

# EC 320 Problem Set 2

Winter 2022

## INSTRUCTIONS:

There are three questions in total. Please answer them all and show the steps of how you derived your answer to receive the full credit

## 1. Simple hypothesis testing with a real example (12 points)

Suppose that  $Y$  is normally distributed with mean  $\mu$  and standard deviation of  $\sigma$ . We set the null hypothesis such that the population mean of  $Y$ , denoted by  $\mu$ , equals 0.

In this section, we will manually compute sample mean, denoted by  $\hat{\mu}$ , compute the standard error of  $\hat{\mu}$ , and calculate the  $t$ -statistic to conduct a hypothesis test at the 5% significance level. We then will compare our computations with the intercept-only regression result, a linear regression model with only an intercept.

$Y$
1
5
10
12

- (a) Calculate the sample mean and call it  $\hat{\mu}$ .
- (b) Calculate the standard error of  $\hat{\mu}$ .
- (c) If the null hypothesis is true, what is the  $t$ -statistic for this test?
- (d) Which of the following is the correct critical value of the  $t$ -distribution to use for the test, where  $t_{1-\alpha}(df)$  is the  $t$ -value below which  $1 - \alpha$  of the data lies with the degrees of freedom  $df$ ?
  - 1)  $t_{0.975}(4)$
  - 2)  $t_{0.975}(3)$
  - 3)  $t_{0.95}(4)$
  - 4)  $t_{0.95}(3)$
- (e) Based on your previous answers, what's your conclusion, do you reject the null or fail to reject the null? Explain your reasoning.

- (f) Compare your answers in (a) and (b) with the following regression estimates from intercept-only model, a linear regression model with only an intercept (i.e., the regression model looks as  $Y_i = \beta_0 + u_i$ ). The number in parenthesis corresponds to standard error of the estimate.

	(1)
(Intercept)	7.000
	(2.483)
Number of observation	4

\*\*\* p < 0.001; \*\* p < 0.01; \* p < 0.05.

## 2. Calculating OLS estimates (30 points)

$X$	30	40	50	80
$Y$	5	10	35	30

Suppose you estimate a regression of the following population model,

$$Y_i = \beta_0 + \beta_1 X_i + u_i.$$

- (a) Find the sample means of  $X$  and  $Y$ .
- (b) Find  $\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})$  and  $\sum_{i=1}^n (X_i - \bar{X})^2$ .
- (c) Use your answer from (b) to calculate  $\hat{\beta}_1$ . Show your steps.
- (d) Use your answer from (c) to calculate  $\hat{\beta}_0$ . Show your steps.
- (e) Explain in your own words what  $\hat{\beta}_1$  means in terms of units of  $X$  and  $Y$ .
- (f) Use your calculations about  $\hat{\beta}_0$  and  $\hat{\beta}_1$  to find the fitted  $Y$ ,  $\hat{Y}_i$ .
- (g) Calculate the residuals,  $\hat{u}_i$ .
- (h) Calculate the Total Sum of Squares (TSS).
- (i) Calculate the Residual Sum of Squares (RSS).
- (j) Calculate  $R^2$ . What does it tell us about the relationship between  $X$  and  $Y$ ?

### 3. Proof (8 points)

- (a) Prove that residuals sum to zero, i.e.,  $\sum_{i=1}^n \hat{u}_i = 0$ .
- (b) Prove that the sample covariance between the independent variable and the residuals is zero, i.e.,  $\sum_{i=1}^n X_i \hat{u}_i$ .