SRM Institute of Science and Technology
Ramapuram Campus

Department of Mathematics

18MAB101T - Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – II

FUNCTIONS OF SEVERAL VARIABLES

Part – C

1. Find the Taylor's series expansion of $f(x,y)=xe^y$ upto first degree terms near the point (0, 0).

Solution:

$$f(x,y)=xe^y \qquad f(0,0)=0$$
$$f_x=e^y \qquad f_x(0,0)=1$$
$$f_y=xe^y \qquad f_y(0,0)=0$$

$$f(x,y)=0+1.(x-0)+0.(y-0)=x$$

2. *If* $x=r\cos\theta$ *and* $y=r\sin\theta$, *then* find $\frac{\partial r}{\partial x}$.

Solution: Given $x=r\cos\theta$, $y=r\sin\theta$, then $r^2=x^2+y^2 \Rightarrow r=\sqrt{x^2+y^2}$

Now $\frac{\partial r}{\partial x}= \frac{1}{2\sqrt{x^2+y^2}}(2x)=\frac{x}{\sqrt{x^2+y^2}}=\frac{x}{r}$

3. Find $\frac{du}{dt}$, if $u = x y^2 + x^2 y$, where $x = a t^2, y = 2 a t$.

Solution:

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$\frac{du}{dt} = (y^2 + 2 x y)(2 a t) + (2 x y + x^2)(2 a) = 16 a^3 t^3 + 10 a^3 t^4.$$

4. Find $\frac{du}{dt}$, if $u = x^3 y^4$, where $x = t^3, y = t^2$.

Solution:

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$\frac{du}{dt} = (3x^2 y^4)(3t^2) + (4 x^3 y^3)(2t) = 17 t^{16}.$$

5. Expand $e^x \sin y$ as Maclaurin's series upto first degree terms.

Solution:

$f(x, y) = e^x \sin y$ $\quad f(0,0) = 0$

$f_x(x, y) = e^x \sin y$ $\quad f_x(0,0) = 0$

$f_y(x, y) = e^x \cos y$ $\quad f_y(0,0) = 1$

Maclaurin's series

$$f(x, y) = 0 + x.0 + y.1 = y$$

6. If $f(x, y) = x^y$, then find $f_{yy}(1,1)$.

Solution:

$$f_y = x^y (\log x)$$

$$f_{yy}(x, y) = x^y (\log x)^2$$

$f_{yy}(1,1) = 0$

7. A rectangular box open at the top is to have a volume of 32 cubic feet. How do you define the auxiliary function using Lagrange's method of multipliers to find the dimensions of the box that requires the least material for its construction?

Solution:

Volume is 32 i.e., xyz = 32

Condition g(x,y,z)=xyz – 32

f(x,y,z) = Total surface area = xy + 2yz + 2zx

$F(x, y, z) = (xy + 2yz + 2zx) + \lambda\,(xyz - 32)$

8. How do you define the auxiliary function using Lagrange's method of multipliers to find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$**?**

Solution:

Volume of the rectangular is 2x .2y .2z = 8xyz =f(x,y,z)

Condition $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1 = g(x,y,z)$$

$$F(x,y,z) = 8xyz + \lambda\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1\right)$$

9. Find the stationary points of the function $f(x,y) = 3(x^2-y^2)-x^3+y^3$**.**

Solution:

$$f(x,y) = 3(x^2-y^2)-x^3+y^3$$

$$p = \frac{\partial f}{\partial x} = 6x-3x^2;\ \mathrm{q} = \frac{\partial f}{\partial y} = -6y+3y^2;$$

$$r = \frac{\partial^2 f}{\partial x^2} = 6-6\mathrm{x};\quad \mathrm{s} = \frac{\partial^2 f}{\partial x \partial y} = 0 \text{ and } t = \frac{\partial^2 f}{\partial y^2} = -6+6y.$$

p = 0 implies x = 0 and x = 2.

and q = 0 implies y = 0 and y = 2

Therefore the stationary points are (0, 0), (0, 2), (2, 0) and (2, 2).

10. Expand e^{xy} in powers of *x* and *y* up to first degree term at the point (0, 0) using Taylor's series expansion.

Solution:

$f(x,y) = e^{xy}$ $\quad\quad$ $f(0,0) = 1$

$f_x(x,y) = ye^{xy}$ $\quad\quad$ $f_x(0,0) = 0$

$f_y(x,y) = xe^{xy}$ $\quad\quad$ $f_y(0,0) = 0$

Taylor's series

$f(x,y) = 1$

* * * * *