Contents

1 Verification of 8NodeBrick elements

1.1 Verification of 8NodeBrick cantilever beams

Problem description: Length=6m, Width=1m, Height=1m, Force=100N, E=1E8Pa, $\nu = 0.0$. The force direction was shown in Figure (1).

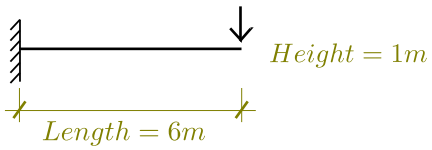
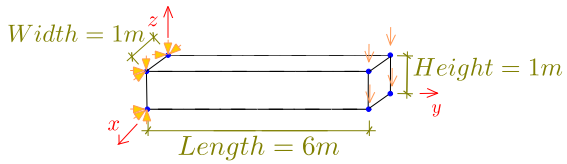


Figure 1: Problem description for cantilever beams

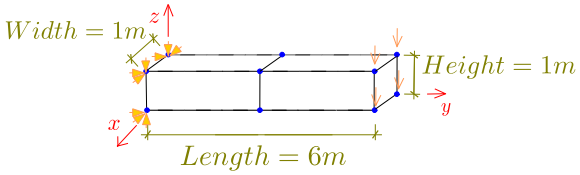
Theoretical displacement (bending and shear deformation):

$$\begin{aligned} d &= \frac{FL^3}{3EI} + \frac{FL}{GA} \\ &= \frac{100N \times 6^3 m^3}{3 \times 10^8 N/m^2 \times \frac{1}{12} m^4} + \frac{100N \times 6m}{5 \times 10^7 N/m^2 \times 1m^2} \\ &= 8.64 \times 10^{-4} m + 0.12 \times 10^{-4} m \\ &= 8.76 \times 10^{-4}\ m \end{aligned} \tag{1}$$

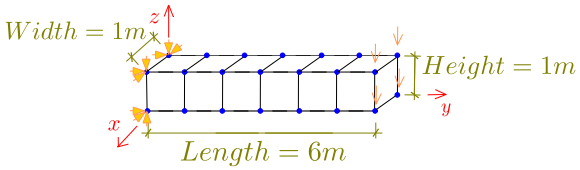
Numerical model:
The 8NodeBrick elements were shown in Figure (2).



(a) One 8NodeBrick element



(b) Two 8NodeBrick elements



(c) Six 8NodeBrick elements

Figure 2: 8NodeBrick elements for cantilever beams

All the ESSI results were listed in Table (1). The theoretical solution is 8.760E-04 m.

Table 1: Results for 8NodeBrick cantilever beams of different element numbers

Element number	1	2	6
8NodeBrick	4.61E-05 m	1.59E-04 m	5.84E-04 m
Error	94.74%	81.82%	33.33%

The errors were plotted in Figure (3).

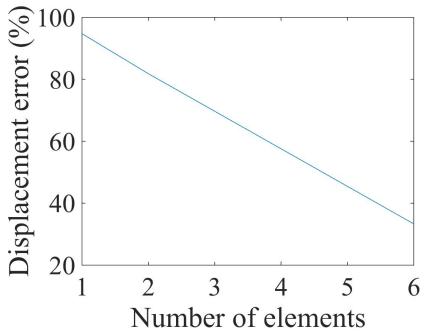


Figure 3: 8NodeBrick cantilever beam for different element number
Displacement error versus Number of elements

The ESSI model fei files for the table above are here

1.2 Verification of 8NodeBrick square plate with four edges clamped

Problem description: Length=20m, Width=20m, Height=1m, Force=100N, E=1E8Pa, $\nu = 0.3$.

The four edges are clamped.

The load is the uniform normal pressure on the whole plate.

The plate flexural rigidity is

$$D = \frac{Eh^3}{12(1-\nu^2)} = \frac{10^8 N/m^2 \times 1^3 m^3}{12 \times (1-0.3^2)} = 9.1575 \times 10^6 \ N \cdot m \tag{2}$$

The theoretical solution is

$$d = \alpha_c \frac{qa^4}{D} = 0.00406 \times \frac{100 N/m^2 \times 20^4 m^4}{9.1575 \times 10^6 \ N \cdot m} = 2.2015 \times 10^{-3} m \tag{3}$$

where α_c is a coefficient, which depends on the ratio of plate length to width. In this problem, the coefficient[1] α_c is 0.00406.

The 8NodeBrick were shown in Figure (4) - (9).

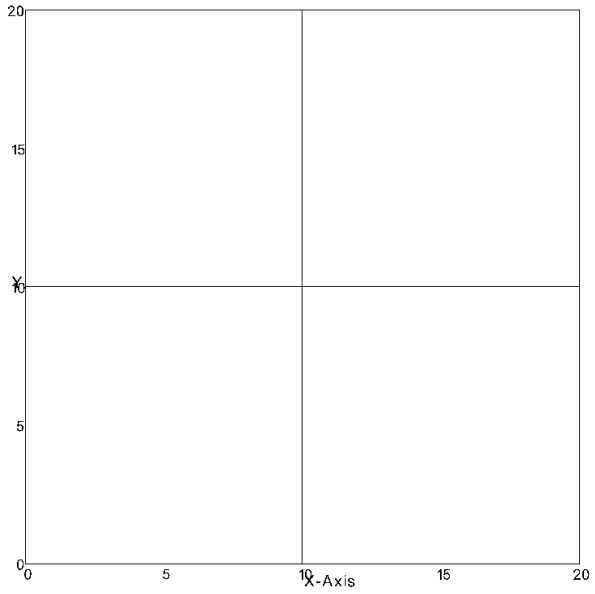


Figure 4: 8NodeBrick edge clamped square plate with element side length 10m

[1] Stephen Timoshenko, Theory of plates and shells (2nd edition). MrGRAW-Hill Inc, page120, 1959.

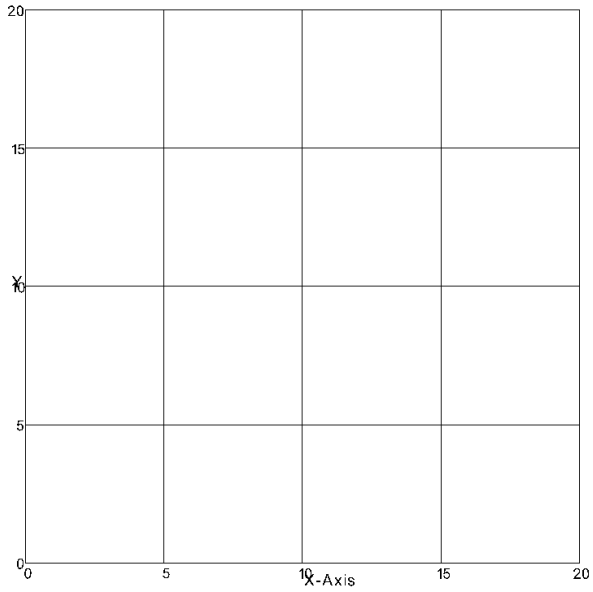


Figure 5: 8NodeBrick edge clamped square plate with element side length 5m

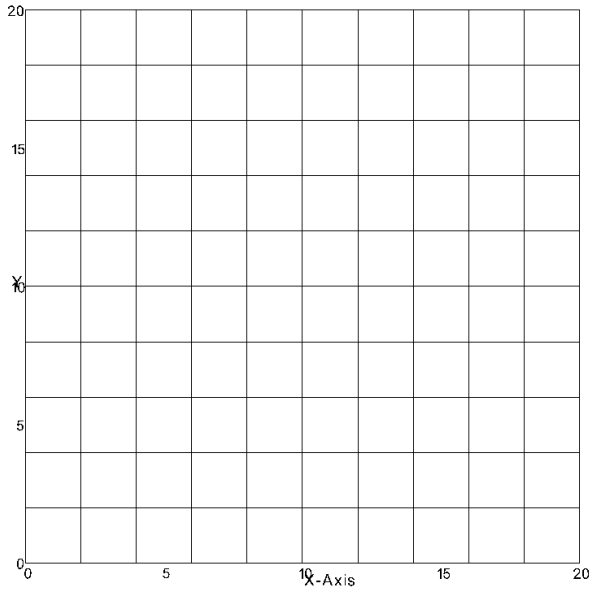


Figure 6: 8NodeBrick edge clamped square plate with element side length 2m

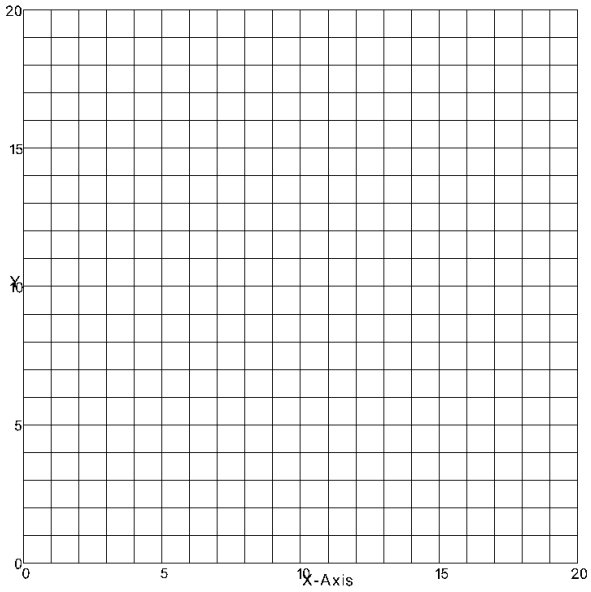


Figure 7: 8NodeBrick edge clamped square plate with element side length 1m

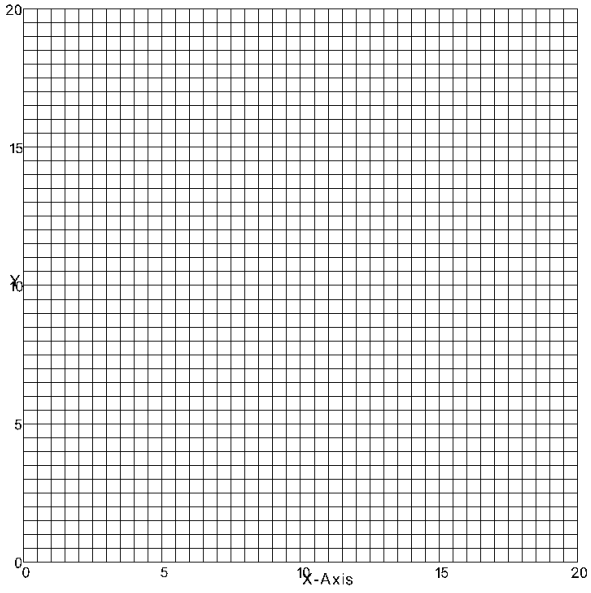


Figure 8: 8NodeBrick edge clamped square plate with element side length 0.5m

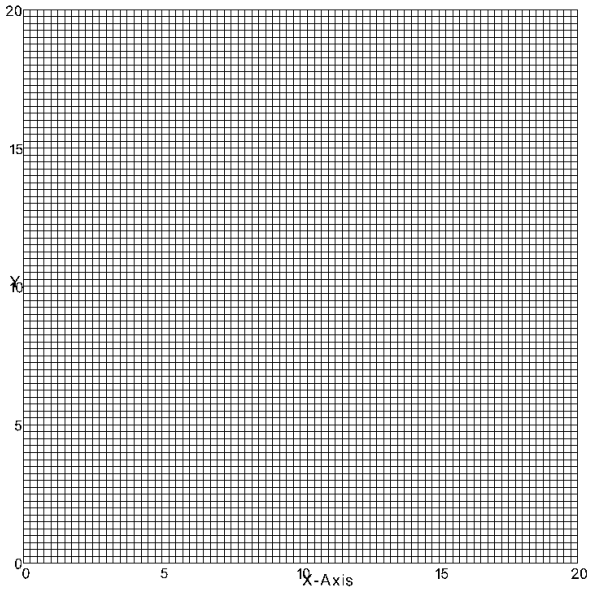


Figure 9: 8NodeBrick edge clamped square plate with element side length 0.25m

The results were listed in Table (2).

Table 2: Results for 8NodeBrick square plate with four edges clamped

Element type	8NodeBrick	8NodeBrick	8NodeBrick	Theoretical displacement
Number of layers	1layer	2layers	4layers	
Element side length	Height:1.00m	Height:0.50m	Height:0.25m	
10m	9.75E-05 m	9.75E-05 m	9.75E-05 m	2.20E-03 m
5m	3.28E-04 m	3.32E-04 m	3.32E-04 m	2.20E-03 m
2m	1.04E-03 m	1.10E-03 m	1.12E-03 m	2.20E-03 m
1m	1.56E-03 m	1.74E-03 m	1.79E-03 m	2.20E-03 m
0.5m	1.80E-03 m	2.30E-03 m	2.12E-03 m	2.20E-03 m
0.25m	1.87E-03 m	2.14E-03 m	2.23E-03 m	2.20E-03 m

The errors were listed in Table (3).

Table 3: Errors for 8NodeBrick square plate with four edges clamped

Element type	8NodeBrick	8NodeBrick	8NodeBrick
Number of layers	1layer	2layers	4layers
Element side length	Height:1.00m	Height:0.50m	Height:0.25m
10m	95.57%	95.57%	95.57%
5m	85.09%	84.94%	84.91%
2m	52.98%	50.09%	49.25%
1m	28.93%	21.17%	18.72%
0.5m	18.26%	4.58%	3.56%
0.25m	15.05%	2.70%	1.37%

The errors were plotted in Figure (10).

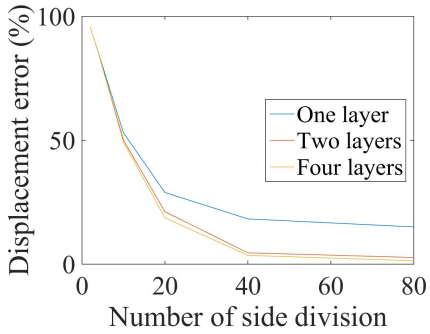


Figure 10: 8NodeBrick square plate with edge clamped
Displacement error versus Number of side division

The ESSI model fei files for the table above are here

2 Verification of 27NodeBrick elements

2.1 Verification of 27NodeBrick cantilever beams

Problem description: Length=6m, Width=1m, Height=1m, Force=100N, E=1E8Pa, $\nu = 0.0$. The force direction was shown in Figure (11).

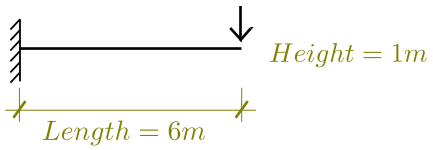
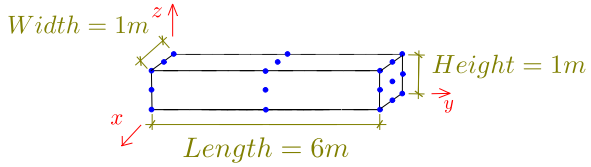


Figure 11: Problem description for cantilever beams

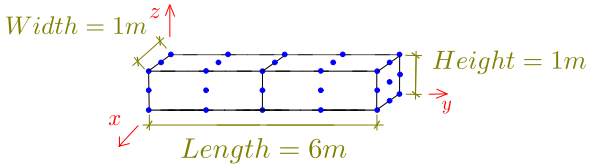
Theoretical displacement (bending and shear deformation):

$$\begin{aligned} d &= \frac{FL^3}{3EI} + \frac{FL}{GA} \\ &= \frac{100N \times 6^3 m^3}{3 \times 10^8 N/m^2 \times \frac{1}{12} m^4} + \frac{100N \times 6m}{5 \times 10^7 N/m^2 \times 1m^2} \\ &= 8.64 \times 10^{-4} m + 0.12 \times 10^{-4} m \\ &= 8.76 \times 10^{-4}\ m \end{aligned} \tag{4}$$

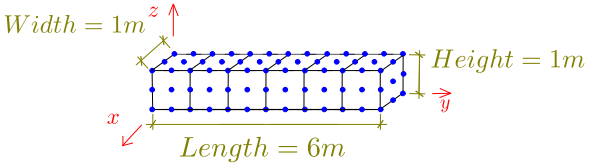
Numerical model:
The 27NodeBrick elements were shown in Figure (12).



(a) One 27NodeBrick element



(b) Two 27NodeBrick elements



(c) Six 27NodeBrick elements

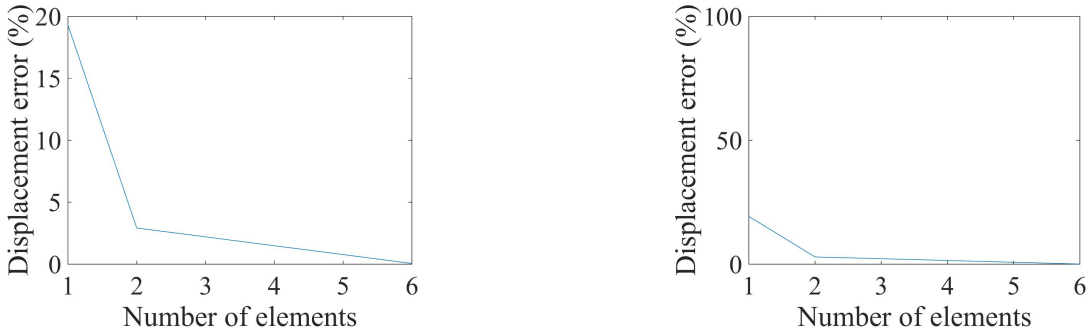
Figure 12: 27NodeBrick elements for cantilever beams

All the ESSI results were listed in Table (4).

Table 4: Results for 27NodeBrick cantilever beams of different element numbers

Element number	1	2	6
27NodeBrick	7.07E-04 m	8.50E-04 m	8.75E-04 m
Error	19.30%	2.92%	0.06%

The errors were plotted in Figure (13).



(a) Error scale 0% - 20%

(b) Error scale 0% - 100%

Figure 13: 27NodeBrick cantilever beam for different element number
Displacement error versus Number of elements

The ESSI model fei files for the table above are here

2.2 Verification of 27NodeBrick circular plate with all edges simply supported

Problem description: Diameter=20m, Height=1m, Force=100N, E=1E8Pa, $\nu = 0.3$.

The four edges are simply supported.

The load is the uniform normal pressure on the whole plate.

The plate flexural rigidity is

$$D = \frac{Eh^3}{12(1-\nu^2)} = \frac{10^8 N/m^2 \times 1^3 m^3}{12 \times (1-0.3^2)} = 9.1575 \times 10^6 \ N \cdot m \tag{5}$$

The theoretical solution[2] is

$$d = \frac{(5+\nu)qa^4}{64(1+\nu)D} = \frac{(5+0.3) \times 100 N/m^2 \times 10^4 m^4}{64 \times (1+0.3) \times 9.1575 \times 10^6 \ N \cdot m} = 6.956 \times 10^{-3} m \tag{6}$$

The 27NodeBrick were shown in Figure (14) - (19).

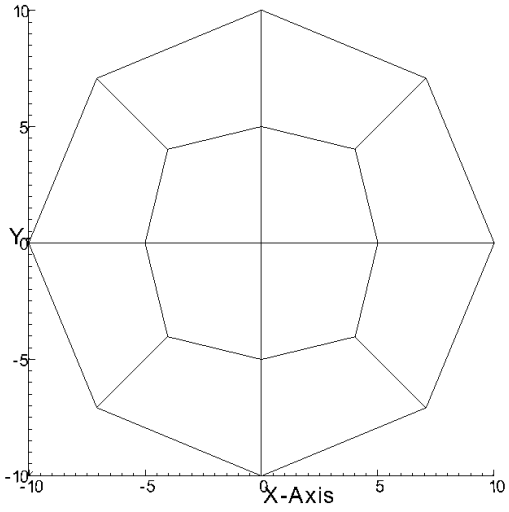


Figure 14: 27NodeBrick edge simply supported circular plate with element side length 10m

[2]Stephen Timoshenko, Theory of plates and shells (2nd edition). MrGRAW-Hill Inc, page55, 1959.

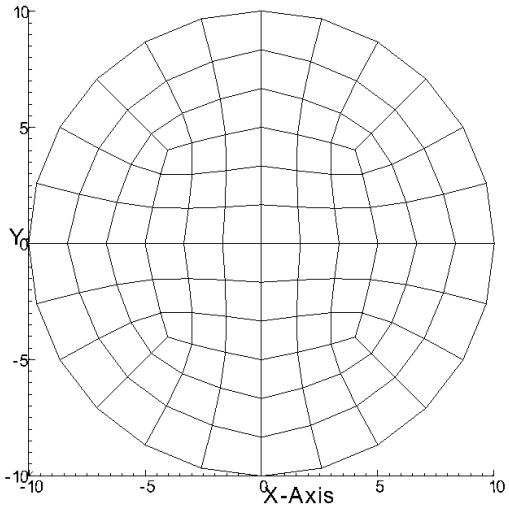


Figure 15: 27NodeBrick edge simply supported circular plate with element side length 5m

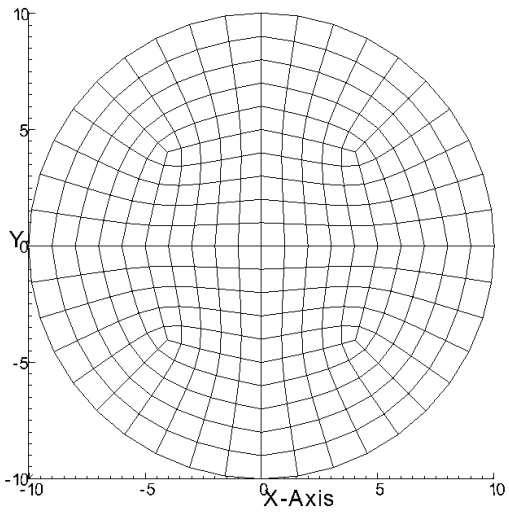


Figure 16: 27NodeBrick edge simply supported circular plate with element side length 2m

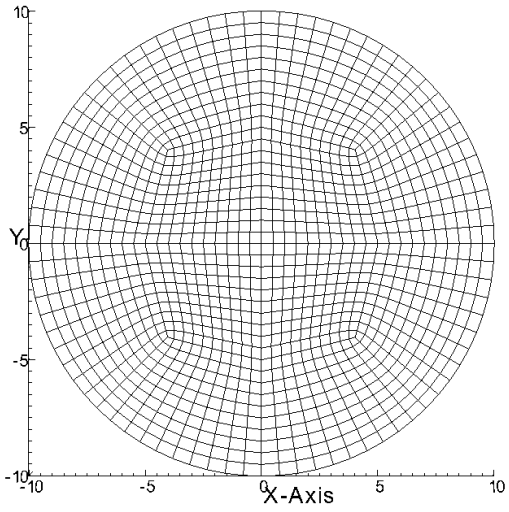


Figure 17: 27NodeBrick edge simply supported circular plate with element side length 1m

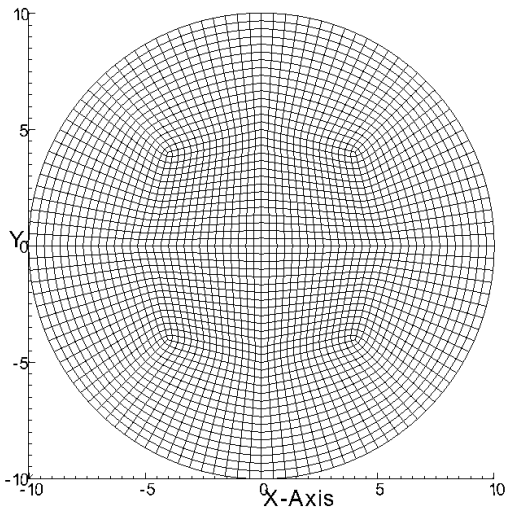


Figure 18: 27NodeBrick edge simply supported circular plate with element side length 0.5m

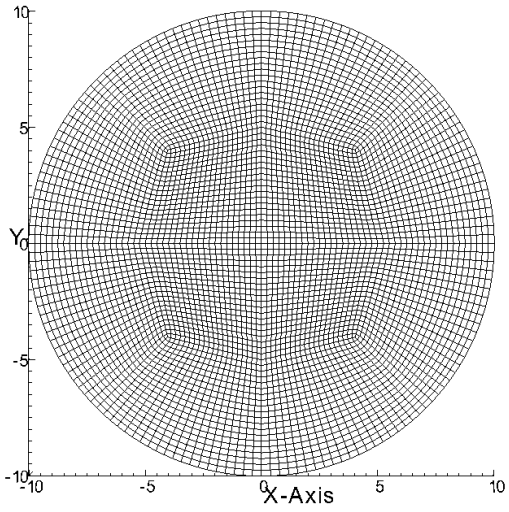


Figure 19: 27NodeBrick edge simply supported circular plate with element side length 0.25m

The results were listed in Table (5).

Table 5: Results for 27NodeBrick cicular plate with four edges simply supported

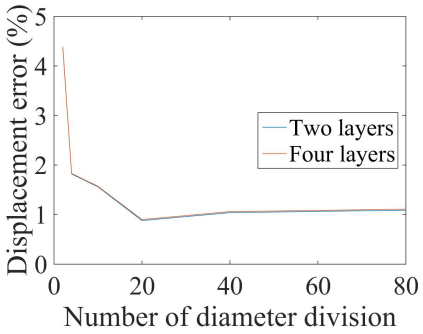
Element type	27NodeBrick	27NodeBrick	Theoretical displacement
Number of layers	2layers	4layers	
Number of diameter divisions	Height:0.50m	Height:0.25m	
4	7.259E-03 m	7.261E-03 m	6.956E-03 m
12	7.083E-03 m	7.084E-03 m	6.956E-03 m
20	7.064E-03 m	7.065E-03 m	6.956E-03 m
40	7.018E-03 m	7.019E-03 m	6.956E-03 m
60	7.029E-03 m	7.030E-03 m	6.956E-03 m
80	7.032E-03 m	7.034E-03 m	6.956E-03 m

The errors were listed in Table (6).

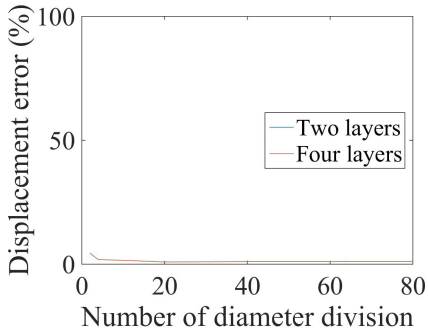
Table 6: Errors for 27NodeBrick cicular plate with four edges simply supported

Element type	27NodeBrick	27NodeBrick
Number of layers	2layers	4layers
Number of diameter divisions	Height:0.50m	Height:0.25m
4	4.36%	4.38%
12	1.82%	1.83%
20	1.56%	1.57%
40	0.88%	0.90%
60	1.04%	1.06%
80	1.09%	1.11%

The errors were plotted in Figure (20).



(a) Error scale 0% - 5%



(b) Error scale 0% - 100%

Figure 20: 27NodeBrick circular plate with edge simply supported
Displacement error versus Number of side division

The ESSI model fei files for the table above are here

Verification for 4NodeANDES

3 Verification of 4NodeANDES elements

3.1 Verification of 4NodeANDES cantilever beams

Problem description: Length=6m, Width=1m, Height=1m, Force=100N, E=1E8Pa, $\nu = 0.0$. The force direction was shown in Figure (21).

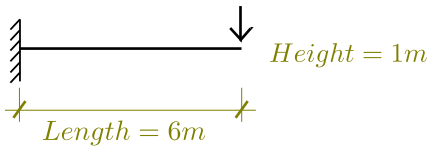


Figure 21: Problem description for cantilever beams

Theoretical displacement (bending and shear deformation):

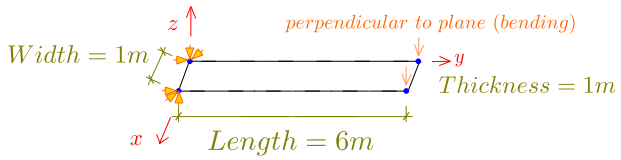
$$\begin{aligned} d &= \frac{FL^3}{3EI} + \frac{FL}{GA} \\ &= \frac{100N \times 6^3 m^3}{3 \times 10^8 N/m^2 \times \frac{1}{12} m^4} + \frac{100N \times 6m}{5 \times 10^7 N/m^2 \times 1m^2} \\ &= 8.64 \times 10^{-4} m + 0.12 \times 10^{-4} m \\ &= 8.76 \times 10^{-4}\ m \end{aligned} \tag{7}$$

4NodeANDES element model:

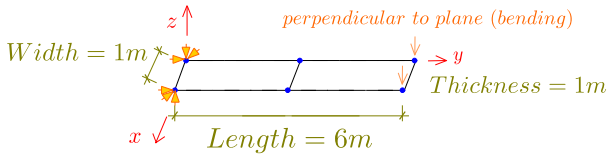
- ***Force direction: perpendicular to plane (bending)***

When the force direction is perpendicular to the plane, only the bending deformation is calculated in 4NodeANDES elements.

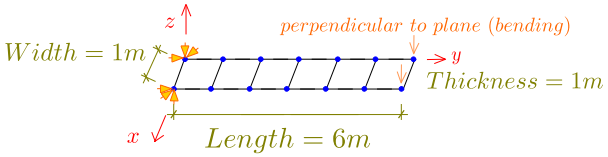
The 4NodeANDES elements were shown in Figure (22).



(a) One 4NodeANDES element



(b) Two 4NodeANDES elements



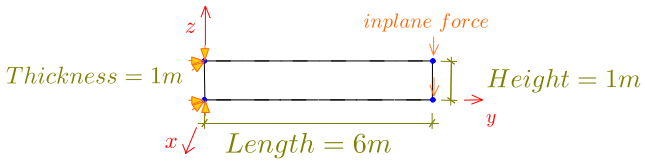
(c) Six 4NodeANDES elements

Figure 22: 4NodeANDES elements for cantilever beams under force perpendicular to plane

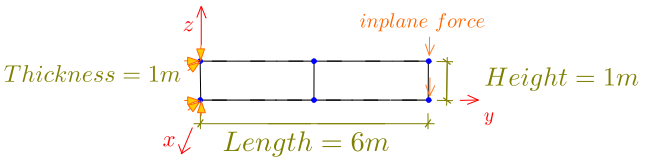
- ***Force direction: inplane force***

When the force direction is inplane, both the bending and shear deformation are calculated in 4NodeANDES elements.

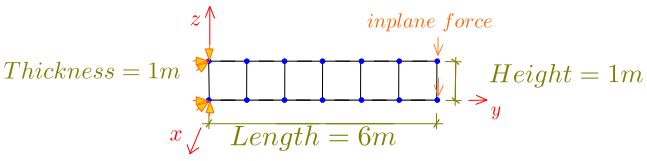
The 4NodeANDES elements under inplane force were shown in Figure (23).



(a) One 4NodeANDES element



(b) Two 4NodeANDES elements



(c) Six 4NodeANDES elements

Figure 23: 4NodeANDES elements for cantilever beams under inplane force

The ESSI results for the force ***perpendicular to plane (bending)*** were listed in Table (7). The theoretical solution is 8.760E-04 m.

Table 7: Results for 4NodeANDES cantilever beams under the force perpendicular to plane (bending)

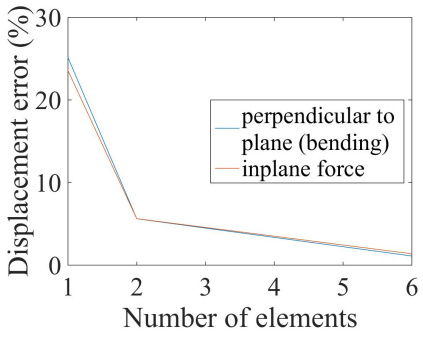
Element number	1	2	6
4NodeANDES	6.56E-04 m	8.27E-04 m	8.86E-04 m
Error	25.14%	5.62%	1.11%

The ESSI results for the ***inplane force*** were listed in Table (8).
The theoretical solution is 8.760E-04 m.

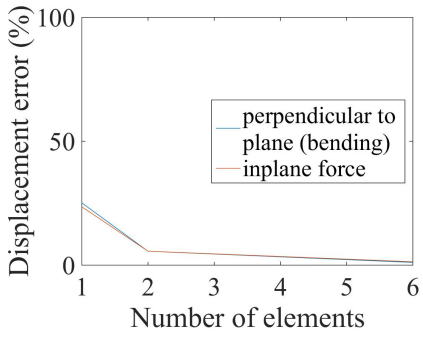
Table 8: Results for 4NodeANDES cantilever beams under the inplane force

Element number	1	2	6
4NodeANDES	6.70E-04 m	8.27E-04 m	8.64E-04 m
Error	23.56%	5.63%	1.38%

The errors were plotted in Figure (24).



(a) Error scale 0% - 30%



(b) Error scale 0% - 100%

Figure 24: 4NodeANDES cantilever beam for different element number
Displacement error versus Number of elements

The ESSI model fei files for the table above are here

3.2 Verification of 4NodeANDES circular plate with all edges simply supported

Problem description: Diameter=20m, Height=1m, Force=100N, E=1E8Pa, $\nu = 0.3$.

The four edges are simply supported.

The load is the uniform normal pressure on the whole plate.

The plate flexural rigidity is

$$D = \frac{Eh^3}{12(1-\nu^2)} = \frac{10^8 N/m^2 \times 1^3 m^3}{12 \times (1-0.3^2)} = 9.1575 \times 10^6 \ N \cdot m \tag{8}$$

The theoretical solution[3] is

$$d = \frac{(5+\nu)qa^4}{64(1+\nu)D} = \frac{(5+0.3) \times 100 N/m^2 \times 10^4 m^4}{64 \times (1+0.3) \times 9.1575 \times 10^6 \ N \cdot m} = 6.956 \times 10^{-3} m \tag{9}$$

The 4NodeANDES were shown in Figure (25) - (30).

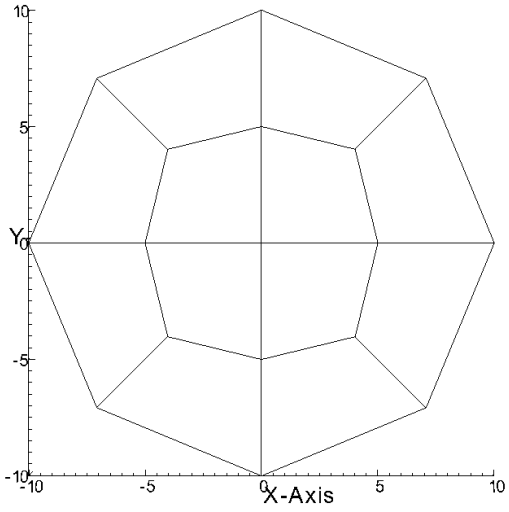


Figure 25: 4NodeANDES edge simply supported circular plate with element side length 10m

[3]Stephen Timoshenko, Theory of plates and shells (2nd edition). MrGRAW-Hill Inc, page55, 1959.

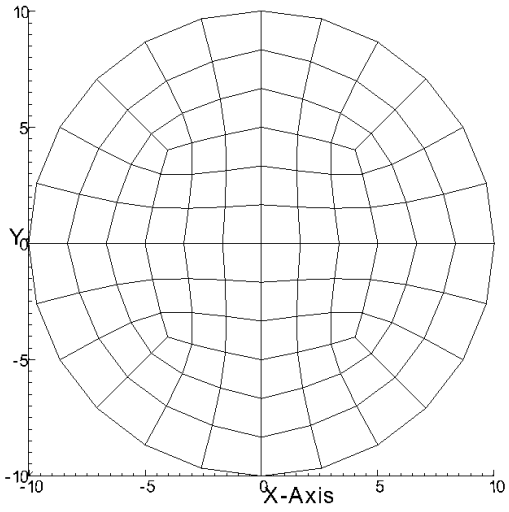


Figure 26: 4NodeANDES edge simply supported circular plate with element side length 5m

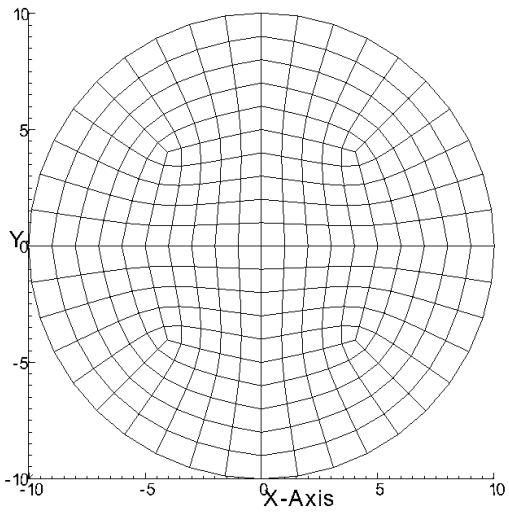


Figure 27: 4NodeANDES edge simply supported circular plate with element side length 2m

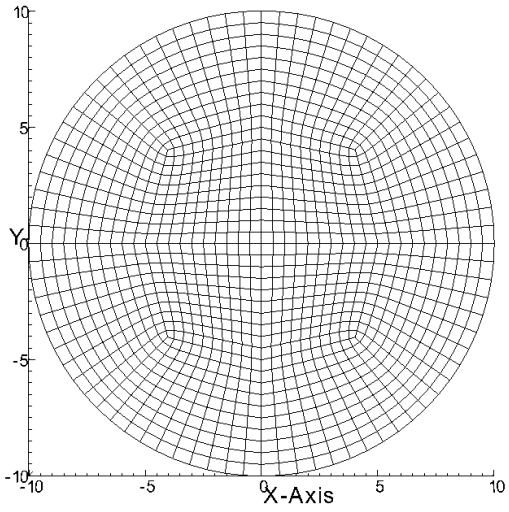


Figure 28: 4NodeANDES edge simply supported circular plate with element side length 1m

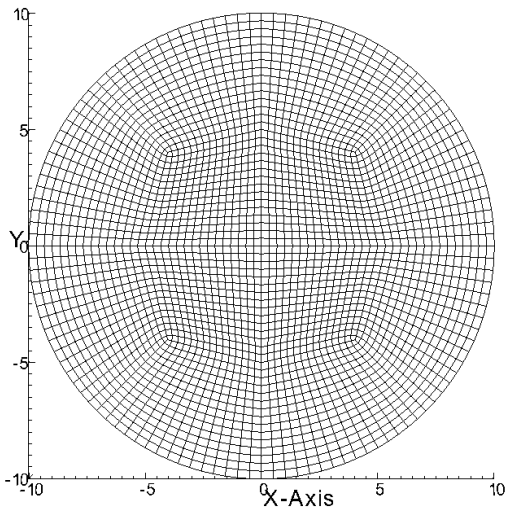


Figure 29: 4NodeANDES edge simply supported circular plate with element side length 0.5m

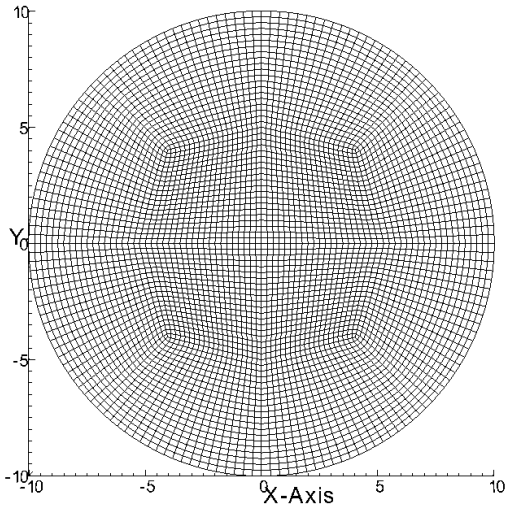


Figure 30: 4NodeANDES edge simply supported circular plate with element side length 0.25m

The results were listed in Table (9).

Table 9: Results for 4NodeANDES cicular plate with four edges simply supported

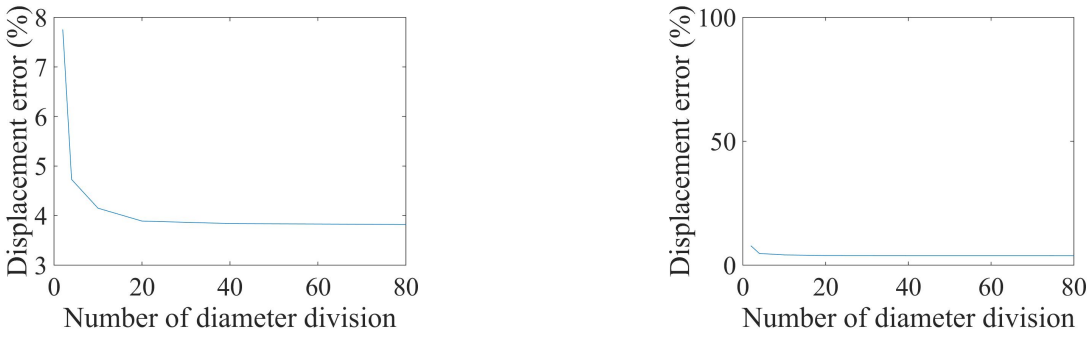
Element type	4NodeANDES	Theoretical
Element side length	Height:1.00m	displacement
10m	7.50E-003 m	6.956E-03 m
5m	7.29E-003 m	6.956E-03 m
2m	7.25E-003 m	6.956E-03 m
1m	7.23E-003 m	6.956E-03 m
0.5m	7.22E-003 m	6.956E-03 m
0.25m	7.22E-003 m	6.956E-03 m

The errors were listed in Table (10).

Table 10: Errors for 4NodeANDES cicular plate with four edges simply supported

Element type	4NodeANDES
Element side length	Height:1.00m
10m	7.75%
5m	4.73%
2m	4.15%
1m	3.89%
0.5m	3.84%
0.25m	3.82%

The errors were plotted in Figure (31).



(a) Error scale 0% - 8%

(b) Error scale 0% - 100%

Figure 31: 4NodeANDES circular plate with edge simply supported
Displacement error versus Number of side division

The ESSI model fei files for the table above are here