63];
double Mp[22][4];
double Yp[22];
int nbRowsP;

// Eliminate the first variable in the original system
bool inconsistency =
  ElimVar3DTime(
    M,
    Y,
    nbRows,
    4,
    Mp,
    Yp,
    &nbRowsP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;

}

// Declare variables to eliminate the second variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
```

```
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mpp[1056][4];
//double Ypp[1056];
double Mpp[57][4];
double Ypp[57];
int nbRowsPP;

// Eliminate the second variable (which is the first in the new system)
inconsistency =
  ElimVar3DTime(
    Mp,
    Yp,
    nbRowsP,
    3,
    Mpp,
    Ypp,
    &nbRowsPP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;

}

// Declare variables to eliminate the third variable
// The size of the array given in the doc is a majoring value.
// Instead I use a smaller value which has proven to be sufficient
// during tests, validation and qualification, to avoid running
// into the heap limit and to optimize slightly the performance
//double Mppp[279840][4];
//double Yppp[279840];
double Mppp[560][4];
double Yppp[560];
int nbRowsPPP;

// Eliminate the third variable (which is the first in the new system)
inconsistency =
  ElimVar3DTime(
    Mpp,
    Ypp,
    nbRowsPP,
    2,
    Mppp,
    Yppp,
    &nbRowsPPP);

// If the system is inconsistent
if (inconsistency == true) {

  // The two Frames are not in intersection
  return false;

}

// Get the bounds for the remaining fourth variable
GetBoundLastVar3DTime(
  FOR_VAR,
  Mppp,
  Yppp,
```

```
    nbRowsPPP,
    &bdgBoxLocal);

  // If the bounds are inconsistent
  if (bdgBoxLocal.min[FOR_VAR] >= bdgBoxLocal.max[FOR_VAR]) {

    // The two Frames are not in intersection
    return false;

  // Else, if the bounds are consistent here it means
  // the two Frames are in intersection.
  // If the user has requested for the resulting bounding box
  } else if (bdgBox != NULL) {

    // Get the bounds of the other variables

    GetBoundVar3DTime(
       THD_VAR,
       Mpp,
       Ypp,
       nbRowsPP,
       2,
       &bdgBoxLocal);

    GetBoundVar3DTime(
       SND_VAR,
       Mp,
       Yp,
       nbRowsP,
       3,
       &bdgBoxLocal);

    GetBoundVar3DTime(
       FST_VAR,
       M,
       Y,
       nbRows,
       4,
       &bdgBoxLocal);

    // Memorize the result
    *bdgBox = bdgBoxLocal;

  }

  // If we've reached here the two Frames are intersecting
  return true;

}
```

5 Minimal example of use

In this section I give a minimal example for each case of how to use the code given in the previous section.

5.1 2D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>

// Include the FMB algorithm library
#include "fmb2d.h"

// Main function
int main(int argc, char** argv) {

  // Create the two objects to be tested for intersection
  double origP2D[2] = {0.0, 0.0};
  double compP2D[2][2] = {
    {1.0, 0.0},  // First component
    {0.0, 1.0}}; // Second component
  Frame2D P2D =
    Frame2DCreateStatic(
      FrameCuboid,
      origP2D,
      compP2D);

  double origQ2D[2] = {0.0, 0.0};
  double compQ2D[2][2] = {
    {1.0, 0.0},
    {0.0, 1.0}};
  Frame2D Q2D =
    Frame2DCreateStatic(
      FrameCuboid,
      origQ2D,
      compQ2D);

  // Declare a variable to memorize the result of the intersection
  // detection
  AABB2D bdgBox2DLocal;

  // Test for intersection between P and Q
  bool isIntersecting2D =
    FMBTestIntersection2D(
      &P2D,
      &Q2D,
      &bdgBox2DLocal);

  // If the two objects are intersecting
  if (isIntersecting2D) {

    printf("Intersection detected in AABB ");

    // Export the local bounding box toward the real coordinates
    // system
    AABB2D bdgBox2D;
    Frame2DExportBdgBox(
      &Q2D,
      &bdgBox2DLocal,
      &bdgBox2D);

    // Clip with the AABB of 'Q2D' and 'P2D' to improve results
    for (int iAxis = 2;
         iAxis--;) {
```

```
      if (bdgBox2D.min[iAxis] < P2D.bdgBox.min[iAxis]) {

        bdgBox2D.min[iAxis] = P2D.bdgBox.min[iAxis];

      }
      if (bdgBox2D.max[iAxis] > P2D.bdgBox.max[iAxis]) {

        bdgBox2D.max[iAxis] = P2D.bdgBox.max[iAxis];

      }

      if (bdgBox2D.min[iAxis] < Q2D.bdgBox.min[iAxis]) {

        bdgBox2D.min[iAxis] = Q2D.bdgBox.min[iAxis];

      }
      if (bdgBox2D.max[iAxis] > Q2D.bdgBox.max[iAxis]) {

        bdgBox2D.max[iAxis] = Q2D.bdgBox.max[iAxis];

      }

    }

    AABB2DPrint(&bdgBox2D);
    printf("\n");

  // Else, the two objects are not intersecting
  } else {

    printf("No intersection.\n");

  }

  return 0;

}
```

5.2 3D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>

// Include the FMB algorithm library
#include "fmb3d.h"

// Main function
int main(int argc, char** argv) {

  // Create the two objects to be tested for intersection
  double origP3D[3] = {0.0, 0.0, 0.0};
  double compP3D[3][3] = {
    {1.0, 0.0, 0.0},  // First component
    {0.0, 1.0, 0.0},  // Second component
    {0.0, 0.0, 1.0}}; // Third component
  Frame3D P3D =
    Frame3DCreateStatic(
      FrameTetrahedron,
```

```
    origP3D,
    compP3D);

double origQ3D[3] = {0.0, 0.0, 0.0};
double compQ3D[3][3] = {
  {1.0, 0.0, 0.0},
  {0.0, 1.0, 0.0},
  {0.0, 0.0, 1.0}};
Frame3D Q3D =
  Frame3DCreateStatic(
    FrameTetrahedron,
    origQ3D,
    compQ3D);

// Declare a variable to memorize the result of the intersection
// detection
AABB3D bdgBox3DLocal;

// Test for intersection between P and Q
bool isIntersecting3D =
  FMBTestIntersection3D(
    &P3D,
    &Q3D,
    &bdgBox3DLocal);

// If the two objects are intersecting
if (isIntersecting3D) {

  printf("Intersection detected in AABB ");

  // Export the local bounding box toward the real coordinates
  // system
  AABB3D bdgBox3D;
  Frame3DExportBdgBox(
    &Q3D,
    &bdgBox3DLocal,
    &bdgBox3D);

  // Clip with the AABB of 'Q3D' and 'P3D' to improve results
  for (int iAxis = 3;
       iAxis--;) {

    if (bdgBox3D.min[iAxis] < P3D.bdgBox.min[iAxis]) {

      bdgBox3D.min[iAxis] = P3D.bdgBox.min[iAxis];

    }
    if (bdgBox3D.max[iAxis] > P3D.bdgBox.max[iAxis]) {

      bdgBox3D.max[iAxis] = P3D.bdgBox.max[iAxis];

    }

    if (bdgBox3D.min[iAxis] < Q3D.bdgBox.min[iAxis]) {

      bdgBox3D.min[iAxis] = Q3D.bdgBox.min[iAxis];

    }
    if (bdgBox3D.max[iAxis] > Q3D.bdgBox.max[iAxis]) {

      bdgBox3D.max[iAxis] = Q3D.bdgBox.max[iAxis];
```

```
      }

    }

    AABB3DPrint(&bdgBox3D);
    printf("\n");

  // Else, the two objects are not intersecting
  } else {

    printf("No intersection.\n");

  }

  return 0;
}
```

5.3 2D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>

// Include the FMB algorithm library
#include "fmb2dt.h"

// Main function
int main(int argc, char** argv) {

  // Create the two objects to be tested for intersection
  double origP2DTime[2] = {0.0, 0.0};
  double speedP2DTime[2] = {0.0, 0.0};
  double compP2DTime[2][2] = {
    {1.0, 0.0},  // First component
    {0.0, 1.0}}; // Second component
  Frame2DTime P2DTime =
    Frame2DTimeCreateStatic(
      FrameCuboid,
      origP2DTime,
      speedP2DTime,
      compP2DTime);

  double origQ2DTime[2] = {0.0,0.0};
  double speedQ2DTime[2] = {0.0,0.0};
  double compQ2DTime[2][2] = {
    {1.0, 0.0},
    {0.0, 1.0}};
  Frame2DTime Q2DTime =
    Frame2DTimeCreateStatic(
      FrameCuboid,
      origQ2DTime,
      speedQ2DTime,
      compQ2DTime);

  // Declare a variable to memorize the result of the intersection
  // detection
  AABB2DTime bdgBox2DTimeLocal;

  // Test for intersection between P and Q
  bool isIntersecting2DTime =
```

```
    FMBTestIntersection2DTime(
      &P2DTime,
      &Q2DTime,
      &bdgBox2DTimeLocal);

  // If the two objects are intersecting
  if (isIntersecting2DTime) {

    printf("Intersection detected in AABB ");

    // Export the local bounding box toward the real coordinates
    // system
    AABB2DTime bdgBox2DTime;
    Frame2DTimeExportBdgBox(
      &Q2DTime,
      &bdgBox2DTimeLocal,
      &bdgBox2DTime);

    // Clip with the AABB of 'Q2DTime' and 'P2DTime' to improve results
    for (int iAxis = 3;
         iAxis--;) {

      if (bdgBox2DTime.min[iAxis] < P2DTime.bdgBox.min[iAxis]) {

        bdgBox2DTime.min[iAxis] = P2DTime.bdgBox.min[iAxis];

      }
      if (bdgBox2DTime.max[iAxis] > P2DTime.bdgBox.max[iAxis]) {

        bdgBox2DTime.max[iAxis] = P2DTime.bdgBox.max[iAxis];

      }

      if (bdgBox2DTime.min[iAxis] < Q2DTime.bdgBox.min[iAxis]) {

        bdgBox2DTime.min[iAxis] = Q2DTime.bdgBox.min[iAxis];

      }
      if (bdgBox2DTime.max[iAxis] > Q2DTime.bdgBox.max[iAxis]) {

        bdgBox2DTime.max[iAxis] = Q2DTime.bdgBox.max[iAxis];

      }

    }

    AABB2DTimePrint(&bdgBox2DTime);
    printf("\n");

  // Else, the two objects are not intersecting
  } else {

    printf("No intersection.\n");

  }

  return 0;
}
```

5.4 3D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>

// Include the FMB algorithm library
#include "fmb3dt.h"

// Main function
int main(int argc, char** argv) {

  // Create the two objects to be tested for intersection
  double origP3DTime[3] = {0.0, 0.0, 0.0};
  double speedP3DTime[3] = {0.0, 0.0, 0.0};
  double compP3DTime[3][3] = {
    {1.0, 0.0, 0.0},  // First component
    {0.0, 1.0, 0.0},  // Second component
    {0.0, 0.0, 1.0}}; // Third component
  Frame3DTime P3DTime =
    Frame3DTimeCreateStatic(
      FrameCuboid,
      origP3DTime,
      speedP3DTime,
      compP3DTime);

  double origQ3DTime[3] = {0.0, 0.0, 0.0};
  double speedQ3DTime[3] = {0.0, 0.0, 0.0};
  double compQ3DTime[3][3] = {
    {1.0, 0.0, 0.0},
    {0.0, 1.0, 0.0},
    {0.0, 0.0, 1.0}};
  Frame3DTime Q3DTime =
    Frame3DTimeCreateStatic(
      FrameCuboid,
      origQ3DTime,
      speedQ3DTime,
      compQ3DTime);

  // Declare a variable to memorize the result of the intersection
  // detection
  AABB3DTime bdgBox3DTimeLocal;

  // Test for intersection between P and Q
  bool isIntersecting3DTime =
    FMBTestIntersection3DTime(
      &P3DTime,
      &Q3DTime,
      &bdgBox3DTimeLocal);

  // If the two objects are intersecting
  if (isIntersecting3DTime) {

    printf("Intersection detected in AABB ");

    // Export the local bounding box toward the real coordinates
    // system
    AABB3DTime bdgBox3DTime;
    Frame3DTimeExportBdgBox(
      &Q3DTime,
      &bdgBox3DTimeLocal,
      &bdgBox3DTime);
```

```
    // Clip with the AABB of 'Q3DTime' and 'P3DTime' to improve results
    for (int iAxis = 3;
         iAxis--;) {

      if (bdgBox3DTime.min[iAxis] < P3DTime.bdgBox.min[iAxis]) {

        bdgBox3DTime.min[iAxis] = P3DTime.bdgBox.min[iAxis];

      }
      if (bdgBox3DTime.max[iAxis] > P3DTime.bdgBox.max[iAxis]) {

        bdgBox3DTime.max[iAxis] = P3DTime.bdgBox.max[iAxis];

      }

      if (bdgBox3DTime.min[iAxis] < Q3DTime.bdgBox.min[iAxis]) {

        bdgBox3DTime.min[iAxis] = Q3DTime.bdgBox.min[iAxis];

      }
      if (bdgBox3DTime.max[iAxis] > Q3DTime.bdgBox.max[iAxis]) {

        bdgBox3DTime.max[iAxis] = Q3DTime.bdgBox.max[iAxis];

      }

    }

    AABB3DTimePrint(&bdgBox3DTime);
    printf("\n");

  // Else, the two objects are not intersecting
  } else {

    printf("No intersection.\n");

  }

  return 0;
}
```

6 Unit tests

In this section I introduce the code I've used to test the algorithm and its implementation. The test consists of running the algorithm on a set of cases for which the solution has been computed by hand. The code of the implementation of the SAT algorithm is given in annex (p.243)

6.1 Code

6.1.1 2D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
```

```
#include <stdbool.h>

// Include the FMB algorithm library
#include "fmb2d.h"

// Epslon for numerical precision
#define EPSILON 0.0001

// Helper structure to pass arguments to the UnitTest function
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
} Param2D;

// Unit test function
// Takes two Frame definitions, the correct answer in term of
// intersection/no intersection and the correct bounding box
// Run the FMB intersection detection alogirhtm on the Frames
// and check against the correct results
void UnitTest2D(
        const Param2D paramP,
        const Param2D paramQ,
         const bool correctAnswer,
  const AABB2D* const correctBdgBox) {

  // Create the two Frames
  Frame2D P =
    Frame2DCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.comp);

  Frame2D Q =
    Frame2DCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.comp);

  // Declare a variable to memorize the resulting bounding box
  AABB2D bdgBoxLocal;

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame2D* that = &P;
  Frame2D* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Display the tested frames
    Frame2DPrint(that);
    printf("\nagainst\n");
    Frame2DPrint(tho);
    printf("\n");

    // Run the FMB intersection test
    bool isIntersecting =
      FMBTestIntersection2D(
        that,
        tho,
        &bdgBoxLocal);
```

```
// If the test hasn't given the expected answer about intersection
if (isIntersecting != correctAnswer) {

  // Display information about the failure
  printf(" Failed\n");
  printf("Expected : ");
  if (correctAnswer == false)
    printf("no ");
  printf("intersection\n");
  printf("Got : ");
  if (isIntersecting == false)
    printf("no ");
  printf("intersection\n");
  exit(0);

// Else, the test has given the expected answer about intersection
} else {

  // If the Frames were intersecting
  if (isIntersecting == true) {

    AABB2D bdgBox;
    Frame2DExportBdgBox(
      tho,
      &bdgBoxLocal,
      &bdgBox);

    for (int iAxis = 2;
         iAxis--;) {

      if (bdgBox.min[iAxis] < that->bdgBox.min[iAxis]) {
        bdgBox.min[iAxis] = that->bdgBox.min[iAxis];
      }
      if (bdgBox.max[iAxis] > that->bdgBox.max[iAxis]) {
        bdgBox.max[iAxis] = that->bdgBox.max[iAxis];
      }
      if (bdgBox.min[iAxis] < tho->bdgBox.min[iAxis]) {
        bdgBox.min[iAxis] = tho->bdgBox.min[iAxis];
      }
      if (bdgBox.max[iAxis] > tho->bdgBox.max[iAxis]) {
        bdgBox.max[iAxis] = tho->bdgBox.max[iAxis];
      }
    }

    // Check the bounding box
    bool flag = true;
    for (int i = 2;
         i--;) {

      if (bdgBox.min[i] > correctBdgBox->min[i] + EPSILON ||
          bdgBox.max[i] < correctBdgBox->max[i] - EPSILON) {

        flag = false;

      }

    }

    // If the bounding box is the expected one
    if (flag == true) {
```

```
          // Display information
          printf("Succeed\n");
          AABB2DPrint(&bdgBox);
          printf("\n");


        // Else, the bounding box wasn't the expected one
        } else {

          // Display information
          printf("Failed\n");
          printf("Expected : ");
          AABB2DPrint(correctBdgBox);
          printf("\n");
          printf("     Got : ");
          AABB2DPrint(&bdgBox);

          // Terminate the unit tests
          exit(0);

        }

      // Else the Frames were not intersected,
      // no need to check the bounding box
      } else {

        // Display information
        printf(" Succeed (no inter)\n");

      }

    }
    printf("\n");

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Test2D(void) {

  // Declare two variables to memozie the arguments to the
  // Validation function
  Param2D paramP;
  Param2D paramQ;

  // Declare a variable to memorize the correct bounding box
  AABB2D correctBdgBox;

  // Execute the unit test on various cases

  // ----------------------------
  paramP = (Param2D)
    {.type = FrameCuboid,
     .orig = {0.0, 0.0},
     .comp =
       {{1.0, 0.0},
        {0.0, 1.0}}
    };
```

```
paramQ = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.0, 0.0},
   .max = {1.0, 1.0}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.5, 0.5},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.5, 0.5},
   .max = {1.0, 1.0}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {-0.5, -0.5},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.5, 0.5},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
UnitTest2D(
  paramP,
  paramQ,
  false,
  NULL);
```

```
// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.25, -0.25},
   .comp =
     {{0.5, 0.0},
      {0.0, 2.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.25, 0.0},
   .max = {0.75, 1.0}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameCuboid,
   .orig = {-0.25, 0.25},
   .comp =
     {{2.0, 0.0},
      {0.0, 0.5}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.0, 0.25},
   .max = {1.0, 0.75}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 1.0},
      {-1.0, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
```

```
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.0, 0.0},
   .max = {1.0, 1.0}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {-0.5, -0.5},
   .comp =
     {{1.0, 1.0},
      {-1.0, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.0, 0.0},
   .max = {0.5, 1.0}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {1.5, 1.5},
   .comp =
     {{1.0, -1.0},
      {-1.0, -1.0}}
  };
paramQ = (Param2D)
  {.type = FrameCuboid,
   .orig = {1.0, 0.0},
   .comp =
     {{-1.0, 0.0},
      {0.0, 1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.5, 0.0},
   .max = {1.0, 1.0}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
```

```
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {1.0, 0.5},
   .comp =
     {{-0.5, 0.5},
      {-0.5, -0.5}}
  };
paramQ = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 1.0},
   .comp =
     {{1.0, 0.0},
      {0.0, -1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.0, 0.0},
   .max = {1.0, 1.0}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {1.0, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameCuboid,
   .orig = {2.0, -1.0},
   .comp =
     {{0.0, 1.0},
      {-0.5, 1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {1.5, 0.5},
   .max = {1.5 + 0.5 / 3.0, 1.0}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.5},
      {0.5, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameCuboid,
```

```
    .orig = {1.0, 1.0},
    .comp =
      {{-0.5, -0.5},
       {0.0, -1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.5, 0.25},
   .max = {1.0, 1.0}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.5},
      {0.5, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameCuboid,
   .orig = {1.0, 2.0},
   .comp =
     {{-0.5, -0.5},
      {0.0, -1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.5, 0.75},
   .max = {1.0, 1.25}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameTetrahedron,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.5},
      {0.5, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameCuboid,
   .orig = {1.0, 2.0},
   .comp =
     {{-0.5, -0.5},
      {0.0, -1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.5, 0.5},
   .max = {0.75, 1.0}
  };
UnitTest2D(
  paramP,
  paramQ,
```

```
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.5},
      {0.5, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameTetrahedron,
   .orig = {1.0, 2.0},
   .comp =
     {{-0.5, -0.5},
      {0.0, -1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.5 + 1.0 / 3.0, 1.0},
   .max = {1.0, 1.0 + 1.0 / 3.0}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameTetrahedron,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.0, 0.0},
   .max = {1.0, 1.0}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
paramQ = (Param2D)
```

```
  {.type = FrameTetrahedron,
   .orig = {0.0, -0.5},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.0, 0.0},
   .max = {0.5, 0.5}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.5, 0.5},
   .comp =
     {{-0.5, 0.0},
      {0.0, -0.5}}
  };
paramQ = (Param2D)
  {.type = FrameTetrahedron,
   .orig = {0.0, -0.5},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.0, 0.0},
   .max = {0.5, 0.5}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.5, 0.5},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameTetrahedron,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
UnitTest2D(
  paramP,
  paramQ,
  false,
  NULL);
```

```
// ----------------------------
paramP = (Param2D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameTetrahedron,
   .orig = {1.5, 1.5},
   .comp =
     {{-1.5, 0.0},
      {0.0, -1.5}}
  };
correctBdgBox = (AABB2D)
  {.min = {0.5, 0.5},
   .max = {1.0, 1.0}
  };
UnitTest2D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2D)
  {.type = FrameTetrahedron,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameTetrahedron,
   .orig = {1.01, 1.01},
   .comp =
     {{-1.0, 0.0},
      {0.0, -1.0}}
  };
UnitTest2D(
  paramP,
  paramQ,
  false,
  NULL);

// ----------------------------
paramP = (Param2D)
  {.type = FrameTetrahedron,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.5},
      {0.5, 1.0}}
  };
paramQ = (Param2D)
  {.type = FrameTetrahedron,
   .orig = {1.0, 1.0},
   .comp =
     {{-0.5, -0.5},
      {0.0, -1.0}}
  };
correctBdgBox = (AABB2D)
```

```
    {.min = {0.5, 0.5 - 1.0 / 6.0},
     .max = {1.0, 0.75}
    };
  UnitTest2D(
    paramP,
    paramQ,
    true,
    &correctBdgBox);

  // ----------------------------
  paramP = (Param2D)
    {.type = FrameTetrahedron,
     .orig = {0.0, 0.0},
     .comp =
       {{1.0, 0.5},
        {0.5, 1.0}}
    };
  paramQ = (Param2D)
    {.type = FrameTetrahedron,
     .orig = {1.01, 1.5},
     .comp =
       {{-0.5, -0.5},
        {0.0, -1.0}}
    };
  UnitTest2D(
    paramP,
    paramQ,
    false,
    NULL);

  // If we reached here, it means all the unit tests succeed
  printf("All unit tests 2D have succeed.\n");

}

// Main function
int main(int argc, char** argv) {

  Test2D();

  return 0;
}
```

6.1.2 3D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>

// Include the FMB algorithm library
#include "fmb3d.h"

// Epslon for numerical precision
#define EPSILON 0.0001

// Helper structure to pass arguments to the UnitTest function
typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
```

```
} Param3D;

// Unit test function
// Takes two Frame definitions, the correct answer in term of
// intersection/no intersection and the correct bounding box
// Run the FMB intersection detection alogirhtm on the Frames
// and check against the correct results
void UnitTest3D(
        const Param3D paramP,
        const Param3D paramQ,
         const bool correctAnswer,
  const AABB3D* const correctBdgBox) {

  // Create the two Frames
  Frame3D P =
    Frame3DCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.comp);

  Frame3D Q =
    Frame3DCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.comp);

  // Declare a variable to memorize the resulting bounding box
  AABB3D bdgBoxLocal;

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame3D* that = &P;
  Frame3D* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Display the tested frames
    Frame3DPrint(that);
    printf("\nagainst\n");
    Frame3DPrint(tho);
    printf("\n");

    // Run the FMB intersection test
    bool isIntersecting =
      FMBTestIntersection3D(
        that,
        tho,
        &bdgBoxLocal);

    // If the test hasn't given the expected answer about intersection
    if (isIntersecting != correctAnswer) {

      // Display information about the failure
      printf(" Failed\n");
      printf("Expected : ");
      if (correctAnswer == false)
        printf("no ");
      printf("intersection\n");
      printf("Got : ");
      if (isIntersecting == false)
        printf("no ");
```

```
  printf("intersection\n");
  exit(0);

// Else, the test has given the expected answer about intersection
} else {

  // If the Frames were intersecting
  if (isIntersecting == true) {

    AABB3D bdgBox;
    Frame3DExportBdgBox(
      tho,
      &bdgBoxLocal,
      &bdgBox);

    for (int iAxis = 3;
         iAxis--;) {

      if (bdgBox.min[iAxis] < that->bdgBox.min[iAxis]) {
        bdgBox.min[iAxis] = that->bdgBox.min[iAxis];
      }
      if (bdgBox.max[iAxis] > that->bdgBox.max[iAxis]) {
        bdgBox.max[iAxis] = that->bdgBox.max[iAxis];
      }
      if (bdgBox.min[iAxis] < tho->bdgBox.min[iAxis]) {
        bdgBox.min[iAxis] = tho->bdgBox.min[iAxis];
      }
      if (bdgBox.max[iAxis] > tho->bdgBox.max[iAxis]) {
        bdgBox.max[iAxis] = tho->bdgBox.max[iAxis];
      }
    }

    // Check the bounding box
    bool flag = true;
    for (int i = 3;
         i--;) {

      if (bdgBox.min[i] > correctBdgBox->min[i] + EPSILON ||
          bdgBox.max[i] < correctBdgBox->max[i] - EPSILON) {

        flag = false;

      }

    }

    // If the bounding box is the expected one
    if (flag == true) {

      // Display information
      printf("Succeed\n");
      AABB3DPrint(&bdgBox);
      printf("\n");

    // Else, the bounding box wasn't the expected one
    } else {

      // Display information
      printf("Failed\n");
      printf("Expected : ");
      AABB3DPrint(correctBdgBox);
      printf("\n");
```

```
          printf("     Got : ");
          AABB3DPrint(&bdgBox);
          printf("\n");

          // Terminate the unit tests
          exit(0);

        }

      // Else the Frames were not intersected,
      // no need to check the bounding box
      } else {

        // Display information
        printf(" Succeed (no inter)\n");

      }

    }
    printf("\n");

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Test3D(void) {

  // Declare two variables to memozie the arguments to the
  // Validation function
  Param3D paramP;
  Param3D paramQ;

  // Declare a variable to memorize the correct bounding box
  AABB3D correctBdgBox;

  // Execute the unit test on various cases

  // ----------------------------
  paramP = (Param3D)
    {.type = FrameCuboid,
     .orig = {0.0, 0.0, 0.0},
     .comp =
       {{1.0, 0.0, 0.0},
        {0.0, 1.0, 0.0},
        {0.0, 0.0, 1.0}}
    };
  paramQ = (Param3D)
    {.type = FrameCuboid,
     .orig = {0.0, 0.0, 0.0},
     .comp =
       {{1.0, 0.0, 0.0},
        {0.0, 1.0, 0.0},
        {0.0, 0.0, 1.0}}
    };
  correctBdgBox = (AABB3D)
    {.min = {0.0, 0.0, 0.0},
     .max = {1.0, 1.0, 1.0}
    };
```

```
UnitTest3D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param3D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}}
  };
paramQ = (Param3D)
  {.type = FrameCuboid,
   .orig = {0.5, 0.5, 0.5},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}}
  };
correctBdgBox = (AABB3D)
  {.min = {0.5, 0.5, 0.5},
   .max = {1.0, 1.0, 1.0}
  };
UnitTest3D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param3D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}}
  };
paramQ = (Param3D)
  {.type = FrameCuboid,
   .orig = {-0.5, -0.5, -0.5},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}}
  };
correctBdgBox = (AABB3D)
  {.min = {0.0, 0.0, 0.0},
   .max = {0.5, 0.5, 0.5}
  };
UnitTest3D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param3D)
```

```
  {.type = FrameCuboid,
   .orig = {0.0, 0.0, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}}
  };
paramQ = (Param3D)
  {.type = FrameCuboid,
   .orig = {1.5, 1.5, 1.5},
   .comp =
     {{-1.0, 0.0, 0.0},
      {0.0, -1.0, 0.0},
      {0.0, 0.0, -1.0}}
  };
correctBdgBox = (AABB3D)
  {.min = {0.5, 0.5, 0.5},
   .max = {1.0, 1.0, 1.0}
  };
UnitTest3D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param3D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}}
  };
paramQ = (Param3D)
  {.type = FrameCuboid,
   .orig = {0.5, 1.5, -1.5},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, -1.0, 0.0},
      {0.0, 0.0, 1.0}}
  };
UnitTest3D(
  paramP,
  paramQ,
  false,
  NULL);

// ----------------------------
paramP = (Param3D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, -1.0}}
  };
paramQ = (Param3D)
  {.type = FrameCuboid,
   .orig = {0.5, 1.5, -1.5},
   .comp =
     {{1.0, 0.0, 0.0},
```

```
      {0.0, -1.0, 0.0},
      {0.0, 0.0, 1.0}}
  };
correctBdgBox = (AABB3D)
  {.min = {0.5, 0.5, -1.0},
   .max = {1.0, 1.0, -0.5}
  };
UnitTest3D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param3D)
  {.type = FrameCuboid,
   .orig = {-1.01, -1.01, -1.01},
   .comp =
     {{1.0, 0.0, 0.0},
      {1.0, 1.0, 1.0},
      {0.0, 0.0, 1.0}}
  };
paramQ = (Param3D)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}}
  };
UnitTest3D(
  paramP,
  paramQ,
  false,
  NULL);

// ----------------------------
paramP = (Param3D)
  {.type = FrameCuboid,
   .orig = {-1.0, -1.0, -1.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {1.0, 1.0, 1.0},
      {0.0, 0.0, 1.0}}
  };
paramQ = (Param3D)
  {.type = FrameCuboid,
   .orig = {0.0, -0.5, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}}
  };
correctBdgBox = (AABB3D)
  {.min = {0.0, -0.5, 0.0},
   .max = {1.0, 0.0, 1.0}
  };
UnitTest3D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);
```

```
// ----------------------------
paramP = (Param3D)
  {.type = FrameTetrahedron,
   .orig = {-1.0, -1.0, -1.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {1.0, 1.0, 1.0},
      {0.0, 0.0, 1.0}}
  };
paramQ = (Param3D)
  {.type = FrameCuboid,
   .orig = {0.0, -0.5, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}}
  };
UnitTest3D(
  paramP,
  paramQ,
  false,
  NULL);

// ----------------------------
paramP = (Param3D)
  {.type = FrameCuboid,
   .orig = {-1.0, -1.0, -1.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {1.0, 1.0, 1.0},
      {0.0, 0.0, 1.0}}
  };
paramQ = (Param3D)
  {.type = FrameTetrahedron,
   .orig = {0.0, -0.5, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}}
  };
correctBdgBox = (AABB3D)
  {.min = {0.0, -0.5, 0.0},
   .max = {0.75, 0.0, 0.75}
  };
UnitTest3D(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param3D)
  {.type = FrameTetrahedron,
   .orig = {-1.0, -1.0, -1.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {1.0, 1.0, 1.0},
      {0.0, 0.0, 1.0}}
  };
paramQ = (Param3D)
  {.type = FrameTetrahedron,
```

```
    .orig = {0.0, -0.5, 0.0},
    .comp =
      {{1.0, 0.0, 0.0},
       {0.0, 1.0, 0.0},
       {0.0, 0.0, 1.0}}
  };
  UnitTest3D(
    paramP,
    paramQ,
    false,
    NULL);

  // ----------------------------
  paramP = (Param3D)
    {.type = FrameTetrahedron,
     .orig = {-0.5, -1.0, -0.5},
     .comp =
       {{1.0, 0.0, 0.0},
        {1.0, 1.0, 1.0},
        {0.0, 0.0, 1.0}}
    };
  paramQ = (Param3D)
    {.type = FrameTetrahedron,
     .orig = {0.0, -0.5, 0.0},
     .comp =
       {{1.0, 0.0, 0.0},
        {0.0, 1.0, 0.0},
        {0.0, 0.0, 1.0}}
    };
  correctBdgBox = (AABB3D)
    {.min = {0.0, -0.5, 0.0},
     .max = {0.5, -1.0 / 3.0, 0.5}
    };
  UnitTest3D(
    paramP,
    paramQ,
    true,
    &correctBdgBox);


  // If we reached here, it means all the unit tests succeed
  printf("All unit tests 3D have succeed.\n");

}

// Main function
int main(int argc, char** argv) {

  Test3D();

  return 0;
}
```

6.1.3 2D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>

// Include the FMB algorithm library
```

```
#include "fmb2dt.h"

// Epslon for numerical precision
#define EPSILON 0.0001

// Helper structure to pass arguments to the UnitTest function
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
  double speed[2];
} Param2DTime;

// Unit test function
// Takes two Frame definitions, the correct answer in term of
// intersection/no intersection and the correct bounding box
// Run the FMB intersection detection alogirhtm on the Frames
// and check against the correct results
void UnitTest2DTime(
        const Param2DTime paramP,
        const Param2DTime paramQ,
         const bool correctAnswer,
  const AABB2DTime* const correctBdgBox) {

  // Create the two Frames
  Frame2DTime P =
    Frame2DTimeCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.speed,
      paramP.comp);

  Frame2DTime Q =
    Frame2DTimeCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.speed,
      paramQ.comp);

  // Declare a variable to memorize the resulting bounding box
  AABB2DTime bdgBoxLocal;

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame2DTime* that = &P;
  Frame2DTime* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Display the tested frames
    Frame2DTimePrint(that);
    printf("\nagainst\n");
    Frame2DTimePrint(tho);
    printf("\n");

    // Run the FMB intersection test
    bool isIntersecting =
      FMBTestIntersection2DTime(
        that,
        tho,
        &bdgBoxLocal);
```

```
// If the test hasn't given the expected answer about intersection
if (isIntersecting != correctAnswer) {

  // Display information about the failure
  printf(" Failed\n");
  printf("Expected : ");
  if (correctAnswer == false)
    printf("no ");
  printf("intersection\n");
  printf("Got : ");
  if (isIntersecting == false)
    printf("no ");
  printf("intersection\n");
  exit(0);

// Else, the test has given the expected answer about intersection
} else {

  // If the Frames were intersecting
  if (isIntersecting == true) {

    AABB2DTime bdgBox;
    Frame2DTimeExportBdgBox(
      tho,
      &bdgBoxLocal,
      &bdgBox);
    // Check the bounding box
    bool flag = true;
    for (int i = 3;
         i--;) {

      if (bdgBox.min[i] > correctBdgBox->min[i] + EPSILON ||
          bdgBox.max[i] < correctBdgBox->max[i] - EPSILON) {

        flag = false;

      }

    }

    // If the bounding box is the expected one
    if (flag == true) {

      // Display information
      printf("Succeed\n");
      AABB2DTimePrint(&bdgBox);
      printf("\n");

    // Else, the bounding box wasn't the expected one
    } else {

      // Display information
      printf("Failed\n");
      printf("Expected : ");
      AABB2DTimePrint(correctBdgBox);
      printf("\n");
      printf("     Got : ");
      AABB2DTimePrint(&bdgBox);
      printf("\n");

      // Terminate the unit tests
```

```
          exit(0);

        }

      // Else the Frames were not intersected,
      // no need to check the bounding box
      } else {

        // Display information
        printf(" Succeed (no inter)\n");

      }

    }
    printf("\n");

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Test2DTime(void) {

  // Declare two variables to memozie the arguments to the
  // Validation function
  Param2DTime paramP;
  Param2DTime paramQ;

  // Declare a variable to memorize the correct bounding box
  AABB2DTime correctBdgBox;

  // Execute the unit test on various cases

  // ----------------------------
  paramP = (Param2DTime)
    {.type = FrameCuboid,
     .orig = {0.0, 0.0},
     .comp =
       {{1.0, 0.0},
        {0.0, 1.0}},
     .speed = {0.0, 0.0}
    };
  paramQ = (Param2DTime)
    {.type = FrameCuboid,
     .orig = {-1.0, 0.0},
     .comp =
       {{1.0, 0.0},
        {0.0, 1.0}},
     .speed = {-1.0, 0.0}
    };
  UnitTest2DTime(
    paramP,
    paramQ,
    false,
    NULL);

  // ----------------------------
  paramP = (Param2DTime)
    {.type = FrameCuboid,
```

```
    .orig = {0.0, 0.0},
    .comp =
      {{1.0, 0.0},
       {0.0, 1.0}},
    .speed = {0.0, 0.0}
  };
paramQ = (Param2DTime)
  {.type = FrameCuboid,
   .orig = {-1.01, -1.01},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}},
   .speed = {1.0, 0.0}
  };
UnitTest2DTime(
  paramP,
  paramQ,
  false,
  NULL);

// ----------------------------
paramP = (Param2DTime)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}},
   .speed = {0.0, 0.0}
  };
paramQ = (Param2DTime)
  {.type = FrameCuboid,
   .orig = {-1.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}},
   .speed = {1.0, 0.0}
  };
correctBdgBox = (AABB2DTime)
  {.min = {0.0, 0.0, 0.0},
   .max = {1.0, 1.0, 1.0}
  };
UnitTest2DTime(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2DTime)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}},
   .speed = {0.0, 0.0}
  };
paramQ = (Param2DTime)
  {.type = FrameCuboid,
   .orig = {-1.0, 0.25},
   .comp =
     {{0.5, 0.0},
      {0.0, 0.5}},
```

```
   .speed = {4.0, 0.0}
  };
correctBdgBox = (AABB2DTime)
  {.min = {0.0, 0.25, 0.125},
   .max = {1.0, 0.75, 0.5}
  };
UnitTest2DTime(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2DTime)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}},
   .speed = {0.0, 0.0}
  };
paramQ = (Param2DTime)
  {.type = FrameCuboid,
   .orig = {0.25, -1.0},
   .comp =
     {{0.5, 0.0},
      {0.0, 0.5}},
   .speed = {0.0, 4.0}
  };
correctBdgBox = (AABB2DTime)
  {.min = {0.25, 0.0, 0.125},
   .max = {0.75, 1.0, 0.5}
  };
UnitTest2DTime(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param2DTime)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0},
   .comp =
     {{1.0, 0.0},
      {0.0, 1.0}},
   .speed = {0.0, 0.0}
  };
paramQ = (Param2DTime)
  {.type = FrameCuboid,
   .orig = {0.9, -1.0},
   .comp =
     {{0.5, 0.0},
      {0.0, 0.5}},
   .speed = {0.0, 4.0}
  };
correctBdgBox = (AABB2DTime)
  {.min = {0.9, 0.0, 0.125},
   .max = {1.0, 1.0, 0.5}
  };
UnitTest2DTime(
  paramP,
```

```
    paramQ,
    true,
    &correctBdgBox);

  // If we reached here, it means all the unit tests succeed
  printf("All unit tests 2DTime have succeed.\n");

}

// Main function
int main(int argc, char** argv) {

  Test2DTime();

  return 0;
}
```

6.1.4 3D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>

// Include the FMB algorithm library
#include "fmb3dt.h"

// Epslon for numerical precision
#define EPSILON 0.0001

// Helper structure to pass arguments to the UnitTest function
typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
  double speed[3];
} Param3DTime;

// Unit test function
// Takes two Frame definitions, the correct answer in term of
// intersection/no intersection and the correct bounding box
// Run the FMB intersection detection alogirhtm on the Frames
// and check against the correct results
void UnitTest3DTime(
        const Param3DTime paramP,
        const Param3DTime paramQ,
         const bool correctAnswer,
  const AABB3DTime* const correctBdgBox) {

  // Create the two Frames
  Frame3DTime P =
    Frame3DTimeCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.speed,
      paramP.comp);

  Frame3DTime Q =
    Frame3DTimeCreateStatic(
      paramQ.type,
      paramQ.orig,
```

```
    paramQ.speed,
    paramQ.comp);

// Declare a variable to memorize the resulting bounding box
AABB3DTime bdgBoxLocal;

// Helper variables to loop on the pair (that, tho) and (tho, that)
Frame3DTime* that = &P;
Frame3DTime* tho = &Q;

// Loop on pairs of Frames
for (int iPair = 2;
     iPair--;) {

  // Display the tested frames
  Frame3DTimePrint(that);
  printf("\nagainst\n");
  Frame3DTimePrint(tho);
  printf("\n");

  // Run the FMB intersection test
  bool isIntersecting =
    FMBTestIntersection3DTime(
      that,
      tho,
      &bdgBoxLocal);

  // If the test hasn't given the expected answer about intersection
  if (isIntersecting != correctAnswer) {

    // Display information about the failure
    printf(" Failed\n");
    printf("Expected : ");
    if (correctAnswer == false)
      printf("no ");
    printf("intersection\n");
    printf("Got : ");
    if (isIntersecting == false)
      printf("no ");
    printf("intersection\n");
    exit(0);

  // Else, the test has given the expected answer about intersection
  } else {

    // If the Frames were intersecting
    if (isIntersecting == true) {

      AABB3DTime bdgBox;
      Frame3DTimeExportBdgBox(
        tho,
        &bdgBoxLocal,
        &bdgBox);
      // Check the bounding box
      bool flag = true;
      for (int i = 4;
           i--;) {

        if (bdgBox.min[i] > correctBdgBox->min[i] + EPSILON ||
            bdgBox.max[i] < correctBdgBox->max[i] - EPSILON) {

          flag = false;
```

```
          }

        }

        // If the bounding box is the expected one
        if (flag == true) {

          // Display information
          printf("Succeed\n");
          AABB3DTimePrint(&bdgBox);
          printf("\n");

        // Else, the bounding box wasn't the expected one
        } else {

          // Display information
          printf("Failed\n");
          printf("Expected : ");
          AABB3DTimePrint(correctBdgBox);
          printf("\n");
          printf("     Got : ");
          AABB3DTimePrint(&bdgBox);
          printf("\n");

          // Terminate the unit tests
          exit(0);

        }

      // Else the Frames were not intersected,
      // no need to check the bounding box
      } else {

        // Display information
        printf(" Succeed (no inter)\n");

      }

    }
    printf("\n");

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Test3DTime(void) {

  // Declare two variables to memozie the arguments to the
  // Validation function
  Param3DTime paramP;
  Param3DTime paramQ;

  // Declare a variable to memorize the correct bounding box
  AABB3DTime correctBdgBox;

  // Execute the unit test on various cases
```

```
// ----------------------------
paramP = (Param3DTime)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}},
   .speed = {0.0, 0.0, 0.0}
  };
paramQ = (Param3DTime)
  {.type = FrameCuboid,
   .orig = {-1.0, 0.0, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}},
   .speed = {-1.0, 0.0, 0.0}
  };
UnitTest3DTime(
  paramP,
  paramQ,
  false,
  NULL);

// ----------------------------
paramP = (Param3DTime)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}},
   .speed = {0.0, 0.0, 0.0}
  };
paramQ = (Param3DTime)
  {.type = FrameCuboid,
   .orig = {-1.01, -1.01, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}},
   .speed = {1.0, 0.0, 0.0}
  };
UnitTest3DTime(
  paramP,
  paramQ,
  false,
  NULL);

// ----------------------------
paramP = (Param3DTime)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}},
   .speed = {0.0, 0.0, 0.0}
  };
paramQ = (Param3DTime)
  {.type = FrameCuboid,
```

```
    .orig = {-1.0, 0.0, 0.0},
    .comp =
      {{1.0, 0.0, 0.0},
       {0.0, 1.0, 0.0},
       {0.0, 0.0, 1.0}},
    .speed = {1.0, 0.0, 0.0}
  };
correctBdgBox = (AABB3DTime)
  {.min = {0.0, 0.0, 0.0, 0.0},
   .max = {1.0, 1.0, 1.0, 1.0}
  };
UnitTest3DTime(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param3DTime)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}},
   .speed = {0.0, 0.0, 0.0}
  };
paramQ = (Param3DTime)
  {.type = FrameCuboid,
   .orig = {-1.0, 0.25, 0.0},
   .comp =
     {{0.5, 0.0, 0.0},
      {0.0, 0.5, 0.0},
      {0.0, 0.0, 1.0}},
   .speed = {4.0, 0.0, 0.0}
  };
correctBdgBox = (AABB3DTime)
  {.min = {0.0, 0.25, 0.0, 0.125},
   .max = {1.0, 0.75, 1.0, 0.5}
  };
UnitTest3DTime(
  paramP,
  paramQ,
  true,
  &correctBdgBox);

// ----------------------------
paramP = (Param3DTime)
  {.type = FrameCuboid,
   .orig = {0.0, 0.0, 0.0},
   .comp =
     {{1.0, 0.0, 0.0},
      {0.0, 1.0, 0.0},
      {0.0, 0.0, 1.0}},
   .speed = {0.0, 0.0, 0.0}
  };
paramQ = (Param3DTime)
  {.type = FrameCuboid,
   .orig = {0.25, -1.0, 0.0},
   .comp =
     {{0.5, 0.0, 0.0},
      {0.0, 0.5, 0.0},
```

```
        {0.0, 0.0, 1.0}},
     .speed = {0.0, 4.0, 0.0}
    };
  correctBdgBox = (AABB3DTime)
    {.min = {0.25, 0.0, 0.0, 0.125},
     .max = {0.75, 1.0, 1.0, 0.5}
    };
  UnitTest3DTime(
    paramP,
    paramQ,
    true,
    &correctBdgBox);

  // ----------------------------
  paramP = (Param3DTime)
    {.type = FrameCuboid,
     .orig = {0.0, 0.0, 0.0},
     .comp =
       {{1.0, 0.0, 0.0},
        {0.0, 1.0, 0.0},
        {0.0, 0.0, 1.0}},
     .speed = {0.0, 0.0, 0.0}
    };
  paramQ = (Param3DTime)
    {.type = FrameCuboid,
     .orig = {0.9, -1.0, 0.0},
     .comp =
       {{0.5, 0.0, 0.0},
        {0.0, 0.5, 0.0},
        {0.0, 0.0, 1.0}},
     .speed = {0.0, 4.0, 0.0}
    };
  correctBdgBox = (AABB3DTime)
    {.min = {0.9, 0.0, 0.0, 0.125},
     .max = {1.0, 1.0, 1.0, 0.5}
    };
  UnitTest3DTime(
    paramP,
    paramQ,
    true,
    &correctBdgBox);

  // If we reached here, it means all the unit tests succeed
  printf("All unit tests 3DTime have succeed.\n");

}

// Main function
int main(int argc, char** argv) {

  Test3DTime();

  return 0;
}
```

6.2 Results

6.2.1 2D static

```
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
```

```
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.500000,0.500000)-maxXY(1.000000,1.000000)

Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.500000,0.500000)-maxXY(1.000000,1.000000)

Co(-0.500000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed (no inter)

Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(-0.500000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.250000,-0.250000) x(0.500000,0.000000) y(0.000000,2.000000)
Succeed
minXY(0.250000,0.000000)-maxXY(0.750000,1.000000)

Co(0.250000,-0.250000) x(0.500000,0.000000) y(0.000000,2.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.250000,0.000000)-maxXY(0.750000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(-0.250000,0.250000) x(2.000000,0.000000) y(0.000000,0.500000)
Succeed
minXY(0.000000,0.250000)-maxXY(1.000000,0.750000)

Co(-0.250000,0.250000) x(2.000000,0.000000) y(0.000000,0.500000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.250000)-maxXY(1.000000,0.750000)

Co(0.000000,0.000000) x(1.000000,1.000000) y(-1.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)
```

```
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,1.000000) y(-1.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

Co(-0.500000,-0.500000) x(1.000000,1.000000) y(-1.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(0.500000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(-0.500000,-0.500000) x(1.000000,1.000000) y(-1.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(0.500000,1.000000)

Co(1.500000,1.500000) x(1.000000,-1.000000) y(-1.000000,-1.000000)
against
Co(1.000000,0.000000) x(-1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.500000,0.000000)-maxXY(1.000000,1.000000)

Co(1.000000,0.000000) x(-1.000000,0.000000) y(0.000000,1.000000)
against
Co(1.500000,1.500000) x(1.000000,-1.000000) y(-1.000000,-1.000000)
Succeed
minXY(0.500000,0.000000)-maxXY(1.000000,1.000000)

Co(1.000000,0.500000) x(-0.500000,0.500000) y(-0.500000,-0.500000)
against
Co(0.000000,1.000000) x(1.000000,0.000000) y(0.000000,-1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,1.000000) x(1.000000,0.000000) y(0.000000,-1.000000)
against
Co(1.000000,0.500000) x(-0.500000,0.500000) y(-0.500000,-0.500000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(1.000000,1.000000)
against
Co(2.000000,-1.000000) x(0.000000,1.000000) y(-0.500000,1.000000)
Succeed
minXY(1.500000,0.000000)-maxXY(1.666667,1.000000)

Co(2.000000,-1.000000) x(0.000000,1.000000) y(-0.500000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(1.000000,1.000000)
Succeed
minXY(1.500000,0.500000)-maxXY(2.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
Co(1.000000,1.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed
minXY(0.500000,0.000000)-maxXY(1.000000,1.000000)

Co(1.000000,1.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
```

```
Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed
minXY(0.500000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
Co(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed
minXY(0.500000,0.500000)-maxXY(1.000000,1.500000)

Co(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed
minXY(0.500000,0.500000)-maxXY(1.000000,1.500000)

To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
Co(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed
minXY(0.500000,0.500000)-maxXY(1.000000,1.000000)

Co(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed
minXY(0.500000,0.500000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
To(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed
minXY(0.500000,1.000000)-maxXY(1.000000,1.500000)

To(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed
minXY(0.500000,1.000000)-maxXY(1.000000,1.500000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(0.000000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,0.500000)

To(0.000000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
```

```
minXY(0.000000,0.000000)-maxXY(1.000000,0.500000)

Co(0.500000,0.500000) x(-0.500000,0.000000) y(0.000000,-0.500000)
against
To(0.000000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(0.500000,0.500000)

To(0.000000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.500000,0.500000) x(-0.500000,0.000000) y(0.000000,-0.500000)
Succeed
minXY(0.000000,0.000000)-maxXY(0.500000,0.500000)

Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed (no inter)

To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(1.500000,1.500000) x(-1.500000,0.000000) y(0.000000,-1.500000)
Succeed
minXY(0.000000,0.500000)-maxXY(1.000000,1.000000)

To(1.500000,1.500000) x(-1.500000,0.000000) y(0.000000,-1.500000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.500000)-maxXY(1.000000,1.000000)

To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(1.010000,1.010000) x(-1.000000,0.000000) y(0.000000,-1.000000)
 Succeed (no inter)

To(1.010000,1.010000) x(-1.000000,0.000000) y(0.000000,-1.000000)
against
To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed (no inter)

To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
To(1.000000,1.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed
minXY(0.500000,0.000000)-maxXY(1.000000,1.000000)

To(1.000000,1.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed
minXY(0.500000,0.000000)-maxXY(1.000000,1.000000)

To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
To(1.010000,1.500000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
 Succeed (no inter)
```

```
To(1.010000,1.500000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
 Succeed (no inter)

All unit tests 2D have succeed.
```

6.2.2 3D static

```
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,0.000000,0.000000)-maxXYZ(1.000000,1.000000,1.000000)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,0.000000,0.000000)-maxXYZ(1.000000,1.000000,1.000000)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.500000,0.500000,0.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.500000,0.500000,0.500000)-maxXYZ(1.000000,1.000000,1.000000)

Co(0.500000,0.500000,0.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.500000,0.500000,0.500000)-maxXYZ(1.000000,1.000000,1.000000)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(-0.500000,-0.500000,-0.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,0.000000,0.000000)-maxXYZ(0.500000,0.500000,0.500000)

Co(-0.500000,-0.500000,-0.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,0.000000,0.000000)-maxXYZ(0.500000,0.500000,0.500000)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
```

```
Co(1.500000,1.500000,1.500000) x(-1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,-1.000000)
Succeed
minXYZ(0.500000,0.500000,0.500000)-maxXYZ(1.000000,1.000000,1.000000)

Co(1.500000,1.500000,1.500000) x(-1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,-1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.500000,0.500000,0.500000)-maxXYZ(1.000000,1.000000,1.000000)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.500000,1.500000,-1.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(0.500000,1.500000,-1.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,-1.000000)
against
Co(0.500000,1.500000,-1.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.500000,0.500000,-1.000000)-maxXYZ(1.000000,1.000000,-0.500000)

Co(0.500000,1.500000,-1.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,-1.000000)
Succeed
minXYZ(0.500000,0.500000,-1.000000)-maxXYZ(1.000000,1.000000,-0.500000)

Co(-1.010000,-1.010000,-1.010000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(-1.010000,-1.010000,-1.010000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
```

```
Succeed
minXYZ(0.000000,-0.500000,0.000000)-maxXYZ(1.000000,0.000000,1.000000)

Co(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,-0.500000,0.000000)-maxXYZ(1.000000,0.000000,1.000000)

To(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
To(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,-0.500000,0.000000)-maxXYZ(1.000000,0.000000,0.750000)

To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,-0.500000,0.000000)-maxXYZ(1.000000,0.000000,1.000000)

To(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
To(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

To(-0.500000,-1.000000,-0.500000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,-0.500000,0.000000)-maxXYZ(0.500000,0.000000,0.500000)
```

```
To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
To(-0.500000,-1.000000,-0.500000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,-0.500000,0.000000)-maxXYZ(0.500000,0.000000,0.500000)

All unit tests 3D have succeed.
```

6.2.3 2D dynamic

```
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(-1.000000,0.000000) s(-1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
 Succeed (no inter)

Co(-1.000000,0.000000) s(-1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(-1.010000,-1.010000) s(1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
 Succeed (no inter)

Co(-1.010000,-1.010000) s(1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(-1.000000,0.000000) s(1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed
minXYT(-1.000000,0.000000,0.000000)-maxXYT(2.000000,1.000000,1.000000)

Co(-1.000000,0.000000) s(1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed
minXYT(-1.000000,0.000000,0.000000)-maxXYT(1.000000,1.000000,1.000000)

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(-1.000000,0.250000) s(4.000000,0.000000) x(0.500000,0.000000) y
```

```
    (0.000000,0.500000)
Succeed
minXYT(-1.500000,0.000000,0.125000)-maxXYT(2.500000,1.000000,0.500000)

Co(-1.000000,0.250000) s(4.000000,0.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed
minXYT(-0.500000,0.000000,0.125000)-maxXYT(1.500000,1.000000,0.500000)

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.250000,-1.000000) s(0.000000,4.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
Succeed
minXYT(0.000000,-1.500000,0.125000)-maxXYT(1.000000,2.500000,0.500000)

Co(0.250000,-1.000000) s(0.000000,4.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed
minXYT(0.000000,-0.500000,0.125000)-maxXYT(1.000000,1.500000,0.500000)

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.900000,-1.000000) s(0.000000,4.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
Succeed
minXYT(0.000000,-1.500000,0.125000)-maxXYT(1.400000,2.500000,0.500000)

Co(0.900000,-1.000000) s(0.000000,4.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed
minXYT(0.000000,-0.500000,0.125000)-maxXYT(1.400000,1.500000,0.500000)

All unit tests 2DTime have succeed.
```

6.2.4 3D dynamic

```
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(-1.000000,0.000000,0.000000) s(-1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(-1.000000,0.000000,0.000000) s(-1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
```

```
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(-1.010000,-1.010000,0.000000) s(1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(-1.010000,-1.010000,0.000000) s(1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(-1.000000,0.000000,0.000000) s(1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(-1.000000,0.000000,0.000000,0.000000)-maxXYZT
    (2.000000,1.000000,1.000000,1.000000)

Co(-1.000000,0.000000,0.000000) s(1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(-1.000000,0.000000,0.000000,0.000000)-maxXYZT
    (1.000000,1.000000,1.000000,1.000000)

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(-1.000000,0.250000,0.000000) s(4.000000,0.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(-1.500000,0.000000,0.000000,0.125000)-maxXYZT
    (2.500000,1.000000,1.000000,0.500000)

Co(-1.000000,0.250000,0.000000) s(4.000000,0.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
```

```
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(-0.500000,0.000000,0.000000,0.125000)-maxXYZT
    (1.500000,1.000000,1.000000,0.500000)

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.250000,-1.000000,0.000000) s(0.000000,4.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(0.000000,-1.500000,0.000000,0.125000)-maxXYZT
    (1.000000,2.500000,1.000000,0.500000)

Co(0.250000,-1.000000,0.000000) s(0.000000,4.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(0.000000,-0.500000,0.000000,0.125000)-maxXYZT
    (1.000000,1.500000,1.000000,0.500000)

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.900000,-1.000000,0.000000) s(0.000000,4.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(0.000000,-1.500000,0.000000,0.125000)-maxXYZT
    (1.400000,2.500000,1.000000,0.500000)

Co(0.900000,-1.000000,0.000000) s(0.000000,4.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(0.000000,-0.500000,0.000000,0.125000)-maxXYZT
    (1.400000,1.500000,1.000000,0.500000)

All unit tests 3DTime have succeed.
```

7 Validation against SAT

In this section I introduce the code I've used to validate the algorithm and its implementation. The validation consists of running the FMB algorithm on randomly generated pairs of Frame and check that its result is equal to the one of running the SAT algorithm on the same pair of Frames. The code of the implementation of the SAT algorithm is given in annex (p.243)

7.1 Code

7.1.1 2D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>

// Include FMB and SAT algorithm library
#include "fmb2d.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of tests of the validation
#define NB_TESTS 1000000

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Global variables to count nb of tests resulting in intersection
// and no intersection
unsigned long int nbInter;
unsigned long int nbNoInter;

// Helper structure to pass arguments to the Validation function
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
} Param2D;

// Validation function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and check the results are identical
void ValidationOnePair2D(
  const Param2D paramP,
  const Param2D paramQ) {

  // Create the two Frames
  Frame2D P =
    Frame2DCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.comp);

  Frame2D Q =
    Frame2DCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
```

```
Frame2D* that = &P;
Frame2D* tho = &Q;

// Loop on pairs of Frames
for (int iPair = 2;
     iPair--;) {

  // Test intersection with FMB
  bool isIntersectingFMB =
    FMBTestIntersection2D(
      that,
      tho,
      NULL);

  // Test intersection with SAT
  bool isIntersectingSAT =
    SATTestIntersection2D(
      that,
      tho);

  // If the results are different
  if (isIntersectingFMB != isIntersectingSAT) {

    // Print the disagreement
    printf("Validation2D has failed\n");
    Frame2DPrint(that);
    printf(" against ");
    Frame2DPrint(tho);
    printf("\n");
    printf("FMB : ");
    if (isIntersectingFMB == false)
      printf("no ");
    printf("intersection\n");
    printf("SAT : ");
    if (isIntersectingSAT == false)
      printf("no ");
    printf("intersection\n");

    // Stop the validation
    exit(0);

  }

  // If the Frames are in intersection
  if (isIntersectingFMB == true) {

    // Update the number of intersection
    nbInter++;

  // If the Frames are not in intersection
  } else {

    // Update the number of no intersection
    nbNoInter++;

  }

  // Flip the pair of Frames
  that = &Q;
  tho = &P;

}
```

```
}

void Validate2D(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Declare two variables to memorize the arguments to the
  // Validation function
  Param2D paramP;
  Param2D paramQ;

  // Initialize the number of intersection and no intersection
  nbInter = 0;
  nbNoInter = 0;

  // Loop on the tests
  for (unsigned long iTest = NB_TESTS;
       iTest--;) {

    // Create two random Frame definitions
    Param2D* param = &paramP;
    for (int iParam = 2;
         iParam--;) {

      // 50% chance of being a Cuboid or a Tetrahedron
      if (rnd() < 0.5)
        param->type = FrameCuboid;
      else
        param->type = FrameTetrahedron;

      for (int iAxis = 2;
           iAxis--;) {

        param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        for (int iComp = 2;
             iComp--;) {

          param->comp[iComp][iAxis] =
            -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        }

      }

      param = &paramQ;

    }

    // Calculate the determinant of the Frames' components matrix
    double detP =
      paramP.comp[0][0] * paramP.comp[1][1] -
      paramP.comp[1][0] * paramP.comp[0][1];

    double detQ =
      paramQ.comp[0][0] * paramQ.comp[1][1] -
      paramQ.comp[1][0] * paramQ.comp[0][1];

    // If the determinants are not null, ie the Frame are not degenerate
    if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {
```

```
      // Run the validation on the two Frames
      ValidationOnePair2D(
        paramP,
        paramQ);

    }

  }

  // If we reached it means the validation was successfull
  // Print results
  printf("Validation2D has succeed.\n");
  printf("Tested %lu intersections ", nbInter);
  printf("and %lu no intersections\n", nbNoInter);

}

int main(int argc, char** argv) {

  printf("===== 2D static ======\n");
  Validate2D();

  return 0;
}
```

7.1.2 3D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>

// Include FMB and SAT algorithm library
#include "fmb3d.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of tests of the validation
#define NB_TESTS 1000000

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Global variables to count nb of tests resulting in intersection
// and no intersection
unsigned long int nbInter;
unsigned long int nbNoInter;

// Helper structure to pass arguments to the Validation function
typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
} Param3D;

// Validation function
```

```
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and check the results are identical
void ValidationOnePair3D(
  const Param3D paramP,
  const Param3D paramQ) {

  // Create the two Frames
  Frame3D P =
    Frame3DCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.comp);

  Frame3D Q =
    Frame3DCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame3D* that = &P;
  Frame3D* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Test intersection with FMB
    bool isIntersectingFMB =
      FMBTestIntersection3D(
        that,
        tho,
        NULL);

    // Test intersection with SAT
    bool isIntersectingSAT =
      SATTestIntersection3D(
        that,
        tho);

    // If the results are different
    if (isIntersectingFMB != isIntersectingSAT) {

      // Print the disagreement
      printf("Validation3D has failed\n");
      Frame3DPrint(that);
      printf(" against ");
      Frame3DPrint(tho);
      printf("\n");
      printf("FMB : ");
      if (isIntersectingFMB == false)
        printf("no ");
      printf("intersection\n");
      printf("SAT : ");
      if (isIntersectingSAT == false)
        printf("no ");
      printf("intersection\n");

      // Stop the validation
      exit(0);

    }
```

```
    // If the Frames are in intersection
    if (isIntersectingFMB == true) {

      // Update the number of intersection
      nbInter++;

    // If the Frames are not in intersection
    } else {

      // Update the number of no intersection
      nbNoInter++;

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Validate3D(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Declare two variables to memorize the arguments to the
  // Validation function
  Param3D paramP;
  Param3D paramQ;

  // Initialize the number of intersection and no intersection
  nbInter = 0;
  nbNoInter = 0;

  // Loop on the tests
  for (unsigned long iTest = NB_TESTS;
       iTest--;) {

    // Create two random Frame definitions
    Param3D* param = &paramP;
    for (int iParam = 2;
         iParam--;) {

      // 50% chance of being a Cuboid or a Tetrahedron
      if (rnd() < 0.5)
        param->type = FrameCuboid;
      else
        param->type = FrameTetrahedron;

      for (int iAxis = 3;
           iAxis--;) {

        param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        for (int iComp = 3;
             iComp--;) {

          param->comp[iComp][iAxis] =
            -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;
```

```
        }

      }

      param = &paramQ;

    }

    // Calculate the determinant of the Frames' components matrix
    double detP =
      paramP.comp[0][0] * (paramP.comp[1][1] * paramP.comp[2][2] -
      paramP.comp[1][2] * paramP.comp[2][1]) -
      paramP.comp[1][0] * (paramP.comp[0][1] * paramP.comp[2][2] -
      paramP.comp[0][2] * paramP.comp[2][1]) +
      paramP.comp[2][0] * (paramP.comp[0][1] * paramP.comp[1][2] -
      paramP.comp[0][2] * paramP.comp[1][1]);

    double detQ =
      paramQ.comp[0][0] * (paramQ.comp[1][1] * paramQ.comp[2][2] -
      paramQ.comp[1][2] * paramQ.comp[2][1]) -
      paramQ.comp[1][0] * (paramQ.comp[0][1] * paramQ.comp[2][2] -
      paramQ.comp[0][2] * paramQ.comp[2][1]) +
      paramQ.comp[2][0] * (paramQ.comp[0][1] * paramQ.comp[1][2] -
      paramQ.comp[0][2] * paramQ.comp[1][1]);

    // If the determinants are not null, ie the Frame are not degenerate
    if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

      // Run the validation on the two Frames
      ValidationOnePair3D(
        paramP,
        paramQ);

    }

  }

  // If we reached it means the validation was successfull
  // Print results
  printf("Validation3D has succeed.\n");
  printf("Tested %lu intersections ", nbInter);
  printf("and %lu no intersections\n", nbNoInter);

}

int main(int argc, char** argv) {

  printf("===== 3D static ======\n");
  Validate3D();

  return 0;
}
```

7.1.3 2D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>
```

```
// Include the FMB and SAT algorithm library
#include "fmb2dt.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of tests of the validation
#define NB_TESTS 1000000

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Global variables to count nb of tests resulting in intersection
// and no intersection
unsigned long int nbInter;
unsigned long int nbNoInter;

// Helper structure to pass arguments to the Validation function
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
  double speed[2];
} Param2DTime;

// Validation function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and check the results are identical
void ValidationOnePair2DTime(
  const Param2DTime paramP,
  const Param2DTime paramQ) {

  // Create the two Frames
  Frame2DTime P =
    Frame2DTimeCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.speed,
      paramP.comp);

  Frame2DTime Q =
    Frame2DTimeCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.speed,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame2DTime* that = &P;
  Frame2DTime* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Test intersection with FMB
    bool isIntersectingFMB =
      FMBTestIntersection2DTime(
        that,
```

```
        tho,
        NULL);

    // Test intersection with SAT
    bool isIntersectingSAT =
      SATTestIntersection2DTime(
        that,
        tho);

    // If the results are different
    if (isIntersectingFMB != isIntersectingSAT) {

      // Print the disagreement
      printf("Validation2D has failed\n");
      Frame2DTimePrint(that);
      printf(" against ");
      Frame2DTimePrint(tho);
      printf("\n");
      printf("FMB : ");
      if (isIntersectingFMB == false)
        printf("no ");
      printf("intersection\n");
      printf("SAT : ");
      if (isIntersectingSAT == false)
        printf("no ");
      printf("intersection\n");

      // Stop the validation
      exit(0);

    }

    // If the Frames are in intersection
    if (isIntersectingFMB == true) {

      // Update the number of intersection
      nbInter++;

    // If the Frames are not in intersection
    } else {

      // Update the number of no intersection
      nbNoInter++;

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Validate2DTime(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Declare two variables to memorize the arguments to the
  // Validation function
  Param2DTime paramP;
```

```
Param2DTime paramQ;

// Initialize the number of intersection and no intersection
nbInter = 0;
nbNoInter = 0;

// Loop on the tests
for (unsigned long iTest = NB_TESTS;
     iTest--;) {

  // Create two random Frame definitions
  Param2DTime* param = &paramP;
  for (int iParam = 2;
       iParam--;) {

    // 50% chance of being a Cuboid or a Tetrahedron
    if (rnd() < 0.5)
      param->type = FrameCuboid;
    else
      param->type = FrameTetrahedron;

    for (int iAxis = 2;
         iAxis--;) {

      param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;
      param->speed[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      for (int iComp = 2;
           iComp--;) {

        param->comp[iComp][iAxis] =
          -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      }

    }

    param = &paramQ;

  }

  // Calculate the determinant of the Frames' components matrix
  double detP =
    paramP.comp[0][0] * paramP.comp[1][1] -
    paramP.comp[1][0] * paramP.comp[0][1];

  double detQ =
    paramQ.comp[0][0] * paramQ.comp[1][1] -
    paramQ.comp[1][0] * paramQ.comp[0][1];

  // If the determinants are not null, ie the Frame are not degenerate
  if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

    // Run the validation on the two Frames
    ValidationOnePair2DTime(
      paramP,
      paramQ);

  }

}
```

```
  // If we reached it means the validation was successfull
  // Print results
  printf("Validation2DTime has succeed.\n");
  printf("Tested %lu intersections ", nbInter);
  printf("and %lu no intersections\n", nbNoInter);

}

int main(int argc, char** argv) {

  printf("===== 2D dynamic ======\n");
  Validate2DTime();

  return 0;
}
```

7.1.4 3D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>

// Include the FMB and SAT algorithm library
#include "fmb3dt.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of tests of the validation
#define NB_TESTS 1000000

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Global variables to count nb of tests resulting in intersection
// and no intersection
unsigned long int nbInter;
unsigned long int nbNoInter;

// Helper structure to pass arguments to the Validation function
typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
  double speed[3];
} Param3DTime;

// Validation function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and check the results are identical
void ValidationOnePair3DTime(
  const Param3DTime paramP,
  const Param3DTime paramQ) {

  // Create the two Frames
  Frame3DTime P =
    Frame3DTimeCreateStatic(
```

```
    paramP.type,
    paramP.orig,
    paramP.speed,
    paramP.comp);

Frame3DTime Q =
  Frame3DTimeCreateStatic(
    paramQ.type,
    paramQ.orig,
    paramQ.speed,
    paramQ.comp);

// Helper variables to loop on the pair (that, tho) and (tho, that)
Frame3DTime* that = &P;
Frame3DTime* tho = &Q;

// Loop on pairs of Frames
for (int iPair = 2;
     iPair--;) {

  // Test intersection with FMB
  bool isIntersectingFMB =
    FMBTestIntersection3DTime(
      that,
      tho,
      NULL);

  // Test intersection with SAT
  bool isIntersectingSAT =
    SATTestIntersection3DTime(
      that,
      tho);

  // If the results are different
  if (isIntersectingFMB != isIntersectingSAT) {

    // Print the disagreement
    printf("Validation3D has failed\n");
    Frame3DTimePrint(that);
    printf(" against ");
    Frame3DTimePrint(tho);
    printf("\n");
    printf("FMB : ");
    if (isIntersectingFMB == false)
      printf("no ");
    printf("intersection\n");
    printf("SAT : ");
    if (isIntersectingSAT == false)
      printf("no ");
    printf("intersection\n");

    // Stop the validation
    exit(0);

  }

  // If the Frames are in intersection
  if (isIntersectingFMB == true) {

    // Update the number of intersection
    nbInter++;
```

```
    // If the Frames are not in intersection
    } else {

      // Update the number of no intersection
      nbNoInter++;

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Validate3DTime(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Declare two variables to memorize the arguments to the
  // Validation function
  Param3DTime paramP;
  Param3DTime paramQ;

  // Initialize the number of intersection and no intersection
  nbInter = 0;
  nbNoInter = 0;

  // Loop on the tests
  for (unsigned long iTest = NB_TESTS;
       iTest--;) {

    // Create two random Frame definitions
    Param3DTime* param = &paramP;
    for (int iParam = 2;
         iParam--;) {

      // 50% chance of being a Cuboid or a Tetrahedron
      if (rnd() < 0.5)
        param->type = FrameCuboid;
      else
        param->type = FrameTetrahedron;

      for (int iAxis = 3;
           iAxis--;) {

        param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;
        param->speed[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        for (int iComp = 3;
             iComp--;) {

          param->comp[iComp][iAxis] =
            -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

        }

      }

      param = &paramQ;
```

```
    }

    // Calculate the determinant of the Frames' components matrix
    double detP =
      paramP.comp[0][0] * (paramP.comp[1][1] * paramP.comp[2][2]-
      paramP.comp[1][2] * paramP.comp[2][1]) -
      paramP.comp[1][0] * (paramP.comp[0][1] * paramP.comp[2][2]-
      paramP.comp[0][2] * paramP.comp[2][1]) +
      paramP.comp[2][0] * (paramP.comp[0][1] * paramP.comp[1][2]-
      paramP.comp[0][2] * paramP.comp[1][1]);

    double detQ =
      paramQ.comp[0][0] * (paramQ.comp[1][1] * paramQ.comp[2][2]-
      paramQ.comp[1][2] * paramQ.comp[2][1]) -
      paramQ.comp[1][0] * (paramQ.comp[0][1] * paramQ.comp[2][2]-
      paramQ.comp[0][2] * paramQ.comp[2][1]) +
      paramQ.comp[2][0] * (paramQ.comp[0][1] * paramQ.comp[1][2]-
      paramQ.comp[0][2] * paramQ.comp[1][1]);

    // If the determinants are not null, ie the Frame are not degenerate
    if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

      // Run the validation on the two Frames
      ValidationOnePair3DTime(
        paramP,
        paramQ);

    }

  }

  // If we reached it means the validation was successfull
  // Print results
  printf("Validation3DTime has succeed.\n");
  printf("Tested %lu intersections ", nbInter);
  printf("and %lu no intersections\n", nbNoInter);

}

int main(int argc, char** argv) {

  printf("===== 3D dynamic ======\n");
  Validate3DTime();

  return 0;
}
```

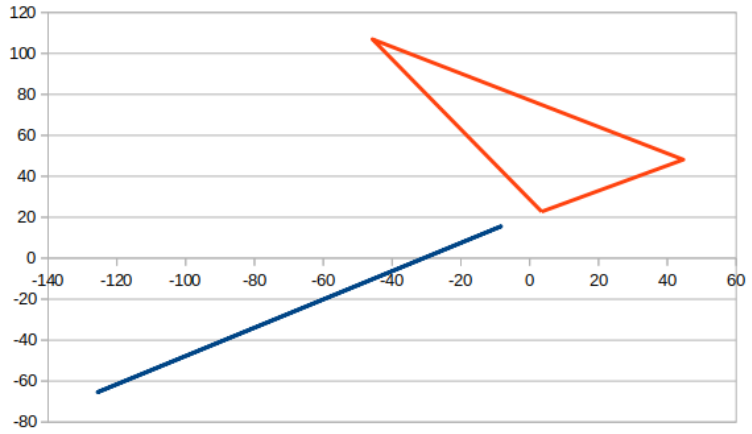
7.2 Results

7.2.1 Failures

Validation has failed in one case: when one or both of the frame are degenerated (at least two of there components are colinear). An example is given below for reference:

```
    ===== 2D static ======
Validation2D has failed
```

```
Co(-63.571705,-22.581119) x(55.239119,38.152177) y(-62.031537,-42.843548) against To(3.474294,22.751011)
x(-49.195251,84.166201) y(41.179031,-95.350316)
FMB : intersection
SAT : no intersection
```



This case can be detected and avoided prior to the intersection test by checking the determinant of the frame: degenerated frames have a null determinant. In the example above the determinant of the first frame is equal to -0.001667.

7.2.2 2D static

```
===== 2D static ======
Validation2D has succeed.
Tested 469546 intersections and 1530372 no intersections
```

7.2.3 2D dynamic

```
===== 2D dynamic ======
Validation2DTime has succeed.
Tested 744628 intersections and 1255300 no intersections
```

7.2.4 3D static

```
===== 3D static ======
Validation3D has succeed.
Tested 315454 intersections and 1684546 no intersections
```

7.2.5 3D dynamic

```
===== 3D dynamic ======
Validation3DTime has succeed.
Tested 524018 intersections and 1475980 no intersections
```

8 Qualification against SAT

In this section I introduce the code I've used to qualify the algorithm and its implementation. The qualification consists of running the FMB algorithm on randomly generated pairs of Frame, and check its execution time against the one of running the SAT algorithm on the same pair of Frames.

8.1 Code

8.1.1 2D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>
#include <sys/time.h>

// Include FMB and SAT algorithm library
#include "fmb2d.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of run
#define NB_RUNS 9
// Nb of tests per run
#define NB_TESTS 100000
// Nb of times the test is run on one pair of frame, used to
// slow down the processus and be able to measure time
#define NB_REPEAT_2D 1500

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Helper structure to pass arguments to the Qualification function
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
} Param2D;

// Global variables to count nb of tests resulting in intersection
// and no intersection, and min/max/total time of execution for each
double minInter;
double maxInter;
double sumInter;
unsigned long countInter;
double minNoInter;
```

```
double maxNoInter;
double sumNoInter;
unsigned long countNoInter;

double minInterCC;
double maxInterCC;
double sumInterCC;
unsigned long countInterCC;
double minNoInterCC;
double maxNoInterCC;
double sumNoInterCC;
unsigned long countNoInterCC;

double minInterCT;
double maxInterCT;
double sumInterCT;
unsigned long countInterCT;
double minNoInterCT;
double maxNoInterCT;
double sumNoInterCT;
unsigned long countNoInterCT;

double minInterTC;
double maxInterTC;
double sumInterTC;
unsigned long countInterTC;
double minNoInterTC;
double maxNoInterTC;
double sumNoInterTC;
unsigned long countNoInterTC;

double minInterTT;
double maxInterTT;
double sumInterTT;
unsigned long countInterTT;
double minNoInterTT;
double maxNoInterTT;
double sumNoInterTT;
unsigned long countNoInterTT;

// Qualification function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and measure the time of execution of each
void Qualification2DStatic(
        const Param2D paramP,
        const Param2D paramQ) {

  // Create the two Frames
  Frame2D P =
    Frame2DCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.comp);

  Frame2D Q =
    Frame2DCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame2D* that = &P;
```

```
Frame2D* tho = &Q;

// Loop on pairs of Frames
for (int iPair = 2;
     iPair--;) {

  // Declare an array to memorize the results of the repeated
  // test on the same pair,
  // to prevent optimization from the compiler to remove the for loop
  bool isIntersectingFMB[NB_REPEAT_2D] = {false};

  // Start measuring time
  struct timeval start;
  gettimeofday(&start, NULL);

  // Run the FMB intersection test
  for (int i = NB_REPEAT_2D;
       i--;) {

    isIntersectingFMB[i] =
      FMBTestIntersection2D(
        that,
        tho,
        NULL);
  }

  // Stop measuring time
  struct timeval stop;
  gettimeofday(&stop, NULL);

  // Calculate the delay of execution
  unsigned long deltausFMB = 0;
  if (stop.tv_sec < start.tv_sec) {
    printf("time warps, try again\n");
    exit(0);
  }
  if (stop.tv_sec > start.tv_sec + 1) {
    printf("deltausFMB >> 1s, decrease NB_REPEAT\n");
    exit(0);
  }
  if (stop.tv_usec < start.tv_usec) {
    deltausFMB = stop.tv_sec - start.tv_sec;
    deltausFMB += stop.tv_usec + 1000000 - start.tv_usec;
  } else {
    deltausFMB = stop.tv_usec - start.tv_usec;
  }

  // Declare an array to memorize the results of the repeated
  // test on the same pair,
  // to prevent optimization from the compiler to remove the for loop
  bool isIntersectingSAT[NB_REPEAT_2D] = {false};

  // Start measuring time
  gettimeofday(&start, NULL);

  // Run the FMB intersection test
  for (int i = NB_REPEAT_2D;
       i--;) {

    isIntersectingSAT[i] =
      SATTestIntersection2D(
        that,
```

```
      tho);

}

// Stop measuring time
gettimeofday(&stop, NULL);

// Calculate the delay of execution
unsigned long deltausSAT = 0;
if (stop.tv_sec < start.tv_sec) {
  printf("time warps, try again\n");
  exit(0);
}
if (stop.tv_sec > start.tv_sec + 1) {
  printf("deltausSAT >> 1s, decrease NB_REPEAT\n");
  exit(0);
}
if (stop.tv_usec < start.tv_usec) {
  deltausSAT = stop.tv_sec - start.tv_sec;
  deltausSAT += stop.tv_usec + 1000000 - start.tv_usec;
} else {
  deltausSAT = stop.tv_usec - start.tv_usec;
}

// If the delays are greater than 10ms
if (deltausFMB >= 10 && deltausSAT >= 10) {

  // If FMB and SAT disagrees
  if (isIntersectingFMB[0] != isIntersectingSAT[0]) {

    printf("Qualification has failed\n");
    Frame2DPrint(that);
    printf(" against ");
    Frame2DPrint(tho);
    printf("\n");
    printf("FMB : ");
    if (isIntersectingFMB[0] == false)
      printf("no ");
    printf("intersection\n");
    printf("SAT : ");
    if (isIntersectingSAT[0] == false)
      printf("no ");
    printf("intersection\n");

    // Stop the qualification test
    exit(0);

  }

  // Get the ratio of execution time
  double ratio = ((double)deltausFMB) / ((double)deltausSAT);

  // If the Frames intersect
  if (isIntersectingSAT[0] == true) {

    // Update the counters
    if (countInter == 0) {

      minInter = ratio;
      maxInter = ratio;

    } else {
```

```
  if (minInter > ratio)
    minInter = ratio;
  if (maxInter < ratio)
    maxInter = ratio;

}
sumInter += ratio;
++countInter;

if (paramP.type == FrameCuboid &&
    paramQ.type == FrameCuboid) {

  if (countInterCC == 0) {

    minInterCC = ratio;
    maxInterCC = ratio;

  } else {

    if (minInterCC > ratio)
      minInterCC = ratio;
    if (maxInterCC < ratio)
      maxInterCC = ratio;

  }
  sumInterCC += ratio;
  ++countInterCC;

} else if (paramP.type == FrameCuboid &&
           paramQ.type == FrameTetrahedron) {

  if (countInterCT == 0) {

    minInterCT = ratio;
    maxInterCT = ratio;

  } else {

    if (minInterCT > ratio)
      minInterCT = ratio;
    if (maxInterCT < ratio)
      maxInterCT = ratio;

  }
  sumInterCT += ratio;
  ++countInterCT;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameCuboid) {

  if (countInterTC == 0) {

    minInterTC = ratio;
    maxInterTC = ratio;

  } else {

    if (minInterTC > ratio)
      minInterTC = ratio;
    if (maxInterTC < ratio)
      maxInterTC = ratio;
```

```
    }
    sumInterTC += ratio;
    ++countInterTC;

  } else if (paramP.type == FrameTetrahedron &&
             paramQ.type == FrameTetrahedron) {

    if (countInterTT == 0) {

      minInterTT = ratio;
      maxInterTT = ratio;

    } else {

      if (minInterTT > ratio)
        minInterTT = ratio;
      if (maxInterTT < ratio)
        maxInterTT = ratio;

    }
    sumInterTT += ratio;
    ++countInterTT;

  }

// Else, the Frames do not intersect
} else {

  // Update the counters
  if (countNoInter == 0) {

    minNoInter = ratio;
    maxNoInter = ratio;

  } else {

    if (minNoInter > ratio)
      minNoInter = ratio;
    if (maxNoInter < ratio)
      maxNoInter = ratio;

  }
  sumNoInter += ratio;
  ++countNoInter;

  if (paramP.type == FrameCuboid &&
      paramQ.type == FrameCuboid) {

    if (countNoInterCC == 0) {

      minNoInterCC = ratio;
      maxNoInterCC = ratio;

    } else {

      if (minNoInterCC > ratio)
        minNoInterCC = ratio;
      if (maxNoInterCC < ratio)
        maxNoInterCC = ratio;

    }
```

```
    sumNoInterCC += ratio;
    ++countNoInterCC;

  } else if (paramP.type == FrameCuboid &&
             paramQ.type == FrameTetrahedron) {

    if (countNoInterCT == 0) {

      minNoInterCT = ratio;
      maxNoInterCT = ratio;

    } else {

      if (minNoInterCT > ratio)
        minNoInterCT = ratio;
      if (maxNoInterCT < ratio)
        maxNoInterCT = ratio;

    }
    sumNoInterCT += ratio;
    ++countNoInterCT;

  } else if (paramP.type == FrameTetrahedron &&
             paramQ.type == FrameCuboid) {

    if (countNoInterTC == 0) {

      minNoInterTC = ratio;
      maxNoInterTC = ratio;

    } else {

      if (minNoInterTC > ratio)
        minNoInterTC = ratio;
      if (maxNoInterTC < ratio)
        maxNoInterTC = ratio;

    }
    sumNoInterTC += ratio;
    ++countNoInterTC;

  } else if (paramP.type == FrameTetrahedron &&
             paramQ.type == FrameTetrahedron) {

    if (countNoInterTT == 0) {

      minNoInterTT = ratio;
      maxNoInterTT = ratio;

    } else {

      if (minNoInterTT > ratio)
        minNoInterTT = ratio;
      if (maxNoInterTT < ratio)
        maxNoInterTT = ratio;

    }
    sumNoInterTT += ratio;
    ++countNoInterTT;

  }
}
```

```
    // Else, if time of execution for FMB was less than a 10ms
    } else if (deltausFMB < 10) {

      printf("deltausFMB < 10ms, increase NB_REPEAT\n");
      exit(0);

    // Else, if time of execution for SAT was less than a 10ms
    } else if (deltausSAT < 10) {

      printf("deltausSAT < 10ms, increase NB_REPEAT\n");
      exit(0);

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Qualify2DStatic(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Open the files to save the results
  FILE* fp = fopen("../Results/qualification2D.txt", "w");
  FILE* fpCC = fopen("../Results/qualification2DCC.txt", "w");
  FILE* fpCT = fopen("../Results/qualification2DCT.txt", "w");
  FILE* fpTC = fopen("../Results/qualification2DTC.txt", "w");
  FILE* fpTT = fopen("../Results/qualification2DTT.txt", "w");

  // Loop on runs
  for (int iRun = 0;
       iRun < NB_RUNS;
       ++iRun) {

    // Ratio intersection/no intersection for the displayed results
    double ratioInter = 0.1 + 0.8 * (double)iRun / (double)(NB_RUNS - 1);

    // Initialize counters
    minInter = 0.0;
    maxInter = 0.0;
    sumInter = 0.0;
    countInter = 0;
    minNoInter = 0.0;
    maxNoInter = 0.0;
    sumNoInter = 0.0;
    countNoInter = 0;

    minInterCC = 0.0;
    maxInterCC = 0.0;
    sumInterCC = 0.0;
    countInterCC = 0;
    minNoInterCC = 0.0;
    maxNoInterCC = 0.0;
    sumNoInterCC = 0.0;
    countNoInterCC = 0;
```

```
minInterCT = 0.0;
maxInterCT = 0.0;
sumInterCT = 0.0;
countInterCT = 0;
minNoInterCT = 0.0;
maxNoInterCT = 0.0;
sumNoInterCT = 0.0;
countNoInterCT = 0;

minInterTC = 0.0;
maxInterTC = 0.0;
sumInterTC = 0.0;
countInterTC = 0;
minNoInterTC = 0.0;
maxNoInterTC = 0.0;
sumNoInterTC = 0.0;
countNoInterTC = 0;

minInterTT = 0.0;
maxInterTT = 0.0;
sumInterTT = 0.0;
countInterTT = 0;
minNoInterTT = 0.0;
maxNoInterTT = 0.0;
sumNoInterTT = 0.0;
countNoInterTT = 0;

// Declare two variables to memorize the arguments to the
// Qualification function
Param2D paramP;
Param2D paramQ;

// Loop on the number of tests
for (unsigned long iTest = NB_TESTS;
     iTest--;) {

  // Create two random Frame definitions
  Param2D* param = &paramP;
  for (int iParam = 2;
       iParam--;) {

    // 50% chance of being a Cuboid or a Tetrahedron
    if (rnd() < 0.5)
      param->type = FrameCuboid;
    else
      param->type = FrameTetrahedron;

    for (int iAxis = 2;
         iAxis--;) {

      param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      for (int iComp = 2;
           iComp--;) {

        param->comp[iComp][iAxis] =
          -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      }

    }
```

```
    param = &paramQ;

  }

  // Calculate the determinant of the Frames' components matrix

  double detP =
    paramP.comp[0][0] * paramP.comp[1][1] -
    paramP.comp[1][0] * paramP.comp[0][1];

  double detQ =
    paramQ.comp[0][0] * paramQ.comp[1][1] -
    paramQ.comp[1][0] * paramQ.comp[0][1];

  // If the determinants are not null, ie the Frame are not degenerate
  if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

    // Run the validation on the two Frames
    Qualification2DStatic(
      paramP,
      paramQ);

  }

}

// Display the results
if (iRun == 0) {

  fprintf(fp, "percPairInter,");
  fprintf(fp, "countInterTo,countNoInterTo,");
  fprintf(fp, "minInterTo,avgInterTo,maxInterTo,");
  fprintf(fp, "minNoInterTo,avgNoInterTo,maxNoInterTo,");
  fprintf(fp, "minTotalTo,avgTotalTo,maxTotalTo\n");

  fprintf(fpCC, "percPairInter,");
  fprintf(fpCC, "countInterCC,countNoInterCC,");
  fprintf(fpCC, "minInterCC,avgInterCC,maxInterCC,");
  fprintf(fpCC, "minNoInterCC,avgNoInterCC,maxNoInterCC,");
  fprintf(fpCC, "minTotalCC,avgTotalCC,maxTotalCC\n");

  fprintf(fpCT, "percPairInter,");
  fprintf(fpCT, "countInterCT,countNoInterCT,");
  fprintf(fpCT, "minInterCT,avgInterCT,maxInterCT,");
  fprintf(fpCT, "minNoInterCT,avgNoInterCT,maxNoInterCT,");
  fprintf(fpCT, "minTotalCT,avgTotalCT,maxTotalCT\n");

  fprintf(fpTC, "percPairInter,");
  fprintf(fpTC, "countInterTC,countNoInterTC,");
  fprintf(fpTC, "minInterTC,avgInterTC,maxInterTC,");
  fprintf(fpTC, "minNoInterTC,avgNoInterTC,maxNoInterTC,");
  fprintf(fpTC, "minTotalTC,avgTotalTC,maxTotalTC\n");

  fprintf(fpTT, "percPairInter,");
  fprintf(fpTT, "countInterTT,countNoInterTT,");
  fprintf(fpTT, "minInterTT,avgInterTT,maxInterTT,");
  fprintf(fpTT, "minNoInterTT,avgNoInterTT,maxNoInterTT,");
  fprintf(fpTT, "minTotalTT,avgTotalTT,maxTotalTT\n");

}

fprintf(fp, "%.1f,", ratioInter);
```

```
fprintf(fp, "%lu,%lu,", countInter, countNoInter);
double avgInter = sumInter / (double)countInter;
fprintf(fp, "%f,%f,%f,", minInter, avgInter, maxInter);
double avgNoInter = sumNoInter / (double)countNoInter;
fprintf(fp, "%f,%f,%f,", minNoInter, avgNoInter, maxNoInter);
double avg =
  ratioInter * avgInter + (1.0 - ratioInter) * avgNoInter;
fprintf(fp, "%f,%f,%f",
  (minNoInter < minInter ? minNoInter : minInter),
  avg,
  (maxNoInter > maxInter ? maxNoInter : maxInter));
if (iRun < NB_RUNS - 1) {
  fprintf(fp, "\n");
}

fprintf(fpCC, "%.1f,", ratioInter);
fprintf(fpCC, "%lu,%lu,", countInterCC, countNoInterCC);
double avgInterCC = sumInterCC / (double)countInterCC;
fprintf(fpCC, "%f,%f,%f,", minInterCC, avgInterCC, maxInterCC);
double avgNoInterCC = sumNoInterCC / (double)countNoInterCC;
fprintf(fpCC, "%f,%f,%f,", minNoInterCC, avgNoInterCC, maxNoInterCC);
double avgCC =
  ratioInter * avgInterCC + (1.0 - ratioInter) * avgNoInterCC;
fprintf(fpCC, "%f,%f,%f",
  (minNoInterCC < minInterCC ? minNoInterCC : minInterCC),
  avgCC,
  (maxNoInterCC > maxInterCC ? maxNoInterCC : maxInterCC));
if (iRun < NB_RUNS - 1) {
  fprintf(fpCC, "\n");
}

fprintf(fpCT, "%.1f,", ratioInter);
fprintf(fpCT, "%lu,%lu,", countInterCT, countNoInterCT);
double avgInterCT = sumInterCT / (double)countInterCT;
fprintf(fpCT, "%f,%f,%f,", minInterCT, avgInterCT, maxInterCT);
double avgNoInterCT = sumNoInterCT / (double)countNoInterCT;
fprintf(fpCT, "%f,%f,%f,", minNoInterCT, avgNoInterCT, maxNoInterCT);
double avgCT =
  ratioInter * avgInterCT + (1.0 - ratioInter) * avgNoInterCT;
fprintf(fpCT, "%f,%f,%f",
  (minNoInterCT < minInterCT ? minNoInterCT : minInterCT),
  avgCT,
  (maxNoInterCT > maxInterCT ? maxNoInterCT : maxInterCT));
if (iRun < NB_RUNS - 1) {
  fprintf(fpCT, "\n");
}

fprintf(fpTC, "%.1f,", ratioInter);
fprintf(fpTC, "%lu,%lu,", countInterTC, countNoInterTC);
double avgInterTC = sumInterTC / (double)countInterTC;
fprintf(fpTC, "%f,%f,%f,", minInterTC, avgInterTC, maxInterTC);
double avgNoInterTC = sumNoInterTC / (double)countNoInterTC;
fprintf(fpTC, "%f,%f,%f,", minNoInterTC, avgNoInterTC, maxNoInterTC);
double avgTC =
  ratioInter * avgInterTC + (1.0 - ratioInter) * avgNoInterTC;
fprintf(fpTC, "%f,%f,%f",
  (minNoInterTC < minInterTC ? minNoInterTC : minInterTC),
  avgTC,
  (maxNoInterTC > maxInterTC ? maxNoInterTC : maxInterTC));
if (iRun < NB_RUNS - 1) {
  fprintf(fpTC, "\n");
}
```

```
    fprintf(fpTT, "%.1f,", ratioInter);
    fprintf(fpTT, "%lu,%lu,", countInterTT, countNoInterTT);
    double avgInterTT = sumInterTT / (double)countInterTT;
    fprintf(fpTT, "%f,%f,%f,", minInterTT, avgInterTT, maxInterTT);
    double avgNoInterTT = sumNoInterTT / (double)countNoInterTT;
    fprintf(fpTT, "%f,%f,%f,", minNoInterTT, avgNoInterTT, maxNoInterTT);
    double avgTT =
      ratioInter * avgInterTT + (1.0 - ratioInter) * avgNoInterTT;
    fprintf(fpTT, "%f,%f,%f",
      (minNoInterTT < minInterTT ? minNoInterTT : minInterTT),
      avgTT,
      (maxNoInterTT > maxInterTT ? maxNoInterTT : maxInterTT));
    if (iRun < NB_RUNS - 1) {
      fprintf(fpTT, "\n");
    }

  }

  // Close the files
  fclose(fp);
  fclose(fpCC);
  fclose(fpCT);
  fclose(fpTC);
  fclose(fpTT);

}

int main(int argc, char** argv) {

  Qualify2DStatic();

  return 0;
}
```

8.1.2 3D static

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>
#include <sys/time.h>

// Include FMB and SAT algorithm library
#include "fmb3d.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of run
#define NB_RUNS 9
// Nb of tests per run
#define NB_TESTS 100000
// Nb of times the test is run on one pair of frame, used to
// slow down the processus and be able to measure time
#define NB_REPEAT_3D 800

// Helper macro to generate random number in [0.0, 1.0]
```

```
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Helper structure to pass arguments to the Qualification function
typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
} Param3D;

// Global variables to count nb of tests resulting in intersection
// and no intersection, and min/max/total time of execution for each
double minInter;
double maxInter;
double sumInter;
unsigned long countInter;
double minNoInter;
double maxNoInter;
double sumNoInter;
unsigned long countNoInter;

double minInterCC;
double maxInterCC;
double sumInterCC;
unsigned long countInterCC;
double minNoInterCC;
double maxNoInterCC;
double sumNoInterCC;
unsigned long countNoInterCC;

double minInterCT;
double maxInterCT;
double sumInterCT;
unsigned long countInterCT;
double minNoInterCT;
double maxNoInterCT;
double sumNoInterCT;
unsigned long countNoInterCT;

double minInterTC;
double maxInterTC;
double sumInterTC;
unsigned long countInterTC;
double minNoInterTC;
double maxNoInterTC;
double sumNoInterTC;
unsigned long countNoInterTC;

double minInterTT;
double maxInterTT;
double sumInterTT;
unsigned long countInterTT;
double minNoInterTT;
double maxNoInterTT;
double sumNoInterTT;
unsigned long countNoInterTT;

// Qualification function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and measure the time of execution of each
void Qualification3DStatic(
        const Param3D paramP,
        const Param3D paramQ) {
```

```
// Create the two Frames
Frame3D P =
  Frame3DCreateStatic(
    paramP.type,
    paramP.orig,
    paramP.comp);

Frame3D Q =
  Frame3DCreateStatic(
    paramQ.type,
    paramQ.orig,
    paramQ.comp);

// Helper variables to loop on the pair (that, tho) and (tho, that)
Frame3D* that = &P;
Frame3D* tho = &Q;

// Loop on pairs of Frames
for (int iPair = 2;
     iPair--;) {

  // Declare an array to memorize the results of the repeated
  // test on the same pair,
  // to prevent optimization from the compiler to remove the for loop
  bool isIntersectingFMB[NB_REPEAT_3D] = {false};

  // Start measuring time
  struct timeval start;
  gettimeofday(&start, NULL);

  // Run the FMB intersection test
  for (int i = NB_REPEAT_3D;
       i--;) {

    isIntersectingFMB[i] =
      FMBTestIntersection3D(
        that,
        tho,
        NULL);
  }

  // Stop measuring time
  struct timeval stop;
  gettimeofday(&stop, NULL);

  // Calculate the delay of execution
  unsigned long deltausFMB = 0;
  if (stop.tv_sec < start.tv_sec) {
    printf("time warps, try again\n");
    exit(0);
  }
  if (stop.tv_sec > start.tv_sec + 1) {
    printf("deltausFMB >> 1s, decrease NB_REPEAT\n");
    exit(0);
  }
  if (stop.tv_usec < start.tv_usec) {
    deltausFMB = stop.tv_sec - start.tv_sec;
    deltausFMB += stop.tv_usec + 1000000 - start.tv_usec;
  } else {
    deltausFMB = stop.tv_usec - start.tv_usec;
  }
```

```
// Declare an array to memorize the results of the repeated
// test on the same pair,
// to prevent optimization from the compiler to remove the for loop
bool isIntersectingSAT[NB_REPEAT_3D] = {false};

// Start measuring time
gettimeofday(&start, NULL);

// Run the FMB intersection test
for (int i = NB_REPEAT_3D;
     i--;) {

  isIntersectingSAT[i] =
    SATTestIntersection3D(
      that,
      tho);

}

// Stop measuring time
gettimeofday(&stop, NULL);

// Calculate the delay of execution
unsigned long deltausSAT = 0;
if (stop.tv_sec < start.tv_sec) {
  printf("time warps, try again\n");
  exit(0);
}
if (stop.tv_sec > start.tv_sec + 1) {
  printf("deltausSAT >> 1s, decrease NB_REPEAT\n");
  exit(0);
}
if (stop.tv_usec < start.tv_usec) {
  deltausSAT = stop.tv_sec - start.tv_sec;
  deltausSAT += stop.tv_usec + 1000000 - start.tv_usec;
} else {
  deltausSAT = stop.tv_usec - start.tv_usec;
}

// If the delays are greater than 10ms
if (deltausFMB >= 10 && deltausSAT >= 10) {

  // If FMB and SAT disagrees
  if (isIntersectingFMB[0] != isIntersectingSAT[0]) {

    printf("Qualification has failed\n");
    Frame3DPrint(that);
    printf(" against ");
    Frame3DPrint(tho);
    printf("\n");
    printf("FMB : ");
    if (isIntersectingFMB[0] == false)
      printf("no ");
    printf("intersection\n");
    printf("SAT : ");
    if (isIntersectingSAT[0] == false)
      printf("no ");
    printf("intersection\n");

    // Stop the qualification test
    exit(0);
```

```
}

// Get the ratio of execution time
double ratio = ((double)deltausFMB) / ((double)deltausSAT);

// If the Frames intersect
if (isIntersectingSAT[0] == true) {

  // Update the counters
  if (countInter == 0) {

    minInter = ratio;
    maxInter = ratio;

  } else {

    if (minInter > ratio)
      minInter = ratio;
    if (maxInter < ratio)
      maxInter = ratio;

  }
  sumInter += ratio;
  ++countInter;

  if (paramP.type == FrameCuboid &&
      paramQ.type == FrameCuboid) {

    if (countInterCC == 0) {

      minInterCC = ratio;
      maxInterCC = ratio;

    } else {

      if (minInterCC > ratio)
        minInterCC = ratio;
      if (maxInterCC < ratio)
        maxInterCC = ratio;

    }
    sumInterCC += ratio;
    ++countInterCC;

  } else if (paramP.type == FrameCuboid &&
             paramQ.type == FrameTetrahedron) {

    if (countInterCT == 0) {

      minInterCT = ratio;
      maxInterCT = ratio;

    } else {

      if (minInterCT > ratio)
        minInterCT = ratio;
      if (maxInterCT < ratio)
        maxInterCT = ratio;

    }
    sumInterCT += ratio;
```

```
    ++countInterCT;

  } else if (paramP.type == FrameTetrahedron &&
             paramQ.type == FrameCuboid) {

    if (countInterTC == 0) {

      minInterTC = ratio;
      maxInterTC = ratio;

    } else {

      if (minInterTC > ratio)
        minInterTC = ratio;
      if (maxInterTC < ratio)
        maxInterTC = ratio;

    }
    sumInterTC += ratio;
    ++countInterTC;

  } else if (paramP.type == FrameTetrahedron &&
             paramQ.type == FrameTetrahedron) {

    if (countInterTT == 0) {

      minInterTT = ratio;
      maxInterTT = ratio;

    } else {

      if (minInterTT > ratio)
        minInterTT = ratio;
      if (maxInterTT < ratio)
        maxInterTT = ratio;

    }
    sumInterTT += ratio;
    ++countInterTT;

  }

// Else, the Frames do not intersect
} else {

  // Update the counters
  if (countNoInter == 0) {

    minNoInter = ratio;
    maxNoInter = ratio;

  } else {

    if (minNoInter > ratio)
      minNoInter = ratio;
    if (maxNoInter < ratio)
      maxNoInter = ratio;

  }
  sumNoInter += ratio;
  ++countNoInter;
```

```
if (paramP.type == FrameCuboid &&
    paramQ.type == FrameCuboid) {

  if (countNoInterCC == 0) {

    minNoInterCC = ratio;
    maxNoInterCC = ratio;

  } else {

    if (minNoInterCC > ratio)
      minNoInterCC = ratio;
    if (maxNoInterCC < ratio)
      maxNoInterCC = ratio;

  }
  sumNoInterCC += ratio;
  ++countNoInterCC;

} else if (paramP.type == FrameCuboid &&
           paramQ.type == FrameTetrahedron) {

  if (countNoInterCT == 0) {

    minNoInterCT = ratio;
    maxNoInterCT = ratio;

  } else {

    if (minNoInterCT > ratio)
      minNoInterCT = ratio;
    if (maxNoInterCT < ratio)
      maxNoInterCT = ratio;

  }
  sumNoInterCT += ratio;
  ++countNoInterCT;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameCuboid) {

  if (countNoInterTC == 0) {

    minNoInterTC = ratio;
    maxNoInterTC = ratio;

  } else {

    if (minNoInterTC > ratio)
      minNoInterTC = ratio;
    if (maxNoInterTC < ratio)
      maxNoInterTC = ratio;

  }
  sumNoInterTC += ratio;
  ++countNoInterTC;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameTetrahedron) {

  if (countNoInterTT == 0) {
```

```
            minNoInterTT = ratio;
            maxNoInterTT = ratio;

          } else {

            if (minNoInterTT > ratio)
              minNoInterTT = ratio;
            if (maxNoInterTT < ratio)
              maxNoInterTT = ratio;

          }
          sumNoInterTT += ratio;
          ++countNoInterTT;

        }
      }

    // Else, if time of execution for FMB was less than a 10ms
    } else if (deltausFMB < 10) {

      printf("deltausFMB < 10ms, increase NB_REPEAT\n");
      exit(0);

    // Else, if time of execution for SAT was less than a 10ms
    } else if (deltausSAT < 10) {

      printf("deltausSAT < 10ms, increase NB_REPEAT\n");
      exit(0);

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Qualify3DStatic(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Open the files to save the results
  FILE* fp = fopen("../Results/qualification3D.txt", "w");
  FILE* fpCC = fopen("../Results/qualification3DCC.txt", "w");
  FILE* fpCT = fopen("../Results/qualification3DCT.txt", "w");
  FILE* fpTC = fopen("../Results/qualification3DTC.txt", "w");
  FILE* fpTT = fopen("../Results/qualification3DTT.txt", "w");

  // Loop on runs
  for (int iRun = 0;
       iRun < NB_RUNS;
       ++iRun) {

    // Ratio intersection/no intersection for the displayed results
    double ratioInter = 0.1 + 0.8 * (double)iRun / (double)(NB_RUNS - 1);

    // Initialize counters
    minInter = 0.0;
    maxInter = 0.0;
```

```
sumInter = 0.0;
countInter = 0;
minNoInter = 0.0;
maxNoInter = 0.0;
sumNoInter = 0.0;
countNoInter = 0;

minInterCC = 0.0;
maxInterCC = 0.0;
sumInterCC = 0.0;
countInterCC = 0;
minNoInterCC = 0.0;
maxNoInterCC = 0.0;
sumNoInterCC = 0.0;
countNoInterCC = 0;

minInterCT = 0.0;
maxInterCT = 0.0;
sumInterCT = 0.0;
countInterCT = 0;
minNoInterCT = 0.0;
maxNoInterCT = 0.0;
sumNoInterCT = 0.0;
countNoInterCT = 0;

minInterTC = 0.0;
maxInterTC = 0.0;
sumInterTC = 0.0;
countInterTC = 0;
minNoInterTC = 0.0;
maxNoInterTC = 0.0;
sumNoInterTC = 0.0;
countNoInterTC = 0;

minInterTT = 0.0;
maxInterTT = 0.0;
sumInterTT = 0.0;
countInterTT = 0;
minNoInterTT = 0.0;
maxNoInterTT = 0.0;
sumNoInterTT = 0.0;
countNoInterTT = 0;

// Declare two variables to memorize the arguments to the
// Qualification function
Param3D paramP;
Param3D paramQ;

// Loop on the number of tests
for (unsigned long iTest = NB_TESTS;
     iTest--;) {

  // Create two random Frame definitions
  Param3D* param = &paramP;
  for (int iParam = 2;
       iParam--;) {

    // 50% chance of being a Cuboid or a Tetrahedron
    if (rnd() < 0.5)
      param->type = FrameCuboid;
    else
      param->type = FrameTetrahedron;
```

```
    for (int iAxis = 3;
         iAxis--;) {

      param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      for (int iComp = 3;
           iComp--;) {

        param->comp[iComp][iAxis] =
          -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      }

    }

    param = &paramQ;

  }

  // Calculate the determinant of the Frames' components matrix
  double detP =
    paramP.comp[0][0] * (paramP.comp[1][1] * paramP.comp[2][2]-
    paramP.comp[1][2] * paramP.comp[2][1]) -
    paramP.comp[1][0] * (paramP.comp[0][1] * paramP.comp[2][2]-
    paramP.comp[0][2] * paramP.comp[2][1]) +
    paramP.comp[2][0] * (paramP.comp[0][1] * paramP.comp[1][2]-
    paramP.comp[0][2] * paramP.comp[1][1]);

  double detQ =
    paramQ.comp[0][0] * (paramQ.comp[1][1] * paramQ.comp[2][2]-
    paramQ.comp[1][2] * paramQ.comp[2][1]) -
    paramQ.comp[1][0] * (paramQ.comp[0][1] * paramQ.comp[2][2]-
    paramQ.comp[0][2] * paramQ.comp[2][1]) +
    paramQ.comp[2][0] * (paramQ.comp[0][1] * paramQ.comp[1][2]-
    paramQ.comp[0][2] * paramQ.comp[1][1]);

  // If the determinants are not null, ie the Frame are not degenerate
  if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

    // Run the validation on the two Frames
    Qualification3DStatic(
      paramP,
      paramQ);

  }

}

// Display the results
if (iRun == 0) {

  fprintf(fp, "percPairInter,");
  fprintf(fp, "countInterTo,countNoInterTo,");
  fprintf(fp, "minInterTo,avgInterTo,maxInterTo,");
  fprintf(fp, "minNoInterTo,avgNoInterTo,maxNoInterTo,");
  fprintf(fp, "minTotalTo,avgTotalTo,maxTotalTo\n");

  fprintf(fpCC, "percPairInter,");
  fprintf(fpCC, "countInterCC,countNoInterCC,");
  fprintf(fpCC, "minInterCC,avgInterCC,maxInterCC,");
  fprintf(fpCC, "minNoInterCC,avgNoInterCC,maxNoInterCC,");
```

```
  fprintf(fpCC, "minTotalCC,avgTotalCC,maxTotalCC\n");

  fprintf(fpCT, "percPairInter,");
  fprintf(fpCT, "countInterCT,countNoInterCT,");
  fprintf(fpCT, "minInterCT,avgInterCT,maxInterCT,");
  fprintf(fpCT, "minNoInterCT,avgNoInterCT,maxNoInterCT,");
  fprintf(fpCT, "minTotalCT,avgTotalCT,maxTotalCT\n");

  fprintf(fpTC, "percPairInter,");
  fprintf(fpTC, "countInterTC,countNoInterTC,");
  fprintf(fpTC, "minInterTC,avgInterTC,maxInterTC,");
  fprintf(fpTC, "minNoInterTC,avgNoInterTC,maxNoInterTC,");
  fprintf(fpTC, "minTotalTC,avgTotalTC,maxTotalTC\n");

  fprintf(fpTT, "percPairInter,");
  fprintf(fpTT, "countInterTT,countNoInterTT,");
  fprintf(fpTT, "minInterTT,avgInterTT,maxInterTT,");
  fprintf(fpTT, "minNoInterTT,avgNoInterTT,maxNoInterTT,");
  fprintf(fpTT, "minTotalTT,avgTotalTT,maxTotalTT\n");

}

fprintf(fp, "%.1f,", ratioInter);
fprintf(fp, "%lu,%lu,", countInter, countNoInter);
double avgInter = sumInter / (double)countInter;
fprintf(fp, "%f,%f,%f,", minInter, avgInter, maxInter);
double avgNoInter = sumNoInter / (double)countNoInter;
fprintf(fp, "%f,%f,%f,", minNoInter, avgNoInter, maxNoInter);
double avg =
  ratioInter * avgInter + (1.0 - ratioInter) * avgNoInter;
fprintf(fp, "%f,%f,%f",
  (minNoInter < minInter ? minNoInter : minInter),
  avg,
  (maxNoInter > maxInter ? maxNoInter : maxInter));
if (iRun < NB_RUNS - 1) {
  fprintf(fp, "\n");
}

fprintf(fpCC, "%.1f,", ratioInter);
fprintf(fpCC, "%lu,%lu,", countInterCC, countNoInterCC);
double avgInterCC = sumInterCC / (double)countInterCC;
fprintf(fpCC, "%f,%f,%f,", minInterCC, avgInterCC, maxInterCC);
double avgNoInterCC = sumNoInterCC / (double)countNoInterCC;
fprintf(fpCC, "%f,%f,%f,", minNoInterCC, avgNoInterCC, maxNoInterCC);
double avgCC =
  ratioInter * avgInterCC + (1.0 - ratioInter) * avgNoInterCC;
fprintf(fpCC, "%f,%f,%f",
  (minNoInterCC < minInterCC ? minNoInterCC : minInterCC),
  avgCC,
  (maxNoInterCC > maxInterCC ? maxNoInterCC : maxInterCC));
if (iRun < NB_RUNS - 1) {
  fprintf(fpCC, "\n");
}

fprintf(fpCT, "%.1f,", ratioInter);
fprintf(fpCT, "%lu,%lu,", countInterCT, countNoInterCT);
double avgInterCT = sumInterCT / (double)countInterCT;
fprintf(fpCT, "%f,%f,%f,", minInterCT, avgInterCT, maxInterCT);
double avgNoInterCT = sumNoInterCT / (double)countNoInterCT;
fprintf(fpCT, "%f,%f,%f,", minNoInterCT, avgNoInterCT, maxNoInterCT);
double avgCT =
  ratioInter * avgInterCT + (1.0 - ratioInter) * avgNoInterCT;
```

```
    fprintf(fpCT, "%f,%f,%f",
      (minNoInterCT < minInterCT ? minNoInterCT : minInterCT),
      avgCT,
      (maxNoInterCT > maxInterCT ? maxNoInterCT : maxInterCT));
    if (iRun < NB_RUNS - 1) {
      fprintf(fpCT, "\n");
    }

    fprintf(fpTC, "%.1f,", ratioInter);
    fprintf(fpTC, "%lu,%lu,", countInterTC, countNoInterTC);
    double avgInterTC = sumInterTC / (double)countInterTC;
    fprintf(fpTC, "%f,%f,%f,", minInterTC, avgInterTC, maxInterTC);
    double avgNoInterTC = sumNoInterTC / (double)countNoInterTC;
    fprintf(fpTC, "%f,%f,%f,", minNoInterTC, avgNoInterTC, maxNoInterTC);
    double avgTC =
      ratioInter * avgInterTC + (1.0 - ratioInter) * avgNoInterTC;
    fprintf(fpTC, "%f,%f,%f",
      (minNoInterTC < minInterTC ? minNoInterTC : minInterTC),
      avgTC,
      (maxNoInterTC > maxInterTC ? maxNoInterTC : maxInterTC));
    if (iRun < NB_RUNS - 1) {
      fprintf(fpTC, "\n");
    }

    fprintf(fpTT, "%.1f,", ratioInter);
    fprintf(fpTT, "%lu,%lu,", countInterTT, countNoInterTT);
    double avgInterTT = sumInterTT / (double)countInterTT;
    fprintf(fpTT, "%f,%f,%f,", minInterTT, avgInterTT, maxInterTT);
    double avgNoInterTT = sumNoInterTT / (double)countNoInterTT;
    fprintf(fpTT, "%f,%f,%f,", minNoInterTT, avgNoInterTT, maxNoInterTT);
    double avgTT =
      ratioInter * avgInterTT + (1.0 - ratioInter) * avgNoInterTT;
    fprintf(fpTT, "%f,%f,%f",
      (minNoInterTT < minInterTT ? minNoInterTT : minInterTT),
      avgTT,
      (maxNoInterTT > maxInterTT ? maxNoInterTT : maxInterTT));
    if (iRun < NB_RUNS - 1) {
      fprintf(fpTT, "\n");
    }

  }

  // Close the files
  fclose(fp);
  fclose(fpCC);
  fclose(fpCT);
  fclose(fpTC);
  fclose(fpTT);

}

int main(int argc, char** argv) {

  Qualify3DStatic();

  return 0;
}
```

8.1.3 2D dynamic

```
// Include standard libraries
```

```
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>
#include <sys/time.h>

// Include FMB and SAT algorithm library
#include "fmb2dt.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of run
#define NB_RUNS 9
// Nb of tests per run
#define NB_TESTS 100000
// Nb of times the test is run on one pair of frame, used to
// slow down the processus and be able to measure time
#define NB_REPEAT_2D 1500

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Helper structure to pass arguments to the Qualification function
typedef struct {
  FrameType type;
  double orig[2];
  double comp[2][2];
  double speed[2];
} Param2DTime;

// Global variables to count nb of tests resulting in intersection
// and no intersection, and min/max/total time of execution for each
double minInter;
double maxInter;
double sumInter;
unsigned long countInter;
double minNoInter;
double maxNoInter;
double sumNoInter;
unsigned long countNoInter;

double minInterCC;
double maxInterCC;
double sumInterCC;
unsigned long countInterCC;
double minNoInterCC;
double maxNoInterCC;
double sumNoInterCC;
unsigned long countNoInterCC;

double minInterCT;
double maxInterCT;
double sumInterCT;
unsigned long countInterCT;
double minNoInterCT;
double maxNoInterCT;
double sumNoInterCT;
unsigned long countNoInterCT;
```

```
double minInterTC;
double maxInterTC;
double sumInterTC;
unsigned long countInterTC;
double minNoInterTC;
double maxNoInterTC;
double sumNoInterTC;
unsigned long countNoInterTC;

double minInterTT;
double maxInterTT;
double sumInterTT;
unsigned long countInterTT;
double minNoInterTT;
double maxNoInterTT;
double sumNoInterTT;
unsigned long countNoInterTT;

// Qualification function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and measure the time of execution of each
void Qualification2DDynamic(
        const Param2DTime paramP,
        const Param2DTime paramQ) {

  // Create the two Frames
  Frame2DTime P =
    Frame2DTimeCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.speed,
      paramP.comp);

  Frame2DTime Q =
    Frame2DTimeCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.speed,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame2DTime* that = &P;
  Frame2DTime* tho = &Q;

  // Loop on pairs of Frames
  for (int iPair = 2;
       iPair--;) {

    // Declare an array to memorize the results of the repeated
    // test on the same pair,
    // to prevent optimization from the compiler to remove the for loop
    bool isIntersectingFMB[NB_REPEAT_2D] = {false};

    // Start measuring time
    struct timeval start;
    gettimeofday(&start, NULL);

    // Run the FMB intersection test
    for (int i = NB_REPEAT_2D;
         i--;) {

      isIntersectingFMB[i] =
```

```
    FMBTestIntersection2DTime(
      that,
      tho,
      NULL);
}

// Stop measuring time
struct timeval stop;
gettimeofday(&stop, NULL);

// Calculate the delay of execution
unsigned long deltausFMB = 0;
if (stop.tv_sec < start.tv_sec) {
  printf("time warps, try again\n");
  exit(0);
}
if (stop.tv_sec > start.tv_sec + 1) {
  printf("deltausFMB >> 1s, decrease NB_REPEAT\n");
  exit(0);
}
if (stop.tv_usec < start.tv_usec) {
  deltausFMB = stop.tv_sec - start.tv_sec;
  deltausFMB += stop.tv_usec + 1000000 - start.tv_usec;
} else {
  deltausFMB = stop.tv_usec - start.tv_usec;
}

// Declare an array to memorize the results of the repeated
// test on the same pair,
// to prevent optimization from the compiler to remove the for loop
bool isIntersectingSAT[NB_REPEAT_2D] = {false};

// Start measuring time
gettimeofday(&start, NULL);

// Run the FMB intersection test
for (int i = NB_REPEAT_2D;
     i--;) {

  isIntersectingSAT[i] =
    SATTestIntersection2DTime(
      that,
      tho);

}

// Stop measuring time
gettimeofday(&stop, NULL);

// Calculate the delay of execution
unsigned long deltausSAT = 0;
if (stop.tv_sec < start.tv_sec) {
  printf("time warps, try again\n");
  exit(0);
}
if (stop.tv_sec > start.tv_sec + 1) {
  printf("deltausSAT >> 1s, decrease NB_REPEAT\n");
  exit(0);
}
if (stop.tv_usec < start.tv_usec) {
  deltausSAT = stop.tv_sec - start.tv_sec;
  deltausSAT += stop.tv_usec + 1000000 - start.tv_usec;
```

```
} else {
  deltausSAT = stop.tv_usec - start.tv_usec;
}

// If the delays are greater than 10ms
if (deltausFMB >= 10 && deltausSAT >= 10) {

  // If FMB and SAT disagrees
  if (isIntersectingFMB[0] != isIntersectingSAT[0]) {

    printf("Qualification has failed\n");
    Frame2DTimePrint(that);
    printf(" against ");
    Frame2DTimePrint(tho);
    printf("\n");
    printf("FMB : ");
    if (isIntersectingFMB[0] == false)
      printf("no ");
    printf("intersection\n");
    printf("SAT : ");
    if (isIntersectingSAT[0] == false)
      printf("no ");
    printf("intersection\n");

    // Stop the qualification test
    exit(0);

  }

  // Get the ratio of execution time
  double ratio = ((double)deltausFMB) / ((double)deltausSAT);

  // If the Frames intersect
  if (isIntersectingSAT[0] == true) {

    // Update the counters
    if (countInter == 0) {

      minInter = ratio;
      maxInter = ratio;

    } else {

      if (minInter > ratio)
        minInter = ratio;
      if (maxInter < ratio)
        maxInter = ratio;

    }
    sumInter += ratio;
    ++countInter;

    if (paramP.type == FrameCuboid &&
        paramQ.type == FrameCuboid) {

      if (countInterCC == 0) {

        minInterCC = ratio;
        maxInterCC = ratio;

      } else {
```

```
    if (minInterCC > ratio)
      minInterCC = ratio;
    if (maxInterCC < ratio)
      maxInterCC = ratio;

  }
  sumInterCC += ratio;
  ++countInterCC;

} else if (paramP.type == FrameCuboid &&
           paramQ.type == FrameTetrahedron) {

  if (countInterCT == 0) {

    minInterCT = ratio;
    maxInterCT = ratio;

  } else {

    if (minInterCT > ratio)
      minInterCT = ratio;
    if (maxInterCT < ratio)
      maxInterCT = ratio;

  }
  sumInterCT += ratio;
  ++countInterCT;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameCuboid) {

  if (countInterTC == 0) {

    minInterTC = ratio;
    maxInterTC = ratio;

  } else {

    if (minInterTC > ratio)
      minInterTC = ratio;
    if (maxInterTC < ratio)
      maxInterTC = ratio;

  }
  sumInterTC += ratio;
  ++countInterTC;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameTetrahedron) {

  if (countInterTT == 0) {

    minInterTT = ratio;
    maxInterTT = ratio;

  } else {

    if (minInterTT > ratio)
      minInterTT = ratio;
    if (maxInterTT < ratio)
      maxInterTT = ratio;
```

```
    }
    sumInterTT += ratio;
    ++countInterTT;

  }

// Else, the Frames do not intersect
} else {

  // Update the counters
  if (countNoInter == 0) {

    minNoInter = ratio;
    maxNoInter = ratio;

  } else {

    if (minNoInter > ratio)
      minNoInter = ratio;
    if (maxNoInter < ratio)
      maxNoInter = ratio;

  }
  sumNoInter += ratio;
  ++countNoInter;

  if (paramP.type == FrameCuboid &&
      paramQ.type == FrameCuboid) {

    if (countNoInterCC == 0) {

      minNoInterCC = ratio;
      maxNoInterCC = ratio;

    } else {

      if (minNoInterCC > ratio)
        minNoInterCC = ratio;
      if (maxNoInterCC < ratio)
        maxNoInterCC = ratio;

    }
    sumNoInterCC += ratio;
    ++countNoInterCC;

  } else if (paramP.type == FrameCuboid &&
             paramQ.type == FrameTetrahedron) {

    if (countNoInterCT == 0) {

      minNoInterCT = ratio;
      maxNoInterCT = ratio;

    } else {

      if (minNoInterCT > ratio)
        minNoInterCT = ratio;
      if (maxNoInterCT < ratio)
        maxNoInterCT = ratio;

    }
    sumNoInterCT += ratio;
```

```
        ++countNoInterCT;

      } else if (paramP.type == FrameTetrahedron &&
                 paramQ.type == FrameCuboid) {

        if (countNoInterTC == 0) {

          minNoInterTC = ratio;
          maxNoInterTC = ratio;

        } else {

          if (minNoInterTC > ratio)
            minNoInterTC = ratio;
          if (maxNoInterTC < ratio)
            maxNoInterTC = ratio;

        }
        sumNoInterTC += ratio;
        ++countNoInterTC;

      } else if (paramP.type == FrameTetrahedron &&
                 paramQ.type == FrameTetrahedron) {

        if (countNoInterTT == 0) {

          minNoInterTT = ratio;
          maxNoInterTT = ratio;

        } else {

          if (minNoInterTT > ratio)
            minNoInterTT = ratio;
          if (maxNoInterTT < ratio)
            maxNoInterTT = ratio;

        }
        sumNoInterTT += ratio;
        ++countNoInterTT;

      }
    }

  // Else, if time of execution for FMB was less than a 10ms
  } else if (deltausFMB < 10) {

    printf("deltausFMB < 10ms, increase NB_REPEAT\n");
    exit(0);

  // Else, if time of execution for SAT was less than a 10ms
  } else if (deltausSAT < 10) {

    printf("deltausSAT < 10ms, increase NB_REPEAT\n");
    exit(0);

  }

  // Flip the pair of Frames
  that = &Q;
  tho = &P;

}
```

```
}

void Qualify2DDynamic(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Open the files to save the results
  FILE* fp = fopen("../Results/qualification2DTime.txt", "w");
  FILE* fpCC = fopen("../Results/qualification2DTimeCC.txt", "w");
  FILE* fpCT = fopen("../Results/qualification2DTimeCT.txt", "w");
  FILE* fpTC = fopen("../Results/qualification2DTimeTC.txt", "w");
  FILE* fpTT = fopen("../Results/qualification2DTimeTT.txt", "w");

  // Loop on runs
  for (int iRun = 0;
       iRun < NB_RUNS;
       ++iRun) {

    // Ratio intersection/no intersection for the displayed results
    double ratioInter = 0.1 + 0.8 * (double)iRun / (double)(NB_RUNS - 1);

    // Initialize counters
    minInter = 0.0;
    maxInter = 0.0;
    sumInter = 0.0;
    countInter = 0;
    minNoInter = 0.0;
    maxNoInter = 0.0;
    sumNoInter = 0.0;
    countNoInter = 0;

    minInterCC = 0.0;
    maxInterCC = 0.0;
    sumInterCC = 0.0;
    countInterCC = 0;
    minNoInterCC = 0.0;
    maxNoInterCC = 0.0;
    sumNoInterCC = 0.0;
    countNoInterCC = 0;

    minInterCT = 0.0;
    maxInterCT = 0.0;
    sumInterCT = 0.0;
    countInterCT = 0;
    minNoInterCT = 0.0;
    maxNoInterCT = 0.0;
    sumNoInterCT = 0.0;
    countNoInterCT = 0;

    minInterTC = 0.0;
    maxInterTC = 0.0;
    sumInterTC = 0.0;
    countInterTC = 0;
    minNoInterTC = 0.0;
    maxNoInterTC = 0.0;
    sumNoInterTC = 0.0;
    countNoInterTC = 0;

    minInterTT = 0.0;
    maxInterTT = 0.0;
```

```
sumInterTT = 0.0;
countInterTT = 0;
minNoInterTT = 0.0;
maxNoInterTT = 0.0;
sumNoInterTT = 0.0;
countNoInterTT = 0;

// Declare two variables to memorize the arguments to the
// Qualification function
Param2DTime paramP;
Param2DTime paramQ;

// Loop on the number of tests
for (unsigned long iTest = NB_TESTS;
     iTest--;) {

  // Create two random Frame definitions
  Param2DTime* param = &paramP;
  for (int iParam = 2;
       iParam--;) {

    // 50% chance of being a Cuboid or a Tetrahedron
    if (rnd() < 0.5)
      param->type = FrameCuboid;
    else
      param->type = FrameTetrahedron;

    for (int iAxis = 2;
         iAxis--;) {

      param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;
      param->speed[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      for (int iComp = 2;
           iComp--;) {

        param->comp[iComp][iAxis] =
          -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      }

    }

    param = &paramQ;

  }

  // Calculate the determinant of the Frames' components matrix

  double detP =
    paramP.comp[0][0] * paramP.comp[1][1] -
    paramP.comp[1][0] * paramP.comp[0][1];

  double detQ =
    paramQ.comp[0][0] * paramQ.comp[1][1] -
    paramQ.comp[1][0] * paramQ.comp[0][1];

  // If the determinants are not null, ie the Frame are not degenerate
  if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

    // Run the validation on the two Frames
    Qualification2DDynamic(
```

```
      paramP,
      paramQ);

  }

}

// Display the results
if (iRun == 0) {

  fprintf(fp, "percPairInter,");
  fprintf(fp, "countInterTo,countNoInterTo,");
  fprintf(fp, "minInterTo,avgInterTo,maxInterTo,");
  fprintf(fp, "minNoInterTo,avgNoInterTo,maxNoInterTo,");
  fprintf(fp, "minTotalTo,avgTotalTo,maxTotalTo\n");

  fprintf(fpCC, "percPairInter,");
  fprintf(fpCC, "countInterCC,countNoInterCC,");
  fprintf(fpCC, "minInterCC,avgInterCC,maxInterCC,");
  fprintf(fpCC, "minNoInterCC,avgNoInterCC,maxNoInterCC,");
  fprintf(fpCC, "minTotalCC,avgTotalCC,maxTotalCC\n");

  fprintf(fpCT, "percPairInter,");
  fprintf(fpCT, "countInterCT,countNoInterCT,");
  fprintf(fpCT, "minInterCT,avgInterCT,maxInterCT,");
  fprintf(fpCT, "minNoInterCT,avgNoInterCT,maxNoInterCT,");
  fprintf(fpCT, "minTotalCT,avgTotalCT,maxTotalCT\n");

  fprintf(fpTC, "percPairInter,");
  fprintf(fpTC, "countInterTC,countNoInterTC,");
  fprintf(fpTC, "minInterTC,avgInterTC,maxInterTC,");
  fprintf(fpTC, "minNoInterTC,avgNoInterTC,maxNoInterTC,");
  fprintf(fpTC, "minTotalTC,avgTotalTC,maxTotalTC\n");

  fprintf(fpTT, "percPairInter,");
  fprintf(fpTT, "countInterTT,countNoInterTT,");
  fprintf(fpTT, "minInterTT,avgInterTT,maxInterTT,");
  fprintf(fpTT, "minNoInterTT,avgNoInterTT,maxNoInterTT,");
  fprintf(fpTT, "minTotalTT,avgTotalTT,maxTotalTT\n");

}

fprintf(fp, "%.1f,", ratioInter);
fprintf(fp, "%lu,%lu,", countInter, countNoInter);
double avgInter = sumInter / (double)countInter;
fprintf(fp, "%f,%f,%f,", minInter, avgInter, maxInter);
double avgNoInter = sumNoInter / (double)countNoInter;
fprintf(fp, "%f,%f,%f,", minNoInter, avgNoInter, maxNoInter);
double avg =
  ratioInter * avgInter + (1.0 - ratioInter) * avgNoInter;
fprintf(fp, "%f,%f,%f",
  (minNoInter < minInter ? minNoInter : minInter),
  avg,
  (maxNoInter > maxInter ? maxNoInter : maxInter));
if (iRun < NB_RUNS - 1) {
  fprintf(fp, "\n");
}

fprintf(fpCC, "%.1f,", ratioInter);
fprintf(fpCC, "%lu,%lu,", countInterCC, countNoInterCC);
double avgInterCC = sumInterCC / (double)countInterCC;
fprintf(fpCC, "%f,%f,%f,", minInterCC, avgInterCC, maxInterCC);
```

```
  double avgNoInterCC = sumNoInterCC / (double)countNoInterCC;
  fprintf(fpCC, "%f,%f,%f,", minNoInterCC, avgNoInterCC, maxNoInterCC);
  double avgCC =
    ratioInter * avgInterCC + (1.0 - ratioInter) * avgNoInterCC;
  fprintf(fpCC, "%f,%f,%f",
    (minNoInterCC < minInterCC ? minNoInterCC : minInterCC),
    avgCC,
    (maxNoInterCC > maxInterCC ? maxNoInterCC : maxInterCC));
  if (iRun < NB_RUNS - 1) {
    fprintf(fpCC, "\n");
  }

  fprintf(fpCT, "%.1f,", ratioInter);
  fprintf(fpCT, "%lu,%lu,", countInterCT, countNoInterCT);
  double avgInterCT = sumInterCT / (double)countInterCT;
  fprintf(fpCT, "%f,%f,%f,", minInterCT, avgInterCT, maxInterCT);
  double avgNoInterCT = sumNoInterCT / (double)countNoInterCT;
  fprintf(fpCT, "%f,%f,%f,", minNoInterCT, avgNoInterCT, maxNoInterCT);
  double avgCT =
    ratioInter * avgInterCT + (1.0 - ratioInter) * avgNoInterCT;
  fprintf(fpCT, "%f,%f,%f",
    (minNoInterCT < minInterCT ? minNoInterCT : minInterCT),
    avgCT,
    (maxNoInterCT > maxInterCT ? maxNoInterCT : maxInterCT));
  if (iRun < NB_RUNS - 1) {
    fprintf(fpCT, "\n");
  }

  fprintf(fpTC, "%.1f,", ratioInter);
  fprintf(fpTC, "%lu,%lu,", countInterTC, countNoInterTC);
  double avgInterTC = sumInterTC / (double)countInterTC;
  fprintf(fpTC, "%f,%f,%f,", minInterTC, avgInterTC, maxInterTC);
  double avgNoInterTC = sumNoInterTC / (double)countNoInterTC;
  fprintf(fpTC, "%f,%f,%f,", minNoInterTC, avgNoInterTC, maxNoInterTC);
  double avgTC =
    ratioInter * avgInterTC + (1.0 - ratioInter) * avgNoInterTC;
  fprintf(fpTC, "%f,%f,%f",
    (minNoInterTC < minInterTC ? minNoInterTC : minInterTC),
    avgTC,
    (maxNoInterTC > maxInterTC ? maxNoInterTC : maxInterTC));
  if (iRun < NB_RUNS - 1) {
    fprintf(fpTC, "\n");
  }

  fprintf(fpTT, "%.1f,", ratioInter);
  fprintf(fpTT, "%lu,%lu,", countInterTT, countNoInterTT);
  double avgInterTT = sumInterTT / (double)countInterTT;
  fprintf(fpTT, "%f,%f,%f,", minInterTT, avgInterTT, maxInterTT);
  double avgNoInterTT = sumNoInterTT / (double)countNoInterTT;
  fprintf(fpTT, "%f,%f,%f,", minNoInterTT, avgNoInterTT, maxNoInterTT);
  double avgTT =
    ratioInter * avgInterTT + (1.0 - ratioInter) * avgNoInterTT;
  fprintf(fpTT, "%f,%f,%f",
    (minNoInterTT < minInterTT ? minNoInterTT : minInterTT),
    avgTT,
    (maxNoInterTT > maxInterTT ? maxNoInterTT : maxInterTT));
  if (iRun < NB_RUNS - 1) {
    fprintf(fpTT, "\n");
  }

}
```

```
  // Close the files
  fclose(fp);
  fclose(fpCC);
  fclose(fpCT);
  fclose(fpTC);
  fclose(fpTT);

}

int main(int argc, char** argv) {

  Qualify2DDynamic();

  return 0;
}
```

8.1.4 3D dynamic

```
// Include standard libraries
#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <time.h>
#include <sys/time.h>

// Include FMB and SAT algorithm library
#include "fmb3dt.h"
#include "sat.h"

// Epsilon to detect degenerated triangles
#define EPSILON 0.1
// Range of values for the random generation of Frames
#define RANGE_AXIS 100.0
// Nb of run
#define NB_RUNS 9
// Nb of tests per run
#define NB_TESTS 100000
// Nb of times the test is run on one pair of frame, used to
// slow down the processus and be able to measure time
#define NB_REPEAT_3D 800

// Helper macro to generate random number in [0.0, 1.0]
#define rnd() (double)(rand())/(double)(RAND_MAX)

// Helper structure to pass arguments to the Qualification function
typedef struct {
  FrameType type;
  double orig[3];
  double comp[3][3];
  double speed[3];
} Param3DTime;

// Global variables to count nb of tests resulting in intersection
// and no intersection, and min/max/total time of execution for each
double minInter;
double maxInter;
double sumInter;
unsigned long countInter;
double minNoInter;
double maxNoInter;
double sumNoInter;
```

```
unsigned long countNoInter;

double minInterCC;
double maxInterCC;
double sumInterCC;
unsigned long countInterCC;
double minNoInterCC;
double maxNoInterCC;
double sumNoInterCC;
unsigned long countNoInterCC;

double minInterCT;
double maxInterCT;
double sumInterCT;
unsigned long countInterCT;
double minNoInterCT;
double maxNoInterCT;
double sumNoInterCT;
unsigned long countNoInterCT;

double minInterTC;
double maxInterTC;
double sumInterTC;
unsigned long countInterTC;
double minNoInterTC;
double maxNoInterTC;
double sumNoInterTC;
unsigned long countNoInterTC;

double minInterTT;
double maxInterTT;
double sumInterTT;
unsigned long countInterTT;
double minNoInterTT;
double maxNoInterTT;
double sumNoInterTT;
unsigned long countNoInterTT;

// Qualification function
// Takes two Frame definition as input, run the intersection test on
// them with FMB and SAT, and measure the time of execution of each
void Qualification3DDynamic(
        const Param3DTime paramP,
        const Param3DTime paramQ) {

  // Create the two Frames
  Frame3DTime P =
    Frame3DTimeCreateStatic(
      paramP.type,
      paramP.orig,
      paramP.speed,
      paramP.comp);

  Frame3DTime Q =
    Frame3DTimeCreateStatic(
      paramQ.type,
      paramQ.orig,
      paramQ.speed,
      paramQ.comp);

  // Helper variables to loop on the pair (that, tho) and (tho, that)
  Frame3DTime* that = &P;
```

```
Frame3DTime* tho = &Q;

// Loop on pairs of Frames
for (int iPair = 2;
     iPair--;) {

  // Declare an array to memorize the results of the repeated
  // test on the same pair,
  // to prevent optimization from the compiler to remove the for loop
  bool isIntersectingFMB[NB_REPEAT_3D] = {false};

  // Start measuring time
  struct timeval start;
  gettimeofday(&start, NULL);

  // Run the FMB intersection test
  for (int i = NB_REPEAT_3D;
       i--;) {

    isIntersectingFMB[i] =
      FMBTestIntersection3DTime(
        that,
        tho,
        NULL);
  }

  // Stop measuring time
  struct timeval stop;
  gettimeofday(&stop, NULL);

  // Calculate the delay of execution
  unsigned long deltausFMB = 0;
  if (stop.tv_sec < start.tv_sec) {
    printf("time warps, try again\n");
    exit(0);
  }
  if (stop.tv_sec > start.tv_sec + 1) {
    printf("deltausFMB >> 1s, decrease NB_REPEAT\n");
    exit(0);
  }
  if (stop.tv_usec < start.tv_usec) {
    deltausFMB = stop.tv_sec - start.tv_sec;
    deltausFMB += stop.tv_usec + 1000000 - start.tv_usec;
  } else {
    deltausFMB = stop.tv_usec - start.tv_usec;
  }

  // Declare an array to memorize the results of the repeated
  // test on the same pair,
  // to prevent optimization from the compiler to remove the for loop
  bool isIntersectingSAT[NB_REPEAT_3D] = {false};

  // Start measuring time
  gettimeofday(&start, NULL);

  // Run the FMB intersection test
  for (int i = NB_REPEAT_3D;
       i--;) {

    isIntersectingSAT[i] =
      SATTestIntersection3DTime(
        that,
```

```
      tho);

}

// Stop measuring time
gettimeofday(&stop, NULL);

// Calculate the delay of execution
unsigned long deltausSAT = 0;
if (stop.tv_sec < start.tv_sec) {
  printf("time warps, try again\n");
  exit(0);
}
if (stop.tv_sec > start.tv_sec + 1) {
  printf("deltausSAT >> 1s, decrease NB_REPEAT\n");
  exit(0);
}
if (stop.tv_usec < start.tv_usec) {
  deltausSAT = stop.tv_sec - start.tv_sec;
  deltausSAT += stop.tv_usec + 1000000 - start.tv_usec;
} else {
  deltausSAT = stop.tv_usec - start.tv_usec;
}

// If the delays are greater than 10ms
if (deltausFMB >= 10 && deltausSAT >= 10) {

  // If FMB and SAT disagrees
  if (isIntersectingFMB[0] != isIntersectingSAT[0]) {

    printf("Qualification has failed\n");
    Frame3DTimePrint(that);
    printf(" against ");
    Frame3DTimePrint(tho);
    printf("\n");
    printf("FMB : ");
    if (isIntersectingFMB[0] == false)
      printf("no ");
    printf("intersection\n");
    printf("SAT : ");
    if (isIntersectingSAT[0] == false)
      printf("no ");
    printf("intersection\n");

    // Stop the qualification test
    exit(0);

  }

  // Get the ratio of execution time
  double ratio = ((double)deltausFMB) / ((double)deltausSAT);

  // If the Frames intersect
  if (isIntersectingSAT[0] == true) {

    // Update the counters
    if (countInter == 0) {

      minInter = ratio;
      maxInter = ratio;

    } else {
```

```
  if (minInter > ratio)
    minInter = ratio;
  if (maxInter < ratio)
    maxInter = ratio;

}
sumInter += ratio;
++countInter;

if (paramP.type == FrameCuboid &&
    paramQ.type == FrameCuboid) {

  if (countInterCC == 0) {

    minInterCC = ratio;
    maxInterCC = ratio;

  } else {

    if (minInterCC > ratio)
      minInterCC = ratio;
    if (maxInterCC < ratio)
      maxInterCC = ratio;

  }
  sumInterCC += ratio;
  ++countInterCC;

} else if (paramP.type == FrameCuboid &&
           paramQ.type == FrameTetrahedron) {

  if (countInterCT == 0) {

    minInterCT = ratio;
    maxInterCT = ratio;

  } else {

    if (minInterCT > ratio)
      minInterCT = ratio;
    if (maxInterCT < ratio)
      maxInterCT = ratio;

  }
  sumInterCT += ratio;
  ++countInterCT;

} else if (paramP.type == FrameTetrahedron &&
           paramQ.type == FrameCuboid) {

  if (countInterTC == 0) {

    minInterTC = ratio;
    maxInterTC = ratio;

  } else {

    if (minInterTC > ratio)
      minInterTC = ratio;
    if (maxInterTC < ratio)
      maxInterTC = ratio;
```

```
    }
    sumInterTC += ratio;
    ++countInterTC;

  } else if (paramP.type == FrameTetrahedron &&
             paramQ.type == FrameTetrahedron) {

    if (countInterTT == 0) {

      minInterTT = ratio;
      maxInterTT = ratio;

    } else {

      if (minInterTT > ratio)
        minInterTT = ratio;
      if (maxInterTT < ratio)
        maxInterTT = ratio;

    }
    sumInterTT += ratio;
    ++countInterTT;

  }

// Else, the Frames do not intersect
} else {

  // Update the counters
  if (countNoInter == 0) {

    minNoInter = ratio;
    maxNoInter = ratio;

  } else {

    if (minNoInter > ratio)
      minNoInter = ratio;
    if (maxNoInter < ratio)
      maxNoInter = ratio;

  }
  sumNoInter += ratio;
  ++countNoInter;

  if (paramP.type == FrameCuboid &&
      paramQ.type == FrameCuboid) {

    if (countNoInterCC == 0) {

      minNoInterCC = ratio;
      maxNoInterCC = ratio;

    } else {

      if (minNoInterCC > ratio)
        minNoInterCC = ratio;
      if (maxNoInterCC < ratio)
        maxNoInterCC = ratio;

    }
```

```
    sumNoInterCC += ratio;
    ++countNoInterCC;

  } else if (paramP.type == FrameCuboid &&
             paramQ.type == FrameTetrahedron) {

    if (countNoInterCT == 0) {

      minNoInterCT = ratio;
      maxNoInterCT = ratio;

    } else {

      if (minNoInterCT > ratio)
        minNoInterCT = ratio;
      if (maxNoInterCT < ratio)
        maxNoInterCT = ratio;

    }
    sumNoInterCT += ratio;
    ++countNoInterCT;

  } else if (paramP.type == FrameTetrahedron &&
             paramQ.type == FrameCuboid) {

    if (countNoInterTC == 0) {

      minNoInterTC = ratio;
      maxNoInterTC = ratio;

    } else {

      if (minNoInterTC > ratio)
        minNoInterTC = ratio;
      if (maxNoInterTC < ratio)
        maxNoInterTC = ratio;

    }
    sumNoInterTC += ratio;
    ++countNoInterTC;

  } else if (paramP.type == FrameTetrahedron &&
             paramQ.type == FrameTetrahedron) {

    if (countNoInterTT == 0) {

      minNoInterTT = ratio;
      maxNoInterTT = ratio;

    } else {

      if (minNoInterTT > ratio)
        minNoInterTT = ratio;
      if (maxNoInterTT < ratio)
        maxNoInterTT = ratio;

    }
    sumNoInterTT += ratio;
    ++countNoInterTT;

  }
}
```

```
    // Else, if time of execution for FMB was less than a 10ms
    } else if (deltausFMB < 10) {

      printf("deltausFMB < 10ms, increase NB_REPEAT\n");
      exit(0);

    // Else, if time of execution for SAT was less than a 10ms
    } else if (deltausSAT < 10) {

      printf("deltausSAT < 10ms, increase NB_REPEAT\n");
      exit(0);

    }

    // Flip the pair of Frames
    that = &Q;
    tho = &P;

  }

}

void Qualify3DDynamic(void) {

  // Initialise the random generator
  srandom(time(NULL));

  // Open the files to save the results
  FILE* fp = fopen("../Results/qualification3DTime.txt", "w");
  FILE* fpCC = fopen("../Results/qualification3DTimeCC.txt", "w");
  FILE* fpCT = fopen("../Results/qualification3DTimeCT.txt", "w");
  FILE* fpTC = fopen("../Results/qualification3DTimeTC.txt", "w");
  FILE* fpTT = fopen("../Results/qualification3DTimeTT.txt", "w");

  // Loop on runs
  for (int iRun = 0;
       iRun < NB_RUNS;
       ++iRun) {

    // Ratio intersection/no intersection for the displayed results
    double ratioInter = 0.1 + 0.8 * (double)iRun / (double)(NB_RUNS - 1);

    // Initialize counters
    minInter = 0.0;
    maxInter = 0.0;
    sumInter = 0.0;
    countInter = 0;
    minNoInter = 0.0;
    maxNoInter = 0.0;
    sumNoInter = 0.0;
    countNoInter = 0;

    minInterCC = 0.0;
    maxInterCC = 0.0;
    sumInterCC = 0.0;
    countInterCC = 0;
    minNoInterCC = 0.0;
    maxNoInterCC = 0.0;
    sumNoInterCC = 0.0;
    countNoInterCC = 0;
```

```
minInterCT = 0.0;
maxInterCT = 0.0;
sumInterCT = 0.0;
countInterCT = 0;
minNoInterCT = 0.0;
maxNoInterCT = 0.0;
sumNoInterCT = 0.0;
countNoInterCT = 0;

minInterTC = 0.0;
maxInterTC = 0.0;
sumInterTC = 0.0;
countInterTC = 0;
minNoInterTC = 0.0;
maxNoInterTC = 0.0;
sumNoInterTC = 0.0;
countNoInterTC = 0;

minInterTT = 0.0;
maxInterTT = 0.0;
sumInterTT = 0.0;
countInterTT = 0;
minNoInterTT = 0.0;
maxNoInterTT = 0.0;
sumNoInterTT = 0.0;
countNoInterTT = 0;

// Declare two variables to memorize the arguments to the
// Qualification function
Param3DTime paramP;
Param3DTime paramQ;

// Loop on the number of tests
for (unsigned long iTest = NB_TESTS;
     iTest--;) {

  // Create two random Frame definitions
  Param3DTime* param = &paramP;
  for (int iParam = 2;
       iParam--;) {

    // 50% chance of being a Cuboid or a Tetrahedron
    if (rnd() < 0.5)
      param->type = FrameCuboid;
    else
      param->type = FrameTetrahedron;

    for (int iAxis = 3;
         iAxis--;) {

      param->orig[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;
      param->speed[iAxis] = -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      for (int iComp = 3;
           iComp--;) {

        param->comp[iComp][iAxis] =
          -RANGE_AXIS + 2.0 * rnd() * RANGE_AXIS;

      }

    }
```

```
    param = &paramQ;

  }

  // Calculate the determinant of the Frames' components matrix

double detP =
  paramP.comp[0][0] * (paramP.comp[1][1] * paramP.comp[2][2]-
  paramP.comp[1][2] * paramP.comp[2][1]) -
  paramP.comp[1][0] * (paramP.comp[0][1] * paramP.comp[2][2]-
  paramP.comp[0][2] * paramP.comp[2][1]) +
  paramP.comp[2][0] * (paramP.comp[0][1] * paramP.comp[1][2]-
  paramP.comp[0][2] * paramP.comp[1][1]);

double detQ =
  paramQ.comp[0][0] * (paramQ.comp[1][1] * paramQ.comp[2][2]-
  paramQ.comp[1][2] * paramQ.comp[2][1]) -
  paramQ.comp[1][0] * (paramQ.comp[0][1] * paramQ.comp[2][2]-
  paramQ.comp[0][2] * paramQ.comp[2][1]) +
  paramQ.comp[2][0] * (paramQ.comp[0][1] * paramQ.comp[1][2]-
  paramQ.comp[0][2] * paramQ.comp[1][1]);

  // If the determinants are not null, ie the Frame are not degenerate
  if (fabs(detP) > EPSILON && fabs(detQ) > EPSILON) {

    // Run the validation on the two Frames
    Qualification3DDynamic(
      paramP,
      paramQ);

  }

}

// Display the results
if (iRun == 0) {

  fprintf(fp, "percPairInter,");
  fprintf(fp, "countInterTo,countNoInterTo,");
  fprintf(fp, "minInterTo,avgInterTo,maxInterTo,");
  fprintf(fp, "minNoInterTo,avgNoInterTo,maxNoInterTo,");
  fprintf(fp, "minTotalTo,avgTotalTo,maxTotalTo\n");

  fprintf(fpCC, "percPairInter,");
  fprintf(fpCC, "countInterCC,countNoInterCC,");
  fprintf(fpCC, "minInterCC,avgInterCC,maxInterCC,");
  fprintf(fpCC, "minNoInterCC,avgNoInterCC,maxNoInterCC,");
  fprintf(fpCC, "minTotalCC,avgTotalCC,maxTotalCC\n");

  fprintf(fpCT, "percPairInter,");
  fprintf(fpCT, "countInterCT,countNoInterCT,");
  fprintf(fpCT, "minInterCT,avgInterCT,maxInterCT,");
  fprintf(fpCT, "minNoInterCT,avgNoInterCT,maxNoInterCT,");
  fprintf(fpCT, "minTotalCT,avgTotalCT,maxTotalCT\n");

  fprintf(fpTC, "percPairInter,");
  fprintf(fpTC, "countInterTC,countNoInterTC,");
  fprintf(fpTC, "minInterTC,avgInterTC,maxInterTC,");
  fprintf(fpTC, "minNoInterTC,avgNoInterTC,maxNoInterTC,");
  fprintf(fpTC, "minTotalTC,avgTotalTC,maxTotalTC\n");
```

```
  fprintf(fpTT, "percPairInter,");
  fprintf(fpTT, "countInterTT,countNoInterTT,");
  fprintf(fpTT, "minInterTT,avgInterTT,maxInterTT,");
  fprintf(fpTT, "minNoInterTT,avgNoInterTT,maxNoInterTT,");
  fprintf(fpTT, "minTotalTT,avgTotalTT,maxTotalTT\n");

}

fprintf(fp, "%.1f,", ratioInter);
fprintf(fp, "%lu,%lu,", countInter, countNoInter);
double avgInter = sumInter / (double)countInter;
fprintf(fp, "%f,%f,%f,", minInter, avgInter, maxInter);
double avgNoInter = sumNoInter / (double)countNoInter;
fprintf(fp, "%f,%f,%f,", minNoInter, avgNoInter, maxNoInter);
double avg =
  ratioInter * avgInter + (1.0 - ratioInter) * avgNoInter;
fprintf(fp, "%f,%f,%f",
  (minNoInter < minInter ? minNoInter : minInter),
  avg,
  (maxNoInter > maxInter ? maxNoInter : maxInter));
if (iRun < NB_RUNS - 1) {
  fprintf(fp, "\n");
}

fprintf(fpCC, "%.1f,", ratioInter);
fprintf(fpCC, "%lu,%lu,", countInterCC, countNoInterCC);
double avgInterCC = sumInterCC / (double)countInterCC;
fprintf(fpCC, "%f,%f,%f,", minInterCC, avgInterCC, maxInterCC);
double avgNoInterCC = sumNoInterCC / (double)countNoInterCC;
fprintf(fpCC, "%f,%f,%f,", minNoInterCC, avgNoInterCC, maxNoInterCC);
double avgCC =
  ratioInter * avgInterCC + (1.0 - ratioInter) * avgNoInterCC;
fprintf(fpCC, "%f,%f,%f",
  (minNoInterCC < minInterCC ? minNoInterCC : minInterCC),
  avgCC,
  (maxNoInterCC > maxInterCC ? maxNoInterCC : maxInterCC));
if (iRun < NB_RUNS - 1) {
  fprintf(fpCC, "\n");
}

fprintf(fpCT, "%.1f,", ratioInter);
fprintf(fpCT, "%lu,%lu,", countInterCT, countNoInterCT);
double avgInterCT = sumInterCT / (double)countInterCT;
fprintf(fpCT, "%f,%f,%f,", minInterCT, avgInterCT, maxInterCT);
double avgNoInterCT = sumNoInterCT / (double)countNoInterCT;
fprintf(fpCT, "%f,%f,%f,", minNoInterCT, avgNoInterCT, maxNoInterCT);
double avgCT =
  ratioInter * avgInterCT + (1.0 - ratioInter) * avgNoInterCT;
fprintf(fpCT, "%f,%f,%f",
  (minNoInterCT < minInterCT ? minNoInterCT : minInterCT),
  avgCT,
  (maxNoInterCT > maxInterCT ? maxNoInterCT : maxInterCT));
if (iRun < NB_RUNS - 1) {
  fprintf(fpCT, "\n");
}

fprintf(fpTC, "%.1f,", ratioInter);
fprintf(fpTC, "%lu,%lu,", countInterTC, countNoInterTC);
double avgInterTC = sumInterTC / (double)countInterTC;
fprintf(fpTC, "%f,%f,%f,", minInterTC, avgInterTC, maxInterTC);
double avgNoInterTC = sumNoInterTC / (double)countNoInterTC;
fprintf(fpTC, "%f,%f,%f,", minNoInterTC, avgNoInterTC, maxNoInterTC);
```

```
    double avgTC =
      ratioInter * avgInterTC + (1.0 - ratioInter) * avgNoInterTC;
    fprintf(fpTC, "%f,%f,%f",
      (minNoInterTC < minInterTC ? minNoInterTC : minInterTC),
      avgTC,
      (maxNoInterTC > maxInterTC ? maxNoInterTC : maxInterTC));
    if (iRun < NB_RUNS - 1) {
      fprintf(fpTC, "\n");
    }

    fprintf(fpTT, "%.1f,", ratioInter);
    fprintf(fpTT, "%lu,%lu,", countInterTT, countNoInterTT);
    double avgInterTT = sumInterTT / (double)countInterTT;
    fprintf(fpTT, "%f,%f,%f,", minInterTT, avgInterTT, maxInterTT);
    double avgNoInterTT = sumNoInterTT / (double)countNoInterTT;
    fprintf(fpTT, "%f,%f,%f,", minNoInterTT, avgNoInterTT, maxNoInterTT);
    double avgTT =
      ratioInter * avgInterTT + (1.0 - ratioInter) * avgNoInterTT;
    fprintf(fpTT, "%f,%f,%f",
      (minNoInterTT < minInterTT ? minNoInterTT : minInterTT),
      avgTT,
      (maxNoInterTT > maxInterTT ? maxNoInterTT : maxInterTT));
    if (iRun < NB_RUNS - 1) {
      fprintf(fpTT, "\n");
    }

  }

  // Close the files
  fclose(fp);
  fclose(fpCC);
  fclose(fpCT);
  fclose(fpTC);
  fclose(fpTT);

}

int main(int argc, char** argv) {

  Qualify3DDynamic();

  return 0;
}
```

8.2 Results

In this subsection I give the results of the qualification for each case. These results are commented in the next section.

8.2.1 2D static

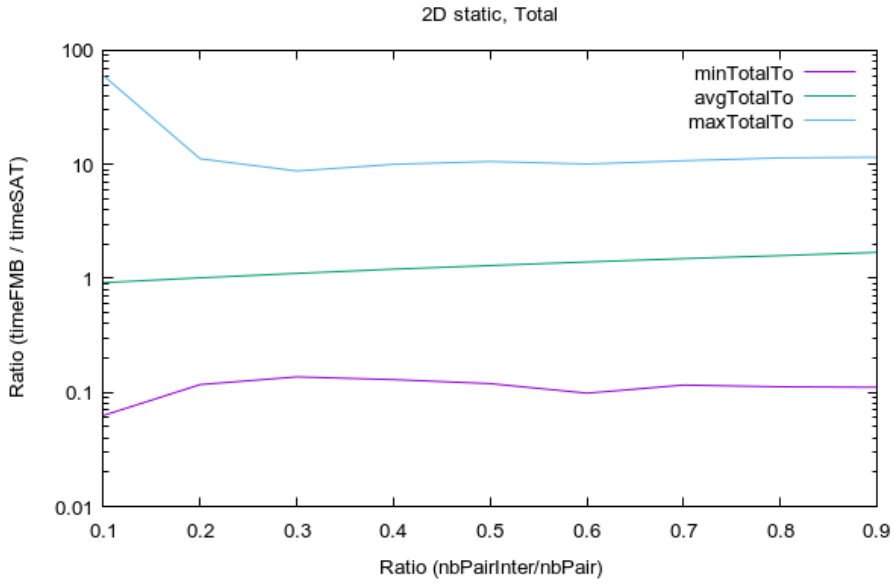
percPairInter	countInterTo	countNoInterTo	minInterTo	avgInterTo	maxInterTo	minNoInterTo	avgNoInterTo	maxNoInterTo	minTotalTo	avgTotalTo	maxTotalTo
0.1	46908	153086	0.261574	1.784204	4.696429	0.062864	0.820909	59.937500	0.062864	0.917238	59.937500
0.2	47114	152882	0.514851	1.783348	4.433333	0.117117	0.819167	11.187500	0.117117	1.012003	11.187500
0.3	46902	153082	0.491429	1.782061	4.622642	0.136842	0.817004	8.750000	0.136842	1.106521	8.750000
0.4	47600	152392	0.535484	1.784899	4.981481	0.129630	0.818987	10.000000	0.129630	1.205352	10.000000
0.5	47016	152976	0.472222	1.779991	5.763636	0.119565	0.814045	10.588235	0.119565	1.297018	10.588235
0.6	46740	153252	0.494186	1.779212	6.759259	0.098485	0.817463	10.062500	0.098485	1.394512	10.062500
0.7	46674	153314	0.562092	1.777920	5.327273	0.116071	0.818380	10.750000	0.116071	1.490058	10.750000
0.8	46832	153162	0.567742	1.777597	4.905660	0.112069	0.818756	11.400000	0.112069	1.585829	11.400000
0.9	47022	152972	0.562092	1.789779	6.068966	0.111111	0.822639	11.533333	0.111111	1.693065	11.533333

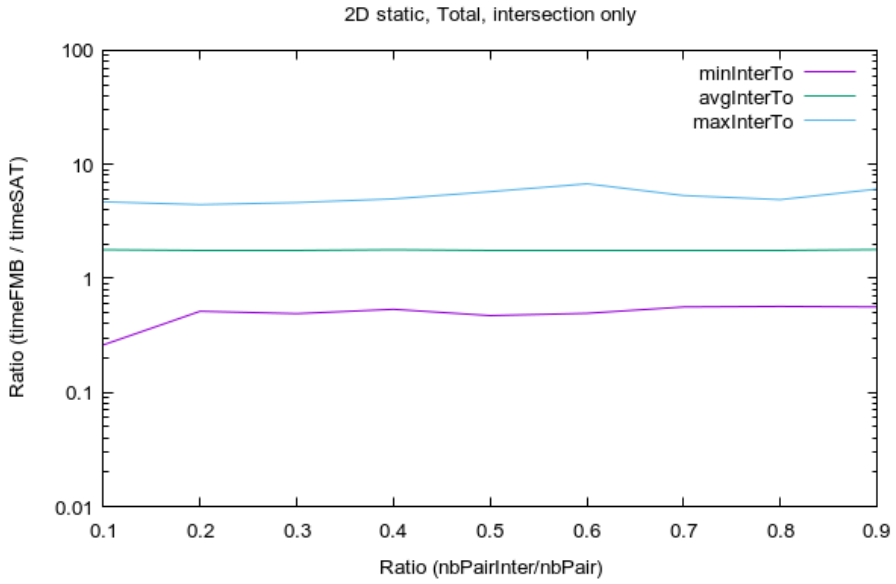
percPairInter	countInterCC	countNoInterCC	minInterCC	avgInterCC	maxInterCC	minNoInterCC	avgNoInterCC	maxNoInterCC	minTotalCC	avgTotalCC	maxTotalCC
0.1	13228	36656	1.000000	2.350122	4.696429	0.153846	0.781352	6.102564	0.153846	0.938229	6.102564
0.2	13398	36946	0.745098	2.349057	4.392157	0.162162	0.776662	5.925000	0.162162	1.091141	5.925000
0.3	13312	36774	0.913669	2.346674	4.622642	0.164557	0.784726	5.825000	0.164557	1.253310	5.825000
0.4	13502	36748	0.815789	2.351671	4.981481	0.149425	0.776886	4.951220	0.149425	1.406800	4.981481
0.5	13270	37046	0.992063	2.347603	5.763636	0.175676	0.775734	4.225000	0.175676	1.561669	5.763636
0.6	13224	36538	0.910256	2.348327	6.759259	0.098485	0.779977	5.300000	0.098485	1.720987	6.759259
0.7	13010	37504	1.050420	2.347744	5.327273	0.180556	0.775275	4.820513	0.180556	1.876004	5.327273
0.8	12858	36754	1.159420	2.352292	4.905660	0.112069	0.776650	4.900000	0.112069	2.037163	4.905660
0.9	13242	36952	0.838926	2.356564	6.068966	0.158537	0.783127	5.073171	0.158537	2.199220	6.068966

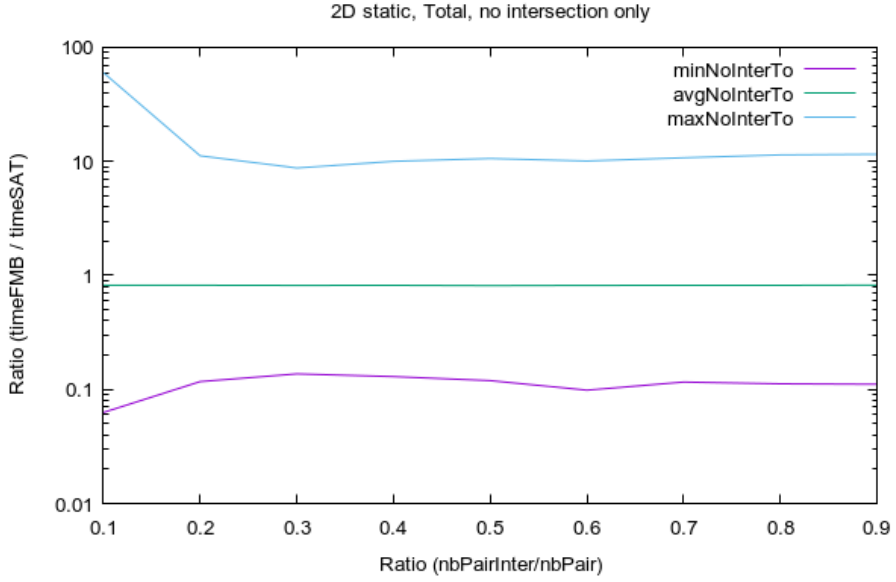
percPairInter	countInterCT	countNoInterCT	minInterCT	avgInterCT	maxInterCT	minNoInterCT	avgNoInterCT	maxNoInterCT	minTotalCT	avgTotalCT	maxTotalCT
0.1	11696	38350	0.831933	1.721398	4.380952	0.138298	0.823503	12.875000	0.138298	0.913293	12.875000
0.2	11550	38088	0.792000	1.718392	4.310345	0.140000	0.824508	8.944444	0.140000	1.003285	8.944444
0.3	11590	37950	0.649351	1.717648	3.919355	0.141304	0.822942	8.250000	0.141304	1.091354	8.250000
0.4	11774	38152	0.832061	1.718451	3.525424	0.129630	0.822479	10.000000	0.129630	1.180868	10.000000
0.5	11784	38266	0.557895	1.715018	3.448276	0.119565	0.817656	9.000000	0.119565	1.266337	9.000000
0.6	11608	38122	0.805970	1.715515	4.300000	0.146341	0.820760	6.733333	0.146341	1.357613	6.733333
0.7	11566	37884	0.666667	1.717445	3.694915	0.116071	0.828445	8.588235	0.116071	1.450745	8.588235
0.8	12032	37910	0.788618	1.717513	4.032258	0.132530	0.824617	8.411765	0.132530	1.538934	8.411765
0.9	11888	38392	0.690323	1.724862	3.786885	0.111111	0.827724	9.062500	0.111111	1.635148	9.062500

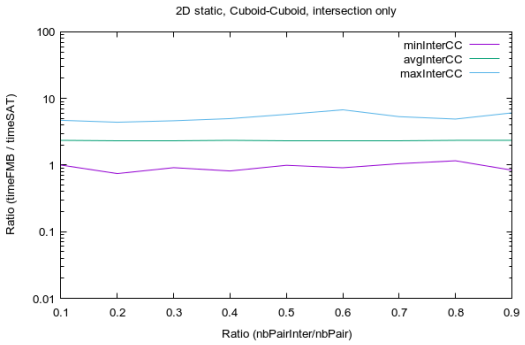
percPairInter	countInterTC	countNoInterTC	minInterTC	avgInterTC	maxInterTC	minNoInterTC	avgNoInterTC	maxNoInterTC	minTotalTC	avgTotalTC	maxTotalTC
0.1	11772	38134	0.261574	1.713987	3.262295	0.062864	0.825331	7.588235	0.062864	0.914197	7.588235
0.2	11800	37952	0.645161	1.713030	4.433333	0.117117	0.825959	11.187500	0.117117	1.003373	11.187500
0.3	11752	38772	0.706250	1.710911	3.666667	0.137931	0.820011	6.800000	0.137931	1.087281	6.800000
0.4	11960	38268	0.815126	1.713402	3.912281	0.176471	0.829550	9.000000	0.176471	1.183091	9.000000
0.5	11696	38288	0.692308	1.710117	4.827586	0.164179	0.822350	6.750000	0.164179	1.266233	6.750000
0.6	11624	39128	0.769231	1.708484	4.700000	0.148936	0.826731	10.062500	0.148936	1.355783	10.062500
0.7	11784	37952	0.746154	1.710892	3.230000	0.149425	0.821293	10.750000	0.149425	1.444012	10.750000
0.8	11732	38678	0.763359	1.710142	4.406780	0.113043	0.828724	11.400000	0.113043	1.533858	11.400000
0.9	11742	37856	0.592814	1.718819	4.322034	0.156627	0.830763	11.533333	0.156627	1.630013	11.533333

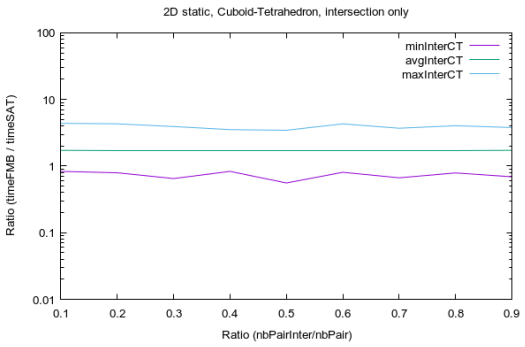
percPairInter	countInterTT	countNoInterTT	minInterTT	avgInterTT	maxInterTT	minNoInterTT	avgNoInterTT	maxNoInterTT	minTotalTT	avgTotalTT	maxTotalTT
0.1	10212	39946	0.527174	1.204025	2.375000	0.115044	0.850495	59.937500	0.115044	0.885848	59.937500
0.2	10366	39896	0.514851	1.204596	2.617647	0.144444	0.846968	8.933333	0.144444	0.918493	8.933333
0.3	10248	39586	0.491429	1.203077	3.130435	0.136842	0.838350	8.750000	0.136842	0.947768	8.750000
0.4	10364	39224	0.535484	1.204518	2.785714	0.134146	0.844729	9.666667	0.134146	0.988644	9.666667
0.5	10266	39376	0.472222	1.200474	3.149254	0.158537	0.838503	10.588235	0.158537	1.019488	10.588235
0.6	10284	39464	0.494186	1.199239	2.814286	0.122449	0.839796	6.000000	0.122449	1.055461	6.000000
0.7	10314	39974	0.562092	1.203547	3.376812	0.146067	0.846517	6.071429	0.146067	1.096438	6.071429
0.8	10210	39820	0.567742	1.202171	2.666667	0.150685	0.842357	7.533333	0.150685	1.130208	7.533333
0.9	10150	39772	0.562092	1.208458	3.233333	0.141026	0.846706	9.133333	0.141026	1.172283	9.133333

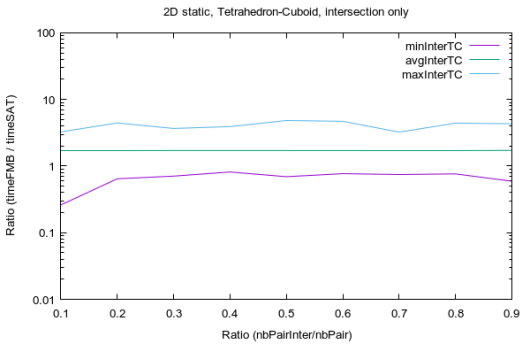

2D static, Total
minTotalTo
avgTotalTo
maxTotalTo
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

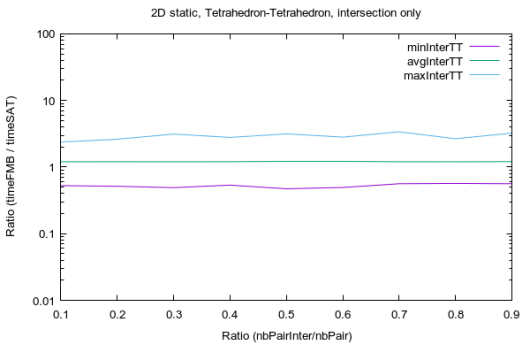

2D static, Total, intersection only
minInterTo
avgInterTo
maxInterTo
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

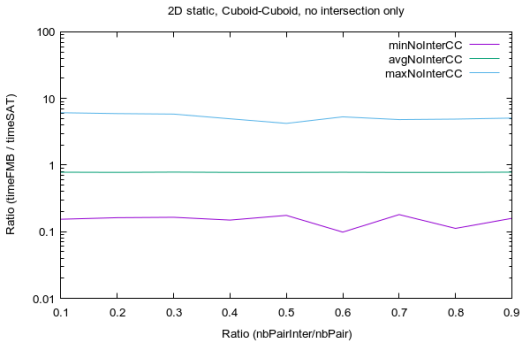

2D static, Total, no intersection only
minNoInterTo
avgNoInterTo
maxNoInterTo
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

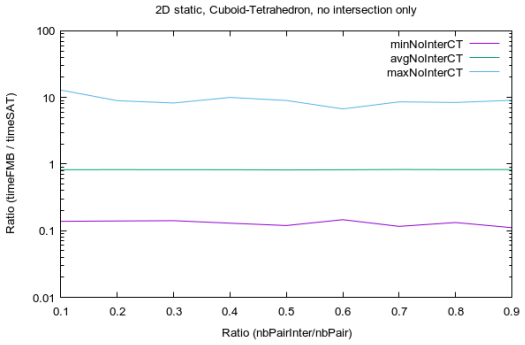

2D static, Cuboid-Cuboid, intersection only
minInterCC
avgInterCC
maxInterCC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

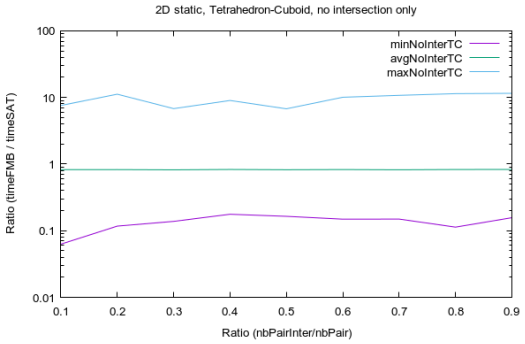

2D static, Cuboid-Tetrahedron, intersection only
minInterCT
avgInterCT
maxInterCT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

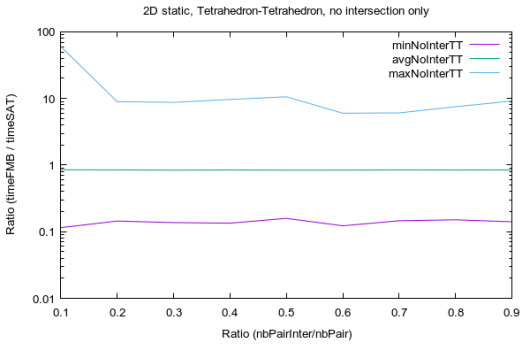

2D static, Tetrahedron-Cuboid, intersection only
minInterTC
avgInterTC
maxInterTC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


2D static, Tetrahedron-Tetrahedron, intersection only
minInterTT
avgInterTT
maxInterTT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


2D static, Cuboid-Cuboid, no intersection only
minNoInterCC
avgNoInterCC
maxNoInterCC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)


2D static, Cuboid-Tetrahedron, no intersection only
minNoInterCT
avgNoInterCT
maxNoInterCT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)


2D static, Tetrahedron-Cuboid, no intersection only
minNoInterTC
avgNoInterTC
maxNoInterTC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)


2D static, Tetrahedron-Tetrahedron, no intersection only
minNoInterTT
avgNoInterTT
maxNoInterTT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)

8.2.2 3D static

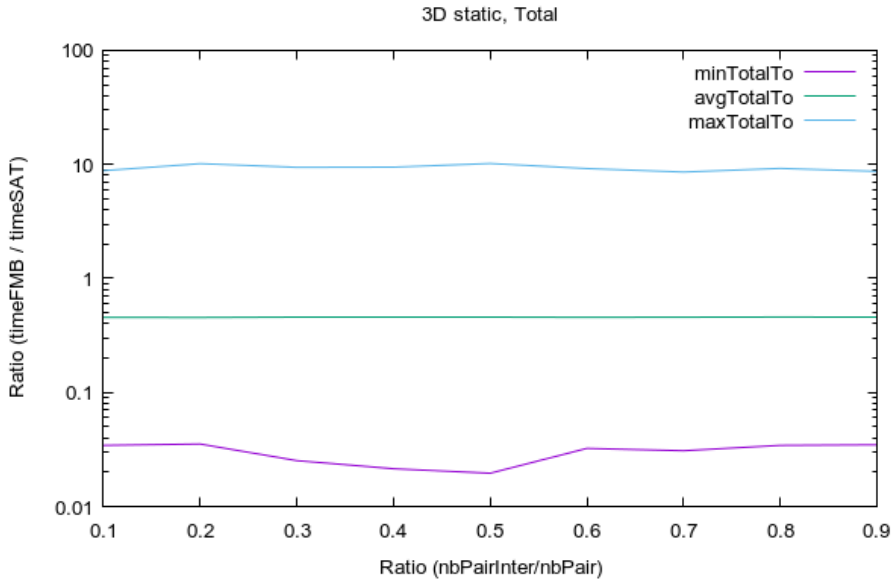
percPairInter	countInterTo	countNoInterTo	minInterTo	avgInterTo	maxInterTo	minNoInterTo	avgNoInterTo	maxNoInterTo	minTotalTo	avgTotalTo	maxTotalTo
0.1	31380	168620	0.074945	0.455603	1.342020	0.034351	0.454517	8.805556	0.034351	0.454626	8.805556
0.2	31316	168684	0.128555	0.456563	1.166937	0.035294	0.453217	10.142857	0.035294	0.453886	10.142857
0.3	31896	168104	0.080882	0.456667	1.558917	0.025225	0.454637	9.416667	0.025225	0.455246	9.416667
0.4	31878	168122	0.053959	0.458715	1.573770	0.021433	0.452469	9.461538	0.021433	0.454967	9.461538
0.5	31350	168650	0.092140	0.458961	1.330579	0.019582	0.455888	10.184211	0.019582	0.457425	10.184211
0.6	31710	168290	0.092078	0.457335	1.238311	0.032330	0.452852	9.184211	0.032330	0.455542	9.184211
0.7	31218	168782	0.080134	0.458905	1.594203	0.030806	0.452355	8.567568	0.030806	0.456940	8.567568
0.8	31720	168280	0.076980	0.459526	1.268571	0.034356	0.453193	9.214286	0.034356	0.458260	9.214286
0.9	32048	167952	0.158805	0.458476	1.037288	0.034704	0.452957	8.685714	0.034704	0.457925	8.685714

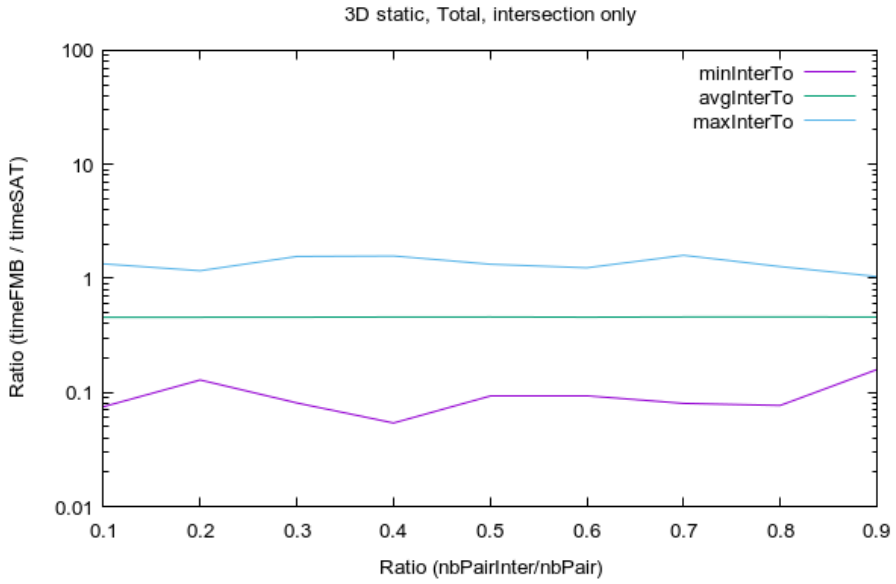
percPairInter	countInterCC	countNoInterCC	minInterCC	avgInterCC	maxInterCC	minNoInterCC	avgNoInterCC	maxNoInterCC	minTotalCC	avgTotalCC	maxTotalCC
0.1	10358	39182	0.181453	0.687653	1.342020	0.039078	0.329691	3.860759	0.039078	0.365487	3.860759
0.2	10408	39316	0.466447	0.686728	1.166937	0.059072	0.329587	3.000000	0.059072	0.401015	3.000000
0.3	10614	39130	0.196906	0.686527	1.558917	0.035194	0.327984	3.706186	0.035194	0.435547	3.706186
0.4	10914	39372	0.204885	0.684934	1.573770	0.034884	0.327088	4.316092	0.034884	0.470227	4.316092
0.5	10604	39234	0.200297	0.687307	1.330579	0.040945	0.329487	3.376471	0.040945	0.508397	3.376471
0.6	10700	39690	0.225341	0.686725	1.238311	0.052288	0.326763	2.976048	0.052288	0.542741	2.976048
0.7	10422	39406	0.234592	0.687736	1.594203	0.039422	0.328036	3.965174	0.039422	0.579826	3.965174
0.8	10708	39312	0.222162	0.687827	1.268571	0.050336	0.327354	2.993789	0.050336	0.615732	2.993789
0.9	10916	39472	0.455457	0.685922	1.037288	0.058586	0.327263	2.621795	0.058586	0.650056	2.621795

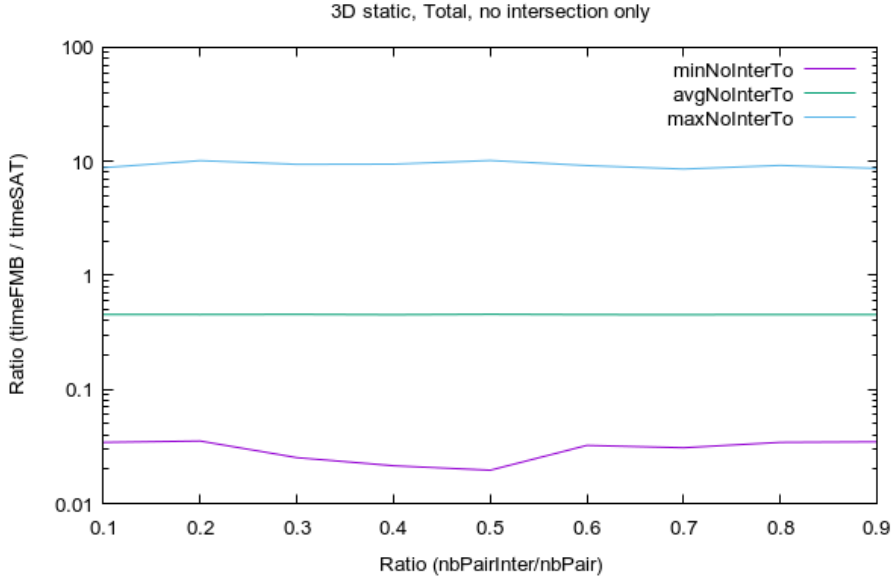
percPairInter	countInterCT	countNoInterCT	minInterCT	avgInterCT	maxInterCT	minNoInterCT	avgNoInterCT	maxNoInterCT	minTotalCT	avgTotalCT	maxTotalCT
0.1	7892	42144	0.111171	0.385748	0.869973	0.036816	0.439313	8.805556	0.036816	0.433957	8.805556
0.2	7828	41916	0.128555	0.384665	0.766169	0.040119	0.438566	8.400000	0.040119	0.427786	8.400000
0.3	7892	42084	0.125253	0.384026	0.857330	0.025225	0.439999	9.416667	0.025225	0.423207	9.416667
0.4	7852	41620	0.113048	0.384068	0.813333	0.034973	0.439404	9.461538	0.034973	0.417269	9.461538
0.5	7650	41696	0.134466	0.385329	0.816327	0.040059	0.440968	8.230769	0.040059	0.413149	8.230769
0.6	7770	42038	0.147664	0.384642	0.835264	0.037227	0.441871	8.171429	0.037227	0.407534	8.171429
0.7	7910	41898	0.127382	0.385705	0.833114	0.040000	0.438432	8.567568	0.040000	0.401523	8.567568
0.8	7968	42468	0.120470	0.385882	0.888466	0.039092	0.440890	8.594595	0.039092	0.396884	8.594595
0.9	7948	42382	0.248521	0.384656	0.748344	0.040299	0.439930	8.685714	0.040299	0.390184	8.685714

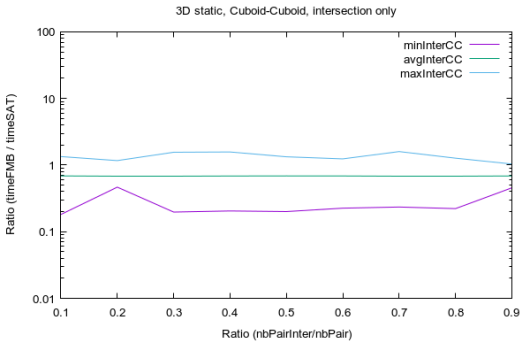
percPairInter	countInterTC	countNoInterTC	minInterTC	avgInterTC	maxInterTC	minNoInterTC	avgNoInterTC	maxNoInterTC	minTotalTC	avgTotalTC	maxTotalTC
0.1	7754	41854	0.113238	0.385359	0.797980	0.038889	0.439189	8.405405	0.038889	0.433806	8.405405
0.2	7888	42468	0.220966	0.384704	0.650773	0.040503	0.440610	8.138889	0.040503	0.429429	8.138889
0.3	8156	42382	0.087447	0.384505	0.910828	0.033835	0.442158	8.194444	0.033835	0.424862	8.194444
0.4	7810	41968	0.078320	0.385179	1.001274	0.021433	0.441748	8.189189	0.021433	0.419121	8.189189
0.5	7900	42004	0.123504	0.385303	0.725955	0.039773	0.444273	10.184211	0.039773	0.414788	10.184211
0.6	7824	42068	0.159537	0.385241	0.695373	0.032330	0.441664	9.184211	0.032330	0.407810	9.184211
0.7	7892	42588	0.080134	0.385825	1.090196	0.038811	0.441218	8.083333	0.038811	0.402442	8.083333
0.8	7852	42418	0.196043	0.386038	0.928082	0.034974	0.442981	8.222222	0.034974	0.397426	8.222222
0.9	7820	41746	0.249392	0.384809	0.537353	0.040230	0.439429	8.138889	0.040230	0.390271	8.138889

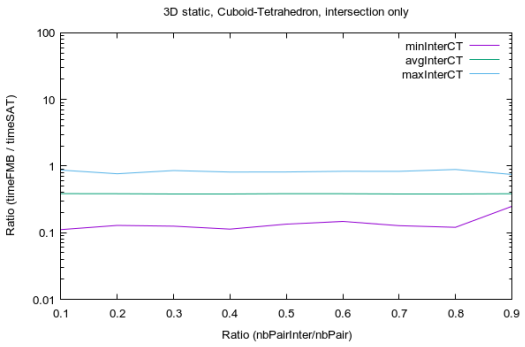
percPairInter	countInterTT	countNoInterTT	minInterTT	avgInterTT	maxInterTT	minNoInterTT	avgNoInterTT	maxNoInterTT	minTotalTT	avgTotalTT	maxTotalTT
0.1	5376	45440	0.074945	0.212375	0.457627	0.034351	0.590372	8.285714	0.034351	0.552572	8.285714
0.2	5192	44984	0.142740	0.212747	0.357062	0.035294	0.586822	10.142857	0.035294	0.512007	10.142857
0.3	5234	44508	0.080882	0.212517	0.540066	0.027211	0.591707	8.964286	0.027211	0.477950	8.964286
0.4	5302	45162	0.053959	0.211920	0.492912	0.032209	0.583777	8.500000	0.032209	0.435034	8.500000
0.5	5196	45716	0.092140	0.213350	0.373695	0.019582	0.588647	7.464286	0.019582	0.400999	7.464286
0.6	5416	44494	0.092078	0.212578	0.449945	0.034749	0.586280	7.481481	0.034749	0.362059	7.481481
0.7	4994	44890	0.087079	0.212786	0.401117	0.030806	0.585048	8.321429	0.030806	0.324464	8.321429
0.8	5192	44082	0.076980	0.212836	0.437030	0.034356	0.587095	9.214286	0.034356	0.287687	9.214286
0.9	5364	44352	0.158805	0.212393	0.244094	0.034704	0.590003	7.481481	0.034704	0.250154	7.481481

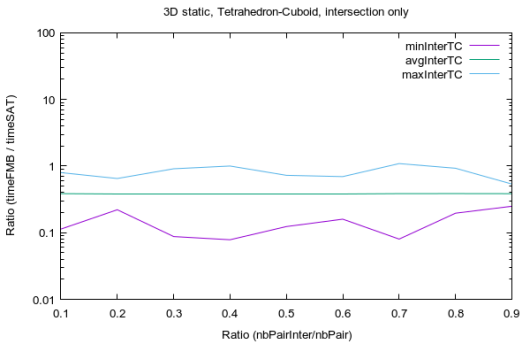

3D static, Total
minTotalTo
avgTotalTo
maxTotalTo
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

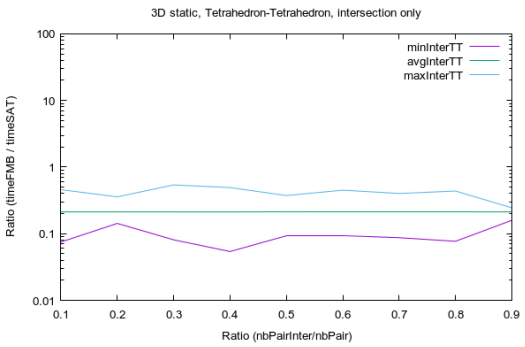

3D static, Total, intersection only
minInterTo
avgInterTo
maxInterTo
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

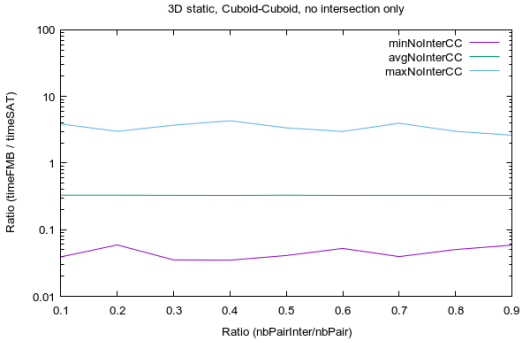

3D static, Total, no intersection only
minNoInterTo
avgNoInterTo
maxNoInterTo
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

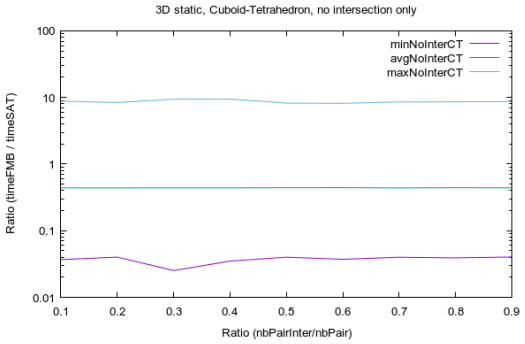

3D static, Cuboid-Cuboid, intersection only
minInterCC
avgInterCC
maxInterCC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

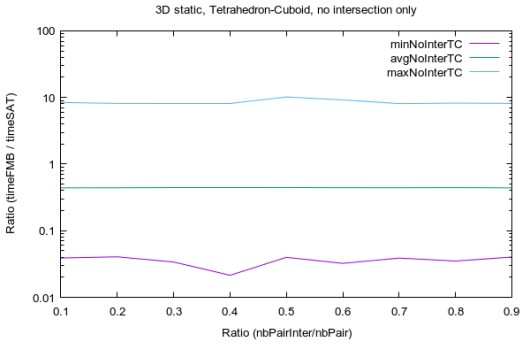

3D static, Cuboid-Tetrahedron, intersection only
minInterCT
avgInterCT
maxInterCT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

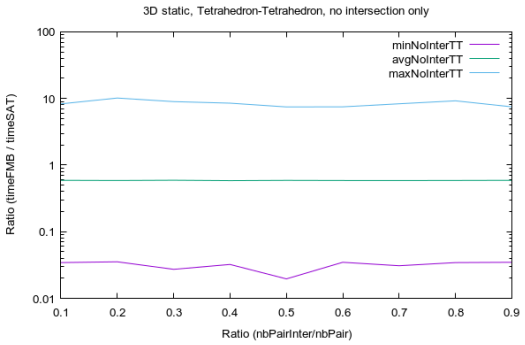

3D static, Tetrahedron-Cuboid, intersection only
minInterTC
avgInterTC
maxInterTC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


3D static, Tetrahedron-Tetrahedron, intersection only
minInterTT
avgInterTT
maxInterTT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


3D static, Cuboid-Cuboid, no intersection only
minNoInterCC
avgNoInterCC
maxNoInterCC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


3D static, Cuboid-Tetrahedron, no intersection only
minNoInterCT
avgNoInterCT
maxNoInterCT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


3D static, Tetrahedron-Cuboid, no intersection only
minNoInterTC
avgNoInterTC
maxNoInterTC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


3D static, Tetrahedron-Tetrahedron, no intersection only
minNoInterTT
avgNoInterTT
maxNoInterTT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

8.2.3 2D dynamic

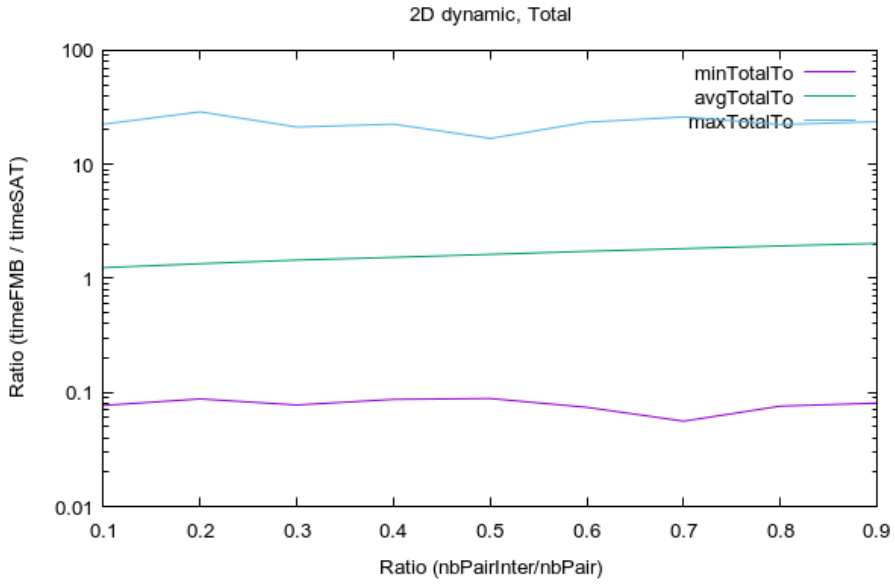
percPairInter	countInterTo	countNoInterTo	minInterTo	avgInterTo	maxInterTo	minNoInterTo	avgNoInterTo	maxNoInterTo	minTotalTo	avgTotalTo	maxTotalTo
0.1	74190	125810	0.735602	2.119511	5.905797	0.076923	1.144130	22.516129	0.076923	1.241668	22.516129
0.2	74610	125384	0.788462	2.119238	5.439759	0.087805	1.150817	28.884615	0.087805	1.344501	28.884615
0.3	74658	125340	0.735437	2.114518	5.741497	0.077626	1.163046	21.230769	0.077626	1.448488	21.230769
0.4	74658	125336	0.776504	2.113837	4.701493	0.086957	1.141893	22.520000	0.086957	1.530670	22.520000
0.5	74876	125110	0.747475	2.108696	5.167939	0.088542	1.143778	16.875000	0.088542	1.626237	16.875000
0.6	74418	125570	0.698895	2.114303	5.180451	0.074074	1.153212	23.413793	0.074074	1.729867	23.413793
0.7	74422	125568	0.710396	2.117589	9.592965	0.055970	1.140577	26.076923	0.055970	1.824485	26.076923
0.8	74854	125138	0.715493	2.122221	5.022727	0.075829	1.145291	22.321429	0.075829	1.926835	22.321429
0.9	74624	125366	0.660574	2.121552	5.218045	0.080569	1.148223	23.629630	0.080569	2.024219	23.629630

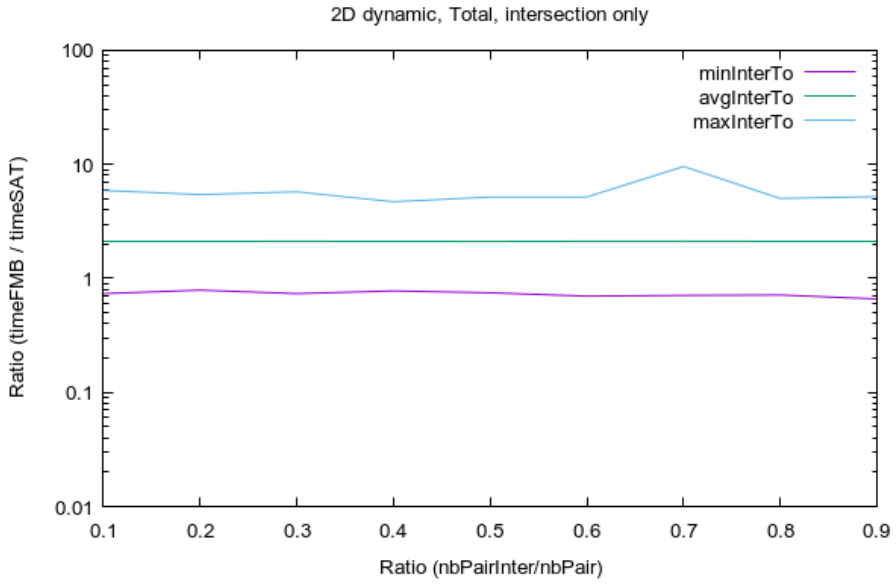
percPairInter	countInterCC	countNoInterCC	minInterCC	avgInterCC	maxInterCC	minNoInterCC	avgNoInterCC	maxNoInterCC	minTotalCC	avgTotalCC	maxTotalCC
0.1	19854	29796	1.154728	2.694981	5.905797	0.095745	1.113978	22.516129	0.095745	1.272079	22.516129
0.2	19770	29836	1.126543	2.699560	5.439759	0.111111	1.096759	19.166667	0.111111	1.417319	19.166667
0.3	20000	29848	1.262411	2.692522	5.261745	0.103659	1.101955	18.275862	0.103659	1.579125	18.275862
0.4	20008	30204	1.305085	2.688182	4.701493	0.107143	1.113436	17.833333	0.107143	1.743334	17.833333
0.5	19812	29974	1.174387	2.689792	5.167939	0.111111	1.113616	16.750000	0.111111	1.901704	16.750000
0.6	19878	30280	1.091743	2.693577	5.180451	0.096591	1.098547	23.413793	0.096591	2.055565	23.413793
0.7	19876	30022	1.226804	2.694843	5.626866	0.093923	1.114890	17.964286	0.093923	2.220857	17.964286
0.8	20398	30216	1.398577	2.696955	5.022727	0.090395	1.100456	22.321429	0.090395	2.377655	22.321429
0.9	19998	29922	1.170520	2.700909	5.218045	0.094444	1.092680	19.517241	0.094444	2.540086	19.517241

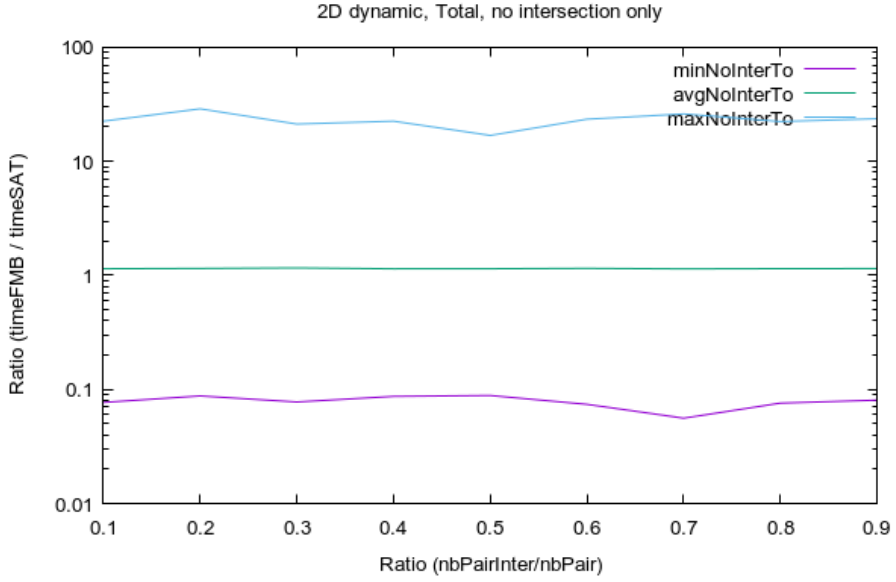
percPairInter	countInterCT	countNoInterCT	minInterCT	avgInterCT	maxInterCT	minNoInterCT	avgNoInterCT	maxNoInterCT	minTotalCT	avgTotalCT	maxTotalCT
0.1	18608	31494	0.735602	2.069174	4.832117	0.076923	1.149664	20.730769	0.076923	1.241615	20.730769
0.2	18678	31688	0.863222	2.070812	5.228395	0.094937	1.178000	21.354839	0.094937	1.356562	21.354839
0.3	18456	31784	0.735437	2.061562	5.116564	0.095808	1.157055	21.000000	0.095808	1.428407	21.000000
0.4	18858	31338	1.050000	2.061545	4.375887	0.100000	1.145613	22.520000	0.100000	1.511986	22.520000
0.5	18702	31150	0.986945	2.061083	4.417808	0.088542	1.140800	15.160000	0.088542	1.600941	15.160000
0.6	18686	31230	0.817109	2.062988	4.410072	0.109756	1.158323	18.461538	0.109756	1.701122	18.461538
0.7	18576	31638	0.878947	2.063309	4.241611	0.077295	1.138800	17.564103	0.077295	1.785957	17.564103
0.8	18600	31140	0.734491	2.066452	4.349650	0.075829	1.153394	20.307692	0.075829	1.883841	20.307692
0.9	18590	31082	0.904605	2.068095	4.621429	0.080569	1.177415	18.576923	0.080569	1.979027	18.576923

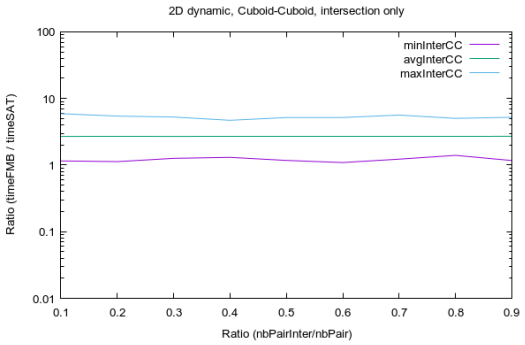
percPairInter	countInterTC	countNoInterTC	minInterTC	avgInterTC	maxInterTC	minNoInterTC	avgNoInterTC	maxNoInterTC	minTotalTC	avgTotalTC	maxTotalTC
0.1	18632	31530	0.791123	2.062074	5.484848	0.091463	1.152822	22.000000	0.091463	1.243747	22.000000
0.2	18686	31404	0.916129	2.063152	5.342857	0.091463	1.155898	28.884615	0.091463	1.337349	28.884615
0.3	18840	31226	0.924157	2.056897	5.741497	0.092025	1.198737	21.230769	0.092025	1.456185	21.230769
0.4	18644	31102	0.910920	2.055191	4.571429	0.086957	1.140241	16.240000	0.086957	1.506221	16.240000
0.5	18732	31368	0.797753	2.053885	4.714286	0.108696	1.149155	16.875000	0.108696	1.601520	16.875000
0.6	18580	31504	0.783934	2.057780	4.600000	0.074074	1.152019	18.142857	0.074074	1.695476	18.142857
0.7	18946	31194	0.729927	2.059781	9.592965	0.055970	1.111971	26.076923	0.055970	1.775438	26.076923
0.8	18498	31336	0.875399	2.062655	4.567376	0.078212	1.138612	18.083333	0.078212	1.877846	18.083333
0.9	18904	31504	0.839080	2.061452	5.192857	0.089005	1.126441	23.629630	0.089005	1.967951	23.629630

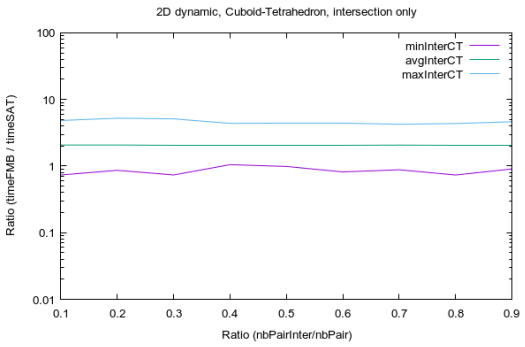
percPairInter	countInterTT	countNoInterTT	minInterTT	avgInterTT	maxInterTT	minNoInterTT	avgNoInterTT	maxNoInterTT	minTotalTT	avgTotalTT	maxTotalTT
0.1	17096	32990	0.757576	1.568590	3.250000	0.088889	1.157774	19.185185	0.088889	1.198855	19.185185
0.2	17476	32456	0.788462	1.574466	3.475309	0.087805	1.169056	20.769231	0.087805	1.250138	20.769231
0.3	17362	32482	0.753807	1.567512	2.910828	0.077626	1.190733	15.185185	0.077626	1.303767	15.185185
0.4	17148	32692	0.776504	1.564971	3.333333	0.119718	1.166189	18.888889	0.119718	1.325702	18.888889
0.5	17630	32618	0.747475	1.564425	3.214724	0.104478	1.169167	13.458333	0.104478	1.366796	13.458333
0.6	17274	32556	0.698895	1.564012	3.467949	0.100629	1.200306	19.600000	0.100629	1.418530	19.600000
0.7	17024	32714	0.710396	1.567192	3.058140	0.101266	1.193145	14.666667	0.101266	1.454978	14.666667
0.8	17358	32446	0.715493	1.570066	3.141975	0.080808	1.185720	17.692308	0.080808	1.493197	17.692308
0.9	17132	32858	0.660574	1.569596	4.044586	0.080808	1.192072	16.423077	0.080808	1.531843	16.423077

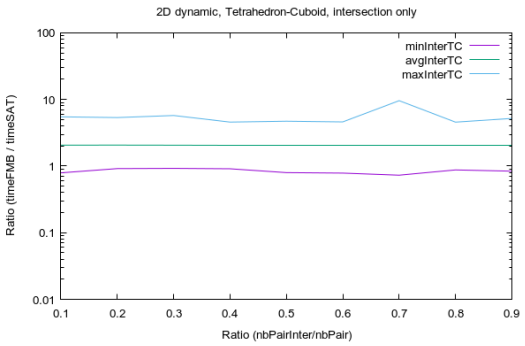

2D dynamic, Total
minTotalTo
avgTotalTo
maxTotalTo
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

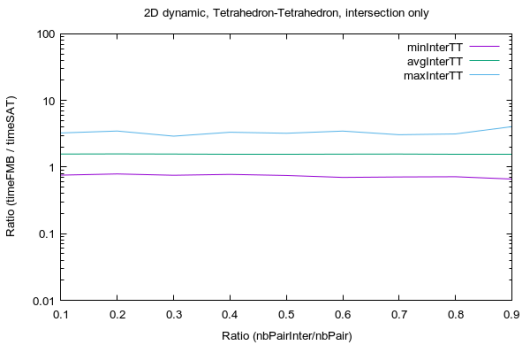

2D dynamic, Total, intersection only
minInterTo
avgInterTo
maxInterTo
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

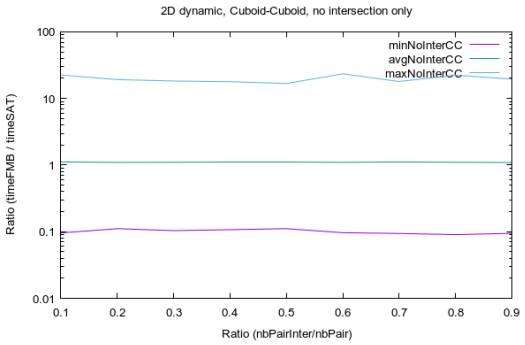

2D dynamic, Total, no intersection only
minNoInterTo
avgNoInterTo
maxNoInterTo
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

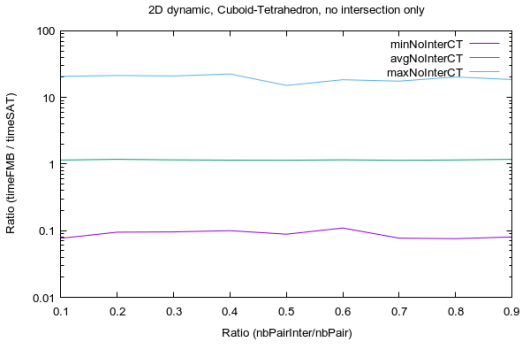

2D dynamic, Cuboid-Cuboid, intersection only
minInterCC
avgInterCC
maxInterCC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

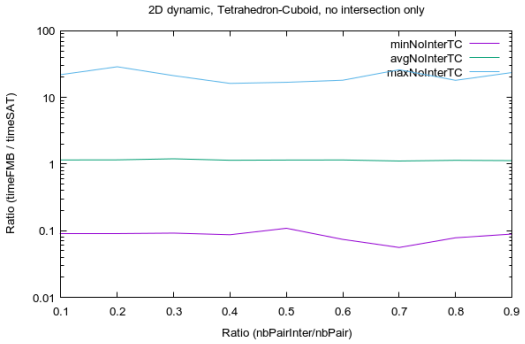

2D dynamic, Cuboid-Tetrahedron, intersection only
minInterCT
avgInterCT
maxInterCT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

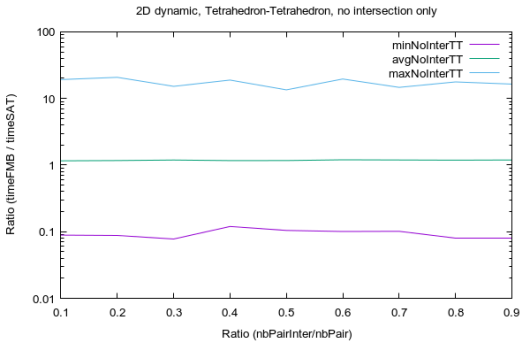

2D dynamic, Tetrahedron-Cuboid, intersection only
minInterTC
avgInterTC
maxInterTC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


2D dynamic, Tetrahedron-Tetrahedron, intersection only
minInterTT
avgInterTT
maxInterTT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


2D dynamic, Cuboid-Cuboid, no intersection only
minNoInterCC
avgNoInterCC
maxNoInterCC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


2D dynamic, Cuboid-Tetrahedron, no intersection only
minNoInterCT
avgNoInterCT
maxNoInterCT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


2D dynamic, Tetrahedron-Cuboid, no intersection only
minNoInterTC
avgNoInterTC
maxNoInterTC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


2D dynamic, Tetrahedron-Tetrahedron, no intersection only
minNoInterTT
avgNoInterTT
maxNoInterTT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

8.2.4 3D dynamic

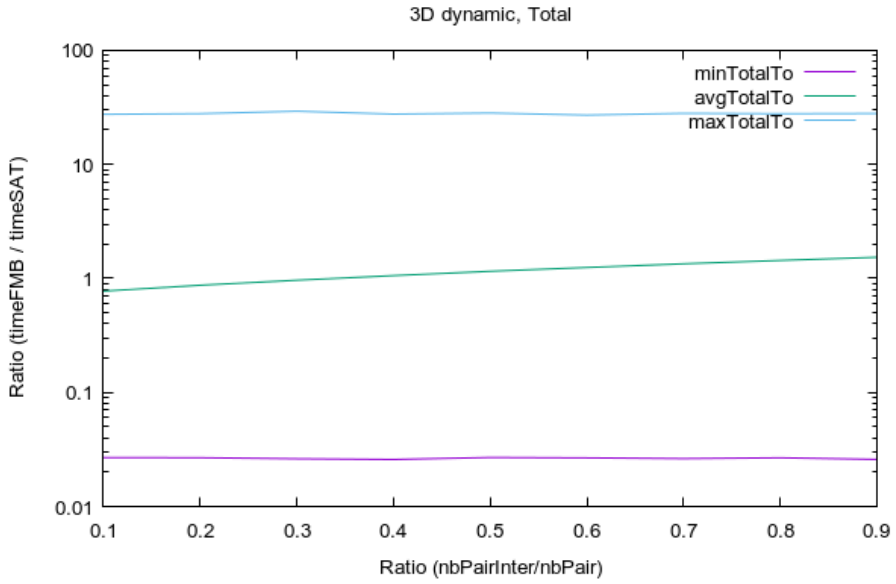
percPairInter	countInterTo	countNoInterTo	minInterTo	avgInterTo	maxInterTo	minNoInterTo	avgNoInterTo	maxNoInterTo	minTotalTo	avgTotalTo	maxTotalTo
0.1	52698	147302	0.300747	1.637476	3.157143	0.026856	0.677828	27.459459	0.026856	0.773792	27.459459
0.2	52622	147378	0.194799	1.625951	3.979298	0.026779	0.682900	27.861111	0.026779	0.871510	27.861111
0.3	52430	147570	0.288567	1.631220	3.240035	0.026214	0.677271	29.228571	0.026214	0.963456	29.228571
0.4	52374	147626	0.288422	1.629524	4.424269	0.025856	0.678658	27.594595	0.025856	1.059004	27.594595
0.5	52130	147870	0.297481	1.632362	3.173479	0.026920	0.675448	28.166667	0.026920	1.153905	28.166667
0.6	53012	146988	0.293688	1.626308	4.066850	0.026688	0.675885	27.027027	0.026688	1.246139	27.027027
0.7	52260	147740	0.289726	1.627865	2.966425	0.026276	0.682515	28.000000	0.026276	1.344260	28.000000
0.8	52084	147916	0.191621	1.627001	4.372177	0.026751	0.674503	27.783784	0.026751	1.436501	27.783784
0.9	52222	147778	0.279582	1.625801	3.465320	0.025895	0.680040	27.944444	0.025895	1.531225	27.944444

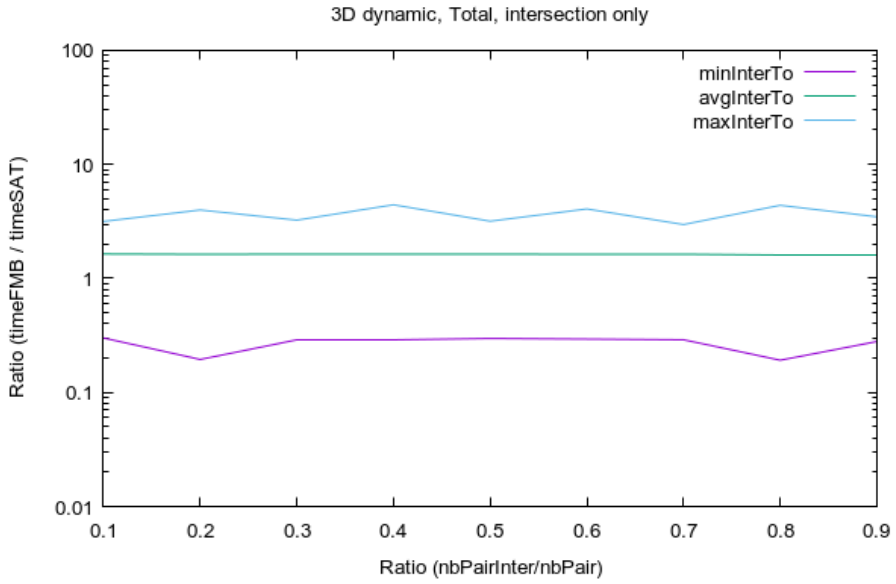
percPairInter	countInterCC	countNoInterCC	minInterCC	avgInterCC	maxInterCC	minNoInterCC	avgNoInterCC	maxNoInterCC	minTotalCC	avgTotalCC	maxTotalCC
0.1	16416	33576	1.728884	2.633187	3.157143	0.040174	0.590040	13.065789	0.040174	0.794355	13.065789
0.2	16060	33534	0.946026	2.632497	3.979298	0.039308	0.597730	14.836910	0.039308	1.004684	14.836910
0.3	16054	34004	1.763317	2.633717	3.240035	0.039828	0.599157	12.986726	0.039828	1.209525	12.986726
0.4	16024	33970	1.916276	2.633041	4.424269	0.040305	0.604536	13.008772	0.040305	1.415938	13.008772
0.5	15988	33972	2.059066	2.633628	3.173479	0.040469	0.612575	13.084821	0.040469	1.623102	13.084821
0.6	16116	33732	1.792553	2.633038	4.066850	0.040685	0.599110	13.097778	0.040685	1.819467	13.097778
0.7	15942	33862	2.367162	2.633075	2.966425	0.040685	0.612614	12.986726	0.040685	2.026936	12.986726
0.8	15886	34324	1.165044	2.633748	4.372177	0.039700	0.596513	13.640167	0.039700	2.226301	13.640167
0.9	15904	33656	1.838129	2.634349	3.465320	0.033473	0.602593	13.285124	0.033473	2.431173	13.285124

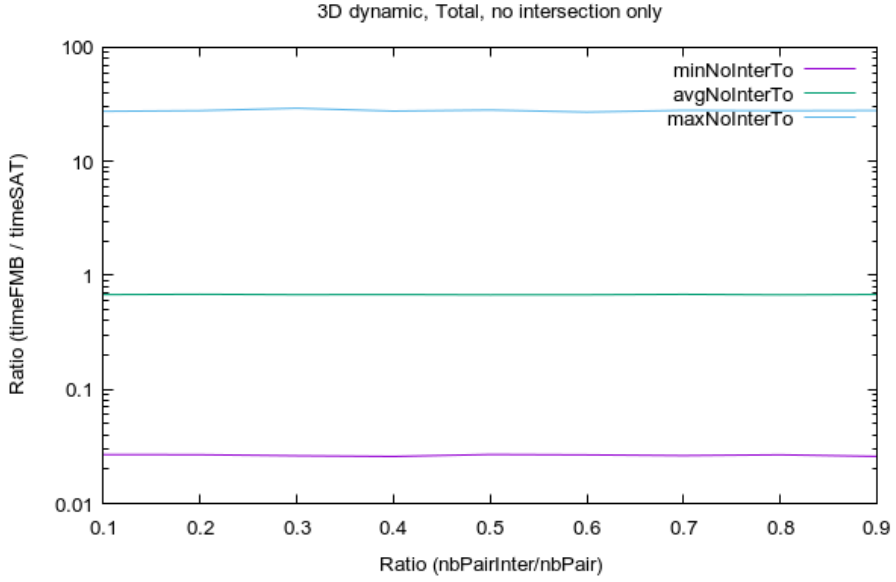
percPairInter	countInterCT	countNoInterCT	minInterCT	avgInterCT	maxInterCT	minNoInterCT	avgNoInterCT	maxNoInterCT	minTotalCT	avgTotalCT	maxTotalCT
0.1	13076	36790	0.460553	1.424992	2.546245	0.026856	0.668576	21.760000	0.026856	0.744218	21.760000
0.2	13186	36774	0.356670	1.425208	2.996403	0.026779	0.685135	21.860000	0.026779	0.833150	21.860000
0.3	13188	37230	0.413209	1.425827	2.790310	0.027244	0.679607	21.176471	0.027244	0.903473	21.176471
0.4	13280	36834	0.406924	1.425854	3.178824	0.027494	0.669581	21.460000	0.027494	0.972090	21.460000
0.5	13292	36786	0.447771	1.423940	2.620444	0.027309	0.683329	22.291667	0.027309	1.053635	22.291667
0.6	13324	36558	0.462799	1.426662	2.574468	0.026688	0.680506	21.187500	0.026688	1.128199	21.187500
0.7	12956	36796	0.468707	1.423738	2.576984	0.026667	0.667209	21.367347	0.026667	1.196779	21.367347
0.8	13046	36660	0.309365	1.424316	3.078067	0.027441	0.675768	20.129630	0.027441	1.274606	20.129630
0.9	13066	37090	0.414118	1.426348	3.140613	0.027113	0.678648	20.834286	0.027113	1.351578	20.834286

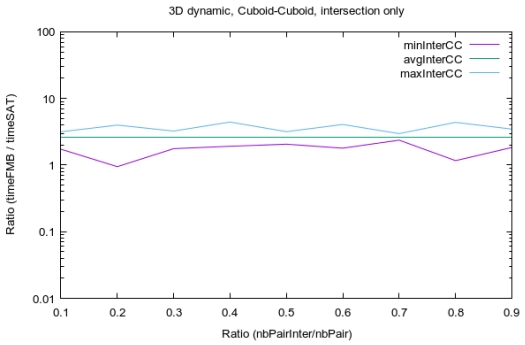
percPairInter	countInterTC	countNoInterTC	minInterTC	avgInterTC	maxInterTC	minNoInterTC	avgNoInterTC	maxNoInterTC	minTotalTC	avgTotalTC	maxTotalTC
0.1	13166	36674	0.313110	1.425090	2.909780	0.027429	0.690067	21.900000	0.027429	0.763569	21.900000
0.2	13146	37114	0.314307	1.423009	3.282780	0.027465	0.683438	21.500000	0.027465	0.831352	21.500000
0.3	13154	36958	0.437149	1.426295	2.791067	0.027553	0.678958	17.885057	0.027553	0.903159	17.885057
0.4	13022	37276	0.470790	1.424033	2.610420	0.027049	0.669714	21.862745	0.027049	0.971442	21.862745
0.5	12960	37004	0.465244	1.425126	2.657914	0.027978	0.655009	20.882353	0.027978	1.040067	20.882353
0.6	13340	36648	0.472358	1.423072	2.560032	0.027200	0.665398	22.395833	0.027200	1.120002	22.395833
0.7	13324	36590	0.454700	1.424460	2.588329	0.026562	0.680810	21.745098	0.026562	1.201365	21.745098
0.8	13074	36820	0.407070	1.423960	3.262875	0.027222	0.670361	21.000000	0.027222	1.273240	21.000000
0.9	13056	37152	0.381030	1.425698	2.949503	0.027961	0.670829	21.117647	0.027961	1.350211	21.117647

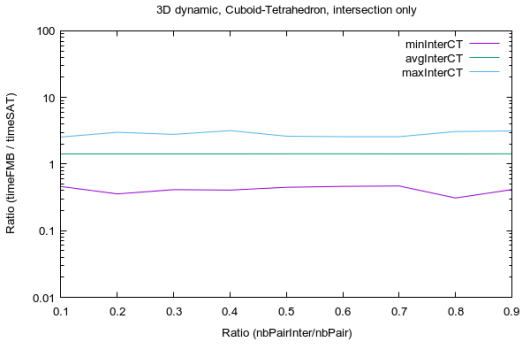
percPairInter	countInterTT	countNoInterTT	minInterTT	avgInterTT	maxInterTT	minNoInterTT	avgNoInterTT	maxNoInterTT	minTotalTT	avgTotalTT	maxTotalTT
0.1	10040	40262	0.300747	0.564679	0.770714	0.026877	0.748342	27.459459	0.026877	0.729976	27.459459
0.2	10230	39956	0.194799	0.565321	0.847841	0.027070	0.751825	27.861111	0.027070	0.714524	27.861111
0.3	10034	39378	0.288567	0.565867	0.840999	0.026214	0.740933	29.228571	0.026214	0.688414	29.228571
0.4	10048	39546	0.288422	0.564663	0.753705	0.025856	0.759213	27.594595	0.025856	0.681393	27.594595
0.5	9890	40108	0.297481	0.565414	0.979322	0.026920	0.740333	28.166667	0.026920	0.652874	28.166667
0.6	10232	40050	0.293688	0.565596	0.764535	0.027027	0.745928	27.027027	0.027027	0.637729	27.027027
0.7	10038	40492	0.289726	0.564884	0.920268	0.026276	0.756422	28.000000	0.026276	0.622346	28.000000
0.8	10078	40112	0.191621	0.565838	1.076491	0.026751	0.743887	27.783784	0.026751	0.601448	27.783784
0.9	10196	39880	0.279582	0.564470	0.977089	0.025895	0.755276	27.944444	0.025895	0.583550	27.944444

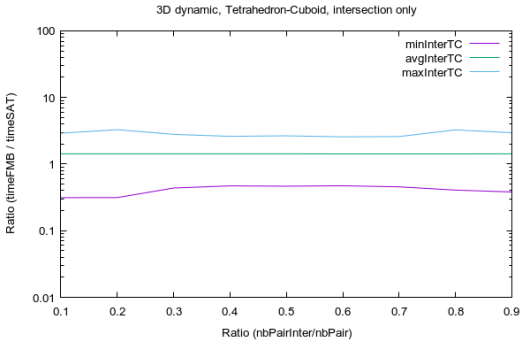

3D dynamic, Total
minTotalTo
avgTotalTo
maxTotalTo
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

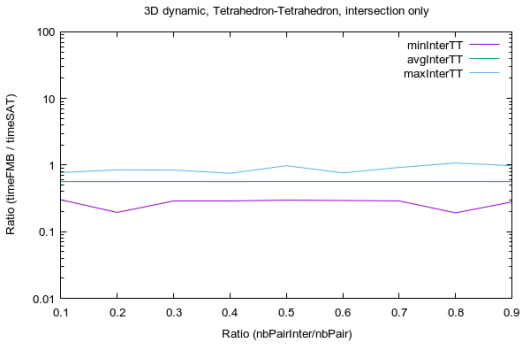

3D dynamic, Total, intersection only
minInterTo
avgInterTo
maxInterTo
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

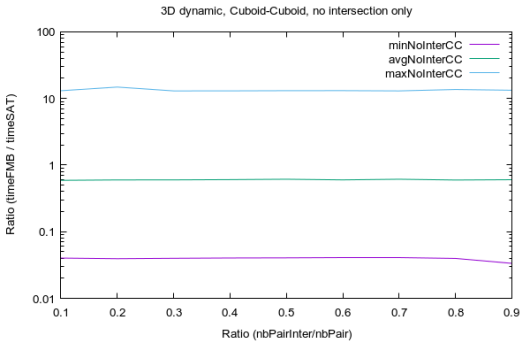

3D dynamic, Total, no intersection only
minNoInterTo
avgNoInterTo
maxNoInterTo
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

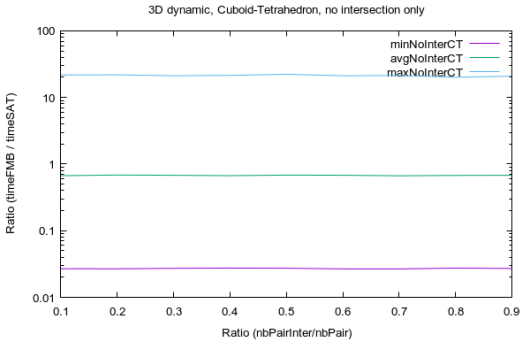

3D dynamic, Cuboid-Cuboid, intersection only
minInterCC
avgInterCC
maxInterCC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)

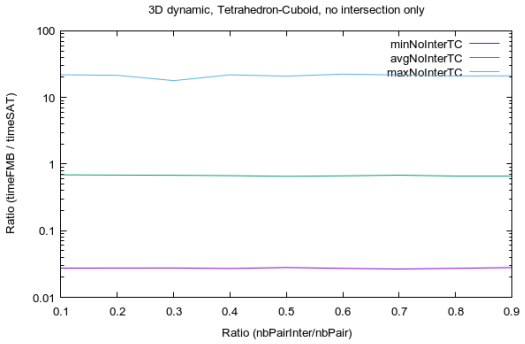

3D dynamic, Cuboid-Tetrahedron, intersection only
minInterCT
avgInterCT
maxInterCT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)

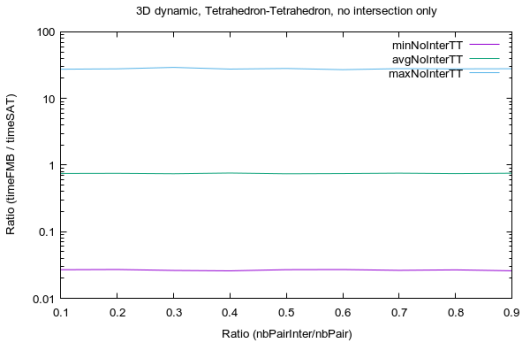

3D dynamic, Tetrahedron-Cuboid, intersection only
minInterTC
avgInterTC
maxInterTC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)


3D dynamic, Tetrahedron-Tetrahedron, intersection only
minInterTT
avgInterTT
maxInterTT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)


3D dynamic, Cuboid-Cuboid, no intersection only
minNoInterCC
avgNoInterCC
maxNoInterCC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


3D dynamic, Cuboid-Tetrahedron, no intersection only
minNoInterCT
avgNoInterCT
maxNoInterCT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


3D dynamic, Tetrahedron-Cuboid, no intersection only
minNoInterTC
avgNoInterTC
maxNoInterTC
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


3D dynamic, Tetrahedron-Tetrahedron, no intersection only
minNoInterTT
avgNoInterTT
maxNoInterTT
Ratio (timeFMB / timeSAT)
Ratio (nbPairInter/nbPair)
100
10
1
0.1
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

9 Comments about the qualification results

10 Conclusion

The validation proves that the FMB algorithm correctly identifies intersection of pairs of Frames in accordance with the results of the SAT algorithm.

The qualification shows that the FMB is 1.2 to 1.8 times slower than the SAT algorithm in the 2D dynamic case. However it is around 2 times faster in the 3D static case, and up to 1.25 times faster in 3D dynamic and up to 1.1 times faster in the 2D static case if the percentage of tested pairs in intersection is less than, respectively, around 40% and 25%.

On one given pair of Frame, the relative speed of the FMB algorithm varies widely, from around 20 times slower to 50 times faster. This is explained by the way the 2 algorithms works: they both make the asumption that the Frames are intersecting and run through a series of tests to try to prove it wrong. This leads to best cases and worst cases for both algorithm: a non interesecting detected right from the first test, or one detected by the last test. These best and worst cases are different for the two algorithms as the tests they performed are completely different. But in average, the FMB algorithm has the advantage for all but the 2D dynamic case.

11 Annex

11.1 Runtime environment

Results introduce in this paper have been produced by compiling and running the corresponding algorithms in the following environment:

```
uname -v

#40~18.04.1-Ubuntu SMP Thu Nov 14 12:06:39 UTC 2019

=============

lshw -short

H/W path       Device      Class          Description
=====================================================
                           system         VC65-C1
/0                         bus            VC65-C1
/0/0                       memory         64KiB BIOS
```

```
/0/2f                       memory         16GiB System Memory
/0/2f/0                     memory         [empty]
/0/2f/1                     memory         16GiB SODIMM DDR4 Synchronous 2400
     MHz (0.4 ns)
/0/39                       memory         384KiB L1 cache
/0/3a                       memory         1536KiB L2 cache
/0/3b                       memory         12MiB L3 cache
/0/3c                       processor      Intel(R) Core(TM) i7-8700T CPU @
    2.40GHz
/0/100                      bridge         8th Gen Core Processor Host Bridge
    /DRAM Registers
/0/100/2                    display        Intel Corporation
/0/100/12                   generic        Cannon Lake PCH Thermal Controller
/0/100/14                   bus            Cannon Lake PCH USB 3.1 xHCI Host
    Controller
/0/100/14/0    usb1         bus            xHCI Host Controller
/0/100/14/0/5               input          ELECOM Wired Keyboard
/0/100/14/0/6               input          PTZ-630
/0/100/14/0/7               generic        USB2.0-CRW
/0/100/14/0/e               communication  Bluetooth wireless interface
/0/100/14/1    usb2         bus            xHCI Host Controller
/0/100/14.2                 memory         RAM memory
/0/100/14.3    wlo1         network        Wireless-AC 9560 [Jefferson Peak]
/0/100/16                   communication  Cannon Lake PCH HECI Controller
/0/100/17                   storage        Cannon Lake PCH SATA AHCI
    Controller
/0/100/1f                   bridge         Intel Corporation
/0/100/1f.3                 multimedia     Cannon Lake PCH cAVS
/0/100/1f.4                 bus            Cannon Lake PCH SMBus Controller
/0/100/1f.5                 bus            Cannon Lake PCH SPI Controller
/0/100/1f.6    eno2         network        Ethernet Connection (7) I219-V
/0/1           scsi0        storage
/0/1/0.0.0     /dev/sda     disk           128GB HFS128G39TND-N21
/0/1/0.0.0/1                volume         99MiB Windows FAT volume
/0/1/0.0.0/2   /dev/sda2    volume         15MiB reserved partition
/0/1/0.0.0/3   /dev/sda3    volume         83GiB Windows NTFS volume
/0/1/0.0.0/4   /dev/sda4    volume         499MiB Windows NTFS volume
/0/1/0.0.0/5   /dev/sda5    volume         35GiB EXT4 volume
/0/2           scsi2        storage
/0/2/0.0.0     /dev/sdb     disk           500GB ST500LM034-2GH17
/0/2/0.0.0/1   /dev/sdb1    volume         463GiB EXT4 volume
/0/2/0.0.0/2   /dev/sdb2    volume         499MiB Windows FAT volume
/0/3           scsi5        storage
/0/3/0.0.0     /dev/cdrom   disk           BD-RE BU50N
/1                          power          To Be Filled By O.E.M.

=============

lscpu

Architecture:        x86_64
CPU op-mode(s):      32-bit, 64-bit
Byte Order:          Little Endian
CPU(s):              12
On-line CPU(s) list: 0-11
Thread(s) per core:  2
Core(s) per socket:  6
Socket(s):           1
NUMA node(s):        1
Vendor ID:           GenuineIntel
CPU family:          6
Model:               158
```

```
Model name:          Intel(R) Core(TM) i7-8700T CPU @ 2.40GHz
Stepping:            10
CPU MHz:             2216.548
CPU max MHz:         4000.0000
CPU min MHz:         800.0000
BogoMIPS:            4800.00
Virtualization:      VT-x
L1d cache:           32K
L1i cache:           32K
L2 cache:            256K
L3 cache:            12288K
NUMA node0 CPU(s):   0-11
Flags:               fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge
    mca cmov pat pse36 clflush dts acpi mmx fxsr sse sse2 ss ht tm pbe
    syscall nx pdpe1gb rdtscp lm constant_tsc art arch_perfmon pebs bts
    rep_good nopl xtopology nonstop_tsc cpuid aperfmperf tsc_known_freq pni
    pclmulqdq dtes64 monitor ds_cpl vmx smx est tm2 ssse3 sdbg fma cx16 xtpr
     pdcm pcid sse4_1 sse4_2 x2apic movbe popcnt tsc_deadline_timer aes
    xsave avx f16c rdrand lahf_lm abm 3dnowprefetch cpuid_fault epb
    invpcid_single pti ssbd ibrs ibpb stibp tpr_shadow vnmi flexpriority ept
     vpid ept_ad fsgsbase tsc_adjust bmi1 hle avx2 smep bmi2 erms invpcid
    rtm mpx rdseed adx smap clflushopt intel_pt xsaveopt xsavec xgetbv1
    xsaves dtherm ida arat pln pts hwp hwp_notify hwp_act_window hwp_epp
    md_clear flush_l1d

=============

gcc -v

Using built-in specs.
COLLECT_GCC=gcc
COLLECT_LTO_WRAPPER=/usr/lib/gcc/x86_64-linux-gnu/7/lto-wrapper
OFFLOAD_TARGET_NAMES=nvptx-none
OFFLOAD_TARGET_DEFAULT=1
Target: x86_64-linux-gnu
Configured with: ../src/configure -v --with-pkgversion='Ubuntu 7.4.0-1
    ubuntu1~18.04.1' --with-bugurl=file:///usr/share/doc/gcc-7/README.Bugs
    --enable-languages=c,ada,c++,go,brig,d,fortran,objc,obj-c++ --prefix=/
    usr --with-gcc-major-version-only --program-suffix=-7 --program-prefix=
    x86_64-linux-gnu- --enable-shared --enable-linker-build-id --libexecdir
    =/usr/lib --without-included-gettext --enable-threads=posix --libdir=/
    usr/lib --enable-nls --with-sysroot=/ --enable-clocale=gnu --enable-
    libstdcxx-debug --enable-libstdcxx-time=yes --with-default-libstdcxx-abi
    =new --enable-gnu-unique-object --disable-vtable-verify --enable-libmpx
    --enable-plugin --enable-default-pie --with-system-zlib --with-target-
    system-zlib --enable-objc-gc=auto --enable-multiarch --disable-werror --
    with-arch-32=i686 --with-abi=m64 --with-multilib-list=m32,m64,mx32 --
    enable-multilib --with-tune=generic --enable-offload-targets=nvptx-none
    --without-cuda-driver --enable-checking=release --build=x86_64-linux-gnu
     --host=x86_64-linux-gnu --target=x86_64-linux-gnu
Thread model: posix
gcc version 7.4.0 (Ubuntu 7.4.0-1ubuntu1~18.04.1)
```

11.2 SAT implementation

In this section I introduce the code of the implementation of the SAT algorithm, used to validate and qualify the FMB algorithm.

11.2.1 Header

```
#ifndef __SAT_H_
#define __SAT_H_

#include <stdbool.h>
#include <string.h>
#include "frame.h"

// ------------- Functions declaration -------------

// Test for intersection between 2D Frame that and 2D Frame tho
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection2D(
  const Frame2D* const that,
  const Frame2D* const tho);

// Test for intersection between moving 2D Frame that and 2D
// Frame tho
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection2DTime(
  const Frame2DTime* const that,
  const Frame2DTime* const tho);

// Test for intersection between 3D Frame that and 3D Frame tho
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection3D(
  const Frame3D* const that,
  const Frame3D* const tho);

// Test for intersection between moving 3D Frame that and 3D
// Frame tho
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection3DTime(
  const Frame3DTime* const that,
  const Frame3DTime* const tho);

#endif
```

11.2.2 Body

```
#include "sat.h"

// ------------- Macros -------------

#define EPSILON 0.0000001

// ------------- Functions declaration -------------

// Check the intersection constraint along one axis for 3D Frames
bool CheckAxis3D(
  const Frame3D* const that,
  const Frame3D* const tho,
  const double* const axis);

// Check the intersection constraint along one axis for moving 3D Frames
bool CheckAxis3DTime(
  const Frame3DTime* const that,
  const Frame3DTime* const tho,
  const double* const axis,
  const double* const relSpeed);
```

```
// ------------- Functions implementation -------------

// Test for intersection between 2D Frame that and 2D Frame tho
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection2D(
  const Frame2D* const that,
  const Frame2D* const tho) {

  // Declare a variable to loop on Frames and commonalize code
  const Frame2D* frameEdge = that;

  // Loop to commonalize code when checking SAT based on that's edges
  // and then tho's edges
  for (int iFrame = 2;
       iFrame--;) {

    // Shortcuts
    FrameType frameEdgeType = frameEdge->type;
    const double* frameEdgeCompA = frameEdge->comp[0];
    const double* frameEdgeCompB = frameEdge->comp[1];

    // Declare a variable to memorize the number of edges, by default 2
    int nbEdges = 2;

    // Declare a variable to memorize the third edge in case of
    // tetrahedron
    double thirdEdge[2];

    // If the frame is a tetrahedron
    if (frameEdgeType == FrameTetrahedron) {

      // Initialise the third edge
      thirdEdge[0] = frameEdgeCompB[0] - frameEdgeCompA[0];
      thirdEdge[1] = frameEdgeCompB[1] - frameEdgeCompA[1];

      // Correct the number of edges
      nbEdges = 3;

    }

    // Loop on the frame's edges
    for (int iEdge = nbEdges;
         iEdge--;) {

      // Get the current edge
      const double* edge =
        (iEdge == 2 ? thirdEdge : frameEdge->comp[iEdge]);

      // Declare variables to memorize the boundaries of projection
      // of the two frames on the current edge
      double bdgBoxA[2];
      double bdgBoxB[2];

      // Declare two variables to loop on Frames and commonalize code
      const Frame2D* frame = that;
      double* bdgBox = bdgBoxA;

      // Loop on Frames
      for (int iFrame = 2;
           iFrame--;) {
```

```
// Shortcuts
const double* frameOrig = frame->orig;
const double* frameCompA = frame->comp[0];
const double* frameCompB = frame->comp[1];
FrameType frameType = frame->type;

// Get the number of vertices of frame
int nbVertices = (frameType == FrameTetrahedron ? 3 : 4);

// Declare a variable to memorize if the current vertex is
// the first in the loop, used to initialize the boundaries
bool firstVertex = true;

// Loop on vertices of the frame
for (int iVertex = nbVertices;
     iVertex--;) {

  // Get the vertex
  double vertex[2];
  vertex[0] = frameOrig[0];
  vertex[1] = frameOrig[1];
  switch (iVertex) {
    case 3:
      vertex[0] += frameCompA[0] + frameCompB[0];
      vertex[1] += frameCompA[1] + frameCompB[1];
      break;
    case 2:
      vertex[0] += frameCompA[0];
      vertex[1] += frameCompA[1];
      break;
    case 1:
      vertex[0] += frameCompB[0];
      vertex[1] += frameCompB[1];
      break;
    default:
      break;
  }

  // Get the projection of the vertex on the normal of the edge
  // Orientation of the normal doesn't matter, so we
  // use arbitrarily the normal (edge[1], -edge[0])
  double proj = vertex[0] * edge[1] - vertex[1] * edge[0];

  // If it's the first vertex
  if (firstVertex == true) {

      // Initialize the boundaries of the projection of the
      // Frame on the edge
      bdgBox[0] = proj;
      bdgBox[1] = proj;

      // Update the flag to memorize we did the first vertex
      firstVertex = false;

  // Else, it's not the first vertex
  } else {

    // Update the boundaries of the projection of the Frame on
    // the edge
    if (bdgBox[0] > proj)
      bdgBox[0] = proj;
```

```
            if (bdgBox[1] < proj)
              bdgBox[1] = proj;

          }

        }

        // Switch the frame to check the vertices of the second Frame
        frame = tho;
        bdgBox = bdgBoxB;

      }

      // If the projections of the two frames on the edge are
      // not intersecting
      if (bdgBoxB[1] < bdgBoxA[0] ||
          bdgBoxA[1] < bdgBoxB[0]) {

         // There exists an axis which separates the Frames,
         // thus they are not in intersection
         return false;

      }

    }

    // Switch the frames to test against the second Frame's edges
    frameEdge = tho;

  }

  // If we reaches here, it means the two Frames are intersecting
  return true;

}

// Test for intersection between moving 2D Frame that and 2D
// Frame tho
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection2DTime(
  const Frame2DTime* const that,
  const Frame2DTime* const tho) {

  // Declare a variable to loop on Frames and commonalize code
  const Frame2DTime* frameEdge = that;

  // Declare a variable to memorize the speed of tho relative to that
  double relSpeed[2];
  relSpeed[0] = tho->speed[0] - that->speed[0];
  relSpeed[1] = tho->speed[1] - that->speed[1];

  // Loop to commonalize code when checking SAT based on that's edges
  // and then tho's edges
  for (int iFrame = 2;
       iFrame--;) {

    // Shortcuts
    FrameType frameEdgeType = frameEdge->type;
    const double* frameEdgeCompA = frameEdge->comp[0];
    const double* frameEdgeCompB = frameEdge->comp[1];

    // Declare a variable to memorize the number of edges, by default 2
```

```
int nbEdges = 2;

// Declare a variable to memorize the third edge in case of
// tetrahedron
double thirdEdge[2];

// If the frame is a tetrahedron
if (frameEdgeType == FrameTetrahedron) {

  // Initialise the third edge
  thirdEdge[0] = frameEdgeCompB[0] - frameEdgeCompA[0];
  thirdEdge[1] = frameEdgeCompB[1] - frameEdgeCompA[1];

  // Correct the number of edges
  nbEdges = 3;

}

// If the current frame is the second frame
if (iFrame == 1) {

  // Add one more edge to take into account the movement
  // of tho relative to that
  ++nbEdges;

}

// Loop on the frame's edges
for (int iEdge = nbEdges;
     iEdge--;) {

  // Get the current edge
  const double* edge =
    (iEdge == 3 ? relSpeed :
      (iEdge == 2 ?
        (frameEdgeType == FrameTetrahedron ? thirdEdge : relSpeed) :
        frameEdge->comp[iEdge]));

  // Declare variables to memorize the boundaries of projection
  // of the two frames on the current edge
  double bdgBoxA[2];
  double bdgBoxB[2];

  // Declare two variables to loop on Frames and commonalize code
  const Frame2DTime* frame = that;
  double* bdgBox = bdgBoxA;

  // Loop on Frames
  for (int iFrame = 2;
       iFrame--;) {

    // Shortcuts
    const double* frameOrig = frame->orig;
    const double* frameCompA = frame->comp[0];
    const double* frameCompB = frame->comp[1];
    FrameType frameType = frame->type;

    // Get the number of vertices of frame
    int nbVertices = (frameType == FrameTetrahedron ? 3 : 4);

    // Declare a variable to memorize if the current vertex is
    // the first in the loop, used to initialize the boundaries
```

```
bool firstVertex = true;

// Loop on vertices of the frame
for (int iVertex = nbVertices;
     iVertex--;) {

  // Get the vertex
  double vertex[2];
  vertex[0] = frameOrig[0];
  vertex[1] = frameOrig[1];
  switch (iVertex) {
    case 3:
      vertex[0] += frameCompA[0] + frameCompB[0];
      vertex[1] += frameCompA[1] + frameCompB[1];
      break;
    case 2:
      vertex[0] += frameCompA[0];
      vertex[1] += frameCompA[1];
      break;
    case 1:
      vertex[0] += frameCompB[0];
      vertex[1] += frameCompB[1];
      break;
    default:
      break;
  }

  // Get the projection of the vertex on the normal of the edge
  // Orientation of the normal doesn't matter, so we
  // use arbitrarily the normal (edge[1], -edge[0])
  double proj = vertex[0] * edge[1] - vertex[1] * edge[0];

  // If it's the first vertex
  if (firstVertex == true) {

      // Initialize the boundaries of the projection of the
      // Frame on the edge
      bdgBox[0] = proj;
      bdgBox[1] = proj;

      // Update the flag to memorize we did the first vertex
      firstVertex = false;

  // Else, it's not the first vertex
  } else {

    // Update the boundaries of the projection of the Frame on
    // the edge
    if (bdgBox[0] > proj)
      bdgBox[0] = proj;

    if (bdgBox[1] < proj)
      bdgBox[1] = proj;

  }

  // If we are checking the second frame's vertices
  if (frame == tho) {

    // Check also the vertices moved by the relative speed
    vertex[0] += relSpeed[0];
    vertex[1] += relSpeed[1];
```

```
            proj = vertex[0] * edge[1] - vertex[1] * edge[0];

            if (bdgBox[0] > proj)
              bdgBox[0] = proj;

            if (bdgBox[1] < proj)
              bdgBox[1] = proj;

          }

        }

        // Switch the frame to check the vertices of the second Frame
        frame = tho;
        bdgBox = bdgBoxB;

      }

      // If the projections of the two frames on the edge are
      // not intersecting
      if (bdgBoxB[1] < bdgBoxA[0] ||
          bdgBoxA[1] < bdgBoxB[0]) {

         // There exists an axis which separates the Frames,
         // thus they are not in intersection
         return false;

      }

    }

    // Switch the frames to test against the second Frame's edges
    frameEdge = tho;

  }

  // If we reaches here, it means the two Frames are intersecting
  return true;

}

// Test for intersection between 3D Frame that and 3D Frame tho
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection3D(
  const Frame3D* const that,
  const Frame3D* const tho) {

  // Declare two variables to memorize the opposite edges in case
  // of tetrahedron
  double oppEdgesThat[3][3];
  double oppEdgesTho[3][3];

  // Declare two variables to memorize the number of edges, by default 3
  int nbEdgesThat = 3;
  int nbEdgesTho = 3;

  // If the first Frame is a tetrahedron
  if (that->type == FrameTetrahedron) {

    // Shortcuts
    const double* frameCompA = that->comp[0];
```

```
  const double* frameCompB = that->comp[1];
  const double* frameCompC = that->comp[2];

  // Initialise the opposite edges
  oppEdgesThat[0][0] = frameCompB[0] - frameCompA[0];
  oppEdgesThat[0][1] = frameCompB[1] - frameCompA[1];
  oppEdgesThat[0][2] = frameCompB[2] - frameCompA[2];

  oppEdgesThat[1][0] = frameCompB[0] - frameCompC[0];
  oppEdgesThat[1][1] = frameCompB[1] - frameCompC[1];
  oppEdgesThat[1][2] = frameCompB[2] - frameCompC[2];

  oppEdgesThat[2][0] = frameCompC[0] - frameCompA[0];
  oppEdgesThat[2][1] = frameCompC[1] - frameCompA[1];
  oppEdgesThat[2][2] = frameCompC[2] - frameCompA[2];

  // Correct the number of edges
  nbEdgesThat = 6;

}

// If the second Frame is a tetrahedron
if (tho->type == FrameTetrahedron) {

  // Shortcuts
  const double* frameCompA = tho->comp[0];
  const double* frameCompB = tho->comp[1];
  const double* frameCompC = tho->comp[2];

  // Initialise the opposite edges
  oppEdgesTho[0][0] = frameCompB[0] - frameCompA[0];
  oppEdgesTho[0][1] = frameCompB[1] - frameCompA[1];
  oppEdgesTho[0][2] = frameCompB[2] - frameCompA[2];

  oppEdgesTho[1][0] = frameCompB[0] - frameCompC[0];
  oppEdgesTho[1][1] = frameCompB[1] - frameCompC[1];
  oppEdgesTho[1][2] = frameCompB[2] - frameCompC[2];

  oppEdgesTho[2][0] = frameCompC[0] - frameCompA[0];
  oppEdgesTho[2][1] = frameCompC[1] - frameCompA[1];
  oppEdgesTho[2][2] = frameCompC[2] - frameCompA[2];

  // Correct the number of edges
  nbEdgesTho = 6;

}

// Declare variables to loop on Frames and commonalize code
const Frame3D* frame = that;
const double (*oppEdgesA)[3] = oppEdgesThat;

// Loop to commonalize code when checking SAT based on that's edges
// and then tho's edges
for (int iFrame = 2;
     iFrame--;) {

  // Shortcuts
  FrameType frameType = frame->type;
  const double* frameCompA = frame->comp[0];
  const double* frameCompB = frame->comp[1];
  const double* frameCompC = frame->comp[2];
```

```
// Declare a variable to memorize the number of faces, by default 3
int nbFaces = 3;

// Declare a variable to memorize the normal to faces
// Arrangement is normFaces[iFace][iAxis]
double normFaces[4][3];

// Initialise the normal to faces
normFaces[0][0] =
  frameCompA[1] * frameCompB[2] -
  frameCompA[2] * frameCompB[1];
normFaces[0][1] =
  frameCompA[2] * frameCompB[0] -
  frameCompA[0] * frameCompB[2];
normFaces[0][2] =
  frameCompA[0] * frameCompB[1] -
  frameCompA[1] * frameCompB[0];

normFaces[1][0] =
  frameCompA[1] * frameCompC[2] -
  frameCompA[2] * frameCompC[1];
normFaces[1][1] =
  frameCompA[2] * frameCompC[0] -
  frameCompA[0] * frameCompC[2];
normFaces[1][2] =
  frameCompA[0] * frameCompC[1] -
  frameCompA[1] * frameCompC[0];

normFaces[2][0] =
  frameCompC[1] * frameCompB[2] -
  frameCompC[2] * frameCompB[1];
normFaces[2][1] =
  frameCompC[2] * frameCompB[0] -
  frameCompC[0] * frameCompB[2];
normFaces[2][2] =
  frameCompC[0] * frameCompB[1] -
  frameCompC[1] * frameCompB[0];

// If the frame is a tetrahedron
if (frameType == FrameTetrahedron) {

  // Shortcuts
  const double* oppEdgeA = oppEdgesA[0];
  const double* oppEdgeB = oppEdgesA[1];

  // Initialise the normal to the opposite face
  normFaces[3][0] =
    oppEdgeA[1] * oppEdgeB[2] -
    oppEdgeA[2] * oppEdgeB[1];
  normFaces[3][1] =
    oppEdgeA[2] * oppEdgeB[0] -
    oppEdgeA[0] * oppEdgeB[2];
  normFaces[3][2] =
    oppEdgeA[0] * oppEdgeB[1] -
    oppEdgeA[1] * oppEdgeB[0];

  // Correct the number of faces
  nbFaces = 4;

}

// Loop on the frame's faces
```

```
  for (int iFace = nbFaces;
       iFace--;) {

    // Check against the current face's normal
    bool isIntersection =
      CheckAxis3D(
        that,
        tho,
        normFaces[iFace]);

    // If the axis is separating the Frames
    if (isIntersection == false) {

      // The Frames are not in intersection,
      // terminate the test
      return false;

    }

  }

  // Switch the frame to test against the second Frame
  frame = tho;
  oppEdgesA = oppEdgesTho;

}

// Loop on the pair of edges between the two frames
for (int iEdgeThat = nbEdgesThat;
     iEdgeThat--;) {

  // Get the first edge
  const double* edgeThat =
    (iEdgeThat < 3 ?
      that->comp[iEdgeThat] :
      oppEdgesThat[iEdgeThat - 3]);

  for (int iEdgeTho = nbEdgesTho;
       iEdgeTho--;) {

    // Get the second edge
    const double* edgeTho =
      (iEdgeTho < 3 ?
        tho->comp[iEdgeTho] :
        oppEdgesTho[iEdgeTho - 3]);

    // Get the cross product of the two edges
    double axis[3];
    axis[0] = edgeThat[1] * edgeTho[2] - edgeThat[2] * edgeTho[1];
    axis[1] = edgeThat[2] * edgeTho[0] - edgeThat[0] * edgeTho[2];
    axis[2] = edgeThat[0] * edgeTho[1] - edgeThat[1] * edgeTho[0];

    // Check against the cross product of the two edges
    bool isIntersection =
      CheckAxis3D(
        that,
        tho,
        axis);

    // If the axis is separating the Frames
    if (isIntersection == false) {
```

```
        // The Frames are not in intersection,
        // terminate the test
        return false;

      }

    }

  }

  // If we reaches here, it means the two Frames are intersecting
  return true;

}

// Test for intersection between moving 3D Frame that and 3D
// Frame tho
// Return true if the two Frames are intersecting, else false
bool SATTestIntersection3DTime(
  const Frame3DTime* const that,
  const Frame3DTime* const tho) {

  // Declare two variables to memorize the opposite edges in case
  // of tetrahedron
  double oppEdgesThat[3][3];
  double oppEdgesTho[3][3];

  // Declare a variable to memorize the speed of tho relative to that
  double relSpeed[3];
  relSpeed[0] = tho->speed[0] - that->speed[0];
  relSpeed[1] = tho->speed[1] - that->speed[1];
  relSpeed[2] = tho->speed[2] - that->speed[2];

  // Declare two variables to memorize the number of edges, by default 3
  int nbEdgesThat = 3;
  int nbEdgesTho = 3;

  // If the first Frame is a tetrahedron
  if (that->type == FrameTetrahedron) {

    // Shortcuts
    const double* frameCompA = that->comp[0];
    const double* frameCompB = that->comp[1];
    const double* frameCompC = that->comp[2];

    // Initialise the opposite edges
    oppEdgesThat[0][0] = frameCompB[0] - frameCompA[0];
    oppEdgesThat[0][1] = frameCompB[1] - frameCompA[1];
    oppEdgesThat[0][2] = frameCompB[2] - frameCompA[2];

    oppEdgesThat[1][0] = frameCompB[0] - frameCompC[0];
    oppEdgesThat[1][1] = frameCompB[1] - frameCompC[1];
    oppEdgesThat[1][2] = frameCompB[2] - frameCompC[2];

    oppEdgesThat[2][0] = frameCompC[0] - frameCompA[0];
    oppEdgesThat[2][1] = frameCompC[1] - frameCompA[1];
    oppEdgesThat[2][2] = frameCompC[2] - frameCompA[2];

    // Correct the number of edges
    nbEdgesThat = 6;

  }
```

```
// If the second Frame is a tetrahedron
if (tho->type == FrameTetrahedron) {

  // Shortcuts
  const double* frameCompA = tho->comp[0];
  const double* frameCompB = tho->comp[1];
  const double* frameCompC = tho->comp[2];

  // Initialise the opposite edges
  oppEdgesTho[0][0] = frameCompB[0] - frameCompA[0];
  oppEdgesTho[0][1] = frameCompB[1] - frameCompA[1];
  oppEdgesTho[0][2] = frameCompB[2] - frameCompA[2];

  oppEdgesTho[1][0] = frameCompB[0] - frameCompC[0];
  oppEdgesTho[1][1] = frameCompB[1] - frameCompC[1];
  oppEdgesTho[1][2] = frameCompB[2] - frameCompC[2];

  oppEdgesTho[2][0] = frameCompC[0] - frameCompA[0];
  oppEdgesTho[2][1] = frameCompC[1] - frameCompA[1];
  oppEdgesTho[2][2] = frameCompC[2] - frameCompA[2];

  // Correct the number of edges
  nbEdgesTho = 6;

}

// Declare variables to loop on Frames and commonalize code
const Frame3DTime* frame = that;
const double (*oppEdgesA)[3] = oppEdgesThat;

// Loop to commonalize code when checking SAT based on that's edges
// and then tho's edges
for (int iFrame = 2;
     iFrame--;) {

  // Shortcuts
  FrameType frameType = frame->type;
  const double* frameCompA = frame->comp[0];
  const double* frameCompB = frame->comp[1];
  const double* frameCompC = frame->comp[2];

  // Declare a variable to memorize the number of faces, by default 3
  int nbFaces = 3;

  // Declare a variable to memorize the normal to faces
  // Arrangement is normFaces[iFace][iAxis]
  double normFaces[10][3];

  // Initialise the normal to faces
  normFaces[0][0] =
    frameCompA[1] * frameCompB[2] -
    frameCompA[2] * frameCompB[1];
  normFaces[0][1] =
    frameCompA[2] * frameCompB[0] -
    frameCompA[0] * frameCompB[2];
  normFaces[0][2] =
    frameCompA[0] * frameCompB[1] -
    frameCompA[1] * frameCompB[0];

  normFaces[1][0] =
    frameCompA[1] * frameCompC[2] -
```

```
  frameCompA[2] * frameCompC[1];
normFaces[1][1] =
  frameCompA[2] * frameCompC[0] -
  frameCompA[0] * frameCompC[2];
normFaces[1][2] =
  frameCompA[0] * frameCompC[1] -
  frameCompA[1] * frameCompC[0];

normFaces[2][0] =
  frameCompC[1] * frameCompB[2] -
  frameCompC[2] * frameCompB[1];
normFaces[2][1] =
  frameCompC[2] * frameCompB[0] -
  frameCompC[0] * frameCompB[2];
normFaces[2][2] =
  frameCompC[0] * frameCompB[1] -
  frameCompC[1] * frameCompB[0];

// If the frame is a tetrahedron
if (frameType == FrameTetrahedron) {

  // Shortcuts
  const double* oppEdgeA = oppEdgesA[0];
  const double* oppEdgeB = oppEdgesA[1];

  // Initialise the normal to the opposite face
  normFaces[3][0] =
    oppEdgeA[1] * oppEdgeB[2] -
    oppEdgeA[2] * oppEdgeB[1];
  normFaces[3][1] =
    oppEdgeA[2] * oppEdgeB[0] -
    oppEdgeA[0] * oppEdgeB[2];
  normFaces[3][2] =
    oppEdgeA[0] * oppEdgeB[1] -
    oppEdgeA[1] * oppEdgeB[0];

  // Correct the number of faces
  nbFaces = 4;

}

// If we are checking the frame 'tho'
if (frame == tho) {

  // Add the normal to the virtual faces created by the speed
  // of tho relative to that

  normFaces[nbFaces][0] =
    relSpeed[1] * frameCompA[2] -
    relSpeed[2] * frameCompA[1];
  normFaces[nbFaces][1] =
    relSpeed[2] * frameCompA[0] -
    relSpeed[0] * frameCompA[2];
  normFaces[nbFaces][2] =
    relSpeed[0] * frameCompA[1] -
    relSpeed[1] * frameCompA[0];
  if (fabs(normFaces[nbFaces][0]) > EPSILON ||
      fabs(normFaces[nbFaces][1]) > EPSILON ||
      fabs(normFaces[nbFaces][2]) > EPSILON)
    ++nbFaces;

  normFaces[nbFaces][0] =
```

```
  relSpeed[1] * frameCompB[2] -
  relSpeed[2] * frameCompB[1];
normFaces[nbFaces][1] =
  relSpeed[2] * frameCompB[0] -
  relSpeed[0] * frameCompB[2];
normFaces[nbFaces][2] =
  relSpeed[0] * frameCompB[1] -
  relSpeed[1] * frameCompB[0];
if (fabs(normFaces[nbFaces][0]) > EPSILON ||
    fabs(normFaces[nbFaces][1]) > EPSILON ||
    fabs(normFaces[nbFaces][2]) > EPSILON)
  ++nbFaces;

normFaces[nbFaces][0] =
  relSpeed[1] * frameCompC[2] -
  relSpeed[2] * frameCompC[1];
normFaces[nbFaces][1] =
  relSpeed[2] * frameCompC[0] -
  relSpeed[0] * frameCompC[2];
normFaces[nbFaces][2] =
  relSpeed[0] * frameCompC[1] -
  relSpeed[1] * frameCompC[0];
if (fabs(normFaces[nbFaces][0]) > EPSILON ||
    fabs(normFaces[nbFaces][1]) > EPSILON ||
    fabs(normFaces[nbFaces][2]) > EPSILON)
  ++nbFaces;

if (frameType == FrameTetrahedron) {

  const double* oppEdgeA = oppEdgesA[0];
  const double* oppEdgeB = oppEdgesA[1];
  const double* oppEdgeC = oppEdgesA[2];

  normFaces[nbFaces][0] =
    relSpeed[1] * oppEdgeA[2] -
    relSpeed[2] * oppEdgeA[1];
  normFaces[nbFaces][1] =
    relSpeed[2] * oppEdgeA[0] -
    relSpeed[0] * oppEdgeA[2];
  normFaces[nbFaces][2] =
    relSpeed[0] * oppEdgeA[1] -
    relSpeed[1] * oppEdgeA[0];
  if (fabs(normFaces[nbFaces][0]) > EPSILON ||
      fabs(normFaces[nbFaces][1]) > EPSILON ||
      fabs(normFaces[nbFaces][2]) > EPSILON)
    ++nbFaces;

  normFaces[nbFaces][0] =
    relSpeed[1] * oppEdgeB[2] -
    relSpeed[2] * oppEdgeB[1];
  normFaces[nbFaces][1] =
    relSpeed[2] * oppEdgeB[0] -
    relSpeed[0] * oppEdgeB[2];
  normFaces[nbFaces][2] =
    relSpeed[0] * oppEdgeB[1] -
    relSpeed[1] * oppEdgeB[0];
  if (fabs(normFaces[nbFaces][0]) > EPSILON ||
      fabs(normFaces[nbFaces][1]) > EPSILON ||
      fabs(normFaces[nbFaces][2]) > EPSILON)
    ++nbFaces;

  normFaces[nbFaces][0] =
```

```
        relSpeed[1] * oppEdgeC[2] -
        relSpeed[2] * oppEdgeC[1];
      normFaces[nbFaces][1] =
        relSpeed[2] * oppEdgeC[0] -
        relSpeed[0] * oppEdgeC[2];
      normFaces[nbFaces][2] =
        relSpeed[0] * oppEdgeC[1] -
        relSpeed[1] * oppEdgeC[0];
      if (fabs(normFaces[nbFaces][0]) > EPSILON ||
          fabs(normFaces[nbFaces][1]) > EPSILON ||
          fabs(normFaces[nbFaces][2]) > EPSILON)
        ++nbFaces;

    }
  }

  // Loop on the frame's faces
  for (int iFace = nbFaces;
       iFace--;) {

    // Check against the current face's normal
    bool isIntersection =
      CheckAxis3DTime(
        that,
        tho,
        normFaces[iFace],
        relSpeed);

    // If the axis is separating the Frames
    if (isIntersection == false) {

      // The Frames are not in intersection,
      // terminate the test
      return false;

    }

  }

  // Switch the frame to test against the second Frame
  frame = tho;
  oppEdgesA = oppEdgesTho;

}

// Loop on the pair of edges between the two frames
for (int iEdgeThat = nbEdgesThat;
     iEdgeThat--;) {

  // Get the first edge
  const double* edgeThat =
    (iEdgeThat < 3 ?
      that->comp[iEdgeThat] :
      oppEdgesThat[iEdgeThat - 3]);

  for (int iEdgeTho = nbEdgesTho + 1;
       iEdgeTho--;) {

    // Get the second edge
    const double* edgeTho =
      (iEdgeTho == nbEdgesTho ?
        relSpeed :
```

```
          (iEdgeTho < 3 ?
            tho->comp[iEdgeTho] :
            oppEdgesTho[iEdgeTho - 3]));

      // Get the cross product of the two edges
      double axis[3];
      axis[0] = edgeThat[1] * edgeTho[2] - edgeThat[2] * edgeTho[1];
      axis[1] = edgeThat[2] * edgeTho[0] - edgeThat[0] * edgeTho[2];
      axis[2] = edgeThat[0] * edgeTho[1] - edgeThat[1] * edgeTho[0];

      // Check against the cross product of the two edges
      bool isIntersection =
        CheckAxis3DTime(
          that,
          tho,
          axis,
          relSpeed);

      // If the axis is separating the Frames
      if (isIntersection == false) {

        // The Frames are not in intersection,
        // terminate the test
        return false;

      }

    }

  }

  // If we reaches here, it means the two Frames are intersecting
  return true;

}

// Check the intersection constraint for Frames that and tho
// relatively to axis
bool CheckAxis3D(
  const Frame3D* const that,
  const Frame3D* const tho,
  const double* const axis) {

  // Declare variables to memorize the boundaries of projection
  // of the two frames on the current edge
  double bdgBoxA[2];
  double bdgBoxB[2];

  // Declare two variables to loop on Frames and commonalize code
  const Frame3D* frame = that;
  double* bdgBox = bdgBoxA;

  // Loop on Frames
  for (int iFrame = 2;
       iFrame--;) {

    // Shortcuts
    const double* frameOrig = frame->orig;
    const double* frameCompA = frame->comp[0];
    const double* frameCompB = frame->comp[1];
    const double* frameCompC = frame->comp[2];
    FrameType frameType = frame->type;
```

```
// Get the number of vertices of frame
int nbVertices = (frameType == FrameTetrahedron ? 4 : 8);

// Declare a variable to memorize if the current vertex is
// the first in the loop, used to initialize the boundaries
bool firstVertex = true;

// Loop on vertices of the frame
for (int iVertex = nbVertices;
     iVertex--;) {

  // Get the vertex
  double vertex[3];
  vertex[0] = frameOrig[0];
  vertex[1] = frameOrig[1];
  vertex[2] = frameOrig[2];
  switch (iVertex) {
    case 7:
      vertex[0] +=
        frameCompA[0] +  frameCompB[0] + frameCompC[0];
      vertex[1] +=
        frameCompA[1] +  frameCompB[1] + frameCompC[1];
      vertex[2] +=
        frameCompA[2] +  frameCompB[2] + frameCompC[2];
      break;
    case 6:
      vertex[0] += frameCompB[0] + frameCompC[0];
      vertex[1] += frameCompB[1] + frameCompC[1];
      vertex[2] += frameCompB[2] + frameCompC[2];
      break;
    case 5:
      vertex[0] += frameCompA[0] + frameCompC[0];
      vertex[1] += frameCompA[1] + frameCompC[1];
      vertex[2] += frameCompA[2] + frameCompC[2];
      break;
    case 4:
      vertex[0] += frameCompA[0] + frameCompB[0];
      vertex[1] += frameCompA[1] + frameCompB[1];
      vertex[2] += frameCompA[2] + frameCompB[2];
      break;
    case 3:
      vertex[0] += frameCompC[0];
      vertex[1] += frameCompC[1];
      vertex[2] += frameCompC[2];
      break;
    case 2:
      vertex[0] += frameCompB[0];
      vertex[1] += frameCompB[1];
      vertex[2] += frameCompB[2];
      break;
    case 1:
      vertex[0] += frameCompA[0];
      vertex[1] += frameCompA[1];
      vertex[2] += frameCompA[2];
      break;
    default:
      break;
  }

  // Get the projection of the vertex on the axis
  double proj =
```

```
        vertex[0] * axis[0] +
        vertex[1] * axis[1] +
        vertex[2] * axis[2];

      // If it's the first vertex
      if (firstVertex == true) {

          // Initialize the boundaries of the projection of the
          // Frame on the edge
          bdgBox[0] = proj;
          bdgBox[1] = proj;

          // Update the flag to memorize we did the first vertex
          firstVertex = false;

      // Else, it's not the first vertex
      } else {

        // Update the boundaries of the projection of the Frame on
        // the edge
        if (bdgBox[0] > proj)
          bdgBox[0] = proj;

        if (bdgBox[1] < proj)
          bdgBox[1] = proj;

      }

    }

    // Switch the frame to check the vertices of the second Frame
    frame = tho;
    bdgBox = bdgBoxB;

  }

  // If the projections of the two frames on the edge are
  // not intersecting
  if (bdgBoxB[1] < bdgBoxA[0] ||
      bdgBoxA[1] < bdgBoxB[0]) {

     // There exists an axis which separates the Frames,
     // thus they are not in intersection
     return false;

  }

  // If we reaches here the two Frames are in intersection
  return true;

}

// Check the intersection constraint for Frames that and tho
// relatively to axis
bool CheckAxis3DTime(
  const Frame3DTime* const that,
  const Frame3DTime* const tho,
  const double* const axis,
  const double* const relSpeed) {

  // Declare variables to memorize the boundaries of projection
  // of the two frames on the current edge
```

```
double bdgBoxA[2];
double bdgBoxB[2];

// Declare two variables to loop on Frames and commonalize code
const Frame3DTime* frame = that;
double* bdgBox = bdgBoxA;

// Loop on Frames
for (int iFrame = 2;
     iFrame--;) {

  // Shortcuts
  const double* frameOrig = frame->orig;
  const double* frameCompA = frame->comp[0];
  const double* frameCompB = frame->comp[1];
  const double* frameCompC = frame->comp[2];
  FrameType frameType = frame->type;

  // Get the number of vertices of frame
  int nbVertices = (frameType == FrameTetrahedron ? 4 : 8);

  // Declare a variable to memorize if the current vertex is
  // the first in the loop, used to initialize the boundaries
  bool firstVertex = true;

  // Loop on vertices of the frame
  for (int iVertex = nbVertices;
       iVertex--;) {

    // Get the vertex
    double vertex[3];
    vertex[0] = frameOrig[0];
    vertex[1] = frameOrig[1];
    vertex[2] = frameOrig[2];
    switch (iVertex) {
      case 7:
        vertex[0] +=
          frameCompA[0] +  frameCompB[0] + frameCompC[0];
        vertex[1] +=
          frameCompA[1] +  frameCompB[1] + frameCompC[1];
        vertex[2] +=
          frameCompA[2] +  frameCompB[2] + frameCompC[2];
        break;
      case 6:
        vertex[0] += frameCompB[0] + frameCompC[0];
        vertex[1] += frameCompB[1] + frameCompC[1];
        vertex[2] += frameCompB[2] + frameCompC[2];
        break;
      case 5:
        vertex[0] += frameCompA[0] + frameCompC[0];
        vertex[1] += frameCompA[1] + frameCompC[1];
        vertex[2] += frameCompA[2] + frameCompC[2];
        break;
      case 4:
        vertex[0] += frameCompA[0] + frameCompB[0];
        vertex[1] += frameCompA[1] + frameCompB[1];
        vertex[2] += frameCompA[2] + frameCompB[2];
        break;
      case 3:
        vertex[0] += frameCompC[0];
        vertex[1] += frameCompC[1];
        vertex[2] += frameCompC[2];
```

```
    break;
  case 2:
    vertex[0] += frameCompB[0];
    vertex[1] += frameCompB[1];
    vertex[2] += frameCompB[2];
    break;
  case 1:
    vertex[0] += frameCompA[0];
    vertex[1] += frameCompA[1];
    vertex[2] += frameCompA[2];
    break;
  default:
    break;
}

// Get the projection of the vertex on the axis
double proj =
  vertex[0] * axis[0] +
  vertex[1] * axis[1] +
  vertex[2] * axis[2];

// If it's the first vertex
if (firstVertex == true) {

    // Initialize the boundaries of the projection of the
    // Frame on the edge
    bdgBox[0] = proj;
    bdgBox[1] = proj;

    // Update the flag to memorize we did the first vertex
    firstVertex = false;

// Else, it's not the first vertex
} else {

  // Update the boundaries of the projection of the Frame on
  // the edge
  if (bdgBox[0] > proj)
    bdgBox[0] = proj;

  if (bdgBox[1] < proj)
    bdgBox[1] = proj;

}

// If we are checking the second frame's vertices
if (frame == tho) {

  // Check also the vertices moved by the relative speed
  vertex[0] += relSpeed[0];
  vertex[1] += relSpeed[1];
  vertex[2] += relSpeed[2];

proj =
  vertex[0] * axis[0] +
  vertex[1] * axis[1] +
  vertex[2] * axis[2];

  if (bdgBox[0] > proj)
    bdgBox[0] = proj;

  if (bdgBox[1] < proj)
```

```
          bdgBox[1] = proj;

      }

    }

    // Switch the frame to check the vertices of the second Frame
    frame = tho;
    bdgBox = bdgBoxB;

  }

  // If the projections of the two frames on the edge are
  // not intersecting
  if (bdgBoxB[1] < bdgBoxA[0] ||
      bdgBoxA[1] < bdgBoxB[0]) {

     // There exists an axis which separates the Frames,
     // thus they are not in intersection
     return false;

  }

  // If we reaches here the two Frames are in intersection
  return true;

}
```

11.3 Makefile

In this section I introduce the Makefile used to compile the code given in the previous sections. It also includes command used to run the unit tests, validation and qualification, and to generate the documentation.

```
COMPILER=gcc
OPTIMIZATION=-O3

all : compile run plot dynamicAnalysis doc

install :
        sudo apt-get install gnuplot

compile : main unitTests validation qualification

main : main2D main2DTime main3D main3DTime

main2D:
        cd 2D; make main OPTIMIZATION=$(OPTIMIZATION); cd -

main2DTime:
        cd 2DTime; make main OPTIMIZATION=$(OPTIMIZATION); cd -

main3D:
        cd 3D; make main OPTIMIZATION=$(OPTIMIZATION); cd -

main3DTime:
        cd 3DTime; make main OPTIMIZATION=$(OPTIMIZATION); cd -

unitTests : unitTests2D unitTests2DTime unitTests3D unitTests3DTime
```

```
unitTests2D:
	cd 2D; make unitTests OPTIMIZATION=$(OPTIMIZATION); cd -

unitTests2DTime:
	cd 2DTime; make unitTests OPTIMIZATION=$(OPTIMIZATION); cd -

unitTests3D:
	cd 3D; make unitTests OPTIMIZATION=$(OPTIMIZATION); cd -

unitTests3DTime:
	cd 3DTime; make unitTests OPTIMIZATION=$(OPTIMIZATION); cd -

validation : validation2D validation2DTime validation3D validation3DTime

validation2D:
	cd 2D; make validation OPTIMIZATION=$(OPTIMIZATION); cd -

validation2DTime:
	cd 2DTime; make validation OPTIMIZATION=$(OPTIMIZATION); cd -

validation3D:
	cd 3D; make validation OPTIMIZATION=$(OPTIMIZATION); cd -

validation3DTime:
	cd 3DTime; make validation OPTIMIZATION=$(OPTIMIZATION); cd -

qualification : qualification2D qualification2DTime qualification3D
    qualification3DTime

qualification2D:
	cd 2D; make qualification OPTIMIZATION=$(OPTIMIZATION); cd -

qualification2DTime:
	cd 2DTime; make qualification OPTIMIZATION=$(OPTIMIZATION); cd -

qualification3D:
	cd 3D; make qualification OPTIMIZATION=$(OPTIMIZATION); cd -

qualification3DTime:
	cd 3DTime; make qualification OPTIMIZATION=$(OPTIMIZATION); cd -

clean : clean2D clean2DTime clean3D clean3DTime

clean2D:
	cd 2D; make clean; cd -

clean2DTime:
	cd 2DTime; make clean; cd -

clean3D:
	cd 3D; make clean; cd -

clean3DTime:
	cd 3DTime; make clean; cd -

valgrind : valgrind2D valgrind2DTime valgrind3D valgrind3DTime

valgrind2D:
	cd 2D; make valgrind; cd -

valgrind2DTime:
```

```
	cd 2DTime; make valgrind; cd -

valgrind3D:
	cd 3D; make valgrind; cd -

valgrind3DTime:
	cd 3DTime; make valgrind; cd -

run : run2D run2DTime run3D run3DTime

run2D:
	cd 2D; ./main > ../Results/main2D.txt; ./unitTests > ../Results/
	    unitTests2D.txt; ./validation > ../Results/validation2D.txt; ./
	    qualification; cd -

run3D:
	cd 3D; ./main > ../Results/main3D.txt; ./unitTests > ../Results/
	    unitTests3D.txt; ./validation > ../Results/validation3D.txt; ./
	    qualification; cd -

run2DTime:
	cd 2DTime; ./main > ../Results/main2DTime.txt; ./unitTests > ../
	    Results/unitTests2DTime.txt; ./validation > ../Results/
	    validation2DTime.txt; ./qualification; cd -

run3DTime:
	cd 3DTime; ./main > ../Results/main3DTime.txt; ./unitTests > ../
	    Results/unitTests3DTime.txt; ./validation > ../Results/
	    validation3DTime.txt; ./qualification; cd -

plot: cleanPlot plot2D plot2DTime plot3D plot3DTime

cleanPlot:
	rm -f Results/*.png

plot2D:
	cd Results; gnuplot qualification2D.gnu; cd -

plot2DTime:
	cd Results; gnuplot qualification2DTime.gnu; cd -

plot3D:
	cd Results; gnuplot qualification3D.gnu; cd -

plot3DTime:
	cd Results; gnuplot qualification3DTime.gnu; cd -

doc:
	cd Doc; make latex; cd -

getRuntimeEnvironment:
	echo "uname -v\n" > runtimeEnv.txt; uname -v >> runtimeEnv.txt; echo
	     "\n=============\n" >> runtimeEnv.txt; echo "lshw -short\n" >>
	    runtimeEnv.txt; sudo lshw -short >> runtimeEnv.txt; echo "\n
	    =============\n" >> runtimeEnv.txt; echo "lscpu\n" >> runtimeEnv
	    .txt; lscpu >> runtimeEnv.txt; echo "\n=============\n" >>
	    runtimeEnv.txt; echo "$(COMPILER) -v\n" >> runtimeEnv.txt; $(
	    COMPILER) -v 1>> runtimeEnv.txt 2>> runtimeEnv.txt

dynamicAnalysis:
	make valgrind 1> dynamicAnalysis.txt 2> dynamicAnalysis.txt
```

11.3.1 2D static

```
all : main unitTests validation qualification

COMPILER?=gcc
OPTIMIZATION?=-O3
BUILD_ARG=$(OPTIMIZATION) -I../SAT -I../Frame

main : main.o fmb2d.o frame.o Makefile
        $(COMPILER) -o main main.o fmb2d.o frame.o

main.o : main.c fmb2d.h ../Frame/frame.h Makefile
        $(COMPILER) -c main.c $(BUILD_ARG)

unitTests : unitTests.o fmb2d.o frame.o Makefile
        $(COMPILER) -o unitTests unitTests.o fmb2d.o frame.o $(LINK_ARG)

unitTests.o : unitTests.c fmb2d.h ../Frame/frame.h Makefile
        $(COMPILER) -c unitTests.c $(BUILD_ARG)

validation : validation.o fmb2d.o sat.o frame.o Makefile
        $(COMPILER) -o validation validation.o fmb2d.o sat.o frame.o

validation.o : validation.c fmb2d.h ../SAT/sat.h ../Frame/frame.h Makefile
        $(COMPILER) -c validation.c $(BUILD_ARG)

qualification : qualification.o fmb2d.o sat.o frame.o Makefile
        $(COMPILER) -o qualification qualification.o fmb2d.o sat.o frame.o $
            (LINK_ARG)

qualification.o : qualification.c fmb2d.h ../SAT/sat.h ../Frame/frame.h
    Makefile
        $(COMPILER) -c qualification.c $(BUILD_ARG)

fmb2d.o : fmb2d.c fmb2d.h ../Frame/frame.h Makefile
        $(COMPILER) -c fmb2d.c $(BUILD_ARG)

sat.o : ../SAT/sat.c ../SAT/sat.h ../Frame/frame.h Makefile
        $(COMPILER) -c ../SAT/sat.c $(BUILD_ARG)

frame.o : ../Frame/frame.c ../Frame/frame.h Makefile
        $(COMPILER) -c ../Frame/frame.c $(BUILD_ARG)

clean :
        rm -f *.o main unitTests validation qualification

valgrind :
        valgrind -v --track-origins=yes --leak-check=full \
        --gen-suppressions=yes --show-leak-kinds=all ./unitTests
```

11.3.2 3D static

```
all : main unitTests validation qualification

COMPILER?=gcc
OPTIMIZATION?=-O3
BUILD_ARG=$(OPTIMIZATION) -I../SAT -I../Frame

main : main.o fmb3d.o frame.o Makefile
        $(COMPILER) -o main main.o fmb3d.o frame.o
```

```
main.o : main.c fmb3d.h ../Frame/frame.h Makefile
        $(COMPILER) -c main.c $(BUILD_ARG)

unitTests : unitTests.o fmb3d.o frame.o Makefile
        $(COMPILER) -o unitTests unitTests.o fmb3d.o frame.o $(LINK_ARG)

unitTests.o : unitTests.c fmb3d.h ../Frame/frame.h Makefile
        $(COMPILER) -c unitTests.c $(BUILD_ARG)

validation : validation.o fmb3d.o sat.o frame.o Makefile
        $(COMPILER) -o validation validation.o fmb3d.o sat.o frame.o

validation.o : validation.c fmb3d.h ../SAT/sat.h ../Frame/frame.h Makefile
        $(COMPILER) -c validation.c $(BUILD_ARG)

qualification : qualification.o fmb3d.o sat.o frame.o Makefile
        $(COMPILER) -o qualification qualification.o fmb3d.o sat.o frame.o $
            (LINK_ARG)

qualification.o : qualification.c fmb3d.h ../SAT/sat.h ../Frame/frame.h
    Makefile
        $(COMPILER) -c qualification.c $(BUILD_ARG)

fmb3d.o : fmb3d.c fmb3d.h ../Frame/frame.h Makefile
        $(COMPILER) -c fmb3d.c $(BUILD_ARG)

sat.o : ../SAT/sat.c ../SAT/sat.h ../Frame/frame.h Makefile
        $(COMPILER) -c ../SAT/sat.c $(BUILD_ARG)

frame.o : ../Frame/frame.c ../Frame/frame.h Makefile
        $(COMPILER) -c ../Frame/frame.c $(BUILD_ARG)

clean :
        rm -f *.o main unitTests validation qualification

valgrind :
        valgrind -v --track-origins=yes --leak-check=full \
        --gen-suppressions=yes --show-leak-kinds=all ./unitTests
```

11.3.3 2D dynamic

```
all : main unitTests validation qualification

COMPILER?=gcc
OPTIMIZATION?=-O3
BUILD_ARG=$(OPTIMIZATION) -I../SAT -I../Frame

main : main.o fmb2dt.o frame.o Makefile
        $(COMPILER) -o main main.o fmb2dt.o frame.o

main.o : main.c fmb2dt.h ../Frame/frame.h Makefile
        $(COMPILER) -c main.c $(BUILD_ARG)

unitTests : unitTests.o fmb2dt.o frame.o Makefile
        $(COMPILER) -o unitTests unitTests.o fmb2dt.o frame.o $(LINK_ARG)

unitTests.o : unitTests.c fmb2dt.h ../Frame/frame.h Makefile
        $(COMPILER) -c unitTests.c $(BUILD_ARG)

validation : validation.o fmb2dt.o sat.o frame.o Makefile
        $(COMPILER) -o validation validation.o fmb2dt.o sat.o frame.o
```

```
validation.o : validation.c fmb2dt.h ../SAT/sat.h ../Frame/frame.h Makefile
	$(COMPILER) -c validation.c $(BUILD_ARG)

qualification : qualification.o fmb2dt.o sat.o frame.o Makefile
	$(COMPILER) -o qualification qualification.o fmb2dt.o sat.o frame.o
	    $(LINK_ARG)

qualification.o : qualification.c fmb2dt.h ../SAT/sat.h ../Frame/frame.h
    Makefile
	$(COMPILER) -c qualification.c $(BUILD_ARG)

fmb2dt.o : fmb2dt.c fmb2dt.h ../Frame/frame.h Makefile
	$(COMPILER) -c fmb2dt.c $(BUILD_ARG)

sat.o : ../SAT/sat.c ../SAT/sat.h ../Frame/frame.h Makefile
	$(COMPILER) -c ../SAT/sat.c $(BUILD_ARG)

frame.o : ../Frame/frame.c ../Frame/frame.h Makefile
	$(COMPILER) -c ../Frame/frame.c $(BUILD_ARG)

clean :
	rm -f *.o main unitTests validation qualification

valgrind :
	valgrind -v --track-origins=yes --leak-check=full \
	--gen-suppressions=yes --show-leak-kinds=all ./unitTests
```

11.3.4 3D dynamic

```
all : main unitTests validation qualification

COMPILER?=gcc
OPTIMIZATION?=-O3
BUILD_ARG=$(OPTIMIZATION) -I../SAT -I../Frame

main : main.o fmb3dt.o frame.o Makefile
	$(COMPILER) -o main main.o fmb3dt.o frame.o

main.o : main.c fmb3dt.h ../Frame/frame.h Makefile
	$(COMPILER) -c main.c $(BUILD_ARG)

unitTests : unitTests.o fmb3dt.o frame.o Makefile
	$(COMPILER) -o unitTests unitTests.o fmb3dt.o frame.o $(LINK_ARG)

unitTests.o : unitTests.c fmb3dt.h ../Frame/frame.h Makefile
	$(COMPILER) -c unitTests.c $(BUILD_ARG)

validation : validation.o fmb3dt.o sat.o frame.o Makefile
	$(COMPILER) -o validation validation.o fmb3dt.o sat.o frame.o

validation.o : validation.c fmb3dt.h ../SAT/sat.h ../Frame/frame.h Makefile
	$(COMPILER) -c validation.c $(BUILD_ARG)

qualification : qualification.o fmb3dt.o sat.o frame.o Makefile
	$(COMPILER) -o qualification qualification.o fmb3dt.o sat.o frame.o
	    $(LINK_ARG)

qualification.o : qualification.c fmb3dt.h ../SAT/sat.h ../Frame/frame.h
    Makefile
	$(COMPILER) -c qualification.c $(BUILD_ARG)
```

```
fmb3dt.o : fmb3dt.c fmb3dt.h ../Frame/frame.h Makefile
	$(COMPILER) -c fmb3dt.c $(BUILD_ARG)

sat.o : ../SAT/sat.c ../SAT/sat.h ../Frame/frame.h Makefile
	$(COMPILER) -c ../SAT/sat.c $(BUILD_ARG)

frame.o : ../Frame/frame.c ../Frame/frame.h Makefile
	$(COMPILER) -c ../Frame/frame.c $(BUILD_ARG)

clean :
	rm -f *.o main unitTests validation qualification

valgrind :
	valgrind -v --track-origins=yes --leak-check=full \
	--gen-suppressions=yes --show-leak-kinds=all ./unitTests
```

11.3.5 Doc

```
latex:
	pdflatex -synctex=1 -interaction=nonstopmode -shell-escape  fmb.tex
```

11.4 Dynamic analysis

```
make[1]: Entering directory '/home/bayashi/GitHub/FMB'
cd 2D; make valgrind; cd -
make[2]: Entering directory '/home/bayashi/GitHub/FMB/2D'
valgrind -v --track-origins=yes --leak-check=full \
--gen-suppressions=yes --show-leak-kinds=all ./unitTests
==10713== Memcheck, a memory error detector
==10713== Copyright (C) 2002-2017, and GNU GPL'd, by Julian Seward et al.
==10713== Using Valgrind-3.13.0 and LibVEX; rerun with -h for copyright info
==10713== Command: ./unitTests
==10713==
--10713-- Valgrind options:
--10713--    -v
--10713--    --track-origins=yes
--10713--    --leak-check=full
--10713--    --gen-suppressions=yes
--10713--    --show-leak-kinds=all
--10713-- Contents of /proc/version:
--10713--   Linux version 5.3.0-26-generic (buildd@lgw01-amd64-039) (gcc
    version 7.4.0 (Ubuntu 7.4.0-1ubuntu1~18.04.1)) #28~18.04.1-Ubuntu SMP
    Wed Dec 18 16:40:14 UTC 2019
--10713--
--10713-- Arch and hwcaps: AMD64, LittleEndian, amd64-cx16-lzcnt-rdtscp-sse3
    -avx-avx2-bmi
--10713-- Page sizes: currently 4096, max supported 4096
--10713-- Valgrind library directory: /usr/lib/valgrind
--10713-- Reading syms from /home/bayashi/GitHub/FMB/2D/unitTests
--10713-- Reading syms from /lib/x86_64-linux-gnu/ld-2.27.so
--10713--   Considering /lib/x86_64-linux-gnu/ld-2.27.so ..
--10713--   .. CRC mismatch (computed 1b7c895e wanted 2943108a)
--10713--   Considering /usr/lib/debug/lib/x86_64-linux-gnu/ld-2.27.so ..
--10713--   .. CRC is valid
--10713-- Reading syms from /usr/lib/valgrind/memcheck-amd64-linux
--10713--   Considering /usr/lib/valgrind/memcheck-amd64-linux ..
--10713--   .. CRC mismatch (computed 41ddb025 wanted 9972f546)
--10713--    object doesn't have a symbol table
--10713--    object doesn't have a dynamic symbol table
```

```
--10713-- Scheduler: using generic scheduler lock implementation.
--10713-- Reading suppressions file: /usr/lib/valgrind/default.supp
==10713== embedded gdbserver: reading from /tmp/vgdb-pipe-from-vgdb-to
    -10713-by-bayashi-on-???
==10713== embedded gdbserver: writing to   /tmp/vgdb-pipe-to-vgdb-from
    -10713-by-bayashi-on-???
==10713== embedded gdbserver: shared mem   /tmp/vgdb-pipe-shared-mem-vgdb
    -10713-by-bayashi-on-???
==10713==
==10713== TO CONTROL THIS PROCESS USING vgdb (which you probably
==10713== don't want to do, unless you know exactly what you're doing,
==10713== or are doing some strange experiment):
==10713==   /usr/lib/valgrind/../../bin/vgdb --pid=10713 ...command...
==10713==
==10713== TO DEBUG THIS PROCESS USING GDB: start GDB like this
==10713==   /path/to/gdb ./unitTests
==10713== and then give GDB the following command
==10713==   target remote | /usr/lib/valgrind/../../bin/vgdb --pid=10713
==10713== --pid is optional if only one valgrind process is running
==10713==
--10713-- REDIR: 0x401f2f0 (ld-linux-x86-64.so.2:strlen) redirected to 0
    x580608c1 (???)
--10713-- REDIR: 0x401f0d0 (ld-linux-x86-64.so.2:index) redirected to 0
    x580608db (???)
--10713-- Reading syms from /usr/lib/valgrind/vgpreload_core-amd64-linux.so
--10713--   Considering /usr/lib/valgrind/vgpreload_core-amd64-linux.so ..
--10713--   .. CRC mismatch (computed 50df1b30 wanted 4800a4cf)
--10713--    object doesn't have a symbol table
--10713-- Reading syms from /usr/lib/valgrind/vgpreload_memcheck-amd64-linux
    .so
--10713--   Considering /usr/lib/valgrind/vgpreload_memcheck-amd64-linux.so
    ..
--10713--   .. CRC mismatch (computed f893b962 wanted 95ee359e)
--10713--    object doesn't have a symbol table
==10713== WARNING: new redirection conflicts with existing -- ignoring it
--10713--     old: 0x0401f2f0 (strlen              ) R-> (0000.0) 0x580608c1
     ???
--10713--     new: 0x0401f2f0 (strlen              ) R-> (2007.0) 0x04c32db0
     strlen
--10713-- REDIR: 0x401d360 (ld-linux-x86-64.so.2:strcmp) redirected to 0
    x4c33ee0 (strcmp)
--10713-- REDIR: 0x401f830 (ld-linux-x86-64.so.2:mempcpy) redirected to 0
    x4c374f0 (mempcpy)
--10713-- Reading syms from /lib/x86_64-linux-gnu/libc-2.27.so
--10713--   Considering /lib/x86_64-linux-gnu/libc-2.27.so ..
--10713--   .. CRC mismatch (computed b1c74187 wanted 042cc048)
--10713--   Considering /usr/lib/debug/lib/x86_64-linux-gnu/libc-2.27.so ..
--10713--   .. CRC is valid
--10713-- REDIR: 0x4edac70 (libc.so.6:memmove) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ed9d40 (libc.so.6:strncpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4edaf50 (libc.so.6:strcasecmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ed9790 (libc.so.6:strcat) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ed9d70 (libc.so.6:rindex) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4edc7c0 (libc.so.6:rawmemchr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4edade0 (libc.so.6:mempcpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
```

```
--10713-- REDIR: 0x4edac10 (libc.so.6:bcmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ed9d00 (libc.so.6:strncmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ed9800 (libc.so.6:strcmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4edad40 (libc.so.6:memset) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ef80f0 (libc.so.6:wcschr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ed9ca0 (libc.so.6:strnlen) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ed9870 (libc.so.6:strcspn) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4edafa0 (libc.so.6:strncasecmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ed9840 (libc.so.6:strcpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4edb0e0 (libc.so.6:memcpy@@GLIBC_2.14) redirected to 0
    x4a2a6e0 (_vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ed9da0 (libc.so.6:strpbrk) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ed97c0 (libc.so.6:index) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ed9c70 (libc.so.6:strlen) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ee46c0 (libc.so.6:memrchr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4edaff0 (libc.so.6:strcasecmp_l) redirected to 0x4a2a6e0
    (_vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4edabe0 (libc.so.6:memchr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4ef8eb0 (libc.so.6:wcslen) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4eda050 (libc.so.6:strspn) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4edaf20 (libc.so.6:stpncpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4edaef0 (libc.so.6:stpcpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4edc7f0 (libc.so.6:strchrnul) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4edb040 (libc.so.6:strncasecmp_l) redirected to 0x4a2a6e0
     (_vgnU_ifunc_wrapper)
--10713-- REDIR: 0x4fca3c0 (libc.so.6:__strrchr_avx2) redirected to 0
    x4c32730 (rindex)
--10713-- REDIR: 0x4ed3070 (libc.so.6:malloc) redirected to 0x4c2faa0 (
    malloc)
--10713-- REDIR: 0x4fca1d0 (libc.so.6:__strchrnul_avx2) redirected to 0
    x4c37020 (strchrnul)
--10713-- REDIR: 0x4fcaab0 (libc.so.6:__mempcpy_avx_unaligned_erms)
    redirected to 0x4c37130 (mempcpy)
--10713-- REDIR: 0x4fca590 (libc.so.6:__strlen_avx2) redirected to 0x4c32cf0
     (strlen)
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
```

```
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.500000,0.500000)-maxXY(1.000000,1.000000)

Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.500000,0.500000)-maxXY(1.000000,1.000000)

Co(-0.500000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed (no inter)

Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(-0.500000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.250000,-0.250000) x(0.500000,0.000000) y(0.000000,2.000000)
Succeed
minXY(0.250000,0.000000)-maxXY(0.750000,1.000000)

Co(0.250000,-0.250000) x(0.500000,0.000000) y(0.000000,2.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.250000,0.000000)-maxXY(0.750000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(-0.250000,0.250000) x(2.000000,0.000000) y(0.000000,0.500000)
Succeed
minXY(0.000000,0.250000)-maxXY(1.000000,0.750000)

Co(-0.250000,0.250000) x(2.000000,0.000000) y(0.000000,0.500000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.250000)-maxXY(1.000000,0.750000)

Co(0.000000,0.000000) x(1.000000,1.000000) y(-1.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,1.000000) y(-1.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)
```

```
Co(-0.500000,-0.500000) x(1.000000,1.000000) y(-1.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(0.500000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(-0.500000,-0.500000) x(1.000000,1.000000) y(-1.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(0.500000,1.000000)

Co(1.500000,1.500000) x(1.000000,-1.000000) y(-1.000000,-1.000000)
against
Co(1.000000,0.000000) x(-1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.500000,0.000000)-maxXY(1.000000,1.000000)

Co(1.000000,0.000000) x(-1.000000,0.000000) y(0.000000,1.000000)
against
Co(1.500000,1.500000) x(1.000000,-1.000000) y(-1.000000,-1.000000)
Succeed
minXY(0.500000,0.000000)-maxXY(1.000000,1.000000)

Co(1.000000,0.500000) x(-0.500000,0.500000) y(-0.500000,-0.500000)
against
Co(0.000000,1.000000) x(1.000000,0.000000) y(0.000000,-1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,1.000000) x(1.000000,0.000000) y(0.000000,-1.000000)
against
Co(1.000000,0.500000) x(-0.500000,0.500000) y(-0.500000,-0.500000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(1.000000,1.000000)
against
Co(2.000000,-1.000000) x(0.000000,1.000000) y(-0.500000,1.000000)
Succeed
minXY(1.500000,0.000000)-maxXY(1.666667,1.000000)

Co(2.000000,-1.000000) x(0.000000,1.000000) y(-0.500000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(1.000000,1.000000)
Succeed
minXY(1.500000,0.500000)-maxXY(2.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
Co(1.000000,1.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed
minXY(0.500000,0.000000)-maxXY(1.000000,1.000000)

Co(1.000000,1.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed
minXY(0.500000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
```

```
Co(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed
minXY(0.500000,0.500000)-maxXY(1.000000,1.500000)

Co(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed
minXY(0.500000,0.500000)-maxXY(1.000000,1.500000)

To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
Co(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed
minXY(0.500000,0.500000)-maxXY(1.000000,1.000000)

Co(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed
minXY(0.500000,0.500000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
To(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed
minXY(0.500000,1.000000)-maxXY(1.000000,1.500000)

To(1.000000,2.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed
minXY(0.500000,1.000000)-maxXY(1.000000,1.500000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,1.000000)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(0.000000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,0.500000)

To(0.000000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.000000)-maxXY(1.000000,0.500000)

Co(0.500000,0.500000) x(-0.500000,0.000000) y(0.000000,-0.500000)
against
To(0.000000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
```

```
minXY(0.000000,0.000000)-maxXY(0.500000,0.500000)

To(0.000000,-0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.500000,0.500000) x(-0.500000,0.000000) y(0.000000,-0.500000)
Succeed
minXY(0.000000,0.000000)-maxXY(0.500000,0.500000)

Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed (no inter)

To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
Co(0.500000,0.500000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(1.500000,1.500000) x(-1.500000,0.000000) y(0.000000,-1.500000)
Succeed
minXY(0.000000,0.500000)-maxXY(1.000000,1.000000)

To(1.500000,1.500000) x(-1.500000,0.000000) y(0.000000,-1.500000)
against
Co(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
Succeed
minXY(0.000000,0.500000)-maxXY(1.000000,1.000000)

To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
against
To(1.010000,1.010000) x(-1.000000,0.000000) y(0.000000,-1.000000)
 Succeed (no inter)

To(1.010000,1.010000) x(-1.000000,0.000000) y(0.000000,-1.000000)
against
To(0.000000,0.000000) x(1.000000,0.000000) y(0.000000,1.000000)
 Succeed (no inter)

To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
To(1.000000,1.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
Succeed
minXY(0.500000,0.000000)-maxXY(1.000000,1.000000)

To(1.000000,1.000000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
Succeed
minXY(0.500000,0.000000)-maxXY(1.000000,1.000000)

To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
against
To(1.010000,1.500000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
 Succeed (no inter)

To(1.010000,1.500000) x(-0.500000,-0.500000) y(0.000000,-1.000000)
against
To(0.000000,0.000000) x(1.000000,0.500000) y(0.500000,1.000000)
 Succeed (no inter)
```

```
All unit tests 2D have succeed.
--10713-- REDIR: 0x4ed3950 (libc.so.6:free) redirected to 0x4c30cd0 (free)
==10713==
==10713== HEAP SUMMARY:
==10713==     in use at exit: 0 bytes in 0 blocks
==10713==   total heap usage: 1 allocs, 1 frees, 4,096 bytes allocated
==10713==
==10713== All heap blocks were freed -- no leaks are possible
==10713==
==10713== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
==10713== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
make[2]: Leaving directory '/home/bayashi/GitHub/FMB/2D'
/home/bayashi/GitHub/FMB
cd 2DTime; make valgrind; cd -
make[2]: Entering directory '/home/bayashi/GitHub/FMB/2DTime'
valgrind -v --track-origins=yes --leak-check=full \
--gen-suppressions=yes --show-leak-kinds=all ./unitTests
==10716== Memcheck, a memory error detector
==10716== Copyright (C) 2002-2017, and GNU GPL'd, by Julian Seward et al.
==10716== Using Valgrind-3.13.0 and LibVEX; rerun with -h for copyright info
==10716== Command: ./unitTests
==10716==
--10716-- Valgrind options:
--10716--    -v
--10716--    --track-origins=yes
--10716--    --leak-check=full
--10716--    --gen-suppressions=yes
--10716--    --show-leak-kinds=all
--10716-- Contents of /proc/version:
--10716--   Linux version 5.3.0-26-generic (buildd@lgw01-amd64-039) (gcc
    version 7.4.0 (Ubuntu 7.4.0-1ubuntu1~18.04.1)) #28~18.04.1-Ubuntu SMP
    Wed Dec 18 16:40:14 UTC 2019
--10716--
--10716-- Arch and hwcaps: AMD64, LittleEndian, amd64-cx16-lzcnt-rdtscp-sse3
    -avx-avx2-bmi
--10716-- Page sizes: currently 4096, max supported 4096
--10716-- Valgrind library directory: /usr/lib/valgrind
--10716-- Reading syms from /home/bayashi/GitHub/FMB/2DTime/unitTests
--10716-- Reading syms from /lib/x86_64-linux-gnu/ld-2.27.so
--10716--   Considering /lib/x86_64-linux-gnu/ld-2.27.so ..
--10716--   .. CRC mismatch (computed 1b7c895e wanted 2943108a)
--10716--   Considering /usr/lib/debug/lib/x86_64-linux-gnu/ld-2.27.so ..
--10716--   .. CRC is valid
--10716-- Reading syms from /usr/lib/valgrind/memcheck-amd64-linux
--10716--   Considering /usr/lib/valgrind/memcheck-amd64-linux ..
--10716--   .. CRC mismatch (computed 41ddb025 wanted 9972f546)
--10716--    object doesn't have a symbol table
--10716--    object doesn't have a dynamic symbol table
--10716-- Scheduler: using generic scheduler lock implementation.
--10716-- Reading suppressions file: /usr/lib/valgrind/default.supp
==10716== embedded gdbserver: reading from /tmp/vgdb-pipe-from-vgdb-to
    -10716-by-bayashi-on-???
==10716== embedded gdbserver: writing to   /tmp/vgdb-pipe-to-vgdb-from
    -10716-by-bayashi-on-???
==10716== embedded gdbserver: shared mem   /tmp/vgdb-pipe-shared-mem-vgdb
    -10716-by-bayashi-on-???
==10716==
==10716== TO CONTROL THIS PROCESS USING vgdb (which you probably
==10716== don't want to do, unless you know exactly what you're doing,
==10716== or are doing some strange experiment):
==10716==   /usr/lib/valgrind/../../bin/vgdb --pid=10716 ...command...
==10716==
```

```
==10716== TO DEBUG THIS PROCESS USING GDB: start GDB like this
==10716==   /path/to/gdb ./unitTests
==10716== and then give GDB the following command
==10716==   target remote | /usr/lib/valgrind/../../bin/vgdb --pid=10716
==10716== --pid is optional if only one valgrind process is running
==10716==
--10716-- REDIR: 0x401f2f0 (ld-linux-x86-64.so.2:strlen) redirected to 0
    x580608c1 (???)
--10716-- REDIR: 0x401f0d0 (ld-linux-x86-64.so.2:index) redirected to 0
    x580608db (???)
--10716-- Reading syms from /usr/lib/valgrind/vgpreload_core-amd64-linux.so
--10716--   Considering /usr/lib/valgrind/vgpreload_core-amd64-linux.so ..
--10716--   .. CRC mismatch (computed 50df1b30 wanted 4800a4cf)
--10716--    object doesn't have a symbol table
--10716-- Reading syms from /usr/lib/valgrind/vgpreload_memcheck-amd64-linux
    .so
--10716--   Considering /usr/lib/valgrind/vgpreload_memcheck-amd64-linux.so
    ..
--10716--   .. CRC mismatch (computed f893b962 wanted 95ee359e)
--10716--    object doesn't have a symbol table
==10716== WARNING: new redirection conflicts with existing -- ignoring it
--10716--     old: 0x0401f2f0 (strlen              ) R-> (0000.0) 0x580608c1
     ???
--10716--     new: 0x0401f2f0 (strlen              ) R-> (2007.0) 0x04c32db0
     strlen
--10716-- REDIR: 0x401d360 (ld-linux-x86-64.so.2:strcmp) redirected to 0
    x4c33ee0 (strcmp)
--10716-- REDIR: 0x401f830 (ld-linux-x86-64.so.2:mempcpy) redirected to 0
    x4c374f0 (mempcpy)
--10716-- Reading syms from /lib/x86_64-linux-gnu/libc-2.27.so
--10716--   Considering /lib/x86_64-linux-gnu/libc-2.27.so ..
--10716--   .. CRC mismatch (computed b1c74187 wanted 042cc048)
--10716--   Considering /usr/lib/debug/lib/x86_64-linux-gnu/libc-2.27.so ..
--10716--   .. CRC is valid
--10716-- REDIR: 0x4edac70 (libc.so.6:memmove) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ed9d40 (libc.so.6:strncpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4edaf50 (libc.so.6:strcasecmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ed9790 (libc.so.6:strcat) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ed9d70 (libc.so.6:rindex) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4edc7c0 (libc.so.6:rawmemchr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4edade0 (libc.so.6:mempcpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4edac10 (libc.so.6:bcmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ed9d00 (libc.so.6:strncmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ed9800 (libc.so.6:strcmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4edad40 (libc.so.6:memset) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ef80f0 (libc.so.6:wcschr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ed9ca0 (libc.so.6:strnlen) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ed9870 (libc.so.6:strcspn) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
```

```
--10716-- REDIR: 0x4edafa0 (libc.so.6:strncasecmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ed9840 (libc.so.6:strcpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4edb0e0 (libc.so.6:memcpy@@GLIBC_2.14) redirected to 0
    x4a2a6e0 (_vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ed9da0 (libc.so.6:strpbrk) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ed97c0 (libc.so.6:index) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ed9c70 (libc.so.6:strlen) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ee46c0 (libc.so.6:memrchr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4edaff0 (libc.so.6:strcasecmp_l) redirected to 0x4a2a6e0
    (_vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4edabe0 (libc.so.6:memchr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4ef8eb0 (libc.so.6:wcslen) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4eda050 (libc.so.6:strspn) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4edaf20 (libc.so.6:stpncpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4edaef0 (libc.so.6:stpcpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4edc7f0 (libc.so.6:strchrnul) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4edb040 (libc.so.6:strncasecmp_l) redirected to 0x4a2a6e0
     (_vgnU_ifunc_wrapper)
--10716-- REDIR: 0x4fca3c0 (libc.so.6:__strrchr_avx2) redirected to 0
    x4c32730 (rindex)
--10716-- REDIR: 0x4ed3070 (libc.so.6:malloc) redirected to 0x4c2faa0 (
    malloc)
--10716-- REDIR: 0x4fca1d0 (libc.so.6:__strchrnul_avx2) redirected to 0
    x4c37020 (strchrnul)
--10716-- REDIR: 0x4fcaab0 (libc.so.6:__mempcpy_avx_unaligned_erms)
    redirected to 0x4c37130 (mempcpy)
--10716-- REDIR: 0x4fca590 (libc.so.6:__strlen_avx2) redirected to 0x4c32cf0
     (strlen)
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(-1.000000,0.000000) s(-1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
 Succeed (no inter)

Co(-1.000000,0.000000) s(-1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(-1.010000,-1.010000) s(1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
 Succeed (no inter)

Co(-1.010000,-1.010000) s(1.000000,0.000000) x(1.000000,0.000000) y
```

```
    (0.000000,1.000000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(-1.000000,0.000000) s(1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed
minXYT(-1.000000,0.000000,0.000000)-maxXYT(2.000000,1.000000,1.000000)

Co(-1.000000,0.000000) s(1.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed
minXYT(-1.000000,0.000000,0.000000)-maxXYT(1.000000,1.000000,1.000000)

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(-1.000000,0.250000) s(4.000000,0.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
Succeed
minXYT(-1.500000,0.000000,0.125000)-maxXYT(2.500000,1.000000,0.500000)

Co(-1.000000,0.250000) s(4.000000,0.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed
minXYT(-0.500000,0.000000,0.125000)-maxXYT(1.500000,1.000000,0.500000)

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.250000,-1.000000) s(0.000000,4.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
Succeed
minXYT(0.000000,-1.500000,0.125000)-maxXYT(1.000000,2.500000,0.500000)

Co(0.250000,-1.000000) s(0.000000,4.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed
minXYT(0.000000,-0.500000,0.125000)-maxXYT(1.000000,1.500000,0.500000)

Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
against
Co(0.900000,-1.000000) s(0.000000,4.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
Succeed
minXYT(0.000000,-1.500000,0.125000)-maxXYT(1.400000,2.500000,0.500000)
```

```
Co(0.900000,-1.000000) s(0.000000,4.000000) x(0.500000,0.000000) y
    (0.000000,0.500000)
against
Co(0.000000,0.000000) s(0.000000,0.000000) x(1.000000,0.000000) y
    (0.000000,1.000000)
Succeed
minXYT(0.000000,-0.500000,0.125000)-maxXYT(1.400000,1.500000,0.500000)

All unit tests 2DTime have succeed.
--10716-- REDIR: 0x4ed3950 (libc.so.6:free) redirected to 0x4c30cd0 (free)
==10716==
==10716== HEAP SUMMARY:
==10716==     in use at exit: 0 bytes in 0 blocks
==10716==   total heap usage: 1 allocs, 1 frees, 4,096 bytes allocated
==10716==
==10716== All heap blocks were freed -- no leaks are possible
==10716==
==10716== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
==10716== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
make[2]: Leaving directory '/home/bayashi/GitHub/FMB/2DTime'
/home/bayashi/GitHub/FMB
cd 3D; make valgrind; cd -
make[2]: Entering directory '/home/bayashi/GitHub/FMB/3D'
valgrind -v --track-origins=yes --leak-check=full \
--gen-suppressions=yes --show-leak-kinds=all ./unitTests
==10719== Memcheck, a memory error detector
==10719== Copyright (C) 2002-2017, and GNU GPL'd, by Julian Seward et al.
==10719== Using Valgrind-3.13.0 and LibVEX; rerun with -h for copyright info
==10719== Command: ./unitTests
==10719==
--10719-- Valgrind options:
--10719--    -v
--10719--    --track-origins=yes
--10719--    --leak-check=full
--10719--    --gen-suppressions=yes
--10719--    --show-leak-kinds=all
--10719-- Contents of /proc/version:
--10719--   Linux version 5.3.0-26-generic (buildd@lgw01-amd64-039) (gcc
    version 7.4.0 (Ubuntu 7.4.0-1ubuntu1~18.04.1)) #28~18.04.1-Ubuntu SMP
    Wed Dec 18 16:40:14 UTC 2019
--10719--
--10719-- Arch and hwcaps: AMD64, LittleEndian, amd64-cx16-lzcnt-rdtscp-sse3
    -avx-avx2-bmi
--10719-- Page sizes: currently 4096, max supported 4096
--10719-- Valgrind library directory: /usr/lib/valgrind
--10719-- Reading syms from /home/bayashi/GitHub/FMB/3D/unitTests
--10719-- Reading syms from /lib/x86_64-linux-gnu/ld-2.27.so
--10719--   Considering /lib/x86_64-linux-gnu/ld-2.27.so ..
--10719--   .. CRC mismatch (computed 1b7c895e wanted 2943108a)
--10719--   Considering /usr/lib/debug/lib/x86_64-linux-gnu/ld-2.27.so ..
--10719--   .. CRC is valid
--10719-- Reading syms from /usr/lib/valgrind/memcheck-amd64-linux
--10719--   Considering /usr/lib/valgrind/memcheck-amd64-linux ..
--10719--   .. CRC mismatch (computed 41ddb025 wanted 9972f546)
--10719--    object doesn't have a symbol table
--10719--    object doesn't have a dynamic symbol table
--10719-- Scheduler: using generic scheduler lock implementation.
--10719-- Reading suppressions file: /usr/lib/valgrind/default.supp
==10719== embedded gdbserver: reading from /tmp/vgdb-pipe-from-vgdb-to
    -10719-by-bayashi-on-???
==10719== embedded gdbserver: writing to   /tmp/vgdb-pipe-to-vgdb-from
    -10719-by-bayashi-on-???
```

```
==10719== embedded gdbserver: shared mem   /tmp/vgdb-pipe-shared-mem-vgdb
    -10719-by-bayashi-on-???
==10719==
==10719== TO CONTROL THIS PROCESS USING vgdb (which you probably
==10719== don't want to do, unless you know exactly what you're doing,
==10719== or are doing some strange experiment):
==10719==   /usr/lib/valgrind/../../bin/vgdb --pid=10719 ...command...
==10719==
==10719== TO DEBUG THIS PROCESS USING GDB: start GDB like this
==10719==   /path/to/gdb ./unitTests
==10719== and then give GDB the following command
==10719==   target remote | /usr/lib/valgrind/../../bin/vgdb --pid=10719
==10719== --pid is optional if only one valgrind process is running
==10719==
--10719-- REDIR: 0x401f2f0 (ld-linux-x86-64.so.2:strlen) redirected to 0
    x580608c1 (???)
--10719-- REDIR: 0x401f0d0 (ld-linux-x86-64.so.2:index) redirected to 0
    x580608db (???)
--10719-- Reading syms from /usr/lib/valgrind/vgpreload_core-amd64-linux.so
--10719--   Considering /usr/lib/valgrind/vgpreload_core-amd64-linux.so ..
--10719--   .. CRC mismatch (computed 50df1b30 wanted 4800a4cf)
--10719--    object doesn't have a symbol table
--10719-- Reading syms from /usr/lib/valgrind/vgpreload_memcheck-amd64-linux
    .so
--10719--   Considering /usr/lib/valgrind/vgpreload_memcheck-amd64-linux.so
    ..
--10719--   .. CRC mismatch (computed f893b962 wanted 95ee359e)
--10719--    object doesn't have a symbol table
==10719== WARNING: new redirection conflicts with existing -- ignoring it
--10719--     old: 0x0401f2f0 (strlen              ) R-> (0000.0) 0x580608c1
     ???
--10719--     new: 0x0401f2f0 (strlen              ) R-> (2007.0) 0x04c32db0
     strlen
--10719-- REDIR: 0x401d360 (ld-linux-x86-64.so.2:strcmp) redirected to 0
    x4c33ee0 (strcmp)
--10719-- REDIR: 0x401f830 (ld-linux-x86-64.so.2:mempcpy) redirected to 0
    x4c374f0 (mempcpy)
--10719-- Reading syms from /lib/x86_64-linux-gnu/libc-2.27.so
--10719--   Considering /lib/x86_64-linux-gnu/libc-2.27.so ..
--10719--   .. CRC mismatch (computed b1c74187 wanted 042cc048)
--10719--   Considering /usr/lib/debug/lib/x86_64-linux-gnu/libc-2.27.so ..
--10719--   .. CRC is valid
--10719-- REDIR: 0x4edac70 (libc.so.6:memmove) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ed9d40 (libc.so.6:strncpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4edaf50 (libc.so.6:strcasecmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ed9790 (libc.so.6:strcat) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ed9d70 (libc.so.6:rindex) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4edc7c0 (libc.so.6:rawmemchr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4edade0 (libc.so.6:mempcpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4edac10 (libc.so.6:bcmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ed9d00 (libc.so.6:strncmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ed9800 (libc.so.6:strcmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
```

```
--10719-- REDIR: 0x4edad40 (libc.so.6:memset) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ef80f0 (libc.so.6:wcschr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ed9ca0 (libc.so.6:strnlen) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ed9870 (libc.so.6:strcspn) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4edafa0 (libc.so.6:strncasecmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ed9840 (libc.so.6:strcpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4edb0e0 (libc.so.6:memcpy@@GLIBC_2.14) redirected to 0
    x4a2a6e0 (_vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ed9da0 (libc.so.6:strpbrk) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ed97c0 (libc.so.6:index) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ed9c70 (libc.so.6:strlen) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ee46c0 (libc.so.6:memrchr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4edaff0 (libc.so.6:strcasecmp_l) redirected to 0x4a2a6e0
    (_vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4edabe0 (libc.so.6:memchr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4ef8eb0 (libc.so.6:wcslen) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4eda050 (libc.so.6:strspn) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4edaf20 (libc.so.6:stpncpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4edaef0 (libc.so.6:stpcpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4edc7f0 (libc.so.6:strchrnul) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4edb040 (libc.so.6:strncasecmp_l) redirected to 0x4a2a6e0
     (_vgnU_ifunc_wrapper)
--10719-- REDIR: 0x4fca3c0 (libc.so.6:__strrchr_avx2) redirected to 0
    x4c32730 (rindex)
--10719-- REDIR: 0x4ed3070 (libc.so.6:malloc) redirected to 0x4c2faa0 (
    malloc)
--10719-- REDIR: 0x4fca1d0 (libc.so.6:__strchrnul_avx2) redirected to 0
    x4c37020 (strchrnul)
--10719-- REDIR: 0x4fcaab0 (libc.so.6:__mempcpy_avx_unaligned_erms)
    redirected to 0x4c37130 (mempcpy)
--10719-- REDIR: 0x4fca590 (libc.so.6:__strlen_avx2) redirected to 0x4c32cf0
     (strlen)
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,0.000000,0.000000)-maxXYZ(1.000000,1.000000,1.000000)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
```

```
minXYZ(0.000000,0.000000,0.000000)-maxXYZ(1.000000,1.000000,1.000000)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.500000,0.500000,0.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.500000,0.500000,0.500000)-maxXYZ(1.000000,1.000000,1.000000)

Co(0.500000,0.500000,0.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.500000,0.500000,0.500000)-maxXYZ(1.000000,1.000000,1.000000)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(-0.500000,-0.500000,-0.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,0.000000,0.000000)-maxXYZ(0.500000,0.500000,0.500000)

Co(-0.500000,-0.500000,-0.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,0.000000,0.000000)-maxXYZ(0.500000,0.500000,0.500000)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(1.500000,1.500000,1.500000) x(-1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,-1.000000)
Succeed
minXYZ(0.500000,0.500000,0.500000)-maxXYZ(1.000000,1.000000,1.000000)

Co(1.500000,1.500000,1.500000) x(-1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,-1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.500000,0.500000,0.500000)-maxXYZ(1.000000,1.000000,1.000000)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.500000,1.500000,-1.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(0.500000,1.500000,-1.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
```

```
 Succeed (no inter)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,-1.000000)
against
Co(0.500000,1.500000,-1.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.500000,0.500000,-1.000000)-maxXYZ(1.000000,1.000000,-0.500000)

Co(0.500000,1.500000,-1.500000) x(1.000000,0.000000,0.000000) y
    (0.000000,-1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,-1.000000)
Succeed
minXYZ(0.500000,0.500000,-1.000000)-maxXYZ(1.000000,1.000000,-0.500000)

Co(-1.010000,-1.010000,-1.010000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(-1.010000,-1.010000,-1.010000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,-0.500000,0.000000)-maxXYZ(1.000000,0.000000,1.000000)

Co(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,-0.500000,0.000000)-maxXYZ(1.000000,0.000000,1.000000)

To(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
Co(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
To(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)
```

```
Co(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,-0.500000,0.000000)-maxXYZ(1.000000,0.000000,0.750000)

To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
Co(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,-0.500000,0.000000)-maxXYZ(1.000000,0.000000,1.000000)

To(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
To(-1.000000,-1.000000,-1.000000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
 Succeed (no inter)

To(-0.500000,-1.000000,-0.500000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
against
To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,-0.500000,0.000000)-maxXYZ(0.500000,0.000000,0.500000)

To(0.000000,-0.500000,0.000000) x(1.000000,0.000000,0.000000) y
    (0.000000,1.000000,0.000000) z(0.000000,0.000000,1.000000)
against
To(-0.500000,-1.000000,-0.500000) x(1.000000,0.000000,0.000000) y
    (1.000000,1.000000,1.000000) z(0.000000,0.000000,1.000000)
Succeed
minXYZ(0.000000,-0.500000,0.000000)-maxXYZ(0.500000,0.000000,0.500000)

All unit tests 3D have succeed.
--10719-- REDIR: 0x4ed3950 (libc.so.6:free) redirected to 0x4c30cd0 (free)
==10719==
==10719== HEAP SUMMARY:
==10719==     in use at exit: 0 bytes in 0 blocks
==10719==   total heap usage: 1 allocs, 1 frees, 4,096 bytes allocated
==10719==
==10719== All heap blocks were freed -- no leaks are possible
==10719==
==10719== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
==10719== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
make[2]: Leaving directory '/home/bayashi/GitHub/FMB/3D'
/home/bayashi/GitHub/FMB
cd 3DTime; make valgrind; cd -
make[2]: Entering directory '/home/bayashi/GitHub/FMB/3DTime'
valgrind -v --track-origins=yes --leak-check=full \
```

```
--gen-suppressions=yes --show-leak-kinds=all ./unitTests
==10722== Memcheck, a memory error detector
==10722== Copyright (C) 2002-2017, and GNU GPL'd, by Julian Seward et al.
==10722== Using Valgrind-3.13.0 and LibVEX; rerun with -h for copyright info
==10722== Command: ./unitTests
==10722==
--10722-- Valgrind options:
--10722--    -v
--10722--    --track-origins=yes
--10722--    --leak-check=full
--10722--    --gen-suppressions=yes
--10722--    --show-leak-kinds=all
--10722-- Contents of /proc/version:
--10722--   Linux version 5.3.0-26-generic (buildd@lgw01-amd64-039) (gcc
    version 7.4.0 (Ubuntu 7.4.0-1ubuntu1~18.04.1)) #28~18.04.1-Ubuntu SMP
    Wed Dec 18 16:40:14 UTC 2019
--10722--
--10722-- Arch and hwcaps: AMD64, LittleEndian, amd64-cx16-lzcnt-rdtscp-sse3
    -avx-avx2-bmi
--10722-- Page sizes: currently 4096, max supported 4096
--10722-- Valgrind library directory: /usr/lib/valgrind
--10722-- Reading syms from /home/bayashi/GitHub/FMB/3DTime/unitTests
--10722-- Reading syms from /lib/x86_64-linux-gnu/ld-2.27.so
--10722--   Considering /lib/x86_64-linux-gnu/ld-2.27.so ..
--10722--   .. CRC mismatch (computed 1b7c895e wanted 2943108a)
--10722--   Considering /usr/lib/debug/lib/x86_64-linux-gnu/ld-2.27.so ..
--10722--   .. CRC is valid
--10722-- Reading syms from /usr/lib/valgrind/memcheck-amd64-linux
--10722--   Considering /usr/lib/valgrind/memcheck-amd64-linux ..
--10722--   .. CRC mismatch (computed 41ddb025 wanted 9972f546)
--10722--    object doesn't have a symbol table
--10722--    object doesn't have a dynamic symbol table
--10722-- Scheduler: using generic scheduler lock implementation.
--10722-- Reading suppressions file: /usr/lib/valgrind/default.supp
==10722== embedded gdbserver: reading from /tmp/vgdb-pipe-from-vgdb-to
    -10722-by-bayashi-on-???
==10722== embedded gdbserver: writing to   /tmp/vgdb-pipe-to-vgdb-from
    -10722-by-bayashi-on-???
==10722== embedded gdbserver: shared mem   /tmp/vgdb-pipe-shared-mem-vgdb
    -10722-by-bayashi-on-???
==10722==
==10722== TO CONTROL THIS PROCESS USING vgdb (which you probably
==10722== don't want to do, unless you know exactly what you're doing,
==10722== or are doing some strange experiment):
==10722==   /usr/lib/valgrind/../../bin/vgdb --pid=10722 ...command...
==10722==
==10722== TO DEBUG THIS PROCESS USING GDB: start GDB like this
==10722==   /path/to/gdb ./unitTests
==10722== and then give GDB the following command
==10722==   target remote | /usr/lib/valgrind/../../bin/vgdb --pid=10722
==10722== --pid is optional if only one valgrind process is running
==10722==
--10722-- REDIR: 0x401f2f0 (ld-linux-x86-64.so.2:strlen) redirected to 0
    x580608c1 (???)
--10722-- REDIR: 0x401f0d0 (ld-linux-x86-64.so.2:index) redirected to 0
    x580608db (???)
--10722-- Reading syms from /usr/lib/valgrind/vgpreload_core-amd64-linux.so
--10722--   Considering /usr/lib/valgrind/vgpreload_core-amd64-linux.so ..
--10722--   .. CRC mismatch (computed 50df1b30 wanted 4800a4cf)
--10722--    object doesn't have a symbol table
--10722-- Reading syms from /usr/lib/valgrind/vgpreload_memcheck-amd64-linux
    .so
```

```
--10722--   Considering /usr/lib/valgrind/vgpreload_memcheck-amd64-linux.so
    ..
--10722--   .. CRC mismatch (computed f893b962 wanted 95ee359e)
--10722--    object doesn't have a symbol table
==10722== WARNING: new redirection conflicts with existing -- ignoring it
--10722--     old: 0x0401f2f0 (strlen              ) R-> (0000.0) 0x580608c1
     ???
--10722--     new: 0x0401f2f0 (strlen              ) R-> (2007.0) 0x04c32db0
     strlen
--10722-- REDIR: 0x401d360 (ld-linux-x86-64.so.2:strcmp) redirected to 0
    x4c33ee0 (strcmp)
--10722-- REDIR: 0x401f830 (ld-linux-x86-64.so.2:mempcpy) redirected to 0
    x4c374f0 (mempcpy)
--10722-- Reading syms from /lib/x86_64-linux-gnu/libc-2.27.so
--10722--   Considering /lib/x86_64-linux-gnu/libc-2.27.so ..
--10722--   .. CRC mismatch (computed b1c74187 wanted 042cc048)
--10722--   Considering /usr/lib/debug/lib/x86_64-linux-gnu/libc-2.27.so ..
--10722--   .. CRC is valid
--10722-- REDIR: 0x4edac70 (libc.so.6:memmove) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ed9d40 (libc.so.6:strncpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4edaf50 (libc.so.6:strcasecmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ed9790 (libc.so.6:strcat) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ed9d70 (libc.so.6:rindex) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4edc7c0 (libc.so.6:rawmemchr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4edade0 (libc.so.6:mempcpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4edac10 (libc.so.6:bcmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ed9d00 (libc.so.6:strncmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ed9800 (libc.so.6:strcmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4edad40 (libc.so.6:memset) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ef80f0 (libc.so.6:wcschr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ed9ca0 (libc.so.6:strnlen) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ed9870 (libc.so.6:strcspn) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4edafa0 (libc.so.6:strncasecmp) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ed9840 (libc.so.6:strcpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4edb0e0 (libc.so.6:memcpy@@GLIBC_2.14) redirected to 0
    x4a2a6e0 (_vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ed9da0 (libc.so.6:strpbrk) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ed97c0 (libc.so.6:index) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ed9c70 (libc.so.6:strlen) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ee46c0 (libc.so.6:memrchr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4edaff0 (libc.so.6:strcasecmp_l) redirected to 0x4a2a6e0
    (_vgnU_ifunc_wrapper)
```

```
--10722-- REDIR: 0x4edabe0 (libc.so.6:memchr) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4ef8eb0 (libc.so.6:wcslen) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4eda050 (libc.so.6:strspn) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4edaf20 (libc.so.6:stpncpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4edaef0 (libc.so.6:stpcpy) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4edc7f0 (libc.so.6:strchrnul) redirected to 0x4a2a6e0 (
    _vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4edb040 (libc.so.6:strncasecmp_l) redirected to 0x4a2a6e0
     (_vgnU_ifunc_wrapper)
--10722-- REDIR: 0x4fca3c0 (libc.so.6:__strrchr_avx2) redirected to 0
    x4c32730 (rindex)
--10722-- REDIR: 0x4ed3070 (libc.so.6:malloc) redirected to 0x4c2faa0 (
    malloc)
--10722-- REDIR: 0x4fca1d0 (libc.so.6:__strchrnul_avx2) redirected to 0
    x4c37020 (strchrnul)
--10722-- REDIR: 0x4fcaab0 (libc.so.6:__mempcpy_avx_unaligned_erms)
    redirected to 0x4c37130 (mempcpy)
--10722-- REDIR: 0x4fca590 (libc.so.6:__strlen_avx2) redirected to 0x4c32cf0
     (strlen)
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(-1.000000,0.000000,0.000000) s(-1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(-1.000000,0.000000,0.000000) s(-1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(-1.010000,-1.010000,0.000000) s(1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(-1.010000,-1.010000,0.000000) s(1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
 Succeed (no inter)

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
```

```
    (0.000000,0.000000,1.000000)
against
Co(-1.000000,0.000000,0.000000) s(1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(-1.000000,0.000000,0.000000,0.000000)-maxXYZT
    (2.000000,1.000000,1.000000,1.000000)

Co(-1.000000,0.000000,0.000000) s(1.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(-1.000000,0.000000,0.000000,0.000000)-maxXYZT
    (1.000000,1.000000,1.000000,1.000000)

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(-1.000000,0.250000,0.000000) s(4.000000,0.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(-1.500000,0.000000,0.000000,0.125000)-maxXYZT
    (2.500000,1.000000,1.000000,0.500000)

Co(-1.000000,0.250000,0.000000) s(4.000000,0.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(-0.500000,0.000000,0.000000,0.125000)-maxXYZT
    (1.500000,1.000000,1.000000,0.500000)

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.250000,-1.000000,0.000000) s(0.000000,4.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(0.000000,-1.500000,0.000000,0.125000)-maxXYZT
    (1.000000,2.500000,1.000000,0.500000)

Co(0.250000,-1.000000,0.000000) s(0.000000,4.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(0.000000,-0.500000,0.000000,0.125000)-maxXYZT
```

```
    (1.000000,1.500000,1.000000,0.500000)

Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.900000,-1.000000,0.000000) s(0.000000,4.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(0.000000,-1.500000,0.000000,0.125000)-maxXYZT
    (1.400000,2.500000,1.000000,0.500000)

Co(0.900000,-1.000000,0.000000) s(0.000000,4.000000,0.000000) x
    (0.500000,0.000000,0.000000) y(0.000000,0.500000,0.000000) z
    (0.000000,0.000000,1.000000)
against
Co(0.000000,0.000000,0.000000) s(0.000000,0.000000,0.000000) x
    (1.000000,0.000000,0.000000) y(0.000000,1.000000,0.000000) z
    (0.000000,0.000000,1.000000)
Succeed
minXYZT(0.000000,-0.500000,0.000000,0.125000)-maxXYZT
    (1.400000,1.500000,1.000000,0.500000)

All unit tests 3DTime have succeed.
--10722-- REDIR: 0x4ed3950 (libc.so.6:free) redirected to 0x4c30cd0 (free)
==10722==
==10722== HEAP SUMMARY:
==10722==     in use at exit: 0 bytes in 0 blocks
==10722==   total heap usage: 1 allocs, 1 frees, 4,096 bytes allocated
==10722==
==10722== All heap blocks were freed -- no leaks are possible
==10722==
==10722== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
==10722== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
make[2]: Leaving directory '/home/bayashi/GitHub/FMB/3DTime'
/home/bayashi/GitHub/FMB
make[1]: Leaving directory '/home/bayashi/GitHub/FMB'
```

References

[1] J.J.-B. Fourier. Oeuvres II. Paris, 1890

[2] T.S. Motzkin. *Beiträge zur Theorie der linearen Ungleichungen*. Thesis, 1936. Reprinted in: *Theodore S. Motzkin: selected papers* (D.Cantor et al., eds,), Birkhäuser, Boston, 1983.