

Problem 1

=====length-3 frequent=====

< {2} {3} {4} >

< {2 5} {3} >

< {3} {4} {5} >

< {1} {2} {3} >

< {1} {2 5} >

< {1} {5} {3} >

< {5} {3 4} >

Joining:

< {2} {3} {4} > remove first item is < {3} {4} >

< {2} {3} {4} > remove last item is < {2} {3} >

< {2} {3} {4} > and < {2} {3} {4} > could not join

< {2} {3} {4} > remove first item is < {3} {4} >

< {2 5} {3} > remove last item is < {2 5} >

< {2} {3} {4} > and < {2 5} {3} > could not join

< {2} {3} {4} > remove first item is < {3} {4} >

< {3} {4} {5} > remove last item is < {3} {4} >

< {2} {3} {4} > and < {3} {4} {5} > could join

length-4 frequent < {2} {3} {4} {5} >

< {2} {3} {4} > remove first item is < {3} {4} >

< {1} {2} {3} > remove last item is < {1} {2} >

< {2} {3} {4} > and < {1} {2} {3} > could not join

< {2} {3} {4} > remove first item is < {3} {4} >

< {1} {2 5} > remove last item is < {1} {2} >

< {2} {3} {4} > and < {1} {2 5} > could not join

< {2} {3} {4} > remove first item is < {3} {4} >

< {1} {5} {3} > remove last item is < {1} {5} >

< {2} {3} {4} > and < {1} {5} {3} > could not join

< {2} {3} {4} > remove first item is < {3} {4} >

< {5} {3 4} > remove last item is < {5} {3} >
< {2} {3} {4} > and < {5} {3 4} > could not join

< {2 5} {3} > remove first item is < {5} {3} >
< {2} {3} {4} > remove last item is < {2} {3} >
< {2 5} {3} > and < {2} {3} {4} > could not join

< {2 5} {3} > remove first item is < {5} {3} >
< {2 5} {3} > remove last item is < {2 5} >
< {2 5} {3} > and < {2 5} {3} > could not join

< {2 5} {3} > remove first item is < {5} {3} >
< {3} {4} {5} > remove last item is < {3} {4} >
< {2 5} {3} > and < {3} {4} {5} > could not join

< {2 5} {3} > remove first item is < {5} {3} >
< {1} {2} {3} > remove last item is < {1} {2} >
< {2 5} {3} > and < {1} {2} {3} > could not join

< {2 5} {3} > remove first item is < {5} {3} >
< {1} {2 5} > remove last item is < {1} {2} >
< {2 5} {3} > and < {1} {2 5} > could not join

< {2 5} {3} > remove first item is < {5} {3} >
< {1} {5} {3} > remove last item is < {1} {5} >
< {2 5} {3} > and < {1} {5} {3} > could not join

< {2 5} {3} > remove first item is < {5} {3} >
< {5} {3 4} > remove last item is < {5} {3} >
< {2 5} {3} > and < {5} {3 4} > could join

length-4 frequent < {2 5} {3 4} >

< {3} {4} {5} > remove first item is < {4} {5} >
< {2} {3} {4} > remove last item is < {2} {3} >
< {3} {4} {5} > and < {2} {3} {4} > could not join

< {3} {4} {5} > remove first item is < {4} {5} >
< {2 5} {3} > remove last item is < {2 5} >

$\langle \{3\} \{4\} \{5\} \rangle$ and $\langle \{2\} \{5\} \{3\} \rangle$ could not join

$\langle \{3\} \{4\} \{5\} \rangle$ remove first item is $\langle \{4\} \{5\} \rangle$

$\langle \{3\} \{4\} \{5\} \rangle$ remove last item is $\langle \{3\} \{4\} \rangle$

$\langle \{3\} \{4\} \{5\} \rangle$ and $\langle \{3\} \{4\} \{5\} \rangle$ could not join

$\langle \{3\} \{4\} \{5\} \rangle$ remove first item is $\langle \{4\} \{5\} \rangle$

$\langle \{1\} \{2\} \{3\} \rangle$ remove last item is $\langle \{1\} \{2\} \rangle$

$\langle \{3\} \{4\} \{5\} \rangle$ and $\langle \{1\} \{2\} \{3\} \rangle$ could not join

$\langle \{3\} \{4\} \{5\} \rangle$ remove first item is $\langle \{4\} \{5\} \rangle$

$\langle \{1\} \{2\} \{5\} \rangle$ remove last item is $\langle \{1\} \{2\} \rangle$

$\langle \{3\} \{4\} \{5\} \rangle$ and $\langle \{1\} \{2\} \{5\} \rangle$ could not join

$\langle \{3\} \{4\} \{5\} \rangle$ remove first item is $\langle \{4\} \{5\} \rangle$

$\langle \{1\} \{5\} \{3\} \rangle$ remove last item is $\langle \{1\} \{5\} \rangle$

$\langle \{3\} \{4\} \{5\} \rangle$ and $\langle \{1\} \{5\} \{3\} \rangle$ could not join

$\langle \{3\} \{4\} \{5\} \rangle$ remove first item is $\langle \{4\} \{5\} \rangle$

$\langle \{5\} \{3\} \{4\} \rangle$ remove last item is $\langle \{5\} \{3\} \rangle$

$\langle \{3\} \{4\} \{5\} \rangle$ and $\langle \{5\} \{3\} \{4\} \rangle$ could not join

$\langle \{1\} \{2\} \{3\} \rangle$ remove first item is $\langle \{2\} \{3\} \rangle$

$\langle \{2\} \{3\} \{4\} \rangle$ remove last item is $\langle \{2\} \{3\} \rangle$

$\langle \{1\} \{2\} \{3\} \rangle$ and $\langle \{2\} \{3\} \{4\} \rangle$ could join

length-4 frequent $\langle \{1\} \{2\} \{3\} \{4\} \rangle$

$\langle \{1\} \{2\} \{3\} \rangle$ remove first item is $\langle \{2\} \{3\} \rangle$

$\langle \{2\} \{5\} \{3\} \rangle$ remove last item is $\langle \{2\} \{5\} \rangle$

$\langle \{1\} \{2\} \{3\} \rangle$ and $\langle \{2\} \{5\} \{3\} \rangle$ could not join

$\langle \{1\} \{2\} \{3\} \rangle$ remove first item is $\langle \{2\} \{3\} \rangle$

$\langle \{3\} \{4\} \{5\} \rangle$ remove last item is $\langle \{3\} \{4\} \rangle$

$\langle \{1\} \{2\} \{3\} \rangle$ and $\langle \{3\} \{4\} \{5\} \rangle$ could not join

$\langle \{1\} \{2\} \{3\} \rangle$ remove first item is $\langle \{2\} \{3\} \rangle$

$\langle \{1\} \{2\} \{3\} \rangle$ remove last item is $\langle \{1\} \{2\} \rangle$

$\langle \{1\} \{2\} \{3\} \rangle$ and $\langle \{1\} \{2\} \{3\} \rangle$ could not join

< {1} {2} {3} > remove first item is < {2} {3} >
< {1} {2 5} > remove last item is < {1} {2} >
< {1} {2} {3} > and < {1} {2 5} > could not join

< {1} {2} {3} > remove first item is < {2} {3} >
< {1} {5} {3} > remove last item is < {1} {5} >
< {1} {2} {3} > and < {1} {5} {3} > could not join

< {1} {2} {3} > remove first item is < {2} {3} >
< {5} {3 4} > remove last item is < {5} {3} >
< {1} {2} {3} > and < {5} {3 4} > could not join

< {1} {2 5} > remove first item is < {2 5} >
< {2} {3} {4} > remove last item is < {2} {3} >
< {1} {2 5} > and < {2} {3} {4} > could not join

< {1} {2 5} > remove first item is < {2 5} >
< {2 5} {3} > remove last item is < {2 5} >
< {1} {2 5} > and < {2 5} {3} > could join

length-4 frequent < {1} {2 5} {3} >

< {1} {2 5} > remove first item is < {2 5} >
< {3} {4} {5} > remove last item is < {3} {4} >
< {1} {2 5} > and < {3} {4} {5} > could not join

< {1} {2 5} > remove first item is < {2 5} >
< {1} {2} {3} > remove last item is < {1} {2} >
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< {2} {3} {4} > remove last item is < {2} {3} >

< {1} {5} {3} > and < {2} {3} {4} > could not join

< {1} {5} {3} > remove first item is < {5} {3} >

< {2 5} {3} > remove last item is < {2 5} >

< {1} {5} {3} > and < {2 5} {3} > could not join

< {1} {5} {3} > remove first item is < {5} {3} >

< {3} {4} {5} > remove last item is < {3} {4} >

< {1} {5} {3} > and < {3} {4} {5} > could not join

< {1} {5} {3} > remove first item is < {5} {3} >

< {1} {2} {3} > remove last item is < {1} {2} >

< {1} {5} {3} > and < {1} {2} {3} > could not join

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< {1} {2 5} > remove last item is < {1} {2} >

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< {1} {5} {3} > remove last item is < {1} {5} >

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< {1} {5} {3} > remove first item is < {5} {3} >

< {5} {3 4} > remove last item is < {5} {3} >

< {1} {5} {3} > and < {5} {3 4} > could join

length-4 frequent < {1} {5} {3 4} >

< {5} {3 4} > remove first item is < {3 4} >

< {2} {3} {4} > remove last item is < {2} {3} >

< {5} {3 4} > and < {2} {3} {4} > could not join

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< {2 5} {3} > remove last item is < {2 5} >

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< {5} {3 4} > remove first item is < {3 4} >
< {1} {2} {3} > remove last item is < {1} {2} >
< {5} {3 4} > and < {1} {2} {3} > could not join

< {5} {3 4} > remove first item is < {3 4} >
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< {5} {3 4} > remove first item is < {3 4} >
< {1} {5} {3} > remove last item is < {1} {5} >
< {5} {3 4} > and < {1} {5} {3} > could not join

< {5} {3 4} > remove first item is < {3 4} >
< {5} {3 4} > remove last item is < {5} {3} >
< {5} {3 4} > and < {5} {3 4} > could not join

=====length-4 frequent=====

< {2} {3} {4} {5} >
< {2 5} {3 4} >
< {1} {2} {3} {4} >
< {1} {2 5} {3} >
< {1} {5} {3 4} >

=====purning=====

all subsequences of < {2} {3} {4} {5} > are ["< {3} {4} {5} >", "< {2} {4} {5} >", "< {2} {3} {5} >", "< {2} {3} {4} >"]

< {3} {4} {5} > in length-3 frequent pattern set
< {2} {4} {5} > not in length-3 frequent pattern set
< {2} {3} {5} > not in length-3 frequent pattern set
< {2} {3} {4} > in length-3 frequent pattern set

all subsequences of < {2 5} {3 4} > are ["< {5} {3 4} >", "< {2} {3 4} >", "< {2 5} {4} >", "< {2 5} {3} >"]

< {5} {3 4} > in length-3 frequent pattern set
< {2} {3 4} > not in length-3 frequent pattern set
< {2 5} {4} > not in length-3 frequent pattern set

$\langle \{2\} \{5\} \{3\} \rangle$ in length-3 frequent pattern set

all subsequences of $\langle \{1\} \{2\} \{3\} \{4\} \rangle$ are [" $\langle \{2\} \{3\} \{4\} \rangle$ ", " $\langle \{1\} \{3\} \{4\} \rangle$ ", " $\langle \{1\} \{2\} \{4\} \rangle$ ", " $\langle \{1\} \{2\} \{3\} \rangle$ "]

$\langle \{2\} \{3\} \{4\} \rangle$ in length-3 frequent pattern set

$\langle \{1\} \{3\} \{4\} \rangle$ not in length-3 frequent pattern set

$\langle \{1\} \{2\} \{4\} \rangle$ not in length-3 frequent pattern set

$\langle \{1\} \{2\} \{3\} \rangle$ in length-3 frequent pattern set

all subsequences of $\langle \{1\} \{2\} \{5\} \{3\} \rangle$ are [" $\langle \{2\} \{5\} \{3\} \rangle$ ", " $\langle \{1\} \{5\} \{3\} \rangle$ ", " $\langle \{1\} \{2\} \{3\} \rangle$ ", " $\langle \{1\} \{2\} \{5\} \rangle$ "]

$\langle \{2\} \{5\} \{3\} \rangle$ in length-3 frequent pattern set

$\langle \{1\} \{5\} \{3\} \rangle$ in length-3 frequent pattern set

$\langle \{1\} \{2\} \{3\} \rangle$ in length-3 frequent pattern set

$\langle \{1\} \{2\} \{5\} \rangle$ in length-3 frequent pattern set

all subsequences of $\langle \{1\} \{5\} \{3\} \{4\} \rangle$ are [" $\langle \{5\} \{3\} \{4\} \rangle$ ", " $\langle \{1\} \{3\} \{4\} \rangle$ ", " $\langle \{1\} \{5\} \{4\} \rangle$ ", " $\langle \{1\} \{5\} \{3\} \rangle$ "]

$\langle \{5\} \{3\} \{4\} \rangle$ in length-3 frequent pattern set

$\langle \{1\} \{3\} \{4\} \rangle$ not in length-3 frequent pattern set

$\langle \{1\} \{5\} \{4\} \rangle$ not in length-3 frequent pattern set

$\langle \{1\} \{5\} \{3\} \rangle$ in length-3 frequent pattern set

=====length-4 frequent=====

$\langle \{1\} \{2\} \{5\} \{3\} \rangle$

Problem 2

 - projected database:

<(_c)(ac)d(cf)>

<(_c)(ae)>

<(df)(cb)>

<c>

items:

a:2

b:0

c:4

d:2

e:1

f:2

_c:2

length-2 sequential pattern having prefix :

<ba>

<bc>

<bd>

<(bc)>

<bf>

Problem 3

(1)

HMM

Alphabet: {g, r, b}

Set of states: {u₁, u₂}

Transition probability:

0.9	0.1
0.1	0.9

Emission probability:

	g	r	b
u ₁	1/3	1/3	1/3
u ₂	1/2	1/4	1/4

Start probability:

[0.8, 0.2]

(2)

According to:

$$f(\text{ggg}) = f_{u1}(\text{ggg}) + f_{u2}(\text{ggg})$$

$$f_{u1}(\text{ggg}) = e_{u1}(g) * (f_{u1}(gg) * a_{u1u1} + f_{u2}(gg) * a_{u2u1})$$

$$f_{u2}(\text{ggg}) = e_{u2}(g) * (f_{u1}(gg) * a_{u1u2} + f_{u2}(gg) * a_{u2u2})$$

$$f_{u1}(gg) = e_{u1}(g) * (f_{u1}(g) * a_{u1u1} + f_{u2}(g) * a_{u2u1})$$

$$f_{u2}(gg) = e_{u2}(g) * (f_{u1}(g) * a_{u1u2} + f_{u2}(g) * a_{u2u2})$$

$$f_{u1}(g) = e_{u1}(g) * (f_0(0) * a_{0u1})$$

$$f_{u2}(g) = e_{u2}(g) * (f_0(0) * a_{0u2})$$

$$f_0(0) = 1$$

$$f_{u1}(g) = e_{u1}(g) * (f_0(0) * a_{0u1}) = 1/3 * 1 * 0.8 = 0.2667$$

$$f_{u2}(g) = e_{u2}(g) * (f_0(0) * a_{0u2}) = 1/2 * 1 * 0.2 = 0.1000$$

$$f_{u1}(gg) = e_{u1}(g) * (f_0(0) * a_{0u1}) = 1/3 * (0.2667 * 0.9 + 0.1 * 0.1) = 0.0803$$

$$f_{u2}(gg) = e_{u2}(g) * (f_0(0) * a_{0u2}) = 1/2 * (0.2667 * 0.1 + 0.1 * 0.9) = 0.0583$$

$$f_{u1}(\text{ggg}) = e_{u1}(g) * (f_{u1}(gg) * a_{u1u1} + f_{u2}(gg) * a_{u2u1}) = 1/3 * (0.0803 * 0.9 + 0.0583 * 0.1) = 0.0260$$

$$f_{u2}(\text{ggg}) = e_{u2}(g) * (f_{u1}(gg) * a_{u1u2} + f_{u2}(gg) * a_{u2u2}) = 1/2 * (0.0803 * 0.1 + 0.0583 * 0.9) = 0.0303$$

$$f(\text{ggg}) = 0.0260 + 0.0303 = 0.0563$$

(3)

According to Viterbi algorithm:

Possibility matrix:

	0	g	g	g
0	1	0	0	0
u ₁	0	0.2667	0.08	0.024
u ₂	0	0.1	0.045	0.0203

Back pointer matrix:

	0	g	g	g
0	null	null	null	null
u ₁	null	(0,0)	(1,1)	(2,1)
u ₂	null	(0,0)	(1,2)	(2,2)

Termination :

(3,1)

Traceback:

(3,1) -> (2,1) -> (1,1)

All the states is u₁ -> u₁ -> u₁

Problem 4
L1 norm = 20

DTW

15	18	20	6	11	12	9	11
14	19	17	5	12	10	8	12
14	15	13	5	12	9	8	12
14	10	9	5	12	8	8	12
14	5	5	9	7	13	18	16
9	5	4	5	10	13	15	17
7	2	3	8	10	16	21	22
2	9	15	17	26	27	29	35

Problem 5

Adjacency matrix

0	1	0	0	1	0
0	1	0	1	0	1
0	1	0	0	0	0
1	0	1	0	0	0
1	0	0	1	0	0
0	1	1	0	0	0

Column stochastic matrix

0	0	0	$1/2$	$1/2$	0
$1/2$	$1/3$	1	0	0	$1/2$
0	0	0	$1/2$	0	$1/2$
0	$1/3$	0	0	$1/2$	0
$1/2$	0	0	0	0	0
0	$1/3$	0	0	0	0

page rank score of each node:

$[0.1627, 0.3599, 0.1472, 0.1605, 0.0789, 0.1270]^T$