晶界相的体积分数：

$$f^{gb}=\frac{(\frac{1}{2})\cdot 4\pi \bar{R}^2\delta}{(\frac{4}{3})\pi \bar{R}^3}=\frac{3\delta}{2\bar{R}}$$

微观组织热力学p135页5.3.3例题
晶界超细化与晶界偏析的关系

$\delta=3$个原子层 0.75nm

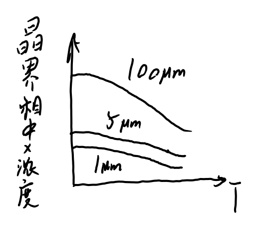
$\bar{R}=5nm$ $f^{gb}=0.225$

$$\begin{cases} x_0=x^{\alpha}(1-f^{gb})+x^{gb}f^{gb} \\ x^{gb}=\dfrac{k_x^{gb}x^{\alpha}}{1+k_x^{gb}x^{\alpha}} \end{cases}$$

当 $\Delta E_x^{gb}=60\ kJ\cdot mol^{-1}$

$\bar{R}=1\mu m$, $f^{gb}=0.0112$

$\bar{R}=3\mu m, 10\mu m$



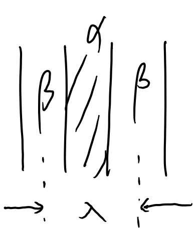
相变驱动力 $\Delta G=\left(\frac{\Delta H}{T_E}\right)\Delta T$ 反映熵大小

共晶、共析能生成α/β层状界面

层状界面能比球状高

微观组织热力学p261页9.3例题共
晶凝固和共析转变的热力学

证明：$\Delta G^{\alpha/\beta}=\dfrac{2\sigma^{\alpha/\beta}V}{\lambda}$ 层片间距



面积 $A^{\alpha/\beta}=\dfrac{2V}{\lambda}$（2个面）

$\Delta G^{\alpha/\beta}=A^{\alpha/\beta}\sigma^{\alpha/\beta}$

需要损耗的一部分驱动力

有效驱动力 $\Delta G_{eff}=\Delta G-\dfrac{2\sigma^{\alpha/\beta}V}{\lambda}$

扩散受化学势梯度驱动

速率 $v=B\cdot F$（B：原子迁移率；F：驱动力）

$F=-\dfrac{\partial\mu_i}{\partial x}$ $J=-D\cdot\dfrac{\partial\rho_i}{\partial x}$（D：扩散系数）

$J=\rho BF=-\rho B\dfrac{\partial\mu_i}{\partial x}$

$D=\rho B\dfrac{\partial\mu_i}{\partial x}\cdot\dfrac{\partial x}{\partial\rho_i}=\rho B\dfrac{\partial\mu_i}{\partial\rho_i}=B\cdot\dfrac{\partial\mu_i}{\partial\ln\rho_i}$

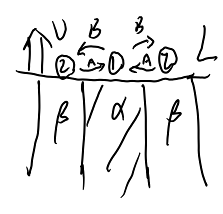
$x_i=\dfrac{\rho_i}{\rho}\Rightarrow\partial\ln\rho_i=\partial\ln x_i$

$D=B\dfrac{RT\partial\ln a_i}{\partial\ln x_i}$ 理想溶液近似、共晶 T_E

$D=B\cdot RT_E$

$B=\dfrac{D}{RT_E}$

共晶



【扩散速率】 $v_B^L(①\to②)=\dfrac{D_{A-B}^L}{RT_E}\cdot\dfrac{\Delta\mu_B^L}{\frac{\lambda}{2}}$

$v_A^L(②\to①)=\dfrac{D_{A-B}^L}{RT_E}\cdot\dfrac{\Delta\mu_A^L}{\frac{\lambda}{2}}$

质量守恒：
$$\begin{cases} v_{B\,(①\to②)}^L=(1-x_B^L)v \\ v_{A\,(②\to①)}^L=(1-x_A^L)v=x_B^L\cdot v \end{cases}$$

$\Delta\mu_{B\,(①\to②)}^L\approx\dfrac{\Delta G_{eff}}{x_B^L}$

$\Delta\mu_{A\,(②\to①)}^L\approx\dfrac{\Delta G_{eff}}{1-x_B^L}$

$v=\dfrac{D_{A-B}^L}{RT_E}\cdot\dfrac{\Delta G_{eff}}{x_B^L(1-x_B^L)\frac{\lambda}{2}}$

$k_E=\dfrac{2D_{A-B}^L}{RT_E x_B^L(1-x_B^L)}$

能量消费明细：$\Delta G(L\to\alpha+\beta)=\underbrace{\dfrac{2\sigma^{\alpha/\beta}V}{\lambda}}_{\text{界面能耗}}+\underbrace{\dfrac{\lambda}{k_E}\cdot v}_{\text{扩散能耗}=\Delta G_{eff}}$

共晶生长速度为最大值时的层间距为λ_0。

此时总驱动力（ΔG）最小

有$\Delta G^{\alpha/\beta}=\Delta G_{eff}$ $\dfrac{\partial v}{\partial\lambda}=0$

$v=\dfrac{k_E}{\lambda}\left(\Delta G-\dfrac{2\sigma^{\alpha/\beta}V}{\lambda}\right)$

$k_E\left(-\dfrac{\Delta G}{\lambda^2}+2*\dfrac{2\sigma^{\alpha/\beta}V}{\lambda^3}\right)=0$

$\Delta G_{min}=\dfrac{4\sigma^{\alpha/\beta}V}{\lambda_0}$

$\dfrac{\partial(\Delta G)}{\partial\lambda}=0$

$\dfrac{\partial\left(\frac{v\lambda}{k_E}+\frac{2\sigma^{\alpha/\beta}V}{\lambda}\right)}{\partial\lambda}=\dfrac{v}{k_E}-\dfrac{2\sigma^{\alpha/\beta}V}{\lambda^2}=0$

$v_0\cdot\lambda_0^2=2\sigma^{\alpha/\beta}Vk_E$

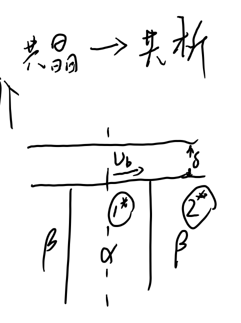
$\Delta G\approx\dfrac{\Delta H}{T_E}\cdot\Delta T=\dfrac{4\sigma^{\alpha/\beta}V}{\lambda_0}$

$\lambda_0=\dfrac{4\sigma^{\alpha/\beta}VT_E}{\Delta H\Delta T}=\dfrac{k_1}{\Delta T}$

$k_1=\dfrac{4\sigma^{\alpha/\beta}V\cdot T_E}{\Delta H}$

$v_0=\dfrac{2\sigma^{\alpha/\beta}\cdot Vk_E}{k_1^2}(\Delta T)^2=k_2(\Delta T)^2$ → 过冷度

v^* 共晶→共析



$1^*\to2^*$

$j_B^i=x_B^\gamma\cdot\delta\cdot v_B^i$

使β v^*生长所需补充的B原子

$j_B^{i\to B}\approx(1-x_B^\gamma)\cdot\dfrac{\lambda_B}{2}\cdot v^*$

$j_B^{i\to B}=j_B^i$

$v^*=\dfrac{\delta x_B^\gamma}{\frac{\lambda_B}{2}(1-x_B^\gamma)}\cdot v_B^i=\dfrac{\delta\cdot v_B^i}{\frac{\lambda}{2}(1-x_B^\gamma)}$

$\lambda_B=x_B^\gamma\cdot\lambda$

$v_B^i(1^*\to2^*)=\dfrac{D_B^i}{RT}\left[\dfrac{\Delta\mu_B^i(1^*\to2^*)}{\frac{\lambda}{2}}\right]$

$=\dfrac{D_B^i}{RT}\dfrac{\Delta\mu_B^\gamma(①\to②)}{\frac{\lambda}{2}}$

$\dfrac{v^*\cdot\frac{\lambda}{2}(1-x_B^\gamma)}{\delta}=\dfrac{D_B^i}{RT}\dfrac{\Delta G_{eff}}{x_B^\gamma\frac{\lambda}{2}}$

$v^*=\dfrac{k_E^*}{\lambda^2}\left(\Delta G-\dfrac{2\sigma^{\alpha/\beta}V}{\lambda}\right)$, $k_E^*=\dfrac{4D_B^i\delta}{RTx_B^\gamma(1-x_B^\gamma)}$

省略了推导 → $\Delta G_{min}^*=\dfrac{3\sigma^{\alpha/\beta}V}{\lambda_0^*}$

西泽 P268

$\lambda_0^*=\dfrac{k_1^*}{RT}$ $v_0^*=k_2^*(\Delta T)^3$

$\Delta G_{eff}^*=\frac{1}{3}\Delta G^*$