数学建模

浙江大学数学系　谈之奕

tanzy@zju.edu.cn

数学建模概论

数学应用

浙江大学 ZheJiang University

数学建模

- 数学在科学、技术和社会中的作用

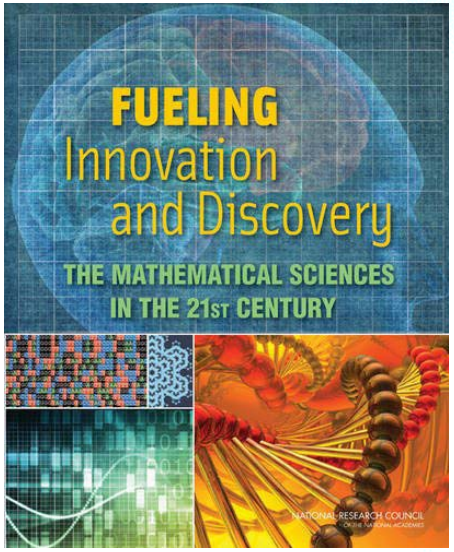
万有引力定律

基因测序

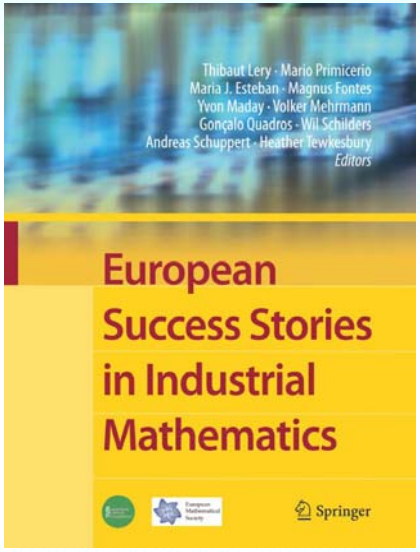
选举理论

文本分析

计算机　　大数据



Committee on the Mathematical Sciences in 2025, National Research Council (USA)



Scientific Committee of the "Mathematics and Industry", European Mathematical Society

数学应用

浙江大学 ZheJiang University

数学建模

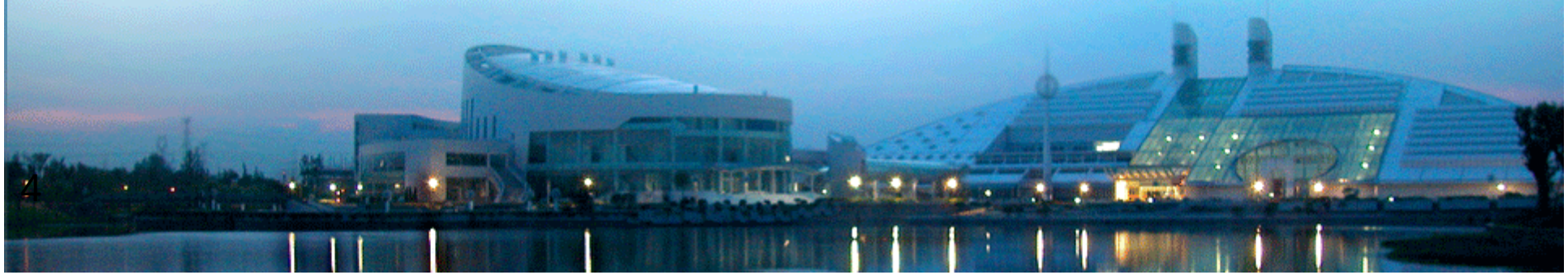
- “数学用不上”?
  - 太专业、太抽象
  - 不理解、不信任
  - 脱离实际、排斥合作
  - 无能为力、无从入手

任. 正如著名数学家 G. B. Dantzig 说的：“对于几乎从来未接触过应用方面的问题，只有纯粹数学背景的人来说，要他懂得如何用数学术语表述一个现实世界的问题，差不多是不可能的. 解决现实问题就更难了.” 后来中国的实践也证明，纯粹数学家大

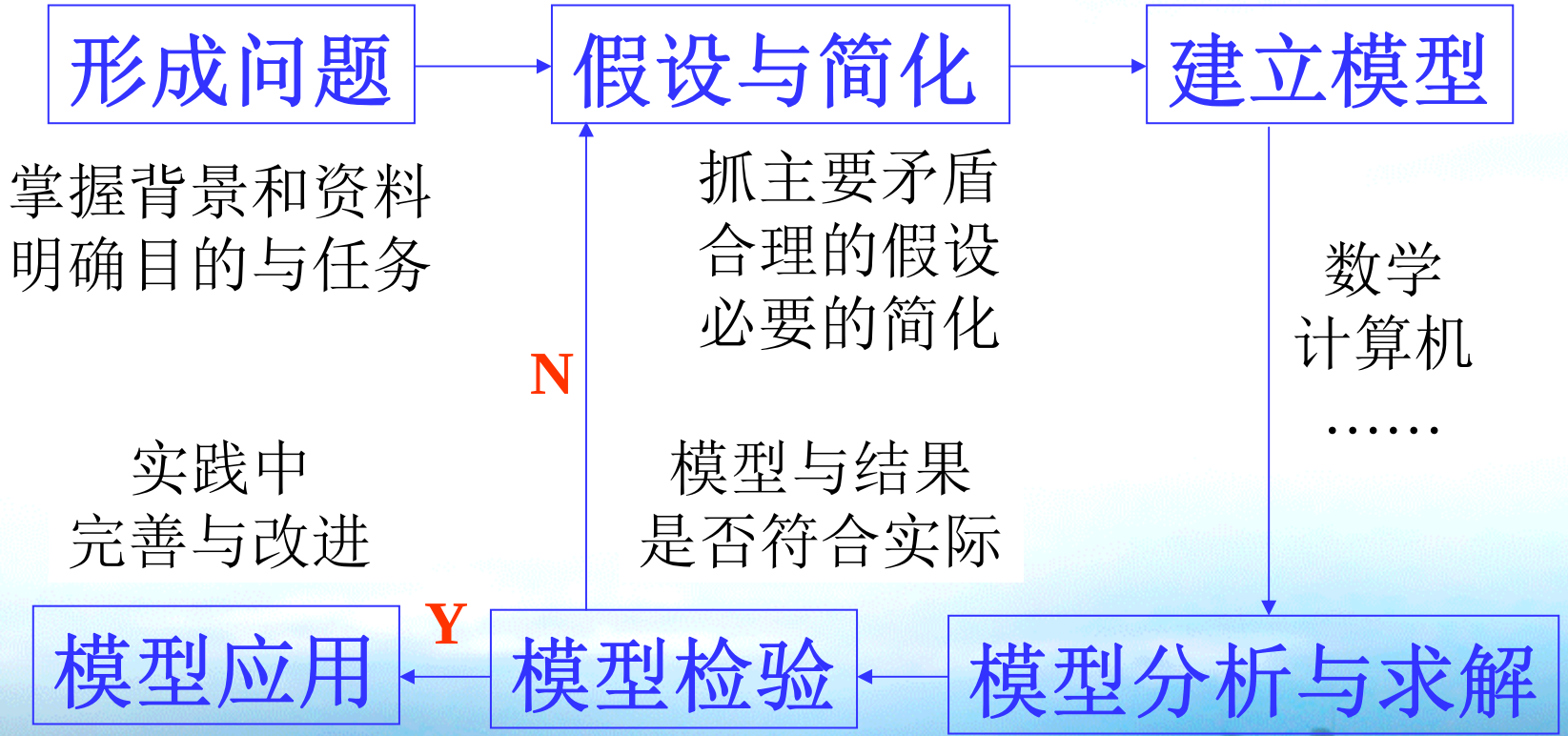
——转引自《华罗庚的数学生涯》

数学模型

- 数学模型是实际问题与数学理论之间的桥梁
  - 模型是对于现实世界的事物、现象、过程和系统的简化描述，或其部分属性的模仿
  - 数学模型（**Mathematical Model**）是针对现实世界的特定对象，为了特定目的，根据特有的内在规律，做出一些必要的简化假设，选用适当的数学工具，得到的一种数学结构
- 建立数学模型的过程，即为数学建模（**Mathematical Modeling**）



数学建模的主要步骤


形成问题
假设与简化
建立模型
掌握背景和资料
明确目的与任务
抓主要矛盾
合理的假设
必要的简化
数学
计算机
……
N
实践中
完善与改进
模型与结果
是否符合实际
Y
模型应用
模型检验
模型分析与求解

数学建模所需的能力

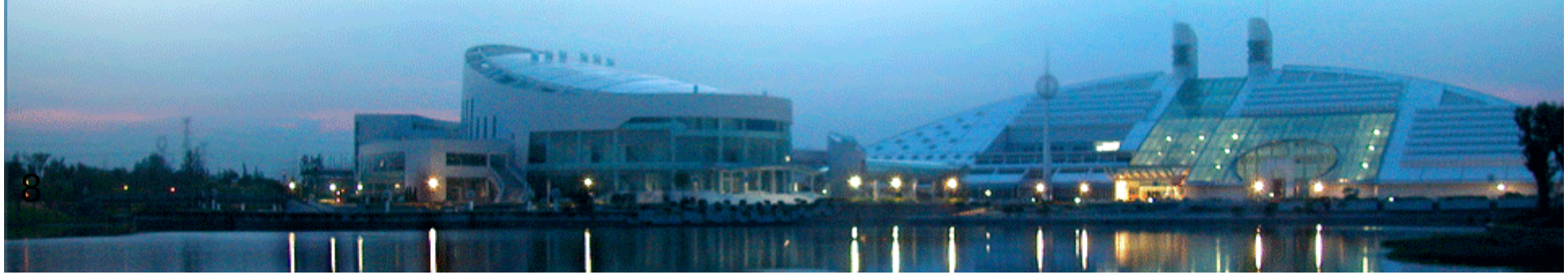
- 通过交流和查阅文献，归纳、抽象问题的能力
- 用数学表述、分析与求解问题的能力
- 使用计算机和数学软件等工具的能力
- 用语言和文字描述成果，推广应用模型的能力

应用数学研究的特点

- 以实际效果为衡量标准，重视理论指导作用
- 充分利用已有成果，创造性地为我所用
- 允许“不严格”，避免不正确
- 多学科协作，团队攻关

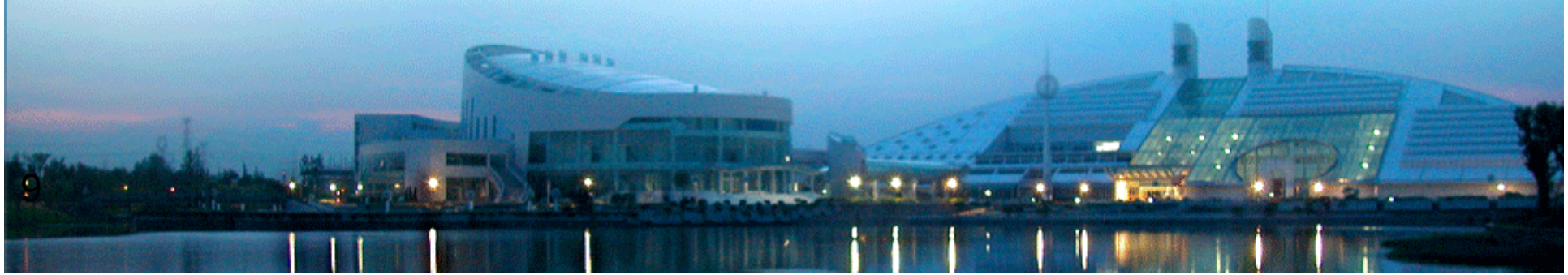
课程概况

- 课程目的与任务
  - 讲授经典数学模型和应用数学方法
  - 介绍数学应用前沿成果，展现数学魅力
  - 培养应用数学知识解决实际问题的能力，加深对数学的理解
  - 通过研究性学习，开展初步科研训练与实践，提升创新能力，提高综合素质
- 课堂讲授内容
  - 数学建模概述
  - 基本数学模型
  - 运筹与统计模型
  - 数学应用专题选讲
- 课程作业和课外实践
  - 一个专题
  - 一项课题



课程概况

- 课程资料
  - **http://estore.zju.edu.cn/share/2755894**
  - 验证码：**1825**
- 期末笔试：开卷考试
- **Email：** **tanzy@zju.edu.cn**



文献查阅

- 书籍
  - **CADAL**（**http://www.cadal.zju.edu.cn**）
- 百科
- 论文
  - **MathSciNet**（**http://www.ams.org/mathscinet/**）
  - **Google学术搜索**（**http://scholar.google.com/**）

综合性全文数据库

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数学建模

- **Nature & Science**
- **Springer**
  （**http://link.springer.com/**）
- **ELSEVIER**
  （**http://www.sciencedirect.com/**）
- **Wiley**
  （**http://onlinelibrary.wiley.com/**）
- **JSTOR**
  （**http://www.jstor.org/**）

专业性全文数据库

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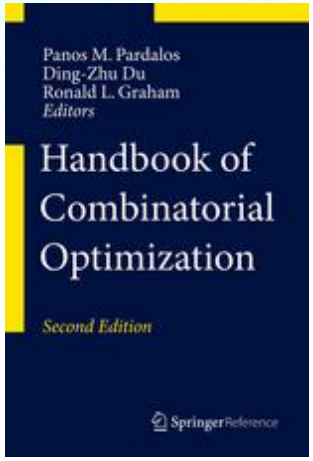
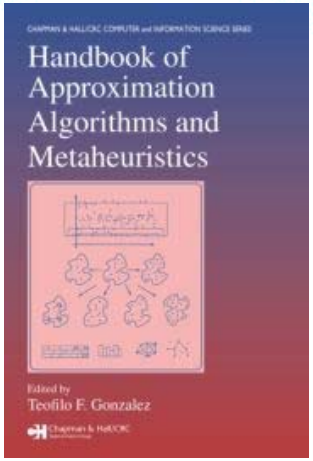
- **IEEE**
  - **The Institute of Electrical and Electronics Engineers（http://ieeexplore.ieee.org）**
- **ACM**
  - **Association for Computing Machinery（http://dl.acm.org/）**
- **SIAM**
  - **The Society for Industrial and Applied Mathematics（http://epubs.siam.org/）**
- **INFORMS**
  - **The Institute for Operations Research and the Management Sciences（http://informsjournals.highwire.org/）**

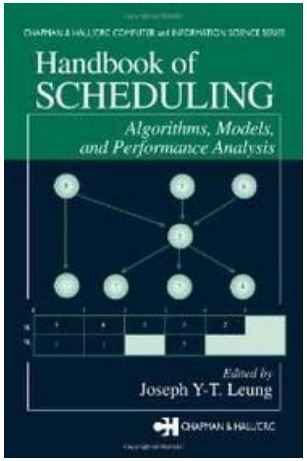
Handbook and Encyclopedia

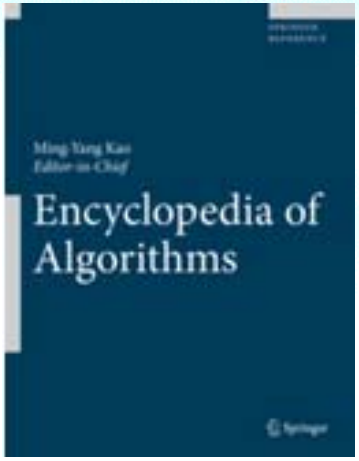
浙江大学 ZheJiang University

数学建模









Handbook of Combinatorial Optimization (2nd), Pardalos, P., Du, D-Z, Graham, R. L.主编, 3409页, Springer, 2013

Handbook of Approximation Algorithms and Metaheuristics, Gonzalez, T. F.主编, 1432页, CRC, 2007

Handbook of Scheduling: Algorithms, Models, and Performance Analysis, Leung, J. Y-T.主编, 1216页, CRC, 2004

Encyclopedia of Optimization Floudas, C. A., Pardalos, P.主编, 1166页, Springer, 2009

Encyclopedia of Algorithms, Kao, M-Y主编, 1166页, Springer, 2008

Handbook and Encyclopedia

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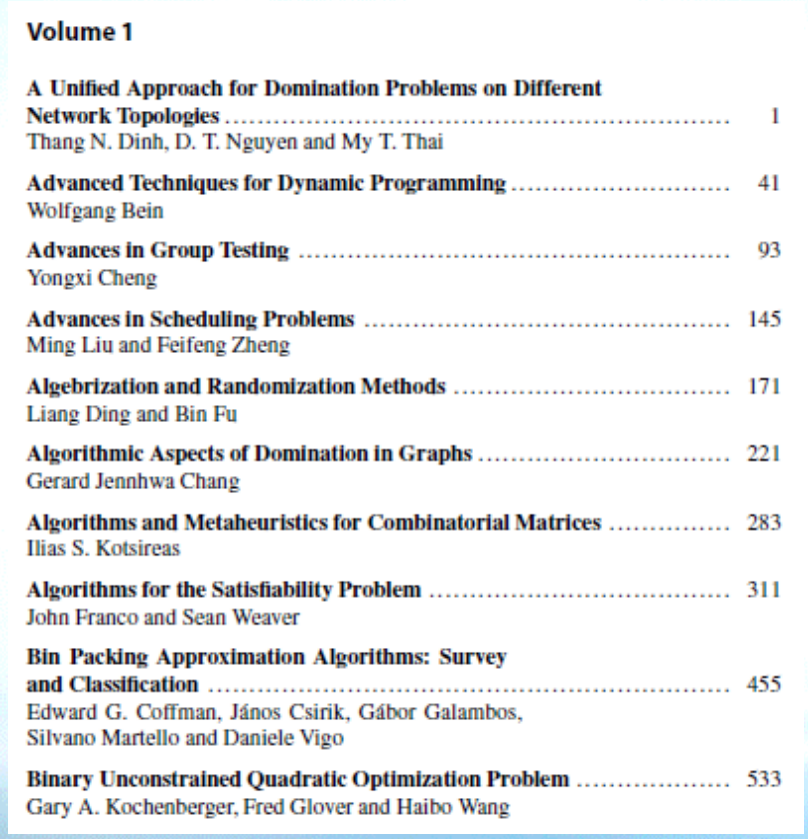
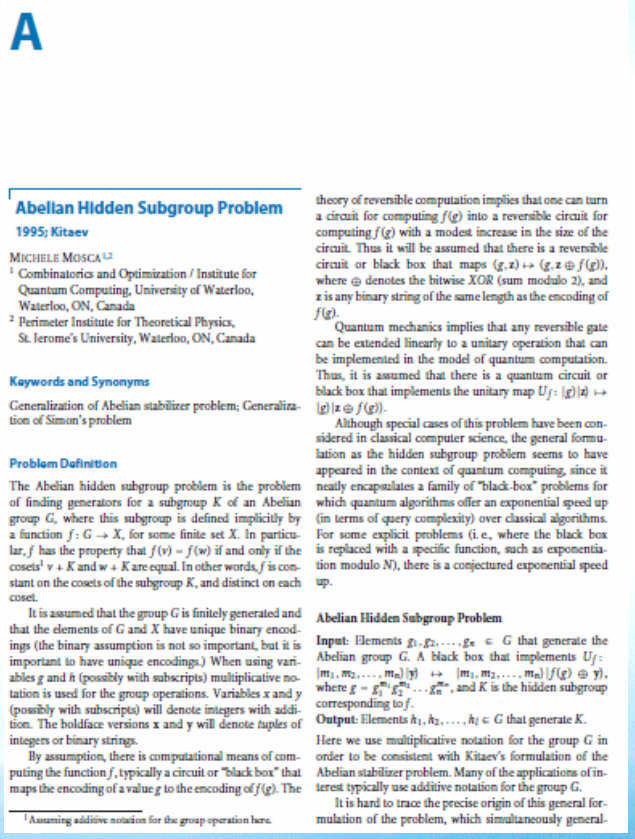
数学建模



Volume 1

Handbook of Combinatorial Optimization目录（部分）



A

Abelian Hidden Subgroup Problem

1995; Kitaev

MICHELE MOSCA[1,2]
[1] Combinatorics and Optimization / Institute for Quantum Computing, University of Waterloo, Waterloo, ON, Canada
[2] Perimeter Institute for Theoretical Physics, St. Jerome's University, Waterloo, ON, Canada

Keywords and Synonyms

Generalization of Abelian stabilizer problem; Generalization of Simon's problem

Problem Definition

The Abelian hidden subgroup problem is the problem of finding generators for a subgroup K of an Abelian group G, where this subgroup is defined implicitly by a function $f: G \to X$, for some finite set X. In particular, f has the property that $f(v) = f(w)$ if and only if the cosets[1] $v + K$ and $w + K$ are equal. In other words, f is constant on the cosets of the subgroup K, and distinct on each coset.

It is assumed that the group G is finitely generated and that the elements of G and X have unique binary encodings (the binary assumption is not so important, but it is important to have unique encodings.) When using variables g and h (possibly with subscripts) multiplicative notation is used for the group operations. Variables x and y (possibly with subscripts) will denote integers with addition. The boldface versions $\mathbf{x}$ and $\mathbf{y}$ will denote *tuples* of integers or binary strings.

By assumption, there is computational means of computing the function f, typically a circuit or "black box" that maps the encoding of a value g to the encoding of $f(g)$. The theory of reversible computation implies that one can turn a circuit for computing $f(g)$ into a reversible circuit for computing $f(g)$ with a modest increase in the size of the circuit. Thus it will be assumed that there is a reversible circuit or black box that maps $(g, z) \mapsto (g, z \oplus f(g))$, where $\oplus$ denotes the bitwise *XOR* (sum modulo 2), and z is any binary string of the same length as the encoding of $f(g)$.

Quantum mechanics implies that any reversible gate can be extended linearly to a unitary operation that can be implemented in the model of quantum computation. Thus, it is assumed that there is a quantum circuit or black box that implements the unitary map $U_f: |g\rangle|z\rangle \mapsto |g\rangle|z \oplus f(g)\rangle$.

Although special cases of this problem have been considered in classical computer science, the general formulation as the hidden subgroup problem seems to have appeared in the context of quantum computing, since it neatly encapsulates a family of "black-box" problems for which quantum algorithms offer an exponential speed up (in terms of query complexity) over classical algorithms. For some explicit problems (i. e., where the black box is replaced with a specific function, such as exponentiation modulo N), there is a conjectured exponential speed up.

Abelian Hidden Subgroup Problem

Input: Elements $g_1, g_2, \ldots, g_n \in G$ that generate the Abelian group G. A black box that implements $U_f: |m_1, m_2, \ldots, m_n\rangle|y\rangle \mapsto |m_1, m_2, \ldots, m_n\rangle|f(g) \oplus y\rangle$, where $g = g_1^{m_1} g_2^{m_2} \cdots g_n^{m_n}$, and K is the hidden subgroup corresponding to f.

Output: Elements $h_1, h_2, \ldots, h_l \in G$ that generate K.

Here we use multiplicative notation for the group G in order to be consistent with Kitaev's formulation of the Abelian stabilizer problem. Many of the applications of interest typically use additive notation for the group G.

It is hard to trace the precise origin of this general formulation of the problem, which simultaneously general-

[1] Assuming additive notation for the group operation here.

Encyclopedia of Algorithms首页

Survey, Review, Tutorial

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Available online at www.sciencedirect.com
ScienceDirect
European Journal of Operational Research 176 (2007) 774–793
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Discrete Optimization

Assignment problems: A golden anniversary survey

David W. Pentico

A.J. Palumbo School of Business, John F. Donahue Graduate School of Business, Duquesne University, Pittsburgh, PA 15282-0180, USA

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Available online 18 November 2005

SIAM REV.
Vol. 40, No. 3, pp. 496–546, September 1998

003

WELL-SOLVABLE SPECIAL CASES OF THE TRAVELING SALESMAN PROBLEM: A SURVEY*

RAINER E. BURKARD†, VLADIMIR G. DEĬNEKO‡, RENÉ VAN DAL§, JACK A. A. VAN DER VEEN¶, AND GERHARD J. WOEGINGER†

Abstract. The traveling salesman problem (TSP) belongs to the most basic, most important, and most investigated problems in combinatorial optimization. Although it is an $\mathcal{NP}$-hard problem, many of its special cases can be solved efficiently in polynomial time. We survey these special cases with emphasis on the results that have been obtained during the decade 1985–1995. This survey complements an earlier survey from 1985 compiled by Gilmore, Lawler, and Shmoys [*The Traveling Salesman Problem—A Guided Tour of Combinatorial Optimization*, Wiley, Chichester, pp. 87–143].

European Journal of Operational Research 232 (2014) 1–8
Contents lists available at SciVerse ScienceDirect
European Journal of Operational Research
journal homepage: www.elsevier.com/locate/ejor

Invited Review

OR analysis of sporting rules – A survey

Mike Wright*

Department of Management Science, Lancaster University, LA1 4YX, UK

Contents lists available at SciVerse ScienceDirect
Surveys in Operations Research and Management Science
journal homepage: www.elsevier.com/locate/sorms

Review

A tutorial on fundamental model structures for railway timetable optimization

Steven S. Harrod*

University of Dayton, United States

ARTICLE INFO

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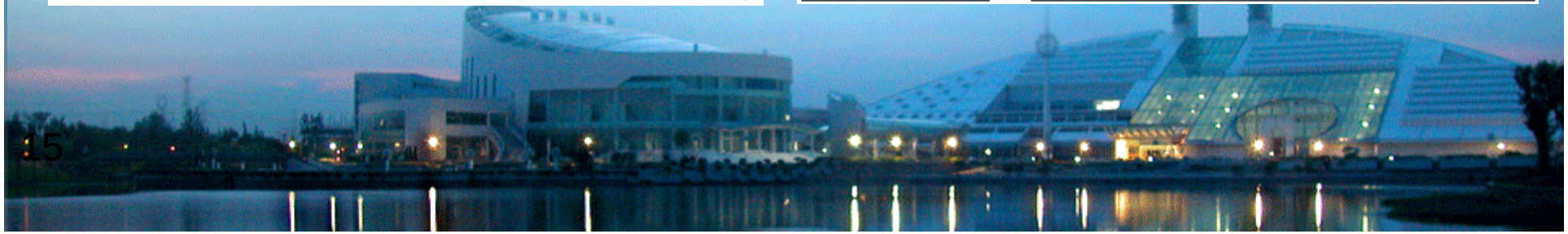
Received 22 June 2012
Accepted 8 August 2012

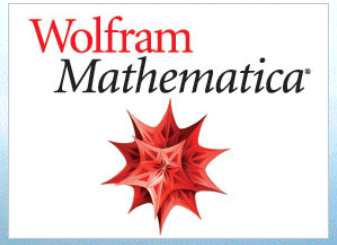
ABSTRACT

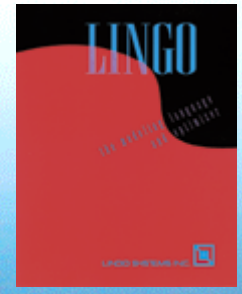
This guide explains the role of railway timetables relative to all other railway scheduling activities, and then presents four fundamental timetable formulations suitable for optimization. Timetabling models may be classified according to whether they explicitly model the track structure, and whether the timetable is intended to be periodic or not (aperiodic). The presentation of models is organized to facilitate the selection of a model by planning objective and available data, regardless of the specific traffic carried or network size.



数学软件

- 程序设计语言
- 综合性数学软件
  - **Mathematica**
  - **Matlab**
  - **Maple**
- 专业性数学软件
  - 统计
    - **SAS**
    - **SPSS**
    - **R**
  - 优化
    - **LINGO**
    - **CPLEX**



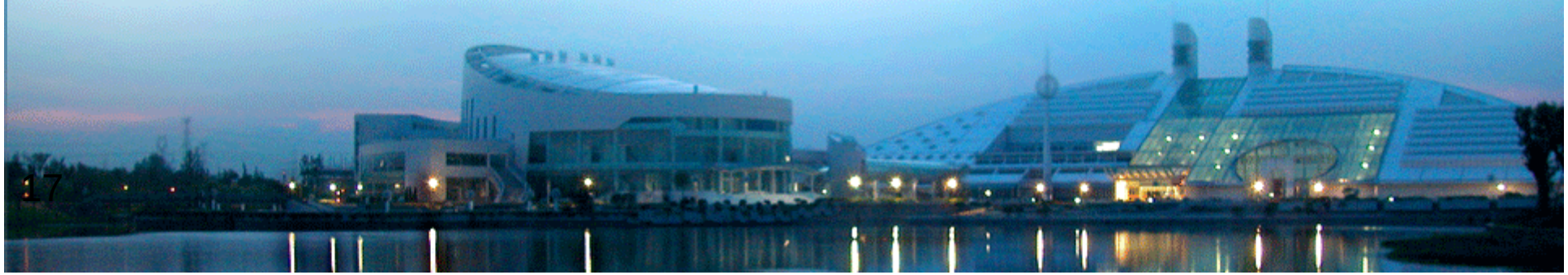


数学建模竞赛

浙江大学 ZheJiang University

数学建模

- 竞赛目的
  - 提高学生建立数学模型和运用计算机技术解决实际问题的综合能力
  - 激励学生学习数学的积极性，鼓励广大学生踊跃参加课外科技活动
  - 开拓知识面，培养创造精神及合作意识
  - 推动大学数学教学体系、教学内容和方法的改革
- 竞赛形式
  - 三人一队、三至四天、通讯竞赛

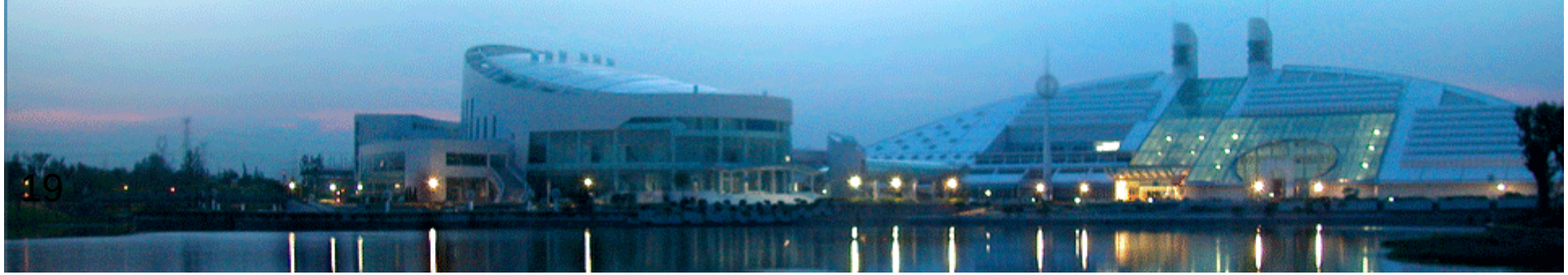


数学建模竞赛

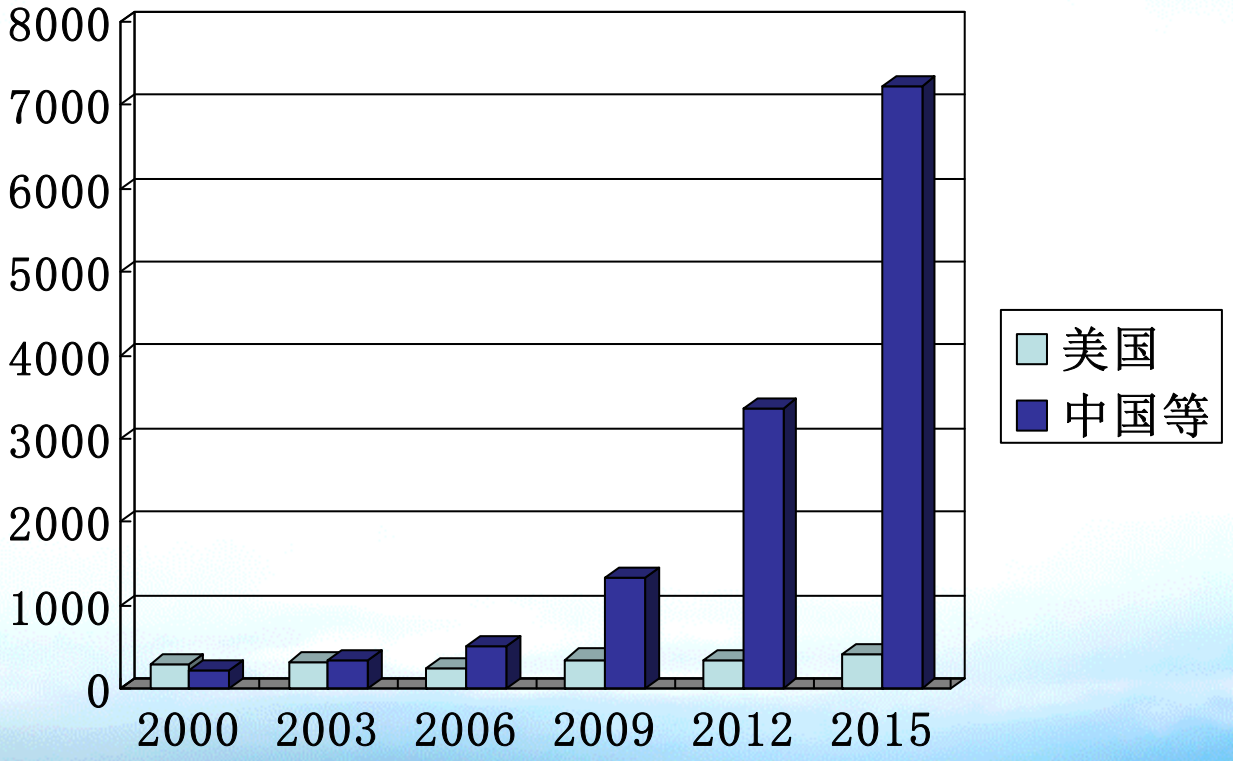
- 竞赛内容
  - 题目一般来源于工程技术和管理科学等方面经过适当简化加工的实际问题，不要求参赛者预先掌握深入的专门知识
  - 参赛者应根据题目要求，完成一篇包括模型的假设、建立和求解、计算方法的设计和计算机实现、结果的分析和检验、模型的改进等方面的论文（即答卷）
  - 评奖以假设的合理性、建模的创造性、结果的正确性和文字表述的清晰程度为主要标准

美国数模竞赛

- **The Mathematical Contest in Modeling（MCM）**
  The Interdisciplinary Contest in Modeling（ICM）
- 主办单位与主要协办单位
  - **COMAP（Consortium for Mathematics and Its Applications）（http://www.comap.com/）**
  - **INFORMS、SIAM、MAA（The Mathematical Association of America）**
- 创办于**1985**年， **2014**年共有**17**个国家**9773**支队伍参加
- 每年一月下旬或二月上旬举行，赛期四天，参赛队可从竞赛网站上公布的四道试题中任选一题



参赛队伍构成

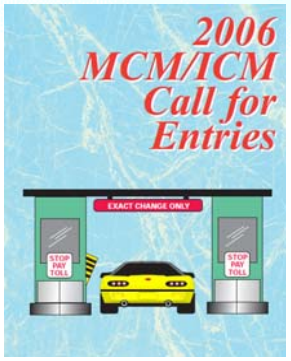


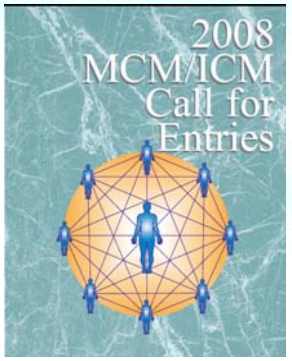
MCM参加队数（2000，2003年数据含ICM参赛队）

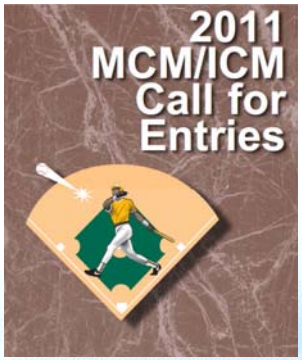
15年来，美国参赛队从300支增加到400支，中国参赛队从200支增加到7000支

竞赛试题

- 部分历年试题
  - **The Stunt Person**
  - **Tollbooths**
  - **Organ Transplant: The Kidney Exchange Problem**
  - **The Sweet Spot**
  - **Snowboard Course**
  - **The Leaves of a Tree**
- **2015年试题**
  - **Eradicating Ebola**
  - **Searching for a lost plane**
  - **Managing Human Capital in Organizations**
  - **Is it sustainable**







设奖情况

奖项	中译名	数目或比例	
		2014	2015
Outstanding Winners	特等奖	**19**	**19**
其中 INFORMS Award		**3**	**4**
SIAM Award		**2**	**2**
MAA Award		**2**	**2**
Finalists	特等奖提名	**17**	**33**
Meritorious Winners	一等奖	**10.11%**	**9.87%**
Honorable Mention	二等奖	**32.57%**	**32.59%**

国内高校获奖情况

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	获特等奖次数	兼获I /S奖次数
浙江大学	8次	5次
清华大学	9次	3次
北京大学	6次	0次
上海交通大学	5次	0次
国防科学技术大学	5次	1次
东南大学	4次	2次
南京大学	2次	1次
复旦大学	2次	0次
西安交通大学	2次	0次
中国科学技术大学	1次	0次

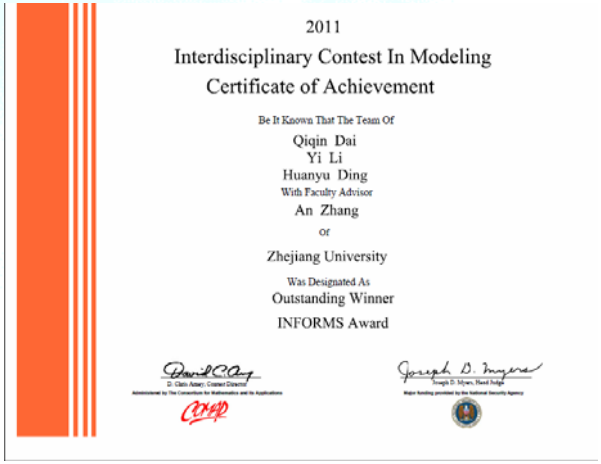
浙江大学历次获奖

1999(I)	沈权，杨振羽，何晓飞
2003(I)	胡煜霄，周恩露、华诤
2010	岳作功，阳宇光，赵聪
2011(I)	戴奇骏，李懿，丁寰宇
2012(S)	傅诚，朱丹亭，赵航琪
2013	朱常友，白伟成，杨文青
2015	陈凡，李锡涵，安毅宁

浙江大学参赛情况

浙江大学 ZheJiang University

数学建模


2011

Interdisciplinary Contest In Modeling

Certificate of Achievement

Be It Known That The Team Of

Qiqin Dai
Yi Li
Huanyu Ding

With Faculty Advisor

An Zhang

or

Zhejiang University

Was Designated As

Outstanding Winner

INFORMS Award

COMAP颁发的
2011年度特等奖奖状

INFORMS协会颁发的
2003年度INFORMS奖奖牌

Fly With Confidence

Hu Yuxiao
Hua Zheng
Zhou Enlu
Zhejiang University
Hangzhou, China

Advisor: Tan Zhiyi

Abstract

We develop a model to design a pile of cardboard boxes to cushion the fall of a stunt motorcycle; the kinetic energy of the motorcycle is consumed through breaking down the boxes.

We ignore the scattering effect of the boxes and begin by analyzing a single box. The energy to break it down has three components: the upper surfaces, the side surfaces, and the vertical edges. When big boxes are used, the upper surface provides the main supporting force; when small ones are used, the vertical edges play a great role.

We extend our analysis to the pile of boxes. Taking into account the maximum impulse that a person can bear, along with the camera effect and cost concerns, we determine the size of a box.

We conceive several stacking strategies and analyze their feasibility. We incorporate their strengths into our final strategy. We also examine several modifications to the box to improve its cushioning effect.

To validate our model, we apply it to different cases and get some encouraging results.

Assumptions

- *The stunt person and the motorcycle are taken as a system*, which we refer to as the motorcycle system or for brevity as the motorcycle. We ignore relative movement and interaction between them.
- *The motorcycle system is a uniform-density block.* We consider only the movement of its mass center, so we consider the motorcycle system as a mass particle.
- *The cardboard boxes are all made of the same material*, single wall S-1 cardboard 4.5 mm thick [Corrugated fiberboard . . . n.d.]

The UMAP Journal 24 (3) (2003) 301–316.

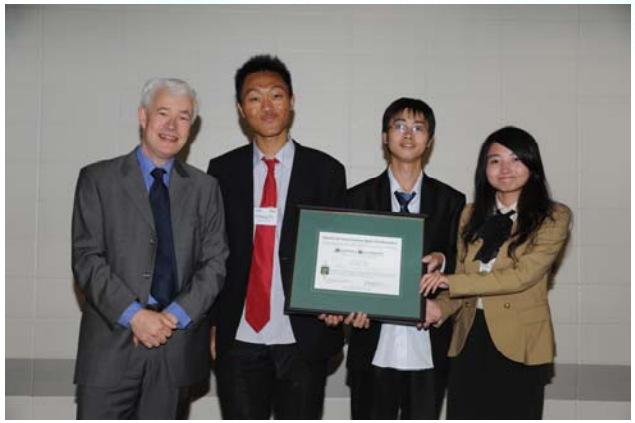
2003年度特等奖兼INFORMS奖论文刊登在
COMAP会刊The UMAP Journal上
（Undergraduate Mathematics and Its Applications）

浙江大学参赛情况

浙江大学 ZheJiang University

数学建模

SIAM协会颁发的
2012年度SIAM奖奖牌



时任**SIAM**主席的
Lloyd Nick Trefethen
教授为浙江大学
获**SIAM**奖学生颁奖


Students

Activities just for you at the 2012 SIAM Annual Meeting

MINN

Student Days Schedule and Other Activities of Interest to Students

Wednesday, July 11, 2012

10:30 AM—12:30 PM SIAM/MCM Award winners presentations (2011, 2012)
4:00 PM—6:00 PM SIAM Student Paper Prize winners presentations (2011, 2012)

美国数模竞赛

- **2016年美国数模竞赛比赛时间为（北京时间）2016年1月29日—2月2日**
  - **MCM Problem A (continuous)**
  - **MCM Problem B (discrete)**
  - **MCM Problem C (data insights)**
  - **ICM Problem D (operations research/network science)**
  - **ICM Problem E (environmental science)**
  - **ICM Problem F (policy)**

全国数模竞赛

浙江大学 ZheJiang University

数学建模

- 全国大学生数学建模竞赛
  China/Contemporary Undergraduate Mathematical Contest in Modeling （CUMCM）
  （ http://www.mcm.edu.cn ）
- 主办单位
  - 教育部高等教育司
  - 中国工业与应用数学学会
- 创办于**1992**年，**2014**年全国及部分海外高校有**1338**所院校、 **25347**个队 、**70000**多名本专科学生参赛
- 每年九月的第二个周末举行，赛期三天，参赛队可从竞赛网站上公布的两道试题中任选一题

CUMCM 中国大学生数学建模竞赛

创新意识
团队精神
重在参与
公平竞争

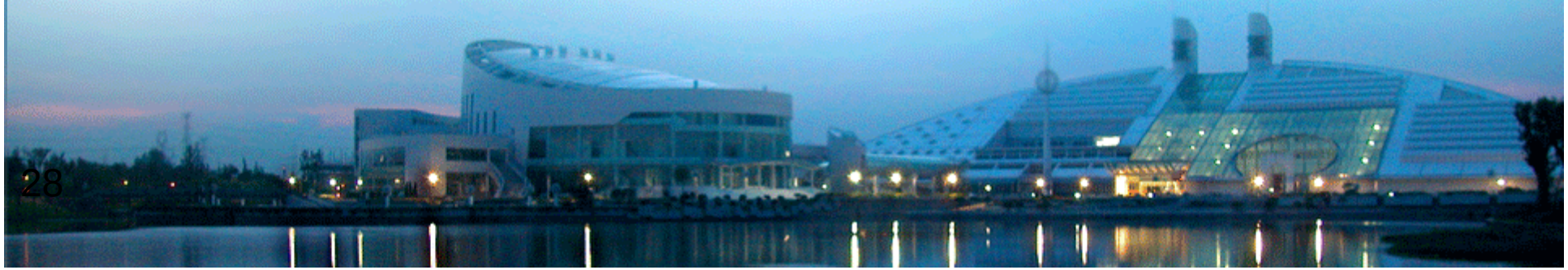
全国数模竞赛

浙江大学 ZheJiang University

数学建模

- **2014年（本科组）评出“高教社杯”1队，MATLAB创新奖1队。全国一等奖293个队、二等奖1256个队，全国一、二等奖分别占1.3%和6.6%。另设省一、二、三等奖若干**

- 部分竞赛试题
  - 彩票中的数学
  - 奥运会临时超市网点设计
  - 长江水质的评价和预测
  - 乘公交，看奥运
  - **2010年上海世博会影响力的定量评估**
  - **（2015A）太阳影子定位**
  - **（2015B）“互联网+”时代的出租车资源配置**



浙江大学参赛情况

浙江大学 ZheJiang University

数学建模

- 浙江大学每年参赛25队，近年来每年约有5～8队获全国一、二等奖
- 2010年浙江大学学生马宇斌、莫璐怡、杨琦获得本科组“高教社杯”


浙 江 大 学

学　生　马宇斌　莫璐怡　杨 琦

指导教师　数模组

荣获二零一零年高教社杯全国大学生数学建模竞赛本科组一等奖，并荣获高教社杯。

教育部高等教育司

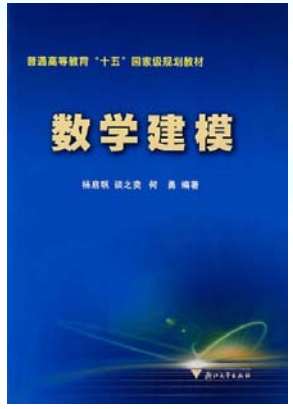
中国工业与应用数学学会

浙江大学数模活动

浙江大学 ZheJiang University

数学建模

- 浙江大学自1982年起开设数学建模课程，是国内最早的高校之一。2003年，数学建模课程被评为首批国家精品课程；2013年入选国家级精品资源共享课立项项目
- 1999年，浙江大学拨款100万元建设全国高校首个数学建模学生课外实践基地



浙江大学数模活动

浙江大学 ZheJiang University

数学建模

- 本科生院教学科研处主管全校各类本科生学科竞赛，浙江大学数学建模实践基地对我校数学建模实践与竞赛活动提供指导
  - 承办每年一度的浙江大学大学生数学建模竞赛，竞赛在每年5月举行，形式和内容基本参照全国数模竞赛，2014年全校共有211队参赛
  - 组织、指导浙江大学学生参加全国数模竞赛和美国数模竞赛

获奖学生

部分在高校和科研机构工作的近年获奖学生

何晓飞	1999美赛特等	浙江大学教授、杰青
曲冬霞	2001美赛一等	劳伦斯·利弗莫尔实验室
周恩露	2003美赛特等	佐治亚理工学院助理教授
华　诤	2003美赛特等	香港大学助理教授
汪琳薇	2004美赛二等	罗彻斯特理工学院助理教授
丁弋川	2004美赛二等	英属哥伦比亚大学助理教授
邬恩信	2005美赛一等	西安大略大学助理教授
黎　娜	2006美赛一等	哈佛大学助理教授


浙江大学
ZheJiang University
谢　谢