

Summary of Asymptotic Notations

Asymptotic Bound	$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = C$	Complete Definition	Asymptotic Family Relationships	Is $g(n)$ an asymptotically tight bound for $f(n)$?
ω	$\infty = C$	$\omega(g(n)) = \left\{ f(n) : \begin{array}{l} \text{for any positive constant} \\ c > 0, \exists \text{ a constant } n_0 > 0 \text{ such that} \\ 0 \leq cg(n) < f(n) \forall n \geq n_0 \end{array} \right\}$	$\omega(g(n)) \subset \Omega(g(n))$	No; asymptotically larger (see pg 52)
Ω	$0 < C \leq \infty$	$\Omega(g(n)) = \left\{ f(n) : \begin{array}{l} \exists \text{ positive constants } c \text{ and } n_0 \\ \text{such that } 0 \leq cg(n) \leq f(n) \forall n \geq n_0 \end{array} \right\}$	$\Theta(g(n)) \subseteq \Omega(g(n))$	Maybe
o	$0 = C$	$o(g(n)) = \left\{ f(n) : \begin{array}{l} \text{for any positive constant} \\ c > 0, \exists \text{ a constant } n_0 > 0 \text{ such} \\ \text{that } 0 \leq f(n) < cg(n) \forall n \geq n_0 \end{array} \right\}$	$o(g(n)) \subset O(g(n))$	No; asymptotically smaller (see pg 52)
O	$0 \leq C < \infty$	$O(g(n)) = \left\{ f(n) : \begin{array}{l} \exists \text{ positive constants } c \text{ and } n_0 \\ \text{such that } 0 \leq f(n) \leq cg(n) \forall n \geq n_0 \end{array} \right\}$	$\Theta(g(n)) \subseteq O(g(n))$	Maybe
Θ	$0 < C < \infty$	$\Theta(g(n)) = \left\{ f(n) : \begin{array}{l} \exists \text{ positive constants } c_1, c_2, \text{ and } n_0 \\ \text{such that } 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \forall n \geq n_0 \end{array} \right\}$	Theorem 3.1 For any two functions $f(n)$ and $g(n)$, we have $f(n) = \Theta(g(n))$ iff $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$	Yes

Notes:

$f(n)$ must be nonnegative whenever n is sufficiently large (asymptotically nonnegative)

$g(n)$ must be asymptotically nonnegative

if $\forall n \geq n_0$ the function $f(n)$ is equal to $g(n)$ to within a constant factor, then $g(n)$ is an asymptotically tight bound for $f(n)$