SRM Institute of Science and Technology

Ramapuram campus

Department of Mathematics

18MAB101T – Calculus and linear algebra

Year/Sem: I/I **Part-A** **Branch: Common to All**

Unit – IV

Differential Calculus

1.	The locus of centre of curvature is called	**1 Mark**	
	(a) Involute (b) Evolute (c) Radius of curvature (d) Envelope	Ans (b)	**(CLO-4 Remember)**
2.	The envelope of the family of curves $A\alpha^2+B\alpha+C=0$ (α is parameter) is	**1 Mark**	
	(a) $B^2+4AC=0$ (b) $B^2-4AC=0$ (c) $B^2+AC=0$ (d) $B^2-AC=0$	Ans (b)	**(CLO-4 Remember)**
3.	The curvature of the straight line is	**1 Mark**	
	(a) 1 (b) 2 (c) -1 (d) 0	Ans (d)	**(CLO-4 Remember)**
4.	Evolute of a curve is ______ of the normals of that curve	**1 Mark**	
	(a) Involute (b) Length (c) Envelope (d) End points	Ans (c)	**(CLO-4 Remember)**
5	The radius of curvature at (3,4) on the curve $x^2+y^2=25$ is	**1 Mark**	
	(a) 5 (b) 4 (c) 0 (d) 2	Ans (a)	**(CLO-4 Remember)**

6.	What is the curvature of a circle of radius 3?	**1 Mark**	
	(a) 3 (b) – 3 (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$	Ans (c)	**(CLO-4 Remember)**
7.	In an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ the radius of curvature at the end of which axis is equal to the semi-latus rectum?	**1 Mark**	
	(a) minor (b) major (c) vertical (d) horizontal	Ans (b)	**(CLO-4 Remember)**
8.	Evolute of a curve is the envelope of ______ of that curve.	**1 Mark**	
	(a) tangent (b) normal (c) parallel (d) locus	Ans (b)	**(CLO-4 Remember)**
9.	The evolute of the cycloid $x = a\,(\theta - \sin\theta)$, $y = a\,(1 - \cos\theta)$ is	**1 Mark**	
	(a) astroid (b) parabola (c) cycloid (d) circle	Ans (c)	**(CLO-4 Remember)**
10.	A curve which touches each member of a family of the curves is called $---$ of that family	**1 Mark**	
	(a) Evolute (b) Envelope (c) Circle of curvature (d) Radius of curvature	Ans (b)	**(CLO-4 Remember)**
11.	Envelope of the curve $y = mx + \frac{a}{m}$ (*where m is the parameter*) is	**1 Mark**	
	(a) $x^2 + ay = 0$ (b) $x + 4ay = 0$ (c) $y^2 - 4ax = 0$ (d) $y^2 + 4ax = 0$	Ans (c)	**(CLO-4 Remember)**

<table>
<tr><td rowspan="2">12.</td><td>If the radius of curvature and curvature of a curve at any point are ρ and k respectively, then</td><td colspan="2">1 Mark</td></tr>
<tr><td>(a) $\rho = \frac{-1}{k}$ (b) $\rho = k$
(c) $\rho = -k$ (d) $\rho = \frac{1}{k}$</td><td>Ans (d)</td><td>(CLO-4 Remember)</td></tr>
<tr><td rowspan="2">13</td><td>The radius of curvature at the point $(0, c)$ of the curve $y = c \cosh\left(\frac{x}{c}\right)$ is</td><td colspan="2">1 Mark</td></tr>
<tr><td>(a) $\rho = c$ (b) $\rho = c^2$
(c) $\rho = kc$ (d) $\rho = kc^2$</td><td>Ans (a)</td><td>(CLO-4 Remember)</td></tr>
<tr><td rowspan="2">14</td><td>The radius of curvature of the curve $y = e^x$ at $x=0$ is</td><td colspan="2">1 Mark</td></tr>
<tr><td>(a) $2\sqrt{2}$ (b) $\sqrt{2}$
(c) 2 (d) 4</td><td>Ans (a)</td><td>(CLO-4 Remember)</td></tr>
<tr><td rowspan="2">15</td><td>The radius of curvature at the point (x, y) of the curve $y = c \log \sec\left(\frac{x}{c}\right)$ is</td><td colspan="2">1 Mark</td></tr>
<tr><td>(a) $\rho = c\sec\left(\frac{x}{c}\right)$ (b) $\rho = c\cos\left(\frac{x}{c}\right)$
(c) $\rho = c\sin\left(\frac{x}{c}\right)$ (d) $\rho = c\tan\left(\frac{x}{c}\right)$</td><td>Ans (a)</td><td>(CLO-4 Remember)</td></tr>
<tr><td rowspan="2">16</td><td>The parametric form of the curve $y^2 = 4ax$ is</td><td colspan="2">1 Mark</td></tr>
<tr><td>(a) $x = at^2; y = 2at$ (b) $x = at; y = 2at$
(c) $x = at^2; y = 2at^2$ (d) $x = 2at^2; y = 2at$</td><td>Ans (a)</td><td>(CLO-4 Remember)</td></tr>
</table>

17	The envelope of the curve $y = mx + \frac{a}{m}$ where m is the parameter is	**1 Mark**	
	(a) $y^2 - 4ax = 0$ (b) $y^2 + 4ax = 0$ (c) $x^2 + y^2 = 1$ (d) $xy = c^2$	Ans (a)	**(CLO-4 Remember)**
18	The radius of curvature of the curve $y = \log \sec x$ at any point on it is	**1 Mark**	
	(a) $\sec x$ (b) $\tan x$ (c) $\cot x$ (d) $\operatorname{cosec} x$	Ans (a)	**(CLO-4 Remember)**
19	The radius of curvature of the curve $x = t^2$, $y = t$ at $t = 1$ is	**1 Mark**	
	(a) $5\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\frac{5}{2}$ (d) $\sqrt{5}$	Ans (a)	**(CLO-4 Remember)**
20	The radius of curvature of the parabola $y^2 = 12x$ at (3, 6) is	**1 Mark**	
	(a) $12\sqrt{2}$ (b) $2\sqrt{2}$ (c) $10\sqrt{2}$ (d) $\sqrt{2}$	Ans (a)	**(CLO-4 Remember)**
21	The radius of curvature of the curve $y = 4 \sin x$ at $x = \frac{\pi}{2}$ is	**1 Mark**	
	(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{-1}{3}$	Ans (c)	**(CLO-4 Remember)**
22	The envelope of family of lines $y = m\,x + a\,m^2$ (where m is the parameter) is	**1 Mark**	
	(a) $x^2 + 2ay = 0$ (b) $x^2 + 4ay = 0$ (c) $y^2 + 2ax = 0$ (d) $x^2 + 4ax = 0$	Ans (b)	**(CLO-4 Remember)**

23	The envelope of the family of lines $\frac{x}{t} + yt = 2c$, t being the parameter is	**1 Mark**	
	(a) $x^2 + y^2 = c^2$ (b) $xy = c^2$ (c) $x^2 y^2 = c^2$ (d) $x^2 - y^2 = c^2$	Ans (b)	**(CLO-4 Remember)**
24	The radius of curvature at any point on the curve $r = e^{\theta}$ is	**1 Mark**	
	(a) $\frac{\sqrt{2}}{r}$ (b) $\frac{r}{\sqrt{2}}$ (c) r (d) $\sqrt{2}\,r$	Ans (d)	**(CLO-4 Remember)**
25	The radius of curvature in Cartesian coordinates is	**1 Mark**	
	(a) $\rho = \frac{(1+y_1^2)^{\frac{3}{2}}}{y_2}$ (b) $\rho = \frac{(1-y_1^2)^{\frac{3}{2}}}{y_2}$ (c) $\rho = \frac{(1+y_1^2)^{\frac{2}{3}}}{y_2}$ (d) $\rho = \frac{(1+y_1^2)^{\frac{3}{2}}}{y_1}$	Ans (a)	**(CLO-4 Remember)**
26	The envelope of $ty - x = at^2$, t is the parameter is	**1 Mark**	
	(a) $x^2 = 4ay$ (b) $y^2 = 4ax$ (c) $x^2 + y^2 = 1$ (d) $x^2 - y^2 = 1$	Ans (b)	**(CLO-4 Remember)**
27	The radius of curvature in polar coordinates is	**1 Mark**	
	(a) $\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r^2 - rr' + 2r'^2}$ (b) $\rho = \frac{\left(r^2 - r'^2\right)^{\frac{3}{2}}}{r^2 - rr' + 2r'^2}$ (c) $\rho = \frac{\left(r^2 - r''^2\right)^{\frac{3}{2}}}{r^2 - rr' + 2r'^2}$ (d) $\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r^2 - rr'' + 2r'^2}$	Ans (d)	**(CLO-4 Remember)**

28	The radius of curvature in parametric coordinates is	**1 Mark**	
	(a) $\rho=\dfrac{\left(x'^2+y'^2\right)^{\frac{3}{2}}}{x'y''-y'x''}$ (b) $\rho=\dfrac{\left(x'^2+y'^2\right)^{\frac{3}{2}}}{x'y''+y'x''}$ (c) $\rho=\dfrac{\left(x'^2-y'^2\right)^{\frac{3}{2}}}{x'y''-y'x''}$ (d) $\rho=\dfrac{\left(x'^2-y'^2\right)^{\frac{3}{2}}}{x'y''+y'x''}$	Ans (a)	**(CLO-4 Remember)**
29	The equation of circle of curvature at any point (x, y) with center of curvature $(\bar{x},\bar{y})$ and radius of curvature ρ is	**1 Mark**	
	(a) $\left(x+\bar{x}\right)^2+\left(y+\bar{y}\right)^2=\rho^2$ (b) $\left(x-\bar{x}\right)^2+\left(y-\bar{y}\right)^2=\rho^2$ (c) $\left(x-\bar{x}\right)^2-\left(y+\bar{y}\right)^2=\rho^2$ (d) $\left(x+\bar{x}\right)^2+\left(y+\bar{y}\right)^2=\rho^2$	Ans (b)	**(CLO-4 Remember)**
30	The curvature at any point of the circle is equal to____ of its radius	**1 Mark**	
	(a) same (b) ellipse (c) reciprocal (d) constant	Ans (c)	**(CLO-4 Remember)**
31	The parametric equations of rectangular hyperbola $xy = c^2$ is	**1 Mark**	
	(a) $x=ct, y=\dfrac{c}{t}$ (b) $x=ct, y=t$ (c) $x=\dfrac{c}{t}, y=t$ (d) $x=ct, y=\dfrac{1}{t}$	Ans (a)	**(CLO-4 Remember)**
32	The value of $\Gamma\left(\dfrac{1}{2}\right)$ is	**1 Mark**	
	(a) π (b) $\dfrac{\pi}{2}$ (c) $\sqrt{\pi}$ (d) $\dfrac{\sqrt{\pi}}{2}$	Ans (c)	**(CLO-4 Remember)**

33	The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if	**1 Mark**	
	(a) p = 0 (b) p =1 (c) p > 1 (d) p < 1	Ans (c)	**(CLO-4 Remember)**
34	As per D' Alembert's ratio test, if $\sum u_n$ is a series of positive terms and $\sum_{n\to\infty} \frac{u_{n+1}}{u_n} = l$, then the series is convergent, if	**1 Mark**	
	(a) l = 0 (b) l =1 (c) l > 1 (d) l < 1	Ans (d)	**(CLO-4 Remember)**
35	If n is a positive integer, then $\Gamma(n+1) =$	**1 Mark**	
	(a) $(n + 1)!$ (b) $n!$ (c) $2n!$ (d) $(n - 1)!$	Ans (b)	**(CLO-4 Remember)**
36	The series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ...$ is	**1 Mark**	
	(a) convergent (b) divergent (c) oscillating (d) monotonic	Ans (b)	**(CLO-4 Remember**

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Unit – IV

DIFFERENTIAL CALCULUS

Part – B

1. Envelope of the curve $y = mx + \frac{a}{m}$ (where m is the parameter) is

(A) $x^2 + a\,y = 0$ (B) $x + 4\,a\,y = 0$

(C) $y^2 - 4\,a\,x = 0$ (D) $y^2 + 4ax = 0$

Solution: Given: $y = mx + \frac{a}{m}$

$$y = \frac{m^2 x + a}{m}$$

$$m^2 x - y\,m + \mathrm{a} = 0$$

The above equation is a quadratic equation in 'm'.

The discriminant is $b^2 - 4ac = 0$.

The envelope of the curve is $y^2 - 4\,a\,x = 0$. **(Option C)**

2. The radius of curvature of the curve $y = e^x$ at $x = 0$ is

(A) $2\sqrt{2}$ (B) $\sqrt{2}$
(C) 2 (D) 4

Solution:

$y_1 = e^x$ at $x = 0$ is 1

$y_2 = e^x$ at $x = 0$ is 1

$$\rho = \frac{\left(1 + y_1^{\,2}\right)^{\frac{3}{2}}}{y_2}$$

$\rho = 2\sqrt{2}$ **(Option A)**

3. The radius of curvature of the curve $y = 4\sin x$ at $x = \frac{\pi}{2}$ is

(A) $\frac{1}{2}$ (B) $\frac{-1}{2}$

(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Solution:

$y_1 = 4\cos x \ at\ x = \frac{\pi}{2} is\ 0$

$y_2 = -4\sin x\, at\ x = \frac{\pi}{2} is - 4$

$$\rho = \frac{\left(1 + y_1^{\,2}\right)^{\frac{3}{2}}}{y_2}$$

$|\rho| = \frac{1}{4}$ **(Option C)**

4. The envelope of family of lines y = $mx + am^2$ (where *m* is the parameter) is

(A) $x^2 + 2ay = 0$ (B) $x^2 + 4\,a\,y = 0$

(C) $y^2 + 2ax = 0$ (D) $x^2 + 4\,a\,x = 0$

Solution:

The given equation is quadratic in 'm'.

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $x^2 + 4ay = 0$. **(Option B)**

5. The envelope of the family of lines $\frac{x}{t}$ + $y\,t$ = 2c, t being the parameter is

(A) $x^2 + y^2 = c^2$ (B) $x\,y = c^2$

(C) $x^2\,y^2 = c^2$ (D) $x^2 - y^2 = c^2$

Solution:

Simplifying the equation $\frac{x}{t} + y\,t = 2c$, we get $yt^2 - 2ct + x = 0$

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $x\,y = c^2$. **(Option B)**

6. The radius of curvature of the curve $r = e^{\theta}$ at any point on it is

(a) $2\sqrt{2}$ (b) $\sqrt{2}\,r$

(c) 2 (d) 4

Solution:

$r' = e^{\theta}$

$r'' = e^{\theta}$

$$\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r^2 - rr'' + 2r'^2}$$

$\rho = \sqrt{2}\,r$

7. The radius of curvature at the point (3, 4) on the curve $x^2 + y^2 = 25$ is

(A) 5 (B) 4 (C) 0 (D) 2

Solution:

We know that the radius of curvature of the circle is equal to its radius.

$\rho = 5$ **(Option A)**

8. B (5/2, 1/2) = _______.

(A) 1 (B) 4

(C) 3π/8 (D) π

Solution:

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$B(5/2,1/2) = \frac{\Gamma(5/2)\Gamma(1/2)}{\Gamma(3)} = \frac{3\pi}{8}$$ **(Option C)**

9. $\Gamma(-5/2)$ **=____________.**

(A) 1 (B) 4

(C) 1/2 (D) $\frac{-8\sqrt{\pi}}{15}$

Solution:

$$\Gamma\left(-n+\frac{1}{2}\right) = \frac{(-2)^n}{1.3.5\cdots(2n-1)}\sqrt{\pi}$$

$$\Gamma\left(-\frac{5}{2}\right) = \Gamma\left(\frac{-6+1}{2}\right)$$

$$= \Gamma\left(-3+\frac{1}{2}\right) = \frac{-8}{15}\sqrt{\pi}$$

(Option D)

10. Evaluate $\int_0^\infty e^{-x} x^4 \, dx$.

(A) 1 (B) 24

(C) 1/2 (D) $\frac{-8\sqrt{\pi}}{3}$

Solution

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} \, dx$$

$$\int_0^\infty e^{-x} x^4 \, dx = \int_0^\infty e^{-x} x^{5-1} \, dx = \Gamma(5) = 4! = 24$$

(Option B)

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DIFFERENTIAL CALCULUS

Part – C

1. Find the radius of curvature of the curve $y^2 = 12x$ at the point (3, 6).

Solution:

$$\frac{dy}{dx} = \frac{6}{y}$$

At (3, 6), $\frac{dy}{dx} = 1$

$$\frac{d^2y}{dx^2} = 6\left(\frac{-1}{y^2}\right)\frac{dy}{dx}$$

At (3, 6), $\frac{d^2y}{dx^2} = 6\left(\frac{-1}{36}\right)1 = \frac{-1}{6}$

$$\rho = \frac{\left(1 + y_1^{\,2}\right)^{\frac{3}{2}}}{y_2}$$

$$\rho = \frac{(1+1)^{\frac{3}{2}}}{-1/6} = -12\sqrt{2}$$

2. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point (a/4, a/4).

Solution:

$$\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$$

At (a/4, a/4), $\frac{dy}{dx} = -1$

$$\frac{d^2y}{dx^2} = -\left[\frac{\sqrt{x}.\frac{1}{2}y^{-1/2}\frac{dy}{dx} - \sqrt{y}.\frac{1}{2}x^{-1/2}}{x}\right]$$

At (a/4, a/4), $\frac{d^2y}{dx^2} = \frac{4}{a}$

$$\rho = \frac{\left(1+y_1^{\,2}\right)^{\frac{3}{2}}}{y_2}$$

$$\rho = \frac{a}{\sqrt{2}}$$

3. Find the radius of curvature of the curve $xy = c^2$ at the point (c, c).
Solution:

$$x.\frac{dy}{dx} + y.1 = 0$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

At (c, c), $\frac{dy}{dx} = -1$

$$\frac{d^2y}{dx^2} = -\left[\frac{x\frac{dy}{dx} - y.1}{x^2}\right]$$

At (c, c), $\frac{d^2y}{dx^2} = -\left(\frac{-2c}{c^2}\right) = \frac{2}{c}$

$$\rho = \frac{\left(1+{y_1}^2\right)^{\frac{3}{2}}}{y_2}$$

$$\rho = \frac{(1+1)^{\frac{3}{2}}}{2/c} = \sqrt{2}\,c$$

4. If $\boldsymbol{x = a\cos}\theta$, $y = \boldsymbol{b\sin}\,\theta$, then find $\dfrac{dy}{dx}$.

Solution:

$$\frac{dx}{d\theta} = -a\sin\theta,\ \frac{dy}{d\theta} = -a\cos\theta,\ \frac{dy}{dx} = -\frac{b}{a}\cot\theta$$

5. Find the envelope of x cos θ + y sin θ = 1, θ being the parameter.

Solution:

$x\cos\theta + y\sin\theta = 1$ ____________________(1)

Differentiate partially w.r.t. θ.

$x(-\sin\theta) + y(\cos\theta) = 0$ ____________________(2)

Squaring and adding (1) and (2)

$x^2 + y^2 = 1$

6. Find the envelope of x cos α + y sin α = a sec α, α being the parameter.

Solution:

$x\cos\alpha + y\sin\alpha = a\sec\alpha$

Divide by cos α.

$x + y\tan\alpha = a\sec^2\alpha$

$x + y\tan\alpha = a(1+\tan^2\alpha)$

$a\tan^2\alpha - y\tan\alpha + (a - x) = 0$

Here A = a, B = $-y$, C = a $-$ x

Envelope is given by $B^2 - 4AC = 0$.

$$y^2 = 4a\,(a - x)$$

7. Find $\int_0^1 x^6 (1-x)^9\, dx$**.**

Solution:

$m = 7$, $n = 10$

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\, dx$$

$$= \int_0^1 x^{7-1}(1-x)^{10-1}\, dx$$

$$= \frac{\Gamma(7)\Gamma(10)}{\Gamma(17)} = \frac{6!9!}{16!}$$

8. Prove that $\dfrac{B(m+1,n)}{B(m,n+1)} = \dfrac{m}{n}$**.**

Solution:

$$\frac{B(m+1,n)}{B(m,n+1)} = \frac{\dfrac{\Gamma(m+1)\Gamma(n)}{\Gamma(m+n+1)}}{\dfrac{\Gamma(m)\Gamma(n+1)}{\Gamma(m+n+1)}} = \frac{m\Gamma(m)\Gamma(n)}{n\Gamma(m)\Gamma(n)} = \frac{m}{n}$$

9. Find $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$**.**

Solution:

$$\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \int_0^{\pi/2} \sqrt{\frac{\sin\theta}{\cos\theta}}\, d\theta = \int_0^{\pi/2} \sin^{1/2}\theta \cos^{-1/2}\theta\, d\theta$$

$$= \frac{1}{2} B\left(\frac{3/2}{2}, \frac{1/2}{2}\right)$$

$$= \frac{1}{2} B\left(\frac{3}{4}, \frac{1}{4}\right)$$

$$= \frac{1}{2}\frac{\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{4}\right)}{\Gamma(1)}$$

Formula $\Gamma(n)\Gamma(1-n) = \dfrac{\pi}{\sin n\pi}$

$$= \frac{\pi}{\sqrt{2}}$$

10. Find $\int_0^{\pi/2} \sin^6\theta \cos^6\theta \, d\theta$ **.**

Solution: m = 6, n = 6

$$\int_0^{\pi/2} \sin^6\theta \cos^6\theta \, d\theta = \frac{1}{2} B\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$$

$$= \frac{1}{2} B\left(\frac{7}{2}, \frac{7}{2}\right)$$

$$= \frac{1}{2}\frac{\Gamma\left(\frac{7}{2}\right)\Gamma\left(\frac{7}{2}\right)}{\Gamma(7)}$$

$$= \frac{1}{2}\frac{\left(\frac{15}{8}\sqrt{\pi}\right)\left(\frac{15}{8}\sqrt{\pi}\right)}{6!}$$

11. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

Solution:

$$B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \, \cos^{2n-1}\theta \, d\theta$$

Put m = ½, n = ½ .

$$B\left(\frac{1}{2}, \frac{1}{2}\right) = 2\int_0^{\pi/2} d\theta$$

$$\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma(1)} = 2.\frac{\pi}{2}$$

$$\left(\Gamma\left(\frac{1}{2}\right)\right)^2 = \pi$$

Hence $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

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