UNIT-4

1] Test statistic $(F) = \dfrac{S_1^2}{S_2^2}$ (or) $\dfrac{S_2^2}{S_1^2}$

$\downarrow$ if, $S_1^2 > S_2^2$ $\qquad$ $\downarrow$ if, $S_2^2 > S_1^2$

where, $S_1^2 = \dfrac{n_1 s_1^2}{n_1 - 1}$ (or) $\dfrac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1}$

$S_2^2 = \dfrac{n_2 s_2^2}{n_2 - 1}$ (or) $\dfrac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1}$

$\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2$
(sum of squares of deviation

2] **Degrees of Freedom (DOF):**

$(n_1 - 1, n_2 - 1)$ or $(n_2 - 1, n_1 - 1)$

NOTE:

The value of $F > 1$

APPLICATION OF F-TEST:

⇒ It is used to test difference b/w variances of 2 small samples.

⇒ It is used to test whether the 2 samples have been drawn from the same population.

note:

$\sigma \rightarrow$ population S.D

$s \rightarrow$ Sample S.D

$\sigma^2 \rightarrow$ population variance

$s^2 \rightarrow$ Sample variance.

$n \rightarrow$ Sample Size.

$\bar{x} \rightarrow$ Mean

$S \rightarrow$ variance (if given)

if, calculated value < Tabular Value

$\therefore$ H_0 is accepted.

if, calculated > Tabular value

$\therefore$ H_0 is rejected.

* $\bar{x}_1 = \frac{\sum x_i}{n_1}$ | * $\bar{x}_2 = \frac{\sum x_2}{n_2}$

TEST-1

3] CHI- SQUARE TEST (χ^2 test) :-

Test statistic $(\chi^2) = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$

where, O_i = Observed Frequency

E_i = Expected Frequency.

APPLICATIONS:

(i) To test the significant difference b/w observed Frequency and Expected Frequency.

(ii) To test the Goodness of Fit.

(iii) Testing independence of Attributes.

NOTE:

1] The no. of observations (N) should be reasonably large, (i.e) $N \geq 50$

2] $4 \leq n \leq 16$

3] Individual frequencies should not be small (i.e) $O_i \geq 10$. If it is not small, then it should be combined with the neighbouring frequencies.

* $H_0 : O_i = E_i$ (uniform)
* $H_1 : O_i \neq E_i$ (Not uniform)
* DOF = $(n-1)$

TEST – 2   TESTING OF GOODNESS OF FIT:

$(\bar{x})$

(MCQ) For Binomial distribution ⇒ DOF $= n-1$

For Poisson's distribution ⇒ DOF $= n-2$

For Normal distribution ⇒ DOF $= n-3$

NOTE:

1] Binomial distribution :- $\boxed{p(x) = {}^nC_x p^x q^{n-x}}$ $x = 0, 1, 2 \ldots$

Mean $= np$ & Mean $= \frac{\Sigma fx}{\Sigma f}$ — ① $q = 1 - p$

↓

② Find p & q

$(\Sigma f = N)$

Expected Frequency $(E_i) = N \times p(x)$

2] Poisson Distribution :- $\boxed{p(x) = \frac{e^{-\lambda}\lambda^x}{x!}}$

Expected Frequency $(E_i) = N \times p(x)$.

TEST–3 : TESTING INDEPENDENCE OF ATTRIBUTES:

Let 'A' and 'B' be 2 characteristic (Attributes).

2X2 CONTIGENCY TABLE:

B \ A	A_1	A_2	
B_1	a	b	a + b
B_2	c	d	c + d
	a + c	b + d	N = a + b + c + d

EXPECTED FREQUENCY :

* $E(a) = \dfrac{(a+c)(a+b)}{N}$
* $E(b) = \dfrac{(b+d)(a+b)}{N}$
* $E(c) = \dfrac{(a+c)(c+d)}{N}$
* $E(d) = \dfrac{(b+d)(c+d)}{N}$

* $\chi^2 = \Sigma \dfrac{(O_i - E_i)^2}{E_i}$

(or)

* For 2x2 contigency table, $\chi^2 = \dfrac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$

(NOTE: Applicable only for 2x2 table)

QUEUEING THEORY :

1] Objective: * To have a good economic balance b/w waiting time cost and service cost.

* To find Optimum solution for minimizing the waiting time & service cost.

Arrival → Queue → Service → Departure

2] QUEUE DISICIPLINE :

1] FIFO / FCFS → service is provided by queueing theory

2] LIFO / LCFS

3] Service at Random

4] Priority

Note: dependent on time → Transient state
If the behaviour of the system is independent of time, then the system is in steady state.

3] CUSTOMER BEHAVIOUR:

* BALKING:- Customer have no intention to join the queue (may leave the queue becoz of lengthy queue)
* Reneging:- Customer may leave the queue due to impatience.
* Jockeying:- Jumping from one queue to another queue

4] ARRIVAL RATE: Arrivals in queue follows poisson distribution
(No. of arrivals per unit time follows Poisson dist.)

⇒ For Poisson distribution, arrival rate = λ (per unit time) (No. of customers)

⇒ For Exponential distribution, arrival rate = $\frac{1}{\lambda}$ (No. of customers during an instante)

5] SERVICE RATE:

⇒ Poisson → service Rate = μ (per unit time)
⇒ Exponential → service Rate = $1/\mu$

6] KENDELL'S NOTATION FOR A QUEUEING SYSTEM:

$\boxed{(a\,|\,b\,|\,c) : (d\,|\,e)}$

a → No. of arrivals per unit time.
b → No. of service per unit time.
c → No. of servers
d → Capacity of the system
e → Queue discipline.

Note:
Interval b/w 2 consecutive arrivals of Poisson process follows Exponential distribution.

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arrival
(M|M|1)

7] TYPES OF QUEUEING MODELS: M → Markov.

(i) $(M|M|1):(\infty|FIFO)$ → single server infinite capacity
(ii) $(M|M|1):(k|FIFO)$ → single server finite capacity
(iii) $(M|M|s):(\infty|FIFO)$ → Multi server infinite capacity
(iv) $(M|M|s):(k|FIFO)$ → Multi Server Finite capacity.

8] NOTATIONS:

* L_s (or) N_s → Avg no. of customers in the system.
* L_q (or) N_q → Avg no. of customers (or) Avg Queue length
* W_s → Avg. waiting time of a customer in the system.
* W_q → Avg. waiting time of a customer in the queue.
* P_0 → The system is idle (i.e) NO customer.

9] LITTLE'S FORMULA:

⇒ Relation b/w L_s, L_q, W_s and W_q

(i) $\underset{(N_q)}{L_q} = \underset{N_s}{L_s} - \frac{\lambda}{\mu}$

(ii) $W_s = \frac{L_s}{\lambda}$ & $L_q = \lambda W_q$ (or N_q)

(iii) $W_q = W_s - \frac{1}{\mu}$

NOTE:

⇒ The no. of customer in the system are always mutually Exclusive.

TYPE - I $(M|M|1):(\infty|FIFO)$ → single server infinite capacity

ρ → utilization factor which represents the proportion of time the servers are busy $= \lambda/\mu$.

1] Traffic Intensity $(\rho) = \frac{\lambda}{\mu}$

2] Avg. no. of customers in the system (L_s or N_s) $= \frac{\lambda}{\mu-\lambda}$

3] Avg no. of customers in the queue / Avg queue length [L_q or N_q] $= \frac{\lambda^2}{\mu(\mu-\lambda)}$

(or)

$L_q = L_s - \frac{\lambda}{\mu}$

4] Avg waiting time of a customer in the system } $W_s = \frac{L_s}{\lambda} = \frac{1}{\mu-\lambda}$

5] Avg. waiting time of customer in the queue } $W_q = W_s - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)}$

6] P[system is busy] $= \lambda/\mu$

7] P[system is ideal] $= P_0 = 1 - P$[system is busy]

$= 1 - \frac{\lambda}{\mu}$

8] P[waiting time of the customer exceeds time 't'] $= P[W_s > t] = e^{-(\mu-\lambda)t}$

9] $P[W_q > t] = \frac{\lambda}{\mu} e^{-(\mu-\lambda)t}$

10] $P[N > k] = \left(\frac{\lambda}{\mu}\right)^{k+1}$ → no. of customers in the system exceeds k.

N → No. of customers in the system.

11] $P[N \geq k] = \left(\frac{\lambda}{\mu}\right)^{k}$

↓

queue length being greater than or equal to 'k'

TYPE - II $(M|M|1):(K|FIFO) \rightarrow$ Single server & Finite capa

1] Traffic intensity $(\rho) = \frac{\lambda}{\mu}$

2] $P_0 = \begin{cases} \frac{1-\rho}{1-\rho^{k+1}}, & \lambda \neq \mu \\ \frac{1}{k+1}, & \lambda = \mu \end{cases}$

(iii) $P_n = \begin{cases} \rho^n P_0, & \lambda \neq \mu \\ \frac{1}{k+1}, & \lambda = \mu \end{cases}$

(4) $L_s = \begin{cases} \frac{\rho}{1-\rho} - \frac{(k+1)\rho^{k+1}}{1-\rho^{k+1}}, & \lambda \neq \mu \\ k/2, & \lambda = \mu \end{cases}$

5] $L_q = L_s - \frac{\lambda'}{\mu}$, where λ' = effective arrival rate $= \mu(1-P_0)$

6] $W_s = \frac{L_s}{\lambda}$, where $\lambda' = \mu(1-P_0)$

7] $W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda'}$

8] P[customer turned away] = $P_k = \rho^k P_0$

(i.e) if $P[N > k]$