

SRM Institute of Science and Technology
Department of Mathematics
18MAB102T-Advanced Calculus and Complex Analysis
2020-2021 Even
Unit – II: Vector Calculus
Tutorial Sheet - I

S.No	Questions	Answers
Part – A [3 Marks]		
1	If $\phi(x, y, z) = x^2y + y^2x + z^2$, find $\nabla\phi$ at the point $(1, 1, 1)$.	$3\vec{i} + 3\vec{j} + 2\vec{k}$
2	Find the directional derivative of $f(x, y, z) = x^2 - 2y^2 + 4z^2$ at the point $(1, 1, -1)$ in the direction of $2\vec{i} + \vec{j} - \vec{k}$	$\frac{8}{\sqrt{6}}$
3	Find the angle between the surfaces $x^2 + yz = 2$ and $x + 2y - z = 2$ at $(1, 1, 1)$	$\frac{\pi}{3}$
4	The temperature of points in space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located at $(1, 1, 2)$ desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move?	$\nabla T = 2\vec{i} + 2\vec{j} - \vec{k}$
5	If $\vec{F} = (x + y + 1)\vec{i} + \vec{j} - (x + y)\vec{k}$ show that $\vec{F} \cdot \text{curl } \vec{F} = 0$	$\vec{F} \cdot \text{curl } \vec{F} = 0$
Part – B [6 Marks]		
6	Find the constants a, b, c so that $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ may be irrotational. For these values of a, b, c find its scalar potential.	$a = 4, b = 2, c = -1$ $\phi = \frac{x^2}{2} - 3\frac{y^2}{2} + z^2 + 2xy + 4zx - yz + c$
7	Prove that $\text{div}(\vec{r}^n \vec{r}) = (n + 3)r^n$. Deduce that $\vec{r}^n \vec{r}$ is solenoidal if and only if $n = -3$	
8	i) If \vec{A} and \vec{B} are irrotational, Prove that $\vec{A} \times \vec{B}$ is solenoidal ii) Prove that $\vec{A} = (2x + yz)\vec{i} + (4y + zx)\vec{j} - (6z - xy)\vec{k}$ is solenoidal as well as irrotational. Also find the scalar potential of \vec{A} .	i) $\nabla \cdot (\vec{A} \times \vec{B}) = 0$ ii) $\phi = x^2 + 2y^2 - 3z^2 + xyz + a$
9	Find the angle between the normals to the surface $x^2 = yz$ at the points $(1, 1, 1)$ and $(2, 4, 1)$	$\theta = \cos^{-1}\left(\frac{13}{3\sqrt{22}}\right)$
10	Find a and b so that the surfaces $ax^2 - byz = (a + 2)x$ and $4x^2y + z^3 = 4$ cut orthogonally at $(1, -1, 2)$	$a = \frac{5}{2}, b = 1$