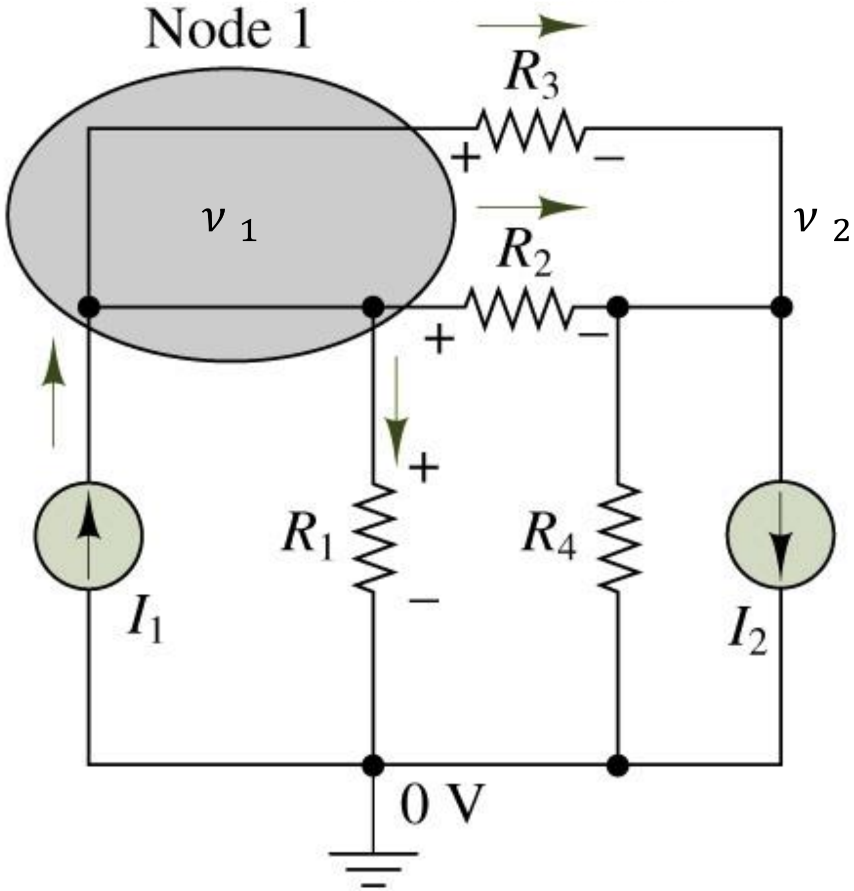
Problem 1 : NVM

The circuit with following quantities is given:

$$I_1 = 10mA,\ I_2 = 50mA$$
$$R_1 = 1\,k\Omega,\quad R_2 = 2\,k\Omega,$$
$$R_3 = 10\,k\Omega,\quad R_4 = 2\,k\Omega,$$



Find the voltages v_1 and v_2.

Problem 1 : NVM

$$I_1 - \frac{v_1 - 0}{R_1} - \frac{v_1 - v_2}{R_2} - \frac{v_1 - v_2}{R_3} = 0 \quad \text{node 1}$$

$$\frac{v_1 - v_2}{R_2} + \frac{v_1 - v_2}{R_3} - \frac{v_2 - 0}{R_4} - I_2 = 0 \quad \text{node 2}$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)v_1 + \left(-\frac{1}{R_2} - \frac{1}{R_3}\right)v_2 = I_1$$

$$\left(-\frac{1}{R_2} - \frac{1}{R_3}\right)v_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)v_2 = -I_2$$

Solving this system of equations, we obtain

$$v_1 = -13.57 \text{ V}$$

$$v_2 = -52.86 \text{ V}$$

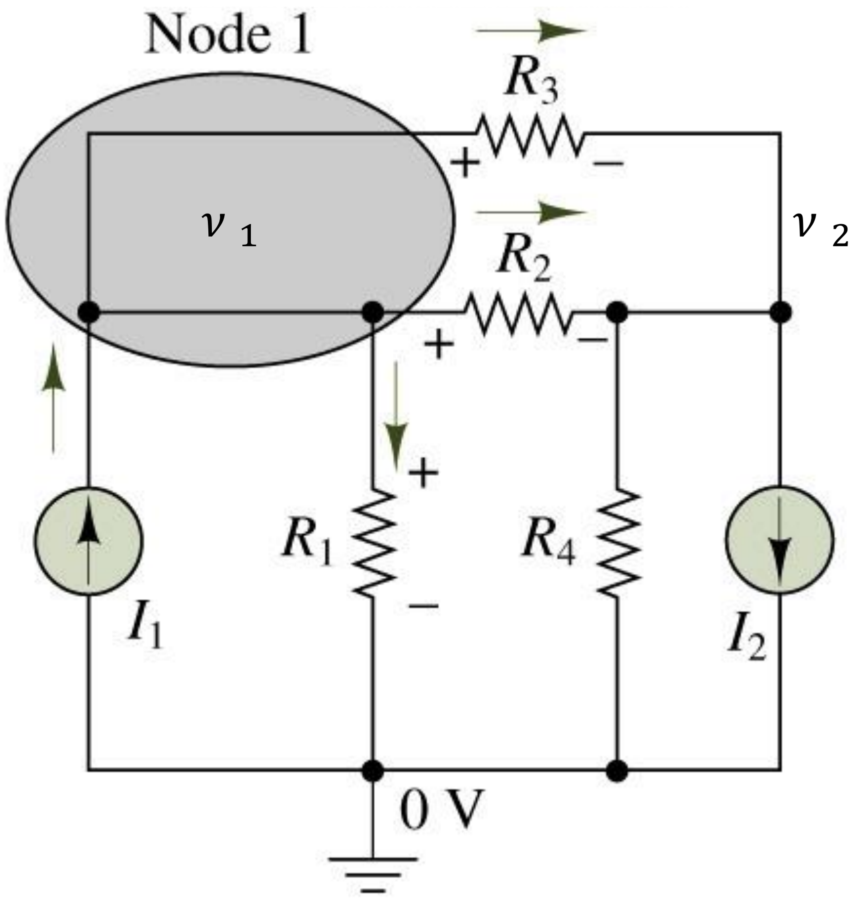
$$i_{R3} = \frac{v_1 - v_2}{10{,}000} = 3.93 \text{ mA}$$

$$i_{R1} = \frac{v_1}{1{,}000} = -13.57 \text{ mA}$$

$I_1 = 10mA, \; I_2 = 50mA$

$R_1 = 1\,k\Omega, \quad R_2 = 2\,k\Omega,$

$R_3 = 10\,k\Omega, \quad R_4 = 2\,k\Omega,$



Problem 2: NVM

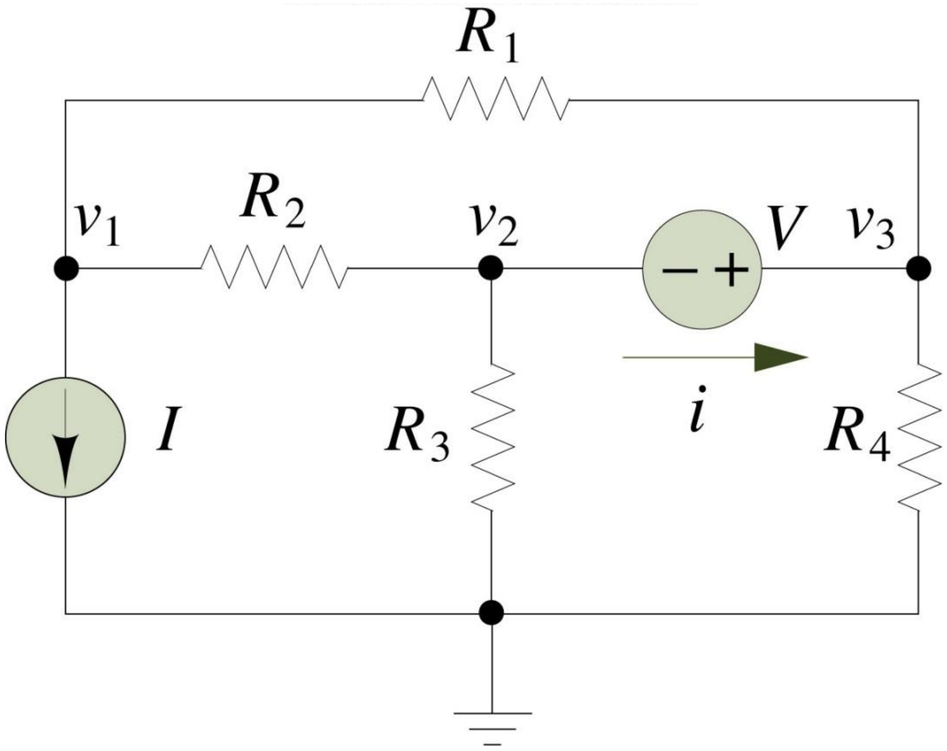
The circuit with following quantities is given:

$$R_1 = R_2 = 2\,\Omega,$$

$$R_3 = 4\,\Omega,\ R_4 = 3\,\Omega$$

$$I = 2\text{ A},\ V = 3\text{ V}$$

Find the current i.



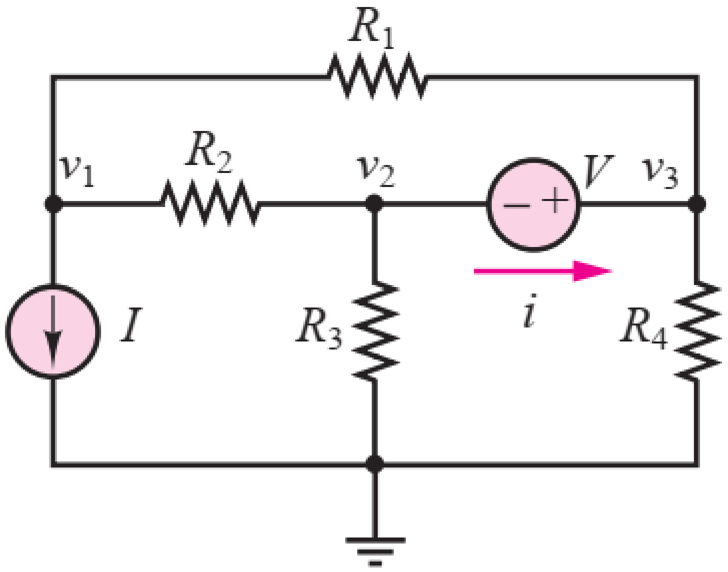
Problem 2: NVM

INNOPOLIS UNIVERSITY

System of equations:

$$\begin{cases} \dfrac{v_3 - v_1}{R_1} + \dfrac{v_2 - v_1}{R_2} - I = 0 \qquad \text{node 1} \\ \dfrac{v_1 - v_2}{R_2} - \dfrac{v_2}{R_3} - i = 0 \qquad \text{node 2} \\ i = \dfrac{v_3 - v_1}{R_1} + \dfrac{v_3}{R_4} \\ v_3 = v_2 + 3\text{ V} \end{cases}$$

$R_1 = R_2 = 2\,\Omega,$
$R_3 = 4\,\Omega,\ R_4 = 3\,\Omega$
$I = 2\text{ A},\ V = 3\text{ V}$



solution:

$$v_1 = -5.64\text{ V}$$

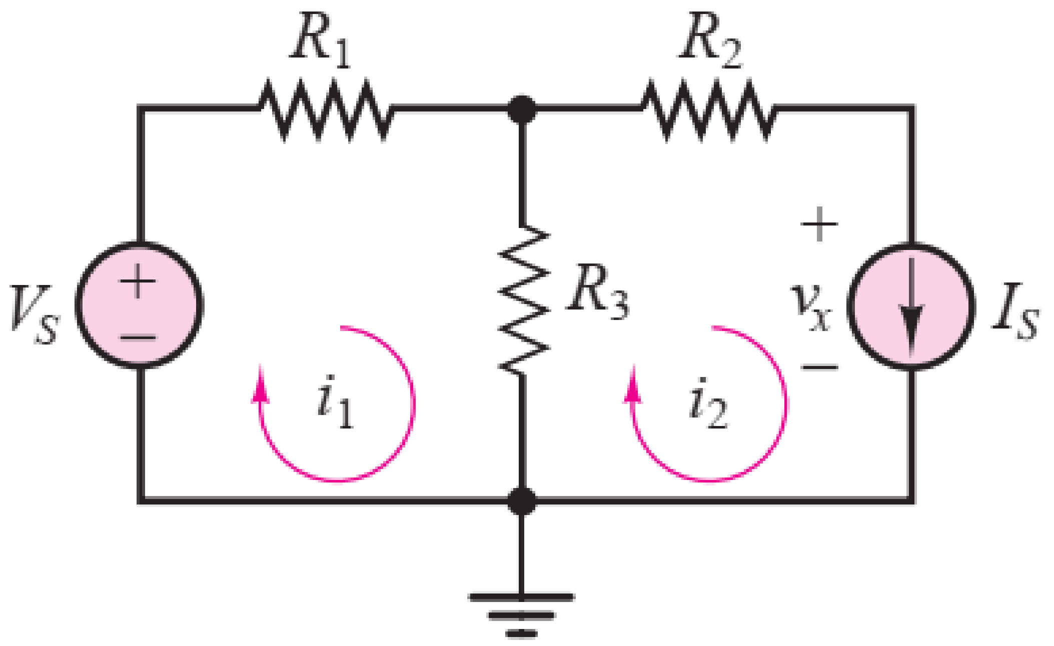
$$v_2 = -5.14\text{ V}$$

$$v_3 = v_2 + 3\text{ V} = -2.14\text{ V}$$

Answer: $i = \dfrac{v_3 - v_1}{R_1} + \dfrac{v_3}{R_4} = \dfrac{-2.14 + 5.64}{2} + \dfrac{-2.14}{3} = 1.04\text{ A}$

Problem 3: MCM

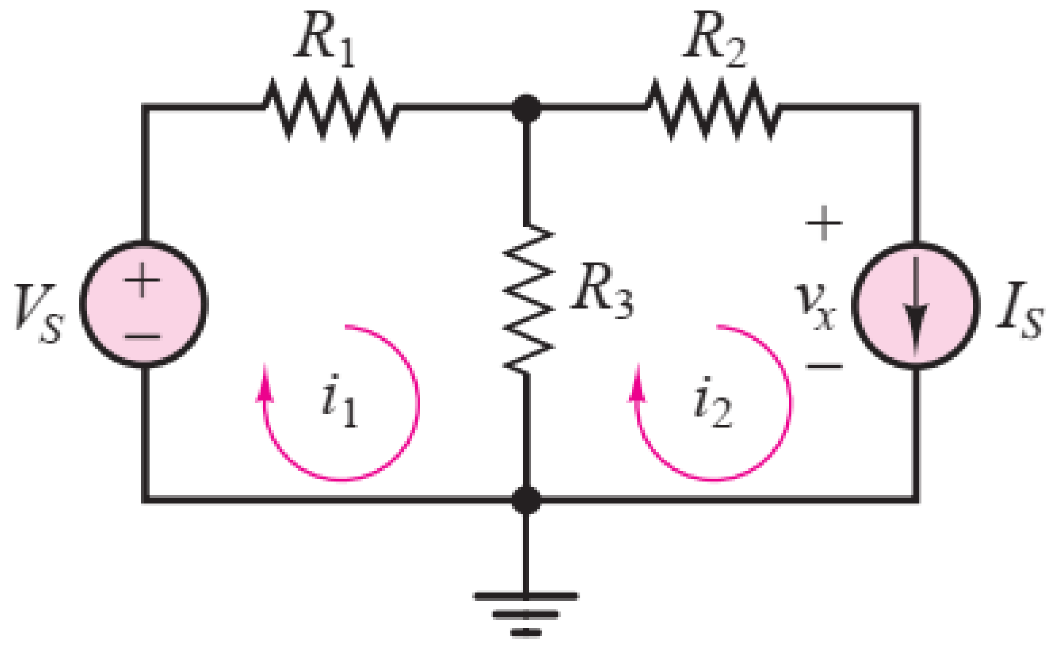
Find unknown current i_1 in the circuit



$V_S = 10$ V; $I_S = 2$ A; $R_1 = 5\ \Omega$; $R_2 = 2\ \Omega$; and $R_3 = 4\ \Omega$.

Problem 3: MCM

Find unknown current i_1 in the circuit



$$i_2 = I_S$$

Thus, the unknown voltage, v_x, can be obtained applying KVL to mesh 2:

$$(i_1 - i_2)R_3 - i_2 R_2 - v_x = 0$$

$$v_x = (i_1 - i_2)R_3 - i_2 R_2 = i_1 R_3 - i_2\,(R_2 + R_3)$$

To find the current i_1 we apply KVL to mesh 1:

$$V_S - i_1 R_1 - (i_1 - i_2)\,R_3 = 0$$

$$V_S + i_2 R_3 = i_1\,(R_1 + R_3)$$

but since $i_2 = I_S$

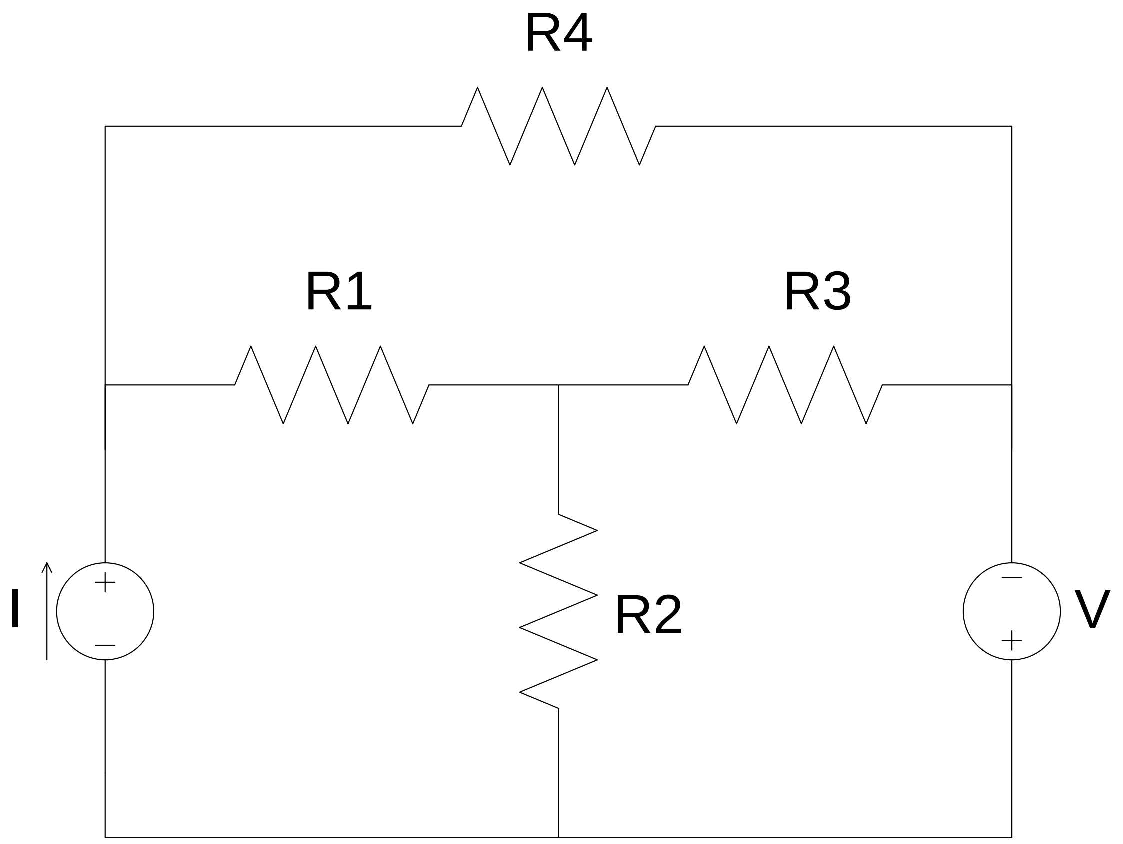
$$i_1 = \frac{V_S + I_S R_3}{(R_1 + R_3)} = \frac{10 + 2 \times 4}{5 + 4} = 2\text{ A}$$

Answer: $i_1 = 2\ A$

$V_S = 10$ V; $I_S = 2$ A; $R_1 = 5\ \Omega$; $R_2 = 2\ \Omega$; and $R_3 = 4\ \Omega$.

Problem 4: MCM

Find the mesh currents



$$I = 0.5\ \mathrm{A};\ V = 6\ \mathrm{V};\ R_1 = 3\ \Omega;\ R_2 = 8\ \Omega;\ R_3 = 6\ \Omega;\ R_4 = 4\ \Omega.$$

Problem 4: MCM

$I = 0.5\ \text{A};\ V = 6\ \text{V};\ R_1 = 3\ \Omega;$

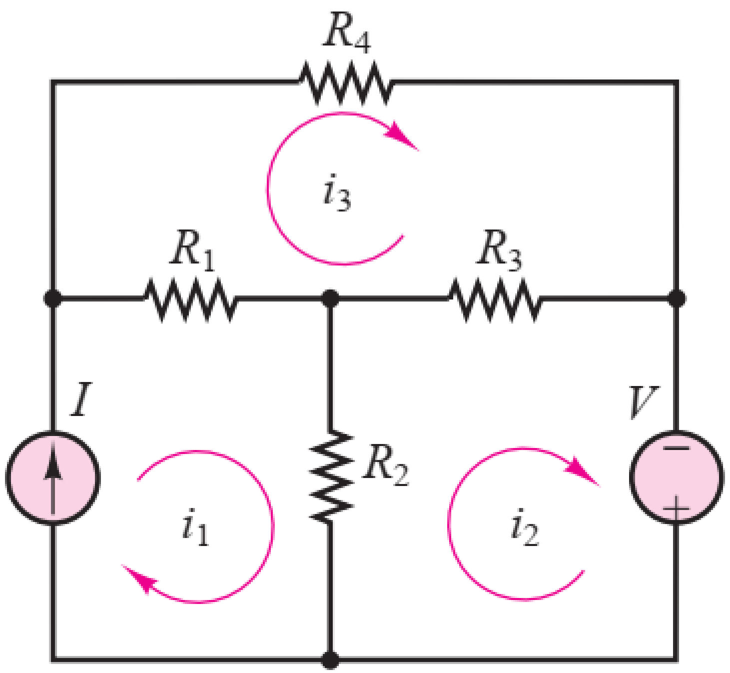
$R_2 = 8\ \Omega;\ R_3 = 6\ \Omega;\ R_4 = 4\ \Omega.$

$$i_1 = I$$

$$-R_2(i_2 - i_1) - R_3(i_2 - i_3) + V = 0 \qquad \text{mesh 2}$$
$$-R_1(i_3 - i_1) - R_4 i_3 - R_3(i_3 - i_2) = 0 \qquad \text{mesh 3}$$

or

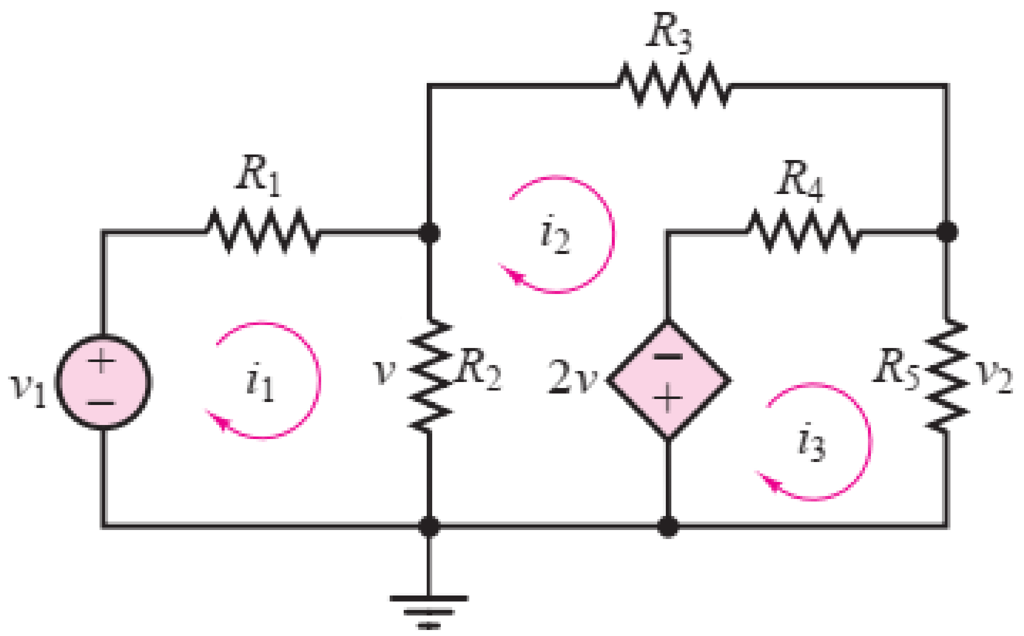
$$14i_2 - 6i_3 = 10$$
$$-6i_2 + 13i_3 = 1.5$$



Answer: $i_2 = 0.95\ \text{A}$ $\quad i_3 = 0.55\ \text{A}$

Problem 5: MCM with Dependent Source

Find the voltage "gain" $A_v = {}^{v_2}/_{v_1}$ if the voltages v and v_2 determined as $v = R_2(i_1 - i_2)$ and $v_2 = R_5 i_3$



$R_1 = 1\ \Omega$; $R_2 = 0.5\ \Omega$; $R_3 = 0.25\ \Omega$; $R_4 = 0.25\ \Omega$; $R_5 = 0.25\ \Omega$.

Problem 5: MCM with Dependent Source

$v = R_2(i_1 - i_2)$, and $v_2 = R_5 i_3$

For mesh 1:

$$v_1 - R_1 i_1 - R_2(i_1 - i_2) = 0$$

or rearranging the equation gives

$$(R_1 + R_2)i_1 + (-R_2)i_2 + (0)i_3 = v_1$$

For mesh 2:

$$v - R_3 i_2 - R_4(i_2 - i_3) + 2v = 0$$

Rearranging the equation and substituting the expression $v = -R_2(i_2 - i_1)$, we obtain

$$-R_2(i_2 - i_1) - R_3 i_2 - R_4(i_2 - i_3) - 2R_2(i_2 - i_1) = 0$$

$$(-3R_2)i_1 + (3R_2 + R_3 + R_4)i_2 - (R_4)i_3 = 0$$

For mesh 3:

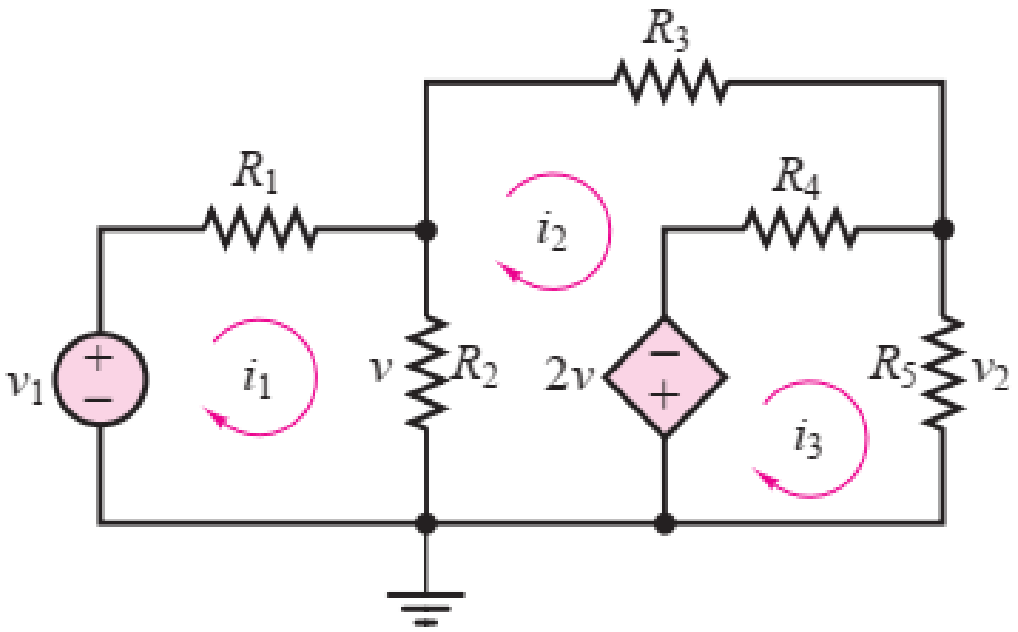
$$-2v - R_4(i_3 - i_2) - R_5 i_3 = 0$$

substituting the expression for $v = R_2(i_1 - i_2)$ and rearranging, we obtain

$$-2R_2(i_1 - i_2) - R_4(i_3 - i_2) - R_5 i_3 = 0$$

$$2R_2 i_1 - (2R_2 + R_4)i_2 + (R_4 + R_5)i_3 = 0$$

$R_1 = 1\ \Omega$; $R_2 = 0.5\ \Omega$; $R_3 = 0.25\ \Omega$;



Problem 5: MCM with Dependent Source

$$R_1 = 1\ \Omega;\ R_2 = 0.5\ \Omega;\ R_3 = 0.25\ \Omega;$$
$$R_4 = 0.25\ \Omega;\ R_5 = 0.25\ \Omega.$$

Physics:

$v = R_2(i_1 - i_2)$, and $v_2 = R_5 i_3$

For mesh 1:

$$v_1 - R_1 i_1 - R_2(i_1 - i_2) = 0$$

or rearranging the equation gives

$$(R_1 + R_2)i_1 + (-R_2)i_2 + (0)i_3 = v_1$$

For mesh 2:

$$v - R_3 i_2 - R_4(i_2 - i_3) + 2v = 0$$

Rearranging the equation and substituting the expression $v = -R_2(i_2 - i_1)$, we obtain

$$-R_2(i_2 - i_1) - R_3 i_2 - R_4(i_2 - i_3) - 2R_2(i_2 - i_1) = 0$$
$$(-3R_2)i_1 + (3R_2 + R_3 + R_4)i_2 - (R_4)i_3 = 0$$

For mesh 3:

$$-2v - R_4(i_3 - i_2) - R_5 i_3 = 0$$

substituting the expression for $v = R_2(i_1 - i_2)$ and rearranging, we obtain

$$-2R_2(i_1 - i_2) - R_4(i_3 - i_2) - R_5 i_3 = 0$$
$$2R_2 i_1 - (2R_2 + R_4)i_2 + (R_4 + R_5)i_3 = 0$$

Mathematics:

$$\begin{bmatrix} (R_1 + R_2) & (-R_2) & 0 \\ (-3R_2) & (3R_2 + R_3 + R_4) & (-R_4) \\ (2R_2) & -(2R_2 + R_4) & (R_4 + R_5) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1.5 & -0.5 & 0 \\ -1.5 & 2 & -0.25 \\ 1 & -1.25 & 0.5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ 0 \\ 0 \end{bmatrix}$$

$$[R][i] = [v] \qquad [i] = [R]^{-1}[v]$$

$$[R]^{-1} = \begin{bmatrix} 0.88 & 0.32 & 0.16 \\ 0.64 & 0.96 & 0.48 \\ -0.16 & 1.76 & 2.88 \end{bmatrix}$$

$$i_1 = 0.88v_1$$
$$i_2 = 0.64v_1$$
$$i_3 = -0.16v_1$$

$$v_2 = R_5 i_3 = R_5(-0.16v_1) = 0.25(-0.16v_1)$$

$$A_v = \frac{v_2}{v_1} = \frac{-0.04v_1}{v_1} = -0.04$$ /Answer