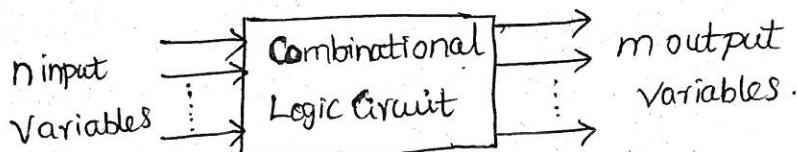


Unit - III - Combinational Logic Circuits.

Combinational Logic Circuit:

→ When logic gates are connected together to produce a specified output for certain specified combinations of input variables, with no storage device.



→ The combinational circuit consists of i/p variables, Logic gates, and output variables. The Logic gates accept signals from the i/p variables and generate output variables.

- (1). Design a Combination Logic circuit with 3 input variables that will produce a logic 1 output when more than one input variables are logic 1.

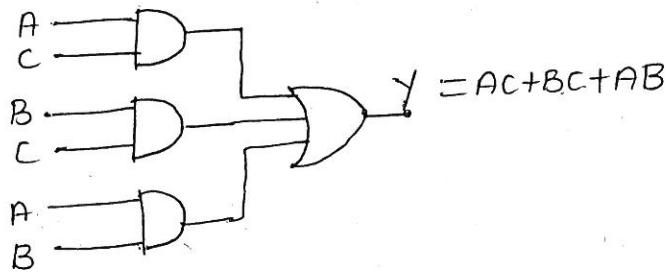
input variables			output (y)
A	B	C	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Simplified Boolean Expression,

		$\bar{B}C$	$\bar{B}C$	BC	$\bar{B}C$
		\bar{A}	0 ₀	0 ₁	1 ₃
		A	0 ₄	1 ₅	1 ₇
					0 ₂
					1 ₆

$$Y = AC + BC + AB$$

Logic diagram,



Adders:- → The simple addition consists of four possible basic operations,

$$0+0=0; 0+1=1; 1+0=1; 1+1=10;$$

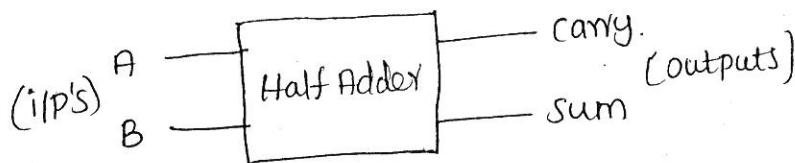
Types:

* Half Adder (The circuit which performs addition of 2 bits)

* Full Adder (The circuit which performs addition of 3 bits)

Half Adder:-

→ Half adder operation needs 2 binary inputs, and 2 binary outputs
(sum & carry).



inputs		outputs	
A	B	carry	sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Kmap Simplification for carry & sum,

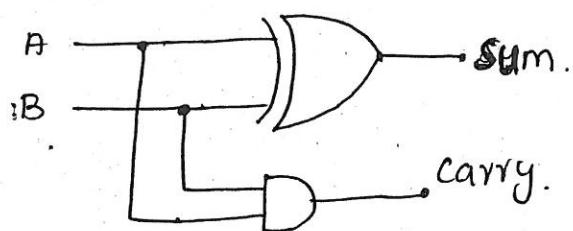
For carry

A	B	0	0
A	\bar{B}	0	1
\bar{A}	B	1	0
Carry = AB			

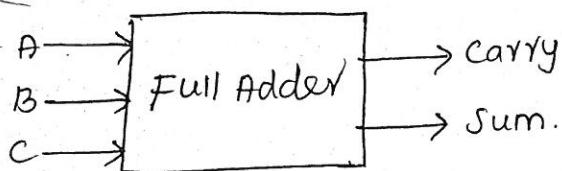
For sum

A	B	0	1
A	\bar{B}	0	1
\bar{A}	B	1	0
Sum = $A\bar{B} + \bar{B}A$ $\Rightarrow A \oplus B$			

Logic diagram,



Full-Adder: - It contains 3 inputs (A, B, C) & Two outputs (sum & carry).



Truth Table

Inputs			Output	
A	B	C	Carry	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

K-map Simplifications,

For carry ($Cout$)

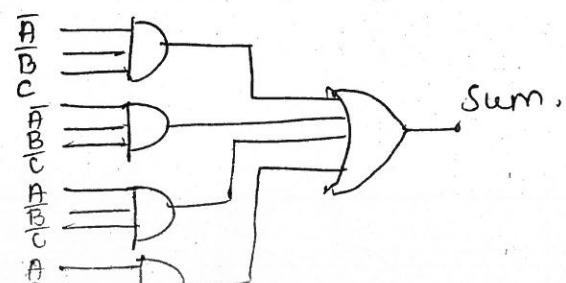
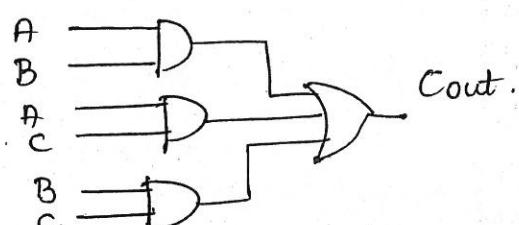
$\bar{A} \backslash BC$		$\bar{B}\bar{C}$		$\bar{B}C$		BC		$B\bar{C}$	
		$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	0	0	1	1	0	0	1	0	0
A	0	1	0	0	1	1	0	1	0

$$Cout = AB + AC + BC$$

For Sum

$\bar{A} \backslash BC$		$\bar{B}\bar{C}$		$\bar{B}C$		BC		$B\bar{C}$	
		$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	0	0	1	0	1	0	1	0	0
A	0	1	0	0	1	1	0	1	0

$$Sum = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$$

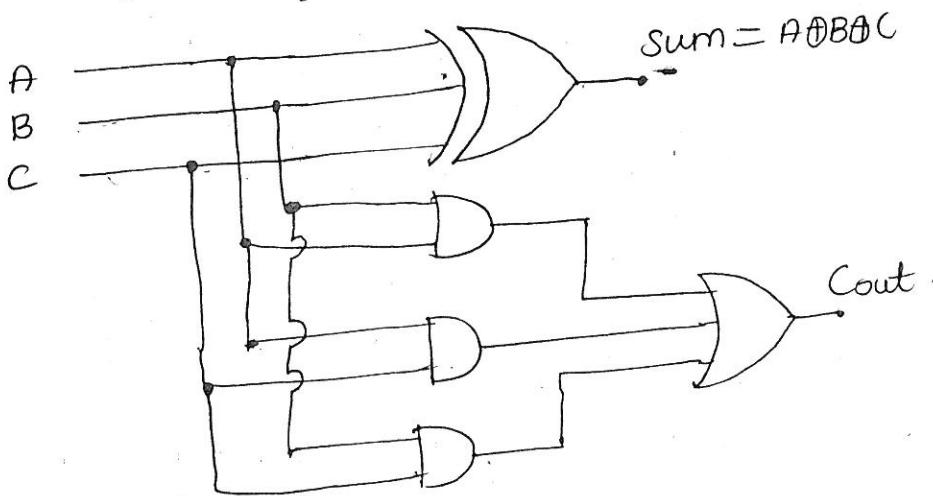


$$\begin{aligned}\text{Sum} &= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC \\ &= C(\bar{A}\bar{B} + AB) + \bar{C}(\bar{A}B + A\bar{B}) \\ &= C(\overline{A \oplus B}) + \bar{C}(A \oplus B)\end{aligned}$$

$$\boxed{\overline{A \oplus B} = \bar{A}\bar{B} + AB}$$

$$\boxed{\text{Sum} = C \oplus A \oplus B}$$

Implementation of Full adder,



i) Implement Full Adder Using 2 Half Adders?

$$\text{Sum} = A \oplus B \oplus C$$

$$\text{Carry} = AB + AC + BC \\ (\text{Cout})$$

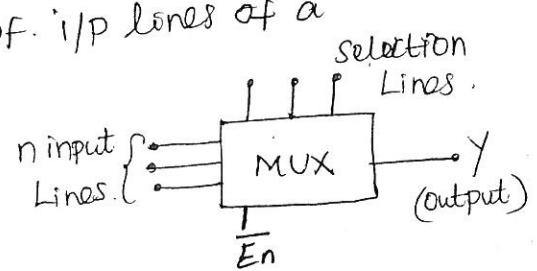
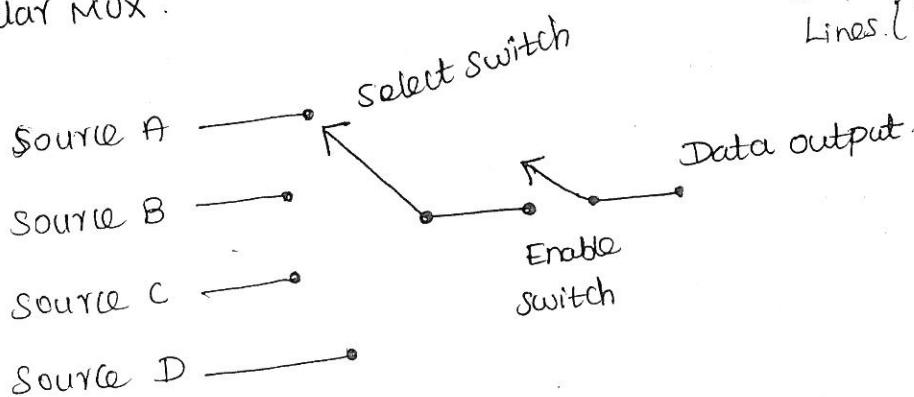
$$\begin{aligned}\text{Cout} &= AB + AC + BC(\bar{A} + \bar{A}) \\ &= AB + AC + BC + \cancel{AB\bar{C}} + \cancel{A\bar{B}C} + \cancel{(A+\bar{A})BC} \\ &= AB + AC + BC + \cancel{AB\bar{C}} + \cancel{ABC} + \cancel{A\bar{B}C} + \cancel{\bar{ABC}} \\ &= ABC + AB\bar{C} + A\bar{B}C + \bar{ABC} \Rightarrow AB(C + \bar{C}) + (\bar{A}B + A\bar{B})C\end{aligned}$$

$$\boxed{\text{Cout} = AB + C(A \oplus B)}$$

Multiplexers :- [Data Selector]

→ Multiplexer is a digital switch. It is a combinational circuit that select one digital information from several sources and transmit the selected information on a single O/P line.

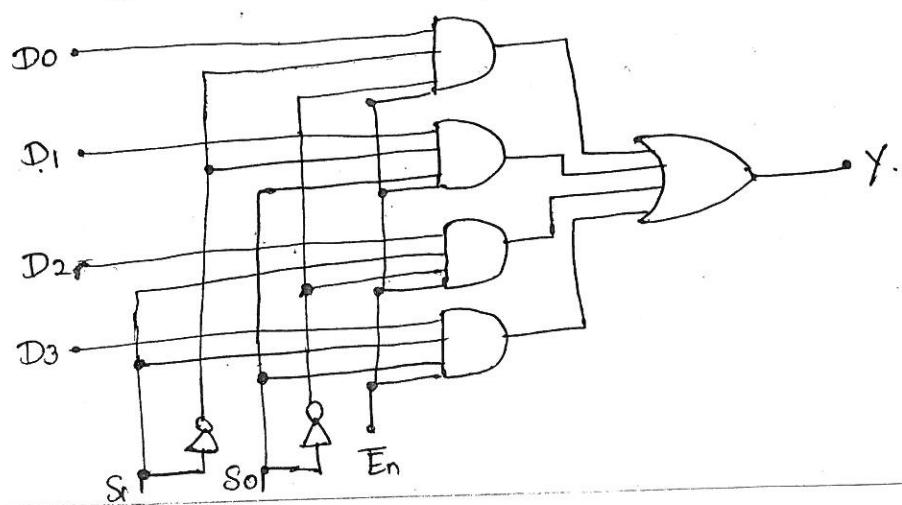
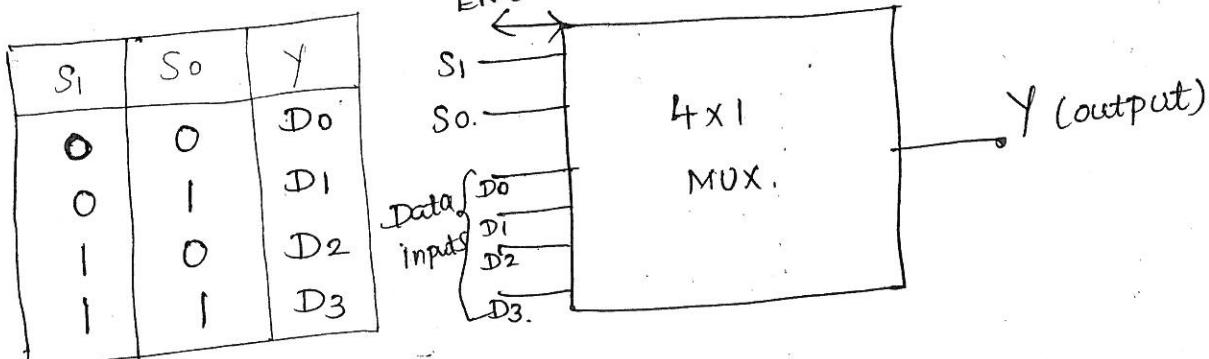
→ The selection line is used to decide the no. of i/p lines of a particular MUX.



4 to 1 MUX

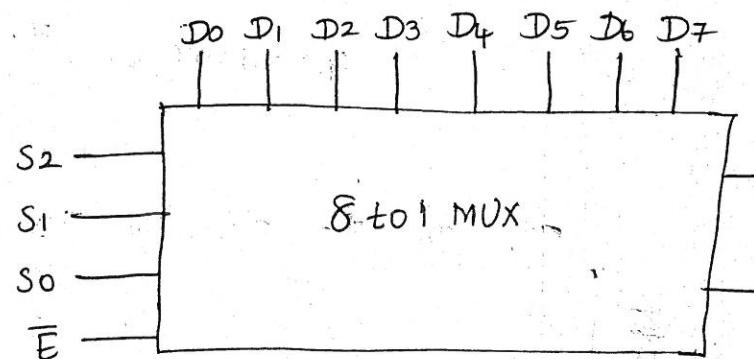
$$n=4 \text{ (} D_0 - D_3 \text{)}, \quad M=2 \text{ (} S_1 \& S_0 \text{), } Y=1.$$

\overline{EN} (enable input).

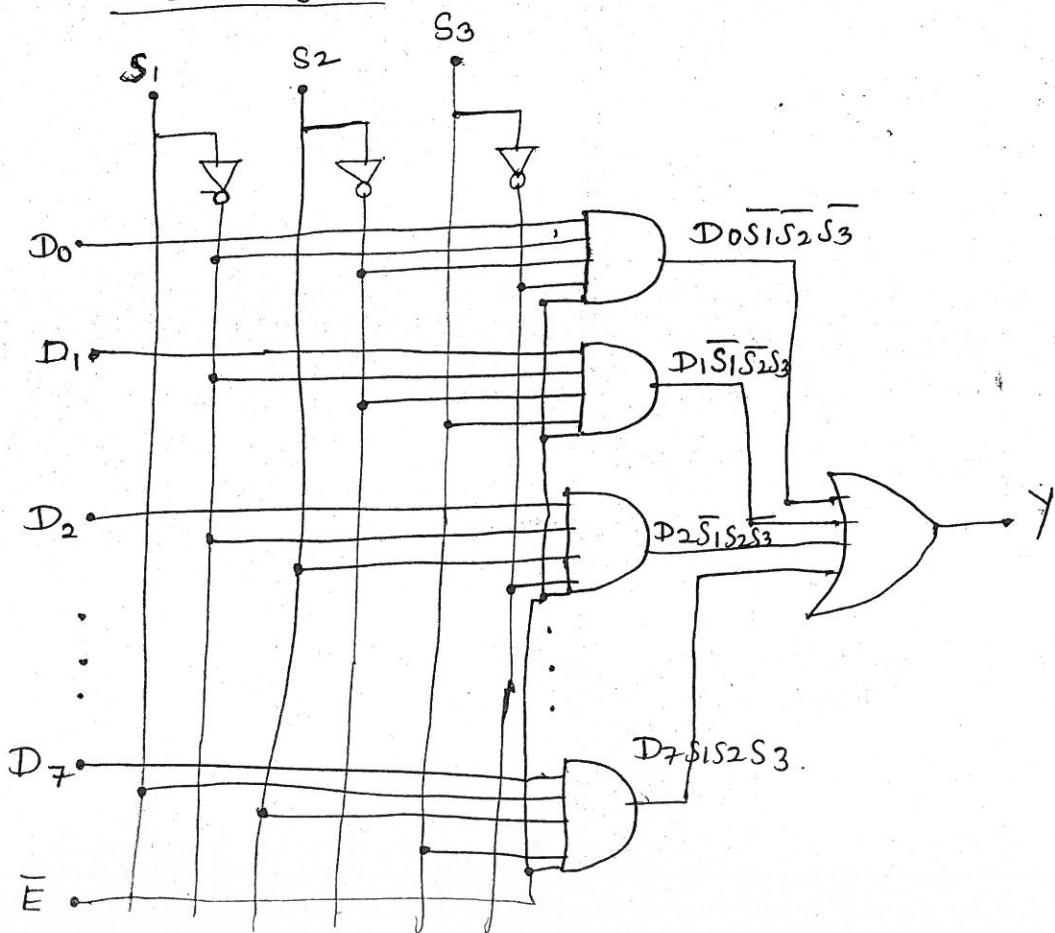


8 to 1 MUX :- $n=8(D_0-D_7)$, $y=1$, $m=2^n=3(S_0, S_1, S_2)$

Inputs			Outputs		
S_2	S_1	S_0	\bar{E}	Y	\bar{Y}
X X X			1	1	0
0 0 0			0	D_0	\bar{D}_0
0 0 1			0	D_1	\bar{D}_1
0 1 0			0	D_2	\bar{D}_2
0 1 1			0	D_3	\bar{D}_3
1 0 0			0	D_4	\bar{D}_4
1 0 1			0	D_5	\bar{D}_5
1 1 0			0	D_6	\bar{D}_6
1 1 1			0	D_7	\bar{D}_7



Logic Diagram



1) Implement the following boolean function with 8:1 MUX,

$$F(A_1B_1C_1D) = \prod M(0, 3, 5, 8, 9, 10, 12, 14)$$

2) Implement the following boolean function with 8:1 MUX

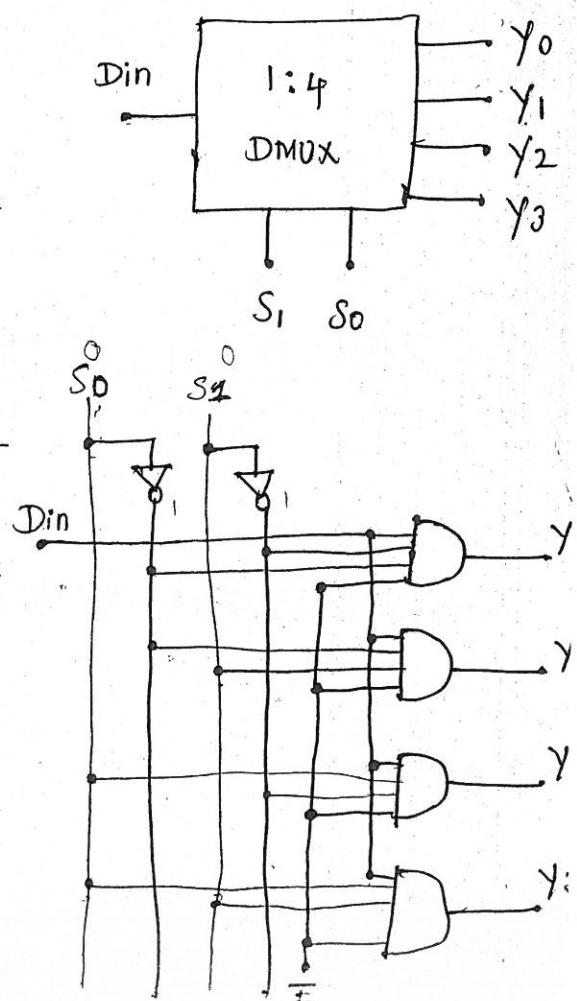
$$F(A_1B_1C_1D) = \sum m(0, 2, 6, 10, 11, 12, 13) + d(3, 8, 14)$$

Demultiplexers :-

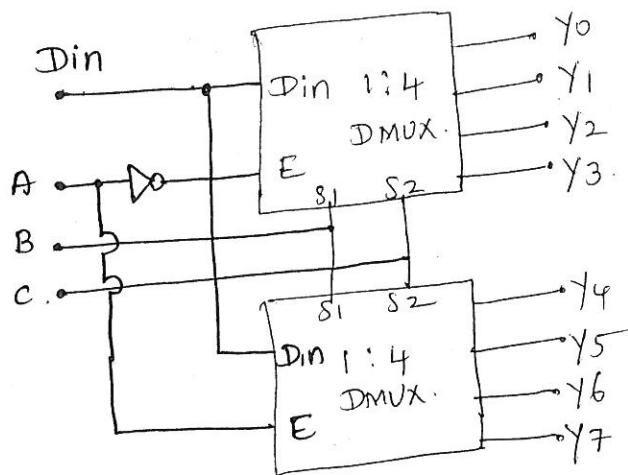
→ It receives the information on a single line and transmits this information on one of 2^n possible output lines. The selection of specific output line is controlled by the values of n selection lines.

1 to 4 DMUX.

Enable	S ₁ S ₀	Din	y ₀ y ₁ y ₂ y ₃
1	X X	X	0 0 0 0
0	0 0	0	0 0 0 0
0	0 0	1	1 0 0 0
0	0 1	0	0 0 0 0
0	0 1	1	0 1 0 0
0	1 0	0	0 0 0 0
0	1 0	1	0 0 1 0
0	1 1	0	0 0 0 0
0	1 1	1	0 0 0 1



2) Design 1:8 demultiplexer using two 1:4 multiplexers.



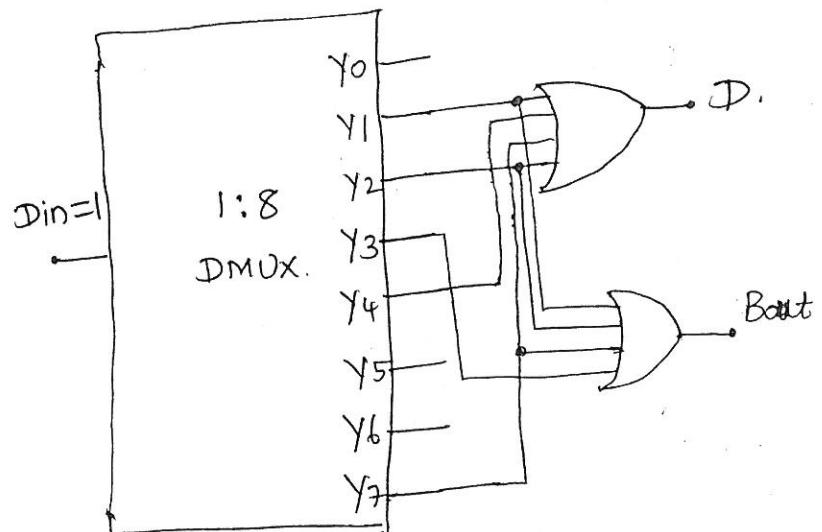
3) Implement full subtractor using demultiplexer.

Truth Table

A	B	Bin	D	Bout
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$\text{D} \Rightarrow f(A_1, B_1, C) = \sum m(1, 2, 4, 7).$$

$$\text{Bout} \Rightarrow f(A_1, B_1, C) = \sum m(1, 2, 3, 7).$$

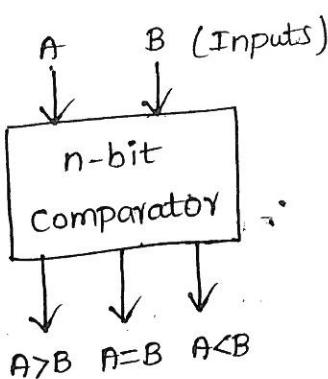


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4, 6, 7, 8, 9, 12, 15, 21, 22, 23, 29, 32, 33, 34, 36, 39, 41, 44, 46, 52,

59, 60,

Magnitude Comparator :-



→ A comparator is a Special Combinational circuit designed primarily to compare the relative magnitude of 2 binary numbers.

→ It receives 2 n-bit numbers A & B as inputs and outputs are $A > B$, $A = B$, & $A < B$.

(outputs). → Depending upon the relative magnitudes of the 2 numbers, one of the outputs will be high.

i). Design 2-bit comparator using gates.

Inputs	Outputs		
$A_1\ A_0\ B_1\ B_0$	$A > B$	$A = B$	$A < B$
0 0 0 0	0	1	0
0 0 0 1	0	0	1
0 0 1 0	0	0	1
0 0 1 1	0	0	1
0 1 0 0	1	0	0
0 1 0 1	0	1	0
0 1 1 0	0	0	1
0 1 1 1	0	0	1
1 0 0 0	1	0	0
1 0 0 1	1	0	0
1 0 1 0	0	1	0
1 0 1 1	0	0	1
1 1 0 0	1	0	0
1 1 0 1	1	0	0
1 1 1 0	1	0	0
1 1 1 1	0	1	0

K-map Simplification,

$(A > B)$

$\overline{B_0}\ \overline{B_1}$	$\overline{B_1}\ B_0$	$\overline{B}_1\ B_0$	$B_1\ B_0$	$B_1\ \overline{B_0}$
$\overline{A_1}\ \overline{A_0}$	0 ₀	0 ₁	0 ₃	0 ₂
$\overline{A}_1\ A_0$	1 ₄	0 ₅	0 ₇	0 ₆
$A_1\ \overline{A_0}$	1 ₁₂	1 ₁₃	0 ₁₅	1 ₁₄
$A_1\ A_0$	1 ₈	1 ₉	0 ₁₁	0 ₁₀

$$(A > B) = \overline{A_1} \overline{B_1} \overline{B_0} + A_1 \overline{B_1} + A_1 A_0 \overline{B_0}$$

(A = B)

	$\bar{B}_1\bar{B}_0$	\bar{B}_1B_0	$B_1\bar{B}_0$	B_1B_0
$\bar{A}_1\bar{A}_0$	0 ₆	0 ₁	0 ₃	0 ₂
\bar{A}_1A_0	0 ₄	0 ₅	0 ₂	0 ₆
$A_1\bar{A}_0$	0 ₁₂	0 ₁₃	0 ₁₅	0 ₁₄
A_1A_0	0 ₈	0 ₉	0 ₁₁	0 ₁₀

$$(A = B) = \bar{A}_1\bar{A}_0\bar{B}_1\bar{B}_0 + \bar{A}_1A_0\bar{B}_1B_0 + A_1\bar{A}_0B_1B_0 + A_1A_0\bar{B}_1\bar{B}_0$$

$$= \bar{A}_1\bar{B}_1(\bar{A}_0\bar{B}_0 + A_0B_0) + A_1B_1(A_0B_0 + \bar{A}_0\bar{B}_0)$$

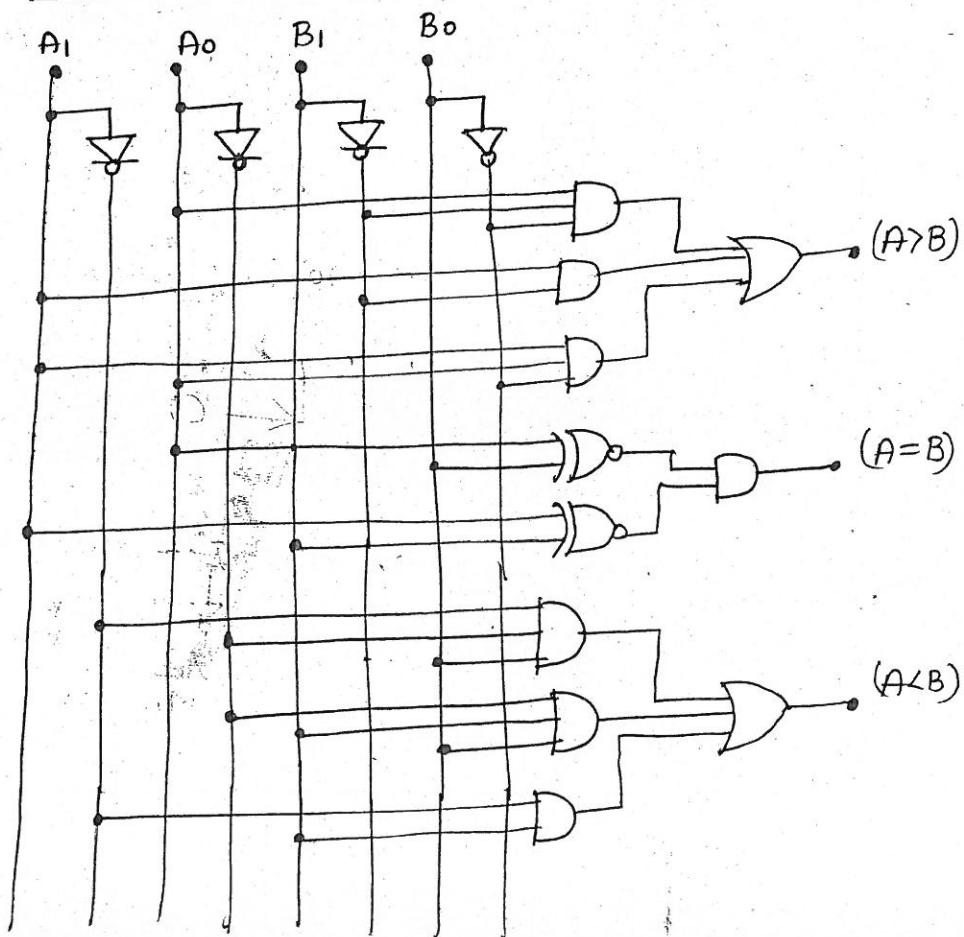
$(A = B) = (A_0 \odot B_0)(A_1 \odot B_1)$

(A < B)

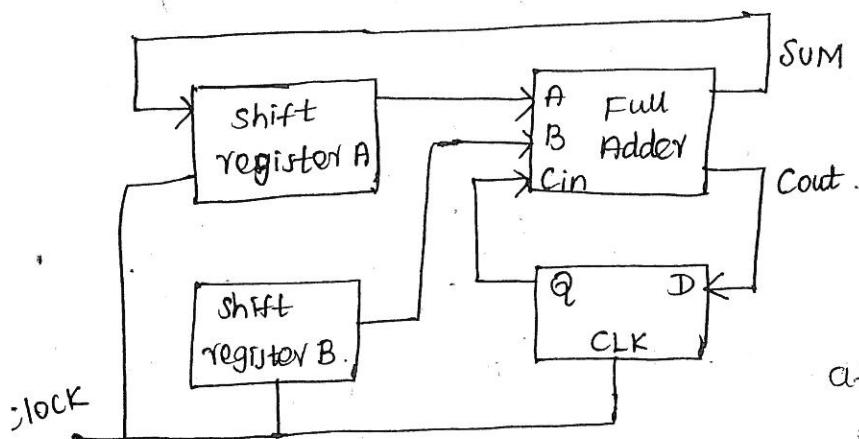
	$\bar{B}_1\bar{B}_0$	\bar{B}_1B_0	$B_1\bar{B}_0$	B_1B_0
$\bar{A}_1\bar{A}_0$	0 ₀	1 ₁	1 ₃	1 ₂
\bar{A}_1A_0	0 ₄	0 ₅	1 ₇	1 ₆
$A_1\bar{A}_0$	0 ₁₂	0 ₁₃	0 ₁₅	0 ₁₄
A_1A_0	0 ₈	0 ₉	1 ₁₁	0 ₁₀

$$(A < B) = \bar{A}_1\bar{A}_0B_0 + \bar{A}_0B_1B_0 + \bar{A}_1$$

Logic Diagram,



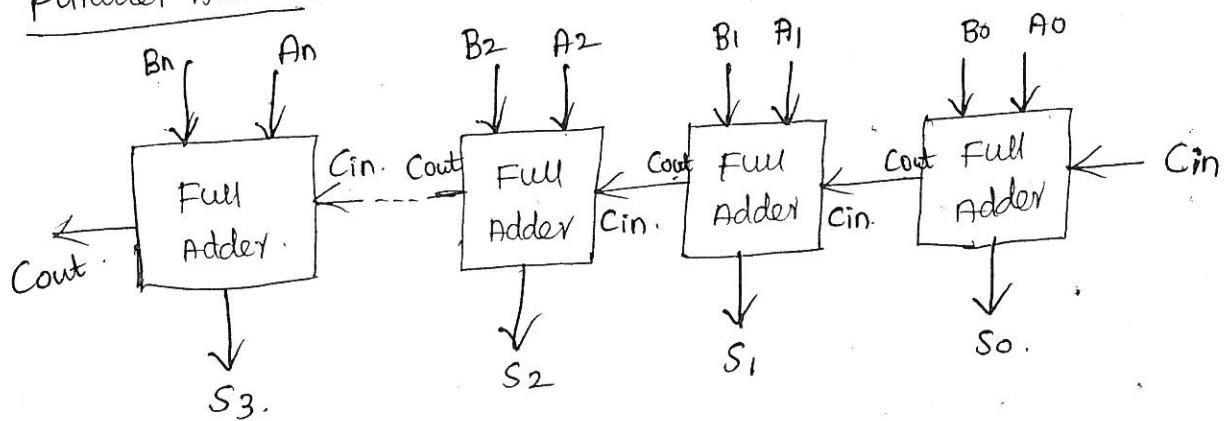
Serial Adder / Subtractor :-



→ Full adder is used to perform bit by bit addition and D-flip flop is used to store the carry output generated after addition.

- This carry is used as carry input for the next addition. Initially, the D-flip flop is cleared and addition starts with the least significant bits of both registers.
- After each clock pulse data within the shift registers are shifted right 1-bit and the result sum is stored bit by bit in the register A.

Parallel Adder :-

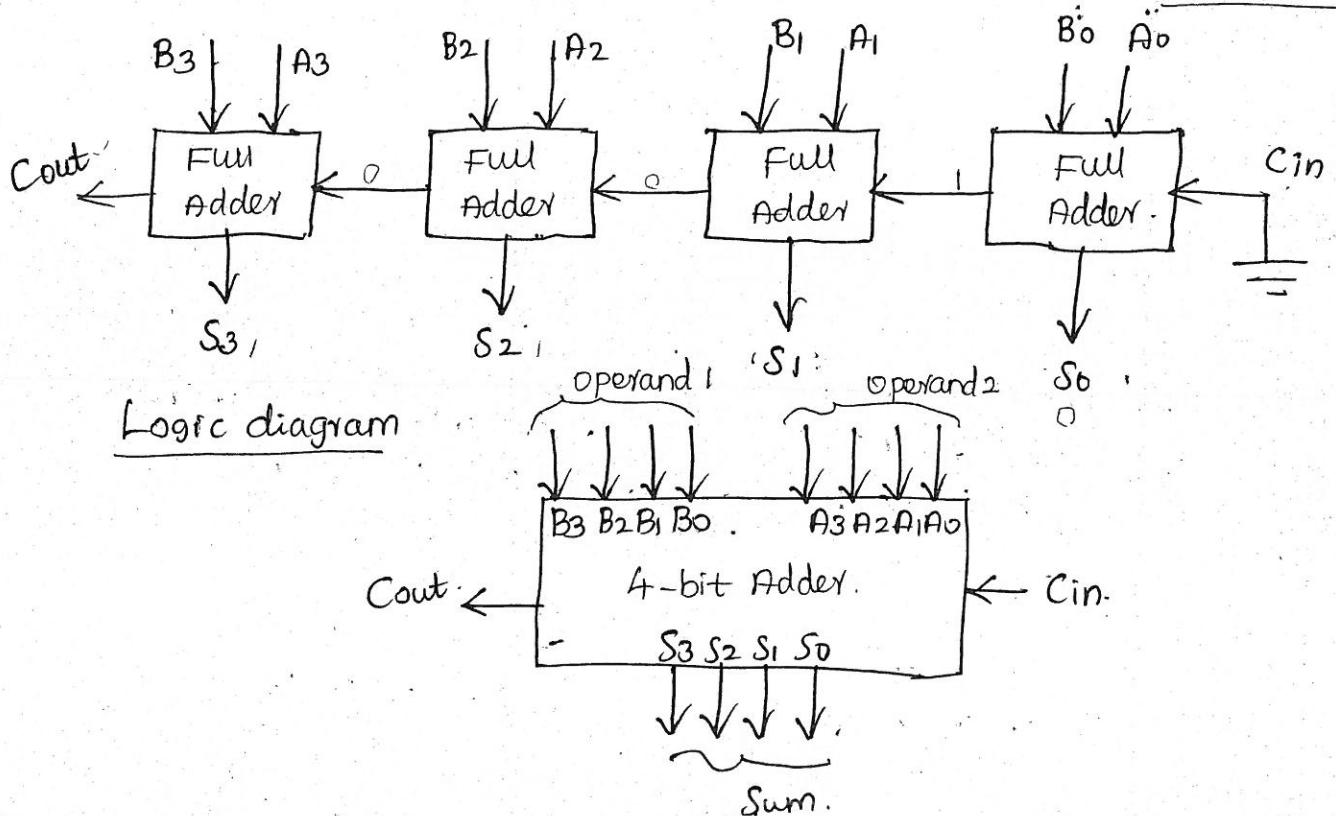


- A single full-adder is capable of adding two one-bit numbers and an input carry. In order to add binary numbers more than one bit, additional full-adders must be employed.

- A n bit, parallel adder can be constructed using no.of. full add circuits connected in parallel.
- ie, the carry output of each adder is connected to the carry in of the next higher-order adder.

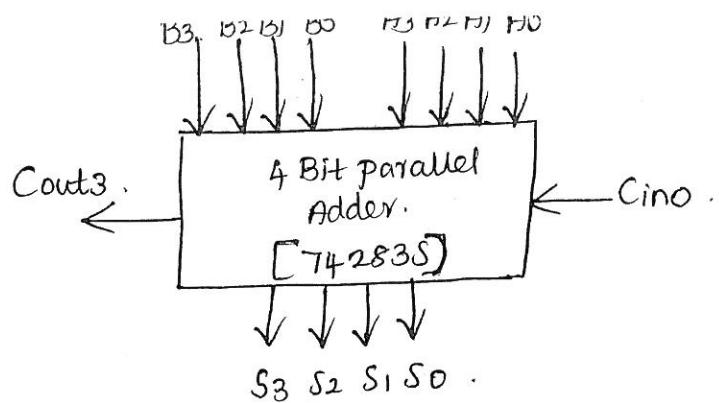
1) Design a 4-bit parallel adder using full-adders.

Block Diagram



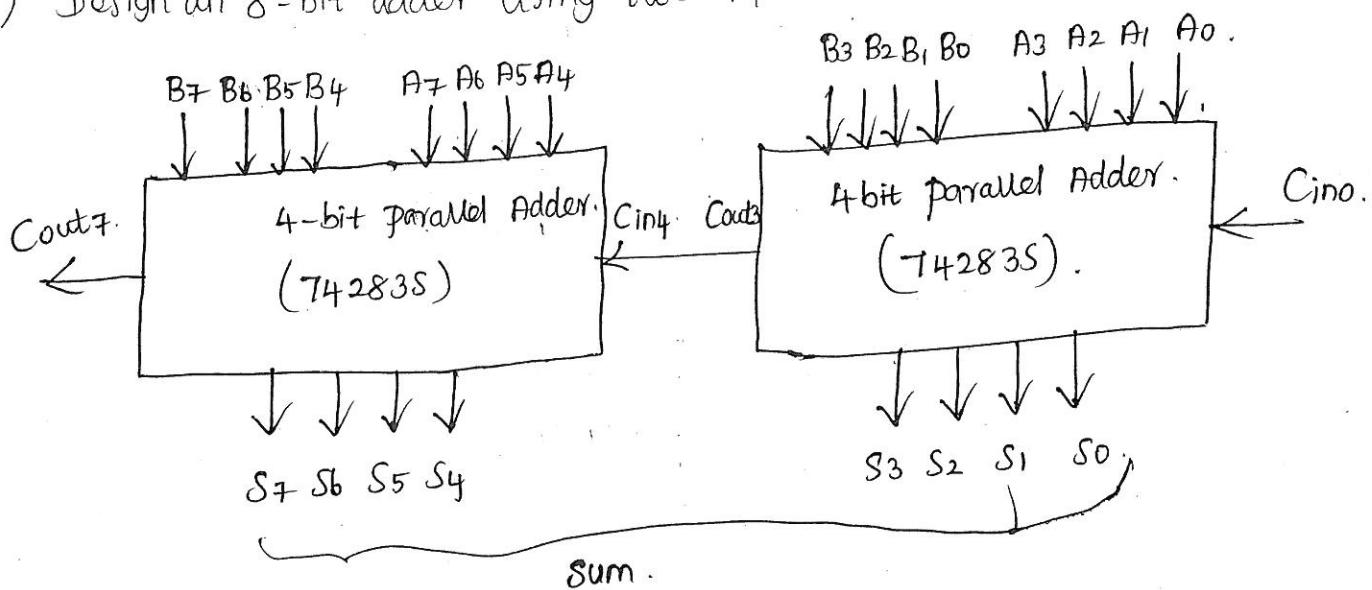
Binary parallel Adder (IC 74LS83 / 74283)

→ Many high-speed adders available in integrated circuit form utilize the look-ahead carry or a similar technique for reducing overall propagation delays. The most common is a 4-bit parallel adder IC (74LS83 / 74283) that contains 4 interconnected full adders and look-ahead carry circuitry needed for high speed operation.

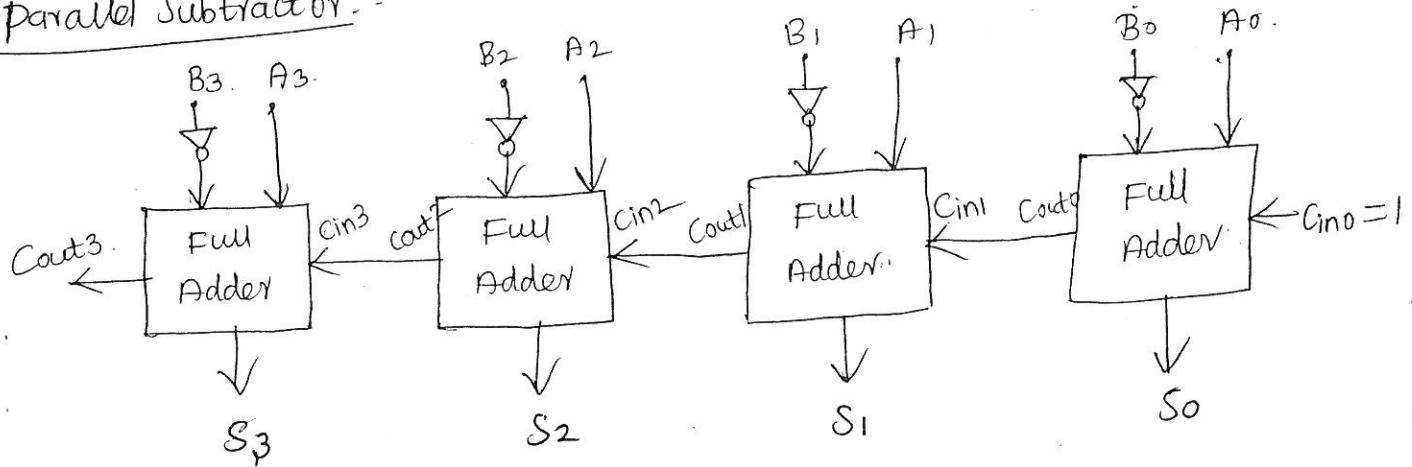


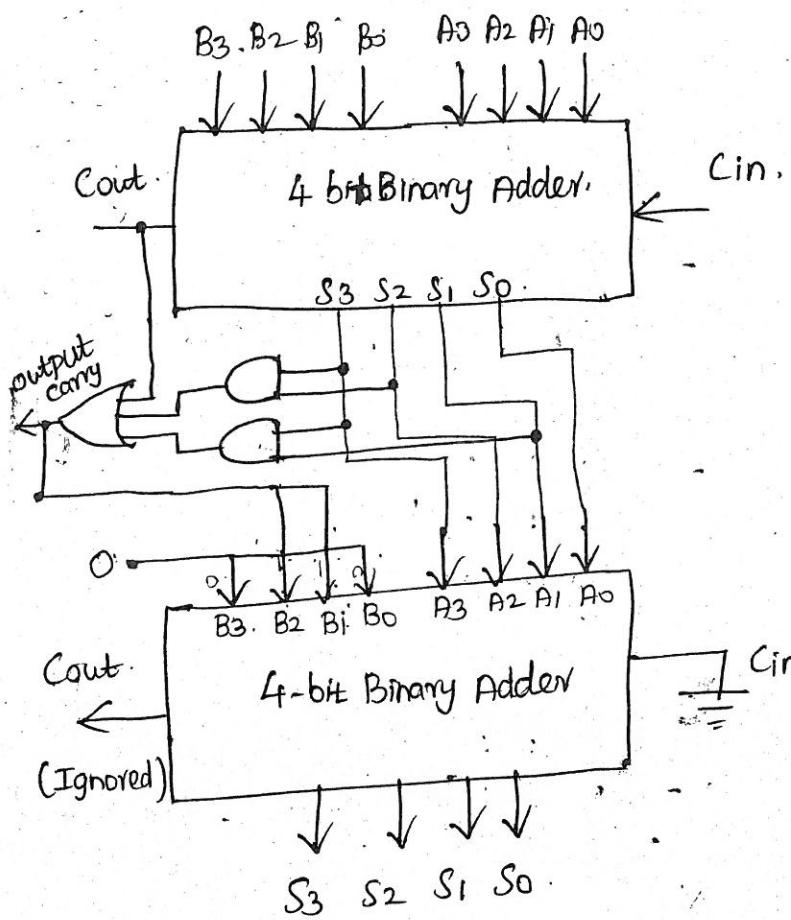
→ Input of IC are two 4-bit numbers, A₃A₂A₁A₀ and B₃B₂B₁B₀ and the carry Cin₀, into the LSB position. The Outputs are the sum bits S₃S₂S₁S₀ and the carry Cout₃, output of the MSB position.

1) Design an 8-bit adder using two 74283s.



Parallel Subtractor:





→ The 2 BCD numbers, together with i/p carry, are first added by the top 4-bit binary adder to produce a binary sum.

→ When the o/p carry is equal to zero. [i.e., when $\text{Sum} \leq 9$], $Cin=0$, $Cout=0$, nothing is added to binary sum.

→ When the o/p carry is equal to 1. (i.e., when $\text{Sum} > 9$ and $Cout=1$)
 binary 0110 is added to the binary sum. And the o/p carry generated from the bottom binary adder can be ignored.

BCD Adder:-

→ A BCD Adder is a circuit that adds the two BCD digits and produces a sum digit also in BCD.

* To Implement BCD adder,

- (i) 4 bit binary adder for initial addition.
- (ii) Logic circuit to detect Sum greater than 9,
- (iii) One more 4 bit adder to add $(10)_2$ in the sum if sum is greater than 9 (or) carry is 1.

Inputs				Output
S_3	S_2	S_1	S_0	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
0	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Kmap Simplification,

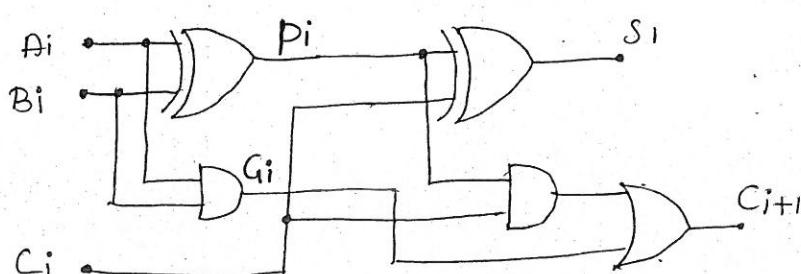
$S_3S_2 \backslash S_1S_0$	00	01	11	10
00	0 ₀	0 ₁	0 ₃	0 ₂
01	0 ₄	0 ₅	0 ₇	0 ₆
11	1 ₁₂	1 ₁₃	1 ₁₅	1 ₁₄
10	0 ₈	0 ₉	1 ₁₁	1 ₁₀

$$Y = S_3S_2 + S_3S_1$$

→ If sum is greater than 9, we add $(0110)_2$ in the sum.

Carry Look Ahead Adder

- In parallel adder, in which the carry output of each full-adder stage connected to the carry input of the next higher-order stage. So, the sum & carry outputs of any stage cannot be produced until the input carry occurs.
- This leads time delay in the addition process → this delay known as carry propagation delay.
- For speeding up this process by eliminating inter stage carry delay is called look ahead carry addition. This method utilizes logic gates to look at the lower-order bits. It uses two functions, (i) carry generate (ii) carry propagate.



$$P_i = A_i \oplus B_i$$

$$G_i = A_i B_i$$

The output sum and carry can be expressed as,

$$S_i = P_i \oplus C_i$$

$$C_{i+1} = G_i + P_i C_i$$

- Here G_i is carry propagate & it produces on carry when both A_i & B_i are one, regardless of the input carry.

→ P_i is called ^{Carry} propagate because it is a term associated with the propagation of the carry from C_i to C_{i+1} .

→ Boolean function for the carry output of each stage,

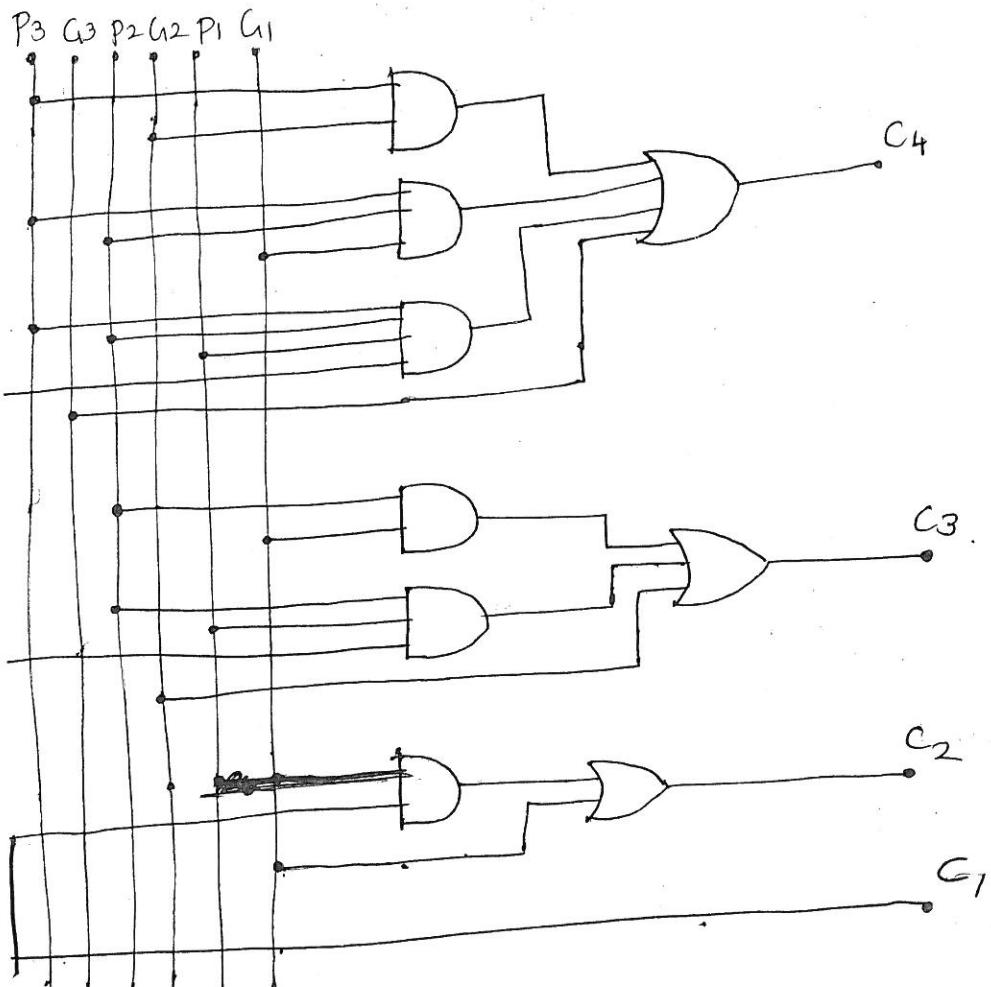
$$C_2 = G_1 + P_1 C_1$$

$$C_3 = G_2 + P_2 C_2 = G_2 + P_2 (G_1 + P_1 C_1) = G_2 + P_2 G_1 + P_2 P_1 C_1$$

$$C_4 = G_3 + P_3 C_3 = G_3 + P_3 (G_2 + P_2 G_1 + P_2 P_1 C_1)$$

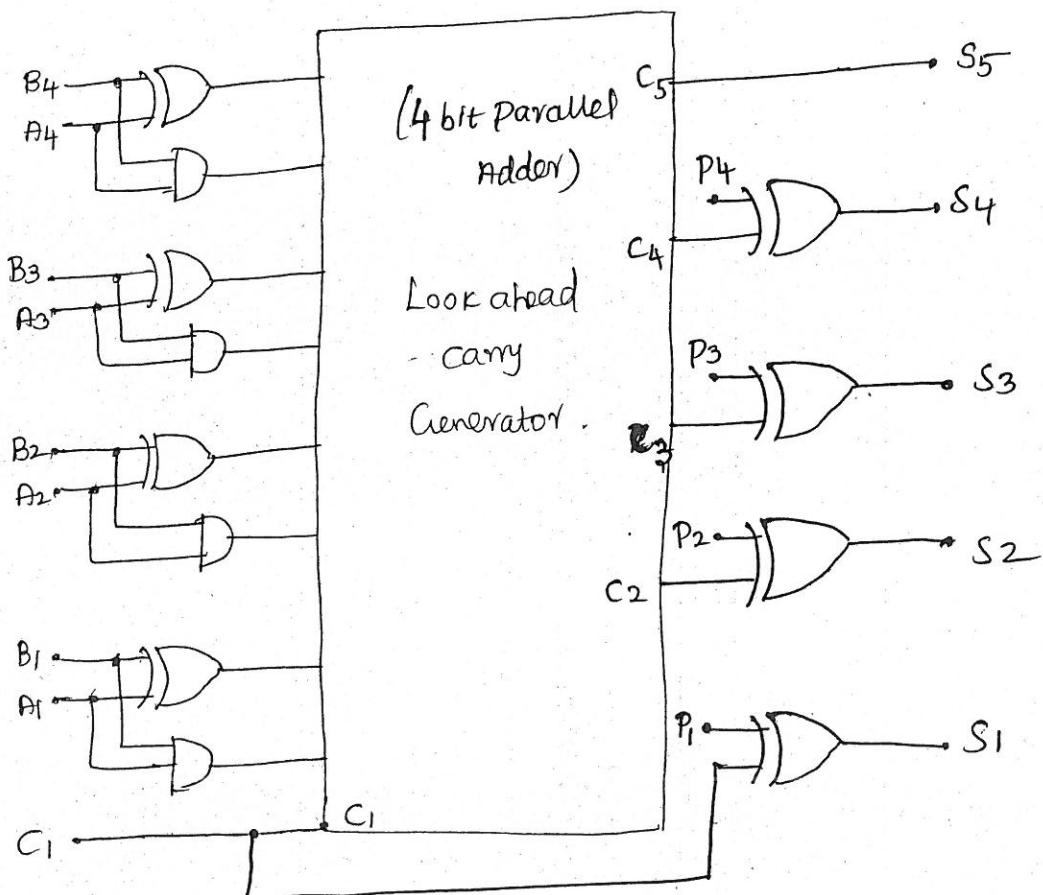
$$= G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 C_1.$$

Logic Diagram:



→ From the above boolean expression seen that C_4 does not have to wait for C_3 and C_2 to propagate; In fact C_4 is propagated at same time as C_2 and C_3 .

4 bit parallel adder with look ahead carry generator.



Quine-McCluskey (or) Tabulation method

→ The minterms whose binary equivalent differ only in one place can be combined to reduce the minterms.

For example: $\bar{A}\bar{B}\bar{C}D$, $\bar{A}\bar{B}C\bar{D}$ $\Rightarrow 0000 + 0010 \Rightarrow$ Resultant term $00-0$
 implicant

→ The terms did not match during the process called → prime implicants

Steps

- (i). List all minterms in the binary form.
 - (ii). Arrange the minterms according to number of 1's.
 - (iii) Compare each binary no. with every term in adjacent next higher category if they differ only by one position.
 - (iv) Continue the process until a single pass through cycles yields no further elimination of variables.
 - (v) List all the prime implicants.
 - (vi) Select the min. number of P.I's which must cover all the minterms.
 - vii). Find the minimal SOP for the boolean expression,
- SOP
- $$f = \Sigma m(1, 2, 3, 7, 8, 9, 10, 11, 14, 15) \text{ using Quine-McCluskey method}$$

TIM Pairs

$$\begin{array}{l} ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} \\ 1111 \quad 0000 \quad 1110 \\ | \quad | \quad | \\ 1110 \quad 0001 \end{array}$$

S-1 List all the minterms in the binary form.

(P)

Minterms	A	B	C	D
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
14	1	1	1	0
15	1	1	1	1

Step-2. Arrange the minterms according to the number of 1's.

Number of 1's	Minterms	A	B	C	D
1	1	0	0	0	1
2	2, 3	0	0	1	0
3	8, 9, 10	1	0	0	1
4	15	1	1	1	1

Step 3:- 2 cell combination: Compare each binary no with every term in the adjacent next higher category & if they differ only by one position.

Combinations	A	B	C	D
(1, 3)	0	0	1	
(1, 9)	-	0	0	1
(2, 3)	0	0	1	-
(2, 10)	-	0	1	0
(8, 9)	1	0	0	-
(8, 10)	1	0	-	0
(3, 7)	0	-	1	1
(3, 11)	-	0	1	1
(9, 11)	1	0	-	1
(10, 11)	1	0	1	-
(10, 14)	1	-	1	0
(7, 15)	-	1	1	1
(11, 15)	1	-	1	1
(11, 15)	1	1	1	-

Step 4: (4 cell combination)

Combinations	A	B	C	D
(1, 3, 9, 11)	-	0	1	
(1, 9, 3, 11)	-	0	-	1
(2, 13, 10, 11)	-	0	1	-
(2, 10, 3, 11)	-	0	1	-
(8, 9, 10, 11)	1	0	-	-
(8, 10, 9, 11)	1	0	-	-
(3, 9, 11, 15)	-	-	1	1
(3, 11, 7, 15)	-	-	1	1
(10, 11, 14, 15)	1	-	1	-
(10, 14, 11, 15)	1	-	1	-

Step 5:- (Prime Implicants)

P.I.	1	2	3	7	8	9	10	11	14	15
(1, 3, 9, 11)	(X)		X			X		X		
(2, 3, 10, 11)		(X)	X				X	X		
(8, 9, 10, 11)				(X)	X	X	X			
(3, 7, 11, 15)			X	(X)				X		X
(10, 11, 14, 15)						X	X	(X)	X	

$$Y = \overline{B}\overline{D} + \overline{B}\overline{C} + A\overline{B} + AC + CD$$

Simplify the boolean function by using a Quine-McCluskey method.

$$F(A, B, C, D) = \Sigma m(0, 2, 3, 6, 7, 8, 10, 12, 13)$$

Step 1 Arrange the minterms in binary form.

Minterms	Variables			
	A	B	C	D
0 → 0 0 0 0				
2 → 0 0 1 0				
3 → 0 0 1 1				
6 → 0 1 1 0				
7 → 0 1 1 1				
8 → 1 0 0 0				
10 → 1 0 1 0				
12 → 1 1 0 0				
13 → 1 1 0 1				

Step 2 - Arrange minterms according to no. of 1's.

No. of 1's	Minterms	Variables			
		A	B	C	D
0	0 → 0 0 0 0				
1	2 → 0 0 1 0 3 → 0 0 1 1 6 → 0 1 1 0 7 → 0 1 1 1				
2	8 → 1 0 0 0 10 → 1 0 1 0 12 → 1 1 0 0				
3	7 → 0 1 1 1 13 → 1 1 0 1				

	Variables			
	A	B	C	D
(0, 8)	-	0	0	0
(0, 12)	0	0	-	0
(2, 3)	0	0	1	-
(2, 6)	0	-	1	0
(2, 10)	-	0	1	0
(8, 10)	1	0	-	0
(8, 12)	1	-	0	0
(3, 7)	0	-	1	1
(6, 7)	0	1	1	-
(12, 13)	1	1	0	-

	Variables			
	A	B	C	D
(0, 8, 12, 10)	-	0	-	0
(0, 12, 8, 10)	-	0	-	0
(2, 3, 6, 7)	0	-	1	-
(2, 6, 3, 7)	0	-	1	-

	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	1	0	1	1
$\bar{A}B$	4	5	1	2
$A\bar{B}$	1	1	1	1
AB	8	9	11	10

Step 5: Prime Implicants:-

$$X = \overline{BD} + \overline{AC} + \overline{ABC}$$

	Minterms								
	0	2	3	6	7	8	10	12	13
(8, 12)						X		X	
(12, 13)								X	(X)
(0, 8, 12, 10)	(X)	X				X	(X)		
(2, 3, 6, 7)		X	(X)	(X)	(X)				

$$Y = ABC\bar{C} + \overline{BD} + \overline{AC}$$

= $\Sigma m(0, 1, 3, 7, 8, 9, 11, 15)$ Simplify using Quine McCluskey method.

Step-1): Write Binary no for the minterms.

Minterms	Variables A B C D
0	0 0 0 0
1	0 0 0 1
3	0 0 1 1
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
11	1 0 1 1
15	1 1 1 1

Step-2) Arrange no. of 1's.

No. of 1's	Minterms	Variables A B C D
0	0	0 0 0 0
1	1	0 0 0 1
2	8	1 0 0 0
3	3	0 0 1 1
3	9	1 0 0 1
3	11	1 0 1 1
3	7	0 1 1 1
4	15	1 1 1 1

Step3: 2 cell combination

Combination	Variables A B C D
$\checkmark(0,8)$	- 0 0 0
$\checkmark(0,1)$	0 0 0 -
$\checkmark(1,9)$	- 0 0 1
$\checkmark(1,3)$	0 0 - 1
$\checkmark(8,9)$	1 0 0 -
$\checkmark(3,11)$	- 0 1 1
$\checkmark(3,7)$	0 - 1 1
$\checkmark(9,11)$	1 0 - 1
$\checkmark(11,15)$	1 - 1 1
$\checkmark(7,15)$	- 1 1 1

Step4:- 4 cell combination

Combination	Variables A B C D
$(0,1,1,8,1,9)$	- 0 0 -
$(0,1,8,11,9)$	- 0 0 -
$(1,1,3,1,9,1,11)$	- 0 - 1
$(1,1,9,1,3,1,11)$	- 0 - 1
$(3,1,7,1,11,1,15)$	- - 1 1
$(3,1,11,1,7,1,15)$	- - 1 1

$$f = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

$$2) f = \Sigma m(1, 3, 4, 5, 9, 11) + \Sigma d(6, 8)$$

, 10, \rightarrow don't care

Step 1

Minterms	Variables A B C D.
1 \rightarrow	0 0 0 1
3 \rightarrow	0 0 1 1
4 \rightarrow	0 1 0 0
5 \rightarrow	0 1 0 1
9 \rightarrow	1 0 0 1
10 \rightarrow	1 0 1 0
11 \rightarrow	1 0 1 1
6 \rightarrow	0 1 1 0
8 \rightarrow	1 0 0 0
10 \rightarrow	1 0 1 0

Step 2:

No. of 1's	Minterms	Variables A B C D.
1	1 \rightarrow 4 \rightarrow 8 \rightarrow	0 0 0 1 0 1 0 0 1 0 0 0
2	3 \rightarrow 5 \rightarrow 6 \rightarrow 9 \rightarrow 10 \rightarrow	0 0 1 1 0 1 0 1 0 1 1 0 1 0 0 1 1 0 1 0
3	1 11 \rightarrow	1 0 1 1

Step 3 2 cell combination:-

Step 4:

Combination	Variables A B C D.
(1, 3) \rightarrow	0 0 - 1
(1, 5) \rightarrow	0 - 0 1
(1, 9) \rightarrow	- 0 0 1
(4, 5) \rightarrow	0 1 0 -
(4, 6) \rightarrow	0 1 - 0
(8, 9) \rightarrow	1 0 0 -
(8, 10) \rightarrow	1 0 - 0
(3, 11) \rightarrow	- 0 1 1
(9, 11) \rightarrow	1 0 - 1
(10, 11) \rightarrow	1 0 1 -

Combination	Variables A B C D.
(1, 3, 9, 11) \rightarrow	- 0 - 1
(8, 9, 10, 11) \rightarrow	1 0 - 1

PI1 \rightarrow (1, 3, 9, 11) \rightarrow - 0 - 1

PI2 \rightarrow (4, 5, 6, 10) \rightarrow 0 1 0 -

PI3 \rightarrow (8, 9, 10, 11) \rightarrow 1 0 - 1

PI4 \rightarrow (3, 11) \rightarrow - 0 1 1

PI5 \rightarrow (9, 11) \rightarrow 1 0 - 1

PI6 \rightarrow (10, 11) \rightarrow 1 0 1 -

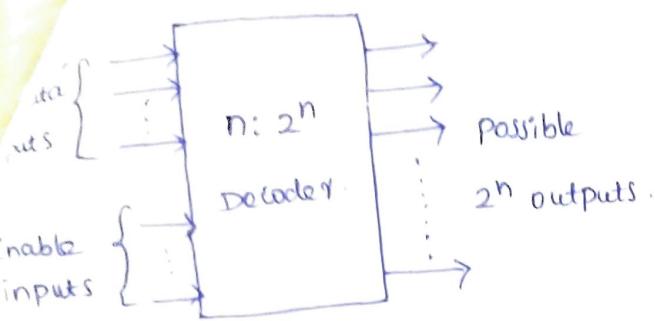
Y = $\overline{AB} + AB\overline{C}$

$\overline{AB} + \overline{BD}$

$\overline{ABC} + \overline{BD}$

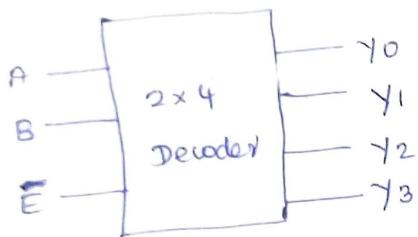
$\overline{AB} + \overline{B}\overline{D}$

Decoders



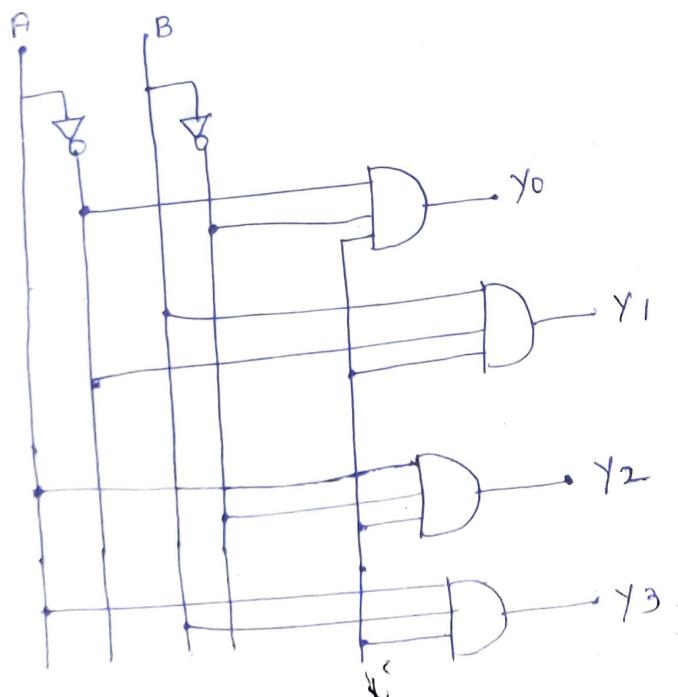
Decoder

i) Design a 2×4 decoder.



Inputs			Outputs			
EN	A	B	Y_0	Y_1	Y_2	Y_3
0	X	X	0	0	0	1
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0

Logic diagram

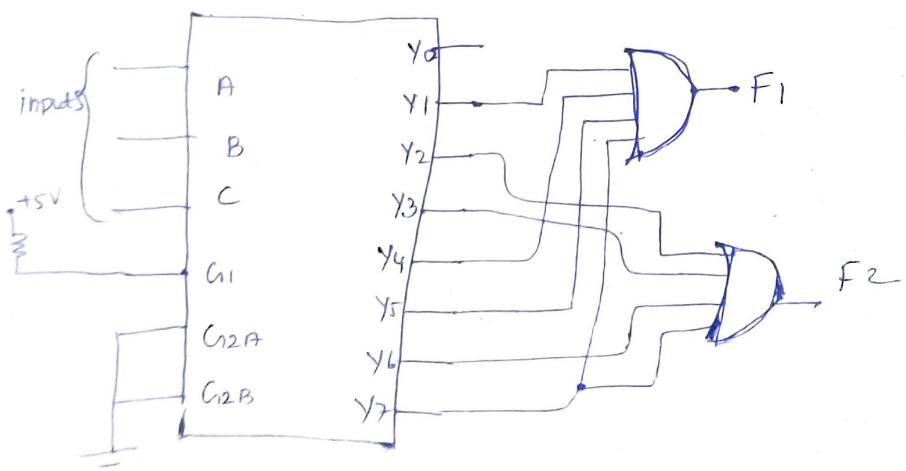


2) Design 3-8 decoder

inputs				outputs							
EN	A	B	C	Y ₇	Y ₆	Y ₅	Y ₄	Y ₃	Y ₂	Y ₁	Y ₀
0	x	x	x	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	1
1	0	0	1	0	0	0	0	0	0	1	0
1	0	1	0	0	0	0	0	0	1	0	0
1	0	1	1	0	0	0	0	1	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	1	0	0	0	0	0
1	1	1	0	0	1	0	0	0	0	0	0
1	1	1	1	1	0	0	0	0	0	0	0

3) Implement $F_1(A, B, C) = \Sigma m(1, 4, 5, 7)$, $F_2(A, B, C) =$

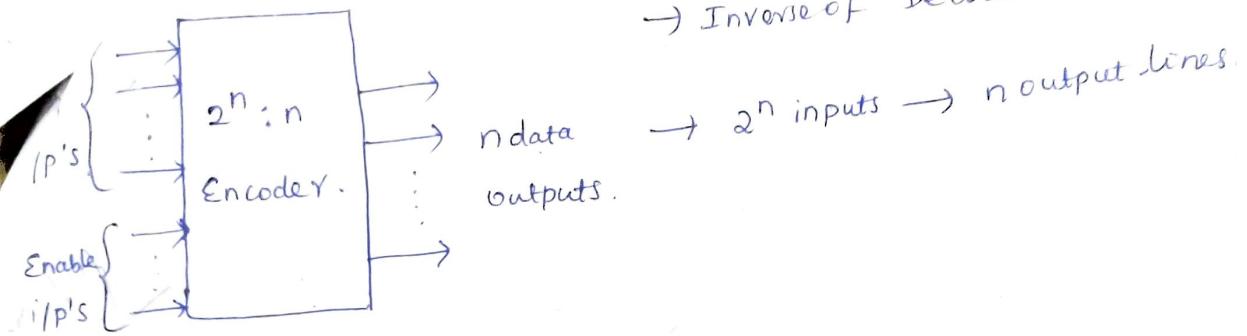
$$\Sigma m(2, 3, 6, 7)$$



Notes

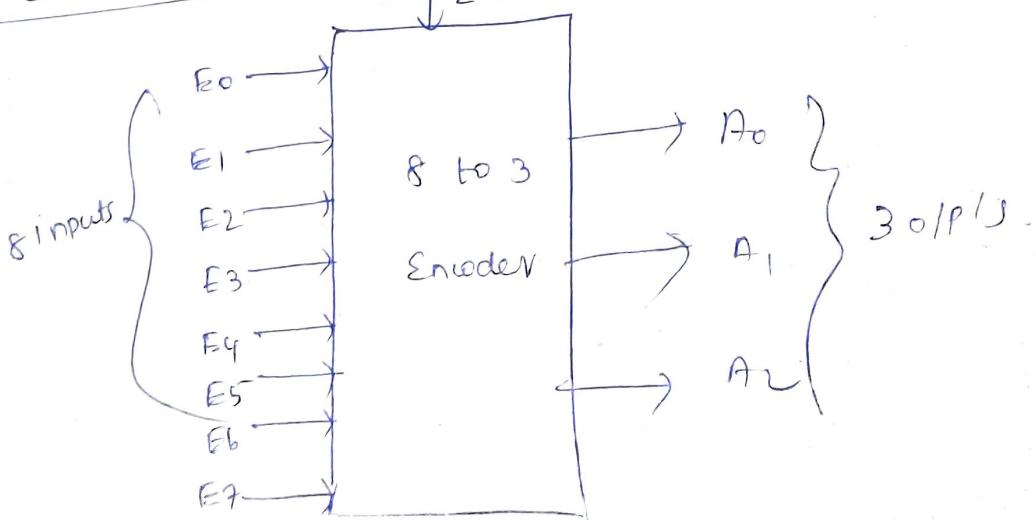
Encoder

→ Inverse of Decoder.



1) Octal to binary encoder. (8 to 3 Encoder).

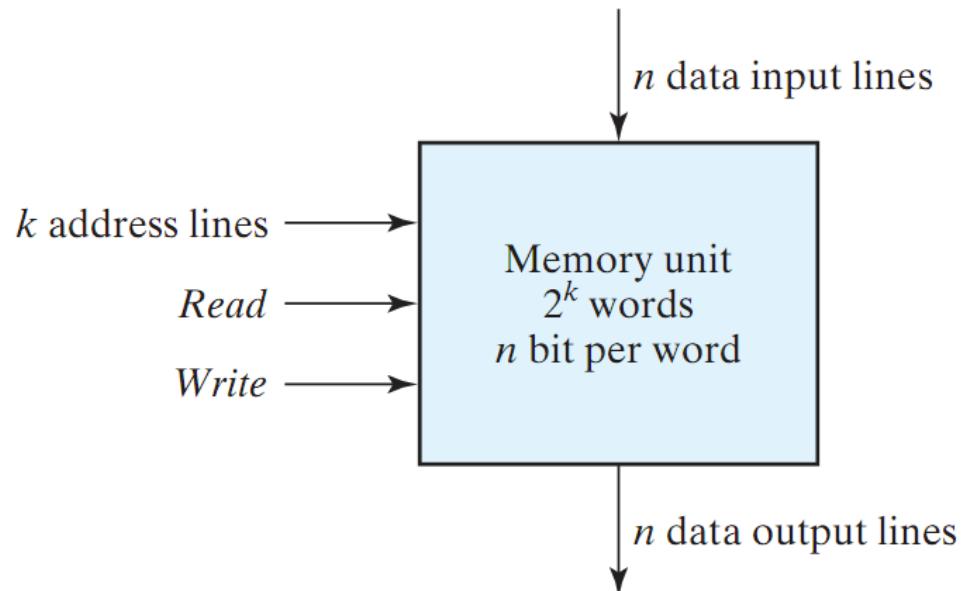
Inputs								outputs		
E_7	E_6	E_5	E_4	E_3	E_2	E_1	E_0	A_0	A_1	A_2
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	1	0	0	0	0	1	0	1
0	0	1	0	0	0	0	0	1	1	0
0	1	0	0	0	0	0	0	1	1	1
1	0	0	0	0	0	0	0	-1	-1	-1



Memory

- **Memory:** A collection of cells capable of storing binary information (1s or 0s)
 - in addition to electronic circuit for storing (writing) and retrieving (reading) information.

- n data lines (input/output)
- k address lines
- 2^k words (data unit)
- Read/Write Control
- Memory size = $2^k \times n$



Memory (cont.)

Two Types of Memory:

- **Random Access Memory (RAM):**
 - Write/Read operations
 - Volatile: Data is lost when power is turned off
- **Read Only Memory (ROM):**
 - Read operation (no write)
 - Non-Volatile: Data is permanent.
 - PROM is programmable (allow special write)

Read-Only Memory (ROM)

- **ROM:** A device in which “permanent” binary information is stored using a special device (programmer)

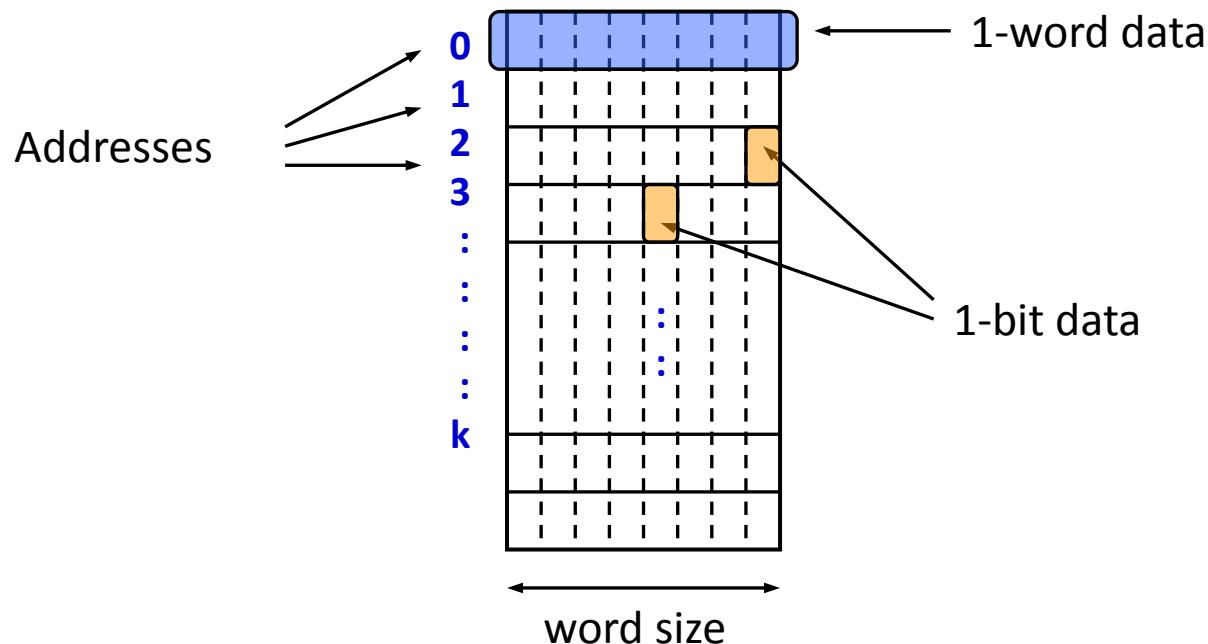


- k inputs (address) → 2^k words each of size n bits (data)
- ROM DOES NOT have a write operation ➔ ROM DOES NOT have data inputs

Word: group of bits stored in one location

Read-Only Memory (ROM)

- A semi-conductor memory is a device where data can be stored and retrieved.
- Logically, this memory device can be regarded as a table of memory cells (data).



Example 1

- **Example:** Design a combinational circuit using ROM (memory size). The circuit accepts a 3-bit number and generates an output binary number equal to the square of the number.

Solution: Derive truth table:

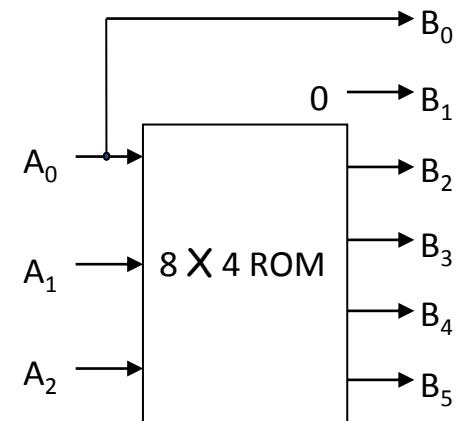
Inputs			Outputs						
A2	A1	A0	B5	B4	B3	B2	B1	B0	SQ
0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	1	1
0	1	0	0	0	0	1	0	0	4
0	1	1	0	0	1	0	0	1	9
1	0	0	0	1	0	0	0	0	16
1	0	1	0	1	1	0	0	1	25
1	1	0	1	0	0	1	0	0	36
1	1	1	1	1	0	0	0	1	49

Example 1 (cont.)

Inputs			Outputs						
A2	A1	A0	B5	B4	B3	B2	B1	B0	SQ
0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	1	1
0	1	0	0	0	0	1	0	0	4
0	1	1	0	0	1	0	0	1	9
1	0	0	0	1	0	0	0	0	16
1	0	1	0	1	1	0	0	1	25
1	1	0	1	0	0	1	0	0	36
1	1	1	1	1	0	0	0	1	49

ROM truth table – specifies the required connections

- B1 is ALWAYS 0 → no need to generate it using the ROM
- B0 is equal to A0 → no need to generate it using the ROM
- Therefore: The minimum size of ROM needed is $2^3 \times 4$ or 8x4



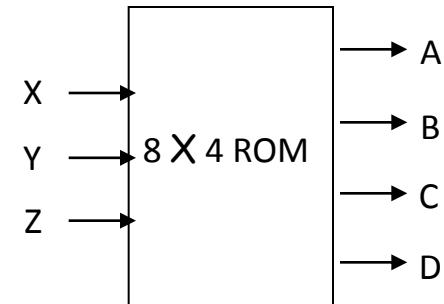
Example 2

- **Problem:** Tabulate the truth for an 8 X 4 ROM that implements the following four Boolean functions:

$$A(X,Y,Z) = \sum m(3,6,7); B(X,Y,Z) = \sum m(0,1,4,5,6)$$

$$C(X,Y,Z) = \sum m(2,3,4); D(X,Y,Z) = \sum m(2,3,4,7)$$

Solution:

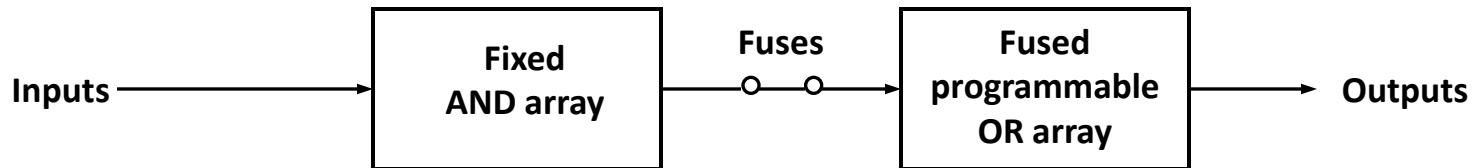


Inputs			Outputs			
X	Y	Z	A	B	C	D
0	0	0	0	1	0	0
0	0	1	0	1	0	0
0	1	0	0	0	1	1
0	1	1	1	0	1	1
1	0	0	0	1	1	1
1	0	1	0	1	0	0
1	1	0	1	1	0	0
1	1	1	1	0	0	1

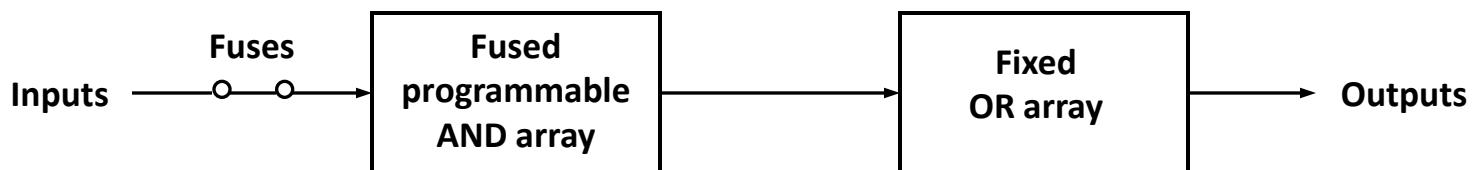
Types of ROMs

- A ROM programmed in four different ways:
- **ROM: Mask Programming**
 - By a semiconductor company
- **PROM (Programmable ROM)**
 - User can connect fuses with a special programming device (PROM programmer)
 - Only programmed once
- **EPROM (Erasable PROM)**
 - Can be erased using Ultraviolet Light
- **Electrically Erasable PROM (EEPROM or E²PROM)**
 - Like an EPROM, but erased with electrical signal

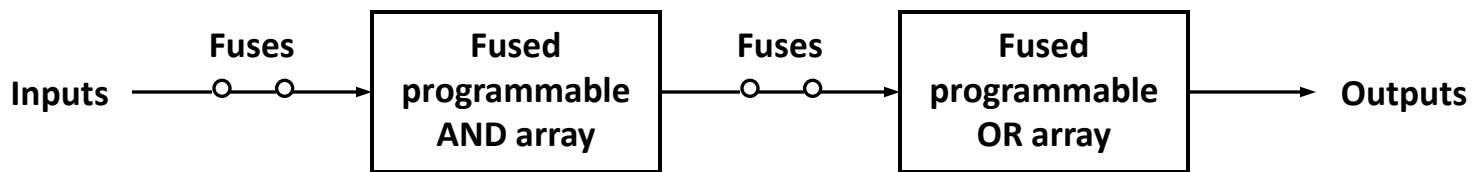
Programmable Logic Devices



Programmable Read Only Memory (PROM)



Programmable Array Logic (PAL)



Programmable Logic Array (PLA)

Programmable Logic Array (PLA)

- Combination of a **programmable AND** array followed by a **programmable OR** array.
- Example: Design a PLA to realise the following three logic functions and show the internal connections.

$$f_1(A,B,C,D,E) = A'.B'.D' + B'.C.D' + A'.B.C.D.E'$$

$$f_2(A,B,C,D,E) = A'.B.E + B'.C.D'.E$$

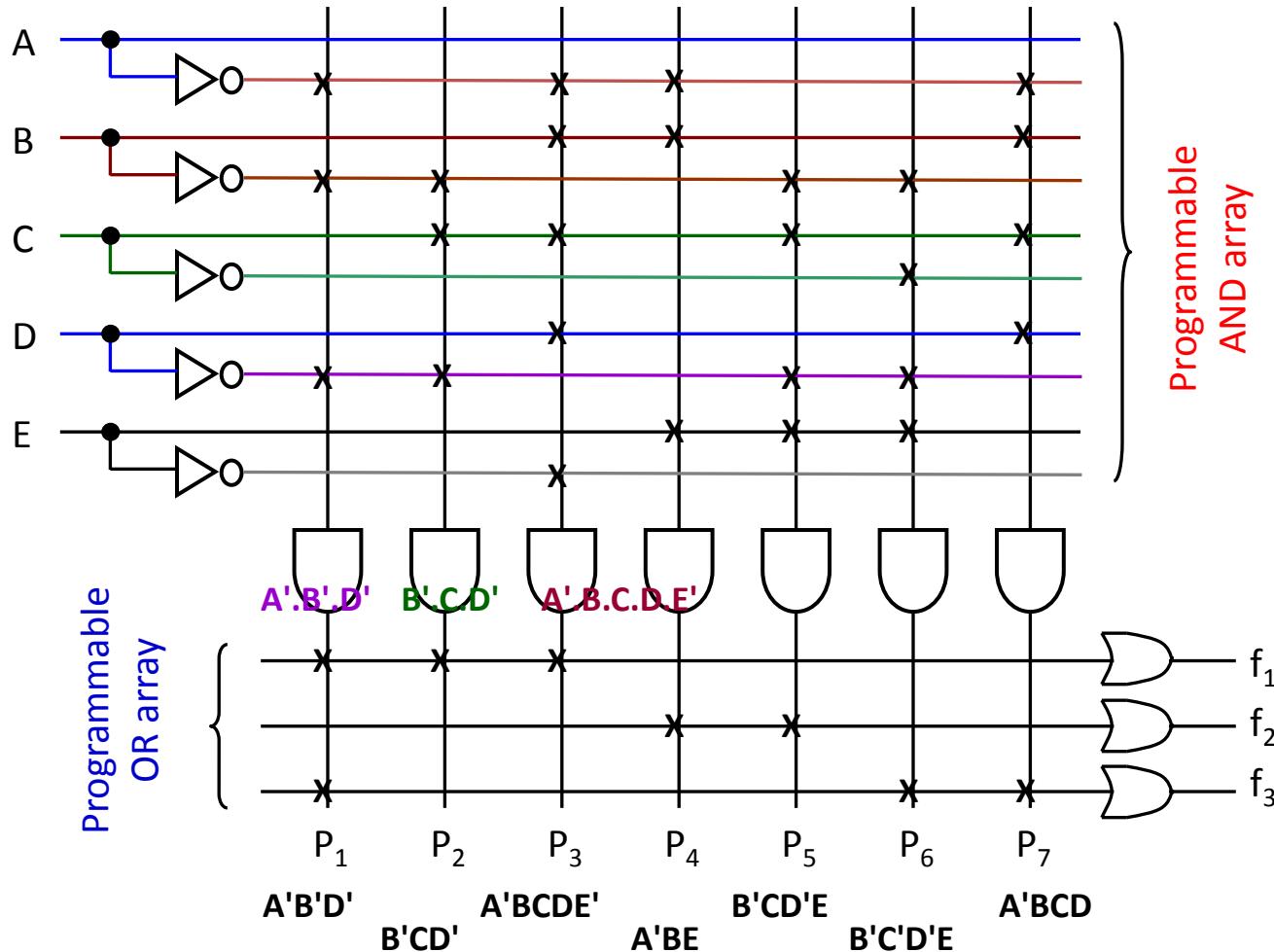
$$f_3(A,B,C,D,E) = A'.B'.D' + B'.C'.D'.E + A'.B.C.D$$

Realising Logic Functions with PLAs

$$f_1(A,B,C,D,E) = A'.B'.D' + B'.C.D' + A'.B.C.D.E'$$

$$f_2(A,B,C,D,E) = A'.B.E + B'.C.D'.E$$

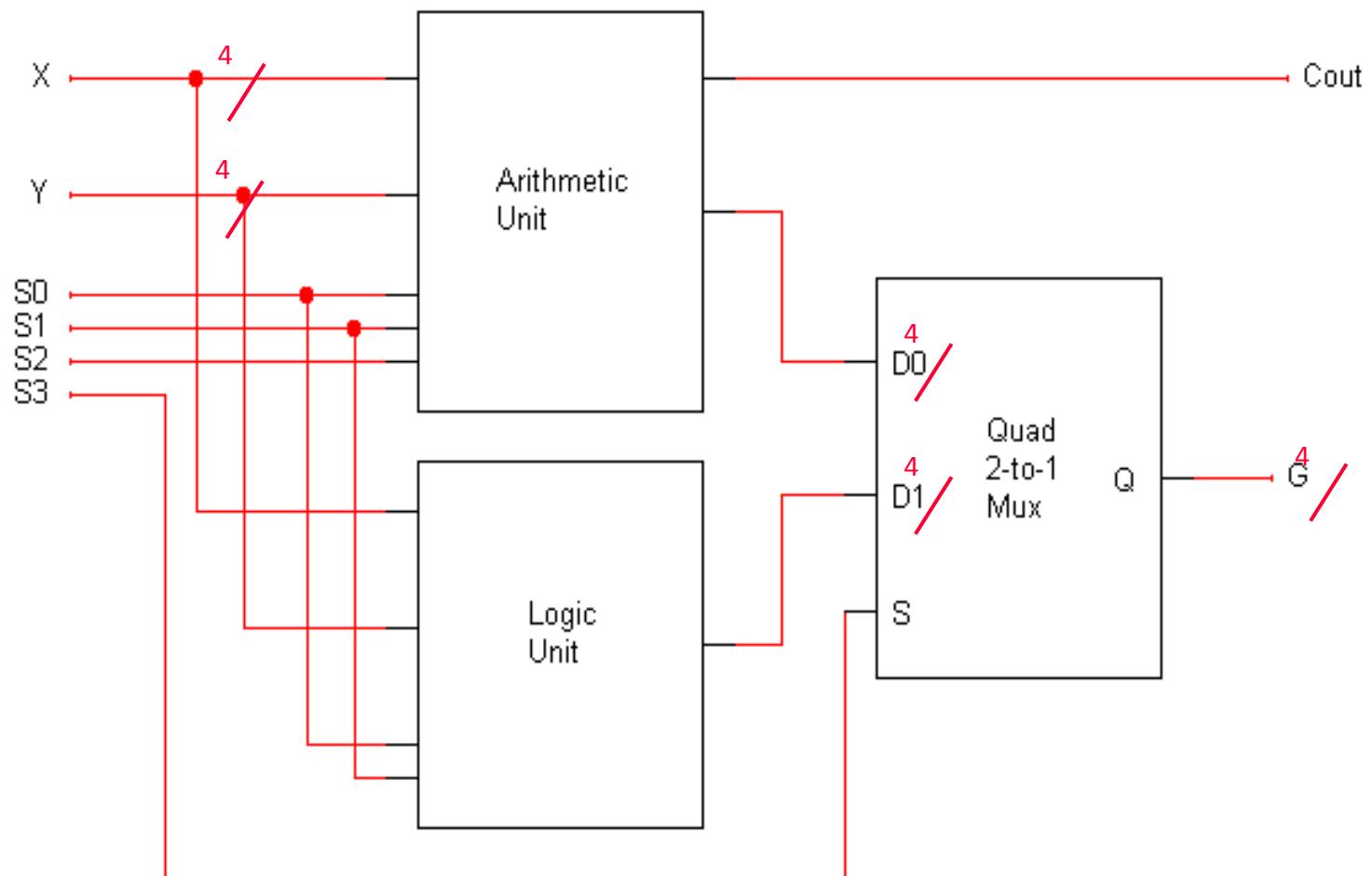
$$f_3(A,B,C,D,E) = A'.B'.D' + B'.C'.D'.E + A'.B.C.D$$



Arithmetic Logic Unit

- An arithmetic-logic unit, or ALU, performs many different arithmetic and logic operations. The ALU is the “heart” of a processor—you could say that everything else in the CPU is there to support the ALU.
- Here’s the plan:
 - We’ll show an arithmetic unit first, by building off ideas from the adder - subtractor circuit.
 - Then we’ll talk about logic operations a bit, and build a logic unit.
 - Finally, we put these pieces together using multiplexers.
- We use some examples from the textbook, but things are re-labeled and treated a little differently.

A complete ALU circuit



What does HDL stand for?

HDL is short for **Hardware Description Language**

VHDL – VHSIC (Very High Speed Integrated Circuit)

Hardware Description Language

Basic Form of VHDL Code

- Every VHDL design description consists of at least **one entity / architecture pair, or one entity with multiple architectures.**
- The entity section is used to declare I/O ports of the circuit. The architecture portion describes the circuit's behavior.
- A behavioral model is similar to a “black box”.
- Standardized design libraries are included before entity declaration.



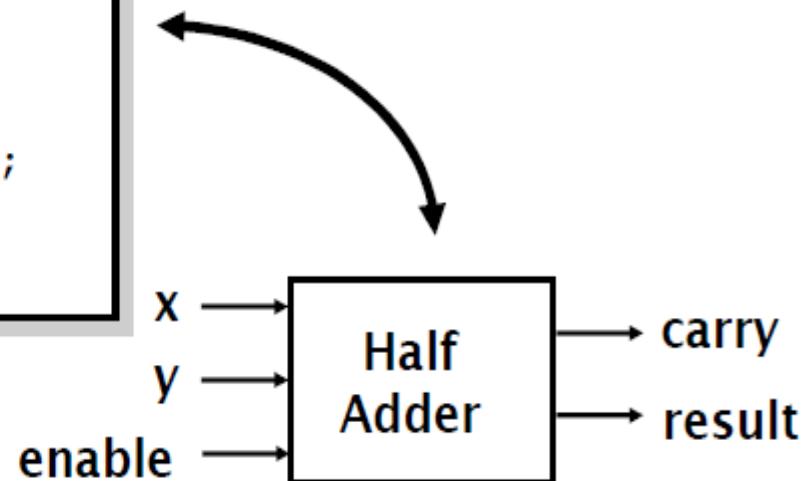
Standard Libraries

- Include *library ieee;* before entity declaration.
- **ieee.std_logic_1164** defines a standard for designers to use in describing interconnection data types used in VHDL modeling.
- **ieee.std_logic_arith** provides a set of arithmetic, conversion, comparison functions for signed, unsigned, std_ulogic, std_logic, std_logic_vector.
- **ieee.std_logic_unsigned** provides a set of unsigned arithmetic, conversion, and comparison functions for std_logic_vector.

Entity Declaration

- An entity declaration describes the interface of the component.
Avoid using Altera's primitive names
- PORT clause indicates input and output ports.
- An entity can be thought of as a symbol for a component.

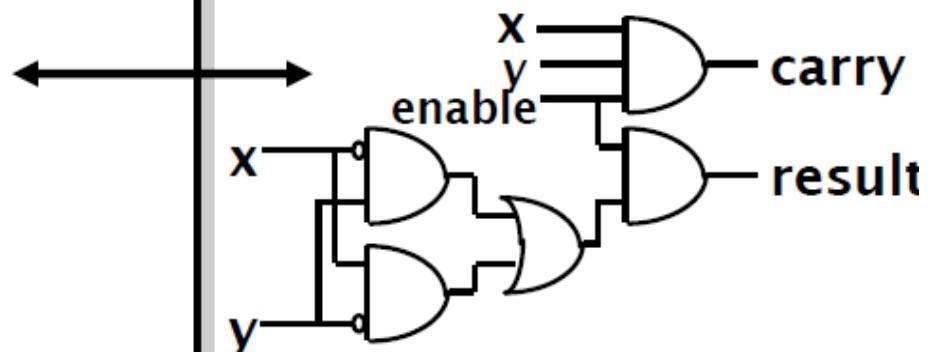
```
ENTITY half_adder IS  
  
    PORT( x, y, enable: IN bit;  
          carry, result: OUT bit);  
  
END half_adder;
```



Architecture Declaration

- Architecture declarations describe the operation of the component.
- Many architectures may exist for one entity, but only one may be active at a time.
- An architecture is similar to a schematic of the component.

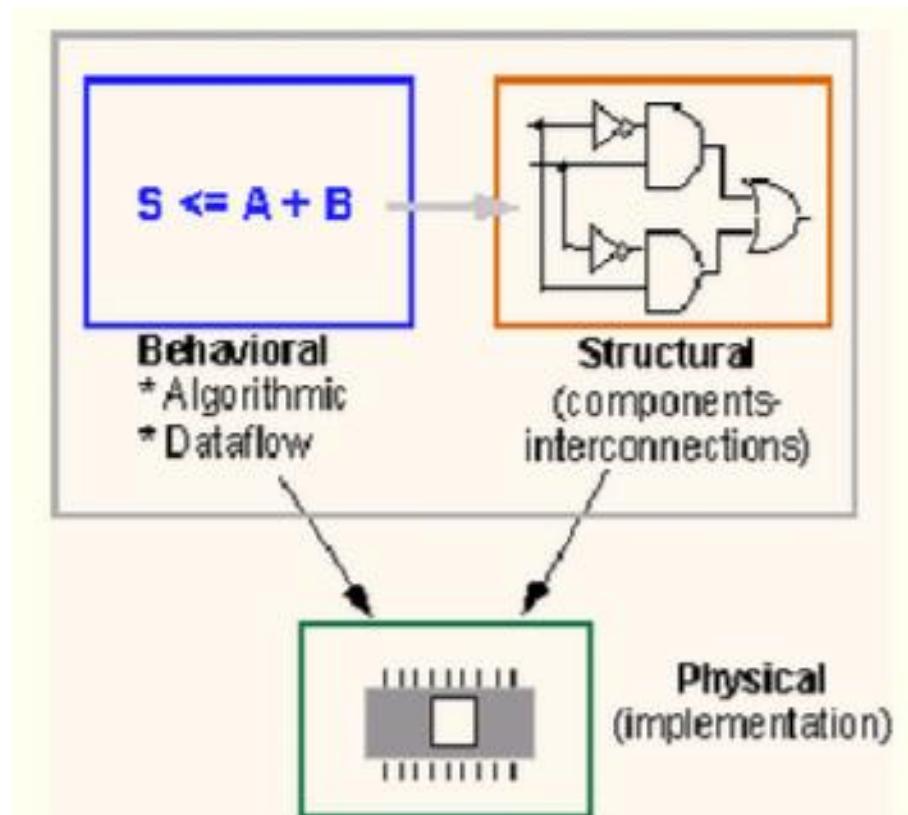
```
ARCHITECTURE behave OF half_adder IS
BEGIN
    PROCESS (enable, x, y)
    BEGIN
        IF (enable = '1') THEN
            result <= x XOR y;
            carry <= x AND y;
        ELSE
            carry <= '0';
            result <= '0';
        END IF;
    END PROCESS;
END behave;
```



Modeling Styles

There are three modeling styles:

- Behavioral (Sequential)
- Data flow
- Structural



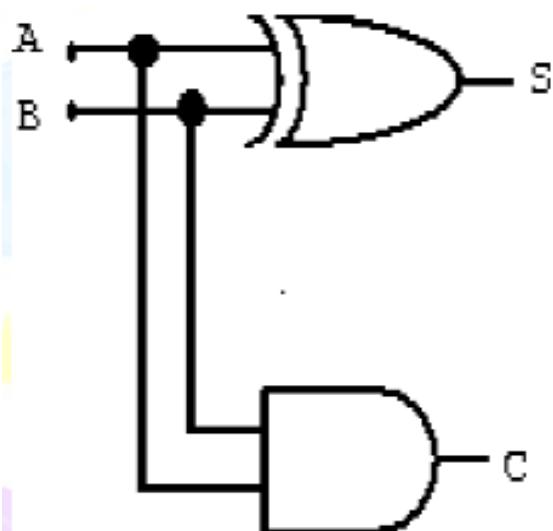
Styles in VHDL

- **Behavioral**
 - High level, algorithmic, sequential execution
 - Hard to synthesize well
 - Easy to write and understand (like high-level language code)
- **Dataflow**
 - Medium level, register-to-register transfers, concurrent execution
 - Easy to synthesize well
 - Harder to write and understand (like assembly code)
- **Structural**
 - Low level, netlist, component instantiations and wiring
 - Trivial to synthesize
 - Hardest to write and understand (very detailed and low level)

Dataflow modeling

- The internal working of an entity can be implemented using concurrent signal assignment.

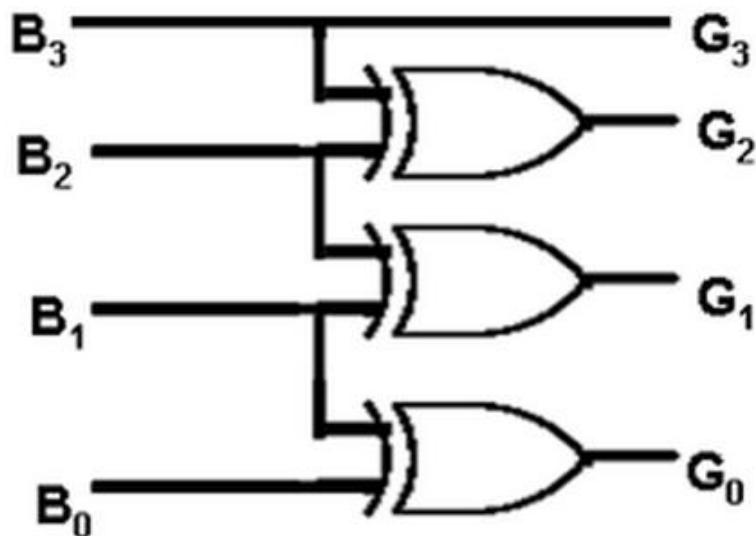
Half adder example



```
Library IEEE;
IEEE.STD_LOGIC_1164.all;
entity ha_en is
    port (A,B:in bit;S,C:out bit);
end ha_en;
architecture dataflow of ha_en is
begin
    S<=A xor B;
    C<=A and B;
end ha_ar;
```

DATAFLOW MODELLING

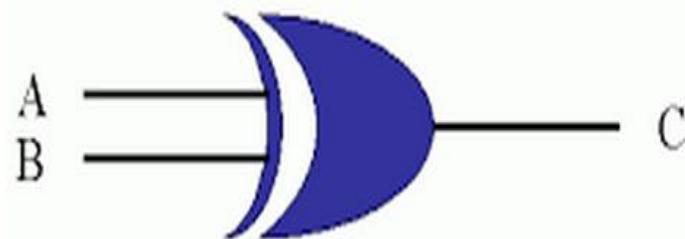
BINARY TO GRAY CODE CONVERTOR



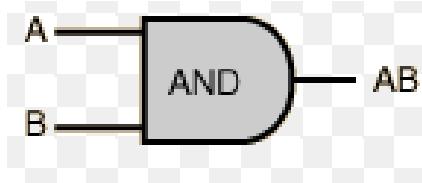
```
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity bintogray is
    Port ( b3 : in STD_LOGIC;
           b2 : in STD_LOGIC;
           b1 : in STD_LOGIC;
           b0 : in STD_LOGIC;
           g3 : out STD_LOGIC;
           g2 : out STD_LOGIC;
           g1 : out STD_LOGIC;
           g0 : out STD_LOGIC);
end bintogray;
architecture dataflow of bintogray is
begin
    g3<=(b3);
    g2<=(b3 xor b2);
    g1<=( b2 xor b1);
    g0<=(b1 xor b0 );
end dataflow;
```

xor gate

```
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity xorgate is
    Port ( a1 : in STD_LOGIC;
           b1 : in STD_LOGIC;
           c1 : out STD_LOGIC);
end xorgate;
architecture dataflow of xorgate is
begin
    c1<=(a1 and (not b1))or(b1 and(not a1));
end dataflow;
```

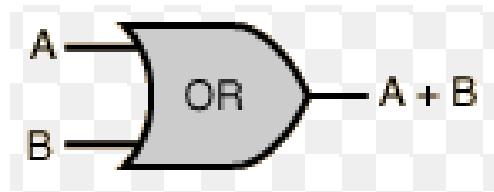


AND gate



```
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity andgate is
    Port ( a2 : in STD_LOGIC;
            b2 : in STD_LOGIC;
            c2 : out STD_LOGIC);
end andgate;
architecture dataflow of andgate is
begin
    c2<= a2 and b2;
end dataflow;
```

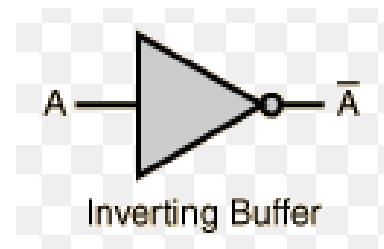
OR gate



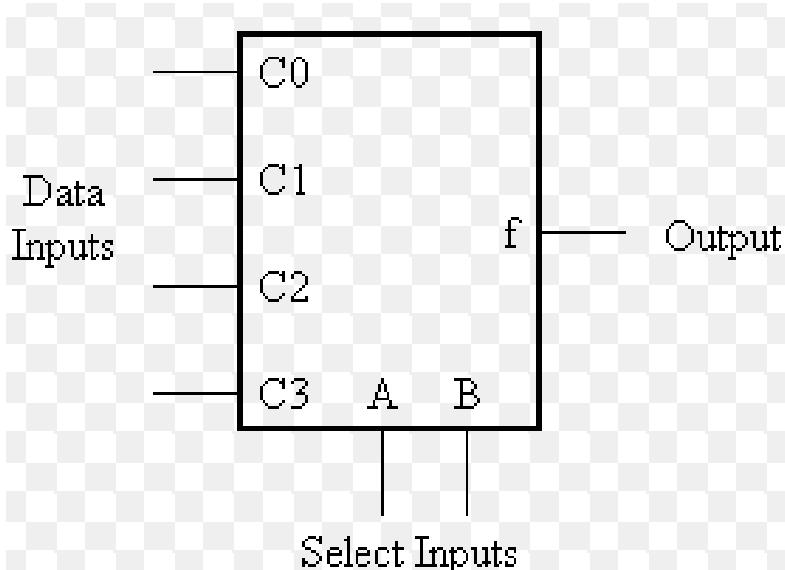
```
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity orgate is
    Port ( a1 : in STD_LOGIC;
           b1 : in STD_LOGIC;
           c1 : out STD_LOGIC);
end orgate;
architecture Dataflow of orgate is
begin
    c1<=( a1 or b1);
end Dataflow;
```

NOT gate

```
library IEEE;  
use IEEE.STD_LOGIC_1164.ALL;  
use IEEE.STD_LOGIC_ARITH.ALL;  
use IEEE.STD_LOGIC_UNSIGNED.ALL;  
entity notgate is  
    Port ( a2 : in STD_LOGIC;  
           c2 : out STD_LOGIC);  
end notgate;  
architecture Dataflow of notgate is  
begin  
    c2<=( not a2);  
end Dataflow;
```



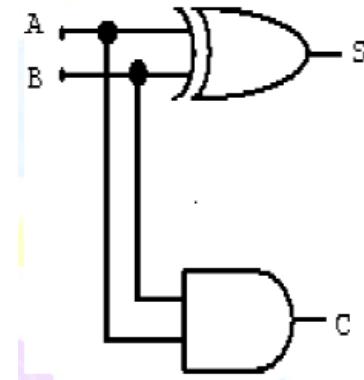
Multiplexer (8*1 MUX)



```
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity mux8_1 is
    Port ( i0 : in STD_LOGIC;
           i1 : in STD_LOGIC;
           i2 : in STD_LOGIC;
           i3 : in STD_LOGIC;
           i4 : in STD_LOGIC;
           i5 : in STD_LOGIC;
           i6 : in STD_LOGIC;
           i7 : in STD_LOGIC;
           s2 : in STD_LOGIC;
           s1 : in STD_LOGIC;
           s0 : in STD_LOGIC;
           y : out STD_LOGIC);
end mux8_1;
architecture dataflow of mux8_1 is
begin
    y<=(((not s2)and (not s1) and (not s0) and i0) or((not s2)and (not s1)
and ( s0) and (i1)) or ((not s2)and ( s1) and (not s0) and (i2)) or
((not s2)and ( s1) and ( s0) and (i3)) or ((s2)and (not s1) and ( s0)
and (i4)) or (( s2)and (not s1) and ( s0) and (i5)) or ((not s2)and ( s1)
and (not s0) and (i6)) or (( s2)and ( s1) and ( s0) and (i7)));
end dataflow;
```

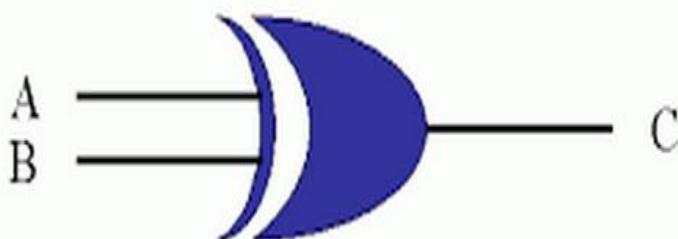
Behavioural modeling

- The internal working of an entity can be implemented using set of statements. It contains:
 - **Process statements**
 - **Sequential statements**
 - **Signal assignment statements**
 - **Wait statements**



BEHAVIORAL MODELLING

XOR GATE



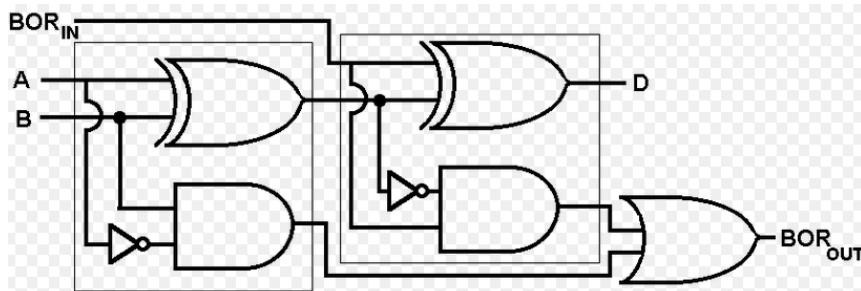
```
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity xorgate is
    Port ( a : in STD_LOGIC;
           b : in STD_LOGIC;
           c : out STD_LOGIC);
end xorgate;
architecture Behav of xorgate is
begin
process(a,b)
begin
if(a='1'and b='0')then
c<='1';
else if(a='0'and b='1')then
c<='1';
else
c<='0';
end if;
end if;
end process;
end xora;
```

FULL SUBTRACTOR

```
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity fullsubst is
    Port ( a : in STD_LOGIC;
            b : in STD_LOGIC;
            bi : in STD_LOGIC;
            d : out STD_LOGIC;
            bo : out STD_LOGIC);
end fullsubst;
architecture Behavioral of fullsubst is
begin
process(a,b,bi)
begin
if((a='0' and b='0' and bi='1')or(a='0' and b='1' and bi='0') or (a='1'
and b='1' and bi='1'))then
d<='1';
bo<='1';
else if(a='0' and b='1' and bi='1')then
d<='0';

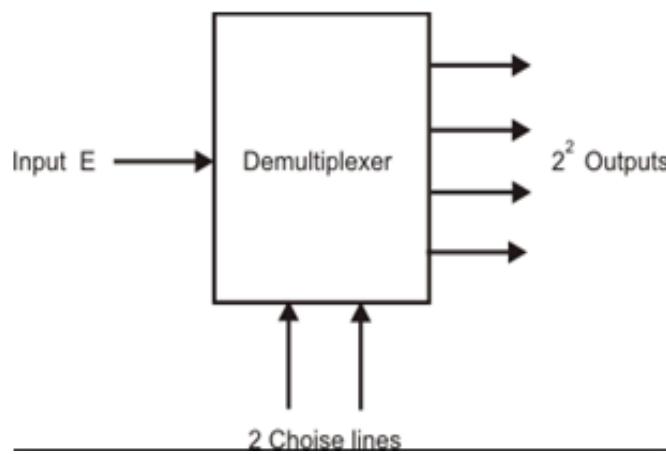
```

```
bo<='1';
else if(a='1' and b='0' and bi='0')then
d<='1';
bo<='0';
else
d<='0';
bo<='0';
end if;
end if;
end if;
end process;
end Behavioral;
```



DEMULTIPLEXER

```
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity decoder is
    Port ( s : in STD_LOGIC_VECTOR (2 downto 0);
           d : out STD_LOGIC_VECTOR (7 downto 0));
end decoder;
architecture Behavioral of decoder is
begin
process(s)
begin
case s is
when "000"=>
d<="00000001";
when "001"=>
d<="00000010";
when "010"=>
d<="00000100";
when "011"=>
d<="00001000";
when "100"=>
d<="00010000";
when "101"=>
d<="00100000";
when "110"=>
d<="01000000";
when "111"=>
d<="10000000";
when others=>
null;
end case;
end process;
end Behavioral;
```



Structural modeling

- The implementation of an entity is done through set of interconnected components. It contains:
 - Signal declaration.
 - Component instances
 - Port maps.
 - Wait statements.

Syntax:

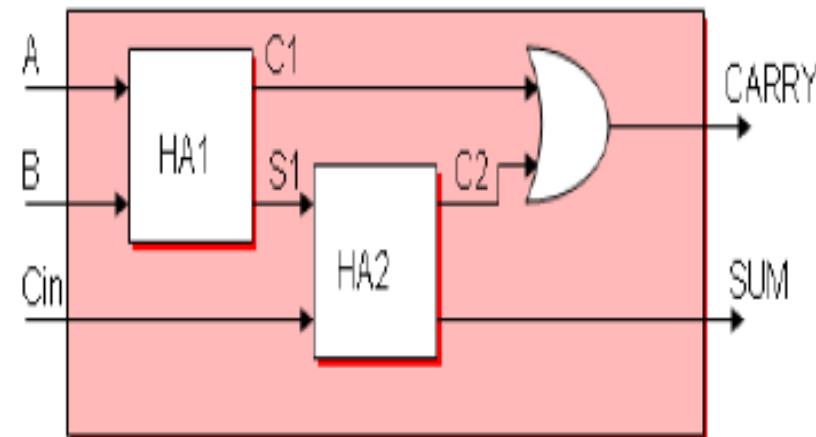
```
Component component_name [is]  
List_of_interface_ports;  
End component component_name;
```

Component declaration:

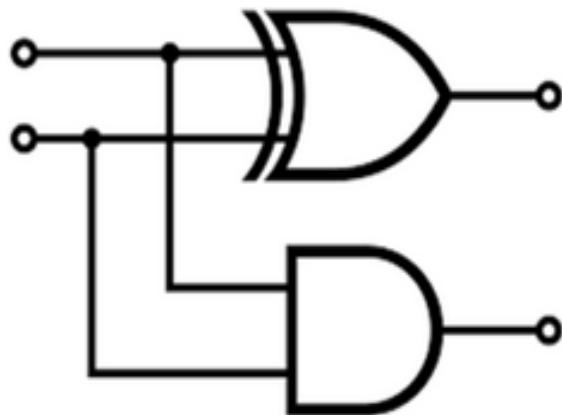
- Before instantiating the component it should be declared Using component declaration as shown above. Component Declaration declares the name of the entity and interface of a component.

Full adder example

```
library IEEE
use IEEE.STD_LOGIC_1164.all;
entity fa_en is
    port(A,B,Cin:in bit; SUM,
CARRY:out bit);
end fa_en;
architecture struct of fa_en is
component ha_en
    port(A,B:in bit;S,C:out bit);
end component;
signal C1,C2,S1:bit;
begin
    HA1:ha_en port
map(A,B,S1,C1);
    HA2:ha_en port
map(S1,Cin,SUM,C2);
    CARRY <= C1 or C2;
end fa_ar;
```



Half adder



```
library IEEE;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity halfadder_str_us_xor is
    Port ( a : in STD_LOGIC;
           b : in STD_LOGIC;
           sum : out STD_LOGIC;
           carry : out STD_LOGIC);
end halfadder_str_us_xor;
architecture Structural of halfadder_str_us_xor is
component xorgate
    Port ( a1 : in STD_LOGIC;
           b1 : in STD_LOGIC;
           c1 : out STD_LOGIC);
end component;
component andgate
    Port ( a2 : in STD_LOGIC;
           b2 : in STD_LOGIC;
           c2 : out STD_LOGIC);
end component;
begin
    L0:xorgate port map(a,b,sum);
    L1:andgate port map(a,b,carry);
end Structural;
```