Chapter 14 - Complex Integration

Selected Problem set 14.1

14.1 1. 5. 7. 11. 13. 23. 25

1–10 **FIND THE PATH** and sketch it.

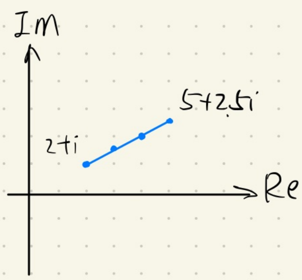
1. $z(t) = (1 + \frac{1}{2}i)t \quad (2 \leqq t \leqq 5)$

$z(2) = 2+i$

$z(3) = 3+1.5i$

$z(4) = 4+2i$

$z(5) = 5+2.5i$



5. $z(t) = 3 - i + \sqrt{10}e^{-it} \quad (0 \leqq t \leqq 2\pi)$

$z(t) = 3 - i + \sqrt{10}\left[\cos(-\theta) + i\sin(-\theta)\right]$

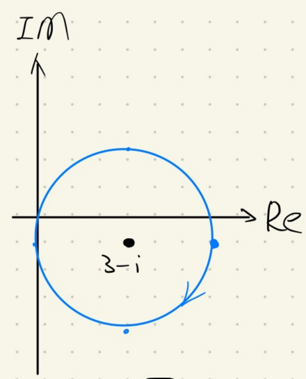
$z(0) = 3+\sqrt{10} - i$

$z(\frac{\pi}{2}) = 3 - (1+\sqrt{10})i$

$z(\pi) = (3-\sqrt{10}) - i$

$z(\frac{3}{2}\pi) = 3 + (\sqrt{10}-1)i$

$z(2\pi) = 3+\sqrt{10} - i$



$r = \sqrt{10}$

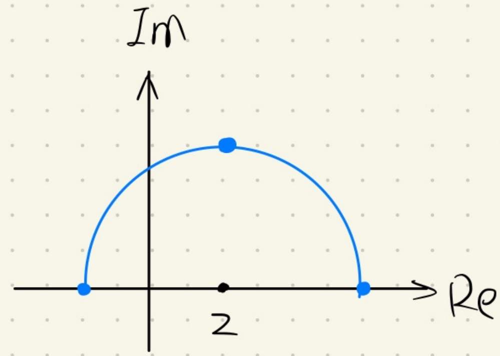
7. $z(t) = 2 + 4e^{\pi i t/2} \quad (0 \leqq t \leqq 2)$

$e^{\frac{\pi t}{2} \cdot i} = \cos\left(\frac{\pi}{2}t\right) + i\,\sin\left(\frac{\pi}{2}t\right)$

$z(0) = 2 + 4(1 + 0i) = 6$

$z(1) = 2 + 4(0 + i) = 2 + 4i$

$z(2) = 2 + 4(-1 + 0i) = -2$

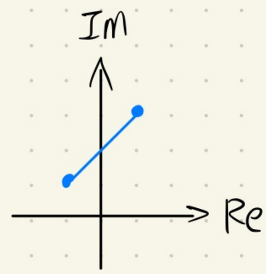


11–20 FIND A PARAMETRIC REPRESENTATION
and sketch the path.

11. Segment from $(-1, 1)$ to $(1, 3)$

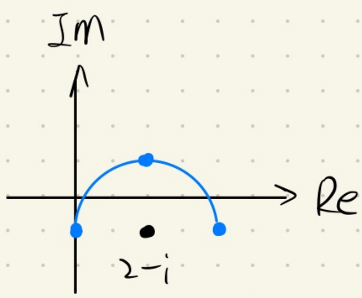
$m = \dfrac{3-1}{1-(-1)} = 1$

$(-1+t,\ 1+t) \qquad 0 \le t \le 2$



13. Upper half of $|z - 2 + i| = 2$ from $(4, -1)$ to $(0, -1)$

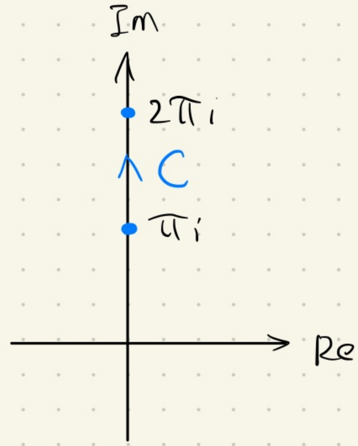
$z(t) = 2 - i + 2e^{it} \qquad 0 \le t \le \pi$



21–30 INTEGRATION

Integrate by the first method or state why it does not apply and use the second method. Show the details.

23. $\int_C e^z\,dz$, C the shortest path from πi to $2\pi i$



C: $z(t) = t\pi i \qquad 1 \le t \le 2$

$\dot{z}(t) = \pi i$

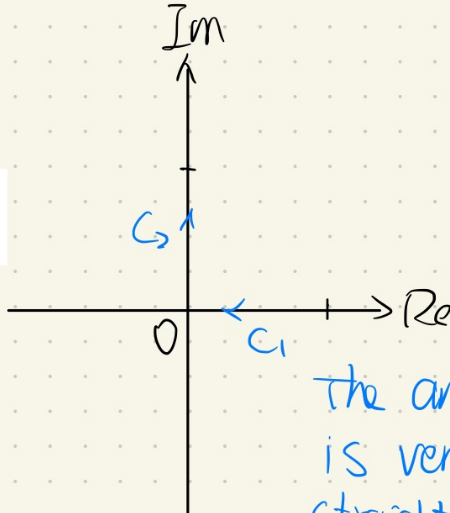
$$\int_1^2 e^{t\pi i} \cdot \pi i \, dt$$

$$= \pi i \int_1^2 [\cos(t\pi) + i \sin(t\pi)] \, dt$$

$$= \pi i \left(0 + -\frac{2}{\pi} \cdot i\right)$$

$$= 2$$

25. $\int_C z \exp(z^2)\,dz$, C from 1 along the axes to i



C_1: $z_1(t) = 1 - t \qquad 0 \le t \le 1$

$\dot{z}_1(t) = -1$

C_2 $z_2(t) = ti \qquad 0 \le t \le 1$

$\dot{z}_2(t) = i$

The answer on PA37 is very simple and straight forward. Not sure the application condition.

$\frac{1}{2} e^{z^2} \Big|_1^i = -\sinh 1$

$$\int_C z \exp(z^2) = \int_{C_1} z_1 \exp(z_1^2) + \int_{C_2} z_2 \exp(z_2^2)$$

$$= \int_0^1 (1-t) \cdot e^{(1-t)^2} \cdot (-1) \, dt + \int_0^1 ti \cdot e^{(ti)^2} \cdot i \, dt$$

$$= \int_0^1 (t-1) \cdot e^{(1-t)^2} - t \cdot e^{-t^2} \, dt = -\sinh 1$$

-1.1752 very complex. numerical approach

Selected Problem set 14.2