PW/TP 13-14 Second-order Linear PDE's

Using methods of ODE's

Exercise 1.

(a) Solve $x\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} = 0$

(b) Find the particular solution for which $z(x,0) = x^5 + x, z(2,y) = 3y^4$

Exercise 2.

Solve for $u = u(x,y)$:

1. $u_{yy} = 0$
2. $u_{xx} + 16\pi^2 u = 0$
3. $25u_{yy} - 4u = 0$
4. $u_y + y^2 u = 0$
5. $2u_{xx} + 9u_x + 4u = -3\cos x - 29\sin x$
6. $u_{yy} + 6u_y + 13u = 4e^{3y}$
7. $u_{xy} = u_x$
8. $x^2 u_x x + 2xu_x - 2u = 0$

Second-order Linear PDE's with constant coefficients

Method 1

Solve the homogeneous equation by assuming $u = e^{ax+by}$, where a and b are constants to be determined.

Solve the non-homogeneous equation by using the method of undetermined coefficients.

Exercise 3.

1. $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$
2. $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$
3. $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = x$
4. $\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^3 \partial y} = 4$
5. $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 3u$
6. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

7. $\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x \sin y$

Method 2. Seperation of variables

Assume that a solution can be expressed as a product of unknown functions each of which depends on only one of the independent variables. $u(x, y) = X(x)Y(y)$.

Exercise 4.

1. $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, \quad u(x, 0) = 4e^{-x}$
2. $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2},\ u(0, t) = 0, u(\pi, t) = 0, \quad u(x, 0) = 2\sin 3x - 4\sin 5x$
3. $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ u_x(0, t) = 0, u(2, t) = 0, \quad u(x, 0) = 8\cos\frac{3\pi x}{4} - 6\cos\frac{9\pi x}{4}$
4. $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y} + u, \quad u(x, 0) = 3e^{-5x} + 2e^{-3x}$
5. $\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2}, \quad y(0, t) = y(5, t) = 0, \quad y(x, 0) = 0, \quad y_t(x, 0) = 5\sin \pi x$
6. $\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2}, \quad y(0, t) = y(5, t) = 0, \quad y(x, 0) = 0, \quad y_t(x, 0) = 3\sin 2\pi x - 2\sin 5\pi x$
7. $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, \quad u(0, t) = u(4, t) = 0, \quad u(x, 0) = 25x \quad \text{where } 0 < x < 4, t > 0$
8. $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad u_x(0, t) = u_x(\pi, t) = 0, \quad u(x, 0) = f(x) \quad \text{where } 0 < x < 4, t > 0$