UMass Boston CS 220
Homework 1
Due 4 PM on February 3, 2022

1. If $p_1, p_2, \ldots, p_n$ are n propositions, explain why

$$\bigwedge_{i=1}^{n-1} \bigwedge_{j=i+1}^{n} (\neg p_i \vee \neg p_j)$$

   is true if and only if at most one of $p_1, p_2, \ldots, p_n$ is true.

2. Construct the truth table of this compound proposition $(p \wedge q) \vee \neg r$.

3. Is the assertion "This statement is false" a proposition?

4. Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.

5. Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

Homework must be typeset, ideally by LaTeX. You can get LaTeX at `https://www.latex-project.org/get/`. Overleaf (`https://www.overleaf.com/`) may ease you into LaTeX. Handwritten answers will be rejected. We use the Linux servers of the Computer Science Department to collect homework. First, you apply for a CS account and join CS 220 Section 01.

- Apply for an account on the CS subnet within UMB at the CS portal `https://portal.cs.umb.edu/registration/register/`
- Enter a username of your choice
- Enter your UMB email — you will receive a link to activate your CS account
- Enter a password of your choice
- After your CS account is activated, join CS 220 Section 01
- You can find step-by-step directions at `https://www.cs.umb.edu/~ghoffman/linux/apply_process.html`

This process of getting your CS account may take a few days to complete, especially when you mix up the passwords for your UMB account, your CS portal account, and your CS Linux account. Your accounts will be locked when you enter the wrong passwords too many times.

Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called `hw1.pdf` — the filename must be exactly `hw1.pdf`, otherwise it will not be collected. Common mistakes for filenames include `Hw1.pdf`, `HW1.pdf`, `hw1.pdf.pdf`, and variations thereof.
- Upload the file to the `cs220` folder in your home directory on the CS Linux server

- If you use a Linux or a Mac, you can do the following in a terminal
  - To upload a file to the server:

    ```
    $ cd to_the_folder_that_contains_your_hw1.pdf
    $ scp hw1.pdf your_CS_username@users.cs.umb.edu:~/cs220
    //enter your password
    ```

  - To ssh into the server:

    ```
    $ ssh your_CS_username@users.cs.umb.edu
    //enter your password
    //after you log on, do the following commands
    $ cd cs220
    $ ls -l
    //this will show you the files in the directory
    $ exit
    //this is how to log off
    ```

- If you use a Windows machine, there are several choices
  - Latest stable Windows 10 has a built-in ssh client
  - WinSCP: `https://winscp.net/eng/index.php`
  - FileZilla: `https://filezilla-project.org/`

  These apps will ask for the username, hostname, and port number. The username is your CS Linux account. The hostname is `users.cs.umb.edu`. The port number is `22`, which is the default.

UMass Boston CS 220
Homework 2
Due 4 PM on February 10, 2022

1. Let $P(x)$ be the predicate "x has a cellphone." Let $Q(x)$ be the predicate "x can solve quadratic equations." Let $R(x)$ be the predicate "x wants to be rich."

   (a) Let the domain be all students in CS 220. Translate the following statements into logical expressions using predicates, quantifiers, and logical connectives.

      i. Everyone in CS 220 has a cellphone.
      ii. Somebody in CS 220 can solve quadratic equations.
      iii. Somebody in CS 220 does not want to be rich.

   (b) Let the domain be all people. Let $C(x)$ be the predicate "x is in CS 220." Translate the statements above again.

2. Show that the argument with premises $(p \wedge t) \to (r \vee s)$, $q \to (u \wedge t)$, $u \to p$, q, and $\neg s$ and conclusion r is valid by using rules of inference.

3. Let S be the conditional statement "If S is true, then unicorns live." Show that if S is a proposition, then "Unicorns live" is true. Show that it follows that S cannot be a proposition.

4. Use existential and universal quantifiers to express the statement "No one has more than two grandmothers" using the predicate $G(x, y)$, which represents "x is the grandmother of y," and the predicate $x \neq y$ for "x is not equal to y."

5. Prove or disprove that if A, B, and C are nonempty sets and $A \times B = A \times C$, then $B = C$.

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called `hw2.pdf` — the filename must be exactly `hw2.pdf` — with your name at the top of the first page
- Upload the file to the `cs220` folder in your home directory on the CS Linux server

UMass Boston CS 220
Homework 3
Due 4 PM on February 17, 2022

1. Let A, B, and C be sets. Show that $(A - B) - C = (A - C) - (B - C)$.

2. Let $f : R \to R$ and let $f(x) > 0$ for all $x \in R$. Show that $f(x)$ is strictly increasing if and only if the function $g(x) = 1/f(x)$ is strictly decreasing.

3. A person deposits \$1,000 in an account that yields 9% interest compounded annually.

   (a) Set up a recurrence relation for the amount in the account at the end of n years.

   (b) Find an explicit formula for the amount in the account at the end of n years.

   (c) How much money will the account contain after 100 years?

4. Telescoping sum is the identity that $\sum_{i=1}^{n}(a_i - a_{i-1}) = a_n - a_0$. Use telescoping sum and the identity
$$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$
to compute
$$\sum_{i=1}^{n} \frac{1}{i(i+1)}.$$

5. Show that the set of functions from the positive integers to the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is uncountable.

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called `hw3.pdf` — the filename must be exactly `hw3.pdf` — with your name at the top of the first page
- Upload the file to the `cs220` folder in your home directory on the CS Linux server

UMass Boston CS 220
Homework 4
Due 4 PM on February 24, 2022

1. Use the definition of $f(n) = \mathrm{O}(g(n))$ to show that $2^n + 17 = \mathrm{O}(3^n)$.
2. Let k be a positive integer. Show that $1^k + 2^k + \cdots + n^k = \mathrm{O}(n^{k+1})$.
3. Arrange the functions $(1.5)^n$, n^{100}, $(\log n)^3$, $\sqrt{n} \log n$, 10^n, $(n!)^2$, and $n^{99} + n^{98}$ in a list so that each function is big-O of the next function.
4. The following C code comes from page 50 and Exercise 2-9 of the C Programming Language by Kernighan and Ritchie, the second edition.

```
int bitCount(unsigned x) {
  int count;

  for (count = 0; x != 0; x &= (x - 1))
    count++;
  return count;
}
```

   (a) Explain why it counts the number of 1 bits in the unsigned integer `x`.

   (b) How many iterations will the for-loop be executed?

5. To calculate the product of three integer matrices ABC, we can parenthesize the calculation as either $(AB)C$ or $A(BC)$. Which parenthesization uses fewer integer multiplications if A, B, and C have dimensions 3×9, 9×4, and 4×2, respectively?

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called `hw4.pdf` — the filename must be exactly `hw4.pdf` — with your name at the top of the first page
- Upload the file to the `cs220` folder in your home directory on the CS Linux server

UMass Boston CS 220
Homework 5
Due 4 PM on March 3, 2022

1. Write out the addition and multiplication tables for Z_7, where the sum $+_7$ and product $\cdot_7$ are *modulo* 7.

$+_7$	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

$\cdot_7$	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

2. Find the sum and product of $(20\text{CBA})_{16}$ and $(\text{A01})_{16}$. Express the answers in hexadecimal.

3. How many zeros are there at the end of 100!?

4. We call a positive integer perfect if it equals the sum of its positive divisors other than itself. Show that 6 and 28 are perfect.

5. Show that if a, b, and m are integers such that $m \geq 2$ and $a \equiv b \pmod{m}$, then $\gcd(a, m) = \gcd(b, m)$.

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called `hw5.pdf` — the filename must be exactly `hw5.pdf` — with your name at the top of the first page
- Upload the file to the `cs220` folder in your home directory on the CS Linux server

UMass Boston CS 220
Homework 6
Due 4 PM on March 10, 2022

1. Let $p_1, p_2, \ldots, p_n$ be the n smallest prime numbers. Prove or disprove that $p_1 p_2 \cdots p_n + 1$ is prime for every n.
2. Using Bézout's theorem, find an inverse of 34 modulo 89 — that is, solve $34a \equiv 1 \pmod{89}$.
3. Use the value of a from the previous question to solve $34x \equiv 77 \pmod{89}$.
4. Show that the positive integers less than 11, except 1 and 10, can be put in pairs such that each pair consists of integers that are inverses of each other modulo 11.
5. Use Fermat's little theorem to find $23^{1002} \bmod 41$.

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called `hw6.pdf` — the filename must be exactly `hw6.pdf` — with your name at the top of the first page
- Upload the file to the `cs220` folder in your home directory on the CS Linux server

UMass Boston CS 220
Homework 7
Due 4 PM on March 24, 2022

1. Prove that 6 divides $n^3 - n$ for all nonnegative integer n.

2. Let A and B be square matrices such that $AB = BA$. Prove that $AB^n = B^n A$ for all positive integer n.

3. Find the flaw with the following "proof" that $a^n = 1$ for all nonnegative integers n, whenever a is a nonzero real number.
   Basis Step: $a^0 = 1$ is true by the definition of a^0.
   Inductive Step: Assume that $a^j = 1$ for all nonnegative integers j with $j \leq k$. Then note that

$$a^{k+1} = \frac{a^k \cdot a^k}{a^{k-1}} = \frac{1 \cdot 1}{1} = 1.$$

4. Give a recursive definition of the set of bit strings that are palindromes.

5. Design a recursive algorithm to find a^{2^n}, where a is a real number and n is a positive integer. Hint: Use the equality $a^{2^{n+1}} = (a^{2^n})^2$.

Homework must be typeset. Handwritten answers will be rejected. Homework is collected at the beginning of class on the due date. Late submission gets zero. To submit your homework:

- Prepare one PDF file called `hw7.pdf` — the filename must be exactly `hw7.pdf` — with your name at the top of the first page
- Upload the file to the `cs220` folder in your home directory on the CS Linux server