Summary of Master Method

Case	Ratio check	Inequality that must be checked	Theorem 4.1 text pg 94	Notes (see pages 94-95)
1	$\frac{f(n)}{n^{\log_b a}} = n^{-\varepsilon},\ \varepsilon > 0$	$f(n) \le cn^{\log_b a}$	If $f(n) \in O\left(n^{\log_b a - \varepsilon}\right)$ for some constant $\varepsilon > 0$, then $T(n) \in \Theta\left(n^{\log_b a}\right)$ Total cost dominated by cost of the recursion tree leaves.	Not only must $f(n)$ be smaller than $n^{\log_b a}$, it must be polynomially smaller
2	$\frac{f(n)}{n^{\log_b a}} = \lg^k n,\ k \ge 0$	$c_1 n^{\log_b a} \le f(n) \le c_2 n^{\log_b a}$	If $f(n) \in \Theta\left(n^{\log_b a}\right)$ then $T(n) \in \Theta\left(n^{\log_b a} \lg n\right)$ Total cost evenly distributed over recursion tree levels.	
Exercise 4.6-2 pg 106	*Not applicable*	*Not applicable*	If $f(n) \in \Theta\left(n^{\log_b a} \lg^k n\right)$ where $k \ge 0$, then $T(n) \in \Theta\left(n^{\log_b a} \lg^{k+1} n\right)$	
3	$\frac{f(n)}{n^{\log_b a}} = n^{\varepsilon},\ \varepsilon > 0$	$cn^{\log_b a} \le f(n)$	If $f(n) \in \Omega\left(n^{\log_b a + \varepsilon}\right)$ for some constant $\varepsilon > 0$, and if $af(n/b) \le cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) \in \Theta\left(f(n)\right)$ Total cost dominated by cost of the recursion tree root.	Not only must $f(n)$ be larger than $n^{\log_b a}$, it must be polynomially larger and satisfy the "regularity" condition $af(n/b) \le cf(n)$ If $f(n)$ has the form n^i $c = \left(a/b^i\right)$ which is < 1

Notes:

Applies only to recurrences of the form $T(n) = aT(n/b) + f(n)$ where $a \ge 1$ and $b > 1$ are constants and $f(n)$ is an asymptotically positive function.

See page 56 for definition $\lg^k n \equiv (\lg n)^k$

Critical exponent $E = \frac{\log a}{\log b} = \log_b a$

If $f(n)$ is smaller than $n^{\log_b a}$ but not polynomially smaller or if $f(n)$ is larger than $n^{\log_b a}$ but not polynomially larger or if the "regularity" condition $af(n/b) \le cf(n)$ is not satisfied the Master method cannot be used to solve the recurrence