SRM Institute of Science and Technology

Ramapuram campus

Department of Mathematics

18MAB101T – Calculus and Linear Algebra

Year/Sem: I/I **Part – A** **Branch: Common to All**

Unit – V

Sequence and Series

1.	A sequence $\{a_n\}$ is said to be convergent if		**1 Mark**
	(a) $\lim_{n\to\infty} a_n = finite$ (b) $\lim_{n\to\infty} a_n = \infty$ (c) $\lim_{n\to\infty} a_n = -\infty$ (d) $\lim_{n\to\infty} a_n = infinite$	Ans (a)	**(CLO 5, Remember)**
2.	The sequence $\{(-1)^n\}$ is		**1 Mark**
	(a) oscillatory (b) monotonic (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (a)	**(CLO 5, Remember)**
3.	A sequence which is monotonic and bounded is		**1 Mark**
	(a) conditionally convergent (b) absolutely convergent (c) convergent (d) divergent	Ans (c)	**(CLO 5, Remember)**
4.	The necessary condition for the convergence of $\sum u_n$ is		**1 Mark**
	(a) $\lim_{n\to\infty} u_n = 0$ (b) $\lim_{n\to\infty} u_n = \infty$ (c) $\lim_{n\to\infty} u_n = -\infty$ (d) $\lim_{n\to\infty} u_n \neq 0$	Ans (a)	**(CLO 5, Remember)**
5.	If $\{a_n\}$ and $\{b_n\}$ are two convergent sequences, then $\{a_n + b_n\}$ is		**1 Mark**
	(a) convergent (b) divergent (c) oscillatory (d) neither convergent nor divergent	Ans (a)	**(CLO 5, Remember)**
6.	The geometric series $1 + x + x^2 + x^3 + \cdots$ converges if		**1 Mark**
	(a) $-1 < x < 1$ (b) $x < 1$ (c) $x > 1$ (d) $x \geq 1$	Ans (a)	**(CLO 5, Remember)**
7.	The series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$ converges if		**1 Mark**
	(a) $p > 0$ (b) $p < 1$ (c) $p > 1$ (d) $p \leq 1$	Ans (c)	**(CLO 5, Remember)**

8.	The series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\cdots$ diverges if		**1 Mark**
	(a) $p>0$ (b) $p<1$ (c) $p>1$ (d) $p\le 1$	Ans (d)	**(CLO 5, Remember)**
9.	If $\sum u_n$ is a convergent series, then $\lim_{n\to\infty} u_n =$		**1 Mark**
	(a) 1 (b) ± 1 (c) 0 (d) ∞	Ans (c)	**(CLO 5, Remember)**
10.	According to D' Alembert's ratio test, if $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=l$, then $\sum u_n$ converges if		**1 Mark**
	(a) $l<1$ (b) $l>1$ (c) $l=1$ (d) $-1<l<1$	Ans (a)	**(CLO 5, Remember)**
11.	By Cauchy's root test, if $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty}(u_n)^{\frac{1}{n}}=l$, then $\sum u_n$ converges if		**1 Mark**
	(a) $l<1$ (b) $l>1$ (c) $l=1$ (d) $-1<l<1$	Ans (a)	**(CLO 5, Remember)**
12.	By Raabe's test, if $\lim\limits_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)=k$, then $\sum u_n$ diverges if		**1 Mark**
	(a) $k<1$ (b) $k>1$ (c) $k=1$ (d) $-1<k<1$	Ans (a)	**(CLO 5, Remember)**
13.	By Logarithmic test, if $\lim\limits_{n\to\infty} n\log\frac{u_n}{u_{n+1}}=l$, then $\sum u_n$ converges if		**1 Mark**
	(a) $l<1$ (b) $l>1$ (c) $l=1$ (d) $-1<l<1$	Ans (b)	**(CLO 5, Remember)**
14.	The series $\sum u_n$ containing positive and negative terms is __________, if $\sum\lvert u_n\rvert$ is divergent but $\sum u_n$ is convergent.		**1 Mark**
	(a) divergent (b) oscillating finitely (c) oscillating infinitely (d) conditionally convergent	Ans (d)	**(CLO 5, Remember)**
15.	The series $\sum u_n$ containing positive and negative terms is absolutely convergent, if $\sum\lvert u_n\rvert$ is		**1 Mark**
	(a) convergent (b) divergent to $-\infty$ (c) divergent to $+\infty$ (d) oscillatory	Ans (a)	**(CLO 5, Remember)**

16.	Every absolutely convergent series is necessarily	**1 Mark**	
	(a) divergent (b) convergent (c) oscillatory (d) conditionally convergent	Ans (b)	**(CLO 5, Remember)**
17.	The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if	**1 Mark**	
	(a) $p = 0$ (b) $p = 1$ (c) $p > 1$ (d) $p < 1$	Ans (c)	**(CLO 5, Remember)**
18.	As per D' Alembert's ratio test, if $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = l$, then the series is divergent if	**1 Mark**	
	(a) $l = 0$ (b) $l = 1$ (c) $l > 1$ (d) $l < 1$	Ans (c)	**(CLO 5, Remember)**
19.	A series of positive terms cannot ____________.	**1 Mark**	
	(a) oscillate (b) absolutely converge (c) converge (d) diverge	Ans (a)	**(CLO 5, Remember)**
20.	The series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ is	**1 Mark**	
	(a) divergent (b) conditionally convergent (c) oscillatory (d) neither convergent nor divergent	Ans (b)	**(CLO 5, Apply)**
21.	The series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$ is	**1 Mark**	
	(a) divergent (b) absolutely convergent (c) oscillatory (d) neither convergent nor divergent	Ans (b)	**(CLO 5, Apply)**
22.	By Raabe's test, if $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - 1\right) = k$, then $\sum u_n$ converges if	**1 Mark**	
	(a) $k < 1$ (b) $k > 1$ (c) $k = 1$ (d) $-1 < k < 1$	Ans (b)	**(CLO 5, Remember)**

23.	By Cauchy's root test, if $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty}(u_n)^{\frac{1}{n}} = l$, then $\sum u_n$ diverges if	**1 Mark**	
	(a) $l < 1$ (b) $l > 1$ (c) $l = 1$ (d) $-1 < l < 1$	Ans (b)	**(CLO 5, Remember)**
24.	By Logarithmic test, if $\lim_{n\to\infty} n \log \frac{u_n}{u_{n+1}} = l$, then $\sum u_n$ diverges if	**1 Mark**	
	(a) $l < 1$ (b) $l > 1$ (c) $l = 1$ (d) $-1 < l < 1$	Ans (a)	**(CLO 5, Remember)**
25.	If $-1 < x < 1$, then the geometric series $1 + x + x^2 + x^3 + \cdots$ converges to	**1 Mark**	
	(a) $\frac{1}{1-x}$ (b) $\frac{1}{1+x}$ (c) e^x (d) $\frac{1}{x!}$	Ans (a)	**(CLO 5, Remember)**
26.	The geometric series $1 + x + x^2 + x^3 + \cdots$ oscillates finitely if	**1 Mark**	
	(a) $x = -1$ (b) $x < 1$ (c) $x > 1$ (d) $x \geq 1$	Ans (a)	**(CLO 5, Remember)**
27.	The geometric series $1 + x + x^2 + x^3 + \cdots$ oscillates infinitely if	**1 Mark**	
	(a) $x < -1$ (b) $x < 1$ (c) $x > 1$ (d) $x \geq 1$	Ans (a)	**(CLO 5, Remember)**
28.	If $\sum u_n$ is convergent, then $\sum k u_n$ (where k is constant) is	**1 Mark**	
	(a) convergent (b) divergent (c) oscillatory (d) neither convergent nor divergent	Ans (a)	**(CLO 5, Remember)**
29.	The series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$ is	**1 Mark**	
	(a) divergent (b) neither convergent nor divergent (c) oscillatory (d) conditionally convergent	Ans (d)	**(CLO 5, Apply)**
30.	The convergence of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ is tested by	**1 Mark**	
	(a) Ratio test (b) Raabe's test (c) Leibnitz test (d) Cauchy Root test	Ans (c)	**(CLO 5, Remember)**

31.	A monotonic increasing sequence which is not bounded above is ______.	1 Mark	
	(a) oscillatory (b) convergent (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (d)	**(CLO 5, Remember)**
32.	A monotonic decreasing sequence which is not bounded below is ______.	1 Mark	
	(a) oscillatory (b) convergent (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (c)	**(CLO 5, Remember)**
33.	The series $\sum u_n$ of positive terms is convergent if	1 Mark	
	(a) $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} < 1$ (b) $\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = 1$ (c) $\lim_{n\to\infty} \frac{u_n}{u_{n+1}} \le 1$ (d) $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} > 1$	Ans (a)	**(CLO 5, Remember)**
34.	The *n*th term of a series in Arithmetic Progression is	1 Mark	
	(a) $t_n = a-(n-1)d$ (b) $t_n = a+(n+1)d$ (c) $t_n = a-(n+1)d$ (d) $t_n = a+(n-1)d$	Ans (d)	**(CLO 5, Remember)**
35.	$\sum_1^\infty \frac{n^3}{3^n}$ is	1 Mark	
	(a) oscillatory (b) convergent (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (b)	**(CLO 5, Apply)**
36.	The series $\sum \frac{1}{n}\sin\left(\frac{1}{n}\right)$ is	1 Mark	
	(a) oscillatory (b) convergent (c) divergent (d) conditionally convergent	Ans (b)	**(CLO 5, Apply)**
37.	If D'Alembert's ratio test fails, then use	1 Mark	
	(a) Comparison test (b) Leibnitz's test (c) Cauchy's integral test (d) Raabe's test	Ans (d)	**(CLO 5, Remember)**
38.	The series $\sum \frac{1}{n!}$ is	1 Mark	
	(a) oscillatory (b) convergent (c) divergent (d) conditionally convergent	Ans (b)	**(CLO 5, Apply)**

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Unit – V

SEQUENCE AND SERIES

Part – B

1. The sequence $\left\{\frac{1}{n}\right\}$ converges to _________.

(A) 0 **(B) 1** **(C) ½** **(D) ∞**

Solution:

$$a_n = \frac{1}{n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$$

$\{a_n\}$ converges to 0. **(Option A)**

2. The sequence $\left\{\frac{n+1}{2n+3}\right\}$ converges to _________.

(A) 0 **(B) 1** **(C) ½** **(D) ∞**

Solution:

$$a_n = \frac{n+1}{2n+3}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n\left(1+\frac{1}{n}\right)}{n\left(2+\frac{3}{n}\right)} = \frac{1}{2}$$

$\{a_n\}$ converges to ½ . **(Option C)**

3. Test the convergence of the series $\sum \frac{1}{\sqrt{n+1}}$ **.**

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{1}{\sqrt{n+1}} = \frac{1}{\sqrt{n\left(1+\frac{1}{n}\right)}}$$

Let $v_n = \frac{1}{\sqrt{n}}$

Now $\frac{u_n}{v_n} = \frac{1}{\sqrt{1+\frac{1}{n}}}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = 1$$

$\sum v_n = \sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$ is divergent.

Hence by comparison test, $\Sigma\, u_n$ is divergent. **(Option B)**

4. Test the convergence of the series $1+\frac{1}{3}+\frac{1}{5}+\cdots$ **.**

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{1}{2n-1} = \frac{1}{n\left(2-\frac{1}{n}\right)}$$

Let $v_n = \frac{1}{n}$

Now $\frac{u_n}{v_n} = \frac{1}{2-\frac{1}{n}}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \frac{1}{2}$$

$\sum v_n = \sum \frac{1}{n}$ is divergent.

Hence by comparison test, $\Sigma\, u_n$ is divergent. **(Option B)**

5. Test the convergence of the series $\sum \frac{x^n}{n\,!}$ where x > 0.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{x^n}{n\,!}, \quad u_{n+1} = \frac{x^{n+1}}{(n+1)\,!}$$

Now $\frac{u_{n+1}}{u_n} = \frac{x}{n+1} = \frac{x}{n\left(1+\frac{1}{n}\right)}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{x}{n\left(1+\frac{1}{n}\right)} = 0 < 1$$

Hence by Ratio test, Σ u_n is convergent. **(Option A)**

6. Test the convergence of the series $\sum \frac{n\,!}{n^n}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{n\,!}{n^n}, \quad u_{n+1} = \frac{(n+1)\,!}{(n+1)^{n+1}}$$

Now $\frac{u_{n+1}}{u_n} = \frac{1}{\left(1+\frac{1}{n}\right)^n}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{1}{e} < 1$$

Hence by Ratio test, Σ u_n is convergent. **(Option A)**

7. The series $1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots$ is ________________.

(A) absolutely convergent **(B) diverges to + ∞**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{x^{n-1}}{(n-1)\,!}, \quad u_{n+1} = \frac{x^n}{n\,!}$$

Now $\dfrac{u_{n+1}}{u_n} = \dfrac{x}{n}$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = 0 < 1$$

Hence the series is absolutely convergent. **(Option A)**

8. Test the convergence of the series $\sum \dfrac{n^3}{3^n}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{n^3}{3^n}, \quad (u_n)^{1/n} = \frac{(n^{1/n})^3}{3}$$

$$\lim_{n\to\infty}(u_n)^{1/n} = \frac{1}{3} < 1$$

Hence by Root test, $\Sigma\, u_n$ is convergent. **(Option A)**

9. Test the convergence of the series $\sum \dfrac{3^n\, n\,!}{n^n}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{3^n\, n\,!}{n^n}, \quad u_{n+1} = \frac{3^{n+1}\,(n+1)\,!}{(n+1)^{n+1}}$$

Now $\dfrac{u_{n+1}}{u_n} = \dfrac{3}{\left(1+\dfrac{1}{n}\right)^n}$

$$\lim_{n\to\infty}\frac{u_{n+1}}{u_n} = \frac{3}{e} > 1$$

Hence by Ratio test, $\Sigma\, u_n$ is divergent. **(Option B)**

10. Test the convergence of the series $\sum \dfrac{1}{n^2}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

By Harmonic Series test or p-test, $\sum \dfrac{1}{n^2}$ converges. **Option (A)**

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Unit – V

SEQUENCE AND SERIES

Part – C

Question 1

Show that the series $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\cdots$ **is convergent.**

Solution

$$u_n=\frac{1}{n(n+1)(n+2)}=\frac{1}{n\,n\left(1+\frac{1}{n}\right)n\left(1+\frac{2}{n}\right)},\text{ and let } v_n=\frac{1}{n^3}.$$

Now $\frac{u_n}{v_n}=\frac{1}{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}$

$$\lim_{n\to\infty}\frac{u_n}{v_n}=1,\text{ which is finite and non}-\text{zero.}$$

$\therefore$ Both $\Sigma\, u_n$ and $\Sigma\, v_n$ converge or diverge together.

But $\sum v_n=\sum\frac{1}{n^3}$ is convergent by Harmonic series test or p-series test.

Hence by comparison test, $\Sigma\, u_n$ is convergent.

Question 2:

Show that the series $1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^3}+\cdots$ **is divergent.**

Solution

Neglect the first term. Then the series is $\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^3}+\cdots$

$$u_n=\frac{n^n}{(n+1)^{n+1}}=\frac{n^n}{n^{n+1}\left(1+\frac{1}{n}\right)^n\left(1+\frac{1}{n}\right)^1} \text{ and let } v_n=\frac{1}{n}.$$

Now $\frac{u_n}{v_n}=\frac{1}{\left(1+\frac{1}{n}\right)^n\left(1+\frac{1}{n}\right)}$

$\lim_{n\to\infty}\frac{u_n}{v_n}=\frac{1}{e}$, which is finite and non-zero.

$\therefore$ Both $\Sigma\, u_n$ and $\Sigma\, v_n$ converge or diverge together.

But $\sum v_n=\sum\frac{1}{n}$ is divergent by Harmonic series test or p-series test.

Hence by comparison test, $\Sigma\, u_n$ is divergent.

Question 3:

Show that the series $\sum_{n=1}^{\infty}\sin\left(\frac{1}{n}\right)$ is divergent.

Solution

$u_n=\sin\left(\frac{1}{n}\right)$

Let $v_n=\frac{1}{n}$

Now $\frac{u_n}{v_n}=\frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}$

$\lim_{n\to\infty}\frac{u_n}{v_n}=\lim_{\frac{1}{n}\to 0}\frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}=1$, which is finite and non-zero. **Formula :** $\lim_{x\to 0}\frac{\sin x}{x}=1$

$\therefore$ Both $\Sigma\, u_n$ and $\Sigma\, v_n$ converge or diverge together.

$\sum v_n = \sum \frac{1}{n}$ is divergent by Harmonic series test or p-series test.

Hence by comparison test, $\Sigma\, u_n$ is divergent.

Question 4:

Show that the series $1+\frac{2^p}{2\,!}+\frac{3^p}{3\,!}+\frac{4^p}{4\,!}+\cdots$ **is convergent for all values of *p*.**

Solution

$$u_n = \frac{n^p}{n\,!}, \quad u_{n+1} = \frac{(n+1)^p}{(n+1)\,!}$$

$$\frac{u_n}{u_{n+1}} = \frac{n^p}{n!} \times \frac{(n+1)!}{(n+1)^p} = \frac{n+1}{\left(1+\frac{1}{n}\right)^p} = \frac{n\left(1+\frac{1}{n}\right)}{\left(1+\frac{1}{n}\right)^p}$$

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \infty > 1$$

Hence by Ratio test, $\Sigma\, u_n$ is convergent.

Question 5:

Establish the convergence of the series $\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+\cdots$

Solution

$$u_n = \left(\frac{n}{2n+1}\right)^n$$

$$\lim_{n\to\infty} (u_n)^{1/n} = \lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2} < 1$$

Hence by Cauchy's root test, $\Sigma\, u_n$ is convergent.

Question 6:

Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n \log n}$.

Solution

$f(n) = \frac{1}{n \log n}$ Clearly $f(n)$ is a monotonic decreasing sequence.

$$\int_2^{\infty} f(x) dx = \int_2^{\infty} \frac{1}{x \log x} dx = \int_2^{\infty} \frac{1/x}{\log x} dx = \left(\log(\log(x)\right)_2^{\infty} = \infty$$

By Cauchy's integral test, the given series is divergent.

Question 7:

Test the convergence of the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$.

Solution

$u_n = \frac{1}{\sqrt{n}}$ $\qquad$ $u_{n+1} = \frac{1}{\sqrt{n+1}}$

(i) Clearly $u_n > u_{n+1}$.

(ii) $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

$\therefore$ By Leibnitz's test, the given series is convergent.

Question 8:

Test the convergence of the series $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \cdots$.

Solution

$u_n = \frac{1}{(2n-1)(2n)}$ $\qquad$ $u_{n+1} = \frac{1}{(2n+1)(2n+2)}$

(i) Since $\frac{1}{(2n-1)(2n)} > \frac{1}{(2n+1)(2n+2)}$ always, clearly $u_n > u_{n+1}$.

(ii) $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{n\left(2-\frac{1}{n}\right)2n} = 0$

$\therefore$ By Leibnitz's test, the given series is convergent.

Question 9:

Prove that the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ **is conditionally convergent.**

Solution

$u_n = \frac{1}{n}$ $\qquad$ $u_{n+1} = \frac{1}{n+1}$

(i) Clearly $u_n > u_{n+1}$.

(ii) $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{n} = 0$

$\therefore$ By Leibnitz's test, the given series is *convergent*.

Also $\Sigma\, |\, u_n | = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots = \sum \frac{1}{n}$ is *divergent* by Harmonic series test (or) p-series test.

Hence the given series is conditionally convergent.

Question 10:

Test the convergence of the series $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots$

Solution

The series of absolute terms $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots = \sum \frac{1}{n^2}$ is clearly *convergent* by Harmonic series test (or) p-series test.

$\therefore$ The series is absolutely convergent.

Since every absolutely convergent series is convergent, the given series is convergent.

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