- W05
  - Counting and Probability
    - Counting
      - Rule of sum and product
      - Strings
        - k-string
        - K-sub string
      - Permutations
        - k-permutation
      - Combinations
        - k-combination
        - Binomial coefficients
          - Binomial expansion
        - Binomial bound
          - $\binom{n}{k} \geq \left(\frac{n}{k}\right)^k$
          - $\binom{n}{k} \leq \left(\frac{en}{k}\right)^k$
    - Probability
      - Axioms of probability
      - Discrete probability distributions
      - Continuous uniform probability distribution
      - Conditional probability and independence
        - Pairwise independent
        - Mutually indepdendent
      - **Bayes's Theorem**
    - Discrete random variable
      - Probability density function
        - Joint probability density function
      - Expected value
        - $\mathrm{E}[X] = \sum_x x \cdot \Pr\{X = x\}$
        - Linearity of expectation
          - $\mathrm{E}[aX + Y] = a\mathrm{E}[X] + \mathrm{E}[Y]$
        - independent
          - E(XY) = E(X)E(Y)
        - convex f(x): Jensen's inequality
          - $\mathrm{E}[f(X)] \geq f(\mathrm{E}[X])$
      - Variance and standard deviation
        - $\mathrm{E}[X^2] = \mathrm{Var}[X] + \mathrm{E}^2[X]$
        - $\mathrm{Var}[aX] = a^2\mathrm{Var}[X]$
    - Geometric and binomial distributions
      - Geometric distribution
        - E(X) = 1/p
        - VAR(X) = q/p^2
      - Binomial distribution
        - E(X) = np
        - VAR(X) = npq
        - ***Lemma C.1***
          Let $n \geq 0$, let $0 < p < 1$, let $q = 1 - p$, and let $0 \leq k \leq n$. Then $b(k; n, p) \leq \left(\frac{np}{k}\right)^k \left(\frac{nq}{n-k}\right)^{n-k}$.
  - The hiring problem
    - Worst-case analysis
    - Probabilistic analysis
      - Average-case running time
      - uniform random permutation
    - Randomized algorithms
      - Expected running time
  - Indicator random variable
    - Linearity of expectation
    - Hiring problem
      - E(X) = sum 1/i = ln n + O(1)
  - Randomized algorithms
    - Randomize in the algorithm, not in the input distribution
    - No particular input elicits its worst-case behavior
    - Common: Assign random priority, sort
      - Uniform random permutation