UNIT : 5 MARKOV PROCESS

⇒ Future value depends on the present value

MARKOV PROCESS :

* The probability of future event depends only on present but not on the past event.

* It is a conditional Probability.

$$= P[x_n = a_n / x_{n-1} = a_{n-1}, x_{n-2} = a_{n-2}, \ldots x_1 = a_1, x_0 = a_0]$$

$$= P[x_n = a_n / x_{n-1} = a_{n-1}]$$

Eg: Prob. of raining today depends only on previous weather conditions existed for the last 2 days & not on past weather conditions.

MARKOV CHAIN :

* A discrete parameter of Markov process is called as the Markov chain, where time is discrete (or) continuous.

* If $P\{x_n = a_n / x_{n-1} = a_{n-1}, x_{n-2} = a_{n-2} = a_{n-2}, \ldots x_0 = a_0\}$

⇒ $P\{x_n = a_n / x_{n-1} = a_{n-1}\}$ for all n.

⇒ The process $\{x_n\}, n = 0, 1, 2, \ldots$ is called Markov chain

⇒ $a_1, a_2, a_3, \ldots \rightarrow$ states of Markov chain.

ONE-STEP TRANSITION PROBABILITY :

$$P[x_n = a_j / x_{n-1} = a_i] = P_{ij}(1)$$

Note:

If 1 step TPM does not depend on the step (i.e) $P_{ij}(n-1, n) = P_{ij}(m-1, m)$ the markov chain is Homogeneous :

N-STEP TRANSITION PROBABILITY:

$$P[x_n = a_i / x_0 = a_j] = P_{ij}^{(n)}$$

where, $a_0, a_1, a_2, \ldots a_n \rightarrow$ states of Markov chain.

TRANSITION PROBABILITY MATRIX (TPM):

The matrix formed by the one-step probabilities denoted by 'P' is called TPM.

STOCHASTIC / REGULAR MATRIX:

The TPM in which every entries are positive and the row sum is equal to one ('1') is called stochastic / regular Matrix.

REGULAR MARKOV CHAIN:

The Markov chain with the stochastic / regular matrix. (Note: Stochastic matrix (P) is regular if all entries of P^m are +ve)

FINITE MARKOV CHAIN:

It is a Markov chain with finite number of steps.

STEADY STATE / LONG RUN / limiting INVORENT PROBABILITY DISTRIBUTION

(*) $\boxed{\pi P = \pi}$, $\pi = (\pi_1, \pi_2)$ and $\pi_1 + \pi_2 = 1$

CHEPMAN - KOLMOGORAV THEOREM:

(*) $\boxed{P_{ij}^{(n)} = [P_{ij}]^n}$

IRREDUCIBLE:

If $P_{ij}^{(n)} > 0$ for some n, for all i, j (i.e) if every state can be accessible (reached) from every other state, then the Markov chain is called Irreducible

$\Rightarrow$ It is finite & non-null persistant.

* ERGODIC: A - non null persistent and aperiodic state

* PERIOD: Let $d_i = \text{Gcd}\{n, P_{ii}^{(n)}\}$

$\Rightarrow$ If $d_i = 1$ then, the state 'i' is aperiodic

(period of order 1)

$\Rightarrow$ If $d_i = n$ then, the state 'i' is periodic (order

TO FIND PERIOD OF STATES:

For state A:

$$P_{AA}^{(1)} = 0, \; P_{AA}^{(2)} = 0, \; P_{AA}^{(3)} = 1/2 > 0, \; P_{AA}^{(4)} = 0, \; P_{AA}^{(5)} = \frac{1}{4} > 0$$

period $= \text{Gcd}\{3, 5, \ldots\} = 1$

$\therefore$ State A is aperiodic.

For state B:

$$P_{BB}^{(1)} = 1 > 0 \; P_{BB}^{(2)} = 1/2, \; P_{BB}^{(3)} = \frac{1}{2} > 0, \; P_{BB}^{(4)} = \frac{1}{4} > 0, \; P_{BB}^{(5)} = 1/4 >$$

period $= \text{Gcd}\{2, 3, 4, 5, \ldots\} = 1$

$\therefore$ state B is aperiodic.

For state C:

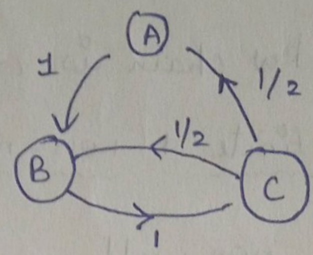
$$P_{CC}^{(1)} = 0, \; P_{CC}^{(2)} = 1 > 0, \; P_{CC}^{(3)} = 0, \; P_{CC}^{(4)} = 1/2 > 0, \; P_{CC}^{(5)} = 1/4 > 0$$

Period $= \text{Gcd}\{2, 4, 5, \ldots\} = 1$

$\therefore$ state C is aperiodic.

Since, each state is irreducible, finite, non-null persistant & aperiodic, the given Markov chain is Ergodic.

TRANSITION DIAGRAM :



X. MCQ's

1] Markov process is the one in which the future value is independent of the past value.

2] Future value depends on present value.

3] Poisson process is a Markov process.

4] Chapman kolmogorov equation is

$$[P_{ij}]^n = (P_{ij})^n \quad \text{or} \quad P_{ij}^{(n)} = P_{ij}^n$$

5] Transition matrix is a square matrix with sum of each row is 1.

6] Ergodic means

* Irreducible → Every state can be reached from every other state. $\boxed{P_{ij}^{(n)} > 0}$
* Period → $\gcd\{n, P_{ij}^{(n)} > 0\}$

  If, period = 1 → aperiodic

  If, period = n → periodic

7] In a TPM, the sum of elements in each row is 1

8] If P is the TPM of a MC then, $\boxed{\pi P = \pi}$

9] In a limiting distribution, $\lim_{n\to\infty} P_{ij}^{(n)} = \underline{\underline{\pi}}$

10] In a steady state / limiting prob. distribution / Invariant distribution $\boxed{\pi P = \pi}$

11] Non-Null persistant → Finite, Irreducible.

12] Ergodic → Irreducible & Aperiodic.

13] Non-null persistent & aperiodic → Ergodic.

14] State i is said to be persistent if return state i is certain

15] " transient " uncertain

16] " non-null persistent, if the mean recurrence time μ_{ii} is finite.

17) null persistent, if $\mu_{ii} = \infty$

18] Absorbing state → if, $\boxed{P_{ii} = 1}$

~~19~~ Limiting Probability $\lim_{n\to\infty} P^n = P$

20] Stationary distribution in Irreducible chain then it is unique.

21] In an absorbing M.C, a state which is not absorbing is transient

22] In a homogeneous M.C is regular, then every sequence of state probability distributions approaches a unique fixed prob. distribution called steady state distribution of M.C.

23] Poisson process is a Markov process.