

Databases - Tutorial 04

Relational Algebra

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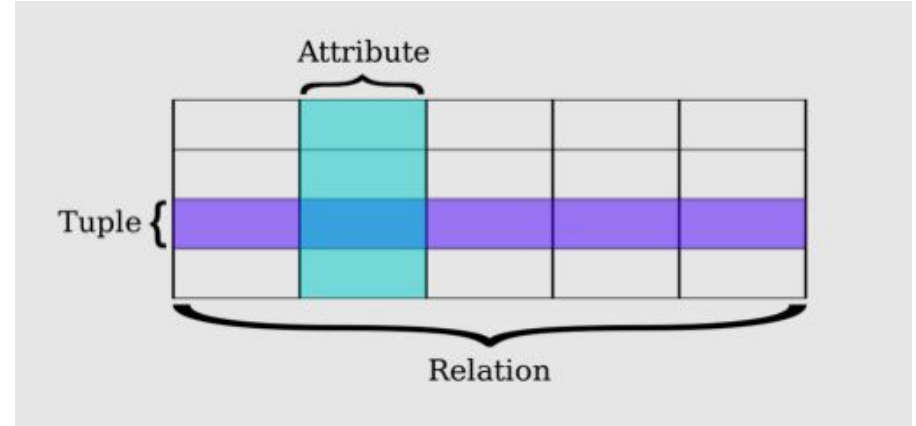
- Relational Algebra



Enhanced ERD

A relation is a set of tuples (d_1, d_2, \dots, d_n), where each element d_j is a member of D_j , a data domain (all the values which a data element may contain)

- No ordering to the elements of the tuples of a relation
- Relation, tuple, and attribute are commonly represented as table, row, and column respectively



Relations

Relations are sets, so we can apply set-theoretic operators + special relational operators

Basic operators

1) Union: \cup

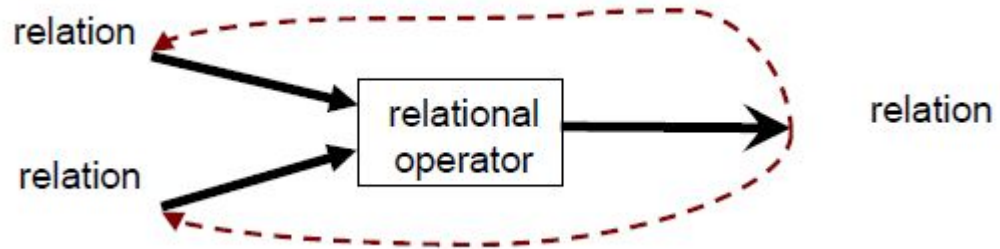
2) Set difference: $-$

3) Cartesian product: \times

4) Select: σ

5) Project: Π

Also Rename, Intersection, Join and Division...



relational algebra

set operations



set union



set intersection



set difference



cartesian product

relational database specific operations



selection



projection



join



set division

set functions



sum

avg

count

any

max

min

Operators



Union

Binary operator

Tuples in relation 1 OR in relation 2

Tuples must be union-compatible

- Same number of columns (attributes)
- 'Corresponding' columns have the same domain (type)

Eliminates duplicates

Notation: $R1 \cup R2$

ID	Firstname	Lastname
125	John	Smith
214	Anna	Kim
336	Leo	Abel

Attends course 1

ID	Firstname	Lastname
231	Maria	Dawn
214	Anna	Kim
255	Jim	White

Attends course 2

Attends course 1 or 2

ID	Firstname	Lastname
125	John	Smith
214	Anna	Kim
336	Leo	Abel
231	Maria	Dawn
255	Jim	White

Set Difference

Binary operator

Tuples in relation 1 AND NOT in relation 2

Tuples must be union-compatible

- Same number of columns (attributes)
- 'Corresponding' columns have the same domain (type)

Non-commutative

Notation: $R1 - R2$ or $R1 \setminus R2$

Set difference (keep the tuples that are in relation 1, but not in relations 2 (binary))

ID	Firstname	Lastname
125	John	Smith
214	Anna	Kim
336	Leo	Abel

Students

ID	Firstname	Lastname
231	Maria	Dawn
214	Anna	Kim
255	Jim	White

Didn't graduate

Graduated students

ID	Firstname	Lastname
125	John	Smith
336	Leo	Abel

$$\{1, 2, 3\} \setminus \{2, 3, 4\} = \{1\}.$$

$$\{2, 3, 4\} \setminus \{1, 2, 3\} = \{4\}.$$

Intersection

Binary operator

Tuples in relation 1 AND in relation 2

Tuples must be union-compatible

- Same number of columns (attributes)
- 'Corresponding' columns have the same domain (type)

commutative

Notation: $R_1 \cap R_2$

Intersection (keep the tuples that are in relation 1 AND in relation 2 (binary))

ID	Firstname	Lastname
125	John	Smith
214	Anna	Kim
336	Leo	Abel

Master students

ID	Firstname	Lastname
231	Maria	Dawn
214	Anna	Kim
255	Jim	White

Graduated students

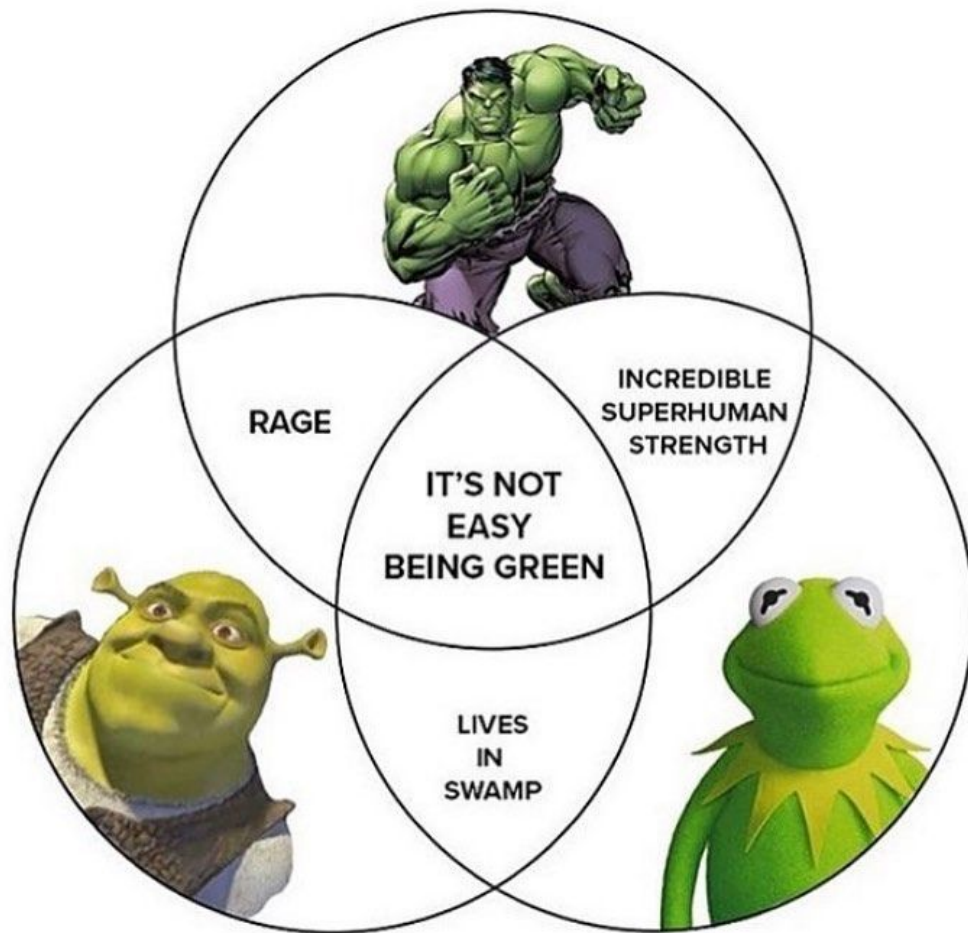
Graduated master students

ID	Firstname	Lastname
214	Anna	Kim



Hulk ____ Shrek =Rage

Hulk ____ Kermit =Rage



Cartesian product

- $S1 \times R1$: Each row of $S1$ paired with each row of $R1$
- Like the cartesian product for mathematical relations
- Every tuple of $S1$ “appended” to every tuple of $R1$
- How many rows in the result?
- No need for the two input relations to be union-compatible
- Result schema has one attribute per attribute of $S1$ and $R1$

Notation: $S1 \times R1$

Students

ID	Firstname	Lastname
125	John	Smith
214	Anna	Kim
336	Leo	Abel

Courses

CID	Course
11	Logic
12	DB

Courses x Students

ID	Firstname	Lastname	CID	Course
125	John	Smith	11	Logic
214	Anna	Kim	11	Logic
336	Leo	Abel	11	Logic
125	John	Smith	12	DB
214	Anna	Kim	12	DB
336	Leo	Abel	12	DB

Renaming

The problem: Father and Mother are different names, but both represent a parent.

The solution: rename attributes!

Paternity

Father	Child
John	Igor
Jim	Eva
Leo	Kate

Mother	Child
Anna	Kate
Maria	Igor
Elena	Andrew

Maternity

Renaming

Rename

- Unary operator
- Changes attribute **names** for a relation without changing any values
- Renaming removes the limitations associated with set operators

Notation: $\rho_{\text{OldName}} \rightarrow \text{NewName}(r)$ (e.g. $\rho_{\text{Father}} \rightarrow \text{Parent}(\text{Paternity})$)

- If there are two or more attributes involved in a renaming operation, then ordering is

meaningful: (e.g., $\rho_{\text{Branch,Salary}} \rightarrow \text{Location,Pay}(\text{Employees})$)

Paternity

Father	Child
John	Igor
Jim	Eva
Leo	Kate

$\rho\text{Father} \rightarrow \text{Parent}(\text{Paternity})$

Parent	Child
John	Igor
Jim	Eva
Leo	Kate

Maternity

Mother	Child
Anna	Kate
Maria	Igor
Elena	Andrew

$\rho\text{Mother} \rightarrow \text{Parent}(\text{Paternity})$

Parent	Child
Anna	Kate
Maria	Igor
Elena	Andrew

Select

- Unary operator
- Selects a subset of rows from a relation that satisfy selection predicate
- Schema of result is same as that of the input relation
- Works like a filter that keeps only those tuples that satisfy a qualifying condition
- The selection condition is a Boolean expression specified on the attributes of relation R

Notation: $\sigma_p(r)$

Select Example: $\sigma_{\text{Age} > 20}(\text{Students})$

Students

ID	Firstname	Lastname	Age
125	John	Smith	21
214	Anna	Kim	19
336	Leo	Abel	22
231	Maria	Dawn	18
255	Jim	White	23

Students with age > 20

ID	Firstname	Lastname	Age
125	John	Smith	21
336	Leo	Abel	22
255	Jim	White	23

How to select students with age greater than 20 and GPA greater than 3.2?

Students

ID	Firstname	Lastname	Age	GPA
125	John	Smith	21	3.1
214	Anna	Kim	19	3.84
336	Leo	Abel	22	3.69
231	Maria	Dawn	18	3.21
255	Jim	White	23	2.9

Projection

- Unary operator
- Deletes unwanted columns from a relation
- Removes duplicated data
- The schema of result has exactly the columns in the projection list, with the same names

that they had in the input relation

Notation: $\Pi_p(r)$

Projection example: $\Pi_{\text{Lastname, Age}}(\text{Students})$

Students

ID	Firstname	Lastname	Age
125	John	Smith	21
214	Anna	Kim	19
336	Leo	Abel	22
231	Maria	Dawn	18
255	Jim	Smith	21

$\Pi_{\text{Lastname, Age}}(\text{Students})$

Lastname	Age
Smith	21
Kim	19
Abel	22
Dawn	18

Extended projection example: $\Pi_{\text{Firstname + Lastname} \rightarrow \text{Name, Age}}(\text{Students})$

Students

ID	Firstname	Lastname	Age
125	John	Smith	21
214	Anna	Kim	19
336	Leo	Abel	22
231	Maria	Dawn	18
255	Jim	Smith	21

Projected table

Name	Age
John Smith	21
Anna Kim	19
Leo Abel	22
Jim Dawn	18
Jim Smith	21

Join

- Binary operator
- Allows us to establish connections among data in different relations, taking advantage of the "value-based" nature of the relational model
- Two versions
 - **"natural" join: takes attribute names into account**
 - **"theta" join.**

Notation: $r1 \bowtie r2$

Natural join (or “just join”)

- Binary operator
- Select rows where attributes that appear in both relations have equal values
- Project all unique attributes and one copy of each of the common ones

Notation: $R \bowtie S$

Attendance

FirstName	Lastname	Course
John	Smith	Logic
John	Smith	DB
Leo	Abel	DB

Courses

CID	Course	Teacher
11	Logic	Pain
12	DB	White
13	English	Gray

Attendance \bowtie Courses

Firstname	Lastname	Course	CID	Teacher
John	Smith	Logic	11	Pain
John	Smith	DB	12	White
Leo	Abel	DB	12	White

Note: Joins can be incomplete or empty

Theta join (or “conditional join”)

- Binary operator
- Results in all combinations of tuples in R and S that satisfy θ (where θ is a binary relational operator in the set $\{<, \leq, =, >, \geq\}$)
- Result schema same as that of cross-product
- In case the operator θ is the equality operator ($=$) then this join is also called an equijoin

Notation: $R \bowtie_{\theta} S = \sigma_{\theta}(R \times S)$

Group A

Lastname	Age
Smith	20
Kim	32
Abel	17

Group B

Lastname	Age
White	21
Gray	32
Li	17

Group A $\bowtie_{A.Age > B.Age}$ Group B

Lastname	Age	Lastname	Age
Kim	32	White	21
Smith	20	Li	17
Kim	32	Li	17

Equijoin

- In case the operator θ is the equality operator ($=$) then this join is also called an equijoin

Students

ID	Lastname	Project
125	Smith	Moon
214	Kim	Solar
336	Abel	Solar

Projects

CID	Name
11	Solar
12	Moon

Students $\bowtie_{\text{Project=Name}}$ Projects

ID	Lastname	Project	CID	Course
125	Smith	Moon	12	Moon
214	Kim	Solar	11	Solar
336	Abel	Solar	11	Solar

Division

The division operator is used for queries which involve the 'all'.

$R1 \div R2$ = tuples of R1 associated with all tuples of R2.

R		S		$R \div S =$	
ColA	ColB	ColB		ColA	
F	1	1		F	
F	2	2		S	
F	3				
E	1				
E	3				
S	1				
S	2				

Let us try together

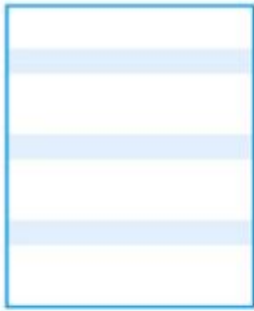
- Suppliers (sid: integer, sname: string, address: string)
- Parts (pid: integer, pname: string, color: string)
- Catalog (sid: integer, pid: integer, cost: real)

1- Find the sids of suppliers who supply some red or green part.

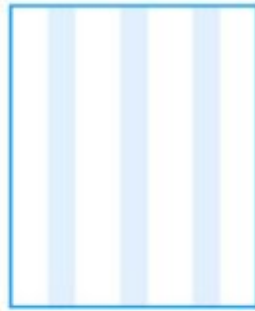
2- Find the sids of suppliers who supply some red part and some green part.

3- Find the sids of suppliers who supply every part.

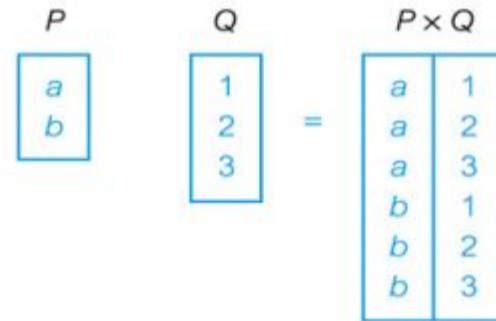
$$\pi_{sid}(\pi_{pid}(\sigma_{color='red' \vee color='green'} Parts) \bowtie catalog)$$
$$\rho(R1, \pi_{sid}((\pi_{pid} \sigma_{color='red'} Parts) \bowtie Catalog))$$
$$\rho(R2, \pi_{sid}((\pi_{pid} \sigma_{color='green'} Parts) \bowtie Catalog))$$
$$R1 \cap R2$$



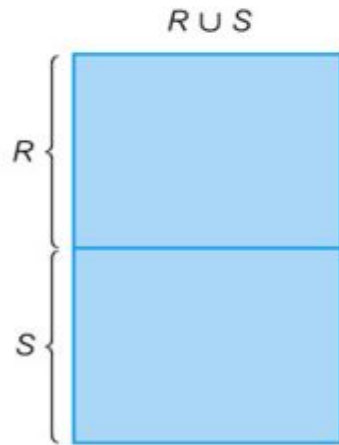
(a) Selection



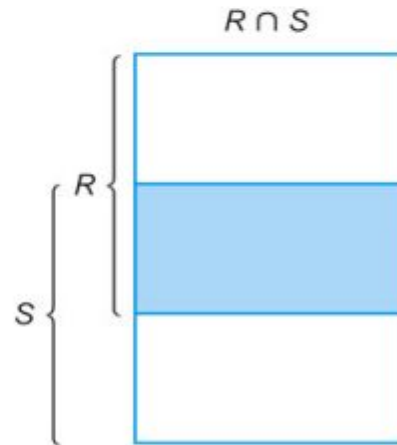
(b) Projection



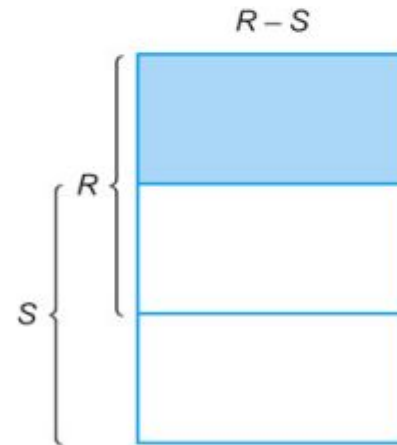
(c) Cartesian product



(d) Union



(e) Intersection



(f) Set difference

	T	
A	B	
a	1	
b	2	

U	
B	C
1	x
1	y
3	z

A	B	C
a	1	x
a	1	y

$T \triangleright_B U$	
A	B
a	1

$T \bowtie_c U$		
A	B	C
a	1	x
a	1	y
b	2	

(g) Natural join

(h) Semijoin

(i) Left Outer join

A diagram of a rectangle divided into four regions by a vertical line and a horizontal line. The top-left region is shaded blue. The top-right region is white and labeled R . The bottom-left region is white. The bottom-right region is white and labeled Remainder.

S

$R \div S$

V	
A	B
a	1
a	2
b	1
b	2
c	1

W	B
	1
	2

$$V \div W$$

A
a b

(j) Divis on (shaded area)

Example of division

Let us try together

- Suppliers (sid: integer, sname: string, address: string)
- Parts (pid: integer, pname: string, color: string)
- Catalog (sid: integer, pid: integer, cost: real)

1- Find the sids of suppliers who supply some red or green part.

2- Find the sids of suppliers who supply some red part and some green part.

3- Find the sids of suppliers who supply every part.

SID	Sname	Address
1	Yosemite Sham	Devil's canyon, AZ
2	Wiley E. Coyote	RR Asylum, NV

Parts

PID	Pname	Color
1	Red1	Red
2	Red2	Red
3	Green1	Green
4	Blue1	Blue
5	Red3	Red

Catalog

SID	PID	Cost
1	1	\$10.00
1	2	\$20.00
1	3	\$30.00
1	4	\$40.00
1	5	\$50.00
2	1	\$9.00
2	3	\$34.00
2	5	\$48.00
3	1	\$11.00

Let us try together

- Suppliers (sid: integer, sname: string, address: string)
- Parts (pid: integer, pname: string, color: string)
- Catalog (sid: integer, pid: integer, cost: real)

$$(\Pi_{\text{sname}}((\sigma_{\text{color}=\text{red}} \text{Parts}) \bowtie (\sigma_{\text{cost}<100} \text{Catalog}) \bowtie \text{Suppliers})) \cap (\Pi_{\text{sname}}((\sigma_{\text{color}=\text{green}} \text{Parts}) \bowtie (\sigma_{\text{cost}<100} \text{Catalog}) \bowtie \text{Suppliers}))$$

Sol : Find the Supplier names of the suppliers who supply a red part that costs less than 100 dollars and a green part that costs less than 100 dollars.

References

- <https://www.guru99.com/relational-algebra-dbms.html>
- <https://www.javatpoint.com/dbms-relational-algebra>
- <https://home.adelphi.edu/~siegfried/cs443/443l9.pdf>