Physics 2. Electrical Engineering

Week 7 Electromagnetism. Inductors

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Objectives

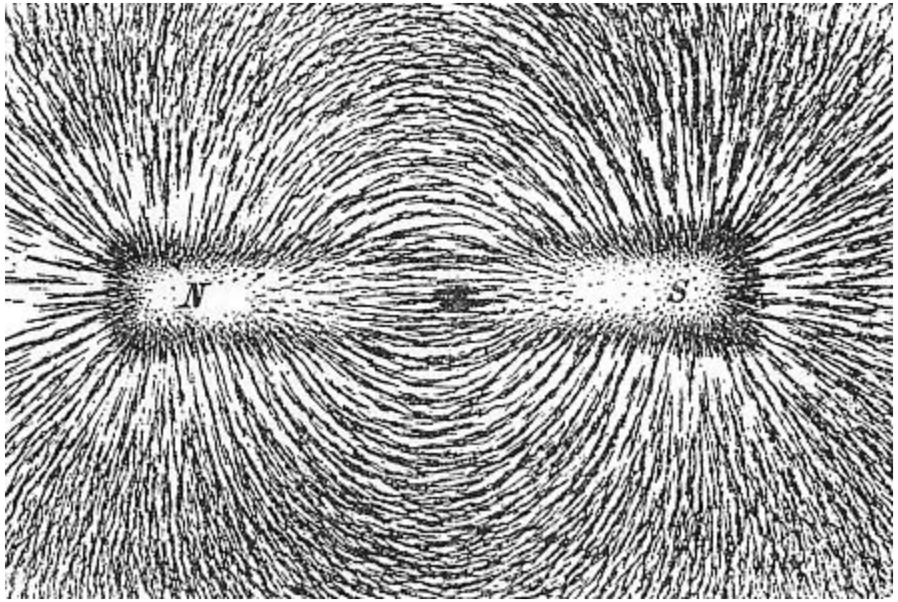
The main objectives of today's lecture are:

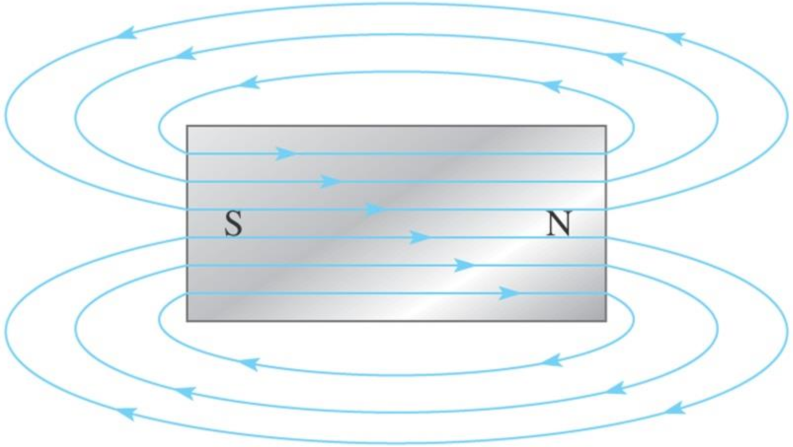
- Review the concepts of electromagnetism
- Learn the basic operation principles of inductors
- Study energy storage in capacitors and inductors

Electromagnetism

Magnetic Field

Magnetic field lines exit a magnet near its north pole and enter near its south pole, but inside the magnet the lines continue through the magnet from the south pole back to the north.



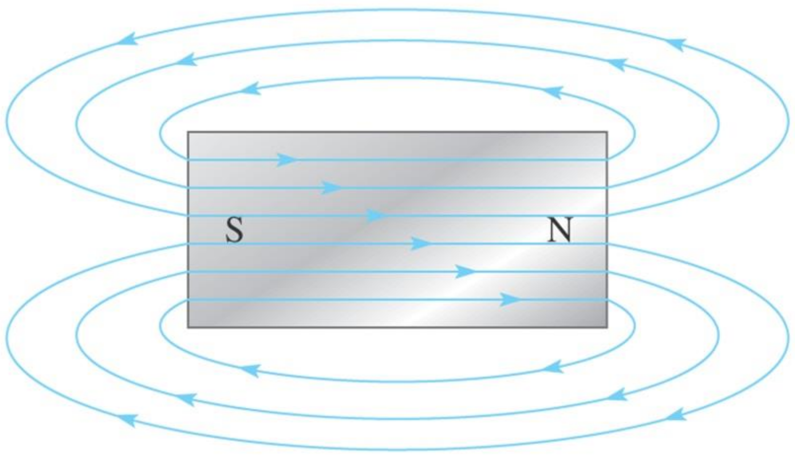


Blue lines represent only a few of the many magnetic lines of force in the magnetic field.

Magnetic Flux

Magnetic flux is a measure of the amount of magnetic field (also called "magnetic flux density") passing through a given surface (such as a conducting coil).

The SI unit of magnetic flux is the weber (Wb) (in derived units: volt-seconds).



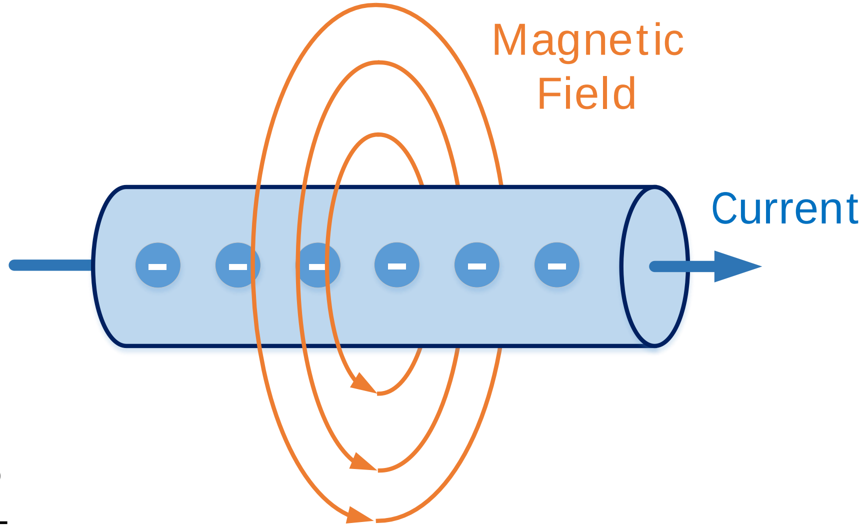
Blue lines represent only a few of the many magnetic lines of force in the magnetic field.

Electromagnetic Field

A magnetic field is generated around a conductor when a current flows through it.

- A change in the magnetic flux passing through a loop of conductive wire will cause an electromotive force (EMF), and therefore an electric current, in the loop.
- The relationship is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi}{dt}$$



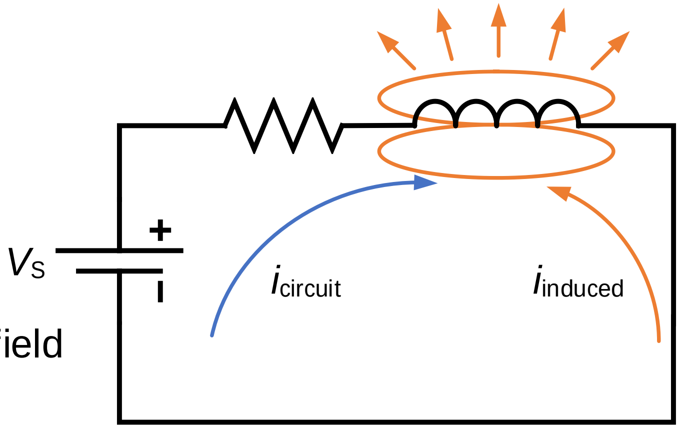
Lenz's law:

- An induced EMF always gives rise to a current whose magnetic field opposes the original change in magnetic flux.

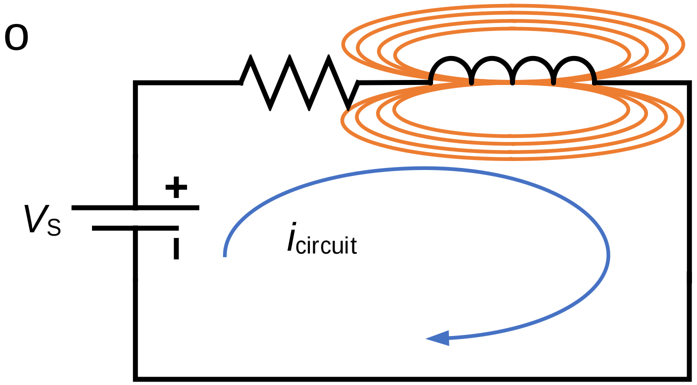
Self-Inductance

Increase in circuit current and, therefore, increase in magnetic field induces voltage in coil. This results in an induced current to oppose circuit current.

Expanding magnetic field

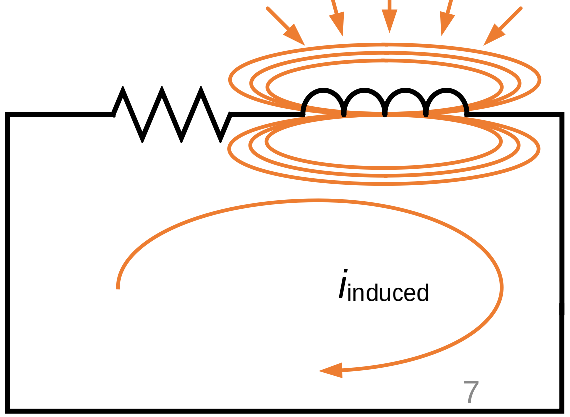


Stationary magnetic field



- Stationary magnetic field means there is no induced voltage or current to oppose the circuit current.

Collapsing magnetic field



- When the power is switched off, the magnetic field collapses and induces a current.

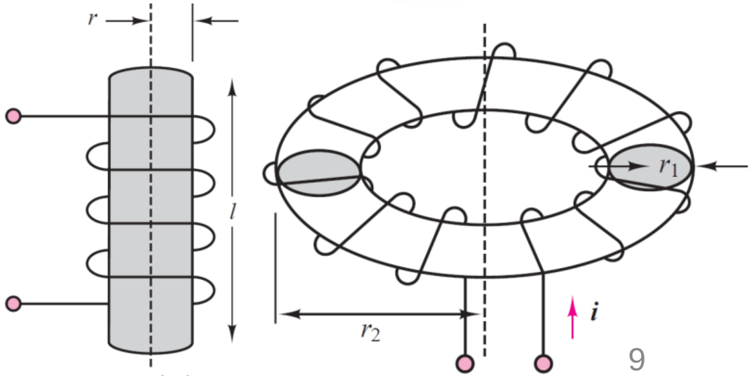
Inductors

Ideal Inductors (1)

The ideal inductor is an element that can store energy in a magnetic field.

- Inductors are typically made by winding a coil of wire around a core, which can be an insulator or a ferromagnetic material.
- When a current flows through the coil, a magnetic field is established.

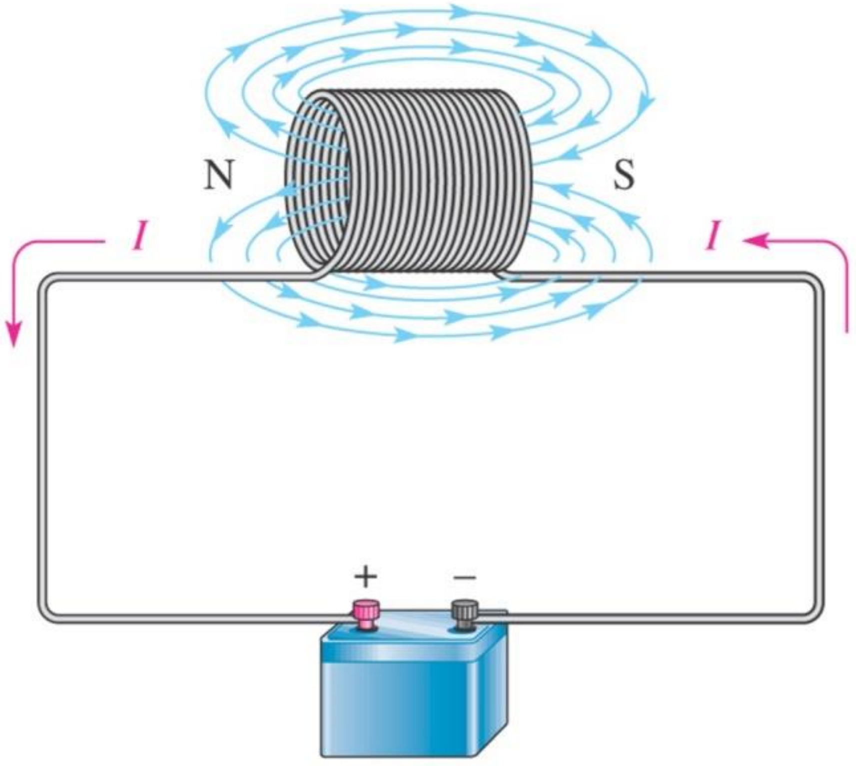
Circuit symbol: 



Ideal Inductors (2)

The inductor works as follows:

- Electric current through the conductor creates a magnetic flux proportional to the current.
- A change in this current creates a corresponding change in magnetic flux which, in turn, by Faraday's Law generates an electromotive force (EMF) that opposes this change in current.
- Thus, inductors resist the change in current.



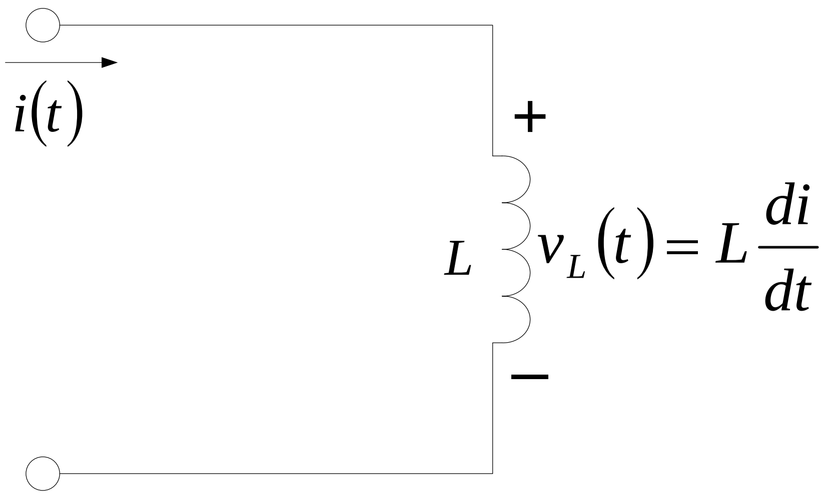
Inductance

Inductance is a measure of the amount of EMF generated per unit change in current.

- Notation: L
- Unit: Henry [H]

1 H =1 Vs/A

- For example, an inductor with an inductance of 1 henry produces an EMF of 1 volt when the current through the inductor changes at the rate of 1 ampere per second.



$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L d\tau$$

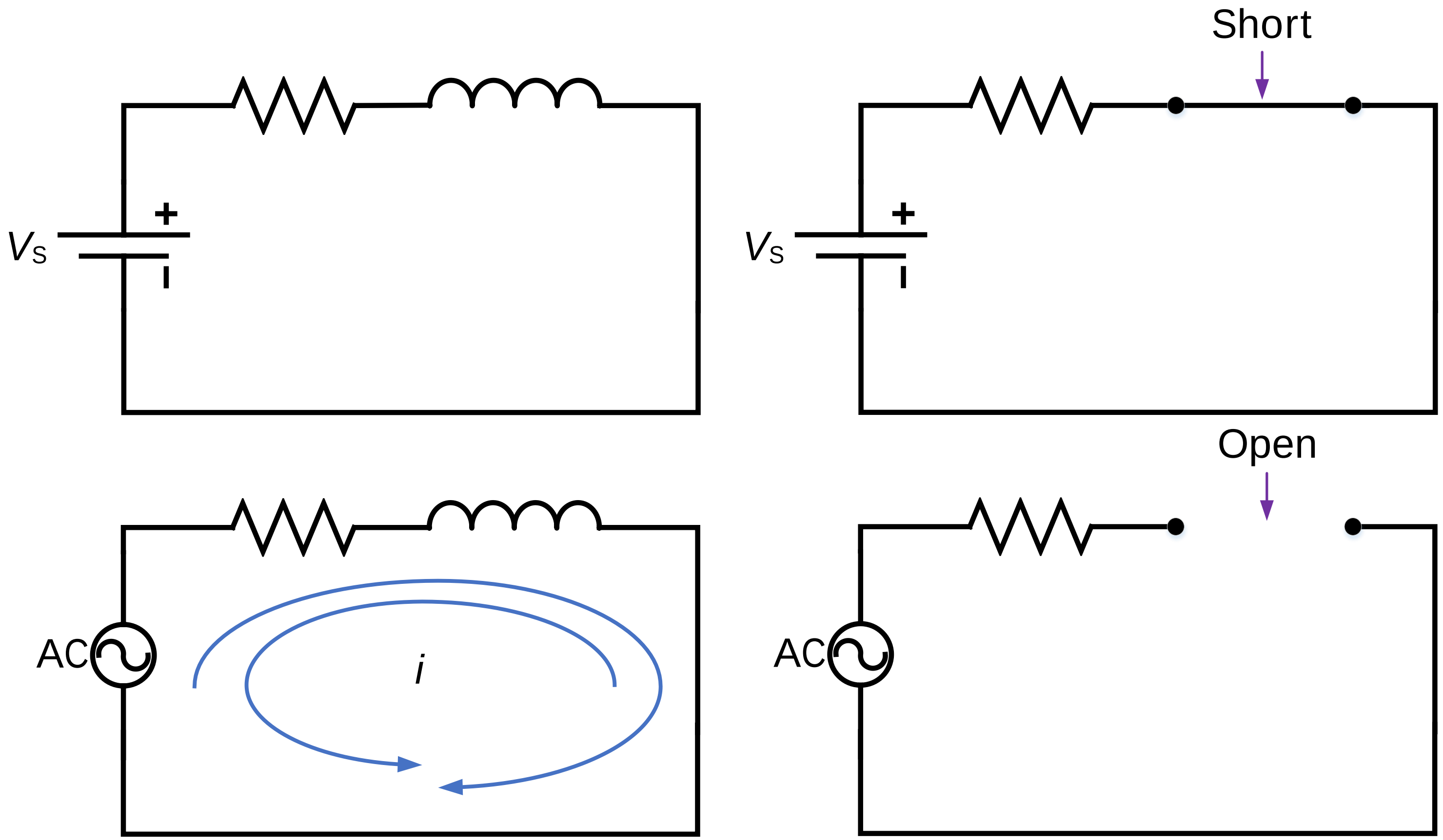
$$i_L(t) = \frac{1}{L}\int_{0}^{t} v_L d\tau + I_0(t_0), \quad t > t_0$$

Inductors in Circuits

The capacitor is

- an short circuit to DC voltage

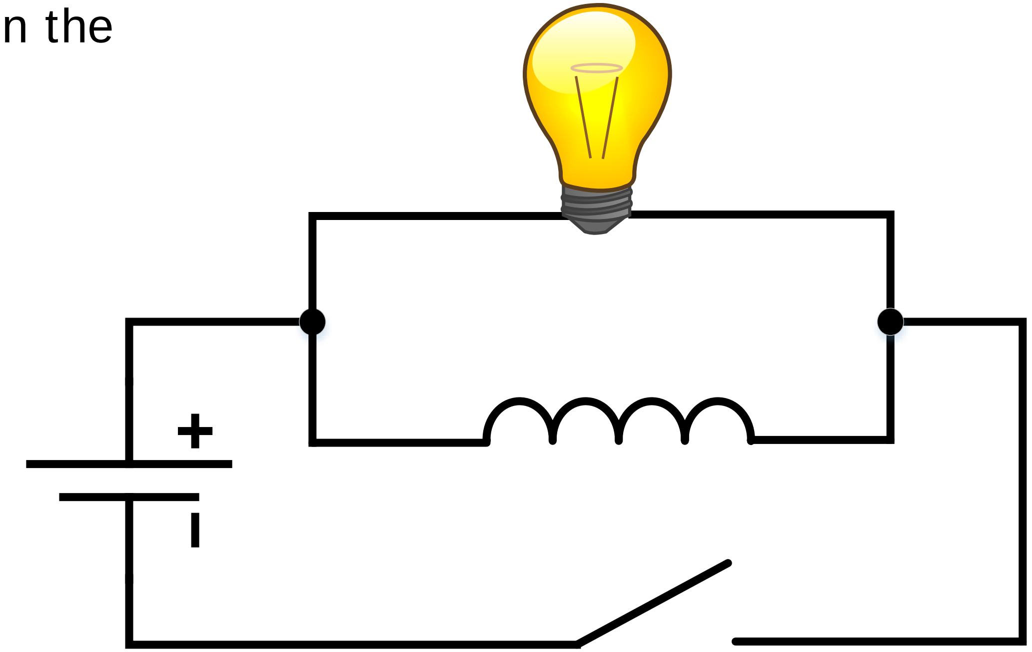
- an open circuit for AC voltage



Inductor and a Lightbulb

Let us try to imagine what will happen to the lightbulb in the circuit shown here once we

1. Close the switch provided it has been open for a long time (the lightbulb is off)
2. Open the switch again after waiting for a long time.



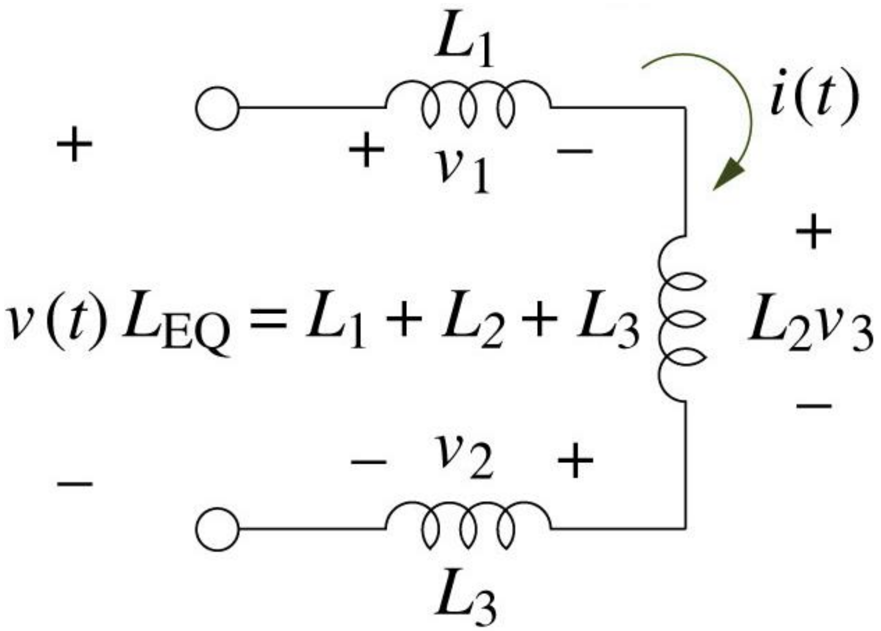
Inductors vs Capacitors

Let us compare the I-V relationships of inductors and capacitors.

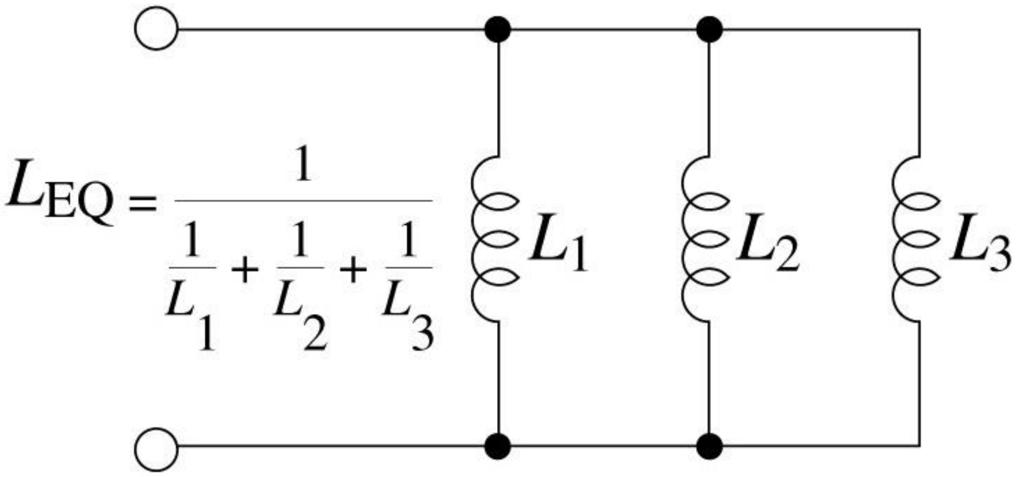
Inductors	Capacitors
$v_L(t) = L\frac{di_L}{dt}$	$i(t) = C\frac{dv(t)}{dt}$
$i_L(t) = \frac{1}{L}\int_0^t v_L(t')dt' + i_0$	$v_C(t) = \frac{1}{C}\int_0^t i_C(t')dt' + v_0$

Inductor Connections

Inductors in series add



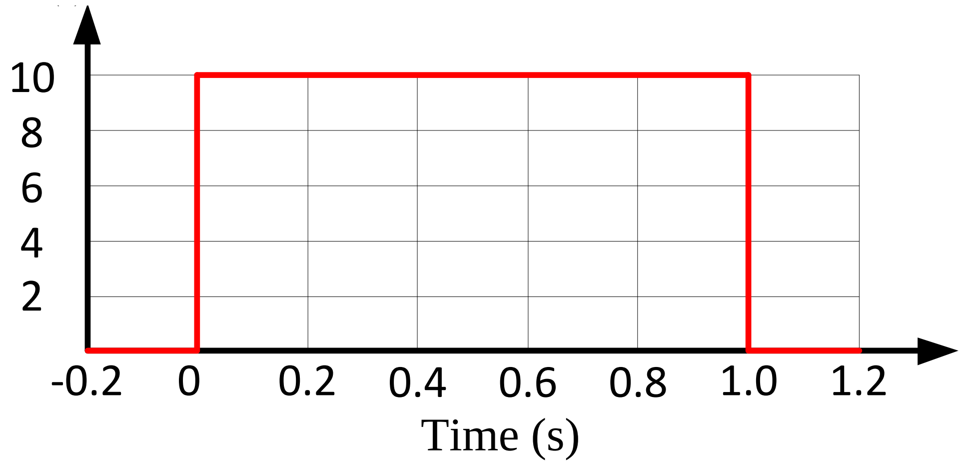
Inductors in parallel combine like resistors in parallel



Inductor Voltage and Current: Exercise

Find the current across an ideal inductor given the voltage profile.

$$v(t) = \begin{cases} 0 & t < 0\text{s} \\ -10\ \text{mV} & 0 \le t \le 1\text{s} \\ 0 & t > 1\text{s} \end{cases}$$



$i_L(t = 0) = 0$ A, $L = 10$ mH.

- Recall that for an inductor, $i_L(t) = \frac{1}{L}\int_0^t v_L(t')dt' + i_0$

Energy Storage

Power

The time rate at which work is done by a force is said to be the power due to the force.

- If a force does an amount of work W in an amount of time Δt, the average power due to the force during that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

- The instantaneous power P is the instantaneous time rate of doing work, which we can write as

$$P = \frac{dW}{dt}$$

Energy Storage

The magnetic energy stored in an ideal inductor may be found from a power calculation.

- The instantaneous power in the inductor is given by

$$P_L(t) = i_L(t)v_L(t) = i_L(t)L\frac{di_L(t)}{dt} = \frac{1}{2}\frac{d}{dt}L \cdot i_L^2(t).$$

- Integrating the power, we obtain the total energy stored in the inductor W_L (compare it with capacitor power, W_C):

$$W_L(t) = \int P_L(t)\,dt = \int \frac{d}{dt}\frac{1}{2}L \cdot i_L^2(t)\,dt = \frac{1}{2}L \cdot i_L^2(t)$$

$$W_C(t) = \frac{1}{2}C \cdot v_C^2(t)$$

Thank you for your attention!

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