EC 320 Problem Set 2

Winter 2022

INSTRUCTIONS:

There are three questions in total. Please answer them all and show the steps of how you derived your answer to receive the full credit

1. Simple hypothesis testing with a real example (12 points)

Suppose that Y is normally distributed with mean μ and standard deviation of σ. We set the null hypothesis such that the population mean of Y, denoted by μ, equals 0.

In this section, we will manually compute sample mean, denoted by $\hat{\mu}$, compute the standard error of $\hat{\mu}$, and calculate the t-statistic to conduct a hypothesis test at the 5% significance level. We then will compare our computations with the intercept-only regression result.

Y
1
5
10
12

(a) Calculate the sample mean and call it $\hat{\mu}$.

(b) Calculate the standard error of $\hat{\mu}$.

(c) If the null hypothesis is true, what is the t-statistic for this test?

(d) Which of the following is the correct critical value of the t-distribution to use for the test, where $t_{1-\alpha}(df)$ is the t-value below which $1-\alpha$ of the data lies with the degrees of freedom df? Recall that the degrees of freedom is calculated as the number of observation - the number of parameters. Recall also that we perform the two-sided t-test, as the null hypothesis is $\mu = 0$.

1) $t_{0.975}(4) \approx 2.78$
2) $t_{0.975}(3) \approx 3.18$
3) $t_{0.95}(4) \approx 2.13$
4) $t_{0.95}(3) \approx 2.35$

(e) Based on your previous answers, what's your conclusion, do you reject the null or fail to reject the null? Explain your reasoning.

(f) Compare your answers in (a) and (b) with the following regression estimates from intercept-only model, a linear regression model with only an intercept (i.e., the regression model looks as $Y_i = \beta_0 + u_i$). The number in parenthesis corresponds to standard error of the estimate.

	(1)
(Intercept)	7.000
	(2.483)
Number of observation	4

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$.

2. Calculating OLS estimates (30 points)

X	30	40	50	80
Y	5	10	35	30

Suppose you estimate a regression of the following population model,

$$Y_i = \beta_0 + \beta_1 X_i + u_i.$$

(a) Find the sample means of X and Y.

(b) Find $\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})$ and $\sum_{i=1}^{n}(X_i - \bar{X})^2$.

(c) Use your answer from (b) to calculate $\hat{\beta}_1$. Show your steps.

(d) Use your answer from (c) to calculate $\hat{\beta}_0$. Show your steps.

(e) Explain in your own words what $\hat{\beta}_1$ means in terms of units of X and Y.

(f) Use your calculations about $\hat{\beta}_0$ and $\hat{\beta}_1$ to find the fitted Y, $\hat{Y}_i$.

(g) Calculate the residuals, $\hat{u}_i$.

(h) Calculate the Total Sum of Squares (TSS).

(i) Calculate the Residual Sum of Squares (RSS).

(j) Calculate R^2. What does it tell us about the relationship between X and Y?

3. Proof (8 points)

(a) Prove that residuals sum to zero, i.e., $\sum_{i=1}^{n} \hat{u}_i = 0$.

(b) Prove that the sample covariance between the independent variable and the residuals is zero, i.e., $\sum_{i=1}^{n} X_i \hat{u}_i$.