1) The electrical resistivity of Copper at 27°C is $1.72\times10^{-8}\,\Omega m$. Compute its thermal Conductivity if the Lorentz number is $2.26\times10^{-8}\,W\Omega K^{-2}$

Given $(\rho) = 1.72\times10^{-8}\,\Omega m$

$T = 27^\circ c \Rightarrow 273+27 = 300\,k$

$L = 2.26\times10^{-8}\,W\Omega K^{-2}$

According to Wiedemann - Franz law using classical free electron theory

$$\frac{k}{\sigma} = LT$$

(or) $k = \sigma LT$

$$k = \frac{LT}{\rho} \qquad \left[\because \sigma = \frac{1}{\rho}\right]$$

Therefore thermal Conductivity $k = \dfrac{2.26\times10^{-8}\times 300}{1.72\times10^{-8}}$

$$k = \boxed{394.18\,Wm^{-1}k^{-1}}$$

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2) Using Fermi dirac distribution function, obtain the Value of $F(E)$ for $E-E_F = 0.01\,eV$ at $200k$

Given $E-E_F = 0.01\,eV$

Temperature $= 200\,k$

Boltzmann's Constant $(k) = 1.38\times10^{-23}\,JK^{-1}$

Using Fermi dira distribution function $F(E) = \dfrac{1}{\left[e^{(E_i-E_F)/kT}\right]+1}$

Then $F(E) = \dfrac{1}{\left[e^{(0.01\times1.6\times10^{-19})/(1.38\times10^{-23}\times200)}\right]+1}$

$$F(E) = \frac{1}{e^{0.5797}+1}$$

$$F(E) = \frac{1}{1.7855+1} = \frac{1}{2.7855} = \boxed{0.359}$$

3) Free electron density of aluminium is $18.10 \times 10^{28}\ m^{-3}$. Calculate its Fermi energy at 0k. Planck's Constant and mass of free electron are 6.62×10^{-34} Js and 9.1×10^{-31} kg.

Given that Electron density of aluminium $(n) = 18.10 \times 10^{28}\ m^{-3}$

Planck's Constant $(h) = 6.62 \times 10^{-34}$ Js

Mass of electron $(m) = 9.1 \times 10^{-31}$ kg

For a given system, (E_F) at 0k is

$$E_F = \frac{h^2}{2m}\left[\frac{3n}{8\pi}\right]^{2/3}$$

$$E_F = \frac{(6.62 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31}}\left[\frac{3 \times 18.10 \times 10^{28}}{8 \times 3.14}\right]^{2/3}$$

$$\boxed{E_F = 1.869 \times 10^{-18}\ J}$$

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4) Calculate the number of states lying in an energy interval of 0.01 eV above the fermi level for a crystal of unit Volume with Fermi energy $E_F = 3.0$ eV

Given $E_F = 3.0\ eV \Rightarrow 3 \times 1.6 \times 10^{-19} = 4.8 \times 10^{-19}$ J

$\Delta E = 0.01$ eV

We know $(h) = 6.63 \times 10^{-34}$ Js & $(m) = 9.1 \times 10^{-31}$ kg

Let $\Delta E = E - E_F$

$$\Delta E + E_F = E = (3 + 0.01)\ eV = E$$

$$= (3.01 \times 1.6 \times 10^{-19})\ J = E$$

$$\Rightarrow 4.816 \times 10^{-19}\ J = E$$

Number of states per unit volume lying between E & E_F is given by

$$n = \frac{4\pi}{h^3} \times (2m)^{3/2} \times \int_{E_F}^{E} (E)^{1/2}\, dE$$

$$= \frac{4\pi}{h^3} \times (2m)^{3/2} \times \left[\frac{2}{3}(E)^{3/2}\right]_{E_F}^{E}$$

$$n = \frac{4\pi}{h^3} \times (2m)^{3/2} \times \frac{2}{3}\left[E^{3/2} - E_F^{3/2}\right]$$

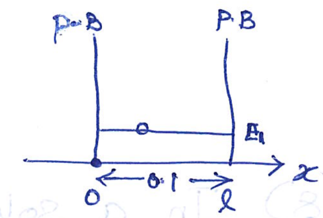
$$n = \frac{4 \times 3.14 \times [2 \times 9.1 \times 10^{-31}]^{3/2}}{(6.63 \times 10^{-34})^3} \times \frac{2}{3}\left[(4.816)^{3/2} - (4.8)^{3/2}\right] \times [10^{-19}]^{3/2}$$

$$n = 3.74 \times 10^{55} \times (1.108 \times 10^{-30})$$

$$\boxed{n = 4.14 \times 10^{25}\ m^{-3}}$$

5) Find the Lowest energy of an electron Confined in One dimensional Potential box seperated by distance 0.1 nm

Given $l = 0.1$ nm & we know $h = 6.62 \times 10^{-34}$



We know Energy of electron in 1-D Box is

$$E_n = \frac{n^2 h^2}{8 m l^2}$$

To find, Lowest energy of an electron $(n = 1)$

$$E_1 = \frac{(1)^2 \times (6.62 \times 10^{-34})^2}{8 \times (9.1 \times 10^{-31}) \times (0.1 \times 10^{-9})^2}$$

$$E_1 = \frac{4.38244 \times 10^{-67}}{7.28 \times 10^{-31} \times 10^{-8}} = \frac{4.38244 \times 10^{-67}}{7.28 \times 10^{-49}}$$

$$\boxed{E_1 = 6.0198 \times 10^{-19}\ J}$$

6) An electron is bound in One dimensional infinite well of width 1×10^{-10} m. Find the energy value in the ground state and first two excited states

Given $l = 1 \times 10^{-10}$ m & we know $h = 6.62 \times 10^{-34}$

We know $E_n = \frac{n^2 h^2}{8 m l^2}$

To find lowest energy of an electron $(n = 1)$

$$E_1 = \frac{(1)^2 \times (6.62 \times 10^{-34})^2}{8 \times (9.1 \times 10^{-31}) \times (1 \times 10^{-10})^2}$$

$$E_1 = 0.6031 \times 10^{-17}\ J$$

Energy of first excited state $= 4 \times 0.6031 \times 10^{-17}$
$= 2.412 \times 10^{-17}$ J

Energy of second excited state $= 9 \times 0.6031 \times 10^{-17}$
$\boxed{= 5.428 \times 10^{-17}\ J}$

7) Evaluate the fermi function for an energy kT above the fermi energy.

Given $E - E_F = KT$

We know $$F(E) = \frac{1}{e^{(E-E_F)/KT} + 1}$$

Therefore $$F(E) = \frac{1}{e^{(KT/KT)}+1} = \frac{1}{e^{(1)}+1} = \frac{1}{2.718+1}$$

$$F(E) = \frac{1}{3.718} = \boxed{0.268}$$

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8) In a solid, Consider the energy level lying 0.01 eV below Fermi level. what is the probability of this level not being occupied by an electron?

Given $E - E_F = 0.01\,eV$ & we know $k = 1.38 \times 10^{-23}\,JK^{-1}$

Assume $T = 300K \Rightarrow KT = 0.026\,eV$

To find probability of an energy level E not being occupied by an electron in given by

$$1 - F(E) = 1 - \frac{1}{1+e^{(E-E_F)/KT}} = \frac{1}{1+e^{(E_F-E)/KT}}$$

$$= \frac{1}{1+e^{0.01/0.026}} = \frac{1}{1+e^{0.385}} = \frac{1}{1+1.47}$$

$$= \boxed{0.405}$$

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9) Find the temperature at which there is 1% probability of a state with an energy 0.5 eV above Fermi energy.

Given $E = E_F + 0.5\,eV$; $F(E) = 1\% = 0.01$

We know $$F(E) = \frac{1}{1+e^{E-E_F/KT}}$$ & $k = 1.38 \times 10^{-23}\,JK^{-1}$

Then $$0.01 = \frac{1}{1+e^{0.5/KT}}$$

$$1 + \exp\left(\frac{0.5}{KT}\right) = \frac{1}{0.01}$$

$$\exp\left(\frac{0.5}{KT}\right) = 99$$

$$\frac{0.5}{KT} = \ln(99)$$

$$T = \frac{0.5 \times 1.6 \times 10^{-19}}{\ln(99) \times 1.38 \times 10^{-23}}$$

$$\boxed{T = 1261\,K}$$

10) The Fermi level for potassium is 2.1 eV. Calculate the velocity of the electron at the Fermi level

Given $E_F = 2.1\,eV$

We know $E_F = \frac{1}{2} m V_F^2 \Rightarrow V_F = \left(\frac{2E_F}{m}\right)^{1/2}$

Then $V_F = \left[\frac{2 \times 2.1 \times 1.602 \times 10^{-19}}{9.11 \times 10^{-31}}\right]^{1/2}$

$$\boxed{V_F = 8.6 \times 10^5\ m/s}$$

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11) Calculate the k-value for an electron in the Conduction band of GaAs having energy of 0.01 eV (measured from band edge). Compare this to the case where the electron in free space. The effective mass of electrons in GaAs is 0.067 m.

We know k-value is given by $k = \frac{\sqrt{2mE}}{\hbar}$

For GaAs appropriate mass in Conduction band is $0.067\,m_0$

This given $k = \frac{[2 \times (0.067 \times 9.1 \times 10^{-31} kg)(0.1 \times 1.6 \times 10^{-19} J)]^{1/2}}{1.05 \times 10^{-34}\ Js}$

$$\boxed{k = 4.2 \times 10^8\ m^{-1}}$$

In free space we get $k = 1.625 \times 10^9\ ms^{-1}$

The two values are quite different since the k-value in the crystal represents an effective momentum

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12) Calculate the energy of an electron and of a hole in the heavy hole band of semi conductor at a k-value of 0.1 Å. The heavy hole mass is $0.5\,m_0$

The electron energy in the valence band is $E_e = E_V - \frac{\hbar^2 k^2}{2 m_h^*}$

After using the parameter given we get

$$E_e = E_V - 1.21 \times 10^{-20}\ J = \boxed{E_V - 0.0755\ eV}$$

The hole energy is inverse of electron energy

$$E_h = \boxed{E_V + 0.0755\ eV}$$