

EC 320 Problem Set 2

Winter 2022

INSTRUCTIONS:

There are three questions in total. Please answer them all and show the steps of how you derived your answer to receive the full credit

1. Simple hypothesis testing with a real example (12 points)

Suppose that Y is normally distributed with mean μ and standard deviation of σ . We set the null hypothesis such that the population mean of Y , denoted by μ , equals 0.

In this section, we will manually compute sample mean, denoted by $\hat{\mu}$, compute the standard error of $\hat{\mu}$, and calculate the t -statistic to conduct a hypothesis test at the 5% significance level. We then will compare our computations with the regression result, where it only take the intercept as an explanatory variable.

Y
1
5
10
12

- (a) Calculate the sample mean and call it $\hat{\mu}$.
- (b) Calculate the standard error of $\hat{\mu}$.
- (c) If the null hypothesis is true, what is the t -statistic for this test?
- (d) Which of the following is the correct critical value of the t -distribution to use for the test, where $t_{1-\alpha}(df)$ is the t -value below which $1 - \alpha$ of the data lies with the degrees of freedom df ?
 - 1) $t_{0.975}(4)$
 - 2) $t_{0.975}(3)$
 - 3) $t_{0.95}(4)$
 - 4) $t_{0.95}(3)$
- (e) Based on your previous answers, what's your conclusion, do you reject the null or fail to reject the null? Explain your reasoning.

- (f) Compare your answers in (a) and (b) with the following regression estimates, where it only takes intercept as an explanatory variable (i.e., the regression model looks as $Y_i = \beta_0 + u_i$). The number in parenthesis corresponds to standard error of the estimate.

	(1)
(Intercept)	7.000
	(2.483)
Number of observation	4

*** p < 0.001; ** p < 0.01; * p < 0.05.

2. Calculating OLS estimates (30 points)

X	30	40	50	80
Y	5	10	35	30

Suppose you estimate a regression of the following population model,

$$Y_i = \beta_0 + \beta_1 X_i + u_i.$$

- (a) Find the sample means of X and Y .
- (b) Find $\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})$ and $\sum_{i=1}^n (X_i - \bar{X})^2$.
- (c) Use your answer from (b) to calculate $\hat{\beta}_1$. Show your steps.
- (d) Use your answer from (c) to calculate $\hat{\beta}_0$. Show your steps.
- (e) Explain in your own words what $\hat{\beta}_1$ means in terms of units of X and Y .
- (f) Use your calculations about $\hat{\beta}_0$ and $\hat{\beta}_1$ to find the fitted Y , \hat{Y}_i .
- (g) Calculate the residuals, \hat{u}_i .
- (h) Calculate the Total Sum of Squares (TSS).
- (i) Calculate the Residual Sum of Squares (RSS).
- (j) Calculate R^2 . What does it tell us about the relationship between X and Y ?

3. Proof (8 points)

- (a) Prove that residuals sum to zero, i.e., $\sum_{i=1}^n \hat{u}_i = 0$.
- (b) Prove that the sample covariance between the independent variable and the residuals is zero, i.e., $\sum_{i=1}^n X_i \hat{u}_i$.