

PW/TP 3-4: Ordinary Differential Equations - Applications (CH2)

Exercises

Exercise 1. An object moves along the x axis, acted upon by a constant force. If its initial velocity in the positive direction is 40 meters/sec while 5 seconds later it is 20 meters/sec, find (a) the velocity at any time, (b) the position at any time assuming the object starts from the origin $x = 0$.

Exercise 2. An electric circuit contains an 9 ohm resistor in series with an inductor of 0.5 henries and a battery of E volts. At $t = 0$ the current is zero. Find the current at any time $t > 0$ and the maximum current if (a) $E = 64$, (c) $E = 32e^{-8t}$.

Exercise 3. Find the orthogonal trajectories of the family of curves $y^2 = cx^2 - 2y$.

Exercise 4. Find the equation of that curve passing through $(0, 1)$ which is orthogonal to each member of the family $x^2 + y^2 = ce^x$.

Exercise 5. A tank contains 100 gallons of water. A salt solution containing 2 lb of salt per gallon flows in at the rate of 3 gallons per minute and the well-stirred mixture flows out at the same rate. (a) How much salt is in the tank at any time? (b) When will the tank have 100 lb of salt?

Exercise 6. The rate at which bacteria multiply is proportional to the instantaneous number present. If the original number doubles in 2 hours, in how many hours will it triple?

Exercise 7. It takes 15 minutes for an object to warm up from 10°C to 20°C in a room whose temperature is 30°C . Assuming Newton's law of cooling, how long would it take to warm up from 20°C to 25°C ?

Exercise 8. A 64lb object falls from rest. The limiting velocity is 4 ft/sec. Find the velocity after t seconds assuming a force of resistance proportional to (a) v (b) v^2 .

Exercise 9. At 1 : 00 P.M. the temperature of a tank of water is 200°F . At 1 : 30 P.M. its temperature is 160°F . Assuming the surrounding temperature is maintained at 80°F , (a) what is the temperature at 2 : 00 P.M. and (b) at what time will the temperature be 100°F ? Assume Newton's law of cooling.

Exercise 10. After 2 days, 10 grams of a radioactive chemical is present. Three days later 5 grams is present. How much of the chemical was present initially assuming the rate of disintegration is proportional to the instantaneous amount which is present?

Exercise 11. An electric circuit contains an 9 ohm resistor in series with an inductor of 0.5 henries and a battery of E volts. At $t = 0$ the current is zero. Find the current at any time $t > 0$ if $E = 64 \sin 8t$. What is the transient current and steady-state current?

Exercise 12. Find the orthogonal trajectories of the family of curves $x^2 + y^2 = cx$.

Exercise 13. An electric circuit contains a 20 ohm resistor in series with a capacitor of 0.05 farads and a battery of E volts. At $t = 0$ there is no charge on the capacitor. Find the charge and current at any time $t > 0$ if $E = 100te^{-2t}$.

Exercise 14. A tank contains 100 gallons of water. A salt solution containing 2 lb of salt per gallon flows in at the rate of 3 gallons per minute. How much salt is in the tank at any time if the well-stirred mixture flows out at the rate of (a) 2 gal/min (b) 4 gal/min?