CS220 Discrete Math - Homework #5

Brendan Nguyen - brendan.nguyen001@umb.edu

March 3, 2022

Question 1

The addition and multiplication tables for Z_7 are shown below. To fill the tables, we use the defined operations for addition and multiplication modulo m, $a +_m b = (a + b) \bmod m$ and $a \cdot_m b = (a \cdot b) \bmod m$ respectively.

$+_7$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

$\cdot_7$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

Examples of above calculations where $m = 7$:

$a = 5, b = 6 : (5 + 6) \bmod 7 = 11 \bmod 7 = 4$
$a = 6, b = 4 : (6 \cdot 4) \bmod 7 = 24 \bmod 7 = 24 \bmod 21 = 3$

Question 2

The sum and product of $(20\text{CBA})_{16}$ and $(\text{A01})_{16}$ are shown below.

$$\begin{array}{lr}
 & {\scriptstyle 1}\phantom{\text{0CBA}_{16}} \\
 & \text{20CBA}_{16} \\
+ & \text{A01}_{16} \\
\hline
 & \text{21 6BB}_{16}
\end{array}$$

$$\begin{array}{lr}
 & {\scriptstyle 7\,7\,6}\phantom{\text{BA}_{16}} \\
 & \text{20CBA}_{16} \\
\times & \text{A01}_{16} \\
\hline
 & \text{20CBA}_{16} \\
 & 0\phantom{\text{A}_{16}} \\
+ & \text{147F4400}_{16} \\
\hline
 & \text{14815 0BA}_{16}
\end{array}$$

The corresponding binary values can be used to double check the above. The sum $(216\text{BB})_{16}$ gets the correct binary value of $(136{,}891)_{10}$ (or $134,330 + 2,561$). The product $(148150\text{BA})_{16}$ gets the correct binary value of $(344{,}019{,}130)_{10}$ (or $134,330 \times 2,561$).

Question 3

Just to establish a preconceived definition of a factorial. The given factorial $100! = 100 \times 99 \times 98 \times 97 \times \cdots \times 3 \times 2 \times 1$. In order to count the number of trailing zeros that exist in the result of 100!, we should find situations (meaning combinations of factors) that could result in an additional trailing zero. We can infer that a trailing zero will be formed by multiplying a multiple of 5 and a multiple of 2 together.

First, we can count the multiples of 5. These consist of $5, 10, 15, 20, 25, \ldots, 95, 100$, 20 multiples of 5. However, the four multiples of 25 $(25, 50, 75, 100)$ need to be counted twice since $25 = 5^2$ (meaning each multiple of 25 is essentially 2 multiples of 5). The final count of multiples of 5 is 24.

Next, we can count the multiples of 2. Getting the initial set of multiples of 2, we get a total of 50 multiples. As we did before, we also need to take into account multiples of 4, 8, etc. We can reasonably infer that the total multiples of 2 will far exceed the initial 50.

Finally, because we have only 24 multiples of 5 and far more multiples of 2, we can say that there will only be 24 trailing zeros in 100! because we can only have that number of unique pairs of multiples of 5 and 2.

Question 4

Listing the factors of 6 and 28 (not including the numbers themselves) and adding them together will show that they are perfect.

6: $1 + 2 + 3 = \mathbf{6}$

28: $1 + 2 + 4 + 7 + 14 = \mathbf{28}$

Question 5

We know that a is congruent to $b \bmod m$ if m divides $a - b$. We also know that a divides b is there's an integer x that satisfies $b = ax$. We can combine these two factors to say that there is an integer x such that $a - b = mx$ or $a = mx + b$. Next, we can define constants that will help us find the gcds: $A = \gcd(a, m)$ and $B = \gcd(b, m)$. Listing the gcds of two integers gets us: $A|a$, $A|m$, $B|b$, and $B|m$.

Since $a = mx + b$, $A|a$ and $A|m$ implies $A|b$. Similarly, $B|b$ and $B|m$ implies $B|a$. Then we can state that if an integer divides two integers, then the integer also divides their gcd.

$$A| \gcd(b, m)$$
$$B| \gcd(a, m)$$

Since $A = \gcd(a, m)$ and $B = \gcd(b, m)$, we can substitute into the two statements above which results in: $A|B$ and $B|A$. If $A|B$ and $B|A$ is true, then you can imply that $A = B$ and therefore, $\gcd(a, m) = \gcd(b, m)$.