

# 数学建模

浙江大学数学系 谈之奕

tanzy@zju.edu.cn



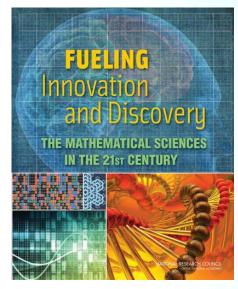
### 数学应用



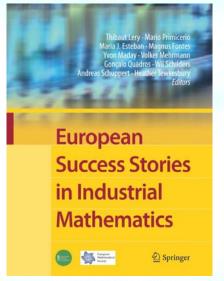
数学建模

• 数学在科学、技术和社会中的作用

万有引力定律 基因测序 选举理论 文本分析 计算机 大数据



Committee on the Mathematical Sciences in 2025, National Research Council (USA)



Scientific Committee of the "Mathematics and Industry", European Mathematical Society



## 数学应用



- "数学用不上"?
  - 太专业、太抽象
  - 不理解、不信任

- 脱离实际、排斥合作
- 无能为力、无从入手

任. 正如著名数学家 G. B. Dantzig 说的: "对于几乎从来未接触过应用方面的问题,只有纯粹数学背景的人来说,要他懂得如何用数学术语表述一个现实世界的问题,差不多是不可能的. 解决现实问题就更难了。"后来中国的实践也证明,纯粹数学家大

——转引自《华罗庚的数学生涯》

## 数学模型



- 数学模型是实际问题与数学理论之间的桥梁
  - 模型是对于现实世界的事物、现象、过程和系统的简 化描述,或其部分属性的模仿
  - 数学模型(Mathematical Model)是针对现实世界的特定对象,为了特定目的,根据特有的内在规律,做出一些必要的简化假设,选用适当的数学工具,得到的一种数学结构
- 建立数学模型的过程,即为数学建模 (Mathematical Modeling)



## 数学建模的主要步骤



#### 形成问题

假设与简化

建立模型

掌握背景和资料明确目的与任务

抓主要矛盾 合理的假设 必要的简化

数学 计算机

实践中 完善与改进 模型与结果 是否符合实际

模型应用

模型检验

模型分析与求解

### 数学建模所需的能力



- 通过交流和查阅文献,归纳、抽象问题的能力
- 用数学表述、分析与求解问题的能力
- 使用计算机和数学软件等工具的能力
- 用语言和文字描述成果,推广应用模型的 能力



### 应用数学研究的特点



- 以实际效果为衡量标准,重视理论指导作用
- 充分利用已有成果,创造性地为我所用
- 允许"不严格",避免不正确
- 多学科协作,团队攻关



### 课程概况



- 课程目的与任务
  - 讲授经典数学模型和应用数 学方法
  - 介绍数学应用前沿成果,展现数学魅力
  - 培养应用数学知识解决实际 问题的能力,加深对数学的 理解
  - 通过研究性学习,开展初步 科研训练与实践,提升创新 能力,提高综合素质

- 课堂讲授内容
  - 数学建模概述
  - 基本数学模型
  - 运筹与统计模型
  - 数学应用专题选讲
- 课程作业和课外实践
  - 一个专题
  - 一项课题



### 课程概况



- 课程资料
  - http://estore.zju.edu.cn/share/2755894
  - 验证码: 1825
- 期末笔试: 开卷考试
- Email: tanzy@zju.edu.cn



### 文献查阅



数学建模

- 书籍
  - CADAL

(http://www.cadal.zju.edu.cn)

- 百科
- 论文
  - MathSciNet
     (http://www.ams.org/mathscinet/)
  - Google学术搜索

(http://scholar.google.com/)







MathSciNet
Mathematical Reviews



### 综合性全文数据库

数学建模

- Nature & Science
- Springer





(http://link.springer.com/)

ELSEVIER





JSTOR

(http://onlinelibrary.wiley.com/)













## 专业性全文数据库



- IEEE
  - The Institute of Electrical and Electronics Engineers (http://ieeexplore.ieee.org)
- **ACM** 
  - Association for Computing Machinery (http://dl.acm.org/)
- SIAM
  - The Society for Industrial and Applied Mathematics (http://epubs.siam.org/)
- INFORMS
  - The Institute for Operations Research and the Management Sciences

(http://informsjournals.highwire.org/)







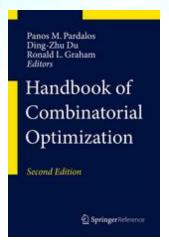


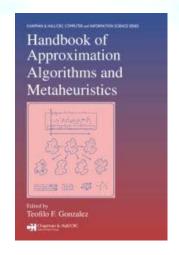


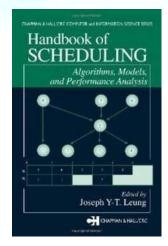
#### Handbook and Encyclopedia

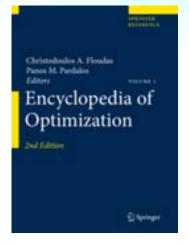


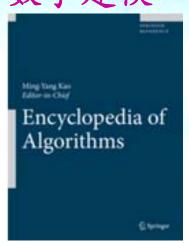












Handbook of Combinatorial Optimization (2<sup>nd</sup>), Pardalos, P., Du, D-Z, Graham, R. L.主编, 3409页, Springer, 2013

Handbook of Approximation Algorithms and Metaheuristics, Gonzalez, T. F.主编, 1432页, CRC, 2007

Handbook of Scheduling: Algorithms, Models, and Performance Analysis, Leung, J. Y-T.主编, 1216页, CRC, 2004

Encyclopedia of Optimization Floudas, C. A., Pardalos, P.主编, 1166页, Springer, 2009 Encyclopedia of Algorithms, Kao, M-Y主编, 1166页, Springer, 2008

#### Handbook and Encyclopedia



#### 数学建模

#### Volume 1

A Unified Approach for Domination Problems on Different Network Topologies	1
Advanced Techniques for Dynamic Programming	41
Advances in Group Testing	93
Advances in Scheduling Problems	145
Algebrization and Randomization Methods	171
Algorithmic Aspects of Domination in Graphs	221
Algorithms and Metaheuristics for Combinatorial Matrices	283
Algorithms for the Satisfiability Problem	311
Bin Packing Approximation Algorithms: Survey and Classification	455
Binary Unconstrained Quadratic Optimization Problem	533

#### Abelian Hidden Subgroup Problem 1995: Kitaev

MICHELE MOSCA 12

- 1 Combinatorics and Optimization / Institute for Quantum Computing, University of Waterloo, Waterloo, ON, Canada
- 2 Perimeter Institute for Theoretical Physics, St. Jerome's University, Waterloo, ON, Canada

#### Keywords and Synonyms

Generalization of Abelian stabilizer problem; Generaliza-  $|g\rangle|z\oplus f(g)\rangle$ . tion of Simon's problem

#### Problem Definition

The Abelian hidden subgroup problem is the problem of finding generators for a subgroup K of an Abelian which quantum algorithms offer an exponential speed up group G, where this subgroup is defined implicitly by a function  $f: G \to X$ , for some finite set X. In particular, f has the property that f(v) = f(w) if and only if the cosets1 v + K and w + K are equal. In other words, f is constant on the cosets of the subgroup K, and distinct on each

It is assumed that the group G is finitely generated and that the elements of G and X have unique binary encodings (the binary assumption is not so important, but it is important to have unique encodings.) When using variables g and h (possibly with subscripts) multiplicative notation is used for the group operations. Variables x and y (possibly with subscripts) will denote integers with addition. The boldface versions x and y will denote tuples of integers or binary strings.

By assumption, there is computational means of commaps the encoding of a value g to the encoding of f(g). The terest typically use additive notation for the group G.

theory of reversible computation implies that one can turn a circuit for computing f(g) into a reversible circuit for computing f(g) with a modest increase in the size of the circuit. Thus it will be assumed that there is a reversible circuit or black box that maps  $(g,z) \mapsto (g,z \oplus f(g))$ , where 

denotes the bitwise XOR (sum modulo 2), and z is any binary string of the same length as the encoding of

Quantum mechanics implies that any reversible gate can be extended linearly to a unitary operation that can be implemented in the model of quantum computation. Thus, it is assumed that there is a quantum circuit or black box that implements the unitary map  $U_f\colon |g\rangle|\mathbf{z}\rangle \mapsto$ 

Although special cases of this problem have been considered in classical computer science, the general formulation as the hidden subgroup problem seems to have appeared in the context of quantum computing, since it neatly encapsulates a family of "black-box" problems for (in terms of query complexity) over classical algorithms. For some explicit problems (i.e., where the black box is replaced with a specific function, such as exponentiation modulo N), there is a conjectured exponential speed

#### Abelian Hidden Subgroup Problem

Input: Elements  $g_1, g_2, \dots, g_n \in G$  that generate the Abelian group G. A black box that implements  $U_f$ :  $|m_1, m_2, \dots, m_n\rangle |\mathbf{y}\rangle \mapsto |m_1, m_2, \dots, m_n\rangle |f(g) \oplus \mathbf{y}\rangle$ where  $g = g_1^{m_1} g_2^{m_2} \dots g_n^{m_n}$ , and K is the hidden subgroup

Output: Elements  $h_1, h_2, ..., h_l \in G$  that generate K.

Here we use multiplicative notation for the group G in order to be consistent with Kitaev's formulation of the puting the function f, typically a circuit or "black box" that Abelian stabilizer problem. Many of the applications of in-

It is hard to trace the precise origin of this general formulation of the problem, which simultaneously general-

**Handbook of Combinatorial** Optimization目录(部分)

Encyclopedia of Algorithms首页

Assuming additive notation for the group operation here.

#### Survey, Review, Tutorial



#### 数学建模



Available online at www.sciencedirect.com



European Journal of Operational Research 176 (2007) 774-793

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

www.elsevier.com/locate/ejor

Discrete Optimization

Assignment problems: A golden anniversary survey

David W. Pentico

A.J. Palumbo School of Business, John F. Donahue Graduate School of Business, Duquesne University, Pittsburgh, PA 15282-0180, USA

Received 19 January 2005; accepted 16 September 2005 Available online 18 November 2005

SIAM REV.
Vol. 40, No. 3, pp. 496–546, September 1998

© 1998 Society for Industrial and Applied Mathematics

#### WELL-SOLVABLE SPECIAL CASES OF THE TRAVELING SALESMAN PROBLEM: A SURVEY\*

RAINER E. BURKARD<sup>†</sup>, VLADIMIR G. DEĬNEKO<sup>‡</sup>, RENÉ VAN DAL<sup>§</sup>,

JACK A. A. VAN DER VEEN<sup>¶</sup>, AND GERHARD J. WOEGINGER<sup>†</sup>

Abstract. The traveling salesman problem (TSP) belongs to the most basic, most important, and most investigated problems in combinatorial optimization. Although it is an  $\mathcal{NP}$ -hard problem, many of its special cases can be solved efficiently in polynomial time. We survey these special cases with emphasis on the results that have been obtained during the decade 1985–1995. This survey complements an earlier survey from 1985 compiled by Gilmore, Lawler, and Shmoys [The Traveling Salesman Problem—A Guided Tour of Combinatorial Optimization, Wiley, Chichester, pp. 87–143].



European Journal of Operational Research 232 (2014) 1-8

Contents lists available at SciVerse ScienceDirect

#### European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor



Invited Review

OR analysis of sporting rules - A survey

Mike Wright\*

Department of Management Science, Lancaster University, LAI 4YX, UK





Contents lists available at SciVerse ScienceDirect

#### Surveys in Operations Research and Management Science

journal homepage: www.elsevier.com/locate/sorms



Reviev

#### A tutorial on fundamental model structures for railway timetable optimization Steven S. Harrod\*

University of Dayton, United States

ARTICLE INFO

Artide history: Received 22 June 2012 Accepted 8 August 2012

#### ABSTRACT

This guide explains the role of railway timetables relative to all other railway scheduling activities, and then presents four fundamental timetable formulations suitable for optimization. Timetabling models may be classified according to whether they explicitly model the track structure, and whether the timetable is intended to be periodic or not (aperiodic). The presentation of models is organized to facilitate the selection of a model by planning objective and available data, regardless of the specific traffic carried or periodic traffic carried or periodic traffic carried or periodic traffic carried or periodic process.

© 2012 Elsevier Ltd, All rights reserved,



## 数学软件



- 程序设计语言
- 综合性数学软件
  - Mathematica
  - Matlab
  - Maple

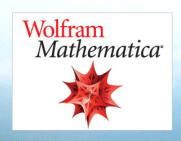


- 统计
  - SAS
  - SPSS
  - R
- 优化
  - LINGO
  - CPLEX













## 数学建模竞赛



- 竞赛目的
  - 提高学生建立数学模型和运用计算机技术解决实际问题的综合能力
  - 激励学生学习数学的积极性,鼓励广大学生踊跃参加课外科技活动
  - 开拓知识面,培养创造精神及合作意识
  - 推动大学数学教学体系、教学内容和方法的改革
- 竞赛形式
  - 三人一队、三至四天、通讯竞赛



## 数学建模竞赛



- 竞赛内容
  - 题目一般来源于工程技术和管理科学等方面经过适当 简化加工的实际问题,不要求参赛者预先掌握深入的 专门知识
  - 参赛者应根据题目要求,完成一篇包括模型的假设、 建立和求解、计算方法的设计和计算机实现、结果的 分析和检验、模型的改进等方面的论文(即答卷)
  - 评奖以假设的合理性、建模的创造性、结果的正确性和文字表述的清晰程度为主要标准



## 美国数模竞赛

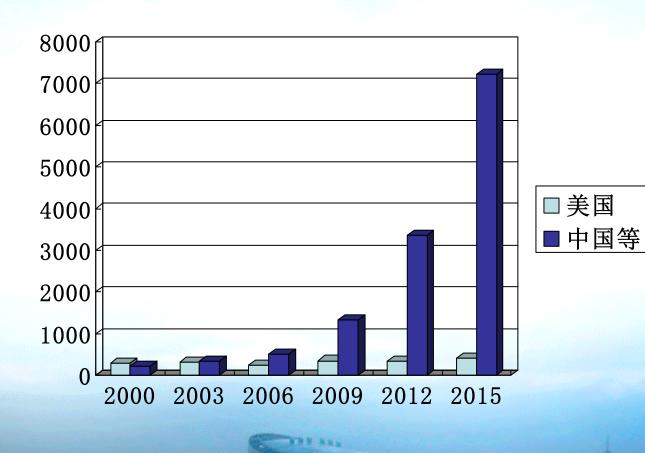


- The Mathematical Contest in Modeling (MCM)
  The Interdisciplinary Contest in Modeling (ICM)
- 主办单位与主要协办单位
  - COMAP (Consortium for Mathematics and Its Applications) (http://www.comap.com/)
  - INFORMS, SIAM, MAA (The Mathematical Association of America)
- 创办于1985年, 2014年共有17个国家9773支队伍参加
- 每年一月下旬或二月上旬举行,赛期四天,参赛队可从竞赛网站上公布的四道试题中任选一题



## 参赛队伍构成





MCM参加队数 (2000, 2003年数 据含ICM参赛队)

15年来,美国参赛队从300 支增加到400 支,中国参赛 队从200支增 加到7000支

### 竞赛试题

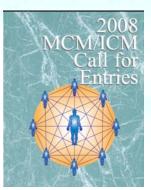


#### 数学建模

- 部分历年试题
  - The Stunt Person
  - Tollbooths
  - Organ Transplant: The Kidney Exchange Problem
  - The Sweet Spot
  - Snowboard Course
  - The Leaves of a Tree
- 2015年试题
  - Eradicating Ebola
  - Searching for a lost plane
  - Managing Human Capital in Organizations
  - Is it sustainable















## 设奖情况



少元	中汉友	数目或比例	
<b>上</b>	中译名	2014	2015
<b>Outstanding Winners</b>	特等奖	19	19
其中 INFORMS Award		3	4
SIAM Award		2	2
MAA Award		2	2
Finalists	特等奖提名	17	33
<b>Meritorious Winners</b>	一等奖	10.11%	9.87%
<b>Honorable Mention</b>	二等奖	32.57%	32.59%

## 国内高校获奖情况



	获特等 奖次数	兼获I/S 奖次数
浙江大学	8次	5次
清华大学	9次	3次
北京大学	6次	0次
上海交通大学	5次	0次
国防科学技术大学	5次	1次
东南大学	4次	2次
南京大学	2次	1次
复旦大学	2次	0次
西安交通大学	2次	0次
中国科学技术大学	1次	0次

#### 浙江大学历次获奖

1999(I)	沈权,杨振羽,何晓飞
2003(I)	胡煜霄,周恩露、华诤
2010	岳作功,阳宇光,赵聪
2011(I)	戴奇骎, 李懿, 丁寰宇
2012(S)	傅诚,朱丹亭,赵航琪
2013	朱常友,白伟成,杨文青
2015	陈凡,李锡涵,安毅宁

### 浙江大学参赛情况





COMAP颁发的 2011年度特等奖奖状

INFORMS协会颁发的 2003年度INFORMS奖奖牌

2003年度特等奖兼INFORMS奖论文刊登在 COMAP会刊The UMAP Journal上 (Undergraduate Mathematics and Its Applications)



#### 数学建模

#### Fly With Confidence

Hu Yuxiao Hua Zheng Zhou Enlu Zhejiang University Hangzhou, China

Advisor: Tan Zhiyi

We develop a model to design a pile of cardboard boxes to cushion the fall of a stunt motorcycle; the kinetic energy of the motorcycle is consumed through breaking down the boxes.

We ignore the scattering effect of the boxes and begin by analyzing a single box. The energy to break it down has three components: the upper surfaces, the side surfaces, and the vertical edges. When big boxes are used, the upper surface provides the main supporting force; when small ones are used, the vertical edges play a great role.

We extend our analysis to the pile of boxes. Taking into account the maximum impulse that a person can bear, along with the camera effect and cost concerns, we determine the size of a box.

We conceive several stacking strategies and analyze their feasibility. We incorporate their strengths into our final strategy. We also examine several modifi cations to the box to improve its cushioning effect.

To validate our model, we apply it to different cases and get some encouraging

#### Assumptions

- . The stunt person and the motorcycle are taken as a system, which we refer to as the motorcycle system or for brevity as the motorcycle. We ignore relative movement and interaction between them.
- . The motorcycle system is a uniform-density block. We consider only the movement of its mass center, so we consider the motorcycle system as a mass
- · The cardboard boxes are all made of the same material, single wall S-1 cardboard 4.5 mm thick [Corrugated fiberboard ... n.d.]

The UMAP Journal 24 (3) (2003) 301-316. (C)Copyright 2003 by COMAP, Inc. All rights reserved. Permission to make digital or hard copies of part or all of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice. Abstracting with credit is permitted, but copyrights for components of this work owned by others than COMAP must be honored. To copy other ror components of this work owned by others than COMAP must be honored. To copy otherwise, to republish, to post on servers, or to redistribute to lists requires prior permission from COMAP.



### 浙江大学参赛情况



#### 数学建模





SIAM协会颁发的 2012年度SIAM奖奖牌 时任SIAM主席的 Lloyd Nick Trefethen 教授为浙江大学 获SIAM奖学生颁奖

Wednesday, July 11, 2012

10:30 AM—12:30 PM 4:00 PM—6:00 PM SIAM/MCM Award winners presentations (2011, 2012) SIAM Student Paper Prize winners presentations (2011, 2012)

#### Students

Activities just for you at the 2012 SIAM Annual Meeting



Studies Day or the SECERAL Annual Markety are the studies and destructions. Cognition these planned sold his and markets where studies are results that the part of productions in their property and the studies of the studies of the studies of the sect of section of the sections of EAM Studies Original from all one of the section.

Equated by the EMB Execution Constitute (Policy E) Policy Carlos (Colore 10), Flucher Days are integrant to recovering in class proteination in EMB, I follow that makes should apply the frankismality and copyrighted industrial to the copyrighted industrial to the Colore and both afficial from Colore and provide in the color to executing the colore proteination in the colore and to the colore and the colore a

Souther Day 1 resident blocks by presentation is a nutrier despite in presentation as so without order adMinistrated by Ministrated Consents in Mandring Ministry and the SWM Studied Physio P19 in. The past one will have been shall be than 5 which have many by the shallow, findingly a possited content between the Studies below of the manufacture of the shallow, findingly a possited content between the Studies of manufacture. It is shallow that which pasted as a small his applied and compositional and the studies of a smallow that which pasted to a small his applied and compositional and the studies.

Including and SIAM (by invitation of the Dy 1991 and the SIAM)

Landwide The meeting place hould, edition such submit depths represented for approving invention has detection related to dissure these for improving stables drapies under a principal for contract or medical control and false stables meeting of the order South Region patients one stables to should find meeting to a ground table in .

Berting, big I, Nesione service to help students field plot and develop that comes helpfulsing principality. Better to Produce to thinking Trains Countries will not religious that what is facile work in thisting. They produced the belowed by a Cases for the oil anapole to the laboral programmation and gradual students. Programmation programmation is a superior to the countries and programmation and be and an other programmation.

Robertoni destignante entry ir Turchy, July 12. In addition o positi decustor, from ull'as an opposit i la rankel ul til professional in the fall. You eight also la interestal in standing third socialism for Woman is Materials. APMI excitato for mallian au dem set incomo PMDs.

Because your cooper of the Fy art. Regardy Microsophic, The hood has a factor number of cooper as effects emparated makes to establish. The designs to make better makes of time in Arms 8, 1997. Since melaced main student cooper others and suit palor to the designs, we incommonate force; as make your microsofters and; if Student Pays Schedule

The title einders loange busined in the Schille Hell .

S FIG. - C.SS FIG. Shelter i Orleranther S FIG. - C.SS FIG. Michons Pacopton

Manulay, July 4, 19149—12879

Universities - Famouch in Applied and Computed Interesting - 1 of 2 Universities - Famouch in Applied and Computed Interesting - 2 of 2 Industry Fuel

ietolog, July 10, IAN-15AN Belo Belos

J. 20, 2012 Seder Chapter making the SAS Landon My Sa Tradition of the Chapter States A RES and THE Control Spatier States Control of the Con

Control of Space (Depter Presentation Copies of Principle (Copies of Principle (Copies of Copies of Principle (Copies of Copies of Copie

Multi-exp Surge-Virtual Methods
Tachay Green White why of this auctivation of Chemistria
A Methods of Approximation for Figure of the Chemistrian
Data Surface, Supporting Onlineary
Declarates of Deformation and Supplification by of Field Res
Surface of Deformation and Supplification by of Field Res
Surface of Chemistrian and Supplification by of Field Res
Surface of Chemistrian and Supplification by of Field Res
Surface of Chemistrian and Supplification by of Field Res
Surface of Chemistrian and Supplification by other Res
Surface of Chemistrian and Supplification by other Res
Surface of Chemistrian and Supplification by the Surface of Chemistry Surface of Ch

Attra Scotte, Cili Chamman Annuag.

Weating of the Cycent Discension of Livraped
Unit festival Luminose Champanins
Burk Catholi, Maltin Jan Facher Witnessey,
Berusting in officers flow in Engineer Research
is Security for officers flow in Engineer Research
is Security Security Function for Out-Occupy, Marchi
1.

An Andy Sai Approach to Green Code Days Treepe, Garage Absorbations Of Falses and America Landbook

Section I of treat analogy will be to be possible to the part of the section (AMER, AMER, THE COMMENT)

July 12, 2012







Register on he as were stancing threating of highest pips. Students can pre-register for the meeting as the reduced rure of \$110 (or \$125 cents) Student Days are Indused.

### 美国数模竞赛



- 2016年美国数模竞赛比赛时间为 (北京时间) 2016年1月29日—2月 2日
  - MCM Problem A (continuous)
  - MCM Problem B (discrete)
  - MCM Problem C (data insights)
  - ICM Problem D (operations research/network science)
  - ICM Problem E (environmental science)
  - ICM Problem F (policy)





## 全国数模竞赛



- 全国大学生数学建模竞赛
  - **China/Contemporary Undergraduate Mathematical Contest in Modeling (CUMCM)**

( http://www.mcm.edu.cn )

- 主办单位
  - 教育部高等教育司
  - 中国工业与应用数学学会
- 创办于1992年,2014年全国及部分海外高校有1338 所院校、25347个队、70000多名本专科学生参赛
- 每年九月的第二个周末举行,赛期三天,参赛队可从竞赛网站上公布的两道试题中任选一题



创新意识 团队精神 重在参与 公平竞争



## 全国数模竞赛



- 2014年(本科组)评出 "高教社杯"1队,
  MATLAB创新奖1队。全 国一等奖293个队、二等 奖1256个队,全国一、二 等奖分别占1.3%和 6.6%。另设省一、二、三 等奖若干
- 部分竞赛试题
  - 彩票中的数学
  - 奥运会临时超市网点设计
  - 长江水质的评价和预测
  - 乘公交,看奥运
  - 2010年上海世博会影响力的定量评估
  - (2015A) 太阳影子定位
  - (2015B)"互联网+"时代的 出租车资源配置

#### 浙江大学参赛情况



- 浙江大学每年参赛25队, 近年来每年约有5~8队获 全国一、二等奖
- 2010年浙江大学学生马宇斌、莫璐怡、杨琦获得本科组"高教社杯"



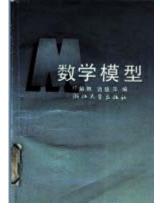


#### 浙江大学数模活动



数学建模

浙江大学自1982年起开设数学建模课程,是国内最早的高校之一。2003年,数学建模课程被评为首批国家精品课程;2013年入选国家级精品资源共享课立项项目





1999年,浙江大学拨款100万元 建设全国高校首个数学建模学 生课外实践基地



### 浙江大学数模活动



- 本科生院教学科研处主管全校各类本科生学科竞赛,浙江大学数学建模实践基地对我校数学建模实践与竞赛活动提供指导
  - 承办每年一度的浙江大学大学生数学建模竞赛,竞赛在每年5月举行,形式和内容基本参照全国数模竞赛,2014年全校共有211队参赛
  - 组织、指导浙江大学学生参加全国数模竞赛和美国数模竞赛







部分在高校和科研机构工作的近年获奖学生

黎娜	2006美赛一等	哈佛大学助理教授
邬恩信	2005美赛一等	西安大略大学助理教授
丁弋川	2004美赛二等	英属哥伦比亚大学助理教授
汪琳薇	2004美赛二等	罗彻斯特理工学院助理教授
华净	2003美赛特等	香港大学助理教授
周恩露	2003美赛特等	佐治亚理工学院助理教授
曲冬霞	2001美赛一等	劳伦斯·利弗莫尔实验室
何晓飞	1999美赛特等	浙江大学教授、杰青

