W9 - Cauchy's Integral Theorem

- 9A: The complex line integral
  - Definition
    - $\dot{z}(t) = \dfrac{dz}{dt} = \dot{x}(t) + i\dot{y}(t)$
    - Closed path!
  - Basic properties
    - Linearity
      - $\int_C [k_1 f_1(z) + k_2 f_2(z)]\,dz = k_1 \int_C f_1(z)\,dz + k_2 \int_C f_2(z)\,dz.$
    - Sense reversal
      - $\int_{z_0}^{Z} f(z)\,dz = -\int_{Z}^{z_0} f(z)\,dz.$
    - Partitioning of path
      - $\int_C f(z)\,dz = \int_{C_1} f(z)\,dz + \int_{C_2} f(z)\,dz.$
  - Existence
  - First: Indefinite Integration
    - f(x) analytics, **in a simply connected domain**
    - $\int_{z_0}^{z_1} f(z)\,dz = F(z_1) - F(z_0) \qquad [F'(z) = f(z)].$
  - Second: Representation of a Path
    - f(z) continuous on a piecewise smooth path
    - $\int_C f(z)\,dz = \int_a^b f[z(t)]\dot{z}(t)\,dt \qquad \left(\dot{z} = \dfrac{dz}{dt}\right).$
    - Steps in Applying Theorem 2
      - **(A)** Represent the path C in the form $z(t)$ $(a \leqq t \leqq b)$.
      - **(B)** Calculate the derivative $\dot{z}(t) = dz/dt$.
      - **(C)** Substitute $z(t)$ for every z in $f(z)$ (hence $x(t)$ for x and $y(t)$ for y).
      - **(D)** Integrate $f[z(t)]\dot{z}(t)$ over t from a to b.
    - $\oint_C \dfrac{dz}{z} = 2\pi i \qquad$ (C the unit circle, counterclockwise).
  - Bounds for Integrals. ML-Inequality
- Cauchy's Integral Theorem
  - Definition
    - A simple closed path
      - does not intersect or touch itself
    - A simply connected domain
      - every simple closed path in D encloses only points of D
    - p-fold connected
  - Cauchy's Integral Theorem
    - f(z) analytic + CLOSED C in simply connected domain D
      - e^z
      - cosz
      - z^n
    - Hole inside? Not apply. => Use second method
    - $\oint_C f(z)\,dz = 0.$
    - contour integral
    - Analyticity Sufficient, Not Necessary
  - Independence of Path
  - Principle of Deformation of Path
  - Existence of Indefinite Integral
  - Multiply Connected Domains