Unit-4 Small Sample Test

χ^2-test (chi-square):

1) Goodness of fit:

$$\chi^2 = \frac{\sum (O-E)^2}{E}$$

$D.O.f = n-1$

Properties of χ^2

1. It is used to test goodness of fit
2. It is used to test independence of attribute
3. Note: In case of binomial distribution

$d.o.f = n-1$

poison $= n-1$

Normal distribution $= n-3$

Q:1 Five coins are tossed 256 times. The number of heads is given below. Examine if the coins are unbiased by employing χ^2 goodness of fit 5% l.o.s.

x	O
0	5
1	35
2	75
3	84
4	45
5	12

ho: no significant diff

h1: There is significant diff.

$n = 5 \quad N = 256$

$p = \frac{1}{2}$

$q = \frac{1}{2}$

Tossing coin comes under binomial distribution

Success — Failure

$P(x) = n_{C_x} p^{x} q^{n-x} \times N$

↳ capital N given in question so use this formula

x	o	$\therefore E[N \times nC_x p^x q^{n-x}]$	O−E	$\frac{(O-E)^2}{E}$
0	5	$256 \times 5C_0 (1/2)^0 (1/2)^{5-0}$ 8	−3	9/8
1	35	$256 \times 5C_1 (1/2)^1 (1/2)^{5-1}$ = 40	−5	25/40
2	75	$256 \times 5C_2 (1/2)^2 (1/2)^{5-2}$ = 80	−5	25/80
3	84	$256 \times 5C_3 (1/2)^3 (1/2)^{5-3}$ = 80	4	16/80
4	45	$256 \times 5C_4 (1/2)^4 (1/2)^{5-4}$ = 40	5	25/40
5	12	$256 \times 5C_5 (1/2)^5 (1/2)^{5-5}$ = 8	4	16/8

$$\chi^2 = \frac{\Sigma (O-E)^2}{E} = 4.8875$$

$$\boxed{\chi^2 = 4.8875}$$

= 9.4

χ^2 table value at 5% los with (5−1 = 4) d.o.f = 9.488

χ^2 cal < χ^2 table

$\therefore$ accept H_0

In a survey of 320 families in 5 children each with following distribution

No. of B :	0	1	2	3	4	5
No. of families:	12	40	88	110	56	14
No. of G :	5	4	3	2	1	0

x	O
0	12
1	40
2	88
3	110
4	56
5	14

x	O	$E = N \times {}^nC_x\, p^x q^{n-x}$	$(O-E)$	$\frac{(O-E)^2}{E}$
0	12	10	2	
1	40	50	-10	
2	88	100	-12	
3.	110	100	10	
4.	56	50	6	
5.	14	10	4	

$$\chi^2 = \frac{\sum(O-E)^2}{E} = 7.16$$

χ^2 table value at 5% los with

$(n-1)$ $5-1 \Rightarrow 4$ d.o.f $= 9.488$.

The theory predicts that the proportion of beans in 4 groups A B C D 9 : 3 : 3 : 1. Out of 1600 beans the number in the 4 groups where 882, 313, 287, 118. Does the experiment support the

x	O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
A	882	$1600 \times \frac{9}{16} = 900$	324	0.36
B	313	$1600 \times \frac{3}{16} = 300$	169	0.563
C	287	$1600 \times \frac{3}{16} = 300$	169	0.563
D	118	$1600 \times \frac{1}{16} = 100$	324	3.24
				4.726

5% los with $(n-1)$ i.e $4-1=3$ d.o.f

table $= 7.82$

cal. value < table value

$\therefore$ Ho accepted.

χ^2 - test to test independent of attributes

			Total
A	a	b	a+b
B	c	d	c+d
Total	a+c	b+d	N

$$E(a) = \frac{(a+b)(a+c)}{N}$$

$$E(b) = \frac{(a+b)(b+d)}{N}$$

$$E(c) = \frac{(a+c)(c+d)}{N}$$

$$E(d) = \frac{(b+d)(c+d)}{N}$$

State whether

	Favourable	Non-favourable	
New	60	30	90
Conventional	40	70	110
Total	100	100	200

Sol:

$$E(60) = \frac{90 \times 100}{200} = 45$$

$$E(30) = \frac{90 \times 100}{200} = 45$$

$$E(40) = \frac{100 \times 110}{200} = 55$$

$$E(70) = \frac{100 \times 110}{200} = 55$$

x	O	E	(O-E)	$(O-E)^2$
a	60	45	15	225/45
b	30	45	-15	225/45
c	40	55	-15	225/55
d	70	55	15	225/55
				18.18

$$D.o.f = (r-1)(c-1)$$
$$= (2-1)(2-1)$$
$$= 1$$

at 5% dof = 3.84

∴ cal > table

Rejected H_0.

In order to relate smoking cigaratte with lung cancer 300 smokers and 400 non smokers randomly ~~rotated~~ selected from a town in which 50 out of 400 non smokers showed sign of cancer and 234 out of 300 smokers showed sign of cancer. On the basis of data can it be suggested that development of lung cancer has strong link on smoking at 5% los

Row total = column total same

otherwise data is incorrect.

	cancer	non-cancer	Total
smokers	234 (a)	66 (b)	300
non-smokers	50 (c)	350 (d)	400
Total	284	416	700

$E(a) = \frac{3\cancel{0}\cancel{0} \times 284}{7\cancel{0}\cancel{0}} = 121.7$

$E(b) = \frac{3\cancel{0}\cancel{0} \times 416}{7\cancel{0}\cancel{0}} = 178.28$

$E(c) = \frac{4\cancel{0}\cancel{0} \times 284}{7\cancel{0}\cancel{0}} = 162.2$

$E(d) = \frac{4\cancel{0}\cancel{0} \times 416}{7\cancel{0}\cancel{0}} = 237.7$

x	O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
a	234	121.7	12611.2	103.62
b	66	178.7	12588.8	70.64
c	50	162.2	12588.8	77.61
d	330	237.7	12611.2	53.05
				304.92

$$\chi^2 = \frac{\sum (O-E)^2}{E} = 304.92$$

D.O.F. $(r-1)(c-1) = (2-1)(2-1) = 1$

at 5% los table value 3.84.

Queuing Theory

Input → service → output.

The principle in Queuing system are

- The customer and the server.
- The customer arrive can get service immediately or should wait ~~then~~ in queue.
- The arrival of customers is defined in terms of inter arrival time (poisson distribution)
- Queue size
- Queue is big
- Queue disciple : The order in which customers are served.
  - → FIFO
  - → LIFO
  - → Priority
- system capacity :
  The max no. of customers allowed in system

Basic characteristics of Queuing Theory:

- Arrival Pattern
- Service pattern
- No. of ~~service~~ servers.
- System capacity
- Queue discipline.

Customer Behaviour:

→ ~~Talkin~~ <u>Balking</u>: customer resists to enter queuing. sys because the queue is too long.

→ <u>Reneging</u>:

customer who leaves a queue without receiving service because of a much waiting time.

→ <u>Jockeying</u>:

when there are parallel queues the customer who jumps from one queue to another queue with shortest length to reduce the waiting time.

State of the system:

1) Transient state
2) Steady state
3) Explosive State

Kendar's notation for representation of queuing theory

$$(a/b/c):(d/e)$$

a: arrival time

b: Service time

c: no. of servers

d: capacity of system

e: queue discipline

$\frac{1}{\lambda} \rightarrow$ interarrival time

$\lambda \rightarrow$ mean arrival rate (avg. no of customer arriving per unit time)

$\frac{1}{\mu} \rightarrow$ inter service time

$\mu \rightarrow$ mean service rate (avg no of customer serviced per unit time)

Model : 1

$(M|M|1):(\infty|FIFO)$

Single server ∞ capacity

i) $L_s = \frac{\lambda}{\mu-\lambda}$ (Average number of customers in the system)

ii) $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$ (Average number of customers in queue)

iii) $W_s = \frac{1}{\mu-\lambda}$ (Average waiting time) in System

iv) $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$ (Average waiting time in queue)

v) $P_0 = 1-\frac{\lambda}{\mu}$ (System is ideal [empty])

(vi) $\rho = \frac{\lambda}{\mu}$ (Traffic intensity)

vii) probability that waiting time exceeds 't'

$$P(W_s>t) = e^{-(\mu-\lambda)t}$$

viii) probability that no. of customers in the system exceeds 'k'

$P(n \geq K) = \left(\frac{\lambda}{\mu}\right)^K$

ix) Steady state

$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1-\frac{\lambda}{\mu}\right) = P^n P_0$

Little's formula:

i) $L_s = \lambda W_s$

ii) $L_q = \lambda W_q$

(iii) $L_s = L_q + \frac{\lambda}{\mu}$

iv) $W_q = W_s - \frac{1}{\mu}$

In the railway yawn goods train arrive at the rate of 30 times per day. Assume that the inter arrival time follows exponential distribution & service time is also exponential with average of 36 mins. Calculate the following.

i) The mean queue size

ii) The probability that the system size is atleast 10.

iii) If the input of the trains increases p avg of 33 per day? what will be the change in above quantity.

$\lambda = 30/\text{day}$

converting to minutes

$$\rightarrow \frac{3\cancel{0}}{24 \times \cancel{60}_{2}} = \frac{1}{48} = 0.021$$

times per minute

Average service time = 36 minutes

$\hookrightarrow$ follows exponential distribution.

So, ~~μ~~ Service time = 36 mins

$$= \frac{1}{\mu} = \frac{1}{36}$$

$$= 0.028$$

i) mean queue size =

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = 2.25 \approx 2 \text{ trains}$$

ii) $P(n \geq K) = \left(\frac{\lambda}{\mu}\right)^K$

$$= \left(\frac{0.021}{0.028}\right)^{10} = 0.0563$$

iii) $\lambda = 33/\text{day}$

converting to minutes

$$= \frac{33}{24 \times 60} = 0.023$$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = 3.7 \simeq 4 \text{ trains}$$

$$P(n \geq k) = \left(\frac{\lambda}{\mu}\right)^k$$

$$P(n \geq 10) = 0.139$$

Q: customers arrive at one man barber shop acc. to poisson process with mean inter arrival time of 20 mins. customer spends 15 mins in barber chair, if an hour is used as unit time then;

i) what is the probability that the customer

ii) expected customers in barbershop and queue.

iii) How much time can a customer expect to spend in the shop.

iv) Find the average waiting time that customer spends in the queue.

v) what is the probability that there will be more than 3 customers in the system one man barber ...

Sol:

$\lambda = \frac{1}{20} = 0.05 \qquad \mu = \frac{1}{15} = 0.067$

i) $P_0 = 1 - \frac{\lambda}{\mu} = 0.253$

ii) $L_s = \frac{\lambda}{\mu - \lambda} = 2.941 \simeq 3$ person

$L_q = 2$ persons

iii) $W_s = 59$ mins

iv) $W_q = 44$ mins

v) $P(n \geq k) = 0.31$

Model II $(M/M/1):(K/FIFO)$

1. $P_0 = \begin{cases} \dfrac{1-\lambda/\mu}{1-(\lambda/\mu)^{K+1}} & \lambda \neq \mu \\ \dfrac{1}{K+1}, & \lambda = \mu \end{cases}$

2. Steady state

$$P_n = \begin{cases} \left(\dfrac{\lambda}{\mu}\right)^n P_0, & n = 0, 1, 2 \dots K \\ 0 & ; n > K \end{cases}$$

3. $L_s = \begin{cases} \dfrac{\lambda}{\mu-\lambda} - \dfrac{(K+1)(\lambda/\mu)^{K+1}}{1-(\lambda/\mu)^{K+1}}, & \lambda = \mu \\ \dfrac{K}{2}, & \lambda = \mu \end{cases}$

4. Effective arrival rate $\lambda' = \mu(1-P_0)$

5. $L_q = L_s - \dfrac{\lambda'}{\mu}$

6. $W_s = \dfrac{L_s}{\lambda'}$

7. $W_q = \dfrac{L_q}{\lambda}$

8. Probability of customer turned away $P_K = (\lambda/\mu)^K P_0$

P:1 An one man barber shop can accomodate 5 people at a time, 4 waiting and 1 getting serviced. ~~The cu~~ A customer arrive at poisson distribution with mean average of 5 per hour. Service is according to exponential distribution on an average of 15 mins.

i) what is the percentage of idle time $P_0 = 0.029$

(ii) probability of customer turned away $P_K = 0.271$

iii) what is the expected no. of customers in the queue. $L_s = 3.13 \Rightarrow L_q = 2.23 \simeq 2$

iv) To find expected time spent in the shop. $\lambda = 0.061$

$\lambda = \frac{1}{12}$ mins, $\mu = \frac{1}{15}$, K system Capacity $= 5(4+1)$ $W_s = \frac{L_s}{\lambda'} = 51.42$ $\lambda' \approx .061$

(i)

$$P_0 = \frac{1 - \lambda/\mu}{1 - (\lambda/\mu)^{K+1}}, \quad \lambda \neq \mu$$

$$= \frac{1 - \dfrac{\frac{1}{12}}{\frac{1}{15}}}{1 - \left(\dfrac{\frac{1}{12}}{1/15}\right)^5} = \frac{1 - \frac{15}{12}}{\left(1 - \frac{15}{12}\right)^5} = \frac{\frac{12-15}{12}}{\frac{12-15}{12}}$$

P.2 Patients arrive at a clinic according to poisson distribution at 30 / hr.

The waiting room cannot wait ~~of~~ more than 14 patients

Examination ~~by~~ time exponential with mean rate of 20 / hr

Find

i) effective arrival rate at the clinic

ii) Probability the arriving patient will not wait (P_0)

(iii) what is the expected waiting time until the patient is discharged from the clinic (W_s).

i)

$\lambda = 30$; $\mu = 20$; $K = 15$ $(14+1)$

$\lambda' \simeq 20$; $P_0 = 0.00076$

$W_s = \frac{L_s}{\lambda'}$

$L_s = 13.024$

$W_s = 0.65$ hr

At a railway station one train is handled at a time, the railway yard is sufficient only for two trains to wait · while the other is given signal to leave the station Trains arrive at the station. with the average rate of 6 per hour. and the station can handle them on an average of 12 per hour, assuming poisson distribution Find the steady state probabilities (P_n) of various no of trains in system. Find average no of trains in system Ls.

Average waiting time in system Ws.

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 = n = 1, 2, 3$$

$P_0 = 0.53$

$P_1 = 0.266$

$P_2 = 0.132$

$P_3 = 0.66$

$L_s = 0.734 \simeq$ train

$W_s = 0.131$