SRM Institute of Science and Technology
Ramapuram Campus

Department of Mathematics

Year / Sem: I / II

Branch: Common to ALL Branches of B.Tech. except B.Tech. (Business Systems)

Unit 5 – Complex Integration

Part – B (Each question carries 3 Marks)

1. Evaluate $\int_C e^{\frac{1}{z}}\,dz$ where C is | z – 2 |=1 by Cauchy's integral theorem.

(A) π i (B) 4 π i (C) 0 (D) 2 π i

Solution

$e^{\frac{1}{z}}$ is analytic inside and on C.

Hence by Cauchy's Integral theorem, $\int_C e^{\frac{1}{z}}\,dz = 0$.

Answer: (C)

2. Evaluate $\int_C \frac{1}{2z-3}\,dz$ where C is | z | = 1 by Cauchy's integral formula.

(A) 1 (B) 4 π i (C) 0 (D) 2 π i

Solution

Here $a = \frac{3}{2}$ lies outside | z | = 2.

By Cauchy's Integral formula,

$$\int_C \frac{1}{2z-3}\,dz = 0$$

Answer: (C)

3. Evaluate $\int_C \frac{1}{(z-3)^2} dz$ where C is | z | = 1 by Cauchy's integral formula.

(A) 1 (B) 4π i (C) 0 (D) 2π i

Solution

Here $a = 3$ lies outside | z | = 1.

By Cauchy's Integral formula,

$$\int_C \frac{1}{(z-3)^2} dz = 0$$

Answer: (C)

4. Evaluate $\int_C \frac{2z}{z-1} dz$ where C is | z | = 2 by Cauchy's integral formula.

(A) 1 (B) 4π i (C) 0 (D) 2π i

Solution

Here $f(z) = 2z$ and $a = 1$ lies inside | z | = 2.

By Cauchy's Integral formula,

$$\int_C \frac{2z}{z-1} dz = 2\pi i \; f(1) = 2\pi i (2) = 4\pi i$$

Answer: (B)

5. Evaluate $\int_C \frac{\cos \pi z}{z-1} dz$ where C is | z | = 3.

(A) -2π i (B) 4π i (C) 0 (D) 2π i

Solution

Here $f(z) = \cos \pi z$ and $a = 1$ lies inside | z | = 3.

By Cauchy's Integral formula,

$$\int_C \frac{\cos \pi z}{z-1} dz = 2\pi i \; f(1) = 2\pi i (-1) = -2\pi i$$

Answer: (A)

6. Evaluate $\int_C \frac{e^{-z}}{z+1} dz$ where C is | z | = 1.5.

(A) $-2\pi i e$ (B) 4π i (C) 0 (D) $2\pi i e$

Solution

Here $f(z) = e^{-z}$ and $a = -1$ lies inside | z | = 1.5.

By Cauchy's Integral formula,

$$\int_C \frac{e^{-z}}{z+1} dz = 2\pi i \ f(-1) = 2\pi i e$$ **Answer: (D)**

7. Evaluate $\int_C \frac{1}{z e^z} dz$ where C is | z | = 1.

(A) $-2\pi i e$ (B) 2π i (C) 0 (D) $2\pi i e$

Solution

Here $f(z) = \frac{1}{e^z}$ and $a = 0$ lies inside | z | = 1.

By Cauchy's Integral formula,

$$\int_C \frac{\frac{1}{e^z}}{z} dz = 2\pi i \ f(0) = 2\pi i 1 = 2\pi i$$ **Answer: (B)**

8. Evaluate $\int_C \frac{z+1}{z(z-2)} dz$ where C is | z | = 1.

(A) $-2\pi i e$ (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $2\pi i e$

Solution

Here $f(z) = \frac{z+1}{z-2}$ and $a = 0$ lies inside | z | = 1.

By Cauchy's Integral formula,

$$\int_C \frac{\frac{z+1}{z-2}}{z}\,dz = 2\pi i\; f(0) = -\frac{1}{2}$$

Answer: (C)

9. Evaluate $\int_C \frac{\cos \pi z^2}{(z-1)(z-2)}\,dz$ where C is | z | = 1.5.

(A) 1 (B) 4π i (C) 0 (D) 2π i

Solution

Here $f(z) = \frac{\cos \pi z^2}{z-2}$ and $a = 1$ lies inside | z | = 1.5.

By Cauchy's Integral formula,

$$\int_C \frac{\frac{\cos \pi z^2}{z-2}}{z-1}\,dz = 2\pi i\; f(1) = 2\pi i\,\frac{\cos \pi}{1-2} = 2\pi i$$

Answer: (D)

10. Evaluate $\int_C \frac{1}{(z+1)(z-2)^2}\,dz$ where C is | z | = 1.5.

(A) 1 (B) $\frac{4\pi i}{9}$ (C) 0 (D) $\frac{2\pi i}{9}$

Solution

Here $f(z) = \frac{1}{(z-2)^2}$ and $a = -1$ lies inside | z | = 1.5.

By Cauchy's Integral formula,

$$\int_C \frac{\frac{1}{(z-2)^2}}{z+1}\,dz = 2\pi i\; f(-1) = 2\pi i\,\frac{1}{9} = \frac{2\pi i}{9}$$

Answer: (D)

11. Evaluate $\int_C \frac{z}{(z-1)^3} dz$ where C is | z | = 2 by Cauchy's integral formula for derivatives.

(A) 1 (B) 4π i (C) 0 (D) 2π i

Solution

Here $f(z) = z$ and $a = 1$ lies inside | z | = 2.

By Cauchy's Integral formula for derivatives,

$$\int_C \frac{z}{(z-1)^3} dz = \frac{2\pi i}{2!} f''(1) = \pi i(0) = 0$$

Answer: (C)

12. Calculate the residue at z = 0 for the function $f(z) = \frac{3-e^{2z}}{z}$.

(A) 1 (B) 2 (C) 3 (D) – 2

Solution

$$\operatorname{Re}s\left[f(z), a\right] = \lim_{z \to a} (z-a) f(z)$$

$$\operatorname{Re}s\left[f(z), 0\right] = \lim_{z \to 0} (z-0) \frac{(3-e^{2z})}{z} = 2$$

Answer: (B)

13. Calculate the residue at z = i for the function $f(z) = \frac{1}{z^2+1}$.

(A) 1 (B) 2 (C) $\frac{1}{2i}$ (D) – 2

Solution

$$\operatorname{Re}s\left[f(z), a\right] = \lim_{z \to a} (z-a) f(z)$$

$$\operatorname{Re}s\left[f(z), i\right] = \lim_{z \to i} (z-i) \frac{1}{(z+i)(z-i)} = \frac{1}{2i}$$

Answer: (C)

14. Calculate the residue at z = $-i$ for the function $f(z)=\dfrac{z}{z^2+1}$.

(A) 1 (B) 2 (C) 1/2 (D) – 2

Solution

$$\operatorname{Re}s\left[f(z),a\right]=\lim_{z\to a}(z-a)\,f(z)$$

$$\operatorname{Re}s\left[f(z),-i\right]=\lim_{z\to -i}\ (z+i)\frac{z}{(z+i)(z-i)}=\frac{1}{2}$$

Answer: (C)

15. Calculate the residue of the function $f(z)=\dfrac{e^{2z}}{(z+1)^2}$ at its pole.

(A) $2e$ (B)$3e$ (C) $2e^{-2}$ (D)$2e^2$

Solution

z = – 1 is a pole of order 2.

$$\operatorname{Re}s\left[f(z),a\right]=\frac{1}{(n-1)!}\lim_{z\to a}\frac{d^{n-1}}{dz^{n-1}}(z-a)^n\ f(z)$$

$$\operatorname{Re}s\left[f(z),-1\right]=\frac{1}{(2-1)!}\lim_{z\to -1}\frac{d^{2-1}}{dz^{2-1}}(z+1)^2\ \frac{e^{2z}}{(z+1)^2}=\frac{1}{1!}\lim_{z\to -1}\frac{d}{dz}e^{2z}\ =2e^{-2}$$

Answer: (C)