Physics 2. Electrical Engineering

Week 6 AC Networks. Capacitors

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Objectives

The main objectives of today's lecture are:

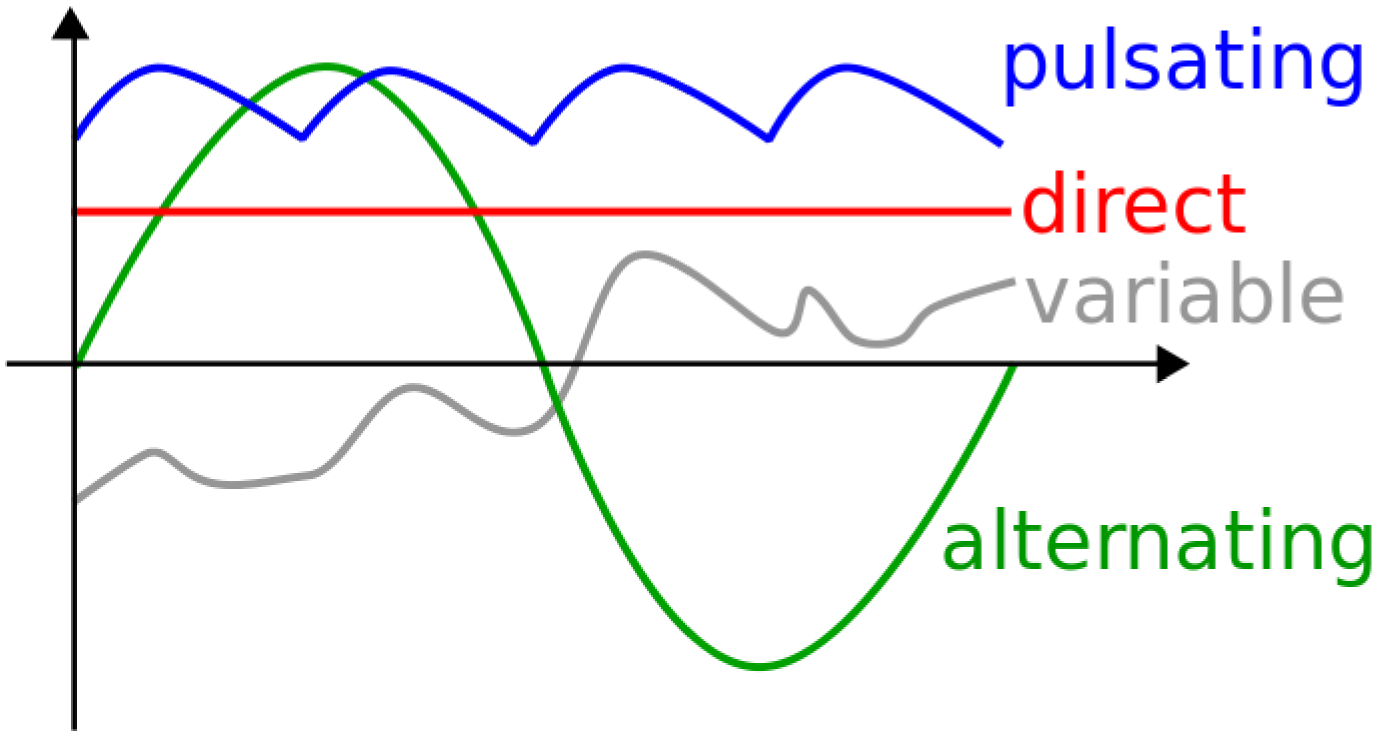
- Learn the fundamentals of alternating current
- Discuss electrical waveforms and their analysis
- Study the operation of capacitors

AC Electricity

AC Electricity (1)

In alternating current (AC or ac) the movement of electric charge periodically reverses direction.

The usual waveform of an AC power circuit is a sine wave.



AC Electricity (2)

But why would one need AC electricity altogether?

- AC voltages can be easily transformed to higher or lower voltage levels, which is difficult to do with DC voltages.
- AC electricity also allows for the use of such electric elements as capacitor and inductor. These elements play significant role in a majority of electric and electronic devices.

Image credit: Electricaldm

AC Electricity : Power Losses

Recall that an electrical power transmitted from one point to another is equal to the product the current and the voltage:

$$P_T = IV$$

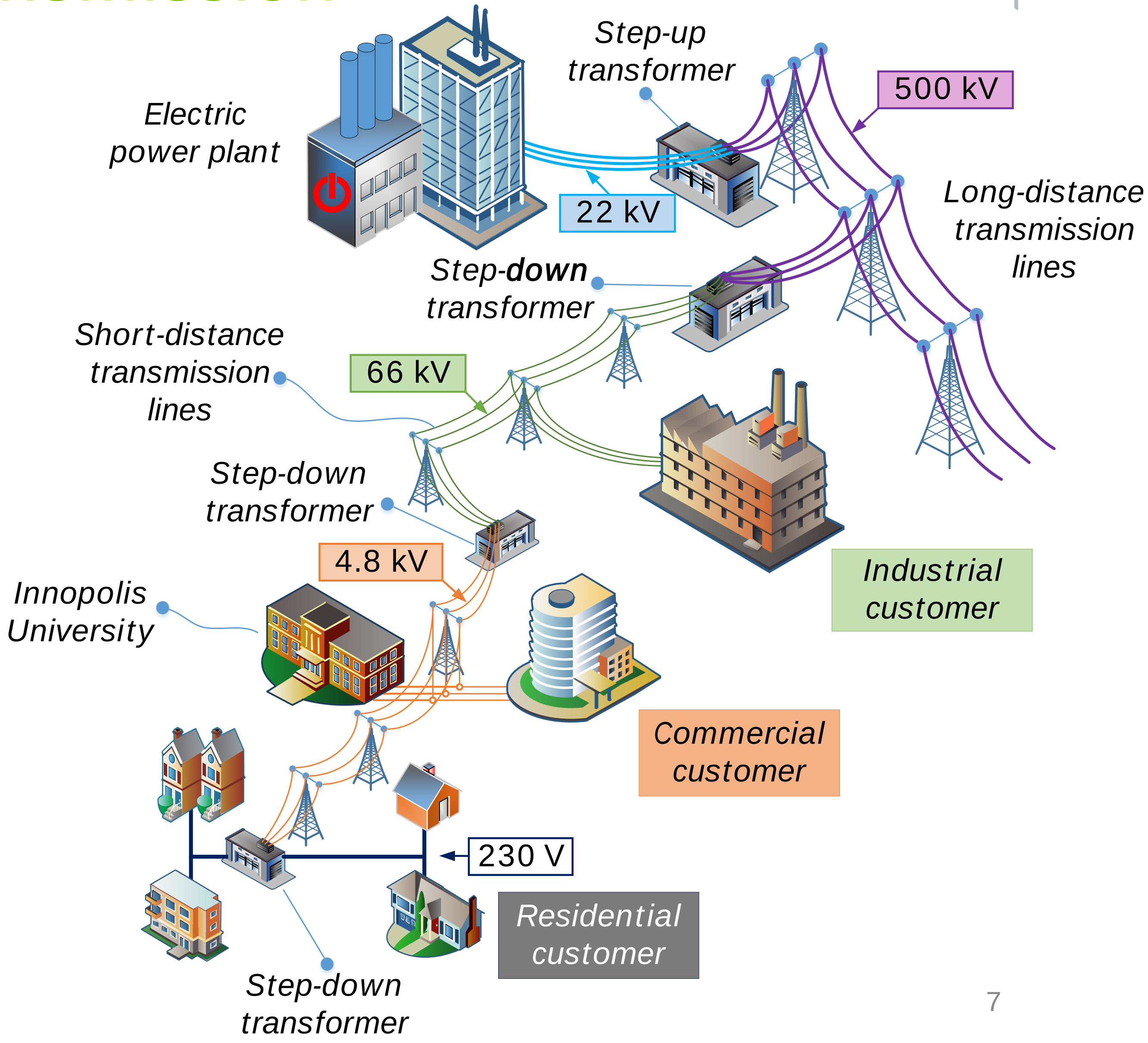
- Thus, the same amount of power can be transmitted with a lower current by increasing the voltage.
- On the other hand, the power losses in a conductor (due to wire heating) are given by

$$P_L = I^2 R$$

It is therefore advantageous when transmitting large amounts of power to distribute the power with high voltages (often hundreds of kilovolts).

AC Electricity : Power Transmission

- Long-distance transmission lines: connect cities and towns to power
- Short-distance transmission lines: distribute power throughout cities and towns
- Industrial customer: AC is used to drive motors, presses, machinery, etc.
- Commercial customer: AC is converted to DC internally for by copiers, computers, etc.
- Residential customer: AC is used for heating, cooling, cooking, and lighting while appliances internally convert AC to DC



AC Electricity: Power Generation&Consumption

Typical power plant outputs (MW):

- Wind turbine: 2
- Geothermal: 50
- Gas-fired: 500
- Coal: 500
- Nuclear: 1,500
- Hydroelectric: 1,500
- Solar panels: 0.0003 (300 W*)

Typical power consumption:

- Industrial customer: 100+ kWh
- Commercial customer: 20 W/m^2.
- Residential customer: 10 kWh/day





Image credits: Wikipedia

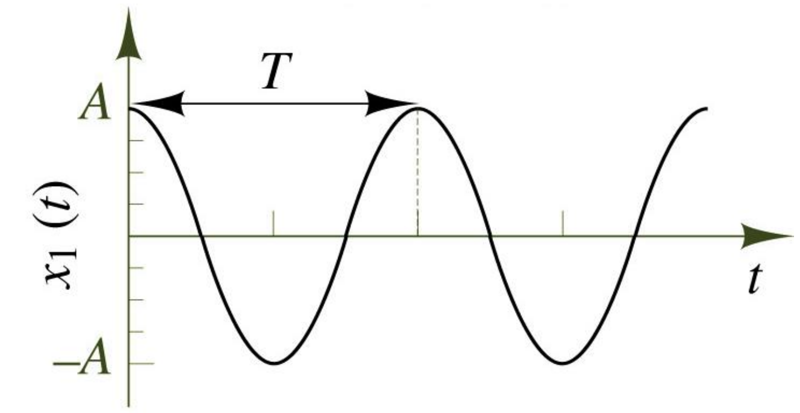
AC Current: Waveform

An arbitrary sine AC waveform can be described as follows:

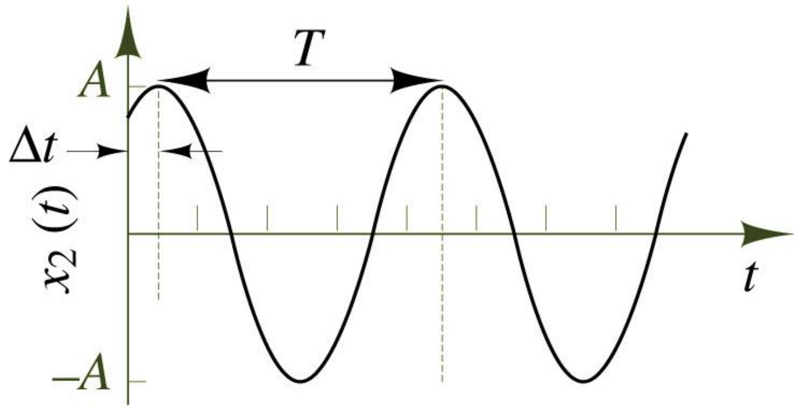
$$x(t) = A\cos(wt + \varphi)$$

The parameters denote:

- T – period (s)
- A – amplitude
- w – radial frequency (rad/s), $w = 2\pi f$
- f – natural frequency (Hz), $f = 1/T$
- φ – phase (rad), $\varphi = 2\pi \frac{\Delta t}{T}$



Reference cosine



Arbitrary sinusoid

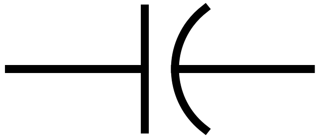
Capacitors

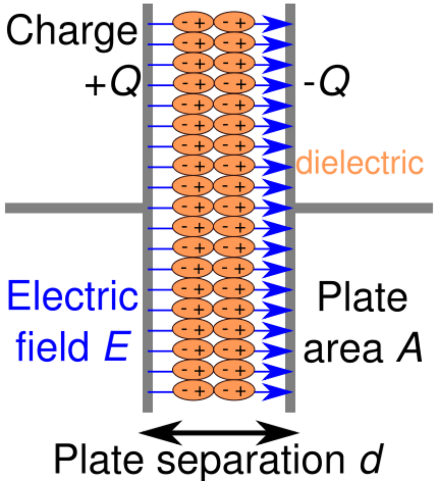
Capacitors (1)

A physical capacitor is a device that can store energy in the form of a charge separation when appropriately polarized by an electric field.

- A capacitor consists of two conductors separated by a non-conductive region.
- The non-conductive region is called the dielectric.

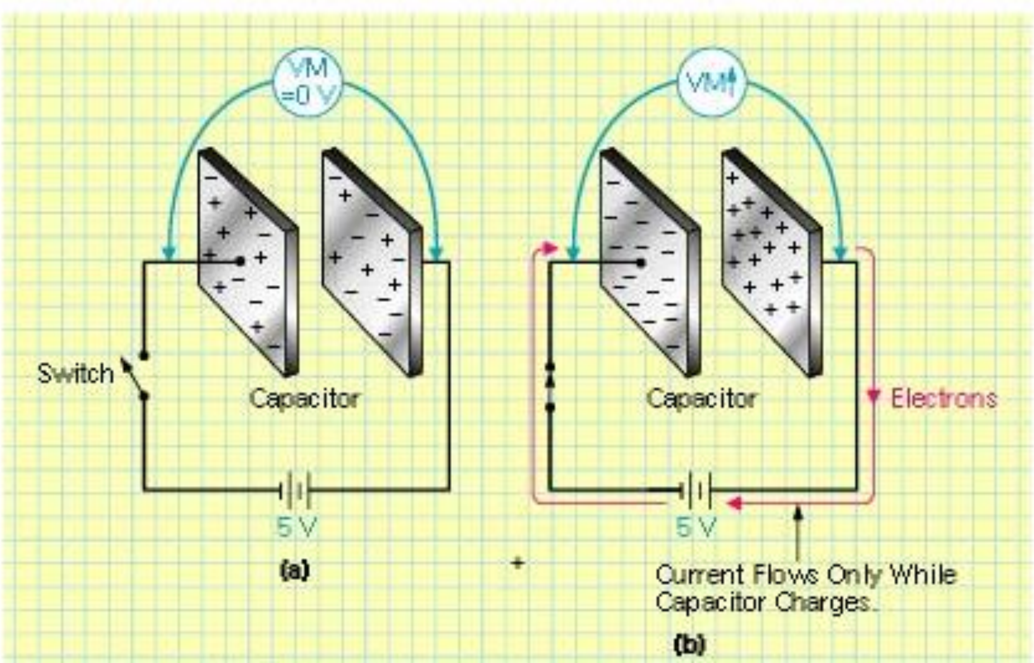
Examples of dielectrics are glass, air, paper, and many others.

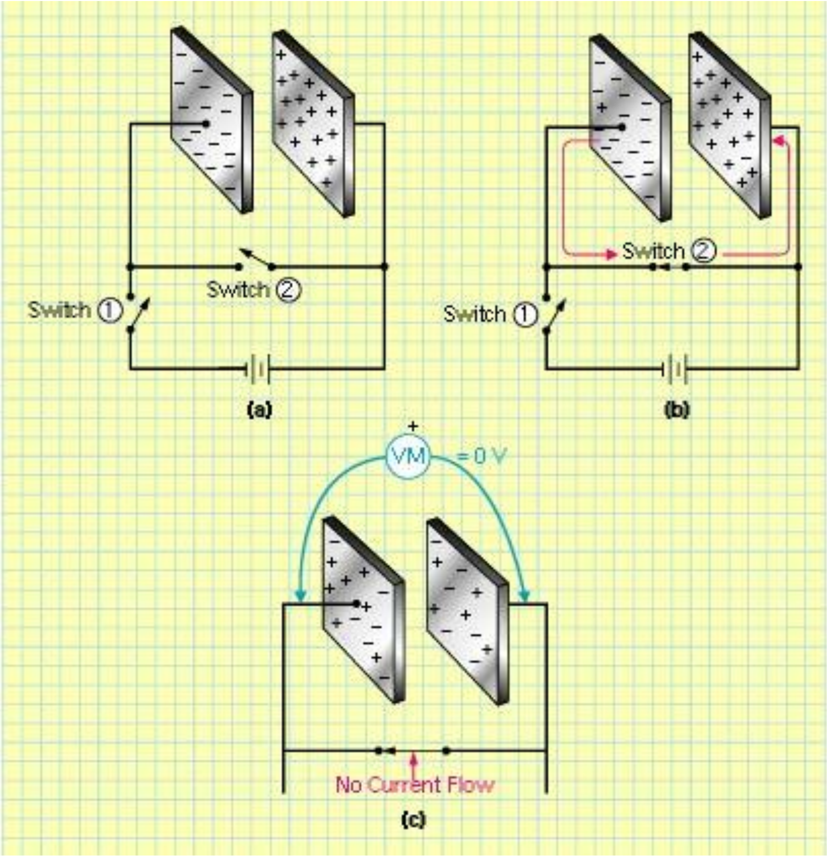
Circuit symbol: 



Capacitors (2)

The conductors hold equal and opposite charges on their facing surfaces, and the dielectric develops an electric field.





Capacitance

Capacitance (unit: Farad) is the ratio of the amount of electric charge stored on a conductor to a difference in electric potential:

$$C = \frac{q}{V}$$

- Capacitance of a parallel-plate capacitor can be also found as

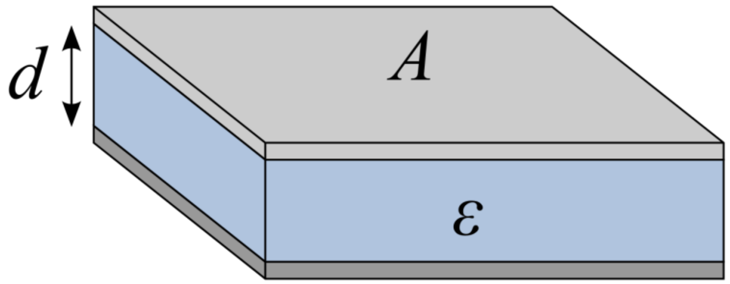
$$C = \varepsilon \frac{A}{d}$$

C – capacitance (Farads)

ε – permittivity

A – area of overlap of the two plates (m^2)

d – separation between the plates (m)



Capacitor Operation (1)

If the external voltage applied to the capacitor plates changes in time, so will the charge that is internally stored by the capacitor:

$$q(t) = C \cdot v(t)$$

- Recall that the electric current is

$$i(t) = \dot{q}(t) = \frac{dq(t)}{dt}$$

- Hence

$$i(t) = C\frac{dv(t)}{dt}$$

Capacitor Operation (2)

Similarly, one can obtain the following relationship for the voltage across a capacitor:

$$i(t) = C\frac{dv(t)}{dt} \Rightarrow v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C\,(t')dt'$$

- It is useful to define the initial voltage for the capacitor as:

$$V_0 = \frac{1}{C}\int_{-\infty}^{t_0} i_C\,(t')dt'$$

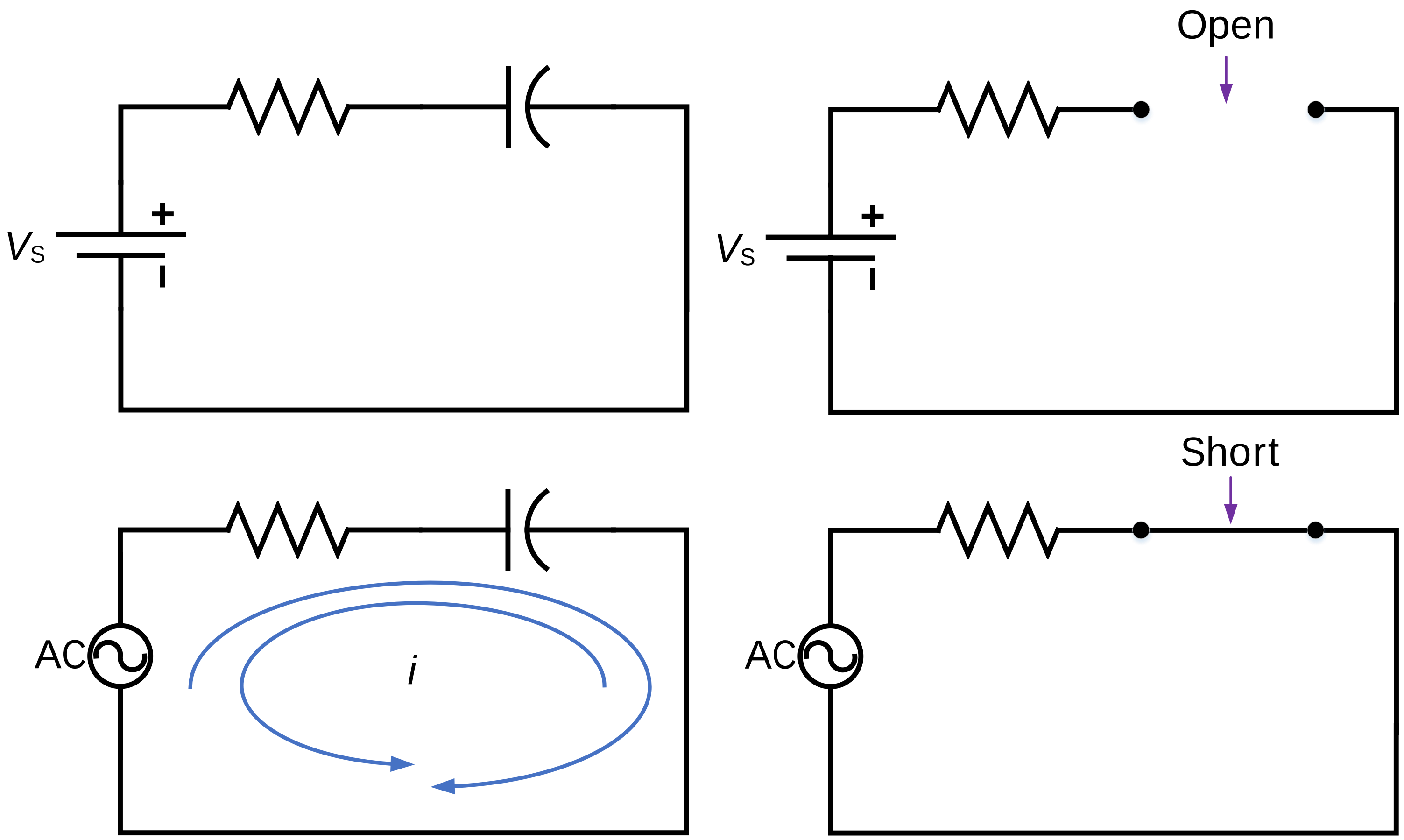
- The capacitor voltage is now given by the expression

$$v_C(t) = \frac{1}{C}\int_{t_0}^{t} i_C\,(t')\,dt' + V_0$$

Capacitors in Circuits

The capacitor is

- an open circuit to DC voltage and
- a short circuit for AC voltage



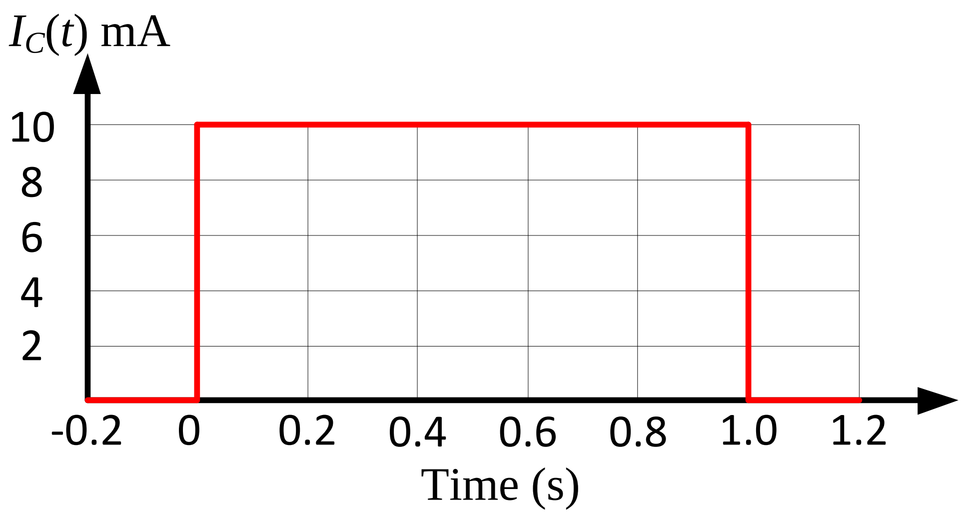
Capacitor Voltage and Current: Exercise

Calculate the voltage across a capacitor from knowledge of its current and initial state of charge.

$$i_C(t) = \begin{cases} 0 & t < 0\text{s} \\ 10\text{mA} & 0 \le t \le 1\text{s} \\ 0 & t > 1\text{s} \end{cases}$$

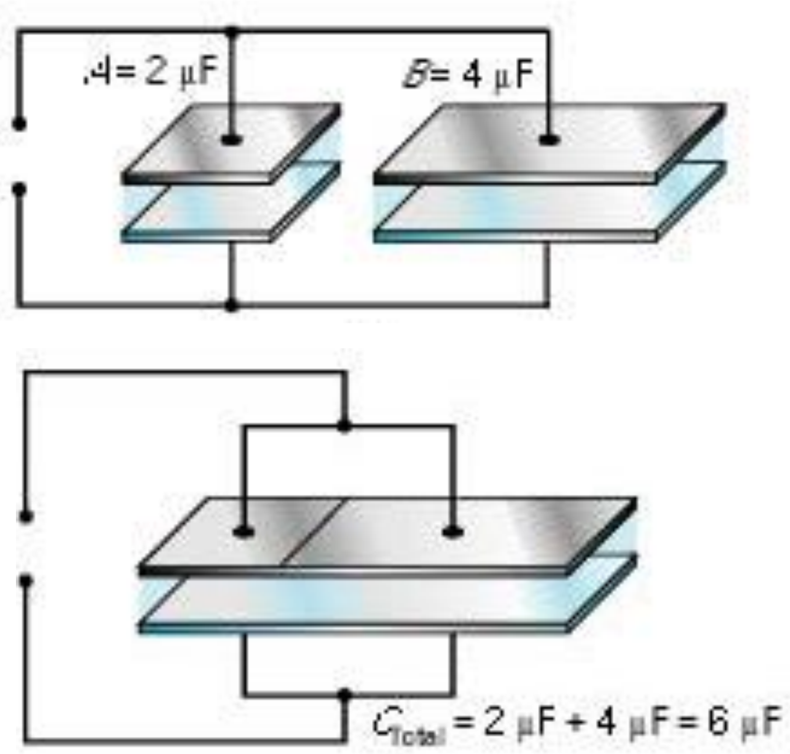
$V_0 = V_C(t = 0) = 2$ V, $C = 1000$ μF.

- Recall that for capacitor, $v_C(t) = \dfrac{1}{C}\displaystyle\int_{t_0}^{t} i_C\,(t')\;dt' + V_0$

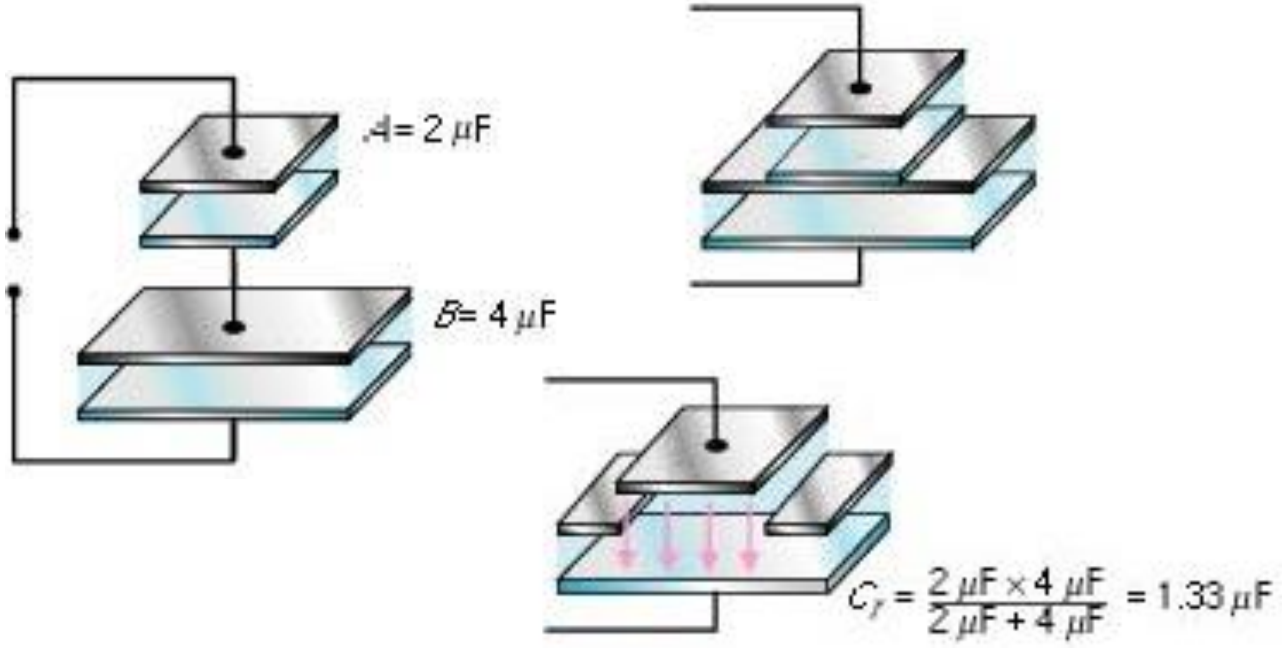


Capacitor Connections

Capacitors in parallel add.



Capacitors in series combine according to the same rules used for resistors connected in parallel



Thank you for your attention!

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