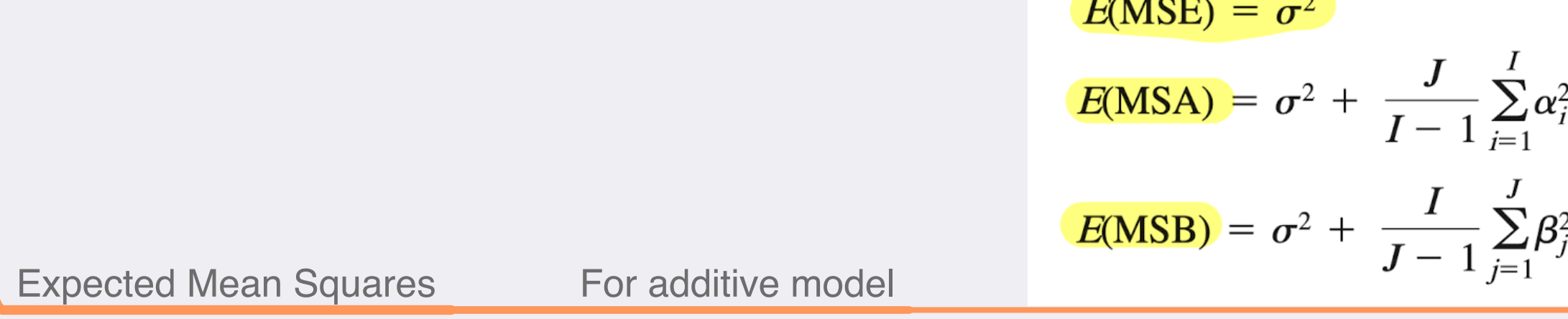


Table 11.1 ANOVA Table for Example 11.3

Source of Variation	df	Sum of Squares	Mean Square	$f$
Factor A (brand)	$I - 1 = 2$	SSA = .1282	MSA = .0641	$f_A = 4.43$
Factor B (wash treatment)	$J - 1 = 3$	SSB = .4797	MSB = .1599	$f_B = 11.05$
Error	$(I - 1)(J - 1) = 6$	SSE = .0868	MSE = .01447	
Total	$IJ - 1 = 11$	SST = .6947		

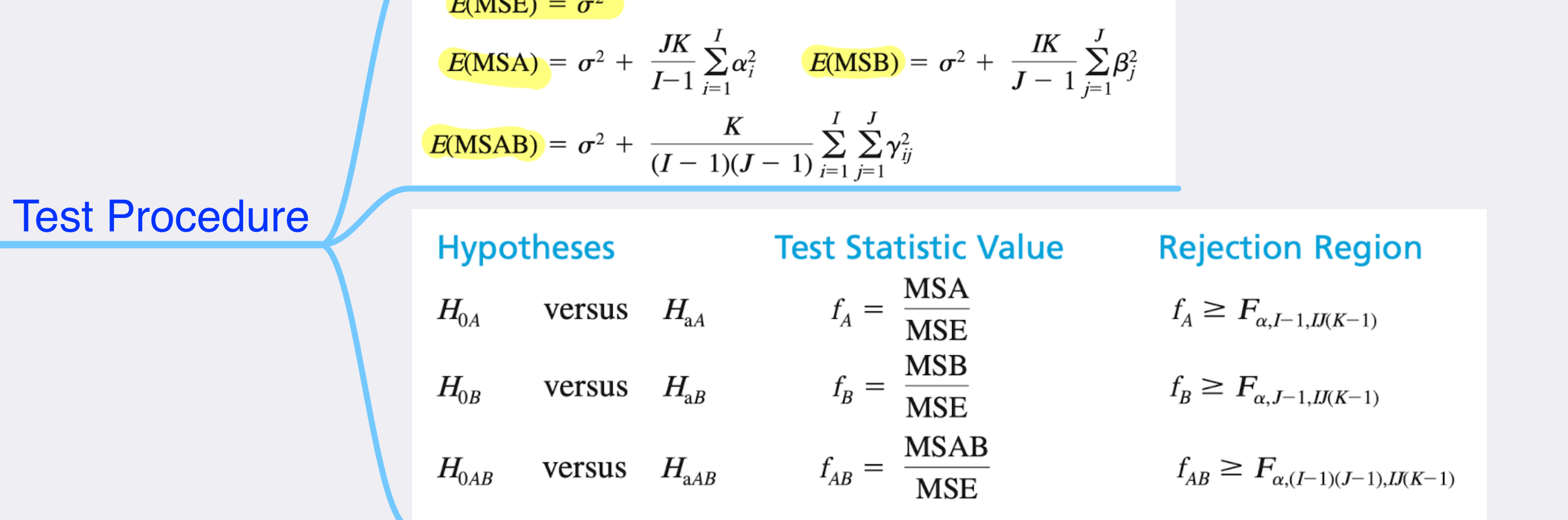
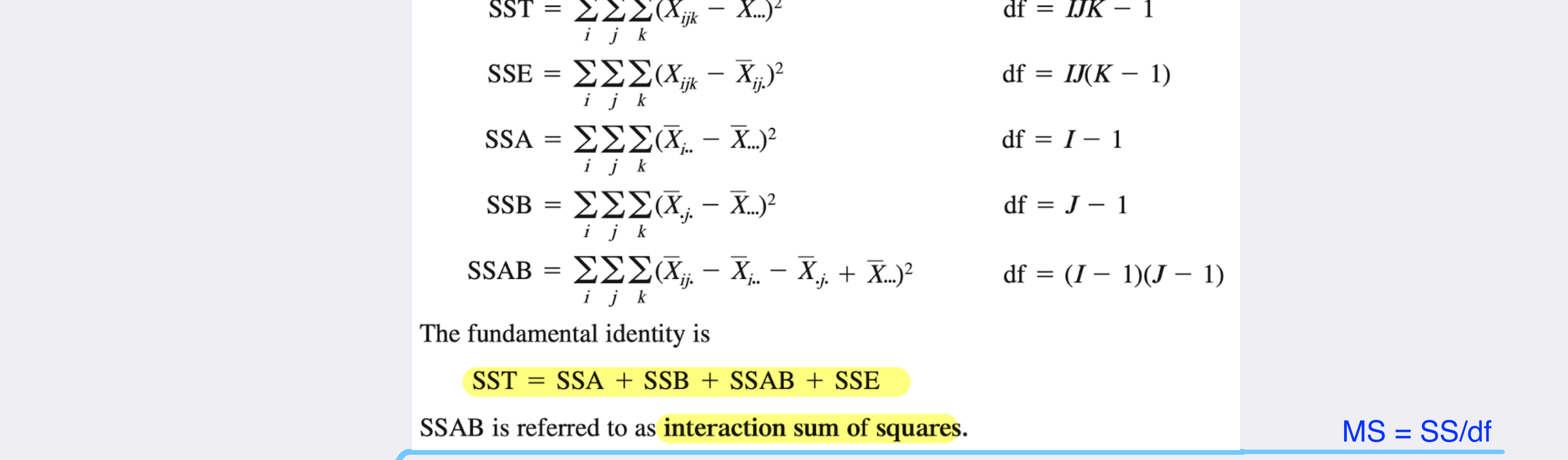
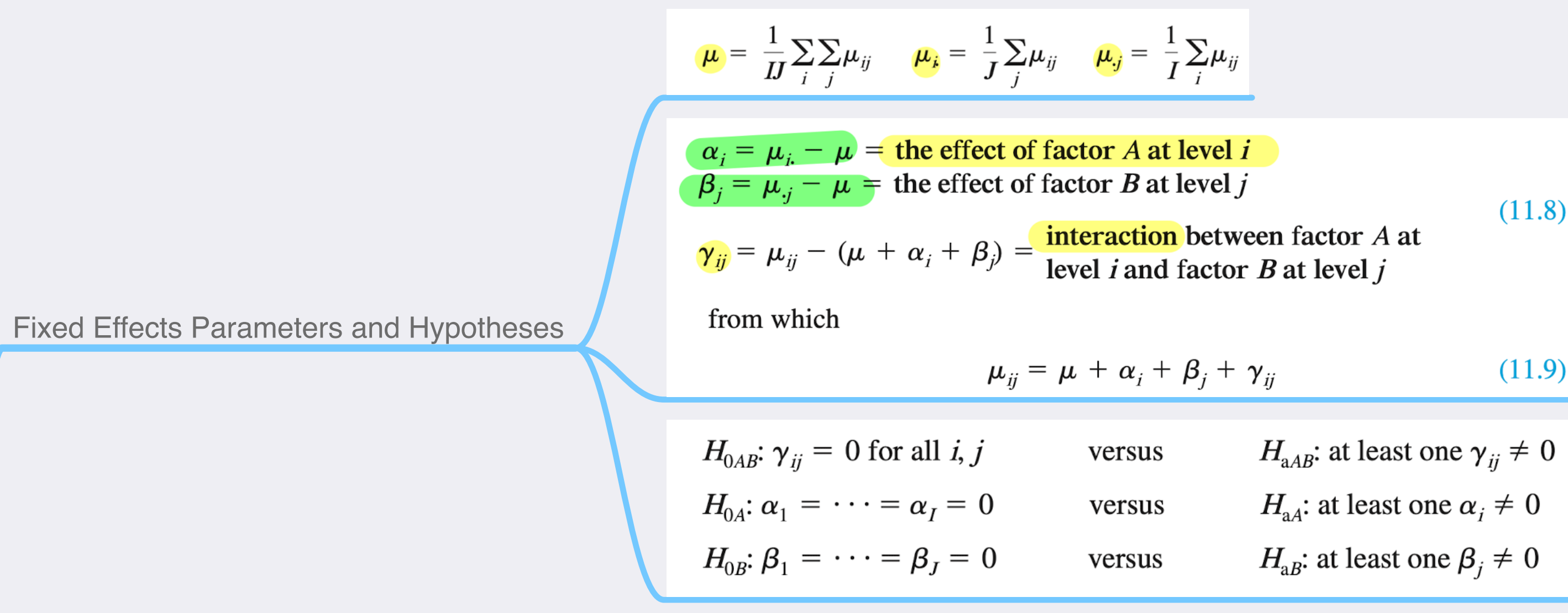


- After rejecting either  $H_{0A}$  or  $H_{0B}$ , Tukey's procedure can be used to identify significant differences between the levels of the factor under investigation.
- For comparing levels of factor  $A$ , obtain  $Q_{\alpha, I(I-1)(J-1)}$ . For comparing levels of factor  $B$ , obtain  $Q_{\alpha, J(I-1)(J-1)}$ .
  - Compute
- $$w = Q \cdot (\text{estimated standard deviation of the sample means being compared})$$
- $$= \begin{cases} Q_{\alpha, I(I-1)(J-1)} \cdot \sqrt{MSE/J} & \text{for factor } A \text{ comparisons} \\ Q_{\alpha, J(I-1)(J-1)} \cdot \sqrt{MSE/I} & \text{for factor } B \text{ comparisons} \end{cases}$$
- (because, e.g., the standard deviation of  $\bar{X}_i$  is  $\sigma/\sqrt{J}$ ).
- Arrange the sample means in increasing order, underscore those pairs differing by less than  $w$ , and identify pairs not underscored by the same line as corresponding to significantly different levels of the given factor.

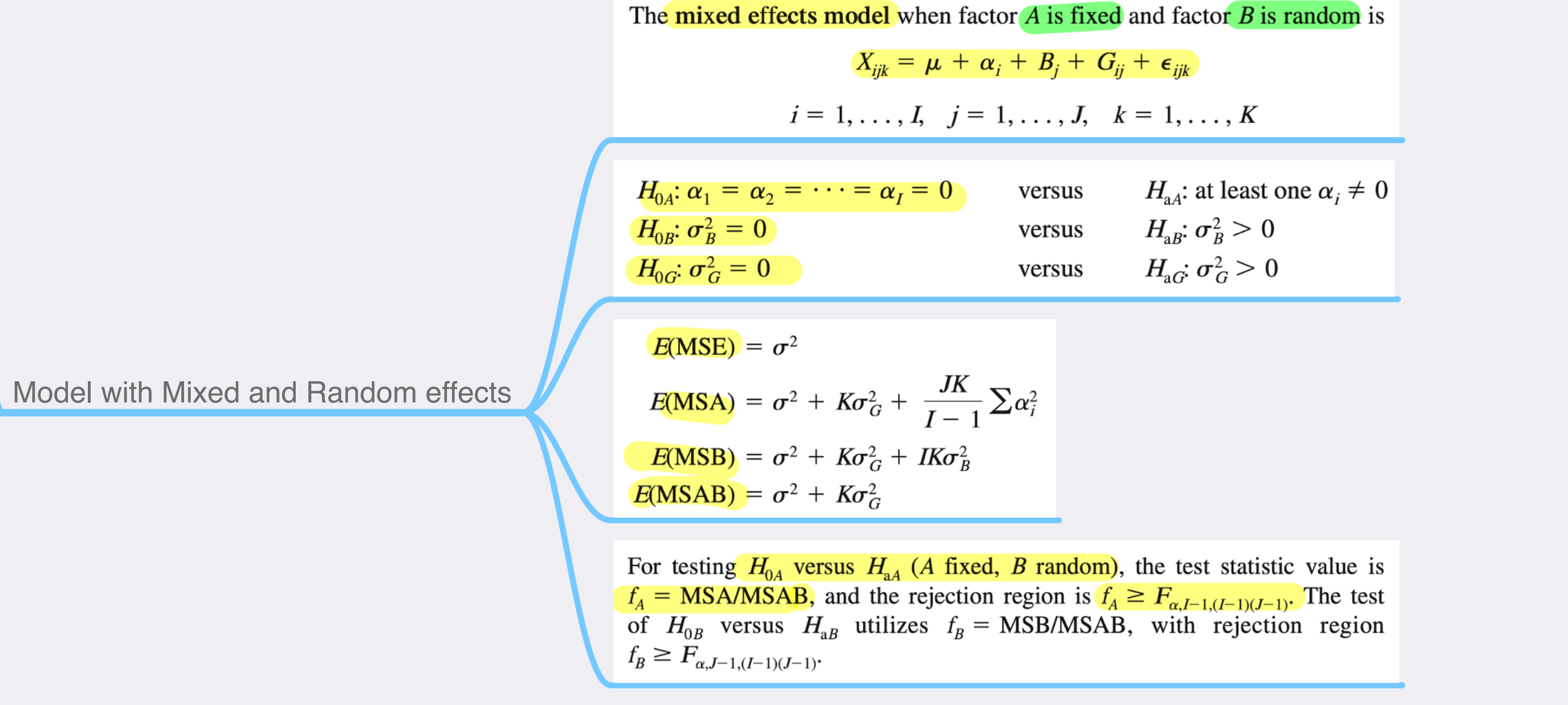
Multiply Comparison

Randomized Block Design P427 Example 11.5, Great example

Models with Random and mixed effects



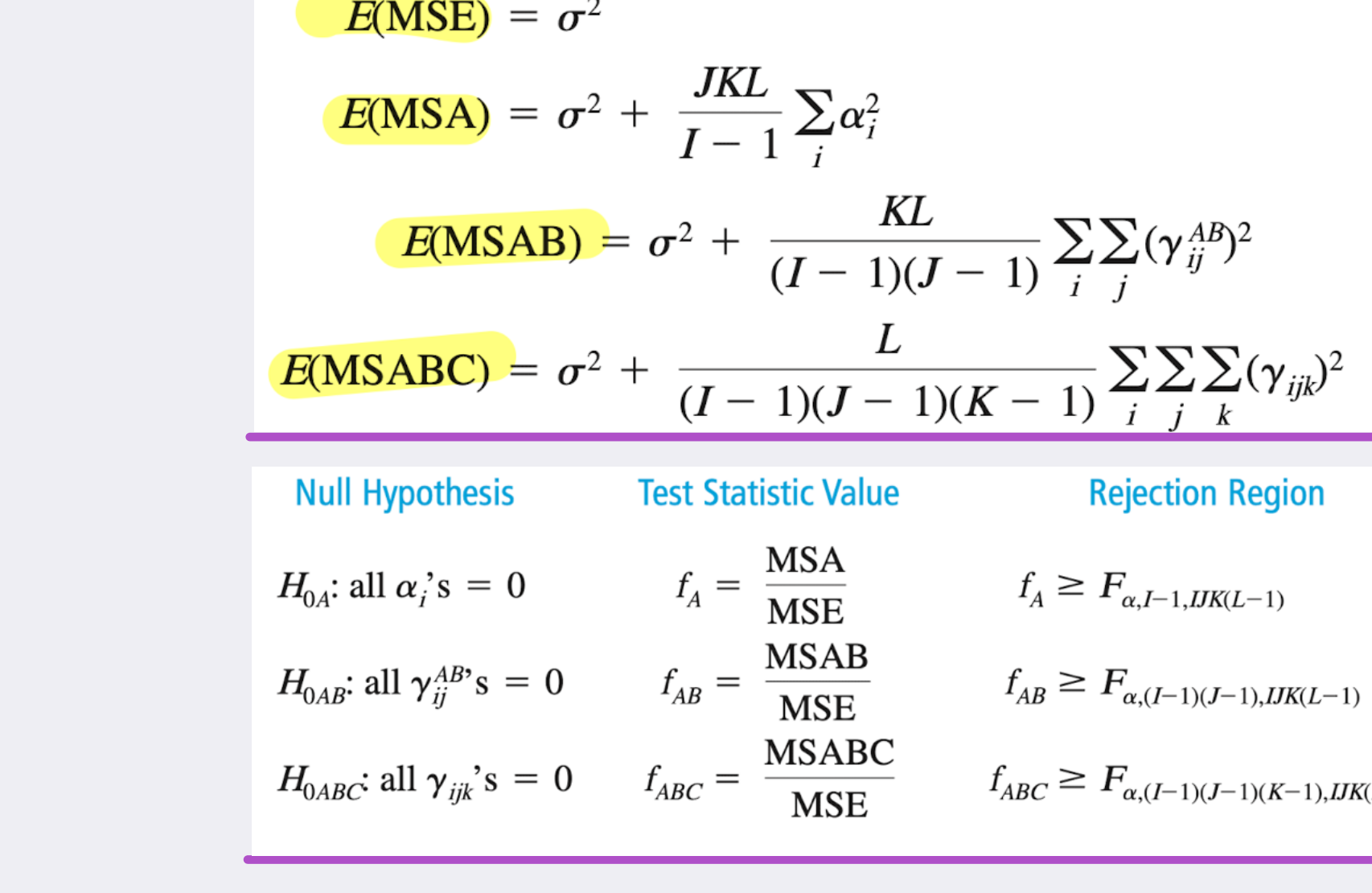
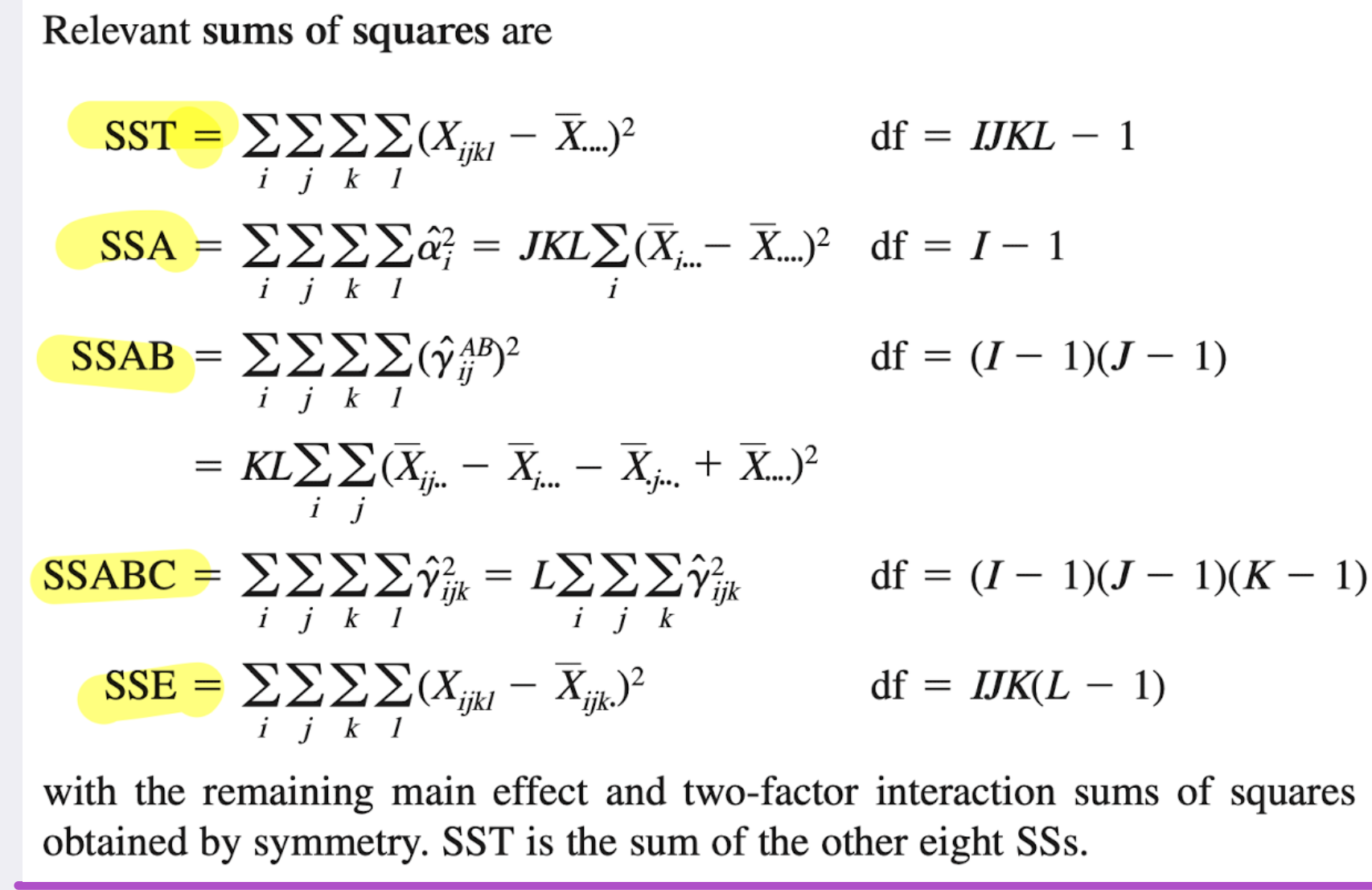
- When the no-interaction hypothesis  $H_{0AB}$  is not rejected and at least one of the two main effect null hypotheses is rejected, Tukey's method can be used to identify significant differences in levels. For identifying differences among the  $\alpha_i$ 's when  $H_{0A}$  is rejected,
- Obtain  $Q_{\alpha, IJ(K-1)}$ , where the second subscript  $I$  identifies the number of levels being compared and the third subscript refers to the number of degrees of freedom for error.
  - Compute  $w = Q\sqrt{MSE/(JK)}$ , where  $JK$  is the number of observations averaged to obtain each of the  $\bar{x}_{.i}$ 's compared in Step 3.
  - Order the  $\bar{x}_{.i}$ 's from smallest to largest and, as before, underscore all pairs that differ by less than  $w$ . Pairs not underscored correspond to significantly different levels of factor  $A$ .



The fixed effects model for three-factor ANOVA with  $L_{ijk} = L$  is

$$X_{ijk} = \mu_{ijk} + \epsilon_{ijk} \quad i = 1, \dots, I, \quad j = 1, \dots, J, \quad k = 1, \dots, K, \quad l = 1, \dots, L \quad (11.12)$$

where the  $\epsilon_{ijk}$ 's are normally distributed with mean 0 and variance  $\sigma^2$ , and

$$\mu_{ijk} = \mu + \alpha_i + \beta_j + \delta_k + \gamma_{ij}^{AB} + \gamma_{ik}^{AC} + \gamma_{jk}^{BC} + \gamma_{ijk} \quad (11.13)$$


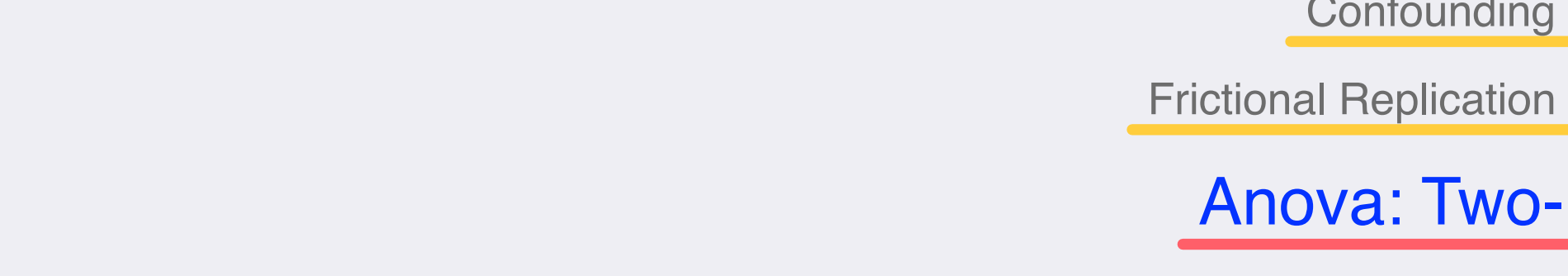
Experimental Condition	Cell Total	A	B	C	Factorial Effect AB    AC	BC	ABC
(1)	$X_{111}$	-	-	-	+	+	+
a	$X_{211}$	+	-	-	-	+	+
b	$X_{121}$	-	+	-	-	+	+
ab	$X_{221}$	+	+	-	+	-	+
c	$X_{112}$	-	-	+	+	-	+
ac	$X_{212}$	+	-	+	+	-	-
bc	$X_{122}$	-	+	+	-	+	-
abc	$X_{222}$	+	+	+	+	+	+

AC contrast =  $+x_{111} - x_{211} + x_{121} - x_{221} - x_{112} + x_{212} - x_{122} + x_{222}$

$SS(\text{effect}) = \frac{(\text{effect contrast})^2}{8n}$

method of Computation

	1	2	Effect Contrast	SS = (contrast) <sup>2</sup> /16
(1) = $x_{111}$	2233	-4771	9735	252.255.06
a = $x_{211}$	2538	4964	2009	28.985.06
b = $x_{121}$	2294	929	681	3.393.06
ab = $x_{221}$	2670	1080	-233	2.328.06
c = $x_{112}$	465	305	193	1.625.06
ac = $x_{212}$	464	376	151	.315.06
bc = $x_{122}$	656	-1	71	-231
abc = $x_{222}$	424	-232	-231	292.036.42



Fixed Effects Models

Three-Factor

Test Procedures

Latin Square Experiment

2^p Factorial Experiements

W11

Two-Factor, K\_ij = 1

Two-Factor, K\_ij > 1