Discretization of 2D Incompressible Governing Equations

$$\nabla . \boldsymbol{u} = 0$$

$$\frac{\partial \phi}{\partial t} + \nabla . (u\phi) = -\frac{1}{\rho} \nabla . (P . n) + \alpha \nabla . (\nabla \phi)$$

$$\int_{cv} -\frac{1}{\rho} \frac{\partial P}{\partial x} \, dV = \int_{cv} -\frac{1}{\rho} \nabla . (P \, \hat{\imath}) \, dV = \int_{cs} -\frac{1}{\rho} P \, \hat{\imath} \, . d\overrightarrow{S_f}$$

FVM Governing equations

$$\frac{V_p\left(u_p^{n+1} - u_p^n\right)}{dt} + \Sigma_f F_f^n u_f^{n+1} + \Sigma_f F_{uf}^{d \, n+1} = -\frac{1}{\rho} \Sigma_f P_f^{n+1} S_{fx}$$

$$\frac{V_p\left(v_p^{n+1} - v_p^n\right)}{dt} + \Sigma_f F_f^n v_f^{n+1} + \Sigma_f F_{vf}^{d \, n+1} = -\frac{1}{\rho} \Sigma_f P_f^{n+1} S_{fy}$$

$$\Sigma F_f^{n+1} = 0$$

$$F_{uf}^d = -\Gamma \, (\nabla u) \, . S_f \; , \; \Gamma = \nu$$

Discretization of convective terms

$$flux_f = \boldsymbol{u_f} \, . S_f \text{ , where } u_f = \frac{V_{nb} u_p + V_p u_{nb}}{V_{nb} + V_p}$$

V_{nb} means Volume of neighbour element.

Use upwind based on the flux value.

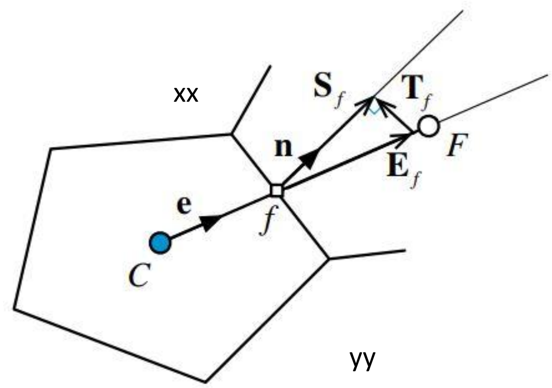
$$u_f^{n+1} = u_f^{n+1} \; if \; flux_f \geq 0$$

$$u_f^{n+1} = u_p^{n+1} \; if \; flux_f < 0$$

We follow the same procedure for solving v-momentum equation

Discretization of diffusive terms

$(\nabla \phi)_f \,.S_f$, where $S_f = \boldsymbol{E_f} + \boldsymbol{T_f}$



$\boldsymbol{E_f}$ is normal direction

$\boldsymbol{T_f}$ is cross direction

$(\nabla \phi)_f \,.S_f = (\nabla \phi)_f \,.\boldsymbol{E_f} + (\nabla \phi)_f \,.\boldsymbol{T_f}$

Using over relaxed approach for better stability.

$$\boldsymbol{E_f} = \frac{S_f \,.\, S_f}{e \,.\, S_f}\, e$$

$$\boldsymbol{E_f} = E_f e \text{ , where } E_f = magnitude\ of\ \boldsymbol{E_f}$$

$$(\nabla \phi)_f \,.\boldsymbol{E_f} = \left(\frac{\partial \phi}{\partial e}\right)_f e \,.E_f\, e = E_f \left(\frac{\partial \phi}{\partial e}\right)_f = E_f (\frac{\phi_f - \phi_c}{d_{cf}})$$

$$\boldsymbol{T_f} = S_f - \boldsymbol{E_f}$$

Similarly, we do for cross diffusion terms

$$(\nabla \phi)_f \,.\boldsymbol{T_f} = T_f (\frac{\phi_{xx} - \phi_{yy}}{|S_f|})$$

ϕ_{xx} *is obtained by volumetric interpolation of all elements sharing point xx*

By using the above discretization we follow the SIMPLE algorithm to solve u,v,p.

For Non-Dimensional case we give $\rho = 1\ and\ \nu = \frac{1}{Re}$