Chapter 9 Vector Differential Calculus. Grad, Div, Curl

P360 - Problem set 9.1

Problem 1-5

PS 9.1

1–5 COMPONENTS AND LENGTH

Find the components of the vector **v** with initial point P and terminal point Q. Find $|\mathbf{v}|$. Sketch $|\mathbf{v}|$. Find the unit vector **u** in the direction of **v**.

1. P: (1, 1, 0), Q: (6, 2, 0)

2. P: (1, 1, 1), Q: (2, 2, 0)

3. P: (−3.0, 4,0, −0.5), Q: (5.5, 0, 1.2)

4. P: (1, 4, 2), Q: (−1, −4, −2)

5. P: (0, 0, 0), Q: (2, 1, −2)

1. $\vec{V} = [5, 1, 0]$

$|V| = \sqrt{26}$

$\vec{u} = [\frac{5\sqrt{26}}{26}, \frac{\sqrt{26}}{26}, 0]$

2. $\vec{V} = [1, 1, -1]$

$|V| = \sqrt{3}$

$\vec{u} = [\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}]$

3. $\vec{V} = [8.5, -4, 1.7]$

$|V| = \sqrt{72.25 + 16 + 2.89}$

$= \sqrt{91.14}$

$\vec{u} = [\frac{8.5}{\sqrt{91.14}}, \frac{-4}{\sqrt{91.14}}, \frac{1.7}{\sqrt{91.14}}]$

4) $\vec{V} = [-2, -8, -4]$

$|V| = \sqrt{4 + 64 + 16}$

$= \sqrt{84}$

$\vec{u} = [-\frac{1}{\sqrt{21}}, -\frac{4}{\sqrt{21}}, -\frac{2}{\sqrt{21}}]$

5) $\vec{V} = [2, 1, -2]$

$|V| = \sqrt{4+1+4} = 3$

$\vec{u} = [\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}]$

6. Q=[4, 2, 13], |v|=4

7. Q=[4, 0. 1/2], |v|=$\sqrt{149}/4$

8. Q=[13.1, 0.8, -2.0], |v|=$\sqrt{171.61+0.64+4}$=$\sqrt{176.25}$
9. Q=[0, 0, 0], |v|=$\sqrt{53}$
10. Q=[0, 0, 0], |v|=$3\sqrt{2}$
11. 2a=[6, 4, 0], 1/2a=[3/2, 1, 0], -a = [-3, -2, 0]
12. (a+b)+c=a+(b+c)=[4, 7, 8] $b\ is\ not\ consistant.$
13. b+c=c+b = [1, 5, 8]
14. 3c-6d=3(c-2d)=[15, -3, 0]
15. 7(c-b)=7c-7b=7*[9, -7, 8]=[63, -49, 56]
16. $\frac{9}{2}a-3c$=$9(\frac{1}{2}a-\frac{1}{3}c)$=[-3/2, 12, -24]
17. (7-3)a=7a-3a=4a=[12, 8, 0]
18. 4a+3b = [0, 26, 0], -4a-3b=-(4a+3b)=[0, -26, 0]
19. 12-associative, 13-commutative, 14-16 scalar multiplication is distributive.
20. $a+b=[a_1,a_2,...,a_n]+[b_1,b_2,...,b_n]=[a_1+b_1,a_2+b_2,...,a_n+b_n]=[b_1+a_1,b_2+a_2,...,b_n+a_n]=b+a$

    The rest can be approved in a simliar way.

6–10 **Find the terminal point Q of the vector $\mathbf{v}$ with** components as given and initial point P. Find $|\mathbf{v}|$.

6. $4, 0, 0$; P: $(0, 2, 13)$

7. $\frac{1}{2}, 3, -\frac{1}{4}$; P: $(\frac{7}{2}, -3, \frac{3}{4})$

8. $13.1, 0.8, -2.0$; P: $(0, 0, 0)$

9. $6, 1, -4$; P: $(-6, -1, -4)$

10. $0, -3, 3$; P: $(0, 3, -3)$

Latex

Mistake

11–18 **ADDITION, SCALAR MULTIPLICATION**

Let $\mathbf{a} = [3, 2, 0] = 3\mathbf{i} + 2\mathbf{j}$; $\mathbf{b} = [-4, 6, 0] = 4\mathbf{i} + 6\mathbf{j}$, $\mathbf{c} = [5, -1, 8] = 5\mathbf{i} - \mathbf{j} + 8\mathbf{k}$, $\mathbf{d} = [0, 0, 4] = 4\mathbf{k}$.

Find:

21–25 FORCES, RESULTANT

Find the resultant in terms of components and its magnitude.

21. $\mathbf{p} = [2, 3, 0]$, $\mathbf{q} = [0, 6, 1]$, $\mathbf{u} = [2, 0, -4]$

22. $\mathbf{p} = [1, -2, 3]$, $\mathbf{q} = [3, 21, -16]$, $\mathbf{u} = [-4, -19, 13]$

23. $\mathbf{u} = [8, -1, 0]$, $\mathbf{v} = [\frac{1}{2}, 0, \frac{4}{3}]$, $\mathbf{w} = [-\frac{17}{2}, 1, \frac{11}{3}]$

24. $\mathbf{p} = [-1, 2, -3]$, $\mathbf{q} = [1, 1, 1]$, $\mathbf{u} = [1, -2, 2]$

25. $\mathbf{u} = [3, 1, -6]$, $\mathbf{v} = [0, 2, 5]$, $\mathbf{w} = [3, -1, -13]$

21) $\vec{r} = \vec{p} + \vec{q} + \vec{u}$
$= [4, 9, -3]$
$|\vec{r}| = \sqrt{16+81+9} = \sqrt{106}$

22) $\vec{r} = \vec{p} + \vec{q} + \vec{u}$
$= [0, 0, 0]$
$|\vec{r}| = 0$

23) $\vec{r} = \vec{u} + \vec{v} + \vec{w}$
$= [0, 0, 5]$
$|\vec{r}| = 5$

24) $\vec{r} = \vec{p} + \vec{q} + \vec{u}$
$= [1, 1, 0]$
$|\vec{r}| = \sqrt{2}$

25) $\vec{r} = \vec{u} + \vec{v} + \vec{w}$
$= [6, 2, -14]$
$|\vec{r}| = \sqrt{36+4+196} = \sqrt{236}$

26–37 FORCES, VELOCITIES

26. Equilibrium. Find **v** such that **p**, **q**, **u** in Prob. 21 and **v** are in equilibrium.

27. Find **p** such that **u**, **v**, **w** in Prob. 23 and **p** are in equilibrium.

28. Unit vector. Find the unit vector in the direction of the resultant in Prob. 24.

26) $\vec{v} = 0 - \vec{r} = [-4, -9, 3]$

27) $\vec{p} = 0 - \vec{r} = [0, 0, -5]$

28) $\vec{u} = \frac{\vec{r}}{|\vec{r}|} = [\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0]$

29. Restricted resultant. Find all **v** such that the resultant of **v**, **p**, **q**, **u** with **p**, **q**, **u** as in Prob. 21 is parallel to the *xy*-plane.

30. Find **v** such that the resultant of **p**, **q**, **u**, **v** with **p**, **q**, **u** as in Prob. 24 has no components in *x*- and *y*-directions.

31. For what *k* is the resultant of [2, 0, −7], [1, 2, −3], and [0, 3, *k*] parallel to the *xy*-plane?

32. If $|\mathbf{p}| = 6$ and $|\mathbf{q}| = 4$, what can you say about the magnitude and direction of the resultant? Can you think of an application to robotics?

33. Same question as in Prob. 32 if $|\mathbf{p}| = 9$, $|\mathbf{q}| = 6$, $|\mathbf{u}| = 3$.

29 Vector start from origin. parallel to x-y means no component in z-direction.

Let result

$\vec{z} = \vec{v} + \vec{p} + \vec{q} + \vec{u} = [x, y, 0]$

So x and y are arbitrary

$\vec{v} = \vec{z} - [4, 9, -3]$

$= [x-4, y-9, 3]$

Let $v_1 = x-4$, $v_2 = y-9$.

So $\vec{v} = [v_1, v_2, 3]$

where v_1 and v_2 are arbitrary

30. $\vec{v} = [0, 0, c] - [1, 1, 0]$

$= [-1, -1, c]$ where c is a constant.

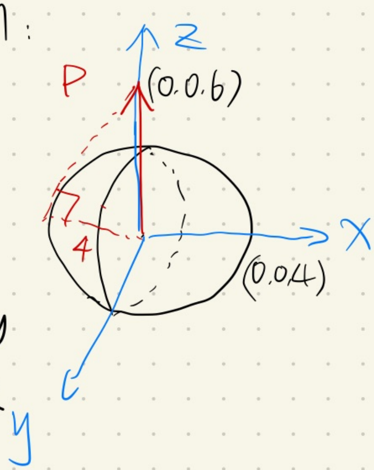
31 $-7-3+k=0$

$k=10$

32. magnitude: min=2, max=10

direction:

A sphere with R=10 and with a r=2 empty sphere inside



Robotic: Vision, the area where Robot can see/detect or scan?

33. magnitude: min=0, max=18

direction

enhanced ice cream without blind spot?

A solid sphere with R=18, No blind point.

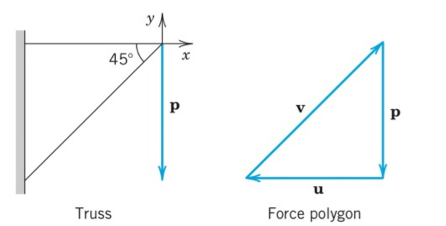
$|p+q+u| \leq 18$

34. Relative velocity. If airplanes A and B are moving southwest with speed $|\mathbf{v}_A| = 550$ mph, and northwest with speed $|\mathbf{v}_B| = 450$ mph, respectively, what is the relative velocity $\mathbf{v} = \mathbf{v}_B - \mathbf{v}_A$ of B with respect to A?

35. Same question as in Prob. 34 for two ships moving northeast with speed $|\mathbf{v}_A| = 22$ knots and west with speed $|\mathbf{v}_B| = 19$ knots.

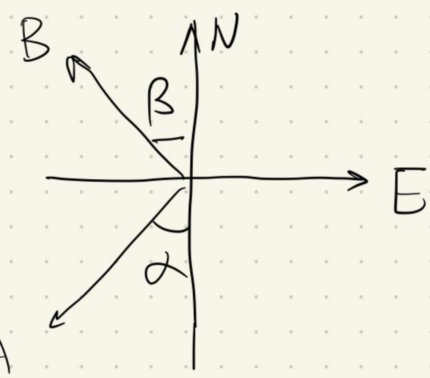
36. Reflection. If a ray of light is reflected once in each of two mutually perpendicular mirrors, what can you say about the reflected ray?

37. Force polygon. Truss. Find the forces in the system of two rods (*truss*) in the figure, where $|\mathbf{p}| = 1000$ nt. *Hint.* Forces in equilibrium form a polygon, the *force polygon*.



Problem 37

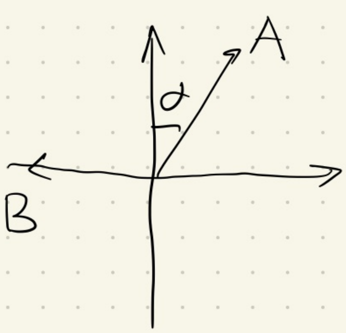
34.



$V = V_B - V_A$ on northeast.

$[550\sin\alpha - 450\sin\beta,$
$550\cos\alpha - 450\cos\beta]$

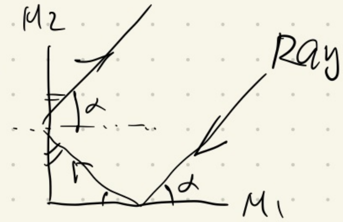
35



$V = V_B - V_A$ on northeast

$[22\sin\alpha - 19 \quad 22\cos\alpha]$

36.



Reverse the direction.

37. $P: [0, -1000]$

$u: [-1000, 0]$

$V: [1000, 1000]$

34. 35: it seems assume α β are 45°

maybe this is by default

38. TEAM PROJECT. Geometric Applications. To increase your skill in dealing with vectors, use vectors to prove the following (see the figures).

(a) The diagonals of a parallelogram bisect each other.

(b) The line through the midpoints of adjacent sides of a parallelogram bisects one of the diagonals in the ratio 1 : 3.

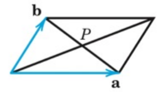
(c) Obtain (b) from (a).

(d) The three medians of a triangle (the segments from a vertex to the midpoint of the opposite side) meet at a single point, which divides the medians in the ratio 2 : 1.

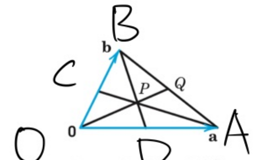
(e) The quadrilateral whose vertices are the midpoints of the sides of an arbitrary quadrilateral is a parallelogram.

(f) The four space diagonals of a parallelepiped meet and bisect each other.

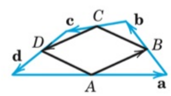
(g) The sum of the vectors drawn from the center of a regular polygon to its vertices is the zero vector.



Team Project 38(a)



Team Project 38(d)



Team Project 38(e)

$\vec{OC} = \frac{1}{2}b$

$\vec{OD} = \frac{1}{2}a$

$\vec{DC} = \frac{1}{2}\vec{AB}$

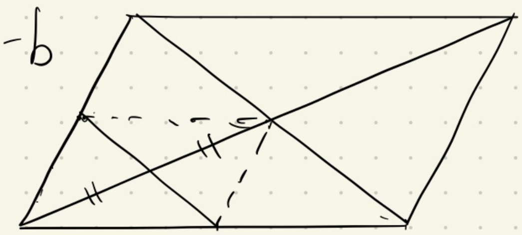
$\triangle CDP \sim \triangle ABP$

$\vec{DP} = \frac{1}{2}\vec{PB}$

38-a $\frac{1}{2}a + \frac{1}{2}b = \frac{1}{2}(a+b)$

$\frac{1}{2}a - \frac{1}{2}b = \frac{1}{2}(a-b)$

38-b



$\frac{1}{4}a - \frac{1}{4}b = \frac{1}{4}(a-b)$

$\frac{1}{4} : \frac{3}{4} = 1:3$

38-c: Obvious.

38-d. $\vec{AB} = a - b$

$\vec{AQ} = \frac{1}{2}(a-b)$

$\vec{QO} = a + \frac{1}{2}(a-b)$

38-e $a - d = c - b$

$\frac{1}{2}(a-d) = \frac{1}{2}(c-b)$

$\vec{AD} = \vec{BC}$

same $\Rightarrow$ $\vec{AB} = \vec{DC}$

38-f.

38-g.

P367 - Problem set 9.2

PS 9-2

1–10 **INNER PRODUCT**

Let $\mathbf{a} = [1, -3, 5]$, $\mathbf{b} = [4, 0, 8]$, $\mathbf{c} = [-2, 9, 1]$. Find:

1. $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{b} \cdot \mathbf{a}$, $\mathbf{b} \cdot \mathbf{c}$
2. $(-3\mathbf{a} + 5\mathbf{c}) \cdot \mathbf{b}$, $15(\mathbf{a} - \mathbf{c}) \cdot \mathbf{b}$
3. $|\mathbf{a}|$, $|2\mathbf{b}|$, $|-\mathbf{c}|$
4. $|\mathbf{a} + \mathbf{b}|$, $|\mathbf{a}| + |\mathbf{b}|$
5. $|\mathbf{b} + \mathbf{c}|$, $|\mathbf{b}| + |\mathbf{c}|$
6. $|\mathbf{a} + \mathbf{c}|^2 + |\mathbf{a} - \mathbf{c}|^2 - 2(|\mathbf{a}|^2 + |\mathbf{c}|^2)$
7. $|\mathbf{a} \cdot \mathbf{c}|$, $|\mathbf{a}||\mathbf{c}|$
8. $5\mathbf{a} \cdot 13\mathbf{b}$, $65\mathbf{a} \cdot \mathbf{b}$
9. $15\mathbf{a} \cdot \mathbf{b} + 15\mathbf{a} \cdot \mathbf{c}$, $15\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$
10. $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c})$, $(\mathbf{a} - \mathbf{b}) \cdot \mathbf{c}$

1. $a \cdot b = 4+0+40 = 44$

$b \cdot a = 44$

$b \cdot c = -8+0+8 = 0$

2. $(-3a+5c) \cdot b = (-13, 54, -10) \cdot b$

$= -52+0-80 = -132$

$15(a-c) \cdot b = 15(-3,-12,4) \cdot b$

$= 15 \cdot (-12+0+32) = 300$

3. $|a| = \sqrt{1+9+25} = \sqrt{35}$

$|2b| = 2 \cdot \sqrt{16+64} = 8\sqrt{5}$

$|-c| = \sqrt{4+81+1} = \sqrt{86}$

4. $|a+b| = \sqrt{25+9+169} = \sqrt{203}$

$|a|+|b| = \sqrt{35} + \sqrt{16+64} = \sqrt{35}+4\sqrt{5}$

5. $|b+c| = \sqrt{2^2+9^2+9^2} = \sqrt{166}$

$|b|+|c| = 4\sqrt{5}+\sqrt{86}$

6. $(1+36+36)+(9+144+16)$

$-2(35+86) = 0$

7. $|a \cdot c| = |-2-27+5| = 24$

$|a||c| = \sqrt{35} \cdot \sqrt{86}$

8. $5a \cdot 13b = 65a \cdot b = 2860$

9. $15a \cdot b + 15a \cdot c = 15a(b+c)$

$= 15 \cdot a \cdot (2,9,9) = 300$

10. $a \cdot (b-c) = a \cdot (6,-9,7)$

$= 68$

$(a-b) \cdot c = (-3,-3,-3)c$

$= 6-27-3 = -24$

11–16 GENERAL PROBLEMS

11. What laws do Probs. 1 and 4–7 illustrate?

12. What does $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$ imply if $\mathbf{u} = \mathbf{0}$? If $\mathbf{u} \neq \mathbf{0}$?

13. Prove the Cauchy–Schwarz inequality.

14. Verify the Cauchy–Schwarz and triangle inequalities for the above $\mathbf{a}$ and $\mathbf{b}$.

15. Prove the parallelogram equality. Explain its name.

16. **Triangle inequality.** Prove Eq. (7). *Hint.* Use Eq. (3) for $|\mathbf{a} + \mathbf{b}|$ and Eq. (6) to prove the square of Eq. (7), then take roots.

11-1 $a \cdot b = b \cdot a$

$ab = 0$. orthogonality

4-5) $|\mathbf{a} + \mathbf{b}| \leqq |\mathbf{a}| + |\mathbf{b}|$ **(Triangle inequality)**.

6) $|\mathbf{a} + \mathbf{b}|^2 + |\mathbf{a} - \mathbf{b}|^2 = 2(|\mathbf{a}|^2 + |\mathbf{b}|^2)$ (*Parallelogram equality*).

7) $|\mathbf{a} \cdot \mathbf{b}| \leqq |\mathbf{a}||\mathbf{b}|$ (*Cauchy–Schwarz inequality*).

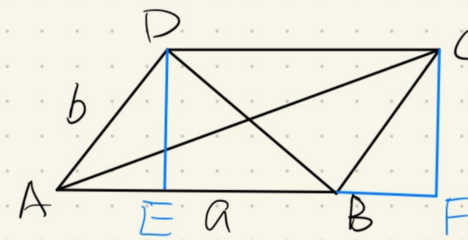
12. if $u=0$, We know nothing about $u.w$

if $u \neq 0$, $p = \frac{a \cdot b}{|b|}$. So $v.w$ belongs to a set B that the projections of u on the vector muliple the magnitude of the vector is a constant

13. projection ≤ itself. $(\cos\theta \leq 1)$

14. 4-5, and 7

15



1) $AE = BF$, $DE = CF$.

2) $AC^2 + BD^2 = (a + BF)^2 + CF^2$

$+ (a - AE)^2 + DE^2$

$= 2(a^2 + b^2)$

16. $|a+b|^2 = (a+b)(a+b)$

$= a^2 + b^2 + 2ab$

$\leq |a|^2 + |b|^2 + 2|a||b|$

$= (|a| + |b|)^2$

17–20 WORK Mistake

Find the work done by a force **p** acting on a body if the body is displaced along the straight segment $\overline{AB}$ from A to B. Sketch $\overline{AB}$ and **p**. Show the details.

17. $\mathbf{p} = [2, 5, 0]$, $A: (1, 3, 3)$, $B: (3, 5, 5)$

18. $\mathbf{p} = [-1, -2, 4]$, $A: (0, 0, 0)$, $B: (6, 7, 5)$

19. $\mathbf{p} = [0, 4, 3]$, $A: (4, 5, -1)$, $B: (1, 3, 0)$

20. $\mathbf{p} = [6, -3, -3]$, $A: (1, 5, 2)$, $B: (3, 4, 1)$

21. Resultant. Is the work done by the resultant of two forces in a displacement the sum of the work done by each of the forces separately? Give proof or counterexample.

17. $W = P \cdot d = P \cdot (2, 2, 2) = 14$

18 $W = p \cdot (6, 7, 5) = 0$

19. $W = p \cdot (-3, -2, 1) = -5$

negative, because of direction.

20. $W = p \cdot (2, -1, -1) = 18$

21. Yes.

$$W = P_1 \cdot d + P_2 \cdot d$$
$$= (P_1 + P_2) \cdot d$$

22–30 ANGLE BETWEEN VECTORS

Let $\mathbf{a} = [1, 1, 0]$, $\mathbf{b} = [3, 2, 1]$, and $\mathbf{c} = [1, 0, 2]$. Find the angle between:

22. **a**, **b**

23. **b**, **c**

24. **a** + **c**, **b** + **c**

22. $\cos\alpha = \frac{ab}{|a||b|} = \frac{3+2}{\sqrt{2}\sqrt{14}}$

$\alpha = \arccos\left(\frac{5\sqrt{7}}{14}\right)$

23. $\cos\alpha = \frac{bc}{|b||c|} = \frac{3+2}{\sqrt{14}\sqrt{5}}$

$\alpha = \arccos\left(\frac{\sqrt{70}}{14}\right)$

24. $\cos\alpha = \frac{(a+c)(b+c)}{|a+c| \cdot |b+c|}$

$= \frac{(4,3,1)(4,2,3)}{\sqrt{16+9+1} \cdot \sqrt{16+4+9}}$

$= \frac{16+6+3}{\sqrt{26} \cdot \sqrt{29}} = \frac{25}{\sqrt{26}\sqrt{29}}$

$\alpha = \arccos\left(\frac{25}{\sqrt{26} \cdot \sqrt{29}}\right)$

25. What will happen to the angle in Prob. 24 if we replace **c** by n**c** with larger and larger n?

26. Cosine law. Deduce the law of cosines by using vectors **a**, **b**, and **a** − **b**.

27. Addition law. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$. Obtain this by using $\mathbf{a} = [\cos\alpha, \sin\alpha]$, $\mathbf{b} = [\cos\beta, \sin\beta]$ where $0 \leqq \alpha \leqq \beta \leqq 2\pi$.

28. Triangle. Find the angles of the triangle with vertices A: (0, 0, 2), B: (3, 0, 2), and C: (1, 1, 1). Sketch the triangle.

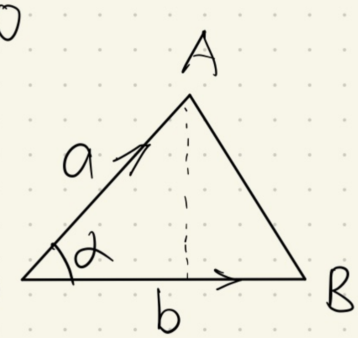
29. Parallelogram. Find the angles if the vertices are (0, 0), (6, 0), (8, 3), and (2, 3).

30. Distance. Find the distance of the point A: (1, 0, 2) from the plane P: $3x + y + z = 9$. Make a sketch.

25. $\lim_{n\to\infty} \frac{(a+nc)(b+nc)}{|a+nc|\cdot|b+nc|} = \frac{c\cdot c}{|c|^2} = 1$

$\alpha \to 0$

26.



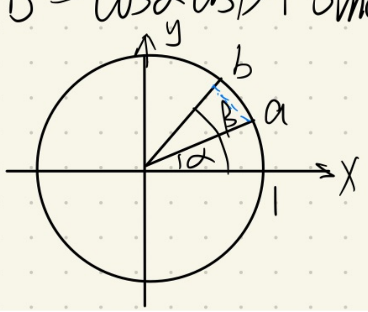
$(\overrightarrow{BA})^2 = (a-b)^2 =$

$(a\sin\alpha)^2 + (b - a\cos\alpha)^2$

$= a^2\sin^2\alpha + b^2 + a^2\cos^2\alpha - 2ab\cos\alpha$

$= a^2 + b^2 - 2ab\cos\alpha$.

27. $a\cdot b = \cos\alpha\cos\beta + \sin\alpha\sin\beta$



the projection of a on b is

$p = a\cdot b = \cos(\beta-\alpha) = \cos(\alpha-\beta)$

28. $\overrightarrow{AB} = (3,0,0) = a$

$\overrightarrow{BC} = (-2,1,-1) = b$

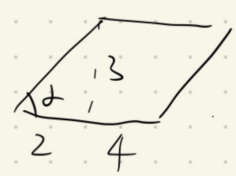
$\overrightarrow{CA} = (-1,-1,1) = c$

$\cos\alpha = \frac{-ab}{|a||b|} = \frac{-6}{3\cdot\sqrt{6}} = \frac{\sqrt{6}}{3}$

$\cos\beta = \frac{-bc}{|b||c|} = \frac{2-1-1}{\sqrt{\ }\sqrt{\ }} = 0$

$\cos\gamma = \frac{-ca}{|c||a|} = \frac{+3}{3\cdot\sqrt{3}} = \frac{\sqrt{3}}{3}$

29. $\tan\alpha = \frac{3}{2}$ $\alpha = \arctan\frac{3}{2}$



$\cos\alpha = \frac{(2.3)(6.0)}{\sqrt{4+9}\cdot 6}$

$= \frac{2}{\sqrt{13}}$

30. moping to $(x-1, y, z-2)$

$A = (0,0,0)$

$P = 3x - 3 + y + z - 2 = 9$

$3x + y + z = 14$

Per example 6,

$P = \frac{c}{|a|} = \frac{14}{\sqrt{11}}$

31–35 **ORTHOGONALITY** is particularly important, mainly because of orthogonal coordinates, such as ***Cartesian coordinates***, whose ***natural basis*** [Eq. (9), Sec. 9.1], consists of three orthogonal unit vectors.

31. For what values of a_1 are $[a_1, 4, 3]$ and $[3, -2, 12]$ orthogonal?

32. Planes. For what c are $3x + z = 5$ and $8x - y + cz = 9$ orthogonal?

33. Unit vectors. Find all unit vectors $\mathbf{a} = [a_1, a_2]$ in the plane orthogonal to $[4, 3]$.

34. Corner reflector. Find the angle between a light ray and its reflection in three orthogonal plane mirrors, known as *corner reflector*.

35. Parallelogram. When will the diagonals be orthogonal? Give a proof.

31. $3a_1 - 8 + 36 = 0$

$a_1 = -\frac{28}{3}$

32. $(3, 0, 1) \cdot (8, -1, c) = 0$

$24 + c = 0 \quad c = -24$

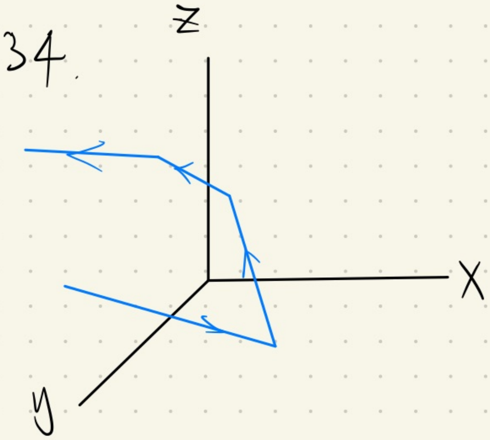
33. $4a_1 + 3a_2 = 0$

$a_2 = -\frac{4}{3}a_1$

$a_1^2 + \frac{16}{9}a_1^2 = 1$

$a_1 = \frac{3}{5}. \quad [\frac{3}{5}, -\frac{4}{5}]$

34.



$[a, b, c]$ Reflect on x-y plane.

$\Rightarrow [a, b, -c]$ Reflect on XZ plane

$\Rightarrow [a, -b, -c]$ R on ZY plane

$[-a, -b, -c]$

35. $(a+b) \cdot (a-b) = 0$

$a^2 - b^2 = 0 \quad a^2 = b^2$

diamond shape. answer is wrong.

36–40 **COMPONENT IN THE DIRECTION OF A VECTOR**

Find the component of **a** in the direction of **b**. Make a sketch.

36. $\mathbf{a} = [1, 1, 1]$, $\mathbf{b} = [2, 1, 3]$

37. $\mathbf{a} = [3, 4, 0]$, $\mathbf{b} = [4, -3, 2]$

38. $\mathbf{a} = [8, 2, 0]$, $\mathbf{b} = [-4, -1, 0]$

39. When will the component (the projection) of **a** in the direction of **b** be equal to the component (the projection) of **b** in the direction of **a**? First guess.

40. What happens to the component of **a** in the direction of **b** if you change the length of **b**?

36. $p = \frac{a \cdot b}{|b|} = \frac{2+1+3}{\sqrt{14}} = \frac{3\sqrt{14}}{7}$

37. $p = \frac{a \cdot b}{|b|} = \frac{12-12+0}{\sqrt{\ }} = 0$ perpendicular

38. $p = \frac{ab}{|b|} = \frac{-32-2}{\sqrt{17}} = -2\sqrt{17}$

On the opp direction.

39. $|a| = |b|$ or $ab = 0$

40. $p = \frac{a \cdot b}{|b|} = \frac{a \cdot kb}{k|b|}$

nothing change.

P374 - Problem set 9.3

PS 9.3

1–10 GENERAL PROBLEMS

1. Give the details of the proofs of Eqs. (4) and (5).
2. What does $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ with $\mathbf{a} \neq \mathbf{0}$ imply?
3. Give the details of the proofs of Eqs. (6) and (11).

1.

$$(4) \qquad (l\mathbf{a}) \times \mathbf{b} = l(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (l\mathbf{b}).$$

$$(2^{**}) \qquad \mathbf{v} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =$$

$la \times b$

$$\begin{vmatrix} i & j & k \\ la_1 & la_2 & la_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= (la_2b_3 - la_3b_2)i - (la_1b_3 - la_3b_1)j + (la_1b_2 - la_2b_1)k$$

$$= l\,[(a_2b_3 - a_3b_2)i - (a_1b_3 - a_3b_1)j + (a_1b_2 - a_2b_1)k$$

Extend the rest, Same.

$$(5) \qquad \begin{array}{ll} (\alpha) & \mathbf{a} \times (\mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{c}), \\ (\beta) & (\mathbf{a} + \mathbf{b}) \times \mathbf{c} = (\mathbf{a} \times \mathbf{c}) + (\mathbf{b} \times \mathbf{c}). \end{array}$$

$a \times (b+c)$

$$\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \end{vmatrix} = [a_2(b_3+c_3) - a_3(b_2+c_2)]\,i - [a_1(b_3+c_3) - a_3(b_1+c_1)]\,j + [a_1(b_2+b_3) - a_2(b_1+c_1)]\,k$$

$a \times b$

$$\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - a_3b_2)\,i - (a_1b_3 - a_3b_1)\,j + (a_1b_2 - a_2b_1)\,k$$

$a \times c$

$$\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \text{Referene above.}$$

$$a \times (b+c) = (a \times b) + (a \times c)$$

(β): Same

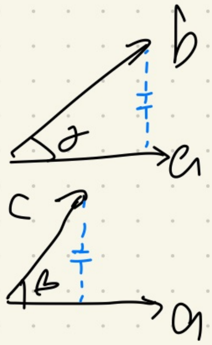
2. What does $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ with $\mathbf{a} \neq \mathbf{0}$ imply?

3. Give the details of the proofs of Eqs. (6) and (11).

② $|v| = |a \times b| = |a||b| \sin\alpha$

$= |a \times c| = |a||c| \sin\beta$

$|b| \sin\alpha = |c| \sin\beta$.



and the direction of the result is same.

① $a = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ x & y & z \end{vmatrix} =$

$(0-y)i - (z-x)j + yk$

$b = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $c = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \cdots$

② $a = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ x & y & z \end{vmatrix} = zi - xk$

$b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $c = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} \cdots$

③

(6) $\qquad \mathbf{b} \times \mathbf{a} = -(\mathbf{a} \times \mathbf{b})$

$\begin{vmatrix} i & j & k \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$ and $\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

exchange row, determinant $\times -1$

P295. Sec 7.7 Theorem 1.

(11) $\qquad (\mathbf{a} \;\; \mathbf{b} \;\; \mathbf{c}) = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}.$

$b \times c = \begin{vmatrix} i & j & k \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (b_2c_3 - b_3c_2)i - (b_1c_3 - b_3c_1)j + (b_1c_2 - b_2c_1)k$

$a \cdot (b \times c) = (a_1b_2c_3 - a_1b_3c_2) - (a_2b_1c_3 - a_2b_3c_1) + (a_3b_1c_2 - a_3b_2c_1)$

$(a \times b) \cdot c \Rightarrow a \to b,\ b \to c,\ c \to a$

$= c_1a_2b_3 - c_1a_3b_2 - (c_2a_1b_3 - c_2a_3b_1) + c_3a_1b_2 - c_3a_2b_1$

Geo perspective, same volume.

4. Lagrange's identity for $|\mathbf{a} \times \mathbf{b}|$. Verify it for $\mathbf{a} = [3, 4, 2]$ and $\mathbf{b} = [1, 0, 2]$. Prove it, using $\sin^2 \gamma = 1 - \cos^2 \gamma$. The identity is

$$(12) \qquad |\mathbf{a} \times \mathbf{b}| = \sqrt{(\mathbf{a} \cdot \mathbf{a})(\mathbf{b} \cdot \mathbf{b}) - (\mathbf{a} \cdot \mathbf{b})^2}.$$

4. $\{3,4,2\}$ cross $\{1,0,2\} = [8, -4, -4]$

$|a \times b| = \sqrt{64+16+16} = \sqrt{96}$

$\sqrt{(a \cdot a)(b \cdot b) - (a \cdot b)^2}$

$= \sqrt{(9+16+4)(1+4) - (3+4)^2}$

$= \sqrt{29 \times 5 - 49} = \sqrt{96}$

Prove: $|a \times b| = |a||b| \sin\alpha$

$= \sqrt{a \cdot a}\sqrt{b \cdot b}\ \sin\alpha$

$\cos\alpha = \dfrac{a \cdot b}{|a||b|} = \dfrac{a \cdot b}{\sqrt{a \cdot a}\sqrt{b \cdot b}}$

$|a \times b| = \sqrt{a \cdot a}\sqrt{b \cdot b}\sqrt{1 - \cos^2\alpha}$

$= \sqrt{a \cdot a}\sqrt{b \cdot b} \cdot \sqrt{1 - \dfrac{(a \cdot b)^2}{(a \cdot a)\, b \cdot b}}$

$= \sqrt{(a \cdot a)(b \cdot b) - (a \cdot b)^2}$

5. What happens in Example 3 of the text if you replace $\mathbf{p}$ by $-\mathbf{p}$?

m change the direction.

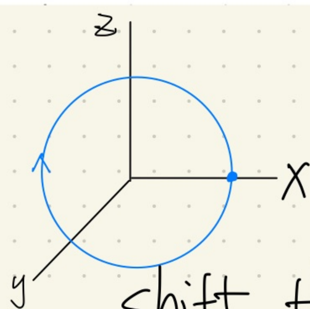
6. What happens in Example 5 if you choose a P at distance $2d$ from the axis of rotation?

W remains the same

$|r|\sin\alpha$ become doubled.

So v get doubled.

7. Rotation. A wheel is rotating about the y-axis with angular speed $\omega = 20\ \mathrm{sec}^{-1}$. The rotation appears clockwise if one looks from the origin in the positive y-direction. Find the velocity and speed at the point $[8, 6, 0]$. Make a sketch.



shift the center of wheel to $[0, 0, 0]$.

So $[8, 6, 0]$ equals to $[8, 0, 0]$

$S = 8 \times 20 = 160 \quad v = [0, 0, -160]$

$v = [0, 20, 0] \times [8, 6, 0]$

$= [0, 0, -160]$

Not quit understand