SRM Institute of Science and Technology

College of Engineering and Technology

Department of Mathematics

Cycle Test-II Answer Key [2020-2021ODD]

Unit: II [Functions of Several Variables]

Subject Code and Title: Calculus and Linear Algebra [18MAB101T]
Slot: C
Date of Exam: 09.12.2020 Time: 9.00am – 10.30 am

1. If $u = f(y-z, z-x, x-y)$ then $u_x + u_y + u_z$ is

(a) 0 (b) 1 (c) 2 (d) 3

Answer (a)

2. If $= (x-y)(y-z)(z-x)$, then $\frac{\partial u}{\partial x}$ is

(a) $y^2 - z^2 - 2yx + 2zx$ (b) $y^2 - z^2 - 2xz + 2yz$

(c) $x^2 - y^2 - 2xz + 2yz$ (d) $x^2 - z^2 - 2yx + 2zx$

Answer (a)

3. If $f(x,y) = x^2 + y^2$, where $x = r\,cos\theta$ and $y = r\,sin\theta$ then $\frac{\partial f}{\partial r}$ is

(a) $cos\theta + sin\theta$ (b) $2\,cos\theta + 2sin\theta$

(c) $2y\,cos\theta + 2xsin\theta$ **(d) $2x\,cos\theta + 2ysin\theta$**

Answer (d)

4. If $f(x,y) = e^x cosy$, then $f_{yyy}(0,0)$ is

(a) 0 (b) 1 (c) -1 (d) 2

Answer (a)

5. If $f(x,y) = tan^{-1}\left(\frac{y}{x}\right)$, then $f_y(1,1)$ is

(a) 0 **(b) $\frac{1}{2}$** (c) 1 (d) -1

Answer (b)

6. The Taylor series expansion of $f(x,y)$ in powers of $(x-a)$ and $(y-b)$ is

(a) $f(x,y) = f(0,0) + \left[(x)f_x(0,0) + (y)f_y(0,0)\right] + \ldots\ldots\ldots\ldots$

(b) $f(x,y) = f(1,1) + \left[(x-1)f_x(1,1) + (y-1)f_y(1,1)\right] + \ldots\ldots\ldots\ldots$

(c) $f(x,y) = f(a,b) + \left[(x-a)f_x(a,b) + (y-b)f_y(a,b)\right] + \ldots\ldots\ldots\ldots$

(d) $f(x,y) = f(a,b) + \left[(x-a)f_{xx}(a,b) + (y-b)f_{yy}(a,b)\right] + \ldots\ldots\ldots\ldots$

Answer (c)

7. If $rt - s^2 > 0$ and $r > 0$ at (a, b) then the point is

(a) Maximum point **(b) Minimum point**

(c) Saddle point (d) No Conclusion

Answer (b)

8. If $rt - s^2 > 0$ and $r < 0$ at (a, b) then the point is

(a) Maximum point (b) Minimum point

(c) Saddle point (d) No Conclusion.

Answer (a)

9. The stationary point of $x^2 + 2y^2 - x$ is

(a)$\left(-\frac{1}{2}, 0\right)$ (b)$(1,1)$ (c)$(1,0)$ **(d)$\left(\frac{1}{2}, 0\right)$**

Answer (d)

10. The stationary points of $x^3 + y^3 - 3axy$ are

(a)$(0,0)$ and $(-1,-1)$ (b)$(0,0)$ and $(1,1)$

(c)$(0,0)$ and $(a,\ a)$ (d)$(1,1)$ and $(a,\ a)$

Answer (c)

11. The stationary point of $xy + \frac{a^3}{x} + \frac{a^3}{y}$ is

(a) $(a, 0)$ (b) $(1,1)$ **(c) (a, a)** (d) $\left(\frac{1}{2}, 0\right)$

Answer (c)

12. Let $f(x, y, z)$ be the function whose extreme values are to be found subject to the restriction $\phi(x, y, z) = 0$, the auxiliary function is

(a) $F(x, y, z, \lambda) = f(x, y, z) + \lambda\phi(x, y, z)$

(b) $F(x, y, z, \lambda) = \phi(x, y, z) + \lambda f(x, y, z)$

(c) $F(x, y, z, \lambda) = f(x, y, z) + \phi(x, y, z)$

(d) $F(x, y, z, \lambda) = \lambda f(x, y, z) + \lambda\phi(x, y, z)$

Answer (a)

13. If J_1 is the Jacobian of u, v with respect to x, y and J_2 is the Jacobian of x, y with respect to u, v then

(a) $J_1 J_2 = 0$ **(b) $J_1 J_2 = 1$** (c) $J_1 J_2 = -1$ (d) $J_1 J_2 = xy$

Answer (b)

14. If u, v are functions of r, s where r, s are functions of x, y then $\frac{\partial(u,v)}{\partial(x,y)}$ is

(a) $\frac{\partial(u,v)}{\partial(r,s)} \cdot \frac{\partial(r,s)}{\partial(x,y)}$ (b) $\frac{\partial(x,y)}{\partial(r,s)} \cdot \frac{\partial(r,s)}{\partial(u,v)}$ (c) 1 (d) 0

Answer (a)

15. If the Jacobian value is zero then u and v are

(a) Functionally independent **(b) Functionally dependent**

(c) Functionally minimum (d) Functionally maximum

Answer (b)

16. If $u = \frac{x+y}{1-xy}$, then $\frac{\partial u}{\partial x}$ is

(a) $\frac{1-y^2}{(1-xy)^2}$ (b) $\frac{x+y^2}{(1-xy)^2}$ (c) $\frac{x-y^2}{(1-xy)^2}$ **(d) $\frac{1+y^2}{(1-xy)^2}$**

Answer: d

17. If $\boldsymbol{u = tan^{-1}\left(\frac{y}{x}\right)}$, find $\boldsymbol{\frac{\partial u}{\partial x}}$ is

(a) $\boldsymbol{\frac{-y}{x^2+y^2}}$ (b) $\frac{-x}{x^2+y^2}$ (c) $\frac{y}{x^2+y^2}$ (d) $\frac{x}{x^2+y^2}$

Answer (a)

18. If $\boldsymbol{x = u^2 - v^2}$ and $\boldsymbol{y = 2uv}$, then the Jacobian of $\boldsymbol{x}$ and $\boldsymbol{y}$ with respect to $\boldsymbol{u}$ and $\boldsymbol{v}$ is

(a) $(x^2 + y^2)$ (b) $(u^2 + v^2)$ (c) $4(x^2 + y^2)$ **(d) $\boldsymbol{4(u^2 + v^2)}$**

Answer (d)

19. If $\boldsymbol{x = r\cos\theta, y = r\sin\theta}$, then $\boldsymbol{\frac{\partial(x,y)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)}}$ is

(a) 0 **(b) 1** (c) 2 (d) 3

Answer (b)

20. Given a rectangular box without a top of maximum capacity whose surface area is 108 sq. cm. If x, y, z are the dimensions of the box then the surface area is

(a) $2xy + 2yz + 2zx = 108$ **(b) $\boldsymbol{xy + 2yz + 2zx = 108}$**

(c) $xy + xz + yz = 108$ (d) $xy + xz + 2yz = 108$

Answer (b)

21. If $\boldsymbol{u = xy + yz + zx}$, where $\boldsymbol{x = e^t, y = e^{-t}, z = \frac{1}{t}}$, find $\boldsymbol{\frac{du}{dt}}$ is

(a) $\boldsymbol{\frac{2}{t}\sin ht - \frac{2}{t^2}\ cosht}$ (b) $\frac{2}{t}\cos ht - \frac{2}{t^2}\ \sin ht$

(c) $\frac{2}{t}\sin ht + \frac{2}{t^2}\ cosht$ (d) $\frac{2}{t}\cos ht + \frac{2}{t^2}\ \sin ht$

Answer (a)

22.If $\boldsymbol{z = x^3 + y^3 - 3axy}$, then $\boldsymbol{\frac{\partial^2 z}{\partial x \partial y}}$ is

(a) 0 **(b) $\boldsymbol{-3a}$** (c) $-2a$ (d) $-a$

Answer (b)

23. If $f(x,y) = \sin(xy)$, then $f_y\left(1,\frac{\pi}{2}\right) =$ ________.

(a) 0 (b) 1 (c) -1 (d) 2

Answer (a)

24. If $f(x,y) = e^{xy}$, then $f_{yyy}(1,1) =$ ________.

(a) e (b) 2e (c) 3e (d) 0

Answer (a)

25. If $u = \log(x+y)$, find $\frac{\partial^3 u}{\partial x^3} =$ _____.

(a) $\frac{-1}{(x+y)^2}$ (b) $\frac{-y}{(x+y)^2}$ **(c) $\frac{2}{(x+y)^3}$** (d) $\frac{2}{(x+y)^2}$

Answer (c)

26. If $f(x,y) = 0$, then $\frac{dy}{dx} =$ _______

(a) $\frac{\partial f/\partial x}{\partial f/\partial y}$ (b) $\frac{\partial f/\partial y}{\partial f/\partial x}$ **(c) $-\frac{\partial f/\partial x}{\partial f/\partial y}$** (d) $-\frac{\partial f/\partial y}{\partial f/\partial x}$

Answer (c)

27. If $xe^{-y} - 2ye^{x} = 1$, then $\frac{dy}{dx} =$ ________

(a) $\frac{e^{-y}-ye^{x}}{xe^{-y}-e^{x}}$ (b) $\frac{e^{-y}-ye^{x}}{xe^{-y}+e^{x}}$ (c) $\frac{e^{-y}-2ye^{x}}{xe^{-y}-2e^{x}}$ **(d) $\frac{e^{-y}-2ye^{x}}{xe^{-y}+2e^{x}}$**

Answer (d)

28. If $(\cos x)^y = (\sin y)^x$, then $\frac{dy}{dx} =$ ________

(a) $\frac{y\tan x+\log(\sin y)}{\log(\cos x)-x\cot y}$ (b) $\frac{x\tan x-\log(\sin y)}{\log(\cos x)-x\cot y}$

(c) $\frac{x\tan x+\log(\sin y)}{\log(\cos x)-x\cot y}$ (d) $\frac{y\tan x-\log(\sin y)}{\log(\cos x)-x\cot y}$

Answer (a)

29. If $u = x+y, y = uv$, find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$.

(a) u (b) v (c) x (d) y

Answer (a)

30. If $\boldsymbol{f(x,y) = x^2y + 3y - 2}$, find $\boldsymbol{f(1,0) =}$ ________

(a) $f(1,0) = 1$ (b) $f(1,0) = -1$

(c) $f(1,0) = 2$ **(d) $\boldsymbol{f(1,0) = -2}$**

Answer (d)

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