1.

Calculate the resistance of a 10m-long wire with the diameter of 0.5 mm made of silver, copper, and aluminum.

TABLE OF ELECTRICAL RESISTIVITY

MATERIAL	ELECTRICAL RESISTIVITY AT 20°C $\times 10^{-8}$ (OHM M)
Aluminum	2.65
Copper	1.72
Gold	2.4
Iron	9.71
Lead	22
Silver	1.59

1.

SOLUTION

d = 0.5 mm = $5 \cdot 10^{-4}$ m / wire diameter

ρ ohm·m /resistivity is given in Tab

$$A = \pi \left(\frac{d}{2}\right)^2$$ m² /cross-sectional area

L = 10 m /wire length

$$R = \frac{L}{A}\rho$$ /resistance

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ANSWER

MATERIAL	ELECTRICAL RESISTANCE AT 20°C (OH M)
Aluminum	1.3496
Copper	0.8760
Silver	0.8098

2.

2.1 Find the mass of a 1km-long overhead power line with d = 30mm if it was made of silver, copper and aluminum, given their densities.

2.2 Find the cost of these wires, provided the price per kg in USD.

2.3 How much would the aluminum line cost if its diameter is to be increased so that the line has the same resistance with the copper line?

TABLE OF CONSTANTS

MATERIAL	ELECTRICAL RESISTIVITY AT 20°C $\times 10^{-8}$ (OH M)	DENSITY (KG/M^3)	PRICE PER KG ($)
Aluminum	2.65	2700	2
Copper	1.72	8940	8
Silver	1.59	10500	819

2.

Find the mass of a 1km-long overhead power line with d = 30 mm if it was made of silver, copper and aluminum.

SOLUTION 2.1

d = 30 mm = $3 \cdot 10^{-2}$ m /wire diameter

$$A = \pi \left(\frac{d}{2}\right)^2$$ m^2 /cross-sectional area

L = 1000 m /wire length

$$V = A \cdot L$$ m^3 /wire volume

D kg/m^3 /density given in Tab

$$m = D \cdot V$$ kg /mass of wire

TABLE OF CONSTANTS			ANSWER
MATERIAL	ELECTRICAL RESISTIVITY AT 20°C $\times 10^{-8}$ (OH M)	DENSITY (KG/M^3)	MASS (TONS)
Aluminum	2.65	2700	1.91
Copper	1.72	8940	6.32
Silver	1.59	10500	7.42

SOLUTION 2.2

2. Find the cost of these wires, provided the price per kg in USD.

Pr \$/kg /wire price given in Tab

$C = Pr \cdot m$ \$ /cost of wire

TABLE OF CONSTANTS				ANSWER
MATERIAL	ELECTRICAL RESISTIVITY AT 20°C $\times 10^{-8}$ (OH M)	PRICE PER KG (\$)	MASS (TONS)	COST (\$ THOUSAND)
Aluminum	2.65	2	1.91	3.82
Copper	1.72	8	6.32	50.55
Silver	1.59	819	7.42	6078.63

SOLUTION 2.3

2.

How much would the aluminum line cost if its diameter is to be increased so that the line has the same resistance with the copper line?

$$R_c = \frac{L}{A_c}\rho_c \quad \text{ohm} \quad \text{/resistance of copper wire}$$

$$R_a = \frac{L}{A_a}\rho_a \quad \text{ohm} \quad \text{/resistance of aluminum wire}$$

$$R_c = R_a; \quad \frac{L}{A_c}\rho_c = \frac{L}{A_a}\rho_a; \quad \frac{A_a}{A_c} = \frac{\rho_a}{\rho_c}; \quad \frac{d_a}{d_c} = \sqrt{\frac{\rho_a}{\rho_c}}; \quad d_a = 37.24; \text{mm} \quad \text{/diameter of aluminum wire}$$

$$C = Pr \cdot m = Pr \cdot D \cdot V = Pr \cdot D \cdot A \cdot L = Pr \cdot D \cdot \pi \left(\frac{d_a}{2}\right)^2 \cdot L = 5.88 \cdot 10^3 \ \$ \quad \text{/cost of aluminum wire}$$

TABLE OF CONSTANTS				ANSWER
MATERIAL	ELECTRICAL RESISTIVITY AT 20°C $\times 10^{-8}$ (OH M)	DENSITY (KG/M³)	PRICE PER KG ($)	COST ($ THOUSAND)
Aluminum	2.65	2700	2	5.88
Copper	1.72	8940	8	50.55

3. Find the total charge in a cylindrical conductor (solid wire) and compute the current flowing in the wire.

List of parameters:

Conductor length:	$L = 1$ m.
Conductor diameter:	$2r = 2 \times 10^{-3}$ m.
Charge density:	$n = 10^{29}$ carriers/m^3.
Charge of one electron:	$q_e = -1.602 \times 10^{-19}$ C.
Charge carrier velocity:	$u = 19.9 \times 10^{-6}$ m/s.

3.

Conductor length:	L = 1 m.
Conductor diameter:	$2r = 2 \times 10^{-3}$ m.
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Charge of one electron:	$q_e = -1.602 \times 10^{-19}$ C.
Charge carrier velocity:	$u = 19.9 \times 10^{-6}$ m/s.

SOLUTION

$$V = L \times \pi r^2 = (1\,\mathrm{m}) \left[\pi \left(\frac{2 \times 10^{-3}}{2} \right)^2 \mathrm{m}^2 \right] = \pi \times 10^{-6}\ \mathrm{m}^3$$ /Volume = length × cross-sectional area

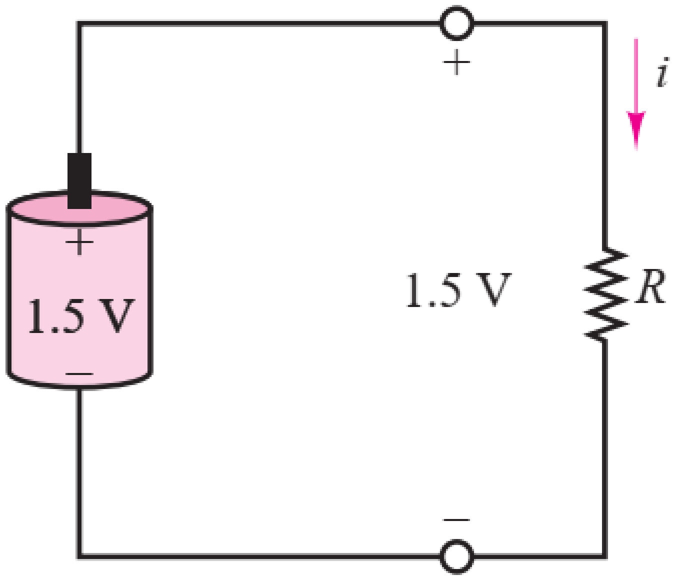
$$N = V \times n = (\pi \times 10^{-6}\,\mathrm{m}^3) \left(10^{29} \frac{\text{carriers}}{\mathrm{m}^3} \right) = \pi \times 10^{23}\ \text{carriers}$$ /Number of carriers

$$Q = N \times q_e = (\pi \times 10^{23}\ \text{carriers}) \times \left(-1.602 \times 10^{-19} \frac{\mathrm{C}}{\text{carrier}} \right) = -50.33 \times 10^3\ \mathrm{C}$$ /Charge

$$I = \left(\frac{Q}{L}\ \frac{\mathrm{C}}{\mathrm{m}} \right) \times \left(u\ \frac{\mathrm{m}}{\mathrm{s}} \right) = \left(-50.33 \times 10^3\ \frac{\mathrm{C}}{\mathrm{m}} \right) \left(19.9 \times 10^{-6}\ \frac{\mathrm{m}}{\mathrm{s}} \right) = -1\ \mathrm{A}$$ /Current

Answer: the total charge is -50.33×10^3 C; the current is -1 A.

4.
Determine the minimum resistor size that can be connected to a given battery without exceeding the resistor’s (1/4) -W power rating.



4.

Determine the minimum resistor size that can be connected to a given battery without exceeding the resistor's (1/4) -W power rating.

Resistor power rating: 0.25 W

Battery voltages: 1.5 V

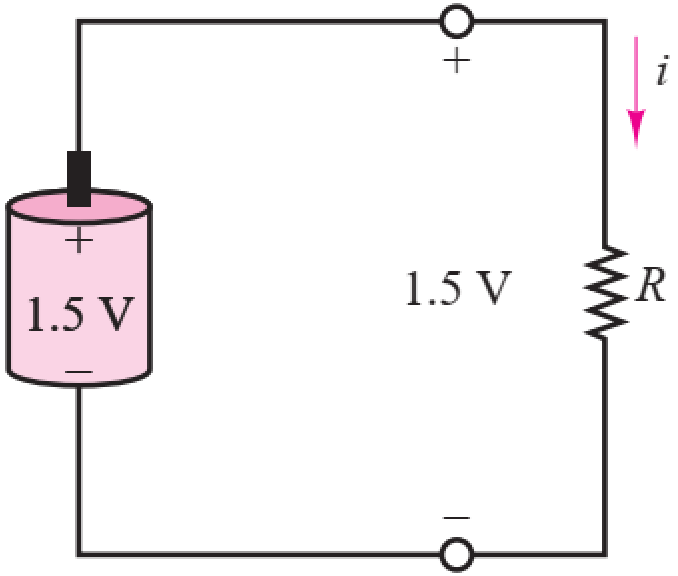
SOLUTION

Since $P = VI$ and that $V = IR$. Thus, the power dissipated by any resistor is

$$P_R = V \times I = V \times \frac{V}{R} = \frac{V^2}{R}$$

$V^2/R \leq 0.25$ / maximum power

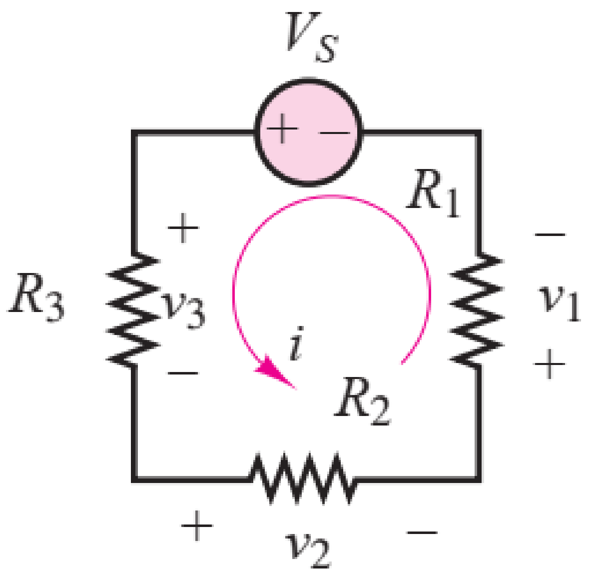
$R = 1.5^2/0.25 = 9\,\Omega$



Answer: the minimum resistor size is 9 ohm

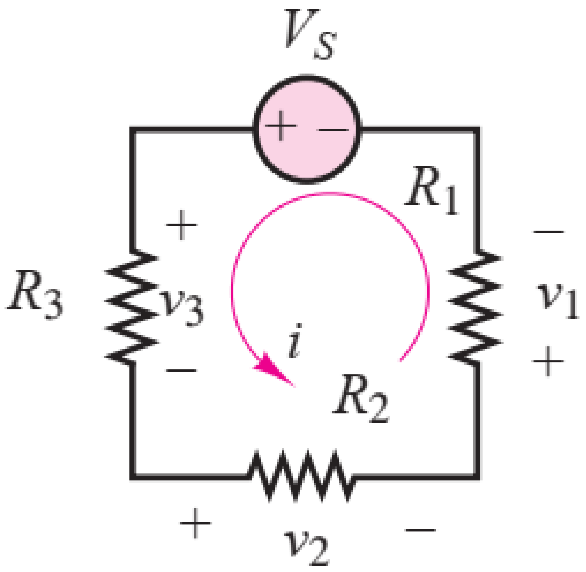
5.

Determine the voltage v_3 in the circuit of Figure, where $R_1 = 10\ \Omega$; $R_2 = 6\ \Omega$; $R_3 = 8\ \Omega$; $V_S = 3$ V.



5.

Determine the voltage v_3 in the circuit of Figure, where $R_1 = 10\ \Omega$; $R_2 = 6\ \Omega$; $R_3 = 8\ \Omega$; $V_S = 3$ V.



SOLUTION

$$v_3 = V_S \times \frac{R_3}{R_1 + R_2 + R_3} = 3 \times \frac{8}{10 + 6 + 8} = 1\ \text{V}$$ /The voltage divider rule

Answer: the voltage v_3 is 1V