SRM Institute of Science and Technology
Ramapuram campus
Department of Mathematics
18MAB101T-Calculus and linear algebra

Year/Sem: I/I **Part-A** **Branch: Common to All**

Unit -II

Functions of several variables

1.	If u and v are functionally dependent then their Jacobian value is	1 Mark	
	a)zero b) one c) non-zero d)greater than zero	Ans (a)	(CLO-2 / Remember)
2.	If $rt - s^2 < 0$ then the point is	1 Mark	
	a)maximum point b) minimum point c) saddle point d) fixed point	Ans (c)	(CLO-2 / Remember)
3.	If $z = x^2 + y^2 + 3xy$ then $\frac{\partial z}{\partial x} =$	1 Mark	
	a)2y+3x b) 3y c) 2x+3y d) 2x	Ans (c)	(CLO-2 / Apply)
4.	If $u = \sin^{-1}(\frac{x^2 + y^2}{x - y})$ is a homogeneous function of degree	1 Mark	
	a) 2 b) 3 c)1 d) 4	Ans (c)	(CLO-2 / Apply)
5.	If f(x,y) is an implicit function then $\frac{dy}{dx} =$	1 Mark	

	a) $-\dfrac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)}$ b) $\dfrac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)}$ c) $\dfrac{\left(\frac{\partial f}{\partial y}\right)}{\left(\frac{\partial f}{\partial x}\right)}$ d) $-\dfrac{\left(\frac{\partial f}{\partial y}\right)}{\left(\frac{\partial f}{\partial x}\right)}$	Ans (a)	(CLO-2 / Remember)
6.	If $x = r\cos\theta$ *and* $y = r\sin\theta$ then $\dfrac{\partial(x,y)}{\partial(r,\theta)} =$	1 Mark	
	a) r b) r^2 c) 2r d) 1/r	Ans (a)	(CLO-2 / Apply)
7.	If u is a homogeneous function of degree n then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} =$	1 Mark	
	a) n b) nu c) u d) n^2u	Ans (b)	(CLO-2 / Remember)
8.	If $x = u^2 - v^2$ *and* $y = 2uv$ then $\dfrac{\partial(x,y)}{\partial(u,v)} =$	1 Mark	
	a) $u^2 + v^2$ b) $2(u^2 + v^2)$ c) $4(u^2 + v^2)$ d) $4v^2$	Ans (c)	(CLO-2 / Apply)
9.	If $J_1 = J\left(\dfrac{x,y}{u,v}\right)$ and $J_2 = J\left(\dfrac{u,v}{x,y}\right)$ then $J_1J_2 =$	1 Mark	
	a) 0 b) -1 c) 2 d) 1	Ans (d)	(CLO-2 / Remember)
10.	The point (0.0) for $f(x,y) = x^3 + y^3 - 3axy$ is	1 Mark	
	a)maximum point b) minimum point	Ans (c)	(CLO-2 / Apply)

	c) saddle point d) fixed point		
11.	If $f(x,y)=x^2y+\sin y+e^x$ then $f_x(1,\pi)$ *is* is	1 Mark	
	a) $2\pi-e$ b) 2π c) $2\pi+e$ d) 0	Ans (c)	(CLO-2 / Apply)
12.	The stationary points of $x^2+y^2+6x+12$ are	1 Mark	
	a) (-3,0) b) (0,3) c) (0, -3) d) (3,0)	Ans (a)	(CLO-2 / Apply)
13.	The stationary points for $f(x,y)=\sin x+\sin y+\sin(x+y)$ are	1 Mark	
	a) $\left(\frac{\pi}{2},\frac{\pi}{3}\right)$ b) $\left(\frac{\pi}{3},\frac{\pi}{3}\right)$ c) $\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ d) $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$	Ans (b)	(CLO-2 / Apply)
14.	If $u=x^2-y^2$ *and* $v=2xy$ then $J\left(\frac{x,y}{u,v}\right) X\ J\left(\frac{u,v}{x,y}\right)=$	1 Mark	
	a) 0 b) -1 c) 2 d) 1	Ans (d)	(CLO-2 / Apply)
15.	If $f(x,y)=e^x\cos y$ then $f_{xy}(0,0)$ *is*	1 Mark	
	a) 0 b) -1 c) 2 d) 1	Ans (a)	(CLO-2 / Apply)
16.	If $u=ax^2+by^2+2hxy$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$	1 Mark	

	a) u b) 2u c) 3u d) 4u	Ans (b)	(CLO-2 / Apply)
17.	If $x^y = y^x$, then $\frac{dy}{dx} =$	1 Mark	
	a) $(x\log y - y)y / x(y\log x - x)$ b) $(x\log x - x) / x(y\log y - y)$ c) $(x\log x - y)y / (y\log x - x)$ d) Does not exists	Ans (a)	(CLO-2 / Apply)
18.	If $f(x,y) = e^{xy}$ then $f_{yyy}(1,1)$ *is*	1 Mark	
	a) $-e$ b) $\frac{1}{e}$ c) e d) -$\frac{1}{e}$	Ans (c)	(CLO-2 / Apply)
19.	If $z = \log(x^2 + y^2 + xy)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$	1 Mark	
	a) 1 b) 2 c) 0 d) 4	Ans (b)	(CLO-2 / Apply)
20.	If $f(x,y) = \tan^{-1}(y/x)$ then $f_x(1,1)$ *is*	1 Mark	
	a) 1/2 b) -1/2 c) 2 d) 1	Ans (b)	(CLO-2 / Apply)
21.	If V= x/y, then $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} =$	1 Mark	
	(a) 2V (b) 3V (c) 4V (d) 0V	Ans (d)	(CLO-2 / Apply)

22.	Saddle points are ------------	1 Mark	
	(a) a minimum (b) a maximum (c) neither a minimum nor a maximum (d) None	Ans (c)	(CLO-2 / Remember)
23.	If u = x+y/1-xy, v= $\tan^{-1}x+\tan^{-1}y$ then the functional relationship between u and v is	1 Mark	
	a) u = tan v (b) v = tan u (c) x = tany (d) y = tan x	Ans (a)	(CLO-2 / Apply)
24.	Lagrange's method of undetermined multipliers is to find the maximum or minimum value of a function of	1 Mark	
	a) Two variables (b) Three or more variables (c) One variable (d) None	Ans (b)	(CLO-2 / Remember)
25.	The condition for a function f(x,y) to have a maximum value is that	1 Mark	
	(a) $rt-s^2$ (b) $rt-s^2>0$, r >0 or s> 0 (c) $rt-s^2>0$, r<0 or s <0 (d) $rt-s^2 = 0$,r >0	Ans (C)	(CLO-2 / Remember)

26.	The condition for a function f(x,y) to have a minimum value is that	1 Mark	
	(a) $rt-s^2$ (b) $rt-s^2>0$, $r>0$ or $s>0$ (c) $rt-s^2>0$, $r<0$ or $s<0$ (d) $rt-s^2=0, r>0$	Ans (b)	(CLO-2 / Remember)
27.	The condition for a function f(x,y) to have neither a maximum nor a minimum value is that	1 Mark	
	(a) $rt-s^2<0$ (b) $rt-s^2>0$, $r>0$ or $s>0$ (c) $rt-s^2>0$, $r<0$ or $s<0$ (d) $rt-s^2=0, r>0$	Ans (a)	(CLO-2 / Remember)
28.	The point (a,b) is called a stationary point if	1 Mark	
	(a) $f_x(a,b)=0$, $f_y(a,b)=0$ (b) $f_{xx}(a,b)=0$ (c) $f_{yy}(a,b)=0$ (d) $f_{xx}(a,b)=0$, $f_{yy}(a,b)=0$	Ans (a)	(CLO-2 / Remember)
29.	The minimum value of the function $x^2+y^2+6x+12$ is	1 Mark	
	(a) $\frac{1}{2}$ (b)2 (c)1 (d)3	Ans (d)	(CLO-2 / Apply)
30.	The maximum value of the function $x^3+y^3-12x-3y+20$ is	1 Mark	
	(a) 75 (b)27 (c)35 (d)38	Ans (d)	(CLO-2 / Apply)
31.	The points at which there is no extreme value are called __________.	1 Mark	

	(a) maximum points (b) minimum points (c) saddle points (d) none	Ans (C)	(CLO-2 / Remember)
32.	If $u = xe^{y}\sin x$ $v = xe^{y}\cos x$ $w = x^2e^{2y}$ then the functional relationship is	1 Mark	
	(a) $u^2+w^2 = v$ (b) $v^2 + w^2 = u$ (c) $x^2+y^2 = u$ (d) $u^2+v^2 = w$	Ans (d)	(CLO-2 / Apply)
33.	In PDE, a real function depends	1 Mark	
	(a) One independent variable (b) More than one independent variable (c) No independent variable (d) None	Ans (b)	(CLO-2 / Remember)
34.	If $z= x^2+y^2+2xy$ then $\partial z/\partial x$ is	1 Mark	
	(a) $2x^2+2y$ (b) $2x+2y$ (c) $2x-2y$ (d) $2y$	Ans (b)	(CLO-2 / Apply)
35.	If $x= r\cos\theta$ $y = r\sin\theta$ then $\frac{\partial(r,\theta)}{\partial(x,y)}$	1 Mark	
	(a) 0 (b) 1 (c) r (d) 1/r	Ans (d)	(CLO-2 / Apply)
36.	If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} =$ ________.	1 Mark	

	(a) 1 (b) 3 u (c) – 1 (d) 0	Ans (d)	(CLO-2 / Apply)

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Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – II

FUNCTIONS OF SEVERAL VARIABLES

Part – B

1. **If $u = (x - y)(y - z)(z - x)$, then find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.**

   (A) 0 **(B) 1** **(C) 2** **(D) 3**

   Solution:

   Given $u = (x - y)(y - z)(z - x)$

$$\frac{\partial u}{\partial x} = (y - z)[(x - y)(-1) + (z - x)(1)]$$
$$= -(x - y)(y - z) + (y - z)(z - x)$$
$$\frac{\partial u}{\partial y} = (z - x)[(x - y)(1) + (y - z)(-1)]$$
$$= (x - y)(z - x) - (y - z)(z - x)$$
$$\frac{\partial u}{\partial z} = (x - y)[(y - z)(1) + (z - x)(-1)]$$
$$= (x - y)(y - z) - (x - y)(z - x)$$
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0 \quad (\textbf{Option A})$$

2. **If $x = r\cos\theta, y = r\sin\theta$ find $\frac{\partial x}{\partial r}, \frac{\partial y}{\partial \theta}$.**

   (A) cos θ, sin θ (B) cos θ, *r* cos θ (C) *r* cos θ, sin θ (D) *r*, θ

   Solution:

$$\frac{\partial x}{\partial r} = \cos\theta$$
$$\frac{\partial y}{\partial \theta} = r\cos\theta \qquad (\textbf{Option B})$$

3. **If $f(x,y) = \sin\left(\frac{x}{y}\right)$, then find $x\frac{\partial f}{\partial x} + y\,\frac{\partial f}{\partial y}$.**

(A) 0 **(B) 1** **(C) 2** **(D) 3**

Solution:

$$\frac{\partial f}{\partial x} = \cos\left(\frac{x}{y}\right)\frac{1}{y}, \quad \frac{\partial f}{\partial y} = \cos\left(\frac{x}{y}\right)\left(-\frac{x}{y^2}\right)$$

$$x\frac{\partial f}{\partial x} + y\,\frac{\partial f}{\partial y} = 0 \text{ (Option A)}$$

4. **$Find\ \frac{dy}{dx}\ when\ x^3 + y^3 = 3axy.$**

$(A) -\frac{x^2-ay}{y^2-ax}$ $(B)\frac{x^2-ay}{y^2-ax}$ $(C)\frac{y^2-ax}{x^2-ay}$ $(D) -\frac{y^2-ax}{x^2-ay}$

Solution:

Let $f(x,y) = x^3 + y^3 - 3axy$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{3x^2 - 3ay}{3y^2 - 3ax}$$

$$= -\frac{x^2 - ay}{y^2 - ax} \quad \text{(Option A)}$$

5. **If $x = uv,\ y = \frac{u}{v}$, find $\frac{\partial(x,y)}{\partial(u,v)}$.**

(A) $\frac{-2u}{v}$ **(B)** $\frac{2u}{v}$ **(C)** $\frac{-2v}{u}$ **(D)** $\frac{2v}{u}$

Solution:

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} v & u \\ \frac{1}{v} & \frac{-u}{v^2} \end{vmatrix} = -\frac{2u}{v} \quad \text{(Option A)}$$

6. If $f(x,y) = e^x \sin y$**, then find** $f_{yy}(0,0)$**.**

(A) 0 **(B) 1** **(C) 2** **(D) 3**

Solution:

$f_y = e^x \cos y$

$f_{yy}(x,y) = e^x(-\sin y)$

$f_{yy}(0,0) = 0$ **(Option A)**

7. If $x^y = y^x$, then find $\frac{dy}{dx}$.

(A) $\frac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} - x^y \log x}$ **(B)** $\frac{y\,x^{y-1} + y^x \log y}{x\,y^{x-1} - x^y \log x}$

(C) $\frac{y\,x^{y-1} + y^x \log y}{x\,y^{x-1} + x^y \log x}$ **(D)** $\frac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} + x^y \log x}$

Solution:

$f(x, y) = x^y - y^x = 0$

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} - x^y \log x}$$ **(Option A)**

8. If $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$**, then find** $f_x(x, y)$ **at the point (1, 1).**

(A) -1/2 **(B) 1** **(C) 1/2** **(D) 3**

Solution:

$f_x(x, y) = \frac{-y}{x^2 + y^2}$ $f_x(1,1) = -\frac{1}{2}$ **(Option A)**

9. If $x = r\cos\theta$, $y = r\sin\theta$ **, then find** $\frac{\partial(x, y)}{\partial(r,\theta)}$**.**

(A) $\boldsymbol{r}$ **(B) 1/** $\boldsymbol{r}$ **(C) 1/2** **(D) 1**

Solution:

$$\text{Now}\quad \frac{\partial(x, y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial v}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\left(\sin^2\theta + \cos^2\theta\right) = r(1) = r$$

(Option A)

10. If $u = 2xy$, $v = x^2 - y^2$, then find $\dfrac{\partial(u,v)}{\partial(x,y)}$.

(A) $-4y^2 - 4x^2$ **(B) $-4y^2 + 4x^2$**

(C) $4y^2 - 4x^2$ **(D) $4y^2 + 4x^2$**

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 2y & 2x \\ 2x & -2y \end{vmatrix} = -4y^2 - 4x^2 \quad \textbf{(Option A)}$$

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Unit – II

FUNCTIONS OF SEVERAL VARIABLES

Part – C

1. Find the Taylor's series expansion of $f(x, y) = xe^y$ upto first degree terms near the point (0, 0).

Solution:

$$f(x, y) = xe^y \qquad f(0,0) = 0$$
$$f_x = e^y \qquad f_x(0,0) = 1$$
$$f_y = xe^y \qquad f_y(0,0) = 0$$

$$f(x, y) = 0 + 1.(x-0) + 0.(y-0) = x$$

2. *If* $x = r\cos\theta$ *and* $y = r\sin\theta$*, then* find $\frac{\partial r}{\partial x}$.

Solution: Given $x = r\cos\theta$, $y = r\sin\theta$, then $r^2 = x^2 + y^2 \Rightarrow r = \sqrt{x^2 + y^2}$

Now $\frac{\partial r}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}}(2x) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{r}$

3. Find $\frac{du}{dt}$, if $u = x y^2 + x^2 y$, where $x = a t^2, y = 2 a t$.

Solution:

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$\frac{du}{dt} = (y^2 + 2 x y)(2 a t) + (2 x y + x^2)(2 a) = 16 a^3 t^3 + 10 a^3 t^4.$$

4. Find $\frac{du}{dt}$, if $u = x^3 y^4$, where $x = t^3, y = t^2$.

Solution:

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$\frac{du}{dt} = (3x^2 y^4)(3t^2) + (4 x^3 y^3)(2t) = 17 t^{16}.$$

5. Expand $e^x \sin y$ as Maclaurin's series upto first degree terms.

Solution:

$f(x,y) = e^x \sin y$ $\quad\quad f(0,0) = 0$

$f_x(x,y) = e^x \sin y$ $\quad\quad f_x(0,0) = 0$

$f_y(x,y) = e^x \cos y$ $\quad\quad f_y(0,0) = 1$

Maclaurin's series

$$f(x,y) = 0 + x.0 + y.1 = y$$

6. If $f(x,y) = x^y$, then find $f_{yy}(1,1)$.

Solution:

$$f_y = x^y (\log x)$$

$$f_{yy}(x,y) = x^y (\log x)^2$$

$f_{yy}(1,1) = 0$

7. A rectangular box open at the top is to have a volume of 32 cubic feet. How do you define the auxiliary function using Lagrange's method of multipliers to find the dimensions of the box that requires the least material for its construction?

Solution:

Volume is 32 i.e., xyz = 32

Condition g(x,y,z)=xyz – 32

f(x,y,z) = Total surface area = xy + 2yz + 2zx

$F(x, y, z) = (xy + 2yz + 2zx) + \lambda (xyz - 32)$

8. How do you define the auxiliary function using Lagrange's method of multipliers to find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$**?**

Solution:

Volume of the rectangular is 2x .2y .2z = 8xyz =f(x,y,z)

Condition $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1 = g(x,y,z)$$

$$F(x,y,z) = 8xyz + \lambda\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1\right)$$

9. Find the stationary points of the function $f(x,y) = 3(x^2 - y^2) - x^3 + y^3$**.**

Solution:

$$f(x,y) = 3(x^2 - y^2) - x^3 + y^3$$

$$p = \frac{\partial f}{\partial x} = 6x - 3x^2;\ \mathrm{q} = \frac{\partial f}{\partial y} = -6y + 3y^2;$$

$$r = \frac{\partial^2 f}{\partial x^2} = 6 - 6x;\ \ \mathrm{s} = \frac{\partial^2 f}{\partial x \partial y} = 0 \text{ and } t = \frac{\partial^2 f}{\partial y^2} = -6 + 6y.$$

p = 0 implies x = 0 and x = 2.

and q = 0 implies y = 0 and y = 2

Therefore the stationary points are (0, 0), (0, 2), (2, 0) and (2, 2).

10. Expand e^{xy} in powers of *x* and *y* up to first degree term at the point (0, 0) using Taylor's series expansion.

Solution:

$f(x,y) = e^{xy}$ $\quad$ $f(0,0) = 1$

$f_x(x,y) = y e^{xy}$ $\quad$ $f_x(0,0) = 0$

$f_y(x,y) = x e^{xy}$ $\quad$ $f_y(0,0) = 0$

Taylor's series

$f(x,y) = 1$

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