**Syntax**

| | | | |
|---|---|---|---|
| $x, y, z$ | Variable | $L ::=$ | Type label |
| $l$ | Value label | $\quad L_c$ | class label |
| $m$ | Method label | $\quad L_a$ | abstract type label |
| $v ::=$ | Value | $S, T, U, V, W ::=$ | Type |
| $\quad x$ | variable | $\quad p.L$ | type selection |
| $t ::=$ | Term | $\quad T\left\{z \Rightarrow \overline{D}\right\}$ | refinement |
| $\quad v$ | value | $\quad T \wedge T$ | intersection type |
| $\quad \mathbf{val}\ x = \mathbf{new}\ c;\ t$ | new instance | $\quad T \vee T$ | union type |
| $\quad t.l$ | field selection | $\quad \top$ | top type |
| $\quad t.m(t)$ | method invocation | $\quad \bot$ | bottom type |
| $p ::=$ | Path | $S_c, T_c ::=$ | Concrete type |
| $\quad x$ | variable | $\quad p.L_c \mid T_c\left\{z \Rightarrow \overline{D}\right\} \mid T_c \wedge T_c \mid \top$ | |
| $\quad p.l$ | selection | $D ::=$ | Declaration |
| $c ::= T_c\left\{\overline{d}\right\}$ | Constructor | $\quad L : S..U$ | type declaration |
| $d ::=$ | Initialization | $\quad l : T$ | value declaration |
| $\quad l = v$ | field initialization | $\quad m : S \to U$ | method declaration |
| $\quad m(x) = t$ | method initialization | | |
| $s ::= \overline{x \mapsto c}$ | Store | $\Gamma ::= \overline{x : T}$ | Environment |

**Reduction** $\boxed{t \mid s \ \to\ t' \mid s'}$

$$\frac{y \mapsto T_c\left\{\overline{l = v'}\ \overline{m(x) = t}\right\} \in s}{y.m_i(v) \mid s \ \to\ [v/x_i]t_i \mid s}\ (\textsc{MSel}) \qquad \mathbf{val}\ x = \mathbf{new}\ c;\ t \mid s \ \to\ t \mid s, x \mapsto c\ (\textsc{New})$$

$$\frac{y \mapsto T_c\left\{\overline{l = v}\ \overline{m(x) = t}\right\} \in s}{y.l_i \mid s \ \to\ v_i \mid s}\ (\textsc{Sel}) \qquad \frac{t \mid s \ \to\ t' \mid s'}{e[t] \mid s \ \to\ e[t'] \mid s'}\ (\textsc{Context})$$

**where** evaluation context $\quad e ::= [\,] \mid e.m(t) \mid v.m(e) \mid e.l$

**Type Assignment** $\boxed{\Gamma \vdash t : T}$

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T}\ (\textsc{Var}) \qquad \frac{\Gamma \vdash t \ni l : T'}{\Gamma \vdash t.l : T'}\ (\textsc{Sel})$$

$$\frac{\Gamma \vdash t \ni m : S \to T \quad \Gamma \vdash t' : T' \,,\ T' <: S}{\Gamma \vdash t.m(t') : T}\ (\textsc{MSel})$$

$$\frac{y \notin fn(T') \quad \Gamma \vdash T_c\ \mathbf{wfe} \,,\ T_c \prec_y \overline{L : S..U}, \overline{D} \quad \Gamma, y : T_c \vdash \overline{S <: U} \,,\ \overline{d : D} \,,\ t' : T'}{\Gamma \vdash \mathbf{val}\ y = \mathbf{new}\ T_c\left\{\overline{d}\right\};\ t' : T'}\ (\textsc{New})$$

**Declaration Assignment** $\boxed{\Gamma \vdash d : D}$

$$\frac{\Gamma \vdash v : V' \,,\ V' <: V}{\Gamma \vdash (l = v) : (l : V)}\ (\textsc{VDecl}) \qquad \frac{\Gamma \vdash S\ \mathbf{wfe} \quad \Gamma, x : S \vdash t : T' \,,\ T' <: T}{\Gamma \vdash (m(x) = t) : (m : S \to T)}\ (\textsc{MDecl})$$

**Figure 1.** The DOT Calculus : Syntax, Reduction, Type / Declaration Assignment

**Well-formed types** $\boxed{\Gamma \vdash T\ \mathbf{wf}}$

$$\frac{\Gamma \vdash T\ \mathbf{wf} \quad \Gamma, z : T\left\{z \Rightarrow \overline{D}\right\} \vdash \overline{D\ \mathbf{wf}}}{\Gamma \vdash T\left\{z \Rightarrow \overline{D}\right\}\ \mathbf{wf}}\ (\textsc{Rfn-Wf}) \qquad \Gamma \vdash \top\ \mathbf{wf}\ (\top\textsc{-Wf}) \qquad \Gamma \vdash \bot\ \mathbf{wf}\ (\bot\textsc{-Wf})$$

$$\frac{\Gamma \vdash p \ni L : S..U \,,\ S\ \mathbf{wf} \,,\ U\ \mathbf{wf}}{\Gamma \vdash p.L\ \mathbf{wf}}\ (\textsc{TSel-Wf}_1) \qquad \frac{\Gamma \vdash p \ni L : \bot..U}{\Gamma \vdash p.L\ \mathbf{wf}}\ (\textsc{TSel-Wf}_2)$$

$$\frac{\Gamma \vdash T\ \mathbf{wf} \,,\ T'\ \mathbf{wf}}{\Gamma \vdash T \wedge T'\ \mathbf{wf}}\ (\wedge\textsc{-Wf}) \qquad \frac{\Gamma \vdash T\ \mathbf{wf} \,,\ T'\ \mathbf{wf}}{\Gamma \vdash T \vee T'\ \mathbf{wf}}\ (\vee\textsc{-Wf})$$

**Well-formed declarations** $\boxed{\Gamma \vdash D\ \mathbf{wf}}$

$$\frac{\Gamma \vdash S\ \mathbf{wf} \,,\ U\ \mathbf{wf}}{\Gamma \vdash L : S..U\ \mathbf{wf}}\ (\textsc{TDecl-Wf}) \qquad \frac{\Gamma \vdash S\ \mathbf{wf} \,,\ U\ \mathbf{wf}}{\Gamma \vdash m : S \to U\ \mathbf{wf}}\ (\textsc{MDecl-Wf})$$

$$\frac{\Gamma \vdash T\ \mathbf{wf}}{\Gamma \vdash l : T\ \mathbf{wf}}\ (\textsc{VDecl-Wf})$$

**Well-formed and expanding types** $\boxed{\Gamma \vdash T\ \mathbf{wfe}}$

$$\frac{\Gamma \vdash T\ \mathbf{wf} \,,\ T \prec_z \overline{D}}{\Gamma \vdash T\ \mathbf{wfe}}\ (\textsc{Wfe})$$

**Figure 5.** The DOT Calculus : Well-Formedness

| | | | |
|---|---|---|---|
| $dom(\overline{D} \wedge \overline{D'})$ | $=$ | $dom(\overline{D}) \cup dom(\overline{D'})$ | |
| $dom(\overline{D} \vee \overline{D'})$ | $=$ | $dom(\overline{D}) \cap dom(\overline{D'})$ | |
| $(D \wedge D')(L)$ | $=$ | $L : (S \vee S')..(U \wedge U')$ | if $(L : S..U) \in \overline{D}$ and $(L : S'..U') \in \overline{D'}$ |
| | $=$ | $D(L)$ | if $L \notin dom(\overline{D'})$ |
| | $=$ | $D'(L)$ | if $L \notin dom(\overline{D})$ |
| $(D \wedge D')(m)$ | $=$ | $m : (S \vee S') \to (U \wedge U')$ | if $(m : S \to U) \in \overline{D}$ and $(m : S' \to U') \in \overline{D'}$ |
| | $=$ | $D(m)$ | if $m \notin dom(\overline{D'})$ |
| | $=$ | $D'(m)$ | if $m \notin dom(\overline{D})$ |
| $(D \wedge D')(l)$ | $=$ | $l : T \wedge T'$ | if $(l : T) \in \overline{D}$ and $(l : T') \in \overline{D'}$ |
| | $=$ | $D(l)$ | if $l \notin dom(\overline{D'})$ |
| | $=$ | $D'(l)$ | if $l \notin dom(\overline{D})$ |
| $(D \vee D')(L)$ | $=$ | $L : (S \wedge S')..(U \vee U')$ | if $(L : S..U) \in \overline{D}$ and $(L : S'..U') \in \overline{D'}$ |
| $(D \vee D')(m)$ | $=$ | $m : (S \wedge S') \to (U \vee U')$ | if $(m : S \to U) \in \overline{D}$ and $(m : S' \to U') \in \overline{D'}$ |
| $(D \vee D')(l)$ | $=$ | $l : T \vee T'$ | if $(l : T) \in \overline{D}$ and $(l : T') \in \overline{D'}$ |

Sets of declarations form a lattice with the given meet $\wedge$ and join $\vee$, the empty set of declarations as the top element, and the bottom element $\overline{D_\bot}$, Here $\overline{D_\bot}$ is the set of declarations that contains for every term label $l$ the declaration $l : \bot$, for every type label $L$ the declaration $L : \top..\bot$ and for every method label $m$ the declaration $m : \top \to \bot$.

**Figure 2.** The DOT Calculus : Declaration Lattice

**Membership** $\boxed{\Gamma \vdash t \ni D}$

$$\frac{\Gamma \vdash p : T \,,\ T \prec_z \overline{D}}{\Gamma \vdash p \ni [p/z]D_i}\ (\textsc{Path-}\ni) \qquad \frac{z \notin fn(D_i) \quad \Gamma \vdash t : T \,,\ T \prec_z \overline{D}}{\Gamma \vdash t \ni D_i}\ (\textsc{Term-}\ni)$$

**Expansion** $\boxed{\Gamma \vdash T \prec_z \overline{D}}$

$$\frac{\Gamma \vdash T \prec_z \overline{D'}}{\Gamma \vdash T\left\{z \Rightarrow \overline{D}\right\} \prec_z \overline{D'} \wedge \overline{D}}\ (\textsc{Rfn-}\prec) \qquad \frac{\Gamma \vdash p \ni L : S..U \,,\ U \prec_z \overline{D}}{\Gamma \vdash p.L \prec_z \overline{D}}\ (\textsc{TSel-}\prec)$$

$$\frac{\Gamma \vdash T_1 \prec_z \overline{D}_1 \,,\ T_2 \prec_z \overline{D}_2}{\Gamma \vdash T_1 \wedge T_2 \prec_z \overline{D}_1 \wedge \overline{D}_2}\ (\wedge\textsc{-}\prec) \qquad \frac{\Gamma \vdash T_1 \prec_z \overline{D}_1 \,,\ T_2 \prec_z \overline{D}_2}{\Gamma \vdash T_1 \vee T_2 \prec_z \overline{D}_1 \vee \overline{D}_2}\ (\vee\textsc{-}\prec)$$

$$\Gamma \vdash \top \prec_z \{\}\ (\top\textsc{-}\prec) \qquad \Gamma \vdash \bot \prec_z \overline{D_\bot}\ (\bot\textsc{-}\prec)$$

**Figure 3.** The DOT Calculus : Membership and Expansion

**Subtyping** $\boxed{\Gamma \vdash S <: T}$

$$\frac{\Gamma \vdash T\ \mathbf{wfe}}{\Gamma \vdash T <: T}\ (\textsc{Refl}) \qquad \frac{\Gamma \vdash T\ \mathbf{wfe}}{\Gamma \vdash \bot <: T}\ (\bot\textsc{-}<:)$$

$$\frac{\Gamma \vdash T\left\{z \Rightarrow \overline{D}\right\}\ \mathbf{wfe} \,,\ S <: T \,,\ S \prec_z \overline{D'} \quad \Gamma, z : S \vdash \overline{D' <: D}}{\Gamma \vdash S <: T\left\{z \Rightarrow \overline{D}\right\}}\ (<:\textsc{-Rfn}) \qquad \frac{\Gamma \vdash T\left\{z \Rightarrow \overline{D}\right\}\ \mathbf{wfe} \,,\ T <: T'}{\Gamma \vdash T\left\{z \Rightarrow \overline{D}\right\} <: T'}\ (\textsc{Rfn-}<:)$$

$$\frac{\Gamma \vdash p \ni L : S..U \,,\ S <: U \,,\ S' <: S}{\Gamma \vdash S' <: p.L}\ (<:\textsc{-TSel}) \qquad \frac{\Gamma \vdash p \ni L : S..U \,,\ S <: U \,,\ U <: U'}{\Gamma \vdash p.L <: U'}\ (\textsc{TSel-}<:)$$

$$\frac{\Gamma \vdash T <: T_1 \,,\ T <: T_2}{\Gamma \vdash T <: T_1 \wedge T_2}\ (<:\textsc{-}\wedge) \qquad \frac{\Gamma \vdash T_1 <: T \,,\ T_2 <: T}{\Gamma \vdash T_1 \vee T_2 <: T}\ (\vee\textsc{-}<:)$$

$$\frac{\Gamma \vdash T_2\ \mathbf{wfe} \,,\ T <: T_1}{\Gamma \vdash T <: T_1 \vee T_2}\ (<:\textsc{-}\vee_1) \qquad \frac{\Gamma \vdash T_2\ \mathbf{wfe} \,,\ T_1 <: T}{\Gamma \vdash T_1 \wedge T_2 <: T}\ (\wedge_1\textsc{-}<:)$$

$$\frac{\Gamma \vdash T_1\ \mathbf{wfe} \,,\ T <: T_2}{\Gamma \vdash T <: T_1 \vee T_2}\ (<:\textsc{-}\vee_2)$$

$$\frac{\Gamma \vdash T\ \mathbf{wfe}}{\Gamma \vdash T <: \top}\ (<:\textsc{-}\top) \qquad \frac{\Gamma \vdash T_1\ \mathbf{wfe} \,,\ T_2 <: T}{\Gamma \vdash T_1 \wedge T_2 <: T}\ (\wedge_2\textsc{-}<:)$$

**Declaration subsumption** $\boxed{\Gamma \vdash D <: D'}$

$$\frac{\Gamma \vdash S' <: S \,,\ T <: T'}{\Gamma \vdash (L : S..T) <: (L : S'..T')}\ (\textsc{TDecl-}<:) \qquad \frac{\Gamma \vdash S' <: S \,,\ T <: T'}{\Gamma \vdash (m : S \to T) <: (m : S' \to T')}\ (\textsc{MDecl-}<:)$$

$$\frac{\Gamma \vdash T <: T'}{\Gamma \vdash (l : T) <: (l : T')}\ (\textsc{VDecl-}<:)$$

**Figure 4.** The DOT Calculus : Subtyping and Declaration Subsumption