	Notation/Formula	Comments
C1: Sample Variance	$s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{S_{xx}}{n-1}$	Because of df, degrees of freedom
Sample standard deviation	$s = \sqrt{s^2}$	
Population variance	$\sigma^2 = \sum_{i=1}^{N}(x_i - \mu)^2/N$	
Population variance	$S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$	Easier
C2: Permutations	$P_{k,n} = \frac{n!}{(n-k)!}$	Ordered subset
Combination	$\binom{n}{k} = \frac{P_{k,n}}{k!} = \frac{n!}{k!(n-k)!}$	Unordered subset
Conditional Probability	$P(A\mid B) = \frac{P(A \cap B)}{P(B)}$	
Bayes' Theorem	$P(A_j\mid B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B\mid A_j)P(A_j)}{\sum_{i=1}^{k} P(B\mid A_i) \cdot P(A_i)} \quad j = 1, \ldots, k$	**Prior probabilities -> Posterior probability**
C3: CDF	$P(a \leq X \leq b) = F(b) - F(a-)$	

Expected Value	$E(X) = \mu_X = \sum_{x \in D} x \cdot p(x)$	
	$E[h(X)] = \sum_D h(x) \cdot p(x)$	
	$E(aX + b) = a \cdot E(X) + b$	Special: b = 0 or a = 1
	$E(aX) = a \cdot E(X)$, $E(X + b) = E(X) + b$	Special: b = 0 or a = 1
Variance	$V(X) = \sum_D (x - \mu)^2 \cdot p(x) = E[(X - \mu)^2]$	Always positive
	$V(X) = \sigma^2 = \left[\sum_D x^2 \cdot p(x)\right] - \mu^2 = E(X^2) - [E(X)]^2$	Shortcut
	$V(aX + b) = \sigma^2_{aX+b} = a^2 \cdot \sigma^2_X$	
	$\sigma_X = \sqrt{\sigma^2_X}$	SD, standard deviation
	$\sigma_{aX+b} = \lvert a \rvert \cdot \sigma_x$	
	$\sigma_{aX} = \lvert a \rvert \cdot \sigma_X,\quad \sigma_{X+b} = \sigma_X$	Special: b = 0 or a = 1

Bernoulli	$p(x) = \begin{cases} p(1-p)^{x-1} & x = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$	Experiment result: 0 and 1
Binomial	$b(x; n, p) = \begin{cases} \binom{n}{x} p^x(1-p)^{n-x} & x = 0, 1, 2, \ldots, n \\ 0 & \text{otherwise} \end{cases}$	n is fixed 2 outcome, S and F independents p is constant Big N, small n (5%), w/o replacement => consider as binomial
	If $X \sim \text{Bin}(n, p)$, then $E(X) = np$, $V(X) = np(1-p) = npq$, and $\sigma_X = \sqrt{npq}$ (where $q = 1 - p$).	
Hypergeometric	$P(X = x) = h(x; n, M, N) = \dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$	N is finite S and F, with **M of successes, N-M of F** sample of n, without replacement
	$\max(0, n - N + M) \leq x \leq \min(n, M)$.	
	$E(X) = n \cdot \dfrac{M}{N} \qquad V(X) = \left(\dfrac{N-n}{N-1}\right) \cdot n \cdot \dfrac{M}{N} \cdot \left(1 - \dfrac{M}{N}\right)$	
	$E(X) = np$ $V(X) = \left(\dfrac{N-n}{N-1}\right) \cdot np(1-p)$	p = M / N

Negative	$nb(x; r, p) = \binom{x+r-1}{r-1} p^r (1-p)^x \quad x = 0, 1, 2, \ldots$	2 outcome, S and F independents p is constant x fail, until a total of r success
	$P(X \leq 10) = \sum_{x=0}^{10} nb(x; 5, .2)$	
	$E(X) = \frac{r(1-p)}{p} \qquad V(X) = \frac{r(1-p)}{p^2}$	
Geometric		nb(x; 1, p)
Poisson	$p(x; \mu) = \frac{e^{-\mu} \cdot \mu^x}{x!} \quad x = 0, 1, 2, 3, \ldots$	$\mu > 0$ binomial pff b(x; n, p), $n \to \infty, p \to 0$ $np \to \mu > 0$
	$b(x; n, p) \approx p(x; \mu)$	n is large and p is small
	$E(X) = V(X) = \mu.$	
	$P_k(t) = e^{-\alpha t} \cdot (\alpha t)^k / k! \quad \mu = \alpha t.$	Poisson process α: expected number during a unit time
C4: Uniform	$f(x; A, B) = \begin{cases} \frac{1}{B-A} & A \leq x \leq B \\ 0 & \text{otherwise} \end{cases}$	
	$P(a \leq X \leq b) = P(a < X < b) = P(a < X \leq b) = P(a \leq X < b)$	

	$F(x) = \begin{cases} 0 & x < A \\ \dfrac{x-A}{B-A} & A \le x < B \\ 1 & x \ge B \end{cases}$	
	$F'(x) = f(x)$	
Percentiles	$p = F(\eta(p)) = \int_{-\infty}^{\eta(p)} f(y)\,dy$	P147 Median: $0.5 = F(\tilde{\mu})$
Expected Value	$\mu_X = E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$	Mean value
	$E[h(X)] = \mu_{h(X)} = \int_{-\infty}^{\infty} h(x) \cdot f(x)\,dx$	
Variance	$\sigma_X^2 = V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)dx = E[(X-\mu)^2]$	
	$V(X) = E(X^2) - [E(X)]^2$	

Normal Distribution	$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)} \quad -\infty < x < \infty$	$X \sim N(\mu, \sigma^2)$. σ: inflection points
	$f(z; 0, 1) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad -\infty < z < \infty$	Standard Normal Distribution CDF: $\Phi(z)$
	$Z = \frac{X - \mu}{\sigma}$ standardized variable is $(X - \mu)/\sigma$.	1 SD: 68% 2 SD: 95% 3 SD: 99.7%
	$P(X \leq a) = \Phi\left(\frac{a - \mu}{\sigma}\right)$	
	$P(X \leq x) = B(x, n, p) \approx \left(\begin{array}{c}\text{area under the normal curve} \\ \text{to the left of } x + .5\end{array}\right) = \Phi\left(\frac{x + .5 - np}{\sqrt{npq}}\right)$	Binomial rv based on n trails with p $np \geq 10$, and $nq \geq 10$ Prefer BINOM.DIST If computer can not handle then use this estimation
Exponential	$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad E(X) = \int_0^\infty x\lambda e^{-\lambda x}\, dx$	Poisson: α the rate of event per 1 unit time exponential $\lambda = \alpha$
	$F(x; \lambda) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\lambda x} & x \geq 0 \end{cases}$	
	$\mu = \frac{1}{\lambda} \quad \sigma^2 = \frac{1}{\lambda^2}$	

Gamma Function	$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\,dx$	
	1. For any $\alpha > 1$, $\Gamma(\alpha) = (\alpha - 1)\cdot\Gamma(\alpha - 1)$ 2. For any positive integer, n, $\Gamma(n) = (n-1)!$ 3. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$	
Gamma Distribution	$f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^\alpha\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$	$\alpha \leq 1$: Strictly decrease $\alpha > 1$: : Raise and then decrease β is scale parameter
	$F(x;\alpha) = \int_0^x \frac{y^{\alpha-1}e^{-y}}{\Gamma(\alpha)}\,dy \qquad x > 0$	
	$P(X \leq x) = F(x;\alpha,\beta) = F\left(\frac{x}{\beta};\alpha\right)$	
	$E(X) = \mu = \alpha\beta \qquad V(X) = \sigma^2 = \alpha\beta^2$	
Chi-Squared		P170
Weibull	$f(x;\alpha,\beta) = \begin{cases} \frac{\alpha}{\beta^\alpha}x^{\alpha-1}e^{-(x/\beta)^\alpha} & x \geq 0 \\ 0 & x < 0 \end{cases}$	$(\alpha > 0, \beta > 0)$ $\alpha = 1$: Exponential distribution, $\lambda = 1/\beta$
	$\mu = \beta\Gamma\left(1 + \frac{1}{\alpha}\right) \qquad \sigma^2 = \beta^2\left\{\Gamma\left(1 + \frac{2}{\alpha}\right) - \left[\Gamma\left(1 + \frac{1}{\alpha}\right)\right]^2\right\}$	

	$$F(x;\alpha,\beta)=\begin{cases}0 & x<0\\ 1-e^{-(x/\beta)^{\alpha}} & x\ge 0\end{cases}$$	
Lognormal	$$f(x;\mu,\sigma)=\begin{cases}\frac{1}{\sqrt{2\pi}\sigma x}e^{-[\ln(x)-\mu]^2/(2\sigma^2)} & x\ge 0\\ 0 & x<0\end{cases}$$	$Y=ln(X)$ $\mu\sigma$ are on $ln(X)$
	$$F(x;\mu,\sigma)=P(X\le x)=P[\ln(X)\le \ln(x)] = P\left(Z\le\frac{\ln(x)-\mu}{\sigma}\right)=\Phi\left(\frac{\ln(x)-\mu}{\sigma}\right)\quad x\ge 0$$	
	$$E(X)=e^{\mu+\sigma^2/2}\qquad V(X)=e^{2\mu+\sigma^2}\cdot(e^{\sigma^2}-1)$$	
Beta	$$f(x;\alpha,\beta,A,B)=\begin{cases}\frac{1}{B-A}\cdot\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)}\left(\frac{x-A}{B-A}\right)^{\alpha-1}\left(\frac{B-x}{B-A}\right)^{\beta-1} & A\le x\le B\\ 0 & \text{otherwise}\end{cases}$$	
	$$\mu=A+(B-A)\cdot\frac{\alpha}{\alpha+\beta}\qquad \sigma^2=\frac{(B-A)^2\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$	
C5: Marginal	$$p_X(x)=\sum_{y:\,p(x,y)>0}p(x,y)\qquad f_X(x)=\int_{-\infty}^{\infty}f(x,y)\,dy$$	
Multinominal	$$p(x_1,\ldots,x_r)=\begin{cases}\frac{n!}{(x_1!)(x_2!)\cdot\cdots\cdot(x_r!)}p_1^{x_1}\cdot\cdots\cdot p_r^{x_r} & x_i=0,1,2,\ldots,\text{ with } x_1+\cdots+x_r=n\\ 0 & \text{otherwise}\end{cases}$$	

Expected Value	$E[h(X,Y)] = \begin{cases} \sum_x \sum_y h(x,y) \cdot p(x,y) & \text{if } X \text{ and } Y \text{ are discrete} \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x,y) \cdot f(x,y)\, dx\, dy & \text{if } X \text{ and } Y \text{ are continuous} \end{cases}$	
Covariance	$\mathrm{Cov}(X,Y) = E[(X-\mu_X)(Y-\mu_Y)]$ $= \begin{cases} \sum_x \sum_y (x-\mu_X)(y-\mu_Y)p(x,y) & X, Y \text{ discrete} \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x-\mu_X)(y-\mu_Y)f(x,y)\, dx\, dy & X, Y \text{ continuous} \end{cases}$	$\mathrm{Cov}(X,X) = E[(X-\mu_X)^2] = V(X).$
	$\mathrm{Cov}(X,Y) = E(XY) - \mu_X \cdot \mu_Y$	
Correlation	$\rho_{X,Y} = \dfrac{\mathrm{Cov}(X,Y)}{\sigma_X \cdot \sigma_Y}$	Standardized covariance $-1 \le \mathrm{Corr}(X,Y) \le 1.$
	$\mathrm{Corr}(aX+b, cY+d) = \mathrm{Corr}(X,Y)$	ρ=0 does not imply independence. Maybe non-linear relation
Distribution of sample mean	$E(\overline{X}) = \mu_{\overline{X}} = \mu$ $V(\overline{X}) = \sigma^2_{\overline{X}} = \sigma^2/n$ and $\sigma_{\overline{X}} = \sigma/\sqrt{n}$ $E(T_o) = n\mu,$ $V(T_o) = n\sigma^2,$ and $\sigma_{T_o} = \sqrt{n}\sigma.$	**Central Limit Theorem, CLT** **N > 30** $np \ge 10, nq \ge 10$

Linear combination	$E(a_1X_1 + a_2X_2 + \cdots + a_nX_n) = a_1E(X_1) + a_2E(X_2) + \cdots + a_nE(X_n)$ $= a_1\mu_1 + \cdots + a_n\mu_n$ (5.8) $V(a_1X_1 + a_2X_2 + \cdots + a_nX_n) = a_1^2V(X_1) + a_2^2V(X_2) + \cdots + a_n^2V(X_n)$ $= a_1^2\sigma_1^2 + \cdots + a_n^2\sigma_n^2$ (5.9) $\sigma_{a_1X_1+\cdots+a_nX_n} = \sqrt{a_1^2\sigma_1^2 + \cdots + a_n^2\sigma_n^2}$	Independent
	$V(a_1X_1 + \cdots + a_nX_n) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_ia_j\text{Cov}(X_i, X_j)$	For any X
2 RV	$E(X_1 - X_2) = E(X_1) - E(X_2)$ for any two rv's X_1 and X_2. $V(X_1 - X_2) = V(X_1) + V(X_2)$ if X_1 and X_2 are independent rv's.	
C6 - Point Estimation		Good guess of the true value of the parameter
	expected or *mean square error* $\text{MSE} = E[(\hat{\theta} - \theta)^2]$.	Typically no possible to fin estimator with smallest MSE
Unbiased Estimator	$E(\hat{\theta}) = \theta$	Prefer
Bias	$E(\hat{\theta}) - \theta$	
	$\hat{\sigma}^2 = S^2 = \dfrac{\sum(X_i - \bar{X})^2}{n-1}$	Unbiased
MVUE	Minimum variance unbiased estimator	
Binomial rv	$E(\hat{p}) = E\left(\dfrac{X}{n}\right) = \dfrac{1}{n}E(X) = \dfrac{1}{n}(np) = p$	

Uniform	$\hat{\theta}_2 = \frac{n+1}{n} \cdot \max(X_1, \ldots, X_n)$	
Normal Distribution	$\hat{\mu} = \overline{X}$ is the MVUE for μ.	
The method of Moments	$E(X) = \mu$ $\Sigma X_i / n = \overline{X}$.	kth population moment, solve equation
	$m = 2$, $E(X)$ and $E(X^2)$ will be functions of θ_1 and θ_2.	$E(X^2) = V(X) + [E(X)]^2$
Maximum Likelihood Estimation	Normal distribution: $f(x_1, \ldots, x_n; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_1-\mu)^2/(2\sigma^2)} \cdot \ldots \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_n-\mu)^2/(2\sigma^2)} = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} e^{-\Sigma(x_i-\mu)^2/(2\sigma^2)}$ $\ln[f(x_1, \ldots, x_n; \mu, \sigma^2)] = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\Sigma(x_i - \mu)^2$ $\hat{\mu} = \overline{X}$ $\hat{\sigma}^2 = \frac{\Sigma(X_i - \overline{X})^2}{n}$	i.i.d if n is large, MLE approximately unbiased, or approximately the MVUE
	$\alpha = \left[\frac{\sum x_i^\alpha \cdot \ln(x_i)}{\sum x_i^\alpha} - \frac{\sum \ln(x_i)}{n}\right]^{-1}$ $\beta = \left(\frac{\sum x_i^\alpha}{n}\right)^{1/\alpha}$	Weibull , solved by iterative numerical procedure
The invariance Principle	The MLE of any function … is the function of the MLE's	