NORMAL DISTRIBUTION

Definition:

A continuous random variable X is said to follow normal distribution, if its probability density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty.$$

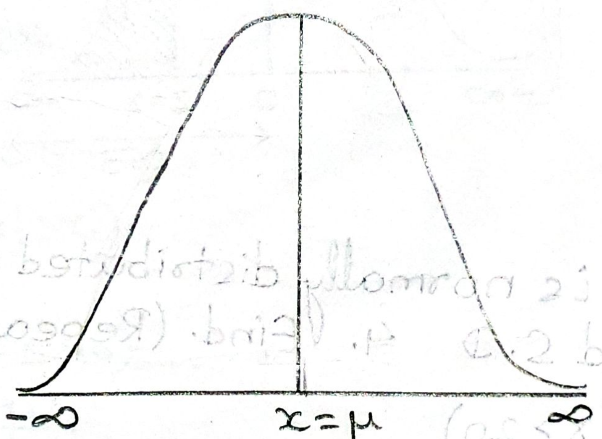
Moment generating function for Mean, variance.

$$M_x(t) = e^{\left(\mu t + \frac{\sigma^2 t^2}{2}\right)}$$

Mean $= \mu$

variance $= \sigma^2$

Properties of normal curve



i.) The curve is bell shaped.

ii.) $f(x)$ approaches zero as $x \to \infty$ and $x \to -\infty$

iii.) The curve is symmetric about the point $x = \mu$.

iv.) Mean = Median = Mode.

v.) Skewness is given.

vi.) The total area under the curve is 1.

28.) X is normally distributed with mean 12 and S.D = 4. Find $P(X \geq 20)$.

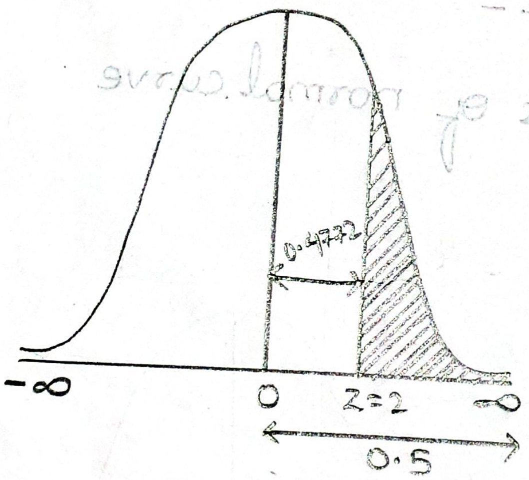
Soln→28.) $Z = \frac{X-\mu}{\sigma}$

Given

$\mu = 12, \sigma = 4$

When $X = 20$, $Z = \frac{20-12}{4} = 2$

$\therefore P(X \geq 20) = P(Z \geq 2) = 0.5 - 0.4772$
$= 0.0228.$



28.) X is normally distributed with mean 12 and S.D 4. Find. (Repeated 28 Question)

(i.) $P(X \geq 20)$

(ii.) $P(X \leq 20)$

(iii.) $P(0 \leq x \leq 12)$

soln→28

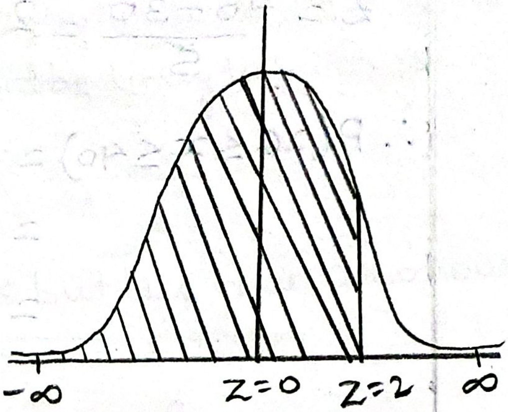
ii) $P(x \le 20)$

Let $z = \frac{x-\mu}{\sigma}$

When $x = 20$,

$z = \frac{20-12}{4}$

$= 2$



$\therefore P(x \le 20) = P(z \le 2)$

$= 0.5 + 0.4772$

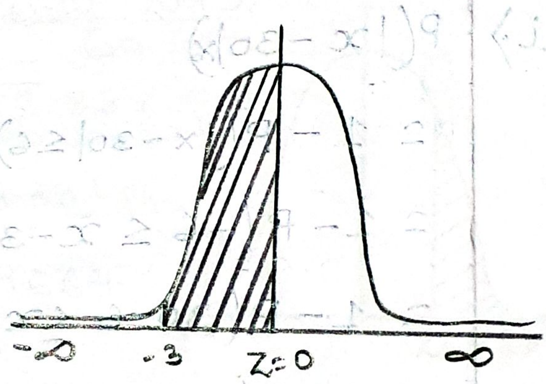
$= 0.9772.$

iii) $P(0 \le x \le 12)$

$z = \frac{x-\mu}{\sigma}$

When $(x = 12)$ $= \frac{12-12}{4} = 0.$

$(x = 0)$ $z = \frac{0-12}{4} = -3.$



$\therefore P(0 \le x \le 12) = P(-3 \le z \le 0)$

$= 0.4987.$

29.) x is normally distributed with mean 30 & variance 25.

Find

i) $P(26 \le x \le 40)$

ii) $P(|x-30| > 6)$

soln→29→ $P(26 \le x \le 40)$

let $z = \frac{x-\mu}{\sigma}$

$\mu = 30$, $\sigma^2 = 25$

$\Rightarrow \sigma = 5.$

When $(x = 26)$

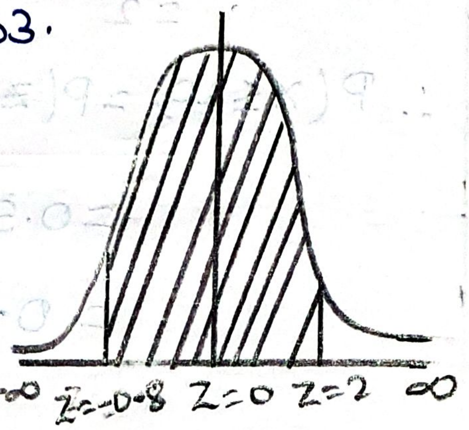
$= \frac{26-30}{5} = \frac{-4}{5} = -0.8$

When

$x = 40$

$$z = \frac{40-30}{5} = 2.$$

$$\therefore P(26 \le x \le 40) = P(-0.8 \le z \le 2)$$
$$= 0.2881 + 0.4772$$
$$= 0.7653.$$



ii.) $P(|x-30| > 6)$

$$= 1 - P(|x-30| \le 6)$$
$$= 1 - P(-6 \le x-30 \le 6)$$
$$= 1 - P(30-6 \le x \le 30+6)$$
$$= 1 - P(24 \le x \le 36) = 1 - P(-1.2 \le x \le 1.2)$$
$$= 1 - (0.3849 + 0.3849)$$
$$= 1 - 0.7698$$
$$= 0.2302.$$

Linear Inequality

$$\boxed{\begin{array}{c} |x| \le a \\ -a \le x \le a \end{array}}$$

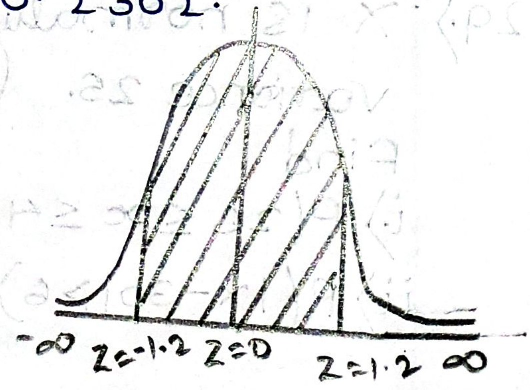
When

$x = 24$

$$z = \frac{24-30}{5} = -\frac{6}{5}$$

When $x = 36$

$$z = \frac{36-30}{5} = \frac{6}{5}$$



30.) In a list of 2000 electric bulbs, it is found that the life of a particular make is normally distributed with an average life of 2040 hours and S.D 60 hours. Estimate the number of bulbs likely to burn for

i.) more than 2150 hours.

ii.) less than 1950 hours.

iii.) more than 1920 hours but less than 2160 hours.

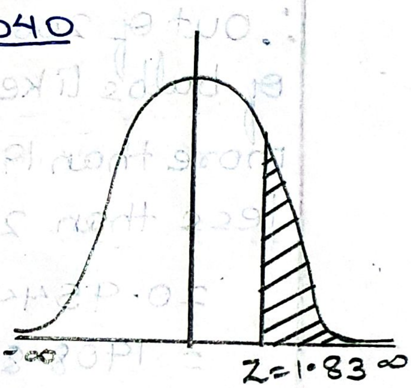
Sln→30→ Mean $= \mu = 2040$, $\sigma = 60$.

Let $z = \frac{x-\mu}{\sigma}$

i.) $P(x > 2150)$

When $x = 2150$, $z = \frac{2150-2040}{60}$

$= 1.83$



$\therefore P(x > 2150) = P(z > 1.83)$

$= 0.5 - 0.4664$

$= 0.036$

$\therefore$ out of 2000 bulbs, number of bulbs likely to burn for more than 2150 hours $= 0.0336 \times 2000$

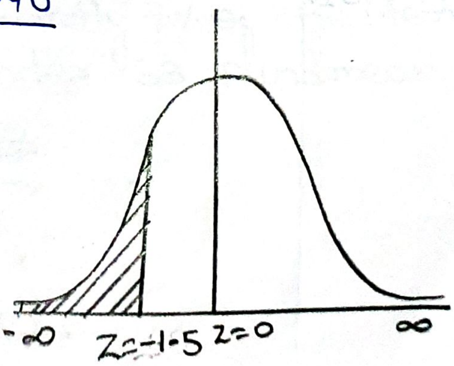
$= 67.2$

$\simeq 67$ bulbs.

ii.) $P(x < 1950)$

When $x = 1950$, $z = \frac{1950-2040}{60}$

$= -1.5$



$\therefore P(x < 1950) = P(z < -1.5)$

$= 0.5 - 0.4332$

$= 0.0668.$

$\therefore$ Out of 2000, bulbs, number of bulbs likely to burn for less than 1950 hours $= 0.0668 \times 2000$

$= 133.6$

$= 134$ bulbs.

iii) $P(1920 < x < 2160)$

When $x = 1920$, $z = \dfrac{1920 - 2040}{60} = -2$

When $x = 2160$, $z = \dfrac{2160 - 2040}{60} = 2$

$\therefore P(1920 < x < 2160) = P(-2 < z < 2)$

$= 0.4772 + 0.4772$

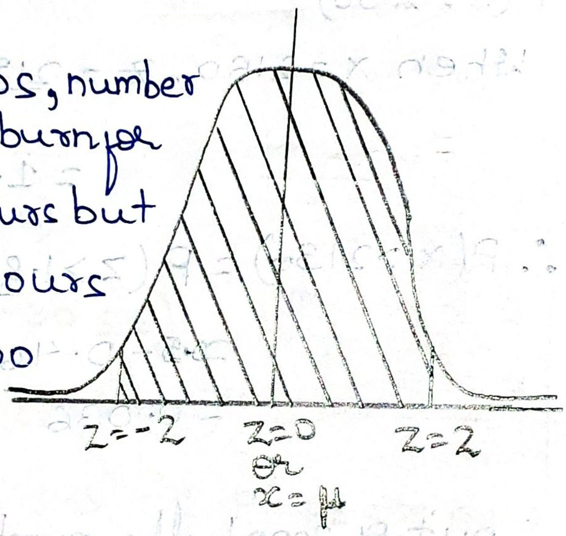
$= 0.9544.$

$\therefore$ out of 2000 bulbs, number of bulb's likely to burn for more than 1920 hours but less than 2160 hours

$= 0.9544 \times 2000$

$= 1908.8$

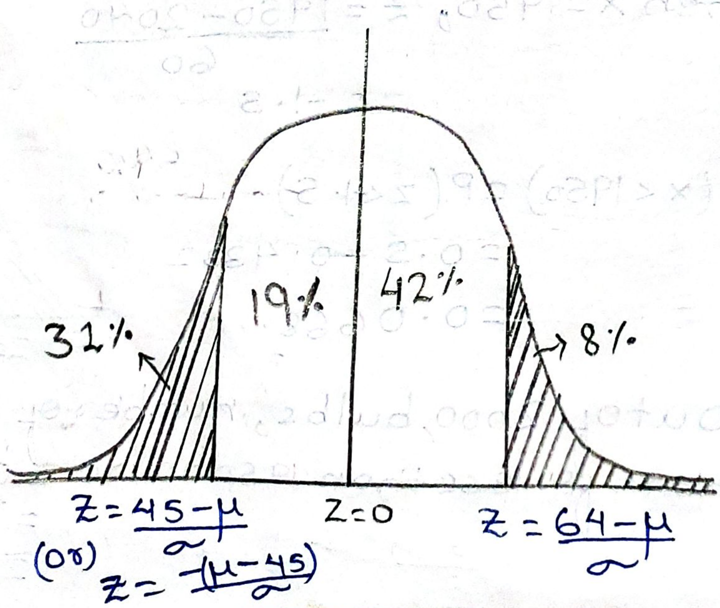
$= 1909$ bulbs.



R.U.Q
V.V.Q

31.) In a normal distribution, 31% of items are under 45 and 8% are over 64. Find the mean and variance.

Soln → 31 →



Given:-

$P(x<45) = 31\%$

let $z = \frac{x-\mu}{\sigma}$

When $x = 45$, $z = \frac{45-\mu}{\sigma}$

$\therefore P(x<45) = P\left(z < \frac{45-\mu}{\sigma}\right)$

$\therefore P\left(-\frac{(\mu-45)}{\sigma} < z < 0\right) = 0.19$

or

$\therefore P\left(0 < z < \frac{\mu-45}{\sigma}\right) = 0.19$

$\Rightarrow \frac{\mu-45}{\sigma} = 0.50$

$\Rightarrow \mu - 45 = 0.50\sigma \rightarrow (1)$

$P(x>64) = 8\%$

When $x = 64$, $z = \frac{64-\mu}{\sigma}$

$\therefore P(x>64) = P\left(z > \frac{64-\mu}{\sigma}\right)$

$\therefore P\left(0<z<\frac{64-\mu}{\sigma}\right) = 0.42$

$\frac{64-\mu}{\sigma} = 1.41$

$64 - \mu = 1.41\sigma \rightarrow (2)$

$$\begin{aligned} \cancel{\mu} - 0.50\sigma &= 45 \\ \cancel{-\mu} - 1.41\sigma &= -64 \\ \hline -1.91\sigma &= -19 \end{aligned}$$

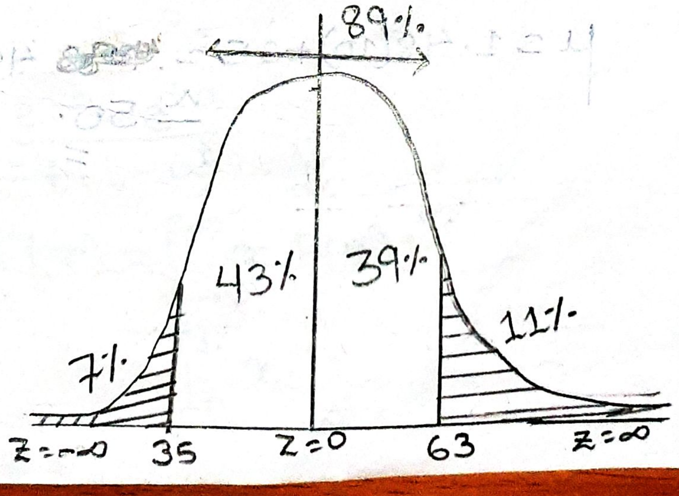
$\sigma = \frac{19}{1.91} = 9.94 \simeq 10$

Variance $= \sigma^2 = 100$

Put $\sigma = 10$ in (1)

$\boxed{\therefore \mu = 50}$

2.) In a normal distribution, exactly 7% of the items are under 35 and 89% are under 63. find mean and S.D.



$Z = \frac{35-\mu}{\sigma}$ $Z = \frac{63-\mu}{\sigma}$

Given:

$P(x < 35) = 7\%$ $P(x<63) = 89\%$

Let $Z = \frac{x-\mu}{\sigma}$

When $x = 35$,

$Z = \frac{35-\mu}{\sigma}$

$P(x<35) = P\left(Z < \frac{35-\mu}{\sigma}\right)$

$P\left(-\left(\frac{\mu-35}{\sigma}\right) < Z < 0\right) = 0.43$

(or)

$P\left(0 < Z < \frac{\mu-35}{\sigma}\right) = 0.43$

$\mu - 35 = 1.48\sigma$ — (1)

When $x = 63$,

$Z = \frac{63-\mu}{\sigma}$

$P(x<63) = P\left(Z < \frac{63-\mu}{\sigma}\right)$

$P\left(0<Z<\frac{63-\mu}{\sigma}\right) = 0.39$

$\frac{63-\mu}{\sigma} = 1.23$

$63 - \mu = 1.23\sigma$

Solve (1) and (2)

$$
\begin{array}{r}
\cancel{\mu} - 1.48\sigma = 35 \\
- \cancel{\mu} - 1.23\sigma = -63 \\
\hline
\cancel{+}2.71\sigma = \cancel{+}28
\end{array}
$$

$\sigma = \frac{28}{2.71}$

$= 10.33$ $\mu = 10$

Put $\sigma = 10$ in the 1^{st} equation

$\mu = 1.48(10) + 35 = 49.8$

$\simeq \underline{\underline{50.}}$