Maximum Ratio Transmission

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Abstract— **This paper presents the concept, principles, and analysis of maximum ratio transmission for wireless communications where multiple antennas are used for both transmission and reception. The principles and analysis are applicable to general cases, including maximum ratio combining. Simulation results agree with the analysis. The analysis shows that the average overall SNR is proportional to the cross-correlation between channel vectors and that error probability decreases inversely with the $(L \times K)$th power of the average SNR.**

I. Introduction

The most adverse propagation effect from which wireless communications systems suffer is the multipath fading. One of the common methods used by wireless communications engineers to combat multipath fading is the antenna diversity technique, where two or more antennas at the receiver and/or transmitter are so separated in space or polarization that their fading envelopes are decorrelated. A classical combining technique is the maximum-ratio combining (MRC) where the signals from the received antenna elements are weighted such that the signal-to-noise ratio (SNR) of the their sum is maximized. The MRC technique has been shown to be optimum if diversity branch signals are mutually uncorrelated and follow a Rayleigh distribution [1]. However, the MRC technique has so far been exclusively for receiving applications. As there are more and more emerging wireless services, more and more applications may require diversity at the transmitter or at both transmitter and receiver to combat severe fading effects. Various transmit diversity techniques have been proposed in the open literature. For example, a delay transmit diversity scheme was proposed by Wittneben [2], [3]. A variation of the delay scheme was suggested by Seshadri and Winters [4], [5], where the replicas of the signal are transmitted through multiple antennas at different times. Another the example of transmit diversity is an simple, but effective scheme proposed by Alamouti [6], where a pair of symbols is transmitted using two antennas at first and the transformed version of the pair is transmitted to obtain the MRC-like diversity. However, these transmit diversity techniques were built on objectives other than to maximize the SNR. That is, they are suboptimum in terms of SNR performance.

Accordingly, the frame work of maximum ratio transmission (MRT) will be established here in terms of concept and principles. It can be considered as the generalization of the maximum ratio algorithm for multiple transmitting antennas and multiple receiving antennas. It also provides a reference for the optimum performance that a system may obtain using both transmit and receive diversity. Therefore, the focus of this document is on the analysis of the MRT scheme rather than on the implementation aspects.

The rest of the document is organized as follows. The system model used in the study is described in the following section. In Section 3, the MRT concept is presented. Discussions are given in terms of average SNR and the order of diversity in Section 4, which is followed by another section with some numerical examples. Conclusions are given in Section 5.

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II. The System Model

In this study, a system is considered, which consists of K antennas for transmission and L antennas for reception. The channel consists of $K \times L$ statistically independent coefficients, as shown in Fig. 1. It can conveniently be represented by a matrix,

$$\mathbf{H} = \begin{bmatrix} h_{11} & \cdots & h_{1K} \\ \vdots & \ddots & \vdots \\ h_{L1} & \cdots & h_{LK} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_L \end{bmatrix} \quad (1)$$

where the entry h_{pk} represents the channel coefficient for Antenna k and Antenna p. It is assumed that the channel coefficients are available to both the transmitter and receiver through some means. It should be pointed out that the feasibility and practicality for obtaining the channel coefficients are important implementation issues. However, these issues are out of the scope of this work, they will not be addressed here.

The system model shown in Fig. 1 is a simple baseband representation. The symbol c to be transmitted is weighted with a transmit weighting vector v to form the transmitted signal vector. The received signal vector is the product of the transmitted signal vector and the channel plus the noise; that is,

$$\mathbf{x} = \mathbf{Hs} + \mathbf{n} \quad (2)$$

where the transmitted signals $\mathbf{s}$ is given by

$$\mathbf{s} = [s_1 \cdots s_K]^T = c\,[v_1 \cdots v_K]^T \quad (3)$$

The noise vector is expressed as

$$\mathbf{n} = [n_1 \cdots n_L]^T \quad (4)$$

Noise is assumed to be white Gaussian and uncorrelated with the signals. The received signals are weighted and summed to produce the estimate of the symbol.

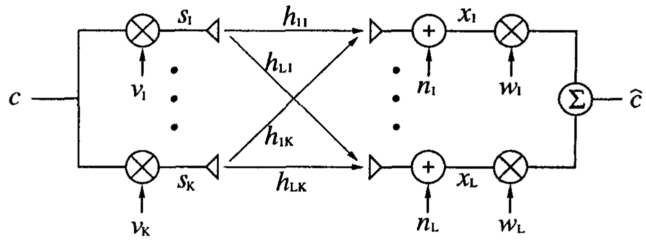

Fig. 1. The system model.

III. Maximum Ratio Transmission

In order to generate the $K \times 1$ transmission weight vector from the channel matrix, a linear transformation is required; that is,

$$\mathbf{v} = \frac{1}{a}(\mathbf{gH})^H \quad (5)$$

where $\mathbf{g} = [g_1 \cdots g_L]$. The transmitted signal vector is then expressed as

$$\mathbf{s} = \frac{c}{a}(\mathbf{gH})^H \quad (6)$$

The normalization factor a is required to be

$$\begin{aligned} a &= |\mathbf{gH}| \\ &= \left(\sum_{p=1}^{L} \sum_{q=1}^{L} g_p g_q^* \sum_{k=1}^{K} h_{pk} h_{qk}^* \right)^{\frac{1}{2}} \end{aligned} \quad (7)$$

The received signal vector is, therefore, given by

$$\mathbf{x} = \frac{c}{a}\mathbf{H}(\mathbf{gH})^H + \mathbf{n} \quad (8)$$

To estimate the transmitted symbol, the receive weight vector $\mathbf{w}$ has to be applied to the received signal vector $\mathbf{x}$. If $\mathbf{w}$ is set to be $\mathbf{g}$, the estimate of the symbol is given by

$$\tilde{c} = \mathbf{gx} = \frac{c}{a}\mathbf{gH}(\mathbf{gH})^H + \mathbf{gn} = ac + \mathbf{gn} \quad (9)$$

with the overall SNR given by

$$\gamma = \frac{a^2}{\mathbf{gg}^H}\gamma_0 = \frac{a^2\gamma_0}{\sum_{p=1}^{L}|g_p|^2} \quad (10)$$

where $\gamma_0 = \frac{\sigma_c^2}{\sigma_n^2}$ denotes the average SNR for the case of a single transmitting antenna (i.e., without diversity). From (10), it can be observed that the overall SNR is a function of $\mathbf{g}$. Thus, it is possible to maximize the SNR by choosing the appropriate values for $\mathbf{g}$. Since h_{qk} are assumed to be statistically identical, the condition that $|g_1| = |g_2| = \cdots = |g_L|$ has to be satisfied for the maximum value of SNR. Without changing the nature of the problem, one can set $|g_p| = 1$ for simplicity. Therefore, the overall SNR is rewritten as

$$\gamma = \frac{a^2}{L}\gamma_0 \tag{11}$$

Maximizing γ is equivalent to maximizing a, which is maximized if

$$\left(g_p g_q^*\right)^* = \frac{\sum_{k=1}^{K} h_{pk}h_{qk}^*}{\left|\sum_{k=1}^{K} h_{pk}h_{qk}^*\right|} \tag{12}$$

Therefore,

$$a = \left(\sum_{p=1}^{L}\sum_{q=1}^{L}\left|\sum_{k=1}^{K} h_{pk}h_{qk}^*\right|\right)^{\frac{1}{2}} \tag{13}$$

which results in the maximum value of γ.

IV. Discussion

A. Average SNR

The expression in (13) can be rewritten as

$$a = \left(\sum_{p=1}^{L}\sum_{k=1}^{K}|h_{pk}|^2 + \sum_{p=1}^{L}\sum_{q=1,q\neq p}^{L}\left|\sum_{k=1}^{K} h_{pk}h_{qk}^*\right|\right)^{\frac{1}{2}} \tag{14}$$

The summation term with respect to K on the right hand side of (14) is actually the inner product of different pairs of channel vectors; namely,

$$\left|\sum_{k=1}^{K} h_{pk}h_{qk}^*\right| = \left|\mathbf{h}_p\mathbf{h}_q^H\right| \tag{15}$$

At one extreme, if $\mathbf{h}_p$ and $\mathbf{h}_q$ are mutually orthogonal (i.e., $\mathbf{h}_p\mathbf{h}_q^H = 0$), then

$$a = \left(\sum_{p=1}^{L}\sum_{k=1}^{K}|h_{pk}|^2\right)^{\frac{1}{2}} \tag{16}$$

and

$$E[a^2] = LK\overline{r^2} \tag{17}$$

On the other extreme, if $\mathbf{h}_p$ and $\mathbf{h}_q$ are fully correlated (i.e., $\mathbf{h}_p\mathbf{h}_q^H = |\mathbf{h}_q|^2$, then

$$a = \left(\sum_{p=1}^{L}\sum_{q=1}^{L}\sum_{k=1}^{K}|h_{qk}|^2\right)^{\frac{1}{2}} \tag{18}$$

and

$$E[a^2] = L^2K\overline{r^2} \tag{19}$$

Therefore, the average overall SNR is bounded by

$$K\overline{r^2}\gamma_0 \leq \bar{\gamma} \leq LK\overline{r^2}\gamma_0 \tag{20}$$

B. Order of Diversity

For a system consisting of $K \times L$ antennas, it is expected that the order of diversity be $K \times L$; that is, the probability of error should decrease inversely with the $(K \times L)$th power of the average SNR. To see this, one may consider the following example.

In the example, a system with BPSK modulation is assumed. The channel coefficients are complex Gaussian and mutually statistically independent. Referring back to (14), its second term on the right hand side is always positive. Therefore, the following inequality holds

$$a^2 \geq \sum_{p=1}^{L}\sum_{k=1}^{K}|h_{pk}|^2 \tag{21}$$

Thus, the worst error probability P is the one evaluated under the equal condition. To determine P, the probability of error conditioned on a set of channel coefficients $\{h_{pk}\}$ must be obtained first. Then the conditional error probability is average over the probability density function of $\{h_{pk}\}$. For Gaussian noise, the conditional error probability is expressed as

$$P(\gamma) = Q(\sqrt{2\gamma}) \tag{22}$$

The probability density function $p(\gamma)$ can be determined via the characteristic function of γ, which turns out to be the characteristic function of a χ^2-distributed random variable with $2 \times L \times K$ degrees of freedom. It follows that $p(\gamma)$ is given by

$$p(\gamma) = \frac{\gamma_b^{LK-1} e^{-\gamma/\bar{\gamma}_a}}{(LK-1)!\bar{\gamma}_a^{LK}} \quad (23)$$

where

$$\bar{\gamma}_a = \gamma_0 E\left[|h_{pk}|^2\right] = \gamma_0 \overline{r^2} \quad (24)$$

The error probability is then given by the following integral

$$P = \int_0^\infty P(\gamma)p(\gamma)d\gamma \quad (25)$$

For $\bar{\gamma}_a \gg 1$,

$$P \approx \left(\frac{1}{4\bar{\gamma}_a}\right)^{LK} \frac{(2LK-1)!}{(LK)!(LK-1)!} \quad (26)$$

which indicates that the error probability decreases inversely with the $(L \times K)$th power of the average SNR γ_0.

V. Numerical Results

The simulations are carried out on the discrete-event (i.e., symbol by symbol) basis. Furthermore, they are performed only at the level of baseband processing. That is, the effects of RF and IF components are not considered here. Without loss of generality, QPSK is used in the simulation for simplicity for most cases. Each value of symbol error rate (SER) is obtained by transmitting one million symbols. A simple channel model is used, where the fading channel coefficients are complex Gaussian; that is, is a random variable with a complex Gaussian distribution. Because the objective of carrying out the simulations is to evaluate the performance, it is assumed that perfect knowledge of channel fading coefficients are available to both transmitting and receiving stations.

The SER performance curves plotted in Fig. 2 show the results using two receiving antennas. Two characteristics that are particularly associated with diversity can be observed here:

1. The improvement becomes greater as SNR increases.
2. The incremental improvement becomes smaller as the diversity order increases.

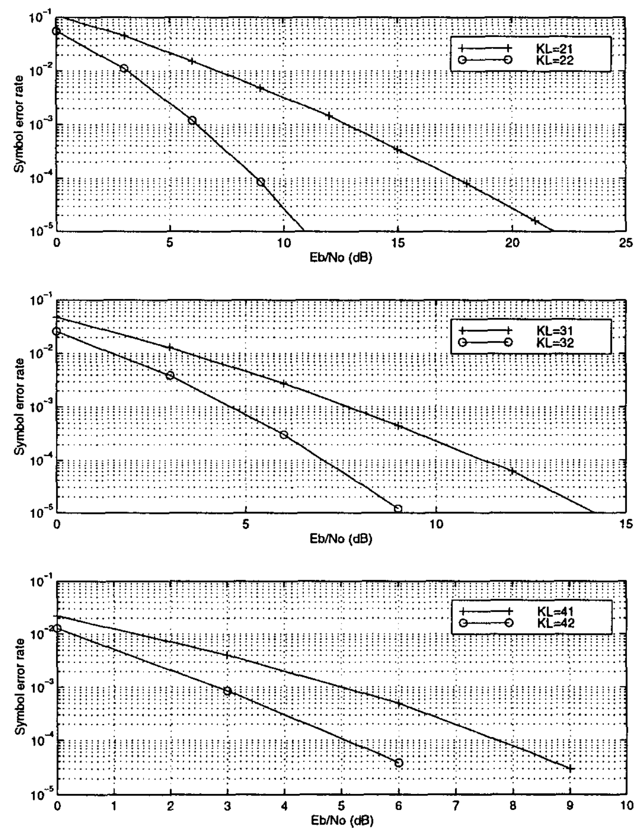

Fig. 2. The comparison of MRT performance curves (SER vs. SNR), showing the effect of adding the second receiving antenna.

In Fig. 3 are given the performance curves for different cases of the 4th-order diversity (i.e., $K \times L = 4$). The results validate the observations made earlier in the previous section. Comparing the curve corresponding to $KL = 41$ with that corresponding to $KL = 22$ and mutually orthogonal channels (OC), one may observe the 3-dB difference in SNR for the same error rate. Furthermore, the performance for $KL = 22$ with random channel (RC) coefficients is somewhere in between, as predicted.

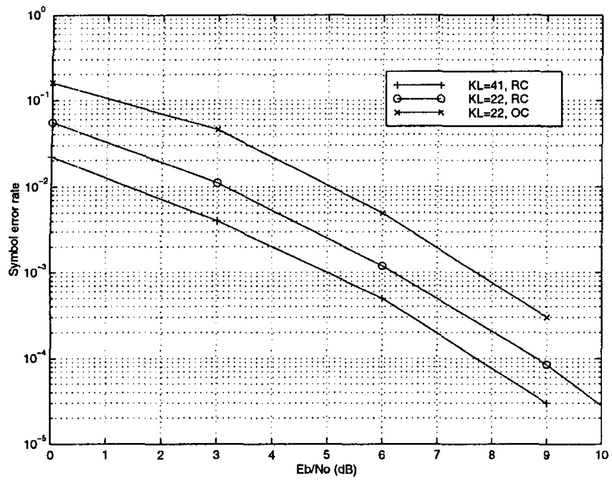

Fig. 3. The comparison of MRT performance curves (SER vs. SNR) for the 4th-order diversity.

VI. Conclusions

In this paper, the concept of maximum ratio transmission has been presented. It has been shown how the maximum signal-to-noise ratio can be obtained in wireless communications where multiple antennas are used for both transmission and reception. The principles and analysis are applicable to general cases, including maximum ratio combining. Simulation results agree with what has been predicted in the analysis. It has been shown that the average overall SNR is proportional to the cross-correlation between channel vectors. It is also observed that the average gain in SNR in the orthogonal-channel case will be $10 \log L$ dB less than the $K \times L$ transmitting antennas and one receiving antenna. Finally, analysis also shows that error probability decreases inversely with the $(L \times K)$th power of the average SNR.

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