

# Learning Theory and Model Selection

- AIC (Akaike Information Criterion)  $\Rightarrow$  赤池信息准则
- BIC (Bayesian Information Criterion)  $\Rightarrow$  贝叶斯信息准则
- RPCL (Rival Penalized Competitive Learning)  $\Rightarrow$  次胜者受罚的竞争性学习规则
- Bayesian method  $\Rightarrow$  贝叶斯方法

AIC, BIC  $\Rightarrow$  metrics for choosing the best model

引入目的: 模型选择问题 (model selection problem) 本质上为模型复杂度 (参数量) 与模型对数据集的描述能力 (以似然函数作为衡量准则, 也即目标函数) 之间的权衡。AIC, BIC 提供了一种定量的指标, 作为衡量统计模型优良性的标准。

$$\begin{aligned} \text{AIC} &= -2 \ln(L) + 2k \Rightarrow \begin{array}{l} \text{似然函数} \swarrow \\ \# \text{ parameters} \end{array} \\ \text{BIC} &= -2 \ln(L) + k \cdot \ln(n) \Rightarrow \begin{array}{l} \# \text{ samples} \swarrow \\ \text{penalty item} \end{array} \end{aligned}$$

△ BIC 的惩罚项  $k \cdot \ln(n)$  要大于 AIC 的惩罚项  $2 \cdot k$ 。

BIC 在维数过大且训练样本数很少的情况下可有效避免维度灾难。

# Competitive Learning

What?

竞争性学习 (Competitive Learning) 是一种学习理论 (learning theory)。Competitors 通过 rules 来争夺 resources,  
↓  
竞争规则, e.g. distance  
近的竞争优势大

并通过 update rules 利用 resources 进行自身更新, 使得自己在下一次的竞争中优势更大。⇒ Intuition: ① 扩大再生产  
② winner takes all/most

CL in clustering:

competitors: cluster center units  
不是 points, 与 k-means  
中均值点类似, 并非 input points

competing rules: winner takes all/most

update rules: gradient update

- punish frequent winner
- punish second winner

What is CCL (Classical Competitive Learning):

- Usually used for making data clustering

- C rules: WTAC (Winner Take All):

$$P_{j,t} = \begin{cases} 1, & \text{if } j = C, \quad C = \underset{k}{\operatorname{argmin}} \left\{ \underbrace{\sum (X_t, u_k)}_{\substack{\uparrow \\ (X_t - u_k)^2}} \right\} \\ 0, & \text{otherwise} \end{cases} \Rightarrow C \text{ is the winner}$$

- U rules:

$$u_j^{\text{new}} = u_j^{\text{old}} + \underbrace{\alpha C}_{\substack{\uparrow \\ P_{j,t}}} P_{j,t} (X_t - u_j^{\text{old}})$$

Note that only the unit  $j$  satisfies  $P_{j,t} = 1$ , it can update its position  $u_j$  closer to the input value.

What is FSCL (Frequency Sensitive Competitive Learning):

- C Rules: punish frequent winner

$$P_{j,t} = \begin{cases} 1, & j = C, \quad C = \underset{k}{\operatorname{argmin}} \left\{ \underbrace{Y_k}_{\substack{\text{frequency that } k\text{th unit won in the past}}} \sum (X_t, u_k) \right\} \\ 0, & \text{otherwise} \end{cases}$$

- U rules:

$$u_j^{\text{new}} = u_j^{\text{old}} + \alpha C P_{j,t} (X_t - u_j^{\text{old}})$$

△ Problem: FSCL can't find exact number of cluster centers.

What is RPCL (Rival Penalized Competitive Learning):

- 性质:
  - 竞争性学习领域算法
  - 用于多主体划分与分治复杂的任务,  
也用于计算 K-means clustering 的 K value
  - 可看成 Bayesian Ying Yang Learning 的简化实现 / FSCL 的扩展

• C Rules: punish second winner

$$P_{j,t} = \begin{cases} 1, & \text{if } j = c, \quad c = \arg \min_k \{ \varepsilon \in L(X_t, u_k) \} \\ -1, & \text{if } i = r, \quad r = \arg \min_{k \neq c} \{ \varepsilon \in L(X_t, u_k) \} \\ 0, & \text{otherwise} \end{cases}$$

次胜竞争者

• U rules:

$$u_j^{\text{new}} = u_j^{\text{old}} + \underbrace{\alpha_c}_{\text{learning rate}} P_{j,t} (X_t - u_j^{\text{old}}) + \underbrace{\alpha_r}_{\text{away rate}} P_{j,t} (X_t - u_j^{\text{old}})$$