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Math
Assignment - II

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1) $u_n = \frac{1}{\sqrt{n+1}} = \frac{1}{\sqrt{n(1+1/n)}}$

Let $v_n = \frac{1}{\sqrt{n}}$

$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{\sqrt{1+1/n}} = 1$ (finite and non zero)

$\therefore$ Both $\sum u_n$ and $\sum v_n$ converges or diverges together.

$\sum u_n = \sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$, $\boxed{P = 1/2 < 1}$ is divergent

By comparison test,

$\therefore$ (B) $\sum u_n \to$ diverges

2) $u_n = \frac{n!}{n^n}$, $u_{n+1} = \frac{(n+1)!}{(n+1)^{n+1}}$

$\frac{u_n}{u_{n+1}} = \frac{n!}{n^n} \times \frac{(n+1)^{n+1}}{(n+1)!} = \frac{(n+1)^n}{n^n}$

$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{n^n}{(n+1)^n} \Rightarrow \lim_{n\to\infty} \frac{\cancel{n^n}}{\cancel{n^n}(1+1/n)^n} \Rightarrow \frac{1}{e} \Rightarrow 0.368 < 1$

$\therefore$ By ratio test, (A) $\sum u_n$ is convergent

3) $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$

$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{(n+1)^3}{3^{n+1}} \times \frac{3^n}{n^3} = \lim_{n\to\infty} \frac{1}{3} \cdot \frac{(n+1)^3}{n^3}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{3} \frac{(1+1/n)^3 \cancel{n^3}}{\cancel{n^3}} \Rightarrow 0.33 < 1$

$\therefore$ By ratio test, (A) $\sum u_n$ converges.

4) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$

Solⁿ: Given, $u_n = \frac{1}{n}$

$u_{n+1} = \frac{1}{n+1}$

i) $u_n - u_{n+1} > 0$ (By Leibnitz's test)

$\Rightarrow \frac{1}{n} - \frac{1}{n+1} > 0$

ii) $\lim_{n \to \infty} \frac{1}{n} = 0$

$\therefore$ series is convergent

$\Sigma |u_n| = 1 + \frac{1}{2} + \frac{1}{3} + \dots$

$= \frac{1}{n^1}$ $(P = 1)$

$\therefore \Sigma |u_n|$ is divergent, it's conditionally convergent.

5) Given, $u_n = \frac{1}{n^2}$

$u_{n+1} = \frac{1}{(n+1)^2}$

i) $u_n - u_{n+1} > 0$ (By Leibnitz's test)

$\frac{1}{n^2} - \frac{1}{(n+1)^2} > 0$

ii) $\lim_{n \to \infty} \frac{1}{n^2} = 0$

$\therefore$ series is convergent

$\Sigma |u_n| = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$

$= \frac{1}{n^2}$ $(P = 2 > 1)$

$\therefore \Sigma |u_n|$ is convergent, it's absolutely convergent

PART-C

1) $\sum_{n=1}^{\infty} \sin(1/n)$

<u>Sol^n</u>: $u_n = \sin(1/n)$

$v_n = 1/n$

$\therefore \lim_{n\to\infty} \frac{u_n}{v_n} = \frac{\sin(1/n)}{1/n}$

since $\sum_{n=1}^{\infty} 1/n = 0$, take $(\frac{1}{n}) = a$

$\therefore \lim_{a\to 0} \frac{u_n}{v_n} = \lim_{a\to 0} \frac{\sin(1/n)}{1/n} = \lim_{a\to 0} \frac{\sin a}{a} = 1$

$\therefore \sum v_n$ diverges, $\sum u_n$ diverges,

the series $\sum_{n=1}^{\infty} \sin(1/n)$ diverges.

2) given,

$f(n) = \frac{1}{n \log n}$

$$\therefore \int_2^\infty f(x)\,dx = \int_2^\infty \frac{1 \times dx}{x \log x}$$

$$= \int_2^\infty \frac{1/x \, dx}{\log x}$$

$$= \left[\log(\log x)\right]_2^\infty$$

$$= \infty$$

By cauchy's integral test,

$\therefore f(n)$ series diverges.

3) $\frac{1}{1\cdot 2} - \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} - \dots$

Sol^n:-

$$u_n = \frac{1}{(2n-1)(2n)}$$

$$u_{n+1} = \frac{1}{(1+2n)(2n+2)}$$

i) By Leibnitz test,

$$u_n - u_{n+1} > 0$$

$$\Rightarrow \frac{1}{2n(2n-1)} - \frac{1}{(2n+1)(2n+2)} > 0$$

ii) $$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{(2n-1)(2n)} = \lim_{n\to\infty} \frac{1}{n} \times \frac{1}{(2-1/n)2n}$$

$$= 0$$

$\therefore$ series converges.