UNIT- 1

ELECTRICAL CIRCUITS

BASICS OF ELECTRICAL ENGINEERING

CHARGE :

* Charge is the most fundamental quantity of electricity. It is the physical property of the matter that experience a force when subjected to electric field.

* The unit of charge is Coulomb (C)

* The symbol is given as (Q)

* One Coulomb of charge is 6.28×10^{18} electrons

* Charge can be of two types,

Positive charge → If protons are more Compare to electrons

Negative charge → If electrons are more Compare to protons.

* One single electron posses a charge of

1.602×10^{-19} C

VOLTAGE

The force to push a charge in a conductor is called Voltage (or) Electric potential.

$$\text{Electric potential, } V = \frac{\text{Work done}}{\text{charge}} = \frac{W}{Q} = \frac{J}{C}$$

$$\boxed{\text{ONE VOLT} = 1\ J/C}$$

CURRENT

* Rate of change of charge is called current

i.e $I = \frac{dQ}{dt}$

* Its unit is called Amps

* $\boxed{\text{ONE AMP} = 1\ C/s}$

POWER:

* Power is the rate at which energy is expended.

* If one joule of energy is expended in transferring one Coulomb of charge through the device in one second, then the rate of energy transfer is one Watt.

$\boxed{P = Vi}$

ENERGY

$\boxed{E = \text{Power} \times \text{time}}$

* The unit of power is "Watt"

ACTIVE AND PASSIVE ELEMENT

* Active elements are the sources of the energy.

* **Eg:** Voltage source
  Current source
  Generator
  Transistor.

* Passive elements are either energy storage or energy dissipative elements.

  Eg: Resistor
  Inductor
  Capacitor.

OHM'S LAW

* Ohm's law gives the relationship between Voltage and current

* According to ohm's law Voltage ~~across~~ a across a resistor is directly proportional to the current through the element.

i.e $\left\{ \begin{array}{l} V \propto I \\ V = IR \end{array} \right\} \rightarrow$ Under Constant temperature.

VOLTAGE CURRENT RELATION FOR RESISTOR, INDUCTOR AND CAPACITOR.

RESISTOR

* A resistor is an energy dissipative element
* It blocks the current flowing through it.
* It is given by

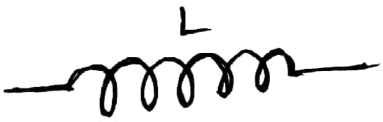
$$\left\{ R = \frac{\rho l}{A} \right\}$$

$\rho \rightarrow$ Resistivity
$l \rightarrow$ length of the element
$A \rightarrow$ Cross sectional area.

* Voltage across the resistor,

$$\left\{ \begin{aligned} V &= iR \\ i &= \frac{V}{R} \end{aligned} \right\}$$

INDUCTOR

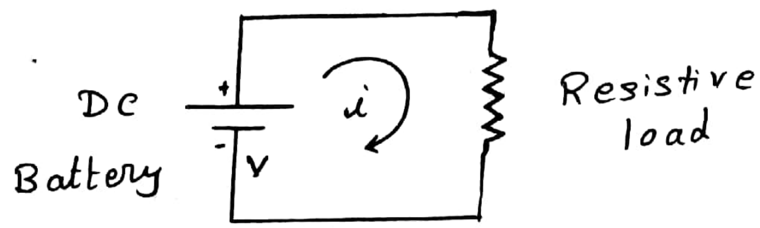


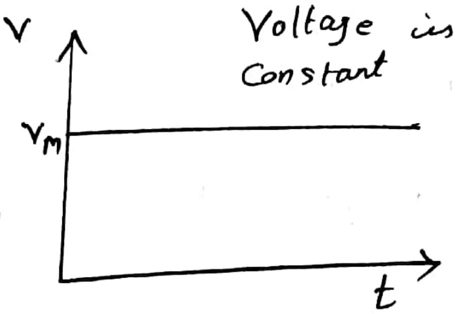
* Inductor is a coil of wire which stores the energy when there is change in current in the form of magnetic field.

* $\left\{ V = L\dfrac{di}{dt} \; ; \; i = \dfrac{1}{L}\int V\,dt \right\}$

DC CIRCUIT :

* DC means Direct Current
* The source for direct current is Battery.
* In DC circuit the flow of electrons will be unidirectional.

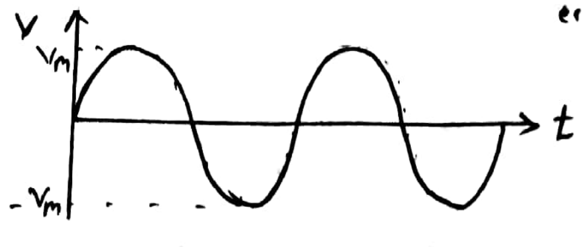




According to Ohms law, $V = iR$ under constant temperature.

AC CIRCUIT

* AC means Alternating Current.
* The source for Alternating current is AC generator.
* In AC circuit the flow of electrons will get alternated for every half cycle
* Since, the conductor is subjected to North pole and south pole alternatively the flow of electrons also gets alternated.



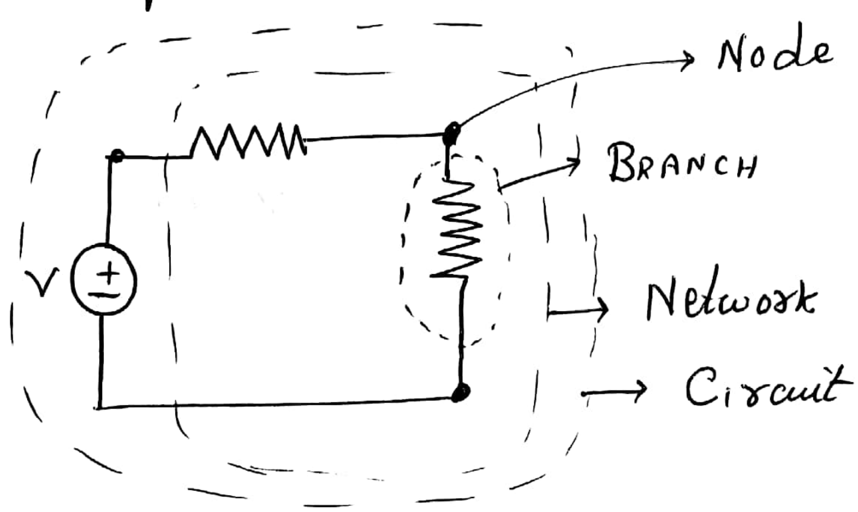
"Voltage is alternating"

NETWORKS AND CIRCUITS

* Interconnection of two or more simple circuit elements is called an electric network
* A network that contains atleast one closed path is called an electric circuit

BRANCH

* All elements of the network is called branch



NODE

* The meeting point of one or two Branches are Called node

LOOP:

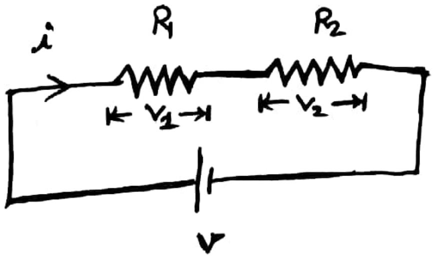
* A closed Path in a circuit is called loop.

MESH:

* A mesh is a loop that does not Contain any other loops

Voltage Division Rule

$$i = \frac{V_1}{R_1} = \frac{V_2}{R_2}$$



$$V_1 = \frac{R_1 V_2}{R_2} \quad ; \quad V_2 = \frac{R_2 V_1}{R_1}$$

In Series circuit

$$V = V_1 + V_2$$

$$V = \frac{R_1 V_2}{R_2} + V_2$$

$$V = \frac{R_1 V_2 + R_2 V_2}{R_2}$$

$$V = V_2 \frac{R_1 + R_2}{R_2}$$

$$\left\{ V_2 = \frac{V R_2}{R_1 + R_2} \right\}$$

To find V_1

$$V = V_1 + V_2$$

$$V = V_1 + \frac{R_2 V_1}{R_1}$$

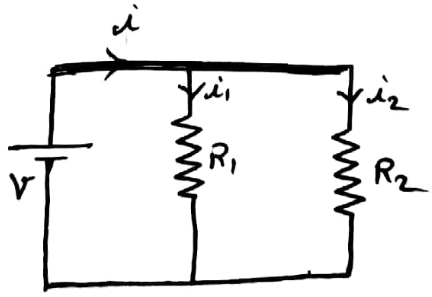
$$V = \frac{R_1 V_1 + R_2 V_1}{R_1}$$

$$V = \frac{V_1 (R_1 + R_2)}{R_1}$$

$$\left\{ V_1 = \frac{V R_1}{R_1 + R_2} \right\}$$

Current Division Rule

$$V = i_1 R_1 = i_2 R_2$$



$$i_1 = i_2 \frac{R_2}{R_1} \quad ; \quad i_2 = \frac{i_1 R_1}{R_2}$$

In parallel circuit

$$i = i_1 + i_2$$

$$i = i_2 \frac{R_2}{R_1} + i_2$$

$$i = \frac{i_2 R_2 + i_2 R_1}{R_1} = \frac{i_2 (R_1 + R_2)}{R_1}$$

$$\left\{ i_2 = \frac{i R_1}{R_1 + R_2} \right\}$$

To find i_1

$$i = i_1 + i_2$$

$$i = i_1 + \frac{i_1 R_1}{R_2}$$

$$i = \frac{i_1 R_2 + i_1 R_1}{R_2} = \frac{i_1 (R_1 + R_2)}{R_2}$$

$$\left\{ i_1 = \frac{i R_2}{R_1 + R_2} \right\}$$

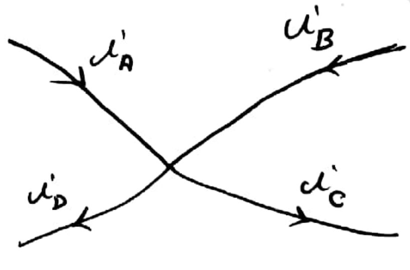
CAPACITOR

* Capacitor is a passive element which stores energy when there is change is voltage.
* It stores energy is electric field.

$$\left\{ V = \frac{1}{C}\int i\, dt \quad ; \quad i = C\frac{dv}{dt} \right\}$$

KIRCHHOFF'S CURRENT LAW :

The algebraic sum of the currents ~~entering~~ at any node is zero

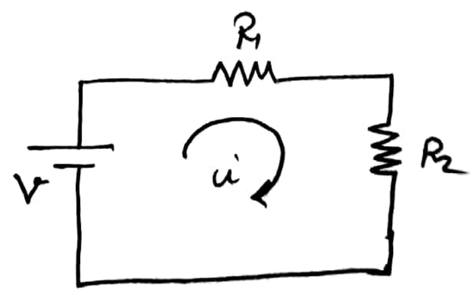


$$i_A + i_B + (-i_C) + (-i_D) = 0$$

$$\boxed{i_A + i_B = i_C + i_D}$$

KIRCHOFF'S VOLTAGE LAW :

The algebraic sum of the voltage around any closed path is zero.



$$V = iR_1 + iR_2$$

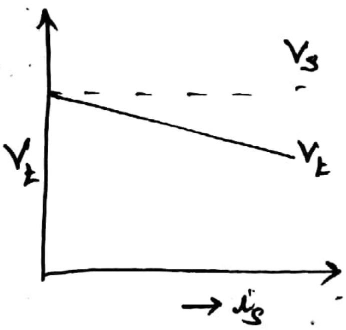
$$\boxed{-V + iR_1 + iR_2 = 0}$$

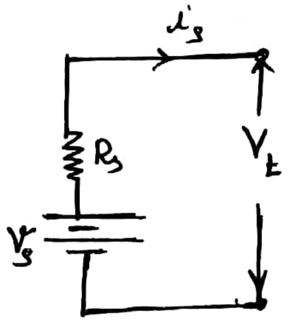
IDEAL SOURCE

In general source can be two types

* Voltage Source
* Current Source

VOLTAGE Source

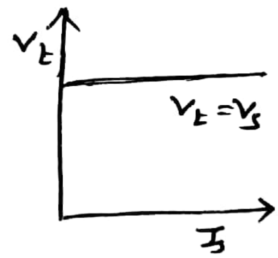




* practical voltage source Consists of internal resistance in series with 'V'
* V_t changes due to change in load current

IDEAL VOLTAGE SOURCE

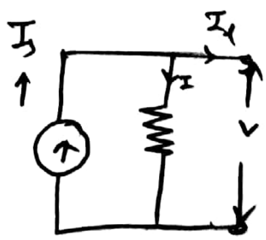
* In ideal Voltage source, there won't be any internal resistor present in Voltage source V_s

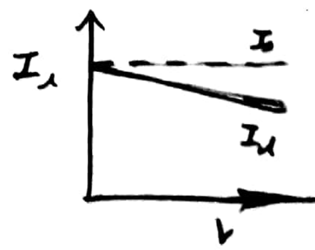


* Here terminal Voltage is equal to source voltage even though load current changes.

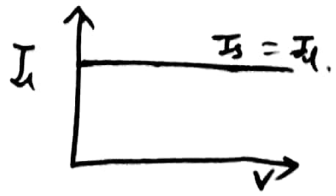
CURRENT SOURCE

* In practical current source, resistance will be parallel to source.



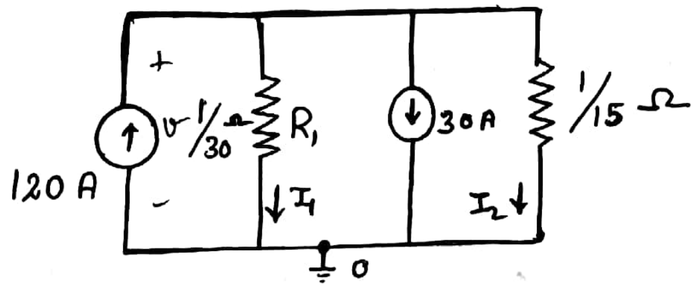


* In Ideal Current source there won't be any parallel resistor across the source
* Hence $\left[I_s = I_L\right]$



UNIV QUES

Find the Current I_1 and Power absorbed by resistance R_1 in circuit shown below



SOLUTION :

Apply KCL to the Reference node '0'

$$\therefore \quad I_1 + 30 + I_2 = 120$$

$$\frac{v}{1/30} + 30 + \frac{v}{1/15} = 120$$

$$30v + 30 + 15v = 120$$

$$45V = +90$$

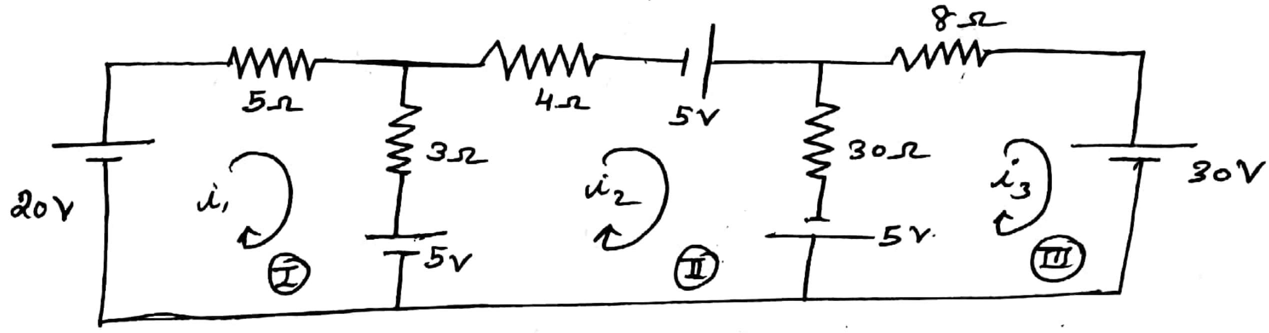
$$\boxed{v = 2\ V}$$

$$I_1 = 2/(1/30) = \underline{\underline{60\ A}}$$

$$I_2 = 2\left(\frac{1}{15}\right) = \underline{\underline{30\ A}}$$

MESH ANALYSIS

Find the mesh currents for the following circuit



SOLUTION

MESH 1

$$20-5 = 5I_1 + 3(I_1 - I_2)$$

$$\{8I_1 - 3I_2 = 15\} \quad — \quad \text{I}$$

MESH II

$$5+5+5 = 4I_2 + 30(I_2 - I_3) + 3(I_2 - I_1)$$

$$15 = 4I_2 + 30I_2 - 30I_3 + 3I_2 - 3I_1$$

$$\{-3I_1 + 37I_2 - 30I_3 = 15\} \quad — \quad \text{II}$$

MESH III

$$-30 - 5 = 30(I_3 - I_2) + 8I_3$$

$$\{-30I_2 + 38I_3 = -35\} \quad — \quad \text{III}$$

Solving eq I, II, III by cramers rule.

$$I_1 = \frac{\Delta_1}{\Delta}; \quad I_2 = \frac{\Delta_2}{\Delta}; \quad I_3 = \frac{\Delta_3}{\Delta}$$

$$\Delta = \begin{vmatrix} 8 & -3 & 0 \\ -3 & 37 & -30 \\ 0 & -30 & 38 \end{vmatrix}$$

$$\boxed{\Delta = 3706}$$

$$\Delta_1 = \begin{vmatrix} 15 & -3 & 0 \\ 15 & 37 & -30 \\ -35 & -30 & 38 \end{vmatrix}; \quad \Delta_1 = 6,150$$

$$\Delta_2 = \begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -30 \\ 0 & -35 & 38 \end{vmatrix}; \quad \Delta_2 = -2,130$$

$$\Delta_3 = \begin{vmatrix} 8 & -3 & 15 \\ -3 & 37 & 15 \\ 0 & -30 & -35 \end{vmatrix}; \quad \Delta_3 = -5,095$$

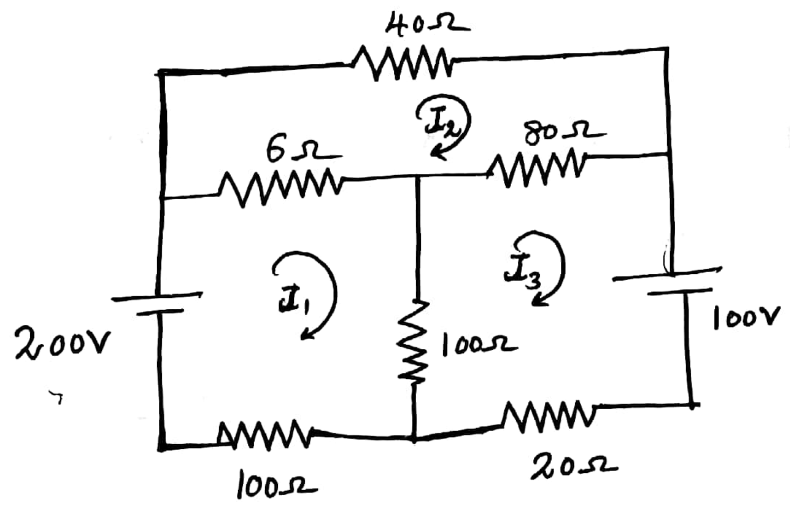
$$I_1 = \frac{6150}{3706} = \underline{\underline{1.66\ A}}$$

$$I_2 = \frac{-2130}{3706} = \underline{\underline{-0.57\ A}}$$

$$I_3 = \frac{-5,095}{3706} = \underline{\underline{-1.37\ A}}$$

MESH ANALYSIS PROBLEMS

For the given circuit find the mesh current and also calculate the power dissipated in 40Ω resistor.



SOLUTION

MESH I

$$200 = 6(I_1 - I_2) + 100(I_1 - I_3) + 100 I_1$$

$$200 = 6I_1 - 6I_2 + 100I_1 - 100I_3 + 100I_1$$

$$206 I_1 - 6 I_2 - 100 I_3 = 200 \quad - \text{(I)}$$

MESH II

$$40 I_2 + 80(I_2 - I_3) + 6(I_2 - I_1) = 0$$

$$40 I_2 + 80 I_2 - 80 I_3 + 6 I_2 - 6 I_1 = 0$$

$$-6 I_1 + 126 I_2 - 80 I_3 = 0 \quad - \text{(II)}$$

MESH III

$$-100 = 20\,I_3 + 100\,(I_3 - I_1) + 80\,(I_3 - I_2)$$

$$-100 = 20\,I_3 + 100\,I_3 - 100\,I_1 + 80\,I_3 - 80\,I_2$$

$$-100\,I_1 - 80\,I_2 + 200\,I_3 = -100$$

Finding roots using Cramers rule.

$$\Delta = \begin{vmatrix} 206 & -6 & -100 \\ -6 & 126 & -80 \\ -100 & -80 & 200 \end{vmatrix} \qquad \Delta = 2509600$$

$$\Delta_1 = \begin{vmatrix} 200 & -6 & -100 \\ 0 & 126 & -80 \\ -100 & -80 & 200 \end{vmatrix} \qquad \Delta_1 = 2452000$$

$$\Delta_2 = \begin{vmatrix} 206 & 200 & -100 \\ -6 & 0 & -80 \\ -100 & -100 & 200 \end{vmatrix} \qquad \Delta_2 = 132000$$

$$\Delta_3 = \begin{vmatrix} 206 & -6 & +200 \\ -6 & 126 & 0 \\ -100 & -80 & \text{[illegible]}00 \end{vmatrix} \qquad \Delta_3 = 24000$$

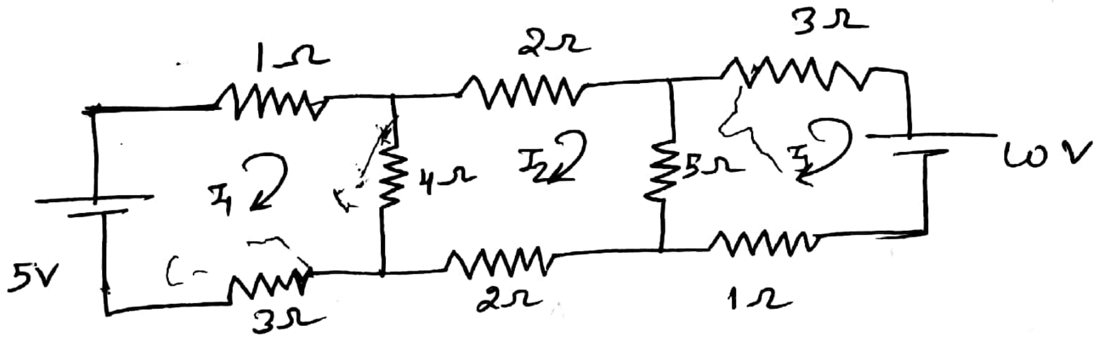
$$I_1 = \frac{\Delta_1}{\Delta} = \underline{0.977\,A}\ ; \quad I_2 = \frac{\Delta_2}{\Delta} = \underline{0.526\,A}\ ; \quad I_3 = \frac{\Delta_3}{\Delta} = \underline{9.563\,mA}$$

$$P_{40} = I_2^2\,R_{40} = (0.526)^2\,(40)$$

$$\boxed{P_{40} = 0.11067}\ W$$

PROBLEM

Find current in meshes for the following circuit shown below.



SOLUTION

$$5 = I_1 + 4(I_1 - I_2) + 3I_1$$

$$5 = I_1 + 4I_1 - 4I_2 + 3I_1$$

$$8I_1 - 4I_2 = 5 \quad - \quad \text{(I)}$$

$$2I_2 + 5(I_2 - I_3) + 2I_2 + 4I_2 - 4I_1 = 0$$

$$-4I_1 + 13I_2 - 5I_3 = 0$$

$$4I_1 - 13I_2 + 5I_3 = 0 \quad - \quad \text{(II)}$$

$$3I_3 + I_3 + 5(I_3 - I_2) = -10$$

$$3I_3 + I_3 + 5I_3 - 5I_2 = -10$$

$$-5I_2 + 9I_3 = -10$$

$$5I_2 - 9I_3 = 10 \quad - \quad \text{(III)}$$

$$\Delta = \begin{vmatrix} 8 & -4 & 0 \\ 4 & -13 & 5 \\ 0 & 5 & -9 \end{vmatrix} \qquad \Delta = 592$$

$$\Delta_1 = \begin{vmatrix} 5 & -4 & 0 \\ 0 & -13 & 5 \\ 10 & 5 & -9 \end{vmatrix} \qquad \Delta_1 = 260$$

$$\Delta_2 = \begin{vmatrix} 8 & 5 & 0 \\ 4 & 0 & 5 \\ 0 & 10 & -9 \end{vmatrix} \qquad \Delta_2 = -220$$

$$\Delta_3 = \begin{vmatrix} 8 & -4 & 5 \\ 4 & -13 & 0 \\ 0 & 5 & 10 \end{vmatrix} \qquad \Delta_3 = -780$$

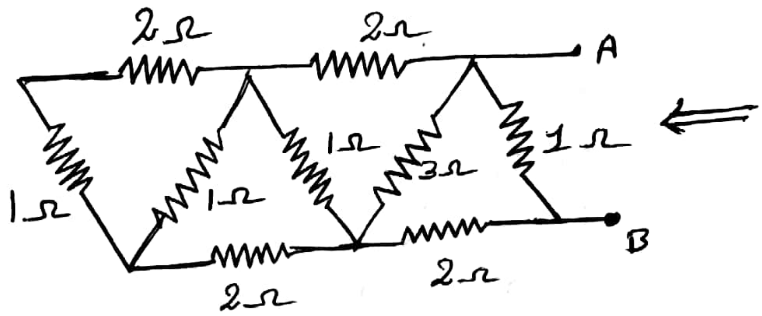
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{260}{592} = \underline{0.4392 \text{ A}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{592} = \underline{-0.3716 \text{ A}}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-780}{592} = -\underline{1.318 \text{ A}}$$

UNIV QUESTION

Determine the resistance between the terminals A & B



SOLUTION

$$(1+2) \parallel 1 = 3 \parallel 1 = \frac{3 \times 1}{3+1} = \frac{3}{4} = 0.75\Omega$$

$$(0.75+2) \parallel 1 = 2.75 \parallel 1 = \frac{2.75 \times 1}{2.75+1} = \frac{2.75}{3.75} = 0.73\Omega$$

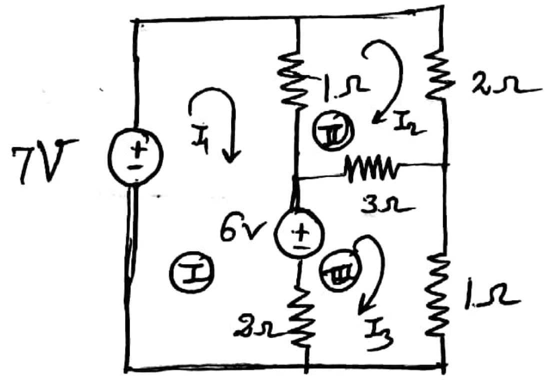
$$(0.73+2) \parallel 3 = 2.73 \parallel 3 = \frac{2.73 \times 3}{2.73+3} = \frac{8.19}{5.73} = 1.43\Omega$$

$$(1.43+2) \parallel 1 = 3.43 \parallel 1 = \frac{3.43 \times 1}{3.43+1} = \frac{3.43}{4.43} \quad 0.77\Omega$$

$$\{ R_{eq} = 0.77\Omega \}$$

UNIV QUESTION

Using mesh analysis to determine the three mesh currents in the circuit shown.



SOLUTION

loop I

$$7 - 6 = (I_1 - I_2) + 2(I_1 - I_3)$$

$$3I_1 - I_2 - 2I_3 = 1 \quad \text{— (I)}$$

Loop II

$$0 = (I_2 - I_1) + 2I_2 + 3(I_2 - I_3)$$

$$-I_1 + 6I_2 - 3I_3 = 0 \quad \text{— (II)}$$

Loop III

$$6 = 3(I_3 - I_2) + I_3 + 2(I_3 - I_1)$$ 117

$$-2I_1 - 3I_2 + 6I_3 = 6$$

$$\left\{ \begin{aligned} i_1 &= 3\ \text{A} \\ i_2 &= 2\ \text{A} \\ i_3 &= 3\ \text{A} \end{aligned} \right\}$$

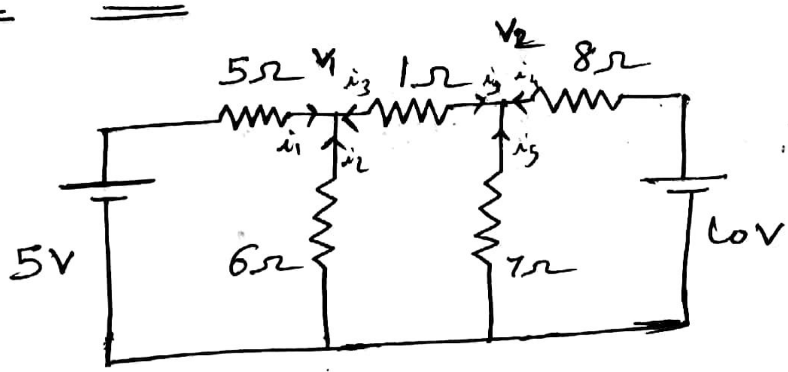
$\Delta = 39$

$\Delta_1 = 117$

$\Delta_2 = 78$

$\Delta_3 = 117$

NODAL ANALYSIS



SOLUTION

$i_1 + i_2 + i_3 = 0$

$$\frac{5-V_1}{5} + \frac{-V_1}{6} + \frac{V_2 - V_1}{1} = 0$$

$$\frac{5}{5} - \frac{V_1}{5} - \frac{V_1}{6} + \frac{V_2}{1} - \frac{V_1}{1} = 0$$

$$V_1\left[-\frac{1}{5} - \frac{1}{6} - \frac{1}{1}\right] + \frac{V_2}{1} = -1$$

$$-1.367V_1 + V_2 = -1$$

$i_3 + i_4 + i_5 = 0$

$$\frac{V_1 - V_2}{1} + \frac{-V_2}{7} + \frac{10 - V_2}{8} = 0$$

$$\frac{V_1}{1} - \frac{V_2}{1} - \frac{V_2}{7} + \frac{10}{8} - \frac{V_2}{8} = 0$$

$$\frac{V_1}{1} - V_2\left[\frac{1}{1} + \frac{1}{7} + \frac{1}{8}\right] = -\frac{10}{8}$$

$$V_1 - 1.2678\,V_2 = -1.25$$

Solving the equation

$$\Delta = \begin{vmatrix} -1.367 & 1 \\ 1 & -1.2678 \end{vmatrix} \quad \Delta = 0.733$$

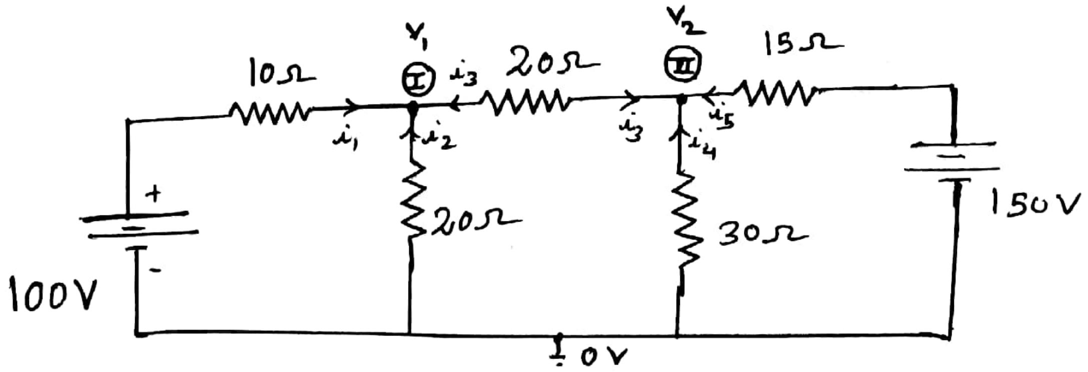
$$\Delta_1 = \begin{vmatrix} -1 & 1 \\ -1.25 & -1.2678 \end{vmatrix} \quad \Delta_1 = 2.5178$$

$$\Delta_2 = \begin{vmatrix} -1.367 & -1 \\ 1 & -1.25 \end{vmatrix}$$

$\Delta_2 = 2.7087$

$$V_1 = \frac{\Delta_1}{\Delta} = \underline{3.43} \qquad V_2 = \frac{\Delta_2}{\Delta} = \underline{3.69\,V}$$

Using nodal analysis, find all node Voltages.



$$i_1 + i_2 + i_3 = 0$$

$$\frac{100 - V_1}{10} + \frac{-V_1}{20} + \frac{V_2 - V_1}{20} = 0$$

$$\frac{100}{10} - \frac{V_1}{10} - \frac{V_1}{20} + \frac{V_2}{20} - \frac{V_1}{20} = 0$$

$$V_1\left[-\frac{1}{10} - \frac{1}{20} - \frac{1}{20}\right] + V_2\left[\frac{1}{20}\right] = -10$$

$$-0.2V_1 + 0.05\,V_2 = -10$$

$$\boxed{0.2\,V_1 - 0.05\,V_2 = 10} \quad - \text{I}$$

$$i_3 + i_4 + i_5 = 0$$

$$\frac{V_1 - V_2}{20} - \frac{V_2}{30} + \frac{150 - V_2}{15} = 0$$

$$\frac{V_1}{20} - \frac{V_2}{20} - \frac{V_2}{30} + \frac{150}{15} - \frac{V_2}{15} = 0$$

$$V_1\left[\frac{1}{20}\right] - V_2\left[\frac{1}{20} + \frac{1}{30} + \frac{1}{15}\right] = -10$$

$$\boxed{0.05V_1 - 0.15V_2 = -10}$$

$$\Delta = \begin{vmatrix} 0.2 & -0.05 \\ 0.05 & -0.15 \end{vmatrix}$$

$$\Delta = 0.2(-0.15) + 0.05(0.05)$$

$$\Delta = -0.0275$$

$$\Delta_1 = \begin{vmatrix} 10 & -0.05 \\ -10 & -0.15 \end{vmatrix}$$

$$\Delta_1 = -2$$

$$\Delta_2 = \begin{vmatrix} 0.2 & 10 \\ 0.05 & -10 \end{vmatrix}$$

$$\Delta_2 = -2.5$$

ON solving

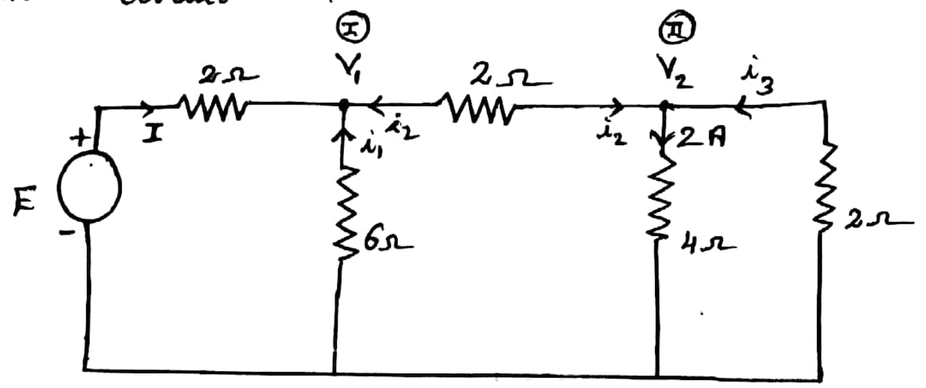
$V_1 = 72.72$ V

$V_2 = 90.9$ V

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{-2}{-0.0275} = 72.72\text{ V}$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-2.5}{-0.0275} = 90.9\text{ V}$$

In the following circuit, find the value of E and I for the circuit shown.



SOLUTION

Solve using nodal analysis

Apply KCL for node I

$$I + i_1 + i_2 = 0$$

$$\frac{E - V_1}{2} + \frac{-6}{V_1} + \frac{V_2 - V_1}{2} = 0$$

$$\frac{E}{2} - \frac{V_1}{2} - \frac{V_1}{6} + \frac{V_2}{2} - \frac{V_1}{2} = 0$$

$$0.5E - V_1\left[\frac{1}{2} + \frac{1}{6} + \frac{1}{2}\right] + 0.5V_2 = 0$$

$$\boxed{0.5E - 1.167V_1 + 0.5V_2 = 0} \quad - \text{I}$$

Apply KCL for node II

$$i_2 + i_3 = 2$$

$$\frac{V_1 - V_2}{2} + \frac{-V_2}{2} = 2$$

$$\frac{V_1}{2} - \frac{V_2}{2} - \frac{V_2}{2} = 2.$$

$$\frac{V_2 - 0}{4} = 2$$

$$\boxed{V_2 = 8\text{ V}}$$

$$\boxed{0.5V_1 - 1V_2 = 2} \Rightarrow \text{(II)}$$

sub $V_2 = 8V$ in (II)

$$0.5V_1 - 8 = 2$$

$$V_1 = \frac{10}{0.5} \qquad \boxed{V_1 = 20\,V}$$

sub $V_1 = 20V$; $V_2 = 8V$ in (I)

$$\therefore \quad 0.5E - 1.167(20) + 0.5(8) = 0$$

$$0.5E = 23.34 - 4$$

$$\boxed{E = 38.68\,V}$$

To find I

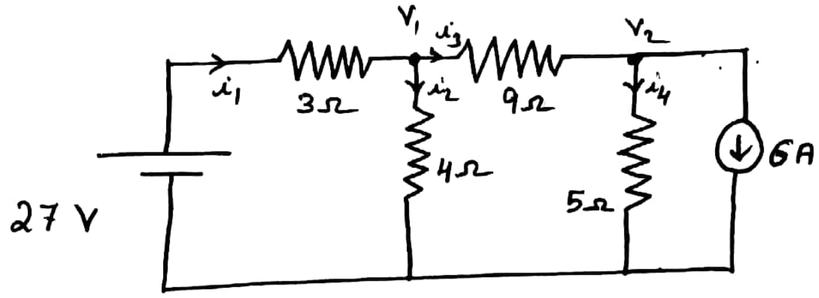
$$I = \frac{E - V_1}{2}$$

$$I = \frac{38.68 - 20}{2}$$

$$\boxed{I = 9.34\,A}$$

UNIV QUESTION

Calculate the power delivered by 9Ω resistor for the network shown below using nodal analysis.



SOLUTION

NODE 1

$$i_1 = i_3 + i_2$$

$$\frac{27 - V_1}{3} = \frac{V_1 - V_2}{9} + \frac{V_1}{4}$$

$$\frac{27}{3} - \frac{V_1}{3} = \frac{V_1}{9} - \frac{V_2}{9} + \frac{V_1}{4}$$

$$\frac{V_1}{9} + \frac{V_1}{4} + \frac{V_1}{3} - \frac{V_2}{9} = 9$$

$$V_1\left[1/9 + 1/4 + 1/3\right] - V_2/9 = 9$$

$$0.6944\, V_1 - 0.11\, V_2 = 9 \quad \text{— (I)}$$

NODE 2

$$i_3 = i_4 + 6$$

$$\frac{V_1 - V_2}{9} = \frac{V_2}{5} + 6$$

$$\frac{V_1}{9} - \frac{V_2}{9} = \frac{V_2}{5} + 6$$

$$V_1\left(\frac{1}{9}\right) - V_2\left(1/9 + 1/5\right) = 6$$

$$0.11V_1 - 0.311\,V_2 = 6 \quad - \text{(II)}$$

$$\Delta = \begin{vmatrix} 0.6944 & -0.11 \\ 0.11 & -0.311 \end{vmatrix} \qquad \Delta = -0.2038$$

$$\Delta_1 = \begin{vmatrix} 9 & -0.11 \\ 6 & -0.311 \end{vmatrix} \qquad \Delta_1 = -2.139$$

$$\Delta_2 = \begin{vmatrix} 0.6944 & +9 \\ 0.11 & 6 \end{vmatrix} \qquad \Delta_2 = 3.1764$$

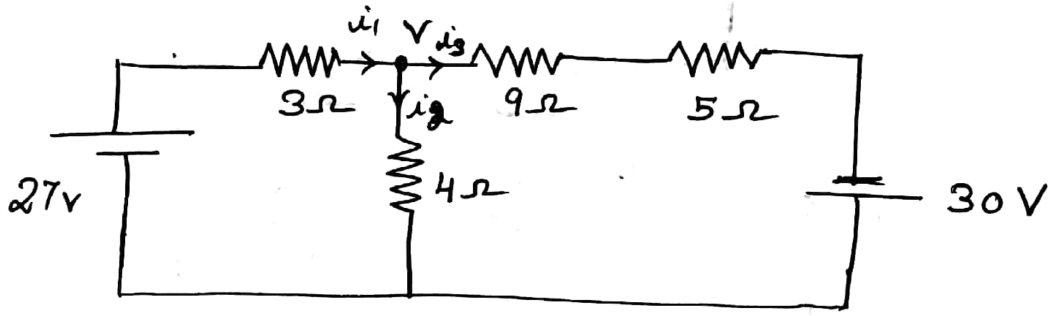
$$V_2 = \frac{\Delta_2}{\Delta} = -15.585\ V$$

$$V_1 = \frac{\Delta_1}{\Delta} = 10.495\ V$$

$$i_3 = \frac{V_1 - V_2}{9} = \frac{10.495 - (-15.585)}{9} = \underline{2.89\ A}$$

$$P_{9\Omega} = i_3^2\,9 = 2.89^2 \times 9 = \underline{75.1689\ W}$$

METHOD : 2 SOURCE TRANSFORMATION



$$i_1 = i_2 + i_3$$

$$\frac{27-V}{3} = \frac{V}{4} + \frac{V-(-30)}{(9+5)}$$

$$\frac{27}{3} - \frac{V}{3} = \frac{V}{4} + \frac{V}{14} + \frac{30}{14}$$

$$\frac{27}{3} - \frac{30}{14} = V\left(\frac{1}{4} + \frac{1}{14} + \frac{1}{3}\right)$$

$$6.857 = 0.6547\,V$$

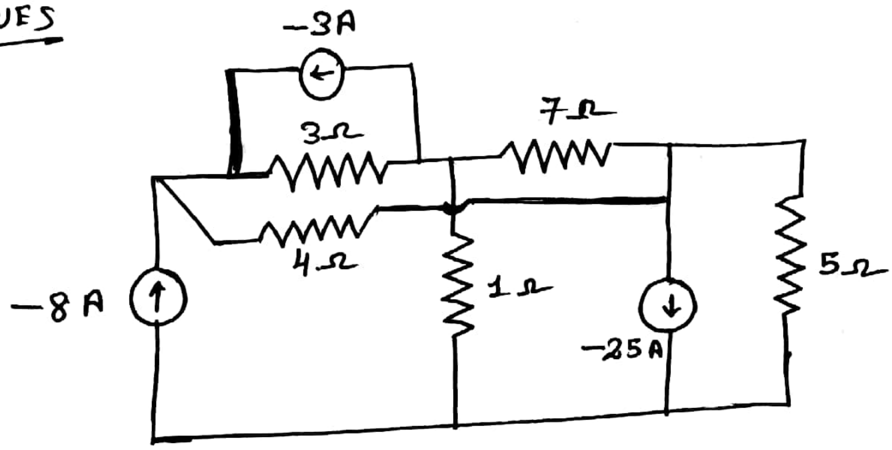
$$\boxed{V = 10.47\ V}$$

$$i_3 = \frac{V+30}{14} = \frac{10.47+30}{14}$$

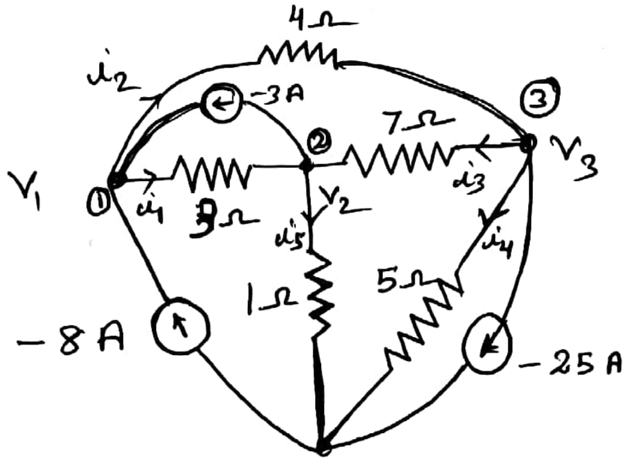
$$\boxed{i_3 = 2.89\ A}$$

$$P_{9\Omega} = 2.89^2(9) = \underline{75.21\ W}$$

UNIV QUES



$\Downarrow$



NODE 1

$$-3-8 = i_1 + i_2$$

$$-11 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4}$$

$$-11 = \frac{V_1}{3} - \frac{V_2}{3} + \frac{V_1}{4} - \frac{V_3}{4}$$

$$V_1\left(\frac{1}{3}+\frac{1}{4}\right) - \frac{V_2}{3} - \frac{V_3}{4} = -11$$

$$0.58V_1 - 0.33V_2 - 0.25V_3 = -11 \quad -(1)$$

Node 2

$$-3 + I_5 = I_1 + I_3$$

$$-3 + \frac{V_2}{1} = \frac{V_1 - V_2}{3} + \frac{V_3 - V_2}{7}$$

$$\frac{V_1}{3} - V_2\left(\frac{1}{3} + \frac{1}{7} + \frac{1}{1}\right) + \frac{V_3}{7} = -3$$

$$0.33V_1 - 1.476V_2 + 0.1428V_3 = -3 \quad - (2)$$

NODE: 3

$$i_2 = i_3 + i_4 - 25$$

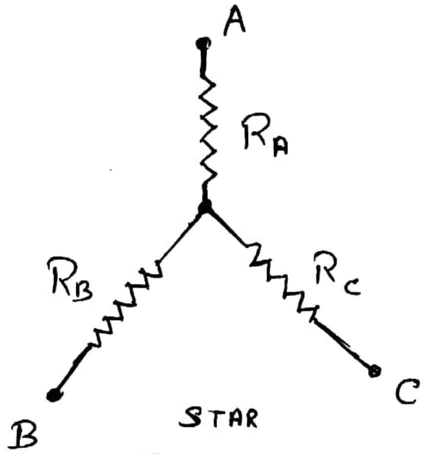
$$\frac{V_1 - V_3}{4} = \frac{V_3 - V_2}{7} + \frac{V_3}{5} - 25$$

$$\frac{V_1}{4} + \frac{V_2}{7} - V_3\left(\frac{1}{4} + \frac{1}{7} + \frac{1}{5}\right) = -25$$

$$0.25V_1 + 0.1428V_2 - 0.5928\,V_3 = -25 \quad - (3)$$

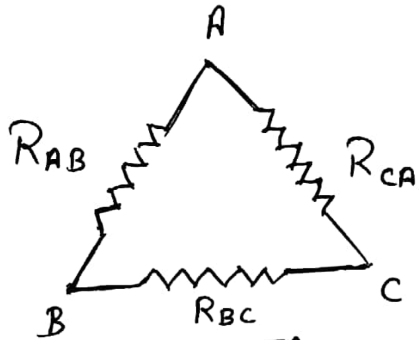
$$\left\{\begin{aligned} V_1 &= 5.38\ V \\ V_2 &= 7.715\ V \\ V_3 &= 46.3\ V \end{aligned}\right\}$$

STAR TO DELTA TRANSFORMATION



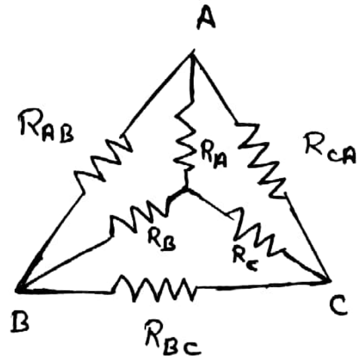
STAR CONNECTION

$\Rightarrow$



DELTA CONNECTION

$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$



$$R_{BC} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

$$R_{CA} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

" If three resistance are equal. $R_{AB} = R_{BC} = R_{CA} = 3R$

DELTA TO STAR TRANSFORMATION

$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C = \frac{R_{BC} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

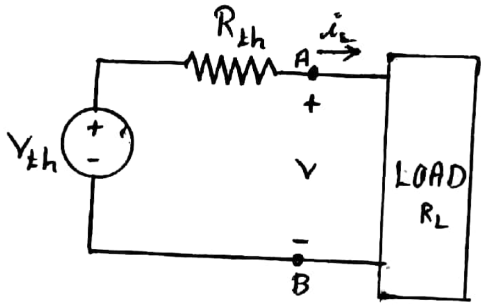
{ " If three resistance are equal.

$R_A = R_B = R_C = \frac{R}{3}$ }

THEVENIN'S THEOREM

* Any linear active network with output terminal A and B can be replaced by a single voltage source ($V_{th} = V_{oc}$) in series with single impedance ($Z_{th} = Z_i$)
* It makes the solution of complicated network quick and easy.

THEVENIN EQUIVALENT CIRCUIT



$$i_L = \frac{V_{th}}{R_{th} + R_L}$$

COMPUTATION OF THEVENIN EQUIVALENT RESISTANCE, R_{TH}

* Remove the load
* Cancel all independent voltage and current source (i.e short circuit the voltage source, open circuit the current source)
* Compute the total resistance with respect to terminal A & B

COMPUTATION OF THEVENIN EQUIVALENT VOLTAGE, V_{TH}

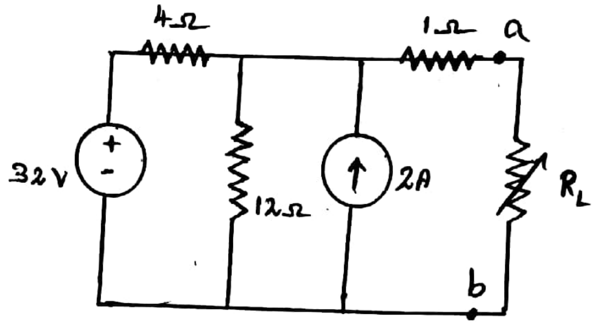
* Remove the load
* Define the open-circuit voltage V_{oc} across the open load terminals.

* Solve for V_{oc}

* The thevenin voltage $V_{th} = V_{oc}$

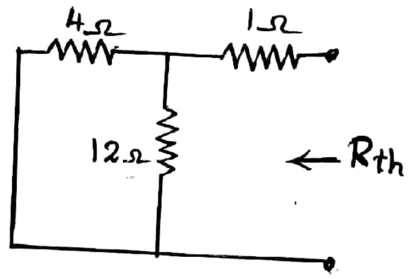
EXAMPLE

1. Find the Thevenin equivalent circuit of the circuit shown.



Solution

To find R_{th}

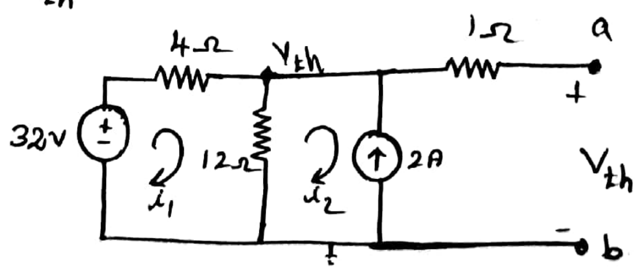


* Voltage source is SC
* Current source is OC
* R_{th} has to calculate from opposite side.

$$R_{th} = 4 \| 12 + 1$$

$$R_{th} = \frac{4 \times 12}{4 + 12} + 1 = 4\Omega$$

To find V_{th}



Using mesh analysis.

$$32 = 4i_1 + 12(i_1 - i_2) \quad - (1)$$

$$i_2 = -2A$$

on solving

$$32 = 4i_1 + 12i_1 - 12(-2)$$

$$= 4i_1 + 12i_1 + 24$$

$$32 = 16i_1 + 24$$

$$\boxed{i_1 = 0.5\ A}$$

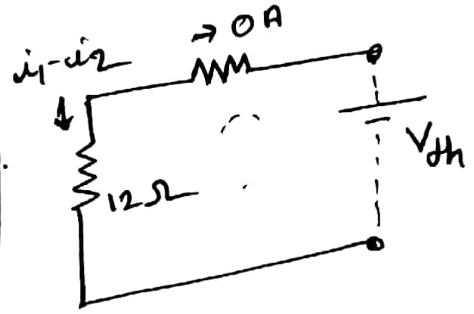
$$V_{th} = 12(i_1 - i_2)$$

$$V_{th} = 12(0.5 - (-2))$$

$$= 12(0.5 + 2)$$

$$\boxed{V_{th} = 30\ V}$$

To Find V_{th}



$$-V_{th} = 12\{-(i_1 - i_2)\}$$

$$-V_{th} = -12(0.5 - (-2))$$

$$-V_{th} = -12(0.5 + 2)$$

$$V_{th} = 12(2.5)$$

$$\boxed{V_{th} = 30\ V}$$

Using Nodal analysis method

* There is only one node.

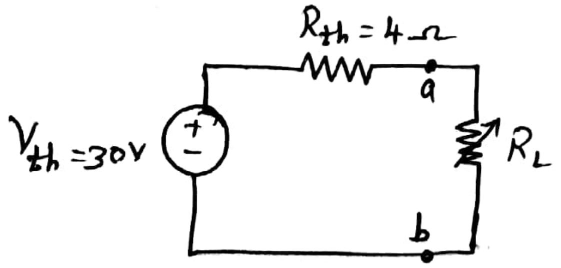
$$\frac{32 - V_{th}}{4} + 2 = \frac{V_{th}}{12}$$

$$96 - 3V_{th} + 24 = V_{th}$$

$$\boxed{V_{th} = 30V}$$

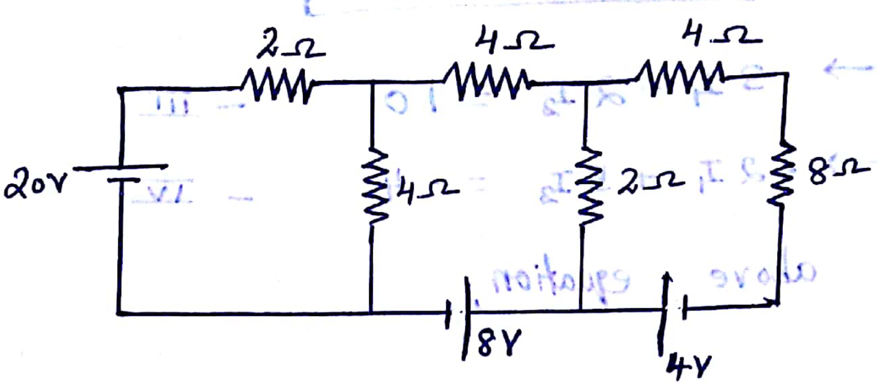
NOTE! While finding V_{th}, don't consider the 2A source.

EQUIVALENT CIRCUIT



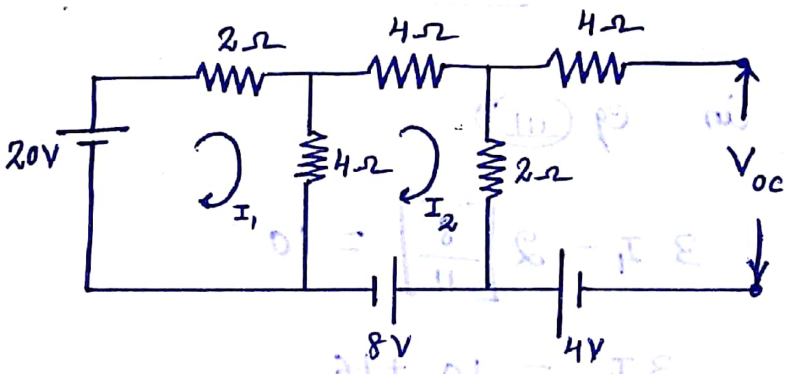
EXAMPLE : 2

Find current flowing through 8Ω resistor in the below circuit using Thevenin's theorem.



STEP - 1

Consider 8Ω resistor as load resistor $[R_L = 8]$ and remove it



STEP - 2

Calculate loop currents I_1 & I_2 using Mesh analysis

$$20 = 2I_1 + 4(I_1 - I_2)$$

$$20 = 2I_1 + 4I_1 - 4I_2$$

$$20 = 6I_1 - 4I_2$$

$$\boxed{3I_1 - 2I_2 = 10} \quad - \text{(I)}$$

$$-8 = 4(I_2 - I_1) + 4I_2 + 2I_2$$

$$-8 = 4I_2 - 4I_1 + 4I_2 + 2I_2$$

$$\boxed{-4I_1 + 10I_2 = -8} \quad - \text{(II)}$$

$$\text{(I)} \rightarrow 3I_1 - 2I_2 = 10 \quad - \text{III}$$

$$\text{(II)} \rightarrow -2I_1 + 5I_2 = -4 \quad - \text{IV}$$

On Solving above equation,

$$\text{(III)} \times 2 \rightarrow \cancel{6I_1} - 4I_2 = 20$$

$$\text{IV} \times 3 \rightarrow \cancel{-6I_1} + 15I_2 = -12$$

$$11I_2 = 8$$

$$\left\{I_2 = \frac{8}{11}\right\}$$

$I_2 = \frac{8}{11}$ in eq (III)

$$3I_1 - 2\left[\frac{8}{11}\right] = 10$$

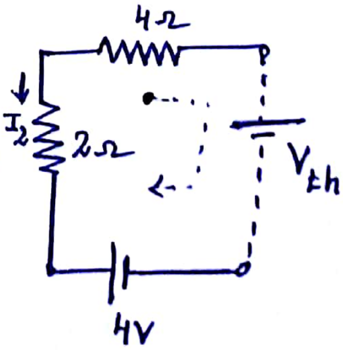
$$3I_1 = 10 + \frac{16}{11}$$

$$I_1 = \frac{126}{33} = \frac{42}{11}$$

$$\left\{I_1 = \frac{42}{11}\right\}$$

STEP-3

Calculate $V_{oc} = V_{th}$



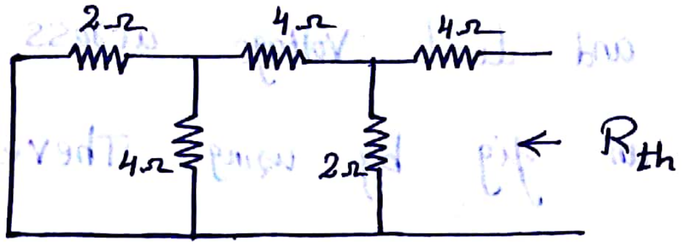
$$-V_{th} + 4 = -2I_2$$

$$V_{th} = 4 + 2I_2 = 4 + 2\left[\frac{8}{11}\right] = \frac{60}{11}$$

$$\boxed{V_{th} = \frac{60}{11}} V$$

STEP-4

To Find R_{th} by Short circuiting the Voltage source.



$$R_{th} = \left[\left[2 \| 4 + 4\right] \| 2\right] + 4$$

$$R_{th} = \left[\left(\frac{8}{6} + 4\right) \| 2\right] + 4$$

$$R_{th} = \left[\frac{32}{6} \| 2\right] + 4$$

$$R_{th} = \frac{\frac{32}{6} \times 2}{\frac{32}{6} + 2} + 4$$

$$= \frac{\frac{32}{3}}{\frac{44}{6}} + 4$$

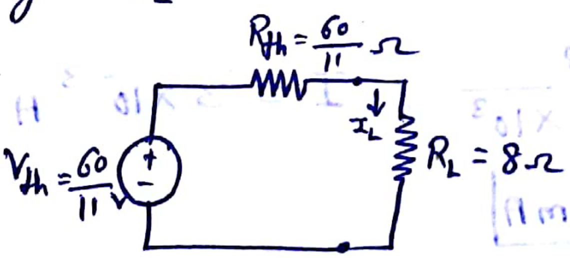
$$= \frac{\overset{16}{\cancel{32}}}{\cancel{3}} \times \frac{\overset{2}{\cancel{6}}}{\underset{\underset{11}{\cancel{22}}}{\cancel{44}}} + 4$$

$$= \frac{16}{11} + 4 = \frac{60}{11}$$

$$\left\{ R_{th} = \frac{60}{11}\Omega \right\}$$

STEP-5

Drawing thevenin Equivalent circuit and finding current through R_L

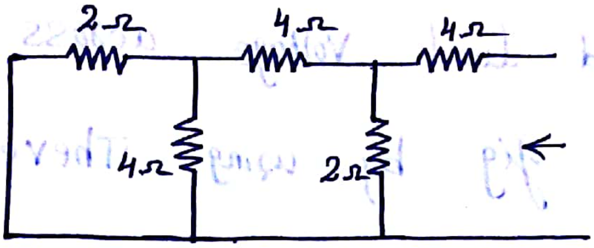


$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{\frac{60}{11}}{\frac{60}{11} + 8} = \frac{\frac{60}{11}}{13.45} = \frac{5.45}{13.45}$$

$$\boxed{I_L = 0.405\ A}$$

STEP-4

To Find R_{th} by short circuiting the voltage source.



← R_{th}

$$R_{th} = \left[\left[2 \| 4 + 4\right] \| 2\right] + 4$$

$$R_{th} = \left[\left(\frac{8}{6} + 4\right) \| 2\right] + 4$$

$$R_{th} = \left[\frac{32}{6} \| 2\right] + 4$$

$$R_{th} = \frac{\frac{32}{6} \times 2}{\frac{32}{6} + 2} + 4$$

$$= \frac{\frac{32}{3}}{\frac{44}{6}} + 4$$

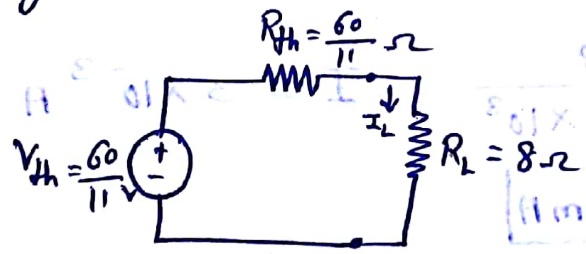
$$= \frac{\cancel{32}^{16}}{\cancel{3}} \times \frac{\cancel{6}^{2}}{\cancel{44}_{\cancel{22}\,11}} + 4$$

$$= \frac{16}{11} + 4 = \frac{60}{11}$$

$$\left\{ R_{th} = \frac{60}{11}\,\Omega \right\}$$

STEP-5

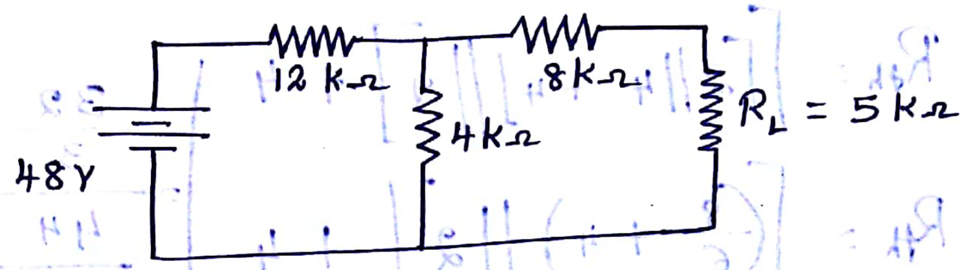
Drawing thevenin Equivalent circuit and finding current through R_L



$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{\frac{60}{11}}{\frac{60}{11} + 8} = \frac{\frac{60}{11}}{13.45} = \frac{5.45}{13.45}$$

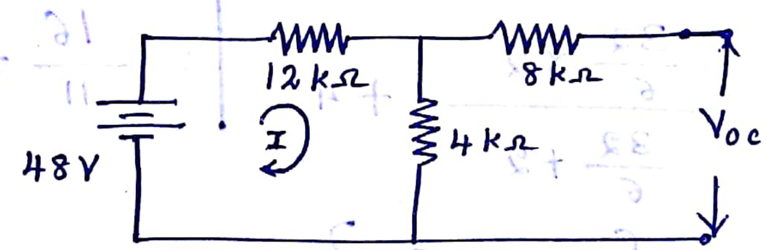
$$\boxed{I_L = 0.405\ A}$$

EXAMPLE: 3 Find V_{th}, R_{th} and the load current flowing through and load voltage across the load resistor in fig by using Thevenin's theorem.



SOLUTION

STEP 1 : Remove the load resistor $R_L = 5\,k\Omega$



STEP 2 : To find load current, 'I'

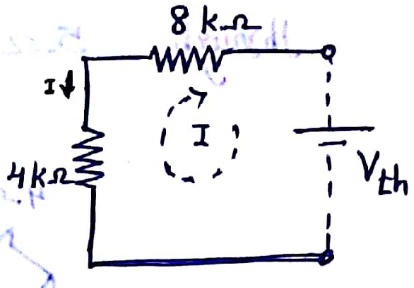
$$48 = 12 \times 10^3\, I + 4 \times 10^3\, I$$

$$48 = 16 \times 10^3\, I$$

$$I = \frac{48}{16 \times 10^3} \;; \; I = 3 \times 10^{-3}\, A$$

$$\boxed{I = 3\,mA}$$

STEP 3: Calculate $V_{oc} = V_{th}$



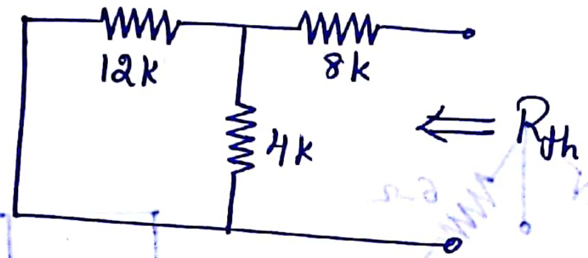
$$-V_{th} = -4\times10^{3}(I)$$

$$V_{th} = 4\times10^{3}(3\times10^{-3})$$

$$\boxed{V_{th} = 12\ V}$$

STEP 4: Calculate "R_{th}"

* short circuiting the voltage source

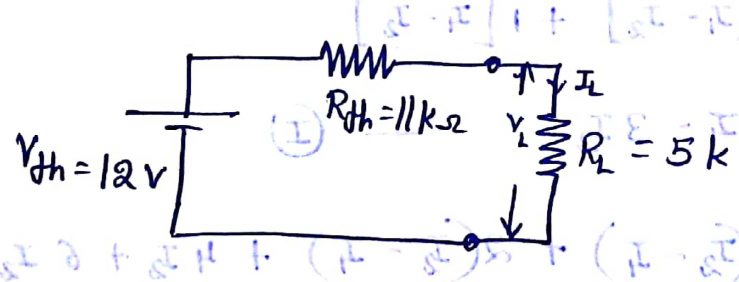


$$R_{th} = \left(\frac{12\times4}{12+4}\right) + 8$$

$$R_{th} = \frac{48}{16} + 8$$

$$\boxed{R_{th} = 11\,k\Omega}$$

STEP 5: Therenin's Equivalent Circuit

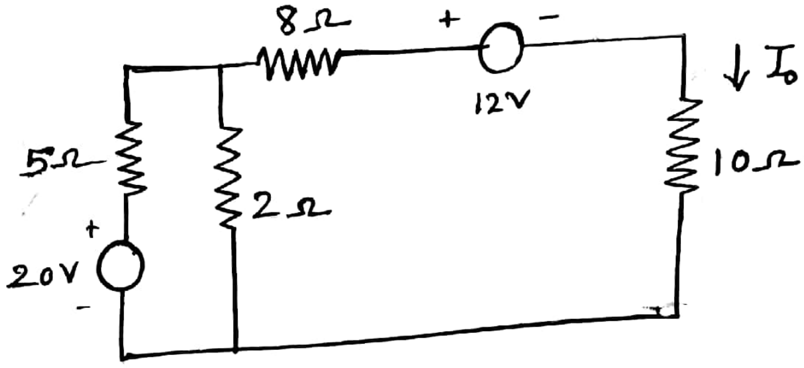


$$I_L = \frac{V_{th}}{R_{th}+R_L} = \frac{12}{(11+5)10^{3}} \qquad \boxed{I_L = 0.75\ mA}$$

$$V_L = I_L R_L = (0.75\times10^{-3}\times5\times10^{3}) \qquad \boxed{V_L = 3.75\ v}$$

UNIV QUESTION

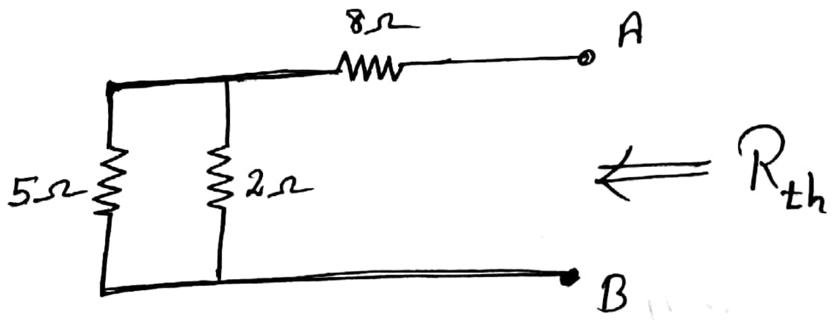
Using Thevenin's theorem find the current I_o for the circuit shown below.



SOLUTION

* Since I_o is the current flowing through 10Ω resistor, Consider it as a load resistor and remove it and mark the terminals as A and B

To find R_{Th} "SC the Voltage Source"



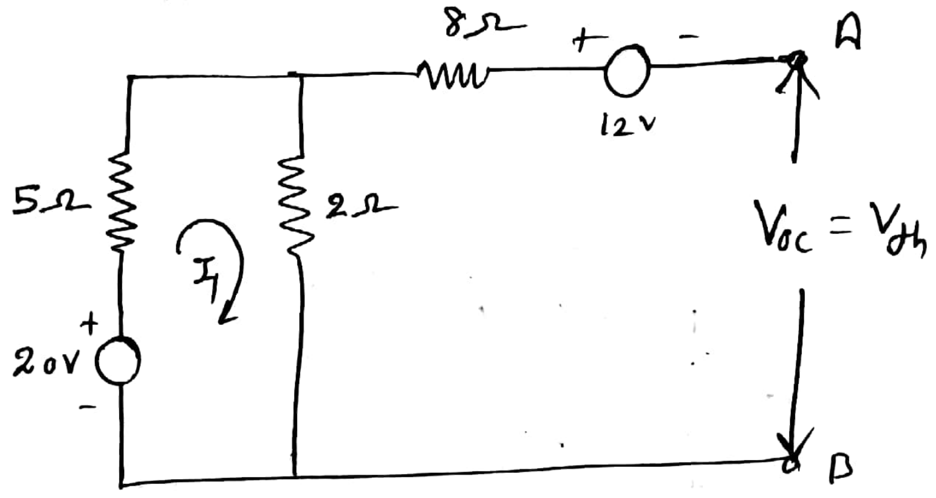
$$R_{th} = [5 \| 2] + 8$$

$$= \frac{5 \times 2}{5+2} + 8$$

$$R_{th} = \frac{10}{7} + 8$$

$$\{R_{th} = 9.43\,\Omega\}$$

To find loop Current

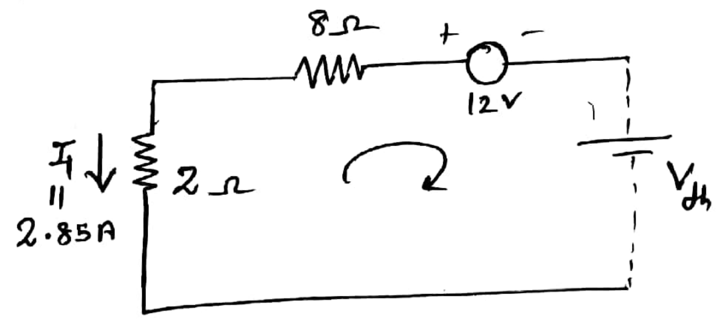


$$20 = 5I_1 + 2I_1$$

$$20 = 7I_1$$

$$\boxed{I_1 = 2.85\ A}$$

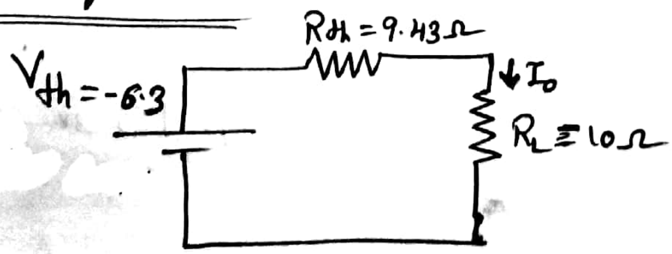
To find V_{th}



$$-12 - V_{th} = -2.85(2)$$

$$\boxed{V_{th} = -6.3\ V}$$

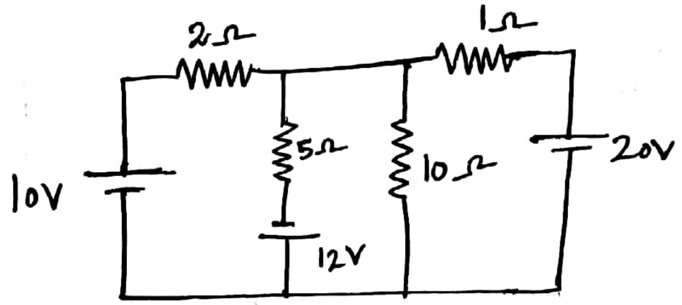
Thevenins Eqv Circuit



$$I_o = \frac{V_{th}}{R_{th} + R_L} = \frac{-6.3}{9.43 + 10}$$

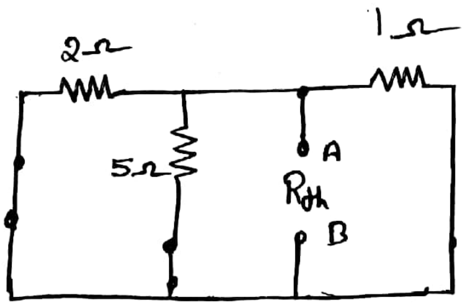
$$\boxed{I_o = -0.324\ A}$$

In the given circuit find the current through 1Ω resistor using Thevenin's theorem.



Solution

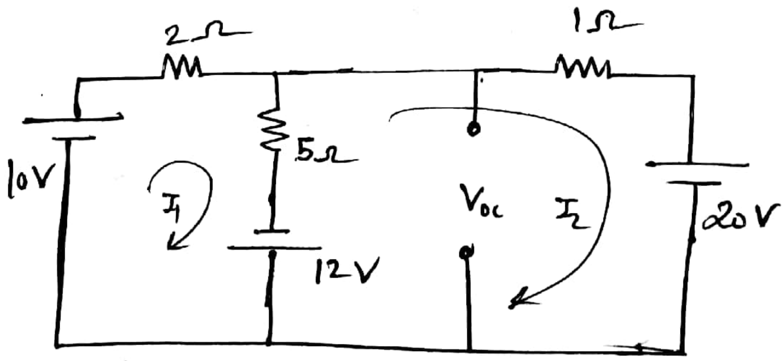
R_{th}



$$\frac{1}{R_{th}} = \frac{1}{2} + \frac{1}{5} + \frac{1}{1}$$

$$\boxed{R_{th} = 0.588\,\Omega}$$

Loop Current



$$10 + 12 = 2I_1 + 5(I_1 - I_2)$$

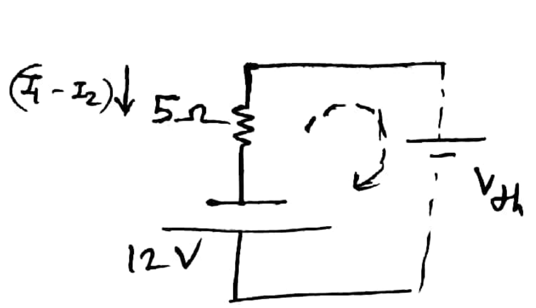
$$7I_1 - 5I_2 = 22 \quad —(1)$$

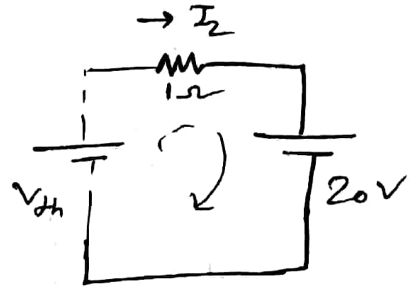
$$-20 - 12 = 5(I_2 - I_1) + I_2$$

$$-5I_1 + 6I_2 = -32 \quad —(2)$$

$$\left\{ \begin{array}{l} I_1 = -1.64\text{ A} \\ I_2 = -6.705\text{ A} \end{array} \right\}$$

To find V_{oc}





$$-12 - V_{th} = -5(I_1 - I_2)$$

$$-12 - V_{th} = -5(-1.64 + 6.705)$$

$$-12 - V_{th} = -25.325$$

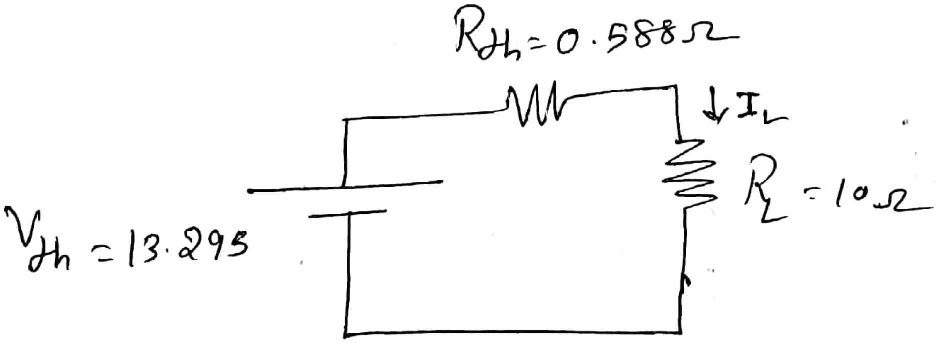
$$\boxed{V_{th} = 13.325\ V}$$

$$V_{th} - 20 = 1(-6.705)$$

$$V_{th} = -6.705 + 20$$

$$\boxed{V_{th} = 13.295\ V}$$

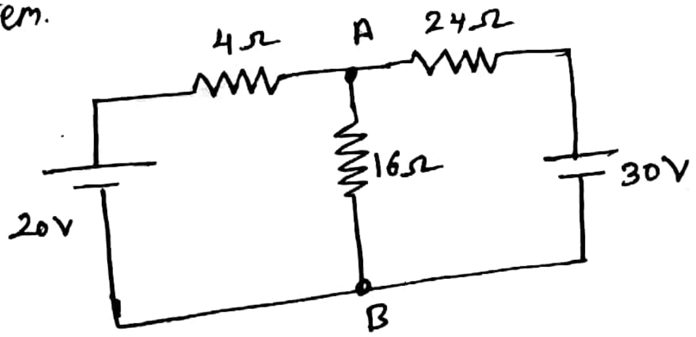
Thevenin's equivalent Circuit



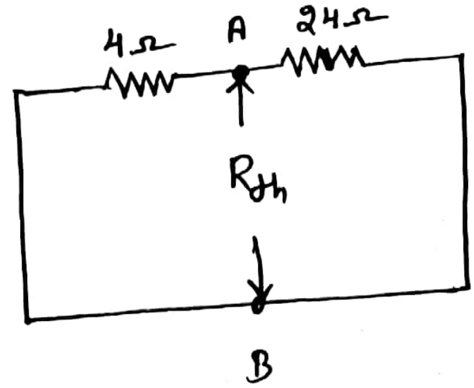
$$I_L = \frac{13.295}{0.588 + 10}$$

$$\boxed{I_L = 1.26\ A}$$

Find current through 16Ω resistor using thevenins theorem.



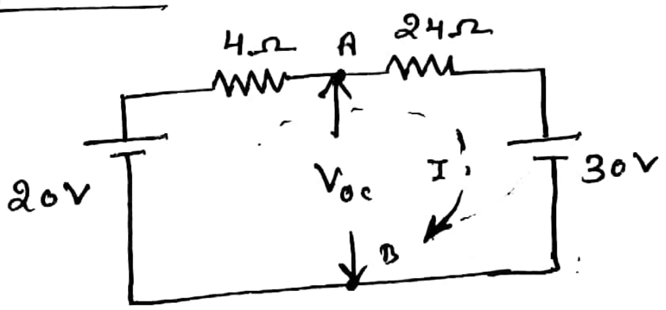
To find R_{th}



$$\frac{1}{R_{th}} = \frac{1}{4} + \frac{1}{24}$$

$$\boxed{R_{th} = 3.43\,\Omega}$$

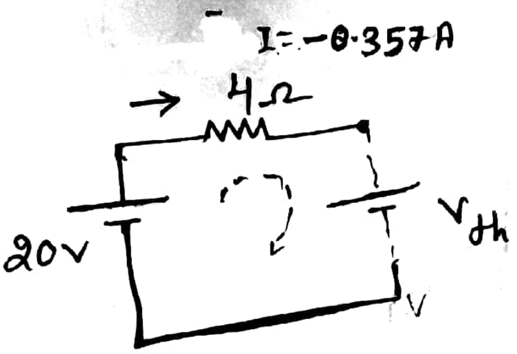
To find loop current



$$20 - 30 = 4I + 24I$$

$$-10 = 28I$$

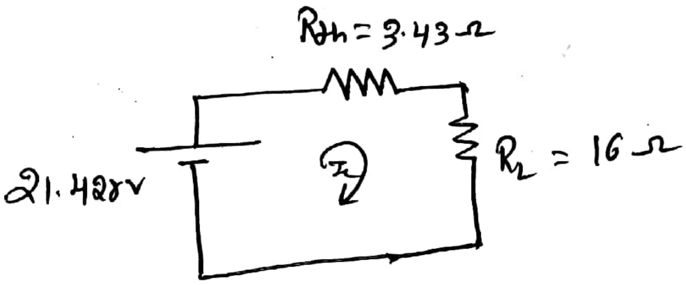
$$I = -\frac{10}{28} = -0.357\text{ A} \qquad \boxed{I = -0.357\text{ A}}$$



$$20 - V_{th} = -4(0.357)$$

$$-V_{th} = -1.428 - 20$$

$$\boxed{V_{th} = 21.428V}$$



$$I_L = \frac{21.428}{3.43 + 16}$$

$$\boxed{I_L = 1.1 \text{ A}}$$

NORTON'S THEOREM

Nortons theorem states that any two terminal linear network with current sources, voltage sources and resistances can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance.

PROCEDURE

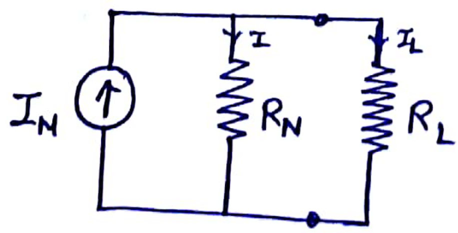
STEP 1 : Remove the load resistor and short circuit that terminal.

STEP 2 : Finding loop current

STEP 3 : Finding current through short circuited terminals and it is called as Nortons current [I_N]

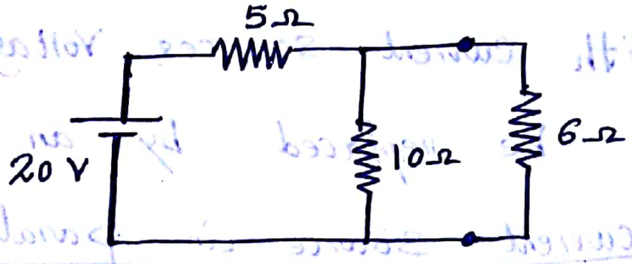
STEP 4 : Finding Norton's resistance by short circuiting the voltage source.

STEP 5 : Draw the Nortons equivalent circuit



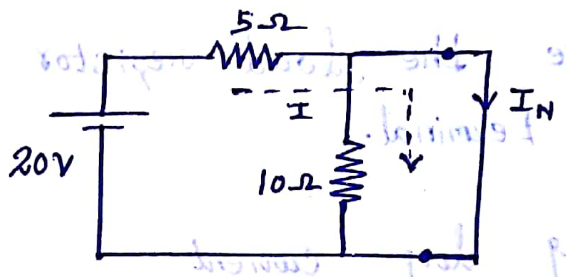
$$I_L = \frac{I_N R_N}{R_N + R_L}$$ //

EXAMPLE 1 : Find the current through load resistor 6Ω using Norton's theorem.



SOLUTION

STEP 1 : Remove 6Ω load resistor and short circuit that terminal



STEP 2 : Finding loop current

$$I = \frac{20}{5}$$

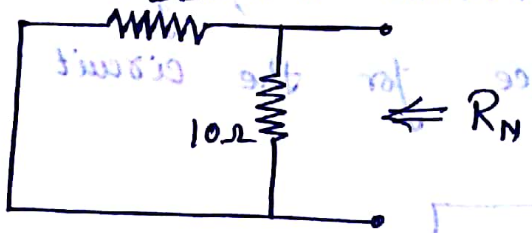
$$\boxed{I = 4\ A}$$

STEP 3 : Finding Norton's current, 'I_N'

* since, across 10Ω resistor, a line is short circuited, no current will flow through 10Ω resistor

$\therefore\ I = I_N = 4A$ $\quad \boxed{I_N = 4\ A}$

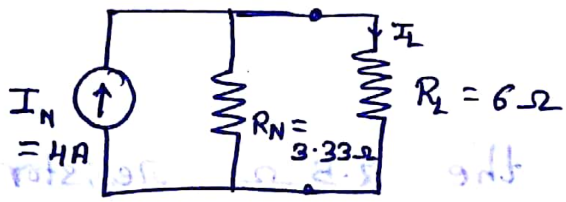
STEP 4 : Finding Nortons resistance R_N



$$R_N = \frac{50}{15}$$

$$\boxed{R_N = 3.33\,\Omega}$$

STEP 5 : Drawing Norton's equivalent circuit



$$I_L = \frac{I_N R_N}{R_N + R_L}$$

$$I_L = \frac{4(3.33)}{3.33 + 6}$$

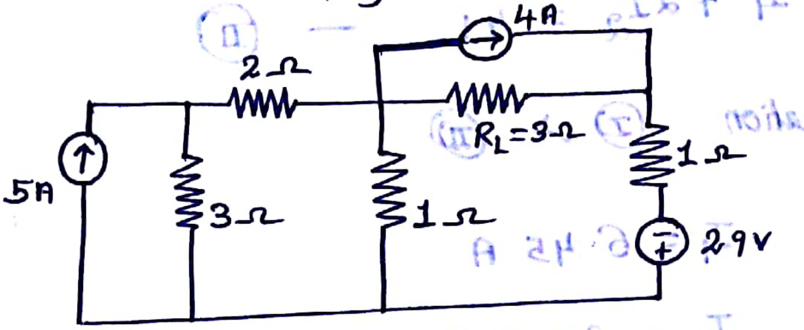
$$\boxed{I_L = 1.43\ A}$$

$$V_L = I_L R_L$$

$$V_L = 1.43(6)$$

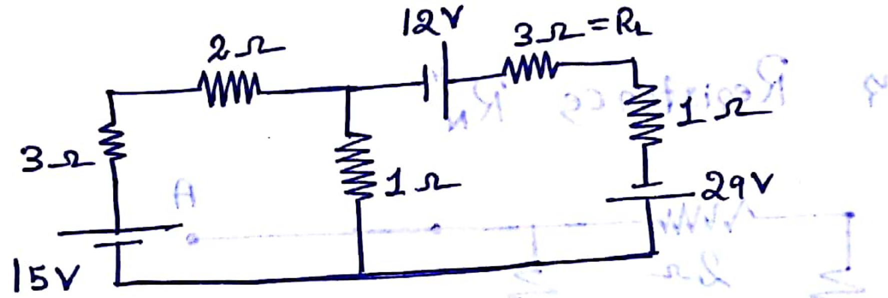
$$\boxed{V_L = 8.5\ V}$$

EXAMPLE: 3 Find the Current flowing through "R_L" for the below circuit using Norton's theorem.



Solution

Simplify the Circuit



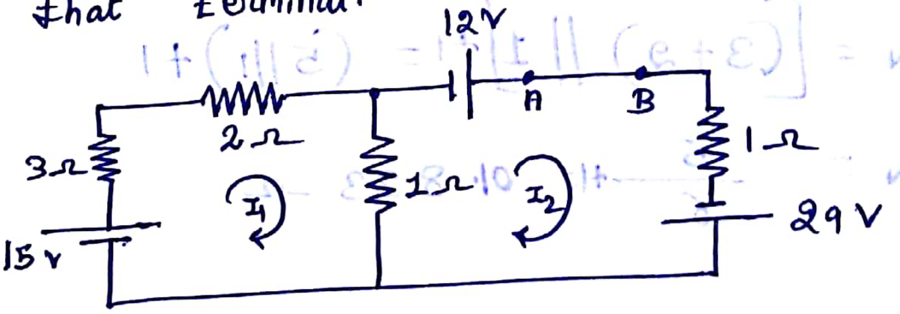
$V = IR$

$V = 5(3)$

$V = 15V$

STEP:1

Remove the load resistance, $R_L = 3Ω$ and short circuit that terminal.



STEP:2 Finding Norton Current, I_N

Here, $I_N = I_2$

$$15 = 3I_1 + 2I_1 + 1(I_1 - I_2)$$

$$6I_1 - I_2 = 15 \quad \text{--- (I)}$$

$$12 + 29 = 1(I_2 - I_1) + I_2$$

$$-I_1 + 2I_2 = 41 \quad \text{— (II)}$$

Solving equation (I) & (II)

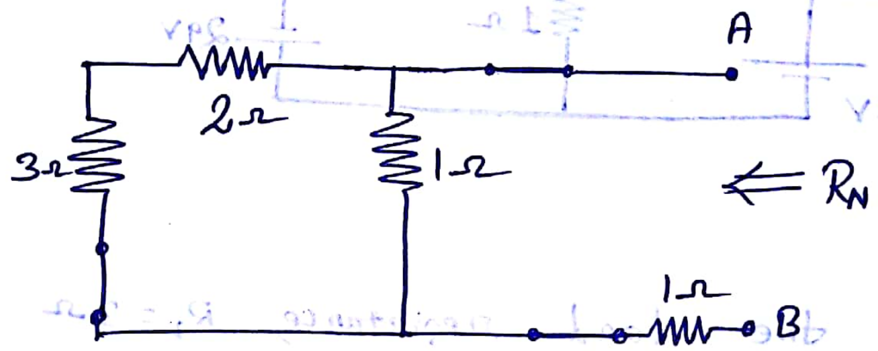
$$I_1 = 6.45 \text{ A}$$

$$I_2 = 23.7 \text{ A}$$

$$\boxed{I_2 = I_N = 23.7 \text{ A}}$$

STEP : 3

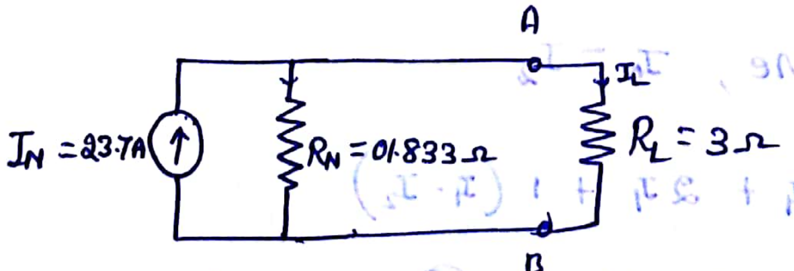
Finding Nortons Resistance "R_N"



$$R_N = [(3+2) \parallel 1] + 1 = (5 \parallel 1) + 1$$

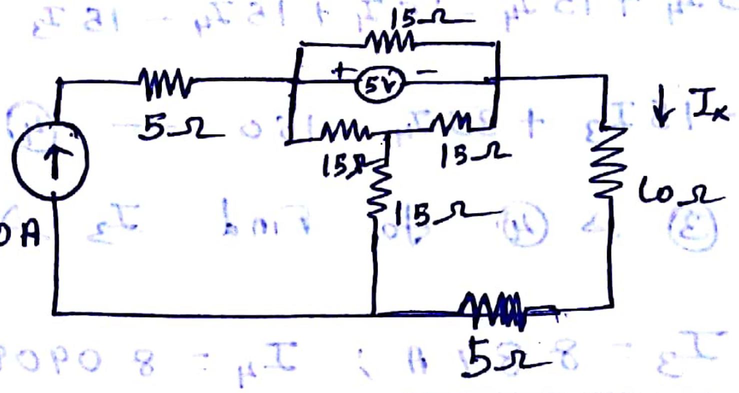
$$R_N = \frac{5}{6} + 1 = 01.833 \ \Omega$$

STEP : 4 Nortons Equivalent Circuit



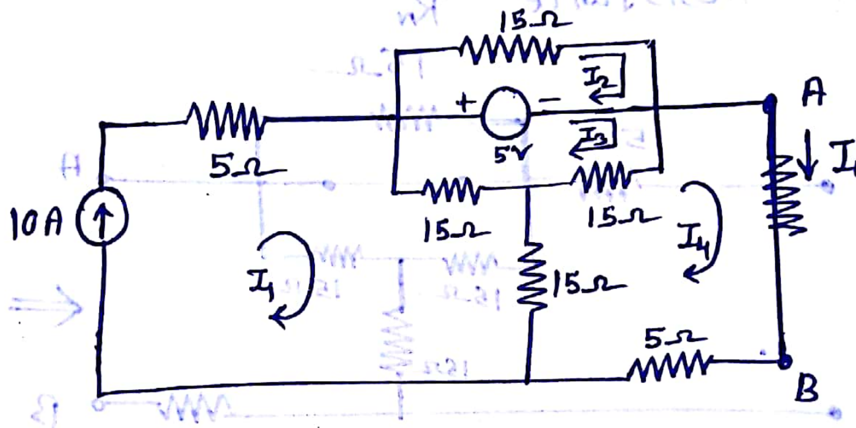
$$\left\{ I_L = \frac{I_N R_N}{R_N + R_L} = \frac{23.7 \times 01.833}{01.833 + 3} = 8.99 \approx 9 \text{ A} \right\}$$

EXAMPLE: 4 Obtain the current I_x in the circuit shown using Norton's theorem.



Solution

STEP 1 : Consider 10Ω resistor as load resistor and remove it. short circuit that terminal



STEP 2 : Finding Norton's Current $I_N = I_4$

$$I_1 = 10\,A \rightarrow (1)$$

$$5 = 15 I_2$$

$$I_2 = \frac{5}{15} = 0.333\,A \rightarrow (2)$$

$$-5 = 15(I_3 - I_4) + 15(I_3 - I_1)$$

$$-5 = 15 I_3 - 15 I_4 + 15 I_3 - 15(10)$$

$$30 I_3 - 15 I_4 = 145 \text{ --- } (3)$$

$$5I_4 + 15(I_4 - I_1) + 15(I_4 - I_3) = 0$$

$$5I_4 + 15I_4 - 15I_1 + 15I_4 - 15I_3 = 0$$

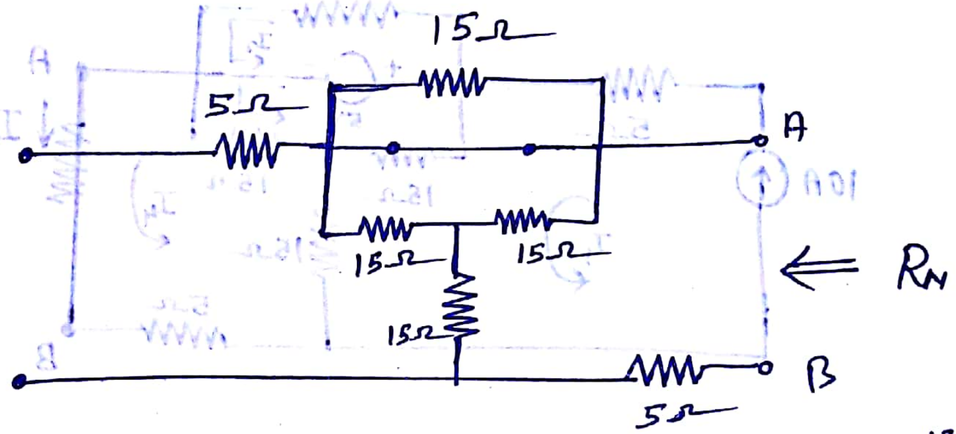
$$-15I_3 + 35I_4 = 150 \quad \text{— (4)}$$

Solving eq (3) & (4) to Find I_3 & I_4

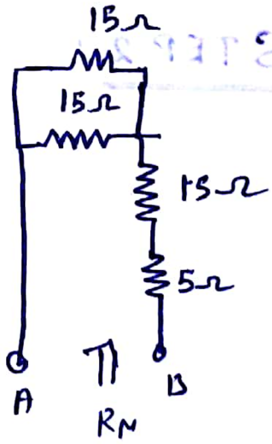
$$I_3 = 8.87\ A \ ; \quad I_4 = 8.0909\ A$$

$$\boxed{I_N = 8.0909\ A}$$

STEP: 3 Norton's Resistance "R_N"



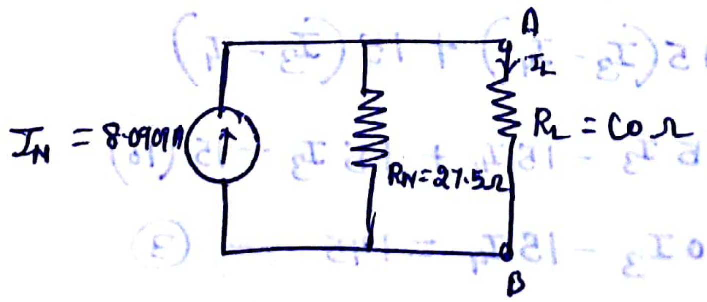
$$R_N = (5+15) + (15 \| 15)$$



$$R_N = 20 + 7.5$$

$$\boxed{R_N = 27.5\ \Omega}$$

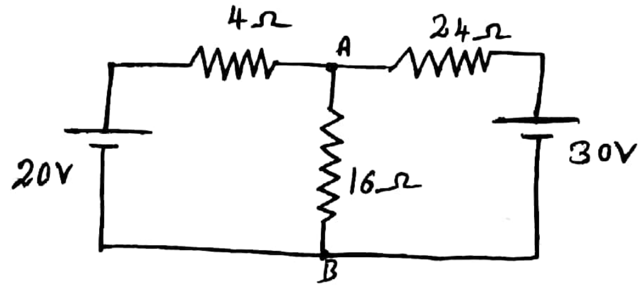
STEP 4 Norton's equivalent circuit



$$I_L = \frac{8.0909 \times 27.5}{27.5 + 60}$$

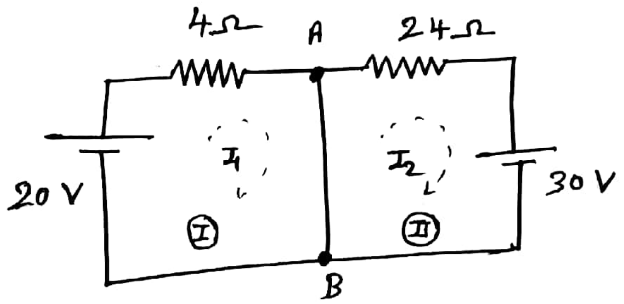
$$\boxed{I_L = 5.93\ A}$$

Find Current through 16Ω resistor using Norton's theorem.



SOLUTION

* Remove the load resistor 16Ω resistance and short circuit it



* Find loop current

Loop 1 $\quad 20 = 4I_1 \quad I_1 = \frac{20}{4} = 5\,A$

Loop 2 $\quad -30 = 24 I_2 \quad I_2 = \frac{-30}{24} = -1.25A$

* Current flowing through short circuited terminal AB is Norton Current

$= I_1 - I_2$ (or) $= I_2 - I_1$

$= 5 - (-1.25)$

$\boxed{I_1 - I_2 = 6.25\,A}$

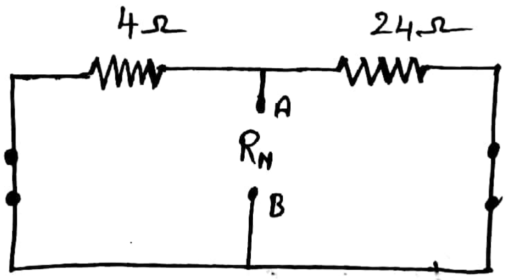
$= -1.25 - 5 = -6.25$

$\boxed{I_2 - I_1 = -6.25}$

$$\boxed{I_N = 6.25\,A}$$

To find Norton's Resistance

Procedure to find Norton's resistance is same a thevenin's resistance. [open circuiting the current source short circuiting the Voltage source]

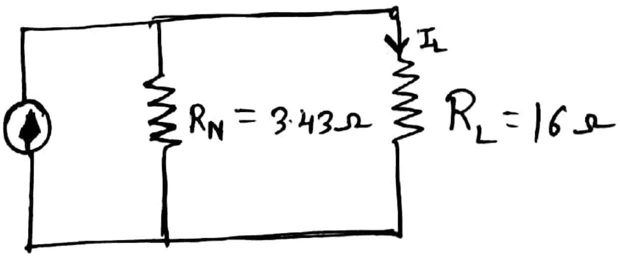


$$R_N = 4 \,||\, 24$$

$$R_N = \frac{4 \times 24}{4+24} \qquad \boxed{R_N = 3.43\,\Omega}$$

Nortons Equivalent Circuit

$I_N = +6.25\,A$

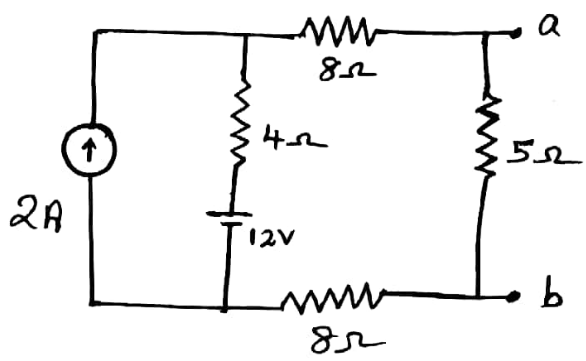


$$I_L = \frac{I_N R_N}{R_N + R_L}$$

$$I_L = \frac{+6.25\,(3.43)}{3.43+16} = \frac{+21.4375}{19.43} = \underline{\underline{1.1\,A}}$$

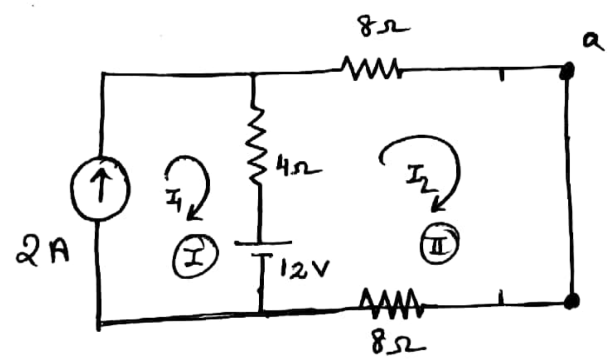
Univ QUESTION

Compute Norton's equivalent circuit for the circuit shown below at terminals a-b.



SOLUTION

* To find Norton's Current.
* shortcircuit the terminals a & b and find loop current.



$$I_1 = 2\,A$$

Apply KVL to loop 2

$$12 = 4(I_2 - I_1) + 8I_2 + 8I_2$$

$$12 = 4I_2 - 4I_1 + 16I_2$$

$$12 = 20I_2 - 4I_1$$

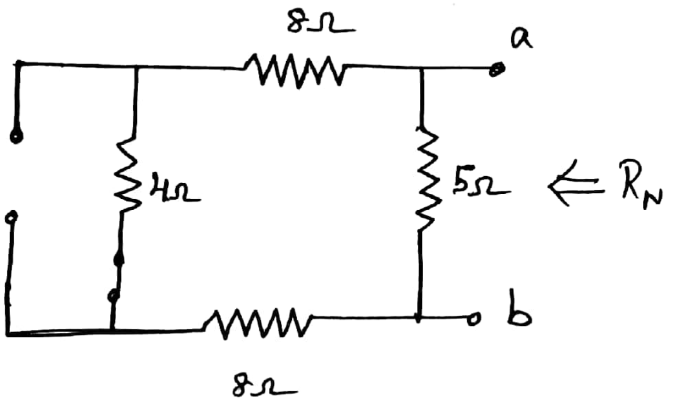
$$12 = 20I_2 - 4(2) \qquad \because I_1 = 2A$$

$$12 + 8 = 20 I_2$$

$$\boxed{I_2 = 1\ A}$$

$$\boxed{I_2 = I_N = 1\ A}$$

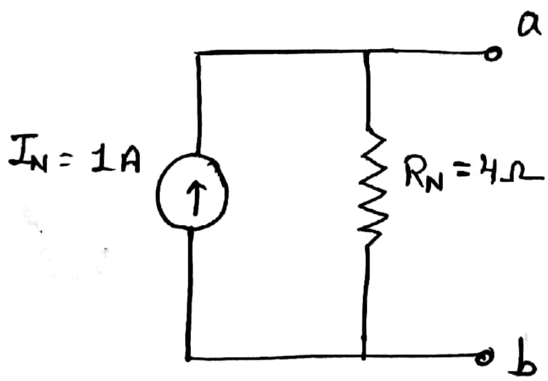
To find Norton's Current.



$$R_N = 5 \parallel 20$$

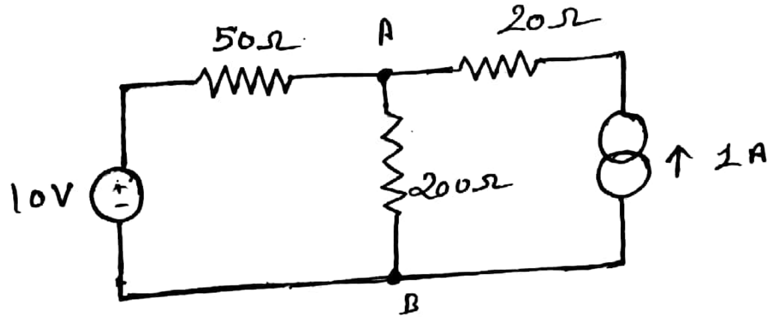
$$R_N = \frac{5 \times 20}{5 + 20} = 4\ \Omega$$

Norton's equivalent circuit



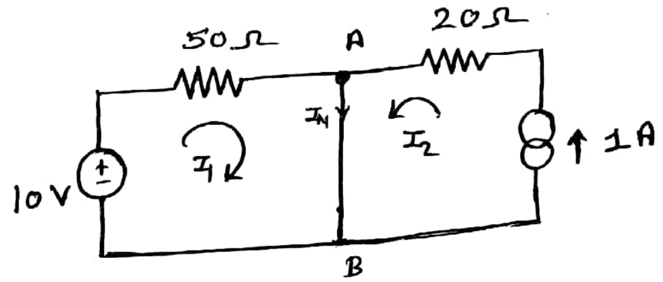
Ex : 2

Determine the Voltage across 200Ω resistor in the circuit by Norton's theorem.



SOLUTION

To find Norton's current



$$I_N = I_1 + I_2$$

KVL to loop 1

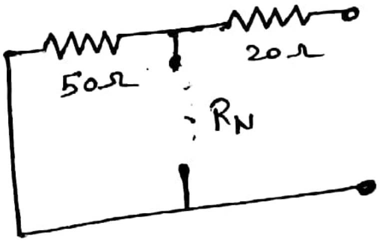
$$10 = 50(I_1)$$

$$I_1 = \frac{10}{50} = 0.2\ A$$

$$I_N = 1 + 0.2$$

$$\boxed{I_N = 1.2\ A}$$

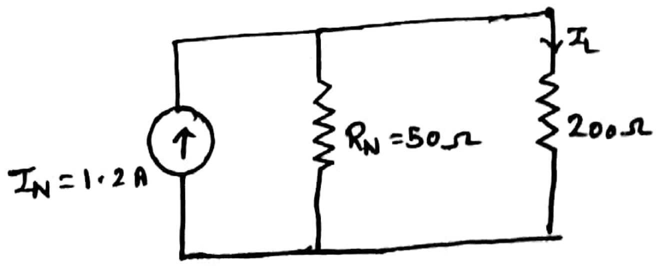
To find Norton's Resistance



$R_N = 50\ \Omega$

∵ 20Ω side is open circuited.

Eq circuit



$$I_L = \frac{I_N\ R_N}{R_N + R_L}$$

$$I_L = \frac{1.2(50)}{200+50} = 0.24A$$

$$V_L = I_L R_L = 0.24(200) = 48V$$