CS220 Discrete Math - Homework #1

Brendan Nguyen - brendan.nguyen001@umb.edu

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Question 1

The expanded form of the given compound composition is:

$$\begin{aligned}\bigwedge_{i=1}^{n-1}\bigwedge_{j=i+1}^{n}(\neg p_i \vee \neg p_j) = &(\neg p_1 \vee \neg p_2) \wedge (\neg p_1 \vee \neg p_3) \wedge (\neg p_1 \vee \neg p_4)\\ &\wedge \ldots \wedge (\neg p_1 \vee \neg p_n) \wedge (\neg p_2 \vee \neg p_3) \wedge \ldots \wedge (\neg p_2 \vee \neg p_n)\\ &\wedge \ldots \wedge (\neg p_{n-1} \vee \neg p_n)\end{aligned}$$

Using what we know:

1. DeMorgan's Law: $(\neg p_i \vee \neg p_j) = \neg(p_i \wedge p_j)$
2. The above should be `true` for all i and j

Statement 2 implies that $(p_i \wedge p_j)$ should be `false` for all i and j, which can only be fulfilled for at most 1 p that is `true`. The case of at most 1 p that is `true` results in $(\neg p_i \vee \neg p_j)$ is always `true`.

Question 2

The truth table is as follows:

p	q	r	$p \wedge q$	$\neg r$	$(p \wedge q) \vee \neg r$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

Table 1: Expanded truth table for $(p \wedge q) \vee \neg r$

Question 3

The statement, “This statement is false,” is not a proposition because it cannot have a truth value. If the statement was `true`, then it would assert that it’s `false`, which is a contradiction. Similarly, if the statement was `false`, it would assert that it’s `true`, another contradiction.

Question 4

Testing the truth values of p and q for the compound proposition, you get:

p	q	$\neg p$	$p \to q$	$\neg p \land (p \to q)$	$\neg q$	$(\neg p \land (p \to q)) \to \neg q$
F	F	T	T	T	T	T
F	T	T	T	T	F	F
T	F	F	F	F	T	T
T	T	F	T	F	F	T

Table 2: Expanded truth table for $(\neg p \land (p \to q)) \to \neg q$

By definition of a tautology, the compound proposition $(\neg p \land (p \to q)) \to \neg q$ is not a tautology because not all truth values are `true` for all p and q.

Question 5

The simplest way to evaluate the logical equivalence of the compound propositions $(p \to q) \lor (p \to r)$ and $p \to (q \lor r)$ is to compare their truth tables.

p	q	r	$p \to q$	$p \to r$	$(p \to q) \lor (p \to r)$	$q \lor r$	$p \to (q \lor r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Table 3: Expanded truth tables for $(p \to q) \lor (p \to r)$ and $p \to (q \lor (p \to r)$

Comparing the 2 truth tables shown in Table 3, you can see that for all p, q, and r, the truth values of the compound propositions are equivalent.