



**SRM Institute of Science and Technology
Ramapuram Campus**

Department of Mathematics

18MAB101T - Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – IV

DIFFERENTIAL CALCULUS

Part – B

1. Envelope of the curve $y = mx + \frac{a}{m}$ (where m is the parameter) is

(A) $x^2 + a y = 0$ (B) $x + 4 a y = 0$

(C) $y^2 - 4 a x = 0$ (D) $y^2 + 4ax = 0$

Solution: Given: $y = mx + \frac{a}{m}$

$$y = \frac{m^2 x + a}{m}$$

$$m^2 x - y m + a = 0$$

The above equation is a quadratic equation in ' m '.

The discriminant is $b^2 - 4ac = 0$.

The envelope of the curve is $y^2 - 4 a x = 0$. **(Option C)**

2. The radius of curvature of the curve $y = e^x$ at $x = 0$ is

(A) $2\sqrt{2}$ (B) $\sqrt{2}$

(C) 2 (D) 4

Solution:

$$y_1 = e^x \text{ at } x = 0 \text{ is } 1$$

$$y_2 = e^x \text{ at } x = 0 \text{ is } 1$$

$$\rho = \frac{(1 + y_1^2)^{\frac{3}{2}}}{y_2}$$

$$\rho = 2\sqrt{2} \quad \text{(Option A)}$$

3. The radius of curvature of the curve $y = 4 \sin x$ at $x = \frac{\pi}{2}$ is

$$(A) \frac{1}{2} \quad (B) \frac{-1}{2}$$

$$(C) \frac{1}{4} \quad (D) \frac{3}{4}$$

Solution:

$$y_1 = 4 \cos x \text{ at } x = \frac{\pi}{2} \text{ is } 0$$

$$y_2 = -4 \sin x \text{ at } x = \frac{\pi}{2} \text{ is } -4$$

$$\rho = \frac{(1 + y_1^2)^{\frac{3}{2}}}{y_2}$$

$$|\rho| = \frac{1}{4} \quad \text{(Option C)}$$

4. The envelope of family of lines $y = mx + am^2$ (where m is the parameter) is

$$(A) x^2 + 2ay = 0 \quad (B) x^2 + 4ay = 0$$

$$(C) y^2 + 2ax = 0 \quad (D) x^2 + 4ax = 0$$

Solution:

The given equation is quadratic in 'm'.

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $x^2 + 4ay = 0$. **(Option B)**

5. The envelope of the family of lines $\frac{x}{t} + y t = 2c$, t being the parameter is

- (A) $x^2 + y^2 = c^2$ (B) $xy = c^2$
 (C) $x^2 y^2 = c^2$ (D) $x^2 - y^2 = c^2$

Solution:

Simplifying the equation $\frac{x}{t} + y t = 2c$, we get $yt^2 - 2ct + x = 0$

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $xy = c^2$. **(Option B)**

6. The radius of curvature of the curve $r = e^\theta$ at any point on it is

- (a) $2\sqrt{2}$ (b) $\sqrt{2}r$
 (c) 2 (d) 4

Solution:

$$r' = e^\theta$$

$$r'' = e^\theta$$

$$\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r^2 - rr'' + 2r'^2}$$

$$\rho = \sqrt{2}r$$

7. The radius of curvature at the point (3, 4) on the curve $x^2 + y^2 = 25$ is

- (A) 5 (B) 4 (C) 0 (D) 2

Solution:

We know that the radius of curvature of the circle is equal to its radius.

$$\rho = 5 \text{ (Option A)}$$

$$8. B(5/2, 1/2) = \text{_____}.$$

$$(A) 1$$

$$(B) 4$$

$$(C) \pi/2$$

$$(D) \pi$$

Solution:

$$B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$B(5/2, 1/2) = \frac{\Gamma(5/2)\Gamma(1/2)}{\Gamma(3)} = \frac{\pi}{2} \text{ (Option C)}$$

$$9. \Gamma(-5/2) = \text{_____}.$$

$$(A) 1$$

$$(B) 4$$

$$(C) 1/2$$

$$(D) \frac{-8\sqrt{\pi}}{3}$$

Solution:

$$\Gamma\left(-n + \frac{1}{2}\right) = \frac{(-2)^n}{1.3.5 \cdots (2n-1)} \sqrt{\pi}$$

$$\Gamma\left(-\frac{5}{2}\right) = \Gamma\left(\frac{-6+1}{2}\right)$$

$$= \Gamma\left(-3 + \frac{1}{2}\right) = \frac{-8}{3} \sqrt{\pi}$$

(Option D)

10. Evaluate $\int_0^{\infty} e^{-x} x^4 dx$.

(A) 1

(B) 24

(C) 1/2

(D) $\frac{-8\sqrt{\pi}}{3}$

Solution

$$\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx$$

$$\int_0^{\infty} e^{-x} x^4 dx = \int_0^{\infty} e^{-x} x^{5-1} dx = \Gamma(5) = 4! = 24$$

(Option B)

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