Two box model description

Bingzhang Chen

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Model description

We assume two regions, one in the tropics (temperature = 25 ºC) and the other one in the temperature zone (temperature = 15 ºC). All individuals share the same birth (β) and death (δ) rates. The death rate increases with the local community abundance:

$$\delta_i = \delta_0 + \delta_1 N_i \tag{1}$$

in which i represents region N_i. N_i represents the number of individuals within region i.
The dynamics of the population size of each species follow:

$$\begin{aligned} \frac{dN}{dt} &= (\beta - \delta)N \\ &= (\beta - \delta_0 - \delta_1 N)N \end{aligned} \tag{2}$$

As such, the carrying capacity (K) can be calculated as: $K = \frac{\beta - \delta_0}{\delta_1}$.
We assume that β, δ_0, δ_1 depend on temperature following the Arrhenuis equation:

$$\beta = \beta_r e^{\frac{E}{k_b}(\frac{1}{T_r} - \frac{1}{T})} \tag{3a}$$

$$\delta_0 = \delta_{0,r} e^{\frac{E}{k_b}(\frac{1}{T_r} - \frac{1}{T})} \tag{3b}$$

$$\delta_1 = \delta_{1,r} e^{\frac{E}{k_b}(\frac{1}{T_r} - \frac{1}{T})} \tag{3c}$$

in which k_b is the Boltzmann constant (8.62×10^{-5} eV K^{-1}). T_r is the reference temperature (288.15 K). T is the environmental temperature (K). E is activation energy (eV). β_r, $\delta_{0,r}$, $\delta_{1,r}$ are normalized constants.

Model step

1. Initialize model

- initialize region temperature;
- initialize individual birth and death rates;
- initialize individual birth and death time (a random variable following exponential distribution with the rates defined above);

2. for each individual at each time step,

    - Advance biological clock;
    - As both time to birth and to death are random variables,
        - If (death time > birth time) then,
        - If (current clock >= birth time), birth occurs, reset the clock, and birth and death times. During birth, there is a small probability (ν) that speciation can occur and a new species appears. Update the birth and speciation rates.

  - Otherwise if (current clock >= death time), death occurs and remove the dead individual. Update the death rate.
- There is a small probability (p_m) that an individual can migrate to the other region.
- Save model results at preferred time step.