

Chapter 8 Linear Algebra: Matrix Eigenvalue Problems

P329 - Problem set 8.1

P.S. 8.1

1-16 EIGENVALUES, EIGENVECTORS

Find the eigenvalues. Find the corresponding eigenvectors.
Use the given λ or factor in Probs. 11 and 15.

1. $\begin{bmatrix} 3.0 & 0 \\ 0 & -0.6 \end{bmatrix}$

2. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 5 & -2 \\ 9 & -6 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$$(A - \lambda I) \cdot X = 0$$

$$1. \det \begin{vmatrix} 3-\lambda & 0 \\ 0 & -0.6-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(-0.6-\lambda) = 0$$

$$-\lambda^2 + 2.4\lambda + 1.8 = 0$$

$$\lambda^2 - 2.4\lambda - 1.8 = 0$$

$$(\lambda - 3)(\lambda + 0.6) = 0$$

1) $\lambda = 3$ 2) $\lambda = -0.6$

$$\begin{bmatrix} 0 & 0 \\ 0 & -3.6 \end{bmatrix} \quad \begin{bmatrix} 3.6 & 0 \\ 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad X = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$2. \det \begin{vmatrix} -\lambda & 0 \\ 0 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 = 0 \quad \lambda = 0$$

?

λ could be any 2×1 vector.

$$3. \det \begin{vmatrix} 5-\lambda & -2 \\ 9 & -6-\lambda \end{vmatrix} = 0$$

$$-(5-\lambda)(6+\lambda) + 18 = 0$$

$$\lambda^2 + \lambda - 30 + 18 = 0$$

$$\lambda^2 + \lambda - 12 = 0$$

$$(\lambda + 4)(\lambda - 3) = 0$$

1) $\lambda = -4$

$$\begin{bmatrix} 9 & -2 \\ 9 & -2 \end{bmatrix} = \begin{bmatrix} 9 & -2 \\ 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

2) $\lambda = 3$

$$\begin{bmatrix} 2 & -2 \\ 9 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$3. \begin{bmatrix} 5 & -2 \\ 9 & -6 \end{bmatrix}$$

$$4. \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$5. \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

$$6. \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

$$4. \det: \begin{vmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{vmatrix} = 0$$

$$(\lambda-1)(\lambda-4) - 4 = 0$$

$$\lambda^2 - 5\lambda = 0$$

$$\lambda = 0 \quad \lambda = 5$$

$$\lambda = 5: \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda = 0: \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$5. \begin{vmatrix} -\lambda & 3 \\ -3 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 + 9 = 0$$

$$\lambda = \pm 3i$$

$$\lambda = 3i: \begin{bmatrix} -3i & 3 \\ -3 & -3i \end{bmatrix} = \begin{bmatrix} 1 & i \\ 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} -i \\ 1 \end{bmatrix}$$

$$\lambda = -3i: \begin{bmatrix} 3i & 3 \\ -3 & 3i \end{bmatrix} = \begin{bmatrix} 1 & -i \\ 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

$$6. \begin{vmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{vmatrix} = 0$$

$$(\lambda-3)(\lambda-1) = 0$$

$$\lambda = 3: \begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda = 1: \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$7. \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$8. \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

$$7. \begin{vmatrix} -\lambda & 1 \\ 0 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 = 0$$

$$\lambda = 0 \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$8. \begin{vmatrix} a-\lambda & b \\ b & a-\lambda \end{vmatrix} = 0$$

$$(a-\lambda)^2 + b^2 = 0$$

$$a-\lambda = \pm bi$$

$$\lambda = a \pm bi$$

$$\lambda = a + bi$$

$$\begin{bmatrix} -bi & b \\ -b & -bi \end{bmatrix} = \begin{bmatrix} 1 & i \\ 0 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\lambda = a - bi \quad \begin{bmatrix} bi & b \\ -b & bi \end{bmatrix} = \begin{bmatrix} 1 & -i \\ 0 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} -1 \\ i \end{bmatrix}$$

$$9. \begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$$

$$10. \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$9. \begin{vmatrix} 0.8-\lambda & -0.6 \\ 0.6 & 0.8-\lambda \end{vmatrix} = 0$$

$$a = 0.8 \quad b = -0.6$$

$$\lambda = 0.8 - 0.6i \quad x = \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\lambda = 0.8 + 0.6i \quad x = \begin{bmatrix} -1 \\ i \end{bmatrix}$$

$$10. \begin{vmatrix} \cos \theta - \lambda & -\sin \theta \\ \sin \theta & \cos \theta - \lambda \end{vmatrix} = 0$$

$$a = \cos \theta \quad b = -\sin \theta$$

$$\lambda = \cos \theta - i \sin \theta \quad x = \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\lambda = \cos \theta + i \sin \theta \quad x = \begin{bmatrix} -1 \\ i \end{bmatrix}$$

$$11. \begin{bmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix}, \lambda = 3$$

$$11. \begin{bmatrix} 3 & 2 & -2 \\ 2 & 2 & 0 \\ -2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 0 \\ 0 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 4 \\ 0 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$12. \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$12. (3-\lambda)(4-\lambda)(1-\lambda)=0$$

$$\lambda = 3$$

$$\begin{bmatrix} 0 & 5 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda = 4$$

$$\begin{bmatrix} -1 & 5 & 3 \\ 0 & 0 & 6 \\ 0 & 0 & -3 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda = 1$$

$$\begin{bmatrix} 2 & 5 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 0 \end{bmatrix} X = \begin{bmatrix} 7 \\ -4 \\ 2 \end{bmatrix}$$

$$13. \begin{bmatrix} 13 & 5 & 2 \\ 2 & 7 & -8 \\ 5 & 4 & 7 \end{bmatrix}$$

$$13. \begin{vmatrix} 13-\lambda & 5 & 2 \\ 2 & 7-\lambda & -8 \\ 5 & 4 & 7-\lambda \end{vmatrix} = 0$$

$$(13-\lambda)(7-\lambda)^2 - 200 + 16 -$$

$$10(7-\lambda) - 32(\lambda+3)$$

$$-10(7-\lambda) = 0$$

$$-\lambda^3 + 27\lambda^2 - 243\lambda + 729 = 0$$

$$(\lambda-9)^3 = 0$$

$$\lambda = 9$$

$$\begin{bmatrix} 4 & 5 & 2 \\ 2 & -2 & -8 \\ 5 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -4 \\ 0 & 0 & 0 \\ 0 & 9 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$