SUPPLEMENTARY MATERIAL

A. Proof of Proposition III.1 on temporal integration

At a discrete rate τ^{-1}, the Fourier transform of ϕ_T is

$$\widehat{\phi_T}(\omega) = \mathscr{Z}\{\phi_T\}(\mathrm{e}^{\mathrm{j}\omega}) = \frac{s\mathrm{e}^{\mathrm{j}\omega}}{\mathrm{e}^{\mathrm{j}\omega} - (1-s)}. \quad (1)$$

The squared magnitude of the denominator simplifies as

$$\begin{aligned} |\mathrm{e}^{\mathrm{j}\omega} - (1-s)|^2 &= (\cos\omega - (1-s))^2 + (\sin\omega)^2 \\ &= 1 + (1-s)^2 - 2(1-s)\cos\omega \\ &= (1-(1-s))^2 + 2(1-s)(1-\cos\omega) \\ &= s^2 + 2(1-s)(1-\cos\omega). \end{aligned} \quad (2)$$

Let $\omega_0 = \frac{s}{\sqrt{1-s}}$. The equivalent $(1-\cos\omega) \sim \frac{\omega^2}{2}$ at the limit $\omega \rightarrow 0$ indicates a quadratic decay in the lowest frequencies, i.e. a sidelobe falloff of 10 dB per decade. On one hand, the double inequality $-1 \leq \cos\omega \leq 1$ leads to

$$\frac{\omega_0^2}{\omega_0^2 + 2} \leq |\widehat{\phi_T}|^2(\omega) \leq 1. \quad (3)$$

The upper (resp. lower) bound is attained if and only if $\omega = 0$ (resp. $\omega = \pi$). Consequently, ϕ_T has a DC gain of 0 dB at 0 Hz, and a gain of $10\log_{10}\frac{\omega_0^2}{2+\omega_0^2}$ (in dB) at the Nyquist frequency. On the other hand, the cutoff frequency at 3 dB ω_{c} is defined by $|\widehat{\phi_T}|^2(\omega_c) = \frac{1}{2}$, which can be reformulated as

$$1 - \cos\omega_{\mathrm{c}} = \frac{s^2}{2(1-s)} = \frac{\omega_0^2}{2}. \quad (4)$$

Solving Equation 4 with $\omega_{\mathrm{c}} > 0$ completes the proof. ■

Remark. For large T, we can approximate ω_c by ω_0, which is polynomial in s. Indeed, applying Taylor's theorem to the cosine function yields the inequality $|1 - \cos\omega - \frac{\omega^2}{2}| \leq \frac{\omega^4}{24}$, of which we deduce

$$\omega_0 \leq \omega_{\mathrm{c}} \leq \frac{\omega_0}{\sqrt{1 - \frac{\omega_{\mathrm{c}}^2}{12}}} \leq \omega_0 + \frac{\omega_0^2}{12} + \frac{\omega_0^3}{12}. \quad (5)$$

B. Proof of Proposition III.2 on adaptive gain control

First, applying Taylor's theorem to $x \mapsto (1+x)^{-\alpha}$ gives $|(\mathbf{M}+\varepsilon)^{-\alpha} - \varepsilon^{-\alpha}| \leq \alpha\frac{\mathbf{M}}{\varepsilon^{1+\alpha}}$, from which we deduce (i) with a relative error of at most $\alpha\frac{\mathbf{M}}{\varepsilon}$. Symmetrically, we have $|(\mathbf{M}+\varepsilon)^{-\alpha} - \mathbf{M}^{-\alpha}| \leq \alpha\frac{\varepsilon}{\mathbf{M}^{1+\alpha}}$ from which we deduce (ii) with a relative error of at most $\alpha\frac{\varepsilon}{\mathbf{M}}$. ■

C. Proof of Proposition III.4 on dynamic range compression

First, applying Taylor's theorem to $x \mapsto (1+x)^r$ gives $|\mathbf{PCEN} - r\delta^{(r-1)}\mathbf{G}| \leq \frac{1-r}{2}r\delta^{(r-1)}\mathbf{G}^2$, of which we deduce (i) with a relative error of at most $\frac{1-r}{2}\mathbf{G}$. Secondly, we have $\left|\left(1+\frac{1}{\mathbf{G}^r}\right)^r - \left(r + \frac{r}{\mathbf{G}^r}\right)\right| \leq \frac{r(1-r)}{2\mathbf{G}^{2r}}$, which, after adding the constant $(1-r)$, leads to (ii) with a relative error of at most $\frac{1-r}{\mathbf{G}^r}(1+\frac{r}{2\mathbf{G}^r})$ by application of the triangular inequality. ■

Remark. In both cases, the relative error is null if and only if $r=1$, i.e. if there is no root compression.

D. Proof of Proposition IV.1 on the link between T and s

Let $\omega_0^2 = 2\left(1 - \cos\frac{2\pi\tau}{T}\right)$. Proposition III.1 yields $\omega_0^2 = \frac{s^2}{(1-s)}$, and thus the quadratic equation $s^2 + \omega_0^2 s - \omega_0^2 = 0$. Its discriminant is $\Delta = \omega_0^4 + 4\times\omega_0^2 = \omega_0^2(\omega_0^2+4) > 0$, and its two real-valued roots are

$$s = \frac{-\omega_0^2 \pm \omega_0\sqrt{\omega_0^2+4}}{2} = \frac{\omega_0}{2}\left(\pm\sqrt{\omega_0^2+4} - \omega_0\right) \quad (6)$$

We retain the positive root $s > 0$. Replacing ω_0 by its definition in the above completes the proof. ■

E. Proof of Proposition III.3 on invariance to impedance curve

Proposition. *Let $\mathbf{s}(t)$ a realization of AWGN with null mean and unit variance. Let $\mathbf{a}(t) > 0$ a deterministic amplitude envelope $\mathbf{h}(t)$ a filter. Let $\mathbf{E_x}(t,f)$ the mel-frequency spectrogram associated to the source-filter model $\mathbf{x}(t) = \mathbf{a}(t)\times(\mathbf{s}*\mathbf{h})(t)$. If*

1) $\forall t_0,\ \dfrac{\mathrm{d}\log\mathbf{a}}{\mathrm{d}t}(t_0) \ll \dfrac{1}{T}$,
2) $\forall t_0,\ \int_{t_0}^{t_0+\tau}\mathbf{h}(t)\,\mathrm{d}t \ll \dfrac{1}{\tau}$, *and*
3) $\forall f_0,\ \dfrac{\mathrm{d}\log|\widehat{\mathbf{h}}|}{\mathrm{d}f}(f_0) \ll \dfrac{1}{\Delta f_0}$,

where Δf_0 is the frequency interval between f_0 and its adjacent subbands on the mel scale, then $\mathbf{PCEN_x}(t,f) \approx \mathbf{PCEN_s}(t,f)$.

Proof. We adapt a result from [25]. The mel-frequency spectrogram $\mathbf{E_x}(t,f)$ can be defined as the complex modulus of the convolution between the signal $\mathbf{x}(t)$ and a filterbank $\psi_f(t)$ of N whose center frequencies f are tuned to the mel scale, ranging betweeen $f_{\min}$ and $f_{\max}$. For a given f, the first hypothesis allows to factorize the amplitude envelope $\mathbf{a}(t)$ out of the convolution $(\mathbf{s}*\mathbf{h}*\psi_f)(t)$ Furthermore, one has:

$$\begin{aligned} \mathbf{E}_{(\mathbf{s}*\mathbf{h})}(t,f) &= |\mathbf{s}*\mathbf{h}*\psi_f|(t) \\ &= \frac{1}{2\pi}\left|\int_{\mathbb{R}}\widehat{\mathbf{s}}(\omega)\widehat{\mathbf{h}}(\omega)\widehat{\psi_f}(\omega)\exp(2\pi\mathrm{i}\omega t)\,\mathrm{d}t\right|. \end{aligned} \quad (7)$$

The second and third hypotheses allow to approximate $\widehat{\mathbf{h}}(\omega)$ by $\widehat{\mathbf{h}}(f)$ in the equation above. This approximation leads to $\mathbf{E}_{(\mathbf{s}*\mathbf{h})}(t,f) \approx |\widehat{\mathbf{h}}|(f)\mathbf{E_s}(t,f)$. Combining the factorization of the amplitude term $\mathbf{a}(t)$ with the factorization of the filter $|\mathbf{h}|(\omega)$ leads to an approximation of the form:

$$\left|\mathbf{E_x}(t,f) - \mathbf{a}(t)|\widehat{\mathbf{h}}|(f)\mathbf{E_s}(t,f)\right| = \eta(t,f). \quad (8)$$

An upper bound on the expectation of the stochastic residual η is given in [25, Appendix]. Likewise, the first hypothesis allows to approximate $\mathbf{M_x}(t,f)$ by $\mathbf{a}(t)|\widehat{\mathbf{h}}(f)|\mathbf{M}_s(t,f)$ with a stochastic residual term $\nu(t,f)$ of bounded expectation. In the active regime ($\mathbf{M} \gg \varepsilon$), AGC cancels $\mathbf{a}(t)$ and $|\widehat{h}(f)|$ in Equation 3. Thus, Proposition III.2 leads to the inequality

$$|\mathbf{G}_{\mathrm{x}} - \mathbf{G}_{\mathrm{s}}|(t,f) \leq \frac{\eta(t,f)}{(\varepsilon + \mathbf{M_s}(t,f))^{\alpha}} + \frac{\alpha\nu(t,f)}{(\varepsilon+\mathbf{M_s}(t,f))^{1+\alpha}}. \quad (9)$$

Because $\delta > 1$ and $r \leq 1$, DRC is nonexpansive (see Proposition III.4). Therefore, it maps each member of the approximate equality $\mathbf{G_x}(t,f) \approx \mathbf{G_s}(t,f)$ to $\mathbf{PCEN_x}(t,f) \approx \mathbf{PCEN_s}(t,f)$ while reducing the expectation of the residual (right-hand side of Equation 9), which completes the proof. ■