UNIT - II

FOURIER SERIES

07/10/2021

Drichlet Condition:

(i) $f(x)$ is single valued.

(ii) $f(x)$ has a finite number of finite discontinuity or no infinite discontinuity

If $\lim_{x \to 0} f(x) = f(x_0)$

$\therefore f(x)$ is continuous at $x = x_0$.

If $\lim_{x \to 0} f(x) \neq f(x_0)$

$f(x)$ is not continuous at $x = x_0$.

(iii) $f(x)$ has almost finite number of maximum and minimum.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$a_0 \rightarrow$ fourier count

a_n, b_n → fourier co-efficient

Interval	a_0	a_n	b_n
$(0, 2\pi)$	$\frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$	$\frac{1}{\pi}\int_0^{2\pi} f(x) \cos nx\,dx$	$\frac{1}{\pi}\int_0^{2\pi} f(x) \sin nx\,dx$
$(0, 2l)$	$\frac{1}{l}\int_0^{2l} f(x)\,dx$	$\frac{1}{l}\int_0^{2l} f(x) \cos\left(\frac{n\pi x}{l}\right)dx$	$\frac{1}{l}\int_0^{2l} f(x) \sin\left(\frac{n\pi x}{l}\right)dx$
$(-\pi, \pi)$ neither odd nor even	$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$	$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\,dx$	$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\,dx$
Even	$\frac{2}{\pi}\int_0^{\pi} f(x)\,dx$	$\frac{2}{\pi}\int_0^{\pi} f(x) \cos nx\,dx$	0
Odd	0	0	$\frac{2}{\pi}\int_0^{\pi} f(x) \sin nx\,dx$

constant function, x^{even} is even

Convergence of Fourier Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

(i) $f(x)$ is continuous at $x = x_0$.
where x_0 is middle / in between the given interval, then sum of $F(S) = f(x_0)$

(ii) If $f(x)$ is discontinuous at $x = x_0$.
where x_0 is in between the given interval, then the sum of $F(S) = \frac{1}{2}[LHL + RHL]$
average of LHL and RHL

(iii) If $f(x)$ is discontinuous at end points of the given interval, then the sum of
$F(S)$ = average of $F(S)$ at the end points.
(i.e) $\frac{1}{2}[f(0) + f(2\pi)]$ $(0, 2\pi)$.

Find the fourier expansion for
$f(x) = x^2$ in $(-\pi, \pi)$; (i) $\sum_{n=1}^{\infty} 1/n^2$, (ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

$f(x) = (-x)^2 = x^2$ (iii) $\sum_{n=odd} 1/n^2$
$f(x)$ is even

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cos nx \, dx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x^2\,dx = \frac{2}{\pi}\left(\frac{x^3}{3}\right)_0^{\pi}$$

$$a_0 = \frac{2\pi^{\cancel{3}2}}{3\cancel{\pi}}$$

$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx\,dx$$

$u = x^2$ $\quad v = \cos nx$

$u' = 2x$ $\quad v_1 = \frac{\sin nx}{n}$

$u'' = 2$ $\quad v_2 = \frac{-\cos nx}{n^2}$

$v_3 = \frac{-\sin nx}{n^3}$

$$a_n = \frac{2}{\pi}\left\{x^2 \frac{\sin nx}{n} + 2x \frac{\cos nx}{n^2} - \frac{2\sin nx}{n^3}\right]_0^{\pi}$$

$$a_n = \frac{2}{\cancel{\pi}}\left[\frac{2\cancel{\pi}(-1)^n}{n^2}\right]$$

$$\boxed{a_n = \frac{4(-1)^n}{n^2}}$$

$$f(x) = \frac{\frac{2\pi^2}{3}}{2} + \sum_{n=1}^{\infty} \frac{4(-1)^2}{n^2}\cos nx$$

$$\boxed{f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2}}$$

(i) $\sum_{n=1}^{\infty} 1/n^2$

$\Rightarrow \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots = ?\quad (-\pi,\pi)$

Put $x=\pi$ in discontinuous point.

$$\frac{f(-\pi)+f(\pi)}{2} = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n(-1)^n}{n^2}$$

$$\left(\frac{\pi^2+\pi^2}{2}\right) - \frac{\pi^2}{3} = 4\sum_{n=1}^{\infty} 1/n^2$$

$$\frac{\cancel{2}\pi^2}{\cancel{2}} - \frac{\pi^2}{3} = 4\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots\infty\right]$$

$$\frac{\pi^2}{6} = \frac{2\pi^2}{3}\times\frac{1}{4} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots\infty$$

$$\frac{3\pi^2-\pi^2}{3\times 4} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots\infty$$

$$\boxed{\frac{\pi^2}{6} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots\infty} \longrightarrow (1)$$

(ii) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\dots\infty$

Put $x=0$ is continuous point.

$$f(0) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

$$0-\frac{\pi^2}{3} = 4\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots\infty\right]$$

$$\cancel{-}\frac{\pi^2}{3}\times\frac{1}{4} = \cancel{-}\left[\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}+\dots\infty\right]$$

$$\boxed{\frac{\pi^2}{12} = \frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}+\dots\infty} \longrightarrow (2)$$

1 + 2 $\Rightarrow$

$$\frac{\pi^2}{6} + \frac{\pi^2}{12} = 2\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$$

$$\frac{2\pi^2 + \pi^2}{12} = 2\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$$

$$\frac{3\pi^2}{\underset{4}{\cancel{12}} \times 2} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

$$\boxed{\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots}$$

2. Find fourier series for the function $f(x) = x(2l - x)$ in the interval $(0, 2l)$. Find the value of the series $1/1^2 - 1/2^2 + \frac{1}{3^2} + \ldots \infty$

Soln:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\, dx$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

$$a_0 = \frac{1}{l}\int_0^{2l} (2lx - x^2)\, dx$$

$$= \frac{1}{l}\left\{\cancel{2}l\frac{x^2}{\cancel{2}} - \frac{x^3}{3}\right\}_0^{2l}$$

$$= \frac{1}{l}\left\{4l^3 - 8l^3/3\right\}$$

$$= \frac{1}{l}\left\{\frac{12l^3 - 8l^3}{3}\right\}$$

$$= \frac{1}{\cancel{l}}\left(\frac{4l^{\cancel{3}\,2}}{3}\right)$$

$$\boxed{a_0 = \frac{4l^2}{3}}$$

$$a_n = \frac{1}{l}\int_0^{2l} (2lx - x^2) \cos\left(\frac{n\pi x}{l}\right) dx$$

$u = 2lx - x^2 \quad \quad v = \cos\left(\frac{n\pi x}{l}\right)$

$u' = 2l - 2x \quad \quad v' = \frac{\sin(n\pi x/l)}{n\pi/l}$

$u'' = -2$

$$v'' = \frac{-\cos(n\pi x/l)}{(n\pi/l)^2}$$

$$v''' = \frac{-\sin(n\pi x/l)}{(n\pi/l)^3}$$

$$a_n = \frac{1}{l}\left[(2lx - x^2)\left(\frac{\sin(n\pi x/l)}{(n\pi/l)}\right) + 2l - 2x\left(\frac{\cos(n\pi x/l)}{(n\pi/l)^2}\right) + 2\left(\frac{\sin(n\pi x/l)}{(n\pi/l)^3}\right)\right]_0^{2l}$$

$$= \frac{1}{l}\left[-2l\left(1/(n\pi/l)^2\right) - 2l \cdot \frac{1}{(n\pi/l)^2}\right]$$

$$= \frac{1}{\cancel{l}}\left[-4\cancel{l}/(n\pi/l)^2\right]$$

$$\boxed{a_n = \frac{-4l^2}{n^2\pi^2}}$$

$$b_n = \frac{1}{l}\int_0^{2l} (2lx - x^2)\sin\left(\frac{n\pi x}{l}\right) dx$$

$u = 2lx - x^2$ $\qquad v = \sin(n\pi x/l)$

$u' = 2l - 2x$ $\qquad v_1 = \frac{-\cos(n\pi x/l)}{(n\pi/l)}$

$u'' = -2$ $\qquad v_2 = \frac{-\sin(n\pi x/l)}{(n\pi/l)^2}$

$\qquad v_3 = \frac{\cos(n\pi x/l)}{(n\pi/l)^3}$

$$b_n = \frac{1}{l}\left[(-2lx - x^2)\frac{\cos(n\pi x/l)}{n\pi/l} + (2l - 2x)\frac{\sin(n\pi x/l)}{(n\pi/l)^2} - 2\frac{\cos(n\pi x/l)}{(n\pi/l)^2}\right]_0^{2l}$$

$$b_n = \frac{1}{l}\left[\left(0 - 2\frac{1}{(n\pi/l)^3}\right) - \left(0 - 2\frac{1}{(n\pi/l)^3}\right)\right]$$

$$\boxed{b_n = 0}$$

$$f(x) = \frac{4l^2/3}{2} + \sum_{n=1}^{\infty} -4l^2/n^2\pi^2 \cos\left(\frac{n\pi x}{l}\right)$$

put $x = l$ in continuous point.

$$f(l) = \frac{2l^2}{3} + \sum_{n=1}^{\infty} \frac{-4l^2}{n^2\pi^2} \cos n\pi$$

$f(x) = x(2l - x)$

$$l^2 - \frac{2l^2}{3} = \frac{-4l^2}{\pi^2} \sum_{n=1}^{\infty} (-1)^n / n^2$$

$$\frac{l^2}{3} \times \frac{-\pi^2}{4l^2} = \left[\frac{-1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \cdots \infty \right]$$

$$\frac{\cancel{l^2}}{3} \times \frac{-\pi^2}{4\cancel{l^2}} = -\left[1/1^2 - 1/2^2 + 1/3^2 - \cdots \infty \right]$$

$$\boxed{\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots \infty}$$

28/10/2021

Half Range, Cosine and Sine Series:

Cosine Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

where, $a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\, dx$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$$

Sine Series:

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

1. Find the sine and cosine series for the function $f(x) = x$ in $(0, l)$

Cosine Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$$

where, $a_0 = \frac{2}{l}\int_0^l f(x)\,dx$

$$a_0 = \frac{2}{l}\int_0^l x\,dx$$

$$a_0 = \frac{2}{l}\left[\frac{x^2}{2}\right]_0^l$$

$$a_0 = \frac{\cancel{2}}{\cancel{l}}\left[l^{\cancel{2}}/\cancel{2}\right]$$

$$\boxed{a_0 = l}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$$

$$a_n = \frac{2}{l}\int_0^l x\cos\frac{n\pi x}{l}\,dx$$

$u = x \qquad v = \frac{\cos n\pi x}{l}$

$u' = 1 \qquad v_1 = \frac{\sin n\pi x/l}{n\pi/l}$

$u'' = 0 \qquad v_2 = \frac{-\cos n\pi x}{n^2\pi^2/l^2}$

$$a_n = \frac{2}{l}\left[\left(x \cdot \frac{\sin n\pi x/l}{n\pi/l}\right) + \frac{\cos n\pi x/l}{n^2\pi^2/l^2}\right]_0^l$$

$$= \frac{2}{l}\left[\frac{\cos n\pi l/l}{n^2\pi^2/l^2} - \frac{\cos 0}{n^2\pi^2/l^2}\right]$$

$$= \frac{2}{\cancel{l}} \times \frac{l^{\cancel{2}}}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$$\boxed{a_n = \begin{cases} \frac{-4l}{n^2\pi^2} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}}$$

$$\boxed{f(x) = \frac{l}{2} + \sum_{n=1}^{\infty} \frac{-4l}{n^2\pi^2} \frac{\cos n\pi x}{l}}$$

Sine Series:

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{l}\int_0^l f(x) \sin \frac{n\pi x}{l}\, dx$$

$$b_n = \frac{2}{l}\int_0^l x \sin \frac{n\pi x}{l}\, dx$$

$u = x \qquad v = \sin \frac{n\pi x}{l}$

$u' = 1 \qquad v_1 = \frac{-\cos n\pi x/l}{n\pi/l}$

$u'' = 0 \qquad v_2 = \frac{-\sin n\pi x/l}{n^2\pi^2/l^2}$

$$b_n = \frac{2}{l}\left[x\left(-\frac{\cos n\pi x/l}{n\pi/l}\right) + \frac{\sin n\pi x/l}{n^2\pi^2/l^2}\right]_0^l$$

$$= \frac{2}{l}\left[-l\left(\frac{\cos n\pi l/l}{n\pi/l}\right)\right]$$

$$= \frac{+2\cancel{l}}{\cancel{l}}\left[\frac{l}{n\pi}(-1)^n\right]$$

$$\cancel{\boxed{b_n = \frac{2l}{n\pi}(-1)^n}}$$

$$\boxed{b_n = \frac{2l}{n\pi}(-1)^{n+1}}$$

$$\boxed{f(x) = \sum_{n=1}^{\infty} \frac{2l}{n\pi}(-1)^{n+1} \sin \frac{n\pi x}{l}}$$

03/11/2021.

1. Find the half range cosine series $f(x) = x^2$ in $(-\pi, \pi)$. Hence deduce the sum.

(i) $\cancel{\frac{1}{\quad}} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots = \frac{\pi^4}{90}$

(ii) $\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \dots$

Soln:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi}\int_{0}^{\pi} f(x)\, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2\, dx$$

$$= \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi} = \frac{2}{\pi}\left[\frac{\pi^3}{3}\right]$$

$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cos nx\, dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx\, dx$$

$u = x^2 \qquad v = \cos nx$

$u' = 2x \qquad v_1 = \frac{\sin nx}{n}$

$u'' = 2 \qquad v_2 = \frac{-\cos nx}{n^2}$

$v_3 = \frac{-\sin nx}{n^3}$

$$a_n = \frac{2}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right) + 2x\left(\frac{\cos nx}{n^2}\right) - 2\left(\frac{\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$\boxed{\cos n\pi = (-1)^n}$$

$$= \frac{2}{\cancel{\pi}}\left[2\cancel{\pi}\left(\frac{\cos n\pi}{n^2}\right)\right] \quad a_n \cancel{\frac{4(-1)^n}{n^2}}$$

$$\boxed{a_n = \frac{4}{n^2}(-1)^n}$$

$$f(x) = \frac{\cancel{2\pi^3}}{\cancel{2\times 3}} \frac{2\pi^2/3}{2} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos nx$$

$$= \frac{\cancel{2}\pi^3}{\cancel{2}\times 3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos nx$$

$$\cancel{f(x) = \frac{\pi^3}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos nx}$$

$$\boxed{f(x) = \frac{\pi^3}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos nx}$$

(i) Using Pascal's Identity.

$$\Rightarrow \bar{y}^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} [f(x)]^2\,dx$$

$$= \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

$$\Rightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} (x^2)^2\,dx$$

$$= \frac{\left(\frac{2\pi^2}{3}\right)^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{4(-1)^n}{n^2}\right)^2$$

$$\Rightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} (x^4)\,dx$$

$$= \frac{\left(\frac{4\pi^4}{9}\right)}{4} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{16}{n^4}$$

$$\Rightarrow \frac{1}{2\pi}\left(\frac{x^5}{5}\right)_{-\pi}^{\pi} = \frac{\pi^4}{9} + 8\sum_{n=1}^{\infty}\frac{1}{n^4}$$

$$\Rightarrow \frac{1}{\cancel{2\pi}}\left(\frac{\cancel{2}\pi^{4}}{5}\right) - \frac{\pi^4}{9} = 8\sum_{n=1}^{\infty} 1/n^4$$

$$\Rightarrow \frac{\pi^4}{5} - \frac{\pi^4}{9} = 8\sum_{n=1}^{\infty} 1/n^4$$

$$\Rightarrow \frac{9\pi^4 - 5\pi^4}{45} = 8\sum_{n=1}^{\infty} 1/n^4$$

$$\Rightarrow \frac{4\pi^4}{45} = \overset{2}{8}\sum_{n=1}^{\infty} 1/n^4$$

$$\Rightarrow \frac{\pi^4}{45\times 2} = \sum_{n=1}^{\infty} 1/n^4$$

$$\Rightarrow \frac{\pi^4}{90} = \sum_{n=1}^{\infty} 1/n^4$$

$$\Rightarrow \boxed{\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots\cdots}$$

(ii)

$$f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$$

$$x^2 = \frac{\pi^2}{3} + 4\left[\frac{-\cos x}{1^2} + \frac{\cos 2x}{2^2} - \frac{\cos 3x}{3^2} + \cdots\cdots\right]$$

Put $\boxed{x = 0}$

$$0 = \frac{\pi^2}{3} - 4\left[\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots\cdots\right]$$

$$-\frac{\pi^3}{3} = -4\left[1 - \frac{1}{2^2} + \frac{1}{3^2} - \cdots\cdots\right]$$

$$\boxed{\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \cdots\cdots}$$

Harmonic Analysis:

Type 1 : π - form:

1. Find the first two harmonics of the fourier series of $f(x)$ given in the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

The values of $y = f(x)$ are spread over the interval $0 \le x \le 2\pi$ and $f(0) = f(2\pi)$. Hence the function is periodic and so we omit the last value of $f(x)$ at $x = 2\pi$.

The first two harmonics of the fourier series $f(x)$ is given by,

$$f(x) = \frac{a_0}{2} + (a_1 \cos x + a_2 \cos 2x) + (b_1 \sin x + b_2 \sin 2x)$$

$$= \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) \longrightarrow (1)$$

x	$y = f(x)$	$\cos x$	$\cos 2x$	$\sin x$	$\sin 2x$	$y \cos x$	$y \cos 2x$
0	1.0	1	1	0	0	1	1
60	1.4	0.5	-0.5	0.866	0.866	0.7	-0.7
120	1.9	-0.5	-0.5	0.866	-0.866	-0.95	-0.95
180	1.7	-1	1	0	0	-1.7	1.7
240	1.5	-0.5	-0.5	-0.866	0.866	-0.75	-0.75
300	1.2	0.5	-0.5	-0.866	-0.866	0.6	-0.6
	$\Sigma = 8.7$					$\Sigma = -1.1$	$\Sigma = -0.3$

	$y \sin x$	$y \sin 2x$
0	0	0
60	1.2124	1.2124
120	1.6454	-1.6454
180	0	0
240	1.299	1.299
300	-1.0392	-1.0392
	$\Sigma = 3.1176$	$\Sigma = -0.1732$

$$a_0 = 2\left[\frac{\Sigma y}{n}\right] = 2\left[\frac{8.7}{6}\right] = 2.9$$

$$a_1 = 2\left[\frac{\Sigma y \cos x}{n}\right] = 2\left[\frac{-1.1}{6}\right] = -0.37$$

$$a_2 = 2\left[\frac{\Sigma y \cos 2x}{n}\right] = 2\left[\frac{-0.3}{6}\right] = -0.1$$

$$b_1 = 2\left[\frac{\Sigma y \sin x}{n}\right] = 2\left[\frac{3.1176}{6}\right] = 1.0392$$

$$b_2 = 2\left[\frac{\Sigma y \sin 2x}{n}\right] = 2\left[\frac{-0.1732}{6}\right] = -0.0577$$

The first two harmonics is given by,

$$f(x) = \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x)$$

$$= \frac{2.9}{2} + (-0.37 \cos x + 1.0392 \sin x) + (-0.1 \cos 2x - 0.0577 \sin 2x)$$

$$f(x) = 1.45 - 0.37 \cos x + 1.0392 \sin x - 0.1 \cos 2x \cancel{-0.0577 \sin} - 0.0577 \sin 2x$$

Type - 2: l - form:

1. Obtain the constant term and coefficient of the first sine and cosine terms in the fourier representation of y as given in the following table.

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

Soln: We know that,

$$0 \le x \le 6$$

$$0 \le x \le 2l$$

$$\Rightarrow 2l = 6$$

$$\boxed{l = 3}$$

formula $\Rightarrow \theta = \frac{\pi x}{l}$

x	θ	y	$\cos\left(\frac{\pi x}{3}\right)$	$\sin\left(\frac{\pi x}{3}\right)$	$y\cos\left(\frac{\pi x}{3}\right)$	$y\sin\left(\frac{\pi x}{3}\right)$
0	0	9	1	0	9	0
1	60°	18	0.5	0.866	9	15.888
2	120°	24	-0.5	0.866	-12	20.888
3	180°	28	-1	0	-28	0
4	240°	26	-0.5	-0.866	-13	-22.817
5	300°	20	0.8	-0.860	0	-17.3021
		$\boxed{\Sigma y = 125}$			$\boxed{\Sigma y\cos\left(\frac{\pi x}{3}\right) = -25}$	

$$\Sigma y\sin\left(\frac{\pi x}{3}\right) = -3.46$$

The first harmonics of $f(x)$ is given by,

$$f(x) = \frac{a_0}{2} + a_1 \cos \frac{\pi x}{3} + b_1 \sin \frac{\pi x}{3}$$

$$a_0 = 2\left[\frac{\Sigma y}{n}\right] = \cancel{} = 41.67$$

$$a_1 = 2\left[\frac{\Sigma y \cos\left(\frac{\pi x}{3}\right)}{n}\right] = -8.33$$

$$b_1 = 2\left[\frac{\Sigma y \sin\left(\frac{\pi x}{3}\right)}{n}\right] = -1.16$$

$$f(x) = \frac{41.67}{2} - 8.33 \cos \frac{\pi x}{3} - 1.16 \sin \frac{\pi x}{3}$$

$$\boxed{f(x) = 20.84 - 8.33 \cos \frac{\pi x}{3} - 1.16 \sin \frac{\pi x}{3}}$$

Note: First harmonics $= a_1 \cos x + b_1 \sin x$

Second harmonics $= a_2 \cos 2x + b_2 \sin 2x$

Type 3 - T- form:

1. The following table gives the variation of a periodic current a over a period t.

$x = t$ (sec)	0	T	T/6	T/3	T/2	2T/3	5T/6	T
$y = A$ (amp)	1.98	1.30	1.05	1.30	-0.88		-0.25	

$x = t$(sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
$y = A$(amp	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a constant part of 0.75 amp. Obtain amplitude of first harmonics.

Soln:

The first harmonics of $f(x)$ is given by,

$$f(x) = \frac{a_0}{2} + a_1 \cos\theta + b_1 \sin\theta$$

$0 \le x \le T$

$0 \le x \le 2l$

$2l = T$

$\boxed{l = T/2}$

when $x = 0$, $\theta = \frac{\pi x}{2l} = 0$

when $x = T/6$, $\theta = \frac{\pi \cancel{x}}{3\cancel{6\times 2}(\cancel{T/2})} = \frac{\pi}{3}$

when $x = \frac{T}{3}$, $\theta = \frac{\pi\left(\cancel{T}\right)}{3 \times 2\left(\cancel{T}/2\right)} = \frac{2\pi}{3}$

when $x = \frac{T}{2}$, $\theta = \frac{\pi \cancel{T}}{2 \times 2\left(\cancel{T}/2\right)} = \pi$

when $x = \frac{2T}{3}$, $\theta = \frac{\pi \times 2\cancel{T}}{3 \times \left(\cancel{T}/2\right)} = \frac{4\pi}{3}$

when $x = \frac{5T}{6}$, $\theta = \frac{\pi \times 5\cancel{T}}{3\cancel{6} \times \cancel{T}/\cancel{2}} = \frac{5\pi}{3}$

x	θ	$y = A$	$\cos\theta$	$\sin\theta$
0	0	1.98	1	0
$T/6$	$\pi/3$	1.30	0.5	0.866
$T/3$	$2\pi/3$	1.05	-0.5	0.866
$T/2$	π	1.30	-1	0
$\frac{2T}{3}$	$\frac{4\pi}{3}$	-0.88	-0.5	-0.866
$\frac{5T}{6}$	$\frac{5\pi}{3}$	-0.25	0.5	-0.866
		$\Sigma y = 4.58$		

$y\cos\theta$	$y\sin\theta$
1.98	0
0.65	1.12
-0.52	0.90
-0.30	0
0.4	0.69
-0.125	0.21
$\Sigma y\cos\theta = 1.09$	$\Sigma y \sin\theta = 2.92$

$$a_0 = (\text{given}) = 0.75$$

$$a_1 = \frac{1.09}{3} = 0.37$$

$$b_1 = \frac{2.92}{3} = 0.97 \simeq 1$$

$$\boxed{f(x) = 0.75 + 0.37 \cos\theta + \sin\theta}$$

X