Physics 2. Electrical and Electronic Circuits

Introduction. Resistors

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Objectives

The main objectives of today's lecture are:

- Review the basic concepts of **conductivity**
- Review the concept of **electric current**
- Study **resistance and resistors**

Atoms (1)

An **atom** is a basic unit of matter that consist of nucleus surrounded by a cloud of negatively charged electrons.

The atomic nucleus contains a mix of

- **positively** charged **protons** and
- electrically **neutral** **neutrons**.

Orbiting the nucleus are the **negatively** charged **electrons**.

Some facts about atoms:

- The number of electrons is always equal to the number of protons
- Their number depends on the element
- Typical atom size is 0.1-0.5 nm, while the nucleus' size is typically 5 orders smaller than that.

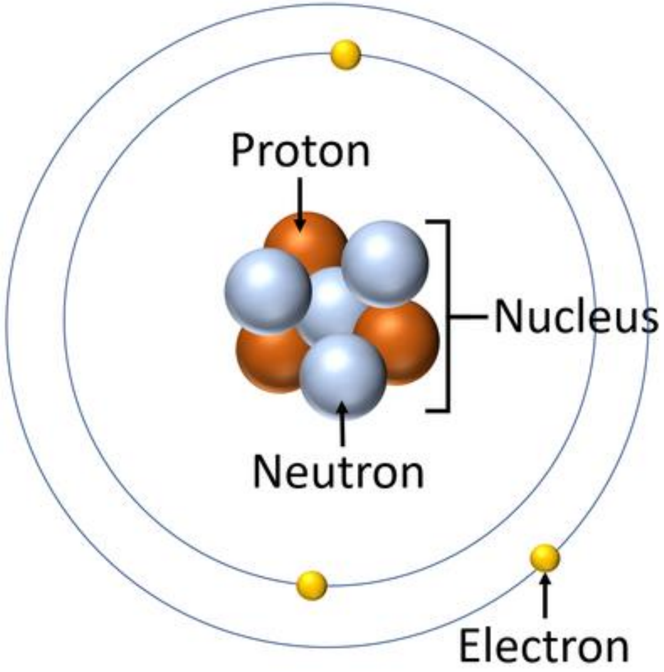
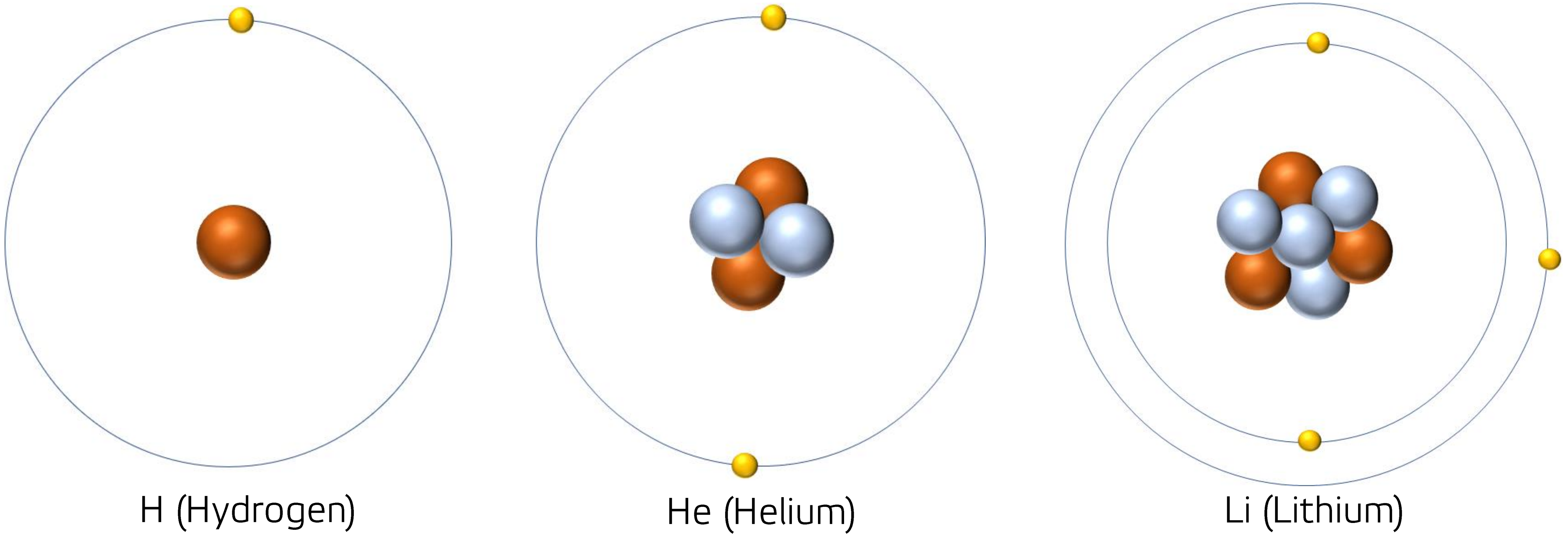


Image credit: Key stage wiki

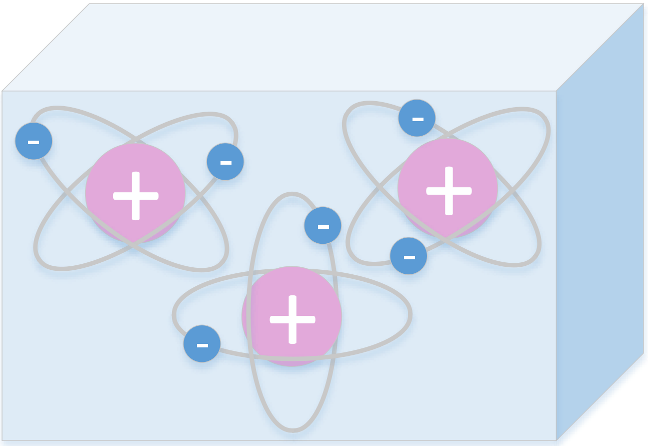
Atoms (2)

The simplest atom is that of **Hydrogen** (1 proton, 1 electron, no neutrons), followed by Helium (2-2-2), Lithium (3-3-4) and so on and so forth.



Diagrams of different atoms
[(Image credit: Key stage wiki)]

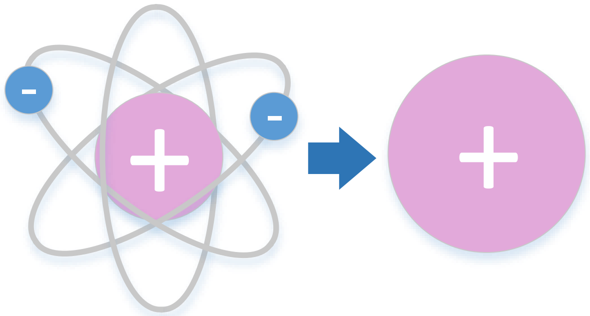
Electric Charge (1)



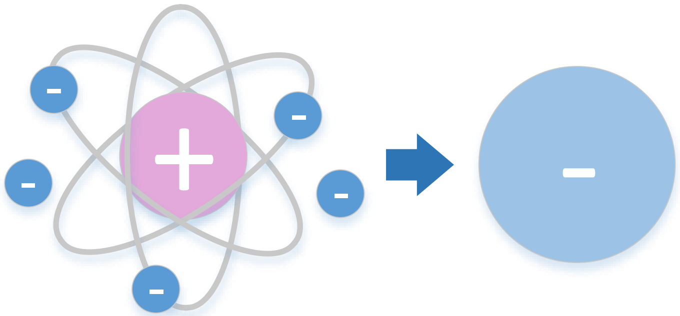
Assume you have a bulk of some material that contains many atoms (6×10^{23} per mole).

Atoms that have either a deficit or a surplus of electrons are called **ions**.

- If there is an excess of positive ions in a bulk of material, its **net charge** Q will be positive, and vice versa.
- Electric charge is measured in coulomb [C].



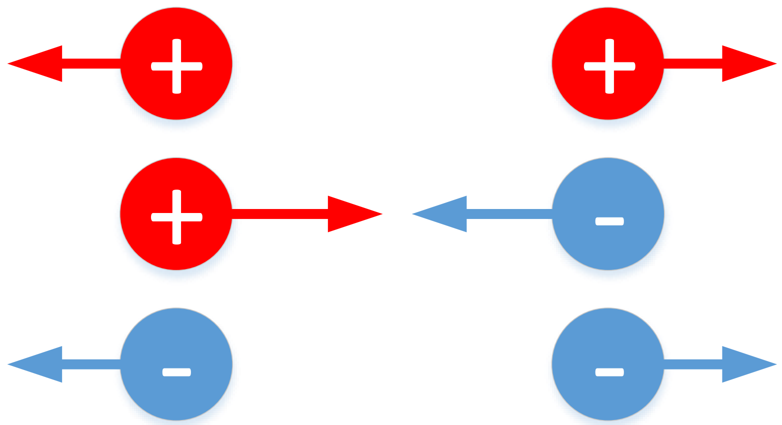
Positive ion



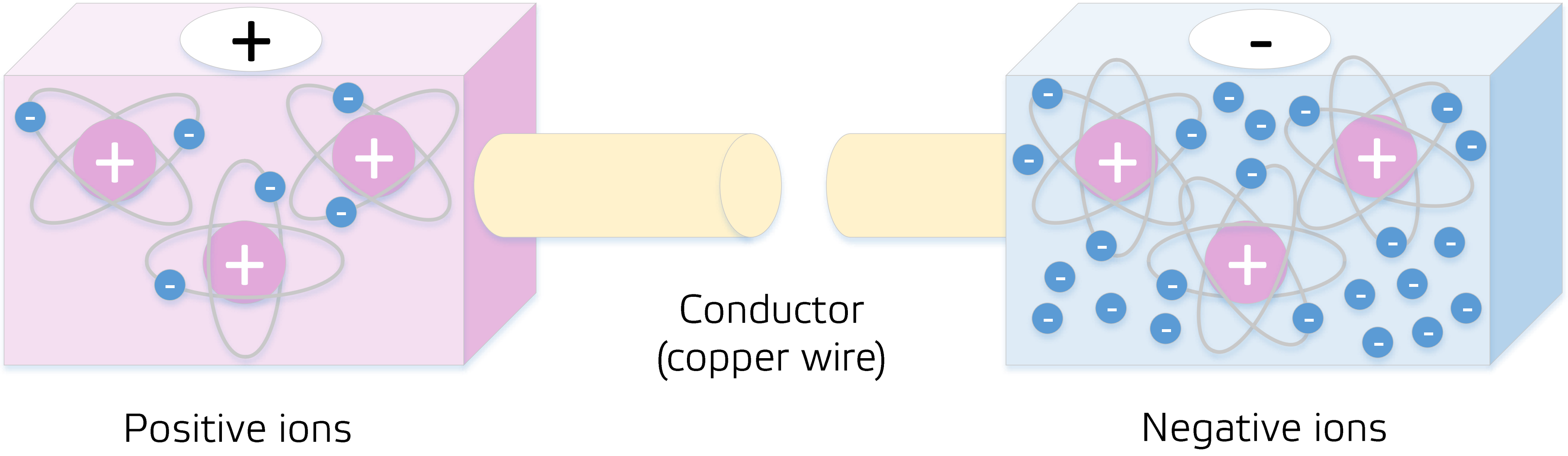
Negative ion

Electric Charge (2)

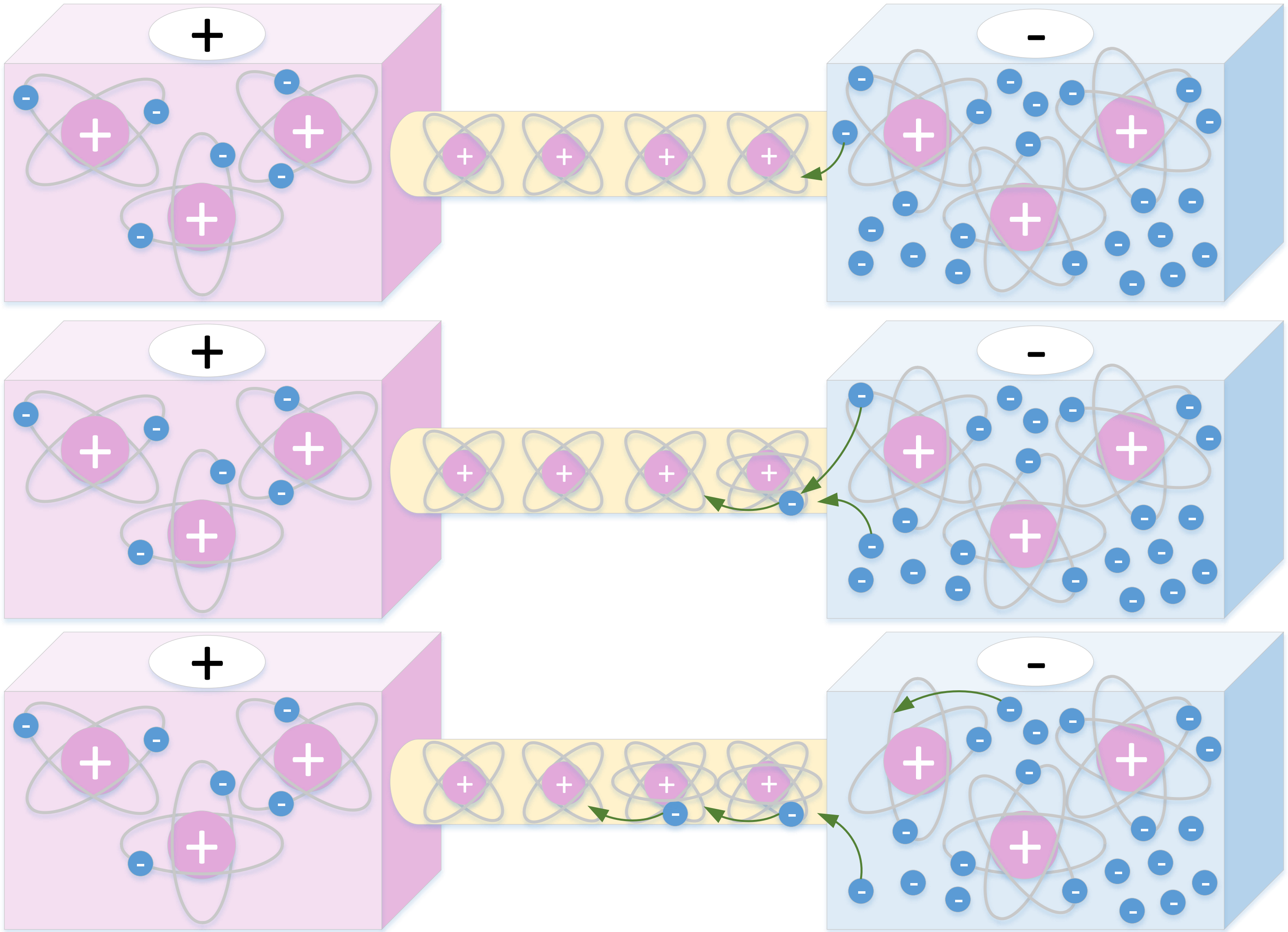
Particles of the same (like) charge repel each other, while particles of **opposite (unlike) charge attract each other**.



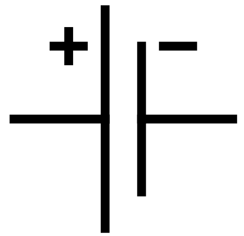
Assume you have 2 pieces of some material with positive and negative net charges, and you connect them with a conductive wire.



Electric Charge (3)



There is a difference in charge levels (**electric potential**)!



Electric Current (1)

Electric **charge can be carried** by subatomic particles, typically – by **moving electrons**.

The stream of such charged particles is called by **electric current** (think water current).

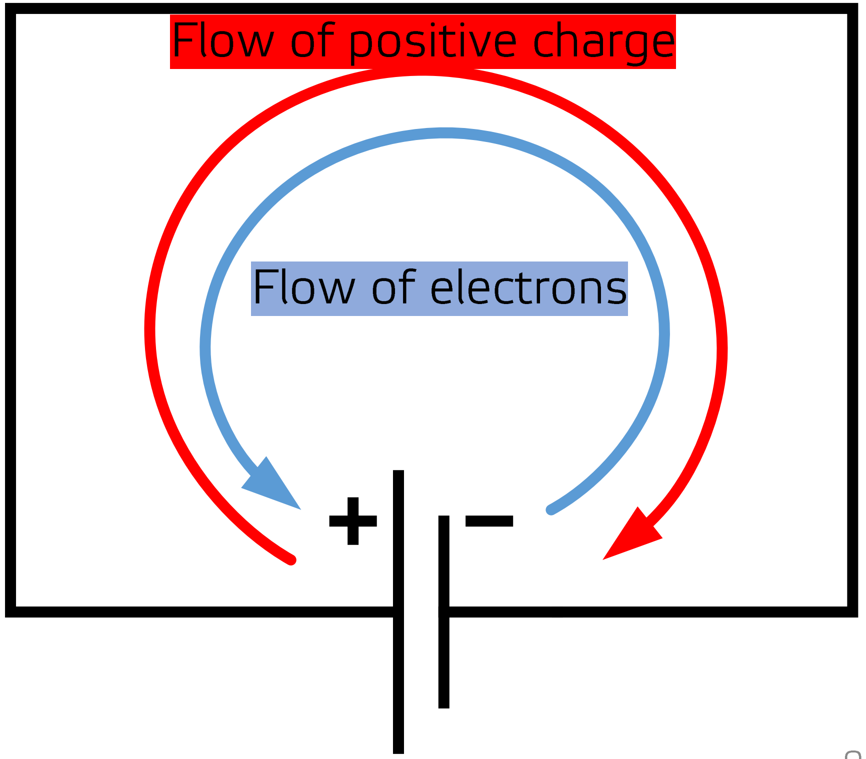
More precisely,

- **Electric current** is the rate of flow of electric charge Q in a region over time t: $I = \frac{Q}{t}$
- Notation: I
- Unit: Ampere [A] (*coulomb per sec*)

Electric Current (2)

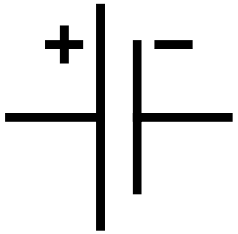
In metals, which are typically used as a conductive material, the positively charged nuclei are held in a fixed position, while the negatively charged electrons can move freely about and carry the charge.

- A flow of positive charges gives the **same electric current**, and has the same effect in a circuit, as an equal flow of negative charges in the opposite direction.

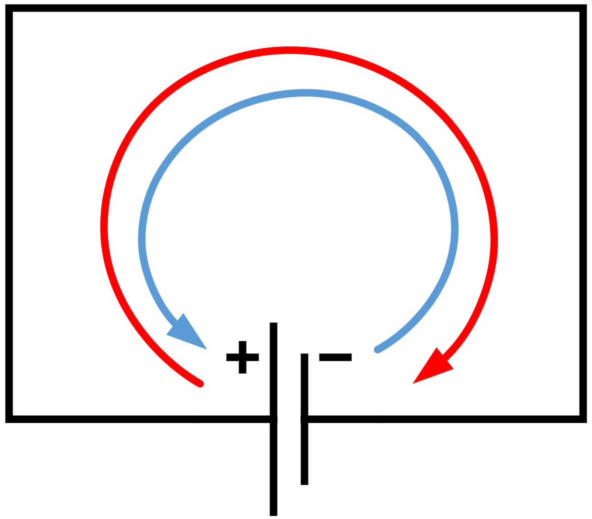


Electric Voltage

Electric voltage is the **difference** in electric potential between two points.



- Notation: V
- Unit: Volt [V]
- The voltage between two points is a short name for the **electrical driving force** (electromotive force: emf) that could determine an electric current between those points.



Electrical Resistivity (1)

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The **electric resistivity** and its inverse, **electrical conductivity**, is a fundamental property of a material that quantifies how **strongly it resists** or conducts electric current.

Various materials have different resistivity and conductivity (shown here for 20 ^{0}C).

Material	Resistivity, ρ	Conductivity
Silver	1.59×10^{-8}	6.30×10^{7}
Copper	1.68×10^{-8}	5.96×10^{7}
Gold	2.44×10^{-8}	4.11×10^{7}
Calcium	3.36×10^{-8}	2.98×10^{7}
Carbon (graphite)	3.10×10^{-3}	3.3×10^{2}
Sea water	2.00×10^{-1}	5
Diamond	10^{12}	10^{-12}
Rubber	10^{13}	10^{-13}
PET	10^{21}	10^{-21}
Teflon	10^{24}	10^{-24}

Electrical Resistance

The **electric resistance** of an electrical element measures its **opposition** to the passage of an electric current.

- Notation: R
- Unit: Ohm $[\Omega]$
- The reciprocal quantity is **electrical conductance** (the ease with which an electric current passes).
- Electrical resistance is a function of conductor's volumetric properties:

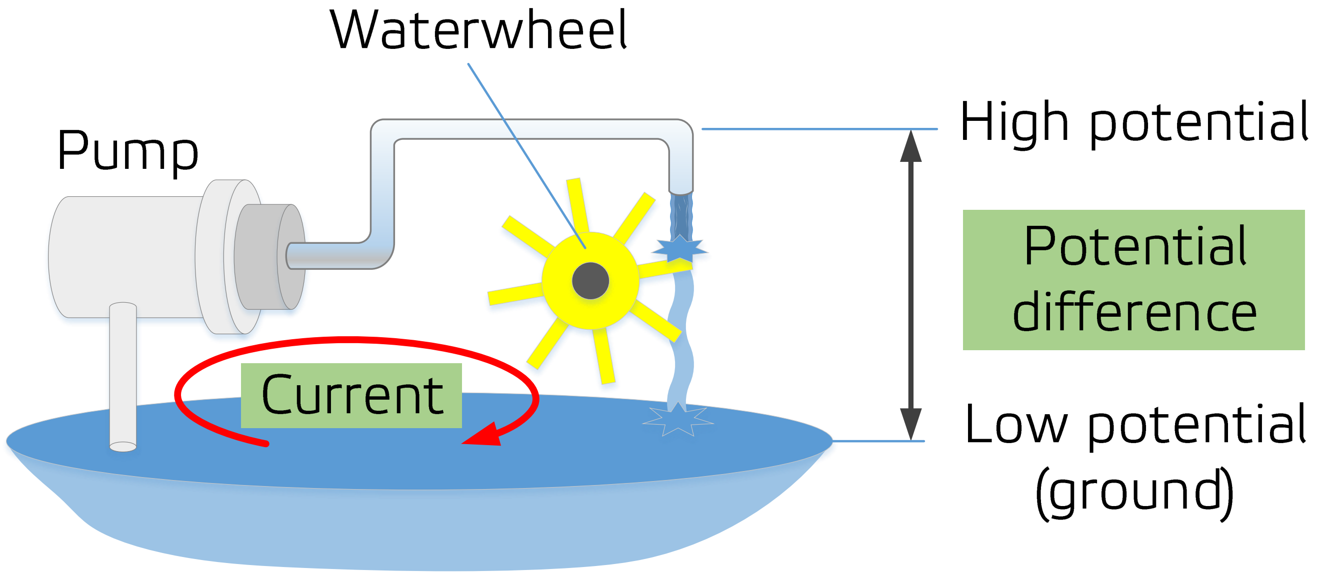
$$R = \rho \frac{l}{A}$$

where ρ is electrical conductivity, l is the length of conductor and A is its cross-sectional area.

Electricity and Mechanics (1)

A simple analogy for an electric circuit is **water flowing** in a closed circuit of pipework, driven by a mechanical pump. This can be called a **water circuit**.

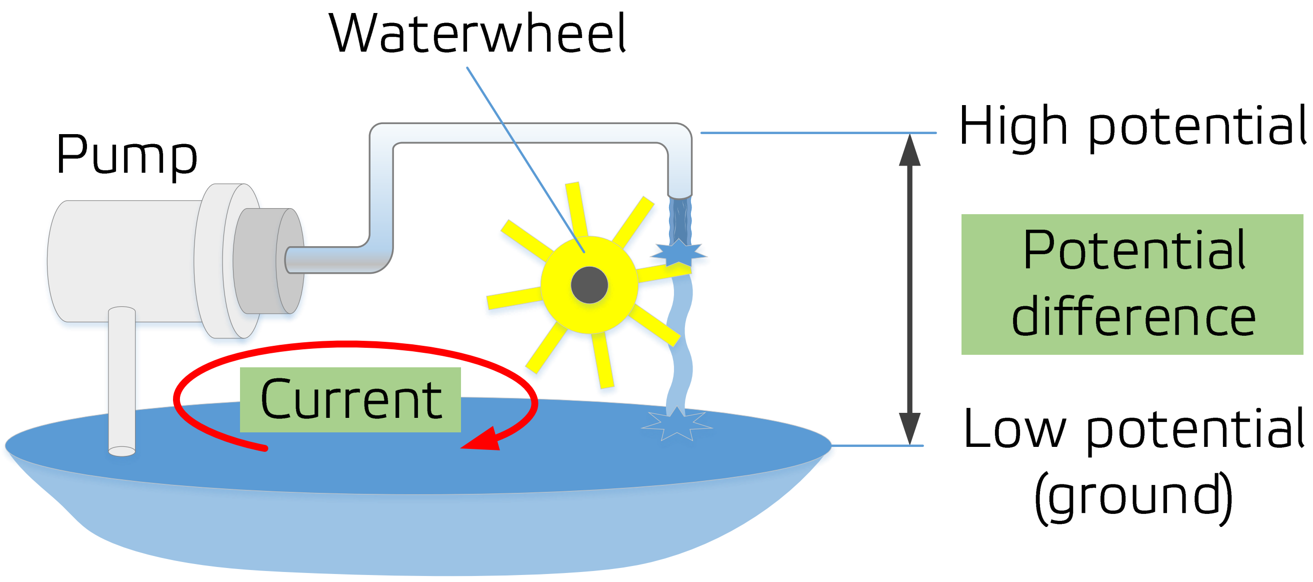
- **Potential difference** between two points corresponds to the water pressure difference between two points.

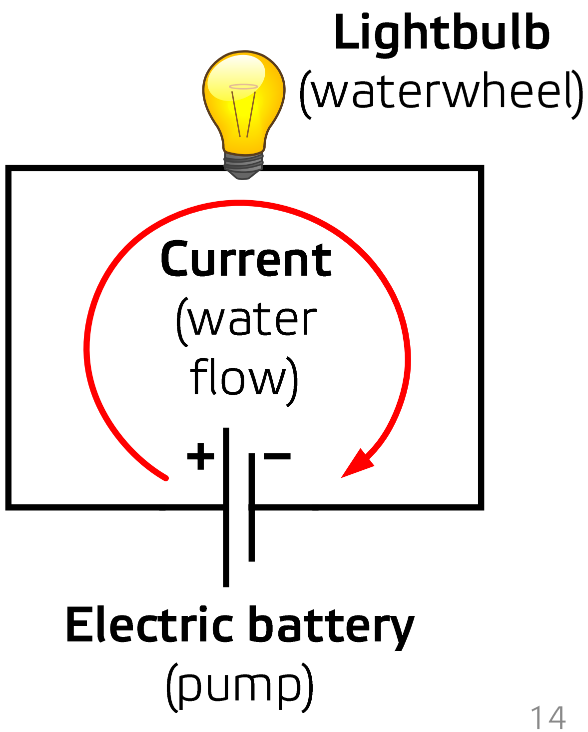


Electricity and Mechanics (2)

If there is a water pressure difference between two points, then

- the water flow (created by the pump) from the will be able to **do work**, such as spinning the waterwheel.
- In a similar way, work can be done by the electric current driven by the potential difference due to an electric battery.

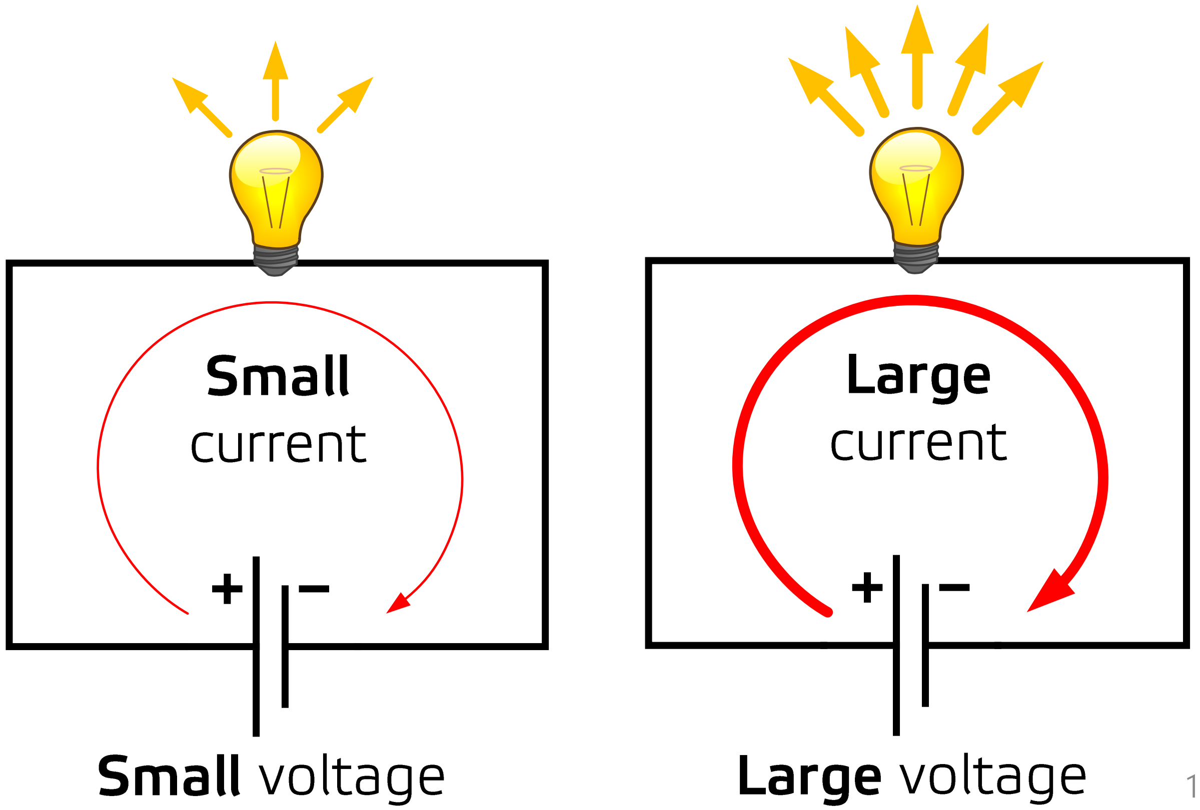




Electric Circuits (1)

When you arrange electrical components forming the network in such a way that it has a closed loop, giving a return path for the current, this network is called **an electrical circuit**.

- Researchers discovered that **increasing the voltage** in a simple circuit shown here resulted in **increased current**.

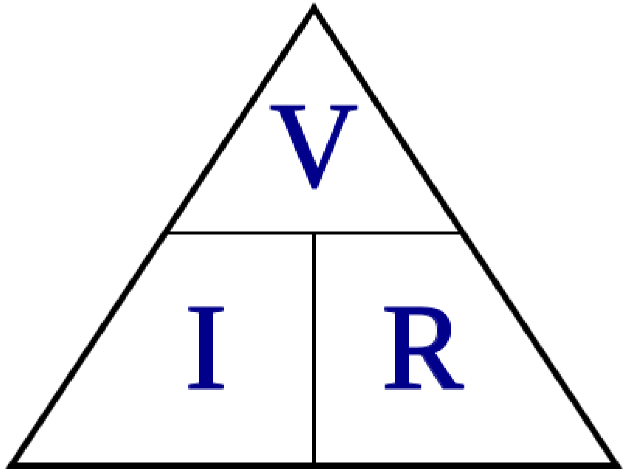


Ohm's Law (1)

This observation forms one of the most fundamental laws of electricity, the Ohm's law:

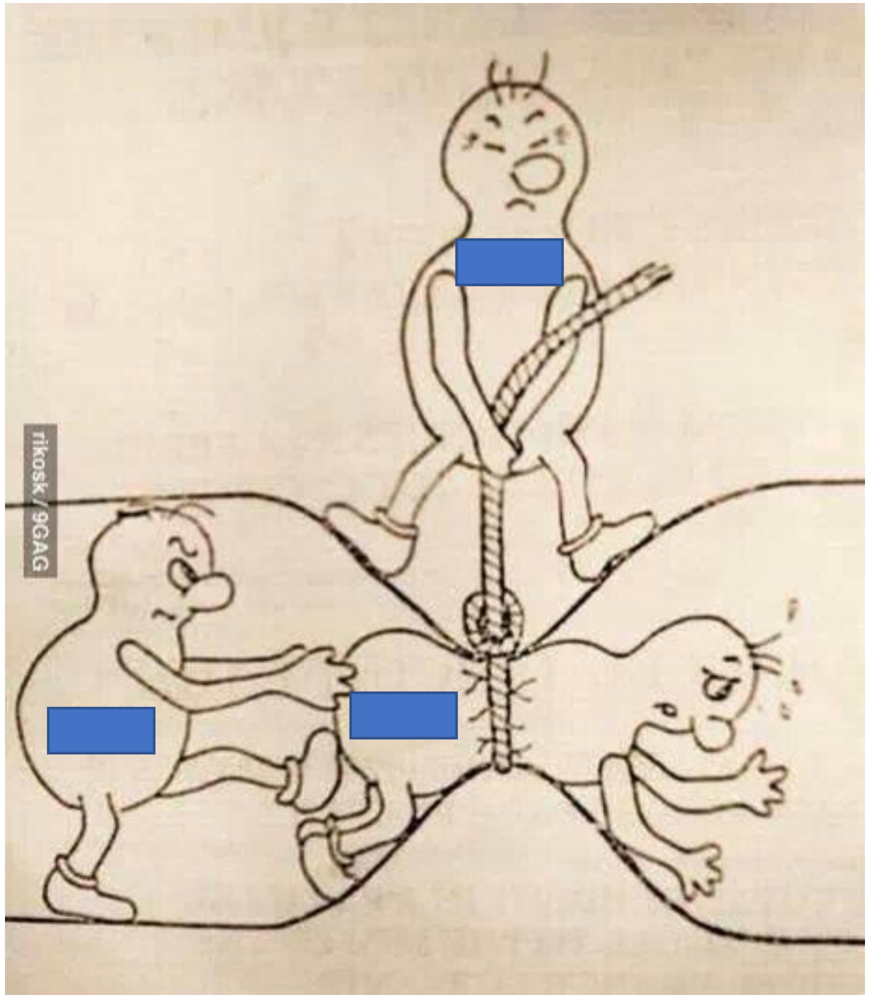
- Current through a conductor between two points is directly proportional to the voltage across the two points, and inversely proportional to the resistance between them.

$$\text{Current}(I) = \frac{\text{Voltage}(V)}{\text{Resistance}(R)}$$



Ohm’s Law (2)

Quiz: whose names is covered by the blocks (electronics-related, of course ☺)?



Electric Power

Electric power is the rate at which electrical energy is transferred by an electric circuit.

- Notation: P
- Unit: Watt [W]

$$\text{Power}(P) = \text{Voltage}(V)\ \text{x}\ \text{Current}(I)$$

- **Q1**: What is the formula for electric power if you substitute the expression for voltage and current derived from the Ohm's law into the equation above?
- **Q2**: Recall the expression of mechanical power. What are the analogies between the 2 worlds?

Resistors

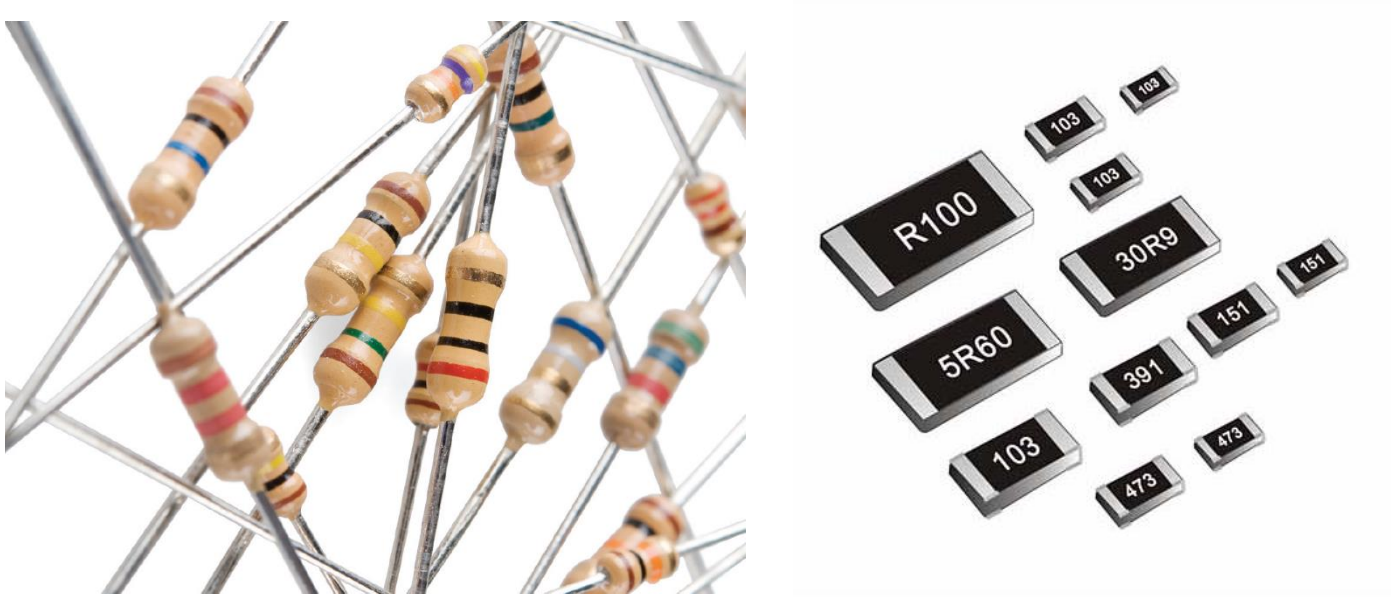
Resistors

A **resistor** is a two-terminal electrical component that implements electrical resistance as a circuit element.

Resistors are common elements of electrical networks and electronic circuits and are ubiquitous in electronic equipment.

- **Q**: Why do we need resistors?

Electronic symbol



Axial-lead vs. SMD resistors

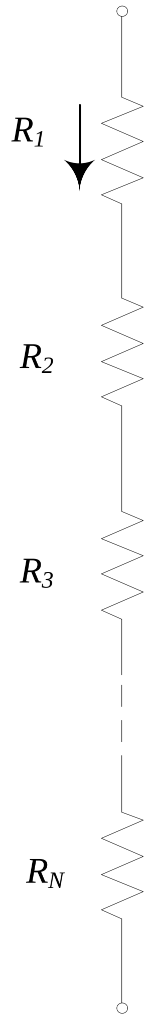
Resistors : Series

The equivalent resistance R_{EQ} of all the resistors in series can be found as

$$R_{EQ} = \sum_{n=1}^{N} R_n$$

- Thus, for instance, for 3 resistors connected in series to the voltage supply V_{supply}, the electrical current is

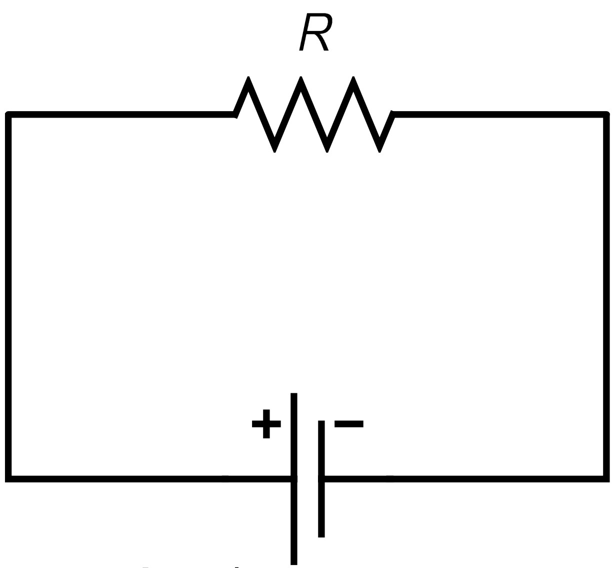
$$i = \frac{V_{\text{supply}}}{R_1 + R_2 + R_3} = \frac{V_{\text{supply}}}{R_{EQ}}$$



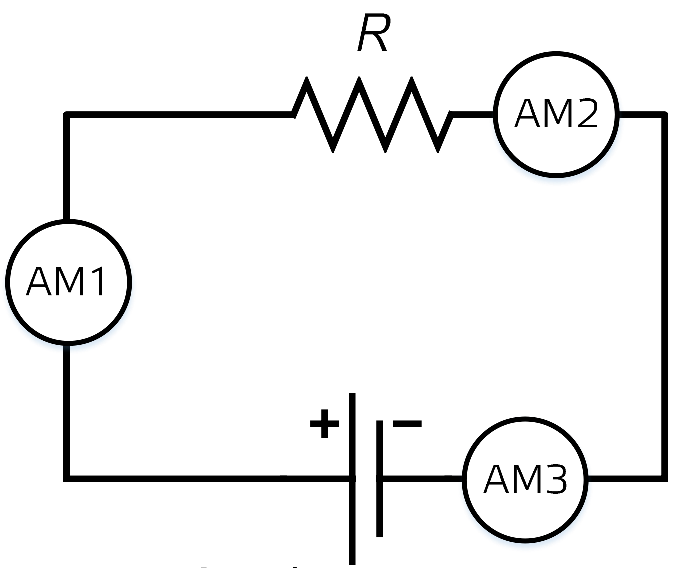
Electric Circuits (2)

Direct current (DC) is a continuous current that flows only in **one direction**.

In a circuit where the elements are placed in **series**, the electrical **current** (measured by ammeters AM1, AM2, AM3 and so on) would be **the same**.



DC voltage source
(battery)

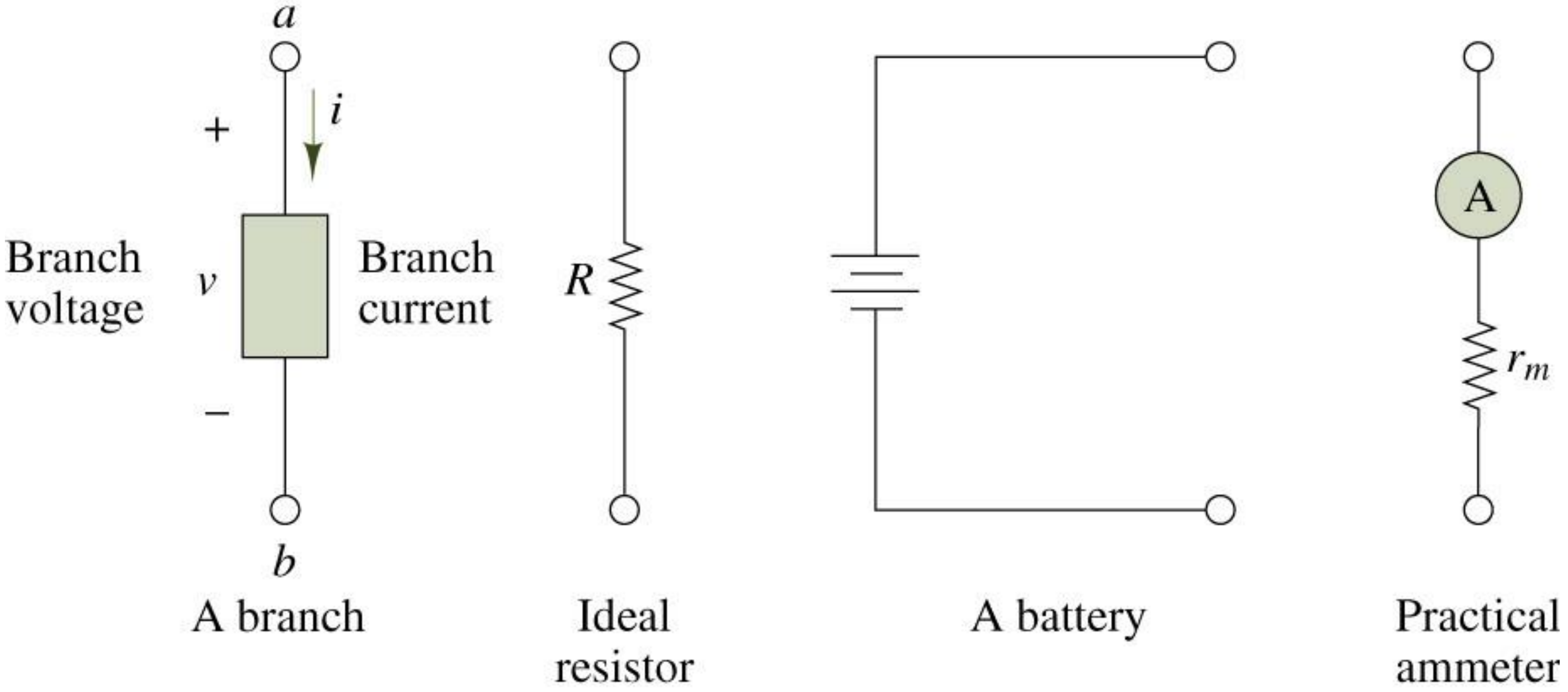


DC voltage source
(battery)

$$I_1 = I_2 = I_3 = \cdots$$

Electric Circuits : Branch

A **branch** is any portion of a circuit with **two terminals** connected to it.



Examples of circuit branches

Electric Circuits : Node

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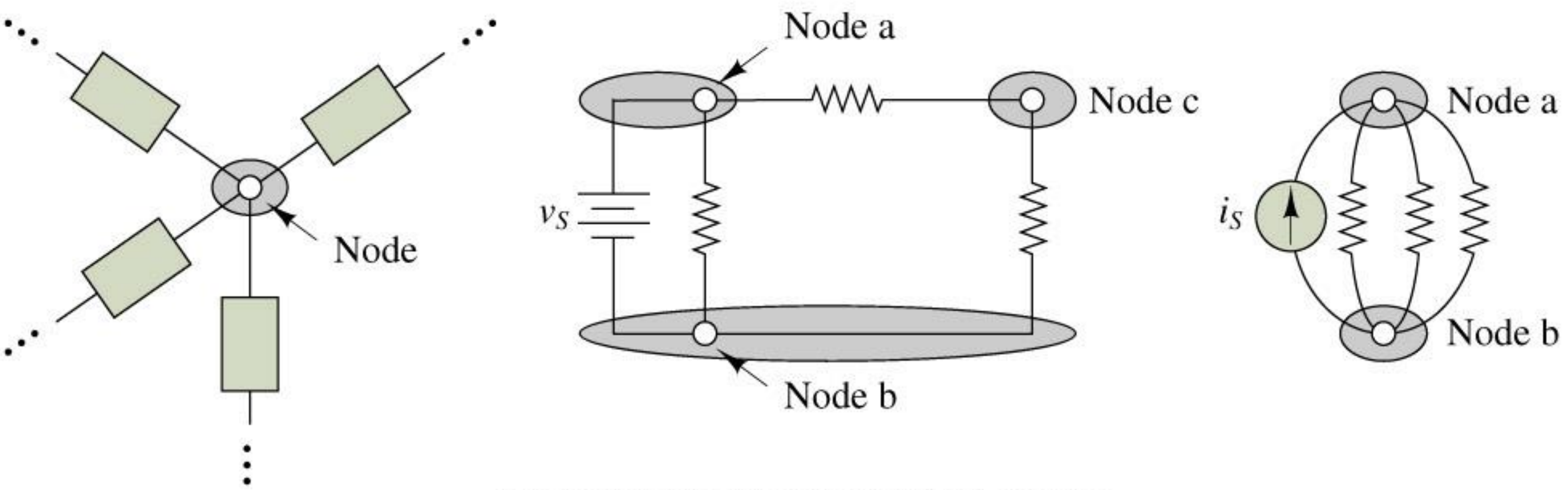
- A **node** is a junction of two or more branches.



Examples of nodes in practical circuits

Kirchhoff's Current Law

KCL: The **sum of the currents** at a node must **equal zero**.

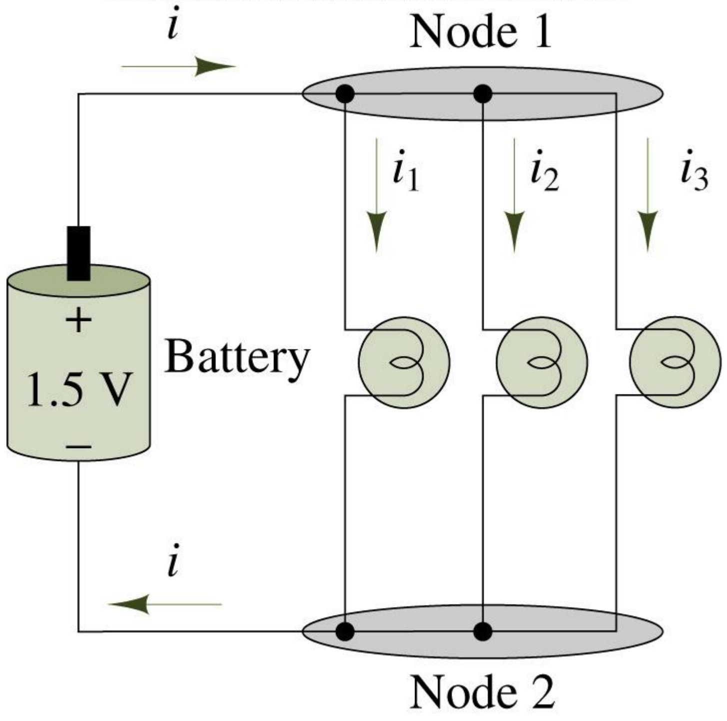


Illustration of KCL at node 1: $-i + i_1 + i_2 + i_3 = 0$

$$\sum_{n=1}^{N} i_n = 0$$

KCL : Example

- Find missing currents in the circuit on the right.

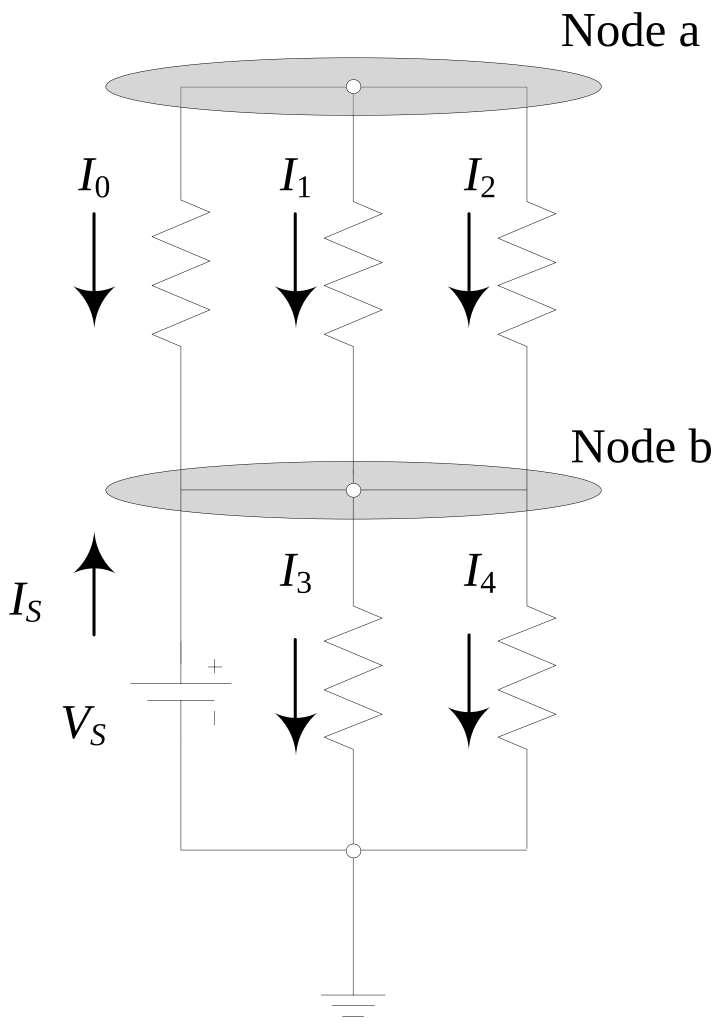
- Known Quantities:

I_S = 5 A; I_1 = 2 A;

I_2 = -3 A; I_3 = 1.5 A.

- Find:

I_0 and I_4



Thank you for your attention!

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Physics 2. Electrical Engineering

Week 2 Resistive Circuits

Igor Gaponov

Professor, Institute of Robotics and Computer Vision

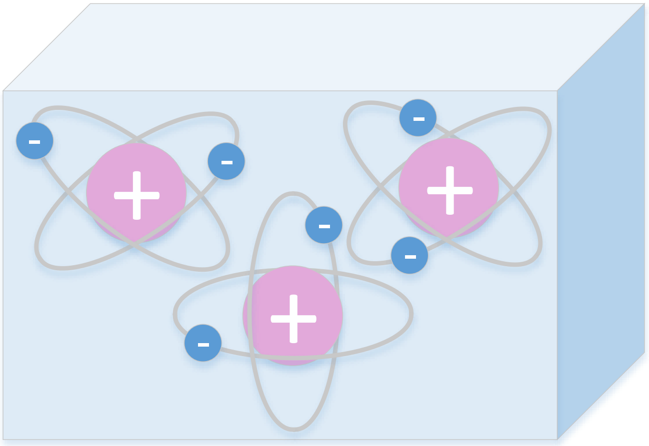
Objectives

The main objectives of today's lecture are:

- Review the basic concepts of open and short circuits
- Study parallel resistors
- Learn to find equivalent resistance

Last Week’s Review: Conductivity & Resistors

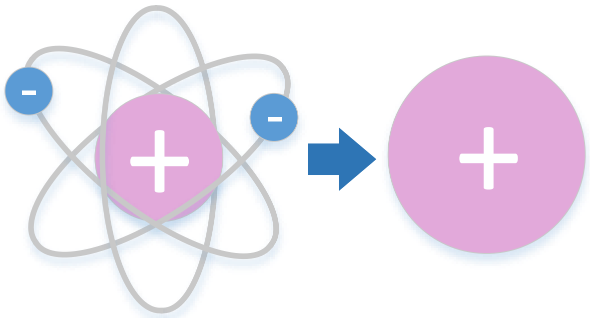
Electric Charge (1)



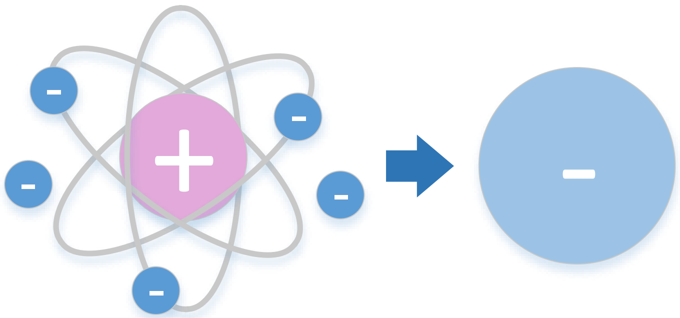
Assume you have a bulk of some material that contains many atoms ($6x10^{23}$ per mole).

Atoms that have either a deficit or a surplus of electrons are called ions.

- If there is an excess of positive ions in a bulk of material, its net charge Q will be positive, and vice versa.
- Electric charge is measured in coulomb [C].

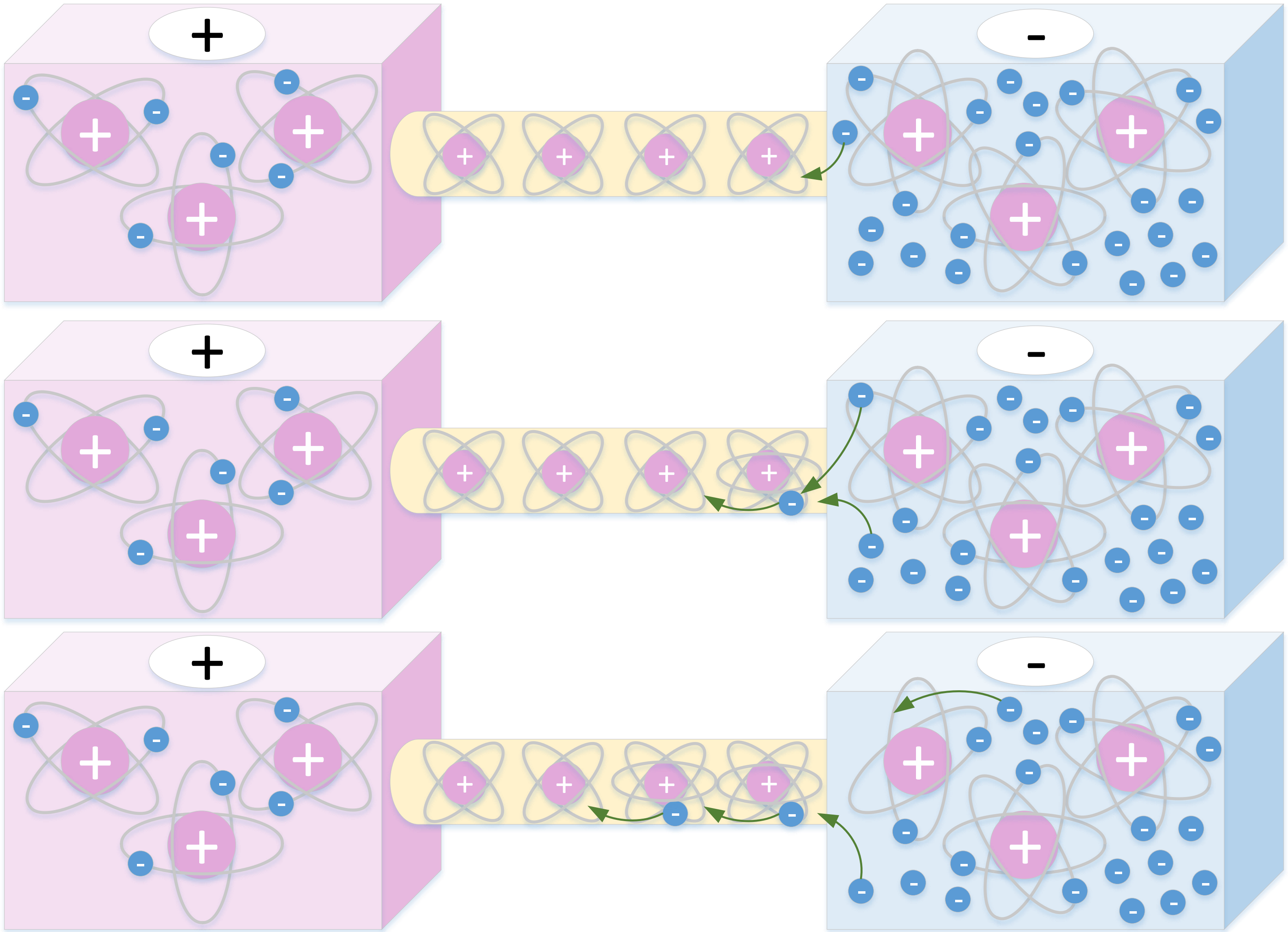


Positive ion

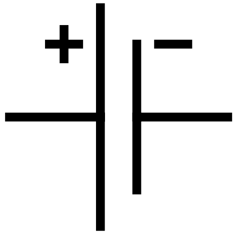


Negative ion

Electric Charge (2)



There is a difference in charge levels (electric potential)!



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More precisely,

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- Notation: I
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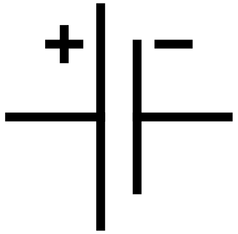
Electric voltage is the difference in electric potential between two points.



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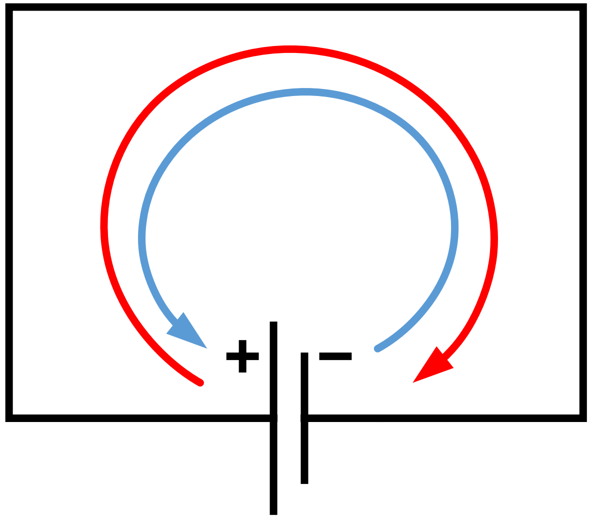

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- Notation: R
- Unit: Ohm [Ω]
- The reciprocal quantity is electrical conductance (the ease with which an electric current passes).
- Electrical resistance is a function of conductor's volumetric properties:

$$R = \rho \frac{l}{A}$$

where ρ is electrical resistivity, l is the length of conductor and A is its cross-sectional area.

Electrical Conductance

The electric conductance is the ability for electricity to flow a certain path.

- Conductance is the inverse of resistance

$$G = \frac{1}{R}$$

Electricity and Mechanics

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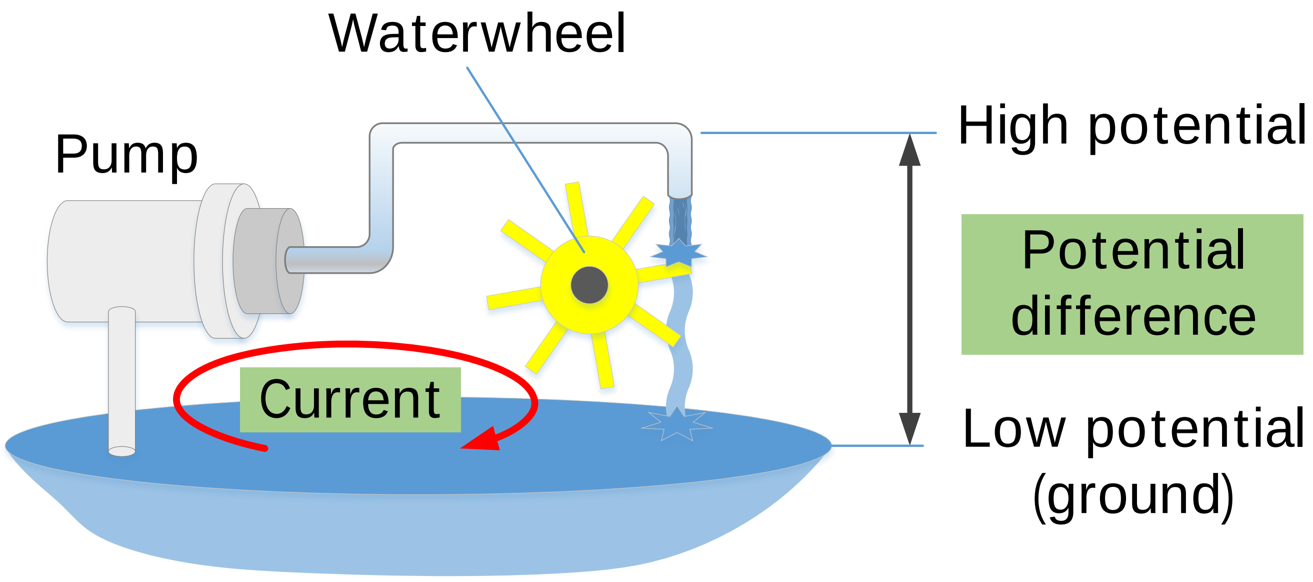
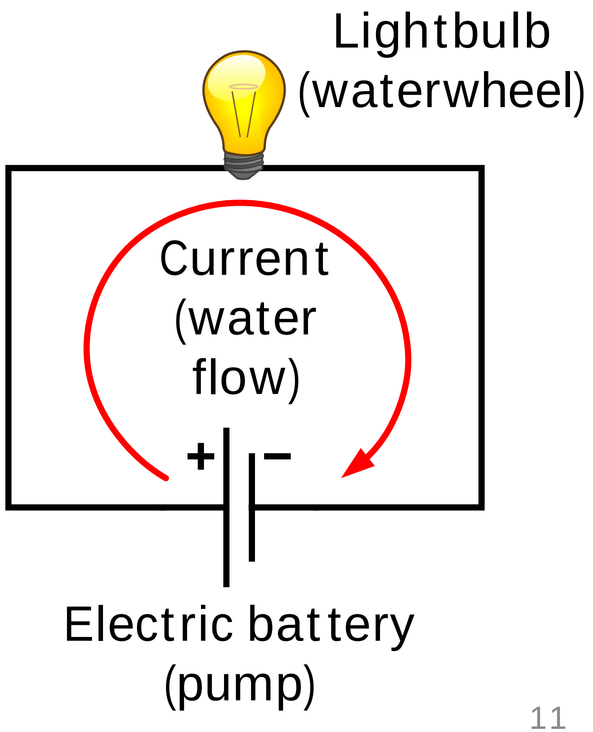


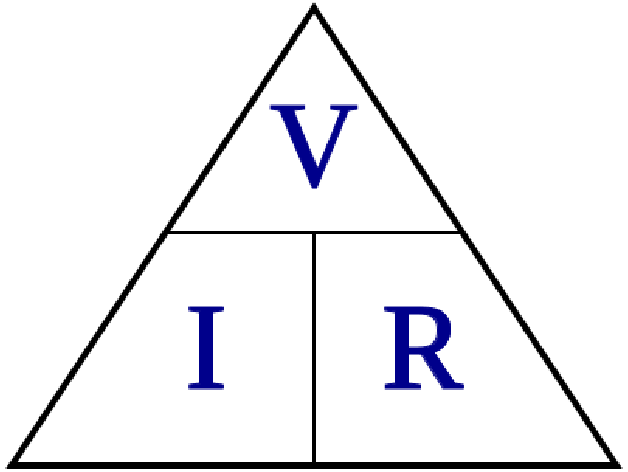


Ohm’s Law

This observation forms one of the most fundamental laws of electricity, the Ohm’s law:

- Current through a conductor between two points is directly proportional to the voltage across the two points, and inversely proportional to the resistance between them.

$$\text{Current}(I) = \frac{\text{Voltage}(V)}{\text{Resistance}(R)}$$



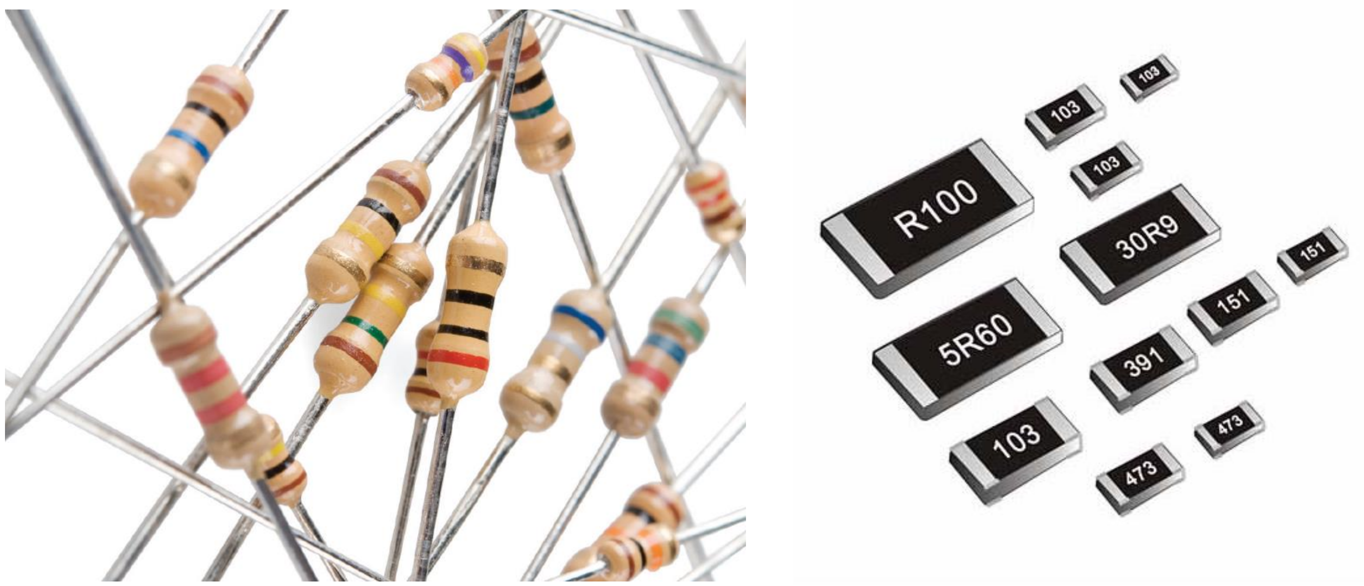
Resistors

A resistor is a two-terminal electrical component that implements electrical resistance as a circuit element.

Resistors are common elements of electrical networks and electronic circuits and are ubiquitous in electronic equipment.

- Q: Why do we need resistors?

Electronic symbol



Axial-lead vs. SMD resistors

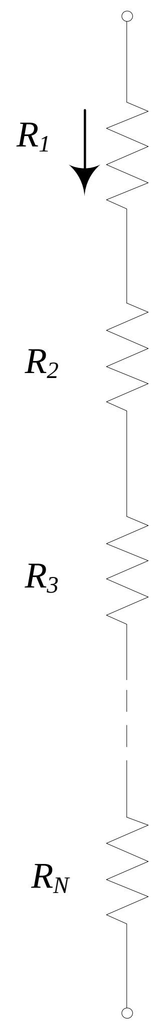
Resistors : Series

The equivalent resistance R_{EQ} of all the resistors in series can be found as

$$R_{EQ} = \sum_{n=1}^{N} R_n$$

- Thus, for instance, for 3 resistors connected in series to the voltage supply V_{supply}, the electrical current is

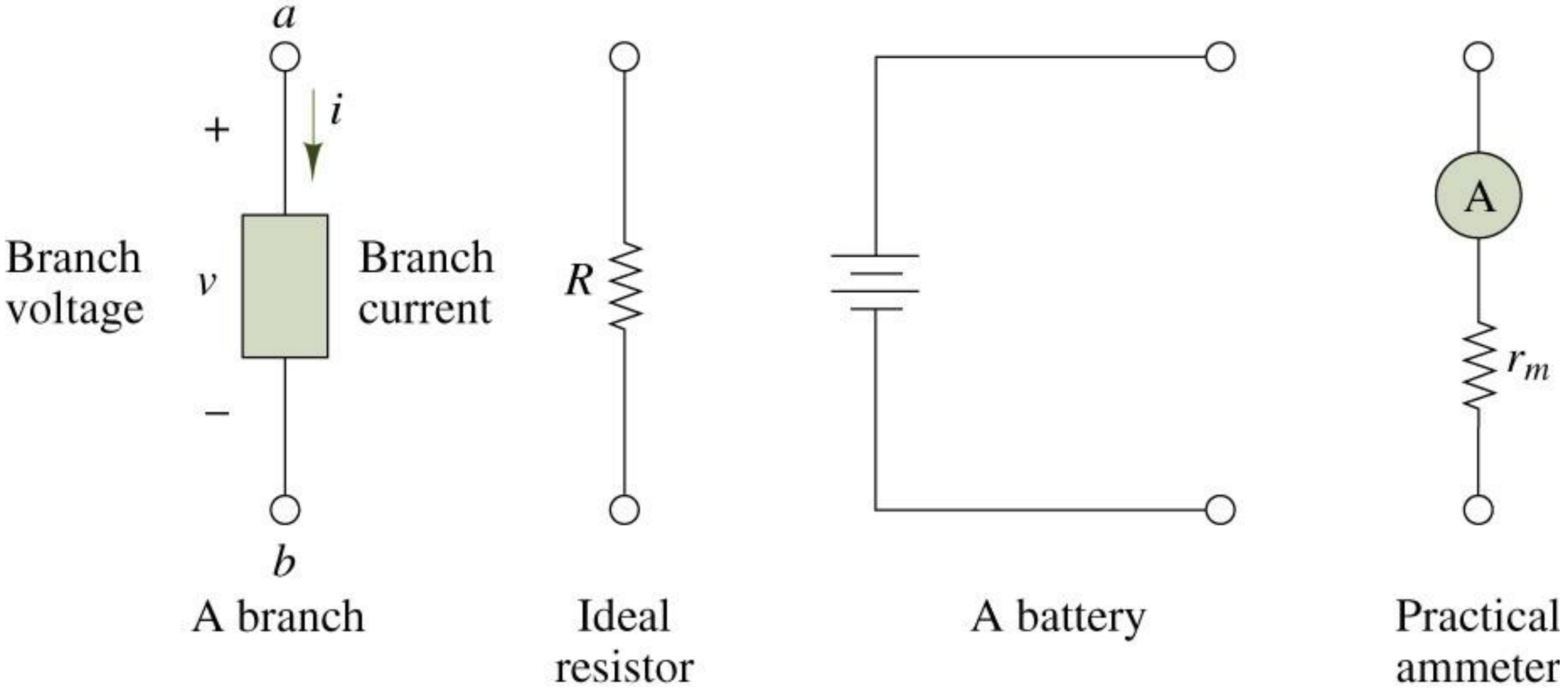
$$i = \frac{V_{\text{supply}}}{R_1 + R_2 + R_3} = \frac{V_{\text{supply}}}{R_{EQ}}$$



Kirchhoff's Laws

Electric Circuits : Branch

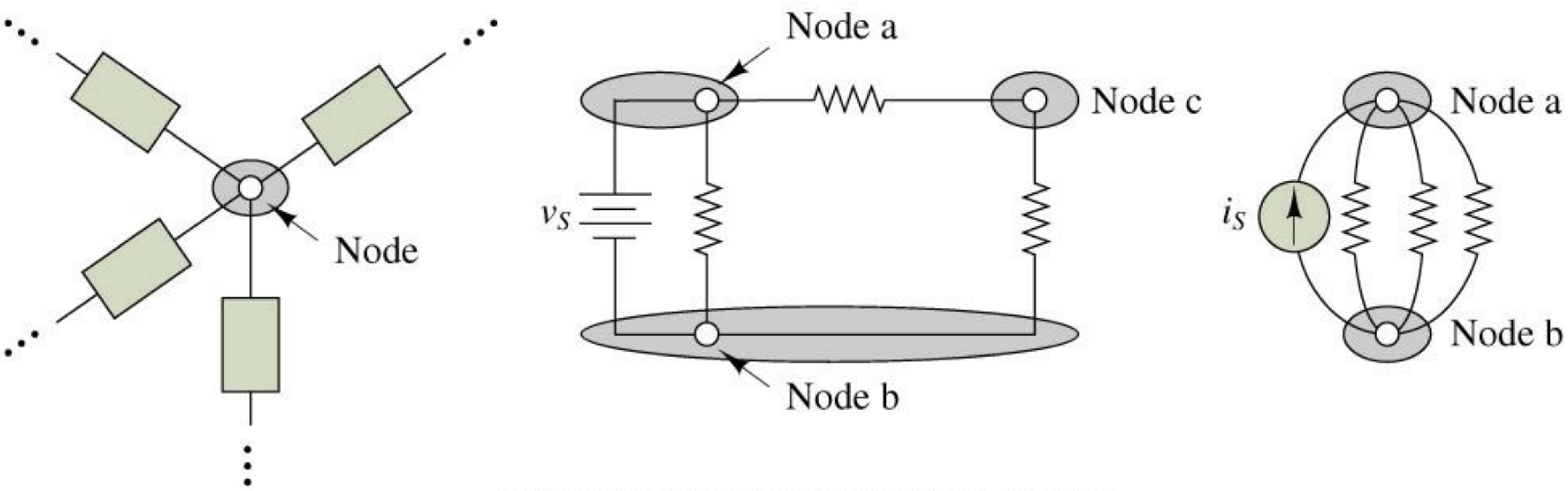
A branch is any portion of a circuit with two terminals connected to it.



Examples of circuit branches

Electric Circuits : Node

- A node is a junction of two or more branches.



Examples of nodes in practical circuits

Kirchhoff's Current Law

KCL: The sum of the currents at a node must equal zero.

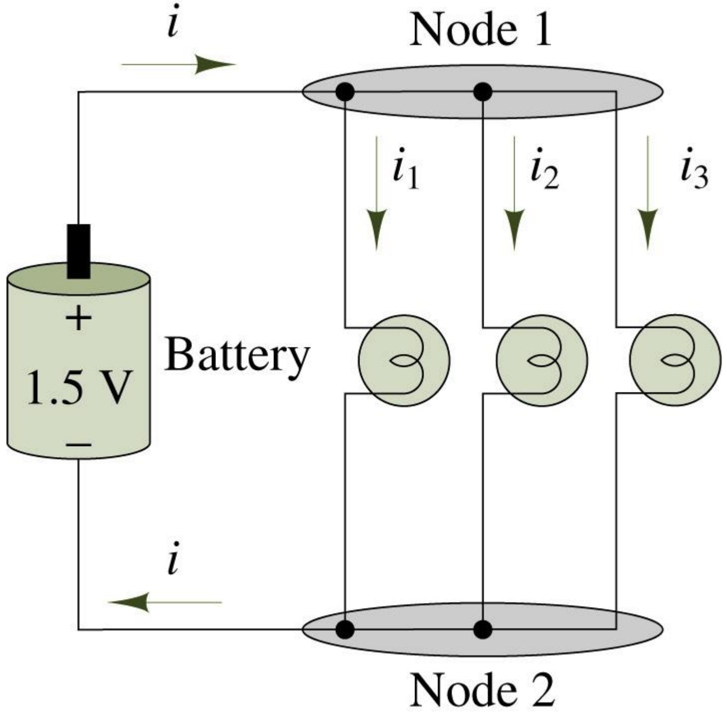
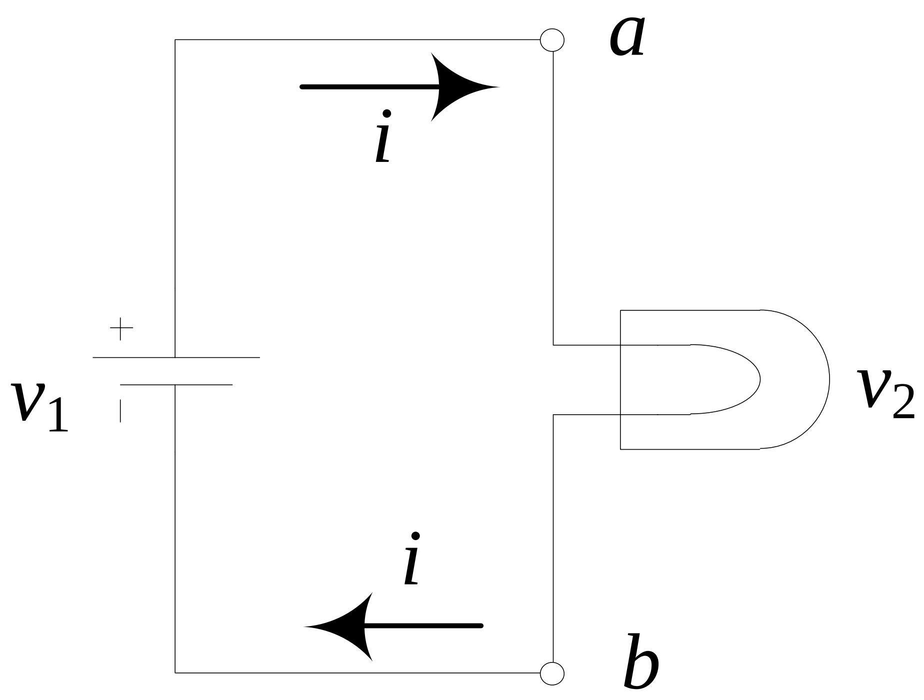


Illustration of KCL at node 1: $-i + i_1 + i_2 + i_3 = 0$

$$\sum_{n=1}^{N} i_n = 0$$

Kirchhoff's Voltage Law

KVL: The sum of all voltages associated with sources must equal the sum of the load voltages, so that the net voltage around a closed circuit is zero.



$$\sum_{n=1}^{N} V_n = 0$$

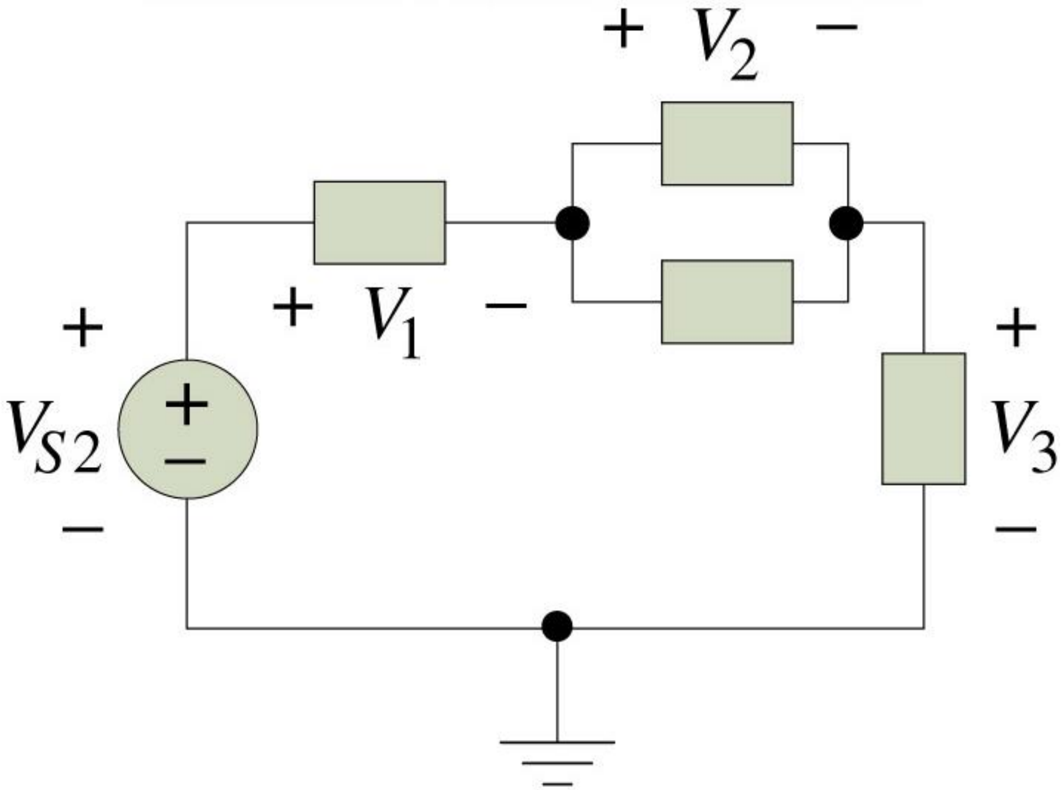
Kirchhoff's Voltage Law : Example

Known Quantities:

V_{S2} = 12 V V_1 = 6 V V_3 = 1 V.

Find:

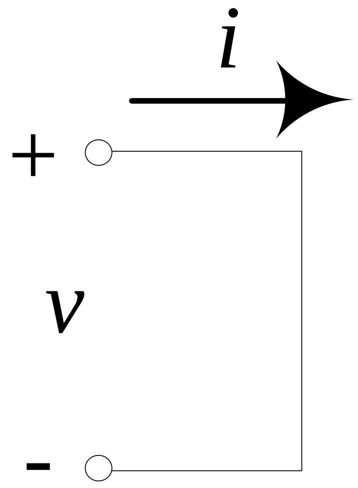
V_2



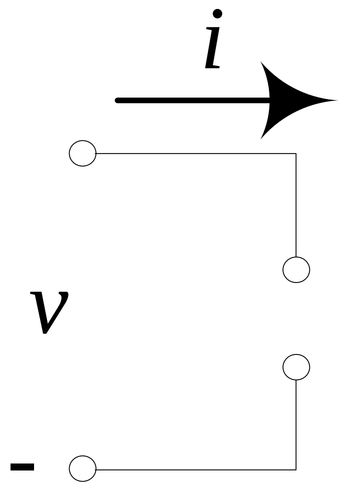
Open and Short Circuits

Open and Short Circuits (1)

- A circuit element with resistance approaching zero is called a short circuit.



- A circuit element whose resistance approaches infinity is called an open circuit.

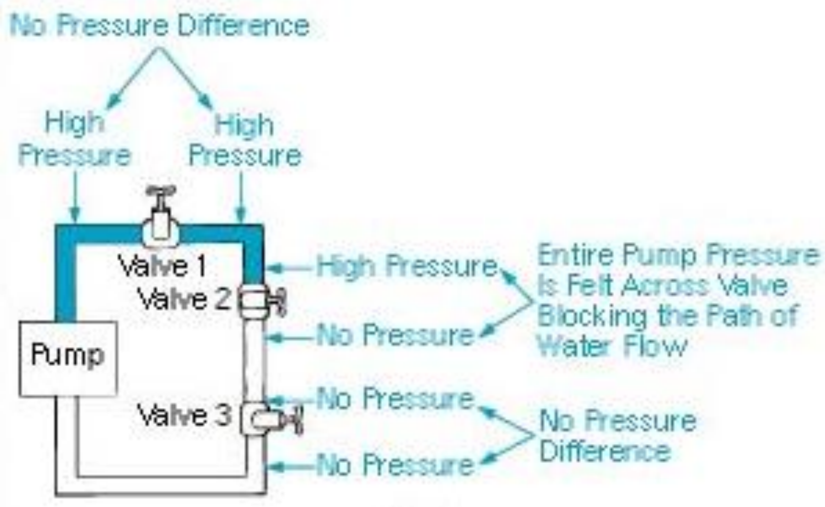


Open and Short Circuits (2)

Imagine a hydraulic circuit with 3 valves connected in series. Initially, all valves are open.

At some point in time, we close valve 2.

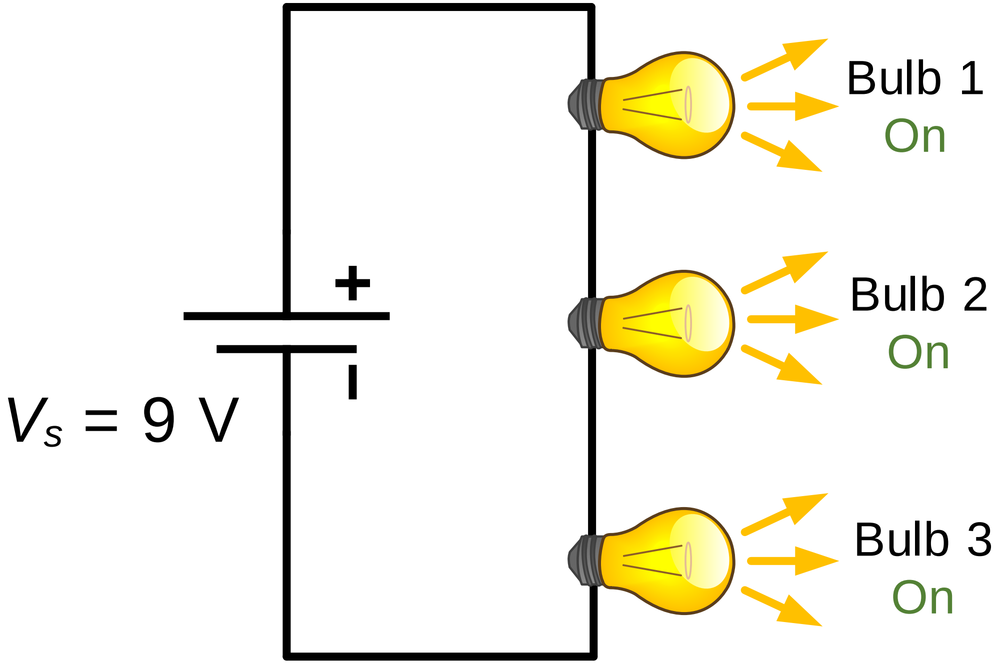
- What will happen with the water flow (current)?

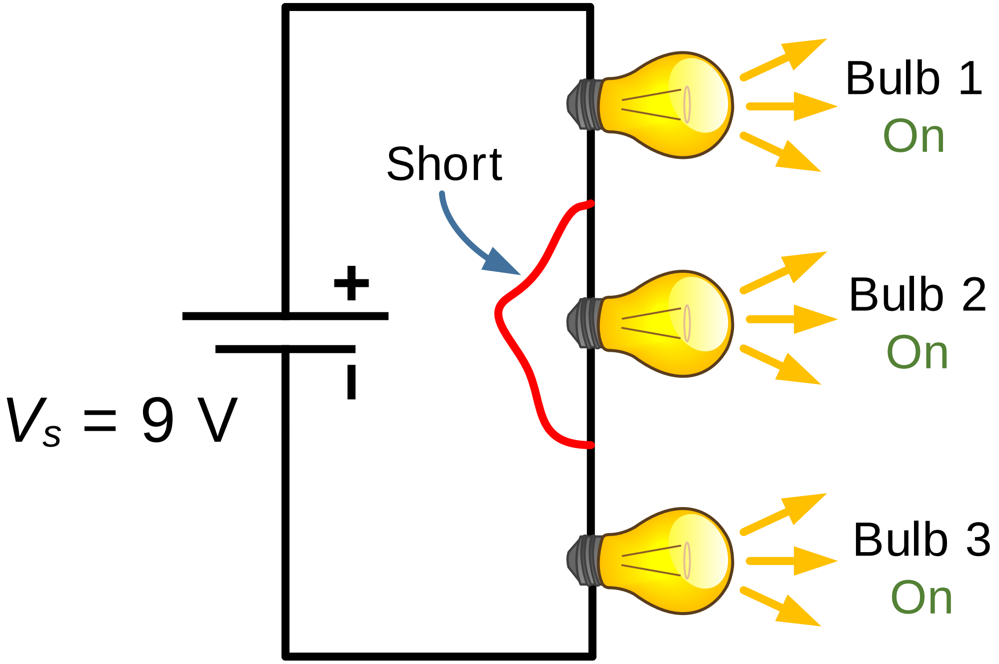


Short Circuits

Assume you have a short Christmas lights line with 3 lightbulbs in series, as shown below.

- What will happen within the circuit if you short circuit lightbulb 2?





Open and Short Circuits (3)

Open circuit **increases** the overall **resistance**. Therefore, the total **current decreases**.

- When the opened branch is in series, the voltage across it becomes equal to the supply voltage.
- In the case of a parallel branch, the voltage across it has a parallel branching voltage.

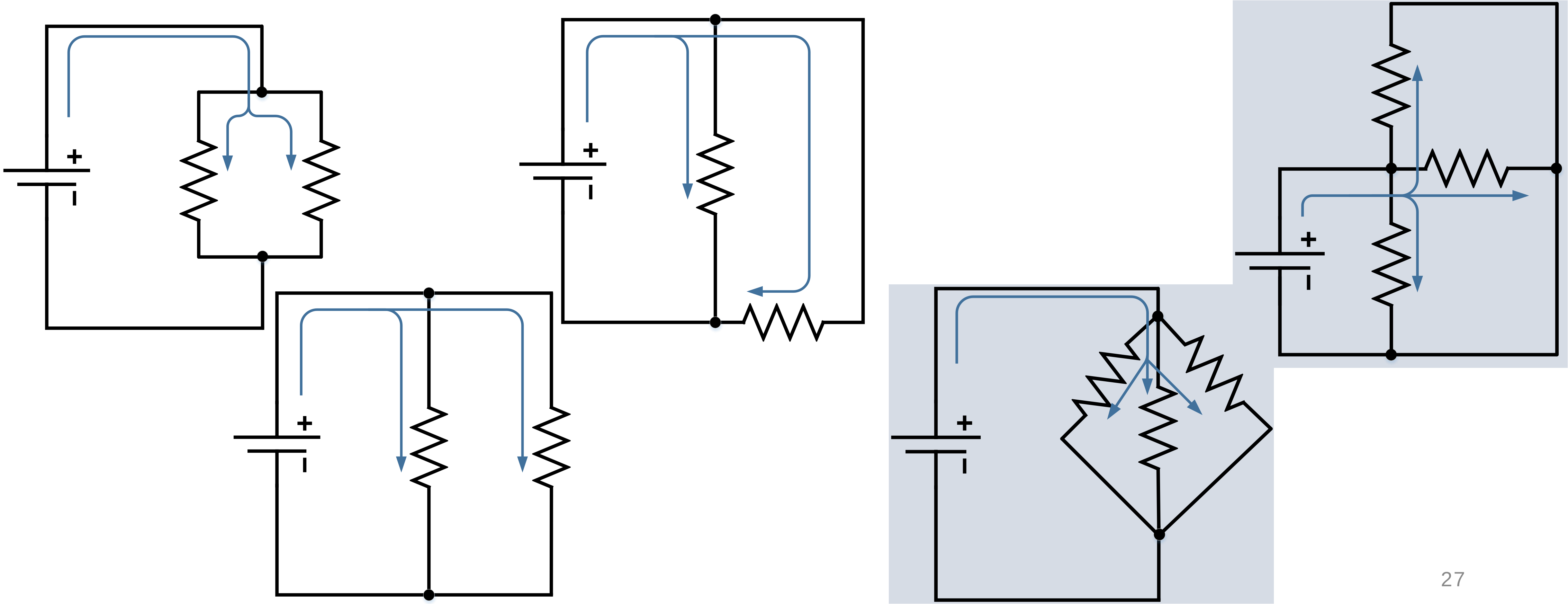
Short circuit **decreases** the overall resistance. Therefore, the total **current increases**.

- When the shorted elements are in series or parallel branches, the voltage drop across them is 0 V.

Parallel Circuits

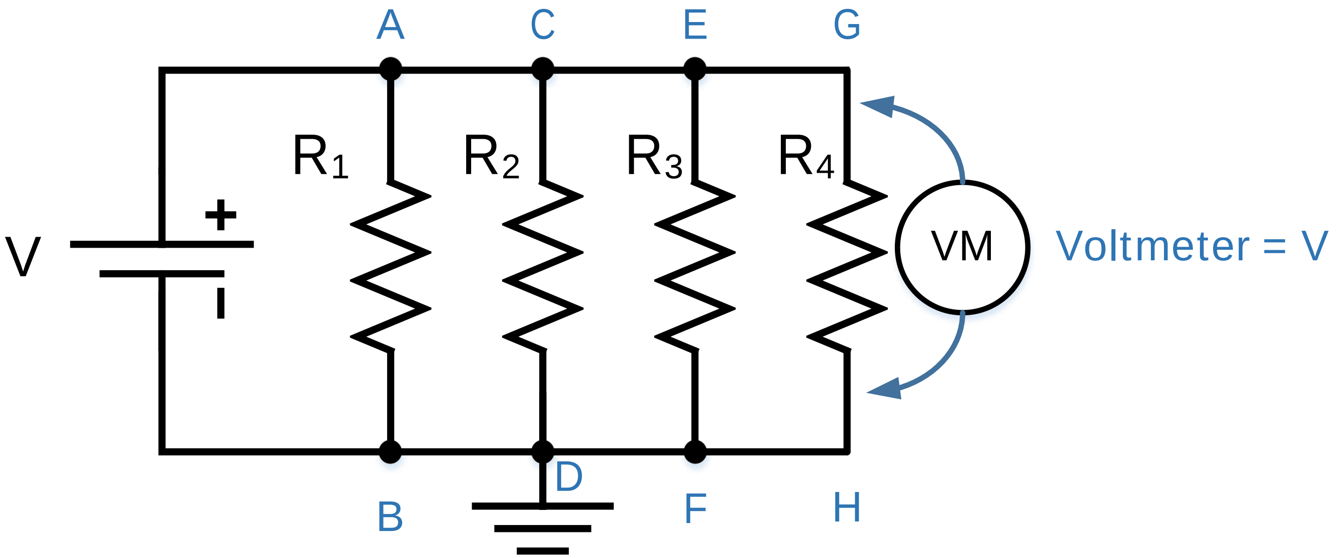
Parallel Circuits (1)

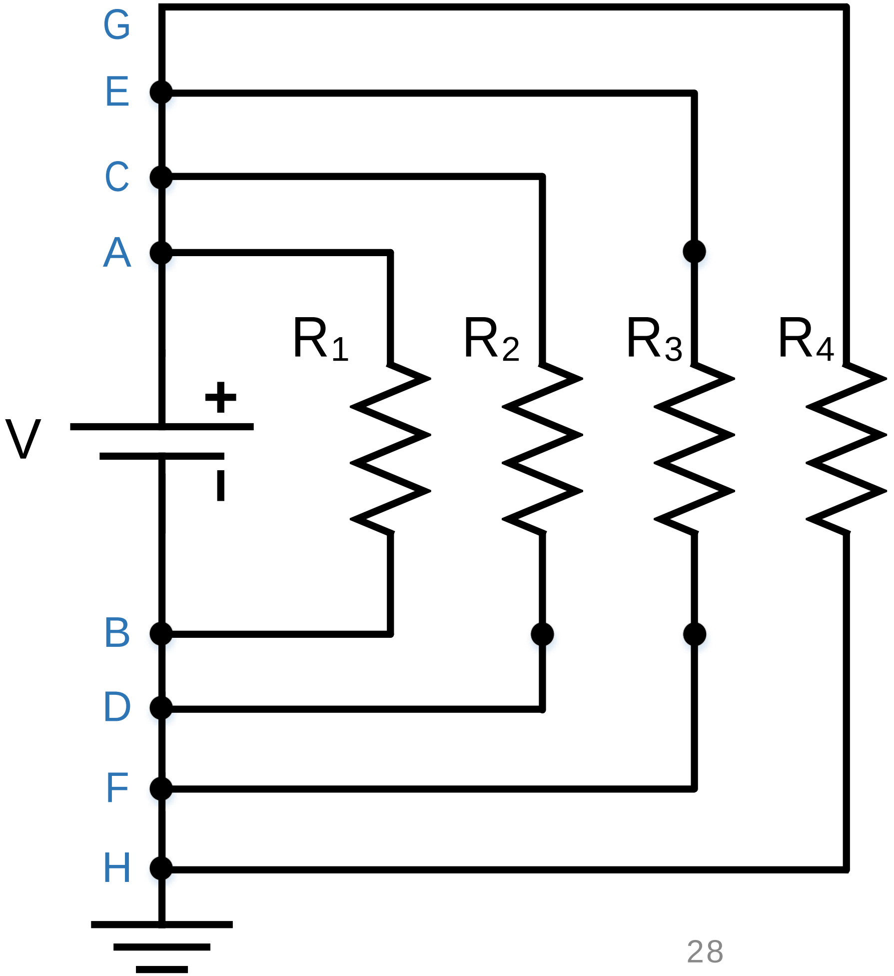
- Circuits with at least 2 paths for the current to flow are call parallel circuits



Parallel Circuits (2)

- Voltage drop across each branch of a parallel circuit is the same





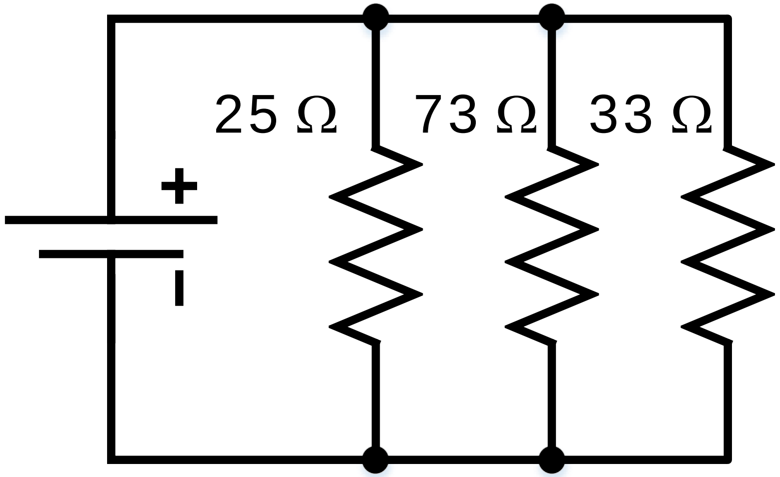
Conductance in Parallel Circuits

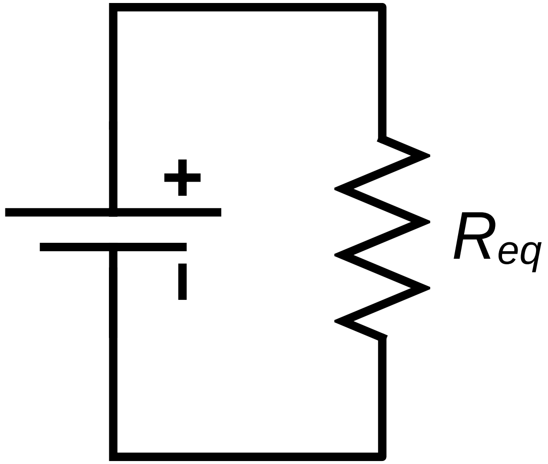
Conductance is the inverse of resistance $G = \frac{1}{R}$

The conductance of a parallel circuit is equal to a sum of conductances of each branch

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots}$$





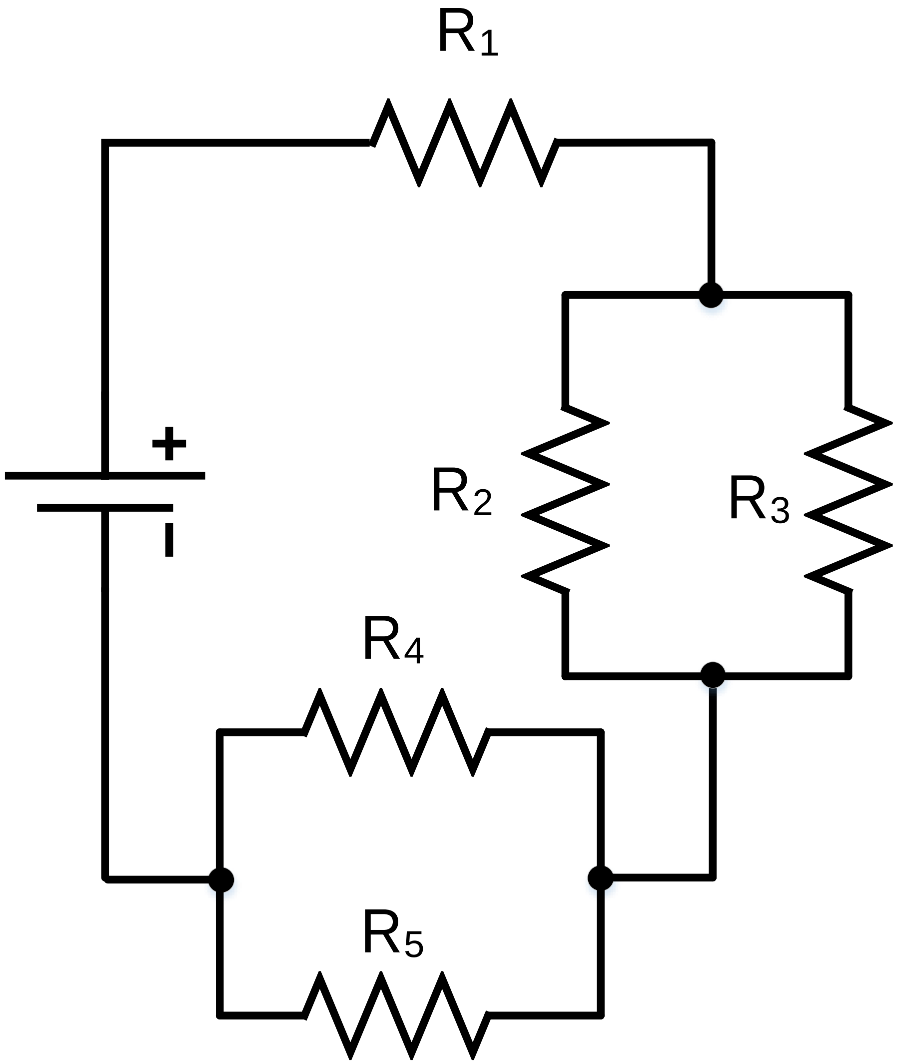
Parallel Circuits (3)

In general, all electronic circuits consist of combinations of series circuit and parallel circuit.

- Series circuit: Current has only 1 path to flow
- Parallel circuit: Current has 2 or more paths

How to find the equivalent resistance of complex circuits:

1. Find the equivalent resistance of all resistors in series
2. Repeat of all resistors in parallel
3. Finally, find the total resistance of resulting series circuit



Thank you for your attention!

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Physics 2. Electrical Engineering

Week 3 Network Analysis 1

Igor Gaponov

Professor, Institute of Robotics and Computer Vision

Objectives

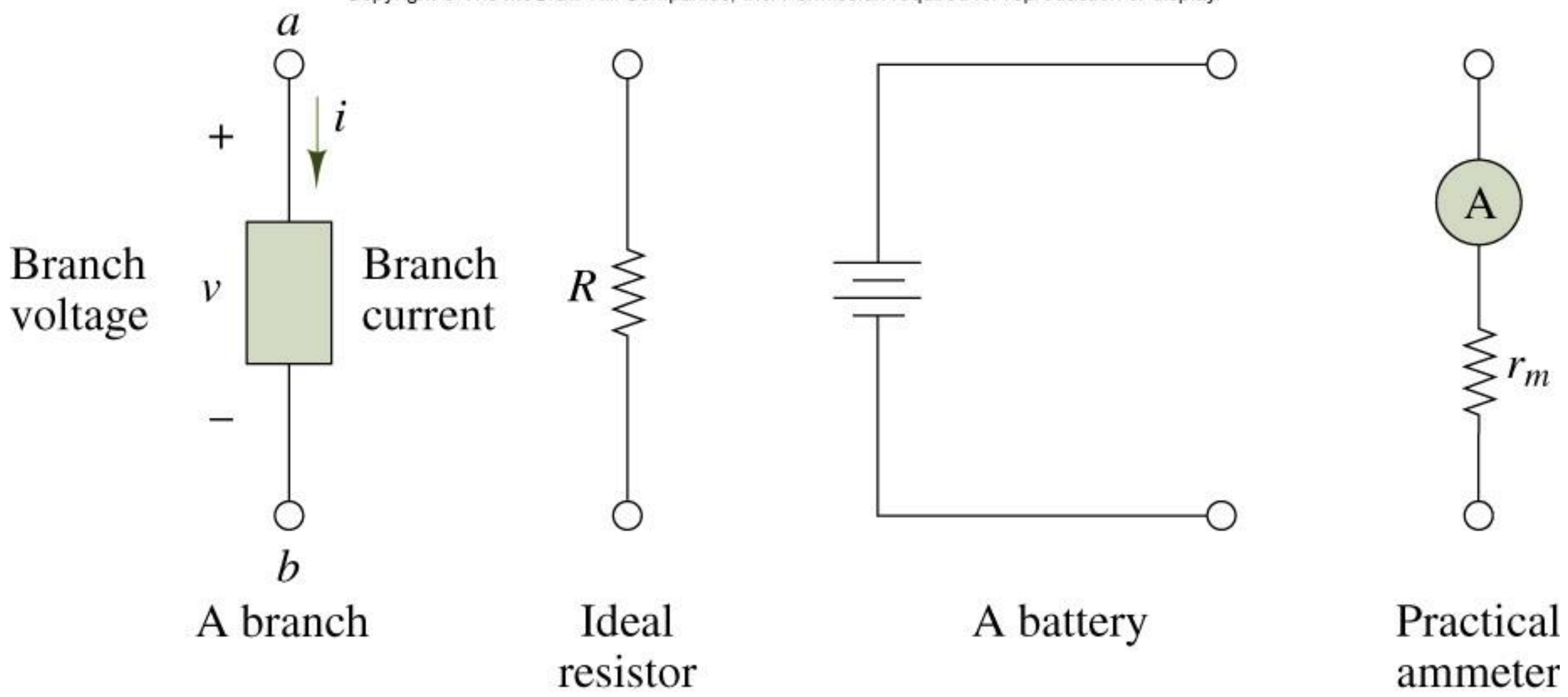
The main objectives of today's lecture are:

- Review the concepts of equivalent resistance
- Learn node voltage method
- Learn mesh current method

Last Week's Review

Electric Circuits : Branch

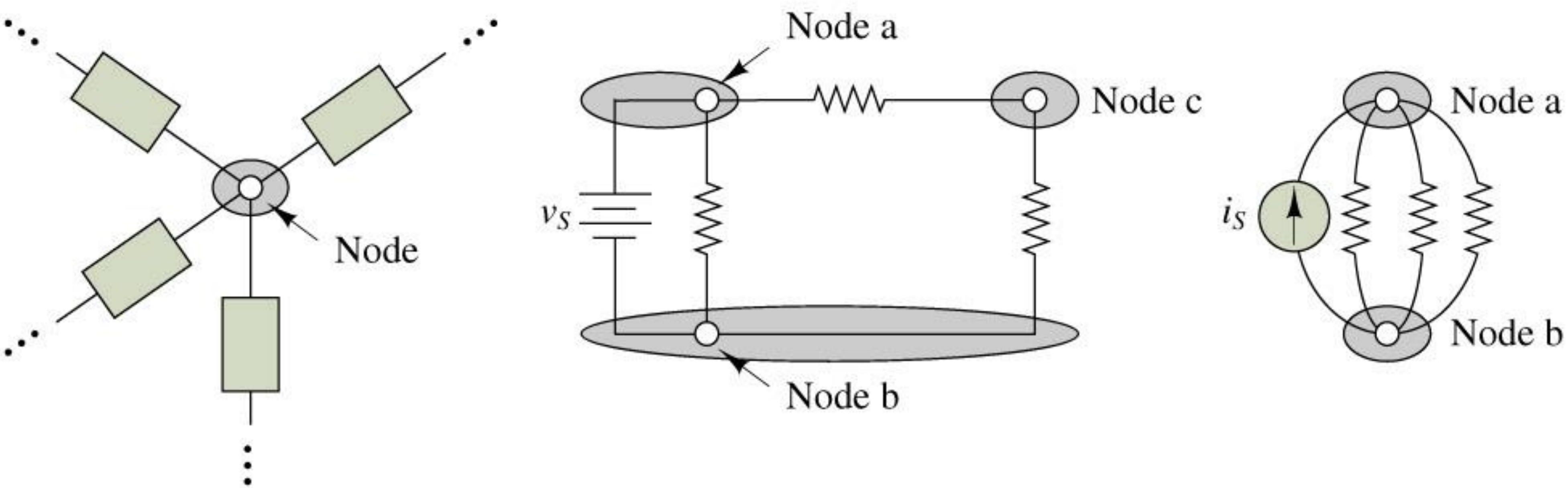
A branch is any portion of a circuit with two terminals connected to it.



Examples of circuit branches

Electric Circuits : Node

- A node is a junction of two or more branches.



Examples of nodes in practical circuits

Kirchhoff's Current Law

KCL: The sum of the currents at a node must equal zero.

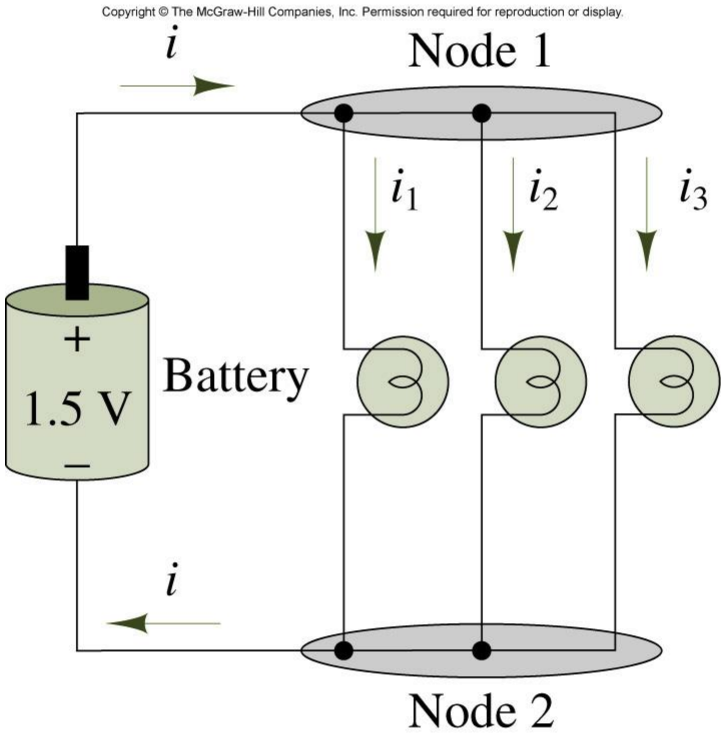


Illustration of KCL at node 1: $-i + i_1 + i_2 + i_3 = 0$

$$\sum_{n=1}^{N} i_n = 0$$

Kirchhoff’s Voltage Law

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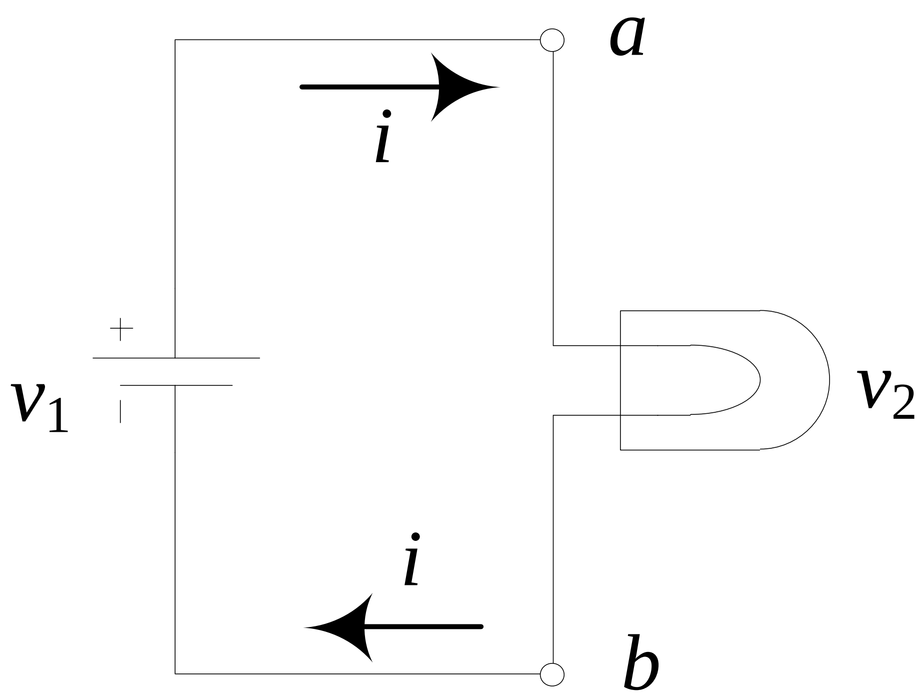
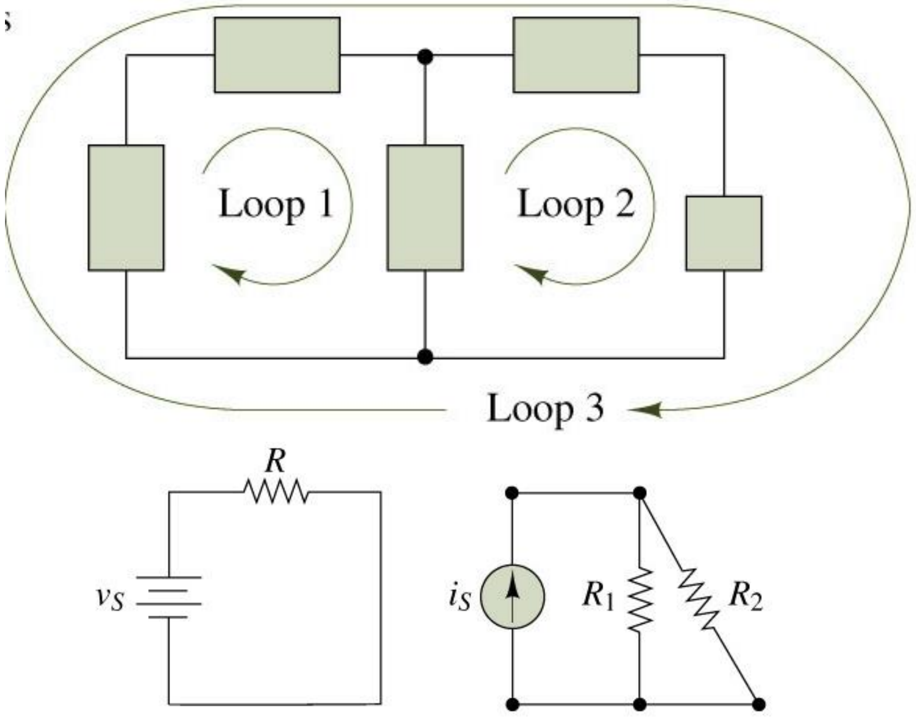
KVL: The sum of all voltages associated with sources must equal the sum of the load voltages, so that the net voltage around a closed circuit is zero.



$$\sum_{n=1}^{N} V_n = 0$$

Loops and Meshes

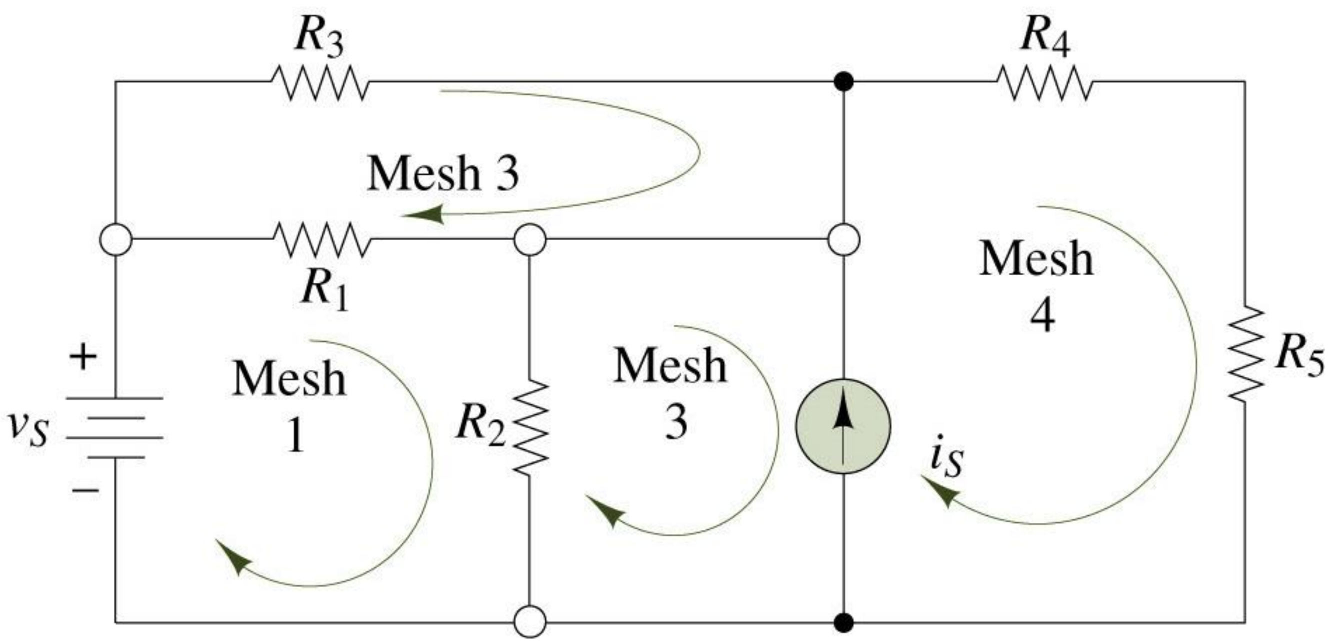
- A loop is any closed connection of branches



1-loop circuit

3-loop circuit
(How many nodes in this circuit?)

- A mesh is a loop that does not contain other loops



Node Voltage Method

Electrical Network Analysis

The analysis of an electrical network consists of determining each of the unknown branch currents and node voltages.

- It is therefore important to define all the relevant variables as clearly as possible, and in systematic fashion.
- Once the known and unknown variables have been identified, a set of equations relating these variables is constructed, and these equations are solved by means of suitable techniques.

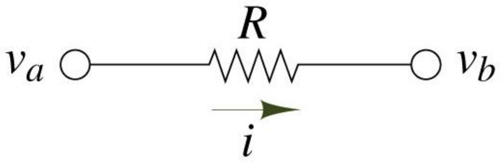
The analysis of electric circuits consists of writing the smallest set of equations sufficient to solve for all the unknown variables.

Node Voltage Method (1)

Node voltage method (NVM) is based of defining the voltage at each node as an independent variable.

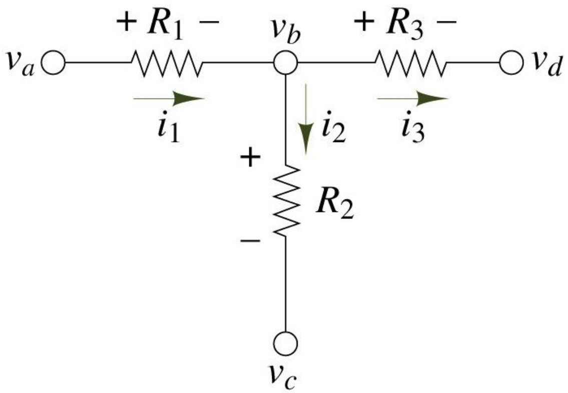
- One of the nodes is selected as a reference node (usually ground).
- Once each node is defined, Ohm's law may be applied between any two adjacent nodes to determine the current flowing in each branch.
- In the NVM, we assign the node voltages v_a and v_b, the branch current flowing from a to be is then expressed in terms of these node voltages:

$$i = \frac{v_a - v_b}{R}$$



- We can then express KCL by

$$\frac{v_a - v_b}{R_1} - \frac{v_b - v_c}{R_2} - \frac{v_b - v_d}{R_3} = 0$$



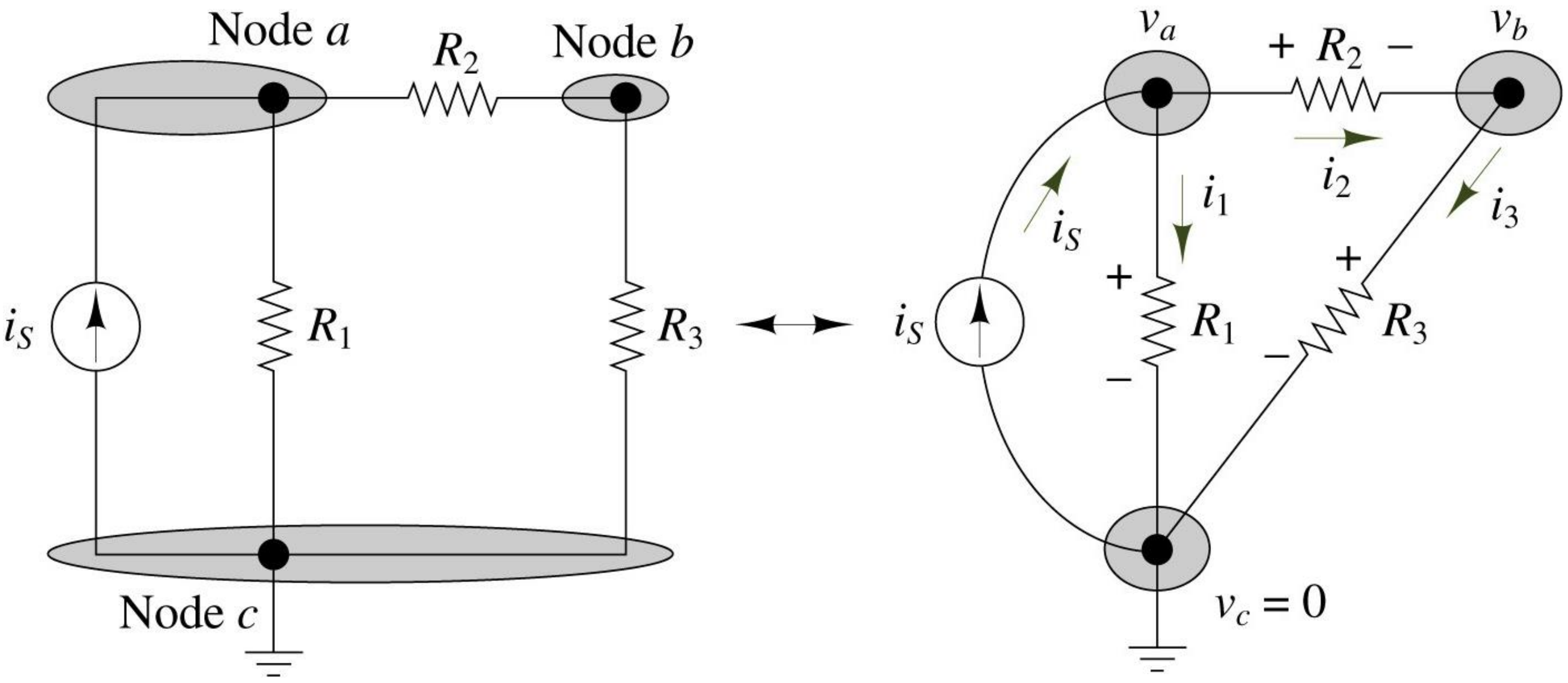
Node Voltage Method (2)

Typical procedure of the NVM is as follows:

1. Select a reference node.
2. Define the remaining n - 1 node voltages as the variables. Each of the m voltage sources in the circuit is associated with a dependent variable. If a node is not connected to a voltage source, then its voltage is treated as an independent variable.
3. Apply KCL at each node labeled as an independent variable.
4. Solve the linear system of n – 1 – m unknowns.

Node Voltage Method : Example (1)

Find all unknown voltages (v_a, v_b, v_c) and currents (i_1, i_2, i_3) in the circuit below.



Node Voltage Method : Example (2)

1. Select a reference node.
2. Define the remaining n - 1 node voltages as the variables.
3. Apply KCL at each node labeled as an independent variable.

$$i_s - i_1 - i_2 = 0$$

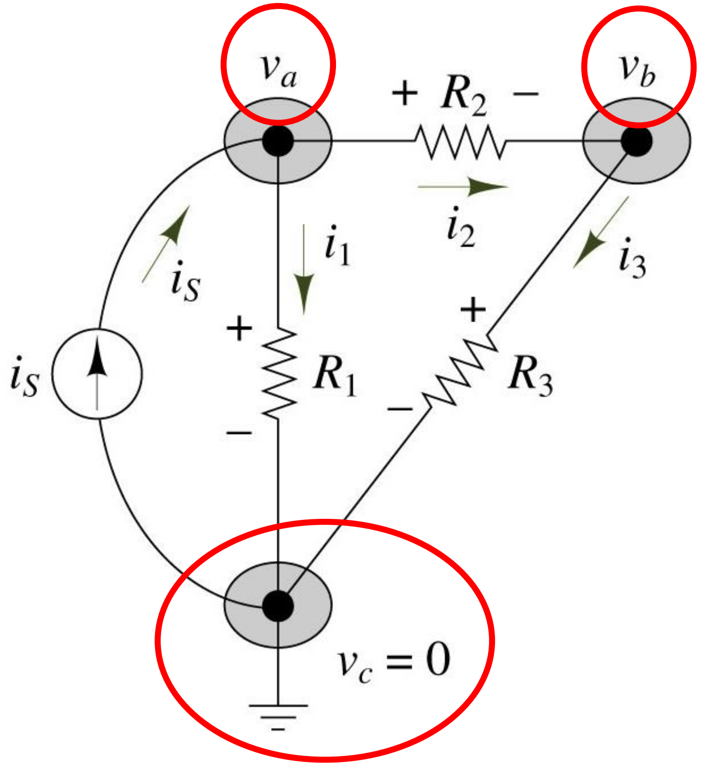
$$i_2 - i_3 = 0$$

$$i_1 + i_3 - i_s = 0 \rightarrow \text{redundant}$$

$$i_s - \frac{v_a}{R_1} - \frac{v_a - v_b}{R_2} = 0$$

$$\frac{v_a - v_b}{R_2} - \frac{v_b}{R_3} = 0$$

4. Solve the linear system of n – 1 – m unknowns.



Node Voltage Method : Example (3)

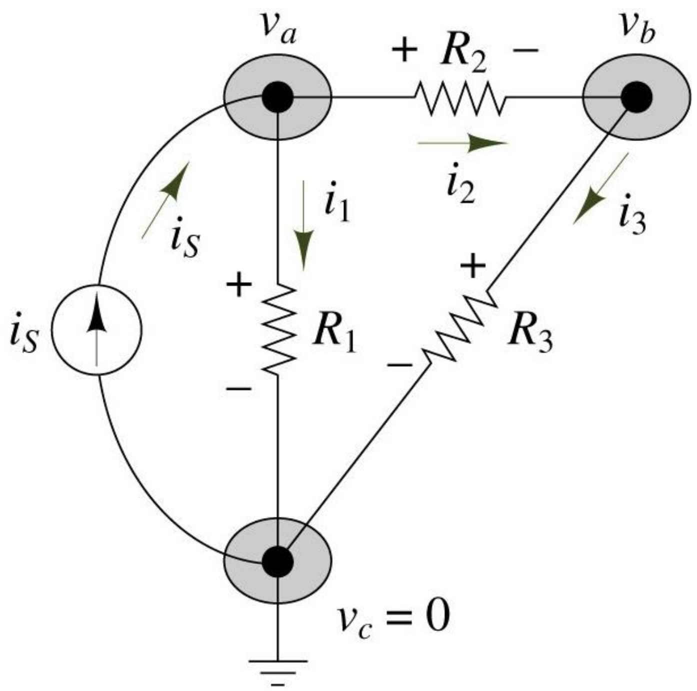
Solve the linear system of n – 1 – m unknowns.

$$i_s - \frac{v_a}{R_1} - \frac{v_a - v_b}{R_2} = 0$$

$$\frac{v_a - v_b}{R_2} - \frac{v_b}{R_3} = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_a + \left(-\frac{1}{R_2}\right) v_b = i_s$$

$$\left(-\frac{1}{R_2}\right) v_a + \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_b = 0$$



Q: But… We have only found voltages, what about the **currents**?

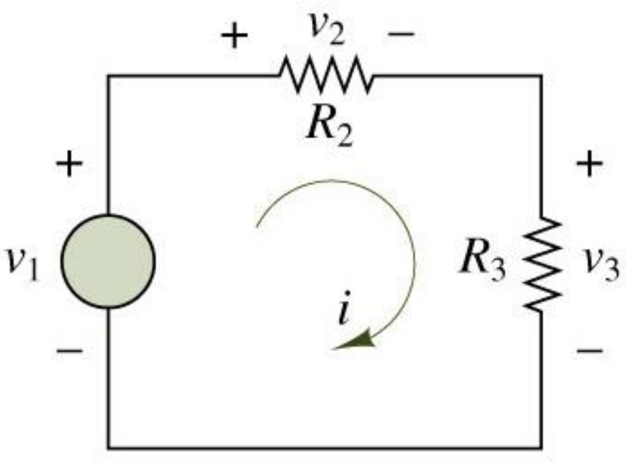
Mesh Current Method

Mesh Current Method (1)

The second method of circuit analysis discussed in this chapter employs mesh currents as the independent variables.

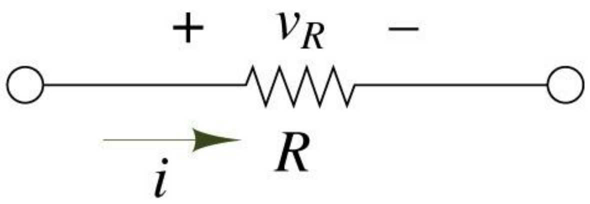
Subsequent application of **Kirchhoff's voltage law** around each mesh provides the desired system of equations.

- Once the direction of current flow has been selected, KVL requires that $v_1 - v_2 - v_3 = 0$



A mesh

- The current I, defined as flowing from left to right, establishes the polarity of the voltage across R :

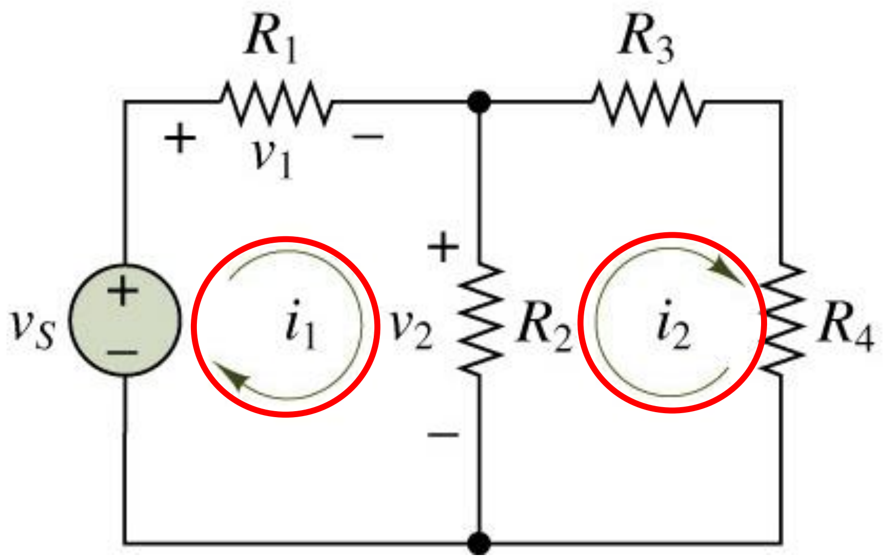


Mesh Current Method (2)

Typical procedure of the MCM is as follows:

1. Define each mesh current consistently. Unknown mesh currents will be always defined in the clockwise direction.
2. In a circuit with n meshes and m current sources, $n - m$ independent equations will result. The unknown mesh currents are $n - m$ independent variables.
3. Apply KVL to each mesh containing an unknown mesh current, expressing each voltage in terms of one or more mesh currents.
4. Solve the linear system of $n - m$ unknowns.

Mesh Current Method : Example 1



Find all voltages and currents in the circuit below.

1. Define each mesh current consistently.
2. In a circuit with n meshes and m current sources, $n - m$ independent equations will result. The unknown mesh currents are $n - m$ independent variables.
3. Apply KVL to each mesh containing an unknown current:

Mesh 1:

$$v_S - v_1 - v_2 = 0$$

$$v_1 = i_1 R_1$$

$$v_2 = (i_1 - i_2)R_2$$

$$v_s - i_1 R_2 - (i_1 - i_2)R_2$$

Mesh 2:

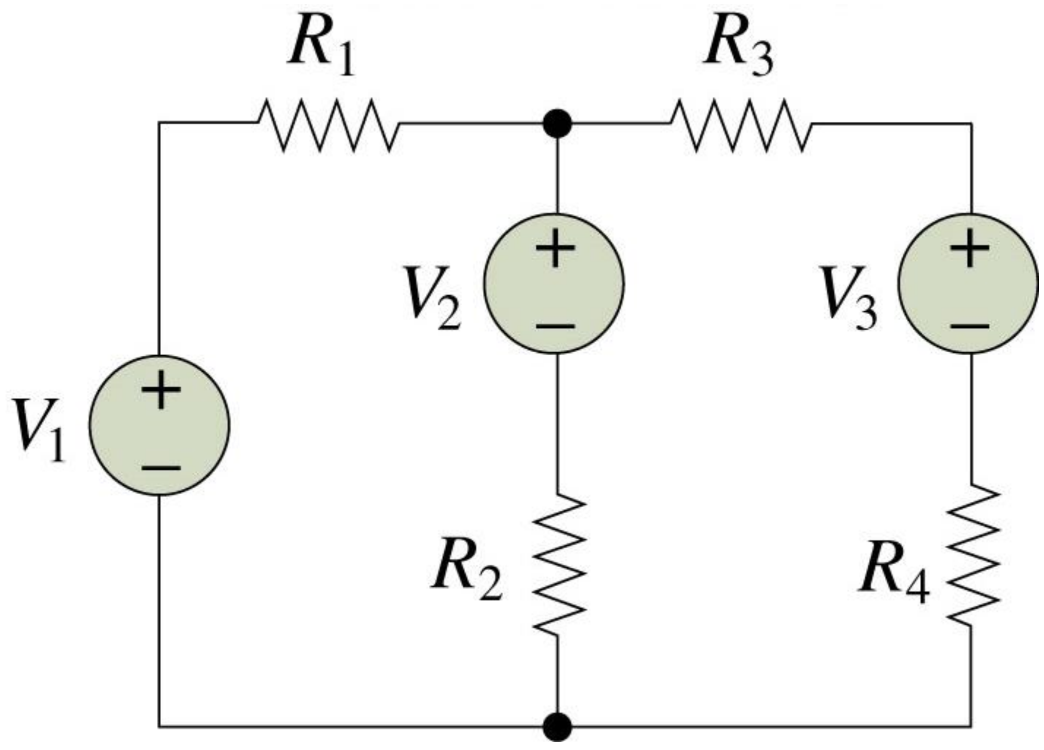
$$v_2 + v_3 + v_4 = 0$$

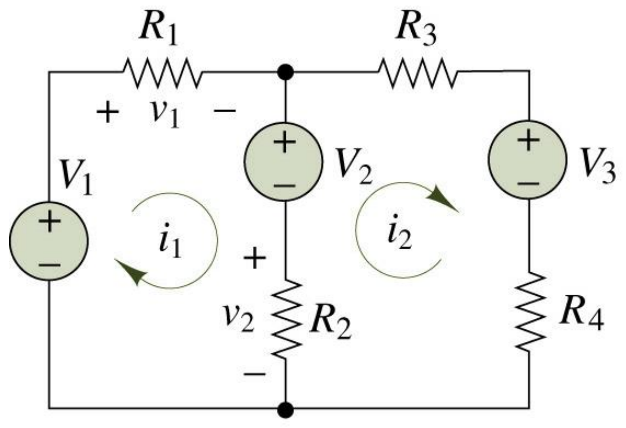
where

$$v_2 = (i_2 - i_1)R_2, \qquad v_3 = i_2 R_3, \qquad v_4 = i_2 R_4$$

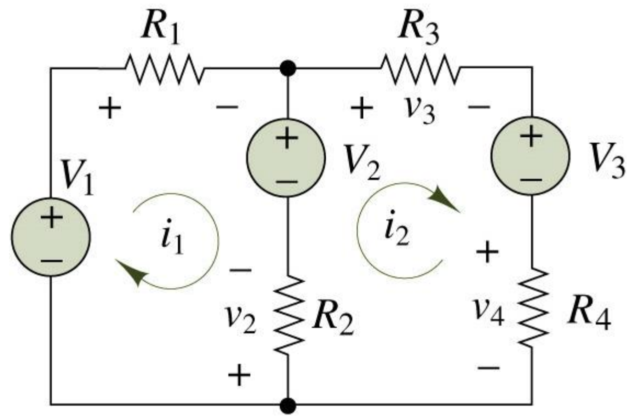
Mesh Current Method : Example 2 (1)

Find all voltages and currents in the circuit below.





Analysis of mesh 1



Analysis of mesh 2

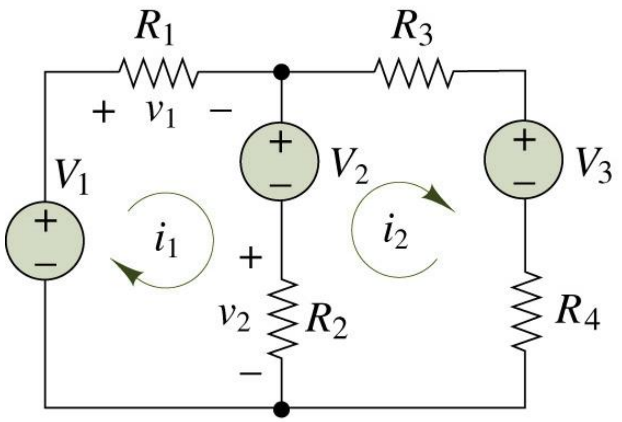
Mesh Current Method : Example 2 (2)

Applying KVL and Ohm's law to Mesh 1 yields:

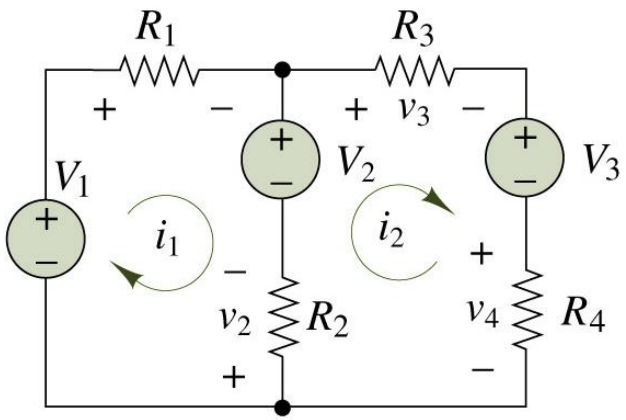
$$V_1 - R_1 i_1 - V_2 - R_2(i_1 - i_2) = 0$$

Applying KVL and Ohm's law to Mesh 2 yields:

$$R_2(i_2 - i_1) - V_2 + R_3 i_2 + V_3 + R_4 i_2 = 0$$



Analysis of mesh 1



Analysis of mesh 2

Thank you for your attention!

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Physics 2. Electrical Engineering

Week 4 Network Analysis 2

Igor Gaponov

Professor, Institute of Robotics and Computer Vision

Objectives

The main objectives of today's lecture are:

- Review the concepts of equivalent resistance
- Learn node voltage method
- Learn mesh current method

Last Week’s Review

Electrical Network Analysis

The analysis of an electrical network consists of determining each of the unknown branch currents and node voltages.

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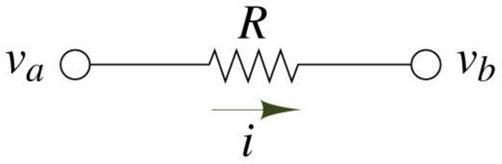
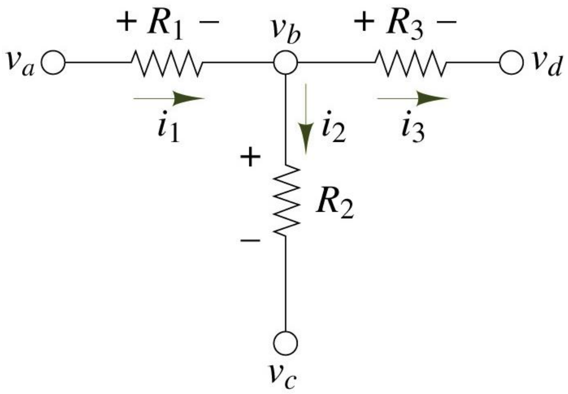


- We can then express KCL by

$$\frac{v_a - v_b}{R_1} - \frac{v_b - v_c}{R_2} - \frac{v_b - v_d}{R_3} = 0$$



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Mesh Current Method (1)

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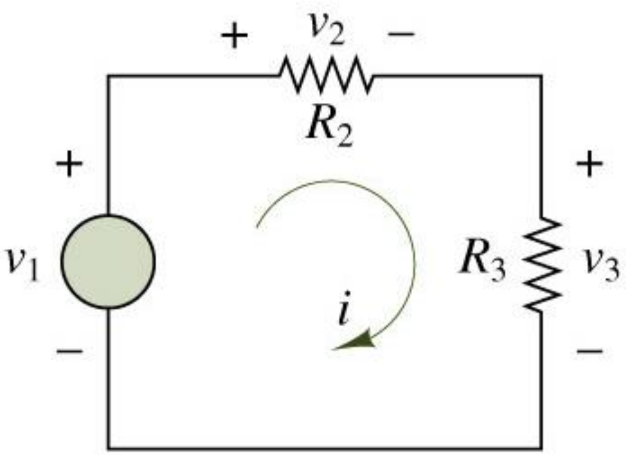

A mesh

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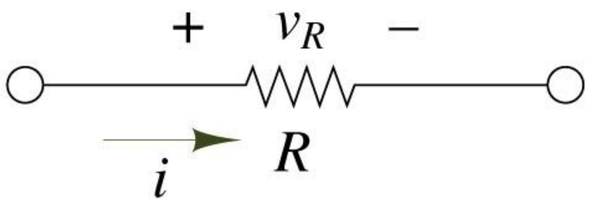

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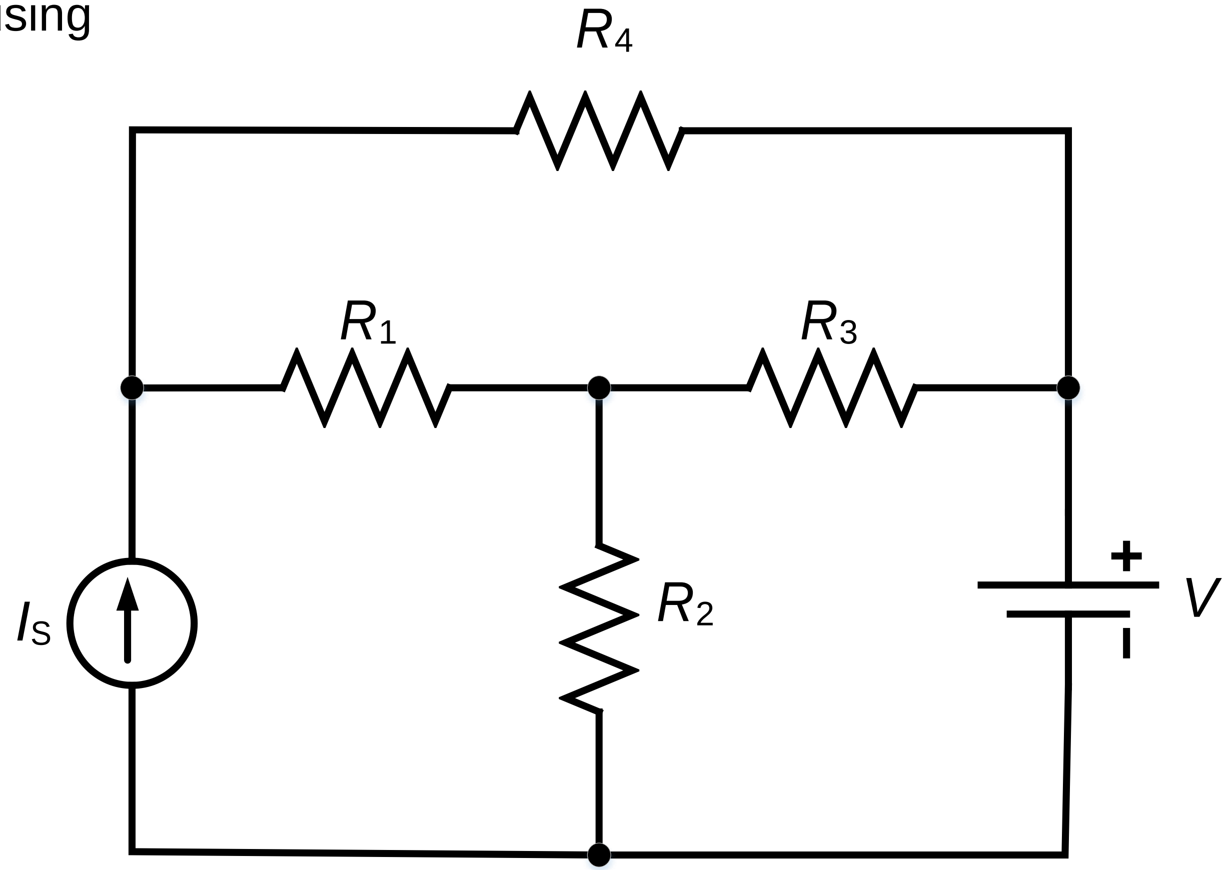
4. Solve the linear system of $n - m$ unknowns.

Electrical Network Analysis : Exercise

Solve for voltages and currents in the given circuit using

- Node voltage method
- Mesh current method



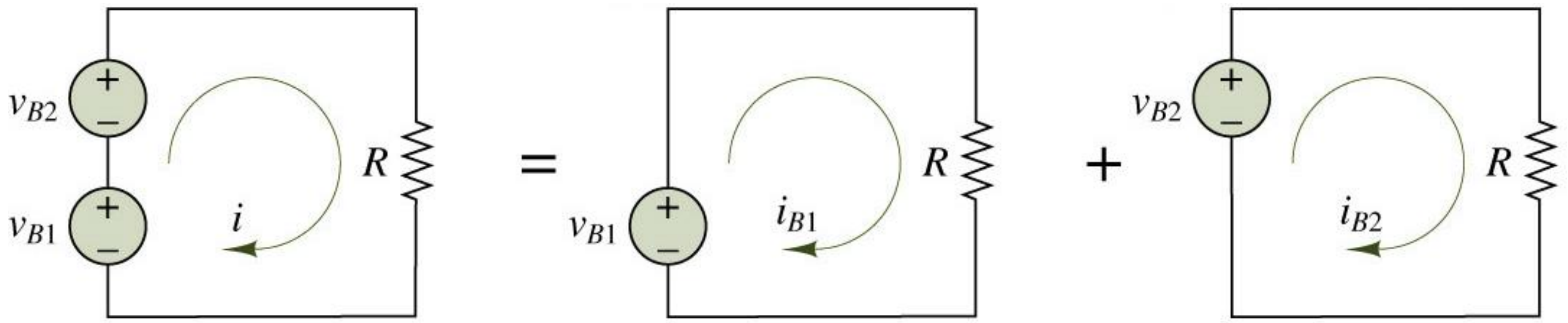
The Principle of Superposition

The Principle of Superposition (1)

Unlike precise analysis technique, like the mesh current and node voltage methods, the principle of superposition is a conceptual aid that can be very useful in visualizing the behavior of a circuit containing multiple sources.

In a circuit containing N sources,

- each branch voltage and current is the sum of N voltages and currents, each of which may be computed by setting all but one source equal to zero and solving the circuit containing that single source.

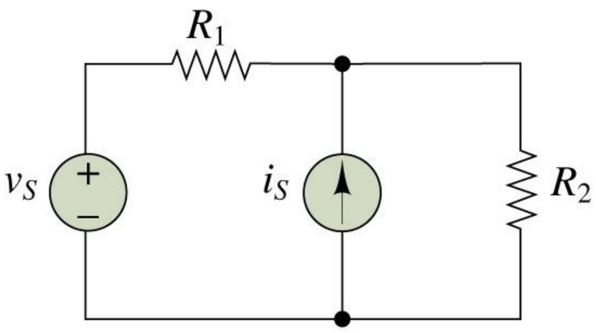


$$i = i_{B1} + i_{B2}$$

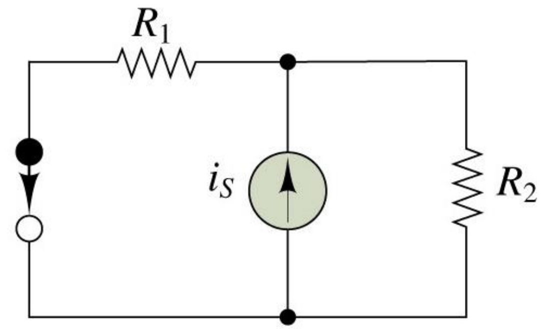
$$= \frac{v_{B1}}{R} + \frac{v_{B2}}{R} = \frac{v_{B1} + v_{B2}}{R}$$

The Principle of Superposition (2)

- In order to set a voltage source equal to zero, we replace it with a short circuit

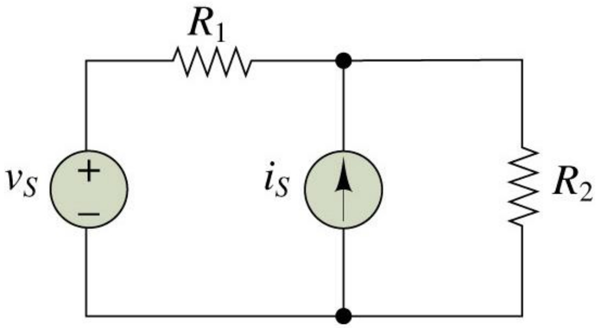


A circuit

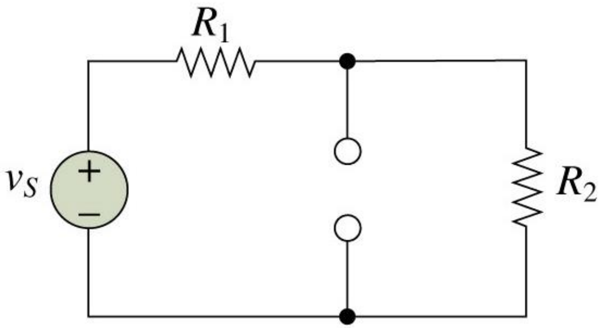


The same circuit with $v_S = 0$

- In order to set a current source equal to zero, we replace it with an open circuit



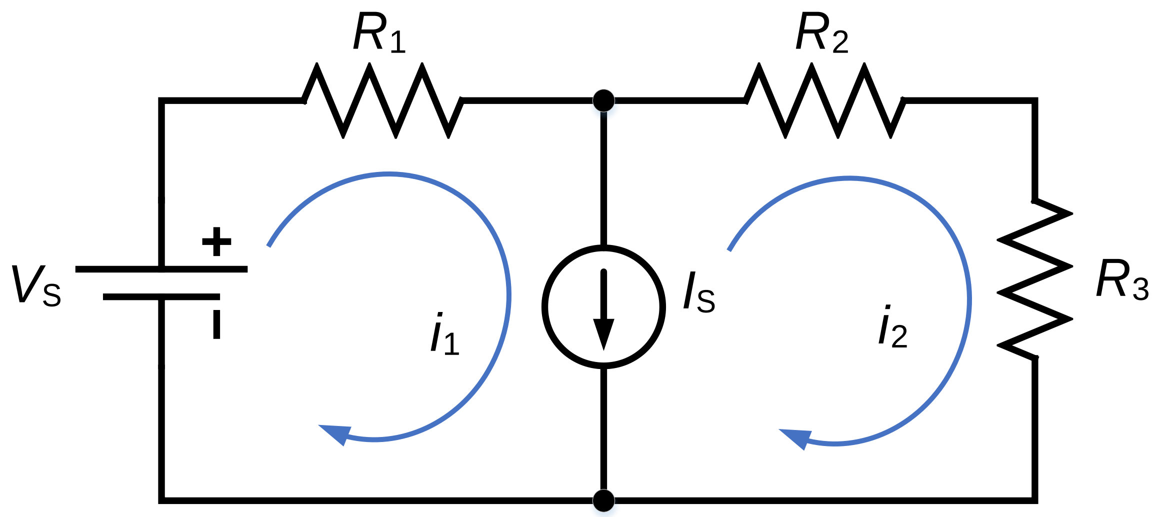
A circuit



The same circuit with $i_S = 0$

The Principle of Superposition : Ex. 1

- Find unknown current i_2



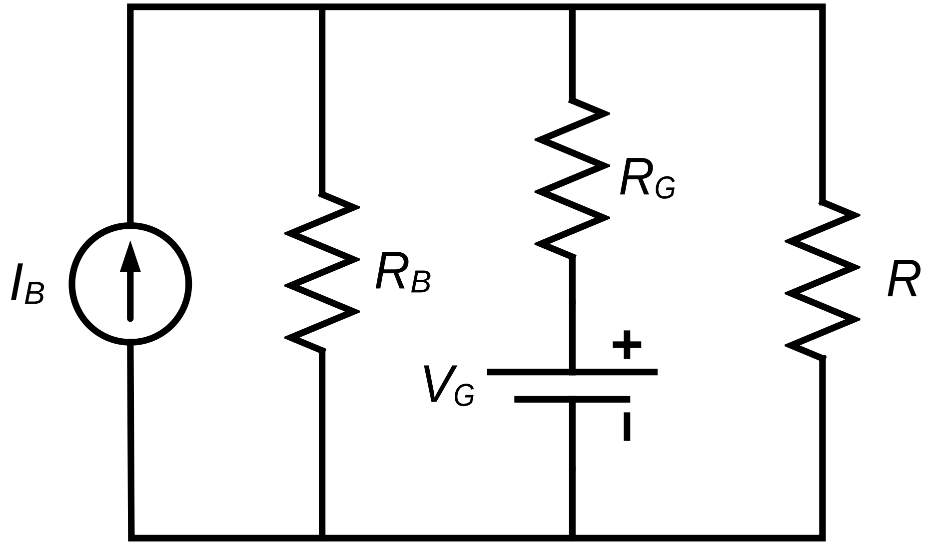
$$i_{2-V} = \frac{V_s}{R_1 + R_2 + R_3}$$

$$i_{2-I} = \frac{1/(R_2 + R_3)}{1/R_1 + 1/(R_2 + R_3)}(-I_s) = \frac{R_1}{R_1 + R_2 + R_3}(-I_s)$$

$$i_2 = i_{2-V} + i_{2-I}$$

The Principle of Superposition : Ex. 2

- Find unknown voltage on resistor R



Superposition:

$$I_B - \frac{V_{R-I}}{R_B} - \frac{V_{R-I}}{R_G} - \frac{V_{R-I}}{R} = 0$$

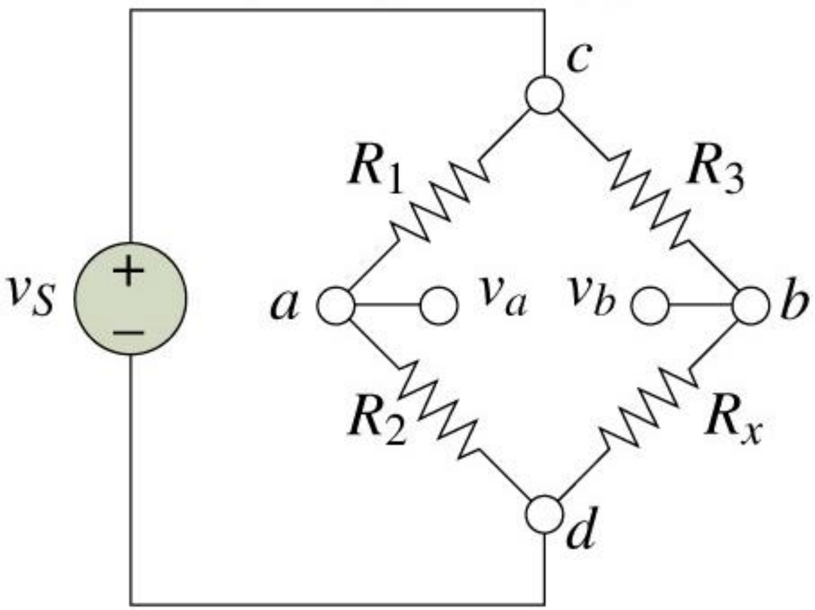
$$\frac{V_{R-V}}{R_B} + \frac{V_{R-V}}{R} + \frac{V_{R-V} - V_G}{R_G} = 0$$

Wheatstone Bridge Circuit

Wheatstone Bridge Circuit (1)

The Wheatstone bridge circuit is frequently encountered in a variety of measurement circuits.

- The Wheatstone bridge circuit is a circuit consisting of 4 resistors, with 1 of them of unknown resistance.
- The general form of the bridge circuit is shown in Figure on the right, where R_1, R_2, and R_3 are known while R_x is an unknown resistance, to be determined.

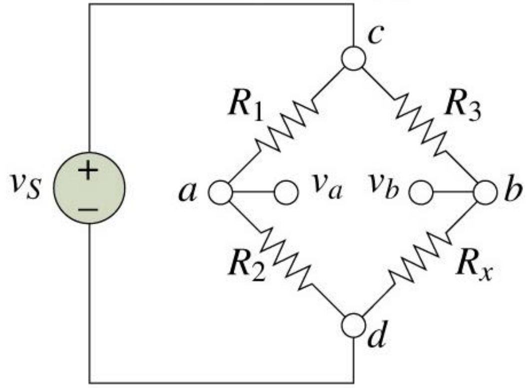


Wheatstone Bridge Circuit (2)

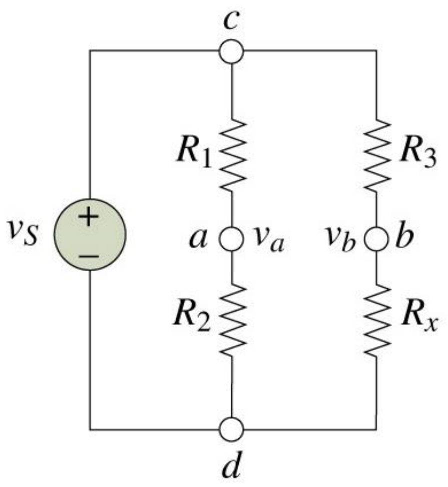
The circuit may also be redrawn as shown in Figure (b) on the right.

- The latter circuit is used to demonstrate the voltage divider rule in a mixed series-parallel circuit
- Finding unknown voltage V_{ab} yields

$$V_{ab} = V_{ad} - V_{bd} = V_s \left(\frac{R_2}{R_1 + R_2} - \frac{R_x}{R_3 + R_x} \right)$$



(a)

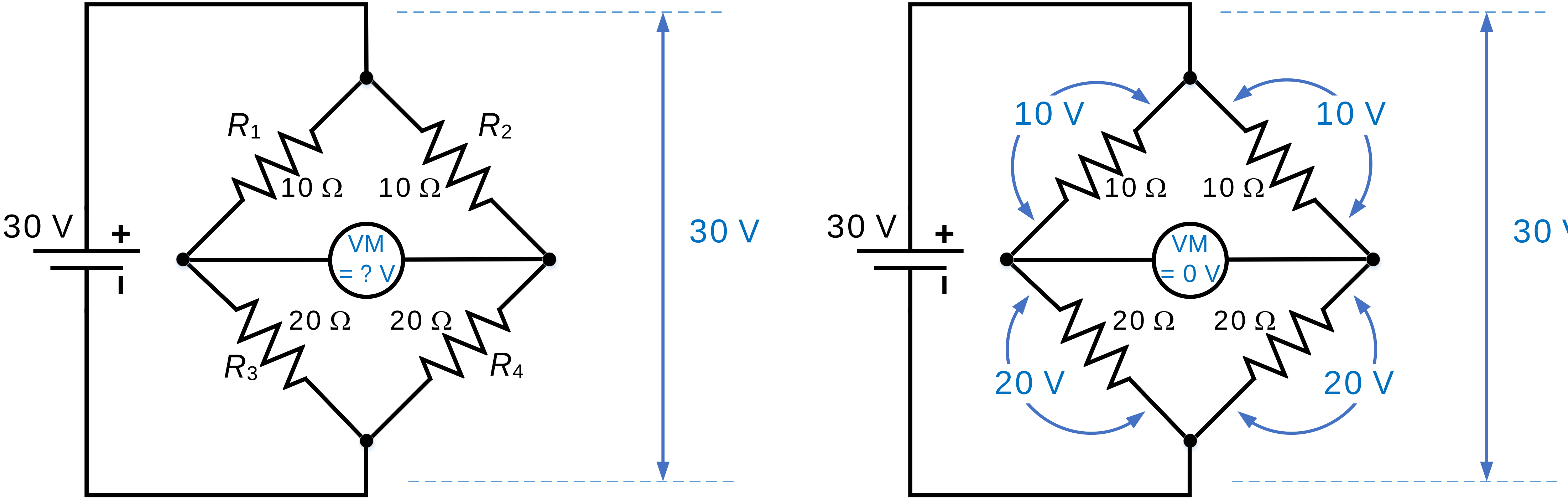


(b)

Wheatstone Bridge Circuit (3)

If the resistances in the upper and bottom halves are equal, there is no voltage difference V_{ab}.

- What happens to V_{ab} if R_3 drops to 10 Ω?



Thank you for your attention!

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