Control Theory

Tutorial 2
ODE ↔ SSE

Stability

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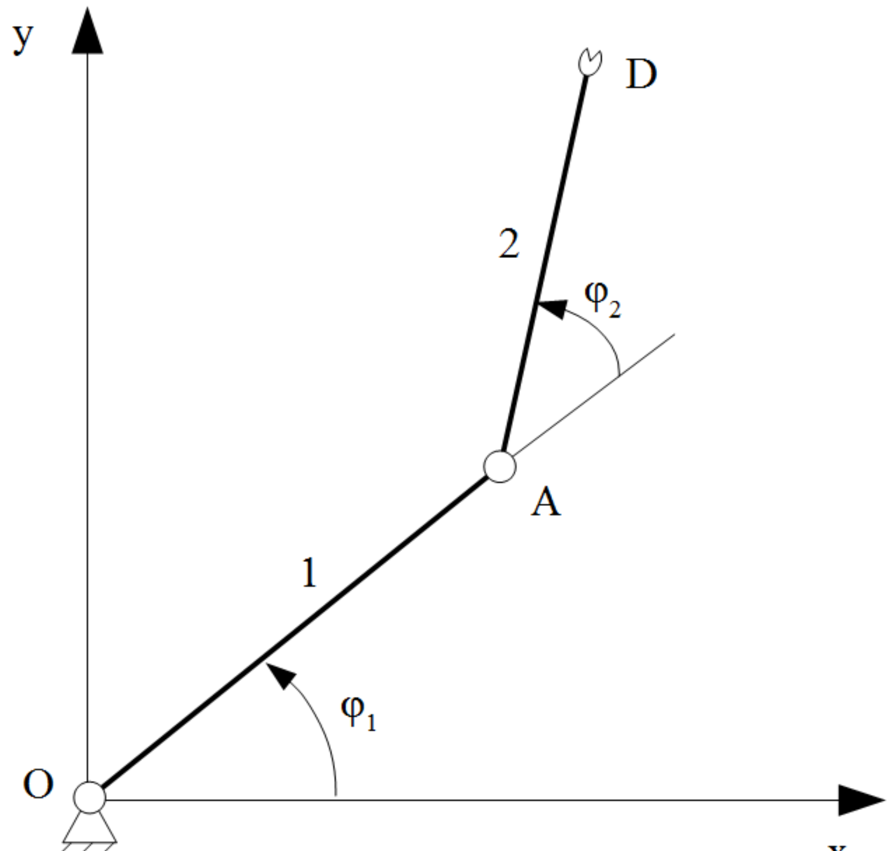
A simple manipulator

Example 4



It is required to make differential equations of the given plane manipulator.

$$\left(J_1+m_2 l_1^2+m_2 l_2^2+2 m_2 l_1 l_2 \cos\varphi_2\right)\ddot{\varphi}_1+\left(m_2 l_2^2+m_2 l_1 l_2 \cos\varphi_2\right)\ddot{\varphi}_2=$$

$$=M_1+2 m_2 l_1 l_2 \dot{\varphi}_1 \dot{\varphi}_2 \sin\varphi_2+m_2 l_1 l_2 \dot{\varphi}_2^2 \sin\varphi_2$$

$$\left(m_2 l_2^2+m_2 l_1 l_2 \cos\varphi_2\right)\ddot{\varphi}_1+\left(m_2 l_2^2+J_2\right)\ddot{\varphi}_2=$$

$$=M_2+m_2 l_1 l_2 \dot{\varphi}_1^2 \sin\varphi_2$$

$$a_{11}\ddot{\varphi}_1+a_{12}\ddot{\varphi}_2=b_1$$
$$a_{21}\ddot{\varphi}_1+a_{22}\ddot{\varphi}_2=b_2$$

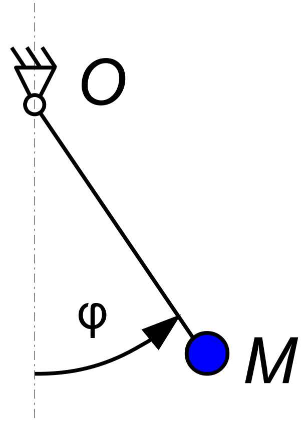
$$a_{11}\ddot{\varphi}_1 + a_{12}\ddot{\varphi}_2 = b_1$$
$$a_{21}\ddot{\varphi}_1 + a_{22}\ddot{\varphi}_2 = b_2$$

$$\dot{\varphi}_1 = \omega_1$$
$$\dot{\varphi}_2 = \omega_2$$
$$a_{11}\dot{\omega}_1 + a_{12}\dot{\omega}_2 = b_1$$
$$a_{21}\dot{\omega}_1 + a_{22}\dot{\omega}_2 = b_2$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a_{11} & a_{12} \\ 0 & 0 & a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ b_1 \\ b_2 \end{bmatrix}$$

A mathematical pendulum

Example 3



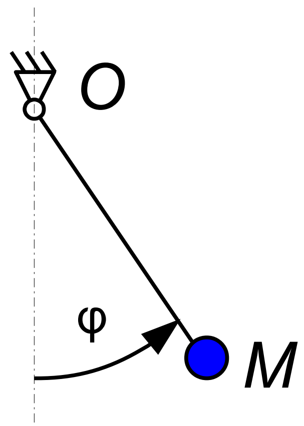
A mathematical pendulum deviated from its vertical position by the angle φ_0 and began to oscillate.
The pendulum length is l = 1 m.

It is required to make and solve differential equations.

$$\ddot{\varphi} = -\frac{g}{l}\sin\varphi \qquad \ddot{\varphi} \approx -\frac{g}{l}\varphi$$

https://colab.research.google.com/drive/1LTHmL096SMKNLL8uzn9SpdR6jT5_6pef?usp=sharing

Example 3a



The same problem with the mathematical pendulum. But now there is a resistance force:
1) linear resistance force with coefficient μ
2) quadratic resistance force with coefficient λ

In this case we must also know the mass of the particle m.

It is required to make and solve differential equations.

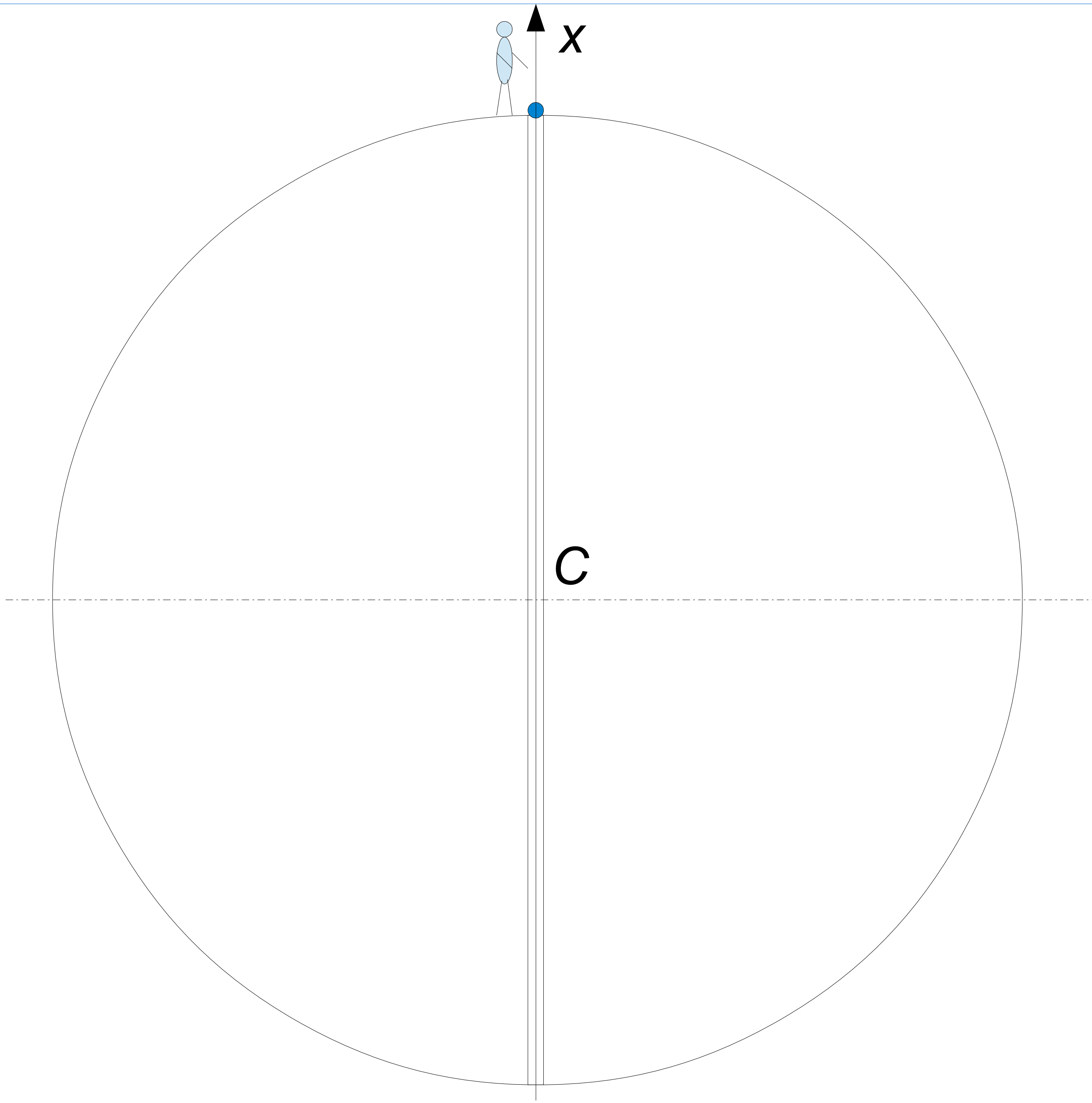
1) $$\ddot{\varphi} = -\frac{g}{l}\sin\varphi - \frac{\mu}{m}\dot{\varphi} \qquad \ddot{\varphi} \approx -\frac{g}{l}\varphi - \frac{\mu}{m}\dot{\varphi}$$

$$\ddot{\varphi} + \frac{\mu}{m}\dot{\varphi} + \frac{g}{l}\varphi = 0$$

2) $$\ddot{\varphi} = -\frac{g}{l}\sin\varphi - \frac{\lambda}{m} l |\dot{\varphi}| \dot{\varphi}$$

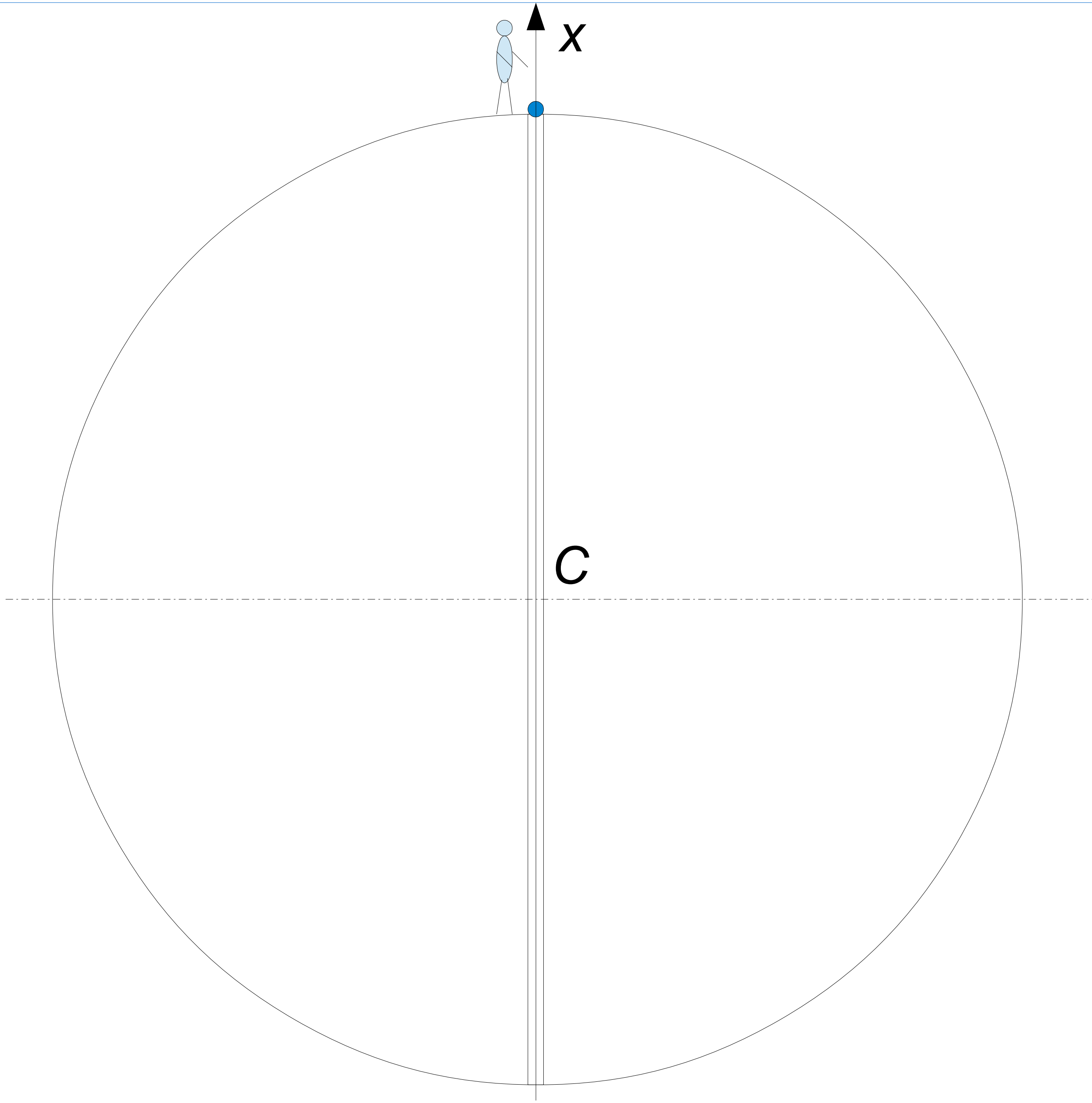
«An asteroid problem»

Example 5



There is an asteroid. Astronauts drilled a through hole in it, passing through the center. Then the astronaut threw a rock into the hole. Describe the motion of the stone if we know that the gravity inside the asteroid is proportional to the distance to the center, and the weight of the stone on the surface equals P.
Radius of the asteroid is R.

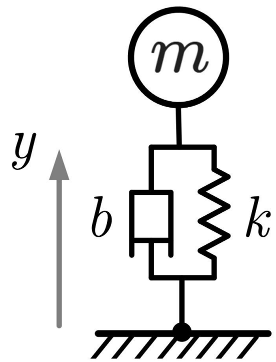
$$\ddot{x} = -\frac{P}{Rm}x$$



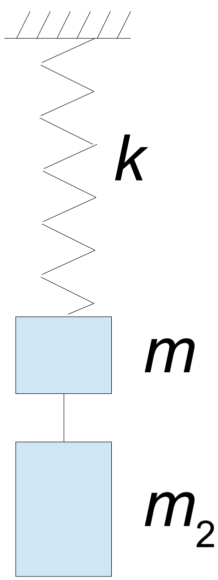
The same problem. But now there is a linear resistance force with coefficient μ

$$\ddot{x} = -\frac{P}{Rm}x - \frac{\mu}{m}\dot{x}$$

Oscillations



This is the standard calculation scheme for oscillations. But let's consider something more realistic.



Two weights of masses *m* and m_2 had been suspended on a spring. Then the lower weight came off, and the upper one started oscillating. Determine the law of motion of the weight.
The spring stiffness is c, the coefficient of viscous resistance is b.

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$\ddot{x} + 2n\dot{x} + \omega^2 x = 0$$

$n < \omega$

$$x = C_1 e^{-nt}\cos(\omega_1 t) + C_2 e^{-nt}\sin(\omega_1 t)$$

$$\omega_1 = \sqrt{\omega^2 - n^2}$$

$n = \omega$

$$x = C_1 e^{-nt} + C_2 t e^{-nt}$$

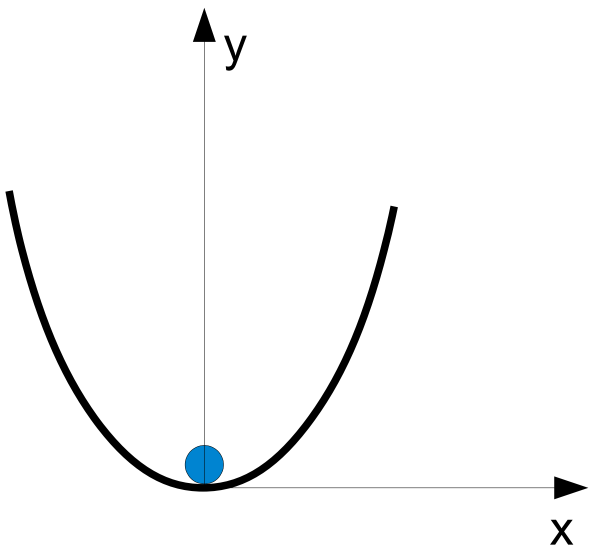
$n > \omega$

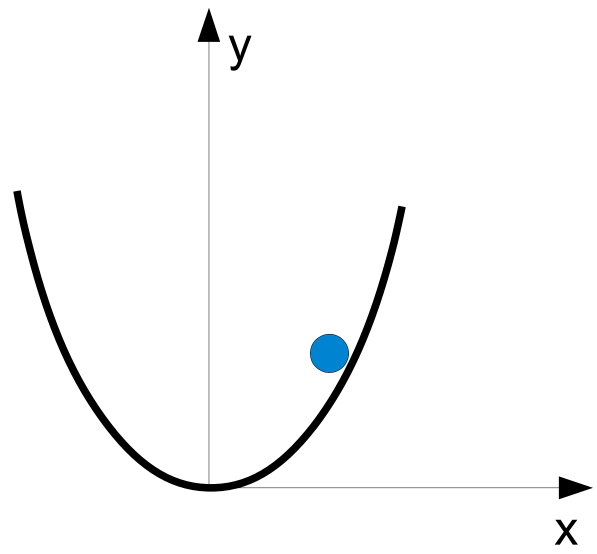
$$x = C_1 e^{-nt} e^{\omega_2 t} + C_2 e^{-nt} e^{-\omega_2 t}$$

$$\omega_2 = \sqrt{n^2 - \omega^2}$$

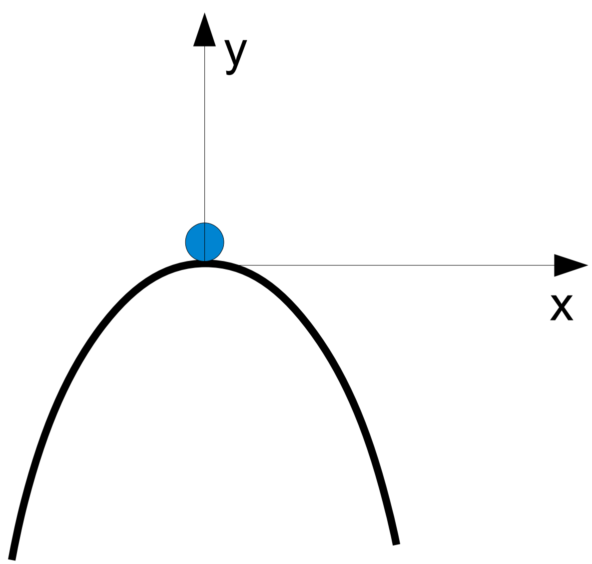
Potencial hill and potencial well

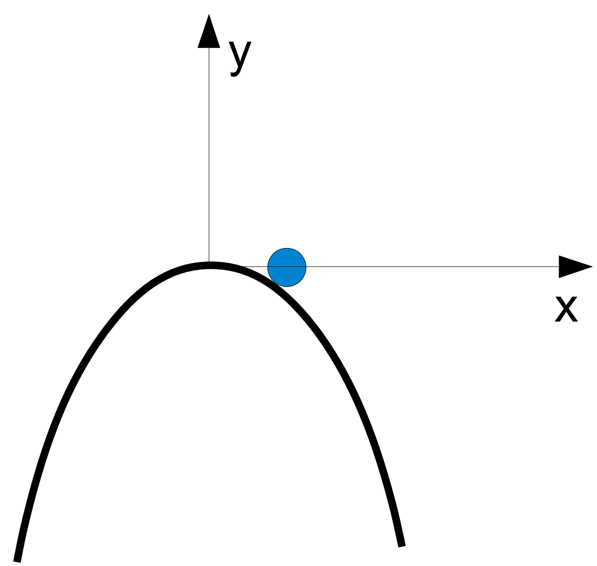
Example 7





$$\ddot{x} = -a\,x$$





$$\ddot{x} = +a\,x$$