

UMass Boston CS 220  
Homework 1  
Due 4 PM on February 3, 2022

1. If  $p_1, p_2, \dots, p_n$  are  $n$  propositions, explain why

$$\bigwedge_{i=1}^{n-1} \bigwedge_{j=i+1}^n (\neg p_i \vee \neg p_j)$$

is true if and only if at most one of  $p_1, p_2, \dots, p_n$  is true.

2. Construct the truth table of this compound proposition  $(p \wedge q) \vee \neg r$ .
3. Is the assertion “This statement is false” a proposition?
4. Determine whether  $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$  is a tautology.
5. Show that  $(p \rightarrow q) \vee (p \rightarrow r)$  and  $p \rightarrow (q \vee r)$  are logically equivalent.

Homework must be typeset, ideally by  $\text{\LaTeX}$ . You can get  $\text{\LaTeX}$  at <https://www.latex-project.org/get/>. Overleaf (<https://www.overleaf.com/>) may ease you into  $\text{\LaTeX}$ . Handwritten answers will be rejected. We use the Linux servers of the Computer Science Department to collect homework. First, you apply for a CS account and join CS 220 Section 01.

- Apply for an account on the CS subnet within UMB at the CS portal  
<https://portal.cs.umb.edu/registration/register/>
- Enter a username of your choice
- Enter your UMB email — you will receive a link to activate your CS account
- Enter a password of your choice
- After your CS account is activated, join CS 220 Section 01
- You can find step-by-step directions at  
[https://www.cs.umb.edu/~ghoffman/linux/apply\\_process.html](https://www.cs.umb.edu/~ghoffman/linux/apply_process.html)

This process of getting your CS account may take a few days to complete, especially when you mix up the passwords for your UMB account, your CS portal account, and your CS Linux account. Your accounts will be locked when you enter the wrong passwords too many times.

Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called `hw1.pdf` — the filename must be exactly `hw1.pdf`, otherwise it will not be collected. Common mistakes for filenames include `Hw1.pdf`, `HW1.pdf`, `hw1.pdf.pdf`, and variations thereof.
- Upload the file to the `cs220` folder in your home directory on the CS Linux server

- If you use a Linux or a Mac, you can do the following in a terminal
  - To upload a file to the server:
 

```
$ cd to_the_folder_that_contains_your_hw1.pdf
$ scp hw1.pdf your_CS_username@users.cs.umb.edu:~/cs220
//enter your password
```
  - To ssh into the server:
 

```
$ ssh your_CS_username@users.cs.umb.edu
//enter your password
//after you log on, do the following commands
$ cd cs220
$ ls -l
//this will show you the files in the directory
$ exit
//this is how to log off
```
- If you use a Windows machine, there are several choices
  - Latest stable Windows 10 has a built-in ssh client
  - WinSCP: <https://winscp.net/eng/index.php>
  - FileZilla: <https://filezilla-project.org/>

These apps will ask for the username, hostname, and port number. The username is your CS Linux account. The hostname is `users.cs.umb.edu`. The port number is 22, which is the default.

UMass Boston CS 220  
Homework 2  
Due 4 PM on February 10, 2022

1. Let  $P(x)$  be the predicate “ $x$  has a cellphone.” Let  $Q(x)$  be the predicate “ $x$  can solve quadratic equations.” Let  $R(x)$  be the predicate “ $x$  wants to be rich.”
  - (a) Let the domain be all students in CS 220. Translate the following statements into logical expressions using predicates, quantifiers, and logical connectives.
    - i. Everyone in CS 220 has a cellphone.
    - ii. Somebody in CS 220 can solve quadratic equations.
    - iii. Somebody in CS 220 does not want to be rich.
  - (b) Let the domain be all people. Let  $C(x)$  be the predicate “ $x$  is in CS 220.” Translate the statements above again.
2. Show that the argument with premises  $(p \wedge t) \rightarrow (r \vee s)$ ,  $q \rightarrow (u \wedge t)$ ,  $u \rightarrow p$ ,  $q$ , and  $\neg s$  and conclusion  $r$  is valid by using rules of inference.
3. Let  $S$  be the conditional statement “If  $S$  is true, then unicorns live.” Show that if  $S$  is a proposition, then “Unicorns live” is true. Show that it follows that  $S$  cannot be a proposition.
4. Use existential and universal quantifiers to express the statement “No one has more than two grandmothers” using the predicate  $G(x, y)$ , which represents “ $x$  is the grandmother of  $y$ ,” and the predicate  $x \neq y$  for “ $x$  is not equal to  $y$ .”
5. Prove or disprove that if  $A$ ,  $B$ , and  $C$  are nonempty sets and  $A \times B = A \times C$ , then  $B = C$ .

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called **hw2.pdf** — the filename must be exactly **hw2.pdf** — with your name at the top of the first page
- Upload the file to the **cs220** folder in your home directory on the CS Linux server

UMass Boston CS 220  
Homework 3  
Due 4 PM on February 17, 2022

1. Let  $A$ ,  $B$ , and  $C$  be sets. Show that  $(A - B) - C = (A - C) - (B - C)$ .
2. Let  $f : R \rightarrow R$  and let  $f(x) > 0$  for all  $x \in R$ . Show that  $f(x)$  is strictly increasing if and only if the function  $g(x) = 1/f(x)$  is strictly decreasing.
3. A person deposits \$1,000 in an account that yields 9% interest compounded annually.
  - (a) Set up a recurrence relation for the amount in the account at the end of  $n$  years.
  - (b) Find an explicit formula for the amount in the account at the end of  $n$  years.
  - (c) How much money will the account contain after 100 years?
4. Telescoping sum is the identity that  $\sum_{i=1}^n (a_i - a_{i-1}) = a_n - a_0$ . Use telescoping sum and the identity

$$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

to compute

$$\sum_{i=1}^n \frac{1}{i(i+1)}.$$

5. Show that the set of functions from the positive integers to the set  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$  is uncountable.

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- Prepare one PDF file called **hw3.pdf** — the filename must be exactly **hw3.pdf** — with your name at the top of the first page
- Upload the file to the **cs220** folder in your home directory on the CS Linux server

UMass Boston CS 220  
Homework 4  
Due 4 PM on February 24, 2022

1. Use the definition of  $f(n) = O(g(n))$  to show that  $2^n + 17 = O(3^n)$ .
2. Let  $k$  be a positive integer. Show that  $1^k + 2^k + \cdots + n^k = O(n^{k+1})$ .
3. Arrange the functions  $(1.5)^n$ ,  $n^{100}$ ,  $(\log n)^3$ ,  $\sqrt{n} \log n$ ,  $10^n$ ,  $(n!)^2$ , and  $n^{99} + n^{98}$  in a list so that each function is big-O of the next function.
4. The following C code comes from page 50 and Exercise 2-9 of the C Programming Language by Kernighan and Ritchie, the second edition.

```
int bitCount(unsigned x) {  
    int count;  
  
    for (count = 0; x != 0; x &= (x - 1))  
        count++;  
    return count;  
}
```

- (a) Explain why it counts the number of 1 bits in the unsigned integer  $x$ .
  - (b) How many iterations will the for-loop be executed?
5. To calculate the product of three integer matrices  $ABC$ , we can parenthesize the calculation as either  $(AB)C$  or  $A(BC)$ . Which parenthesization uses fewer integer multiplications if  $A$ ,  $B$ , and  $C$  have dimensions  $3 \times 9$ ,  $9 \times 4$ , and  $4 \times 2$ , respectively?

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called **hw4.pdf** — the filename must be exactly **hw4.pdf** — with your name at the top of the first page
- Upload the file to the **cs220** folder in your home directory on the CS Linux server

UMass Boston CS 220  
Homework 5  
Due 4 PM on March 3, 2022

1. Write out the addition and multiplication tables for  $Z_7$ , where the sum  $+_7$  and product  $\cdot_7$  are *modulo 7*.

$+_7$	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

$\cdot_7$	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

2. Find the sum and product of  $(20CBA)_{16}$  and  $(A01)_{16}$ . Express the answers in hexadecimal.
3. How many zeros are there at the end of  $100!$ ?
4. We call a positive integer perfect if it equals the sum of its positive divisors other than itself. Show that 6 and 28 are perfect.
5. Show that if  $a$ ,  $b$ , and  $m$  are integers such that  $m \geq 2$  and  $a \equiv b \pmod{m}$ , then  $\gcd(a, m) = \gcd(b, m)$ .

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called **hw5.pdf** — the filename must be exactly **hw5.pdf** — with your name at the top of the first page
- Upload the file to the **cs220** folder in your home directory on the CS Linux server

UMass Boston CS 220  
Homework 6  
Due 4 PM on March 10, 2022

1. Let  $p_1, p_2, \dots, p_n$  be the  $n$  smallest prime numbers. Prove or disprove that  $p_1 p_2 \cdots p_n + 1$  is prime for every  $n$ .
2. Using Bézout's theorem, find an inverse of 34 modulo 89 — that is, solve  $34a \equiv 1 \pmod{89}$ .
3. Use the value of  $a$  from the previous question to solve  $34x \equiv 77 \pmod{89}$ .
4. Show that the positive integers less than 11, except 1 and 10, can be put in pairs such that each pair consists of integers that are inverses of each other modulo 11.
5. Use Fermat's little theorem to find  $23^{1002} \pmod{41}$ .

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called **hw6.pdf** — the filename must be exactly **hw6.pdf** — with your name at the top of the first page
- Upload the file to the **cs220** folder in your home directory on the CS Linux server

UMass Boston CS 220  
Homework 7  
Due 4 PM on March 24, 2022

1. Prove that 6 divides  $n^3 - n$  for all nonnegative integer  $n$ .
2. Let  $A$  and  $B$  be square matrices such that  $AB = BA$ . Prove that  $AB^n = B^n A$  for all positive integer  $n$ .
3. Find the flaw with the following “proof” that  $a^n = 1$  for all nonnegative integers  $n$ , whenever  $a$  is a nonzero real number.  
Basis Step:  $a^0 = 1$  is true by the definition of  $a^0$ .  
Inductive Step: Assume that  $a^j = 1$  for all nonnegative integers  $j$  with  $j \leq k$ . Then note that

$$a^{k+1} = \frac{a^k \cdot a^k}{a^{k-1}} = \frac{1 \cdot 1}{1} = 1.$$

4. Give a recursive definition of the set of bit strings that are palindromes.
5. Design a recursive algorithm to find  $a^{2^n}$ , where  $a$  is a real number and  $n$  is a positive integer.  
Hint: Use the equality  $a^{2^{n+1}} = (a^{2^n})^2$ .

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called **hw7.pdf** — the filename must be exactly **hw7.pdf** — with your name at the top of the first page
- Upload the file to the **cs220** folder in your home directory on the CS Linux server



UMass Boston CS 220  
Homework 8  
Due 4 PM on April 7, 2022

1. Consider truth tables for a compound proposition of  $n$  variables. How many rows do the truth tables have? How many different truth tables exist? Use the product rule of counting to justify your answers.
2. How many ordered pairs of integers  $(a, b)$  are needed to guarantee that there are two ordered pairs  $(a_1, b_1)$  and  $(a_2, b_2)$  such that  $a_1 \bmod 5 = a_2 \bmod 5$  and  $b_1 \bmod 5 = b_2 \bmod 5$ ?
3. Let  $n_1, n_2, \dots, n_t$  be positive integers. Show that if  $n_1 + n_2 + \dots + n_t - t + 1$  objects are placed into  $t$  boxes, then for some  $i$ ,  $i = 1, 2, \dots, t$ , the  $i$ -th box contains at least  $n_i$  objects.
4. How many permutations of the letters ABCDEFGH contain
  - (a) the string ED?
  - (b) the string CDE?
  - (c) the strings BA and FGH?
  - (d) the strings AB, DE, and GH?
  - (e) the strings CAB and BED?
  - (f) the strings BCA and ABF?
5. Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men?

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called **hw8.pdf** — the filename must be exactly **hw8.pdf** — with your name at the top of the first page
- Upload the file to the **cs220** folder in your home directory on the CS Linux server

UMass Boston CS 220  
Homework 9  
Due 4 PM on April 14, 2022

1. Consider the expansion of  $(x + 1/x)^{15}$ . What are the coefficients of  $x^7$  and  $x^8$ ?
2. Use a combinatorial argument to show that, for a positive integer  $n$ ,

$$\binom{2n}{2} = 2 \binom{n}{2} + n^2.$$

3. In bridge, the 52 cards of a standard deck are dealt to four players. How many different ways are there to deal bridge hands to four players?
4. A shelf holds 12 books in a row. How many ways are there to choose five books so that no two adjacent books are chosen? Hint: Represent the books that are chosen by bars and the books not chosen by stars. Count the number of sequences of five bars and seven stars so that no two bars are adjacent.
5. We distribute 5 balls into 7 distinguishable boxes. Each box can hold at most 1 ball. How many ways can we distribute the balls if
  - (a) the balls are distinguishable
  - (b) the balls are indistinguishable

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called **hw9.pdf** — the filename must be exactly **hw9.pdf** — with your name at the top of the first page
- Upload the file to the **cs220** folder in your home directory on the CS Linux server

UMass Boston CS 220  
Homework 10  
Due 4 PM on April 21, 2022

1. What is the probability that a five-card poker hand contains a royal flush, that is, the 10, jack, queen, king, and ace of one suit?
2. Suppose that instead of three doors, there are four doors in the Monty Hall puzzle. What is the probability that you win by not changing once the host, who knows what is behind each door, opens a losing door and gives you the chance to change doors? What is the probability that you win by changing the door you select to one of the two remaining doors among the three that you did not select?
3. When we randomly select a permutation of  $\{1, 2, 3\}$ , what is the probability of the following events?
  - (a) 1 precedes 3.
  - (b) 3 precedes 1 and 3 precedes 2.
4. What is the probability of these events when we randomly permute the 26 lowercase letters?
  - (a) The first 13 letters of the permutation are in alphabetical order.
  - (b) a and z are next to each other in the permutation.
  - (c) a and z are not next to each other in the permutation.
5. Let  $E$  be the event that a randomly generated bit string of length three contains an odd number of 1s, and let  $F$  be the event that the string starts with 1. Are  $E$  and  $F$  independent?

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called `hw10.pdf` — the filename must be exactly `hw10.pdf` — with your name at the top of the first page
- Upload the file to the `cs220` folder in your home directory on the CS Linux server

UMass Boston CS 220  
Homework 11  
Due 4 PM on April 28, 2022

1.  $E$  and  $F$  are events in a sample space, and  $p(E) = 2/3$ ,  $p(F) = 3/4$ , and  $p(F|E) = 5/8$ . Find  $p(E|F)$ .
2. Suppose that  $E$ ,  $F_1$ ,  $F_2$ , and  $F_3$  are events from a sample space  $S$  and that  $F_1$ ,  $F_2$ , and  $F_3$  are pairwise disjoint and their union is  $S$ . Find  $p(F_2|E)$  if  $p(E|F_1) = 2/7$ ,  $p(E|F_2) = 3/8$ ,  $p(E|F_3) = 1/2$ ,  $p(F_1) = 1/6$ ,  $p(F_2) = 1/2$ , and  $p(F_3) = 1/3$ .
3. What is the expected value when a \$1 lottery ticket is bought in which the purchaser wins exactly \$10 million if the ticket contains the six winning numbers chosen from the set  $\{1, 2, 3, \dots, 50\}$  and the purchaser wins nothing otherwise?
4. What is the variance of the number of times a 6 appears when a fair die is rolled 10 times?
5. Use Chebyshev's inequality to find an upper bound on the probability that the number of tails that come up when a biased coin with probability of heads equal to 0.6 is tossed  $n$  times deviates from the mean by more than  $\sqrt{n}$ .

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called `hw11.pdf` — the filename must be exactly `hw11.pdf` — with your name at the top of the first page
- Upload the file to the `cs220` folder in your home directory on the CS Linux server