

Queuing Theory

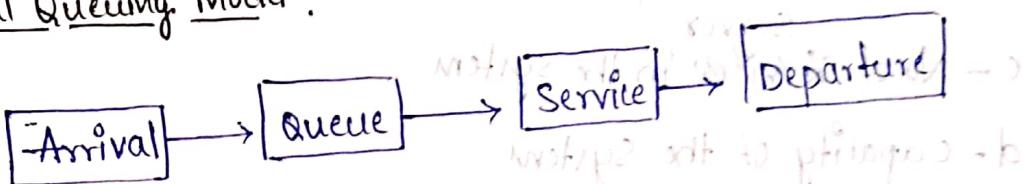
Queuing Theory Objective:

The objective is to have good economic balance.

Waiting time cost and service cost

It is used to find optimum solution for minimising the waiting time & service cost.

General Queuing Model:



Queue discipline:

- * FIFO (FCFS) (First in first out)

- * LIFO (LCFS)

- * Service in Random Order

- * priority

Customer Behaviour:

Balking: - Customers has no intention to join Queue

Raneging: - Customers may leave the Queue due to impatience

Jockeying: - Customers jump from one queue to other queue.

Arriving / Arrival rate of customers:

For poisson distribution, arrival rate = λ (no of customers)

For exponential distribution, arrival rate = $\frac{1}{\lambda}$ (one customer during an interval)

Service Cost:

For poisson, service rate = μ (no of customers served)

For exponential, service rate = $\frac{1}{\mu}$ (no of customers served in an interval instant)

Queuing Theory

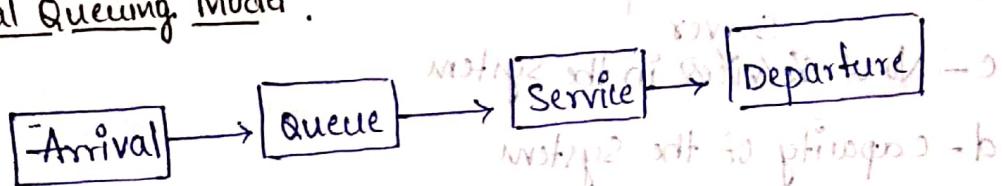
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Service Cost:

For poisson, service rate = μ (no of customers service)

For exponential, service rate = $\frac{1}{\mu}$ (No of customers Served in an instant)

System Capacity: Maximum no. of customers accommodated for service.

Kendall's Notation for a queuing System:

Notation: $(a/b/c):(d/e)$

a - no. of arrivals per unit time

b - no. of services per unit time

c - No. of servers in the system

d - capacity of the system

e - queueing discipline

Type I: $(M/M/1):(\infty/FIFO)$

$P_0 \rightarrow$ the system is idle

$P_1 \rightarrow$ the system is busy

n → no. of customers

N → Queueing capacity (avg no. of customers)

$L_s \rightarrow$ Average Queue length in System

$L_q \rightarrow$ Average Queue length

$W_s \rightarrow$ Waiting time in System

$W_q \rightarrow$ Waiting time in queue

System → queue + service

(X) (3)

Little's Formulae:

$$L_q = L_s - \frac{\lambda}{\mu}$$

$$W_s = \frac{L_s}{\lambda} \quad W_q = \text{average waiting time}$$

$$W_q = W_s - \frac{1}{\mu} \quad W_s = \text{average service time}$$

Type I Formulae

1) $P = \frac{\lambda}{\mu}$ (Traffic Intensity)

(row)

$$2) L_s = \frac{\lambda}{\mu - \lambda}$$

3) $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$

4) $W_s = \frac{L_s}{\lambda} = \frac{1}{\mu - \lambda}$

5) $W_q = W_s - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu - \lambda)}$

6) P (the system is busy)

$$= P_1 = \frac{\lambda}{\mu}$$

7) P (the system is idle) = $P_0 = 1 - P_1$

$$P_0 = 1 - \frac{\lambda}{\mu}$$

8) $P[W_s > t] = e^{-(\mu - \lambda)t}$

9) $P[W_q > t] = \frac{\lambda}{\mu} e^{-(\mu - \lambda)t}$

10) $P[N > k] = \left(\frac{\lambda}{\mu}\right)^{k+1}$

11) $P[N \geq k] = \left(\frac{\lambda}{\mu}\right)^k$

Problems On Type I ($M/M/1$): ($\infty/FIFO$) \rightarrow (single Server, infinite capacity)

a) A departmental store has a single cashier during the rush hrs
customers arrive at the rate of 20 per hr. The avg no of customers
than can be process by the cashier is 24 per hr.

(i) What does the prob that cashier is idle?

(ii) What is the avg no of customer in the queuing system

(iii) What does the avg time the customer spend in the system -

- (iv) What is the avg no of customers in a queue
- (v) Avg time spends by customer in queue ~~before waiting for service~~
- (vi) What is the probability that waiting time in system exceeds 30hrs.

So: $\lambda = 20 \text{ hr.} \rightarrow \text{Arrival rate}$

$\mu = 24 \text{ /hr} \rightarrow \text{Server rate.}$

$$\text{Traffic Intensity } f = \frac{\lambda}{\mu} = \frac{20}{24} = 0.83$$

$$\begin{aligned} (i) \quad P_0 &= 1 - P_1 \\ &= 1 - \frac{\lambda}{\mu} \\ &= 1 - 0.83 \\ &= 0.17 \end{aligned}$$

(ii) System = Queue + Service

$$\begin{aligned} \text{Avg customers} \quad L_s &= \frac{\lambda}{\mu - \lambda} \\ &= \frac{20}{4} \\ &= 5 \end{aligned}$$

$$\text{Avg queue length} \quad L_q = L_s - \frac{\lambda}{\mu}$$

we have got $L_s = 5$ which is 5 times of arrival rate so we have to wait for another 5 hrs on avg. since 0.83 is for one to serve 20 customers

$$(iii) \quad W_s = \frac{L_s}{\lambda} = \frac{5}{20} = \frac{1}{4} = 0.25 \text{ hr}$$

which means with all customers to be served with 2 servers (ii)

$$(IV) \text{ Let } L_s = \frac{\lambda}{\mu - \lambda}$$

$$\frac{1}{D_s} = \frac{1}{\mu - \lambda}$$

$$Lq = 4$$

$$\frac{1}{\mu - \lambda} = \frac{1}{24}$$

$$\frac{\lambda}{\mu - \lambda} = \frac{4}{24}$$

$$(V) W_q = W_s - \frac{1}{\mu}$$

$$= 0.25 - \frac{1}{24}$$

$$= 0.25 - 0.04$$

$$= 0.21 \text{ or } 5/24 \text{ hrs}$$

$$+ X_{st} =$$

$$dV =$$

$$80.0 =$$

$$(VI) P(W_s > 30)$$

$$P(W_s > t) = e^{-(\mu-\lambda)t}$$

$$= e^{-(24-20)t}$$

$$= e^{-4t}$$

$$= e^{-4(30)}$$

$$= e^{-120}$$

$$\frac{dV}{80.0 - 24.0} = \frac{t}{24 - 20} = 30$$

$$(part 1) \rightarrow (part 2) \rightarrow (iii)$$

$$80.0 - 79.9$$

(iv)

$$3(X-N) \rightarrow (31-27) \rightarrow (iii)$$

Q) Arrivals at telephone booth are considered to be poisson with the avg time 12 min b/w the arrivals and length of the phone call is consumed to be distributed exponentially with a mean 4min.

i) Find Avg no of persons waiting in a system

ii) What is the prob that a person arriving at a booth will have to wait in the queue.

iii) What is the prob that if he takes more than 12 min all together wait for the phone and complete this call

iv) Estimate the fraction of day with the phone is been used

v) The telephone department will instal a second booth when convince that arrival has to wait on the avg atleast 30 min

By how much flow of arrival increase in order to justify a 2nd booth

Sol: $\lambda = \frac{1}{12}$, $\mu = \frac{1}{4}$

Mean of Exponential = $\frac{1}{\mu}$

Traffic Intensity, $\rho = \frac{\lambda}{\mu}$

$$= \frac{1}{12} \times 4$$

$$= \frac{1}{3}$$

$$= 0.33$$

$$\frac{1-\rho}{\mu} = \rho W \quad (i)$$

$$\frac{1-\frac{1}{3}}{\frac{1}{4}} = \frac{2}{3} =$$

$$0.8 - 0.33 =$$

(i) L_s (Avg no of customers)

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{\frac{1}{12}}{\frac{1}{4} - \frac{1}{12}}$$

$$(0.8 - 0.33) \frac{1}{3} =$$

$$=$$

$$\frac{1}{3} = (0.8 - 0.33) \frac{1}{3} =$$

(ii) P (system is busy)

$$P_b = P_{\text{busy}} = 0.33$$

$$0.8 - \frac{1}{3} =$$

$$(0.8 - 0.33) \frac{1}{3} =$$

$$0.8 - \frac{1}{3} =$$

(iii) $P(W_s > 10) = e^{-(\mu - \lambda)t}$

prob of having 10 or more arrivals in 10 min = $e^{-\frac{10}{(\frac{1}{4} - \frac{1}{12})10}}$

if 100 arriving in 10 min then sloving with $\lambda = 10$ and $\mu = 1$

given $\lambda = 10$ and $\mu = 1$ then $\lambda - \mu = 9$ so e^{-9} is probability of having 10 or more arrivals in 10 min

$= e^{-9} = e^{-5/3}$ according to our previous notes

then obtained by giving $t = 10$ in formula $e^{-(\mu - \lambda)t}$

$$= e^{-10} = 0.37$$

(iv) phone is free \rightarrow system is busy during off peak time

$$P_b = P = \frac{\lambda}{\mu} = 0.33$$

so need to multiply with prob of additional off peak time

and under off peak hours $\lambda = 1$ because from graph probability of 1000 calls less than 1000 calls in 10 min is 0.33

(iv) $W_q > 3$

$$\frac{\lambda}{\mu(\mu-\lambda)} > 3$$

$$\frac{\lambda}{1/4(1/4-\lambda)} > 3$$

$$4\lambda > 3(1/4 - \lambda)$$

$$4\lambda > 3/4 - 3\lambda$$

$$7\lambda > 3/4$$

$$\lambda > 3/28$$

Diff b/w $3/28 - 1/12$:

$$= \frac{3-2}{42}$$

$$= 1/42$$

The flow of arrival when the customers atleast 3min = $1/42$

(Q) Customers arrive at single terminal computer centre to check their mail according to a poisson process the mean interval arrival time of 20min. The customers spend avg of 15min in the computer. If an hr is used as unit of time

(i) What is the probability that the customer need not to wait to check the mail

(ii) what is the expected number of customers and in queue

(iii) How much time can a customer expect to spend in queue

(iv) find the avg time the customer spend in queue

(v) find the prob that centre is idle

$$\text{Sol: } \lambda = 1/20, \mu = 1/15$$

$$(I) P_0 = \frac{1-\lambda}{\mu} = 1 - \frac{1}{20} \times 15 = 1 - \frac{3}{4} = 0.25$$

$$(II) L_s = \frac{\lambda}{\mu - \lambda} = \frac{1/20}{1/15 - 1/20} = \frac{1/20}{\cancel{60}/\cancel{4-3}} = \frac{1/20}{60} = 1/20 \times 60 = 3$$

$$L_s = 3$$

$$L_q = L_s - \frac{\lambda}{\mu} = 3 - 0.75 = 2.25 \approx 2$$

$$L_q = 2$$

$$(III) W_s = \frac{L_s}{\lambda} = \frac{3}{1/20} = 60 \text{ min} = 1 \text{ hr}$$

$$W_s = 1 \text{ hr}$$

\Rightarrow Nine hours to complete one cycle required to wash off all foreign bodies from mouth & nose.

 $W_q = W_s - \frac{1}{\mu}$ of time taken by mouth & nose to wash off all foreign bodies.
 $= 60 - \frac{1}{1/15} = 60 - 15 = 45 \text{ min}$ for mouth & nose to wash off all foreign bodies.
 $= 60 - 0.064 \times 60 - 15 = 40.56 \text{ min}$ for mouth & nose to wash off all foreign bodies.
 $= 60 - 40.56 = 19.44 \text{ min}$ for mouth & nose to wash off all foreign bodies.
 $= 19.44 \times 60 = 1166.4 \text{ sec}$ for mouth & nose to wash off all foreign bodies.
 $W_q = 3/4 \text{ hrs}$

(IV) $P_w = 1 - \frac{\lambda}{\mu}$ time required to admit patients with 25 beds.

Admission rate of patients per hour is same as admission rate of patients per hour.

 $= 0.25$

Time required to admit patients with 25 beds.

Time required to admit patients with 25 beds.

$$\therefore P_0 = 0.25$$

$$L_s = 3$$

$$L_q = 2$$

$$W_s = 1 \text{ hr}$$

$$W_q = 3/4 \text{ hrs}$$

$$E(W_q) = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{3}{10} = 0.3 \text{ hrs}$$

$$E(W_q) = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{3}{10} = 0.3 \text{ hrs}$$

(Q) Customers arrive at 1 man barber shop according to poisson's process with the mean

customers spend an avg of 10ms in the barber chair.

(i) What is the expected number of customers in barber shop and in queue?

(ii) calculate the % of time an arrival can walk straightly to barber chair if he without having to wait

(iii) How much time can a customer expect to spend in barber shop?

(iv) Management will provide other chair and their other barber when the customers waiting time exceeds 1.25hr. How much

(v) what is the avg time customers spend in queue?

(vi) what is the probability that waiting time exceeds t?

(vii) calculate the % of customers

(viii) What is the probability that more than 3 customers in system.

$$\text{Sol} \quad \lambda = \frac{1}{12}, \quad \mu = \frac{1}{10}$$

$$\text{Traffic intensity } \rho = \frac{\lambda}{\mu} = 0.83$$

$$(i) L_s = \frac{\lambda}{\mu - \lambda} = 5$$

$$L_q = L_s - \frac{\lambda}{\mu} = 5 - 0.83 \\ = 4.17 \approx 4$$

(ii) $P(\text{customer need not wait}) = P(\text{system is idle})$

$$\Rightarrow P_0 = 1 - P_i$$

$$= 1 - 0.83$$

$$= 0.17 \text{ or } 17\%$$

$$\text{percentage} = 0.17 \times 100$$

$$= 17\%$$

$$(iii) W_s = \frac{L_s}{\lambda} = \frac{5}{1/12} = 60 \text{ min} = 1 \text{ hr}$$

$$W_s > 1.25 \text{ hrs}$$

$$W_s > (1.25)(60)$$

$$W_s > 75 \text{ min}$$

$$\frac{1}{\mu - \lambda_s} > 75$$

$$\frac{1}{10\lambda_r} > 75$$

$$\frac{10}{10\lambda_r} > 75$$

$$10 > 75 - 750\lambda_r$$

$$\frac{65}{750} > \lambda_r \Rightarrow \lambda_r < \frac{-65}{750} = -\frac{13}{150}$$

$$-750\lambda_r < 10 - 75$$

$$-750\lambda_r < -65$$

$$\lambda_r > \frac{65}{750}$$

$$(V) W_q = \frac{\lambda}{\mu(\mu-\lambda)} = W_s - \frac{1}{\mu} = 60 - \frac{1}{10} = 59.9 \text{ min}$$

$$(VI) P(W_s > t) = e^{-\lambda t} = e^{-(\mu-\lambda)t} = e^{-(\gamma_{10} - \gamma_{12})t} = e^{-(0.3)t} = 0.6065$$

$$(VII) P(\text{customer to wait}) = P(\text{system is busy})$$

$$P_1 = 1 - P_0 = 1 - \frac{1}{1+\lambda} = \frac{\lambda}{1+\lambda} = \frac{0.3}{1+0.3} = 0.83$$

percentage = 83%

$$(VIII) P(N > k) = (\lambda/\mu)^{k+1}$$

$$P(N > 3) = (0.83)^{3+1} = (0.83)^4 = 0.4823$$

(Q) In a city airport flights arrive at the rate of 24 per day. It is known that inter arrival time follows an exponential distribution and service type also follows ED. with an avg of 30 mins.

(i) what is the prob. that airport is idle

(ii) find the avg no of flights in queue

(iii) find prob that queue size exceeds 7

(iv) If the input of flight increases to an avg 30 time per day.

What will be the changes of question 1, 2, 3.

Sol) $\lambda = \frac{24}{30} = 1 \text{ per hr}$ $\mu = \frac{1}{30} \text{ per min} = \frac{1}{30} \times 60 = 2 \text{ per hr}$

$$P = \frac{1}{\frac{\lambda}{\mu}} = \frac{1}{\frac{1}{2}} = 0.5 \quad (\text{Traffic Intensity})$$

(i) $P(\text{system is idle}) = P_0$

$$P_0 = 1 - P_1 \\ = 1 - 0.5$$

$$= 0.5 \quad (\text{prob of not being in service or queue})$$

(ii) $L_s = \frac{\lambda}{(\lambda - \mu)} = \frac{1}{1-1} = 1$

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = L_s - \frac{\lambda}{\mu}$$

$$= 1 - 0.5 \quad (\text{prob of queueing})$$

$$= 0.5 \quad (\text{prob of queueing})$$

(iii) $P(N > 7) = \left(\frac{\lambda}{\mu}\right)^{k+1} = (0.5)^8 = 3.90 \times 10^{-8} = 0.00039$

(iv) $\lambda = \frac{30}{24} = 1.25 \text{ per hr}$ $\mu = \frac{1}{30} \text{ per min}$

$\lambda = 1.25$ $\mu = \frac{1}{30} \times 60 = 2 \text{ per hr}$ $P = \frac{\lambda}{\mu} = 0.625 \text{ hr}^{-1}$

(i) $P_0 = 1 - 0.625 = 0.375$

(ii) $L_s = 1.041 \approx 1$

(iii) $P(N > 7) = (0.625)^8 = 0.0232$

Type II: $(M/M/1)$: $(K/FIFO)$ (single server, infinite capacity)
 Probability of finding n customers in system = P_n
 $P_0 = \frac{\lambda}{\mu}$ (system busy) \rightarrow $\lambda < \mu$

2. Traffic Intensity $\rho = \frac{\lambda}{\mu}$ will be ≤ 1 if traffic is
 $\lambda < \mu$ \rightarrow $P_0 = \rho P_0$

3. $P_1 = \rho P_0$ \rightarrow $\lambda < \mu$ \rightarrow $\lambda = \mu - \rho \mu$

4. $P_0 = \begin{cases} \frac{1-\rho}{1-\rho^{k+1}}, & \lambda \neq \mu \\ \frac{1}{k+1}, & \lambda = \mu \end{cases}$ \rightarrow $\lambda = \mu$ \rightarrow $\lambda = \mu$

4. $P_q =$ the customer need not wait $(0.1111) : (1/M/1)$

$$P_n = \begin{cases} \rho^n P_0, & \lambda \neq \mu \\ \frac{1}{k+1}, & \lambda = \mu \end{cases}$$

$$5. L_s = \frac{\frac{\rho}{1-\rho}}{1-\rho} = \frac{\rho}{1-\rho^{k+1}}, \lambda \neq \mu$$

$$= \frac{\frac{\lambda}{\lambda-\mu}}{1-\frac{\lambda}{\lambda-\mu}} = \frac{\lambda}{\lambda-\mu} = \frac{\lambda}{\lambda-\lambda+\rho\mu} = \frac{1}{1-\rho} = 880.0$$

6. $P(\text{customer turned away}) = P_k = \rho^k P_0$

Little's formulae:

1. Effective arrival state $\lambda' = \mu(1-P_0)$

$$2. L_q = L_s - \frac{\lambda'}{\mu}$$

$$3. W_s = \frac{L_s}{\lambda'} = \frac{(880.0)(20.0)}{2800.0} = 64.0$$

$$4. W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda'}$$

(Q) The local one person barber shop can accommodate a maximum of 5 people at a time. Customer arrives according to Poisson distribution with mean 5 per hr. The barber cuts hair at average rate of 4 per hr.

- (i) What is the % of time is barber is idle?
- (ii) What fraction of day the potential customers turned away?
- (iii) What is the expected no. of customers waiting for haircut?
- (iv) How much time can a customer expect to spend in Barber shop?

Sol: The given problem is type II

(M/M/I): (k/FIFO) Thus, formulae are used with $\lambda = \mu$

$$\lambda = 5 \quad \mu = 4 \quad k = 5, \lambda \neq \mu$$

$$f = \frac{\lambda}{\mu} = \frac{5}{4} = 1.25$$

$$(1) P_0 = \frac{1-f}{1-f^{k+1}} = \frac{1-1.25}{1-(1.25)^6} = \frac{1-\frac{0.25}{F(1.25)}}{1-(1.25)^6} = \frac{\frac{-0.25}{F(1.25)}}{1-(1.25)^6} = \frac{-0.25}{1-(1.25)^6} = \frac{-0.25}{1-1.488} = \frac{-0.25}{-0.488} = 0.51$$

$$= 0.888$$

percentage = 188% or 8.8% (approximate number)

(ii) P(customers turned away)

$$P_5 = f^5 P_0$$

$$= (1.25)^5 (0.088)$$

$$= (3.05) (0.088)$$

$$= 0.2685$$

percentage = 26.85% .

$$(iii) L_q = \frac{L_s}{\mu} - \frac{\lambda^1}{\mu}$$

Arrivals & service times follow exponential distribution with λ & μ respectively.

$$L_s = \frac{\rho}{1-\rho} = \frac{(k+1) \cdot \rho^{k+1}}{1-\rho^{k+1}}$$

$$= \frac{1.25}{1-1.25} = \frac{6(1.25)^6}{1-(1.25)^6}$$

$$= 3.13$$

$$\lambda^1 = \mu(1-P_0)$$

$$= 4(1-0.25) = 4(0.75) = 3.00$$

$$= 4(1-0.088) = 4(0.912) = 3.65$$

$(1-\lambda)\mu = k$ after finding waiting

$$L_q = 3.13 - \frac{3.65}{4}$$

$$= 3.13 - 0.912$$

$$= 2.218 \approx 2 \text{ per hr}$$

$$(iv) W_{qs} = \frac{L_s}{\lambda^1}$$

$$= \frac{3.13}{3.65}$$

$$= 0.857 \text{ hr}$$

$$= 51.5 \text{ mints.}$$

(Q) The patients arrived at clinic with 1 consultant according to poisson distribution 30 per hr. The waiting queue does not accommodate more than 14 patients. Examination time for patient is exponential with mean rate 20 per hr.

(1) find effective arrival rate

(2) what is the probability that at arriving patients will not wait

(3) How much time can a patient expect to spend in centre

(4) Find the average time the patient spent in waiting room

(5) What is the expected waiting time until the patient is

discharge from the clinic

(6) what is the probability that patient has to wait

Sol: The given problem is of $(M/M/1) : (K/FIFO)$

$$\lambda = 30 \quad (\text{Arrival rate}) \quad K = 14+1 = 15, \lambda \neq \mu$$

$$(I) \rho = \frac{\lambda}{\mu} = \frac{30}{40} = 0.75$$

Effective arrival rate $\lambda' = \mu(1 - \rho)$

$$P_0 = \frac{1-\rho}{1-\rho^{K+1}} \quad P_0 + \lambda' t - P_0$$

$$\Rightarrow \frac{1-0.75}{1-(1.5)^{16}} = \frac{-0.5}{1-(1.5)^{16}} = \frac{-0.5}{-656.84} = 0.00078$$

$$\lambda' = 20(1 - 0.00078) = 19.9844 \approx 20 \text{ per hr}$$

(II) $P(\text{customer need not wait})$

$$P_0 = 0.00078$$

$$(III) W_s = \frac{L_s}{\lambda'} \quad \text{Waiting time for service after arrival to service starting point}$$
$$L_s = \frac{\rho}{1-\rho} \cdot \frac{(K+1)\rho^{K+1}}{1-\rho^{K+1}} \quad \text{Waiting time for service starting point}$$
$$\text{for } W_s = \frac{1.5}{1-0.5} \cdot \frac{(16)(1.5)^{16}}{1-(1.5)^{16}} \quad \text{Shortest waiting time for service starting point}$$

$$\text{Waiting time} \Rightarrow \frac{1.5}{0.5} \cdot \frac{1(16)(656.84)}{1-656.84} = \frac{1.5}{0.5} \cdot \frac{10509.45}{1-656.84} \quad \text{Waiting time}$$

Waiting time for service starting point = 13

Waiting time for service starting point = 13

$$W_s = \frac{13}{40} = 0.65 \text{ hrs}$$

$$(IV) W_q = W_s - \frac{1}{\mu} = 0.65 - 0.05 \\ = 0.6$$

$$(V) W_s = 0.65 \text{ hr}$$

$$(VI) P(\text{patient has to wait}) = 1 - P_0$$

$$= 1 - 0.00078$$

$$= 0.99922 //$$

(Q) At railway station only 1 train is handled at a time. The railway yard is sufficient only for 2 trains to wait. While other is given signal to leave the station. Trains arrives at a station at a avg rate of 6 per hr and the railway station can handle them on the avg of 6 per hr. Assume poisson arrivals and exponential distribution. Find

(i) Find the probability that no of trains in the system

(ii) Find avg waiting time of new train coming into the yard

(iii) If the handling wait is double how will the above results

get modified.

$$\lambda = 6 \text{ per hr} \quad \mu = 6 \text{ /hr} \quad k = 2+1 = 3$$

$$f = \frac{\lambda}{\mu} = 1$$

$$\boxed{\lambda = \mu}$$

$$(i) P_n = \frac{1}{k+1} = \frac{1}{4} = 0.25$$

$$(ii) W_s = \frac{L_s}{\lambda^1} \quad \lambda = \mu(1 - P_0) \quad P_0 = \frac{1}{k+1}$$

$$= \frac{1.5}{4.5} = \frac{1}{3} \text{ hr} \quad = 6 \times \frac{3}{4} \quad = \frac{1}{4}$$

$$= 20 \text{ units} = 4.5 \text{ /hr.}$$

$$(iii) \mu = 12 \text{ per hr}, \lambda = 6 \text{ per hr} \quad k = \frac{3}{20.0 - 12.0} = \frac{3}{8} = 0.375$$

$$\boxed{\lambda \neq \mu}$$

$$20.0 - 12.0 = \frac{1}{k} - \text{idle} = \text{busy}$$

$$8.0 = \frac{1}{k}$$

$$(i) P_n = f^n P_0 \quad f = \frac{6}{12} = \frac{1}{2} < 0.5 \text{ per hr}$$

$$= (0.5)^n (P_0)$$

$$\lambda' = \mu (1 - P_0) \text{ at steady state}$$

$$P_0 = \frac{1-f}{1-f^{k+1}} = \frac{1-0.5}{1-0.5^{12}} = 12(1-8/15) = 12(7/15)$$

wait to for halftime b/w 2 arrivals = $\frac{8.0}{12} = 0.6667$ hours
 $P(\text{no of trains}) = P_n$ after first arrival is busy position after
 the arrival instant available with prob. $= 5.6/12 = 0.4667$ chance of getting a train
 $P_n = f^n P_0$ after first arrival with prob. $= 0.5^n$ to get a train
 $\text{mean} = (0.5)^n \frac{8}{15}$ min. to prob. get no wait between nos
 b/w consecutive train arrivals has above prob.

(ii) $L_s = \frac{f}{1-f} = \frac{(k+1) P^{k+1}}{1-f^{k+1}} = \frac{11}{1-0.5^{12}} = 0.73$ minutes
 busy with $1-f$ prob. $\frac{11}{1-0.5^{12}} = 0.73$ to wait between two arrivals
 where waiting time used also to be busy with 0.5^n chance of getting a train

$$W_s = \frac{L_s}{\lambda'}$$

$$\Rightarrow \frac{0.73}{5.6} = \frac{11}{84} \text{ hrs} = 0.9 \text{ minutes}$$

$$d = \frac{L_s}{\lambda} = 0.73$$

$$20.0 = \frac{1}{f} = \frac{1}{1+f} = 0.9$$

$$\frac{1}{1+f} = 0.9 \quad (1+f) = \frac{1}{0.9} = 1.1111$$

$$\frac{1}{f} = \frac{1}{1+f} = \frac{1}{1+1.1111} = 0.5555$$

$$1.1111 = 1.1111$$

Arithematic Progression:

$a, a+d, a+2d, a+3d$ are said to be in A.P

last term = $t_n = n^{\text{th}}$ term

$$\rightarrow \text{Sum of } n \text{ terms} = \frac{n}{2} [2a + (n-1)d]$$

$$\rightarrow \text{Sum of } n \text{ terms} = \frac{n}{2} [a + l] \text{ where } l \text{ is last term.}$$

Geometric progression:

a, ar^2, ar^3, \dots are said to be in G.P in which

$$\rightarrow n^{\text{th}} \text{ term} = a \cdot r^{n-1}$$

first term = a

common ratio = r

$$\rightarrow \text{Sum of } n \text{ terms} = \frac{a(1-r^n)}{1-r}, \text{ where } r < 1$$

$$\frac{a(r^n-1)}{r-1}, \quad r > 1$$

* How many natural numbers b/w $\underline{78}$ and 80 are divisible by 6.

$$a + (n-1)d = 78 \\ a = 12, d = 6$$

$$12 + (11)6 = 78$$

$$= 12 + 66$$

$$= 78$$

Arithmetic Progression:

$a, a+d, a+2d, a+3d$ are said to be in A.P

last term = $t_n = n^{\text{th}} \text{ term}$

$$\rightarrow \text{Sum of } n \text{ terms} = \frac{n}{2} [2a + (n-1)d]$$

$$\rightarrow \text{sum of } n \text{ terms} = \frac{n}{2} [a + l] \text{ where } l \text{ is last term.}$$

Geometric progression:

a, ar^2, ar^3 are said to be in G.P in which

first term = a

common ratio = r

$$\rightarrow n^{\text{th}} \text{ term} = a \cdot r^{n-1}$$

$$\rightarrow \text{sum of } n \text{ terms} = \frac{a(1-r^n)}{1-r}, \text{ where } r < 1$$

$$\underbrace{a(r^n-1)}, \quad r > 1$$

* How many natural numbers b/w ± 8 and 80 are divisible by 6.

$$a + (n-1)d = \pm 8$$

$$a = \pm 12, d = 8$$

$$12 + (11)8 = \pm 8$$

$$= 12 + 66$$

$$= \pm 8$$