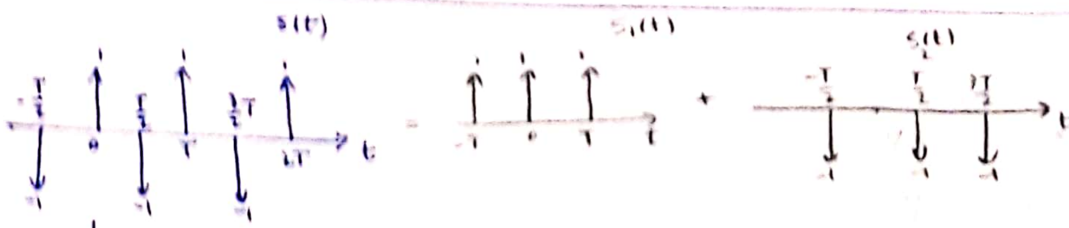
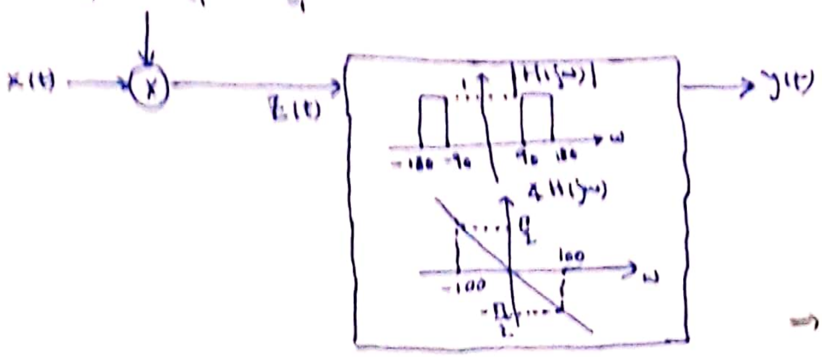
۱) الف)



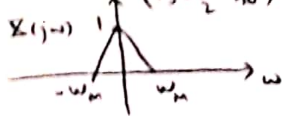


$$\Rightarrow s(t) = s_1(t) + s_2(t)$$
$$= \sum_n \delta(t-nT) - \sum_n \delta\left(t-nT-\frac{T}{2}\right)$$
$$\Rightarrow S(j\omega) = \frac{2\pi}{T}\sum_K \delta(\omega-K\omega_s) - \frac{2\pi}{T}e^{-j\omega\frac{T}{2}}\sum_K \delta(\omega-K\omega_s)$$
$$\Rightarrow S(j\omega) = \frac{2\pi}{T}\left(1-e^{-j\omega\frac{T}{2}}\right)\sum_K \delta(\omega-K\omega_s)$$

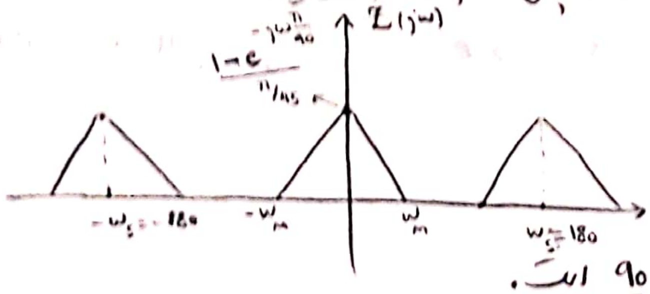
$$z(t) = x(t)\times s(t) \Longrightarrow Z(j\omega) = \frac{1}{2\pi}X(j\omega) * S(j\omega)$$

$$\Rightarrow \boxed{Z(j\omega) = \frac{1-e^{-j\omega\frac{T}{2}}}{T}\sum_{K=-\infty}^{+\infty} X(j(\omega-K\omega_s))} \xRightarrow{T=\frac{\pi}{45}} \boxed{Z(j\omega) = \frac{1-e^{-j\omega\frac{\pi}{90}}}{\pi/45}\sum_{K=-\infty}^{+\infty} X(j(\omega-180K))}$$

* حالا اگر فرض کنیم طیف سیگنال $x(t)$ $\left(T_s = \frac{T}{2} = \frac{\pi}{90}\right)$



باشد ، پس $Z(j\omega)$ به شکل زیر است ،



→ طبق قضیه نمونه برداری $\Rightarrow \omega_s > 2\omega_M$

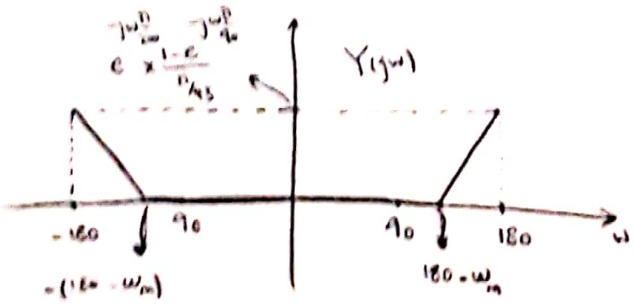
$$\Rightarrow 180 > 2\omega_M \Rightarrow \boxed{\omega_M < 90}$$

پس حداکثر ω_M برای جلوگیری از پدیده‌ی aliasing ، برابر با 90 است.

→ طبق تصاویر داده شده ،
$$H(j\omega) = \begin{cases} e^{-j\omega\frac{\pi}{100}} & 90<|\omega|<180 \\ 0 & \text{o.w.} \end{cases}$$

$$y(t) = z(t) * h(t) \Rightarrow Y(j\omega) = Z(j\omega)H(j\omega)$$

طبق دیاگرام $Z(j\omega)$ ، دیاگرام $Y(j\omega)$ به صورت زیر است (همچنین $\omega_M<90$)



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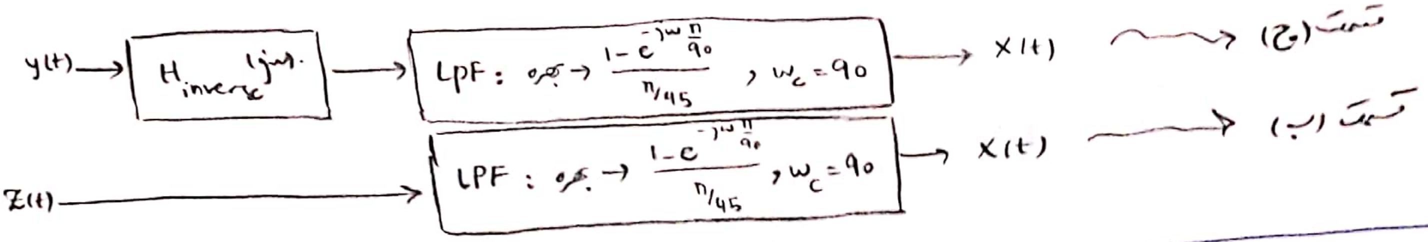
ب) بازسازی $x(t)$ از روی $z(t)$؟ به یک LPF با بهره $\dfrac{1-e^{-j\omega\frac{\pi}{90}}}{\pi/45}$ و $w_c = \dfrac{w_s}{2} = 90$ احتیاج داریم.

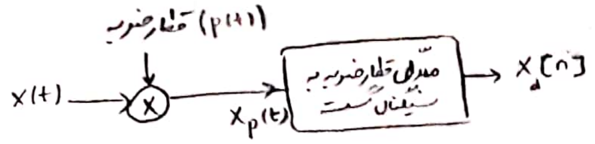
ج) " " " $y(t)$؟

$$H_{inverse}(j\omega)H(j\omega) = 1 \Rightarrow H_{inverse}(j\omega) = \frac{1}{H(j\omega)} = \begin{cases} \dfrac{1}{e^{-j\omega\frac{\pi}{100}}} & 90<|\omega|<180 \\ 0 & \text{o.w.} \end{cases}$$



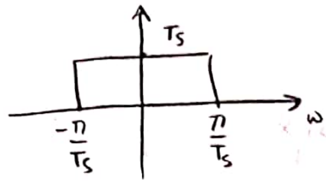
(۲)

$x(nT_s)$

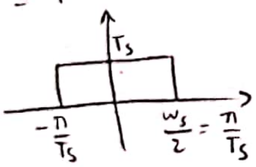


$$x_p(t) = x_c(t)p(t) = x_c(t)\sum_{n=-\infty}^{+\infty}\delta(t-nT_s) = \sum_n x_c(nT_s)\delta(t-nT_s) = \delta(t)$$

$\xRightarrow{\text{تبدیل فوریه}}$ $X_p(j\omega) = 1$ $\left\{ \xrightarrow[\text{فیلتر}]{\text{بعد از اعمال}} \right.$ $X(j\omega) =$



LPF فیلتر :

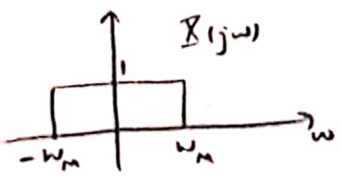


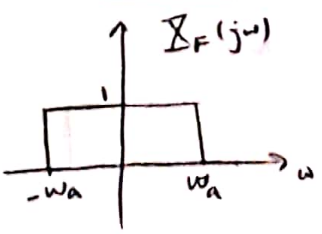
$$\boxed{x(t) = \mathcal{F}^{-1}\{X(j\omega)\} = T_s \times \frac{\sin(\frac{\pi}{T_s}t)}{\pi t} = \frac{\sin(\frac{\pi}{T_s}t)}{\frac{\pi}{T_s}t} = \text{Sinc}(\frac{\pi}{T_s}t)}$$

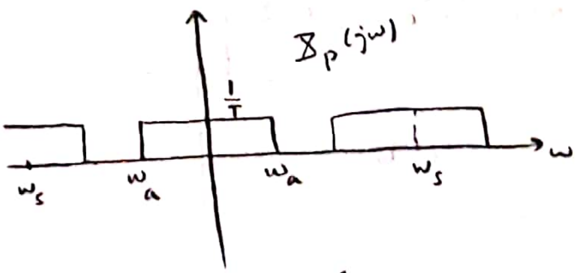
۳) (الف) LPF اولیه که روی $x(t)$ اعمال شده صرفاً به جهت محدود کردن طیف فرکانسی $x(t)$ است (برای صدق کردن در قضیه نمونه برداری) پس $x_F(t)$ حداکثر دارای فرکانس w_a است. حالا جهت جلوگیری از پدیده‌ی aliasing بایستی تا $w_s > 2w_a$ باشد.

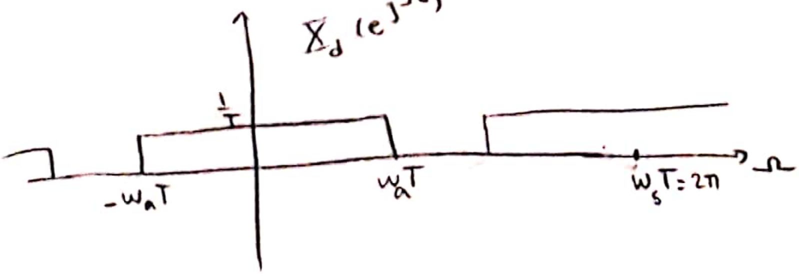
$w_s > 2w_a \rightarrow \frac{2\pi}{T} > 2w_a \rightarrow \boxed{w_a < \frac{\pi}{T}} \rightarrow$ حداکثر تابش قبول برای w_a برابر با $\frac{\pi}{T}$ است.

(ب)









از اینجا به بعد حل مسئله برای $w_a > \frac{w_s}{4}$ و $w_a < \frac{w_s}{4}$ متفاوت است.

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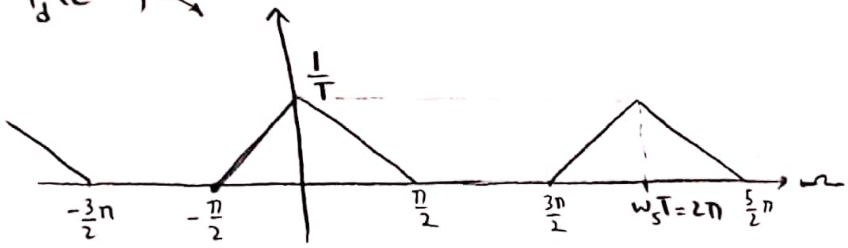
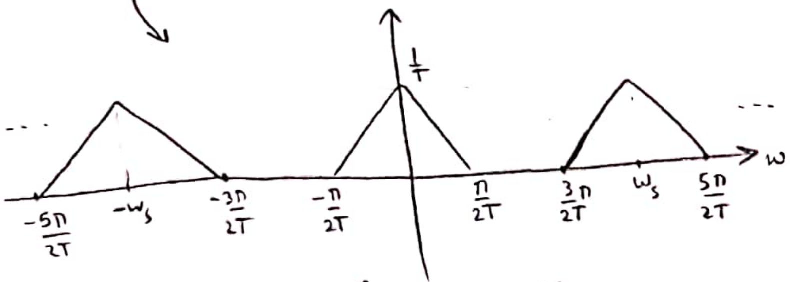
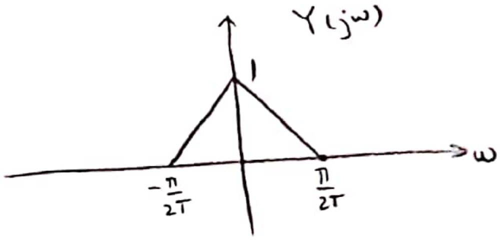
* اگر $\omega_a > \frac{\omega_s}{4} = \frac{\pi}{2T}$:

$Y_d(e^{j\Omega})$ →

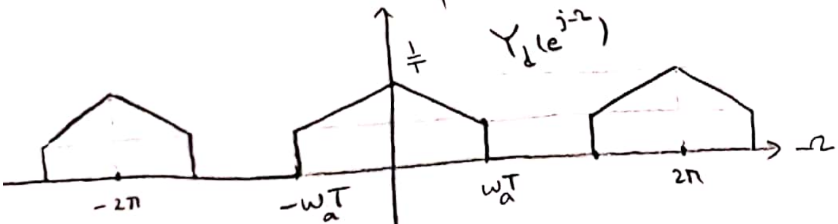


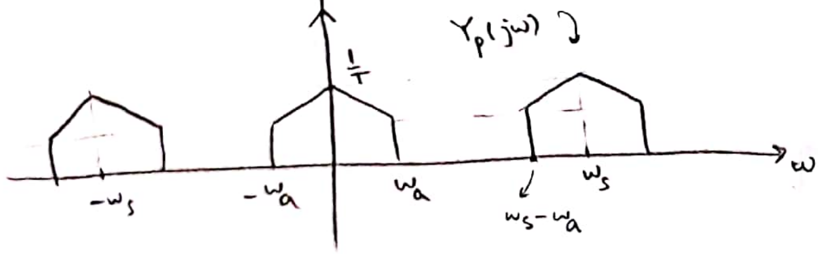
$$Y_p(j\omega) = Y_d(e^{j\Omega})\Big|_{\Omega=\omega T} = Y_d(e^{j\frac{\omega}{T}})$$

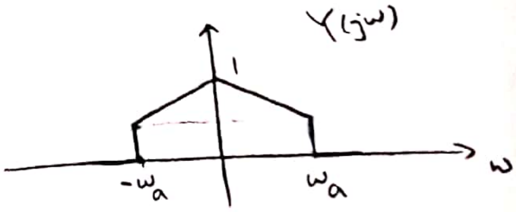




* اگر $\omega_a < \frac{\omega_s}{4} = \frac{\pi}{2T}$:

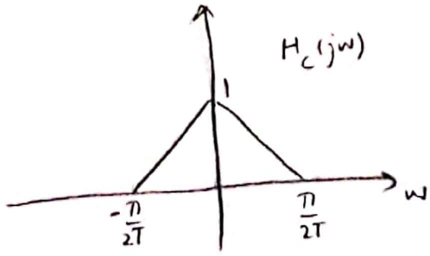




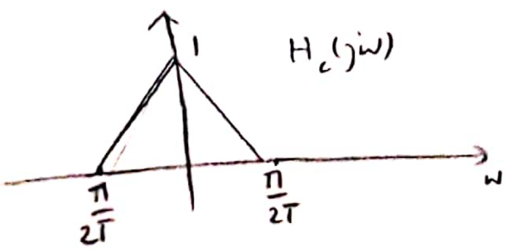


ج) $H_c(j\omega) = ?$

الف) $\omega_a > \frac{\omega_s}{4} = \frac{\pi}{2T}$:



ب) $\omega_a < \frac{\omega_s}{4} = \frac{\pi}{2T}$:



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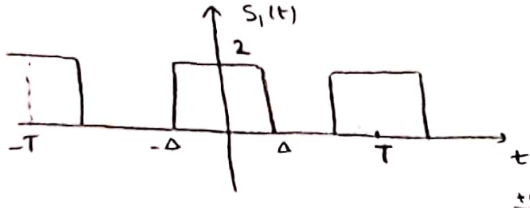
$s(t) = s_1(t) - 1$ (7.24)



$$\Rightarrow S_1(j\omega) = \sum_{k=-\infty}^{+\infty} \frac{4\sin(2k\pi\frac{\Delta}{T})}{k}\,\delta(\omega - k\tfrac{2\pi}{T})$$

$$\Rightarrow S(j\omega) = S_1(j\omega) - 2\pi\delta(\omega) = \sum_{k=-\infty}^{+\infty} \frac{4\sin(2k\pi\frac{\Delta}{T})}{k}\,\delta(\omega - k\tfrac{2\pi}{T}) - 2\pi\delta(\omega)$$

الف) $: \Delta = \frac{T}{3}$

$$S(j\omega) = \sum_k \frac{4\sin(\frac{2k\pi}{3})}{k}\,\delta(\omega - k\tfrac{2\pi}{T}) - 2\pi\delta(\omega)$$

$$w(t) = s(t)x(t) \Rightarrow W(j\omega) = \frac{1}{2\pi}\sum_k \frac{4\sin(\frac{2\pi k}{3})}{k}\,X(j(\omega - k\tfrac{2\pi}{T})) - 2\pi X(j\omega)$$

دوره‌ی تناوب $= \frac{2\pi}{T}$

$$\Rightarrow \omega_M < \frac{\pi}{T} \Rightarrow \boxed{T_{max} = \frac{\pi}{\omega_M}}$$

ب) $: \Delta = \frac{T}{4}$

$$S(j\omega) = \sum_k \frac{4\sin(\frac{2k\pi}{4})}{k}\,\delta(\omega - k\tfrac{2\pi}{T}) - 2\pi\delta(\omega) \longrightarrow$$ برای kهای زوج، $S(j\omega) = 0$

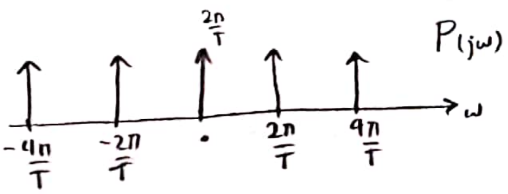
در نتیجه، دوره‌ی تناوب $= \frac{4\pi}{T}$

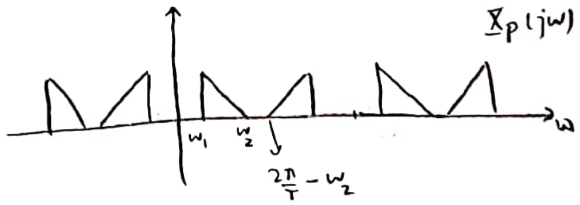
$$\Rightarrow \omega_M < \frac{2\pi}{T} \Rightarrow \boxed{T_{max} = \frac{2\pi}{\omega_M}}$$

(7.26)

$$P(j\omega) = \frac{2\pi}{T}\,\delta(\omega - k\tfrac{2\pi}{T})$$

$$x_p(t) = x(t)\cdot p(t) \xrightarrow{\mathcal{F}} X_p(j\omega) = \frac{1}{2\pi}X(j\omega) * P(j\omega) = \frac{1}{T}X(j(\omega - k\tfrac{2\pi}{T}))$$





$$\Rightarrow 0 < 2\omega_1 - \omega_2 < \frac{2\pi}{T} - \omega_2 < \omega_2 \Rightarrow \left.\begin{array}{l}\{\frac{2\pi}{T} - \omega_2\}\downarrow \equiv T\uparrow \\ \min\{\frac{2\pi}{T} - \omega_2\} = 0\end{array}\right\} \Rightarrow \boxed{T_{max} = \frac{2\pi}{\omega_2}}$$

(7.28)

(a)

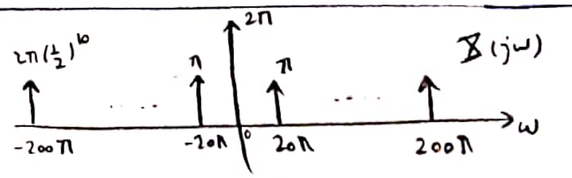
$$x(t) \xrightarrow{\text{فرکانس اصلی}} \omega_0 = 20\pi$$

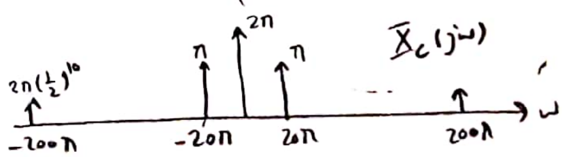
$$X(j\omega) = 2\pi\sum_k a_k\,\delta(\omega - 20\pi k)$$

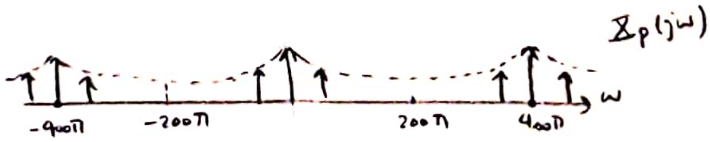
$$P(j\omega) = \frac{2\pi}{5\times10^{-3}}\sum_k \delta(\omega - k\frac{2\pi}{5\times10^{-3}})$$

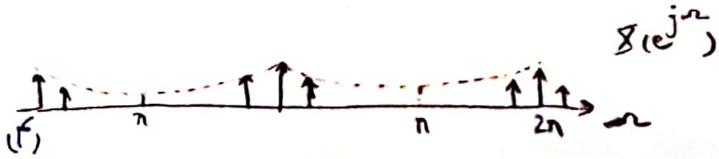
$$\Rightarrow X_p(j\omega) = \frac{1}{2\pi}X_c(j\omega) * P(j\omega)$$

$$X(e^{j\Omega}) = X_p(j\omega)\Big|_{\omega = \Omega T}$$









(ب)

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دوره‌ی تناوب اصلی : $\frac{2\pi}{0.1\pi} = 20$

(ه) ضرایب سری فوریه $x[n]$ عبارت‌اند از :

$$a_k = \begin{cases} \frac{2\pi}{T}\left(\frac{1}{2}\right)^k & ; k = 0, \pm 1, \dots, \pm 9 \\ \frac{4\pi}{T}\left(\frac{1}{2}\right)^k & ; k = 10 \end{cases}$$

(7.37)

$$p(t) = p_1(t) + p_1(t-\Delta) \quad ; \quad p_1(t) = \sum_k \delta\left(t - \frac{2k\pi}{W}\right)$$

$$\Rightarrow P(j\omega) = (1 + e^{-j\Delta\omega}) P_1(j\omega) \quad ; \quad P_1(j\omega) = W \sum_k \delta(\omega - kW)$$

$$g(t) = p(t).f(t) = p_1(t).f(t) + p_1(t+\Delta).f(t)$$

$$= a p_1(t) + b p_1(t-\Delta)$$

$$\Rightarrow G(j\omega) = W \sum_k (a + b e^{-jk\Delta W}) \delta(\omega - kW)$$

$$y_1(t) = x(t) p(t) f(t) \Rightarrow Y_1(j\omega) = \frac{1}{2\pi} G(j\omega) * X(j\omega)$$

$$\Rightarrow Y_1(j\omega) = \frac{W}{2\pi} \sum_k (a + b e^{-jk\Delta W}) X(j(\omega - kW))$$

$$0 < \omega < W \; : \; Y_1(j\omega) = \frac{W}{2\pi}\left[(a+b) X(j\omega) + (a + b e^{-j\Delta W}) X(j(\omega - W))\right]$$

$$Y_2(j\omega) = Y_1(j\omega) H_1(j\omega)$$

$$0 < \omega < W \; : \; Y_2(j\omega) = \frac{jW}{2\pi}\left[(a+b) X(j\omega) + (a + b e^{-j\Delta W}) X(j(\omega - W))\right]$$

$$y_3(t) = x(t) p(t)$$

$$0 < \omega < W \; : \; Y_3(j\omega) = \frac{W}{2\pi}\left[2 X(j\omega) + (1 + e^{-j\Delta W}) X(j(\omega - W))\right]$$

$$0 < W\Delta < \pi \Rightarrow Y_2(j\omega) + Y_3(j\omega) = K X(j\omega) .$$

$$0 < \omega < W \; : \; \frac{W}{2\pi}\left[(2 + ja + jb) X(j\omega)\right] + \frac{W}{2\pi}\left[(1 + e^{-j\Delta W} + ja + jb e^{-j\Delta W}) X(j(\omega - W))\right]$$

$$= K X(j\omega)$$

$$\Rightarrow 1 + e^{-j\Delta W} + ja + jb e^{-j\Delta W} = 0 \Rightarrow \boxed{a = 1 , b = -1}$$

$$W\Delta = \frac{\pi}{2} \Rightarrow \quad a = \sin(W\Delta) + \frac{1 + \cos(W\Delta)}{\tan(W\Delta)} \quad , \quad b = -\frac{1 + \cos(W\Delta)}{\sin(W\Delta)} \quad , W\Delta \neq \frac{\pi}{2}$$

$$\Rightarrow K = \frac{2\pi}{W}\left[\frac{1}{2 + ja + jb}\right]$$

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