Physics 2. Electrical Engineering

# Week 4 Network Analysis 2

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# Objectives

The main objectives of today's lecture are:

- Review the concepts of equivalent resistance
- Learn node voltage method
- Learn mesh current method

# Last Week’s Review

# Electrical Network Analysis

The analysis of an electrical network consists of determining each of the unknown branch currents and node voltages.

- It is therefore important to define all the relevant variables as clearly as possible, and in systematic fashion.
- Once the known and unknown variables have been identified, a set of equations relating these variables is constructed, and these equations are solved by means of suitable techniques.

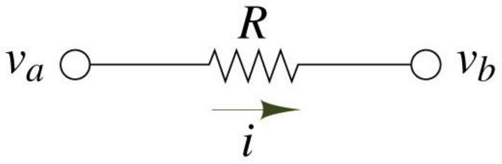
The analysis of electric circuits consists of writing the smallest set of equations sufficient to solve for all the unknown variables.

# Node Voltage Method (1)

Node voltage method (NVM) is based of defining the voltage at each node as an independent variable.

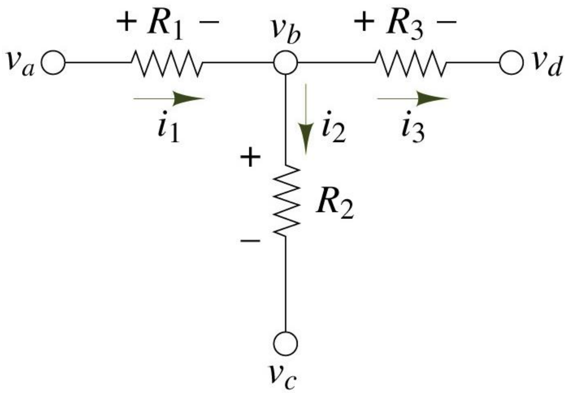
- One of the nodes is selected as a reference node (usually ground).
- Once each node is defined, Ohm's law may be applied between any two adjacent nodes to determine the current flowing in each branch.
- In the NVM, we assign the node voltages $v_a$ and $v_b$, the branch current flowing from a to be is then expressed in terms of these node voltages:

$$i = \frac{v_a - v_b}{R}$$




- We can then express KCL by

$$\frac{v_a - v_b}{R_1} - \frac{v_b - v_c}{R_2} - \frac{v_b - v_d}{R_3} = 0$$



# Node Voltage Method (2)

Typical procedure of the NVM is as follows:

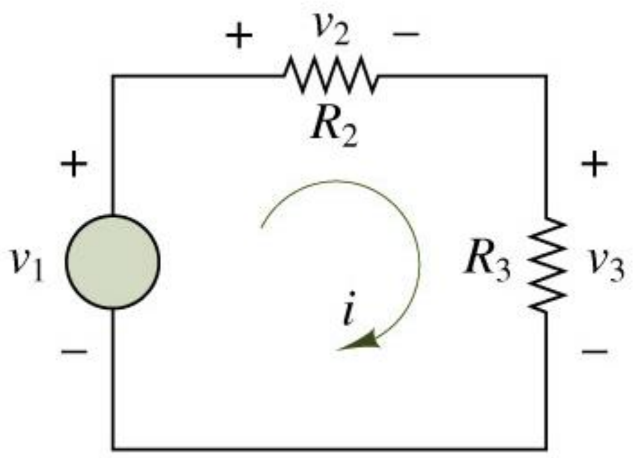
1. Select a reference node.
2. Define the remaining $n$ - 1 node voltages as the variables. Each of the m voltage sources in the circuit is associated with a dependent variable. If a node is not connected to a voltage source, then its voltage is treated as an independent variable.
3. Apply KCL at each node labeled as an independent variable.
4. Solve the linear system of $n$ – 1 – $m$ unknowns.

# Mesh Current Method (1)

The second method of circuit analysis discussed in this chapter employs mesh currents as the independent variables.

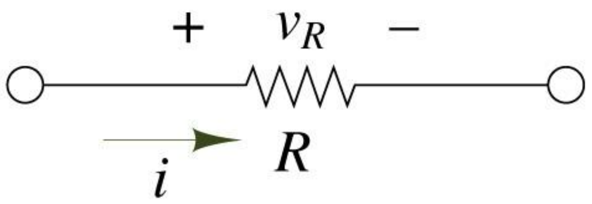
Subsequent application of **Kirchhoff's voltage law** around each mesh provides the desired system of equations.

- Once the direction of current flow has been selected, KVL requires that $v_1 - v_2 - v_3 = 0$



A mesh

- The current $I$, defined as flowing from left to right, establishes the polarity of the voltage across $R$ :



# Mesh Current Method (2)

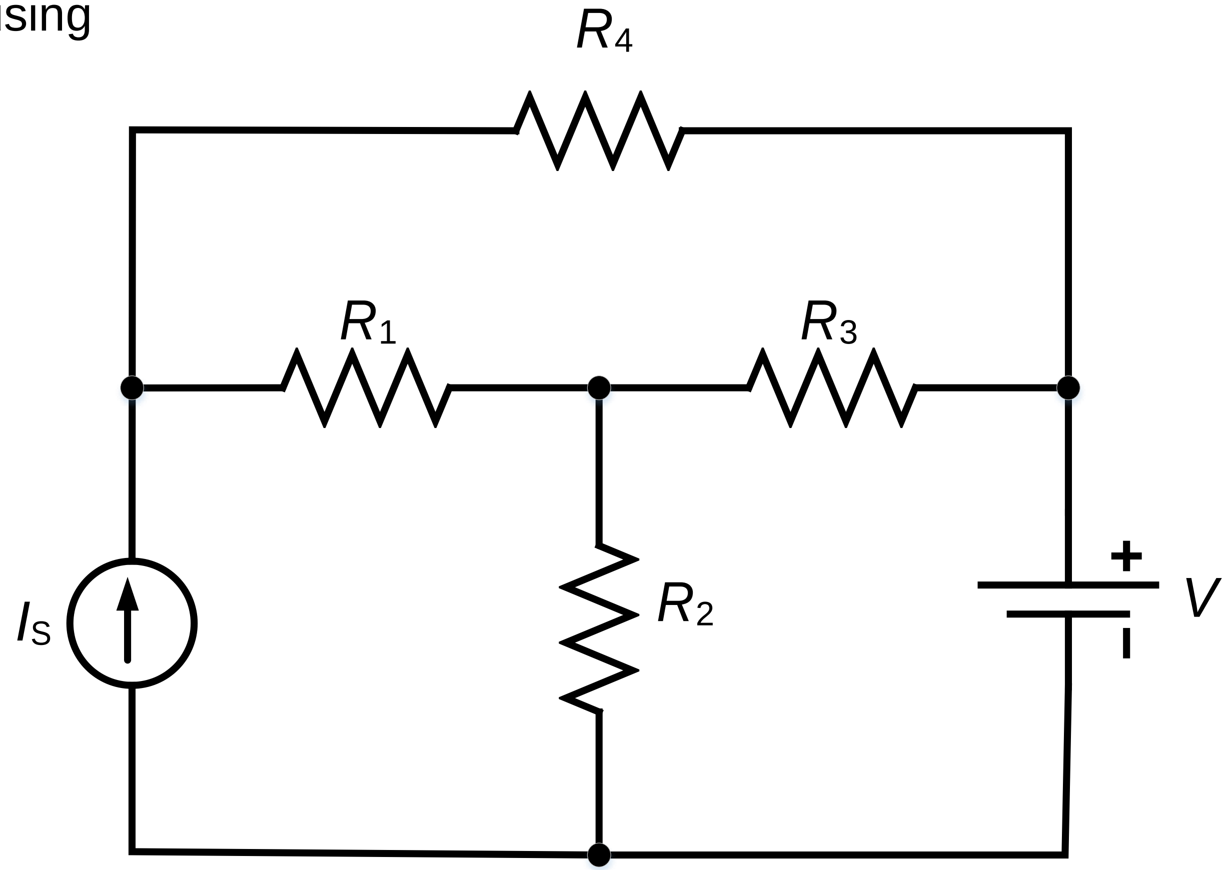
Typical procedure of the MCM is as follows:

1. Define each mesh current consistently. Unknown mesh currents will be always defined in the clockwise direction.
2. In a circuit with $n$ meshes and $m$ current sources, $n - m$ independent equations will result. The unknown mesh currents are $n - m$ independent variables.
3. Apply KVL to each mesh containing an unknown mesh current, expressing each voltage in terms of one or more mesh currents.
4. Solve the linear system of $n - m$ unknowns.

# Electrical Network Analysis : Exercise

Solve for voltages and currents in the given circuit using

- Node voltage method
- Mesh current method



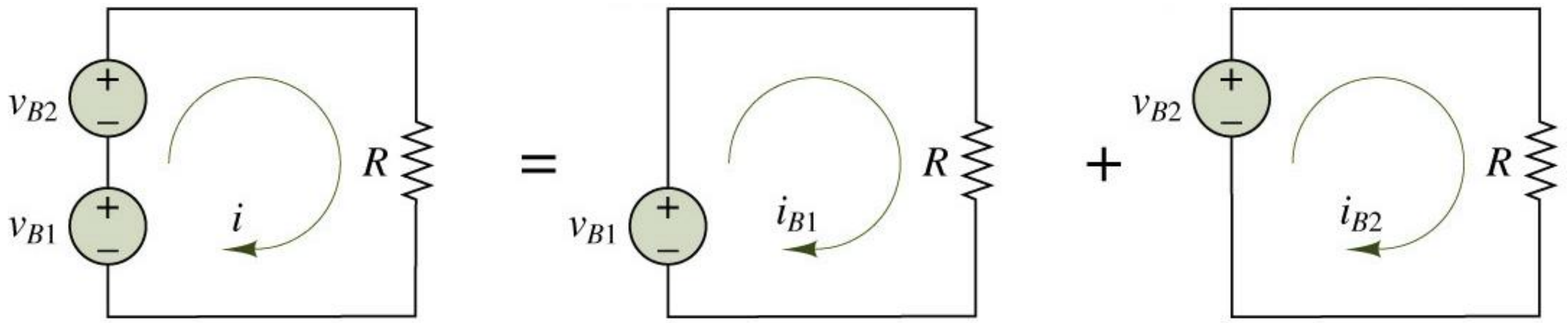
# The Principle of Superposition

# The Principle of Superposition (1)

Unlike precise analysis technique, like the mesh current and node voltage methods, the principle of superposition is a conceptual aid that can be very useful in visualizing the behavior of a circuit containing multiple sources.

In a circuit containing $N$ sources,

- each branch voltage and current is the sum of $N$ voltages and currents, each of which may be computed by setting all but one source equal to zero and solving the circuit containing that single source.

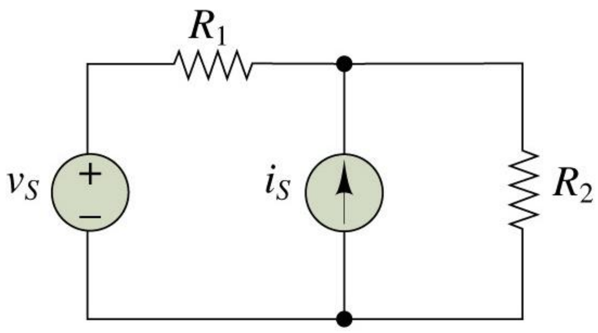



$$i = i_{B1} + i_{B2}$$

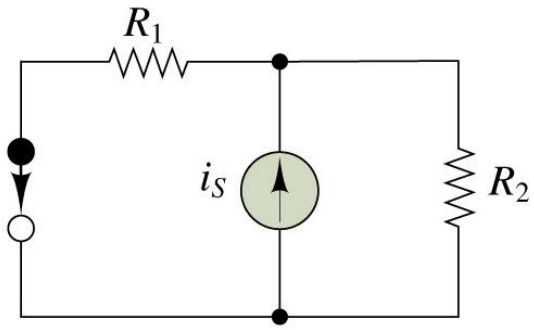
$$= \frac{v_{B1}}{R} + \frac{v_{B2}}{R} = \frac{v_{B1} + v_{B2}}{R}$$

# The Principle of Superposition (2)

- In order to set a voltage source equal to zero, we replace it with a short circuit

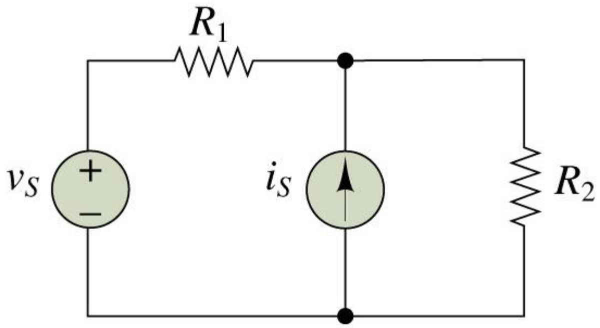



A circuit

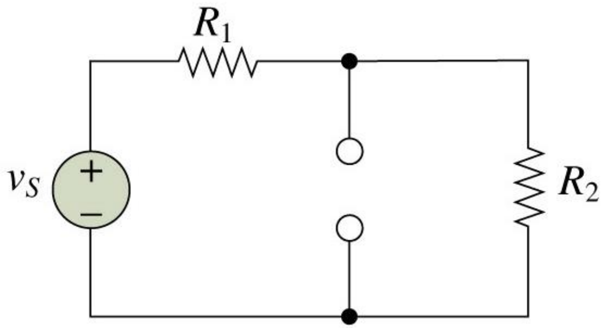



The same circuit with $v_S = 0$

- In order to set a current source equal to zero, we replace it with an open circuit



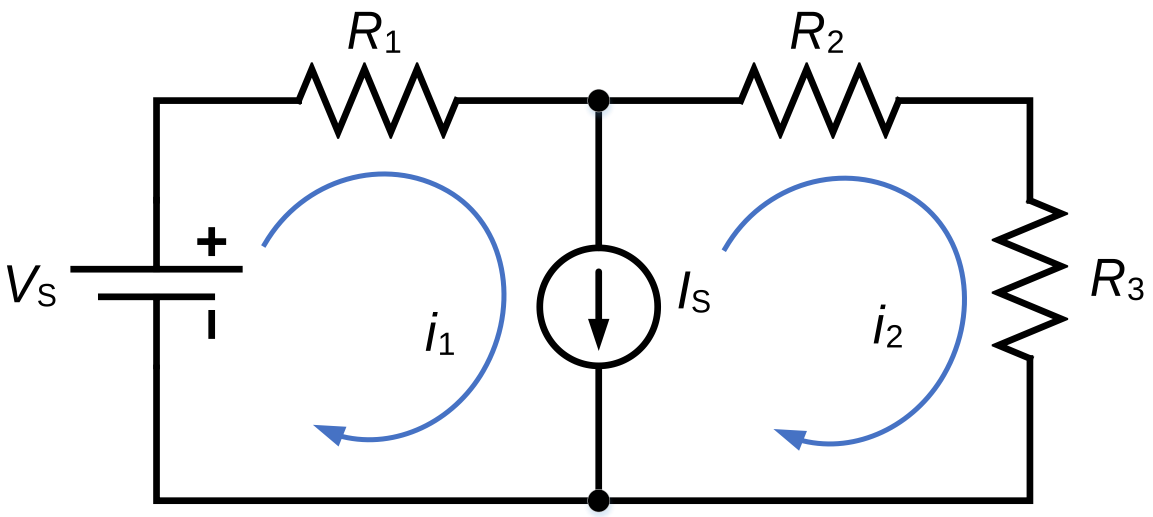

A circuit




The same circuit with $i_S = 0$

# The Principle of Superposition : Ex. 1

- Find unknown current $i_2$



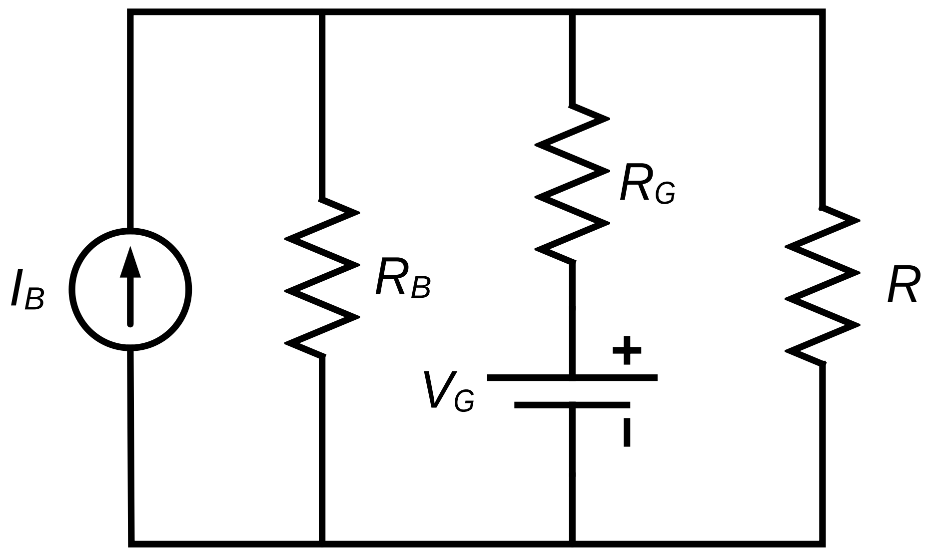

$$i_{2-V} = \frac{V_s}{R_1 + R_2 + R_3}$$

$$i_{2-I} = \frac{1/(R_2 + R_3)}{1/R_1 + 1/(R_2 + R_3)}(-I_s) = \frac{R_1}{R_1 + R_2 + R_3}(-I_s)$$

$$i_2 = i_{2-V} + i_{2-I}$$

# The Principle of Superposition : Ex. 2

- Find unknown voltage on resistor $R$




Superposition:

$$I_B - \frac{V_{R-I}}{R_B} - \frac{V_{R-I}}{R_G} - \frac{V_{R-I}}{R} = 0$$

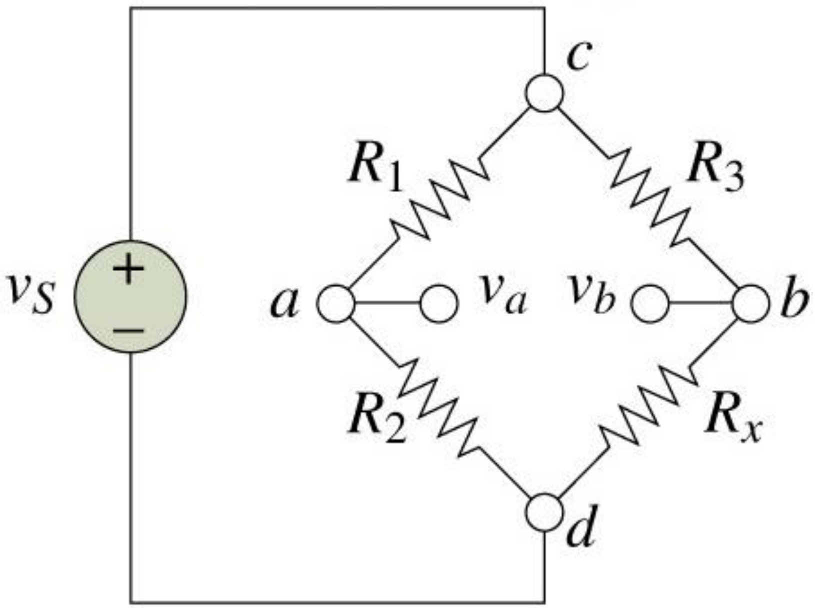
$$\frac{V_{R-V}}{R_B} + \frac{V_{R-V}}{R} + \frac{V_{R-V} - V_G}{R_G} = 0$$

# Wheatstone Bridge Circuit

# Wheatstone Bridge Circuit (1)

The Wheatstone bridge circuit is frequently encountered in a variety of measurement circuits.

- The Wheatstone bridge circuit is a circuit consisting of 4 resistors, with 1 of them of unknown resistance.
- The general form of the bridge circuit is shown in Figure on the right, where $R_1$, $R_2$, and $R_3$ are known while $R_x$ is an unknown resistance, to be determined.

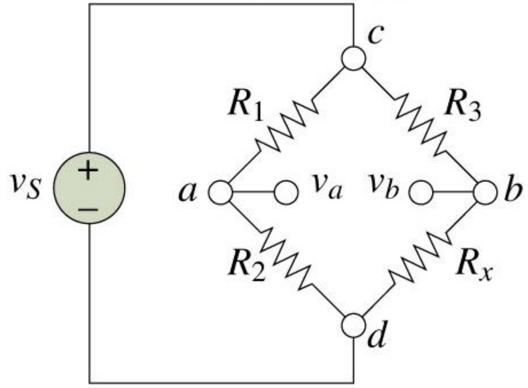


# Wheatstone Bridge Circuit (2)

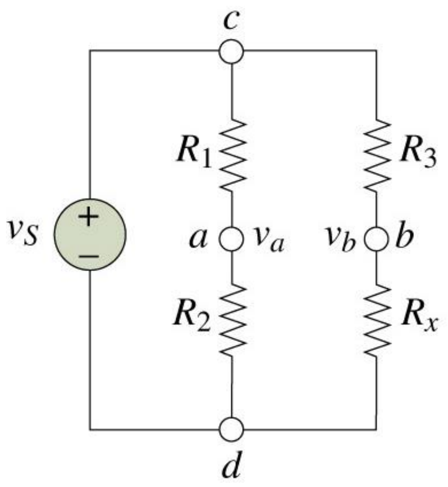
The circuit may also be redrawn as shown in Figure (b) on the right.

- The latter circuit is used to demonstrate the voltage divider rule in a mixed series-parallel circuit
- Finding unknown voltage $V_{ab}$ yields

$$V_{ab} = V_{ad} - V_{bd} = V_s\left(\frac{R_2}{R_1 + R_2} - \frac{R_x}{R_3 + R_x}\right)$$




(a)

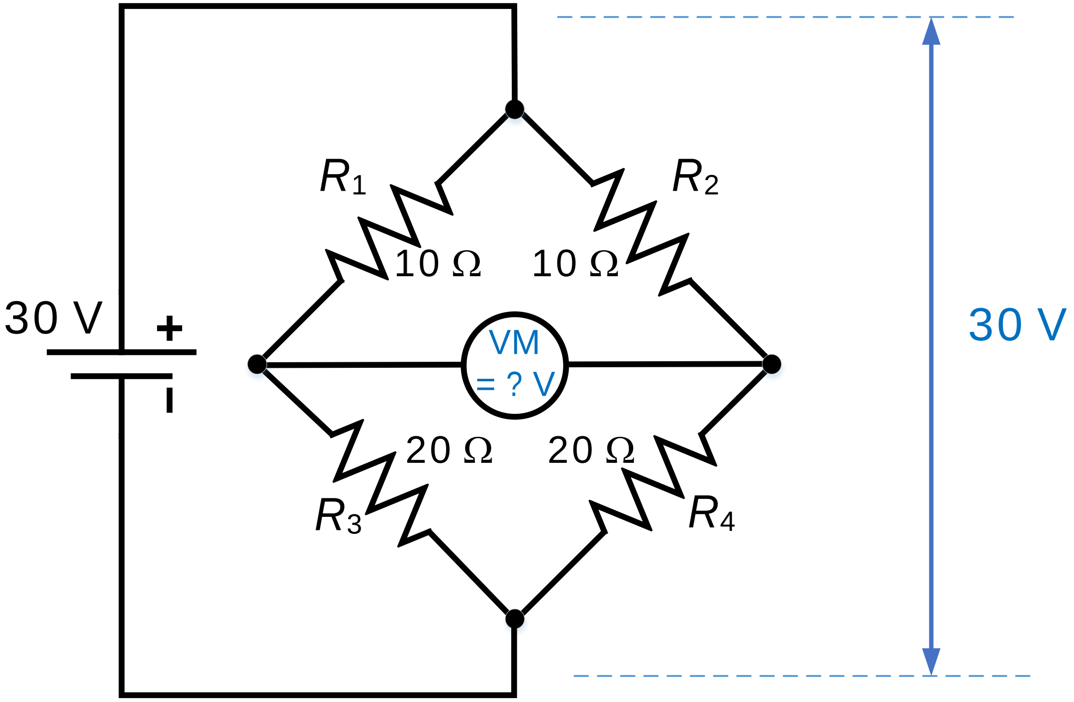


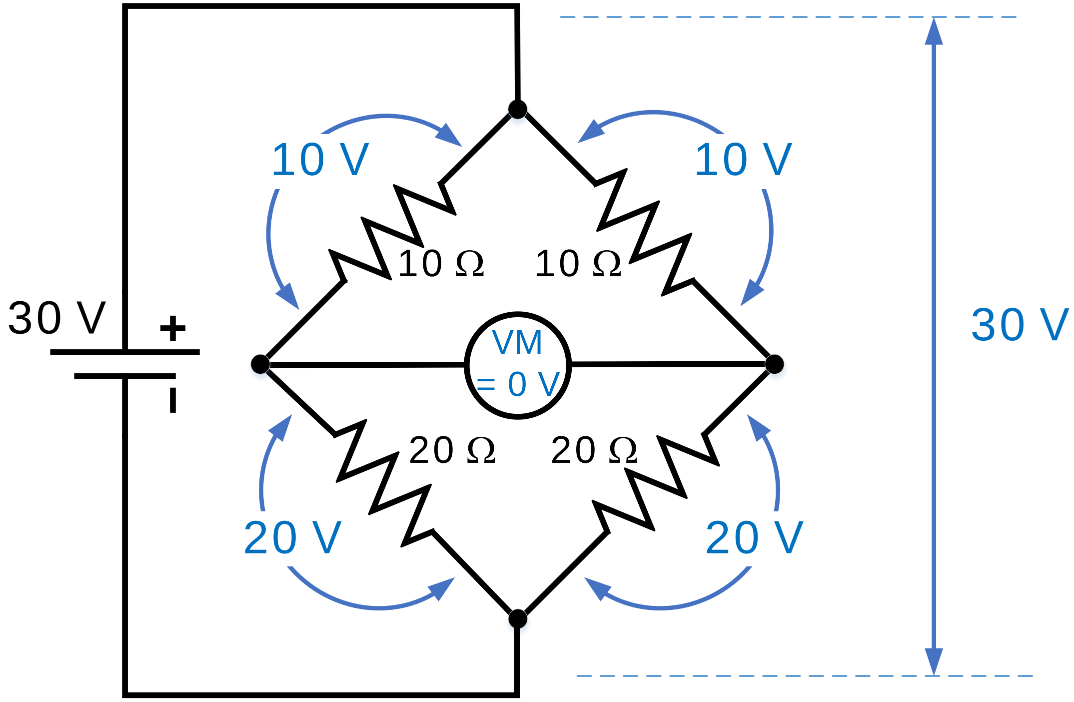

(b)

# Wheatstone Bridge Circuit (3)

If the resistances in the upper and bottom halves are equal, there is no voltage difference $V_{ab}$.

- What happens to $V_{ab}$ if $R_3$ drops to 10 Ω?



# Thank you for your attention!

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