Exercise 4 - Continuous random variable

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The content of this script is only as a supplementary illustration to the exercise, it is not necessary to know at the exam. It is important to be able to calculate manually.

Numerical integration in R

R function **integrate**

integrate(f, a, b)=$\int_a^b f(x)dx$

- **f** is a R function(defined by us) which has one input argument - a vector of values in which to return its values
- **a** lower integration limit
- **b** upper integration limit

In [1]:

```
f = function(x){return(x*x)} # x ^ 2
a = -1
b = 2
integrate(f, a, b)
```

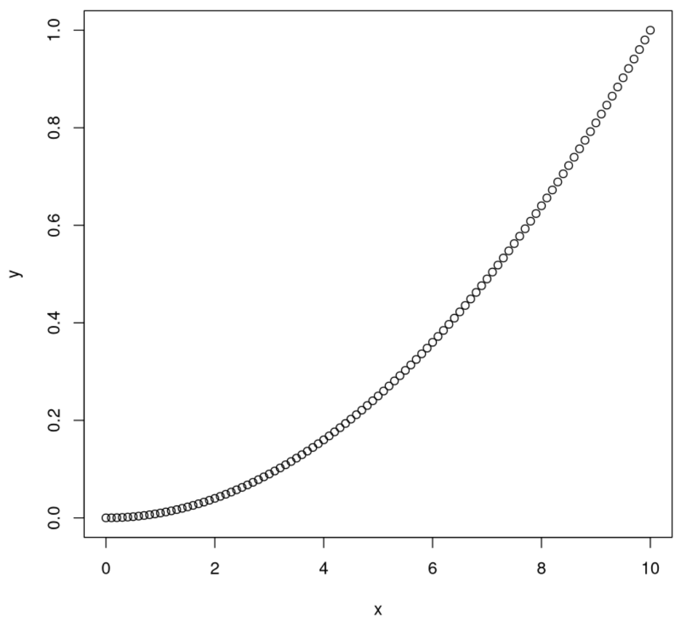
```
3 with absolute error < 3.3e-14
```

In [2]:

```
x = seq(0,10,0.1)
```

In [3]:

```
y=x*x/100
plot(x,y)
```



In [4]:

```
(9*50-20^2)/9
```

5.55555555555556

Examples

Example 1.

Random variable X has distribution function

$$F(x) = \begin{cases} 0 & x \le 0 \\ cx^2 & 0 < x \le 1 \\ 1 & 1 < x \end{cases}$$

What values can the constant c take?

In [5]:

```
# derivative of F(x) is the density of density f(x)
# corresponding probability density at interval<0.1>
f = function(x){return(2*x)} # f(x)=2x
a = 0
b = 1
integrate(f, a, b)$value
```

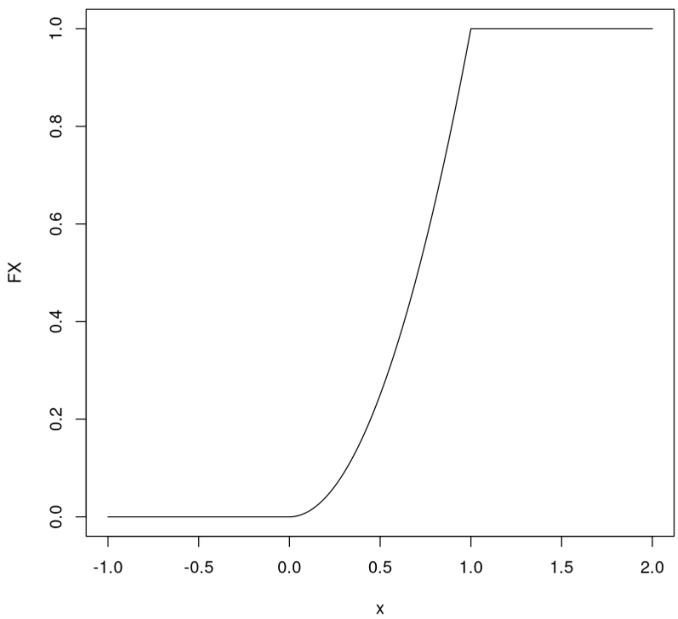
1

In [6]:

```
# c=1, so the distribution function looks like this:
F.dist = function(x){
    res = x*x      # x ^ 2
    res[x<=0] = 0 # 0 for x<=0
    res[x>1] = 1  # 1 for x>1
    return(res)
}
```

In [7]:

```
x = seq(from = -1, to = 2, by = 0.01) # points on the x-axis
FX = F.dist(x) # values of F(x)
plot(x, FX, type = 'l') # draw as a line
```



Example 2.

The distribution of a random variable X is given by the density

$$f(x) = \begin{cases} 2x+2 & x \in <-1;0> \\ 0 & x \notin <-1;0> \end{cases}$$

Specify:

2. a)

$F(x)$,

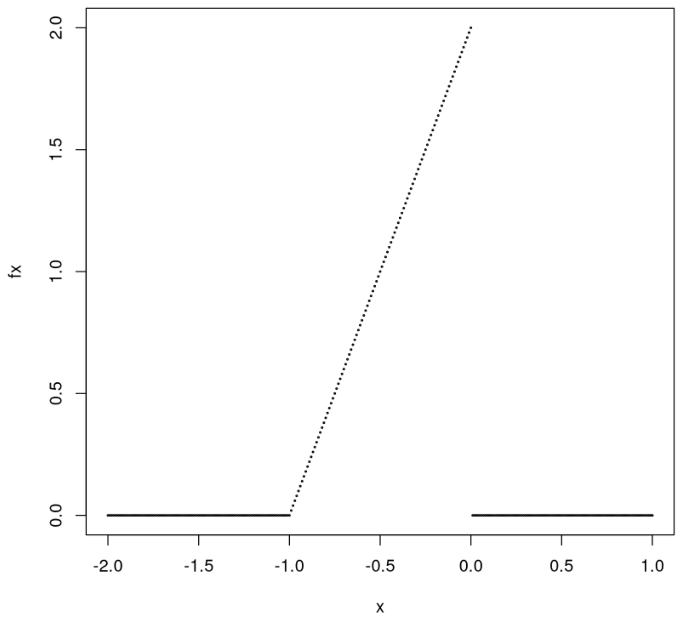
In [8]:

```
f.dens = function(x){
```

```
    res = 2*x + 2
    # watch out for x<-1 because '<-' is in the assignment line
    res[x < -1] = 0 # 0 for x<=0
    res[x > 0] = 0  # 1 for x>1
    return(res)
}
```

In [9]:

```
x = seq(from = -2, to = 1, by = 0.01) # points on the x-axis
fx = f.dens(x) # values of f(x)
plot(x, fx, cex=0.2) # draw dots(cex is the size)
```

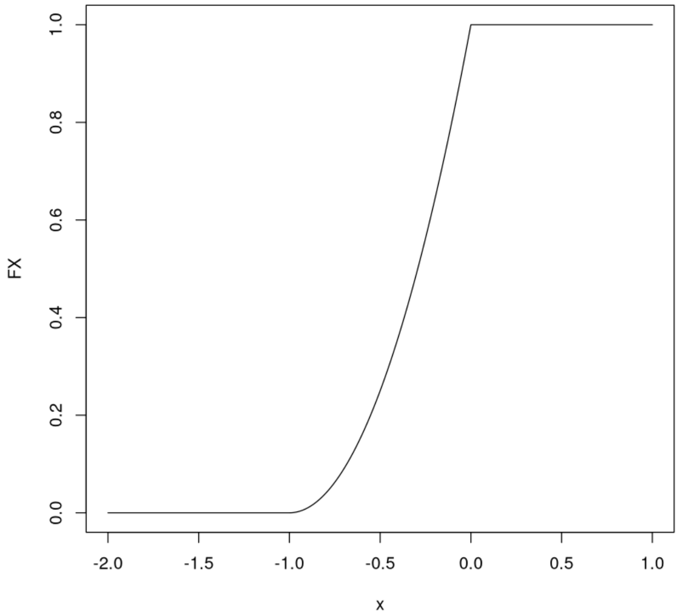


In [10]:

```
F.dist = function(x){
    res = x*x+2*x+1     # x ^ 2 + 2x + 1
    res[x < -1] = 0 # 0 for x<=0
    res[x > 0] = 1  # 1 for x>1
    return(res)
}
```

In [11]:

```
x = seq(from = -2, to = 1, by = 0.01) # points on the x-axis
FX = F.dist(x) # values of f(x)
plot(x, FX, type='l') # draw dots(cex is the size)
```



2. b)

P(−2 ≤ X ≤ 0.5), P(−2 ≤ X ≤ −1), P(X>0.5), P(X=0.3)

In [12]:
```
# P(-2 ≤ X ≤.50.5)
integrate(f.dens, -2, -0.5)$value
integrate(f.dens, -1, -0.5)$value
```

0.25

0.25

In [13]:
```
# P(-2 ≤ X - -1)
integrate(f.dens, -2, -1)$value
```

0

In [14]:
```
# P(X>0.5)
integrate(f.dens, 0.5, 1e16)$value # This will not always work
```

0

In [15]:
```
# P(X=0.3)
integrate(f.dens, 0.3, 0.3)$value
# it is clear that this probability is 0
# corresponds to the integral sa=b, ie with zero size on the x-axis
```

0

2. c)

mean, variance and standard deviation of the random variable X.

In [16]:
```
# E(X)
x_fx = function(x){
    fx = f.dens(x)
    return(x*fx)
}
# we integrate only where we know that f(x) is nonzero
E_X = integrate(x_fx, -1, 0)$value
E_X
-1/3
```

-0.333333333333333

-0.333333333333333

In [17]:

```
# E(X ^ 2)
xx_fx = function(x){
    fx = f.dens(x)
    return(x*x*fx)
}
# we integrate only where we know that f(x) is nonzero
E_XX = integrate(xx_fx, -1, 0)$value
E_XX
1/6
```

0.166666666666667

0.166666666666667

In [18]:

```
# D(X)
D_X = E_XX - E_X^2
D_X
1/18
```

0.0555555555555556

0.0555555555555556

In [19]:

```
# sigma(x)
std_X = sqrt(D_X)
std_X
sqrt(2)/6
```

0.235702260395516

0.235702260395516

2. d)

mode $\hat{x}$

In [20]:

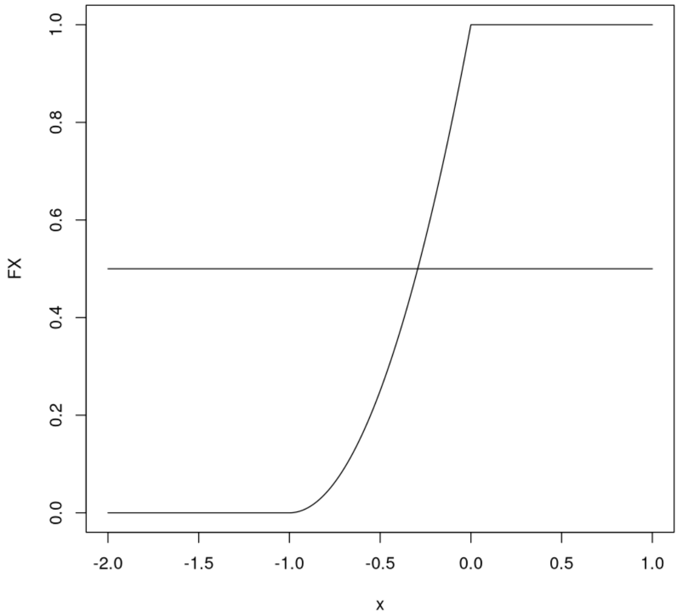
```
# mode=0
```

2. e)

median $x_{0,5}$

In [21]:

```
x = seq(from = -2, to = 1, by = 0.001) # points on the x-axis
FX = F.dist(x)
plot(x, FX, type='l')
lines(c(-2, 1),c(0.5, 0.5))
```



In [22]:

```
x[FX >= 0.5][1] # first element zx for which F(x)>=0.5
(-2+sqrt(2))/2
```

-0.292

-0.292893218813452

Example 3.

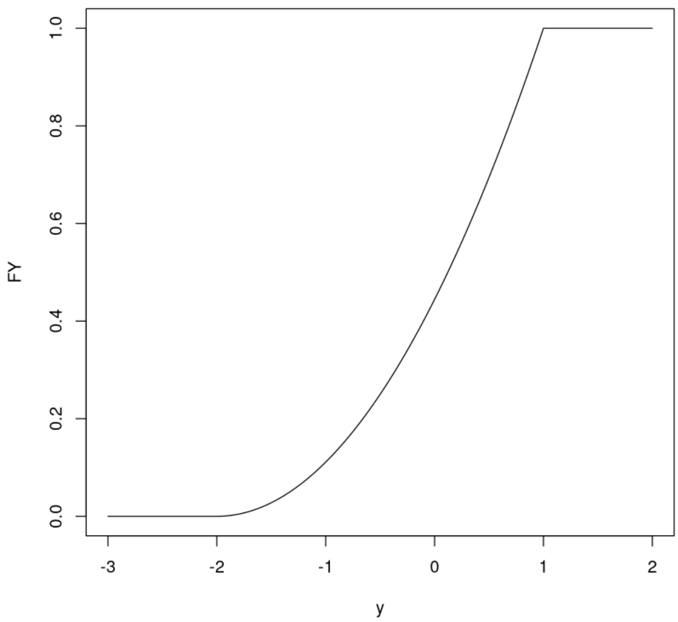
The random variable Y is defined as: Y=3X + 1, where X is the random variable from the previous example. Specify:

3. a)

$F_Y(y)$

In [23]:

```
FY.dist = function(y){
    # calculated from the relation FY(y)=P(Y<y)=P(3X + 1<y)=...
    x = (y-1)/3
    FY = F.dist(x)
    return(FY)
}
y = seq(from = -3, to = 2, by = 0.001) # points on the x-axis
FY = FY.dist(y)
plot(y, FY, type='l')
```



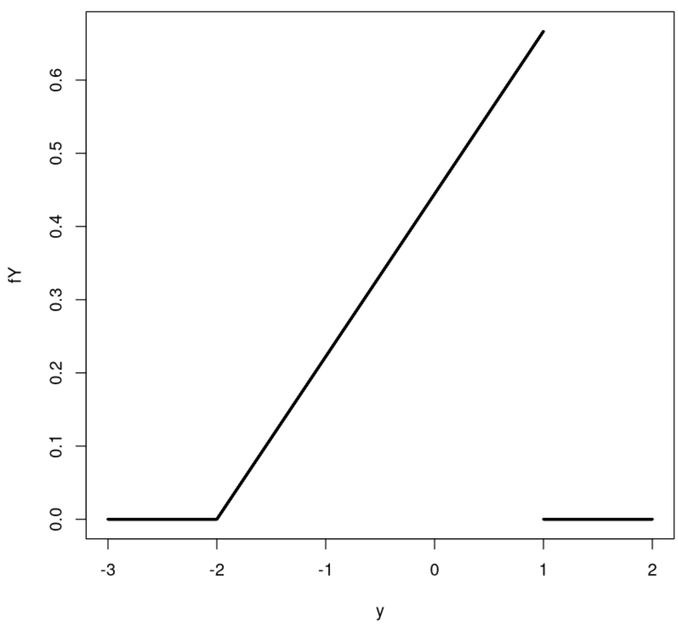
3. b)

$f_Y(y)$

In [24]:

```
# derivation of F_Y
fY.dens = function(y){
    res = 2/9*(y + 2)
    res[y < -2] = 0 # 0 for x<-2
    res[y > 1] = 0  # 1 for x>1
    return(res)
}
integrate(fY.dens,-2,1)$value # total integral check
y = seq(from = -3, to = 2, by = 0.001) # points on the x-axis
fY = fY.dens(y)
plot(y, fY, cex=0.2)
```

1



3. c)

E(Y), D(Y), σ(Y)

In [25]:
```
# E(Y)
y_fy = function(y){
    fy = fY.dens(y)
    return(y*fy)
}
# we integrate only where we know that f(y) is nonzero
E_Y = integrate(y_fy, -2, 1)$value
E_Y
0
```

1.04083408558608e-17

0

In [26]:
```
# alternatively
E_Y = 3*E_X + 1
E_Y
```

0

In [27]:
```
# E(Y ^ 2)
yy_fy = function(y){
    fy = fY.dens(y)
    return(y*y*fy)
}
# we integrate only where we know that f(y) is nonzero
E_YY = integrate(yy_fy, -2, 1)$value
E_YY
1/2
```

0.5

0.5

In [28]:
```
# D(Y)
D_Y = E_YY - E_Y^2
D_Y
1/2
```

0.5

0.5

In [29]:
```
# alternatively
D_Y = 3^2*D_X
D_Y
```

0.5

In [30]:
```
# sigma(Y)
sqrt(D_Y)
sqrt(2)/2
```

0.707106781186548

0.707106781186548

Example 4

Calculate ω such that a random variable X with probability density:

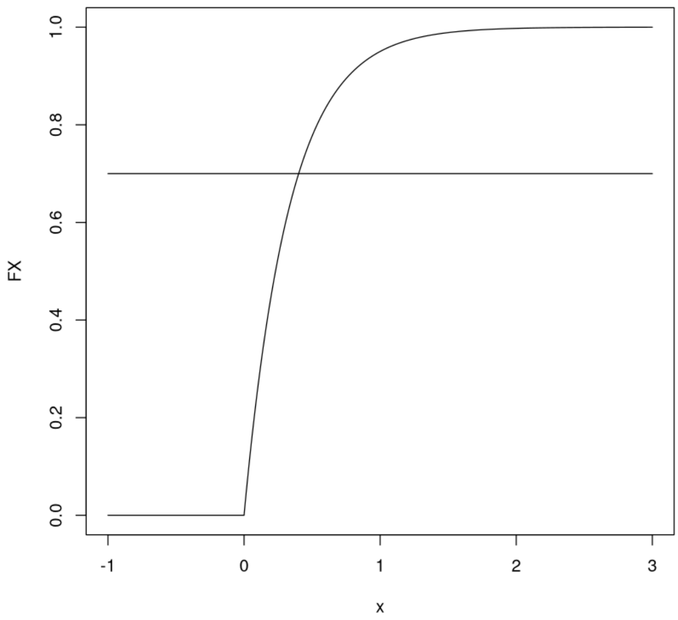
$$f(x) = \begin{cases} 0 & x < 0 \\ 3e^{-3x} & x \geq 0 \end{cases}$$

is greater than ω with probability 0.3.

In [31]:
```
F.dist = function(x){
    res = 1 - exp(-3*x)
    res[x < 0] = 0 # 0 for x<=0
    return(res)
}
```

In [32]:
```
x = seq(from = -1, to = 3, by = 0.001) # points on the x-axis
```

```
FX = F.dist(x)
plot(x, FX, type='l')
lines(c(-1, 3),c(0.7, 0.7))
```



In [33]:

```
x[FX >= 0.7][1]
```

0.402

In [34]:

```
-1/3*log(0.3)
```

0.401324268108645