SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics
18MAB101T - Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – III

ORDINARY DIFFERENTIAL EQUATIONS

Part – C

1. Find the particular integral of $(D^2+3D+2)y=e^{-x}$

Ans:

$$PI=\frac{1}{D^2+3D+2}e^{-x}$$
$$=\frac{1}{1-3+2}e^{-x}$$
$$=x.\frac{1}{2D+3}e^{-x}$$
$$=x.\frac{1}{1}e^{-x}$$
$$PI=xe^{-x}$$

2. Solve $x^2\frac{d^2y}{dx^2}-7x\frac{dy}{dx}+12y=0$

Ans:

$Let\ x=e^z \Rightarrow z=\log x$

$subsitute\ \mathrm{xD}=\mathrm{D}';x^2D^2=\mathrm{D}'(\mathrm{D}'-1)$

$(\mathrm{D}'^2-8\mathrm{D}'+12)\mathrm{y}=0$

$\mathrm{m}^2-8m+12=0\Rightarrow m=2,6$

$C.F=Ae^{2z}+Be^{6z}$

$y=Ax^2+Bx^6$

3. Find the particular integral of $\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=e^x$.

Ans:

$$\text{P.I.}=\frac{1}{f(D)}F(x)=\frac{1}{f(D)}e^x \text{, putting } D=1, f(1)=0$$

$$\therefore \text{P.I}= x\frac{1}{f'(D)}e^x \qquad \because \text{P. I.}= x\frac{1}{f'(a)}e^{ax} \text{ if } f(a)=0$$

$$\Rightarrow \text{P.I.}= x\frac{1}{2D+1}e^x=\frac{1}{f'(1)}e^x \text{, } f'(1)\neq 0$$

$$\Rightarrow \text{P.I.}=\frac{xe^x}{3}$$

4. Find the particular integral of $(D^2+4)y=Cos2x$

Ans:

$$PI=\frac{1}{D^2+4}\cos 2x$$

$$=\frac{1}{-4+4}\cos 2x$$

$$=x.\frac{1}{2D}\cos 2x$$

$$=\frac{x}{4}\sin 2x$$

$$PI=\frac{x}{4}\sin 2x$$

5. Find the particular integral of $(D^2+9)y = x\cos x$

Ans:

$$\text{P.I.} = \frac{1}{f(D)}F(x) = \frac{1}{D^2+9}x\cos x$$

$$= x\frac{1}{D^2+9}\cos x + \frac{-2D}{(D^2+9)^2}\cos x$$

$$= x\frac{1}{-1+9}\cos x + \frac{-2D}{(-1+9)^2}\cos x, \qquad \text{Putting } D^2 = -1$$

$$= \frac{x\cos x}{8} - \frac{2D\cos x}{64}$$

$$= \frac{x\cos x}{8} - \frac{2D\cos x}{64}$$

$$\therefore \text{P.I.} = \frac{x\cos x}{8} + \frac{\sin x}{32}$$

6. Find the particular integral of $\frac{d^2y}{dx^2} - y = 5x - 2.$

Ans:

$$\text{P.I.} = \frac{1}{f(D)}F(x) = \frac{1}{D^2-1}(5x-2)$$

$$= \frac{1}{-(1-D^2)}(5x-2)$$

$$= -(1-D^2)^{-1}(5x-2)$$

$$= -[1+D^2+\cdots](5x-2)$$

$$= -(5x-2)$$

7. Solve $x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=0$

Ans:

$Let\ x=e^z \Rightarrow z=\log x$

$subsitute\ \mathrm{xD}=\mathrm{D}';x^2D^2=\mathrm{D}'(\mathrm{D}'-1)$

$(\mathrm{D}'^2-5\mathrm{D}'+6)\mathrm{y}=0$

$\mathrm{m}^2-5m+6=0\Rightarrow m=-2,-3$

$C.F=Ae^{-2z}+Be^{-3z}$

$y=\frac{A}{x^2}+\frac{A}{x^3}$

8. Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$.

Ans:

$(\mathrm{D}'^2-2\mathrm{D}'+1)\mathrm{y}=0$

$\mathrm{m}^2-2m+1=0\Rightarrow m=1,1$

$C.F=(A+Bz)e^z$

$y=(A+B\log x)x$

9. Solve $x\frac{dy}{dx}+2y=0$

Ans:

$(\mathrm{D}'+2)\mathrm{y}=0$

$m+2=0\Rightarrow m=-2$

$y=\frac{A}{x^2}$

10. Find the particular integral of $(x^2D^2 + xD - 1)y = \sin(\log\ x)$.

Ans

$$PI = \frac{1}{D'^2 - 1}\sin z$$
$$= \frac{\sin z}{-2}$$
$$= -\frac{\sin(\log\ x)}{2}$$

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