SRM Institute of Science and Technology
Ramapuram Campus.

Department of Mathematics

ASSIGNMENT QUESTIONS

Sub. Code: 18MAB101T — Date: 22.01.2021
Sub. Title: Calculus and Linear Algebra — Max. Marks: 19
Year : I Year B. Tech. (Common to all Branches) — Semester : I

Unit – 4

Part – B (5 × 2 = 10 Marks)
(Solution with Full Explanation is Needed.)

1. The radius of curvature of the curve $y = 4\sin x$ at $x = \frac{\pi}{2}$ is

   (A) $\frac{1}{2}$ (B) $\frac{-1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

2. The radius of curvature of the curve $r = e^{\theta}$ at any point on it is

   (A) $2\sqrt{2}$ (B) $\sqrt{2}\,r$ (C) 1 (D) 2

3. Envelope of the curve $y = mx + \frac{a}{m}$ (where m is the parameter) is

   (A) $x^2 + a\,y = 0$ (B) $x + 4\,a\,y = 0$
   (C) $y^2 - 4\,a\,x = 0$ (D) $y^2 + 4ax = 0$

4. The value of $\Gamma\left(-\frac{5}{2}\right)$ is ________.

   (A) $\frac{15}{8}\sqrt{\pi}$ (B) $\frac{8}{15}\sqrt{\pi}$ (C) $\frac{15}{8}\pi$ (D) $\frac{-8}{15}\sqrt{\pi}$

5. The value of $B\left(\frac{5}{2},\frac{1}{2}\right)$ is _______.

   (A) $\frac{3}{8}\pi$ (B) $\frac{5}{8}\pi$ (C) $\frac{5}{8}\sqrt{\pi}$ (D) $\frac{3}{8}\sqrt{\pi}$

Part – C (3 × 3 = 09 Marks)
(Solution with Full Explanation is Needed.)

1. Find the envelope of the family of straight lines represented by $x\cos\alpha + y\sin\alpha = a\sec\alpha$, where α is the parameter.
2. Evaluate $\int_0^1 x^6 (1-x)^9\, dx$ using Beta Gamma functions.
3. Evaluate $\int_0^{\pi/2} \sin^6\theta \cos^6\theta\, d\theta$ using Beta Gamma functions.

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22/01

Unit IV - Assignment (solution)

Part B

Q1) $y = 4\sin x$ at $x = \pi/2$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$y_1 = \frac{dy}{dx} = 4\cos x$; $y_2 = \frac{d^2y}{dx^2} = -4\sin x$

$\Rightarrow y_1(x=\pi/2) = 4\cos\pi/2 = 0$ $\Rightarrow y_2(x=\pi/2) = -4\sin\pi/2 = -4$

$$\therefore \rho = \frac{(1+0)^{3/2}}{-4} = \frac{-1}{4}$$

Q2) $r = e^{\theta}$

$r_1 = \frac{dr}{d\theta} = e^{\theta} = r$, $r_2 = \frac{d^2r}{d\theta^2} = e^{\theta} = r$

$$\rho = \frac{(r^2+r_1^2)^{3/2}}{r^2+2r_1^2+rr_2} = \frac{(r^2+r^2)^{3/2}}{r^2+2r^2+r^2} = \frac{(2r^2)^{3/2}}{4r^2}$$

$$\Rightarrow \frac{(2)^{3/2} r^{\cancel{3}1}}{4\cancel{r^2}} = \frac{r}{\sqrt{2}} = \frac{e^{\theta}}{\sqrt{2}}$$

Q3) $y = mx + \frac{a}{m}$

$\Rightarrow my = m^2x + a \Rightarrow m^2x - my + a = 0$

$\therefore A = x$, $B = -y$, $C = a$

$D = B^2 - 4AC = 0$

$\Rightarrow y^2 - 4xa = 0$

Q4) $\boxed{\Gamma(n+1) = n\Gamma n}$

$\Gamma(-\frac{5}{2}+1) = \frac{-5}{2}\Gamma_{-5/2} \Rightarrow \Gamma_{-3/2} = \frac{-5}{2}\Gamma_{-5/2} \Rightarrow \Gamma_{-5/2} = \frac{-2}{5}\Gamma_{-3/2}$ — (1)

$\Gamma(-3/2+1) = \frac{-3}{2}\Gamma_{-3/2} \Rightarrow \Gamma_{-1/2} = \frac{-3}{2}\Gamma_{-3/2} \Rightarrow \Gamma_{-3/2} = \frac{-2}{3}\Gamma_{-1/2}$ — (2)

$\Gamma(-1/2+1) = \frac{-1}{2}\Gamma_{-1/2} \Rightarrow \Gamma_{1/2} = -1/2\,\Gamma_{-1/2} \Rightarrow \Gamma_{-1/2} = -2\Gamma_{1/2}$ — (3)

Using (1), (2), (3)

$\Rightarrow \Gamma_{-5/2} = \frac{-2}{5} \times \frac{-2}{3} \times -2\sqrt{\pi} = \frac{-8\sqrt{\pi}}{15} \rightarrow$ option (D)

05) $\beta(\frac{5}{2}, \frac{1}{2})$

$= 2\int_0^{\pi/2} \sin^{2\times\frac{5}{2}-1}(\theta) \cos^{2\times\frac{1}{2}-1}(\theta)\, d\theta$

$= 2\int_0^{\pi/2} \sin^4\theta\, d\theta = 2\int_0^{\pi/2} (\sin^2\theta)^2 d\theta$

$[\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)]$

$= 2\int_0^{\pi/2} \frac{1}{4}(1-\cos 2\theta)^2 d\theta$

$= \frac{1}{2}\int_0^{\pi/2} (1+\cos^2 2\theta - 2\cos 2\theta)\, d\theta$

$[\cos^2 2\theta = \frac{1}{2}(1+\cos 4\theta)]$

$= \frac{1}{2}\int_0^{\pi/2} (1 + \frac{1}{2} + \frac{\cos 4\theta}{2} - 2\cos 2\theta)\, d\theta$

$= \frac{1}{2}\int_0^{\pi/2} (\frac{3}{2} + \frac{\cos 4\theta}{2} - 2\cos 2\theta)\, d\theta$

$= \frac{1}{2}\left[\frac{3}{2}\theta + \frac{\sin 4\theta}{8} - \frac{2\sin 2\theta}{2}\right]_0^{\pi/2}$

$= \frac{1}{2}\left[\frac{3\pi}{4} + \cancel{\frac{\sin 2\pi}{8}} - \cancel{\frac{2\sin\pi}{2}}\right]$

$= \frac{3\pi}{8}$

$\therefore$ option (A) is correct.

PART-C

01) $x\cos\alpha + y\sin\alpha = a\sec\alpha$

(Divide by $\cos\alpha$ in both sides)

$\Rightarrow x + y\tan\alpha = a\sec^2\alpha$

$\Rightarrow x + y\tan\alpha = a(1+\tan^2\alpha)$

$\Rightarrow a\tan^2\alpha - y\tan\alpha + (a-x) = 0$

$\therefore A = a,\ B = -y,\ C = (a-x)$

$B^2 - 4AC = 0 \Rightarrow \boxed{y^2 - 4a(a-x) = 0}$

02) $\int_0^1 x^6(1-x)^9\,dx$ — ①

using formula,

$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ — ②

Comparing both ① and ②

$\Rightarrow \boxed{m = 7 \text{ and } n = 10}$ ␣

03) $\int_0^{\pi/2} \sin^6\theta \cos^6\theta \, d\theta$ — ①

using formula,

$\beta(m,n) = \int_0^{\pi/2} \sin^{2m-1}(\theta) \cos^{2n-1}(\theta)\, d\theta$ — ②

Comparing both ① and ②

$\Rightarrow \boxed{m = 7/2 \text{ and } n = 7/2}$ ␣