CS220 Discrete Math - Homework #3

Brendan Nguyen - brendan.nguyen001@umb.edu

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Question 1

A, B, and C are sets.

$$\begin{aligned}
(A-C)-(B-C) &= (A \cap C^c) \cap (B \cap C^c)^c \\
&= (A \cap C^c) \cap (B^c \cup C) \\
&= ((A \cap C^c) \cap B^c) \cup ((A \cap C^c) \cap C) \\
&= ((A \cap B^c) \cap C^c) \cup (A \cap (C^c \cap C)) \\
&= ((A \cap B^c) \cap C^c) \cup (A \cap \varnothing) \\
&= ((A-B)-C) \cup \varnothing \\
&= (A-B)-C
\end{aligned}$$

Question 2

By definition, $f(x)$ is strictly increasing if:

$$\forall x \forall y (x < y \to f(x) < f(y))$$

Dividing the inequality $f(x) < f(y)$ by the inequality $f(x)f(y) > 0$ results in:

$$\frac{1}{f(y)} < \frac{1}{f(x)}$$

The above inequality is equal to $g(y) < g(x)$, therefore:

$$\forall x \forall y (x < y \to g(x) > g(y))$$

Conversely, we can prove the inverse by testing $g(x) = \frac{1}{f(x)}$ which is strictly decreasing:

$$\forall x \forall y (x < y \to g(x) > g(y))$$

Using $g(x) > g(y) \stackrel{\text{def}}{=} \frac{1}{f(x)} < \frac{1}{f(y)}$ that we proved previously, we get:

$$\forall x \forall y (x < y \to f(x) < f(y))$$

Meaning that $f(x)$ is strictly increasing.

Question 3

(a) $A_n = 1.09 \cdot A_{n-1}$
denotes the recurrence relation for the amount in the account at the end of n years.

(b) $A_n = 1000 \cdot 1.09^n$
denotes the explicit formula for the amount in the account at the end of n years.

(c) $A_{100} = 1000 \cdot 1.09^{100} = \$5,529,040.79$
is the amount of money in the account after 100 years.

Question 4

$$\begin{aligned}
\sum_{i=1}^{n} \frac{1}{i(i+1)} &= \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1} \right) \\
&= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{n} - \frac{1}{n+1} \\
&= 1 - \frac{1}{n+1}
\end{aligned}$$

Question 5

To show that the set of functions $\{0,1,2,3,4,5,6,7,8,9\}$ is uncountable, we can use the fact that the set of all subsets of $\mathbb{N}$, $F(\mathbb{N})$, is uncountable. We see that the set of functions from $\mathbb{N}$ to $\{0,1,2,3,4,5,6,7,8,9\}$ contains the set $\{0,1\}^{\mathbb{N}}$ of functions from $\mathbb{N}$ to $\{0,1\}$ using injection. Therefore, you can say that there is a bijection between $F(\mathbb{N})$ and $\{0,1\}^{\mathbb{N}}$. In conclusion, since the set $\{0,1\}^{\mathbb{N}}$ is uncountable and the set is a subset in the set $\{0,1,2,3,4,5,6,7,8,9\}$, then we can say that the set $\{0,1,2,3,4,5,6,7,8,9\}$ is also uncountable.