SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics
18MAB101T – Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – I

MATRICES

Part – A

1.	The sum of the eigen values of $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ is	1 Mark	
	(a) 2 (b) 4 (c) – 3 (d) 0	Ans (a)	(CLO – 1Apply)
2.	The eigen values of A^{-1}, if $A = \begin{pmatrix} 1 & 5 & -1 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{pmatrix}$ are	1 Mark	
	(a) 2, 3, 4 (b) 2, 5, – 1 (c) 0, 0, 0 (d) $1, \frac{1}{3}, \frac{1}{4}$	Ans (d)	(CLO -1Apply)
3.	If two eigen values of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ are 3 and 15, then the third eigen value is __________.	1 Mark	
	(a) 1 (b) 0 (c) 2 (d) 3	Ans (b)	(CLO -1 Apply)
4.	If – 1, – 1, 2 are the eigen values of a matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, then the eigen values of A^T are	1 Mark	
	(a) – 1, – 1, 2 (b) 1, 1, 1/2 (c) 1,1,4 (d) -1,-1,-2	Ans (a)	(CLO - 1 Apply)

5.	The sum of eigen values of the identity matrix of order 3 is	1 Mark	
	(a) 0 (b) 1 (c) 2 (d) 3	Ans (d)	(CLO - 1 Remember)
6.	The product of the two eigen values of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 4.Then the third eigen value is	1 Mark	
	(a) 2 (b) 1 (c) 3 (d) 8	Ans (a)	(CLO - 1 Apply)
7.	The index of the canonical form $Q = -{y_1}^2 + {y_2}^2 + 4{y_3}^2$ is	1 Mark	
	a) 3 (b) 2 (c) 1 (d) 0	Ans (b)	(CLO -1 Apply)
8.	If the eigen values of the matrix of the quadratic form $2{x_1}^2 + 6{x_2}^2 + 2{x_3}^2 + 8x_1x_3$ are – 2, 6, 6, then the nature of the quadratic form is ______.	1 Mark	
	(a) positive semi-definite (b) indefinite (c) negative definite (d) positive definite	Ans (a)	(CLO - 1 Apply)
9.	The matrix corresponding to the quadratic form ${x_1}^2 + 5{x_2}^2 + {x_3}^2 + 2x_2x_3 + 6x_3x_1 + 2x_1x_2$ is	1 Mark	
	(a) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 5 & 2 \\ 3 & 2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 4 & 4 \\ 4 & 5 & 3 \\ 4 & 3 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 4 & 3 \\ 4 & 5 & 4 \\ 3 & 4 & 1 \end{pmatrix}$	Ans (a)	(CLO - 1 Apply)
10.	A homogeneous polynomial of the _________ degree in any number of variables is called a quadratic form.	1 Mark	

	(a) first (b) second (c) third (d) fourth	Ans (b)	(CLO - 1 Remember)
11.	A square matrix A is called orthogonal if	1 Mark	
	(a) $A = A^2$ (b) $A = A^{-1}$ (c) $A^T = A^{-1}$ (d) $AA^{-1} = I$	Ans (c)	(CLO - 1 Remember)
12.	The sum of the squares of the eigen values $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ is	1 Mark	
	(a) 10 (b) 38 (c) 45 (d) 20	Ans (b)	(CLO - 1 Apply)
13.	All the eigen values of a symmetric matrix with real elements are	1 Mark	
	(a) distinct (b) real (c) equal (d) conjugate complex numbers	Ans (a)	(CLO - 1 Remember)
14.	If the sum of two eigen values and trace of a 3 x 3 matrix A are equal, then the value of det (A) is	1 Mark	
	(a) 0 (b) 1 (c) -1 (d) 2	Ans (a)	(CLO - 1 Apply)
15.	If the canonical form of a quadratic form is $-{y_1}^2 + {y_2}^2 + 2{y_3}^2$, then the signature of the quadratic form is	1 Mark	
	(a) 2 (b) 1 (c) 0 (d) 3	Ans (b)	(CLO - 1Apply)
16.	Find the sum and product of the eigen values of A = $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$	1 Mark	

	(a) 5, 3 (b) 3, 5 (c) 2, 1 (d) 0, 1	Ans (b)	(CLO - 1 Apply)
17.	The eigen vectors corresponding to the distinct eigen values of a real symmetric matrix are	1 Mark	
	(a) imaginary (b) non-orthogonal (c) real (d) orthogonal	Ans (d)	(CLO - 1 Remember)
18.	The eigen values of a skew symmetric matrix are	1 Mark	
	(a) real (b) imaginary (c) unitary (d) orthogonal	Ans (b)	(CLO - 1 Remember)
19.	Find the characteristic equation of the matrix A = $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$	1 Mark	
	(a) $\lambda^2 - 7\lambda + 6 = 0$ (b) $\lambda^2 + 7\lambda + 6 = 0$ (c) $\lambda^2 - 7\lambda - 6 = 0$ (d) $\lambda^2 - 7\lambda + 5 = 0$	Ans (a)	(CLO - 1 Apply)
20.	The eigen values of an orthogonal matrix have the absolute value	1 Mark	
	(a) 0 (b) 1 (c) 2 (d) 3	Ans (b)	(CLO -1 Remember)
21.	The number of positive terms in the canonical form is called	1 Mark	
	(a) Signature (b) Index (c)quadratic (d)positive definite	Ans (b)	(CLO - 1Remember)
22.	The difference between the positive terms and negative terms in the canonical form is called	1 Mark	
	(a) Signature (b) Index (c)quadratic (d)positive definite	Ans (a)	(CLO - 1 Remember)

23.	Find the eigen values of A^2 if $A = \begin{pmatrix} 3 & 2 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$.	1 Mark	
	(a) 6, 4, 10 (b) 9, 4, 25 (c) 9, 2, 5 (d) 3, 2, 5	Ans (b)	(CLO - 1 Apply)
24.	Find the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$	1 Mark	
	(a)Positive definite (b) Negative definite (c) Positive semi-definite (d) Indefinite	Ans (b)	(CLO – 1 Apply)
25.	Find the eigen values of A^{10} if $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$	1 Mark	
	(a) $1, 3^{10}$ (b) 1,3 (c) $3^2, 1^{10}$ (d)1,10	Ans (a)	(CLO - 1 Apply)
26.	Find the eigen values of the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.	1 Mark	
	(a) 1, -3 (b) 3, 1 (c) 2, 1 (d) 1, 2	Ans (b)	(CLO - 1 Apply)
27.	If the sum of two eigen values and trace of a 3 x 3 matrix A are equal,then the value of determinant of A is	1 Mark	
	(a) 0 (b) 1 (c) -1 (d) 2	Ans (a)	(CLO - 1 Apply)
28.	Find the eigen values of the matrix $A^3 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.	1 Mark	

	(a) 1, -3 (b) 3, 1 (c) 1,9 (d) 1, -9	Ans (c)	(CLO - 1Apply)
29.	The eigen values of the matrix $A^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$	1 Mark	
	(a) 4,3 (b) 3, 1 (c) -2, 1 (d) 1, 2	Ans (a)	(CLO - 1 Apply)
30.	Find the sum and product of the eigen values of the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.	1 Mark	
	(a) 4,3 (b) 3, 1 (c) -2, 1 (d) 1, 2	Ans (a)	(CLO - 1 Apply)
31.	Find the eigen values of $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.	1 Mark	
	(a) $1, 3, -4$ (b) $1, -3, -4$ (c) $1, -3, 4$ (d) $-1, 3, -4$	Ans (a)	(CLO - 1 Apply)
32.	Two eigen values of $A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{pmatrix}$ are equal and they are double the third. Find them.	1 Mark	
	(a) 1, 2, 2 (b) 2, 1, 1 (c) 2, 0, 1 (d) 1, 2, 3	Ans (a)	(CLO - 1Apply)
33.	The eigen values of a diagonal matrix are the _______ elements of the matrix	1 Mark	

	(a) diagonal (b) upper triangular (c) zero (d) unity	Ans (a)	(CLO - 1 Remember)
34.	Cayley-Hamilton theorem states that “Every ________ matrix satisfies its own characteristic equation”.	1 Mark	
	(a) square (b) column (c) row (d) zero	Ans (a)	(CLO - 1Remember)
35.	Find rank and index of the QF whose canonical form is $3x^2 – 3y^2$.	1 Mark	
	(a) 2, 1 (b) 1, 2 (c) 0, 1 (d) 0, 2	Ans (a)	(CLO – 1 Apply)
36.	Write the Q.F. defined by the matrix $A = \begin{pmatrix} 6 & 1 & -7 \\ 1 & 2 & 0 \\ -7 & 0 & 1 \end{pmatrix}$	1 Mark	
	(a) $6x^2 + 2y^2 + z^2 + 2xy - 14xz$ (b) $6x + y^2 + 6z^2 + x\,y - 7xz$ (c) $6x^2 + 2y^2 + z^2 + 2xy + 14xz$ (b) $6x + y^2 + 6z^2 + x\,y - 14xz$	Ans (a)	(CLO -1Apply)

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Unit – I

MATRICES

Part – B

1. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.

(A) $\lambda^2 - 3\lambda + 2 = 0$ **(B)** $\lambda^2 + 3\lambda + 2 = 0$

(C) $\lambda^2 - 3\lambda - 2 = 0$ **(D)** $\lambda^2 + 3\lambda - 2 = 0$

__Solution__: Let A = $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$. Its characteristic equation is $\lambda^2 - S_1\lambda + S_2 = 0$ where $S_1 =$ $sum\ of\ the\ main\ diagonal\ elements = 1 + 2 = 3$,

$S_2 = Determinant\ of\ A = |A|$ = 1(2) – 2(0) = 2

Therefore, the characteristic equation is $\lambda^2 - 3\lambda + 2 = 0$ **(Option A)**

2. Find the characteristic equation of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(A) $\lambda^3 - 18\lambda^2 + 45\lambda = 0$ **(B)** $\lambda^3 - 28\lambda^2 + 45\lambda = 0$

(C) $\lambda^3 - 18\lambda^2 + 35\lambda = 0$ **(D)** $\lambda^3 - 18\lambda^2 - 45\lambda = 0$

__Solution__: Its characteristic equation is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$, where $S_1 =$ $sum\ of\ the\ main\ diagonal\ elements$ = 8 + 7 + 3 = 18, $S_2 =$ $Sum\ of\ the\ minors\ of\ the\ main\ diagonal\ elements = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix} = 5 + 20 + 20 = 45$, $S_3 = Determinant\ of\ A = |A|$ = 8(5)+6(-10)+2(10) = 40 – 60 + 20 = 0

Therefore, the characteristic equation is $\lambda^3 - 18\lambda^2 + 45\lambda = 0$ **(Option A)**

3. Find the eigen values of the matrix $A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$.

(A) $2, -2$ **(B) $1, -1$**

(C) $3, -3$ **(D) $2, 2$**

<u>Solution</u>: Let A = $\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$ which is a non-symmetric matrix.

The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$ where $S_1 =$ $sum\ of\ the\ main\ diagonal\ elements = 1 - 1 = 0,$

$S_2 = Determinant\ of\ A = |A|$ = 1(-1) – 1(3) = - 4

Therefore, the characteristic equation is $\lambda^2 - 4 = 0$ i.e., $\lambda^2 = 4$ or $\lambda = \pm 2$

Therefore, the eigen values are 2, -2. **(Option A)**

4. Find the sum and product of the eigen values of the matrix $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

(A) $-3, 4$ **(B) $-3, -4$**

(C) $3, 4$ **(D) $-3, -4$**

<u>Solution</u>: Sum of the eigen values = Sum of the main diagonal elements = -3

Product of the eigen values = $|A|$ = -1 (1 – 1) -1(-1 – 1) + 1(1- (-1)) = 2 + 2 = 4

(Option A)

5. Find the sum and product of eigen values of the matrix A^T where $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(A) $18, 0$ **(B) $18, 2$**

(C) $28, 0$ **(D) $18, -2$**

<u>Solution</u>: Since matrix A is symmetric , A and A^T have same eigen values.

Sum of Eigen value of A^T = trace(A) = 8+7+3=18

Product of Eigen value of $A^T = |A|$ = 8(5) + 6(−10)+2(10) = 40 −60 + 20 = 0 **(Option A)**

6. If 1, 1, 5 are the eigen values of A = $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, find the eigen values of 5A.

(A) $5, 5, 2$ **(B) $5, 5, 25$** **(C) $2, 3, 2$** **(D) $7, 8, 7$**

Solution: By the property "If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of A, then $k\lambda_1, k\lambda_2, k\lambda_3$ are the eigen values of kA, the eigen values of 5A are 5(1), 5(1), 5(5) ie., 5,5,25**.** **(Option B)**

7. Find the eigen values of the matrix $2A^{-1}$ where A= $\begin{pmatrix} 3 & 8 & 9 \\ 0 & 1 & 3 \\ 0 & 0 & -2 \end{pmatrix}$.

(A) $\frac{2}{3}, 2, -1$ **(B) $\frac{1}{3}, 2, -4$** **(C) $\frac{2}{3}, 2, 1$** **(D) $\frac{2}{3}, 1, -2$**

Solution: Since given matrix is triangular matrix, the Eigen values are its diagonal elements.

$$\therefore \quad \lambda_1 = 3,\ \lambda_2 = 1,\ \lambda_3 = -2$$

Eigen values of $2A^{-1}$ are $\frac{2}{3}$, 2 , -1 **(Option A)**

8. If the sum of two eigen values and trace of a 3 × 3 matrix A are equal, find the value of $|A|$.

(A) 5 **(B) 25** **(C) 2** **(D) 0**

Solution: Sum of the eigen values = $\lambda_1 + \lambda_2 + \lambda_3$ = sum of the diagonal elements

Given $\lambda_1 + \lambda_2$ = trace of A.

i.e., $\lambda_1 + \lambda_2 = \lambda_1 + \lambda_2 + \lambda_3$

Therefore $\lambda_3 = 0$. Then $|A|$ = Product of Eigen values = $\lambda_1 \lambda_2 \lambda_3 = 0$ **(Option D)**

9. Write the matrix corresponding to the quadratic form $x^2 + 2yz$.

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

Solution: Given $\boldsymbol{X^T A X = x^2 + 2yz}$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \textbf{ (Option C)}$$

10. If 2, 3, –1 are the eigen values of 3 × 3 matrix, find rank, index and signature of the quadratic form.

(A) $\mathbf{5, 5, 2}$ **(B)** $\mathbf{5, 5, 25}$ **(C)** $\mathbf{3, 2, 1}$ **(D)** $\mathbf{1, 2, 3}$

Solution: Rank (r) = number of non zero terms in canonical form = 3

Index (p) = Number of positive terms in canonical form = 2

Signature (s) = Difference between number of positive terms and negative terms

= 2p – r

= 4 – 3

= 1 **(Option C)**

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Unit – I

MATRICES

Part – C

1. Find the eigen values of $A = \begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$.

<u>Solution</u>:

Its characteristic equation can be written as $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$ where

$S_1 = sum\ of\ the\ main\ diagonal\ elements = 2 + 1 - 3 = 0$

$S_2 = Sum\ of\ the\ minors\ of\ the\ main\ diagonal\ elements$

$$= \begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix} + \begin{vmatrix} 2 & -7 \\ 0 & -3 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} = -5 + (-6) + (-2) = -5 - 6 - 2 = -13$$

$S_3 = Determinant\ of\ A = |A|$ = 2 (-5)-2 (-6) $-7(2)$ = $-10 + 12 - 14 = -12$

Therefore, the characteristic equation of A is $\lambda^3 - 13\lambda + 12 = 0$

3	1	0	−13	12
	0	3	9	−12
	1	3	−4	0

$(\lambda - 3)(\lambda^2 + 3\lambda - 4) = 0$

$$\Rightarrow \lambda = 3\ , \lambda = \frac{-3 \pm \sqrt{3^2 - 4(1)(-4)}}{2(1)} = \frac{-3 \pm \sqrt{25}}{2} = \frac{-3 \pm 5}{2} = \frac{-3+5}{2}, \frac{-3-5}{2} = 1, -4$$

Therefore, the eigen values are $\lambda = 3$, 1 and -4.

2. The product of two eigen values of the matrix A = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigen value.

Solution: Let the eigen values of the matrix be $\lambda_1, \lambda_2, \lambda_3$.

Given $\lambda_1\lambda_2 = 16$

We know that $\lambda_1\lambda_2\lambda_3 = |A|$ (Since product of the eigen values is equal to the determinant of the matrix)

$\lambda_1\lambda_2\lambda_3 = \begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$ = 6(9-1)+2(-6+2) +2(2-6) = 48-8-8 = 32

$Therefore, \lambda_1\lambda_2\lambda_3 = 32 \Rightarrow 16\lambda_3 = 32 \Rightarrow \lambda_3 = 2$

3. Show that the matrix $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ satisfies its own characteristic equation.

Solution: Let A $= \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$. The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$ where

$S_1 = Sum\ of\ the\ main\ diagonal\ elements = 1 + 1 = 2,$

$S_2 = |A| = 1 - (-4) = 5$

The characteristic equation is $\lambda^2 - 2\lambda + 5 = 0$

To prove $A^2 - 2A + 5I = 0$

$A^2 = A(A) = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ 4 & -3 \end{bmatrix}$

$A^2 - 2A + 5I = \begin{bmatrix} -3 & -4 \\ 4 & -3 \end{bmatrix} - \begin{bmatrix} 2 & -4 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Therefore, the given matrix satisfies its own characteristic equation.

4. If A = $\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$ write $\mathbf{A^2}$ interms of A and I, using Cayley – Hamilton theorem.

Solution: Cayley-Hamilton theorem states that every square matrix satisfies its own characteristic equation.

The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$ where

$S_1 = Sum\ of\ the\ main\ diagonal\ elements = 6$, $S_2 = |A| = 5$

Therefore, the characteristic equation is $\lambda^2 - 6\lambda + 5 = 0$

By Cayley-Hamilton theorem, $A^2 - 6A + 5I = 0$

$$\text{(i.e.) } A^2 = 6A - 5I$$

5. Determine A^4 If A = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, using Cayley – Hamilton theorem.

<u>Solution</u>: Cayley-Hamilton theorem states that every square matrix satisfies its own characteristic equation.

The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$ where

$S_1 = Sum\ of\ the\ main\ diagonal\ elements = 0$

$S_2 = |A| = -5$

Therefore, the characteristic equation is $\lambda^2 - 5 = 0$

By Cayley-Hamilton theorem, $A^2 - 5I = 0$ (i.e.) $A^2 = 5I$

$$A^2 = \begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$$

$$\text{Therefore } A^4 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}$$

6. Given A = $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, Find A^{-1} using Cayley – Hamilton theorem.

<u>Solution:</u> The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$,

Here, $S_1 = 4$ and $S_2 = -5 \Rightarrow \lambda^2 - 4\lambda - 5 = 0$.

By Cayley – Hamilton theorem $A^2 - 4A - 5I = 0$.

Multiply by A^{-1}, we get $A - 4I - 5A^{-1} = 0$ $\quad \therefore A^{-1} = \frac{1}{5}[A - 4I] = \begin{bmatrix} \frac{-3}{5} & \frac{2}{5} \\ \frac{4}{5} & \frac{-1}{5} \end{bmatrix}$

7. Determine the nature of the following quadratic form $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2$.

<u>Solution</u>: The matrix of the quadratic form is Q = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

The eigen values of the matrix are 1, 2, 0

Therefore, the quadratic form is Positive Semi-definite.

8. Discuss the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$ without reducing it to canonical form.

Solution: The matrix of the quadratic form is Q = $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

$D_1 = 2(+ve)$

$D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5(+ve)$

$D_3 = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 2(6-0) - 1(2-0) + 0 = 12 - 2 = 10(+ve)$

Therefore, the quadratic form is positive definite.

9. Find the quadratic form of the matrix A= $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 2 \\ -1 & 2 & 5 \end{bmatrix}$.

Solution: Let A = $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 2 \\ -1 & 2 & 5 \end{bmatrix}$

Quadratic form is $X^T A X$, where $X^T = (x, y, z)$

Therefore, Q.F= $(x \;\; y \;\; z)\begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 2 \\ -1 & 2 & 5 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2x^2 + 3y^2 + 5z^2 - 2zx + 4yz$

10. If the eigen vectors of a 2 × 2 matrix A are $X_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $X_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, then verify that they are mutually orthogonal. Also find normalized matrix N.

Solution: X_1 and X_2 are said to be mutually orthogonal if $X_1^T X_2 = 0$.

$$(1 \;\; -2)\begin{pmatrix} 2 \\ 1 \end{pmatrix} = 0$$

Modal matrix M=$\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$. Normalized matrix N=$\begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$

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