CHAPTER 2

AC FUNDAMENTALS

2.1. INTRODUCTION TO ALTERNATING CURRENT

An alternating current or voltage is the one which changes its value (or magnitude) and direction with time. The wave shapes of such a current are shown in Fig. 2.1.

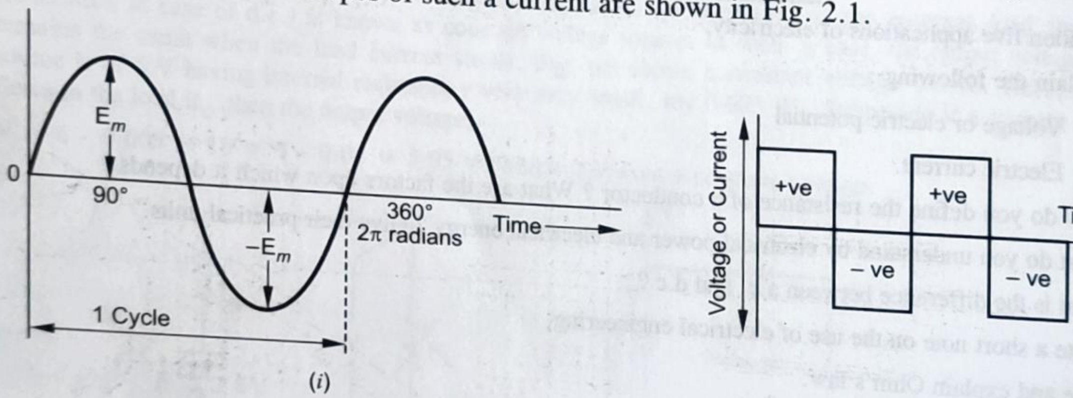


Fig. 2.1

Second Definition. *Alternating current is that which changes its direction regularly, rising from zero to maximum, returning to zero and then going through similar variation in strength in the opposite direction.* A complete set of these variations comprises the cycle as shown in Fig. 2.1 *(i)* and *(ii)*.

Generation of Alternating Voltage and Current

Principle : *When a coil or conductor is moved in magnetic field, an e.m.f. is induced in the coil or conductor.*

When a conductor is moved in magnetic field, magnetic lines of force are cut. In other words, flux is cut by the conductor or flux linking the conductor changes. According to Faraday's first law of electromagnetic induction an *e.m.f.* is induced in the conductor when magnetic flux is cut by the conductor.

The alternating voltage reverses its polarity and the current reverses its direction in the circuit.

Alternating voltage and current can be generated by :

(i) Rotating a coil in a magnetic field.

(ii) Rotating magnetic field within a stationary coil.

A.C. Generator. A simple *a.c.* generator consists of a rectangular coil made up of large number of turns and mounted on a shaft which can rotate. This arrangement is called **Armature**. The armature is placed in a very strong magnetic field and rotated by an engine or a turbine.

2.2. EQUATION OF ALTERNATING VOLTAGE (E.M.F.)

Consider a rectangular coil of N turns rotating in a uniform magnetic field in the anti-clockwise direction with an angular velocity ω radians per second as shown in Fig. 3.8*(i)*. To start with, let the plane of coil coincides with X-axis as shown in Fig. 3.8*(ii)*. In this case, maximum flux (ϕ_{max}) is cut by the coil. As the coil rotates in the anti-clockwise direction, the flux is cut by it and hence an *e.m.f.* is induced in it. Let the coil turns through an angle θ in t seconds and reaches in position as shown in Fig. 2.2*(iii)*. Thus, $\theta = \omega t$.

In this position of the coil, maximum flux (ϕ_{max}) acting vertically downwards can be resolved into two perpendicular components.

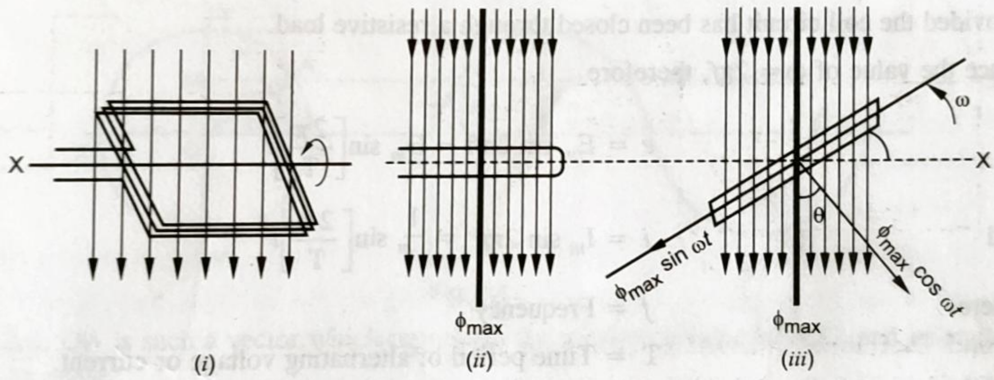


Fig. 2.2

(i) Component 'ϕ_{max} sin ωt' parallel to the plane of the coil. As the coil moves parallel to the flux, therefore, there is no cutting of flux. Thus, this component induces no *e.m.f.* in the coil.

(ii) Component 'ϕ_{max} cos ωt' perpendicular to the plane of the coil. As the coil moves perpendicular to the flux, therefore this component induces *e.m.f.* in the coil.

$\therefore$ Flux linkage of the coil at this instant = No. of turns × Flux cut (when coil has turned through θ°)

According to Faraday's laws of electromagnetic induction, *e.m.f.* induced in the coil is equal to the rate of change of flux cut by the coil. Therefore, the *e.m.f.* E induced in the coil at this instant is given by differentiating flux linkage w.r.t. time. This is the instantaneous value of *e.m.f.*

$$\therefore \qquad \boxed{e = -\frac{d}{dt}\text{(Flux linkage)}}$$

The negative sign has been taken due to the reason that the direction of induced *e.m.f.* is such that it opposes the cause producing it (Lenz's law).

$$\therefore \quad e = -\frac{d}{dt}(N\,\phi_{max} \cos \omega t)$$

or $$e = -N\,\phi_{max}\,\frac{d}{dt}(\cos \omega t)$$

or $$e = -N\,\phi_{max}\,(-\sin \omega t) \times \omega$$

or $$e = N\omega\,\phi_{max} \sin \omega t \text{ volt} \qquad \ldots(i)$$

The value of e will be maximum (E_m) when the coil has turned through 90° $\quad (\because \sin 90° = 1)$

$$\therefore \quad E_m = N\,\omega\,\phi_{max} \text{ volt} \qquad \ldots(ii)$$

Substituting the value of $N\omega\phi_{max}$ in equation *(i)* we get,

$$e = E_m \sin \omega t \text{ or } e = E_m \sin \theta$$

Similarly, the equation of the induced alternating current is

$$i = I_m \sin \omega t$$

Provided the coil circuit has been closed through a resistive load.

Since the value of $\omega = 2\pi f$, therefore

$$e = E_m \sin 2\pi ft = E_m \sin\left[\frac{2\pi}{T}\right]t$$

and $$i = I_m \sin 2\pi ft = \frac{1}{m} \sin\left[\frac{2\pi}{T}\right]t$$

where $$f = \text{Frequency}$$

$$T = \text{Time period of alternating voltage or current.}$$

and $$T = \frac{I}{f}$$

From the above equations, it is clear that the magnitude of induced *e.m.f.* and current varies according to the sine of the angle θ or ωt. The wave shape of induced *e.m.f.* is as shown in Fig. 2.3*(a)*.

If this voltage is applied across a resistor, an alternating current will pass through it varying sinusoidally *i.e.* following a sine law.

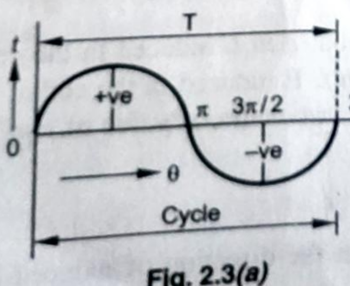


Fig. 2.3*(a)*

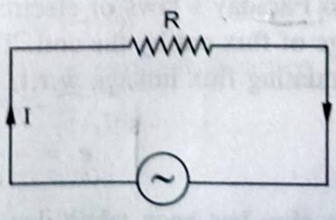


Fig. 2.3*(b)*

Then, the equation of alternating current is

$$i = I_m \sin \omega t$$

or
$$i = I_m \sin \theta$$

2.3. REPRESENTATION OF SINUSIODAL QUANTITY

A scalar quantity has only magnitude and no direction such as 10 metres, 15 seconds and 10 rupees etc.

A vector quantity is that which has both magnitude and direction. Such quantities are known only when particulars of their magnitude, direction or the sense in which they act, are given. They are represented by straight lines called vectors. The length of line gives the magnitude of alternating quantity, the inclination of the line with respect to some reference axis gives the direction and arrowhead placed at one end represents the direction in which that quantity acts. The A.C. quantities like voltage and currents are represented by such vectors rotating anti-clockwise with the same frequency as that of alternating quantity.

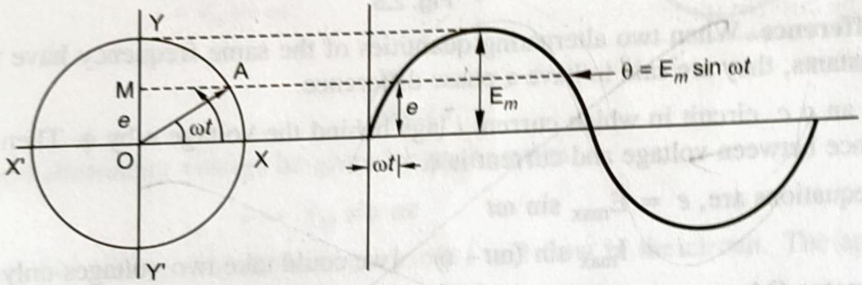


Fig. 2.4

In Fig. 2.4, OA is such a vector which represents the maximum value of A.C. and its angle with x-axis gives its phase.

Let the alternating voltage be represented by $e = E_m \sin \omega t$, then the projection of OA on the y-axis at any instant gives the instantaneous value of that voltage.

$$OM = OA \sin \omega t$$

$\therefore$
$$e = OA \sin \omega t$$

or
$$e = E_m \sin \omega t$$

Here it should be noted that a line OA can be made to represent alternating voltage or current, if it satisfies the following conditions :

(i) Its length should be equal to the maximum value of sinusoidal alternating voltage to a suitable scale.

(ii) It should be in the horizontal position at the same instant when the alternating quantity is zero and increasing positively.

(iii) Its angular velocity (ω) should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

Phase. *The phase of an alternating quantity (current or voltage) at any particular instant is defined as the fractional part of a cycle through which the quantity has advanced from selected origin.*

In practice, we are more concerned with the phase difference between the two alternating quantities.

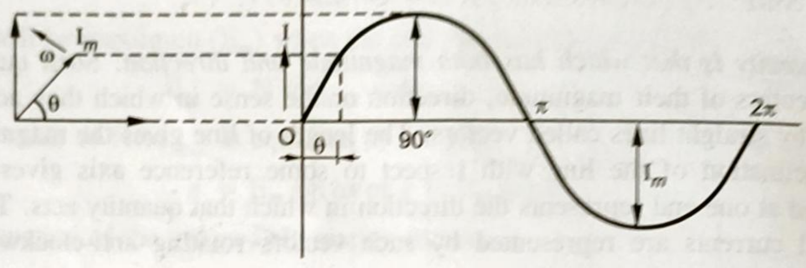


Fig. 2.5

Phase difference. When two alternating quantities of the same frequency have their values zero at different instants, they are said to have a phase difference.

Consider an *a.c.* circuit in which current i lags behind the voltage e by ϕ. Then, we say that the phase difference between voltage and current is ϕ.

Their equations are, $e = E_{max} \sin \omega t$

$i = I_{max} \sin (\omega t - \phi)$ [we could take two voltages only or two currents]

Let the vector OA represents voltage and OB represents current. Since the vectors are taken to rotate anti-clockwise so OB lags behind OA (– angle) by an angle ϕ. $[\because \omega t - (\omega t - \phi) = \phi]$

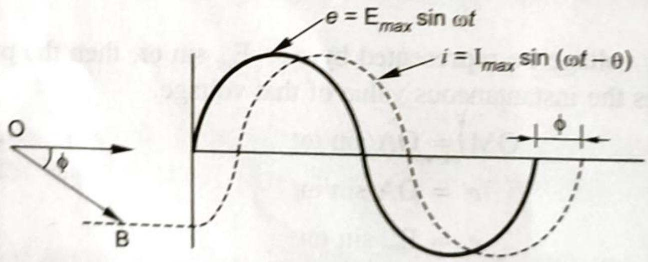


Fig. 2.6

It means OA will reach its maximum value first and OB afterwards. Thus, $\overrightarrow{OA}$ (vector OA) leads $\overrightarrow{OB}$ (vector OB) by an angle ϕ, or $\overrightarrow{OB}$ lags behind $\overrightarrow{OA}$ by an angle ϕ.

2.4. SIMPLE RLC CIRCUITS

2.4.1. A.C. Through Resistance, Inductance and Capacitance

Alternating supply is used in our domestic as well as industrial installations. The path taken by an alternating current in a circuit is known as A.C. circuit. In A.C. circuits, the opposition to the flow of current may be due to Resistance (R), Inductive reactance (X_L) and Capacitive reactance (X_C) whereas in *d.c.* circuits, the opposition to the flow of current was due to resistance only. We shall now consider the phase angle introduced between alternating voltage and current when the circuit contains resistance only, inductance only and capacitance only.

(i) A.C. Through Pure Resistance Only :

The circuit containing pure resistance R Ω is shown in Fig. 2.7.

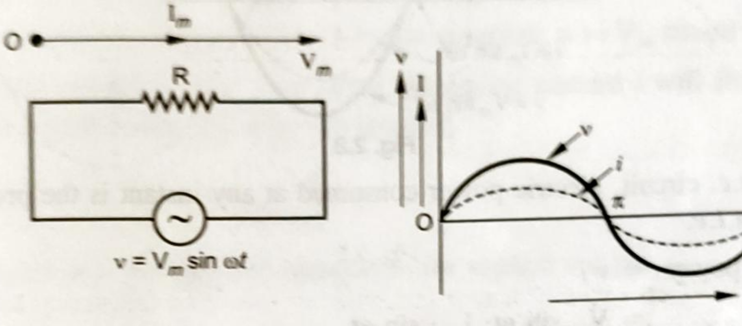


Fig. 2.7

Let the applied alternating voltage be given by the equation,

$$v = V_m \sin \omega t \quad \ldots(i)$$

As a result of this voltage, an alternating current *i* will flow in the circuit. The applied voltage has to overcome the voltage drop in the resistance only *i.e.*

$$v = iR \quad \text{or} \quad i = \frac{v}{R}$$

Putting the value of *v* from above in *(i)*, we get,

$$V_m \sin \omega t = iR$$

$$\therefore \quad i = \frac{V_m \sin \omega t}{R} \quad \ldots(ii)$$

Current *i* is maximum when ωt is 90° or sin ωt = 1

$$\therefore \quad I_m = \frac{V_m}{R} \quad \ldots(iii)$$

From equations *(ii)* and *(iii)*, we have,

$$i = I_m \sin \omega t$$

By comparing equations *(i)* and *(iii)*, we find that alternating voltage and current are in phase with each other as shown in Fig. 2.8.

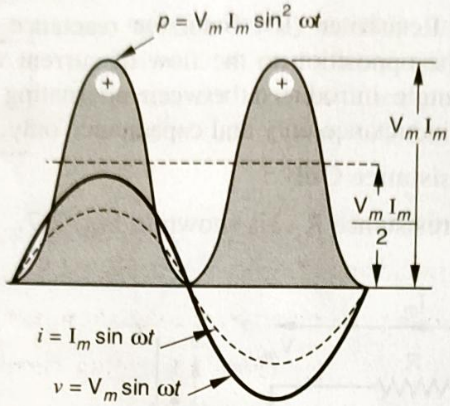


Fig. 2.8

Power : In an *a.c.* circuit, electric power consumed at any instant is the product of voltage and current at that instant *i.e.*

$$\begin{aligned}
\text{Instantaneous power} &= v \cdot i \\
&= V_m \sin \omega t \cdot I_m \cdot \sin \omega t \\
&= V_m \cdot I_m \cdot \sin^2 \omega t \\
&= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot 2 \sin^2 \omega t \\
&= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot (1 - \cos 2\omega t) \\
&= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t
\end{aligned}$$

Power consists of two parts, constant part $\left(\frac{V_m I_m}{2}\right)$ and a fluctuating part $\left(\frac{V_m I_m}{2}\right) \cos 2\omega t$. For a complete cycle, the average value of $\frac{V_m I_m}{2} \cos 2\omega t$ is zero.

Hence, power for whole cycle is $\frac{V_m I_m}{2}$

$$\therefore \qquad P = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V \times I \text{ watt}$$

where V is *r.m.s.* value of applied voltage and I is *r.m.s.* value of the current.

Hence power, $P = V \times I$ watt

It is seen from the power curve that no part of the power cycle becomes negative at any time. In other words in a purely resistive circuit, power is never zero. So, it is clear that power is always positive. This is so because the instantaneous values of voltage and current are always either both positive or negative and hence, the product is always positive.

Phase Angle. It is seen from equations *(i)* and *(ii)* that the voltage and current are in the same phase with each other *i.e.*, they pass through their zero values at the same time and also attain their +ve and −ve peak values at the same time.

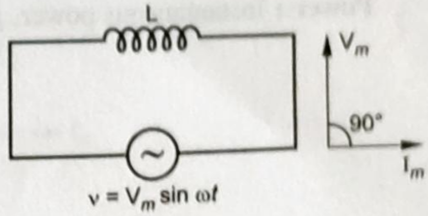


Fig. 2.9

(ii) **A.C. Through Pure Inductance :**

The circuit containing pure inductance L is shown in Fig. 2.9.

Let the applied alternating voltage be given by the equation, $v = V_m \sin \omega t$ ...*(i)*

As a result of this voltage $v = V_m \sin \omega t$ an alternating current i will flow in the circuit and induce an *e.m.f.* in the inductance, given by the relation.

$$e = -L\frac{di}{dt}$$

Since this induced *e.m.f.* is equal and opposite to the applied voltage,

$$\therefore \quad v = -e = -\left(-L\frac{di}{dt}\right) = L\frac{di}{dt}$$

$$\therefore \quad V_m \sin \omega t = L\frac{di}{dt}$$

or
$$di = \frac{V_m}{L} \cdot \sin \omega t \cdot dt$$

Integrating both sides, w.r.t. t, we get,

$$i = \frac{V_m}{L}\int \sin \omega t \cdot dt$$

or
$$i = \frac{V_m}{\omega L}(-\cos \omega t)$$

or
$$i = \frac{V_m}{\omega L}\sin(\omega t - \pi/2) \quad \text{...(ii)}$$

The value of current will be maximum when $\sin(\omega t - \pi/2)$ is unity.

Hence the value of maximum current,

$$I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}, \text{ where } \omega L = X_L$$

Then by substituting the value of $\frac{V_m}{\omega L} = I_m$ in equation *(ii)*, we get

$$i = I_m \sin(\omega t - \pi/2) \quad \text{...(iii)}$$

Power : Instantaneous power, $p = v \cdot i$

$$= V_m \sin \omega t \cdot I_m \sin(\omega t - \pi/2)$$

$$= -V_m I_m \sin \omega t \cdot \cos \omega t$$

$$= -\frac{V_m I_m}{2} \cdot \sin 2\omega t$$

$\therefore$ Average power, P = Average of p over one cycle

$$= \frac{1}{2\pi} \int_0^{2\pi} -\frac{V_m I_m}{2} . \sin 2\omega t \, d(\omega t) = 0$$

Hence, power consumed in a pure inductive circuit is zero.

Phase angle. From equations *(i)* and *(iii)*, it is found that current flowing through a pure inductive circuit lags behind the applied voltage by $\pi/2$ radian as shown in Fig. 2.10.

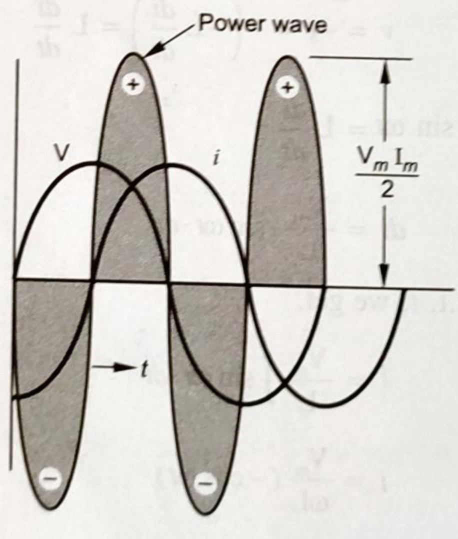


Fig. 2.10

Hence in an *a.c.* circuit containing pure inductance, current lags behind the applied voltage by 90°.

Power curve. From the power curve diagram, it is clear that the average power in a half cycle is zero, as the negative and positive loop areas under power curve are the same. Fig. 2.10 shows the power curve for a pure inductive circuit. During the first quarter cycle (90°), the power supplied by the source that is stored in the magnetic field set up around the coil. In the next quarter cycle (*i.e.* 90°), the magnetic field collapses and the power stored in the field is returned to the source. This process is repeated in each and every half cycle. Hence, no power is consumed in any pure inductive circuit.

(iii) A.C. Through Pure Capacitance :

The circuit containing pure capacitance C is shown in Fig. 2.11.

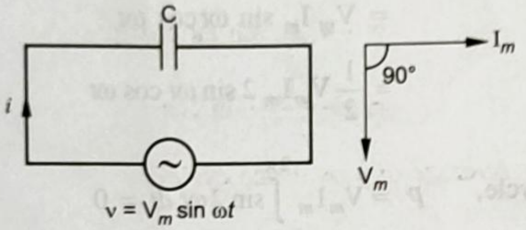


Fig. 2.11

Let the applied alternating voltage be given by the equation,

$$v = V_m \sin \omega t \qquad \ldots(i)$$

Charge on the plates at that instant

i.e., $q = Cv$ (where C is the capacitance).

$$= CV_m \sin \omega t$$

Now current i is given by the rate of flow of charge.

$$\therefore \quad i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin \omega t) = \omega CV_m \cos \omega t$$

or

$$i = \frac{V_m}{\frac{1}{\omega C}} \cdot \cos \omega t$$

or

$$= \frac{V_m}{\frac{1}{\omega C}} . \sin(\omega t + \pi / 2) \qquad \ldots(ii)$$

Now current i will be maximum when $\sin(\omega t + \pi/2)$ is unity

$$\therefore \quad I_m = \frac{V_m}{\frac{1}{\omega C}} = \frac{V_m}{X_C} \qquad \left(\because X_C = \frac{1}{\omega C}\right)$$

The denominator $\frac{1}{\omega C}$ is known as capacitive reactance and is in ohm. If C is in farad and ω is in radian/second, it is denoted by X_C.

$\therefore$Putting the value in equation *(ii)*, we get,

$$\therefore \quad i = I_m \sin(\omega t + \pi/2) \qquad \ldots(iii)$$

Comparing equations *(i)* and *(iii)*, it is found that current leads the applied voltage by an angle $\pi/2$ radian or 90°.

Hence, we see that current in a pure capacitive circuit leads its voltage by 90° as shown in Fig. 2.11.

Power. Instantaneous power, $p = vi$

$$= V_m \sin \omega t \, I_m \sin (\omega t + \pi/2)$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{1}{2} V_m I_m 2 \sin \omega t \cos \omega t$$

Power for the whole cycle, $\quad p = V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt = 0$

So, we see that in a pure capacitive circuit the power consumed is zero.

Power Curve : The power curve for a pure capacitive circuit is shown in Fig. 2.12.

It is clear from the curve, that the average power in half cycle is zero because the positive and negative loop areas under power curve are same.

In the first quarter, the power (energy) supplied by the source is stored in the electric field set up between the capacitor plates. In the next quarter, the electric field collapses and the power (energy) stored in the electric field is repeated in each and every half cycle. Hence, no power is consumed by the circuit.

Power angle : From equations *(i)* and *(iii)*, it is clear that the current passing in any pure capacitive circuit leads the applied voltage by 90° as shown in Fig. 2.12.

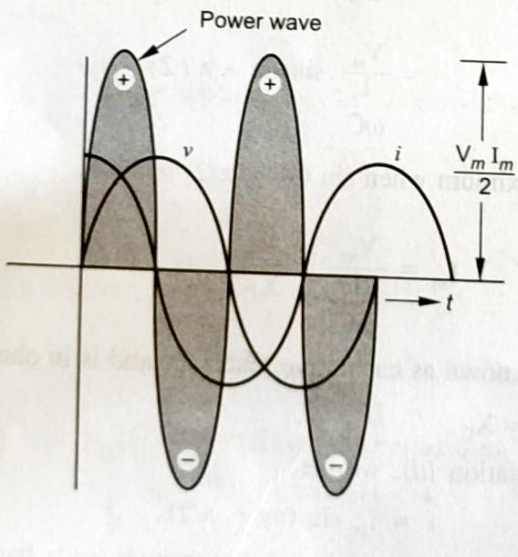


Fig. 2.12

Problem 2.1. An a.c. circuit consists of a pure resistance of 25 ohm and is connected across an a.c. supply of 250 V, 50 Hz. Calculate ;

***(i)* Current**

***(ii)* Power consumed**

***(iii)* Write down the equations of voltage and current.**

Sol. Resistance of the circuit, R = 25 Ω

$$\text{Applied voltage, } V = 250 \text{ V}$$

$$\text{Supply frequency, } f = 50 \text{ Hz}$$

(i) Current in the circuit, $I = \frac{V}{R} = \frac{250}{25} = \mathbf{10\ A\ (Ans.)}$

(ii) Power consumed, $P = V \cdot I = 250 \times 10 = \mathbf{2500\ W\ (Ans.)}$

(iii) Max. value of voltage, $V_m = \sqrt{2} \times I_{r.m.s.} = 1 \cdot 4142 \times 250 = 353 \cdot 5 \text{ V}$

$\therefore$ $\mathbf{v = 353 \cdot 5 \sin 314\, t\ (Ans.)}$

Maximum value of current, $I_m = \sqrt{2} \times I_{r.m.s.} = 1 \cdot 414 \times 10 = \mathbf{14 \cdot 142\ A}$

$\therefore$ $\mathbf{i = 14 \cdot 142 \sin 314\, t\ (Ans.)}$

Problem 2.2. An inductive coil has negligible resistance and inductance of 0·1 henry. It is connected across 220 volt, 100 hertz supply. Find ;

***(i)* Current**

***(ii)* Power**

***(iii)* Write down the equations of voltage and current.**

Sol. Inductance of coil, L = 0·1 H

$$\text{Applied voltage, } V = 220 \text{ V}$$

$$\text{Supply frequency, } f = 100 \text{ Hz}$$

$$\text{Inductive reactance, } X_L = \omega L = 2 \times 3 \cdot 14 \times 100 \times 0 \cdot 1 = 62 \cdot 8\ \Omega$$

(i) Current, $I = \frac{220}{62 \cdot 8} = \mathbf{3 \cdot 5\ A\ (Ans.)}$

(ii) Power consumed in a pure inductive circuit is zero

$\therefore$ $\mathbf{P = 0\ (Ans.)}$

(iii) Max. value of voltage $= \sqrt{2} \times 220$

$$V_m = 311 \text{ V}$$

$\therefore$ $\mathbf{v = 311 \sin 62 \cdot 8\, t\ (Ans.)}$

Max. value of current $= \sqrt{2} \times 3{\cdot}5 = 4{\cdot}95$ A

$\therefore \quad i = \mathbf{4{\cdot}95 \sin (62{\cdot}8\, t - \pi/2)}$ **(Ans.)**

Problem 2.3. A capacitor of capacitance 100 μF is connected across a 250 volt, 50 hertz supply. Calculate ;

(i) **Current**

(ii) **Power**

(iii) **Write down the equations of voltage and current.**

Sol. Capacitance of the circuit, C = 100 μF

Voltage applied, V = 250 V

Frequency, f = 50 Hz

$$\text{Capacitive reactance, } X_C = \frac{1}{\omega C} = \frac{10^{-6}}{2 \times 3{\cdot}14 \times 50 \times 100}$$

$$X_C = 31{\cdot}76\ \Omega$$

(i)
$$\text{Current, } I = \frac{V}{X_C} = \frac{250}{31{\cdot}76} = \mathbf{7{\cdot}852\ A\ (Ans.)}$$

(ii) Power consumed in any pure capacitive circuit is zero

$\therefore \quad$ P = **0 (Ans.)**

(iii) Max. value of voltage, $V_m = \sqrt{2} \times 250 = 353{\cdot}5$ V

$\therefore \quad v = \mathbf{353{\cdot}5 \sin 314\, t}$ **(Ans.)**

Max. value of current, $I_m = \sqrt{2} \times 7{\cdot}852 = 11{\cdot}1$ A

$\therefore \quad i = \mathbf{11{\cdot}1 \sin (314\, t + \pi/2)}$ **(Ans.)**

2.4.2. Resistance and Inductance in Series

The Fig. 2.13 shows a simple circuit containing resistance of R ohm and pure inductance of L henry in series.

A voltage of R.M.S. value V is applied across the combination. Let the current flowing through the circuit is I ampere.

Let V_R be the voltage across R and V_L is the voltage across inductance L.

The circuit can be easily solved by drawing the phasor diagram.

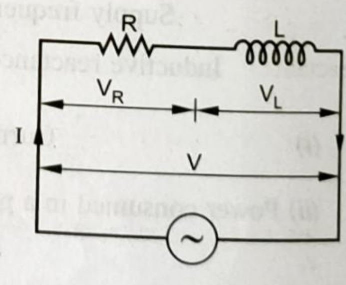


Fig. 2.13

Drawing Phasor diagram : Since in series circuit the current is constant or common to both resistance and inductance, so it should be chosen as reference vector.

The phasor diagram is shown in Fig. 2.14.

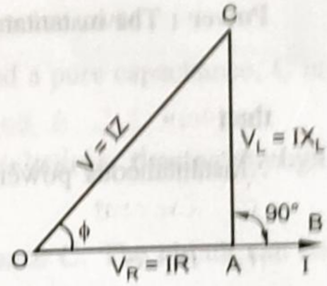


Fig. 2.14

Taking I, as the reference vector, it is shown as OB drawn along x-axis. The voltage across pure resistance R is $V_R = I \cdot R$ and acts in phase with the current as shown in Fig. 2.14.

It is taken as $\overrightarrow{OA}$ in the same line of current.

The voltage across pure inductance is $V_L = I \cdot X_L$ and leads the current by 90° (shown in Fig. 3.30) and it is represented by $\overrightarrow{AC}$ to the same scale.

The applied voltage V is the sum of phasors OA and AC. This is given by OC. $\overrightarrow{OC}$ can be measured and checked as per scale.

From right angled triangle OAC,

$$OC^2 = OA^2 + AC^2$$

or

$$V^2 = (V_R)^2 + (V_L)^2$$

$$(V)^2 = (I \cdot R)^2 + (I \cdot X_L)^2$$

$$V^2 = I^2(R^2 + X_L^2)$$

$$V = I\sqrt{R^2 + X_L^2}$$

$$V = I \cdot Z \text{ or } I = V/Z \text{ or } Z = V/I$$

where $Z = \sqrt{R^2 + X_L^2}$ is known as impedance of the circuit and is measured in ohm.

The Impedance triangle : Since the current is common to all V_R, V_L and V, so it may be omitted and impedance triangle constructed with only R, X_L and Z is as shown in Fig. 2.15.

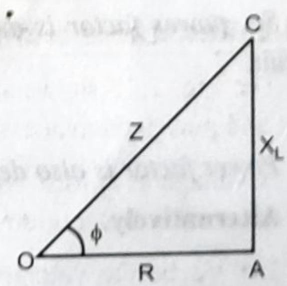


Fig. 2.15

From the impedance triangle, impedance of the circuit

$Z = \sqrt{R^2 + X_L^2}$, where $X_L = \omega L = 2\pi f \cdot L$ is the inductive reactance.

$$\therefore \quad \tan \phi = \frac{X_L}{R}$$

or $\phi = \tan^{-1} \dfrac{X_L}{R}$ is known as the phase angle.

The current lags behind the voltage by this angle ϕ.

$\cos \phi = \dfrac{R}{Z}$ is known as the power factor of the circuit and is only a ratio.

Power : The instantaneous value of the applied voltage is given by the expression,

$$v = V_m \sin \omega t$$

then $$i = I_m \sin(\omega t - \phi)$$

$\therefore$Instantaneous power, $p = vi$

$$= V_m \sin \omega t \cdot I_m \sin(\omega t - \phi)$$

$$= \frac{V_m \cdot I_m}{2} 2 \cdot \sin \omega t \sin(\omega t - \phi)$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} [\cos \phi - \cos(2\omega t - \phi)]$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos \phi - \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos(2\omega t - \phi)$$

Average power consumed in the circuit over a complete cycle,

$$P = \text{Average of } \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos \phi - \text{Average of } \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos(2\omega t - \phi)$$

or $$P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos \phi - 0$$

or $$P = V_{r.m.s.} I_{r.m.s.} \cos \phi$$

or $$P = VI \cos \phi$$

Here, $\cos \phi$ is called power factor of the circuit.

From phasor diagram, $\cos \phi = \frac{V_R}{V} = \frac{I \cdot R}{I \cdot Z} = \frac{R}{Z}$

So, power factor is defined as the choice of the angle between voltage and current in an a.c. circuit.

Or

Power factor is also defined as the ratio of resistance to impedance of an a.c. circuit.

Alternatively.

$$\text{Power} = V \cdot I \cdot \cos \phi$$

$$= (I \cdot Z) \cdot I \cdot \left(\frac{R}{Z}\right) = I^2 \cdot R$$

From the above, it is clear that power is consumed in resistance only and inductance does not consume any power.

2.4.3. Resistance and Capacitance in Series

The Fig. 2.16(*a*) shows a circuit containing pure resistance, R ohm and a pure capacitance, C in series.

A voltage of *r.m.s.* value V is applied across the combination. Let the current flowing through the circuit is I ampere.

Let V_R be the voltage across R and V_C is the voltage across capacitance C. The circuit can be shown by drawing the phasor diagram [Fig. 2.16(*b*)].

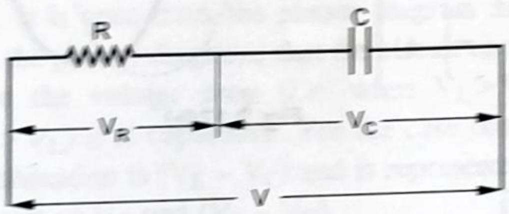


Fig. 2.16(*a*)

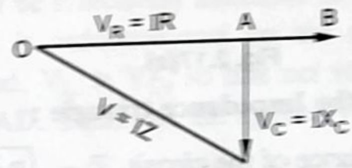


Fig. 2.16(*b*)

Drawing of Phasor Diagram : In series circuits, the current is common to both resistance and capacitance, so it should be taken as a reference vector. The phasor diagram is shown in Fig. 2.16(*b*). Taking I, as the reference vector, it is shown as OB drawn along *x*-axis. The voltage across pure resistance R is $V_R = I R$ and acts in phase with the current as shown in Fig. 2.16(*b*). It is taken as $\overrightarrow{OA}$ in the same line of the current.

The voltage across pure capacitance is $V_C = I X_C$, lags the current by 90° [shown in Fig. 2.16(*b*)] and is represented by $\overrightarrow{AC}$ to the same scale.

The applied voltage V is the sum of phasors OA and OC. This is given by $\overrightarrow{OC}$. OC can be measured and checked as per scale.

From right angled triangle OAC :

$$OC^2 = OA^2 + AC^2$$

or

$$V^2 = (V_R)^2 + (V_C)^2$$

$$V^2 = (IR)^2 + (IX_C)^2$$

$$V = I\sqrt{R^2 + X_C^2}$$

$$\therefore \quad V = I Z \text{ or } I = \frac{V}{Z} \text{ or } Z = \frac{V}{I}$$

Here, $\sqrt{R^2 + X_C^2}$ is known as impedance of the circuit. It is denoted by Z and is measured in ohm.

The Impedance Triangle : Since the current is common to all V_R, V_C and V, so it may be omitted and impedance triangle is constructed with only R, X_C and Z as shown in Fig. 2.17*(a)*.

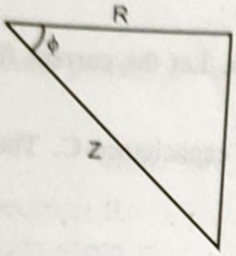


Fig. 2.17*(a)*

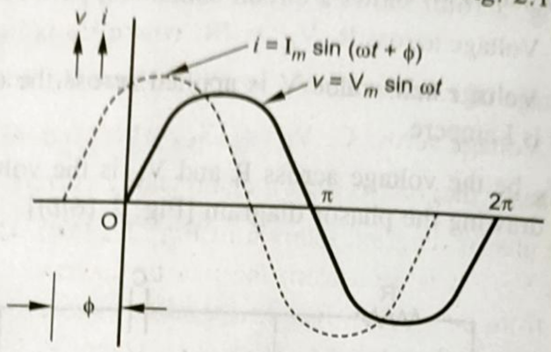


Fig. 2.17*(b)*

From the Impedance triangle :

Impedance of the circuit, $Z = \sqrt{R^2 + X_C^2}$

where, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$ is the capacitive reactance

$\tan \phi = \dfrac{X_C}{R} \Rightarrow \phi = \tan^{-1} \dfrac{X_C}{R}$ is known as the phase angle.

The current leads the voltage by this angle ϕ.

Power : The instantaneous value of v is given by the expression

$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t + \phi)$$

Average power, $P = \text{Average of } vi = VI \cos \phi$

2.4.4. R–L–C Series Circuit

A circuit which contains a pure resistance R, a pure inductance L and pure capacitance C connected in series is called an R-L-C series circuit. The Fig. 2.18 *(a)* shows an R-L-C series circuit.

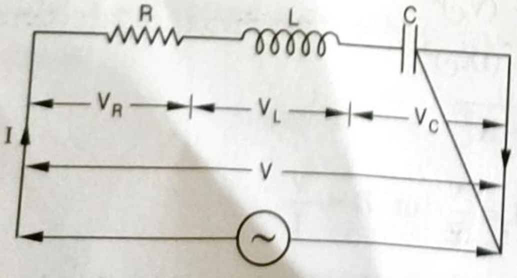


Fig. 2.18*(a)*

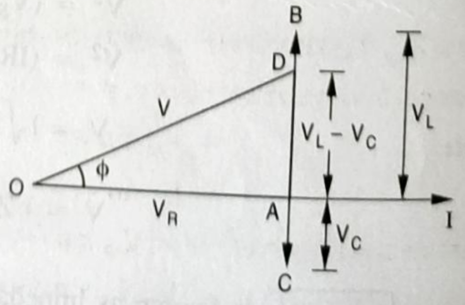


Fig. 2.18*(b)*

If V (r.m.s.) is the voltage across the circuit and I (*r.m.s*) is the current flowing in the circuit. Then the voltages across each component will be as below.

Voltage across R, $V_R = IR$, where V_R is in phase with I.

Voltage across L, $V_L = IX_L$, where V_L leads I by 90°.

Voltage across C, $V_C = IX_C$, where V_C lags I by 90°.

The phasor diagram is drawn again taking current as the reference vector.

In the phasor diagram (shown in Fig. 2.18*(b)*). OA represents V_R, AB represents V_L and AC represents V_C. It is seen from the phasor diagram that phasor V_L is in phase opposition to V_C. It is clear from the phasor diagram, that the circuit can either be effectively inductive or capacitive, depending upon the voltage drop (*i.e.* when $V_L > V_C$) it is inductive and on the other hand (*i.e.* when $V_C > V_L$) it is capacitive. For the case considered, $V_L > V_C$, so that net voltage drop across L–C combination is $(V_L - V_C)$ and is represented by AD. Therefore the applied voltage V is the the phasor sum of V_R and $(V_L - V_C)$.

$$\therefore \quad V = \sqrt{(V_R^2) + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$V = IZ$$

Where $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and offers opposition to the flow of current and is called impedance of the circuit.

Phase angle : From phasor diagram :

$$\text{Circuit power factor, } \cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{(X_L - X_C)}{R}$$

Since X_L, X_C and R are known, therefore, phase angle ϕ of the circuit can be determined.

Power : Instantaneous power, $p = v \cdot i$

Here, $\quad v = V_m \sin \omega t$

$$i = I_m \sin(\omega t - \phi)$$

Instantaneous power, $p = V_m \sin(\omega t - \phi)\, I_m \sin(\omega t - \phi)$

or $\quad p = V_m . I_m . \sin \omega t \sin(\omega t - \phi)$

Average power consumed in the circuit can be derived as in case of R–L circuit.

$\therefore$ Average Power, $P = V\, I \cos\phi$.

Three cases of R-L-C Series Circuit. In the previous article, we have seen that the impedance of an R-L-C series circuit is given by :

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Case I. When $(X_L - X_C)$ is positive *i.e.* if $X_L > X_C$, then phase angle ϕ is positive and the circuit will be inductive. In such cases, current lags the voltage by an angle ϕ.

Case II. When $(X_L - X_C)$ is negative *i.e.* if $(X_L < X_C)$, then the phase angle ϕ is negative and the circuit is capacitive. In such circuits, current leads the voltage by an angle ϕ.

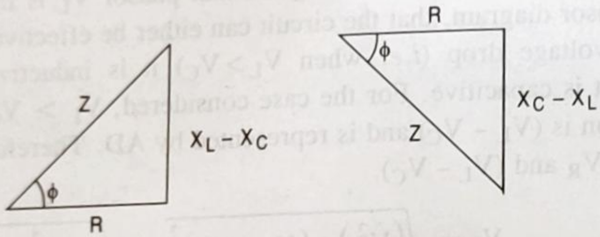


Fig. 2.19*(a)* Fig. 2.19*(b)*

Case III. When $(X_L - X_C)$ is zero (*i.e.* $X_L = X_C$), then the phase angle $\phi = 0°$ and the circuit is pure resistive In such cases, the current is in phase with the voltage. The power factor of such circuits is unity. If the equation for the applied voltage is $v = V_{max} \sin \omega t$.

Then the equations of the currents can be :

(*i*) $i = I_{max} \sin(\omega t - \phi)$... case (*i*) ; $X_L > X_C$

(*ii*) $i = I_{max} \sin(\omega t + \phi)$... case (*ii*) ; $X_L < X_C$

(*iii*) $i = I_{max} \sin \omega t$ ... .case (*iii*) ; $X_L = X_C$

Important and Expected Questions

Q.1. Describe the principle of generation of alternating voltage and current.

Ans. Principle : *When a coil or conductor is moved in magnetic field, an e.m.f. is induced in the coil or conductor.*

When a conductor is moved in magnetic field, magnetic lines of force are cut. In other words, flux is cut by the conductor or flux linking the conductor changes. According to Faraday's first law of electromagnetic induction an *e.m.f.* is induced in the conductor when magnetic flux is cut by the conductor.

The alternating voltage reverses its polarity and the current reverses its direction in the circuit.

Alternating voltage and current can be generated by :

(*i*) Rotating a coil in a magnetic field.

(*ii*) Rotating magnetic field within a stationary coil.

A.C. Generator. A simple *a.c.* generator consists of a rectangular coil made up of large number of turns and mounted on a shaft which can rotate. This arrangement is called **Armature**. The armature is placed in a very strong magnetic field and rotated by an engine or a turbine.

Q.2. Explain the concept of phase difference between two sinusoidal a.c. quantities.

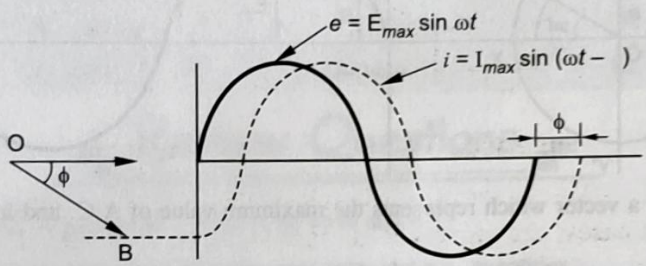
Ans. When two alternating quantities of the same frequency have their values zero at different instants, they are said to have a phase difference.

Consider an *a.c.* circuit in which current i lags behind the voltage e by ϕ. Then, we say that the phase difference between voltage and current is ϕ.

Their equations are, $e = E_{max} \sin \omega t$

$i = I_{max} \sin (\omega t - \phi)$ [we could take two voltages only or two currents]

Let the vector OA represents voltage and OB represents current. Since the vectors are taken to rotate anti-clockwise so OB lags behind OA (– angle) by an angle ϕ. [$\because \omega t - (\omega t - \phi) = \phi$]



It means OA will reach its maximum value first and OB afterwards. Thus, $\overrightarrow{OA}$ (vector OA) leads $\overrightarrow{OB}$ (vector OB) by an angle ϕ, or $\overrightarrow{OB}$ lags behind $\overrightarrow{OA}$ by an angle ϕ.

Q.3. Derive an expression for the instantaneous value of alternating voltage varying sinusoidally.

Ans. According to Faraday's laws of electromagnetic induction, *e.m.f.* induced in the coil is equal to the rate of change of flux cut by the coil. Therefore, the *e.m.f.* E induced in the coil at this instant is given by differentiating flux linkage w.r.t. time. This is the instantaneous value of *e.m.f.*

$$\therefore \quad e = -\frac{d}{dt}(\text{Flux linkage})$$

The negative sign has been taken due to the reason that the direction of induced *e.m.f.* is such that it opposes the cause producing it (Lenz's law).

$$\therefore \quad e = -\frac{d}{dt}(N\,\phi_{max} \cos \omega t)$$

$$\text{or} \quad e = -N\,\phi_{max} \frac{d}{dt}(\cos \omega t)$$

$$\text{or} \quad e = -N\,\phi_{max}(-\sin \omega t) \times \omega$$

$$\text{or} \quad e = N\omega\,\phi_{max} \sin \omega t \text{ volt} \quad \ldots(i)$$

The value of e will be maximum (E_m) when the coil has turned through 90° ($\because \sin 90° = 1$)

$$\therefore \quad E_m = N\,\omega\phi_{max} \text{ volt} \quad \ldots(ii)$$

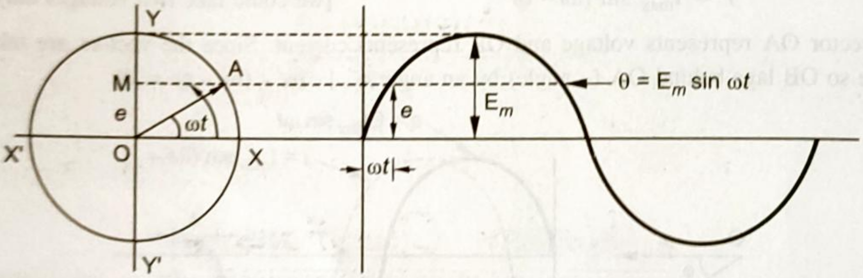
Substituting the value of $N\omega\phi_{max}$ in equation *(i)* we get,

$$e = E_m \sin \omega t \text{ or } e = E_m \sin \theta$$

Q.4. Explain how sinusoidal quantities can be represented by vectors.

Ans. A scalar quantity has only magnitude and no direction such as 10 metres, 15 seconds and 10 rupees etc.

A vector quantity is that which has both magnitude and direction. Such quantities are known only when particulars of their magnitude, direction or the sense in which they act, are given. They are represented by straight lines called vectors. The length of line gives the magnitude of alternating quantity, the inclination of the line with respect to some reference axis gives the direction and arrowhead placed at one end represents the direction in which that quantity acts. The A.C. quantities like voltage and currents are represented by such vectors rotating anti-clockwise with the same frequency as that of alternating quantity.



In Fig., OA is such a vector which represents the maximum value of A.C. and its angle with x-axis gives its phase.

Let the alternating voltage be represented by $e = E_m \sin \omega t$, then the projection of OA on the y-axis at any instant gives the instantaneous value of that voltage.

$$OM = OA \sin \omega t$$

$$\therefore \quad e = OA \sin \omega t$$

or $$e = E_m \sin \omega t$$

Here it should be noted that a line OA can be made to represent alternating voltage or current, if it satisfies the following conditions :

(i) Its length should be equal to the maximum value of sinusoidal alternating voltage to a suitable scale.

(ii) It should be in the horizontal position at the same instant when the alternating quantity is zero and increasing positively.

(iii) Its angular velocity (ω) should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

Objective Type Questions

1. The maximum value of power factor in an a.c. circuit can be ....
2. The power consumed by a pure inductive circuit is ....
3. In an a.c. circuit, the ratio of resistance to impedance is called ....
4. The power factor of a pure capacitive circuit is ....

5. The power factor of a pure resistive circuit is ....
6. In R-L series circuits, the power factor is ....
7. Power loss in an electrical circuit takes place in .... only.
8. For a series R-C circuit, V_R is measured to be 4 V and V_C is measured as 3 V, the a.c. source voltage will be ....
9. The power factor of an ordinary electric bulb is ....
10. kW = kVA × ....
11. In R-C series circuit, the power will be ....
12. While drawing phase diagram for a series circuit. The reference vector is ....

Answers

1. one	2. zero	3. Power Factor	4. zero
5. unity	6. lagging	7. resistance	8. 5 V
9. unity	10. cost	11. leading	12. current.

Review Questions

1. How an alternating *e.m.f.* is generated ? What is its wave shape ?
2. Explain the concept of phase difference between two sinusoidal a.c. quantities.
3. Explain the following :
   (a) A.C. through resistance
   (b) A.C. through inductance
   (c) A.C. through capacitance.
4. Write short notes on the following :
   (a) Resistance and inductance in series
   (b) Resistance and capacitance in series.
5. Explain RLC series circuit briefly.

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