

PW/TP 5-6: Linear Differential Equations - Constant coefficients (CH3)

Solutions

Exercise 1. Solve

1. (3.59 a) $y'' + 8y' + 12y = 0$

```
syms y(x) x
eq = diff(diff(y,x),x) + 8*diff(y,x)+12*y == 0;
dsolve(eq)
```

$$\text{ans} = C_2 e^{-2x} + C_1 e^{-6x}$$

$$m^2 + 8m + 12 = 0 \Rightarrow D = 64 - 48 = 16 \Rightarrow m = \frac{-8 \pm 4}{2} = -2 \vee -6$$

$$y_H = c_1 e^{-2x} + c_2 e^{-6x}$$

2. (3.60 a) $(D^2 + 25)y = 0; y(0) = 2; y'(0) = -5$

```
syms y(x) x
eq = diff(diff(y,x),x) + 25*y == 0;
cond1 = y(0)==2;
df(x) = diff(y,x);
cond2 = df(0) == -5;
dsolve(eq,cond1,cond2)
```

$$\text{ans} = 2 \cos(5x) - \sin(5x)$$

$$m^2 + 25 = 0 \Rightarrow m = \pm 5i$$

$$y_H = c_1 \cos 5x + c_2 \sin 5x$$

$$y_H(0) = c_1 \cos 0 + c_2 \sin 0 \Rightarrow c_1 = 2$$

$$y_H'(0) = -10 \sin 0 + 5c_2 \cos 0 \Rightarrow c_2 = -1$$

$$\Rightarrow y_H = 2 \cos 5x - \sin 5x$$

3. (3.61 a) $y'' + 4y' + 4y = 0$

```
syms y(x) x
eq = diff(diff(y,x),x) + 4*diff(y,x)+4*y == 0;
dsolve(eq)
```

$$\text{ans} = C_1 e^{-2x} + C_2 x e^{-2x}$$

$$m^2 + 4m + 4 = 0 \Rightarrow (m + 2)^2 = 0 \rightarrow m = -2$$

$$y_H = (c_1 + c_2 x)e^{-2x}$$

Exercise 2. Solve

1. (3.63 a) $y'' - 5y' + 6y = 50 \sin 4x$

```
syms y(x) x
eq = diff(diff(y,x),x) - 5*diff(y,x)+6*y == 50*sin(4*x);
dsolve(eq)
```

ans =

$$\sqrt{5} \cos\left(4x + \operatorname{atan}\left(\frac{1}{2}\right)\right) + C_1 e^{2x} + C_2 e^{3x}$$

$$m^2 - 5m + 6 = 0 \Rightarrow m = 3 \vee 2 \Rightarrow y_H = c_1 e^{3x} + c_2 e^{2x}$$

Take a trial solution: $y_P = a \sin 4x + b \cos 4x$

```
clear;
syms x a b
y = a*sin(4*x)+b*cos(4*x);
eq = diff(y,x,2) - 5*diff(y,x)+6*y == 50*sin(4*x);
eq
```

$$\text{eq} = 20b \sin(4x) - 10a \sin(4x) - 20a \cos(4x) - 10b \cos(4x) = 50 \sin(4x)$$

$$\Rightarrow \begin{cases} a = -1 \\ b = 2 \end{cases}$$

$$y_P = 2 \cos 4x - \sin 4x$$

2. (3.64 a) $y'' + 3y' + 2y = 4e^{-2x}$

```
syms y(x) x
eq = diff(diff(y,x),x) + 3*diff(y,x)+2*y == 4*exp(-2*x);
dsolve(eq)
```

$$\text{ans} = C_1 e^{-2x} - 4x e^{-2x} - 4e^{-2x} + C_2 e^{-x}$$

$$m^2 + 3m + 2 = 0 \Rightarrow m = -1 \vee -2 \Rightarrow y_H = c_1 e^{-x} + c_2 e^{-2x}$$

Take a trial solution: $y_P = ax e^{-2x}$

```
syms yp(x) x a
yp(x) = a*x*exp(-2*x);
simplify(subs(eq,y,yp))
```

$$\text{ans}(x) = a = -4$$

$$\Rightarrow y_P = -4x e^{-2x}$$

3. (3.64 b) $(D^3 + 3D^2)y = 180x^3 + 24x$

```
syms y(x) x
eq = diff(diff(diff(y,x),x),x) + 3*diff(diff(y,x),x) == 180*x^3+24*x;
dsolve(eq)
```

ans =

$$\frac{C_1}{9} + C_2 - 8x^2 + 8x^3 - 5x^4 + 3x^5 - x \left(\frac{C_1}{3} - \frac{16}{3} \right) + C_3 e^{-3x} - \frac{16}{9}$$

$$m^3 + 3m^2 = 0 \Rightarrow m = 0 \vee -3 \Rightarrow y_H = (c_1 + c_2 x) + c_3 e^{-3x}$$

Exercise 3. (3.66 a) Solve $y'' + 2y' - 3y = xe^{-x}$ using variation of parameters.

```
syms y(x) x
eq = diff(y,x,2)+2*diff(y,x)-3*y == x*exp(-x);
simplify(dsolve(eq))
```

ans =

$$C_2 e^x - \frac{x e^{-x}}{4} + C_1 e^{-3x}$$

Exercise 4. Evaluate the following:

1. (3.67 b) $\frac{1}{D^2 + D - 12} \{9e^{5x} - 4e^{-x}\}$

```
syms y(x) x
eq = diff(y,x,2)+diff(y,x)-12*y == 9*exp(5*x)-4*exp(-x);
simplify(dsolve(eq))
```

ans =

$$\frac{e^{-x}}{3} + \frac{e^{5x}}{2} + C_1 e^{-4x} + C_2 e^{3x}$$

2. (3.67 c) $\frac{1}{(D+1)^2} \{4 \sin 2x + 3 \cos 2x\}$

```
syms y(x) x
eq = diff(y,x,2)+2*diff(y,x)+y == 4*sin(2*x)+3*cos(2*x);
simplify(dsolve(eq))
```

$$\text{ans} = C_1 e^{-x} - \cos(2x) + C_2 x e^{-x}$$

3. (3.67 e) $\frac{1}{(D-4)^5} \{x e^{4x}\}$

4. (3.67 f) $\frac{1}{D^2 - 4} \{16x^3\}$

```
syms y(x) x
```

```
eq = diff(y,x,2)-4*y == 16*x^3;
simplify(dsolve(eq))
```

$$\text{ans} = C_1 e^{-2x} - 4x^3 - 6x + C_2 e^{2x}$$

Exercise 5. Solve

1. (3.60 c) $(D^3 + 8)y = 0$
2. (3.61 c) $(D + 6)^4(D - 3)^2y = 0$
3. (3.62) $D^4(D + 1)^2(D^2 + 4D + 5)^2(D^2 + 4)y = 0$

Exercise 6. (3.65) Solve $(D^6 - 2D^5 + D^4)y = 120x + 8e^x$

Exercise 7. Solve using variation of parameters:

1. (3.66 b) $y'' + 4y = \csc 2x$
2. (3.66 d) $(D^3 + D)y = 4 \tan x$

Exercise 8. Evaluate the following:

1. (3.67 a) $\frac{1}{D+3}\{e^{-2x}\}$
2. (3.67 d) $\frac{D-1}{D^4+D^2+1}\{8 \cos x\}$
3. (3.67 g) $\frac{1}{D^2+D-2}\{x^2 e^{2x}\}$
4. (3.67 h) $\frac{1}{D^2-1}\{e^x(\sin x + \cos x)\}$
5. (3.67 j) $\frac{1}{(D-4)(D+3)(D+1)}\{e^{-2x} \cos 2x\}$

Exercise 9. (3.68 a) Solve $(D^2 + 4D + 4)y = 18e^x - 8 \sin 2x$ using the operator technique