CS220 Discrete Math - Homework #3

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Question 1

A, B, and C are sets.

$$\begin{aligned}
(A - C) - (B - C) &= (A \cap C^c) \cap (B \cap C^c)^c \\
&= (A \cap C^c) \cap (B^c \cup C) \\
&= ((A \cap C^c) \cap B^c) \cup ((A \cap C^c) \cap C) \\
&= ((A \cap B^c) \cap C^c) \cup (A \cap (C^c \cap C)) \\
&= ((A \cap B^c) \cap C^c) \cup (A \cap \varnothing) \\
&= ((A - B) - C) \cup \varnothing \\
&= (A - B) - C
\end{aligned}$$

Question 2

By definition, $f(x)$ is strictly increasing if:

$$\forall x \forall y (x < y \rightarrow f(x) < f(y))$$

Dividing the inequality $f(x) < f(y)$ by the inequality $f(x)f(y) > 0$ results in:

$$\frac{1}{f(y)} < \frac{1}{f(x)}$$

The above inequality is equal to $g(y) < g(x)$, therefore:

$$\forall x \forall y (x < y \rightarrow g(x) > g(y))$$

Conversely, we can prove the inverse by testing $g(x) = \frac{1}{f(x)}$ which is strictly decreasing:

$$\forall x \forall y (x < y \rightarrow g(x) > g(y))$$

Using $g(x) > g(y) \stackrel{\text{def}}{=} \frac{1}{f(x)} < \frac{1}{f(y)}$ that we proved previously, we get:

$$\forall x \forall y (x < y \rightarrow f(x) < f(y))$$

Meaning that $f(x)$ is strictly increasing.

Question 3

(a) $A_n = 1.09 \cdot A_{n-1}$
denotes the recurrence relation for the amount in the account at the end of n years.

(b) $A_n = 1000 \cdot 1.09^n$
denotes the explicit formula for the amount in the account at the end of n years.

(c) $A_{100} = 1000 \cdot 1.09^{100} = \$5,529,040.79$
is the amount of money in the account after 100 years.

Question 4

Question 5