

# EC 320 Problem Set 3

Winter 2022

## INSTRUCTIONS:

There are three questions in total. Please answer them all and show the steps of how you derived your answer to receive the full credit

## 1. Simple linear regression (15 points)

Suppose that the underlying population model is

$$Y_i = \beta_0 + \beta_1 X_i + u_i.$$

Let's suppose we use ordinary least squares (OLS) to estimate  $\beta_0$  and  $\beta_1$  with 20 observations, and find the estimates  $\hat{\beta}_1 = 2$  and  $\text{s.e.}(\beta_1) = 1$ . You are considering two different hypothesis tests.

The first is a two-sided test where

$$\begin{aligned} H_0 : \beta_1 &= 0 \\ H_a : \beta_1 &\neq 0. \end{aligned}$$

The second is a one-sided test where

$$\begin{aligned} H_0 : \beta_1 &= 0 \\ H_a : \beta_1 &> 0. \end{aligned}$$

Suppose we set the significance level at 5%, i.e.  $\alpha = 0.05$ .

Below is some information about critical values from the t-distribution, where  $t_x(df)$  gives t-score below which  $x\%$  of the data falls when the degrees of freedom is equal to  $df$ . For example  $t_{0.975}(100)$  is t-score below which 97.5% of the data falls when the degrees of freedom is 100.

$$t_{0.975}(20) = 2.086$$

$$t_{0.95}(20) = 1.725$$

$$t_{0.975}(18) = 2.101$$

$$t_{0.95}(18) = 1.734$$

- Calculate t-statistics of the first hypothesis test.
- Run the first hypothesis test at 5% significance level and state your conclusion.
- Calculate t-statistics of the second hypothesis test.
- Run the second hypothesis test at 5% significance level and state your conclusion.
- Construct a 95% confidence interval of  $\beta_1$ .

## 2. Classical assumptions (10 points)

- (a) Provide me a quick scatter plot that violates sample variation assumption.
- (b) Provide me a quick scatter plot that violates homoskedasticity assumption.
- (c) Explain in your own words what Gauss-Markov Theorem is. Make sure that you reference the classical assumptions.

## 3. Multiple linear regression (15 points)

Suppose that the underlying population model as well as the regression model is

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

Using OLS, it is reported that

$$\hat{\beta}_0 = -2.62, \text{ s.e.}(\hat{\beta}_0) = 3.39$$

$$\hat{\beta}_1 = 0.98, \text{ s.e.}(\hat{\beta}_1) = 3.20$$

$$\hat{\beta}_2 = -1.26, \text{ s.e.}(\hat{\beta}_2) = 0.62$$

Information about critical values from the t-distribution is given below:

$$t_{0.995}(997) = 2.581$$

$$t_{0.99}(998) = 2.33$$

$$t_{0.975}(997) = 1.962$$

$$t_{0.95}(998) = 1.646$$

Suppose we conduct two-sided hypothesis test.

- (a) Is  $\hat{\beta}_2$  statistically significant at 5% significance level?
- (b) Is  $\hat{\beta}_2$  statistically significant at 1% significance level?
- (c) Interpret  $\hat{\beta}_1$  as in terms of units of  $Y$  and  $X_1$ .
- (d) Suppose instead that we estimate the following regression model using OLS

$$Y_i = \beta_0 + \beta_1 X_{1i} + u_i,$$

and find  $\hat{\beta}_0 = -5.09$  and  $\hat{\beta}_1 = -5.32$ . Find the omitted variable bias.

- (e) Continuing on (d), what can you infer about the sign of the covariance between  $X_1$  and  $X_2$ ?