

AVL Tree

Self balancing binary search tree \hookrightarrow AVL Tree
Adelson, Velsky, Landis. (AVL)

Height of a node.

Height of tree.

Balancing factor

$$= \text{Height}(\text{left subtree}) - \text{Height}(\text{right subtree})$$

Balance factor must be $-1, 0, 1$
but not more than $(-1, 0, 1)$

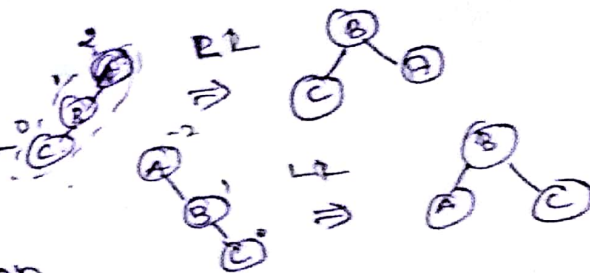
4 - Rotation

Right Rotation \Rightarrow

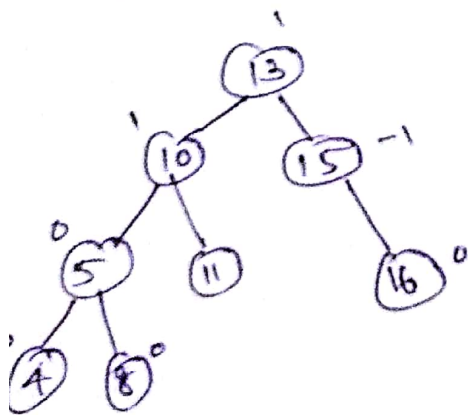
Left Rotation \Rightarrow

Left-Right Rotation

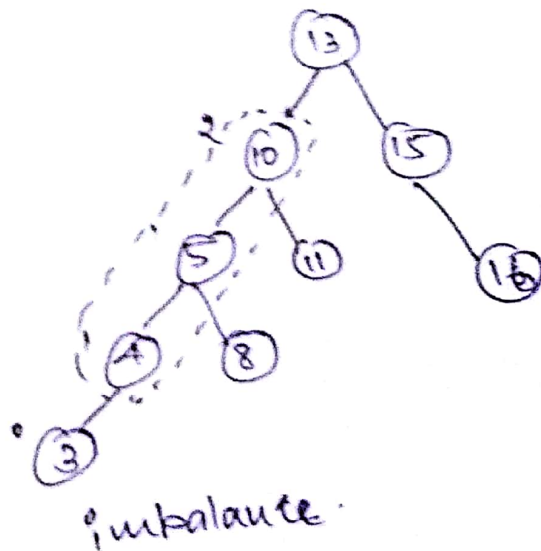
Right-Left Rotation



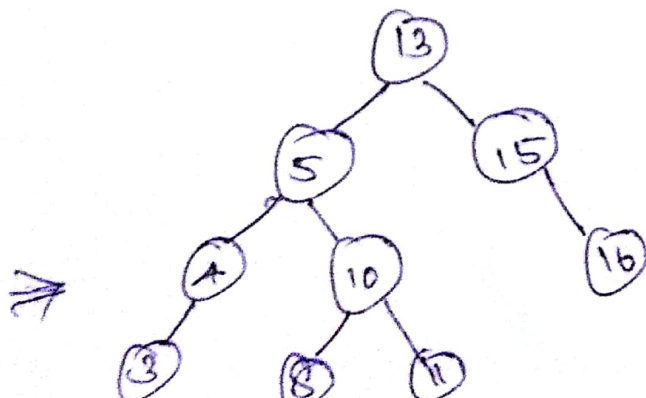
Right Rotation:



insert 3

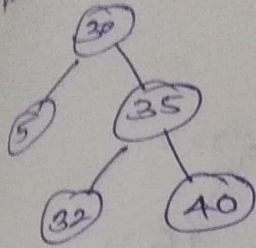


unbalance.

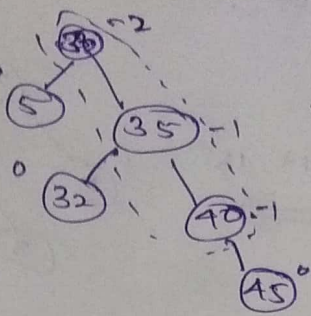


Left Rotation

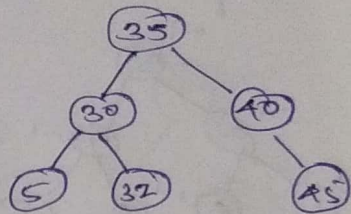
Tree



insert 45

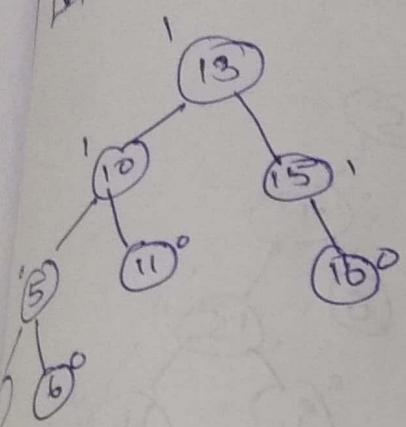


Left Rotate

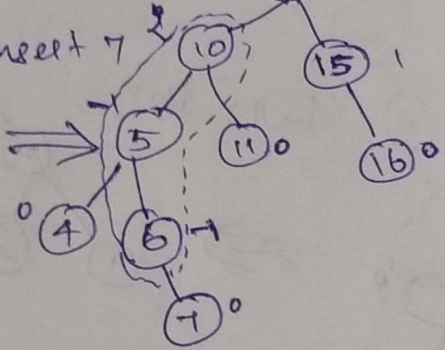


Unbalance

Left-Right Rotation:

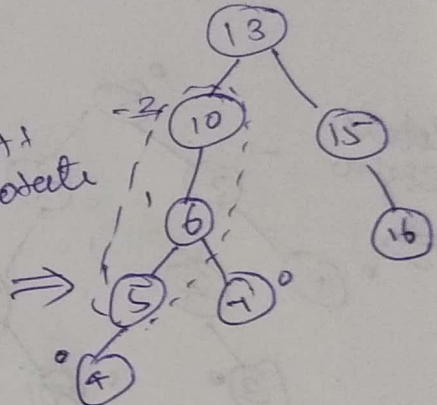


insert 7



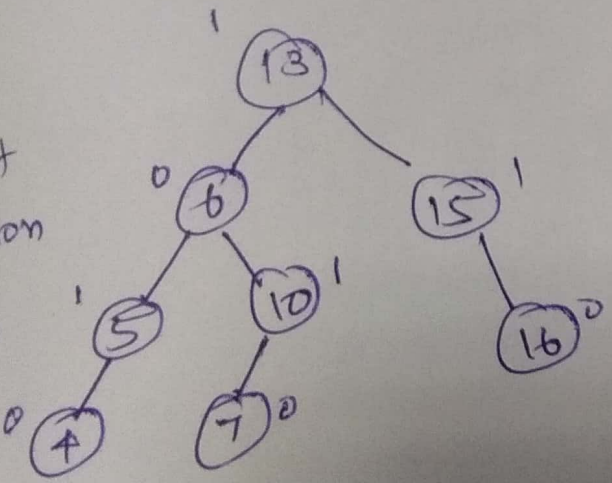
Unbalanced

Left Rotate



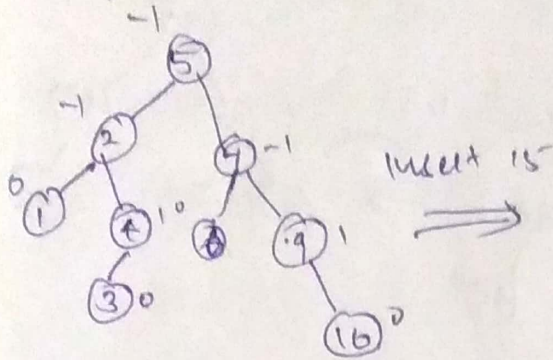
Unbalance

Right Rotation

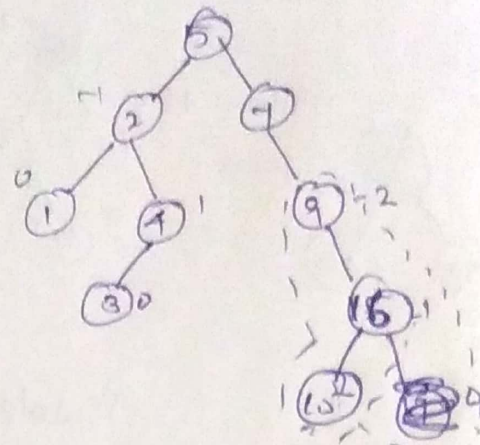


balanced

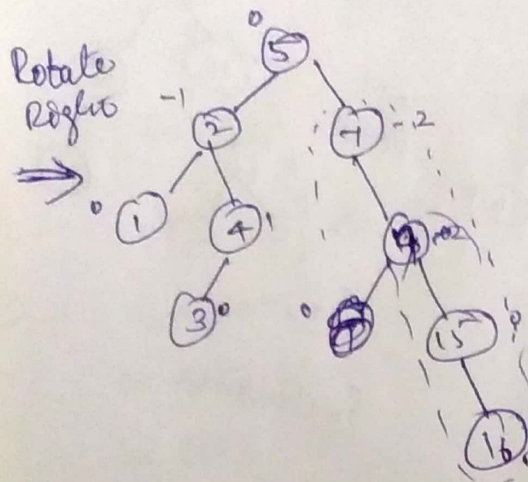
Right-Left Rotation:



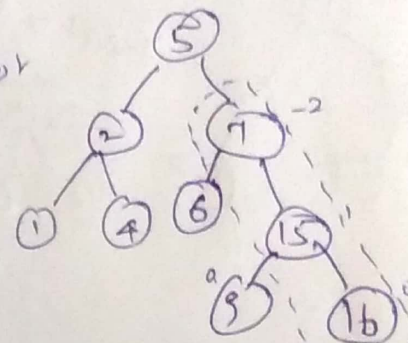
balanced



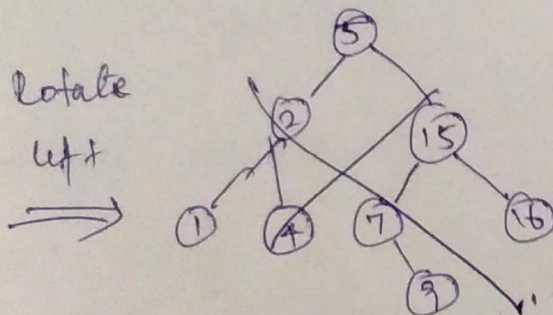
imbalance



Left Rot



imbalance

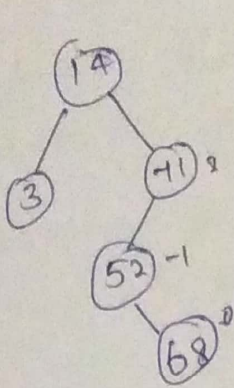


balanced

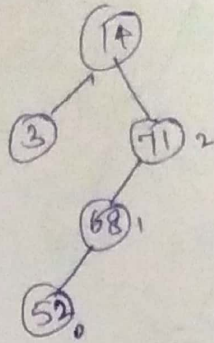
Right subtree of left child \Rightarrow Left Rotate
 Left subtree of right child \Rightarrow Right Rotate

Example:

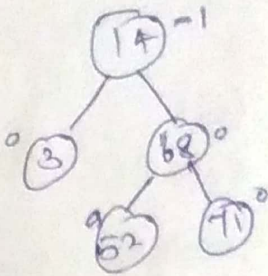
14, 71, 3, 52, 68, 92, 59, 37, 22



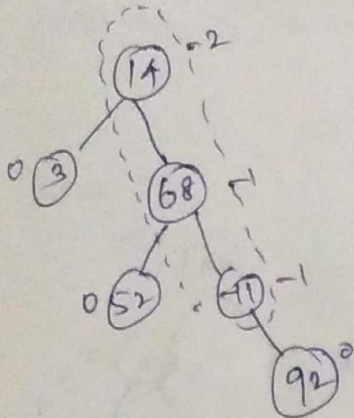
Left Rotate
⇒



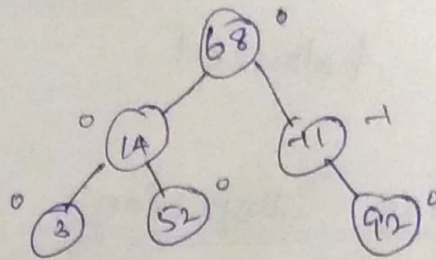
Right Rotate
⇒



balanced.

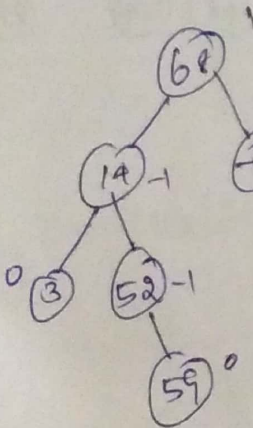


Left Rotate
⇒

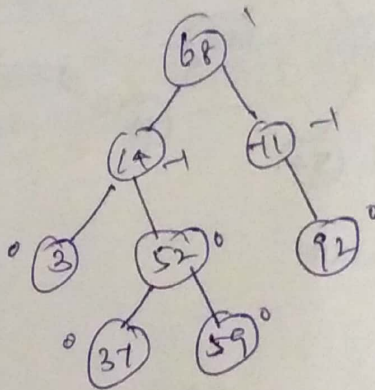


balanced.

Unbalance

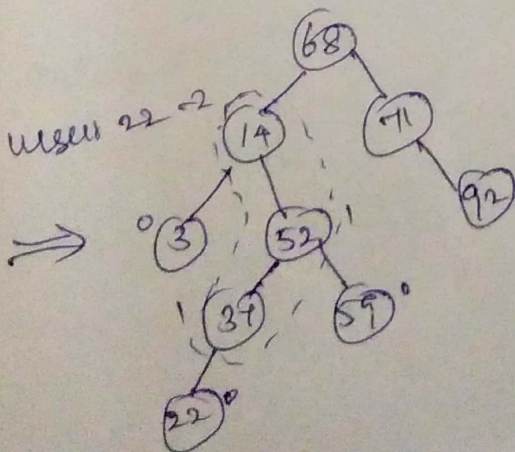


insert 37

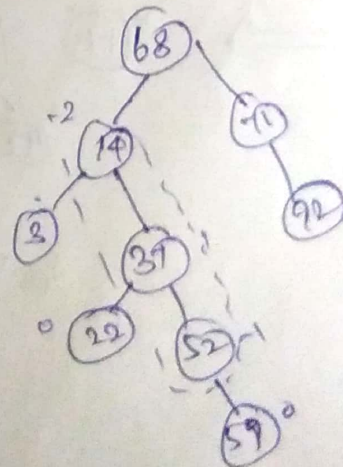


balanced

balanced

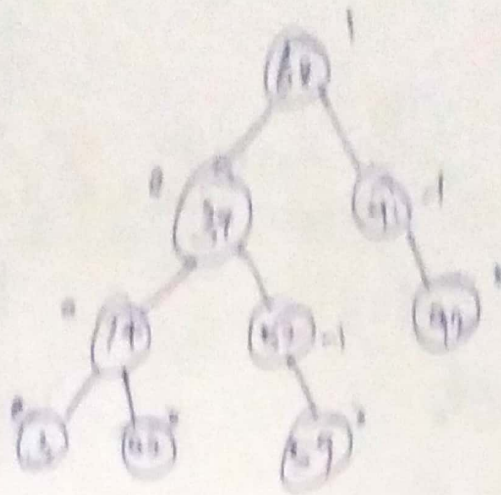


Right Rotate
⇒



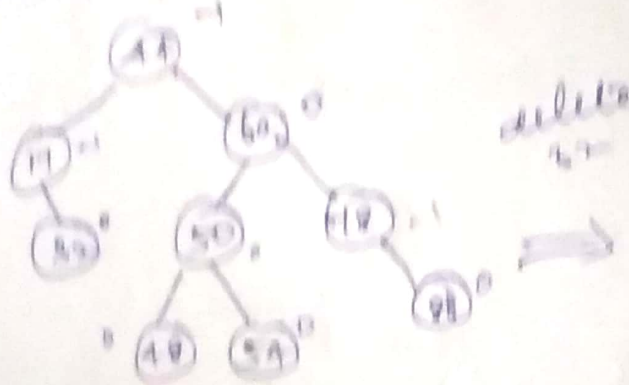
unbalance

Left
Rotate



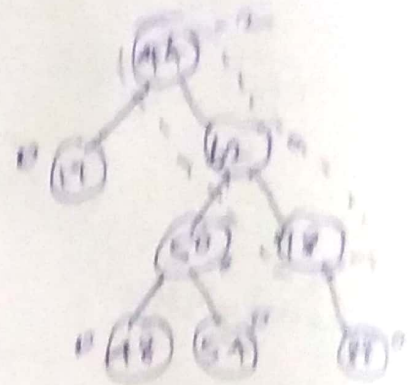
balanced.

AVL - tree . Deletion

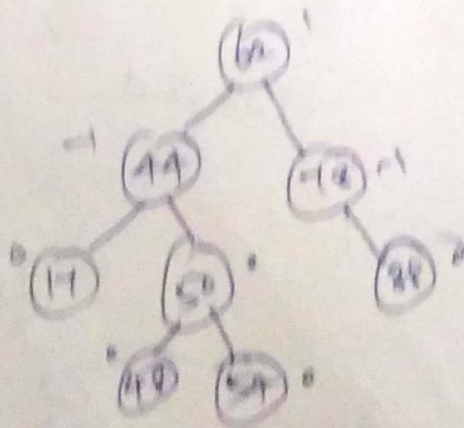
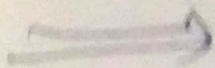


balanced

delete
22



Rotate
Right



balanced.

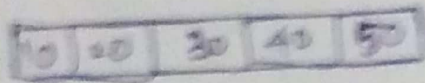
B-Tree

Construct B-tree with elements

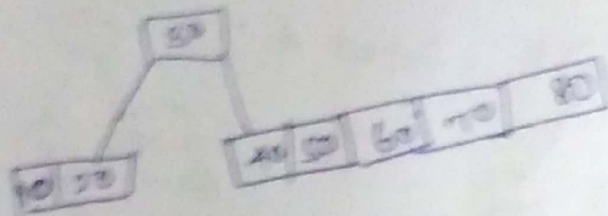
10, 20, 30, 40, 50, 60, 70, 80, 90. min degree is 3

✓ maximum no. of keys $= d * t - 1 = 5$
in a node

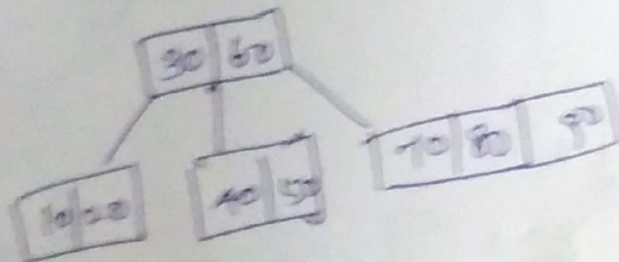
Step 1:



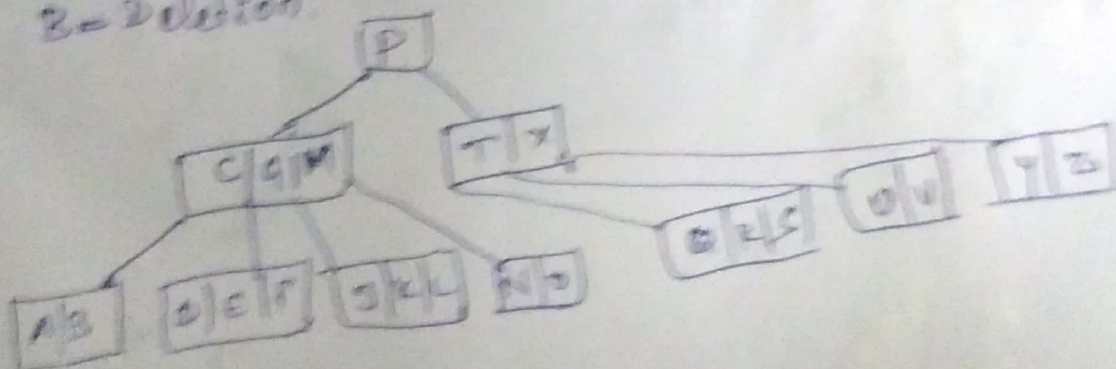
Step 2:



Step 3:



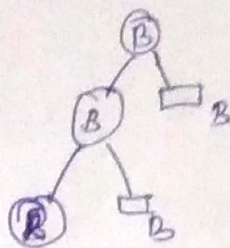
2 = Deletion:



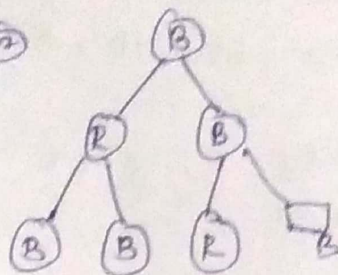
Delete 50:

RED-BLACK TREE

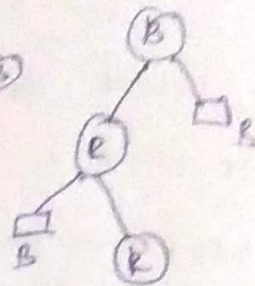
①



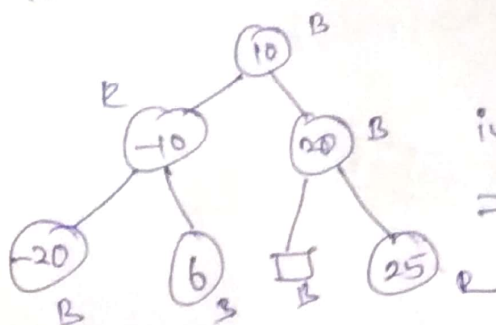
②



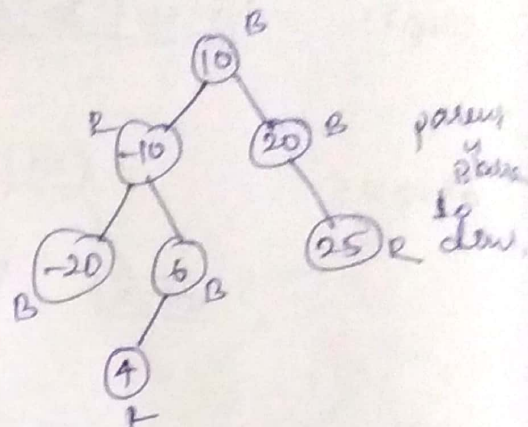
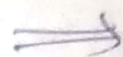
③



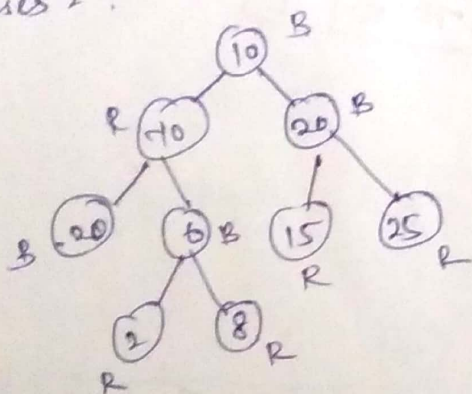
Insertion:



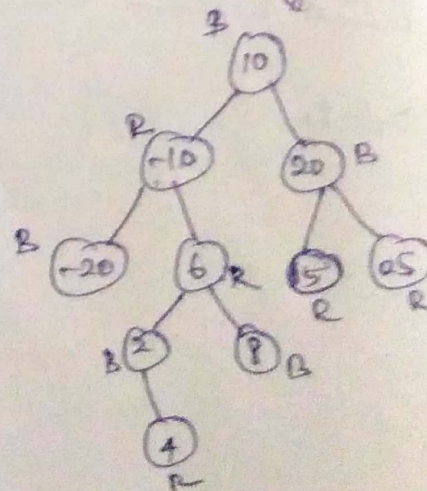
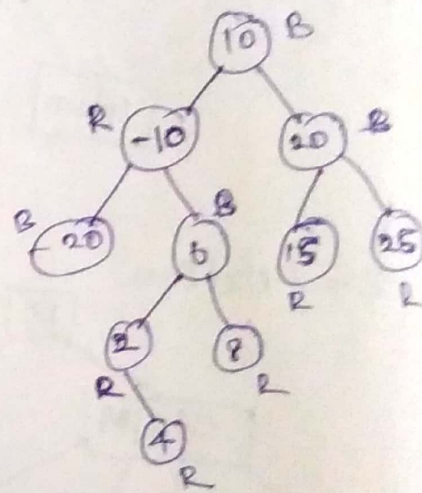
insert 4



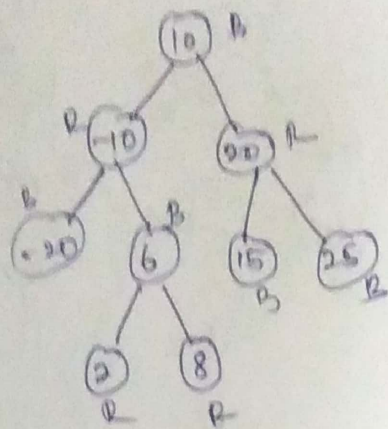
Cases 2 :



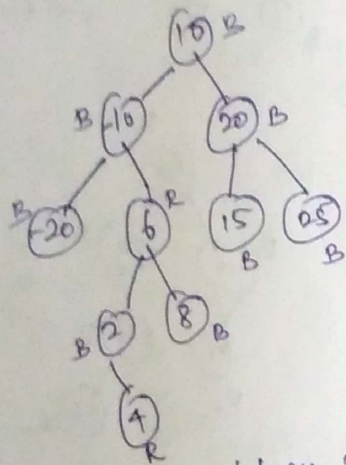
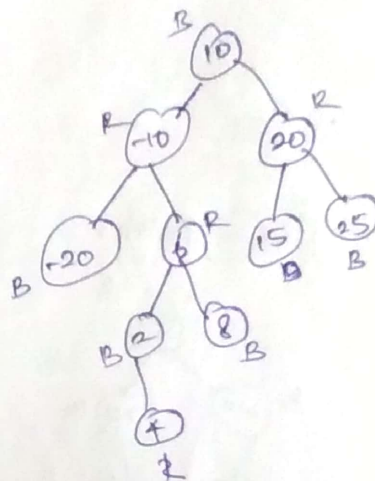
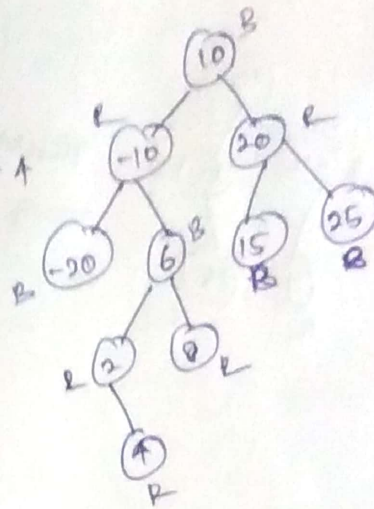
insert 4



Case 3:

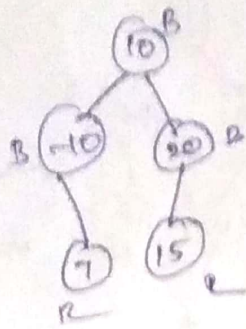


Insert 4

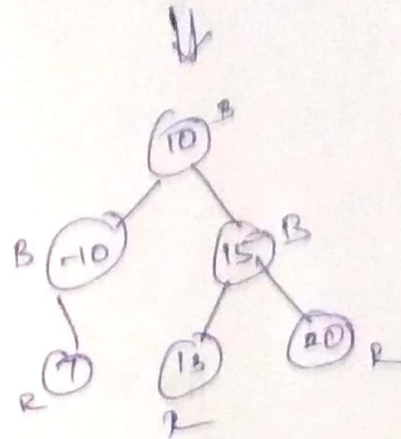
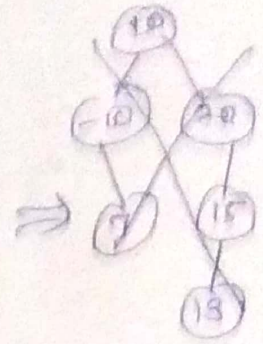
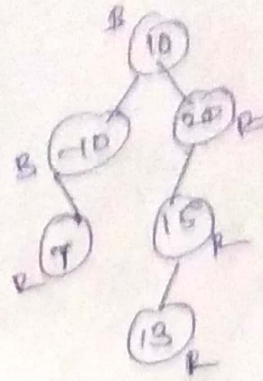


Red-black tree.

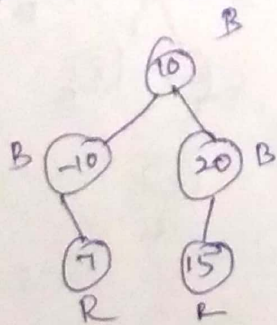
Insertion with Rotation:



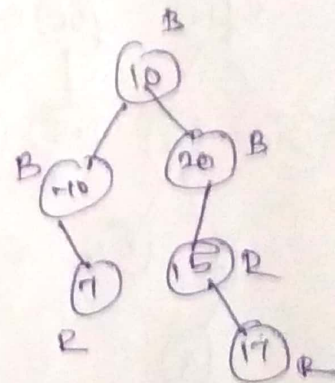
Insert 13



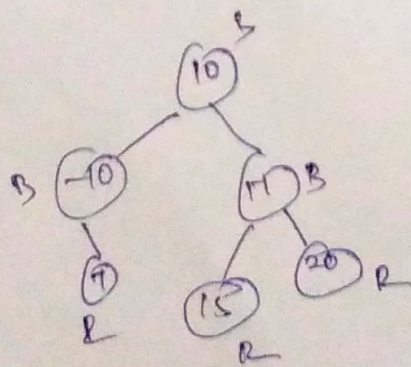
L-R Rotations



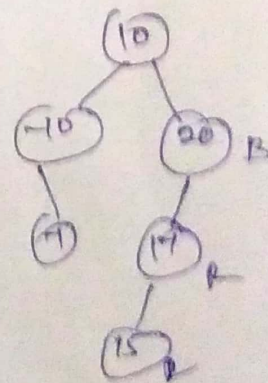
Insert 17



rotate Left

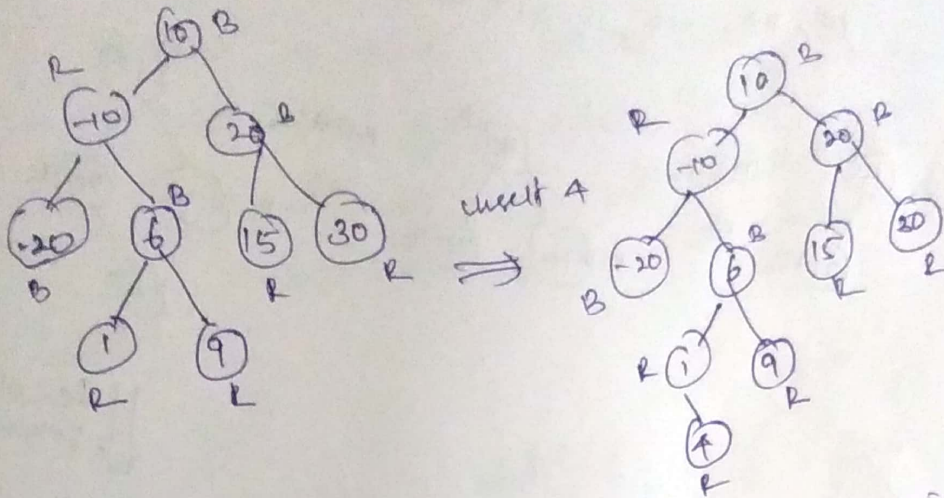


rotate Right

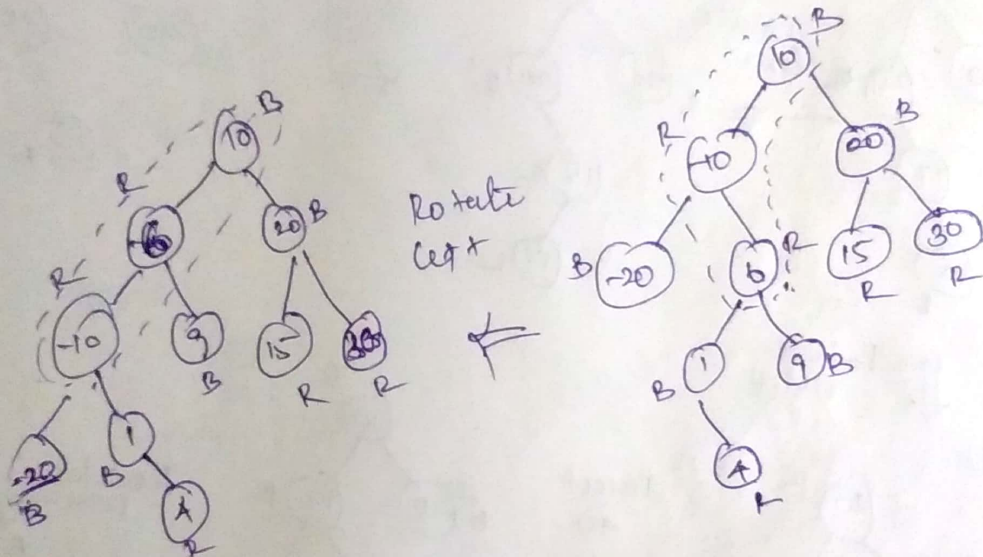


R-B free

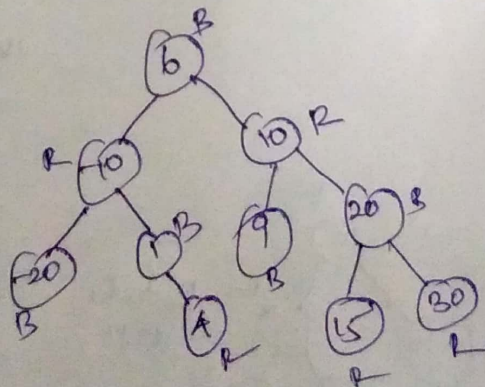
parent sibling is black:



Recoloring.



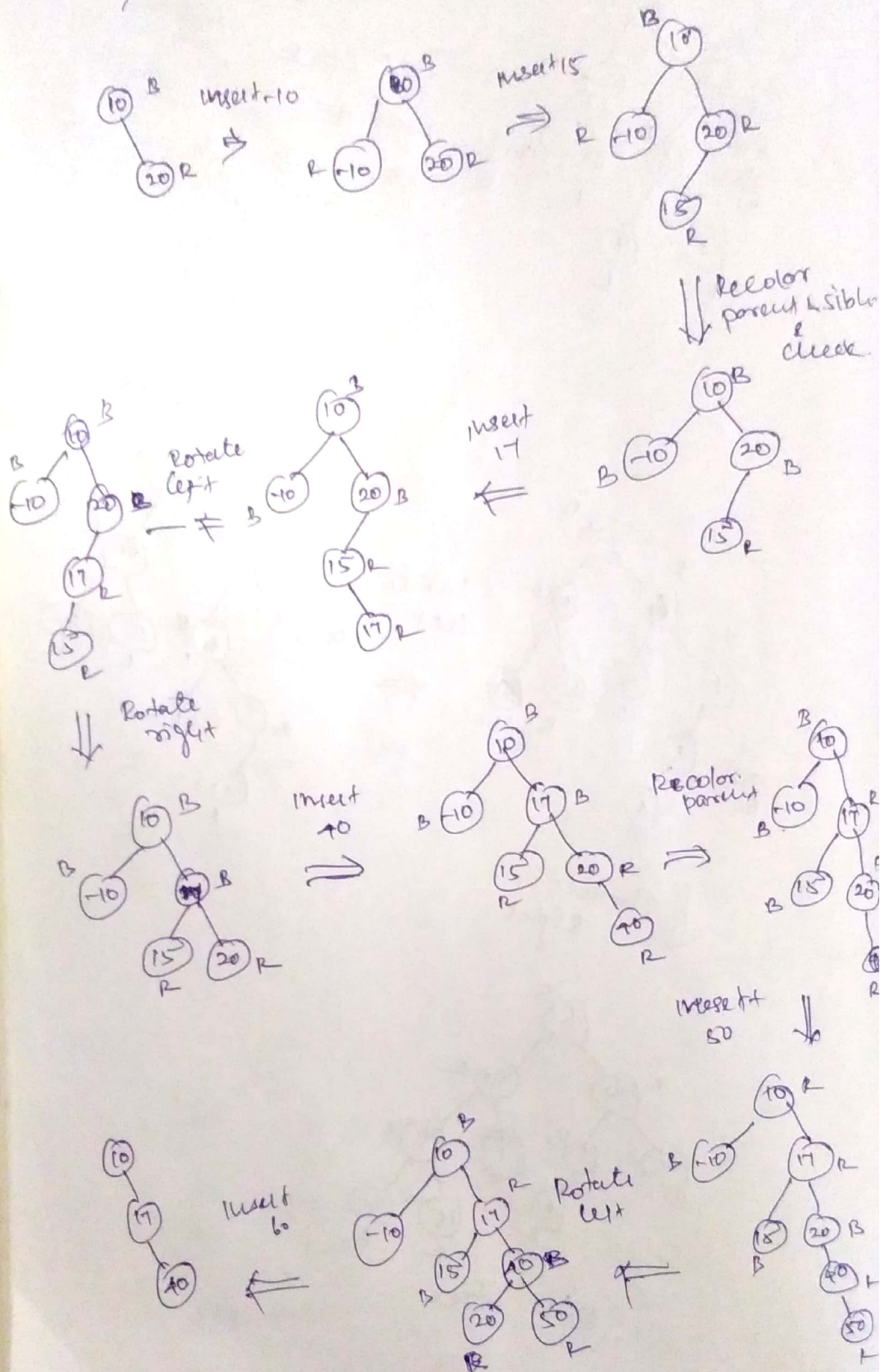
Rotate Right.



R-D. tree.

Construct Red-Black Tree:

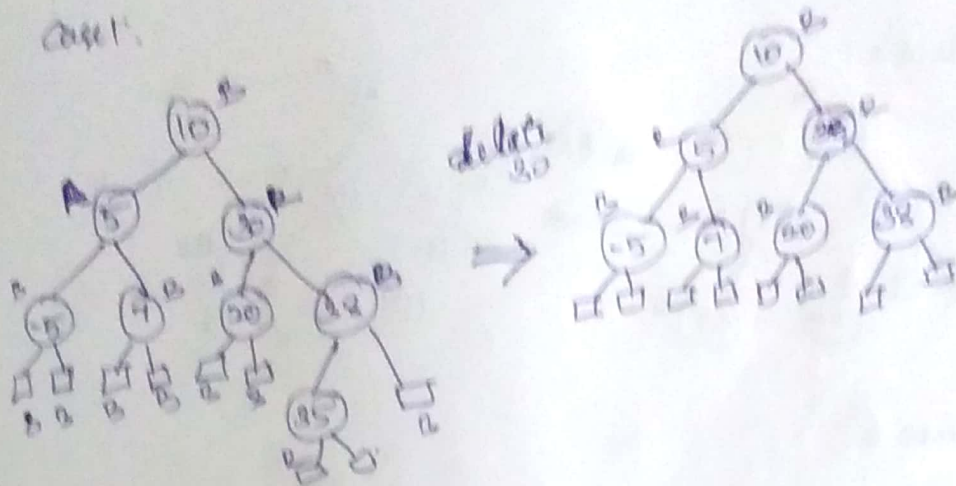
10, 20, -10, 15, 17, 40, 50, 60



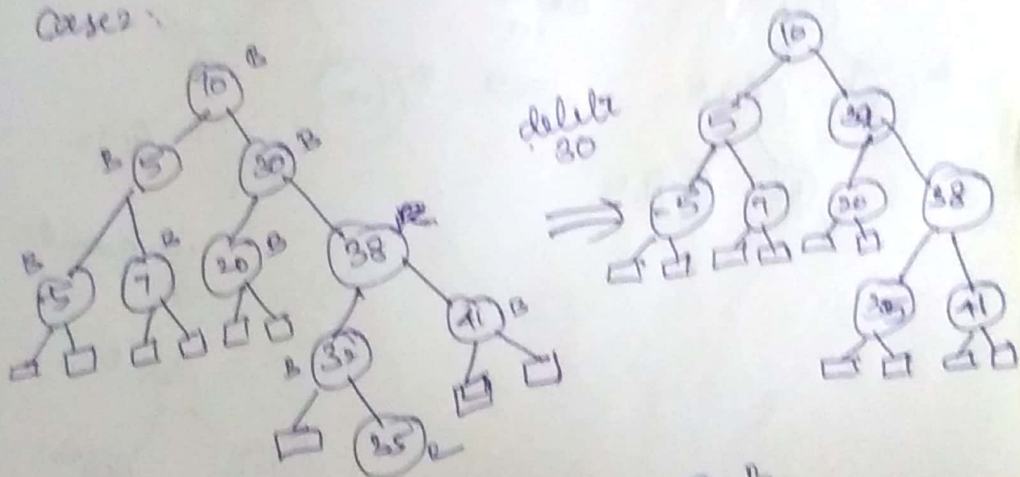
Red-Black Tree Deletion:

- convert to a ^{not null} ~~delete~~ case.
- If root node just delete it.
- If black node with red child then delete black & turn red into black node.

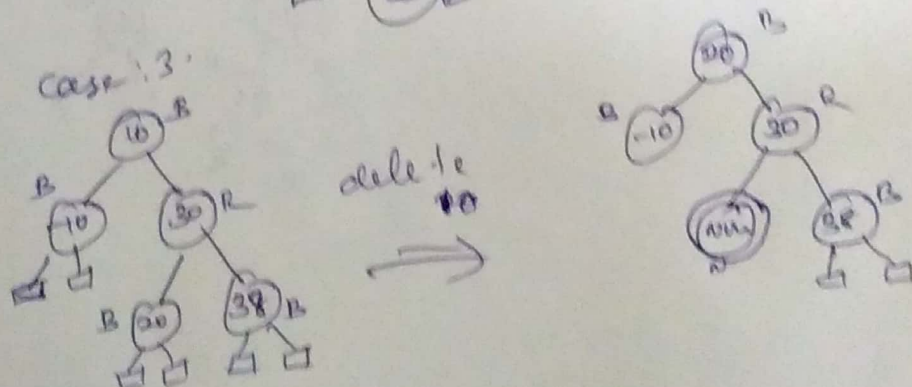
Case 1:

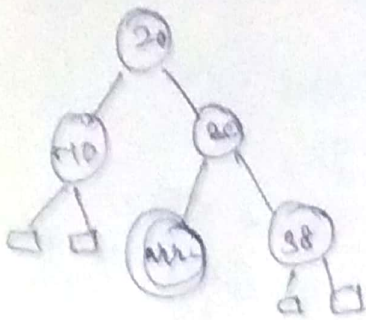


Case 2:

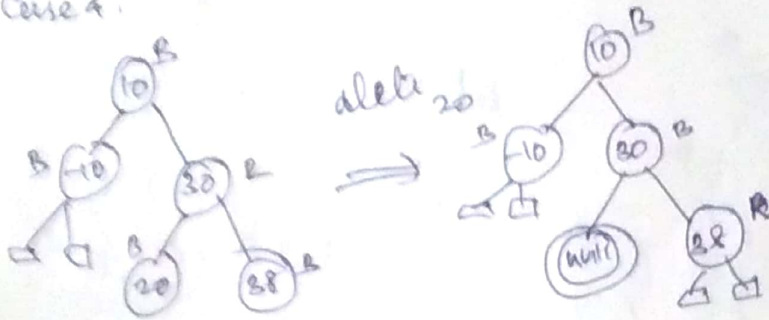


Case 3:

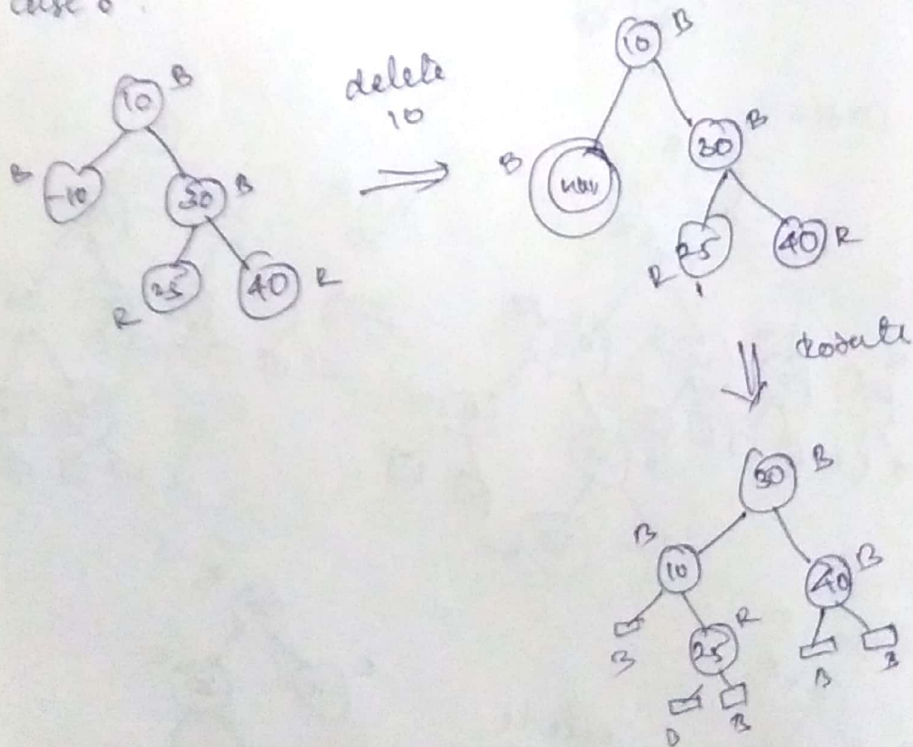




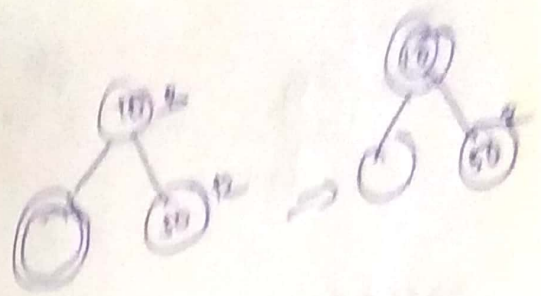
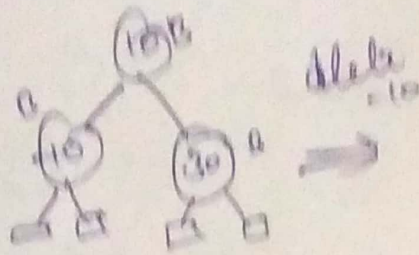
Case 4:



case 6:



Case 2



Case 5

