Chapter 7 Linear Algebra: Matrices, Vectors, Determinant. Linear Systems

P261 - Problem set 7.1

1. 2x2: $a_{11} \neq b_{11}, b_{12} \neq c_{12}$, 2x3: $d_{11} \neq e_{11}$
2. $a_{31} = 10, \quad a_{13} = 81 \quad, a_{26} = 96, \quad a_{33} = 0$
3. E1: 3x3, 3x4,
   E2: 3x7,
   E3: 2x2, 2x2, 2x2, 2x3, 2x3,
   E5:3x2
4. 1A: 4, 0, 1
   3A: a_{11}, a_{22}
   3B: 4, -1
5. $B = \frac{1}{5}A$,
   $B = \frac{1}{10}A$
6. $B = \frac{1}{1.609}A$
7. No. No(1x1 as exception?). Yes. Maybe not in math (how about 1x1?) but OK in python. No.
8. $2A + 4B = 4B + 2A = \begin{bmatrix} 0 & 24 & 16 \\ 32 & 22 & 26 \\ -6 & 16 & -14 \end{bmatrix}$
   B
   $0.4B - 4.2A = \begin{bmatrix} 0 & -6.4 & -16 \\ -23.2 & -19.8 & -19.4 \\ -5 & 1.6 & 11.8 \end{bmatrix}$
9. $3A = \begin{bmatrix} 0 & 6 & 12 \\ 18 & 15 & 15 \\ 3 & 0 & -9 \end{bmatrix}$
   $0.5B = \begin{bmatrix} 0 & 2.5 & 1 \\ 2.5 & 1.5 & 2 \\ -1 & 2 & -1 \end{bmatrix}$
   $3A + 0.5B = \begin{bmatrix} 0 & 8.5 & 13 \\ 20.5 & 16.5 & 17 \\ 2 & 2 & -10 \end{bmatrix}$
   3A+0.5B+C is not defined.

10. $(4 \bullet 3)A = 4(3A) = \begin{bmatrix} 0 & 24 & 48 \\ 72 & 60 & 60 \\ 12 & 0 & -36 \end{bmatrix}$

$14B - 3B = 11B = \begin{bmatrix} 0 & 55 & 22 \\ 55 & 33 & 44 \\ -22 & 44 & -22 \end{bmatrix}$

11. $8C + 10D = 2(5D + 4C) = \begin{bmatrix} 0 & 26 \\ 34 & 32 \\ 28 & -10 \end{bmatrix}$

$0.6C - 0.6D = 0.6(C - D) = \begin{bmatrix} 5.4 & 0.6 \\ -4.2 & 2.4 \\ -0.6 & 0.6 \end{bmatrix}$

12. $(C + D) + E = (D + E) + C = \begin{bmatrix} 1 & 5 \\ 6 & 8 \\ 6 & -2 \end{bmatrix}$

$0(C - E) + 4D = 4D = \begin{bmatrix} -16 & 4 \\ 20 & 0 \\ 8 & -4 \end{bmatrix}$

$A - 0C$: 3x3 can not minus 3x2, not defined

13. $(2 \bullet 7)C = 2(7C) = \begin{bmatrix} 70 & 28 \\ -28 & 56 \\ 14 & 0 \end{bmatrix}$

$-D + 0E = -D = \begin{bmatrix} 4 & -1 \\ -5 & 0 \\ -2 & 1 \end{bmatrix}$

$E - D + C + u$: Since EDC are 3x2 but u is 3x1, not defined.

14. $(5u + 5v) - \frac{1}{2}w = \begin{bmatrix} 5 \\ 30 \\ -10 \end{bmatrix}$

$-20(u + v) + 2w = -4[(5u + 5v) - \frac{1}{2}w] = \begin{bmatrix} -20 \\ -120 \\ 40 \end{bmatrix}$

$E - (u + v)$: 3x2 can not minus 3x1, not defined

$10(u + v) + w = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

15. $(u + v) - w = u + (v - w) = \begin{bmatrix} 5.5 \\ 33 \\ -11 \end{bmatrix}$

$C + 0w$: 3x2 can not minus 3x1, not defined

$0E + u - v$: 3x2 can not minus 3x1, not defined

16. $15v - 3w - 0u = -3w + 15v = \begin{bmatrix} 0 \\ 135 \\ 0 \end{bmatrix}$

$D - u + 3C$: 3x2 can not minus 3x1, not defined

$8.5w - 11.1u + 0.4v = \begin{bmatrix} 25.45 \\ 256.2 \\ 119.1 \end{bmatrix}$

17. $u + v + w = \begin{bmatrix} -4.5 \\ -27 \\ 9 \end{bmatrix}$

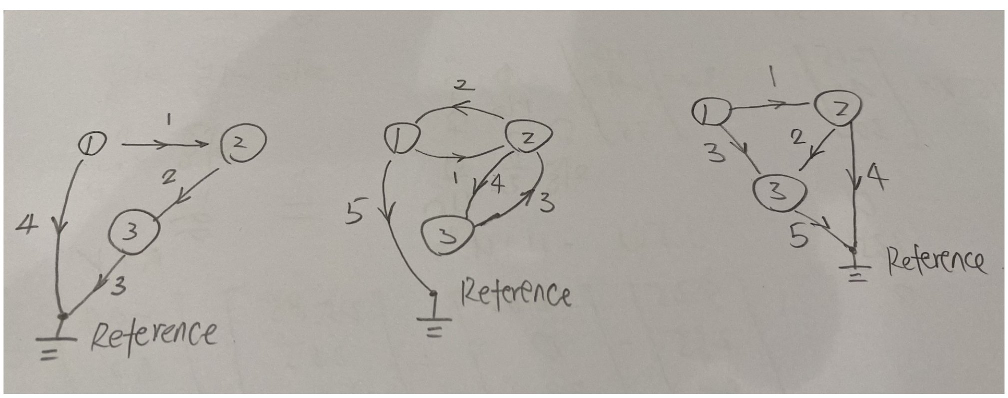
18. $p = 0 - u - v - w = \begin{bmatrix} 4.5 \\ 27 \\ -9 \end{bmatrix}$

19. Expand metrics with entries a_{ij}, then follow the basic arithmetic rule.

20. b-1: $\begin{bmatrix} -1 & 1 & 0 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

b-2: $\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 1 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$

c:



P270 - Problem set 7.2

Example13. In the final stable situation(limit),

$I + C + R = 100$

$0.7C + 0.1I = C$

$0.2C + 0.9I + 0.2R = I$

$0.1C + 0.8R = R$

So we can get C=200/9, I=200/3, R=100/9.
Will revisit it after Sec. 8.2

1. Per definition, the number of the entries in the columns of the second matrix have to be same as the number of the entries in the rows of the first matrix. In short, if mxn matrix multiple pxq, then n=p. Or you won't be able to perform the dot product.
2. All entries or components are 0
3. No. All rows are proportional.
4. Min is 1 which is 0, and max is $n(n-1)+1$
   Take 3x3 as example, $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$
5. Min is 1 which is 0, and max is $\frac{n(n+1)}{2}$
   Take 3x3 as example, $\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$
6. U_1+U_2, U_1U_2, U_1^2 are upper triangular matrices.L_1+L_2 is lower triangular.
7. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
8. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ for any $m \geq 1, m \in N$. $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ for any $m \geq 2, m \in N$.
9. Expand metrics with entries a_{ij}, then follow the basic arithmetic rule.
10. Expand metrics with entries a_{ij}, then follow the basic arithmetic rule.
11. $AB = AB^T = \begin{bmatrix} 10 & -14 & -6 \\ -5 & 7 & -12 \\ -5 & -1 & -4 \end{bmatrix}$

$$BA = B^TA = \begin{bmatrix} 10 & -5 & -15 \\ -14 & 7 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

12. $AA^T = \begin{bmatrix} 29 & 8 & 6 \\ 8 & 41 & 12 \\ 6 & 12 & 9 \end{bmatrix}, A^2 = \begin{bmatrix} 23 & -4 & 6 \\ -4 & 17 & 12 \\ 2 & 4 & 19 \end{bmatrix}, BB^T = B^2 = \begin{bmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

13. $CC^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 13 & -6 \\ 0 & -6 & 4 \end{bmatrix}, BC = \begin{bmatrix} -9 & -5 \\ 3 & -1 \\ 4 & 0 \end{bmatrix}$, CB not defined, $C^TB = \begin{bmatrix} -9 & 3 & 4 \\ -5 & -1 & 0 \end{bmatrix}$

14. $3A-2B=\begin{bmatrix}10 & 0 & 9\\ 0 & 1 & 18\\ 3 & 6 & 10\end{bmatrix}$, $(3A-2B)^T=3A^T-2B^T=\begin{bmatrix}10 & 0 & 3\\ 0 & 1 & 6\\ 9 & 18 & 10\end{bmatrix}$,

$(3A-2B)^Ta^T=\begin{bmatrix}10\\ -2\\ -27\end{bmatrix}$

15. Aa not defined, $Aa^T=\begin{bmatrix}8\\ -4\\ -3\end{bmatrix}$, $(Ab)^T=b^TA^T=\begin{bmatrix}7 & -11 & 3\end{bmatrix}$

16. $BC=Problem13.2=\begin{bmatrix}-9 & -5\\ 3 & -1\\ 4 & 0\end{bmatrix}$, BC^T not defined, $Bb=\begin{bmatrix}0\\ -8\\ 2\end{bmatrix}$, $b^TB=$ $\begin{bmatrix}0 & -8 & 2\end{bmatrix}$

17. $ABC=\begin{bmatrix}-30 & -18\\ 45 & 9\\ 5 & -7\end{bmatrix}$, ABa not defined, $ABb=\begin{bmatrix}22\\ 4\\ -12\end{bmatrix}$, Ca^T= not defined.

18. $ab=1$, $ba=\begin{bmatrix}3 & -6 & 0\\ 1 & -2 & 0\\ -1 & 2 & 0\end{bmatrix}$, $aA=\begin{bmatrix}8 & -4 & -9\end{bmatrix}$, $Bb=problem16.3=\begin{bmatrix}0\\ -8\\ 2\end{bmatrix}$

19. $1.5a+3.0b$ not defined. $1.5a^T+3.0b=\begin{bmatrix}10.5\\ 0\\ -3\end{bmatrix}$, $(A-B)b=Ab-Bb=\begin{bmatrix}7\\ -3\\ 1\end{bmatrix}$

20. b^TAb=7, aBa^T=17, $aCC^T=\begin{bmatrix}-3 & -24 & 12\end{bmatrix}$, $C^Tba=\begin{bmatrix}5 & -10 & 0\\ 5 & -10 & 0\end{bmatrix}$

21. Expand metrics with entries a_{ij}, then follow the basic arithmetic rule.

22. $A=\begin{bmatrix}a_1\\ a_2\\ a_3\end{bmatrix}$, $B=\begin{bmatrix}b_1 & b_2 & b_3\end{bmatrix}$, $AB=\begin{bmatrix}a_1b_1 & a_1b_2 & a_1b_3\\ a_2b_1 & a_2b_2 & a_2b_3\\ a_3b_1 & a_3b_2 & a_3b_3\end{bmatrix}$

23. $AB=A\begin{bmatrix}b_1 & b_2 & b_3\end{bmatrix}=\begin{bmatrix}Ab_1 & Ab_2 & Ab_3\end{bmatrix}$

24. $AB=BA$, $\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}2 & 3\\ 3 & 4\end{bmatrix}=\begin{bmatrix}2 & 3\\ 3 & 4\end{bmatrix}\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$,

$2a_{11}+3a_{12}=2a_{11}+3a_{21} \Rightarrow a_{12}=a_{21}$

$3a_{11}+4a_{12}=2a_{12}+3a_{22} \Rightarrow 3a_{11}+2a_{12}=3a_{22}$

$2a_{21}+3a_{22}=3a_{11}+4a_{21}$,

$3a_{21}+4a_{22}=3a_{12}+4a_{22}$

Let $A=\begin{bmatrix}x & y\\ y & \frac{3x+2y}{3}\end{bmatrix}$,

Check: $AB=BA=\begin{bmatrix}2x+3y & 3x+4y\\ 3x+4y & 4x+5\frac{2}{3}y\end{bmatrix}$

25. a) Obvious.

b) $C=[c_{ij}]$, $C^T=[c_{ji}]$

$D = C + C^T = [d_{ij}] = [c_{ij} + c_{ji}] = [c_{ji} + c_{ij}] = [d_{ji}]$, so D is symmetric
$E = C - C^T = [e_{ij}] = [c_{ij} - c_{ji}] = -[c_{ji} - c_{ij}] = -[e_{ji}]$, so E is skew-symmetric.
Let $S = \frac{1}{2}D, T = \frac{1}{2}E$
$S + T = \frac{1}{2}(D + E) = \frac{1}{2}(C + C^T + C - C^T) = C$

$$A = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix}, A^T = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 1 & 2 \\ 3 & 6 & 2 \end{bmatrix},$$

$$S = \frac{1}{2}(A + A^T) = \begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & 4 \\ 2 & 4 & 2 \end{bmatrix}, T = \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ -1 & -2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, B^T = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$S = \frac{1}{2}(B + B^T) = B = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, T = \frac{1}{2}(B - B^T) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

c) symmetric: $A = [a_{ij}] = [a_{ji}], B = [b_{ij}] = [b_{ji}], ..., M = [m_{ij}] = [m_{ji}]$
$aA + bB + ... + mM = a[a_{ij}] + b[b_{ij}] + ... + m[m_{ij}] = a[a_{ji}] + b[b_{ji}] + +... + m[m_{ji}]$.
Skew-symmetric: $A = [a_{ij}] = -[a_{ji}], B = [b_{ij}] = -[b_{ji}], ..., M = [m_{ij}] = -[m_{ji}]$
$aA + bB + ... + mM = a[a_{ij}] + b[b_{ij}] + ... + m[m_{ij}] = -(a[a_{ji}] + b[b_{ji}] + +... + m[m_{ji}])$
.

d) $A = [a_{ij}] = [a_{ji}], B = [b_{ij}] = [b_{ji}]$
$AB = [a_p b_q]$, if AB is symmetric, then $AB = [a_p b_q] = [a_q b_p] = [b_p a_q] = BA$
vice verse.

e)$A = [a_{ij}] = -[a_{ji}], B = [b_{ij}] = -[b_{ji}]$
$AB = [a_p b_q]$, if AB is skew-symmetric, then $AB = [a_p b_q] = -[a_q b_p] = -[b_p a_q] = -BA$
vice verse.

26. First day, status = $\begin{bmatrix} N \\ T \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, stochastic matrix = $\begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}$

Second day = $\begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}$

Two days after today = $\begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 0.74 \\ 0.26 \end{bmatrix}$

Three days after today = $\begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} 0.74 \\ 0.26 \end{bmatrix} = \begin{bmatrix} 0.722 \\ 0.278 \end{bmatrix}$

The limit of N is $\frac{5}{7}$

27. Reserve for future

28. Present = $\begin{bmatrix} Subs. \\ Not \end{bmatrix} = \begin{bmatrix} 1200 \\ 98800 \end{bmatrix}$, stochastic matrix = $\begin{bmatrix} 0.9 & 0.002 \\ 0.1 & 0.998 \end{bmatrix}$

After 1 season = $\begin{bmatrix} Subs. \\ Not \end{bmatrix} = \begin{bmatrix} 0.9 & 0.002 \\ 0.1 & 0.998 \end{bmatrix} \begin{bmatrix} 1200 \\ 98800 \end{bmatrix} = \begin{bmatrix} 1278 \\ 98722 \end{bmatrix}$, increase

After 2 seasons = $\begin{bmatrix} Subs. \\ Not \end{bmatrix} = \begin{bmatrix} 0.9 & 0.002 \\ 0.1 & 0.998 \end{bmatrix} \begin{bmatrix} 1278 \\ 98722 \end{bmatrix} = \begin{bmatrix} 1344 \\ 98656 \end{bmatrix}$, increase

After 3 seasons = $\begin{bmatrix} Subs. \\ Not \end{bmatrix} = \begin{bmatrix} 0.9 & 0.002 \\ 0.1 & 0.998 \end{bmatrix} \begin{bmatrix} 1344 \\ 98656 \end{bmatrix} = \begin{bmatrix} 1407 \\ 98593 \end{bmatrix}$, increase

29. $p = \begin{bmatrix} 35 \\ 62 \\ 30 \end{bmatrix}$

$v = Ap = \begin{bmatrix} 24,920 \\ 25,940 \end{bmatrix}$

30.$y = Ax$
$y_1 = x_1 \cos\theta - x_2 \sin\theta, y_2 = x_1 \sin\theta + x_2 \cos\theta$
$|y|^2 = (x_1 \cos\theta - x_2 \sin\theta)^2 + (x_1 \sin\theta + x_2 \cos\theta)^2 = x_1^2 + x_2^2 = |x|^2$
$\cos\alpha = \frac{x*y}{|x||y|} = \frac{x_1^2 \cos\theta + x_2^2 \cos\theta}{x_1^2 + x_2^2} = \cos\theta$
so x and y have the same length, and from x to y is counterclockwise rotate of θ

b)$AA = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} =$
$\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

c)$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} =$
$\begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & -\cos\alpha\sin\beta - \sin\alpha\cos\beta \\ \sin\alpha\cos\beta + \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos\beta \end{bmatrix} = \begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$

d)$[x_1, x_2, x_3] \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = [3x_1, x_2, \frac{1}{2}x_3]$

$[x_1, x_2, x_3] \begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix} = [cx_1, cx_2, cx_3]$, Scalar matrix will amplify or squeeze the picture by c.

e) $[x_1, x_2, x_3]\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{bmatrix} = [x_1, x_2\cos\theta + x_3\sin\theta, -x_2\sin\theta + x_3\cos\theta]$

$x_1$remain the same. counterclockwise rotation of the Cartesian coordinate system x_2x_3 in the plane about the origin by angle of θ

$$[x_1, x_2, x_3]\begin{bmatrix}\cos\varphi & 0 & -\sin\varphi\\ 0 & 1 & 0\\ \sin\varphi & 0 & \cos\varphi\end{bmatrix} = [x_1\cos\varphi + x_3\sin\varphi, x_2, -x_1\sin\varphi + x_3\cos\varphi]$$

$x_2$remain the same. counterclockwise rotation of the Cartesian coordinate system x_1x_3 in the plane about the origin by angle of φ

$$[x_1, x_2, x_3]\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix} = [x_1\cos\psi + x_2\sin\psi, -x_1\sin\psi + x_2\cos\psi, x_3]$$

$x_3$remain the same. counterclockwise rotation of the Cartesian coordinate system x_1x_2 in the plane about the origin by angle of ψ

P280 - Problem set 7.3

1. $\begin{bmatrix}1 & 0 & -2\\ 0 & 1 & \frac{1}{2}\end{bmatrix}$

2. $\begin{bmatrix}1 & 0 & \frac{2}{5}\\ 0 & 1 & \frac{6}{5}\end{bmatrix}$

3. $\begin{bmatrix}1 & 0 & 0 & 1\\ 0 & 1 & 0 & 3\\ 0 & 0 & 1 & -5\end{bmatrix}$

4. $\begin{bmatrix}1 & -4 & 1 & -2\\ 0 & 17 & -4 & 12\\ 0 & -34 & 8 & -13\end{bmatrix}$ = No solution

5. $\begin{bmatrix}1 & 33 & -225\\ 0 & 139 & -973\\ 0 & -376 & 2632\end{bmatrix} = \begin{bmatrix}1 & 0 & 6\\ 0 & 1 & -7\\ 0 & 1 & -7\end{bmatrix}$

6. $\begin{bmatrix} 1 & -2 & 2 & 9 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & -5 & -20 \end{bmatrix} = \begin{bmatrix} 2t+1 \\ t \\ 4 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 5 & -1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -3t \\ t \\ 2t \end{bmatrix}$

8. $\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 4 & 3 & 8 \\ 0 & 4 & 3 & 4 \end{bmatrix}$ = No solution

9. $\begin{bmatrix} 3 & 4 & -5 & 13 \\ 0 & 1 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 3t-1 \\ 4-t \\ t \end{bmatrix}$

10. $\begin{bmatrix} 5 & -7 & 3 & 17 \\ 5 & -7 & 3 & -50/3 \end{bmatrix}$ = No solution

11. $\begin{bmatrix} 2 & -3 & -3 & 6 & 2 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, so we can get $\begin{bmatrix} 1 \\ 2m-n \\ n \\ m \end{bmatrix}$

12. $\begin{bmatrix} 1 & -1 & 2 & 0 & 0 \\ 1 & -1 & 2 & 0 & 0 \\ 1 & -1 & 2 & -5/3 & -5 \end{bmatrix}$, so we can get $\begin{bmatrix} n-2m \\ n \\ m \\ 3 \end{bmatrix}$

13. $$\begin{bmatrix} 0 & 10 & 4 & -2 & -4 \\ -3 & -17 & 1 & 2 & 2 \\ 1 & 1 & 1 & 0 & 6 \\ 8 & -34 & 16 & -10 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 6 \\ 0 & 10 & 4 & -2 & -4 \\ 0 & -14 & 4 & 2 & 20 \\ 0 & 21 & -4 & 5 & 22 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 & 1 & 0 & 6 \\ 0 & 1 & -12 & 9 & 30 \\ 0 & 0 & -41 & 32 & 110 \\ 0 & 0 & -31 & 23 & 76 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix}$$

14. $\begin{bmatrix} 1 & -1 & 3 & -3 & 3 \\ 2 & 3 & 1 & -11 & 1 \\ 5 & -2 & 5 & -4 & 5 \\ 3 & 4 & -7 & 2 & -7 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 3 & -3 & 3 \\ 0 & 1 & -1 & -1 & -1 \\ 0 & 3 & -10 & 11 & -10 \\ 0 & 7 & -16 & 11 & -16 \end{bmatrix} =$
$\begin{bmatrix} 1 & -1 & 3 & -3 & 3 \\ 0 & 1 & -1 & -1 & -1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 & 1 \end{bmatrix}$

so we can get $\begin{bmatrix} 0 \\ 3t \\ 1+2t \\ t \end{bmatrix}$

15. Expand metrics with entries a_{ij}, then follow the basic arithmetic rule.
16. Reserve for future
17. $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 0 & 1 & 16 \\ 0 & 4 & 1 & 32 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 4 & 1 & 32 \\ 0 & 0 & 6 & 48 \end{bmatrix}$

so we can get $\begin{bmatrix} 2 \\ 6 \\ 8 \end{bmatrix}$
18. $\color{red}{Mark}$: I think I get it.
$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 4 & 12 & 36 \\ 0 & 12 & -8 & 24 \end{bmatrix}$

so we can get $\begin{bmatrix} 27/11 \\ 24/11 \\ 3/11 \end{bmatrix}$
19. $\begin{bmatrix} \frac{E_0}{R_2} + \frac{E_0}{R_1} \\ -\frac{E_0}{R_1} \\ \frac{E_0}{R_2} \end{bmatrix}$
20. $I_3 = I_x, I_1 = I_2$
$I_1 R_1 = I_x R_x, I_3 R_3 = I_2 R_2$, so we can get
$R_x = R_3 R_1 / R_2$
21. $\begin{bmatrix} 1 & 1 & 0 & 0 & 1600 \\ 1 & 0 & 0 & 1 & 1000 \\ 0 & 1 & 1 & 0 & 2200 \\ 0 & 0 & 1 & 1 & 1600 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1600 \\ 0 & 1 & 0 & -1 & 600 \\ 0 & 0 & 1 & 1 & 1600 \\ 0 & 0 & 1 & 1 & 1600 \end{bmatrix}$

Rank=3 < N=4, So the solutin is not unique.

22. $40-2P_1-P2=4P_1-P_2+4, 6P_1=36, P_1=6$
$5P_1-2P_2+16=3P_2-4, P_1=P_2-4, P_2=2$

23. $\begin{bmatrix}3 & 0 & -1 & 0 & 0\\ 8 & 0 & 0 & -2 & 0\\ 0 & 2 & -2 & -1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0 & -3 & 2 & 0\\ 0 & 2 & -2 & -1 & 0\\ 0 & 0 & 4 & -3 & 0\end{bmatrix}$

so we can get $\begin{bmatrix}t\\ 5t\\ 3t\\ 4t\end{bmatrix}$

The smallest positive integers are 1, 5, 3, 4

24. a) $\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\\ 10 & 11 & 12\end{bmatrix}=\begin{bmatrix}1 & 2 & 3\\ 7 & 8 & 9\\ 4 & 5 & 6\\ 10 & 11 & 12\end{bmatrix}$

$$A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\\ a_{31} & a_{32}\\ a_{41} & a_{42}\end{bmatrix}$$
$$B=\begin{bmatrix}a_{11} & a_{12}\\ a_{31} & a_{32}\\ -5a_{11}+a_{21} & -5a_{12}+a_{22}\\ 8a_{41} & 8a_{42}\end{bmatrix}$$
$$C=\begin{bmatrix}a_{11} & a_{12}\\ -5a_{11}+a_{31} & -5a_{12}+a_{32}\\ a_{21} & a_{22}\\ 8a_{41} & 8a_{42}\end{bmatrix}$$

So $B\neq C$

b)Natually.

Row switch: reference E1

Row multiplication: reference E3 (replace by c)

Row addition and subtraction: reference E2.

Expand metrics with entries a_{ij}, then follow the basic arithmetic rule.

P287 - Problem Set 7.4

1. $\begin{bmatrix}2 & -1 & 3\\ 0 & 0 & 0\end{bmatrix}$, Rank=1, $\{[2,-1,3]\}$

$$A^T = \begin{bmatrix} 2 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \{[2,-1]^T\}$$

2. $\begin{bmatrix} a & b \\ a & \frac{a^2}{b} \end{bmatrix}$,
   if $a = b = 0$, rank = 0, $\{0\}$, $\{0\}$
   if $b = \pm a$, rank = 1, $\{[1,-1]\}$, $\{[1,-1]^T\}$
   The rest, rank = 2, $\{[a,b],[b,a]\}$, $\{[a,b]^T,[b,a]^T\}$

3. $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 5 \\ 0 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1 \end{bmatrix}$, rank = 3, $\{[1,0,0],[0,1,0],[0,0,1]\}$,
   $\{[1,0,0]^T,[0,1,0]^T,[0,0,1]^T\}$,

4. $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 3 \\ 6 & -4 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$, rank = 3, $\{[1,0,0],[0,1,0],[0,0,1]\}$,
   $\{[1,0,0]^T,[0,1,0]^T,[0,0,1]^T\}$,

5. $\begin{bmatrix} 1 & 0 & -21 \\ 0 & 11 & -3 \\ 2 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -21 \\ 0 & 11 & -3 \\ 0 & 0 & 1 \end{bmatrix}$, rank = 3, $\{[1,0,0],[0,1,0],[0,0,1]\}$,
   $\{[1,0,0]^T,[0,1,0]^T,[0,0,1]^T\}$,

6. $\begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 0 \\ 0 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, rank = 2, $\{[1,1,4],[0,1,0]\}$,
   $A^T = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 4 \\ 0 & -4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,
   $\{[1,-1,4]^T,[0,1,0]^T,\}$,

7. $\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, rank = 2, $\{[2,0,1,0],[0,1,0,2]\}$,
   $A^T = \begin{bmatrix} 8 & 0 & 4 \\ 0 & 2 & 0 \\ 4 & 0 & 2 \\ 0 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\{[2,0,1]^T,[0,1,0]^T\}$,

8. $\begin{bmatrix}1 & 2 & 4 & 8\\ 0 & 12 & 30 & 63\\ 0 & 0 & 0 & 1\\ 0 & 6 & 0 & -6\end{bmatrix} = \begin{bmatrix}1 & 2 & 4 & 8\\ 0 & 1 & 0 & -1\\ 0 & 12 & 30 & 63\\ 0 & 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 2 & 4 & 8\\ 0 & 1 & 0 & -1\\ 0 & 0 & 30 & 75\\ 0 & 0 & 0 & 1\end{bmatrix}$, rank = 4,
$\{[1,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1]\}$,
$\{[1,0,0,0]^T,[0,1,0,0]^T,[0,0,1,0]^T,[0,0,0,1]^T\}$

9. $\begin{bmatrix}1 & 1 & 1 & 1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 1 & 0\\ 9 & 0 & 1 & 0\end{bmatrix} = \begin{bmatrix}1 & 1 & 1 & 1\\ 0 & 9 & 8 & 9\\ 0 & 0 & 1 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$, rank = 3, $\{[1,1,1,1],[0,9,8,9],[0,0,1,0]\}$,

$A^T = \begin{bmatrix}9 & 0 & 1 & 0\\ 0 & 0 & 1 & 0\\ 1 & 1 & 1 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}$, $\{[9,0,1,0]^T,[0,0,1,0]^T,[1,1,1,1]^T\}$

10. $\begin{bmatrix}1 & -4 & -11 & 2\\ 0 & 1 & 2 & 0\\ 5 & -2 & 1 & 0\\ -2 & 0 & -4 & 1\end{bmatrix} = \begin{bmatrix}1 & -4 & -11 & 2\\ 0 & 1 & 2 & 0\\ 0 & 0 & 2 & -1\\ 0 & 0 & 2 & -1\end{bmatrix}$, rank = 3,
$\{[1,-4,-11,2],[0,1,2,0],[0,0,2,-1]\}$,

$A^T = A$, $\{[1,-4,-11,2]^T,[0,1,2,0]^T,[0,0,2,-1]^T\}$

11. New row 1 = row 2 - row 1 = [1, 1, ... , 1]
    Add new row 1 to row k will get row k+1. so rank = 2, base is row 1 and row 2.
    b) Same
    c) All rows similar to row 1, just matter of factor 2^k. So rank = 1
12. $Rank(AB) = Rank[(AB)^T] = Rank(B^T A^T)$
13. $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$

$\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

14. Let A is a $m \times n$ matrix, and assume $m > n$
    $Rank(A) \le n < m$. so A is linearly dependent on the row vectors
    verse vise, L.D on the column vectors
15. n = Rank of row = rank of column
16. Matrix A, B, AB.
    Let A as the base of the vector space V(A), then V(AB) is the subset of V(A).
    $Rank(A) = dim[V(A)] \ge dim[V(AB)] = Rank(AB)$

If B is nonsingular, then Rank(A)=Rank(AB)

Vise verse on B.

17. $\begin{bmatrix} 1 & 16 & -12 & -22 \\ 3 & 4 & 0 & 2 \\ 2 & -1 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 16 & -12 & -22 \\ 0 & 11 & -9 & -17 \\ 0 & 33 & -37 & -51 \end{bmatrix} = \begin{bmatrix} 1 & 16 & -12 & -22 \\ 0 & 11 & -9 & -17 \\ 0 & 0 & 10 & 0 \end{bmatrix}$

Linear independent.

18. $\begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 30 & 20 & 15 & 12 \\ 20 & 15 & 12 & 10 \\ 105 & 84 & 70 & 60 \end{bmatrix} = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 0 & 1 & 1 & 0.9 \\ 0 & 15 & 16 & 15 \\ 0 & 126 & 140 & 135 \end{bmatrix} = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 0 & 1 & 1 & 0.9 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 14 & 21.6 \end{bmatrix}$

Rank = 4, Linear independent.

19. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Rank = 3, Linear independent.

20. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Rank = 2, Linear Dependent.

21. $\begin{bmatrix} 2 & 0 & 0 & 7 \\ 2 & 0 & 0 & 8 \\ 2 & 0 & 0 & 9 \\ 2 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -7 \end{bmatrix}$

Rank = 3, Linear Dependent.

22. V1 * 30/4 = V3, rank=1. Linear Dependent.

23. $\begin{bmatrix} 9 & 8 & 7 & 6 & 5 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$

Rank = 2, Linear independent.

24. 4 rows 3 column, Linear Dependent.

25. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 6 & 0 & -1 & 3 \\ 2 & 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 6 & 7 & 3 \\ 0 & 0 & 3 & 0 \end{bmatrix}$

Rank = 3, Linear independent.

26. $V_4 = 2V_1$, discard V_4

$\begin{bmatrix} 3 & 0 & 1 & 2 \\ 6 & 1 & 0 & 0 \\ 12 & 1 & 2 & 4 \\ 9 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 & 2 \\ 0 & 1 & -2 & -4 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & -2 & -4 \end{bmatrix} discardV_3

27. Yes, dimension=2, {[-2, 0, 1], [0, 2, 1]}

28. k=0, Yes, dimension=2, {[1, 0, 0], [0, 1, -3]}
    if k!=0, No. 2*V is not in the set.
29. No. -1*V not in the set.
30. n = 2, dimension = 1, {0}.
    $n > 2$. dimension = 2, {[0, 0 ... 1, 0], [0, 0 ... 0, 1]}
31. No. -1 * V not include in the set.
32. Yes, dimension=1, {[-5/4, 1, -23/4]}
33. Yes, dimension=1, {[1, 10/3, 3]}
34. No. 2 * V not in the set.
35. Yes, dimension=1, {[1, 1/2, 1/3, 1/4]}

P300 - Problem set 7.7

Theorems 1-a: we change from right handed to the left handed, so we get -1?

1. Theorems 1-a) $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$
   $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 0 - 1 = -1$

Theorems 1-b) $\begin{vmatrix} 1 & 0 \\ c & 1 \end{vmatrix} = 1 - 0 = 1$

Theorems 1-c) $\begin{vmatrix} 1 & 0 \\ 0 & c \end{vmatrix} = c - 0 = c$

Theorems 2-a) $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 0 - 1 = -1$

Theorems 2-b) $\begin{vmatrix} 1 & c \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

Theorems 2-c) $\begin{vmatrix} 1 & 0 \\ 0 & c \end{vmatrix} = c - 0 = c$

Theorems 2-d) In this example, $A = A^T = 1$

Theorems 2-e) $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$

Theorems 2-f) $\begin{vmatrix} 1 & 2 \\ a & 2a \end{vmatrix} = 2a - 2a = 0$

2. $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$
$= a_{11}|a_{22}| - a_{12}|a_{21}| = a_{11}a_{22} - a_{12}a_{21}$
$= a_{11}|a_{22}| - a_{21}|a_{12}| = a_{11}a_{22} - a_{12}a_{21}$
$= a_{22}|a_{11}| - a_{12}|a_{21}| = a_{11}a_{22} - a_{12}a_{21}$
$= a_{22}|a_{11}| - a_{21}|a_{12}| = a_{11}a_{22} - a_{12}a_{21}$
3. My guess is Example 2 but not Theoream 2? $\color{red}{Mark}$
4. Gauss elimination obviously a better option. It takes n^3 (I heard it can improve), which is way better than $n!$
5. $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1, \begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix} = k^2 - 0 = k^2$
6. $M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}, M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}, M_{32} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$
7. $\cos\alpha\cos\beta - \sin\alpha\sin\beta = \cos(\alpha+\beta)$
8. -7.87
9. $\cos(n\theta)\cos(n\theta) + \sin(n\theta)\sin(n\theta) = \cos(n\theta - n\theta) = 1$
10. $\cosh t \cosh t - \sinh t \sinh t = \cosh(t-t) = \frac{1}{2}(e^0 + e^{-0}) = 1$
11. 40
12. $a^3 + b^3 + c^3 - 3abc$
13. $0 \cdot (0 + 6 + -6 - 0 - 0 - 0) - 4 \cdot (0 + -15 + 2 - 0 - 0 - 4) + (-1) \cdot (0 + 0 + -4 - -30 - 0 - -8) - 5 \cdot (-12 + 0 + 6 - 45 - 0 - 0) = 0 - 4 \cdot (-17) + (-1) \cdot (34) - 5 \cdot (-51) = 289$
14. Question: I feel we can do it in the below way, with certain condition. Can not remember what exactly it is, and it does not apply for 13. The result is same while I expand the 4th order Determinant.
$\begin{vmatrix} 4 & 7 \\ 2 & 8 \end{vmatrix} \begin{vmatrix} 1 & 5 \\ -2 & 2 \end{vmatrix} = (32-14)(2+10) = 216$
$\color{red}{Mark}$
P.S: it is called block matrices. for upper (lower) triangular block matrix, diagonal blocks $A_1, A_2..A_n$, and we will get $det = det(A_1)det(A_2)..det(A_n)$.
15. $\begin{vmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 2 & 0 \\ 0 & 2 & 9 & 2 \\ 0 & 0 & 2 & 16 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 9 & 2 \\ 0 & 0 & 2 & 16 \end{vmatrix} = -\begin{vmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & 9 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 16 \end{vmatrix} = -\begin{vmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & 9 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 16 \end{vmatrix} = -64$
16. $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 0 - 1 = -1$
$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 2$

$$\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -3$$

So I would assume this special n order matrix have Determinant $(-1)^{n-1}(n-1)$

Try to prove it by induction - $Mark$

Incidence Matrices ?? $Mark$

17. $\begin{vmatrix} 4 & 9 \\ -8 & -6 \end{vmatrix} = -24 + 72 \neq 0$

$\begin{bmatrix} 4 & 9 \\ 0 & 12 \\ 0 & 24 \end{bmatrix}$, rank = 2

18. $\begin{vmatrix} 4 & 4 & 4 \\ 4 & 0 & 10 \\ -6 & 10 & 0 \end{vmatrix} = 0 + (-240) + (-240) - 0 - 0 - 0 > 0$

$\begin{bmatrix} 4 & 4 & 4 \\ 4 & 0 & 10 \\ -6 & 10 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 4 \\ 0 & 4 & -6 \\ 0 & 16 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 4 \\ 0 & 4 & -6 \\ 0 & 0 & 30 \end{bmatrix}$, rank=3

19. $\begin{vmatrix} 1 & 5 & 2 \\ 1 & 3 & 2 \\ 4 & 0 & 8 \end{vmatrix} = 24 + 40 + 0 - 24 - 40 - 0 = 0$

$\begin{vmatrix} 5 & 2 & 2 \\ 3 & 2 & 6 \\ 0 & 8 & 48 \end{vmatrix} = 480 + 0 + 48 - 0 - 48 * 5 - 48 * 6 = 0$

$\begin{vmatrix} 1 & 5 \\ 1 & 3 \end{vmatrix} = 3 - 5 = -2 \neq 0$

$\begin{bmatrix} 1 & 5 & 2 & 2 \\ 1 & 3 & 2 & 6 \\ 4 & 0 & 8 & 48 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 2 & 2 \\ 0 & 2 & 0 & -4 \\ 0 & 20 & 0 & -40 \end{bmatrix}$, rank=2

20. b) $\begin{cases} ax + by + cz + d * 1 = 0 \\ ax_1 + by_1 + cz_1 + d * 1 = 0 \\ ax_2 + by_2 + cz_2 + d * 1 = 0 \\ ax_3 + by_3 + cz_3 + d * 1 = 0 \end{cases}$

PS 7.7

20. TEAM PROJECT. Geometric Applications: Curves and Surfaces Through Given Points. The idea is to get an equation from the vanishing of the determinant of a homogeneous linear system as the condition for a nontrivial solution in Cramer's theorem. We explain the trick for obtaining such a system for the case of a line L through two given points P_1: (x_1, y_1) and P_2: (x_2, y_2). The unknown line is $ax + by = -c$, say. We write it as $ax + by + c \cdot 1 = 0$. To get a nontrivial solution a, b, c, the determinant of the "coefficients" $x, y, 1$ must be zero. The system is

$$
\begin{aligned}
&ax + by + c \cdot 1 = 0 \quad &&(\text{Line } L)\\
(12)\qquad &ax_1 + by_1 + c \cdot 1 = 0 \quad &&(P_1 \text{ on } L)\\
&ax_2 + by_2 + c \cdot 1 = 0 \quad &&(P_2 \text{ on } L).
\end{aligned}
$$

(a) Line through two points. Derive from $D = 0$ in (12) the familiar formula

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}.$$

(b) Plane. Find the analog of (12) for a plane through three given points. Apply it when the points are (1, 1, 1), (3, 2, 6), (5, 0, 5).

(c) Circle. Find a similar formula for a circle in the plane through three given points. Find and sketch the circle through (2, 6), (6, 4), (7, 1).

(d) Sphere. Find the analog of the formula in (c) for a sphere through four given points. Find the sphere through (0, 0, 5), (4, 0, 1), (0, 4, 1), (0, 0, −3) by this formula or by inspection.

(e) General conic section. Find a formula for a general conic section (the vanishing of a determinant of 6th order). Try it out for a quadratic parabola and for a more general conic section of your own choice.

b)

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

$D=0 \Rightarrow$ Non-trivial solutions.

$$\begin{vmatrix} x & y & z & 1 \\ 1 & 1 & 1 & 1 \\ 3 & 2 & 6 & 1 \\ 5 & 0 & 5 & 1 \end{vmatrix} = 0$$

$x \cdot (6+10+0-0-2-5)$

$-y\,(6+15+5-30-3-5)$

$+z\,(2+5+0-10-3-0)$

$-(10+30+0-10-0-15)=0$

$9x+12y-6z-15=0$

a)

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = xy_1 + yx_2 + x_1y_2$$

$$-x_2y_1 - xy_2 - x_1y$$

$$(xy_1 - xy_2) - (x_1y_1 - x_1y_2) = (x_1y - x_1y_1) - (x_2y - x_2y_1)$$

$$(x-x_1)(y_1-y_2) = (x_1-x_2)(y-y_1)$$

$$\Rightarrow \frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2}$$

b) check:

$a = P_2 - P_1 = (2, 1, 5)$ $\quad$ $b = P_3 - P_1 = (4, -1, 4)$

or $a \times b = \begin{vmatrix} i & j & k \\ 2 & 1 & 5 \\ 4 & -1 & 4 \end{vmatrix}$

$= i(4+5) - j(8-20) + k(-2-4)$

$= 9i + 12j - 6k = 3i + 4j - 2k$

$3(x-1) + 4(y-1) - 2(z-1) = 0$

$3x - 3 + 4y - 4 - 2z + 2 = 0$

$3x + 4y - 2z - 5 = 0$

Mark

21. $D = \begin{vmatrix} 3 & -5 \\ 6 & 16 \end{vmatrix} = 78$

$x = \frac{1}{78}\begin{vmatrix} 15.5 & -5 \\ 5 & 16 \end{vmatrix} = 3.5$

$y = \frac{1}{78}\begin{vmatrix} 3 & 15.5 \\ 6 & 5 \end{vmatrix} = -1$

$\begin{bmatrix} 3 & -5 & 15.5 \\ 6 & 16 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -5 & 15.5 \\ 0 & 26 & -26 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3.5 \\ 0 & 1 & -1 \end{bmatrix}$

22. $D = \begin{vmatrix} 2 & -4 \\ 5 & 2 \end{vmatrix} = 24$

$x = \frac{1}{24}\begin{vmatrix} -24 & -4 \\ 0 & 2 \end{vmatrix} = -2$

$y = \frac{1}{24}\begin{vmatrix} 2 & -24 \\ 5 & 0 \end{vmatrix} = 5$

$\begin{bmatrix} 2 & -4 & -24 \\ 5 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -12 \\ 0 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 5 \end{bmatrix}$

23. $D = \begin{vmatrix} 0 & 3 & -4 \\ 2 & -5 & 7 \\ -1 & 0 & -9 \end{vmatrix} = 0 - 21 + 0 - (-20) - 0 - (-54) = 53$

$x = \frac{1}{53}\begin{vmatrix} 16 & 3 & -4 \\ -27 & -5 & 7 \\ 9 & 0 & -9 \end{vmatrix} = \frac{1}{53}(80 * 9 + 21 * 9 + 0 - 20 * 9 - 0 - 81 * 9) = 0$

$y = \frac{1}{53}\begin{vmatrix} 0 & 16 & -4 \\ 2 & -27 & 7 \\ -1 & 9 & -9 \end{vmatrix} = \frac{1}{53}(0 - 112 - 72 + 108 - 0 + 288) = 212/53 = 4$

$z = \frac{1}{53}\begin{vmatrix} 0 & 3 & 16 \\ 2 & -5 & -27 \\ -1 & 0 & 9 \end{vmatrix} = \frac{1}{53}(0 - +81 + 0 - 80 - 0 - 54) = -1$

$\begin{bmatrix} 0 & 3 & -4 & 16 \\ 2 & -5 & 7 & -27 \\ -1 & 0 & -9 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 11 & 3 & 41 \\ 0 & 3 & -4 & 16 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 2 & 15 & -7 \\ 0 & 0 & 53 & -53 \end{bmatrix} =$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

24. $[1, -3, 4]^T$

25. $[3, 0, 2, -2]^T$

P308 - Problem set 7.8

1. $[A\ I] = \begin{bmatrix} 1.8 & -2.32 & 1 & 0 \\ -0.25 & 0.6 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -58/45 & 15/9 & 0 \\ -0.25 & 0.6 & 0 & 1 \end{bmatrix} =$

$\begin{bmatrix} 1 & -58/45 & 15/9 & 0 \\ 0 & 10/9 & 5/9 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -58/45 & 15/9 & 0 \\ 0 & 1 & 1/2 & 18/5 \end{bmatrix} =$

$\begin{bmatrix} 1 & 0 & 6/5 & 116/25 \\ 0 & 1 & 1/2 & 18/5 \end{bmatrix} = [I\ A^{-1}]$

$\begin{bmatrix} 1.8 & -2.32 \\ -0.25 & 0.6 \end{bmatrix}\begin{bmatrix} 6/5 & 116/25 \\ 1/2 & 18/5 \end{bmatrix} = \begin{bmatrix} 6/5 & 116/25 \\ 1/2 & 18/5 \end{bmatrix}\begin{bmatrix} 1.8 & -2.32 \\ -0.25 & 0.6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} \cos 2\theta & \sin 2\theta & 1 & 0 \\ -\sin 2\theta & \cos 2\theta & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & s/c & 1/c & 0 \\ 0 & 1/c & s/c & 1 \end{bmatrix} = \begin{bmatrix} 1 & s/c & 1/c & 0 \\ 0 & 1 & s & c \end{bmatrix} =$

$\begin{bmatrix} 1 & 0 & c & -s \\ 0 & 1 & s & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cos 2\theta & -\sin 2\theta \\ 0 & 1 & \sin 2\theta & \cos 2\theta \end{bmatrix}$

3. $\begin{bmatrix} 0.3 & -0.1 & 0.5 & 1 & 0 & 0 \\ 2 & 6 & 4 & 0 & 1 & 0 \\ 5 & 0 & 9 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1/3 & 5/3 & 10/3 & 0 & 0 \\ 0 & 1 & 1/10 & -1 & 3/20 & 0 \\ 0 & 0 & 1 & -30 & -1/2 & 2 \end{bmatrix} =$
$\begin{bmatrix} 1 & -1/3 & 5/3 & 10/3 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1/5 & -1/5 \\ 0 & 0 & 1 & -30 & -1/2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 54 & 9/10 & -17/5 \\ 0 & 1 & 0 & 2 & 1/5 & -1/5 \\ 0 & 0 & 1 & -30 & -1/2 & 2 \end{bmatrix}$

4. $\begin{bmatrix} 0 & 0 & 0.1 & 1 & 0 & 0 \\ 0 & -0.4 & 0 & 0 & 1 & 0 \\ 2.5 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2.5 & 0 & 0.1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -5/2 & 0 \\ 2.5 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} =$
$\begin{bmatrix} 2.5 & 0 & 0.1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -5/2 & 0 \\ 0 & 0 & 0.1 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2/5 \\ 0 & 1 & 0 & 0 & -5/2 & 0 \\ 0 & 0 & 1 & 10 & 0 & 0 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 5 & 4 & 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 4 & 1 & -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 3 & -4 & 1 \end{bmatrix}$

6. $\begin{bmatrix} -4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 8 & 13 & 0 & 1 & 0 \\ 0 & 3 & 5 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1/4 & 0 & 0 \\ 0 & 1 & 2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 0 & -3 & 8 \end{bmatrix} =$
$\begin{bmatrix} 1 & 0 & 0 & -1/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 5 & -13 \\ 0 & 0 & 1 & 0 & -3 & 8 \end{bmatrix}$

7. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ We can exchange 2 lines in Matrix. Determinant will * -1.

8. $R_3 - R_2 = R_2 - R_1$, Rank<3. Inverse matrix does not exist.

9. $\begin{bmatrix} 0 & 0 & 1/2 \\ 1/8 & 0 & 0 \\ 0 & 1/4 & 0 \end{bmatrix}$

10. $\begin{bmatrix} 2 & 1 & 2 & 3 & 0 & 0 \\ -2 & 2 & 1 & 0 & 3 & 0 \\ 1 & 2 & -2 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -2 & 0 & 0 & 3 \\ 0 & 1 & -2 & -10 & 0 & 2 \\ 0 & 0 & 3 & 2 & 1 & -2 \end{bmatrix} =$
$\begin{bmatrix} 1 & 0 & 0 & 2/3 & -2/3 & 1/3 \\ 0 & 1 & 0 & 1/3 & 2/3 & 2/3 \\ 0 & 0 & 1 & 2/3 & 1/3 & -2/3 \end{bmatrix}$

11. $AA = \begin{bmatrix} 1.8 & -2.32 \\ -0.25 & 0.6 \end{bmatrix} \begin{bmatrix} 1.8 & -2.32 \\ -0.25 & 0.6 \end{bmatrix} = \begin{bmatrix} 3.82 & -5.568 \\ -0.6 & 0.94 \end{bmatrix}$

$[(AA)^{-1}\ I] = \begin{bmatrix} 3.82 & -5.568 & 1 & 0 \\ -0.6 & 0.94 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3.76 & 22.272 \\ 0 & 1 & 2.4 & 15.28 \end{bmatrix}$

$(A^{-1})^2 = \begin{bmatrix} 6/5 & 116/25 \\ 1/2 & 18/5 \end{bmatrix} \begin{bmatrix} 6/5 & 116/25 \\ 1/2 & 18/5 \end{bmatrix} = \begin{bmatrix} 94/25 & 2784/25 \\ 12/5 & 382/25 \end{bmatrix}$

12. $A^{-1} = A^{-1}I = A^{-1}(AA)(AA)^{-1} = A(AA)^{-1}$ multiple A^{-1} on both side

$A^{-1}A^{-1} = A^{-1}A(AA)^{-1}$, so $(A^{-1})^2 = (A^2)^{-1}$

13. $A = \begin{bmatrix} 1.8 & -2.32 \\ -0.25 & 0.6 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1.2 & 4.64 \\ 0.5 & 3.6 \end{bmatrix}$,

$A^T = \begin{bmatrix} 1.8 & -0.25 \\ -2.32 & 0.6 \end{bmatrix}$, $(A^T)^{-1} = \begin{bmatrix} 1.2 & 4.64 \\ 0.5 & 3.6 \end{bmatrix}$

14. $(A^T)^{-1}A^T = I = I^T = (AA^{-1})^T = (A^{-1})^T A^T$, multiply $(A^T)^{-1}$ on both side,

$(A^T)^{-1}A^T(A^T)^{-1} = (A^{-1})^T A^T (A^T)^{-1}$

$(A^T)^{-1} = (A^{-1})^T$

15. $(A^{-1})^{-1} = (A^{-1})^{-1}I = (A^{-1})^{-1}(A^{-1}A) = A$

16. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ rotate the matrix $[x_1, x_2]^T$ by θ

so $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ rotate the matrix $[x_1, x_2]^T$ by $-\theta$

So $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} I = I$ and $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} I = I$

17. Take upper triangular as an example. We can use the back substution from the last row and move up, all the transation will not impact the 0 in the lower part in the Unix matrix. So the inverse matrix is also an upper triangular Matrix.

18. $Mark \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

19. $det(A) = \begin{vmatrix} 0.3 & -0.1 & 0.5 \\ 2 & 6 & 4 \\ 5 & 0 & 9 \end{vmatrix} = 16.2 + (-2) + 0 - 15 - 0 - (-1.8) = 1$

$A^{-1} = \frac{1}{det(A)}[C_{jk}]^T$

$C_{11} = \begin{vmatrix} 6 & 4 \\ 0 & 9 \end{vmatrix} = 54$

$C_{12} = -\begin{vmatrix} 2 & 4 \\ 5 & 9 \end{vmatrix} = -(18 - 20) = 2$

$C_{13} = \begin{vmatrix} 2 & 6 \\ 5 & 0 \end{vmatrix} = -30$

$C_{21} = -\begin{vmatrix} -0.1 & 0.5 \\ 0 & 9 \end{vmatrix} = 0.9$

$$C_{22} = \begin{vmatrix} 0.3 & 0.5 \\ 5 & 9 \end{vmatrix} = 2.7 - 2.5 = 0.2$$
$$C_{23} = -\begin{vmatrix} 0.3 & -0.1 \\ 5 & 0 \end{vmatrix} = -0.5$$
$$C_{31} = \begin{vmatrix} -0.1 & 0.5 \\ 6 & 4 \end{vmatrix} = -0.4 - 3 = -3.4$$
$$C_{32} = -\begin{vmatrix} 0.3 & 0.5 \\ 2 & 4 \end{vmatrix} = -0.2$$
$$C_{33} = \begin{vmatrix} 0.3 & -0.1 \\ 2 & 6 \end{vmatrix} = 2$$

So $A^{-1} = \begin{bmatrix} -54 & 0.9 & -3.4 \\ 2 & 0.2 & -0.2 \\ -30 & -0.5 & 2 \end{bmatrix}$

20. $\color{red}{Mark}$ Same as 19. Leave for the future.

P318 - Problem set 7.9

1. {[1, 0],   [0, 1]}, {[2, 0],   [0, 1]}, {[2, 0],   [1, 1]}
2. $a_{(1)}, ..., a_{(n)}$ are linearly indepdent.
   If $v = c_1 a_{(1)} + ... + c_n a_{(n)} = d_1 a_{(1)} + ... + d_n a_{(n)}$
   $(c_1 - d_1) a_{(1)} + ... + (c_n - d_n) a_{(n)} = 0$
   per page 311 (1), impliess $(c_1 - d_1) = 0, ...(c_n - d_n) = 0.$
3. The solution to the 2 equations is: [t, 11t, -7t]. so it is a vector space, the dimention is 1 and basis is $[t, 11t, -7t]^T, t \neq 0$
4. All skew-symmetric can present as $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$, so dimentino is 3 and a basis could be
   $\begin{bmatrix} 0 & a & 0 \\ -a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & b \\ 0 & 0 & 0 \\ -b & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & c \\ 0 & -c & 0 \end{bmatrix}, (a, b, c \neq 0)$
5. No. -1*v not in the set.
6. If we consider the function as the object in the vector space, then it is dimention 2, $\cos(2x), \sin(2x)$ is a set of basis.

If we evalute each function's vector space, then
if $a = b = 0$, it is a {0}, with dimention 0 (I would assume)
if one of a b equals 0, assume a=0, then it is a vector space. dimension =1 and basis is $\sin(2x)$
if none of a b is 0, then $|y(x)| < |a| + |b|$, so it has upper and lower limit. Not a vector space.

7. dimension = 2, $\{xe^{-x}, x\}$

8. No.
$\begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}+\begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$, which has determinant 1
9. Yes. $\begin{bmatrix}a & b\\c & -a\end{bmatrix}$, so dimention=3, $\{\begin{bmatrix}a & 0\\0 & -a\end{bmatrix},\begin{bmatrix}0 & b\\0 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\c & 0\end{bmatrix}\}, (a,b,c\neq 0)$
10. $Mark$: The first column is a 3x1 vector, can not multiply with a 3x1 vector.
If it means the objects in the first column multiple the 3, 0, -5 separately, then it means a matrix as
$\begin{bmatrix}a & b\\0 & c\\d & e\end{bmatrix}$, and a, b, c, d, e $\in R$. it is a vector space. with a set of basis
$\begin{bmatrix}a & 0\\0 & 0\\0 & 0\end{bmatrix},\begin{bmatrix}0 & b\\0 & 0\\0 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\0 & c\\0 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\0 & 0\\d & 0\end{bmatrix},\begin{bmatrix}0 & 0\\0 & 0\\0 & e\end{bmatrix}$, and a, b, c, d, e $\neq 0$

11. $\begin{bmatrix}0.5 & -0.5 & 1 & 0\\1.5 & -2.5 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -1 & 2 & 0\\0 & 1 & 3 & -1\end{bmatrix}=\begin{bmatrix}1 & 0 & 5 & -1\\0 & 1 & 3 & -1\end{bmatrix}$
$x_1=5y_1-y_2$
$x_2=3y_1-y_2$

12. $\begin{bmatrix}3 & 2 & 1 & 0\\4 & 1 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -1 & -1 & 1\\0 & 5 & 4 & -3\end{bmatrix}=\begin{bmatrix}1 & 0 & -1/5 & 2/5\\0 & 1 & 4/5 & -3/5\end{bmatrix}$

13. $\begin{bmatrix}5 & 3 & -3 & 1 & 0 & 0\\3 & 2 & -2 & 0 & 1 & 0\\2 & 01 & 2 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 1 & -1 & -1 & 2 & 0\\0 & 1 & -1 & -3 & 5 & 0\\0 & 3 & -4 & -12 & 4 & 1\end{bmatrix}=$
$\begin{bmatrix}1 & 0 & 0 & 2 & -3 & 0\\0 & 1 & 0 & -10 & 16 & 1\\0 & 0 & 1 & -7 & 11 & 1\end{bmatrix}$

14. $\begin{bmatrix}0.2 & -0.1 & 0 & 1 & 0 & 0\\0 & -0.2 & 0.1 & 0 & 1 & 0\\0.1 & 0 & 0.1 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -0.5 & 0 & 5 & 0 & 10\\0 & 1 & -0.5 & 0 & -5 & 0\\1 & 0 & 1 & 0 & 0 & 10\end{bmatrix}=$
$\begin{bmatrix}1 & 0 & 0 & 4 & -2 & 1\\0 & 1 & 0 & -2 & -4 & 2\\0 & 0 & 1 & -4 & 2 & 4\end{bmatrix}$

15. $\sqrt{26}$
16. $\sqrt{26}/6$
17. $\sqrt{5}$
18. 9

19. 1
20. 1
21. -20
22. $2x + z = 0$, Yes, it is a vector space
23. $||(a, b)|| = || - 12 + 8 + 4|| = 0$
    $||a|| + ||b|| = \sqrt{26} + 9$
24. $|(a, b)| = 1/3 + 2/9 - 1/6 = 7/18$
    $||a|| \, ||b|| = \sqrt{2}6/6$
    $7/18 = \sqrt{49}/18 < \sqrt{26 * 9}/18 = \sqrt{26}/6$
25. $64 + 25 + 1 + 4 + 1 + 9 = 2(25 + 9 + 4 + 9 + 4 + 1) = 104$

P318 - Chapter 7, Review questions and problems

Maybe leave for the practice before the mid-term exam. It is pretty time consuming and I should move on