MATHS ASSIGNMENT - 4

1. S.T the function $f(z) = e^z$ is analytic

$z = x + iy$

$e^z = e^{x+iy} = e^x . e^{iy}$

$= e^x (\cos y + i \sin y)$

$= e^x \cos y + i e^x \sin y$

$\Rightarrow u = e^x \cos y, \; v = e^x \sin y$

$\frac{\partial u}{\partial x} = e^x \cos y, \quad \frac{\partial v}{\partial y} = e^x \cos y$

$\frac{\partial u}{\partial y} = -e^x \sin y, \quad \frac{\partial v}{\partial x} = e^x \sin y$

$\Rightarrow \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} ; \; \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$\therefore f(z) = e^z$ is analytic

2. Show that $u = 2xy + 3y$ is harmonic

$$\frac{\partial u}{\partial x} = 2y \; ; \; \frac{\partial u}{\partial y} = 2x + 3$$

$$\frac{\partial^2 u}{\partial x^2} = 0 \; ; \; \frac{\partial^2 u}{\partial y^2} = 0$$

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \Rightarrow u = 2xy + 3y \text{ is harmonic}$$

3. Find the harmonic Conjugate of $u = e^x \cos y$

we need v such that $V = u + iv$ is analytic

$$\frac{\partial u}{\partial x} = e^x \cos y, \; \frac{\partial u}{\partial y} = -e^x \sin y$$

By Cauchy remans equation: $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$$\Rightarrow \frac{\partial v}{\partial y} = e^x \cos y, \; \frac{\partial v}{\partial x} = e^x \sin y$$

Integrating on both Sides, we have.

$V = e^x \sin y + c$ is the harmonic Conjugate of

$u = e^x \cos y$

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4. Find the image of circle $|z| = 2$ under the transformation $w = 3z$

$$u + iv = 3(x + iy)$$

$$u = 3x, \quad v = 3y$$

$$x = \frac{u}{3}, \quad y = \frac{v}{3}$$

$$|z| = 3 \Rightarrow x^2 + y^2 = z^2$$

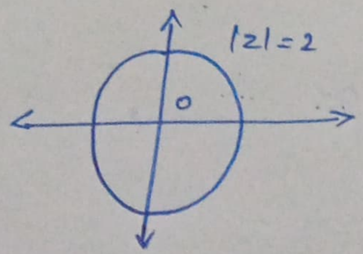
$$\left(\frac{u}{3}\right)^2 + \left(\frac{v}{3}\right)^2 = z^2$$

$$u^2 + v^2 = 4 \times 9 = 36$$

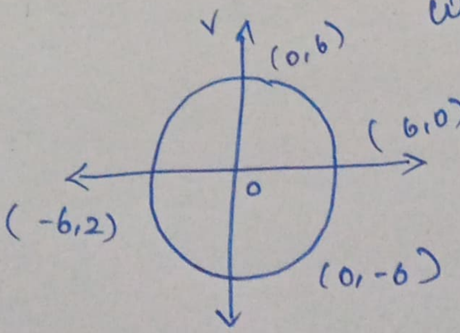
$$u^2 + v^2 = 6^2$$

It is a circle with centre as origin & radius 6 units.

z plane



w plane



5 Find the fixed or invariant points of the transformation. $w = \frac{z-1}{z+1}$

$$w = \frac{z-1}{z+1}$$

$$z = \frac{z-1}{z+1}$$

$$(z+1)(z) = z-1$$

$$z^2 + z = z - 1$$

$$z^2 = -1$$

$$z = \pm\sqrt{-1}$$

$$z = \pm i$$

$$\therefore z = i, -i$$