

Combinatorial Optimisation Coursework

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Definition of neighbourhood

The definition of my neighbourhood N is as follows.

For a ranking R_1 we can obtain a new ranking by first selecting a random player in the ranking as a pivot, then swap that player with an adjacent player in the ranking.

For example:

1. Let $R_1 = [1, 2, 3, 4, 5]$.
2. Select 3 as the pivot element.
3. We can swap 3 with either 2 or 4, let's choose 4.
4. From this we can obtain $R_2 = [1, 2, 4, 3, 5]$

In practice we always swap the selected element with the one to its right to ensure we only ever have to generate one random number per neighbourhood.

The cost of R_2 , $C(R_2)$ can be easily computed from the swaps made to create R_2 from R_1 and $C(R_1)$.

1. let $R_1 = [1, 2, 3, \dots, k, k+1, \dots, k+n]$
2. From R_1 obtain a new ranking R_2 by swapping k and $k+1$.
3. Set new score $C(R_1)'$ to $C(R_1)$.
4. If in the tournament k beat $k+1$:
 - Then add the matchup $(k, k+1)$'s weight to $C(R_1)'$
5. If in the tournament $k+1$ beat k :
 - Then subtract the matchup $(k+1, k)$'s weight from $C(R_1)'$
6. Set $C(R_2)$ to $C(R_1)'$

This runs in constant time.

Simulated Annealing Parameter Selection

I found that the best parameters for me are:

- $TI = 0.45$
- $TL = 162500$
- a in $f(T) = a \times T = 0.99$
- `num_non_improve (nni) = 1`

With these we can get an average score of 72.88 with an average runtime of 303ms

I tried a range of parameters, and the one that had the biggest effect was changing the temperature length. I've tested with grid search the ranges $TL \rightarrow [1000, 200000]$ and `nni` $\rightarrow [100, 100000]$

What I've found is that there are 2 main approaches. You can have a large temperature length and a low `nni`, or a large `nni` and a small temperature length.

I tried both approaches and found that you can achieve good scores with really quick runtimes (25ms - 80ms) with the latter. However, this approach also give higher variance in kemeny score and runtime. Scores hover around 71 to 75, but up to 90 on certain runs.

In contrast, setting a low `nni` with a high temperature length achieves the opposite. We can obtain consistently good scores (71 to 75) with high runtimes, with variance decreasing as we increase the temperature length. In fact we can achieve an average of 71.25 score with the parameter `TL = 500000`, `nni = 1`, but with runtimes of 1000ms. With these parameters we're only doing one pass of the outer loop. Therefore, with this configuration of the parameters the cooling ratio doesn't matter and we must set the initial temperature to a low number. However, too low of a `TI` and we don't make enough uphill moves, I found that a `TI` of 0.475 gives good results.

As for the existence of local optima, when running my program, even for a large number of iterations, with many uphill moves it can get stuck on non-optimal solutions. After observing these non-optimal solutions, almost all of the time they're unlike. This indicates that they're unique local optimas. Therefore I speculate that there are many local optimas.

Figures

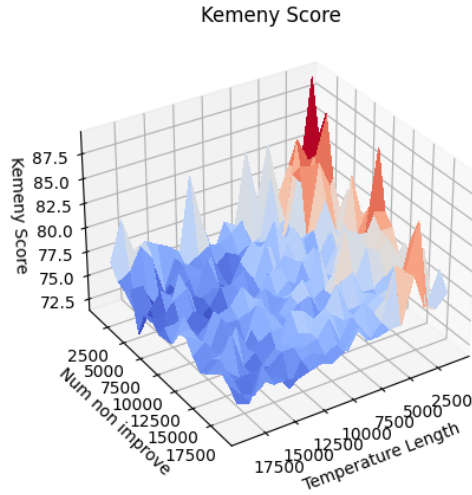


Figure 1: Kemeny score. $TI=0.9$, $a=0.99$

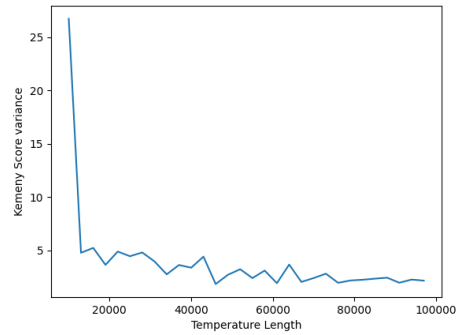


Figure 2: Kemeny Score Variance ($n = 25$). $TI = 0.45$, $a=0.95$, `nni`=1

Screenshots

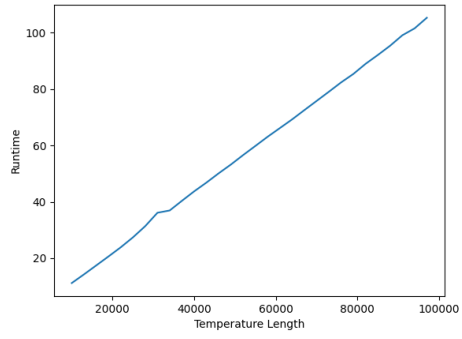


Figure 3: Runtime averaged over 25 runs. $TI = 0.45$, $a = 0.95$, $nmi = 1$

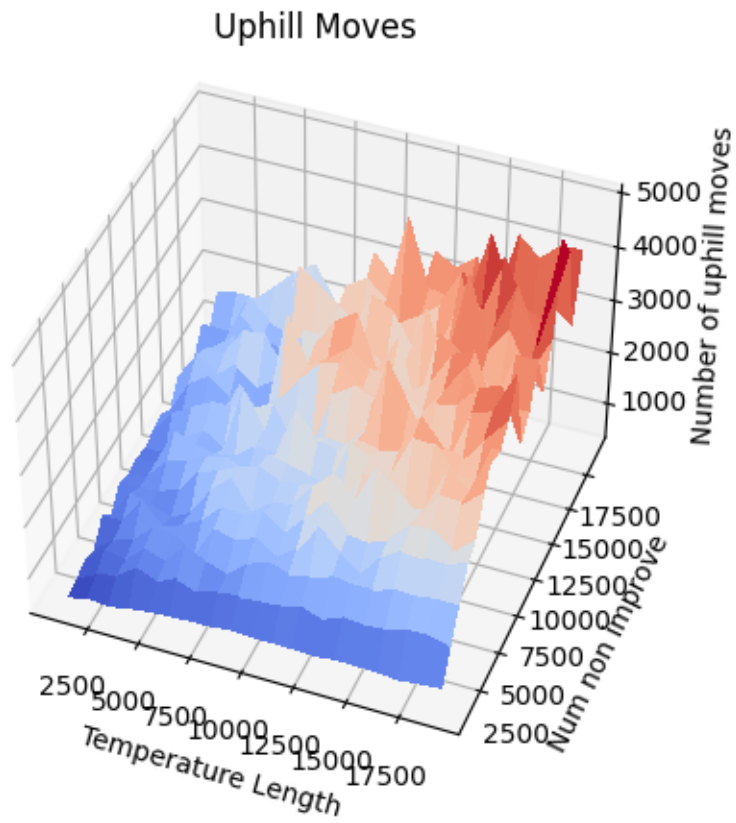


Figure 4: Number of uphill moves. $TI=0.9$, $a=0.99$

```
1.      Michael Schumacher
2.      Damon Hill
3.      Nigel Mansell
4.      Gerhard Berger
5.      David Coulthard
6.      Jean Alesi
7.      Mika Hakkinen
8.      Olivier Panis
9.      Rubens Barrichello
10.     Karl Wendlinger
11.     Eric Bernard
12.     Christian Fittipaldi
13.     Martin Brundle
14.     Mark Blundell
15.     Pierluigi Martini
16.     J J Lehto
17.     Heinz-Harald Frentzen
18.     Franck Lagorce
19.     Jean-Denis Deletraz
20.     Mika Salo
21.     Johnny Herbert
22.     Erik Comas
23.     Pedro Lamy
24.     Jos Verstappen
25.     Michele Alboreto
26.     Aguri Suzuki
27.     Olivier Beretta
28.     Jean-Marc Gounon
29.     David Brabham
30.     Ukyo Katayama
31.     Eddie Irvine
32.     Yannick Dalmas
33.     Alex Zanardi
34.     Domenico Schiattarella
35.     Gianni Morbidelli
36.     Taki Inoue
37.     Philippe Adams
38.     Nicola Larini
39.     Andrea de Cesaris
40.     Ayrton Senna
41.     Hideki Noda
42.     Philippe Alliot
43.     Bertrand Gachot
44.     Roland Ratzenberger
45.     Paul Belmondo
46.     Andrea Montermini
Elapsed time: 306.916ms
Kemeny Score: 76
uphill moves: 3852
```

Figure 5: Screenshot 1
4

```
1.      Michael Schumacher
2.      Damon Hill
3.      David Coulthard
4.      Nigel Mansell
5.      Gerhard Berger
6.      Jean Alesi
7.      Mika Hakkinen
8.      Olivier Panis
9.      Rubens Barrichello
10.     Karl Wendlinger
11.     Eric Bernard
12.     Jos Verstappen
13.     Martin Brundle
14.     Christian Fittipaldi
15.     Nicola Larini
16.     Mark Blundell
17.     Pierluigi Martini
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19.     Heinz-Harald Frentzen
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27.     Pedro Lamy
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31.     Olivier Beretta
32.     Eddie Irvine
33.     Yannick Dalmas
34.     Alex Zanardi
35.     Domenico Schiattarella
36.     Gianni Morbidelli
37.     Taki Inoue
38.     Andrea de Cesaris
39.     Philippe Adams
40.     Hideki Noda
41.     Philippe Alliot
42.     Ayrton Senna
43.     Bertrand Gachot
44.     Roland Ratzenberger
45.     Paul Belmondo
46.     Andrea Montermini
Elapsed time: 304.62899999999996ms
Kemeny Score: 72
uphill moves: 3901
```

Figure 6: Screenshot 2
5

```
1. Michael Schumacher
2. Damon Hill
3. David Coulthard
4. Gerhard Berger
5. Jean Alesi
6. Nigel Mansell
7. Mika Hakkinen
8. Olivier Panis
9. Rubens Barrichello
10. Eric Bernard
11. Karl Wendlinger
12. Jos Verstappen
13. Christian Fittipaldi
14. Nicola Larini
15. Martin Brundle
16. Mark Blundell
17. Pierluigi Martini
18. J J Lehto
19. Heinz-Harald Frentzen
20. Franck Lagorce
21. Jean-Denis Deletraz
22. Mika Salo
23. Johnny Herbert
24. Michele Alboreto
25. Erik Comas
26. Pedro Lamy
27. Aguri Suzuki
28. Olivier Beretta
29. Jean-Marc Gounon
30. David Brabham
31. Ukyo Katayama
32. Eddie Irvine
33. Yannick Dalmas
34. Alex Zanardi
35. Gianni Morbidelli
36. Taki Inoue
37. Domenico Schiattarella
38. Philippe Adams
39. Andrea de Cesaris
40. Ayrton Senna
41. Hideki Noda
42. Philippe Alliot
43. Bertrand Gachot
44. Roland Ratzenberger
45. Paul Belmondo
46. Andrea Montermini
Elapsed time: 299.4939999999997ms
Kemeny Score: 71
```

Figure 7: Screenshot 3