Underactuated Robot Control: Comparing LQR, Subspace Stabilization, and Combined Error Metric Approaches

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***Abstract*—In this paper, three techniques for robust control of underactuated robots are experimentally compared on the classical ball and beam system. An adaptive tracking controller is first designed and implemented to identify the nominal friction characteristic. Then, designs for a linear quadratic regulator (LQR), subspace stabilization controller, and combined error metric controller are presented. Step response tests confirm that both nonlinear approaches exhibit better stability properties than the standard LQR design. In addition, the subspace stabilization approach permits a much more aggressive beam motion, resulting in shorter settling time with excellent control of overshoot.**

***Index Terms*—Friction compensation, linear quadratic regulator (LQR), Lyapunov methods, sliding mode control, subspace stabilization.**

I. Introduction

RECENT research and development in the field of humanoid robotics has motivated great interest in the control of under-actuated systems. Under-actuated systems abound in robotics, including flexible joint and flexible link robots, wheeled robots, and walking robots. Such systems can be inherently unstable and may also possess complicated internal dynamics. Furthermore, under-actuated robotic systems may not be feedback linearizable. Well known examples of under-actuated research robots include the Acrobot [1], [2], Pendubot [3]–[5], and double inverted pendulum [6]. In this paper, the authors review design procedures and compare experimental performances for three robust control approaches: LQR control, nonlinear subspace stabilization, and nonlinear combined error metric control. Experimental evaluations are performed on a ball and beam system.

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A. System Model

The ball and beam system is a classical under-actuated mechanical system with two degrees of freedom. The beam rotates in the vertical plane, driven by a torque at the center of rotation (usually generated in the laboratory by an electrical drive). The ball rolls freely along the beam and in contact with the beam. Despite its mechanical simplicity, the ball and beam system presents significant challenges under large motion dynamics; the system is nonlinear, unstable, and has complex structural properties, including a poorly defined relative degree.

Assuming that the friction that appears in the transmission between the drive and the beam is Coulombic, the equations of motion of the plant are given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= B(\underline{x}) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= U \end{aligned} \tag{1}$$

where

$$B(\underline{x}) = \frac{M}{M/R^2 + J + J_B}\left(x_1 x_4^2 + g\sin(x_3)\right) \tag{2}$$

and the input torque τ is related to U by

$$\tau = 2Mx_1x_2x_4 + Mgx_1\cos(x_3) + (Mx_1^2 + J + J_B)U + F_C\mathrm{sgn}(x_4). \tag{3}$$

The following notations are used: x_1 for ball position along the beam; x_2 for ball velocity; x_3 for beam angle; x_4 for beam angular velocity; τ for input torque. Parameters of the model are: J for beam moment of inertia; J_B for ball moment of inertia; M for ball mass; R for ball radius; g for gravitational acceleration; F_C for Coulomb friction coefficient. Underlined symbols represent matrices, e.g., $\underline{x} = [\,x_1 \quad \cdots \quad x_4\,]^T$.

B. Overview of Other Relevant Works

Because the ball and beam dynamics are nonlinear, natural choices to formulate the control law may be either the input–output linearization method or the feedback linearization method. Unfortunately, the relative degree of the system is not well defined because of the centrifugal term $x_1x_4^2$ in the expression for $B(\underline{x})$ [7]. Thus, the conventional input–output linearization approach is not applicable for this system. The standard Jacobian linearization method can approximate well the nonlinear model only when the states are near the linearization point.

The control of ball and beam dynamics has been recently examined by many researchers. In [8] the control problem was locally solved using approximate input–output feedback linearization. Using nonlinear transformations and neglecting the centrifugal term $x_1x_4^2$ in the function (2), the authors arrived at an approximate system having well-defined relative degree. Independently, a fuzzy systems approach has been proposed to blend two control approaches: exact feedback linearization when the system is far from the singularities of $(x_1x_4^2 = 0)$, and approximate linearization near the singularities. The control law is a weighted average of the two control laws that were defined over different subspaces [9]. In [10], the idea was further developed using a switching algorithm to choose between approximate and exact feedback linearization algorithms. Instead of switching, a variable structure switching algorithm to avoid the singular states is also proposed in [11].

In [12], a model based control algorithm was proposed for the ball and beam system which exploits the plant model satisfying the so called matching condition. This condition guarantees that the control law will induce some specified structure on the closed loop system. In the paper [13], a passivity-based controller that was developed for underactuated mechanical systems was tested on the ball and beam system. A model based adaptive controller was proposed in [14] for the nonlinear ball and beam system with unknown parameters.

The study [15] presents the problem of synchronized stabilization of two ball and beam systems. Another approach for stabilizing single input nonlinear systems was presented in [16] in which switching control was applied for designing a controller that simultaneously stabilizes a collection of single-input nonlinear systems.

Model free approaches are also very popular for ball and beam control. In [17] and [18] the system was controlled using dynamic neural networks and in [19] an adaptive fuzzy approach was treated. The paper [20] proposes a support vector machine based fuzzy rule-based modeling for the approximation of the ball beam system nonlinearities in the control algorithm. In the study [21] an adaptive neural network-based sliding-mode controller, designed for fourth-order nonlinear systems, was applied for ball and beam control.

In summary, control of under-actuated system remains an open issue, as evidenced by the continuing study of systems like the ball and beam problem. In this paper, several robust nonlinear control methods are presented with experimental comparisons. The remainder of the paper is organized as follows. Section II presents an adaptive control experiment that is used to identify the friction characteristic of the test system. Section III presents two nonlinear control algorithms for robust stabilization of the ball and beam system. Section IV presents the experimental apparatus and the experimental results based on the proposed algorithms. Finally, Section V sums up the conclusions drawn from the experimental results.

II. Adaptive Friction Identification

It is well known that if precise positioning is desired in mechanical control systems, then friction effects must also be taken into consideration in the control algorithm [22]. If friction compensation is omitted, then steady-state error can appear. In the case of the ball and beam system, the ball will not precisely reach the desired equilibrium point (center of the beam). The dominant friction appears in the mechanical transmission between the driving motor and the beam.

In this paper, an adaptive control approach is used to identify the friction characteristic. Specifically, the unknown friction parameter is estimated using an adaptive control law designed for a trajectory tracking task. The steady-state value of the estimated parameter is then used for fixed friction friction compensation in the model based nonlinear control algorithms. To solve the tracking problem of the beam with unknown Coulomb friction, a simplified version of the algorithm presented in [23] is applied.

Assuming that the friction can be described with a Coulomb friction model, the dynamics of the beam without the ball is given by

$$J\dot{x}_4 = \tau - F_C\mathrm{sgn}(x_4). \quad (4)$$

In the model above, the beam moment of inertia J is known, but the Coulomb friction coefficient F_C is unknown. Denote with $\hat{F}_C$ the unknown (estimated) parameter and let the estimation error be $\tilde{F}_C = F_C - \hat{F}_C$.

Define the tracking error metric for the beam

$$S_B = (x_4 - \dot{x}_{3ref}) + \lambda_B(x_3 - x_{3ref}) \quad (5)$$

where x_{3ref} is the prescribed beam angle, a twice differentiable function of time.

Choose the control law

$$\tau = \widehat{F}_C\mathrm{sgn}(x_4) + J\left(\ddot{x}_{3ref} - \lambda_B(x_4 - \dot{x}_{3ref})\right) - K_S\, S_B,\ K_S > 0. \quad (6)$$

To obtain the unknown friction parameter choose the adaptation law

$$\dot{\hat{F}} = -\gamma_F S_B\mathrm{sgn}(x_4). \quad (7)$$

For stability analysis of the adaptive control law, define the following Lyapunov function candidate:

$$V_B = \frac{1}{2}S_B^2 + \frac{1}{2\gamma_F}\tilde{F}_C^2, \quad \gamma_F > 0. \quad (8)$$

It can be shown that with control law (6) and adaptation law (7), the time derivative of the Lyapunov function is negative definite

$$\dot{V}_B = -K_S S_B^2 < 0, \quad \forall S_B \neq 0. \quad (9)$$

According to (9) the tracking error metric converges to zero: $S_B \to 0$ as $t \to \infty$. From the Lyapunov analysis one cannot directly conclude that the friction parameter estimation error converges to zero. However, from the closed loop system dynamics

$$\dot{S}_B + K_S S_B = -F_C \mathrm{sgn}(x_3) + \widehat{F}_C \mathrm{sgn}(x_3)$$

it can be shown that if S_B and $\dot{S}_B$ converge to zero, then the value of the estimated friction parameter will converge to the real friction parameter value so long as $\dot{x}_4 \neq 0$; this is an expression of LaSalle's theorem. Experimental confirmation is presented in Section IV-B.

A nominal, fixed value of the friction coefficient is required for the robust control design methods. A value for the nominal friction coefficient is computed as the mean value of the estimated parameter in steady state. This mean value is used in implementations of the various stabilizing control laws.

III. Stabilizing Ball and Beam Control

In this section, two recently developed approaches are presented as candidate robust controllers for the nonlinear ball and beam system. One is the nonlinear subspace stabilization approach, and the second is the nonlinear combined error metric method. Both methods are experimentally compared to the classical LQR control in Section IV.

A. Subspace Stabilization Approach

The subspace stabilization approach [24], [25] is a "divide-and-conquer" approach for nonlinear systems, and constructs a control having two terms

$$U = u_{ss} + v. \tag{10}$$

First, one picks a subspace containing the dynamics that contribute to structural problems. Next, a stabilizing controller u_{ss} is designed to minimize the effects of the subspace dynamics. The compensated system becomes the model for designing the second term v that stabilizes overall dynamics.

In the case of the ball and beam system, it is well known that the centripetal term $x_1 x_4^4$ poses challenges to standard nonlinear design techniques—it is the term addressed in works such as [8]–[10]. That term is associated with the beam velocity, so the subspace stabilization approach will first stabilize beam dynamics. A Lyapunov method is chosen to stabilize the beam dynamics, which make up the (x_3, x_4) subspace. The system model having stabilized beam is then stabilized by an LQR approach.

Define the Lyapunov function for beam stabilization

$$V(x_3, x_4) = \frac{1}{2}\left(x_3^2 + x_4^2\right). \tag{11}$$

The time derivative of the Lyapunov function yields

$$\dot{V} = x_3 x_4 + x_4 U. \tag{12}$$

Design the subspace stabilizing control as

$$u_{ss} = -x_4 - x_3 - x_4 x_3^2. \tag{13}$$

With u_{ss} given above and $v = 0$, the time derivative of the Lyapunov function is negative definite for $x_3, x_4 \neq 0$

$$\dot{V}(x_3, x_4) = -x_4^2 - x_4^2 x_3^2. \tag{14}$$

The system model with stabilized beam is given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= B(\underline{x}) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -x_4 - x_3 - x_4 x_3^2 + v. \end{aligned} \tag{15}$$

To apply the LQR design approach, the subspace stabilized system (15) must be linearized. Use the following approximations:

$$\begin{aligned} B(\underline{x}) &= \frac{M}{M/R^2 + J + J_B}\left(x_1 x_4^2 + g\sin(x_3)\right) \\ &\approx \frac{Mg}{M/R^2 + J + J_B} x_3 \end{aligned} \tag{16}$$

$$-x_4 - x_3 - x_4 x_3^2 \approx -x_4 - x_3. \tag{17}$$

Then, the linearized approximation of (15) is given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{M}{M/R_2 + J + J_B} x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -x_4 - x_3 + v. \end{aligned} \tag{18}$$

Design v for the linear system above using the `lqr` function in MATLAB

$$v = -\underline{K}_{ss}\underline{x}. \tag{19}$$

The complete control (10) is specified by (13) and (19).

It should be noted that other combinations of subspace stabilization are possible. For example, one could apply a sliding mode approach to design u_{ss}, followed by a model reference adaptive control for the control v.

B. Nonlinear Combined Error Metric Control Approach

The second nonlinear controller tested is based on the sliding control experiments originally reported in [14]. To develop a control law for the ball and beam system, define the following combined error metric:

$$S = \alpha(x_2 + \lambda_1 x_1) + (x_4 + \lambda_2 x_3) \tag{20}$$

where α, λ_1, $\lambda_2 > 0$.

The first term component of the error is meant to stabilize the ball in the center of the beam. The second term is introduced to avoid the large motions of the beam that could introduce instability during the control.

The control task can be formulated as follows: find a control law U such that $S(t) \to 0$ as $t \to \infty$.

Fig. 1. Experimental ball-and-beam system.

It is well known that the function $S(t)$ will converge to zero if it satisfies

$$\dot{S} + K_S S = 0, \quad K_S > 0. \tag{21}$$

Calculate the time derivative of S

$$\dot{S} = \alpha\left(B(\underline{x}) + \lambda_1 x_2\right) + (U + \lambda_2 x_4). \tag{22}$$

Formulate the control law as follows:

$$U = -\alpha\left(B(\underline{x}) + \lambda_1 x_2\right) - \lambda_2 x_4 - K_S S. \tag{23}$$

Introducing (23) into (22) yields (21). Hence, with U given by (23) the proposed control problem is solved.

The control law (23) guarantees that the weighted sum of the states converges to zero. However, the parameters $\lambda_1 > 0$, $\lambda_2 > 0$, $\alpha > 0$ do not depend on the controlled system dynamics. Therefore, convergence of the combined error metric (20) holds for arbitrarily chosen controller parameters so long as the state converges to zero.

The main advantage of this approach is that the plant nonlinearity appears as an additive term in the dynamics (22) of the error metric. Hence, it can easily be compensated with the term "$-\alpha B(\underline{x})$" of the control law (23) and the stabilization of the function S can be guaranteed. The remaining terms of the control law (23) are equivalent to a linear state feedback.

The stabilizing control laws can be extended with the $\widehat{F}_C \mathrm{sgn}(x_4)$ term to compensate for friction. During the stabilizing control the mean value of the estimated parameter $\widehat{F}_C$ in steady state is applied, which can be obtained from an *a priori* identification process as described in Section II.

IV. Experimental Evaluations

A. Experimental Setup

The proposed control algorithms were tested on the AMIRA BW500 ball and beam system [26] controlled by a computer equipped with a dSpace-1102 control board (see Fig. 1). The angular position of the beam is measured by an incremental encoder. The position of the ball is sensed by a CCD camera placed over the beam. The exact position of the ball along the beam is calculated by combining the CCD camera signal and the angular position signal. With these measurements, the achievable position resolution of the ball on the beam is 1 mm. The ball velocity and angular velocity are obtained by differentiating the respective position signals; high frequency noise is limited by using second order digital filters.

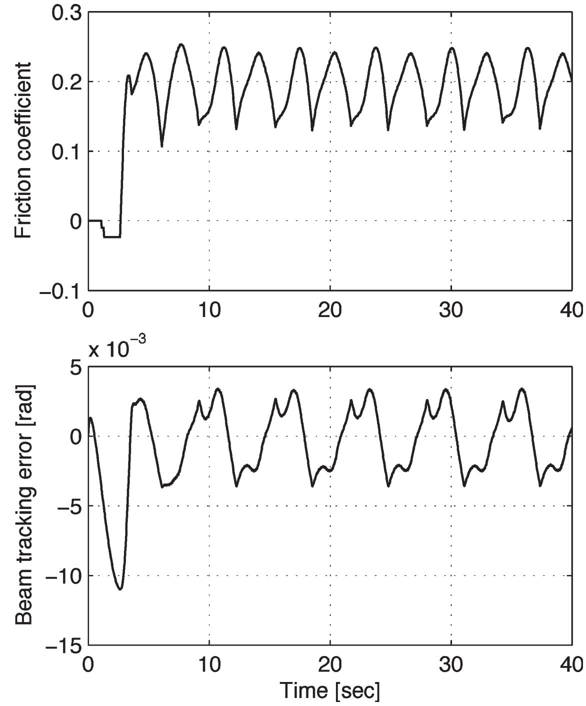

Fig. 2. Adaptive friction compensation in the beam.

The parameters of the controlled mechanical system are as follows: beam moment of inertia $J = 13.875 \times 10^{-2}$ [kg-m^2], ball mass $M = 0.025$ [kg], ball radius $R = 0.02$ [m], gravitational acceleration $g = 9.81$ [m/s^2]. The ball moment of inertia is calculated using the analytical relation $J_B = (2/5)MR^2$.

The control algorithms were implemented on the dSpace-1102 board equipped with a TMS-320 type digital signal processor (DSP). To implement the control algorithm, a MATLAB-SIMULINK model was developed. A part of the control algorithm (input calibration and filtering) was programmed using SIMULINK blocks while the control laws are programmed in C language and they are introduced in the SIMULINK model as S-functions. The built model is compiled and downloaded to the DSP. The sample time was chosen 10 ms and the fixed step Euler method was used for numerical integration.

B. Experimental Measurements: Friction Estimation

The adaptive controller described in Section II was used to identify the friction model. A sinusoidal reference trajectory was chosen: $x_{3ref} = 0.1\sin(2\pi t)$, and the following controller parameters were used: $\lambda_B = 10$, $K_S = 2$, $\gamma_F = 0.05$. Shown in Fig. 2 are the identified friction coefficient and beam tracking error histories. The mean value of the friction

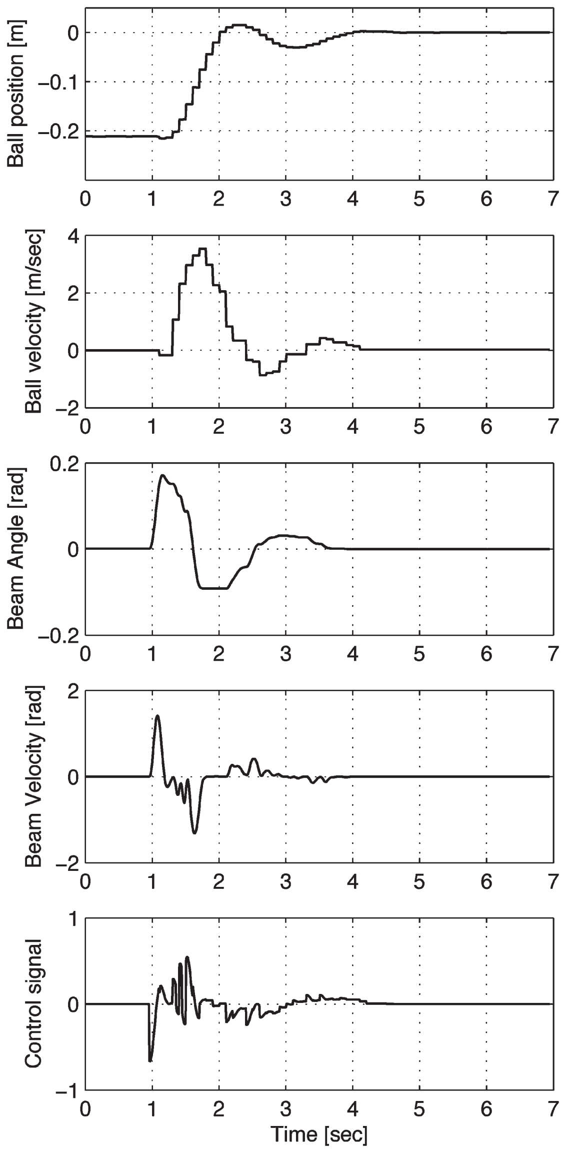

Fig. 3. LQR control, with friction compensation.

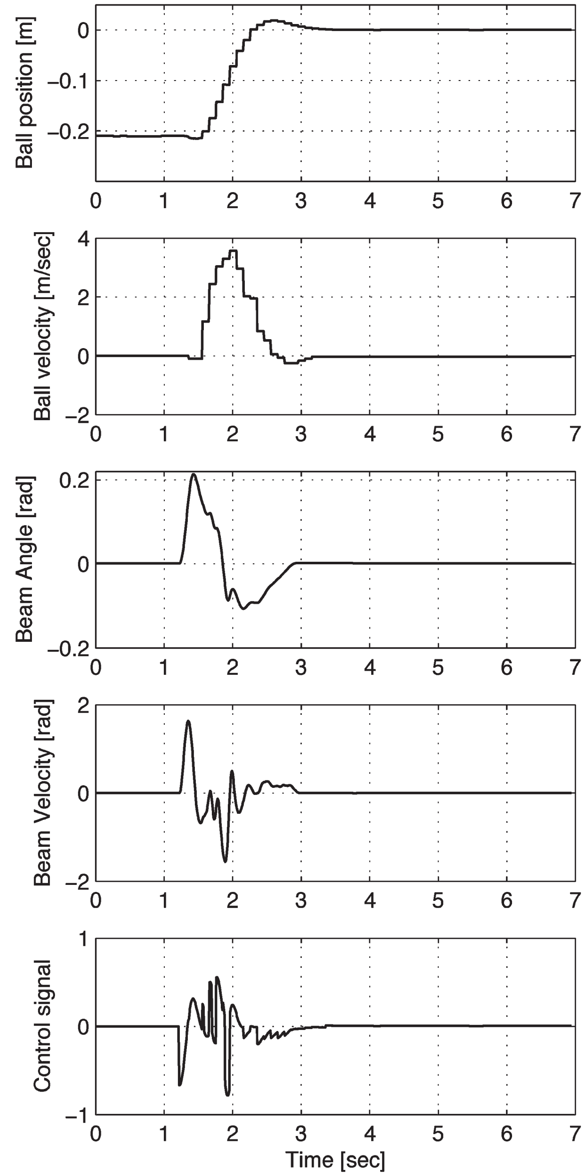

Fig. 4. LQR control combined with subspace stabilization and friction compensation.

coefficient in sinusoidal steady state was computed to be $F_C = 0.19$. This value is used in the remaining robust controller experiments.

C. Experimental Measurements: Subspace Stabilization Based Control

In the first experiment the subspace stabilization controller was compared with the standard LQR control. First, an LQR controller was implemented for the original system (1), linearized using (16). The LQR controller parameters were obtained using the system parameters enumerated in Section IV-A, and Q and R matrices were taken as follows: $Q = \mathrm{diag}(1000,\ 0.1,\ 0.1,\ 0.1)$, $R = 1$. The obtained controller parameters are

$$\begin{aligned} \underline{K}_{LQR} &= [K_1 \quad K_2 \quad K_3 \quad K_4] \\ &= [31.6228 \quad 21.5383 \quad 50.3577 \quad 10.0407]. \end{aligned} \tag{24}$$

Then, an LQR controller for the subspace stabilized system (18) with same Q and R matrices is designed. The obtained controller parameters are

$$\begin{aligned} \underline{K}_{ss} &= [K_{1ss} \quad K_{2ss} \quad K_{3ss} \quad K_{4ss}] \\ &= [31.6228 \quad 21.4671 \quad 49.0254 \quad 8.9575]. \end{aligned} \tag{25}$$

The applied control law in this case is given by (10).

The experimental results are presented in the Figs. 3 and 4. It can be seen that if the LQR controller is extended with subspace stabilization, then much better transient behavior can be obtained. The peak beam angle is larger, yet the settling time is shorter. In other words, a more aggressive beam motion can be employed when the subspace stabilization approach is used.

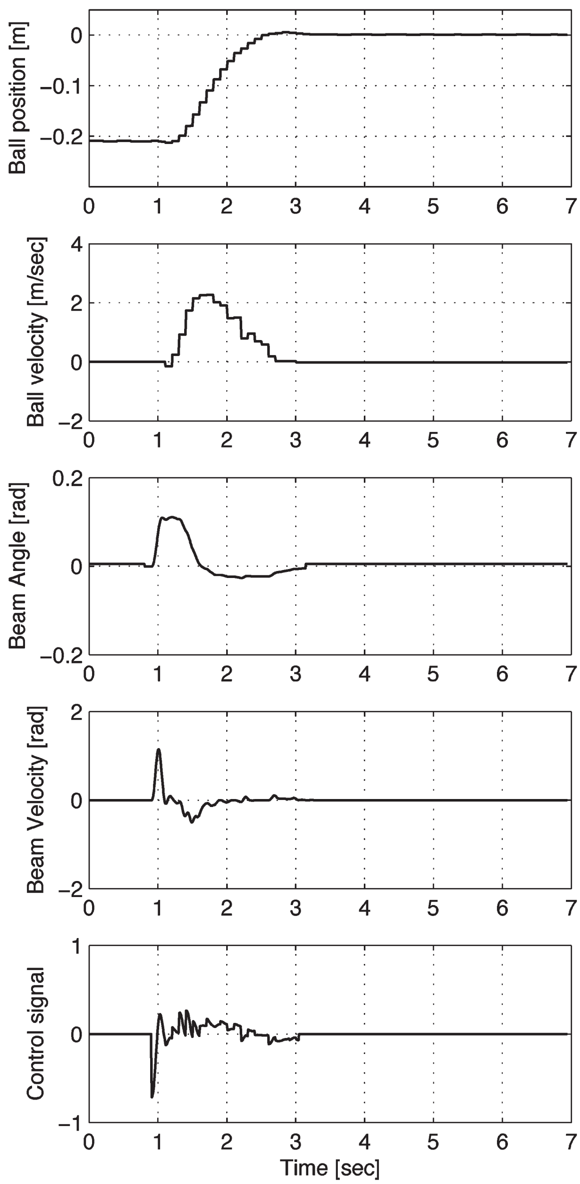

Fig. 5. Nonlinear combined error metric control, with friction compensation.

D. Experimental Measurements: Nonlinear Combined Error Metric Control

In the second experiment the nonlinear combined error metric control law was tested. The applied the control law was (23).

The gains in linear part of the control law was chosen in such way to be equivalent with the LQR control from previous subsection given by (24). Based on (20) and (23) the equivalent gains can be calculated using the following relations $K_1 = K_S\alpha\lambda_1$, $K_2 = K_S\alpha$, $K_3 = K_S\lambda_2$, $K_4 = K_S$.

The experimental results are presented in the Fig. 5. The experiment shows that the nonlinear combined error metric control approach also guarantees better transient response than the standard linear LQR control. Settling time is shorter, and overshoot is also much reduced.

The choice of the sampling period is critical during controller implementation. If the sampling period is too large, the the closed loop system may became uncontrollable. The sampling period can be chosen by taking into account the bandwidth and time constants of the closed loop system. Reasonable sampling rates are 10 to 30 times the closed loop bandwidth, assuming the dynamics can be approximated by a first order linear system [27]. The sampling frequency in the experiments was 100 Hz (approximately ten times the closed loop bandwidth).

V. Conclusion

Detailed design procedures for nonlinear subspace stabilization and nonlinear combined error metric mode control have been reviewed in this work. The experimental performances of these nonlinear robust controllers were compared to the classical LQR controller on a ball and beam system, which is an unstable, under-actuated system having poorly defined relative degree. Friction compensation was used in all experiments, and an adaptive control experiment was used to identify the friction characteristic. Both nonlinear controllers extend the range of stable operation when compared to LQR control. The subspace stabilization approach results in a control that can rotate the beam more aggressively while maintaining stability, yielding the shortest settling time and also low overshoot. The lowest overshoot was achieved by the nonlinear combined error metric controller, and the settling time was nearly comparable to that achieved by nonlinear subspace stabilization. The experimental results are encouraging validation of robust nonlinear control methods for under-actuated robotic systems that often exhibit challenging, complex dynamics.

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