Physics 2. Electrical Engineering

Week 3 Network Analysis 1

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Objectives

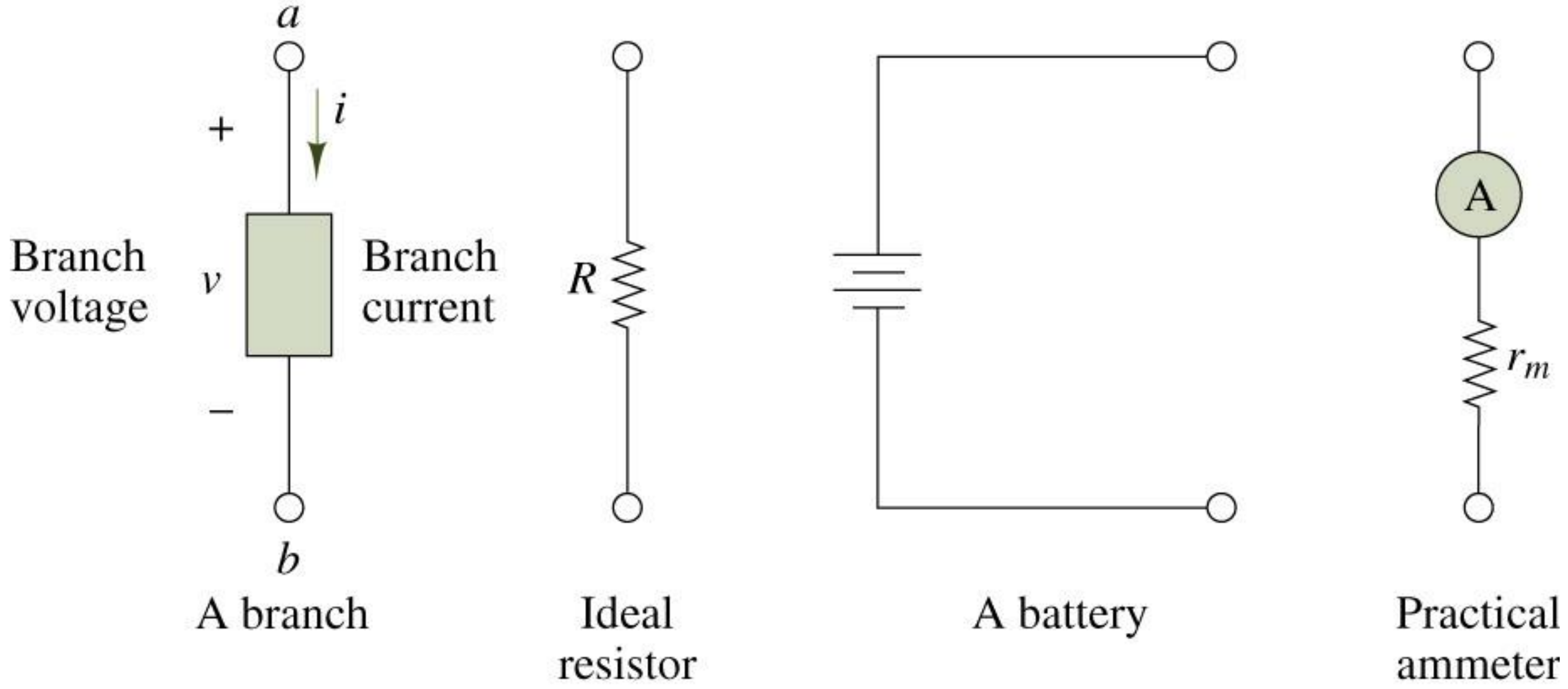
The main objectives of today's lecture are:

- Review the concepts of equivalent resistance
- Learn node voltage method
- Learn mesh current method

Last Week's Review

Electric Circuits : Branch

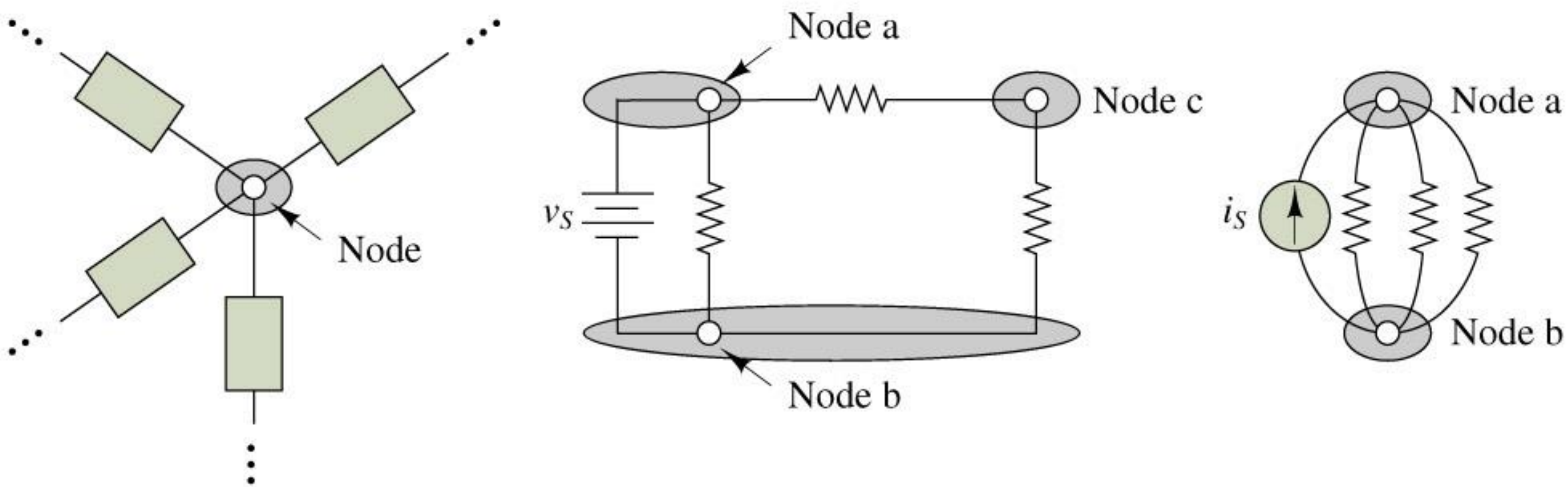
A branch is any portion of a circuit with two terminals connected to it.



Examples of circuit branches

Electric Circuits : Node

- A node is a junction of two or more branches.



Examples of nodes in practical circuits

Kirchhoff's Current Law

KCL: The sum of the currents at a node must equal zero.

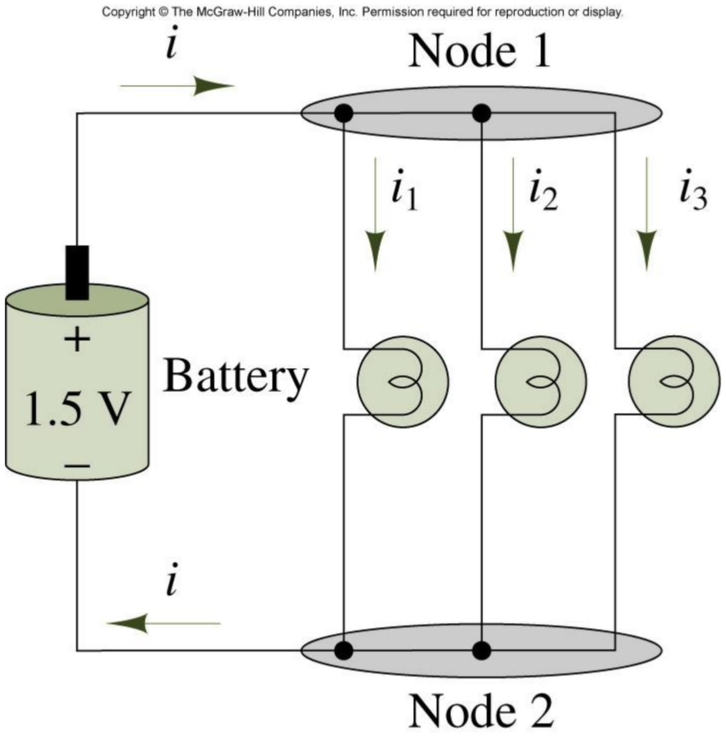
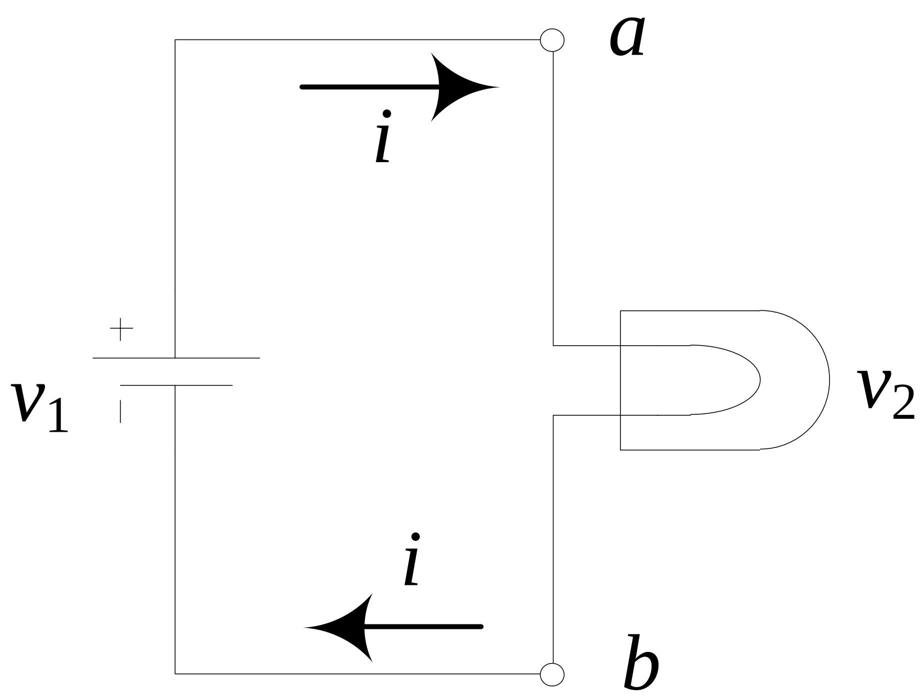


Illustration of KCL at
node 1: $-i + i_1 + i_2 + i_3 = 0$

$$\sum_{n=1}^{N} i_n = 0$$

Kirchhoff's Voltage Law

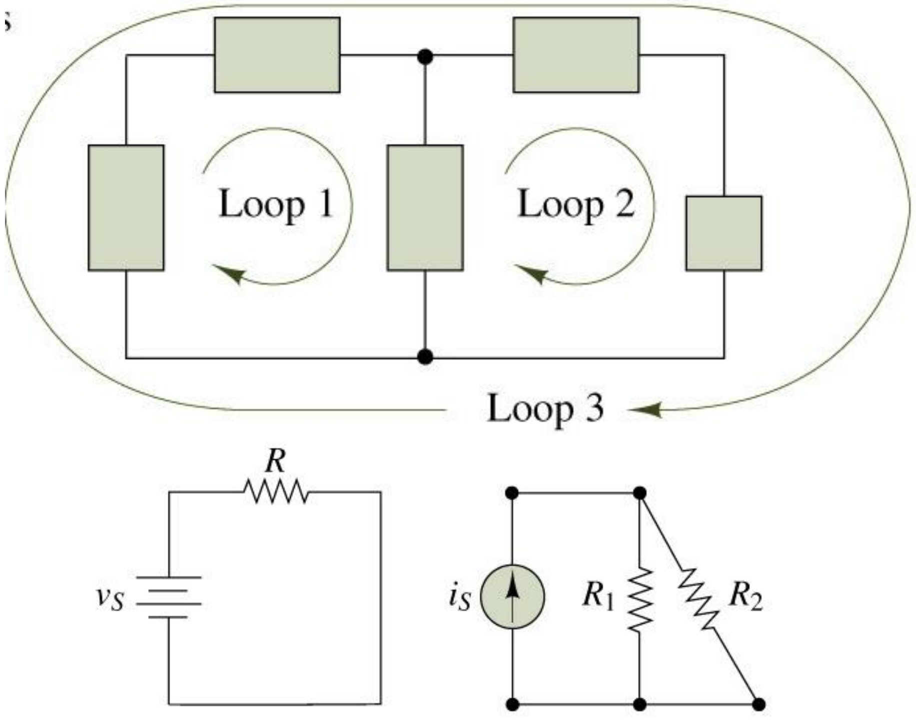
KVL: The sum of all voltages associated with sources must equal the sum of the load voltages, so that the net voltage around a closed circuit is zero.



$$\sum_{n=1}^{N} V_n = 0$$

Loops and Meshes

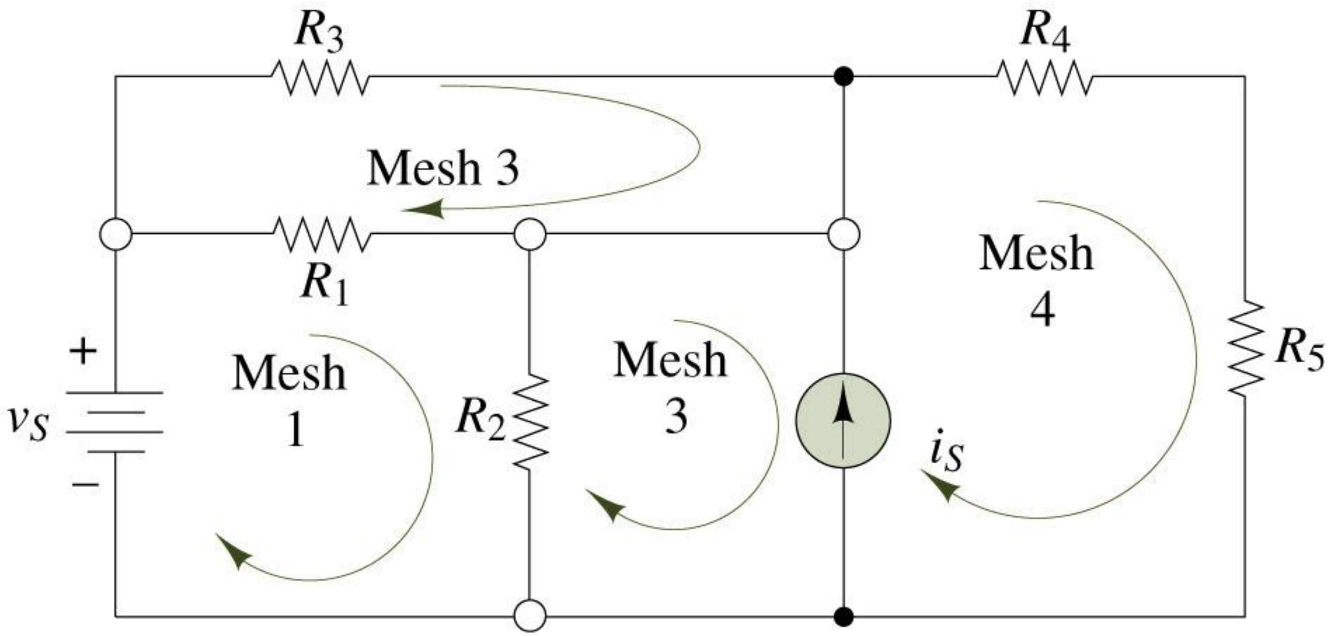
- A loop is any closed connection of branches



1-loop circuit

3-loop circuit
(How many nodes in this circuit?)

- A mesh is a loop that does not contain other loops



Node Voltage Method

Electrical Network Analysis

The analysis of an electrical network consists of determining each of the unknown branch currents and node voltages.

- It is therefore important to define all the relevant variables as clearly as possible, and in systematic fashion.
- Once the known and unknown variables have been identified, a set of equations relating these variables is constructed, and these equations are solved by means of suitable techniques.

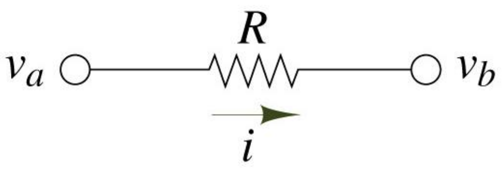
The analysis of electric circuits consists of writing the smallest set of equations sufficient to solve for all the unknown variables.

Node Voltage Method (1)

Node voltage method (NVM) is based of defining the voltage at each node as an independent variable.

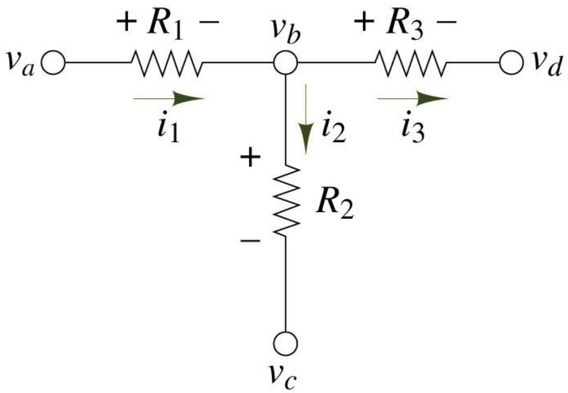
- One of the nodes is selected as a reference node (usually ground).
- Once each node is defined, Ohm's law may be applied between any two adjacent nodes to determine the current flowing in each branch.
- In the NVM, we assign the node voltages v_a and v_b, the branch current flowing from a to be is then expressed in terms of these node voltages:

$$i = \frac{v_a - v_b}{R}$$



- We can then express KCL by

$$\frac{v_a - v_b}{R_1} - \frac{v_b - v_c}{R_2} - \frac{v_b - v_d}{R_3} = 0$$



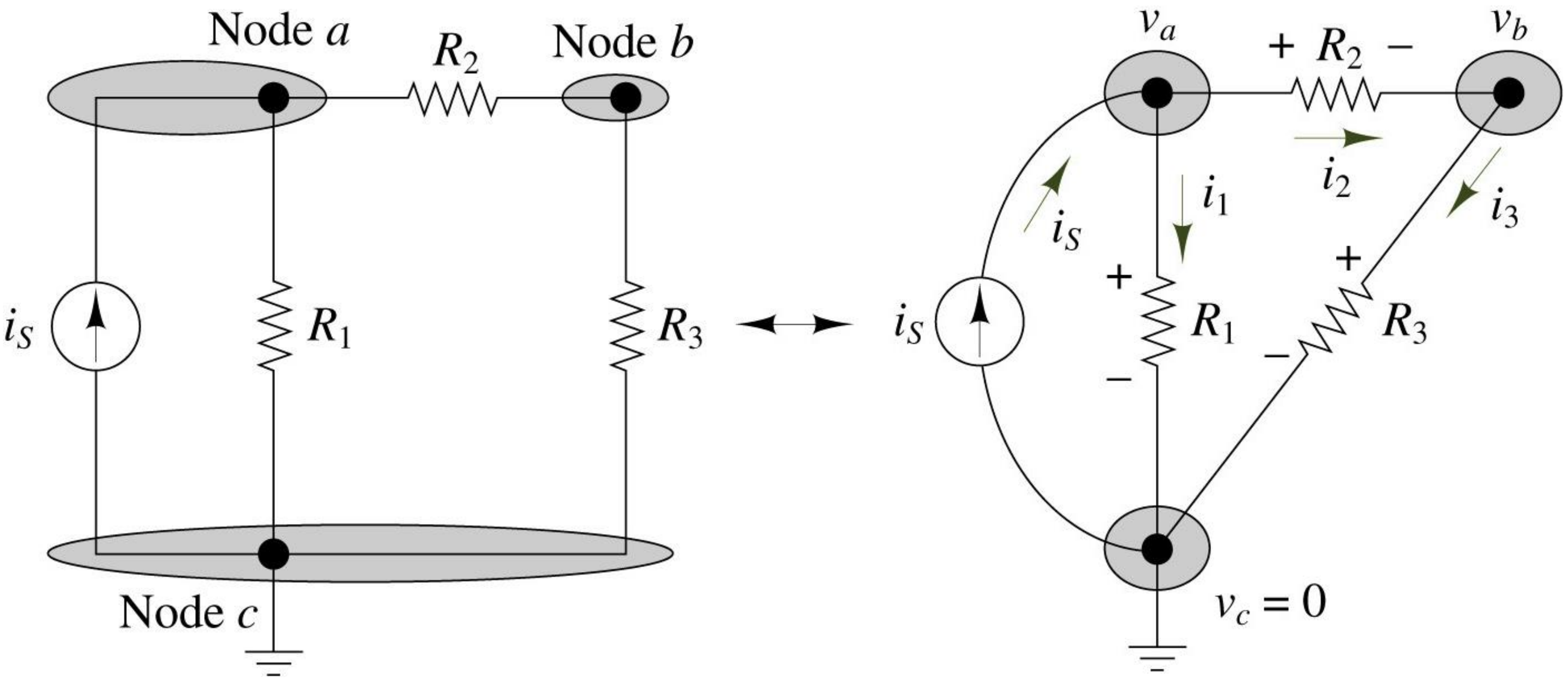
Node Voltage Method (2)

Typical procedure of the NVM is as follows:

1. Select a reference node.
2. Define the remaining n - 1 node voltages as the variables. Each of the m voltage sources in the circuit is associated with a dependent variable. If a node is not connected to a voltage source, then its voltage is treated as an independent variable.
3. Apply KCL at each node labeled as an independent variable.
4. Solve the linear system of n – 1 – m unknowns.

Node Voltage Method : Example (1)

Find all unknown voltages (v_a, v_b, v_c) and currents (i_1, i_2, i_3) in the circuit below.



Node Voltage Method : Example (2)

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1. Select a reference node.
2. Define the remaining n - 1 node voltages as the variables.
3. Apply KCL at each node labeled as an independent variable.

$$i_s - i_1 - i_2 = 0$$

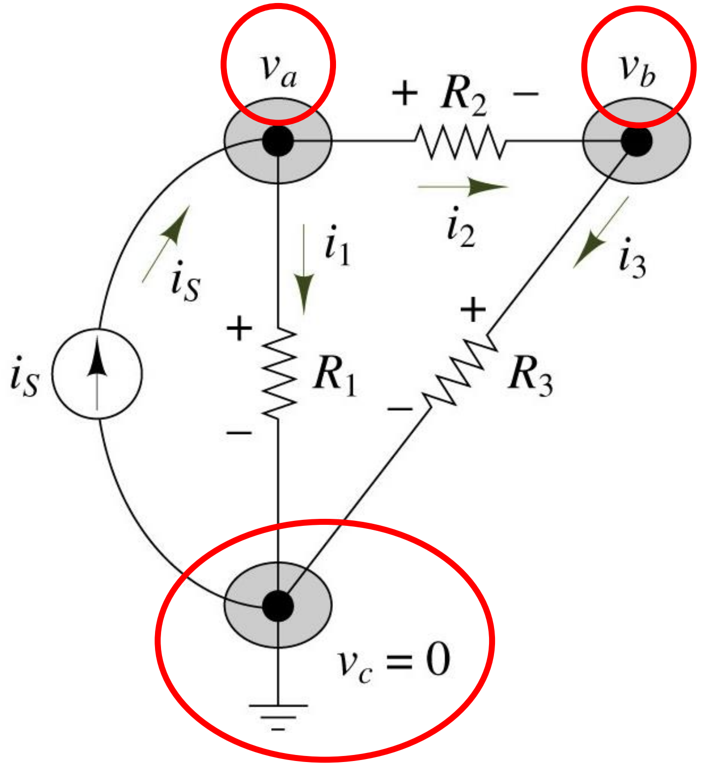
$$i_2 - i_3 = 0$$

$$i_1 + i_3 - i_s = 0 \rightarrow \text{redundant}$$

$$i_s - \frac{v_a}{R_1} - \frac{v_a - v_b}{R_2} = 0$$

$$\frac{v_a - v_b}{R_2} - \frac{v_b}{R_3} = 0$$

4. Solve the linear system of n – 1 – m unknowns.



Node Voltage Method : Example (3)

Solve the linear system of n – 1 – m unknowns.

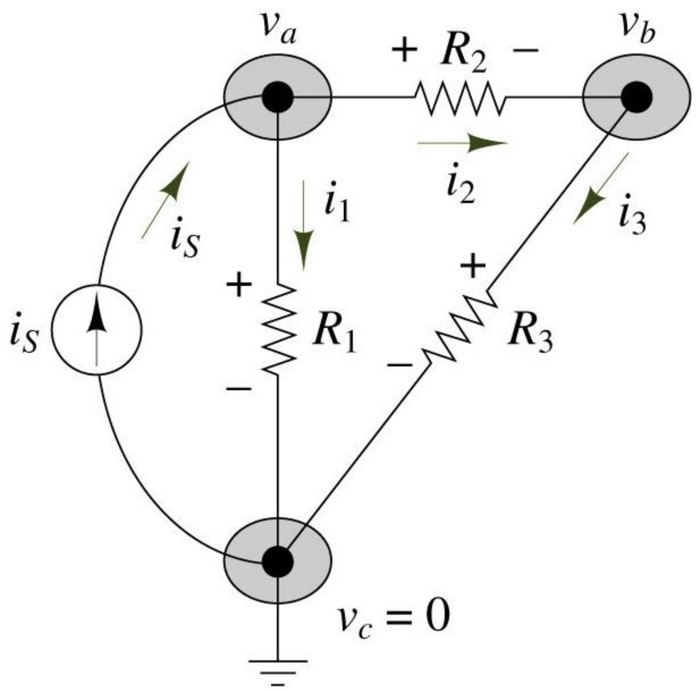
$$i_s - \frac{v_a}{R_1} - \frac{v_a - v_b}{R_2} = 0$$

$$\frac{v_a - v_b}{R_2} - \frac{v_b}{R_3} = 0$$

$$\Rightarrow$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_a + \left(-\frac{1}{R_2}\right) v_b = i_s$$

$$\left(-\frac{1}{R_2}\right) v_a + \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_b = 0$$



Q: But… We have only found voltages, what about the **currents**?

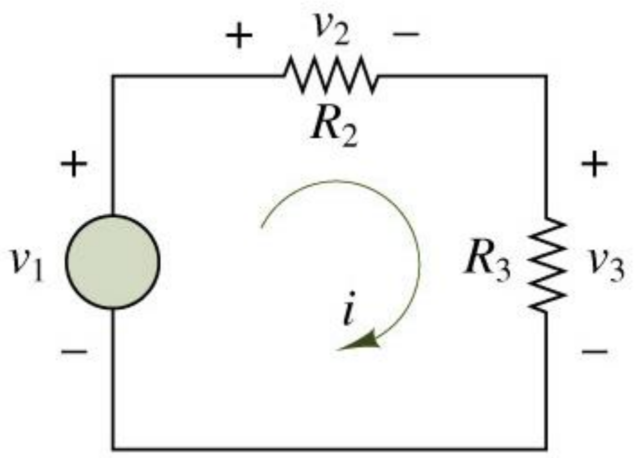
Mesh Current Method

Mesh Current Method (1)

The second method of circuit analysis discussed in this chapter employs mesh currents as the independent variables.

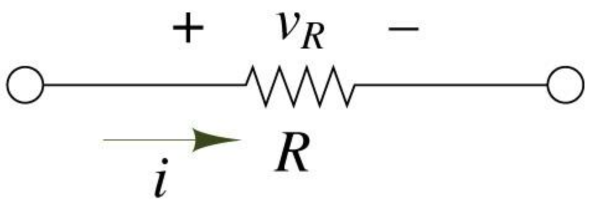
Subsequent application of **Kirchhoff's voltage law** around each mesh provides the desired system of equations.

- Once the direction of current flow has been selected, KVL requires that $v_1 - v_2 - v_3 = 0$



A mesh

- The current I, defined as flowing from left to right, establishes the polarity of the voltage across R :



Mesh Current Method (2)

Typical procedure of the MCM is as follows:

1. Define each mesh current consistently. Unknown mesh currents will be always defined in the clockwise direction.
2. In a circuit with n meshes and m current sources, $n - m$ independent equations will result. The unknown mesh currents are $n - m$ independent variables.
3. Apply KVL to each mesh containing an unknown mesh current, expressing each voltage in terms of one or more mesh currents.
4. Solve the linear system of $n - m$ unknowns.

Mesh Current Method : Example 1

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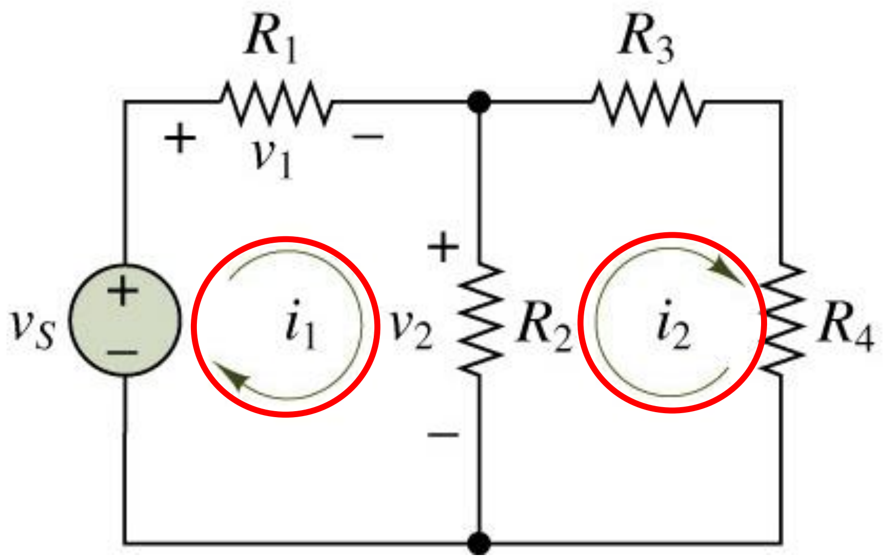


Find all voltages and currents in the circuit below.

1. Define each mesh current consistently.
2. In a circuit with n meshes and m current sources, $n - m$ independent equations will result. The unknown mesh currents are $n - m$ independent variables.
3. Apply KVL to each mesh containing an unknown current:

Mesh 1:

$$v_S - v_1 - v_2 = 0$$

$$v_1 = i_1 R_1$$

$$v_2 = (i_1 - i_2)R_2$$

$$v_s - i_1 R_2 - (i_1 - i_2)R_2$$

Mesh 2:

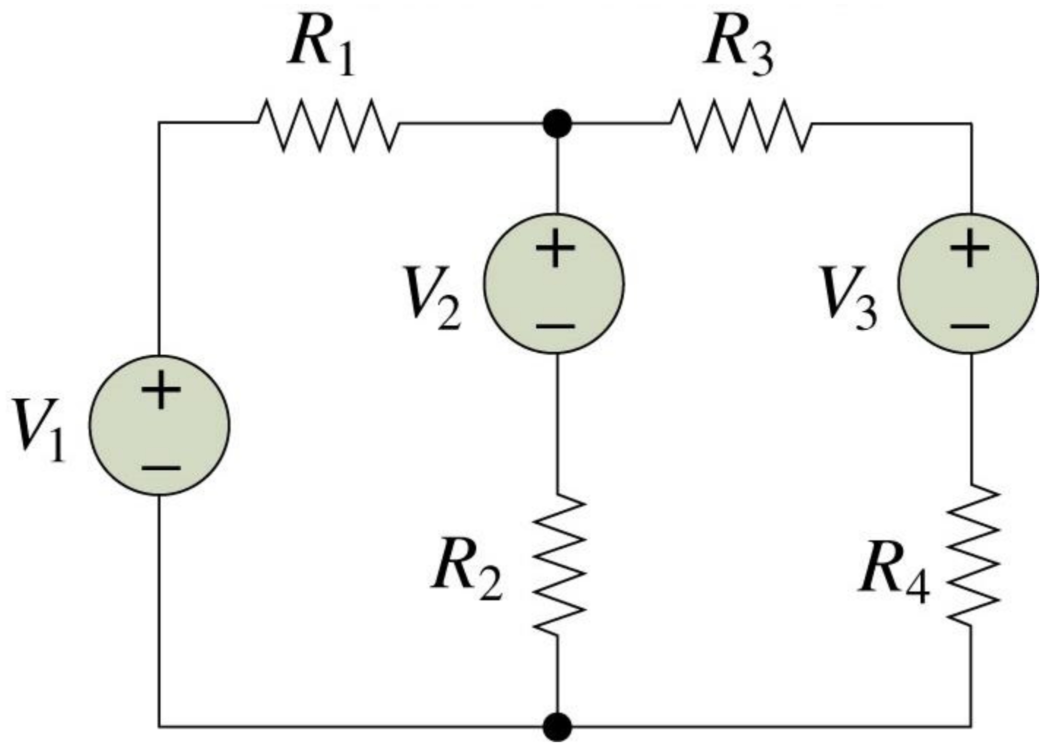
$$v_2 + v_3 + v_4 = 0$$

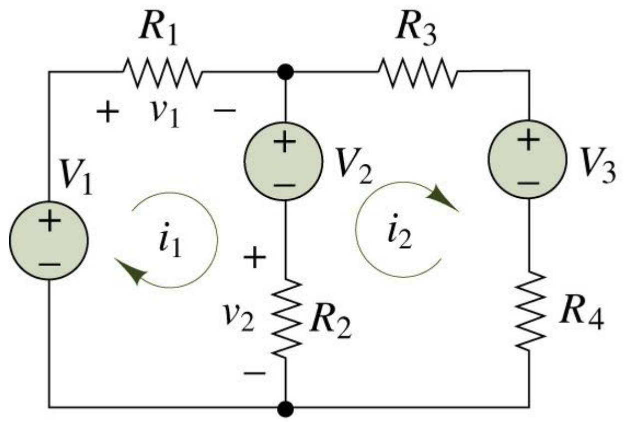
where

$$v_2 = (i_2 - i_1)R_2, \qquad v_3 = i_2 R_3, \qquad v_4 = i_2 R_4$$

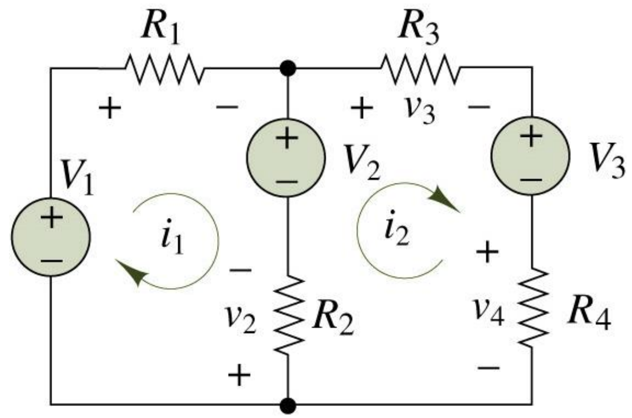
Mesh Current Method : Example 2 (1)

Find all voltages and currents in the circuit below.





Analysis of mesh 1



Analysis of mesh 2

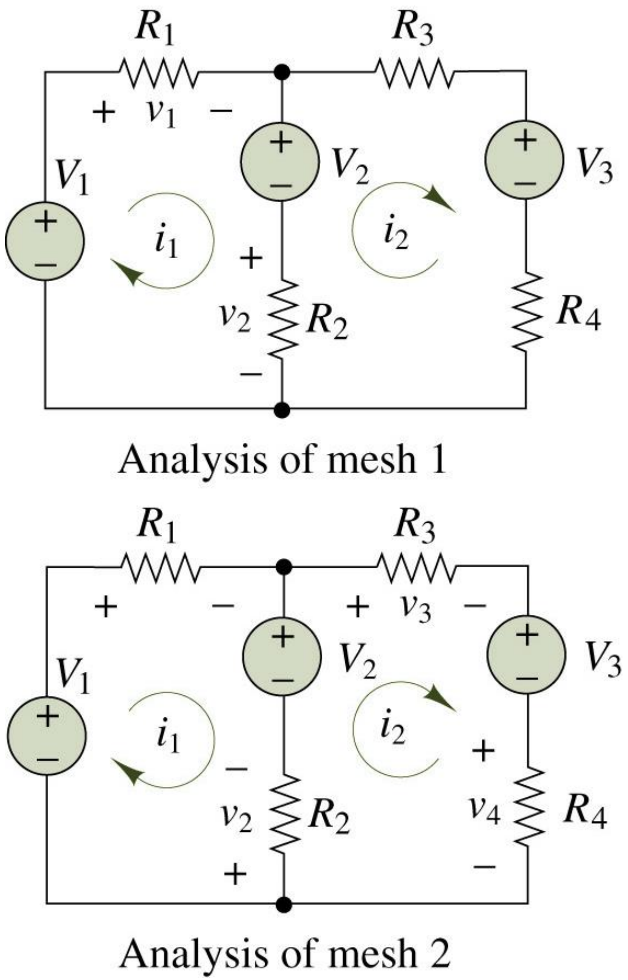
Mesh Current Method : Example 2 (2)

Applying KVL and Ohm's law to Mesh 1 yields:

$$V_1 - R_1 i_1 - V_2 - R_2(i_1 - i_2) = 0$$

Applying KVL and Ohm's law to Mesh 2 yields:

$$R_2(i_2 - i_1) - V_2 + R_3 i_2 + V_3 + R_4 i_2 = 0$$



Thank you for your attention!

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