



## تکلیف سوم

مریم سعیدمهر  
شماره دانشجویی: ۹۶۲۹۳۷۳

### فهرست مطالب

- ۱ چند سال طول میکشد تا مبلغی که در حساب سپرده با نرخ بهره اسمی سالیانه ۱۸ درصد سرمایه گذاری شده ۳ برابر گردد اگر دوره مرکب شدن ماهیانه باشد  
۲
- ۲ دو دستگاه A و B با اطلاعات زیر را در نظر بگیرید. اگر  $MARR = 15\%$  باشد، انتخاب کدام دستگاه اقتصادی تر است؟  
۲
- ۳ سه پروژه A و B و C را با استفاده از نرخ بازگشت سرمایه با هم مقایسه کنید. عمر مفید پروژه ها ۲۰ سال و حداقل نرخ جذب ۶٪ فرض میشود.  
۳
- ۴ پیوست  
۶

۱ چند سال طول میکشد تا مبلغی که در حساب سپرده با نرخ بهره اسمی سالیانه ۱۸ درصد سرمایه گذاری شده ۳ برابر گردد اگر دوره مرکب شدن ماهیانه باشد

$$\left. \begin{array}{l} F = 3P \\ r = 18\% \\ t = 12 \end{array} \right\} \Rightarrow i_e = \left(1 + \frac{r}{t}\right)^t - 1 = \left(1 + \frac{0.18}{12}\right)^{12} - 1 = 19.5618171\%$$

$$\left(\frac{F}{P}, 19.56, n\right) = 3$$

با توجه به جدول ۱۴ مقدار n احتمالا باید بین ۶ و ۷ سال باشد. حال تست میکنیم این جواب درست است یا ن :

$$F = P(1 + i_e)^n$$

$$3P = P(1 + 0.1956)^n$$

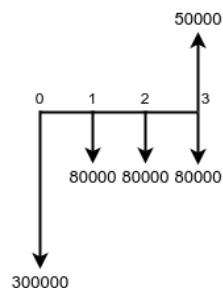
$$3 = (1.1956)^n \Rightarrow n \cong 6 \text{ or } 7 \Rightarrow \begin{cases} n = 6: & (1.1956)^6 = 2.921157954 \\ n = 7: & (1.1956)^7 = 3.49258953 \end{cases}$$

در نهایت مقدار کمی دقیق تر n برابر است با 6.15 یعنی بعد از 6.15 سال سرمایه ما ۳ برابر سرمایه اولیه میشود(با نرخ بهره اسمی ۱۸ درصد و دوره مرکب شدن ماهانه)

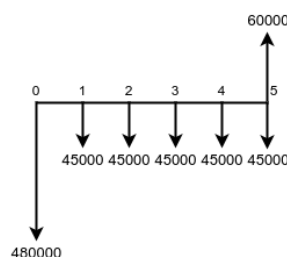
۲ دو دستگاه A و B با اطلاعات زیر را در نظر بگیرید. اگر  $MARR = 15\%$  باشد ، انتخاب کدام دستگاه اقتصادی تر است؟

B	A	.
300000	480000	هزینه اولیه
80000	45000	هزینه سالیانه
50000	60000	ارزش اسقاط
3	5	عمر مفید

براساس جدول فوق ، جریان مالی دو دستگاه A و B به صورت زیر است :

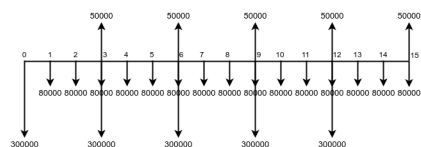


شکل ۲: جریان مالی دستگاه B در طول عمر ۳ ساله اش

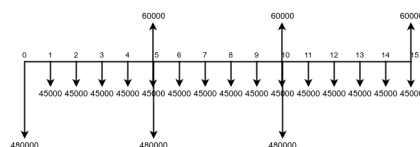


شکل ۱: جریان مالی دستگاه A در طول عمر ۵ ساله اش

کوچکترین مضرب مشترک طول عمرهای این دو دستگاه  $\frac{3 \times 5}{gcd(3,5)} = 15$  است به این ترتیب جریان مالی این دو دستگاه برای ۱۵ سال به شکل زیر خواهد بود :



شکل ۴: جریان مالی دستگاه B در طول دوره ۱۵ ساله



شکل ۳: جریان مالی دستگاه A در طول دوره ۱۵ ساله

از آنجایی که تنهای درآمدان فقط ارزش اسقاطی است و مابقی همگی هزینه هستند لذا کافیت که ارزش کنونی هزینه ها را برای این دو دستگاه مقایسه کنیم و دستگاهی اقتصادی تر است که ارزش کنونی هایش کمتر باشد.  
به این ترتیب داریم :

$$\begin{aligned} PWC_A &= 480000 + 480000 \left( \frac{P}{F}, 15\%, 5 \right) + 480000 \left( \frac{P}{F}, 15\%, 10 \right) \\ &\quad + 45000 \left( \frac{P}{A}, 15\%, 15 \right) \\ &\quad - 60000 \left( \frac{P}{F}, 15\%, 5 \right) - 60000 \left( \frac{P}{F}, 15\%, 10 \right) - 60000 \left( \frac{P}{F}, 15\%, 15 \right) \\ &= 480000 + 480000 \times 0.4972 + 480000 \times 0.2472 \\ &\quad + 45000 \times 5.847 \\ &\quad - 60000 \times 0.4972 - 60000 \times 0.2472 - 60000 \times 0.1229 \\ &= 1048389 \end{aligned}$$

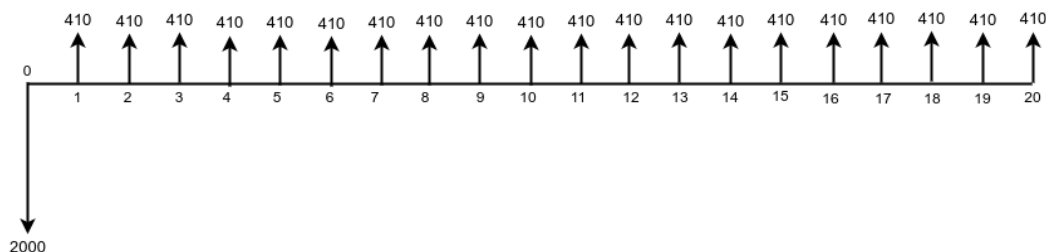
$$\begin{aligned} PWC_B &= 300000 + 300000 \left( \frac{P}{F}, 15\%, 3 \right) + 300000 \left( \frac{P}{F}, 15\%, 6 \right) + 300000 \left( \frac{P}{F}, 15\%, 9 \right) + 300000 \left( \frac{P}{F}, 15\%, 12 \right) \\ &\quad + 80000 \left( \frac{P}{A}, 15\%, 15 \right) \\ &\quad - 50000 \left( \frac{P}{F}, 15\%, 3 \right) - 50000 \left( \frac{P}{F}, 15\%, 6 \right) - 50000 \left( \frac{P}{F}, 15\%, 9 \right) - 50000 \left( \frac{P}{F}, 15\%, 12 \right) - 50000 \left( \frac{P}{F}, 15\%, 15 \right) \\ &= 300000 + 300000 \times 0.6575 + 300000 \times 0.4323 + 300000 \times 0.2843 + 300000 \times 0.1869 \\ &\quad + 80000 \times 5.847 \\ &\quad - 50000 \times 0.6575 - 50000 \times 0.4323 - 50000 \times 0.2843 - 50000 \times 0.1869 - 50000 \times 0.1229 \\ &= 730881 \end{aligned}$$

با توجه به محاسبات فوق ، ارزش هزینه های کنونی دستگاه B کمتر از دستگاه A است لذا خرید دستگاه B اقتصادی تر می باشد

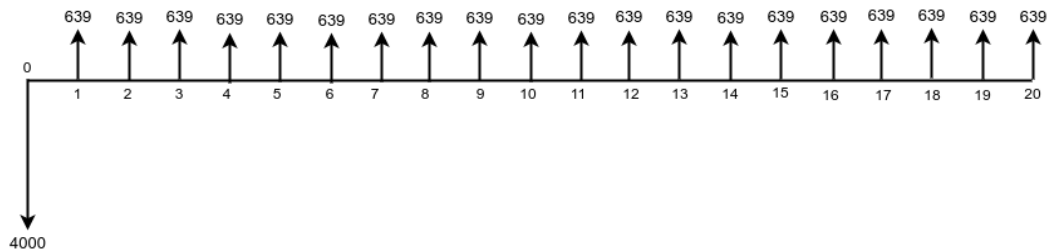
۳ سه پروژه A و B و C را با استفاده از نرخ بازگشت سرمایه با هم مقایسه کنید. عمر مفید پروژه ها ۲۰ سال و حداقل نرخ جذب 6% فرض میشود.

C	B	A	.
5000	4000	2000	هزینه اولیه
700	639	410	درآمد سالیانه

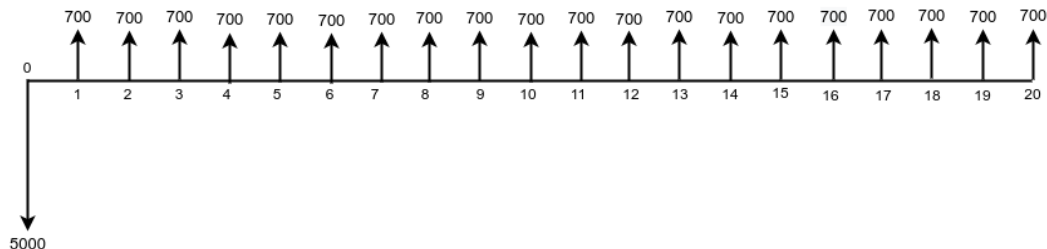
جریان های مالی مشابه زیر است :



شکل ۵: جریان مالی پروژه A در طول دوره ۲۰ ساله



شکل ۶: جریان مالی پروژه B در طول دوره ۲۰ ساله



شکل ۷: جریان مالی پروژه C در طول دوره ۲۰ ساله

به این ترتیب داریم :

$$\left. \begin{aligned} PWC_A &= 2000 \\ PWB_A &= 410 \left( \frac{P}{A}, i\%, 20 \right) \\ PWC_A &= PWB_A \end{aligned} \right\} \Rightarrow 410 \left( \frac{P}{A}, i\%, 20 \right) = 2000 \Rightarrow \left( \frac{P}{A}, i\%, 20 \right) = 4.87804878$$

طبق جدول ۱۵ مقدار  $i$  برای پروژه A برابر با  $i = 20\%$  است.  
برای سایر پروژه ها نیز همین روند را تکرار میکنم :

$$\left. \begin{aligned} PWC_B &= 4000 \\ PWB_B &= 639 \left( \frac{P}{A}, i\%, 20 \right) \\ PWC_B &= PWB_B \end{aligned} \right\} \Rightarrow 639 \left( \frac{P}{A}, i\%, 20 \right) = 4000 \Rightarrow \left( \frac{P}{A}, i\%, 20 \right) = 6.259780908$$

با جدول ۱۳ مقدار  $i$  برای پروژه B برابر با  $i = 15\%$  است.

$$\left. \begin{aligned} PWC_B &= 5000 \\ PWB_B &= 700 \left( \frac{P}{A}, i\%, 20 \right) \\ PWC_B &= PWB_B \end{aligned} \right\} \Rightarrow 700 \left( \frac{P}{A}, i\%, 20 \right) = 5000 \Rightarrow \left( \frac{P}{A}, i\%, 20 \right) = 7.142857143$$

با جدول ۱۲ مقدار  $i$  برای پروژه C برابر با  $i \approx 12\%$  است.  
حال برای مقایسه پروژه ها باید الگوریتم مطرح شده در کلاس (۴ گام) را اجرا کنیم:

۱. گام اول : پروژه ها را برحسب هزینه اولیه به صورت صعودی مرتب کنید.

• این مورد در جدول صورت سوال رعایت شده.

۲. گام دوم : مقدار ROR را برای هر پروژه حساب کنید.

• محاسبات انجام شد و جدول نهایی تا این گام به شکل زیر است :

C	B	A	.
5000	4000	2000	هزینه اولیه
700	639	410	درآمد سالانه
12%	15%	20%	نرخ بازگشت سرمایه

۳. گام سوم: اگر ROR پروژه از MARR کمتر بود آن پروژه را از روند مقایسه ها حذف کنید.

• مقدار  $MARR = 6\%$  است و هیچ پروژه ای در این گام حذف نمیشود.

۴. گام چهارم: برای پروژه های باقیمانده با تحلیل سرمایه گذاری اضافی، پروژه ها را دو به دو مقایسه کنید تا اقتصادی ترین پروژه شناسایی شود.

• حالا شروع میکنیم دو به دو پروژه ها را بررسی کنیم:

B - A	.
2000	هزینه اولیه
229	درآمد سالانه

$$\left. \begin{aligned} PWC_{B-A} &= 2000 \\ PWB_{B-A} &= 229 \left( \frac{P}{A}, i\%, 20 \right) \\ PWC_{B-A} &= PWB_{B-A} \end{aligned} \right\} \Rightarrow 229 \left( \frac{P}{A}, i\%, 20 \right) = 2000 \Rightarrow \left( \frac{P}{A}, i\%, 20 \right) = 8.733624454$$

طبق جداول ۱۰ و ۱۱ مقدار  $i$  بین ۹ و ۱۰ درصد خواهد شد که در هر حال از MARR بیشتر است لذا میران سرمایه گذاری اضافه در پروژه B نسبت به A توجیه اقتصادی دارد و نتیجتاً پروژه B نسبت به A اقتصادی تر است. پس پروژه A حذف میشود. حال مقایسه را بین پروژه های B و C ادامه میدهم.

C - B	.
1000	هزینه اولیه
61	درآمد سالانه

$$\left. \begin{aligned} PWC_{C-B} &= 1000 \\ PWB_{C-B} &= 61 \left( \frac{P}{A}, i\%, 20 \right) \\ PWC_{C-B} &= PWB_{C-B} \end{aligned} \right\} \Rightarrow 61 \left( \frac{P}{A}, i\%, 20 \right) = 1000 \Rightarrow \left( \frac{P}{A}, i\%, 20 \right) = 16.393442623$$

طبق جداول ۸ و ۹ مقدار  $i$  بین ۱۰.۷۵ و ۲ درصد خواهد شد که در هر حال از MARR کمتر است لذا میران سرمایه گذاری اضافه در پروژه C نسبت به B توجیه اقتصادی ندارد و نتیجتاً پروژه B نسبت به C اقتصادی تر است. پس پروژه C حذف میشود و در نهایت پروژه B اقتصادی ترین پروژه بین این سه مورد بود.

Compound Interest Factors									
Single Payment		Uniform Payment Series					Arithmetic Gradient		
n	Compound Amount Factor Find F Given P	Present Worth Factor Find P Given F	Sinking Fund Factor Find A Given F	Capital Recovery Factor Find A Given P	Compound Amount Factor Find F Given A	Present Worth Factor Find P Given A	Gradient Uniform Series Find A Given G	Gradient Present Worth Find P Given G	n
	F/P	P/F	A/F	A/P	F/A	P/A	A/G	P/G	
1	1.018	.9828	1.0000	1.0175	1.000	0.983	0	0	1
2	1.035	.9659	.4957	.5132	2.018	1.949	0.496	0.966	2
3	1.053	.9493	.3276	.3451	3.053	2.898	0.989	2.865	3
4	1.072	.9330	.2435	.2610	4.106	3.831	1.478	5.664	4
5	1.091	.9169	.1931	.2106	5.178	4.748	1.965	9.332	5
6	1.110	.9011	.1595	.1770	6.269	5.649	2.450	13.837	6
7	1.129	.8856	.1355	.1530	7.378	6.535	2.931	19.152	7
8	1.149	.8704	.1175	.1350	8.508	7.405	3.409	25.245	8
9	1.169	.8554	.1036	.1211	9.656	8.261	3.885	32.088	9
10	1.189	.8407	.0924	.1099	10.825	9.101	4.357	39.655	10
11	1.210	.8263	.0832	.1007	12.015	9.928	4.827	47.918	11
12	1.231	.8121	.0756	.0931	13.225	10.740	5.294	56.851	12
13	1.253	.7981	.0692	.0867	14.457	11.538	5.758	66.428	13
14	1.275	.7844	.0637	.0812	15.710	12.322	6.219	76.625	14
15	1.297	.7709	.0589	.0764	16.985	13.093	6.677	87.417	15
16	1.320	.7576	.0547	.0722	18.282	13.851	7.132	98.782	16
17	1.343	.7446	.0510	.0685	19.602	14.595	7.584	110.695	17
18	1.367	.7318	.0477	.0652	20.945	15.327	8.034	123.136	18
19	1.390	.7192	.0448	.0623	22.311	16.046	8.481	136.081	19
20	1.415	.7068	.0422	.0597	23.702	16.753	8.924	149.511	20
21	1.440	.6947	.0398	.0573	25.116	17.448	9.365	163.405	21
22	1.465	.6827	.0377	.0552	26.556	18.130	9.804	177.742	22
23	1.490	.6710	.0357	.0532	28.021	18.801	10.239	192.503	23
24	1.516	.6594	.0339	.0514	29.511	19.461	10.671	207.671	24
25	1.543	.6481	.0322	.0497	31.028	20.109	11.101	223.225	25
26	1.570	.6369	.0307	.0482	32.571	20.746	11.528	239.149	26
27	1.597	.6260	.0293	.0468	34.141	21.372	11.952	255.425	27
28	1.625	.6152	.0280	.0455	35.738	21.987	12.373	272.036	28
29	1.654	.6046	.0268	.0443	37.363	22.592	12.791	288.967	29
30	1.683	.5942	.0256	.0431	39.017	23.186	13.206	306.200	30
36	1.867	.5355	.0202	.0377	49.566	26.543	15.640	415.130	36
40	2.002	.4996	.0175	.0350	57.234	28.594	17.207	492.017	40
48	2.300	.4349	.0135	.0310	74.263	32.294	20.209	652.612	48
50	2.381	.4200	.0127	.0302	78.903	33.141	20.932	693.708	50
52	2.465	.4057	.0119	.0294	83.706	33.960	21.644	735.039	52
60	2.832	.3531	.00955	.0271	104.676	36.964	24.389	901.503	60
70	3.368	.2969	.00739	.0249	135.331	40.178	27.586	1 108.34	70
72	3.487	.2868	.00704	.0245	142.127	40.757	28.195	1 149.12	72
80	4.006	.2496	.00582	.0233	171.795	42.880	30.533	1 309.25	80
84	4.294	.2329	.00531	.0228	188.246	43.836	31.644	1 387.16	84
90	4.765	.2098	.00465	.0221	215.166	45.152	33.241	1 500.88	90
96	5.288	.1891	.00408	.0216	245.039	46.337	34.756	1 610.48	96
100	5.668	.1764	.00375	.0212	266.753	47.062	35.721	1 681.09	100
104	6.075	.1646	.00345	.0209	290.028	47.737	36.652	1 749.68	104
120	8.019	.1247	.00249	.0200	401.099	50.017	40.047	2 003.03	120
240	64.308	.0156	.00028	.0178	3 617.6	56.254	53.352	3 001.27	240
360	515.702	.00194	.00003	.0175	29 411.5	57.032	56.443	3 219.08	360
480	4 135.5	.00024		.0175	236 259.0	57.129	57.027	3 257.88	480

شکل ۸: Compute Interest Factor for  $i = 1.75\%$

2%		Compound Interest Factors								2%
n	Single Payment		Uniform Payment Series				Arithmetic Gradient		n	
	Compound Amount Factor Find F Given P	Present Worth Factor Find P Given F	Sinking Fund Factor Find A Given F	Capital Recovery Factor Find A Given P	Compound Amount Factor Find F Given A	Present Worth Factor Find P Given A	Gradient Uniform Series Find A Given G	Gradient Present Worth Find P Given G		
	F/P	P/F	A/F	A/P	F/A	P/A	A/G	P/G		
1	1.020	.9804	1.0000	1.0200	1.000	0.980	0	0	1	
2	1.040	.9612	.4951	.5151	2.020	1.942	0.495	0.961	2	
3	1.061	.9423	.3268	.3468	3.060	2.884	0.987	2.846	3	
4	1.082	.9238	.2426	.2626	4.122	3.808	1.475	5.617	4	
5	1.104	.9057	.1922	.2122	5.204	4.713	1.960	9.240	5	
6	1.126	.8880	.1585	.1785	6.308	5.601	2.442	13.679	6	
7	1.149	.8706	.1345	.1545	7.434	6.472	2.921	18.903	7	
8	1.172	.8535	.1165	.1365	8.583	7.325	3.396	24.877	8	
9	1.195	.8368	.1025	.1225	9.755	8.162	3.868	31.571	9	
10	1.219	.8203	.0913	.1113	10.950	8.983	4.337	38.954	10	
11	1.243	.8043	.0822	.1022	12.169	9.787	4.802	46.996	11	
12	1.268	.7885	.0746	.0946	13.412	10.575	5.264	55.669	12	
13	1.294	.7730	.0681	.0881	14.680	11.348	5.723	64.946	13	
14	1.319	.7579	.0626	.0826	15.974	12.106	6.178	74.798	14	
15	1.346	.7430	.0578	.0778	17.293	12.849	6.631	85.200	15	
16	1.373	.7284	.0537	.0737	18.639	13.578	7.080	96.127	16	
17	1.400	.7142	.0500	.0700	20.012	14.292	7.526	107.553	17	
18	1.428	.7002	.0467	.0667	21.412	14.992	7.968	119.456	18	
19	1.457	.6864	.0438	.0638	22.840	15.678	8.407	131.812	19	
20	1.486	.6730	.0412	.0612	24.297	16.351	8.843	144.598	20	
21	1.516	.6598	.0388	.0588	25.783	17.011	9.276	157.793	21	
22	1.546	.6468	.0366	.0566	27.299	17.658	9.705	171.377	22	
23	1.577	.6342	.0347	.0547	28.845	18.292	10.132	185.328	23	
24	1.608	.6217	.0329	.0529	30.422	18.914	10.555	199.628	24	
25	1.641	.6095	.0312	.0512	32.030	19.523	10.974	214.256	25	
26	1.673	.5976	.0297	.0497	33.671	20.121	11.391	229.196	26	
27	1.707	.5859	.0283	.0483	35.344	20.707	11.804	244.428	27	
28	1.741	.5744	.0270	.0470	37.051	21.281	12.214	259.936	28	
29	1.776	.5631	.0258	.0458	38.792	21.844	12.621	275.703	29	
30	1.811	.5521	.0247	.0447	40.568	22.396	13.025	291.713	30	
36	2.040	.4902	.0192	.0392	51.994	25.489	15.381	392.036	36	
40	2.208	.4529	.0166	.0366	60.402	27.355	16.888	461.989	40	
48	2.587	.3865	.0126	.0326	79.353	30.673	19.755	605.961	48	
50	2.692	.3715	.0118	.0318	84.579	31.424	20.442	642.355	50	
52	2.800	.3571	.0111	.0311	90.016	32.145	21.116	678.779	52	
60	3.281	.3048	.00877	.0288	114.051	34.761	23.696	823.692	60	
70	4.000	.2500	.00667	.0267	149.977	37.499	26.663	999.829	70	
72	4.161	.2403	.00633	.0263	158.056	37.984	27.223	1 034.050	72	
80	4.875	.2051	.00516	.0252	193.771	39.744	29.357	1 166.781	80	
84	5.277	.1895	.00468	.0247	213.865	40.525	30.361	1 230.413	84	
90	5.943	.1683	.00405	.0240	247.155	41.587	31.793	1 322.164	90	
96	6.693	.1494	.00351	.0235	284.645	42.529	33.137	1 409.291	96	
100	7.245	.1380	.00320	.0232	312.230	43.098	33.986	1 464.747	100	
104	7.842	.1275	.00292	.0229	342.090	43.624	34.799	1 518.082	104	
120	10.765	.0929	.00205	.0220	488.255	45.355	37.711	1 710.411	120	
240	115.887	.00863	.00017	.0202	5 744.4	49.569	47.911	2 374.878	240	
360	1 247.5	.00080	.00002	.0200	62 326.8	49.960	49.711	2 483.567	360	
480	13 429.8	.00007		.0200	671 442.0	49.996	49.964	2 498.027	480	

شکل ۹: Compute Interest Factore for  $i = 2\%$

9%		Compound Interest Factors							9%	
		Single Payment		Uniform Payment Series				Arithmetic Gradient		
		Compound Amount Factor Find <i>F</i> Given <i>P</i> <i>F/P</i>	Present Worth Factor Find <i>P</i> Given <i>F</i> <i>P/F</i>	Sinking Fund Factor Find <i>A</i> Given <i>F</i> <i>A/F</i>	Capital Recovery Factor Find <i>A</i> Given <i>P</i> <i>A/P</i>	Compound Amount Factor Find <i>F</i> Given <i>A</i> <i>F/A</i>	Present Worth Factor Find <i>P</i> Given <i>A</i> <i>P/A</i>	Gradient Uniform Series Find <i>A</i> Given <i>G</i> <i>A/G</i>	Gradient Present Worth Find <i>P</i> Given <i>G</i> <i>P/G</i>	<i>n</i>
<i>n</i>										<i>n</i>
1	1.090	.9174	1.0000	1.0900	1.000	0.917	0	0		1
2	1.188	.8417	.4785	.5685	2.090	1.759	0.478	0.842		2
3	1.295	.7722	.3051	.3951	3.278	2.531	0.943	2.386		3
4	1.412	.7084	.2187	.3087	4.573	3.240	1.393	4.511		4
5	1.539	.6499	.1671	.2571	5.985	3.890	1.828	7.111		5
6	1.677	.5963	.1329	.2229	7.523	4.486	2.250	10.092		6
7	1.828	.5470	.1087	.1987	9.200	5.033	2.657	13.375		7
8	1.993	.5019	.0907	.1807	11.028	5.535	3.051	16.888		8
9	2.172	.4604	.0768	.1668	13.021	5.995	3.431	20.571		9
10	2.367	.4224	.0658	.1558	15.193	6.418	3.798	24.373		10
11	2.580	.3875	.0569	.1469	17.560	6.805	4.151	28.248		11
12	2.813	.3555	.0497	.1397	20.141	7.161	4.491	32.159		12
13	3.066	.3262	.0436	.1336	22.953	7.487	4.818	36.073		13
14	3.342	.2992	.0384	.1284	26.019	7.786	5.133	39.963		14
15	3.642	.2745	.0341	.1241	29.361	8.061	5.435	43.807		15
16	3.970	.2519	.0303	.1203	33.003	8.313	5.724	47.585		16
17	4.328	.2311	.0270	.1170	36.974	8.544	6.002	51.282		17
18	4.717	.2120	.0242	.1142	41.301	8.756	6.269	54.886		18
19	5.142	.1945	.0217	.1117	46.019	8.950	6.524	58.387		19
20	5.604	.1784	.0195	.1095	51.160	9.129	6.767	61.777		20
21	6.109	.1637	.0176	.1076	56.765	9.292	7.001	65.051		21
22	6.659	.1502	.0159	.1059	62.873	9.442	7.223	68.205		22
23	7.258	.1378	.0144	.1044	69.532	9.580	7.436	71.236		23
24	7.911	.1264	.0130	.1030	76.790	9.707	7.638	74.143		24
25	8.623	.1160	.0118	.1018	84.701	9.823	7.832	76.927		25
26	9.399	.1064	.0107	.1007	93.324	9.929	8.016	79.586		26
27	10.245	.0976	.00973	.0997	102.723	10.027	8.191	82.124		27
28	11.167	.0895	.00885	.0989	112.968	10.116	8.357	84.542		28
29	12.172	.0822	.00806	.0981	124.136	10.198	8.515	86.842		29
30	13.268	.0754	.00734	.0973	136.308	10.274	8.666	89.028		30
31	14.462	.0691	.00669	.0967	149.575	10.343	8.808	91.102		31
32	15.763	.0634	.00610	.0961	164.037	10.406	8.944	93.069		32
33	17.182	.0582	.00556	.0956	179.801	10.464	9.072	94.931		33
34	18.728	.0534	.00508	.0951	196.983	10.518	9.193	96.693		34
35	20.414	.0490	.00464	.0946	215.711	10.567	9.308	98.359		35
40	31.409	.0318	.00296	.0930	337.883	10.757	9.796	105.376		40
45	48.327	.0207	.00190	.0919	525.860	10.881	10.160	110.556		45
50	74.358	.0134	.00123	.0912	815.085	10.962	10.430	114.325		50
55	114.409	.00874	.00079	.0908	1260.1	11.014	10.626	117.036		55
60	176.032	.00568	.00051	.0905	1944.8	11.048	10.768	118.968		60
65	270.847	.00369	.00033	.0903	2998.3	11.070	10.870	120.334		65
70	416.731	.00240	.00022	.0902	4619.2	11.084	10.943	121.294		70
75	641.193	.00156	.00014	.0901	7113.3	11.094	10.994	121.965		75
80	986.555	.00101	.00009	.0901	10950.6	11.100	11.030	122.431		80
85	1517.9	.00066	.00006	.0901	16854.9	11.104	11.055	122.753		85
90	2335.5	.00043	.00004	.0900	25939.3	11.106	11.073	122.976		90
95	3593.5	.00028	.00003	.0900	39916.8	11.108	11.085	123.129		95
100	5529.1	.00018	.00002	.0900	61422.9	11.109	11.093	123.233		100

شكل ١٠: Compute Interest Factor for  $i = 9\%$



10%		Compound Interest Factors							10%
n	Single Payment		Uniform Payment Series				Arithmetic Gradient		n
	Compound Amount Factor Find F Given P F/P	Present Worth Factor Find P Given F P/F	Sinking Fund Factor Find A Given F A/F	Capital Recovery Factor Find A Given P A/P	Compound Amount Factor Find F Given A F/A	Present Worth Factor Find P Given A P/A	Gradient Uniform Series Find A Given G A/G	Gradient Present Worth Find P Given G P/G	
1	1.100	.9091	1.0000	1.1000	1.000	0.909	0	0	1
2	1.210	.8264	.4762	.5762	2.100	1.736	0.476	0.826	2
3	1.331	.7513	.3021	.4021	3.310	2.487	0.937	2.329	3
4	1.464	.6830	.2155	.3155	4.641	3.170	1.381	4.378	4
5	1.611	.6209	.1638	.2638	6.105	3.791	1.810	6.862	5
6	1.772	.5645	.1296	.2296	7.716	4.355	2.224	9.684	6
7	1.949	.5132	.1054	.2054	9.487	4.868	2.622	12.763	7
8	2.144	.4665	.0874	.1874	11.436	5.335	3.004	16.029	8
9	2.358	.4241	.0736	.1736	13.579	5.759	3.372	19.421	9
10	2.594	.3855	.0627	.1627	15.937	6.145	3.725	22.891	10
11	2.853	.3505	.0540	.1540	18.531	6.495	4.064	26.396	11
12	3.138	.3186	.0468	.1468	21.384	6.814	4.388	29.901	12
13	3.452	.2897	.0408	.1408	24.523	7.103	4.699	33.377	13
14	3.797	.2633	.0357	.1357	27.975	7.367	4.996	36.801	14
15	4.177	.2394	.0315	.1315	31.772	7.606	5.279	40.152	15
16	4.595	.2176	.0278	.1278	35.950	7.824	5.549	43.416	16
17	5.054	.1978	.0247	.1247	40.545	8.022	5.807	46.582	17
18	5.560	.1799	.0219	.1219	45.599	8.201	6.053	49.640	18
19	6.116	.1635	.0195	.1195	51.159	8.365	6.286	52.583	19
20	6.728	.1486	.0175	.1175	57.275	8.514	6.508	55.407	20
21	7.400	.1351	.0156	.1156	64.003	8.649	6.719	58.110	21
22	8.140	.1228	.0140	.1140	71.403	8.772	6.919	60.689	22
23	8.954	.1117	.0126	.1126	79.543	8.883	7.108	63.146	23
24	9.850	.1015	.0113	.1113	88.497	8.985	7.288	65.481	24
25	10.835	.0923	.0102	.1102	98.347	9.077	7.458	67.696	25
26	11.918	.0839	.00916	.1092	109.182	9.161	7.619	69.794	26
27	13.110	.0763	.00826	.1083	121.100	9.237	7.770	71.777	27
28	14.421	.0693	.00745	.1075	134.210	9.307	7.914	73.650	28
29	15.863	.0630	.00673	.1067	148.631	9.370	8.049	75.415	29
30	17.449	.0573	.00608	.1061	164.494	9.427	8.176	77.077	30
31	19.194	.0521	.00550	.1055	181.944	9.479	8.296	78.640	31
32	21.114	.0474	.00497	.1050	201.138	9.526	8.409	80.108	32
33	23.225	.0431	.00450	.1045	222.252	9.569	8.515	81.486	33
34	25.548	.0391	.00407	.1041	245.477	9.609	8.615	82.777	34
35	28.102	.0356	.00369	.1037	271.025	9.644	8.709	83.987	35
40	45.259	.0221	.00226	.1023	442.593	9.779	9.096	88.953	40
45	72.891	.0137	.00139	.1014	718.905	9.863	9.374	92.454	45
50	117.391	.00852	.00086	.1009	1 163.9	9.915	9.570	94.889	50
55	189.059	.00529	.00053	.1005	1 880.6	9.947	9.708	96.562	55
60	304.482	.00328	.00033	.1003	3 034.8	9.967	9.802	97.701	60
65	490.371	.00204	.00020	.1002	4 893.7	9.980	9.867	98.471	65
70	789.748	.00127	.00013	.1001	7 887.5	9.987	9.911	98.987	70
75	1 271.9	.00079	.00008	.1001	12 709.0	9.992	9.941	99.332	75
80	2 048.4	.00049	.00005	.1000	20 474.0	9.995	9.961	99.561	80
85	3 299.0	.00030	.00003	.1000	32 979.7	9.997	9.974	99.712	85
90	5 313.0	.00019	.00002	.1000	53 120.3	9.998	9.983	99.812	90
95	8 556.7	.00012	.00001	.1000	85 556.9	9.999	9.989	99.877	95
100	13 780.6	.00007	.00001	.1000	137 796.3	9.999	9.993	99.920	100

شكل ١١: Compute Interest Factor for  $i = 10\%$

12%		Compound Interest Factors							12%	
Single Payment		Uniform Payment Series					Arithmetic Gradient			
	Compound Amount Factor Find <i>F</i> Given <i>P</i> <i>F/P</i>	Present Worth Factor Find <i>P</i> Given <i>F</i> <i>P/F</i>	Sinking Fund Factor Find <i>A</i> Given <i>F</i> <i>A/F</i>	Capital Recovery Factor Find <i>A</i> Given <i>P</i> <i>A/P</i>	Compound Amount Factor Find <i>F</i> Given <i>A</i> <i>F/A</i>	Present Worth Factor Find <i>P</i> Given <i>A</i> <i>P/A</i>	Gradient Uniform Series Find <i>A</i> Given <i>G</i> <i>A/G</i>	Gradient Present Worth Find <i>P</i> Given <i>G</i> <i>P/G</i>		
<i>n</i>									<i>n</i>	
1	1.120	.8929	1.0000	1.1200	1.000	0.893	0	0	1	
2	1.254	.7972	.4717	.5917	2.120	1.690	0.472	0.797	2	
3	1.405	.7118	.2963	.4163	3.374	2.402	0.925	2.221	3	
4	1.574	.6355	.2092	.3292	4.779	3.037	1.359	4.127	4	
5	1.762	.5674	.1574	.2774	6.353	3.605	1.775	6.397	5	
6	1.974	.5066	.1232	.2432	8.115	4.111	2.172	8.930	6	
7	2.211	.4523	.0991	.2191	10.089	4.564	2.551	11.644	7	
8	2.476	.4039	.0813	.2013	12.300	4.968	2.913	14.471	8	
9	2.773	.3606	.0677	.1877	14.776	5.328	3.257	17.356	9	
10	3.106	.3220	.0570	.1770	17.549	5.650	3.585	20.254	10	
11	3.479	.2875	.0484	.1684	20.655	5.938	3.895	23.129	11	
12	3.896	.2567	.0414	.1614	24.133	6.194	4.190	25.952	12	
13	4.363	.2292	.0357	.1557	28.029	6.424	4.468	28.702	13	
14	4.887	.2046	.0309	.1509	32.393	6.628	4.732	31.362	14	
15	5.474	.1827	.0268	.1468	37.280	6.811	4.980	33.920	15	
16	6.130	.1631	.0234	.1434	42.753	6.974	5.215	36.367	16	
17	6.866	.1456	.0205	.1405	48.884	7.120	5.435	38.697	17	
18	7.690	.1300	.0179	.1379	55.750	7.250	5.643	40.908	18	
19	8.613	.1161	.0158	.1358	63.440	7.366	5.838	42.998	19	
20	9.646	.1037	.0139	.1339	72.052	7.469	6.020	44.968	20	
21	10.804	.0926	.0122	.1322	81.699	7.562	6.191	46.819	21	
22	12.100	.0826	.0108	.1308	92.503	7.645	6.351	48.554	22	
23	13.552	.0738	.00956	.1296	104.603	7.718	6.501	50.178	23	
24	15.179	.0659	.00846	.1285	118.155	7.784	6.641	51.693	24	
25	17.000	.0588	.00750	.1275	133.334	7.843	6.771	53.105	25	
26	19.040	.0525	.00665	.1267	150.334	7.896	6.892	54.418	26	
27	21.325	.0469	.00590	.1259	169.374	7.943	7.005	55.637	27	
28	23.884	.0419	.00524	.1252	190.699	7.984	7.110	56.767	28	
29	26.750	.0374	.00466	.1247	214.583	8.022	7.207	57.814	29	
30	29.960	.0334	.00414	.1241	241.333	8.055	7.297	58.782	30	
31	33.555	.0298	.00369	.1237	271.293	8.085	7.381	59.676	31	
32	37.582	.0266	.00328	.1233	304.848	8.112	7.459	60.501	32	
33	42.092	.0238	.00292	.1229	342.429	8.135	7.530	61.261	33	
34	47.143	.0212	.00260	.1226	384.521	8.157	7.596	61.961	34	
35	52.800	.0189	.00232	.1223	431.663	8.176	7.658	62.605	35	
40	93.051	.0107	.00130	.1213	767.091	8.244	7.899	65.116	40	
45	163.988	.00610	.00074	.1207	1358.2	8.283	8.057	66.734	45	
50	289.002	.00346	.00042	.1204	2400.0	8.304	8.160	67.762	50	
55	509.321	.00196	.00024	.1202	4236.0	8.317	8.225	68.408	55	
60	897.597	.00111	.00013	.1201	7471.6	8.324	8.266	68.810	60	
65	1581.9	.00063	.00008	.1201	13173.9	8.328	8.292	69.058	65	
70	2787.8	.00036	.00004	.1200	23223.3	8.330	8.308	69.210	70	
75	4913.1	.00020	.00002	.1200	40933.8	8.332	8.318	69.303	75	
80	8658.5	.00012	.00001	.1200	72145.7	8.332	8.324	69.359	80	
85	15259.2	.00007	.00001	.1200	127151.7	8.333	8.328	69.393	85	
90	26891.9	.00004		.1200	224091.1	8.333	8.330	69.414	90	
95	47392.8	.00002		.1200	394931.4	8.333	8.331	69.426	95	
100	83522.3	.00001		.1200	696010.5	8.333	8.332	69.434	100	

شکل ۱۲: Compute Interest Factor for  $i = 12\%$

15%		Compound Interest Factors							15%	
n	Single Payment		Uniform Payment Series				Arithmetic Gradient			
	Compound Amount Factor Find F Given P	Present Worth Factor Find P Given F	Sinking Fund Factor Find A Given F	Capital Recovery Factor Find A Given P	Compound Amount Factor Find F Given A	Present Worth Factor Find P Given A	Gradient Uniform Series Find A Given G	Gradient Present Worth Find P Given G		
	F/P	P/F	A/F	A/P	F/A	P/A	A/G	P/G		
1	1.150	.8696	1.0000	1.1500	1.000	0.870	0	0		
2	1.322	.7561	.4651	.6151	2.150	1.626	0.465	0.756		
3	1.521	.6575	.2880	.4380	3.472	2.283	0.907	2.071		
4	1.749	.5718	.2003	.3503	4.993	2.855	1.326	3.786		
5	2.011	.4972	.1483	.2983	6.742	3.352	1.723	5.775		
6	2.313	.4323	.1142	.2642	8.754	3.784	2.097	7.937		
7	2.660	.3759	.0904	.2404	11.067	4.160	2.450	10.192		
8	3.059	.3269	.0729	.2229	13.727	4.487	2.781	12.481		
9	3.518	.2843	.0596	.2096	16.786	4.772	3.092	14.755		
10	4.046	.2472	.0493	.1993	20.304	5.019	3.383	16.979		
11	4.652	.2149	.0411	.1911	24.349	5.234	3.655	19.129		
12	5.350	.1869	.0345	.1845	29.002	5.421	3.908	21.185		
13	6.153	.1625	.0291	.1791	34.352	5.583	4.144	23.135		
14	7.076	.1413	.0247	.1747	40.505	5.724	4.362	24.972		
15	8.137	.1229	.0210	.1710	47.580	5.847	4.565	26.693		
16	9.358	.1069	.0179	.1679	55.717	5.954	4.752	28.296		
17	10.761	.0929	.0154	.1654	65.075	6.047	4.925	29.783		
18	12.375	.0808	.0132	.1632	75.836	6.128	5.084	31.156		
19	14.232	.0703	.0113	.1613	88.212	6.198	5.231	32.421		
20	16.367	.0611	.00976	.1598	102.444	6.259	5.365	33.582		
21	18.822	.0531	.00842	.1584	118.810	6.312	5.488	34.645		
22	21.645	.0462	.00727	.1573	137.632	6.359	5.601	35.615		
23	24.891	.0402	.00628	.1563	159.276	6.399	5.704	36.499		
24	28.625	.0349	.00543	.1554	184.168	6.434	5.798	37.302		
25	32.919	.0304	.00470	.1547	212.793	6.464	5.883	38.031		
26	37.857	.0264	.00407	.1541	245.712	6.491	5.961	38.692		
27	43.535	.0230	.00353	.1535	283.569	6.514	6.032	39.289		
28	50.066	.0200	.00306	.1531	327.104	6.534	6.096	39.828		
29	57.575	.0174	.00265	.1527	377.170	6.551	6.154	40.315		
30	66.212	.0151	.00230	.1523	434.745	6.566	6.207	40.753		
31	76.144	.0131	.00200	.1520	500.957	6.579	6.254	41.147		
32	87.565	.0114	.00173	.1517	577.100	6.591	6.297	41.501		
33	100.700	.00993	.00150	.1515	664.666	6.600	6.336	41.818		
34	115.805	.00864	.00131	.1513	765.365	6.609	6.371	42.103		
35	133.176	.00751	.00113	.1511	881.170	6.617	6.402	42.359		
40	267.864	.00373	.00056	.1506	1 779.1	6.642	6.517	43.283		
45	538.769	.00186	.00028	.1503	3 585.1	6.654	6.583	43.805		
50	1 083.7	.00092	.00014	.1501	7 217.7	6.661	6.620	44.096		
55	2 179.6	.00046	.00007	.1501	14 524.1	6.664	6.641	44.256		
60	4 384.0	.00023	.00003	.1500	29 220.0	6.665	6.653	44.343		
65	8 817.8	.00011	.00002	.1500	58 778.6	6.666	6.659	44.390		
70	17 735.7	.00006	.00001	.1500	118 231.5	6.666	6.663	44.416		
75	35 672.9	.00003		.1500	237 812.5	6.666	6.665	44.429		
80	71 750.9	.00001		.1500	478 332.6	6.667	6.666	44.436		
85	144 316.7	.00001		.1500	962 104.4	6.667	6.666	44.440		

شکل ۱۳: Compute Interest Factor for  $i = 15\%$

18%		Compound Interest Factors							18%	
Single Payment			Uniform Payment Series				Arithmetic Gradient			
	Compound Amount Factor Find <i>F</i> Given <i>P</i>	Present Worth Factor Find <i>P</i> Given <i>F</i>	Sinking Fund Factor Find <i>A</i> Given <i>F</i>	Capital Recovery Factor Find <i>A</i> Given <i>P</i>	Compound Amount Factor Find <i>F</i> Given <i>A</i>	Present Worth Factor Find <i>P</i> Given <i>A</i>	Gradient Uniform Series Find <i>A</i> Given <i>G</i>	Gradient Present Worth Find <i>P</i> Given <i>G</i>		
<i>n</i>	<i>F/P</i>	<i>P/F</i>	<i>A/F</i>	<i>A/P</i>	<i>F/A</i>	<i>P/A</i>	<i>A/G</i>	<i>P/G</i>	<i>n</i>	
1	1.180	.8475	1.0000	1.1800	1.000	0.847	0	0	1	
2	1.392	.7182	.4587	.6387	2.180	1.566	0.459	0.718	2	
3	1.643	.6086	.2799	.4599	3.572	2.174	0.890	1.935	3	
4	1.939	.5158	.1917	.3717	5.215	2.690	1.295	3.483	4	
5	2.288	.4371	.1398	.3198	7.154	3.127	1.673	5.231	5	
6	2.700	.3704	.1059	.2859	9.442	3.498	2.025	7.083	6	
7	3.185	.3139	.0824	.2624	12.142	3.812	2.353	8.967	7	
8	3.759	.2660	.0652	.2452	15.327	4.078	2.656	10.829	8	
9	4.435	.2255	.0524	.2324	19.086	4.303	2.936	12.633	9	
10	5.234	.1911	.0425	.2225	23.521	4.494	3.194	14.352	10	
11	6.176	.1619	.0348	.2148	28.755	4.656	3.430	15.972	11	
12	7.288	.1372	.0286	.2086	34.931	4.793	3.647	17.481	12	
13	8.599	.1163	.0237	.2037	42.219	4.910	3.845	18.877	13	
14	10.147	.0985	.0197	.1997	50.818	5.008	4.025	20.158	14	
15	11.974	.0835	.0164	.1964	60.965	5.092	4.189	21.327	15	
16	14.129	.0708	.0137	.1937	72.939	5.162	4.337	22.389	16	
17	16.672	.0600	.0115	.1915	87.068	5.222	4.471	23.348	17	
18	19.673	.0508	.00964	.1896	103.740	5.273	4.592	24.212	18	
19	23.214	.0431	.00810	.1881	123.413	5.316	4.700	24.988	19	
20	27.393	.0365	.00682	.1868	146.628	5.353	4.798	25.681	20	
21	32.324	.0309	.00575	.1857	174.021	5.384	4.885	26.300	21	
22	38.142	.0262	.00485	.1848	206.345	5.410	4.963	26.851	22	
23	45.008	.0222	.00409	.1841	244.487	5.432	5.033	27.339	23	
24	53.109	.0188	.00345	.1835	289.494	5.451	5.095	27.772	24	
25	62.669	.0160	.00292	.1829	342.603	5.467	5.150	28.155	25	
26	73.949	.0135	.00247	.1825	405.272	5.480	5.199	28.494	26	
27	87.260	.0115	.00209	.1821	479.221	5.492	5.243	28.791	27	
28	102.966	.00971	.00177	.1818	566.480	5.502	5.281	29.054	28	
29	121.500	.00823	.00149	.1815	669.447	5.510	5.315	29.284	29	
30	143.370	.00697	.00126	.1813	790.947	5.517	5.345	29.486	30	
31	169.177	.00591	.00107	.1811	934.317	5.523	5.371	29.664	31	
32	199.629	.00501	.00091	.1809	1103.5	5.528	5.394	29.819	32	
33	235.562	.00425	.00077	.1808	1303.1	5.532	5.415	29.955	33	
34	277.963	.00360	.00065	.1806	1538.7	5.536	5.433	30.074	34	
35	327.997	.00305	.00055	.1806	1816.6	5.539	5.449	30.177	35	
40	750.377	.00133	.00024	.1802	4163.2	5.548	5.502	30.527	40	
45	1716.7	.00058	.00010	.1801	9531.6	5.552	5.529	30.701	45	
50	3927.3	.00025	.00005	.1800	21813.0	5.554	5.543	30.786	50	
55	8984.8	.00011	.00002	.1800	49910.1	5.555	5.549	30.827	55	
60	20555.1	.00005	.00001	.1800	114189.4	5.555	5.553	30.846	60	
65	47025.1	.00002		.1800	261244.7	5.555	5.554	30.856	65	
70	107581.9	.00001		.1800	597671.7	5.556	5.555	30.860	70	

شکل ۱۴: Compute Interest Factor for  $i = 18\%$

20%		Compound Interest Factors							20%
n	Single Payment		Uniform Payment Series				Arithmetic Gradient		n
	Compound Amount Factor Find F Given P F/P	Present Worth Factor Find P Given F P/F	Sinking Fund Factor Find A Given F A/F	Capital Recovery Factor Find A Given P A/P	Compound Amount Factor Find F Given A F/A	Present Worth Factor Find P Given A P/A	Gradient Uniform Series Find A Given G A/G	Gradient Present Worth Find P Given G P/G	
1	1.200	.8333	1.0000	1.2000	1.000	0.833	0	0	1
2	1.440	.6944	.4545	.6545	2.200	1.528	0.455	0.694	2
3	1.728	.5787	.2747	.4747	3.640	2.106	0.879	1.852	3
4	2.074	.4823	.1863	.3863	5.368	2.589	1.274	3.299	4
5	2.488	.4019	.1344	.3344	7.442	2.991	1.641	4.906	5
6	2.986	.3349	.1007	.3007	9.930	3.326	1.979	6.581	6
7	3.583	.2791	.0774	.2774	12.916	3.605	2.290	8.255	7
8	4.300	.2326	.0606	.2606	16.499	3.837	2.576	9.883	8
9	5.160	.1938	.0481	.2481	20.799	4.031	2.836	11.434	9
10	6.192	.1615	.0385	.2385	25.959	4.192	3.074	12.887	10
11	7.430	.1346	.0311	.2311	32.150	4.327	3.289	14.233	11
12	8.916	.1122	.0253	.2253	39.581	4.439	3.484	15.467	12
13	10.699	.0935	.0206	.2206	48.497	4.533	3.660	16.588	13
14	12.839	.0779	.0169	.2169	59.196	4.611	3.817	17.601	14
15	15.407	.0649	.0139	.2139	72.035	4.675	3.959	18.509	15
16	18.488	.0541	.0114	.2114	87.442	4.730	4.085	19.321	16
17	22.186	.0451	.00944	.2094	105.931	4.775	4.198	20.042	17
18	26.623	.0376	.00781	.2078	128.117	4.812	4.298	20.680	18
19	31.948	.0313	.00646	.2065	154.740	4.843	4.386	21.244	19
20	38.338	.0261	.00536	.2054	186.688	4.870	4.464	21.739	20
21	46.005	.0217	.00444	.2044	225.026	4.891	4.533	22.174	21
22	55.206	.0181	.00369	.2037	271.031	4.909	4.594	22.555	22
23	66.247	.0151	.00307	.2031	326.237	4.925	4.647	22.887	23
24	79.497	.0126	.00255	.2025	392.484	4.937	4.694	23.176	24
25	95.396	.0105	.00212	.2021	471.981	4.948	4.735	23.428	25
26	114.475	.00874	.00176	.2018	567.377	4.956	4.771	23.646	26
27	137.371	.00728	.00147	.2015	681.853	4.964	4.802	23.835	27
28	164.845	.00607	.00122	.2012	819.223	4.970	4.829	23.999	28
29	197.814	.00506	.00102	.2010	984.068	4.975	4.853	24.141	29
30	237.376	.00421	.00085	.2008	1 181.9	4.979	4.873	24.263	30
31	284.852	.00351	.00070	.2007	1 419.3	4.982	4.891	24.368	31
32	341.822	.00293	.00059	.2006	1 704.1	4.985	4.906	24.459	32
33	410.186	.00244	.00049	.2005	2 045.9	4.988	4.919	24.537	33
34	492.224	.00203	.00041	.2004	2 456.1	4.990	4.931	24.604	34
35	590.668	.00169	.00034	.2003	2 948.3	4.992	4.941	24.661	35
40	1 469.8	.00068	.00014	.2001	7 343.9	4.997	4.973	24.847	40
45	3 657.3	.00027	.00005	.2001	18 281.3	4.999	4.988	24.932	45
50	9 100.4	.00011	.00002	.2000	45 497.2	4.999	4.995	24.970	50
55	22 644.8	.00004	.00001	.2000	113 219.0	5.000	4.998	24.987	55
60	56 347.5	.00002		.2000	281 732.6	5.000	4.999	24.994	60

شکل ۱۵: Compute Interest Factor for  $i = 20\%$