Chapter 13 - Complex Numbers and Functions. Complex Differentiation

Selected Problem set 13.1

13.1 1. 5. 11. 17

1. Powers of *i*. Show that $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, $i^5 = i, \cdots$ and $1/i = -i$, $1/i^2 = -1$, $1/i^3 = i, \cdots$.

$$z_1z_2 = (x_1, y_1)(x_2, y_2) = (x_1x_2 - y_1y_2, \quad x_1y_2 + x_2y_1).$$

$i^2 = (0,1)(0,1) = (0-1,\ 0+0) = (-1,0) = -1 + 0i = -1$

$i^3 = i^2 \cdot i = -1 \cdot i = (-1,0)(0,1) = (0-0,\ -1+0) = (0,-1) = -i$

$i^4 = (i^2)(i)^2 = -1 \cdot -1 = 1$

$i^5 = i^4 \cdot i = 1 \cdot i = i$

$i \cdot (-i) = -i^2 = 1 \Rightarrow 1/i = -i$.

$1/i^2 = 1/-1 = -1$.

$1/i^3 = 1/i^2 \cdot \frac{1}{i} = (-i)\cdot(-1) = i$.

5. Pure imaginary number. Show that $z = x + iy$ is pure imaginary if and only if $\bar{z} = -z$.

$\bar{z} = x - iy = -z = -x - iy$.

$x = -x \Rightarrow x = 0 \Rightarrow z = iy$

z is a pure imaginary number

8–15 COMPLEX ARITHMETIC

Let $z_1 = -2 + 11i$, $z_2 = 2 - i$. Showing the details of your work, find, in the form $x + iy$:

8. $z_1 z_2$, $\overline{(z_1 z_2)}$ **9.** $\text{Re}\,(z_1^2)$, $(\text{Re}\,z_1)^2$

10. $\text{Re}\,(1/z_2^2)$, $1/\text{Re}\,(z_2^2)$

11. $(z_1 - z_2)^2/16$, $(z_1/4 - z_2/4)^2$

11. $(z_1 - z_2)^2/16 = (-4+12i)^2/16 = (16-144-96i)/16$

$= (-128-96i)/16 = -8-6i$

$(z_1/4 - z_2/4)^2 = \left[\left(-\frac{1}{2}+\frac{11}{4}i\right) - \left(\frac{1}{2} - \frac{1}{4}i\right)\right]^2$

$= (-1+3i)^2 = 1-9-6i = -8-6i$

16–20 Let $z = x + iy$. Showing details, find, in terms of x and y:

16. $\text{Im}\,(1/z)$, $\text{Im}\,(1/z^2)$ **17.** $\text{Re}\,z^4 - (\text{Re}\,z^2)^2$

17. $z^2 = (x^2-y^2) + 2xy\,i$

$z^4 = [(x^2-y^2)^2 - 4x^2y^2] + 4xy(x^2-y^2)\,i$

$\text{Re}\,z^4 = (x^2-y^2)^2 - 4x^2y^2$

$(\text{Re}\,z^2)^2 = (x^2-y^2)^2$

$\text{Re}\,z^4 - (\text{Re}\,z^2)^2 = -4x^2y^2$