Applied Linear Algebra
Quiz #1

1. Which item is true? If it is true give a proof, otherwise give a counterexample.
   (i) If W_1 and W_2 are two subspaces of $\mathbb{R}^4$ such that $\dim W_1 = \dim W_2 = 3$ then $\dim(W_1 \cap W_2) \geq 2$.
   (ii) If W_1 and W_2 are two subspaces of $\mathbb{R}^4$ such that $\dim W_1 = \dim W_2 = 2$ then $W_1 + W_2 = \mathbb{R}^4$.
   (ii) Planes xoy and xoz are orthogonal subspaces of $\mathbb{R}^3$.

2. Let $v_1, \ldots, v_n$ be orthonormal vectors. Also, let $x = \alpha_1 v_1 + \cdots + \alpha_n v_n$. Prove that $\|x\|^2 = |\alpha_1|^2 + \cdots + |\alpha_n|^2$.

3. Let $W \subset \mathbb{R}^4$ be the span of the vectors $\{x_1, x_2, x_3, x_4\}$. We know that $\dim W = 3$. Find an orthonormal basis for W.

$$x_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}, x_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_4 = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

4. Let $u = [5, -6, 7]^T$ and W be the set of all vectors in $\mathbb{R}^3$ orthogonal to u.
   (i) Find the projection of $x = [1, 2, 3]^T$ onto W.
   (ii) Find the distance from the vector $x = [1, 2, 3]^T$ to the subspace W.