

# Chapter 15 - Power Series, Taylor Series

## Selected Problem set 15.1

15.1 1. 5, 9, 17, 19, 23, 25

bound.  
convergent  
limit

Convergent, hence bounded

### 1-10 SEQUENCES

Is the given sequence  $z_1, z_2, \dots, z_n, \dots$  bounded? Convergent? Find its limit points. Show your work in detail.

1.  $z_n = (1+i)^{2n}/2^n$       2.  $z_n = (3+4i)^n/n!$

$$1. \frac{z_{n+1}}{z_n} = \frac{(1+i)^{2(n+1)}}{2^{n+1}} \cdot \frac{2^n}{(1+i)^n} = \frac{(1+i)^2}{2} = \frac{1+i+2i}{2} = i$$

$$z_1 = \frac{(1+i)^2}{2} = i, \quad z_2 = -1, \quad z_3 = -i, \quad z_4 = 1, \quad z_5 = i,$$

bounded.

Theorem 7.  $\left| \frac{z_{n+1}}{z_n} \right| = |i| \geq 1$  diverges

5.  $z_n = (-1)^n + 10i$

$$z_1 = -1 + 10i, \quad z_2 = 1 + 10i, \quad z_3 = z_1, \quad z_4 = z_2, \dots$$

bounded.

for  $t = 1, 2, \dots$

$$\left| \frac{z_{2t+1}}{z_{2t}} \right| = \left| \frac{((-1)^{2t+1})^2}{(-1)^{2t+1} + 10i} \right| = \left| \frac{1 - 100 + 20i}{-100 - 1} \right| = \left| + \frac{99}{101} - \frac{20}{101}i \right| \geq 1$$

diverges.

$$9. z_n = (3 + 3i)^{-n}$$

$$|z| = \left| \frac{1}{6} (1-i) \right| = \frac{\sqrt{2}}{6}$$

$$|z_1| = \left| \frac{i}{18} \right| = \frac{1}{18}$$

$$|z_3| = \left| -\frac{1}{108} (1+i) \right| = \frac{\sqrt{2}}{108}$$

$$|z_{n+1}| = |z_n \cdot \left( \frac{1}{3+3i} \right)| = |z_n| \cdot \frac{\sqrt{2}}{6} \leq |z| = \frac{\sqrt{2}}{6} \text{ bounded.}$$

$$\left| \frac{z_{n+1}}{z_n} \right| = \left| \frac{(3+3i)^{-(n+1)}}{(3+3i)^{-n}} \right| = \left| (3+3i)^{-1} \right| = \frac{\sqrt{2}}{6} < 1 \text{ converges}$$

### 16-25 SERIES

Is the given series convergent or divergent? Give a reason.  
Show details.

$$16. \sum_{n=0}^{\infty} \frac{(20 + 30i)^n}{n!}$$

$$17. \sum_{n=2}^{\infty} \frac{(-i)^n}{\ln n}$$

$$17. \left| \frac{z_{n+1}}{z_n} \right| = \left| \frac{(-1)^{n+1}}{\ln(n+1)} \cdot \frac{\ln n}{(-1)^n} \right| = (-1) \cdot \left| \frac{\ln n}{\ln(n+1)} \right| = \left| \frac{\ln n}{\ln(n+1)} \right|$$

$$n^{n+1} > (n+1)^n \quad \text{for } n \geq 3 \Rightarrow \frac{\ln n}{\ln(n+1)} > \frac{n}{n+1}$$

$$\Rightarrow \left| \frac{z_{n+1}}{z_n} \right| > \left| \frac{n}{n+1} \right| \text{ diverges}$$

$$19. \sum_{n=0}^{\infty} \frac{i^n}{n^2 - i}$$

$$\begin{aligned}
\left| \frac{z_{n+1}}{z_n} \right| &= \left| \frac{\frac{1}{(n+1)^2 - i} \cdot \frac{n^2 - i}{i^n}}{1} \right| = \left| i \cdot \frac{n^2 - i}{(n+1)^2 - i} \right| \\
&= \left| \frac{(n^2 - i)[(n+1)^2 + i]}{[(n+1)^2 - i][(n+1)^2 + i]} \right| \\
&= \left| \frac{(n^2 - i)(n^2 + 2n + 1 + i)}{(n+1)^4 + 1} \right| \\
&= \left| \frac{n^4 + 2n^3 + n^2 + i n^2 - i n^2 - 2n i - i + 1}{(n+1)^4 + 1} \right| \\
&= \left| \frac{1 + \frac{2}{n} + \frac{1}{n^2} - \frac{2i}{n^3} + \frac{1-i}{n^4}}{\left(1 + \frac{1}{n}\right)^4 + \frac{1}{n}} \right|
\end{aligned}$$

$i \rightarrow 1 \Rightarrow$  don't know

$$\left| z_n \right|^{\frac{1}{n}} = \left| \frac{i}{n^2 - i} \right| = \left| \frac{i}{(n^2 + 1)^{\frac{1}{n}}} \right| < \left| \frac{i}{n^{\frac{2}{n}}} \right| = 1$$

Converge absolutely

$$\left| z_n \right| = \left| \frac{i^n}{n^2 - i} \right| = \frac{|i^n|}{|n^2 - i|} = \frac{1}{\sqrt{n^4 + 1}} \leq \frac{1}{\sqrt{n^4}} = \frac{1}{n^2} \quad \text{Converge.}$$

$$23. \sum_{n=0}^{\infty} \frac{(-1)^n (1+i)^{2n}}{(2n)!}$$

$$\left| \frac{z_{n+1}}{z_n} \right| = \left| \frac{(-1)^{n+1} (1+i)^{2n+2}}{(2n+2)!} \cdot \frac{2n!}{(-1)^n (1+i)^{2n}} \right| \\ = \left| \frac{-1 \cdot (1+i)^2}{(2n+2)(2n+1)} \right|$$

$\lim_{n \rightarrow \infty} \left| \frac{z_{n+1}}{z_n} \right| = 0 < 1$  converge absolutely

$$25. \sum_{n=1}^{\infty} \frac{i^n}{n}$$

$$\left| \frac{z_{n+1}}{z_n} \right| = \left| \frac{i^{n+1}}{n+1} \cdot \frac{n}{i^n} \right| = \left| \frac{n i}{n+1} \right| = \left| \frac{n}{n+1} \right| \quad \begin{matrix} \lim \rightarrow 1 \\ \text{not sure} \end{matrix}$$

$$\left| z_n^{\frac{1}{n}} \right| = \left| \sqrt[n]{i} \right| \Rightarrow \lim_{n \rightarrow \infty} |z_n|^{\frac{1}{n}} = 1 \quad \text{not sure}$$

Per example 3.  $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$  ... conditionally converge.

so  $i, -1, -i, 1$

$$\left\{ z_{4t+1} + z_{4t+3} \right\} \quad t \text{ is even} \Rightarrow \stackrel{n=2m}{\sum_{m=1}^{\infty} \frac{(-1)^m}{m}} \Rightarrow 0$$

$$\left\{ z_{4t+2} + z_{4t+4} \right\} \quad t \text{ is odd} \Rightarrow \stackrel{n=2m+1}{i \sum_{m=0}^{\infty} \frac{(-1)^m}{2m+1}} \Rightarrow 0$$

$\Rightarrow$  converge to 0

# Selected Problem set 15.2

15.2

7.9. (1, 13, 17)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \left(z - \frac{\pi}{2}\right)^n$$

$\downarrow$

$R$   
 $\downarrow$

$\sqrt{R}$

$$\text{Center} = \frac{\pi}{2}$$

$$7. \sqrt{\left| \frac{a_{n+1} (z - \frac{\pi}{2})^{2n+2}}{a_n (z - \frac{\pi}{2})^{2n}} \right|} = \sqrt{\left| \frac{a_{n+1}}{a_n} \cdot \left| (z - \frac{\pi}{2})^2 \right| \right|}$$

$$= \sqrt{\left| \frac{(-1)^{n+1}}{(2n+2)!} \cdot \frac{2n!}{(-1)^n} \right| \cdot \left| (z - \frac{\pi}{2})^2 \right|}$$

$$= \sqrt{\left| \frac{1}{(2n+2)(2n+1)} \right| \cdot \left| (z - \frac{\pi}{2})^2 \right|}$$

$$L = L^* \cdot \left| z - \frac{\pi}{2} \right|, \quad \lim_{n \rightarrow \infty} L^* = 0, \quad \lim_{n \rightarrow \infty} L = 0$$

Converge for all  $z$  by the ratio test

$$9. \sum_{n=0}^{\infty} \frac{n(n-1)}{3^n} (z - i)^{2n}$$

$$\sqrt{\left| \frac{a_{n+1} (z - i)^{2n+2}}{a_n (z - i)^{2n}} \right|} = \sqrt{\left| \frac{a_{n+1}}{a_n} \cdot |z - i|^2 \right|}$$

$$= \sqrt{\frac{(n+1) \cdot n}{3^{n+1}} \cdot \frac{3^n}{n(n-1)} \cdot |z - i|^2}$$

$$= \sqrt{\frac{n+1}{n-1} \cdot \frac{1}{3} \cdot |z - i|^2}$$

$$L = L^* \cdot |z - i|, \quad \lim_{n \rightarrow \infty} L^* = \frac{\sqrt{3}}{3}, \quad \lim_{n \rightarrow \infty} L = \sqrt{3} \quad \text{Center} = i$$

$$11. \sum_{n=0}^{\infty} \left( \frac{2-i}{1+5i} \right) z^n$$

$$\sum_{n=0}^{\infty} \left( \frac{2-i}{1+5i} \cdot z \right)^n \Rightarrow \text{Radius } \sqrt{\frac{26}{5}}$$

$|z| \geq 1$ , divergence.

$|z| < 1$  Converges

$$S_n = \left( \frac{2-i}{1+5i} \right) z^0 + \left( \frac{2-i}{1+5i} \right) z + \dots \left( \frac{2-i}{1+5i} \right) z^n$$

$$z S_n = \left( \frac{2-i}{1+5i} \right) z^1 + \dots \left( \frac{2-i}{1+5i} \right) z^{n+1}$$

$$(1-z) S_n = \frac{2-i}{1+5i} \cdot 1 - \left( \frac{2-i}{1+5i} \right) z^{n+1}$$

$$n \rightarrow +\infty \quad z^{n+1} \rightarrow 0$$

$$S_n = \frac{2-i}{(1-z)(1+5i)}$$

Center: 0  
Radius: 1

$$13. \sum_{n=0}^{\infty} 16^n (z+i)^{4n}$$

$$\begin{aligned} \sqrt[4]{\left| \frac{a_{n+1}(z+i)^{4n+4}}{a_n(z+i)^{4n}} \right|} &= \sqrt[4]{\frac{|a_{n+1}|}{|a_n|}} \cdot |z+i| \\ &= \sqrt[4]{16} \cdot |z+i| = 2|z+i| \end{aligned}$$

$$\lim_{n \rightarrow \infty} L^* \rightarrow \lim_{n \rightarrow \infty} L = \frac{1}{z} \quad \text{Center} - i$$

$$17. \sum_{n=1}^{\infty} \frac{2^n}{n(n+1)} z^{2n+1}$$

$$\begin{aligned}\sqrt{\frac{a_{n+1} \cdot z^{2n+3}}{a_n \cdot z^{2n+1}}} &= \sqrt{\frac{a_{n+1}}{a_n} \cdot |z|} \\&= \sqrt{\frac{2^{n+1}}{(n+1)(n+2)} \cdot \frac{n(n+1)}{2^n}} (z) \\&= \sqrt{\frac{2n}{n+2}} \cdot |z|\end{aligned}$$

$$\lim_{n \rightarrow \infty} L^* = \sqrt{2} \quad \lim_{n \rightarrow \infty} L = \frac{\sqrt{2}}{2} \text{ center: } 0$$

# Selected Problem set 15.3

(5.3) 5, 7, 9, 11, 15

## 5-15 RADIUS OF CONVERGENCE BY DIFFERENTIATION OR INTEGRATION

Find the radius of convergence in two ways: (a) directly by the Cauchy–Hadamard formula in Sec. 15.2, and (b) from a series of simpler terms by using Theorem 3 or Theorem 4.

$$5. \sum_{n=2}^{\infty} \frac{n(n-1)}{2^n} (z-2i)^n \quad 6. \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \left(\frac{z}{2\pi}\right)^{2n+1}$$

5.(a)

$$R = \frac{1}{L^*} = \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right| \quad (\text{Cauchy–Hadamard formula}^1).$$

$$R = \lim_{n \rightarrow \infty} \left| \frac{n(n-1)}{2^n} \cdot \frac{2^{n+1}}{(n+1) \cdot n} \right| = \lim_{n \rightarrow \infty} \left| \frac{2(n-1)}{n+1} \right| = 2$$

$$(b) f(z) = \sum_{n=2}^{\infty} \frac{n(n-1)}{2^n} (z-2i)^n = \sum_{n=2}^{\infty} n(n-1) \cdot \left(\frac{z-2i}{2}\right)^n$$

$$g(x) = \sum_{n=2}^{\infty} x^n \Rightarrow R=1$$

$$g^{(2)}(x) = \sum_{n=2}^{\infty} n(n-1) \cdot x^{n-2} \Rightarrow R=1.$$

$$\left| \frac{z-2i}{2} \right| < 1 \quad |z-2i| < 2 \Rightarrow R=2$$

$$7. \sum_{n=1}^{\infty} \frac{n}{3^n} (z + 2i)^{2n}$$

a) Per P683.

$$\left| \frac{\frac{n+1}{3^{n+1}} (z+2i)^{2(n+1)}}{\frac{n}{3^n} (z+2i)^{2n}} \right| = \left| \frac{n+1}{3^{n+1}} \cdot \frac{3^n}{n} \right| \cdot \left| (z+2i)^2 \right| = \left| \frac{n+1}{3n} \right| \cdot \left| (z+2i)^2 \right|$$

$$\text{The limit is } L = L^* |z+2i| = \frac{\sqrt{3}}{3} |z+2i|$$

$$R = \frac{1}{L^*} = \sqrt{3} \quad \text{or} \quad \frac{\sqrt{3}}{3} |z+2i| < 1, |z+2i| < \underline{\sqrt{3}}$$

$$b) f(x) = 3 \cdot \left(\frac{x}{3}\right)^n = \frac{1}{3^{n-1}} \cdot x^n$$

$$f'(x) = \frac{1}{3^{n-1}} \cdot x^{n-1}$$

$$f'(x) \cdot \frac{x}{3} = \frac{1}{3^n} x^n$$

$$f(x); R = \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right| = \left| \frac{3^n}{3^{n-1}} \right| = 3$$

$$f'(x); R = 3 \quad f'(x) \cdot \frac{x}{3}; R = 3.$$

$$\Rightarrow R = \sqrt{3}$$

$$9. \sum_{n=1}^{\infty} \frac{(-2)^n}{n(n+1)(n+2)} z^{2n}$$

$$\text{a) } \left| \frac{(-2)^{n+1} \cdot z^{2(n+1)}}{(n+1)(n+2)(n+3)} \cdot \frac{n(n+1)(n+2)}{(-2)^n \cdot z^{2n}} \right| = \left| \frac{n}{n+3} \cdot (-2) \cdot z^2 \right| \\ = \left| \frac{2n}{n+3} \right| \cdot |z^2|$$

The lim is  $|z^2| < 1$ ,  $|z| < \frac{\sqrt{2}}{2}$ ,  $R = \frac{\sqrt{2}}{2}$

$$\text{b). } f(x) = x^{n+2} \quad |x| < 1$$

$$f'(x) = n(n+1)(n+2)x^n \quad |x| < 1$$

$$|-2z^2| < 1 \quad |z| < \frac{\sqrt{2}}{2} \quad R = \frac{\sqrt{2}}{2}$$

$$11. \sum_{n=1}^{\infty} \frac{3^n n(n+1)}{7^n} (z+2)^{2n}$$

$$\text{a) } \left| \frac{\frac{3^{n+1}}{7^{n+1}} \cdot (n+1)(n+2) \cdot (z+2)^{2n+2}}{(z+2)^{2n} \cdot 3^n \cdot n(n+1)} \right| \\ = \left| \frac{3(n+2)}{7n} \right| \cdot \left| (z+2)^2 \right|$$

The limit is  $\frac{3}{7} \cdot |(z+2)^2| < 1$   $R: \sqrt{\frac{7}{3}}$

$$\text{b). } f(x) = \left(\frac{3}{7}x\right)^{n+1} = \left(\frac{3}{7}\right)^{n+1} x^{n+1} \quad \left|\frac{3}{7}x\right| < 1$$

$$f'(x) = n(n+1) \left(\frac{3}{7}\right)^{n+1} x^{n-1} = \frac{3^{n+1} \cdot n(n+1)}{7^{n+1}} \cdot x^{n-1}$$

$$f'(x) \cdot \frac{7}{3} \cdot x = \frac{3^n n(n+1)}{7^n} \cdot x^n$$

$$\left| (z+2)^2 \cdot \frac{3}{7} \right| < 1 \quad \left| (z+2)^2 \right| < \frac{7}{3} \quad R: \sqrt{\frac{7}{3}}$$

$$15. \sum_{n=2}^{\infty} \frac{4^n n(n-1)}{3^n} (z-i)^n$$

a)  $R = \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right| = \lim_{n \rightarrow \infty} \left| \frac{4^n n(n-1)}{3^n} \cdot \frac{3^{n+1}}{4^{n+1}(n+1)} \right| = \lim_{n \rightarrow \infty} \left| \frac{3(n-1)}{4(n+1)} \right| = \frac{3}{4}$

b)  $f(x) = \left( \frac{4}{3}x \right)^n = \left( \frac{4}{3} \right)^n \cdot x^n \quad \left| \frac{4}{3}x \right| < 1$

$$f'(x) = \left( \frac{4}{3} \right)^n \cdot n(n-1) \cdot x^{n-2}$$

$$f''(x) \cdot x^2 = \frac{4^n n(n-1)}{3^n} \cdot x^n$$

$$\left| \frac{4}{3} (z-i) \right| < 1 \quad R = \frac{3}{4}$$

# Selected Problem set 15.4

15.4 3, 5, 7, 19, 21, 23

**3-10 MACLAURIN SERIES**

Find the Maclaurin series and its radius of convergence.

3.  $\sin 2z^2$

4.  $\frac{z+2}{1-z^2}$

$$3. \sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\sin 2z^2 = \sum_{n=0}^{\infty} (-1)^n \frac{(2z^2)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n+1} z^{4n+2}}{(2n+1)!} = 2z^2 - \frac{8z^6}{3!} + \frac{32z^{10}}{5!} - \dots$$

$R = \infty$

5.  $\frac{1}{2+z^4}$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1+x+x^2+\dots \quad |x| < 1$$

$$\begin{aligned} \frac{1}{2+z^4} &= \frac{1}{2} \cdot \frac{1}{1-(\frac{1}{2}z^4)} = \frac{1}{2} \sum_{n=0}^{\infty} \left(-\frac{1}{2}z^4\right)^n = -\left(\frac{1}{2}\right)^{n+1} z^{4n} \\ &= \frac{1}{2} - \frac{1}{4}z^4 + \frac{1}{8}z^8 - \dots \end{aligned}$$

7.  $\cos^2 \frac{1}{2}z$

$$\cos^2 x = \frac{1}{2}[\cos(2x) + 1] = 1 - x^2 + \frac{x^4}{3} + \dots$$

$|z| < \sqrt[4]{2}$

$$\begin{aligned} \cos^2 \frac{1}{2}z &= 1 - \left(\frac{1}{2}z\right)^2 + \frac{\left(\frac{1}{2}z\right)^4}{3} + \dots \\ &= 1 - \frac{z^2}{4} + \frac{z^4}{48} - \dots \end{aligned}$$

$R = \infty$

**18-25 TAYLOR SERIES**

Find the Taylor series with center  $z_0$  and its radius of convergence.

18.  $1/z, z_0 = i$       19.  $1/(1-z), z_0 = i$   
 20.  $\cos^2 z, z_0 = \pi/2$       21.  $\sin z, z_0 = \pi/2$   
 22.  $\cosh(z - \pi i), z_0 = \pi i$       23.  $1/(z+i)^2, z_0 = i$       24.  $e^{z(z-2)}, z_0 = 1$

| 9.

$$f(z) = a_0 + a_1(z - z_0) + a_2(z - z_0)^2 + a_3(z - z_0)^3 + \dots$$

$$\begin{aligned} f(z) &= f(z_0) + \frac{z - z_0}{1!} f'(z_0) + \frac{(z - z_0)^2}{2!} f''(z_0) + \dots \\ &\quad + \frac{(z - z_0)^n}{n!} f^{(n)}(z_0) + R_n(z). \end{aligned}$$

$$\begin{aligned} \text{Or: } \frac{1}{1-z} &= \frac{1}{1-(z-i)-i} \\ &= \frac{1}{(1-i)-(z-i)} \\ &= \frac{1}{1-i} \cdot \frac{1}{1-\frac{z-i}{1-i}} \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} f(z) &= \frac{1}{1-z} = \frac{1}{1-i} + \frac{z-z_0}{1!} f'(z_0) + \frac{(z-z_0)^2}{2!} f''(z_0) + \dots \\ &= \frac{1}{1-i} + \frac{z-z_0}{1} \cdot \frac{1}{(1-i)^2} + \frac{(z-z_0)^2}{2!} \cdot \frac{2!}{(1-i)^3} \\ &\quad + \frac{(z-z_0)^3}{3!} \cdot \frac{3!}{(1-i)^4} + \dots \\ &= \frac{1+i}{2} + \frac{i}{2}(z-i) + \left(-\frac{1-i}{4}\right)(z-i)^2 + -\frac{1}{4}(z-i)^3 + \dots \\ &= \sum_{n=0}^{\infty} \frac{1}{(1-i)^{n+1}} \cdot (z-z_0)^n \\ R &= \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right| = \left( \lim_{n \rightarrow \infty} \left| \frac{(1-i)^{n+2}}{(1-i)^{n+1}} \right| \right) = \left| 1-i \right| = \sqrt{2} \end{aligned}$$

21.  $\sin z, z_0 = \pi/2$

$$\begin{aligned} f(z) &= \sin z = f\left(\frac{\pi}{2}\right) + \frac{z - \frac{\pi}{2}}{1!} \cdot f'\left(\frac{\pi}{2}\right) + \frac{(z - \frac{\pi}{2})^2}{2!} \cdot f''\left(\frac{\pi}{2}\right) + \dots \\ &= 1 + \frac{z - \frac{\pi}{2}}{1!} \cdot (\cos\frac{\pi}{2}) + \frac{(z - \frac{\pi}{2})^2}{2!} \cdot (-\sin\frac{\pi}{2}) + \frac{(z - \frac{\pi}{2})^3}{3!} \cdot (-\cos\frac{\pi}{2}) + \dots \\ &= 1 + 0 - \frac{1}{2!} (z - \frac{\pi}{2})^2 + 0 + \frac{1}{4!} (z - \frac{\pi}{2})^4 + 0 - \frac{(z - \frac{\pi}{2})^6}{6!} \end{aligned}$$

OR:

$$\begin{aligned} \sin z &= \sin\left(z - \frac{\pi}{2} + \frac{\pi}{2}\right) = \sin\left(z - \frac{\pi}{2}\right)\cos\frac{\pi}{2} + \cos\left(z - \frac{\pi}{2}\right)\sin\frac{\pi}{2} \\ &= \cos\left(z - \frac{\pi}{2}\right) \end{aligned}$$

$$\cos z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!} = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(z - \frac{\pi}{2})^{2n}}{2n!}$$

$$\cos z, R = \infty, \Rightarrow R = \infty.$$

23.  $1/(z + i)^2, z_0 = i$

$$\frac{1}{(z+i)^2} = \frac{1}{[(z-i)+2i]^2} = \frac{1}{(2i)^2} \cdot \frac{1}{\left(\frac{z-i}{2i}+1\right)^2}$$

P696 :

$$\begin{aligned} \frac{1}{(1+z)^m} &= (1+z)^{-m} = \sum_{n=0}^{\infty} \binom{-m}{n} z^n \\ &= 1 - mz + \frac{m(m+1)}{2!} z^2 - \frac{m(m+1)(m+2)}{3!} z^3 + \dots \end{aligned}$$

$$= -\frac{1}{4} \sum_{n=0}^{\infty} \binom{-2}{n} \left(\frac{z-i}{2i}\right)^n$$

$$\left| \frac{z-i}{2i} \right| < 1 \quad |z-i| < 2 \quad R = 2$$