1.

Add the sinusoidal voltages $v_1(t) = A\cos(\omega t + \phi)$ and $v_2(t) = B\cos(\omega t + \theta)$ using phasor notation, and then convert back to time-domain form.

$$A = 1.5 \text{ V}, \phi = 10°; B = 3.2 \text{ V}, \theta = 25°.$$

1.

Add the sinusoidal voltages $v_1(t) = A\cos(\omega t + \phi)$ and $v_2(t) = B\cos(\omega t + \theta)$ using phasor notation, and then convert back to time-domain form.

$$A = 1.5\text{ V},\ \phi = 10^\circ;\ B = 3.2\text{ V},\ \theta = 25^\circ.$$

$$v_1(t) \rightarrow 1.5\big(cos(10^0) + j \cdot sin(10^0)\big) = 1.48 + j \cdot 0.26$$

$$v_2(t) \rightarrow 3.2\big(cos(25^0) + j \cdot sin(25^0)\big) = 2.9 + j \cdot 1.35$$

$$v_1(t) + v_2(t) \rightarrow 4.38 + j \cdot 1.61 = 4.66\angle 20.21^0$$

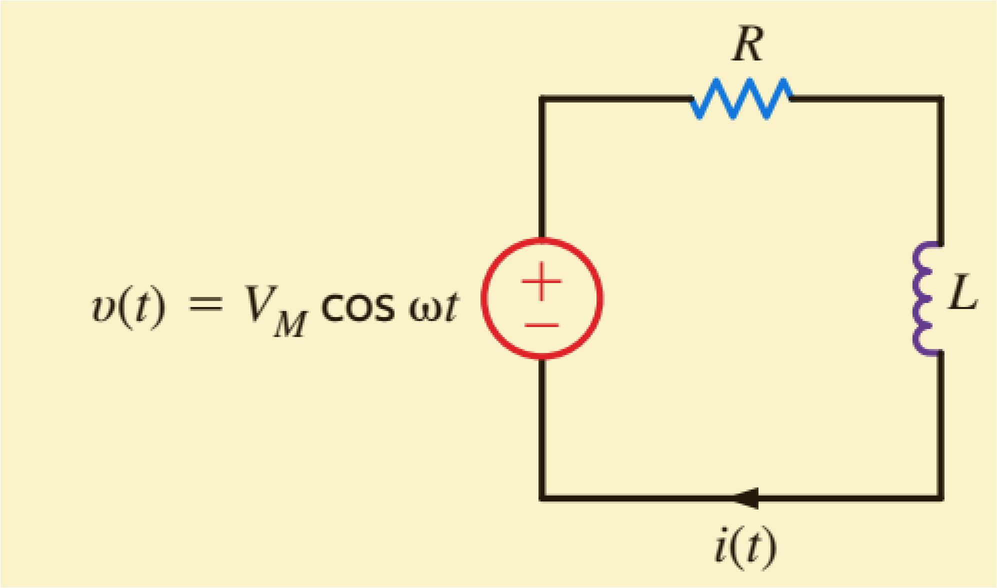
Answer: $v_1(t) + v_2(t) = 4.66 \cdot \cos(\omega t + 20.21^0)$

$$\sqrt{4.38^2 + 1.61^2} = 4.66$$

$$\text{atan}\left(\frac{1.612}{4.377}\right) \cdot \frac{180}{\pi} = 20.21^{0}$$

2.

Derive the expression for the current.



2.

Derive the expression for the current.

$$L\frac{di(t)}{dt} + Ri(t) = V_M \cos \omega t$$ /KVL

$$i(t) = A \cos(\omega t + \phi)$$ /we assume this form of current function

$$i(t) = A\cos\phi \cos\omega t - A\sin\phi\sin\omega t$$
$$= A_1 \cos\omega t + A_2 \sin\omega t$$

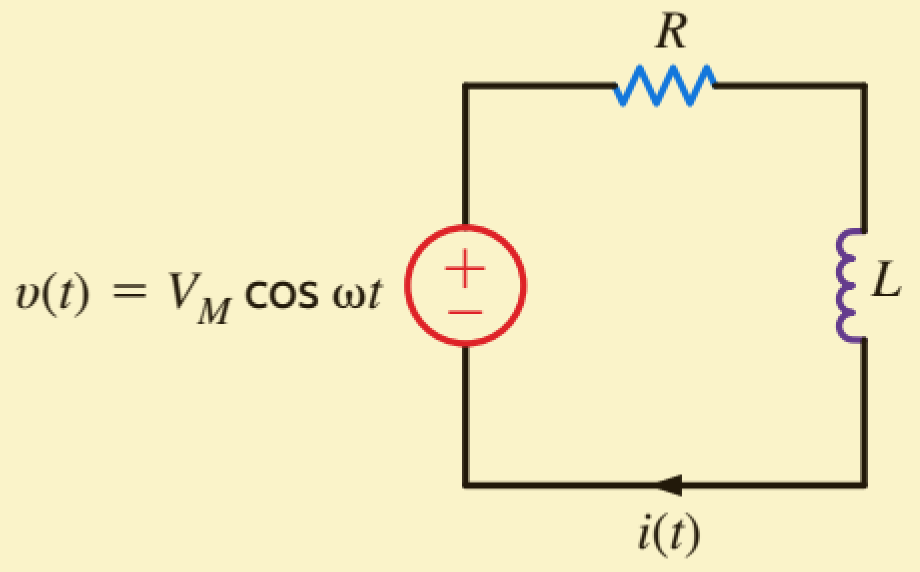
$$L\frac{d}{dt}(A_1 \cos\omega t + A_2 \sin\omega t) + R(A_1\cos\omega t + A_2\sin\omega t) = V_M\cos\omega t$$

$$-A_1\omega L\sin\omega t + A_2\omega L\cos\omega t + RA_1\cos\omega t + RA_2\sin\omega t = V_M\cos\omega t$$

$$-A_1\omega L + A_2 R = 0$$ /coefficients of sine function

$$A_1 R + A_2\omega L = V_M$$ /coefficients of cosine function

$$A_1 = \frac{RV_M}{R^2 + \omega^2 L^2}$$
$$A_2 = \frac{\omega L V_M}{R^2 + \omega^2 L^2}$$
/solution for A_1 and A_2



Derived expression for current

$$i(t) = \frac{RV_M}{R^2 + \omega^2 L^2}\cos\omega t + \frac{\omega L V_M}{R^2 + \omega^2 L^2}\sin\omega t$$

Derive the expression for the current.

Derived expression for current

$$i(t) = \frac{RV_M}{R^2 + \omega^2 L^2} \cos \omega t + \frac{\omega L V_M}{R^2 + \omega^2 L^2} \sin \omega t \quad \Leftrightarrow \quad i(t) = A \cos(\omega t + \phi)$$

$$A \cos \phi = \frac{RV_M}{R^2 + \omega^2 L^2}$$

$$A \sin \phi = \frac{-\omega L V_M}{R^2 + \omega^2 L^2}$$

$$\tan \phi = \frac{A \sin \phi}{A \cos \phi} = -\frac{\omega L}{R}$$

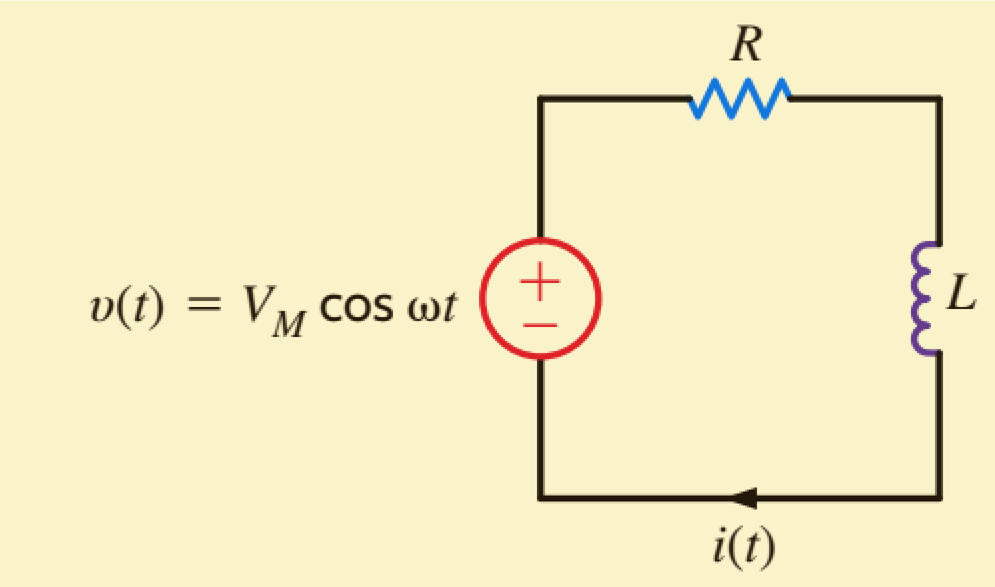
$$\phi = -\tan^{-1} \frac{\omega L}{R}$$

$$(A \cos \phi)^2 + (A \sin \phi)^2 = A^2(\cos^2\phi + \sin^2\phi) = A^2$$

$$A^2 = \frac{R^2 V^2_M}{(R^2 + \omega^2 L^2)^2} + \frac{(\omega L)^2 V^2_M}{(R^2 + \omega^2 L^2)^2}$$

$$= \frac{V^2_M}{R^2 + \omega^2 L^2}$$

$$A = \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}}$$

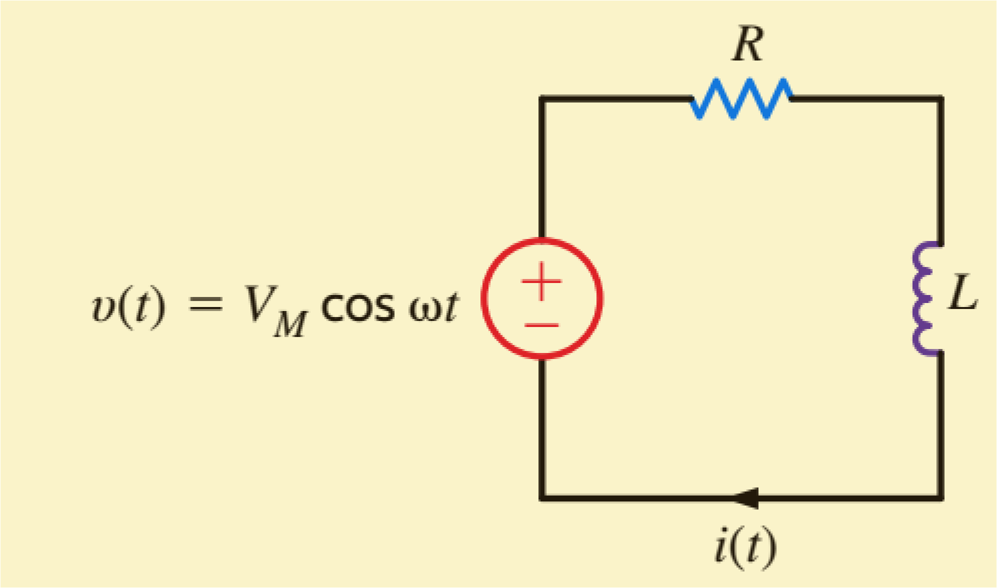


Final expression for current

$$i(t) = \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1} \frac{\omega L}{R}\right)$$

2.

Derive the expression for the current with phasor notation.



HINT

Technique for taking the reciprocal:

$$\frac{1}{R + jX} = \frac{R - jX}{(R + jX)(R - jX)}$$

$$= \frac{R - jX}{R^2 + X^2}$$

2.

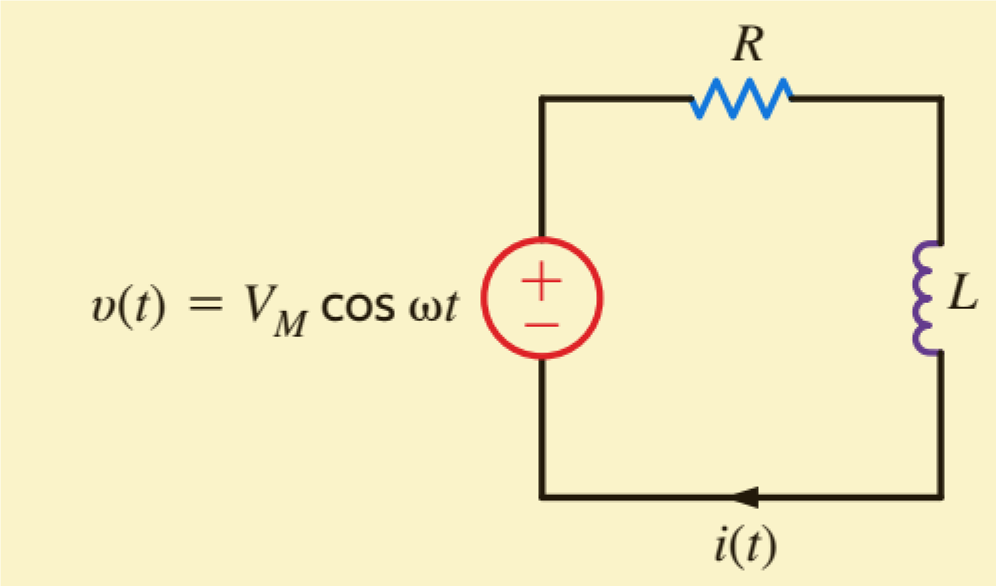
Derive the expression for the current with phasor notation.

$$L\frac{di(t)}{dt} + Ri(t) = V_M \cos \omega t$$ /KVL

$$L\frac{d}{dt}\left(\mathbf{I}e^{j\omega t}\right) + R\mathbf{I}e^{j\omega t} = \mathbf{V}e^{j\omega t}$$

$$j\omega L\mathbf{I}e^{j\omega t} + R\mathbf{I}e^{j\omega t} = \mathbf{V}e^{j\omega t}$$

$$j\omega L\mathbf{I} + R\mathbf{I} = \mathbf{V}$$ /linear equation with one unknown $\mathbf{I}$



$$\mathbf{I} = \frac{\mathbf{V}}{R + j\omega L} = I_M \angle \phi = \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}} \angle -\tan^{-1}\frac{\omega L}{R}$$ /Answer for phasor current

$$i(t) = \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$ /Final answer

HINT

Technique for taking the reciprocal:

$$\frac{1}{R + jX} = \frac{R - jX}{(R + jX)(R - jX)} = \frac{R - jX}{R^2 + X^2}$$

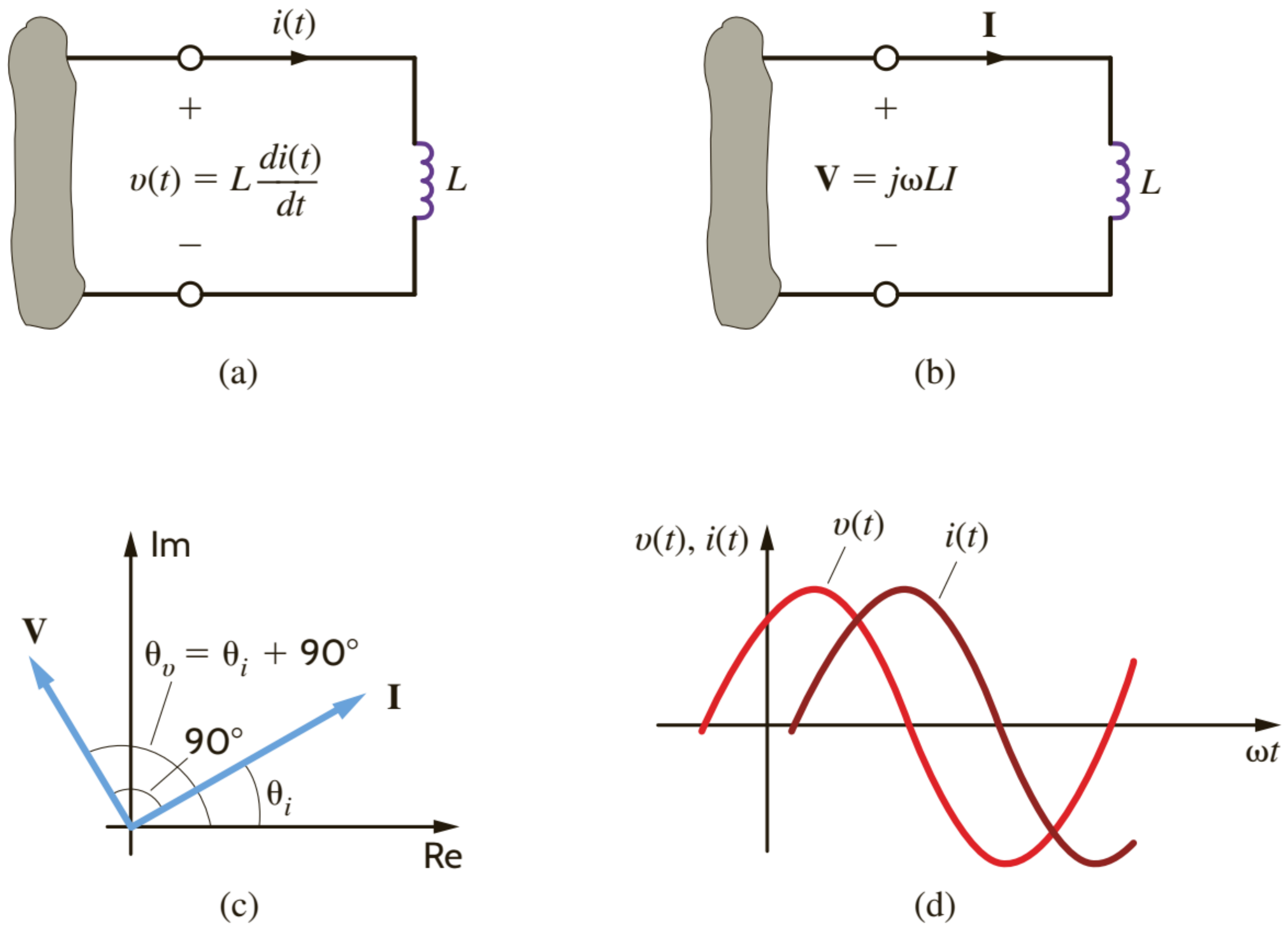
3.

The voltage u(*t*) = 12 cos (377*t* + 20°) V is applied to a 20-mH inductor. Find the resultant current.

HINT

Applying $\mathbf{V} = j\omega L\mathbf{I}$

$$\frac{x_1 \angle \theta_1}{x_2 \angle \theta_2} = \frac{x_1}{x_2} \angle \theta_1 - \theta_2$$



3.

The voltage $u(t) = 12 \cos(377t + 20°)$ V is applied to a 20-mH inductor. Find the resultant current.

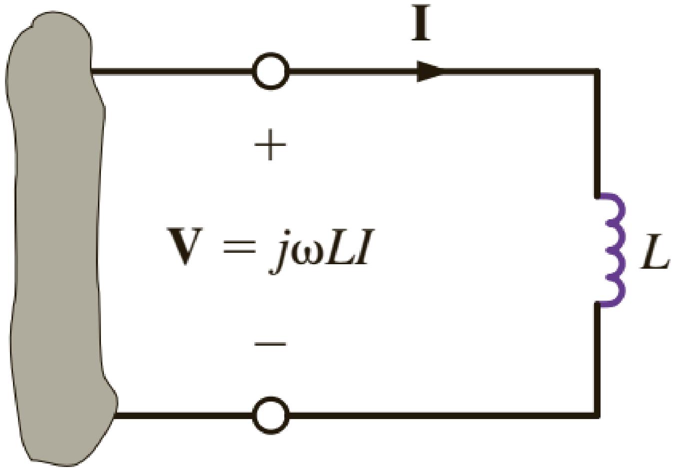
Solution:

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{12\,\underline{/20°}}{\omega L\,\underline{/90°}}$$

$$= \frac{12\,\underline{/20°}}{(377)(20 \times 10^{-3})\,\underline{/90°}}$$

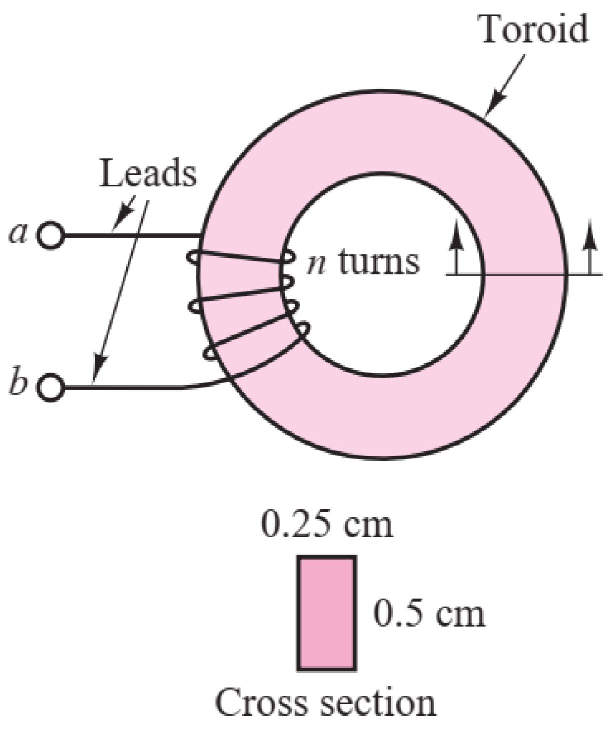
$$= 1.59\,\underline{/-70°}\text{ A}$$

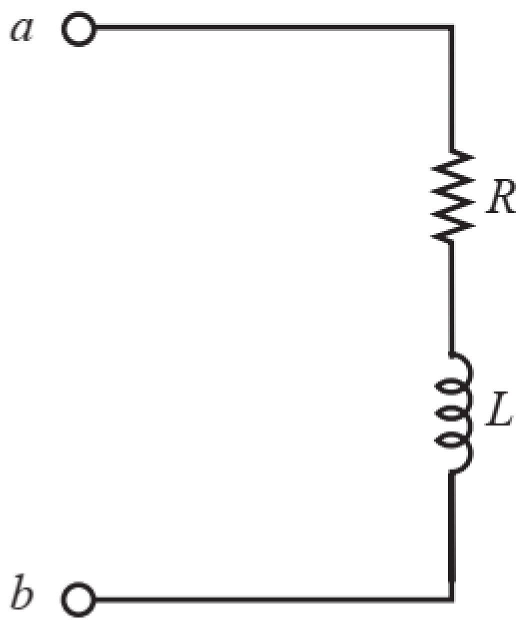
Answer: $i(t) = 1.59 \cos(377t - 70°)$ A



4.

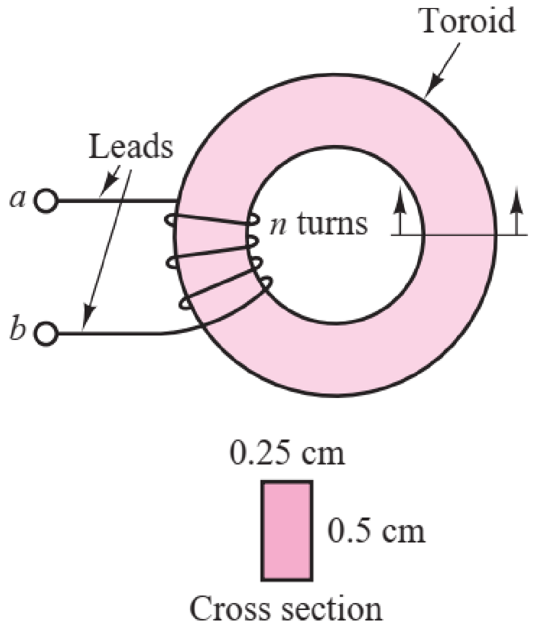
Figure shows a *toroidal* (doughnut-shaped) inductor. The series resistance represents the resistance of the coil wire and is usually small. Find the range of frequencies over which the impedance of this practical inductor is largely *inductive* (i.e., due to the inductance in the circuit). We shall consider the impedance to be inductive if the impedance of the inductor in the circuit is at least 10 times as large as that of the resistor.





L = 0.098 H; lead length = l_c = 2 × 10 cm; n = 250 turns. Resistance of 30-gauge wire = 0.344 Ω/m.

4. $L = 0.098$ H; lead length = $l_c = 2 \times 10$ cm; $n = 250$ turns. Resistance of 30-gauge wire = 0.344 Ω/m.



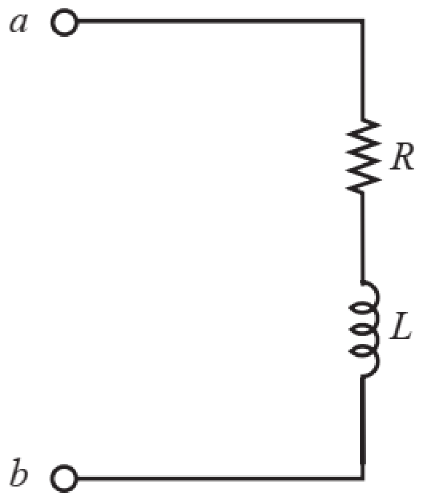
Resistance impedance:

$$l_w = 250(2 \times 0.25 + 2 \times 0.5) = 375 \text{ cm}$$ /Length of coil wire

$$l = \text{ total length} = l_w + l_c = 375 + 20 = 395 \text{ cm}$$

$$R = 0.344(\Omega/m) \cdot 3.95m = 1.36\Omega$$ /Wire resistance

Inductor impedance:



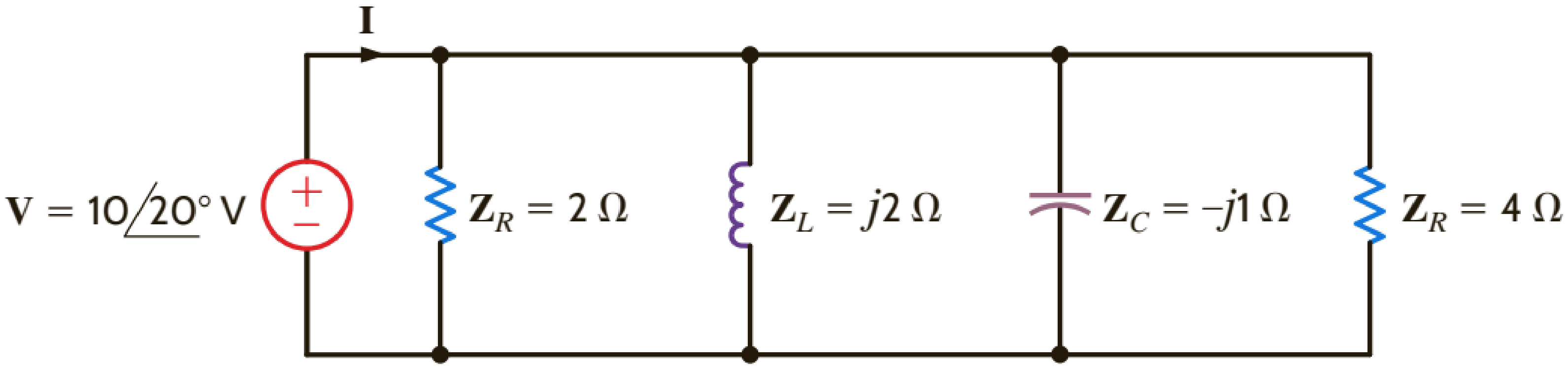
Thus, we wish to determine the range of radian frequencies, ω, over which the magnitude of $j\omega L$ is greater than $10 \cdot 1.36\Omega$:

$$\omega L > 13.6\Omega \quad \text{or} \quad \omega > 138.7 \text{ rad/s}$$

Alternatively, the range is $f = \omega/2\pi > 22$ Hz. /Answer

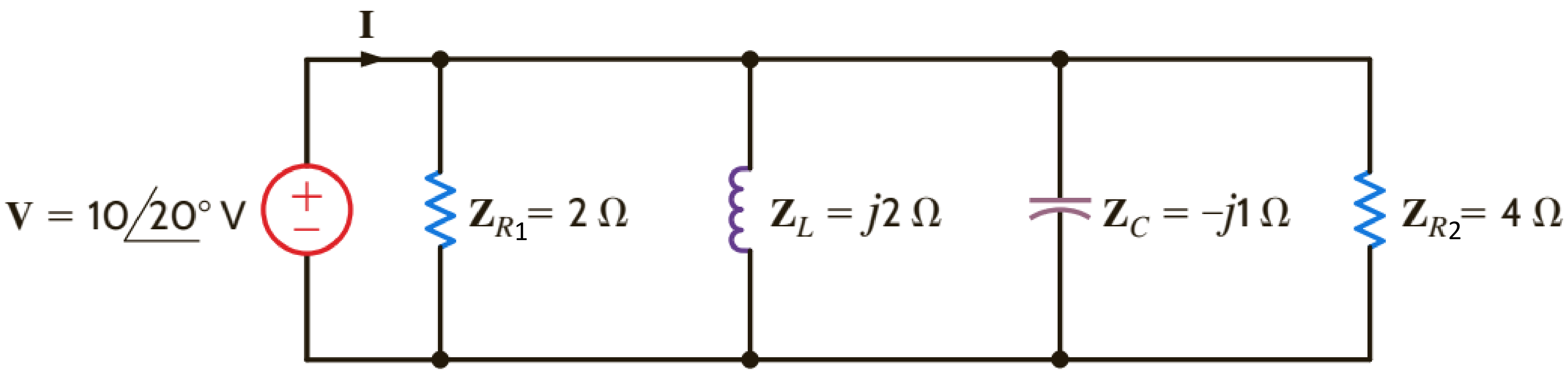
5.

Find the current in the network



5.

Find the current in the network



$$Y = Z^{-1} = \left(\frac{1}{Z_{R1}} + \frac{1}{Z_L} + \frac{1}{Z_C} + \frac{1}{Z_{R2}}\right) = \left(\frac{1}{2} + \frac{1}{j2} + \frac{1}{-j1} + \frac{1}{4}\right) = \left(\frac{3}{4} + j\frac{1}{2}\right) \text{ S}$$

$$\left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{2}\right)^2\right]^{\frac{1}{2}} \rightarrow \frac{\sqrt{13}}{4} = 0.901 \quad \text{S}$$

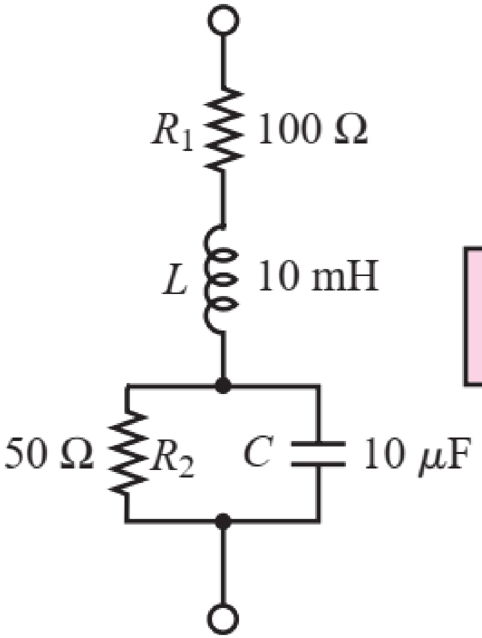
$$\text{atan}\left(\frac{\frac{1}{2}}{\frac{3}{4}}\right) \cdot \frac{180}{\pi} = 33.69 \quad {}^{0}$$

$$Y = 0.901\angle 33.69\ ^o$$

Answer: $I = VY = 10\angle 20\ ^o \cdot 0.901\angle 33.69\ ^o = 9.01\angle 53.69\ ^o$

6.

Find the equivalent impedance of the circuit operating at $\omega = 10^4\,\mathrm{rad}/s$



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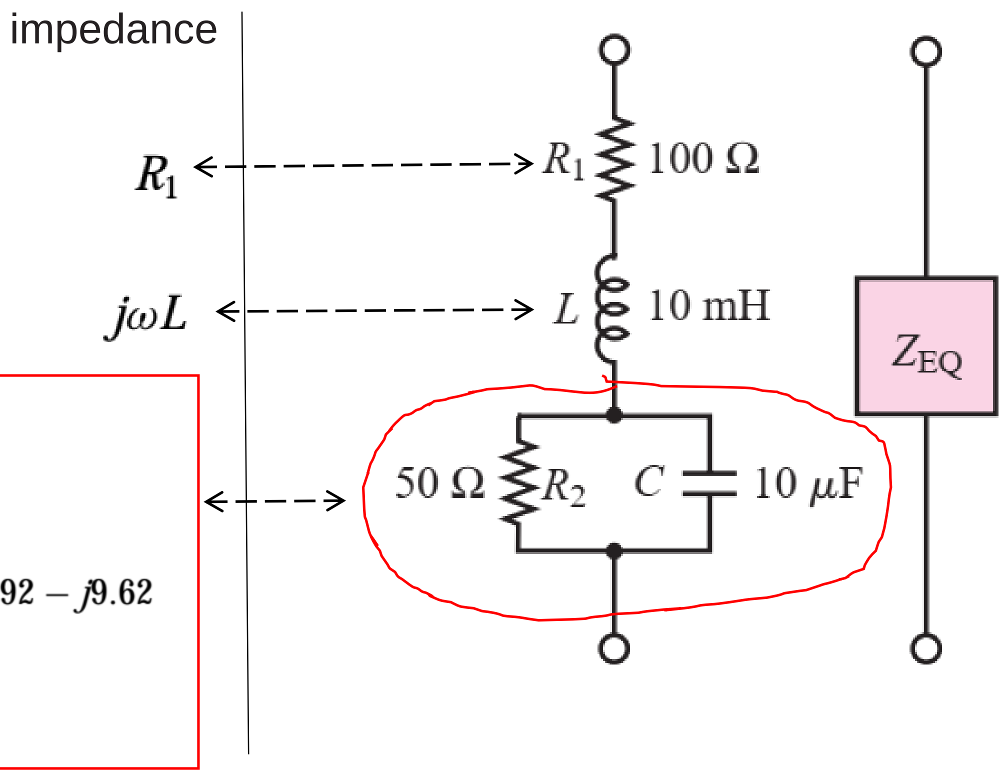

$$Z_{||} = R_2 \left\| \frac{1}{j\omega C} \right. = \frac{R_2(1/j\omega C)}{R_2 + 1/j\omega C} = \frac{R_2}{1 + j\omega C R_2}$$

$$= \frac{50}{1 + j10^4 \times 10 \times 10^{-6} \times 50} = \frac{50}{1 + j5} = 1.92 - j9.62$$

$$= 9.81\angle(-1.3734)\ \Omega$$

Answer:
$$Z_{eq} = R_1 + j\omega L + Z_{||} = 100 + j10^4 \times 10^{-2} + 1.92 - j9.62$$
$$= 101.92 + j90.38 = 136.2\angle 0.723\ \Omega$$