تکلیف دوم

مریم سعیدمهر
شماره دانشجویی : ۹۶۲۹۳۷۳

فهرست مطالب

۱ روابط زیر را اثبات کنید.

$$\left(\frac{P}{A},i,n\right)+\left(\frac{P}{F},i,n+1\right)=\left(\frac{P}{A},i,n+1\right) \quad ۱.۱$$

در ویدیو درس اثبات شد که :

$$f(\frac{P}{F},i,n)=(1+i)^{-n} \implies P=F\left[\frac{1}{(1+i)^n}\right] \qquad (۱)$$

حالا اگر فرض کنیم هر پرداخت یا دریافت مساوی A ، نقش F را ایفا کند، بنابراین با استفاده از رابطه (۱) خواهیم داشت :

$$P=\frac{A}{1+i}+\frac{A}{(1+i)^2}+\ldots+\frac{A}{(1+i)^n}$$

طرفین این رابطه را در $\frac{1}{1+i}$ ضرب میکنیم :

$$\frac{P}{1+i}=A\left[\frac{1}{(1+i)^2}+\frac{1}{(1+i)^3}+\ldots+\frac{1}{(1+i)^{n+1}}\right]$$

از تفاضل دو رابطه فوق خواهیم داشت :

$$\frac{P}{1+i}-P=A\left[\frac{-1}{1+i}+\frac{1}{(1+i)^{n+1}}\right]$$

از P فاکتور گرفته و رابطه فوق را مرتب میکنیم :

$$P\left[\frac{1}{1+i}-1\right]=A\left[\frac{1}{(1+i)^{n+1}}-\frac{1}{1+i}\right] \implies P\left[\frac{-i}{1+i}\right]=A\left(\frac{1}{1+i}\right)\left[\frac{1}{(1+i)^{n+1}}-1\right]$$

رابطه اخیر را بر $\frac{-i}{1+i}$ تقسیم نموده ، پس از ساده کردن خواهیم داشت :

$$P=A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] \implies f\left(\frac{P}{A},i,n\right)=\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] \qquad (۲)$$

حال با توجه به دو رابطه (۱) و (۲) خواهیم داشت :

$$\begin{aligned}\left(\frac{P}{A},i,n\right)+\left(\frac{P}{F},i,n+1\right)&=\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]+\frac{1}{(1+i)^{n+1}}\\&=\left(\frac{1+i}{1+i}\right)\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]+\left(\frac{i}{i}\right)\frac{1}{(1+i)^{n+1}}\\&=\left[\frac{\left((1+i)^{n+1}-(1+i)\right)+i}{i(1+i)^{n+1}}\right]\\&=\left[\frac{(1+i)^{n+1}-1}{i(1+i)^{n+1}}\right]\\&=\left(\frac{P}{A},i,n+1\right) \qquad \because 2 \quad \blacksquare\end{aligned}$$

لذا حکم اثبات شد.

$$\left(\frac{P}{A},i,n\right)\left(\frac{F}{P},i,n\right)=\left(\frac{F}{A},i,n\right) \quad ۲.۱$$

در ویدیو درس اثبات شد که :

$$f(\frac{F}{P},i,n)=(1+i)^n \implies F=P\left[(1+i)^n\right] \qquad (۳)$$

اگر در رابطه (۳) به جای P رابطه ۲ را قرار دهیم، خواهیم داشت :

$$F=A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right](1+i)^n$$

و بنابراین :

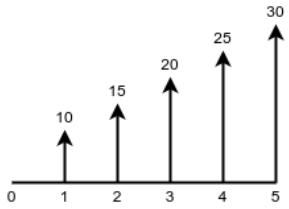
$$F = A\left[\frac{(1+i)^n - 1}{i}\right] \Longrightarrow f(\frac{F}{A}, i, n) = \frac{(1+i)^n - 1}{i} \qquad (۴)$$

حال با توجه به روابط (۲) و (۳) و (۴) خواهیم داشت :

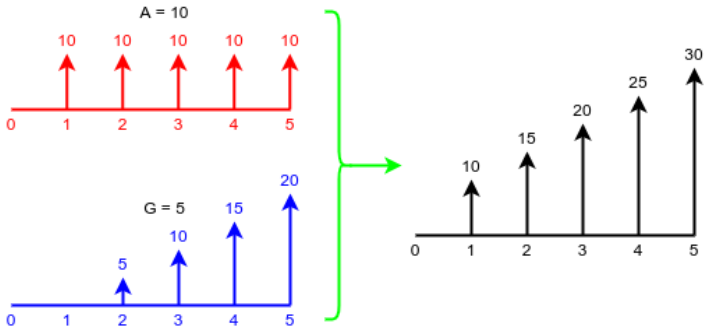
$$\begin{aligned}\left(\frac{P}{A}, i, n\right)\left(\frac{F}{P}, i, n\right) &= \left[\frac{(1+i)^n - 1}{i\,(1+i)^n}\right](1+i)^n \\ &= \frac{(1+i)^n - 1}{i} \\ &= \left(\frac{F}{A}, i, n\right) \qquad \because 4 \quad \blacksquare\end{aligned}$$

لذا حکم اثبات شد.

۲ ارزش فعلی فرایند مالی زیر را محاسبه کنید($\mathbf{i = 10.5\%}$)



کافیست فرایند مالی فوق را به شکل زیر تفکیک کنیم :



همچنین داریم :

$$P = A\left(\frac{P}{A}, i, n\right) \qquad (۵)$$

$$P = G\left(\frac{P}{G}, i, n\right) \qquad (۶)$$

طبق جدول های ۲ و ۳ و برون یابی خطی مقادیر $(\frac{P}{A}, 10.5, 5)$ و $(\frac{P}{G}, 10.5, 5)$ را محاسبه میکنم :

$$\left.\begin{matrix}(\frac{P}{A}, 10, 5) = 3.791 \\ (\frac{P}{A}, 12, 5) = 3.605\end{matrix}\right\} \Longrightarrow \frac{2}{0.5} = \frac{-0.186}{?} \Longrightarrow \left(\frac{P}{A}, 10.5, 5\right) = 3.791 - 0.0465 = 3.7445$$

$$\left.\begin{matrix}(\frac{P}{G}, 10, 5) = 6.682 \\ (\frac{P}{G}, 12, 5) = 6.397\end{matrix}\right\} \Longrightarrow \frac{2}{0.5} = \frac{-0.285}{?} \Longrightarrow \left(\frac{P}{G}, 10.5, 5\right) = 6.682 - 0.07125 = 6.61075$$

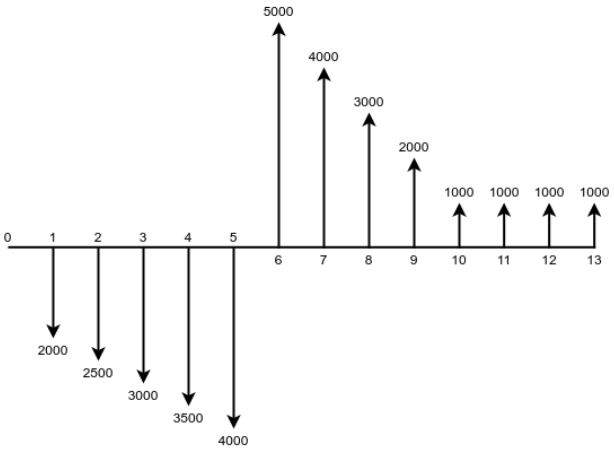
پس به این ترتیب ، ارزش فعلی این فرایند مالی عبارت است از :

$$\begin{aligned} P &= A\left(\frac{P}{A}, i, n\right) + G\left(\frac{P}{G}, i, n\right) \\ &= 10\left(\frac{P}{A}, 10.5, 5\right) + 5\left(\frac{P}{G}, 10.5, 5\right) \\ &= 10 \times 3.7445 + 5 \times 6.61075 \\ &= 70.49875 \end{aligned}$$

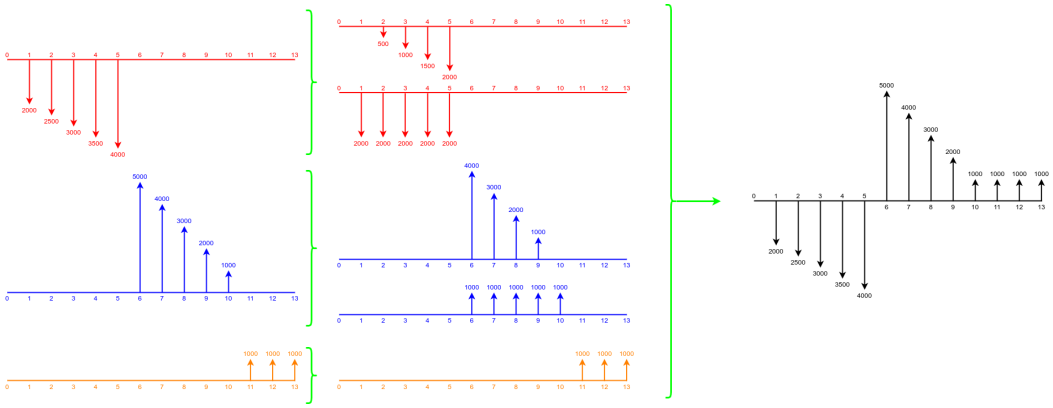
توجه! اگر منظور سوال همان ''ارزش نهایی'' باشد آنگاه روابط فوق به شکل زیر میشود :

$$\begin{aligned} F &= A\left(\frac{F}{A}, i, n\right) + G\left(\frac{F}{G}, i, n\right) \\ &= 10\left(\frac{F}{A}, 10.5, 5\right) + 5\left(\frac{F}{G}, 10.5, 5\right) \\ &= 10 \times 3.7445 + ? \end{aligned}$$

۳ ارزش فعلی (ارزش سال صفر) فرایند مالی زیر را محاسبه نمایید.($\mathbf{MARR = 7\%}$)



کافیست فرایند مالی فوق را به شکل زیر تفکیک کنیم :



حالا تقریبا مشابه سوال قبل و به کمک جدول ۱ خواهیم داشت :

$$P_1 = A\left(\frac{P}{A}, i, n\right) + G\left(\frac{P}{G}, i, n\right) \qquad \textit{Red Diagram}$$
$$= 2000\left(\frac{P}{A}, 7\%, 5\right) + 500\left(\frac{P}{G}, 7\%, 5\right)$$
$$= 2000 \times 4.100 + 500 \times 7.647$$
$$= 8200 + 3823.5 = 12023.5$$

$$P_2 = \left[A\left(\frac{P}{A}, i, n\right) - G\left(\frac{P}{G}, i, n\right)\right]\left(\frac{P}{F}, i, n\right) \qquad \textit{Blue Diagram}$$
$$= \left[5000\left(\frac{P}{A}, 7\%, 5\right) - 1000\left(\frac{P}{G}, 7\%, 5\right)\right]\left(\frac{P}{F}, 7\%, 5\right)$$
$$= [5000 \times 4.100 - 1000 \times 7.647] \times 0.713 = 9164.189$$

$$P_3 = A\left(\frac{P}{A}, i, n\right)\left(\frac{P}{F}, i, n\right) \qquad \textit{Orange Diagram}$$
$$= 1000\left(\frac{P}{A}, 7\%, 3\right)\left(\frac{P}{F}, 7\%, 10\right)$$
$$= 1000 \times 2.624 \times 0.5083 = 1333.7792$$

$$P_{total} = P_2 + P_3 - P_1$$
$$= 9164.189 + 1333.7792 - 12023.5$$
$$= -1525.5318$$

7%	Compound Interest Factors								7%
	Single Payment		Uniform Payment Series				Arithmetic Gradient		
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*	Compound Amount Factor Find *F* Given *A* *F/A*	Present Worth Factor Find *P* Given *A* *P/A*	Gradient Uniform Series Find *A* Given *G* *A/G*	Gradient Present Worth Find *P* Given *G* *P/G*	n
1	1.070	.9346	1.0000	1.0700	1.000	0.935	0	0	1
2	1.145	.8734	.4831	.5531	2.070	1.808	0.483	0.873	2
3	1.225	.8163	.3111	.3811	3.215	2.624	0.955	2.506	3
4	1.311	.7629	.2252	.2952	4.440	3.387	1.416	4.795	4
5	1.403	.7130	.1739	.2439	5.751	4.100	1.865	7.647	5
6	1.501	.6663	.1398	.2098	7.153	4.767	2.303	10.978	6
7	1.606	.6227	.1156	.1856	8.654	5.389	2.730	14.715	7
8	1.718	.5820	.0975	.1675	10.260	5.971	3.147	18.789	8
9	1.838	.5439	.0835	.1535	11.978	6.515	3.552	23.140	9
10	1.967	.5083	.0724	.1424	13.816	7.024	3.946	27.716	10
11	2.105	.4751	.0634	.1334	15.784	7.499	4.330	32.467	11
12	2.252	.4440	.0559	.1259	17.888	7.943	4.703	37.351	12
13	2.410	.4150	.0497	.1197	20.141	8.358	5.065	42.330	13
14	2.579	.3878	.0443	.1143	22.551	8.745	5.417	47.372	14
15	2.759	.3624	.0398	.1098	25.129	9.108	5.758	52.446	15
16	2.952	.3387	.0359	.1059	27.888	9.447	6.090	57.527	16
17	3.159	.3166	.0324	.1024	30.840	9.763	6.411	62.592	17
18	3.380	.2959	.0294	.0994	33.999	10.059	6.722	67.622	18
19	3.617	.2765	.0268	.0968	37.379	10.336	7.024	72.599	19
20	3.870	.2584	.0244	.0944	40.996	10.594	7.316	77.509	20
21	4.141	.2415	.0223	.0923	44.865	10.836	7.599	82.339	21
22	4.430	.2257	.0204	.0904	49.006	11.061	7.872	87.079	22
23	4.741	.2109	.0187	.0887	53.436	11.272	8.137	91.720	23
24	5.072	.1971	.0172	.0872	58.177	11.469	8.392	96.255	24
25	5.427	.1842	.0158	.0858	63.249	11.654	8.639	100.677	25
26	5.807	.1722	.0146	.0846	68.677	11.826	8.877	104.981	26
27	6.214	.1609	.0134	.0834	74.484	11.987	9.107	109.166	27
28	6.649	.1504	.0124	.0824	80.698	12.137	9.329	113.227	28
29	7.114	.1406	.0114	.0814	87.347	12.278	9.543	117.162	29
30	7.612	.1314	.0106	.0806	94.461	12.409	9.749	120.972	30
31	8.145	.1228	.00980	.0798	102.073	12.532	9.947	124.655	31
32	8.715	.1147	.00907	.0791	110.218	12.647	10.138	128.212	32
33	9.325	.1072	.00841	.0784	118.934	12.754	10.322	131.644	33
34	9.978	.1002	.00780	.0778	128.259	12.854	10.499	134.951	34
35	10.677	.0937	.00723	.0772	138.237	12.948	10.669	138.135	35
40	14.974	.0668	.00501	.0750	199.636	13.332	11.423	152.293	40
45	21.002	.0476	.00350	.0735	285.750	13.606	12.036	163.756	45
50	29.457	.0339	.00246	.0725	406.530	13.801	12.529	172.905	50
55	41.315	.0242	.00174	.0717	575.930	13.940	12.921	180.124	55
60	57.947	.0173	.00123	.0712	813.523	14.039	13.232	185.768	60
65	81.273	.0123	.00087	.0709	1 146.8	14.110	13.476	190.145	65
70	113.990	.00877	.00062	.0706	1 614.1	14.160	13.666	193.519	70
75	159.877	.00625	.00044	.0704	2 269.7	14.196	13.814	196.104	75
80	224.235	.00446	.00031	.0703	3 189.1	14.222	13.927	198.075	80
85	314.502	.00318	.00022	.0702	4 478.6	14.240	14.015	199.572	85
90	441.105	.00227	.00016	.0702	6 287.2	14.253	14.081	200.704	90
95	618.673	.00162	.00011	.0701	8 823.9	14.263	14.132	201.558	95
100	867.720	.00115	.00008	.0701	12 381.7	14.269	14.170	202.200	100

شکل ۱: Compute Interest Factore for i = 7%

10%	Compound Interest Factors								10%
	Single Payment		Uniform Payment Series				Arithmetic Gradient		
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*	Compound Amount Factor Find *F* Given *A* *F/A*	Present Worth Factor Find *P* Given *A* *P/A*	Gradient Uniform Series Find *A* Given *G* *A/G*	Gradient Present Worth Find *P* Given *G* *P/G*	n
1	1.100	.9091	1.0000	1.1000	1.000	0.909	0	0	1
2	1.210	.8264	.4762	.5762	2.100	1.736	0.476	0.826	2
3	1.331	.7513	.3021	.4021	3.310	2.487	0.937	2.329	3
4	1.464	.6830	.2155	.3155	4.641	3.170	1.381	4.378	4
5	1.611	.6209	.1638	.2638	6.105	3.791	1.810	6.862	5
6	1.772	.5645	.1296	.2296	7.716	4.355	2.224	9.684	6
7	1.949	.5132	.1054	.2054	9.487	4.868	2.622	12.763	7
8	2.144	.4665	.0874	.1874	11.436	5.335	3.004	16.029	8
9	2.358	.4241	.0736	.1736	13.579	5.759	3.372	19.421	9
10	2.594	.3855	.0627	.1627	15.937	6.145	3.725	22.891	10
11	2.853	.3505	.0540	.1540	18.531	6.495	4.064	26.396	11
12	3.138	.3186	.0468	.1468	21.384	6.814	4.388	29.901	12
13	3.452	.2897	.0408	.1408	24.523	7.103	4.699	33.377	13
14	3.797	.2633	.0357	.1357	27.975	7.367	4.996	36.801	14
15	4.177	.2394	.0315	.1315	31.772	7.606	5.279	40.152	15
16	4.595	.2176	.0278	.1278	35.950	7.824	5.549	43.416	16
17	5.054	.1978	.0247	.1247	40.545	8.022	5.807	46.582	17
18	5.560	.1799	.0219	.1219	45.599	8.201	6.053	49.640	18
19	6.116	.1635	.0195	.1195	51.159	8.365	6.286	52.583	19
20	6.728	.1486	.0175	.1175	57.275	8.514	6.508	55.407	20
21	7.400	.1351	.0156	.1156	64.003	8.649	6.719	58.110	21
22	8.140	.1228	.0140	.1140	71.403	8.772	6.919	60.689	22
23	8.954	.1117	.0126	.1126	79.543	8.883	7.108	63.146	23
24	9.850	.1015	.0113	.1113	88.497	8.985	7.288	65.481	24
25	10.835	.0923	.0102	.1102	98.347	9.077	7.458	67.696	25
26	11.918	.0839	.00916	.1092	109.182	9.161	7.619	69.794	26
27	13.110	.0763	.00826	.1083	121.100	9.237	7.770	71.777	27
28	14.421	.0693	.00745	.1075	134.210	9.307	7.914	73.650	28
29	15.863	.0630	.00673	.1067	148.631	9.370	8.049	75.415	29
30	17.449	.0573	.00608	.1061	164.494	9.427	8.176	77.077	30
31	19.194	.0521	.00550	.1055	181.944	9.479	8.296	78.640	31
32	21.114	.0474	.00497	.1050	201.138	9.526	8.409	80.108	32
33	23.225	.0431	.00450	.1045	222.252	9.569	8.515	81.486	33
34	25.548	.0391	.00407	.1041	245.477	9.609	8.615	82.777	34
35	28.102	.0356	.00369	.1037	271.025	9.644	8.709	83.987	35
40	45.259	.0221	.00226	.1023	442.593	9.779	9.096	88.953	40
45	72.891	.0137	.00139	.1014	718.905	9.863	9.374	92.454	45
50	117.391	.00852	.00086	.1009	1 163.9	9.915	9.570	94.889	50
55	189.059	.00529	.00053	.1005	1 880.6	9.947	9.708	96.562	55
60	304.482	.00328	.00033	.1003	3 034.8	9.967	9.802	97.701	60
65	490.371	.00204	.00020	.1002	4 893.7	9.980	9.867	98.471	65
70	789.748	.00127	.00013	.1001	7 887.5	9.987	9.911	98.987	70
75	1 271.9	.00079	.00008	.1001	12 709.0	9.992	9.941	99.332	75
80	2 048.4	.00049	.00005	.1000	20 474.0	9.995	9.961	99.561	80
85	3 299.0	.00030	.00003	.1000	32 979.7	9.997	9.974	99.712	85
90	5 313.0	.00019	.00002	.1000	53 120.3	9.998	9.983	99.812	90
95	8 556.7	.00012	.00001	.1000	85 556.9	9.999	9.989	99.877	95
100	13 780.6	.00007	.00001	.1000	137 796.3	9.999	9.993	99.920	100

شکل ۲: Compute Interest Factore for i = 10%

12%	Compound Interest Factors								12%
	Single Payment		Uniform Payment Series				Arithmetic Gradient		
n	Compound Amount Factor Find F Given P F/P	Present Worth Factor Find P Given F P/F	Sinking Fund Factor Find A Given F A/F	Capital Recovery Factor Find A Given P A/P	Compound Amount Factor Find F Given A F/A	Present Worth Factor Find P Given A P/A	Gradient Uniform Series Find A Given G A/G	Gradient Present Worth Find P Given G P/G	n
1	1.120	.8929	1.0000	1.1200	1.000	0.893	0	0	1
2	1.254	.7972	.4717	.5917	2.120	1.690	0.472	0.797	2
3	1.405	.7118	.2963	.4163	3.374	2.402	0.925	2.221	3
4	1.574	.6355	.2092	.3292	4.779	3.037	1.359	4.127	4
5	1.762	.5674	.1574	.2774	6.353	3.605	1.775	6.397	5
6	1.974	.5066	.1232	.2432	8.115	4.111	2.172	8.930	6
7	2.211	.4523	.0991	.2191	10.089	4.564	2.551	11.644	7
8	2.476	.4039	.0813	.2013	12.300	4.968	2.913	14.471	8
9	2.773	.3606	.0677	.1877	14.776	5.328	3.257	17.356	9
10	3.106	.3220	.0570	.1770	17.549	5.650	3.585	20.254	10
11	3.479	.2875	.0484	.1684	20.655	5.938	3.895	23.129	11
12	3.896	.2567	.0414	.1614	24.133	6.194	4.190	25.952	12
13	4.363	.2292	.0357	.1557	28.029	6.424	4.468	28.702	13
14	4.887	.2046	.0309	.1509	32.393	6.628	4.732	31.362	14
15	5.474	.1827	.0268	.1468	37.280	6.811	4.980	33.920	15
16	6.130	.1631	.0234	.1434	42.753	6.974	5.215	36.367	16
17	6.866	.1456	.0205	.1405	48.884	7.120	5.435	38.697	17
18	7.690	.1300	.0179	.1379	55.750	7.250	5.643	40.908	18
19	8.613	.1161	.0158	.1358	63.440	7.366	5.838	42.998	19
20	9.646	.1037	.0139	.1339	72.052	7.469	6.020	44.968	20
21	10.804	.0926	.0122	.1322	81.699	7.562	6.191	46.819	21
22	12.100	.0826	.0108	.1308	92.503	7.645	6.351	48.554	22
23	13.552	.0738	.00956	.1296	104.603	7.718	6.501	50.178	23
24	15.179	.0659	.00846	.1285	118.155	7.784	6.641	51.693	24
25	17.000	.0588	.00750	.1275	133.334	7.843	6.771	53.105	25
26	19.040	.0525	.00665	.1267	150.334	7.896	6.892	54.418	26
27	21.325	.0469	.00590	.1259	169.374	7.943	7.005	55.637	27
28	23.884	.0419	.00524	.1252	190.699	7.984	7.110	56.767	28
29	26.750	.0374	.00466	.1247	214.583	8.022	7.207	57.814	29
30	29.960	.0334	.00414	.1241	241.333	8.055	7.297	58.782	30
31	33.555	.0298	.00369	.1237	271.293	8.085	7.381	59.676	31
32	37.582	.0266	.00328	.1233	304.848	8.112	7.459	60.501	32
33	42.092	.0238	.00292	.1229	342.429	8.135	7.530	61.261	33
34	47.143	.0212	.00260	.1226	384.521	8.157	7.596	61.961	34
35	52.800	.0189	.00232	.1223	431.663	8.176	7.658	62.605	35
40	93.051	.0107	.00130	.1213	767.091	8.244	7.899	65.116	40
45	163.988	.00610	.00074	.1207	1 358.2	8.283	8.057	66.734	45
50	289.002	.00346	.00042	.1204	2 400.0	8.304	8.160	67.762	50
55	509.321	.00196	.00024	.1202	4 236.0	8.317	8.225	68.408	55
60	897.597	.00111	.00013	.1201	7 471.6	8.324	8.266	68.810	60
65	1 581.9	.00063	.00008	.1201	13 173.9	8.328	8.292	69.058	65
70	2 787.8	.00036	.00004	.1200	23 223.3	8.330	8.308	69.210	70
75	4 913.1	.00020	.00002	.1200	40 933.8	8.332	8.318	69.303	75
80	8 658.5	.00012	.00001	.1200	72 145.7	8.332	8.324	69.359	80
85	15 259.2	.00007	.00001	.1200	127 151.7	8.333	8.328	69.393	85
90	26 891.9	.00004		.1200	224 091.1	8.333	8.330	69.414	90
95	47 392.8	.00002		.1200	394 931.4	8.333	8.331	69.426	95
100	83 522.3	.00001		.1200	696 010.5	8.333	8.332	69.434	100

شکل ۳: Compute Interest Factore for i = 12%