CS310: Advanced Data Structures and Algorithms

Fall 2022 Assignment 5

Due: Thursday, Dec. 15, 2022 on Gradescope

Goals

Huffman's coding, network flow

Questions

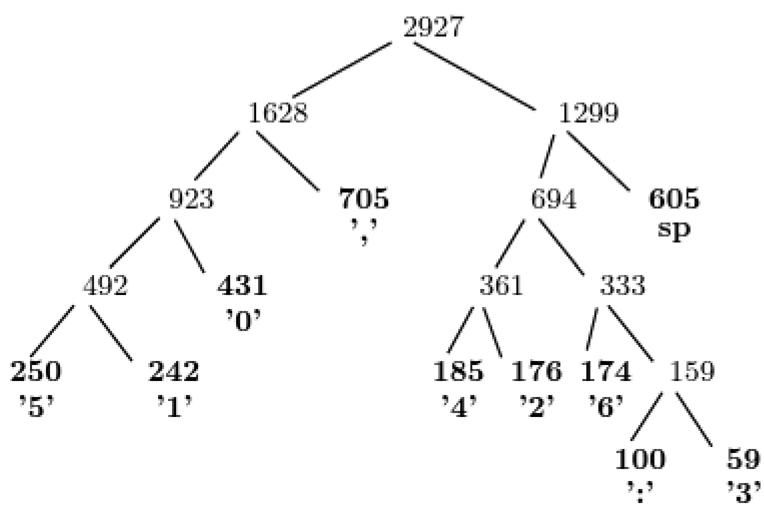
1. **Huffman's Coding:**

   (a) Show the Huffman trie that results from the following distributions (frequencies in parentheses): colon (100), space (605), comma (705), 0 (431), 1 (242), 2 (176), 3 (59), 4 (185), 5 (250), 6 (174). (To make the trie fully-specified, put the lower-weight subtree on the right on each merge operation. Start by listing the weights in increasing order from left to right, with their symbols below them on the next line, leaving space above to build trees.)

   The sorted characters:

Character	3	:	6	2	4	1	5	0	' '	,
Frequency	59	100	174	176	185	242	250	431	605	705

   The resulting tree is:



   The resulting code table and analysis:

character	code	frequency	total bits
:	10110	100	500
”	11	605	1210
,	01	705	1410
0	001	431	1293
1	0001	242	968
2	1001	176	704
3	10111	59	295
4	1000	185	740
5	0000	250	1000
6	1010	174	696

(b) What is the resulting binary code for the most frequent symbol? The least frequent?

From the table above, we see that the most frequent is ’,’ and the least frequent is ’3’.

(c) With the coding scheme above, code the 7-symbol text “04: 12,”. Show the binary string and the bytes in hex. The code would be `001 1000 10110 11 0001 1001 01` which translates to `31 6c 65`

(d) With the coding scheme above, decode 011101001011001.

, ’ ’ , 0 , 2

2. **LZW compression:** Compress the following text: aababacbaacbaadaaa using LZW. Show the output and the compression table.

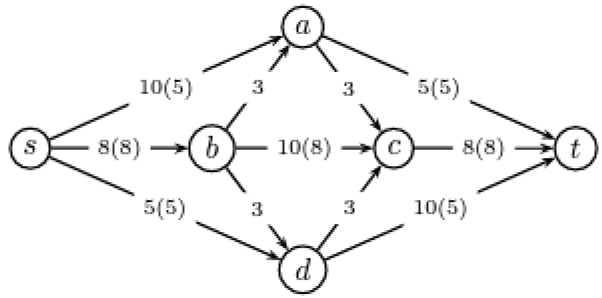
61 61 62 82 61 63 83 85 87 64 81 61 80

The compression table is as follows:

key	value
a	61
b	62
c	63
d	64
...	
aa	81
ab	82
ac	83
aba	84
ac	85
cb	86
baa	87
acb	88
baad	89
da	8A
aaa	8b

3. **Max-Flow-Min-Cut:** K& T 7.2: Given the following flow network on which an s-t flow has been computed.

The capacity of each edge appears as a label on the edge, and the numbers in parentheses give the amount of flow sent on each edge. (Edges without parentheses– specifically, the four edges of capacity 3– have no flow being sent on them.)

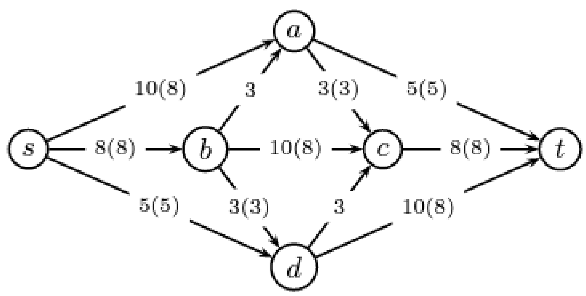


(a) What is the value of this flow? Is this a maximum (s,t) flow in this graph?

The value of the flow is 18, as seen by the sum of the flow on the edges leaving s or entering t.

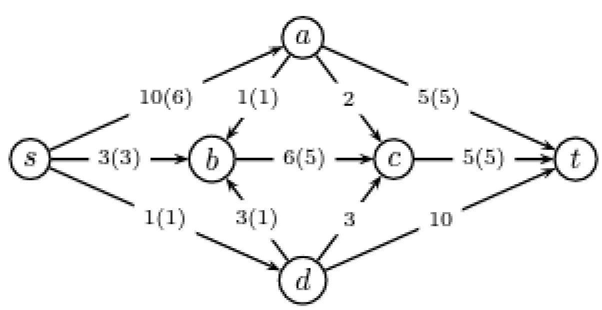
(b) Find a minimum s-t cut in the flow network and also say what its capacity is.

The minimum cut is $\{s, a\}$ and its capacity is 21. This is also the value of the maximum flow. It can be obtained by the following residual network:



The augmenting paths are: $x \to a \to t$ (capacity: 5), $s \to b \to c \to t$ (capacity: 5), $s \to a \to c \to t$ (capacity: 3), $s \to d \to t$ (capacity: 5), $s \to b \to d \to t$ (capacity: 3).

4. **Max-Flow-Min-Cut:** K& T 7.3: Given the following flow network on which an s-t flow has been computed. The capacity of each edge appears as a label on the edge, and the numbers in parentheses give the amount of flow sent on each edge. (as before, edges with no parentheses have no flow being sent on them.)

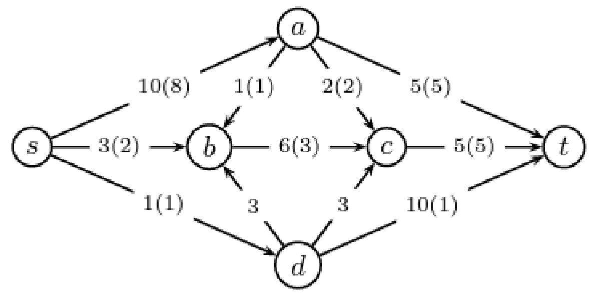


(a) What is the value of this flow? Is this a maximum (s,t) flow in this graph?

The value of the flow is 10, as seen by the sum of the flow on the edges leaving s or entering t.

(b) Find a minimum s-t cut in the flow network and also say what its capacity is.

The minimum cut is $\{s, a, b, c\}$ and its capacity is 11, which is also the value of the maximum flow. It can be obtained by the following residual network:



5. **Max-Flow-Min-Cut:** K & T 7.4: Decide whether the following statement is true or false. If it is true, give a short explanation. If it is false, give a counter example:

   Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity $c(e)$ on every edge e. If f is a maximum s-t flow in G, then f saturates every edge out of s with flow (i.e., for all edges e out of s, we have $f(e) = c(e)$).

   It is false. The examples from the previous questions provide counter-examples for this.