CS294-112 Deep Reinforcement Learning HW2: Policy Gradients
Solution by Benjamin Liu@Berkeley

Problem 1. State-dependent baseline: In lecture we saw that the policy gradient is unbiased if the baseline is a constant with respect to τ (Equation **??**). The purpose of this problem is to help convince ourselves that subtracting a state-dependent baseline from the return keeps the policy gradient unbiased. Using the law of iterated expectations show that the policy gradient is still unbiased if the baseline b is function of a state at a particular timestep of τ (Equation **??**). Please answer the questions below in LaTeXin your report.

1. Solution to (a):
   Denote $\nabla_\theta \log \pi(a_t|s_t)b(s_t)$ as $g(a_t, s_t; \theta)$; and $\pi_\theta(a_t|s_t)p(s_t|a_{t-1}, s_{t-1})$ as $q(a_t, s_t|a_{t-1}, s_{t-1})$ where $p(s_t|a_{t-1}, s_{t-1})$ is the transition dynamics. W.L.O.G, let's assume a_t, s_t are discrete variable.
   Using the chain rule, we can express $p_\theta(\tau)$ as a product of the state-action marginal (s_t, a_t) and the probability of the rest of the trajectory conditioned on (s_t, a_t). The derivation for the conditional expectation as follows:

$$
\begin{aligned}
&\mathbb{E}_{p_\theta(\tau)}[\nabla_\theta \log \pi_\theta(a_t|s_t)b(s_t)] \\
&= \sum_{(a_1,s_1)} ... \sum_{(a_t,s_t)} ... \sum_{(a_T,s_T)} g(a_t, s_t; \theta) q(a_T, s_T|a_{T-1}, s_{T-1})...q(a_t, s_t|a_{t-1}, s_{t-1})...q(a_1, s_1) \\
&= \sum_{(a_1,s_1)} ... \sum_{(a_{t-1},s_{t-1})} \sum_{(a_{t+1},s_{t+1})} ... \sum_{(a_T,s_T)} (\sum_{(a_t,s_t)} g(a_t, s_t; \theta) q(a_t, s_t|a_{t-1}, s_{t-1})) \\
&\quad q(a_T, s_T|a_{T-1}, s_{T-1})...q(a_1, s_1) \\
&= \sum_{(a_1,s_1)} ... \sum_{(a_{t-1},s_{t-1})} \sum_{(a_{t+1},s_{t+1})} ... \sum_{a_T,s_T} q(a_T, s_T|a_{T-1}, s_{T-1})...q(a_1, s_1) \\
&\quad \sum_{(a_t,s_t)} g(a_t, s_t; \theta) q(a_t, s_t|a_{t-1}, s_{t-1})
\end{aligned}
$$

And conditioned on a_t:

$$
\begin{aligned}
&\sum_{(a_t,s_t)} g(a_t, s_t; \theta) q(a_t, s_t | a_{t-1}, s_{t-1}) \\
&= \sum_{s_t} \sum_{a_t} \nabla_\theta \log \pi(a_t|s_t) b(s_t) \pi_\theta(a_t|s_t) p(s_t|a_{t-1}, s_{t-1}) \\
&= \sum_{s_t} \sum_{a_t} b(s_t) p(s_t|a_{t-1}, s_{t-1}) \nabla_\theta \log \pi(a_t|s_t) \pi_\theta(a_t|s_t) \\
&= \sum_{s_t} b(s_t) p(s_t|a_{t-1}, s_{t-1}) \sum_{a_t} \nabla_\theta \log \pi(a_t|s_t) \pi_\theta(a_t|s_t) \\
&= \sum_{s_t} b(s_t) p(s_t|a_{t-1}, s_{t-1}) \sum_{a_t} \nabla_\theta \pi_\theta(a_t|s_t) \\
&= \sum_{s_t} b(s_t) p(s_t|a_{t-1}, s_{t-1}) \nabla_\theta \sum_{a_t} \pi_\theta(a_t|s_t) \\
&= \sum_{s_t} b(s_t) p(s_t|a_{t-1}, s_{t-1}) \nabla_\theta 1 \\
&= \sum_{s_t} b(s_t) p(s_t|a_{t-1}, s_{t-1}) 0 = 0
\end{aligned}
$$

Hence, $\mathbb{E}_{p_\theta(\tau)}[\nabla_\theta \log \pi_\theta(a_t|s_t) b(s_t)] = 0$
Similarly, all the arguments can be applied on cases when s_t, a_t are continuous variables.

2. Solution to (b):

   (a) Due to Markov Property of MDP, the future states only depend on the current state and the past is irrelevant.

   (b) With the same notation in (a), consider expectaion over $\tau^* = (s_1, a_1, ..., s_t, a_t)$, and then conditioned on (a_t, s_t)

$$
\begin{aligned}
&\mathbb{E}_{p_\theta(\tau^*)}[\nabla_\theta \log \pi_\theta(a_t|s_t) b(s_t)] \\
&= \sum_{(a_1,s_1)} ... \sum_{(a_t,s_t)} g(a_t, s_t; \theta) q(a_t, s_t|a_{t-1}, s_{t-1}) ... q(a_1, s_1) \\
&= \sum_{(a_1,s_1)} ... \sum_{(a_{t-1},s_{t-1})} \sum_{(a_t,s_t)} g(a_t, s_t; \theta) q(a_t, s_t|a_{t-1}, s_{t-1}) q(a_{t-1}, s_t|a_{t-1}, s_{t-1}) ... q(a_1, s_1) \\
&= \sum_{(a_1,s_1)} ... \sum_{(a_{t-1},s_{t-1})} q(a_{t-1}, s_t|a_{t-1}, s_{t-1}) ... q(a_1, s_1) \\
&\quad \sum_{(a_t,s_t)} g(a_t, s_t; \theta) q(a_t, s_t|a_{t-1}, s_{t-1})
\end{aligned}
$$

And again, conditioned on a_t and the same argument in (a):

$$
\begin{aligned}
&\sum_{(a_t,s_t)} g(a_t, s_t; \theta) q(a_t, s_t | a_{t-1}, s_{t-1}) \\
&= \sum_{s_t} \sum_{a_t} \nabla_\theta \log \pi(a_t|s_t) b(s_t) \pi_\theta(a_t|s_t) p(s_t|a_{t-1}, s_{t-1}) \\
&= \sum_{s_t} b(s_t) p(s_t|a_{t-1}, s_{t-1}) \sum_{a_t} \nabla_\theta \pi_\theta(a_t|s_t) \\
&= \sum_{s_t} b(s_t) p(s_t|a_{t-1}, s_{t-1}) 0 = 0
\end{aligned}
$$

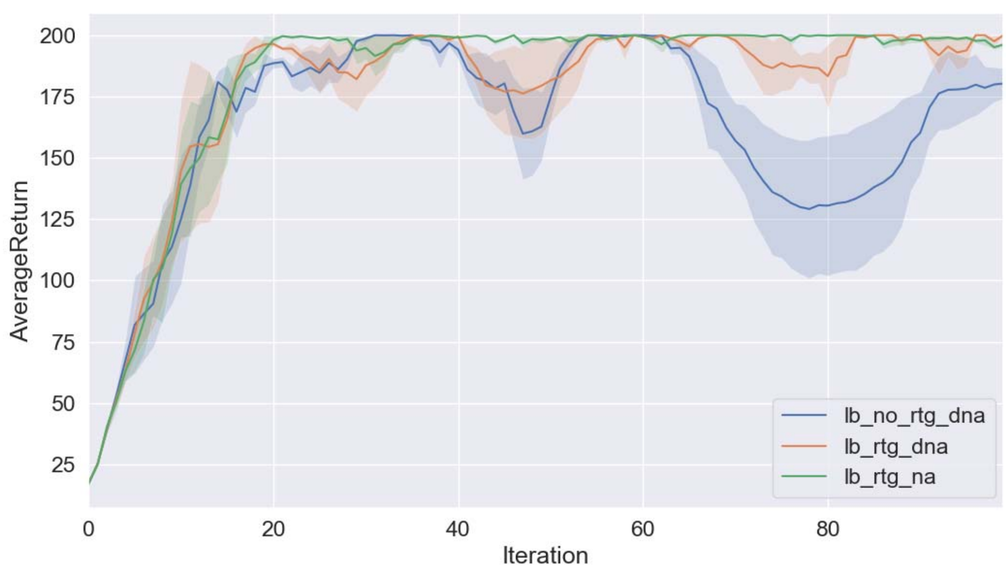
Similarly, all the arguments can be applied on cases when s_t, a_t are continuous variables.

Problem 4 - CartPole

Graph the results

- In the first graph, compare the learning curves (average return at each iteration) for the experiments prefixed with sb_. (The small batch experiments.)

- In the second graph, compare the learning curves for the experiments prefixed with lb_. (The large batch experiments.)



Answer the following questions briefly:

- Which gradient estimator has better performance without advantage-centering—the trajectory-centric one, or the one using reward-to-go?

In the first graph, the blue curve represents estimator without reward-to-go and advantage-centering; the red curve represents estimator without advantage-centering but with reward-to-go. The red learning curve indicates better learning for the reward-to-go estimator.

- Did advantage centering help?

In the first graph, the red curve represents estimator without advantage-centering but with reward-to-go; the green curve represents estimator with advantage-centering and reward-to-go. The green learning curve indicates better learning for the advantage-centering estimator.

- Did the batch size make an impact?

The first graph represents training with small batch; the second graph represents training with large batch. In the second graph, all of the three learner can reach ideal reward-level (~200) and achieve lower variance, hence large batch size, in this case, is good for learning.

Provide the exact command line configurations you used to run your experiments

Codes are include in `q4_main.bash`

```
python train_pg_f18.py CartPole-v0 -n 100 -b 1000 -e 3 -dna --exp_name sb_no_rtg_dna

python train_pg_f18.py CartPole-v0 -n 100 -b 1000 -e 3 -rtg -dna --exp_name sb_rtg_dna

python train_pg_f18.py CartPole-v0 -n 100 -b 1000 -e 3 -rtg --exp_name sb_rtg_na

python train_pg_f18.py CartPole-v0 -n 100 -b 5000 -e 3 -dna --exp_name lb_no_rtg_dna

python train_pg_f18.py CartPole-v0 -n 100 -b 5000 -e 3 -rtg -dna --exp_name lb_rtg_dna

python train_pg_f18.py CartPole-v0 -n 100 -b 5000 -e 3 -rtg --exp_name lb_rtg_na

python plot.py data/sb_no_rtg_dna_CartPole-v0_12-09-2018_00-23-34 data/sb_rtg_dna_CartPole-v0_15-09-2018_19-28-58 data/sb_rtg_na_CartPole-v0_15-09-2018_19-44-28 --value AverageReturn

python plot.py data/lb_no_rtg_dna_CartPole-v0_15-09-2018_19-45-55 data/lb_rtg_dna_CartPole-v0_15-09-2018_19-52-24 data/lb_rtg_na_CartPole-v0_15-09-2018_19-57-13 --value AverageReturn
```

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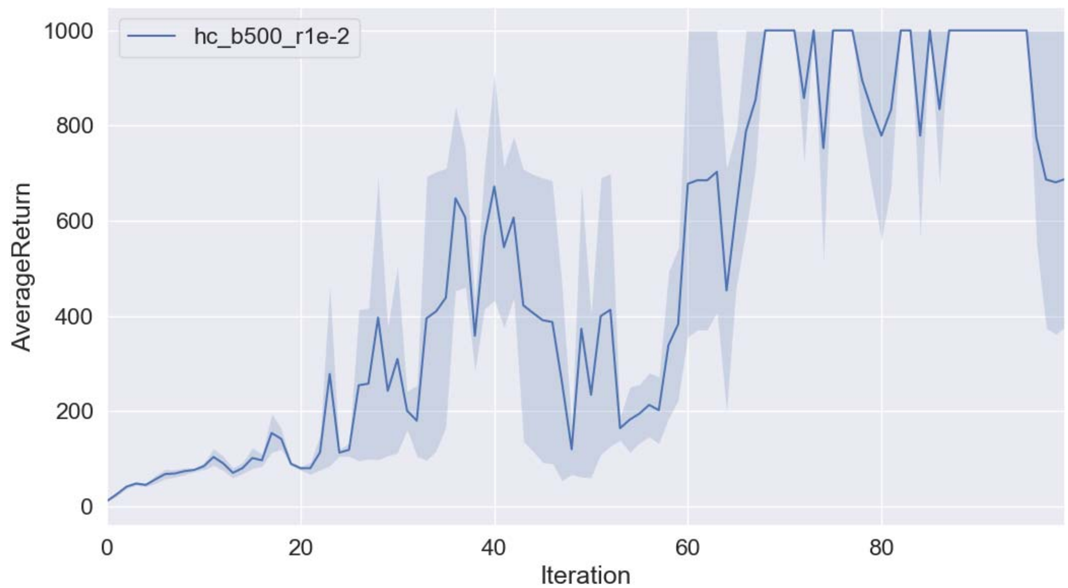
Problem 5 - Inverted Pendulum

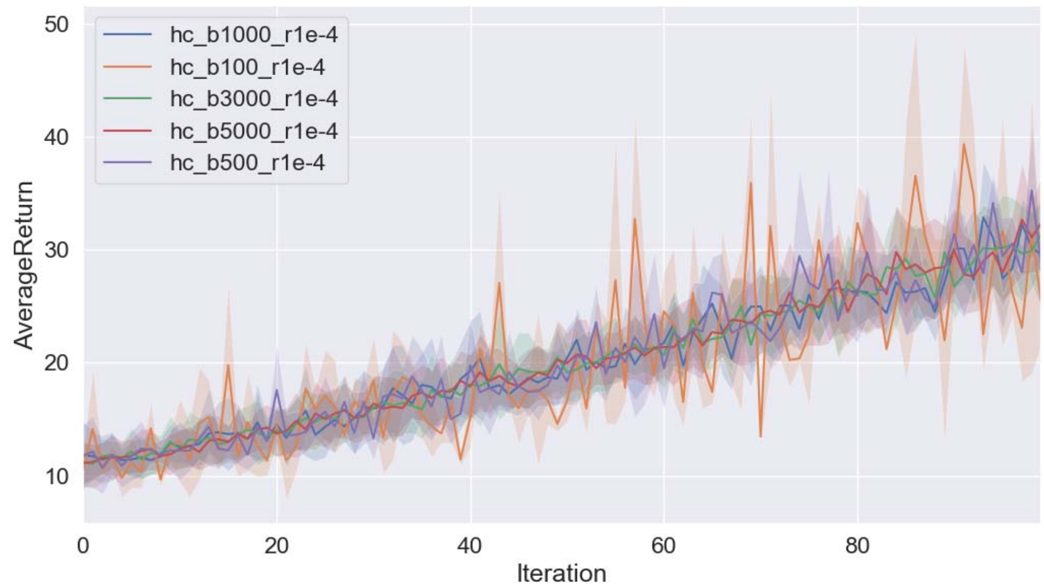
Find the optimal b* and r*, plot the learning curve

The optimal `b*=500` , `r*=0.01 (or 0.02)` . The corresponding learning curve is

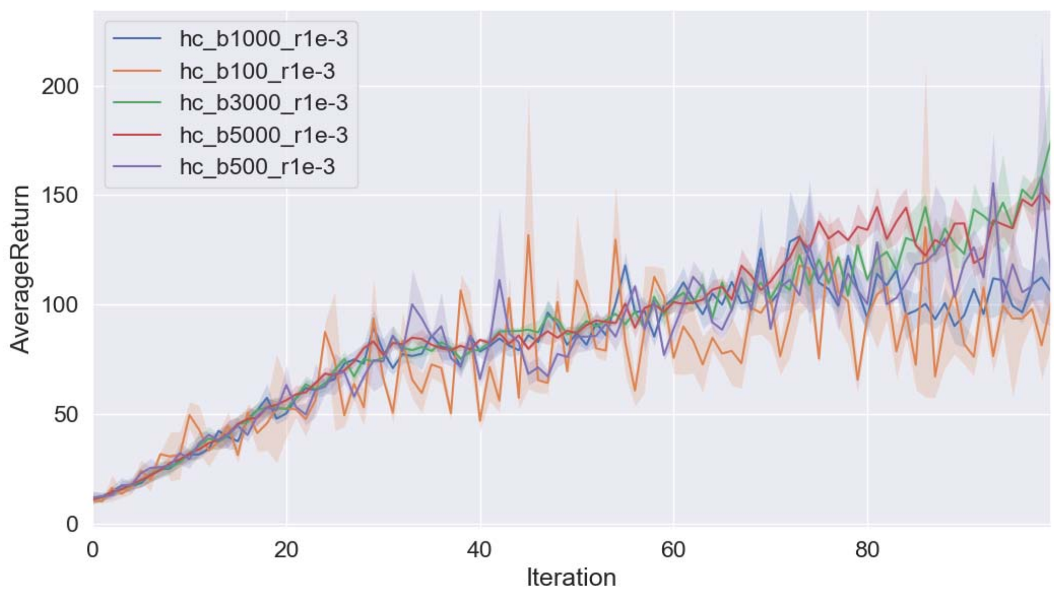


I have explored the `b=[100, 500, 1000, 3000, 5000]` and `lr=[2e-2, 1e-2, 5e-3, 1e-3, 1e-4]` and the learning curve under different learning rates are displayed below. I find that when `lr=1e-2` (last second figure), under all scenarios, the optimal value 1000 can be reached and in most cases, optimal value is reached in the first 50 iterations. When `batch_size=500` (corresponding to all purple curves), the 1000-reward can be reached earlier than `batch_size=100` and stay at the 1000 level. If we increase the batch size beyond 500, there is no significant improvement in the learning curve and sometimes the learner rewards fluctuate a lot from the 1000 level.

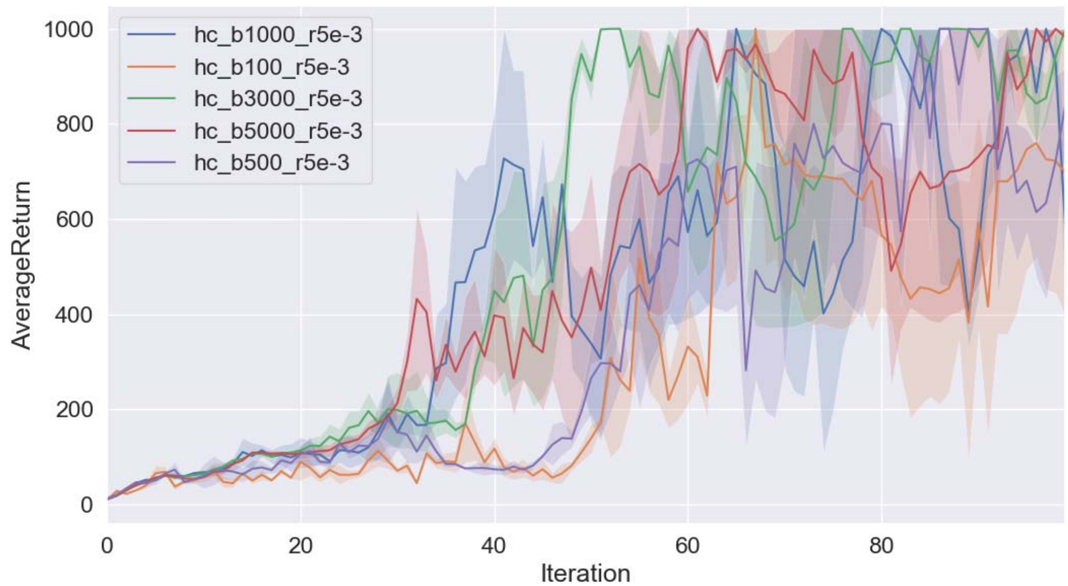
`lr=1e-4` Learning Curve under Different Training Batches



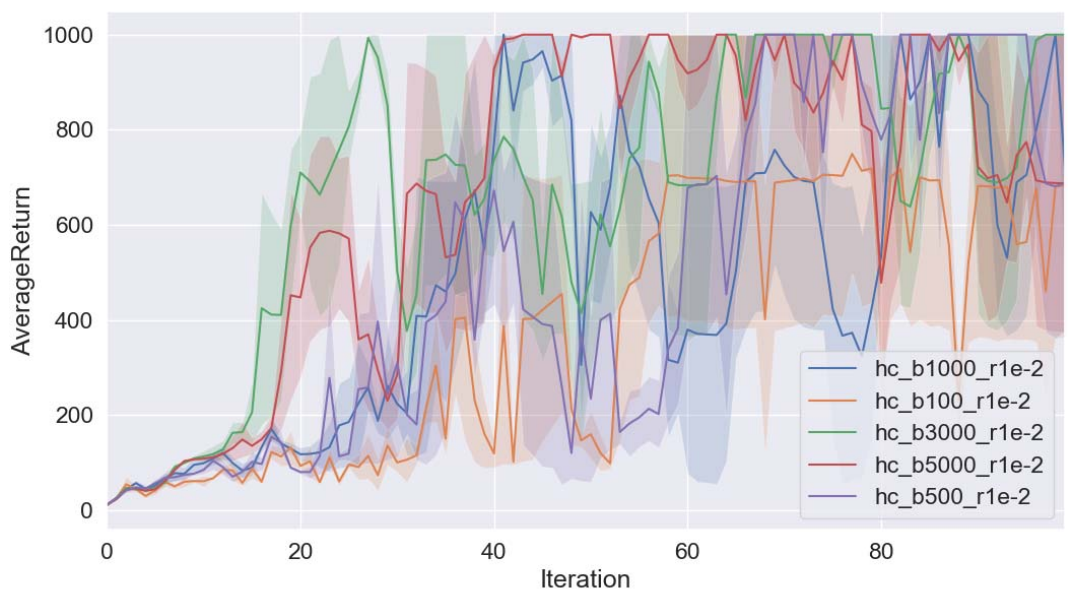
`lr=1e-3` Learning Curve under Different Training Batches



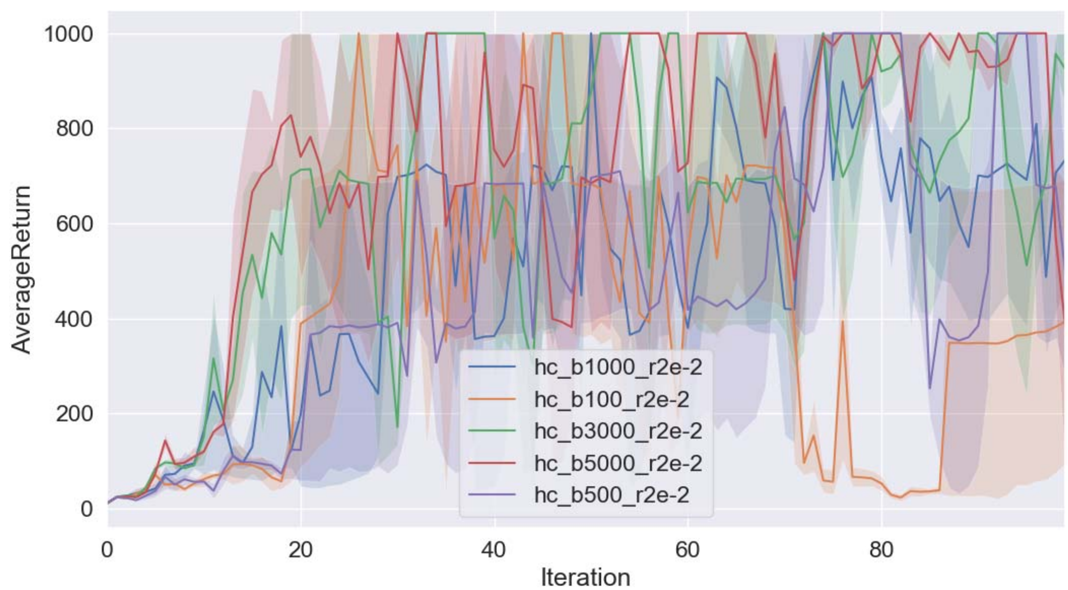
`lr=5e-3` Learning Curve under Different Training Batches



`lr=1e-2` Learning Curve under Different Training Batches



`lr=2e-2` Learning Curve under Different Training Batches



Provide the exact command line configurations you used to run your experiments

Codes are include in `q5_main.bash`

```
for batch in 100 500 1000 3000 5000
do
    for lr in 2e-2 1e-2 5e-3 1e-3 1e-4
    do
        py train_pg_f18.py InvertedPendulum-v2 -ep 1000 --discount 0.9 -n
 100 -e 3 -l 2 -s 64 -b $batch -lr $lr -rtg --exp_name hc_b${batch}_r${l
r}
    done
done

py plot.py data/hc_b1000_r1e-2_InvertedPendulum-v2_16-09-2018_00-45-21 da
ta/hc_b100_r1e-2_InvertedPendulum-v2_16-09-2018_00-33-44 data/hc_b3000_r1
e-2_InvertedPendulum-v2_16-09-2018_00-54-45 data/hc_b5000_r1e-2_InvertedP
endulum-v2_16-09-2018_01-11-36 data/hc_b500_r1e-2_InvertedPendulum-v2_16-
09-2018_00-38-46 --value AverageReturn

py plot.py data/hc_b1000_r1e-3_InvertedPendulum-v2_16-09-2018_00-48-48 da
ta/hc_b100_r1e-3_InvertedPendulum-v2_16-09-2018_00-35-48 data/hc_b3000_r1
e-3_InvertedPendulum-v2_16-09-2018_01-00-40 data/hc_b5000_r1e-3_InvertedP
endulum-v2_16-09-2018_01-26-49 data/hc_b500_r1e-3_InvertedPendulum-v2_16-
09-2018_00-41-37 --value AverageReturn

py plot.py data/hc_b1000_r1e-4_InvertedPendulum-v2_16-09-2018_00-50-12 da
ta/hc_b100_r1e-4_InvertedPendulum-v2_16-09-2018_00-36-35 data/hc_b3000_r1
e-4_InvertedPendulum-v2_16-09-2018_01-03-28 data/hc_b5000_r1e-4_InvertedP
endulum-v2_16-09-2018_01-35-03 data/hc_b500_r1e-4_InvertedPendulum-v2_16-
09-2018_00-42-33 --value AverageReturn

py plot.py data/hc_b1000_r2e-2_InvertedPendulum-v2_16-09-2018_00-43-32 da
ta/hc_b100_r2e-2_InvertedPendulum-v2_16-09-2018_00-32-34 data/hc_b3000_r2
e-2_InvertedPendulum-v2_16-09-2018_00-51-39 data/hc_b5000_r2e-2_InvertedP
endulum-v2_16-09-2018_01-06-47 data/hc_b500_r2e-2_InvertedPendulum-v2_16-
09-2018_00-37-17 --value AverageReturn

py plot.py data/hc_b1000_r5e-3_InvertedPendulum-v2_16-09-2018_00-47-05 da
ta/hc_b100_r5e-3_InvertedPendulum-v2_16-09-2018_00-34-49 data/hc_b3000_r5
e-3_InvertedPendulum-v2_16-09-2018_00-57-44 data/hc_b5000_r5e-3_InvertedP
endulum-v2_16-09-2018_01-17-25 data/hc_b500_r5e-3_InvertedPendulum-v2_16-
09-2018_00-40-14 --value AverageReturn

py plot.py data/hc_b500_r1e-2_InvertedPendulum-v2_16-09-2018_00-38-46 --v
alue AverageReturn
```

In [ ]:

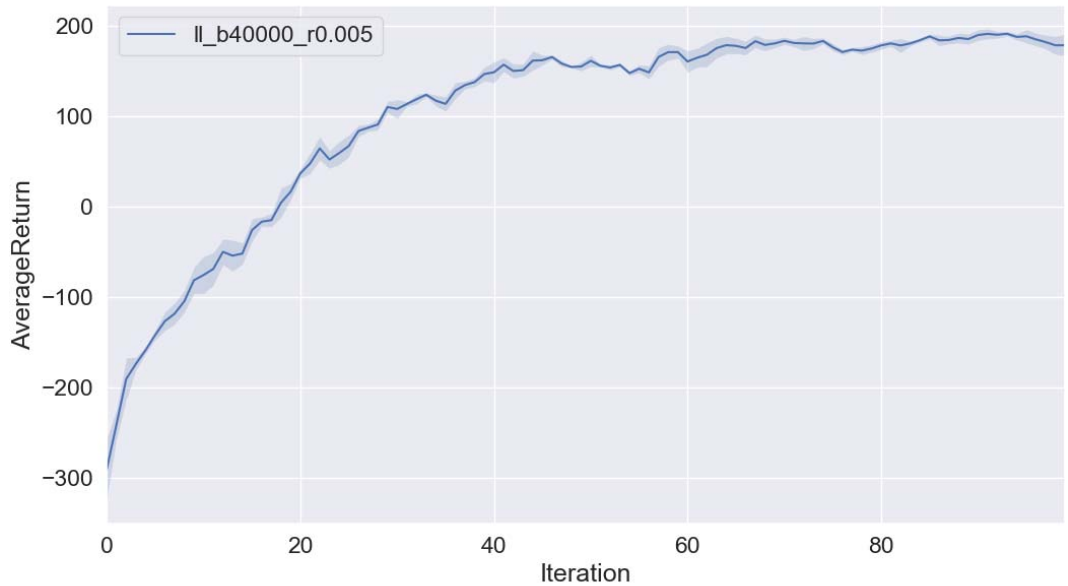
```

```

Problem 7 - Lunar Lander

Graph the learning curve

The learning curve for the configuration specified by homwork is



Provide the exact command line configurations you used to run your experiments

Codes are include in `q7_main.bash`

```
python train_pg_f18.py LunarLanderContinuous-v2 -ep 1000 --discount 0.99 -n 100 -e 3 -l 2 -s 64 -b 40000 -lr 0.005 -rtg --nn_baseline --exp_name ll_b40000_r0.005
```

```
python plot.py data/ll_b40000_r0.005_LunarLanderContinuous-v2_15-09-2018_21-41-36 --value AverageReturn
```

In [ ]:

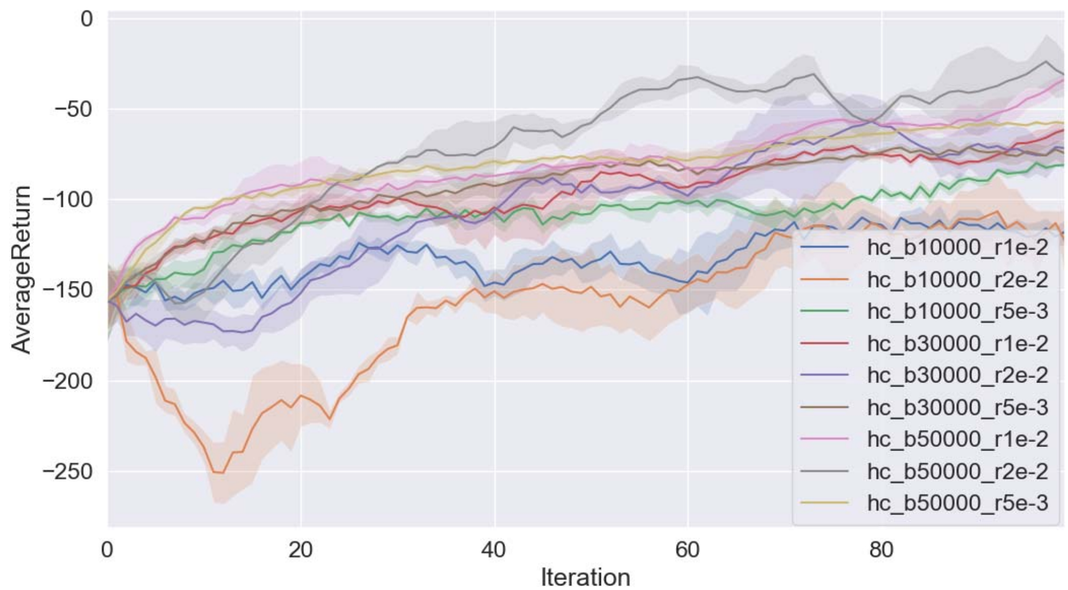
```

```

Problem 8 - HalfCheetah

How did the batch size and learning rate affect the performance?

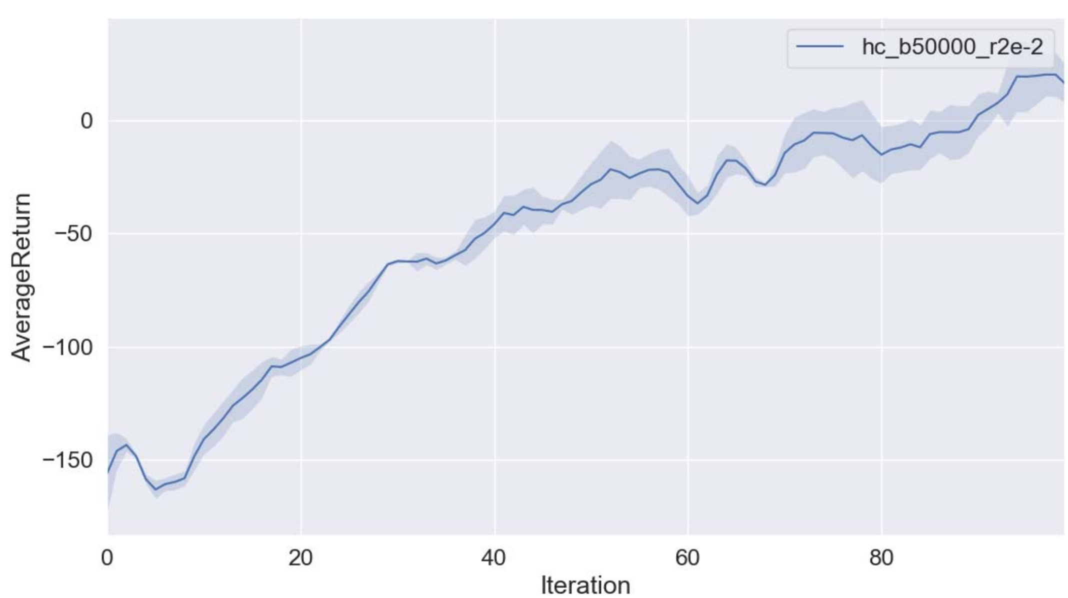
The learning curves under different batches and learning rates are plotted. From 10000 to 50000, as the batch size increases, the learning curve reaches better results. With different batch sizes, the learning rates have different influence on the learning performance.



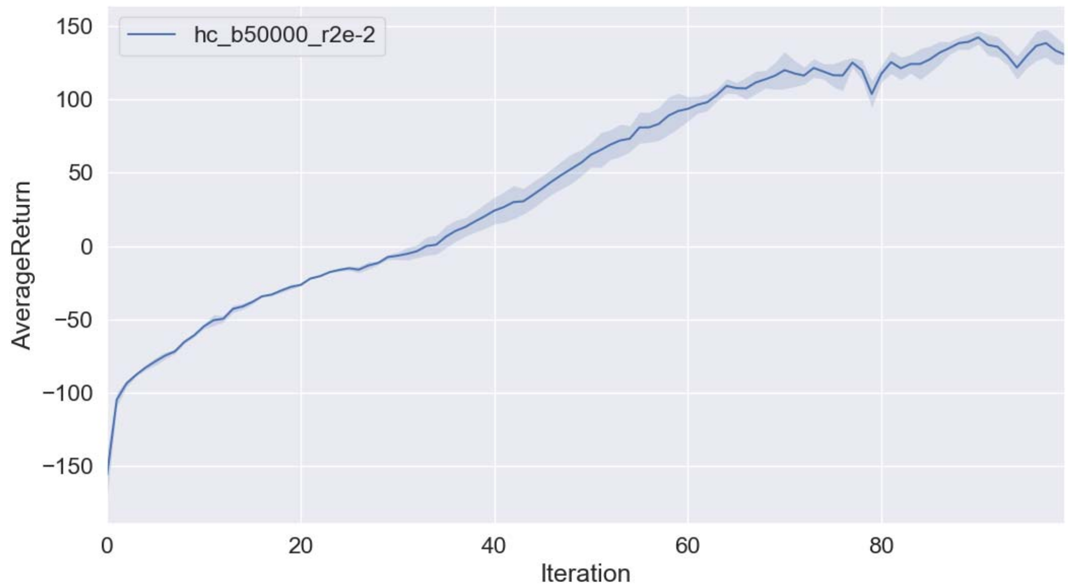
Find the optimal b* and r* and run the following commands

We can see that the optimal `b*=50000` and `r*=2e-2`

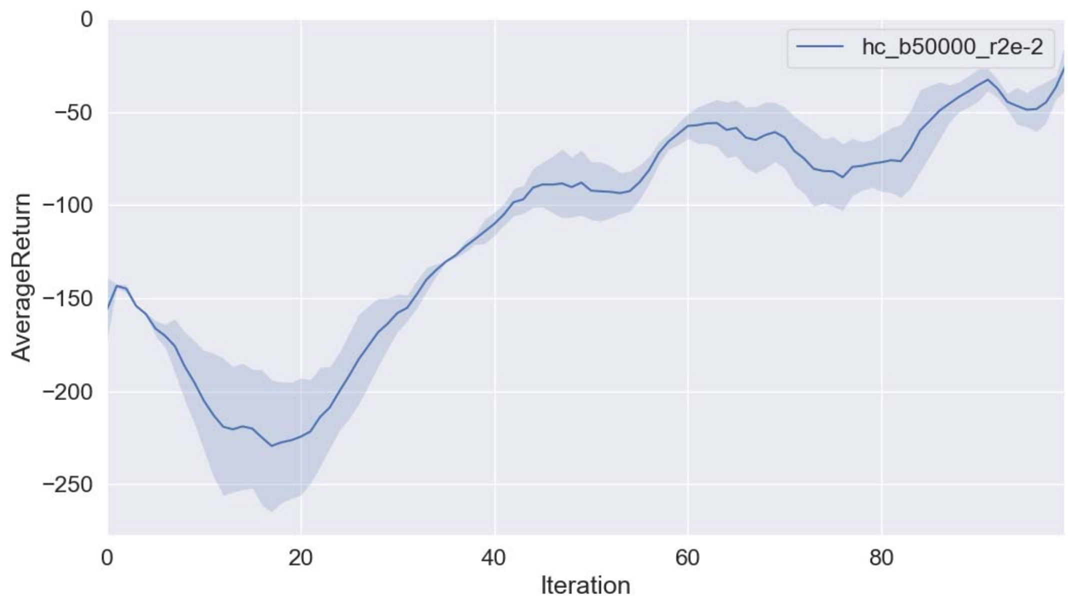
Results `py train_pg_f18.py HalfCheetah-v2 -ep 150 --discount 0.95 -n 100 -e 3 -l 2 -s 32 -b <b*> -lr <r*> --exp_name hc_b<b*>_r<r*>`



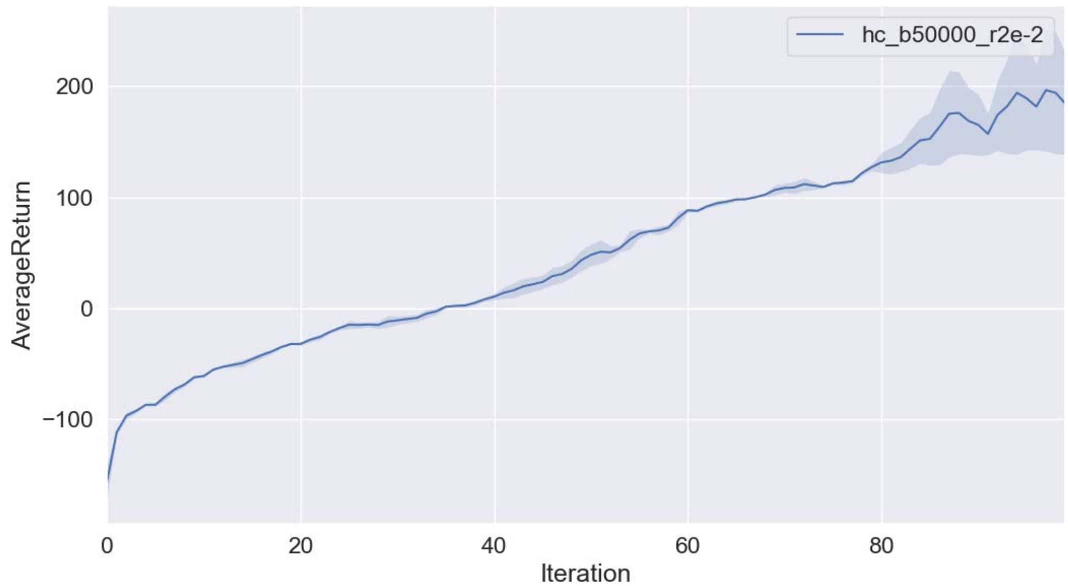
Results with reward-to-go `py train_pg_f18.py HalfCheetah-v2 -ep 150 --discount 0.95 -n 100 -e 3 -l 2 -s 32 -b <b*> -lr <r*> -rtg --exp_name hc_b<b*>_r<r*>`



Results with baseline `py train_pg_f18.py HalfCheetah-v2 -ep 150 --discount 0.95 -n 100 -e 3 -l 2 -s 32 -b <b*> -lr <r*> --nn_baseline --exp_name hc_b<b*>_r<r*>`



Results with reward-to-go and baseline `py train_pg_f18.py HalfCheetah-v2 -ep 150 --discount 0.9 -n 100 -e 3 -l 2 -s 32 -b <b*> -lr <r*> -rtg --nn_baseline --exp_name hc_b<b*>_r<r*>`



Provide the exact command line configurations you used to run your experiments

Codes are include in `q8_main.bash`

```
for batch in 10000 30000 50000
do
    for lr in 2e-2 1e-2 5e-3
    do
        py train_pg_f18.py HalfCheetah-v2 -ep 150 --discount 0.9 -n 100 -
e 3 -l 2 -s 32 -b $batch -lr $lr --exp_name hc_b${batch}_r${lr}
    done
done
```

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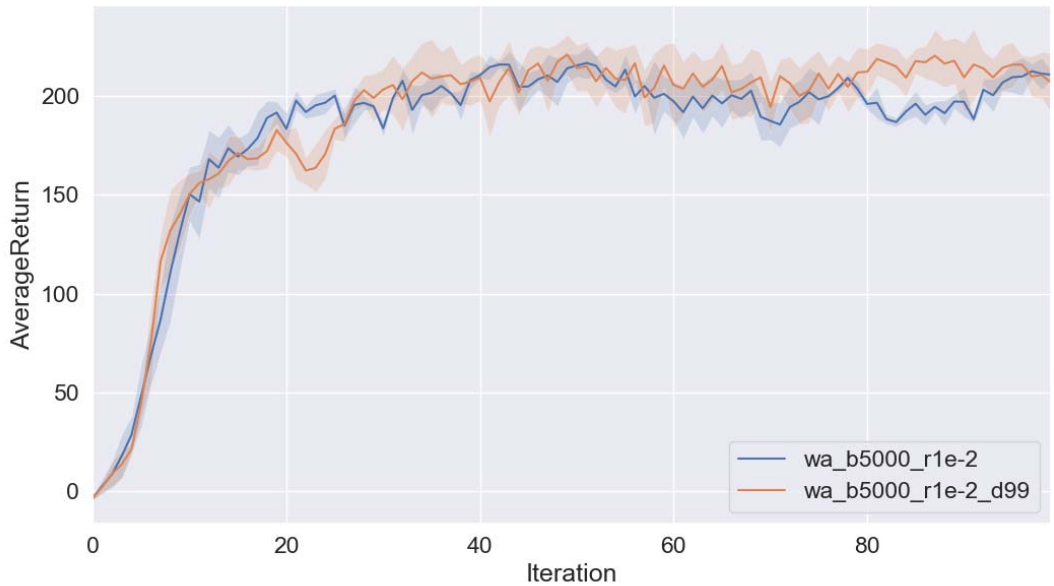
Bonus Question - Implementation of GAE_lambda

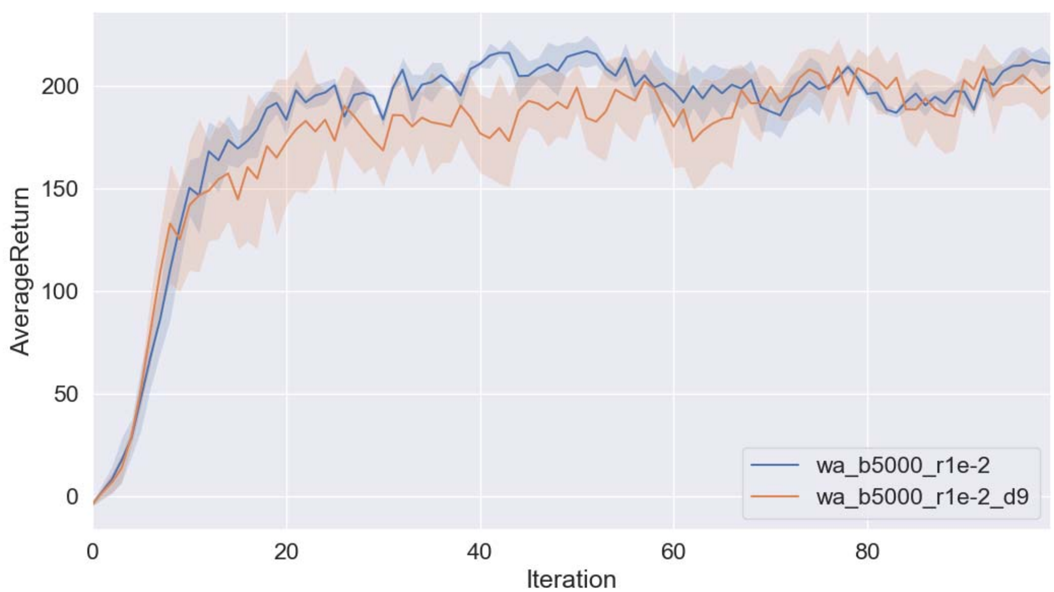
Learning performance comparison of

- Learner 1: with baseline and reward to go
- Learner 2: with bl, rtg and GAE with `lambda=0.9`
- Learner 3: with bl, rtg and GAE with `lambda=0.99`

Details see file `train_pg_f18.py` line 501-line 516.

Trained date one `batch_size=[5000, 50000]` , `learning_rate=[1e-2, 2e-2]` and `nn_size=[32, 64]` have been tested out and the resulting images are in folder `data` . The results for `batch_size=5000` , `learning_rate=2e-2` and `nn_size=32` are displayed.



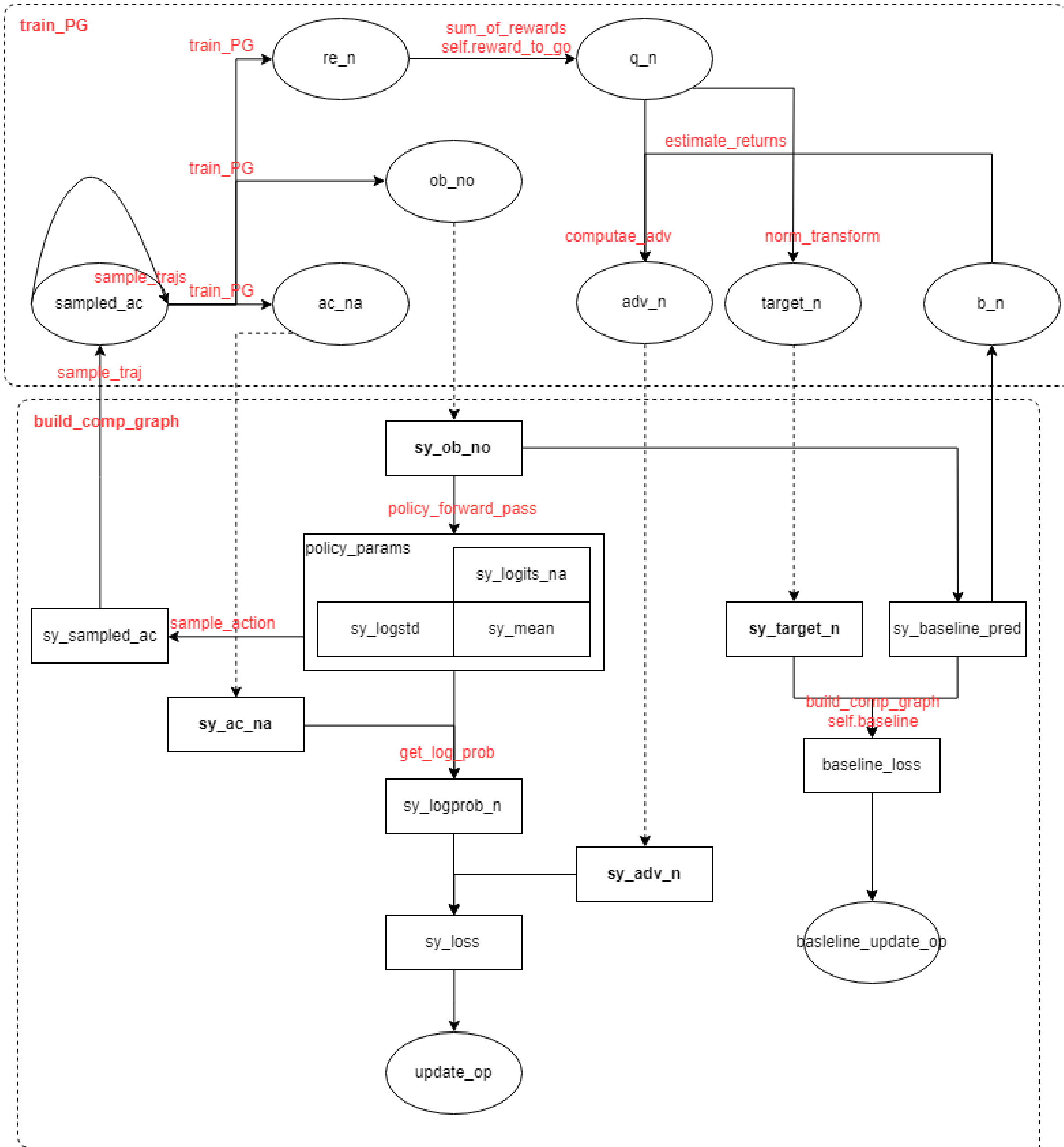


Provide the exact command line configurations you used to run your experiments

Codes are include in `gae_main.bash`

Overview of the Architecture && Computational Graph

Agent


train_PG
train_PG
re_n
sum_of_rewards
self.reward_to_go
q_n
train_PG
ob_no
estimate_returns
computae_adv
norm_transform
sample_trajs
train_PG
sampled_ac
ac_na
adv_n
target_n
b_n
sample_traj
build_comp_graph
sy_ob_no
policy_forward_pass
policy_params
sy_logits_na
sy_logstd
sy_mean
sy_sampled_ac
sample_action
sy_target_n
sy_baseline_pred
sy_ac_na
build_comp_graph
self.baseline
get_log_prob
baseline_loss
sy_logprob_n
sy_adv_n
sy_loss
basleline_update_op
update_op