## Exercise 7 (2.56)

$$x + x = y = 1$$

$$x + x = y = 0$$

$$c = -\frac{x}{2}$$

$$C = -\frac{x}{yy'}$$

$$x^{2} - \frac{xy^{2}}{yy'} = 1 \quad \Leftrightarrow \quad x^{2}y' - xy = y'$$

$$\Leftrightarrow \quad \boxed{0 = (1 - x^{2})y' + xy}$$

(b) 
$$y^2 = ax + b$$

$$\frac{\partial}{\partial y}$$
  $\frac{\partial}{\partial x} = \frac{\partial}{\partial y} + \frac{\partial}{\partial x} = \frac{\partial}{\partial y} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}$ 

1) 
$$(2.59 \text{ b}) \propto^3 y''' = 1 + \chi^4 ; y(1) = v'(1) = y''(1) = 0$$
  
=> Separable

$$\Rightarrow \text{Separable}$$

$$\int dy'' = \int \frac{1+x^4}{x^4} dx$$

 $\int dx' = \left( -\frac{4}{2x^2} + \frac{x^2}{9} \right) dx$ 

 $\int dy = \left(\frac{\Delta}{2} + \frac{x^3}{3} - \frac{\partial}{\partial y}\right) dx$ 

$$(2.53 b) x^{3}y = 1 + x^{4} y (1) = 0$$
=> Separable
$$\int dy'' = \int \frac{1 + x^{4}}{x^{3}} dx$$

$$y'' = -\frac{1}{2x^{2}} + \frac{x^{2}}{x^{2}} + C_{1} \quad y'(1) = 0$$

 $y' = -\frac{4}{2}\left(-\frac{4}{2} - \frac{x^{3}}{3} + C_{2}\right) \quad y'(1) = 0$   $0 = -\frac{4}{2}\left(-1 - \frac{4}{3} + C_{2}\right)$   $= \sqrt{\frac{4}{2}} + \frac{x^{3}}{C} - \frac{4}{3}\right) dx \qquad C_{2} = \frac{4}{3}$ 

 $y = \frac{4}{2} \ln x + \frac{x^4}{94} - \frac{2}{3} x + C_3$  y(x) = 0

g= = 1 lnx + x4 - 2x+ 5

0=- + + + + = + == 0

 $0 = 0 + \frac{1}{34} - \frac{2}{3} + C_3$ 

C3= 15 = 5

1) 
$$(2.59 \text{ b}) x^3 y'' = 1 + x^4 ; y(1) = y'(1) = y''(1) = 0$$
  
=> Separable
$$\int dy'' = \int \frac{1+x^4}{y^3} dx$$

$$\frac{2)(3.64b)(x^{2}+x-y^{2})dx-ydy=0}{M}=-2y\neq\frac{2N}{8x}=0$$

$$R(x) = \frac{1}{N} \left( \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial M}{\partial y} \right) = -\frac{1}{y} \left( -\frac{\partial M}{\partial y} - \frac{\partial M}{\partial y} \right) = -\frac{1}{y} \left( -\frac{$$

$$R(x) = \frac{1}{N} \left( \frac{\partial H}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{1}{y} \left( -2y - 0 \right) = 2$$

$$F(x) = e^{\int 2dx} = e^{2x}$$

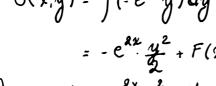
$$\frac{2}{N} \left( \frac{2}{N} - \frac{2}{N} \right) dy = 0$$

$$e^{2x} \left( x^{2} + x - y^{2} \right) dx - e^{2x} y dy = 0$$

$$\frac{\partial}{\partial y} = -2e^{2x}y$$

$$\frac{\partial}{\partial x} = -2e^{2x}y$$

$$U(x,y) = \int (-e^{2x}y)dy + F(x)$$



U(x,4) = = = x2e2x == = exxy2+C

ce = x2-y2

$$\frac{\partial}{\partial x} U(x,y) = -e^{2x}y^2 + F'(x) = \Lambda$$

$$\int F'(x) = \int e^{2x} (x^2 + x) dx$$

$$\frac{3}{3x} U(x,y) = -c^{2x} y^2 + F'(x) = M = e^{2x} (x^2 + x - y^2)$$

F(x) = 1 x2e2x+c

$$= -e^{2x} \frac{y^2}{2} + F(x)$$

$$+ F(x)$$

Jx exx u= x2 du= 2xclx

du=e2xdx v= 4 e2x

1 1 x 2 e2x - 5x e2 x dx

3) 
$$(2.67 \text{ b}) 2x^2y' = xy + y$$

$$\frac{dy}{dy} = \frac{4}{3}y^3$$

$$\frac{dy}{dx} - \frac{1}{2x}y = \frac{1}{2x^2}y^3$$

$$\frac{dy}{dx} - \frac{1}{2x}y = \frac{1}{2x^2}y^3 + \frac{1}{2x^2}y^{-2} + \frac{1}{2x^2}y^{-3} + \frac{1}{2x^2$$

dv + y = - 4

 $\frac{dv}{dx} + \frac{1}{x} v = -\frac{1}{x^2}$ 

 $\frac{x}{y^2} = -\ln x + C$   $\begin{cases} \frac{x}{y^2} = -C \chi^{-1} \end{cases}$ 

N= e = X

 $x \cdot v = \int x \cdot \left(-\frac{1}{x^2}\right) dx + C$ 

 $x \cdot v = -\ln x + c$  (with  $v = y^{-2}$ )

$$\frac{dy}{dx} - \frac{4}{2x}y = \frac{4}{2x^2}y^3 + \frac{3}{2y^2}$$

$$\frac{dy}{dx} - \frac{\Delta}{2x} y = \frac{\Delta}{2x^2} y^3$$

3) 
$$(2.67 \text{ b}) 2x^2y' = xy + y^3$$
  
 $\frac{dy}{dx} - \frac{4}{2x}y = \frac{4}{2x^2}y^3$ 

$$y' = xy + y^3$$
  
=  $\frac{1}{9} \times 2 y^3 + x^3$ 



4) 
$$(2.72 a) xy'' - 3y' = x^2$$

$$\frac{4)(3.12a) \times y - 3y = x}{y \text{ is missing : } y' = P}$$

y is missing: 
$$y' = P$$
  $y'' = \frac{dp}{dx}$   
 $\times \frac{dp}{dy} - 3p = x^2$ 

$$x \frac{dp}{dx} - 3p = x^{2}$$

$$\frac{dp}{dx} - \frac{3}{x}p = x \quad (= linear first order equation)$$

$$p = e^{-\int \frac{3}{x}x} \frac{dx}{dx} = \frac{3}{x} e^{-3} e^{-x} = x^{3}$$

$$x^{3}p = \int x^{3} \times dx + c_{n} = -\frac{1}{x} + c_{n}$$

$$\frac{dy}{dx} = a = x^{2} + C_{n}x^{3}$$

$$\frac{dy}{dx} = p = -x^{2} + C_{1}x^{3}$$

$$y = -\frac{4}{3}x^{3} + \frac{C_{1}}{4}x^{4} + C_{2}$$

$$y = -\frac{4}{3} x^3 + \frac{C_1}{4} x^4 + C_2$$

2.61 b) 
$$\frac{dy}{dx} = \frac{3 - 4 \times y^2}{4 \times^2 y + 6 y}$$

2.61 b) 
$$\frac{dy}{dx} = \frac{3-4xy}{4x^2y+6}$$

$$\frac{\left(4x^{2}y+6y^{2}\right)dy+\left(4xy^{2}-3\right)dx=0}{N}$$

$$\frac{\partial H}{\partial x}=8xy$$

$$\frac{\partial N}{\partial y}=8xy$$

2x2y2+2y3-3x= c with y(1)=-1 2> c=-3

 $\int 2x^2y^2 + 2y^3 - 3x = -3$ 

$$\frac{\partial H}{\partial x} = 8xy \qquad \frac{\partial U}{\partial y} = 8xy$$

$$\int M dy = \int (4x^2y + 6y^2) dy = 4x^2y^2 + 3y^3$$

$$\frac{\partial}{\partial x} (2x^2y^2 + 2y^3) = 4xy^2$$

$$(V - 4xy^2) dx = \int (-3) dx = -3x$$

5) 
$$(2.61 \text{ b}) \frac{dy}{dx} = \frac{3-4xy^2}{4x^2y+6y^2}; y(1)=-1$$
  
 $(4x^2y+6y^2) dy + (4xy^2-3) dx = 0$   
 $\frac{9H}{9x} = 8xy$   $\frac{3U}{3y} = 8xy$ 

$$\frac{(4x^2y + 6y^2) dy + (4xy^2 - 3)}{N}$$

$$\frac{\partial H}{\partial x} = 8xy \qquad \frac{\partial U}{\partial y} = 8x$$

$$\int M du = \int (4x^2u + 6u^2) dy = 8x$$

$$\frac{(4x^2y + 6y^2)}{N} dy + (4xy^2 - 3)$$

$$\frac{9H}{9x} = 8xy$$

$$\frac{3U}{3y} = 89$$

$$e^{-\frac{1}{2} \int_{X}^{2} x \, dx} = e^{-\frac{3}{2} \int_{X}^{2} x \, dx} = x$$

$$= p = -x^{2} + C_{1} x^{3}$$

6) 
$$\frac{(2.70 \text{ a}) y'^2 + (y-1)y'-y=0}{y'^2 + yy'-y'-y=0}$$
  
 $y(y'-1)=y'-y'^2$ 

$$y'^{2} + yy' - y' - y = 0$$

$$y(y'-1) = y' - y'^{2}$$

$$y = \frac{y'(1-y')}{y'-1} = -y' \quad \text{if} \quad y'' = 1$$

$$y' + yy - y' - y'^{2}$$

$$y' = y' + (1 - y')^{2}$$

$$y' = y' + (1 - y')^{2}$$

$$y' = -x' + y' + 1$$

$$y' = 1$$

$$y' = 1$$

$$y = \frac{y'(1-y')}{y'-1} = -y'$$
 if  $y'-1 = y'$ 
 $y = \frac{y'(1-y')}{y'-1} = -y'$  if  $y'-1 = y'$ 
 $y'=1$ 
 $y = x + c_1$ 

$$\int dx = -\int \frac{1}{y} dy$$

$$x + c_{x} = -\ln y$$

$$\frac{(3.65 b) \times y}{y' - \frac{4}{x}y} = 1$$

u = e = x

 $x^{-1}y = \int x^{-4}dx + C$ 

3y = - x + c x4

 $y = -\frac{1}{3}x^{-3}.x^{4} + cx^{4}$ 









8) 
$$(2.63 \text{ a}) (3y - 2xy^3) dx + (4x - 3x^2y^2) dy = 0$$
  
 $\frac{2}{3y} = 3 - 6xy^2 + \frac{2}{3x} = 4 - 6xy^2$ 

8) 
$$(2.63 \text{ a})$$
  $(3y - xxy) + (4x - 3x y) + (4y - 3x y)$ 

$$\frac{3y^{2} + 3y^{2}}{2y^{2}} = 0$$

$$\frac{3y - 2xy^{3}}{x^{2}y^{2}} = 0$$

$$\frac{3y - 2xy^{3}}{x^{2}y^{2}} = 0$$

$$\frac{x^{2}y^{2}}{x^{2}y^{2}} = 0$$

$$\frac{3N}{99} = 3 \times \frac{7}{9} y^{-9} (1-q) - 2 \times \frac{x^{1-9} y^{2-9}}{3} (3-q)$$

$$\frac{\partial N}{\partial y} = 3 \frac{x^{-p} y^{-q}}{(1-q)} - 3 \frac{x^{-p} y^{2-q}}{(3-q)}$$

$$\frac{\partial M}{\partial x} = 4(1-p) \frac{x^{-p} x^{-q}}{(3-q)} - 3(2-p) \frac{x^{4-p} y^{2-q}}{(3-q)}$$

$$\frac{3N}{3y} = 3x^{-1}y^{-1}(1-q) - 2x^{-1}y^{-1}(3-q)$$

$$\frac{3M}{3x} = 4(1-p)x^{-p}x^{-q} - 3(2-p)x^{-p}y^{2-q}$$

$$\begin{cases} 3(1-q) = 4(1-p) & (p=-2) \\ -2(3-q) = -3(2-p) & (q=-3) \end{cases}$$

$$\begin{cases} -2(3-q) = -3(2-p) \\ -3(3-q) = -3(2-p) \\ -3$$

$$\frac{dy}{dx} = \frac{dp}{dx} \cdot x + p + 4p\frac{dp}{dx}$$

$$\times \frac{dp}{dx} + 4p\frac{dp}{dx} = 0$$

$$\frac{P}{x \frac{dp}{dx}} + 4p \frac{dp}{dx} = c$$

$$(x+4p)\frac{dp}{dx}=0$$

$$(x+4p)\frac{\partial}{\partial x}=0$$

Case 1, 
$$x+4p \neq 0$$
.  $\frac{dp}{dx} = 0$ 

$$y'=p=c$$

$$y=cx+2c^2$$
 (general)

case 2, 
$$x_1 + p = 0$$
.  $p = -\frac{x}{4}$   $y = -\frac{1}{8}x^2$ 

$$\frac{\partial}{\partial y} = \lambda \sin x + 1 \lambda y^{3} \sin x \cos x \qquad \frac{\partial}{\partial x} = \lambda \cos x \sin x + 4 y^{3} + \sin x$$

$$R(x) = \frac{-1}{4y^3 \cos^2 x + \cos x} \left( \sin x + 4y^3 \sin x \cos x \right)$$

$$= -\frac{1}{\cos x} \cdot \sin x \left( \frac{1 + 4y^3 \cos x}{4y^3 \cos x + 1} \right) = -\tan x$$

$$F(x) = e^{-\int t dx} dx$$

$$= \int \frac{\sin x}{\cos x} dx \quad dt = -\sin x dx$$

$$= \int \frac{1}{t} dt = \ln(t)$$

=) 
$$(2y\sin x\cos x + 3y^4 \sin x\cos^2 x) dx - (4y^3\cos^3 x + \cos^2 x) dy = 0$$
  

$$\frac{3}{3y} = 2\sin x\cos x + 12y^3\sin x\cos^2 x$$

$$\frac{3}{3y} = 12y^3\cos^2 x \cdot \sin x + 2\cos x \sin x$$

$$\int aysin \times \cos x + 3y^4 \sin x \cos^2 x \, dx = -\int (ayt + 3y^4 t^2) \, dt$$

$$\cos x = t \rightarrow dt = -\sin x \, dx$$

$$\frac{\partial}{\partial y} \left( -y \cos^2 x - y^4 \cos^3 x + F(y) \right) = -\cos^2 x - 4y^3 \cos^3 x + F(y)$$

$$= -4y^3 \cos^3 x - \cos^2 x$$

$$C = -y\cos^2x - y^4\cos^3x$$

M) 
$$(2.66 \text{ a}) \frac{dy}{dx} = \frac{2y}{x} - \frac{y^2}{x^2}$$

$$(y = \frac{y}{x}) \quad y + x \frac{dy}{dx} = 2y - y^2$$

$$\frac{dx}{dx} = \frac{dy}{x}$$

$$v + x \frac{dv}{dx} = 2v - v^{2}$$

$$\int_{v-v^{2}} \frac{1}{v^{2}} dv = \int_{x} \frac{1}{v^{2}} dx$$

$$\int_{V-V^2}^{\frac{1}{2}} dv = \int_{X}^{\frac{1}{2}} dx$$

$$\int_{V-V^2}^{\frac{1}{2}} dv = \int_{X}^{\frac{1}{2}} dx$$

$$\int_{V-V^2}^{\frac{1}{2}} dv = \ln(v) - \ln(1-v) = \ln x + C$$

$$\int_{V-V^2}^{\frac{1}{2}} dv = \int_{\frac{1}{X}}^{\frac{1}{2}} dx$$

$$\int_{V-V^2}^{\frac{1}{2}} dv = \ln(v) - \ln(1-v) = \ln x + C$$

$$\frac{v}{1-v} = C x$$

$$\int_{V-V^{2}}^{\frac{1}{2}} dv = \int_{X}^{\frac{1}{2}} dx$$

$$\int_{V-V^{2}}^{\frac{1}{2}} dv = \ln(v) - \ln(1-v) = \ln x + C$$

$$\frac{V}{1-V} = C \times C$$

$$\frac{y|v}{1-V} = \frac{y}{x-y} = C \times C$$

y = Cx2-Cxy

$$\frac{(2) = \chi^{2} - \chi y}{(2.66 d)} (\chi - y) y' + 3y - 5\chi = 0$$

$$\frac{dy}{d\chi} = \frac{5\chi - 3y}{\chi - y} (y - y\chi) + \chi \frac{dy}{d\chi} = \frac{5 - 3y}{\Lambda - y}$$

$$\chi \frac{dy}{d\chi} = \frac{5 - 4y + y^{2}}{1 - y}$$

$$\int \frac{1 - y}{5 - 4y + y^{2}} dy = \int \frac{1}{\chi} d\chi$$

$$\int \frac{2y - 4}{5 - 4y + y^{2}} dy + \int \frac{2}{5 - 4y + y^{2}} dy = -2 \int \frac{1}{\chi} d\chi$$

ln (5-4v+v2) + 2 tan-2(v-2) = - 2 ln x + C

lm (y2-4xy+5x2)+2tan-2(y-2x)-c)

 $m(5-4(\frac{1}{x})+(\frac{1}{x})^2)+m(x^2)+2\tan^{-2}(\frac{1}{x}-2)=C$ 

13) 
$$\frac{(2.72 c) y'' + 4y = 0}{missing a: y' = p y'' = dx} = \frac{dp}{dx} = \frac{dp}{dx} = \frac{dp}{dy}$$

$$\frac{dp}{dy} = 4y = 0$$

 $\int_{\sqrt{\frac{C_1}{2} - y^2}} \frac{1}{dy} = \int_{\mathcal{X}} dx$ 

Sin-4 ( 2 ) = 27 + C2

 $y = \frac{2}{c} \sin(2x + c_2)$ 

y= C1 Sin (2x + C2)

14) (2.69) = x2+ 2xy+y2+2x+2y; y(0)=0

 $\frac{1}{x+y+1} = x + C \text{ with } y(0) = 0 \iff C = -1$  1 = (1-x)(x+y+1)

U= X + Y  $\frac{dv}{dz} = 1 + \frac{dv}{dz}$ 

 $\frac{dy}{dy} = (x+y)^2 + 2(x+y)$ 

 $\frac{dv}{dv} - 1 = v^2 + 2v$ 

 $\int_{(1)+4)^2} dv = \int dx$ 

- 1 = x+C

missing 
$$a : y' = p$$
  $y'' = \frac{dp}{dx} = \frac{dp}{dx} = \frac{dp}{dx}$ 

$$P \frac{dp}{dy} = 4y$$

$$P \frac{dp}{dy} = -4y$$

$$\int p dp = \int -4y \, dy$$

$$P \frac{dP}{dy} = 4y = 0$$

$$P \frac{dP}{dy} = -4y$$

$$\int_{P} dP = \int_{-4y} dy$$

$$\frac{P^{2}}{2} = -8y^{2} + C_{1}$$

$$\frac{9.71 \text{ a)} \times y = x p - yp^2, with p=y^2}{x^2y + p^2y = x^3p}$$

 $P\left((x^{2}+p^{2})\frac{(x^{2}+p^{2}-3x^{2})+2x^{4}}{p^{2}-2x^{2}}\right)+2x^{4}=(x^{5}-x^{3}p^{2})\frac{dp}{dx}$ 

Case 2,  $p^3 + x^3 \frac{dp}{dx} = 0$  |  $Ap^2 = \frac{4x^2}{dcx^2 - 1}$   $p = \frac{\pm x}{\sqrt{acy^2 - 1}}$ 

 $p^{3}(p^{2}-x^{2})+x^{3}(p^{2}-x^{2})\frac{dp}{dx}=0$ 

 $\left(p^2-\chi^2\right)\left(p^3+\chi^3\frac{dp}{dp}\right)=0$ 

2 x 3 y = ± x 4

 $\frac{1}{X^3} dx = -\frac{1}{D^3} dp$ 

 $-\frac{1}{8x^2} + C = \frac{1}{8p^2}$ 

Cose 1, p2-x2=0. p=±x

P4 - p2 x2

 $P(p^2x^2 + p^4 = 2x^4 - 2p^2x^2 + 2x^4) - x^3(x^2 - p^2)\frac{dp}{dx} = 0$ 

15) 
$$\frac{(2.71 \text{ d}) \times^{2}y = \times^{2}p - yp^{2}, \text{ with } p = y'}{x^{2}y + p^{2}y = x^{3}p}$$

$$y = \frac{x^{3}p}{x^{2} + p^{2}}$$

$$p = \frac{dy}{dx} = \frac{(x^{2} + p^{2})}{(x^{2} + p^{2})} \frac{d(x^{3}p)}{dx} - x^{3}p \frac{d(x^{2} + p^{2})}{dx}$$

$$\frac{(3.47) \text{ a)} \times \text{g}^{2} \times \text{p}^{2} \text{g}^{2} \times \text{g}^{3} \text{p}}{\text{x}^{3} \text{p}}$$

$$\frac{1 \text{ d) } x^2y = x^3p - yp^2, \text{ with } p = y'$$

$$p^2y = x^3p$$

$$\frac{1 \, d}{p^2 y} = x^3 p$$

 $(x^2+p^2)^2$ 

 $p(x^{2}+p^{2})^{2}-(x^{2}+p^{2})3x^{2}p+2x^{4}p=((x^{2}+p^{2})x^{3}-2x^{3}p^{2})\frac{dp}{dx}$ 

P(x2+p2)2 = (x2+p2)(3x2p+x3dp)-x3p(2x+2pdx

 $x^{2}y = x^{3}(\pm x) - yx^{2}$   $\Rightarrow y = \pm \frac{x^{2}}{2}$  (singular)

$$\frac{1 \, d}{p^2 \, y} = x^3 p - y p^2, with p = y'$$

$$\frac{1}{p^{2}y} = x^{3}p - yp^{2}$$
, with  $p = y^{3}$ 

15) 
$$\frac{(2.71 \text{ d}) \times^2 y = \times^3 p - y p^2, \text{ with } p = y'}{x^2 y + p^2 y} = x^3 p$$

$$\frac{\pm x}{|x|^{2}-1} x^{2}y = x^{3}p - yp^{2}$$

$$p = \frac{\pm x}{\sqrt{2cy^2-1}}$$

$$x^2y = x^2p - y^2$$

$$p = \frac{\pm x}{\sqrt{2cy^2-1}}$$

$$\frac{\pm x}{\sqrt{2} + x^2} = x^2 p - y p^2$$

$$= \frac{\pm x}{\sqrt{2} + x^2}$$

 $x^2y + \frac{x^2y}{2cx^2-1} = \frac{\pm x^4}{\sqrt{2cx^2-1}}$ 

 $\frac{x^2y}{g_{Cx^2}} \left( g_{Cx^2} \right) = \frac{J x^7}{\sqrt{g_{Cx^2}}}$ 

 $\frac{\mathcal{A}(\mathcal{Y})}{\mathcal{A}(x^2-1)} = \frac{\pm 1}{\sqrt{\mathcal{A}(x^2-1)}}$ 

 $\frac{4c^2y^2}{(2(x^2-1)^2-2cx^2-1)}$ 

4c2 y2 = 2cx2-1

16) (2.65 d) dy + 2y cot x = csc x N= e /2cot x dx = sin2 x

sin2x y = Sin2x 1 dx + C

y = - cos x · sin-2 x + c· sin-2 x

 $y^2 = \frac{x^2}{9c} - \frac{1}{4c^2}$  (c =  $\frac{1}{9c}$ )

 $y^2 = Cx^2 - C^2 \quad (general)$ 

 $\rho = \frac{\pm \alpha}{\sqrt{3cy^2-1}}, \quad \alpha^2 y = \alpha^3 \rho - y \rho^2$  $x^2y = \frac{1}{\sqrt{3}cv^2} - \frac{1}{\sqrt{2}} \frac{x^2}{\sqrt{2}}$ 

$$H(a.60 d) \frac{dy}{dx} = \frac{y\sqrt{1-y^2}}{y\sqrt{1-x^2}}$$

$$\int \frac{y}{dy} dy = \int \frac{x}{y}$$

$$\int \frac{y}{\sqrt{1-y^2}} \, dy = \int \frac{x}{\sqrt{1-x^2}} \, dx$$

-3( that = - 1 ) find du

1-y2 = 1-x2 +C

 $\sqrt{1-x^2}-\sqrt{1-y^2}=C$ 

18) (2.61c) (yex-ey)dx + (xey + ex)dy =0

\( \( \times e^{y} + e^{x} - e^{y} \) dy = - x e^{-y}

yex-xe-y = c

 $\frac{3(ye^{x}-e^{-y})}{3y}:e^{x}+e^{-y}$   $\frac{3(xe^{-y}+e^{x})}{3x}=e^{-y}+e^{x}$ 

$$\int \frac{y}{\sqrt{1-y^2}}$$

$$1-y^2=t$$

$$-2y\,dy=dt$$