1 Introduction

This document presets the mathematical model used to implement the ship-in-transit-simulator".

2 Ship dynamics

The ship is modelled in three degrees of freedom (surge, sway and yaw) according to the following equations:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{R}_z(\psi)\boldsymbol{v} \\ \boldsymbol{M}\dot{\boldsymbol{v}} &= -\boldsymbol{C}_{RB}(\boldsymbol{v})\boldsymbol{v} - \boldsymbol{C}_A(\boldsymbol{v}_R)\boldsymbol{v}_R - \boldsymbol{D}_L\boldsymbol{v}_R - \boldsymbol{D}_{NL}(\boldsymbol{v}_R)\boldsymbol{v}_R + \boldsymbol{\tau}_w + \boldsymbol{\tau}_r + \boldsymbol{\tau}_p \end{aligned} \tag{1}$$

where:

- $\boldsymbol{x} = [N,\, E,\, psi]^T$ is the north and east position, and the yaw angle.
- $\boldsymbol{x} = [u,\, v,\, r]^T$ is the velocity vector representing the forward and sideways speed, and the yaw rate.
- $\boldsymbol{R}_z(\psi)$ is the rotation matrix about the vertical axis, used to express the velocity vector in terms of a reference frame oriented north-east-down.
- $\boldsymbol{M} = \boldsymbol{M}_{RB} + \boldsymbol{M}_A$ is the ship´s mass matrix including the rigid body mass and added mass.
- $\boldsymbol{C}_{RB}(\boldsymbol{v})\boldsymbol{c}$ is the centripetal and Coriolis forces.
- $\boldsymbol{C}_A(\boldsymbol{v}_R)\boldsymbol{v}_R$ are the centripetal and Coriolis forces due to the added mass.
- $\boldsymbol{D}_L\boldsymbol{v}_R$ is the linear damping forces.Only the diagonal of the linear damping matrix is non-zero.
- $\boldsymbol{D}_{NL}(\boldsymbol{v}_R)\boldsymbol{v}_R$ is the non-linear damping forces. Only the diagonal of the non-linear damping matrix is non-zero.
- $\boldsymbol{\tau}_w$ is the forces acting on the ship from the wind. The wind forces are calculated based on the relative wind speed, based on the ship´s projected area towards the wind, according to $F = 0.5\rho_a v_w^2 c_d A_p$ where F is the force in some direction, ρ_a is the density of the air, v_w is the relative wind speed in the direction of F, c_d is the drag coefficient in air and A_p is the projected area of the ship in the same direction.
- $\boldsymbol{\tau}_r = [0,\, F_v,\, F_r]^T$ is the rudder force where $F_v = c_v \cdot \delta(u - u_c)$ and $F_r = c_r \cdot \delta(u - u_c)$ where c_v and c_r are coefficients, $u - u_c$ is the velocity of a water particle in the direction of the ship´s longitudinal axis relative to the ship´s surge speed, and δ is the rudder angle.

- $\boldsymbol{\tau}_p = [F_p,\, 0,\, 0]^T$ is the propulsion force. The provided classes gives at least two ways of generating the propulsion force.

3 Machinery system

The machinery system can be modelled either using a model that includes shaft dynamics, or as a simplified model where the the dynamics between engine load and propulsion force is modelled as a transfer function. In either case the engine load is the input which determines the propulsion force (output).

3.1 Model with shaft dynamics

The shaft dynamics is described as:

$$J_p \dot{\omega}_p = \frac{1}{r_{ME}}(\tau_{ME} - d_{ME}\omega_p) + \frac{1}{r_{HSG}}(\tau_{HSG} - d_{HSG}\omega_p) - k_p \omega_p^2 \tag{2}$$

where...

The thrust force F_p is given as

$$F_p = D_p^4 K_T \omega_p |\omega_p| \tag{3}$$

where D_p is the diameter of the propeller, K_T is a constant, and ω_p is the rotation speed of the propeller.

3.2 Simplified model

The simplified propeller force dynamics is described as

$$\dot{F_p} = -\frac{P_{max}}{T \cdot F_{max}} F_p + \frac{1}{T} P \tag{4}$$

where F_p is the propeller force in the surge direction, P_{max} is the maximum conntinous rated power of the main engine (or the set of engines used to generate propulsion force), $F_m ax$ is the maximal propulsion force, T is a time constant, and u is the engine load in kW.