① A rectangular box open at the top is to have volume of 32 cubic feet. Find the dimensions of the box in order that total surface area is minimum.

Sol: STEP 1

Let x, y, z be the dimensions of the rectangular box.

x = length

y = breadth

z = height

Given volume = 32

$\Rightarrow xyz = 32$ [$\because$ l×b×h]

$\Rightarrow xyz - 32 = 0$.

C.F : $g(x,y,z) = xyz - 32$.

f = Least material of construction [least T.S.A]

= $2lb + 2bh + 2hl$. [as upper side is opened].

$f \Rightarrow xy + 2yz + 2zx$.

$F = (f + \lambda g)$

$= [xy + 2yz + 2zx) + \lambda(xyz - 32)$. I.

STEP 2

$F_x = 0 = \frac{\partial}{\partial x}\left[(xy + 2yz + 2zx) + \lambda(xyz - 32)\right]$.

$= (y + 0 + 2z) + \lambda(yz)$

$\boxed{= y + 2z + \lambda yz = 0}$ — ①

$F_y = 0 = \frac{\partial}{\partial y}\left[(xy + 2yz + 2zx) + \lambda(xyz - 32)\right]$

$= (x + 2z) + \lambda(xz)$.

$$= \boxed{x + 2z + \lambda xz = 0} \quad \text{— (2)}$$

$$F_z = 0 = \frac{\partial}{\partial z}\left[(xy + 2yz + 2zx) + \lambda(xyz - 32)\right].$$

$$= (2y + 2x) + \lambda(xy)$$

$$= \boxed{2y + 2x + \lambda xy = 0} \quad \text{— (3)}$$

$$F_\lambda = 0 = \frac{\partial}{\partial \lambda}\left[(xy + 2yz + 2zx) + \lambda(xyz - 32)\right]$$

$$= \boxed{xyz - 32 = 0} \quad \text{— (4)}$$

STEP 3

eq (1) × x ⇒ $xy + 2xz + \lambda xyz$

eq (2) × y ⇒ $xy + 2yz + \lambda xyz$

$$\Rightarrow 2xz - 2yz = 0$$

$$2xz = 2yz$$

$$\boxed{x = y} \quad \text{— (5)}$$

STEP 4

eq (2) × y ⇒ $xy + 2yz + \lambda xyz$

eq (3) × z ⇒ $2yz + 2xz + \lambda xyz$

$$x - 2z + y - x$$

$$\Rightarrow x - 2z + y - x$$

$$-2z = -y$$

$$\boxed{z = y/2} \quad \text{— (6)}$$

put ⑤ & ⑥ in ④

$$xyz - 32 = 0$$

$$y \cdot y \cdot \frac{y}{2} - 32 = 0$$

$$y^3 = 64$$

$$\boxed{y = 4.}$$

as $y = x = y$ so

$$\boxed{x = 4.}$$

from ⑥

$$z = \frac{y}{2} = 4/2 \Rightarrow 2$$

$$\Rightarrow \boxed{z = 2}$$

The dimensions of the rectangular box are

$x = 4$
$y = 4$
$z = 2$.

Least T.S.A = $xy + 2yz + 2zx$

$= (4 \times 4) + 2(4 \times 2) + 2 \times (2 \times 4)$

$= 16 + 16 + 16.$

$= 48$ sqm.

Hence solved