

→ Quick Sort

→ is a sorting algorithm that uses the divide & conquer strategy.

3 steps of quick sort are as follows,

- Divide :

Split the array based on pivot element. All the ele. that are less than pivot should be in left sub array & all the ele. that are more than pivot should be in right sub array.

- Conquer :

Recursively sort the 2 sub arrays.

- Combine :

Combine all the ele. in a group to form a list of sorted ele.

Consider an array $A[i]$ whose indices ranging from 0 to $n-1$ then we can formalize the dimension of array elements.

$\underbrace{A[0] \dots A[m-1]}_{\text{ele. are less than } A[m]}, A[m], \underbrace{A[m+1] \dots A[n-1]}_{\text{ele. are greater than } A[m]}$

→ Algorithm Quick($A[0 \dots n-1]$, low, high)

// Pblm descriptn : Sorting of ele.

// i/p: Array $A[0 \dots n-1]$ unsorted ele.

// o/p: Ascending order.

if ($\text{low} < \text{high}$) then

 // Split the array into 2 sub arrays

$m \leftarrow \text{partition}(A[\text{low} \dots \text{high}])$ // m is mid

 Quick($A[\text{low} \dots m-1]$) of the array

 Quick($A[m+1 \dots \text{high}]$)

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Algorithm Partition(A[low...high])

// Pblm description : partitions the subarray using pivot.

Hip: $A \rightarrow \text{left floor}, \text{right floor}$

11 clp : Pilot occupies the right position.

$\text{pirol} \leftarrow A[\text{loc}]$

c ← loc()

$j \leftarrow \text{high} + 1$

while ($c < = j$) do

۳

while ($A[i] \leq pivot$) do

← it

while ($A[i] \geq pivot$) do

$$= j + j - 1$$

if (c <= p). then

$\text{swap}(A[i], A[j]) // \text{swap}$

swap(A[low], A[j]) // when $i > j$
swap A[low] & A[j]

swap A[loop] & A[i]

Julian 11 rightmost index

→ Example: (50, 30, 10, 90, 80, 20, 40, 70)

50	30	10	90	80	20	40	70
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→ choose pivot & split the array into sub

Pivot element: 50

50	30	10	90	80	20	40	70
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pivot i j

Left side ① $\Rightarrow 50 > 30$ moves the ptr i to right

② $\Rightarrow 50 > 10$

③ $\Rightarrow 50 < 90$ stop incrementing.

Right side ④ $\Rightarrow 50 < 70$ moves ptr j left.

$50 > 40$ so swap $A[i] \leftrightarrow A[j]$

$$A[i] = 90 \rightarrow A[j] = 40$$

50	30	10	40	80	20	90	70
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pivot i j

$\hookleftarrow 50 > 40$, increment

$\hookleftarrow 50 < 80$ stop incrementing

right

$50 < 90$ decrement by 1

$50 > 20$ stop decrementing j.

Swap $A[i] + A[j]$

i) 80 4 20

[50] [30] [10] [40] [20] [80] [90] [70]

j ↑ⁱ --- j ↓ⁱ

$A[i] <$ pivot

$A[j] >$ pivot

So continue incrementing i & decreasj

[50] [30] [10] [40] [20] [80] [90] [70]

i, j

[50] [30] [10] [40] [20] [80] [90] [70]

j i

$A[j] < 50$ & j has crossed i.

So swap $A[i] + A[j]$ & pivot.

do & 50

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20	30	10	40	50	80	90	70
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left sublist ↓
 pivot is shifted
 at its position right
 sublist.

→ Take left sublist:

pivot ↓
 i j j
 20 30 10 40 50 old 80 90 70
 pivot occupied its position.

↳ $20 < 30 \rightarrow$ stop : increase

↳ $20 < 40 \rightarrow$ swap decrease by 1

$20 > 10 \rightarrow$ stop

swap $A[i] \leftrightarrow A[j]$

$A[i]$ $A[j]$
 \downarrow
 $30 \leftrightarrow 10$

\downarrow

80	10	30	90	50	80	90	70
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$20 > 10$ so insertion

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20 10 30 40 50 80 90 70

i j.

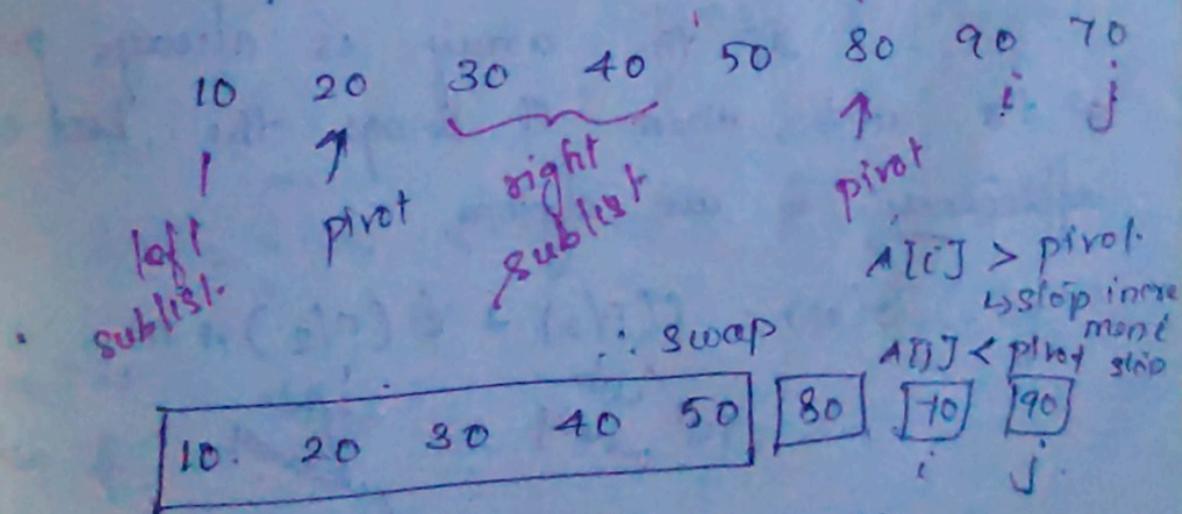
$A[i:j] > \text{pivot}$ or $A[i:j] < \text{pivot}$ depending j .

20 10 30 40 50 80 90 70

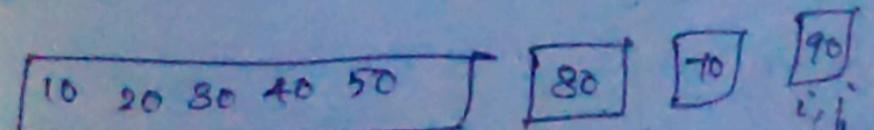
j i

$A[i:j] < \text{pivot}$ & can't decrement j

So Swap.



$A[i:j] < 80$ increment j .



$A[i:j] > \text{pivot}$ decrement j .

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10 20 30 40 50 80 70 90
j i

So swap A[j] & A[i] -

10 20 30 40 50 70 80 90

This is a sorted list.

→ Analysis:

Best case (split in the middle)

If the array is always partitioned at the mid, then it brings the best case efficiency of an algorithm.

$$T(n) = T(n/2) + T(n/2) + n$$

left subarray right subarray ← Time req.
for partitioning the subarray

$$\text{if } T(1) = 0$$

\rightarrow using Master theorem

$$T(n) = 2T(n/2) + n.$$

$$\text{Here, } f(n) \in n^1 \quad \therefore d=1$$

$$\text{Now, } a=2 \quad b=2,$$

As from case 2 we get $a=b^d$ i.e. $2=2^1$,
we get,

$$T(n) \Rightarrow c(n) = \Theta(n^d \log n)$$

$$c(n) = \Theta(n \log n).$$

Thus,

Best time complexity of quick sort
is $\Theta(n \log_2 n)$.

Q) Input 5 3 1 9 8 2 4 7

Ans. 8 3 2 9 7 1 5 4