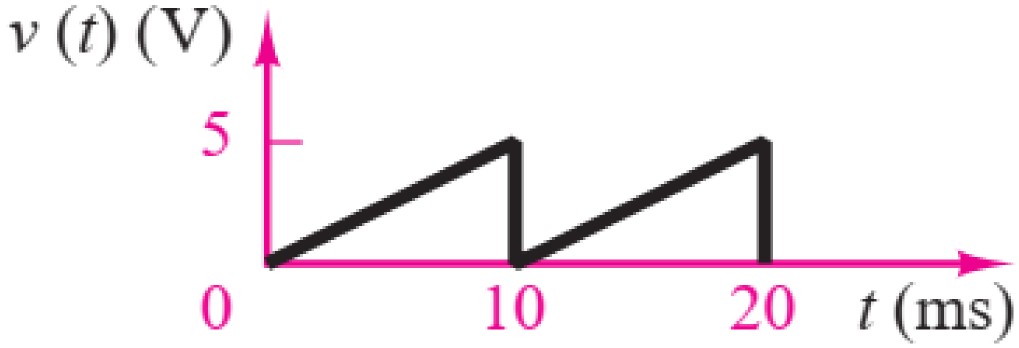
1.

$$x_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t')\,dt'} \qquad \text{Root-mean-square value}$$

Find the rms value of the waves shown in the figure:



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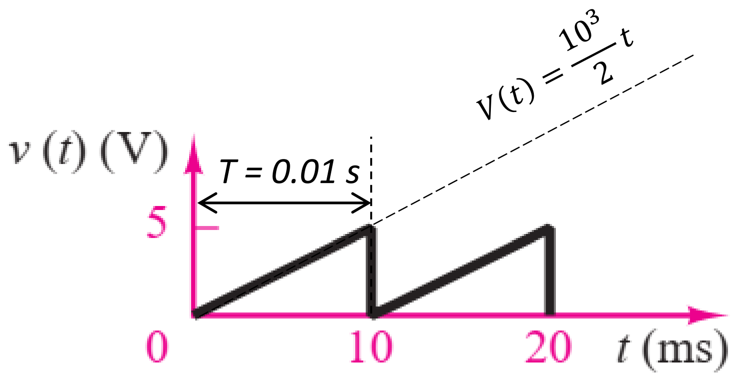

Answer: $$\left[\frac{1}{0.01}\cdot\int_0^{0.01}\left(\frac{10^3}{2}\cdot t\right)^2 dt\right]^{\frac{1}{2}} = 2.887 \quad V$$

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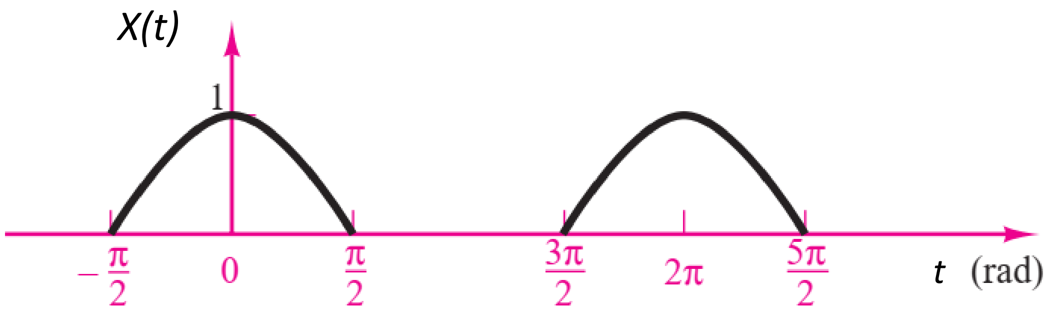
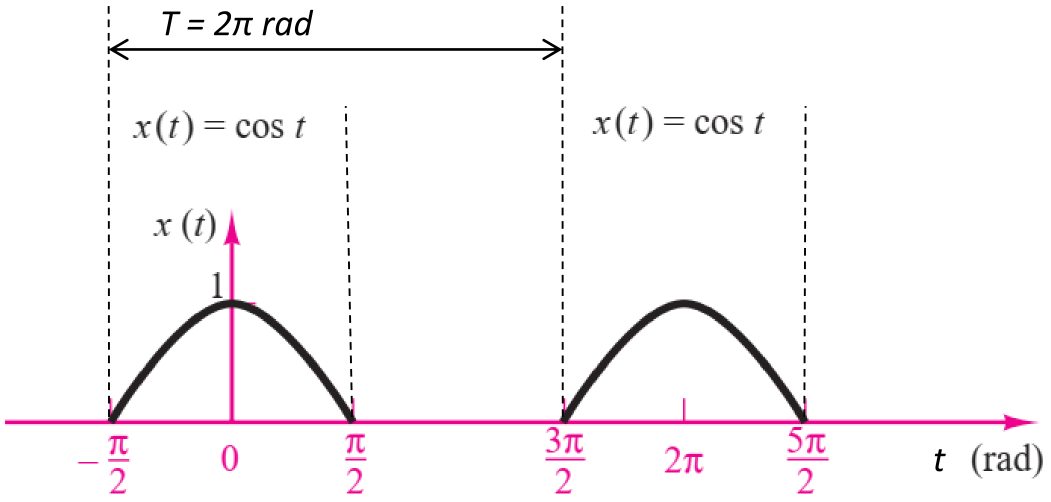
Find the rms value of the waves shown in the figure:



1.

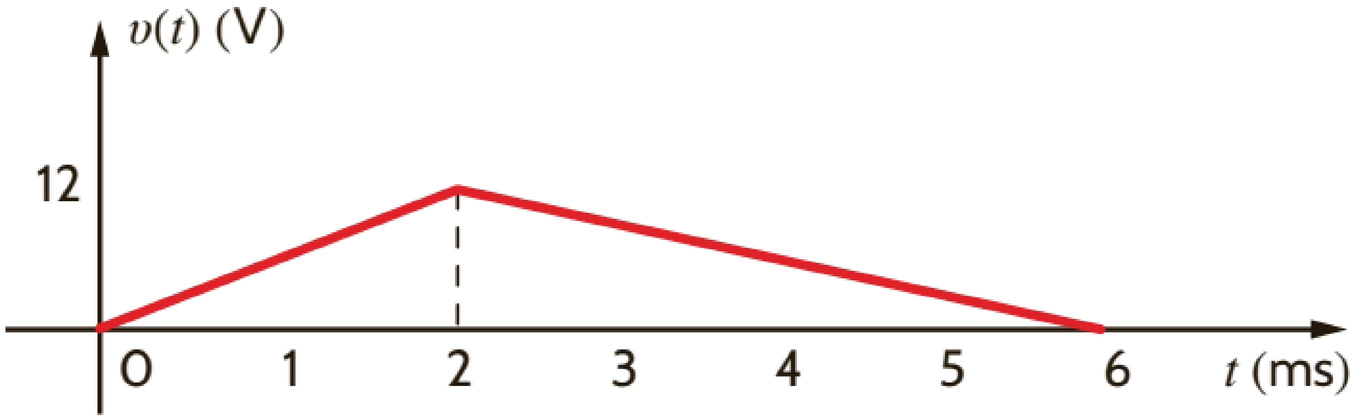
$$x_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t')\,dt'}$$ Root-mean-square value



Answer: $$\left(\frac{1}{2\cdot\pi}\cdot\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\cos(t)^2\,dt\right)^{\frac{1}{2}} = 0.5\ \ V$$

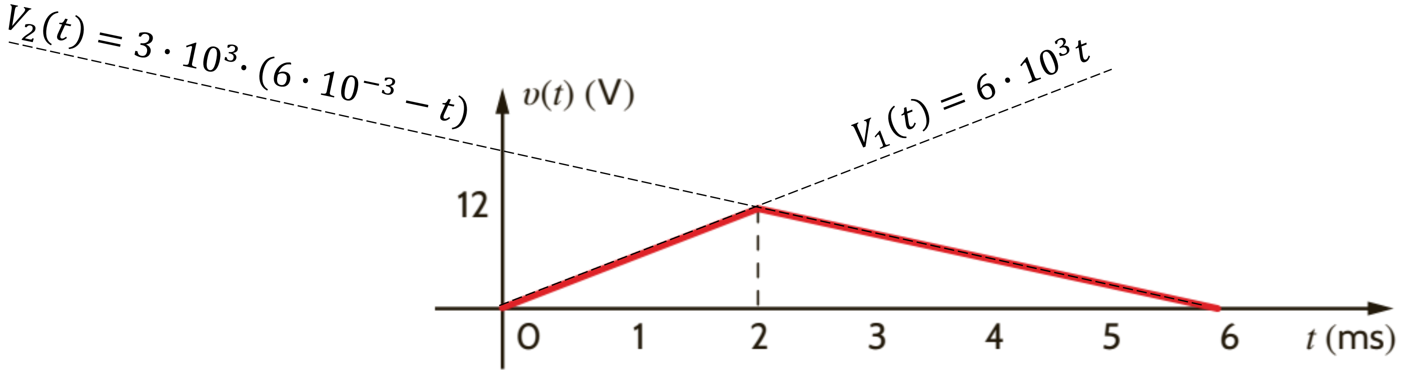
2.

The voltage across a 2-µF capacitor is shown in plot below. Determine the waveform for the capacitor current and compute the energy stored in the electric field of the capacitor at t = 2 ms.



2.

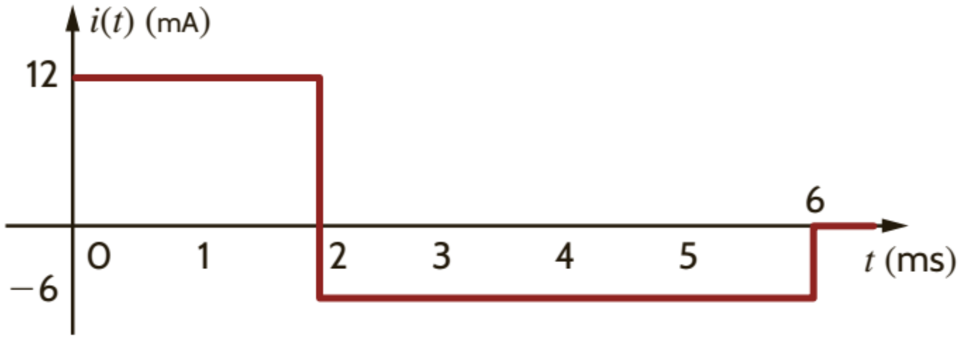
The voltage across a 2-μF capacitor is shown in plot below. Determine the waveform for the capacitor current and compute the energy stored in the electric field of the capacitor at t = 2 ms.



$$i(t) = C\frac{dV}{dt} \qquad \begin{cases} i_1(t) = 2 \cdot 10^{-6} \cdot 6 \cdot 10^3 \text{ A} \\ i_2(t) = -2 \cdot 10^{-6} \cdot 3 \cdot 10^3 \text{ A} \end{cases}$$

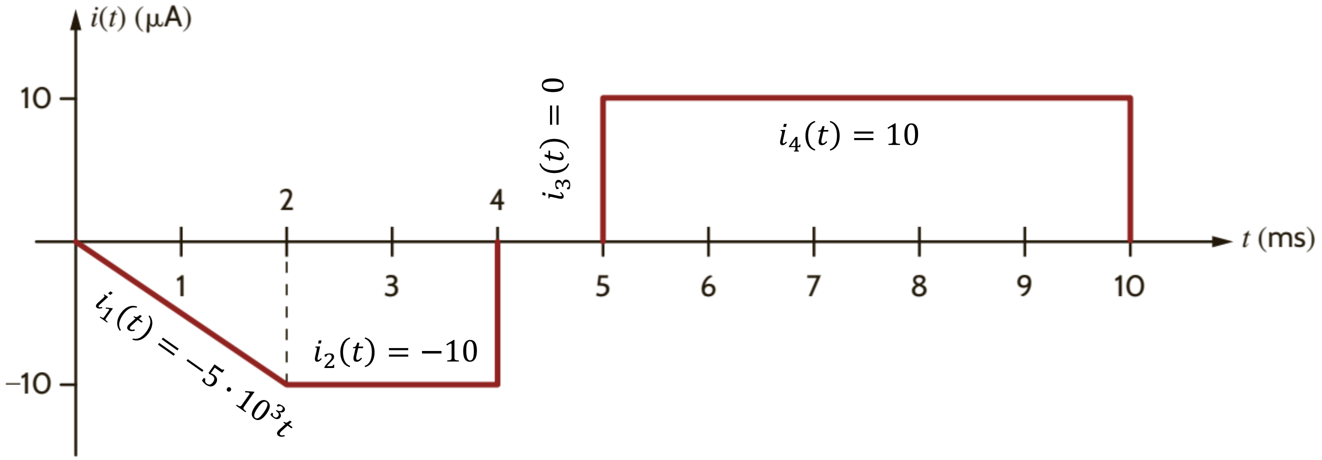
$$W(T) = \int_0^T I \cdot V dt = \int_0^T C \cdot \left(\frac{d}{dt}V\right) \cdot V dt = \int_0^T C \cdot V dV = \frac{C \cdot \left(V(T)^2 - V(0)^2\right)}{2} = \frac{C \cdot V(T)^2}{2} \;\Bigg|\; \longrightarrow W(T = 2\,ms) = \frac{2 \cdot 10^{-6} \cdot 12^2}{2}$$

Answer:

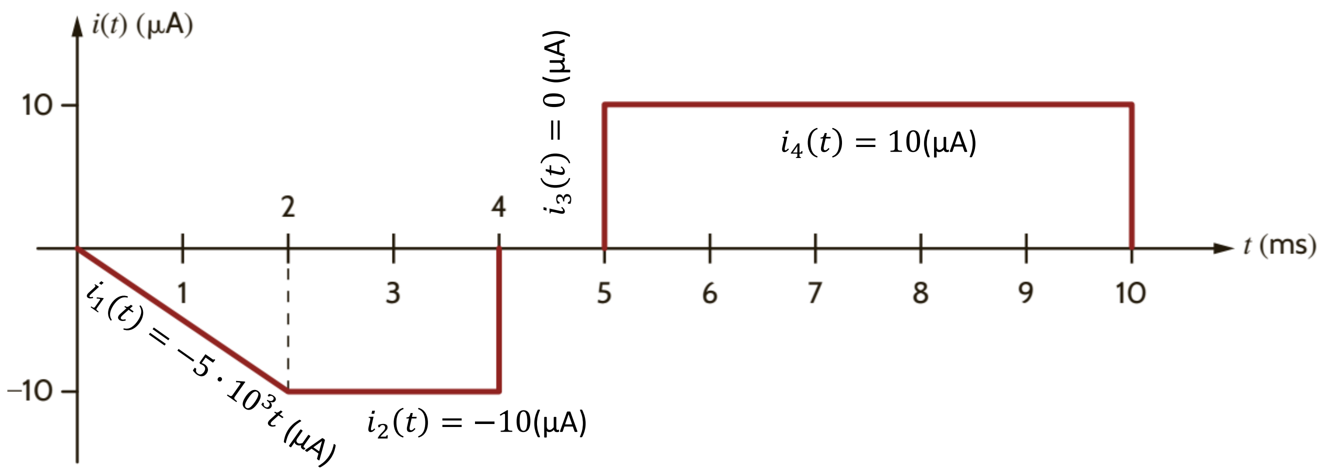


; $W(T = 2\,ms) = 144\ \mu J$

3. The waveform for the current in a 1-nF capacitor is presented below. If the capacitor has an initial voltage of -5 V, determine the waveform for the capacitor voltage. How much energy is stored in the capacitor at t = 6 ms?



3. The waveform for the current in a 1-nF capacitor is presented below. If the capacitor has an initial voltage of -5 V, determine the waveform for the capacitor voltage. How much energy is stored in the capacitor at t = 6 ms?



$$i(t) = C\frac{dV}{dt}$$

$$V(t) = \frac{1}{C}\int_{t_i}^{t} i(t')dt' + V_0$$

$$\begin{cases} V_1(t) = \dfrac{1}{1 \cdot 10^{-9}}\displaystyle\int_0^t i_1(t')dt' - 5 = -2.5 \cdot 10^6 \cdot t^2 - 5 \\ V_2(t) = \dfrac{-10^{-5}}{1 \cdot 10^{-9}}\displaystyle\int_{2\cdot 10^{-3}}^{t} dt' - 15 = 5 - 10^4 t \\ V_3(t) = V_2(t = 4\,ms) = -35\,V \\ V_4(t) = \dfrac{-10^{-5}}{1 \cdot 10^{-9}}\displaystyle\int_{5\cdot 10^{-3}}^{t} dt' - 35 = 10^4 t - 85 \end{cases}$$

$$V_1(t = 2\,ms) = -15\,V$$

$$V_2(t = 4\,ms) = -35\,V$$

$$V_4(t = 10\,ms) = 15\,V$$

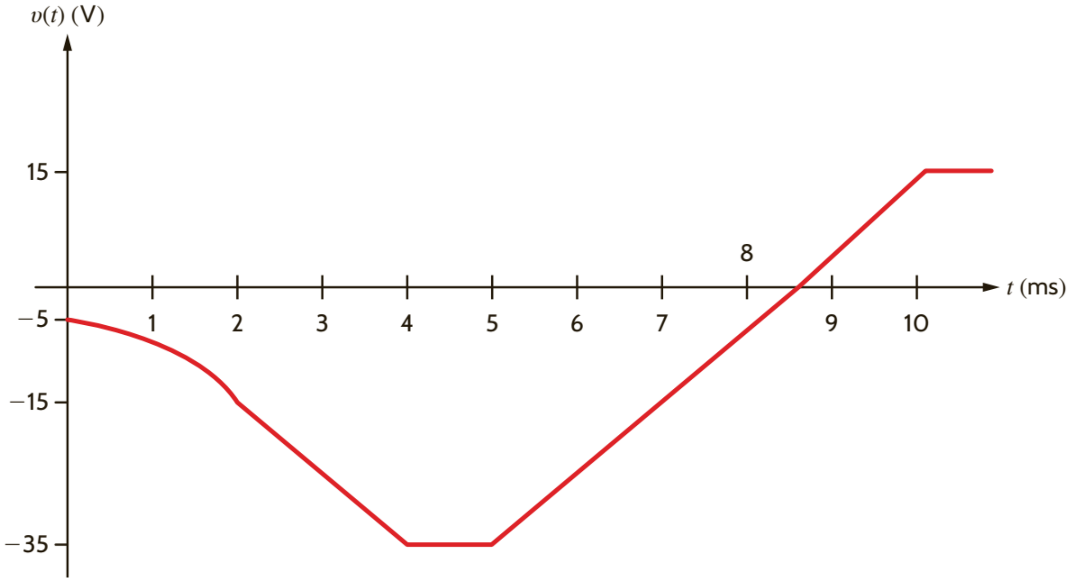
3. The waveform for the current in a 1-nF capacitor is presented below. If the capacitor has an initial voltage of -5 V, determine the waveform for the capacitor voltage. How much energy is stored in the capacitor at *t* = 6 ms?

$$V_1(t) = \frac{1}{1\cdot 10^{-9}}\int_0^t i_1(t')dt' - 5 = -2.5\cdot 10^6 \cdot t^2 - 5$$

$$V_2(t) = \frac{-10^{-5}}{1\cdot 10^{-9}}\int_{2\cdot 10^{-3}}^t i_2(t')dt' - 15 = 5 - 10^4 t$$

$$V_3(t) = V_2(t = 4\,ms) = -35\,V$$

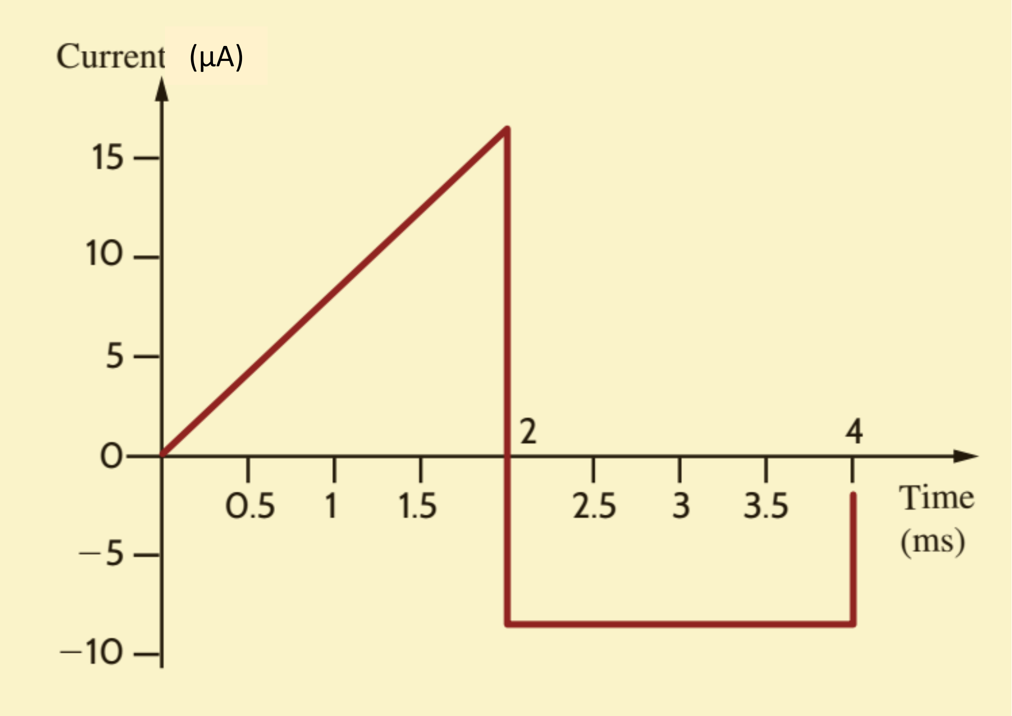
$$V_4(t) = \frac{-10^{-5}}{1\cdot 10^{-9}}\int_{5\cdot 10^{-3}}^t i_4(t')dt' - 35 = 10^4 t - 85$$



Answer: $$W(6\,ms) = \frac{CV_4(6\,ms)^2}{2} = 312.5\,nJ$$

4.

The current in an initially uncharged 4-µF capacitor is shown in figure below. Plot the waveforms for the voltage, power, and energy and compute the energy stored in the electric field of the capacitor at t = 2 ms.



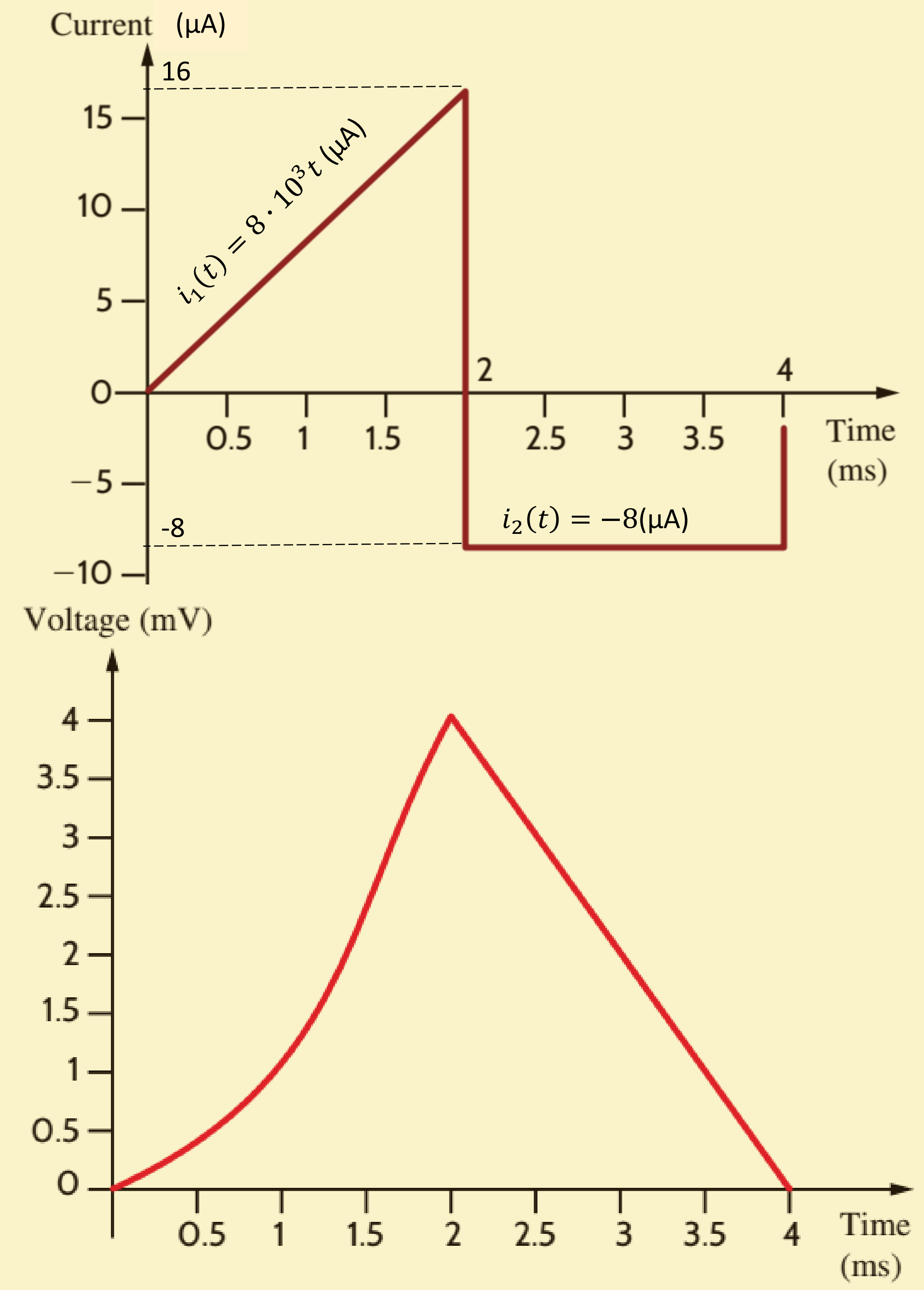
4.

Voltage

$$V(t) = \frac{1}{C}\int_{t_i}^{t} i(t')dt' + V_0$$

$$V_1(t) = \frac{1}{4\cdot 10^{-6}}\int_0^t i_1(t')dt' + 0 = 10^3 \cdot t^2 \qquad V_1(2\ ms) = 4\ mV$$

$$V_2(t) = \frac{1}{4\cdot 10^{-6}}\int_{2\ ms}^{t} i_2(t')dt' + V_1(2\ ms) = \frac{1}{125} - 2t \qquad V_2(4\ ms) = 0\ mV$$



4.

Voltage

$$V(t) = \frac{1}{C}\int_{t_i}^{t} i(t')dt' + V_0$$

$$V_1(t) = \frac{1}{4\cdot 10^{-6}}\int_0^t i_1(t')dt' + 0 = 10^3 \cdot t^2 \qquad V_1(2\,ms) = 4\,mV$$

$$V_2(t) = \frac{1}{4\cdot 10^{-6}}\int_{2\,ms}^t i_2(t')dt' + V_1(2\,ms) = \frac{1}{125} - 2t \qquad V_2(4\,ms) = 0\,mV$$

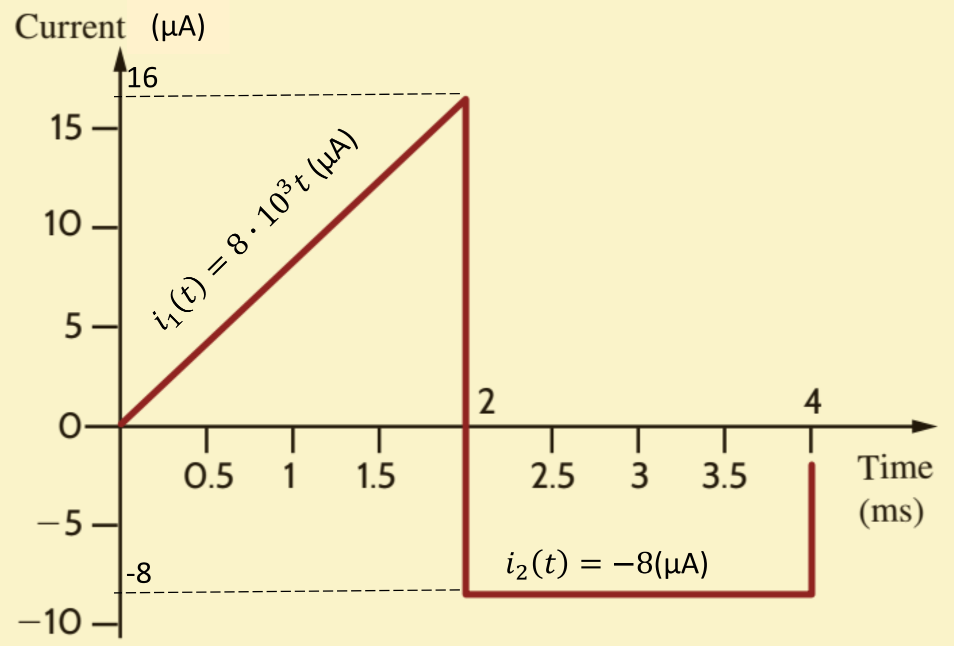
Power

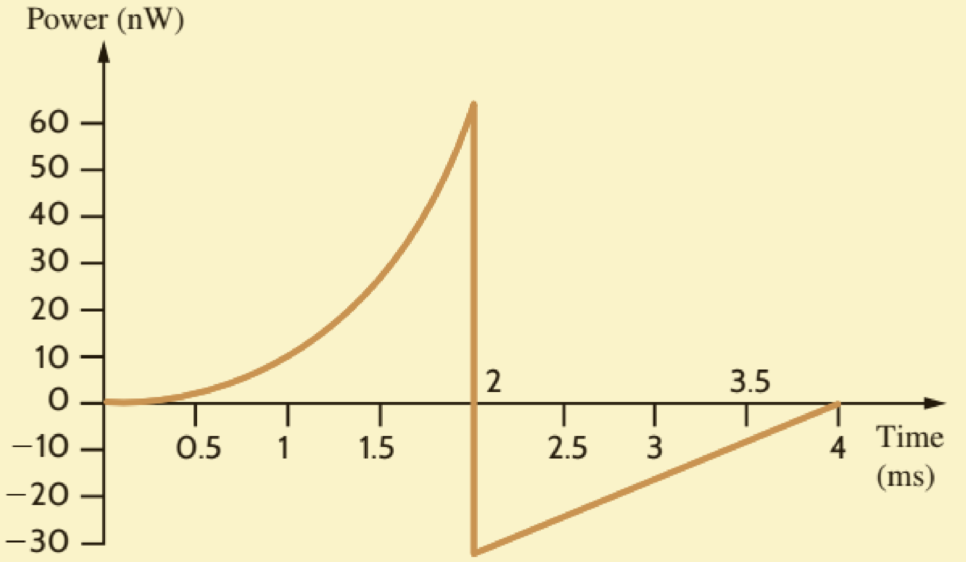
$$P(t) = V(t)i(t)$$

$$P_1(t) = V_1(t)i_1(t) = 8\cdot t^3 \qquad P_1(2\,ms) = 64\text{ nW}$$

$$P_2(t) = V_2(t)i_2(t) = -8\cdot 10^{-6}\cdot\left(\frac{1}{125} - 2t\right) \qquad P_2(2\,ms) = -32\text{ nW}$$

$$P_2(4\,ms) = 0\text{ nW}$$





4.

Voltage

$$V(t) = \frac{1}{C}\int_{t_i}^{t} i(t')dt' + V_0$$

$$V_1(t) = \frac{1}{4\cdot 10^{-6}}\int_0^t i_1(t')dt' + 0 = 10^3 \cdot t^2 \qquad V_1(2\,ms) = 4\,mV$$

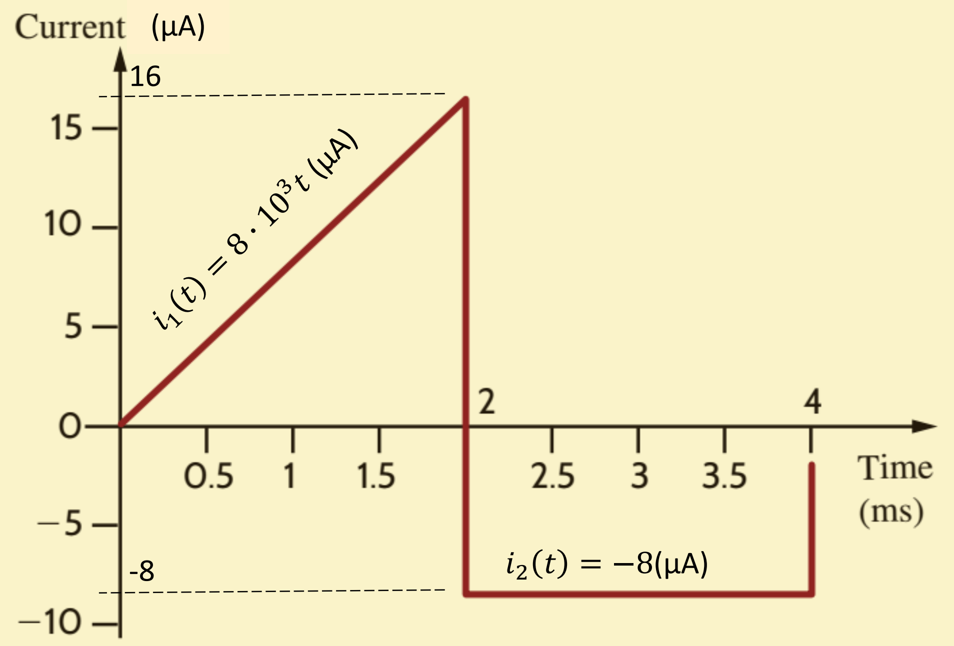
$$V_2(t) = \frac{1}{4\cdot 10^{-6}}\int_{2\,ms}^t i_2(t')dt' + V_1(2\,ms) = \frac{1}{125} - 2t \qquad V_2(4\,ms) = 0\,mV$$

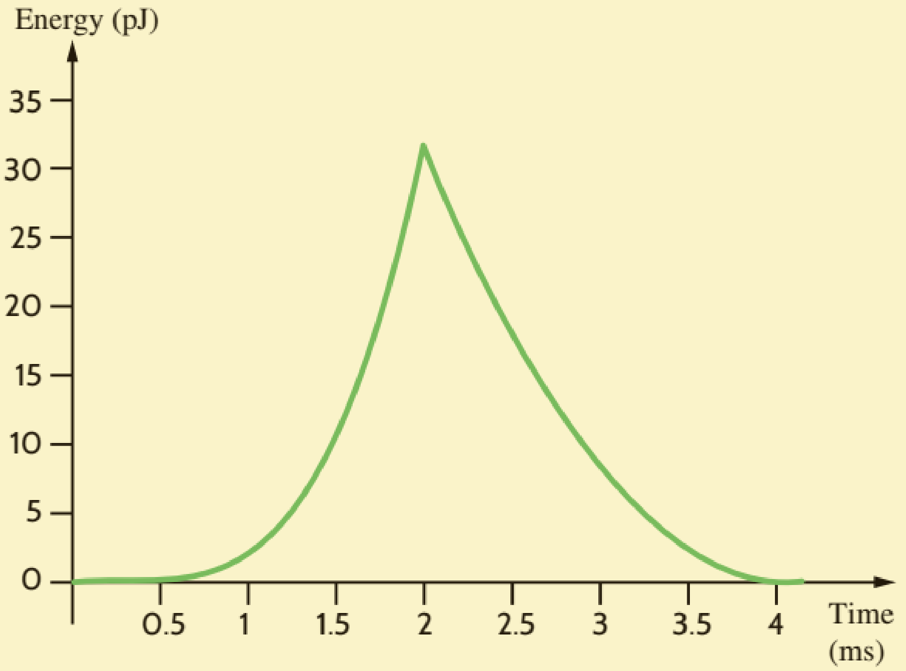
Energy

$$W(t) = \frac{CV(t)^2}{2}$$

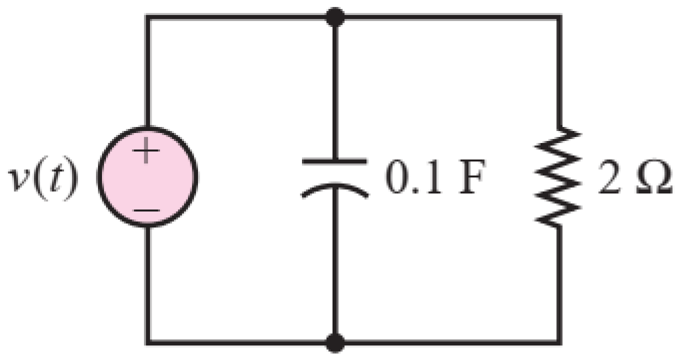
$$W_1(t) = \frac{CV_1(t)^2}{2} = 2\cdot t^4 \qquad W_1(2\,ms) = 32\,pJ$$

$$W_2(t) = \frac{CV_2(t)^2}{2} = 2\cdot 10^{-6}\left(\frac{1}{125} - 2t\right)^2 \qquad W_2(4\,ms) = 0\,pJ$$

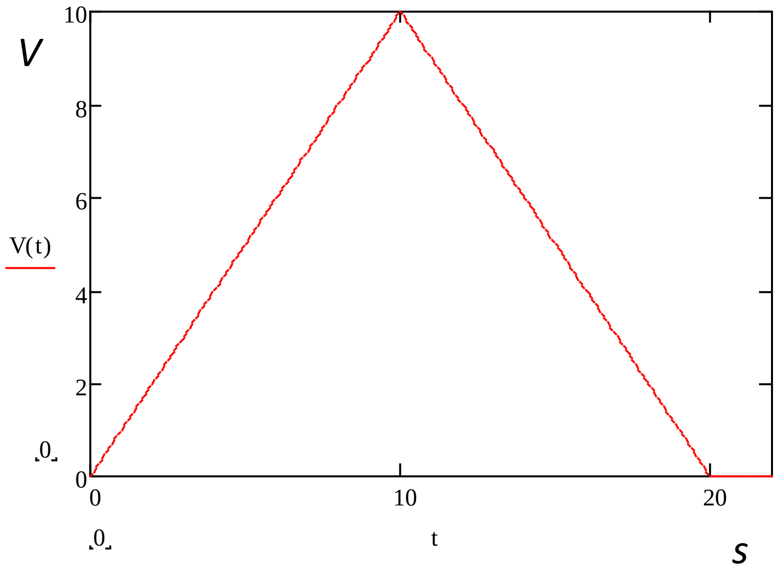




5.



$$v(t) = \begin{cases} 0 & \text{for } -\infty < t < 0 \\ t & \text{for } 0 \le t < 10 \text{ s} \\ 20 - t & \text{for } 10 \le t < 20 \text{ s} \\ 0 & \text{for } 20 \text{ s} \le t < \infty \end{cases}$$



Find

a. The energy stored in the capacitor for all time

b. The energy delivered by the source for all time

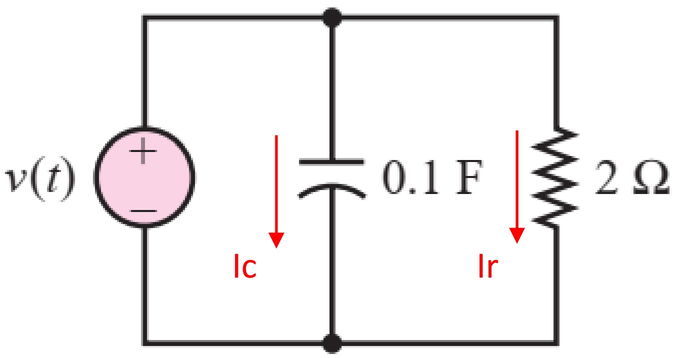
5.

Energy stored in the capacitor:

$$Wc(T) = \int_0^T Ic \cdot Vc\, dt = \int_0^T C \cdot \left(\frac{d}{dt} Vc\right) \cdot Vc\, dt = \int_0^T C \cdot Vc\, dVc = \frac{C \cdot \left(Vc(T)^2 - Vc(0)^2\right)}{2} = \frac{C \cdot Vc(T)^2}{2}$$

$$Ic = C \cdot \left(\frac{d}{dt} Vc\right)$$

$$Vc(t) = V(t)$$



Energy delivered by the source:

$$W(T) = \left[\int_0^T I \cdot V dt = \int_0^T (Ic + Ir) \cdot V dt = \frac{C \cdot V(T)^2}{2} + \int_0^T \frac{V(t)^2}{R} dt\right]$$

$$v(t) = \begin{cases} 0 & \text{for } -\infty < t < 0 \\ t & \text{for } 0 \le t < 10 \text{ s} \\ 20 - t & \text{for } 10 \le t < 20 \text{ s} \\ 0 & \text{for } 20 \text{ s} \le t < \infty \end{cases}$$