

$$\text{minimize } \sum_n a_n N_n^{-1} \quad \sigma_{H_n}^2 = a_n \quad \text{Marika}$$

$$\text{s.t. } \sum_n N_n - N_T = 0$$

$$\mathcal{L} = \sum_n a_n N_n^{-1} + \lambda (\sum_n N_n - N_T)$$

$$\frac{\partial \mathcal{L}}{\partial N_n} = -\frac{a_n}{N_n^2} + \lambda = 0 \quad \lambda = \frac{a_n}{N_n^2} \text{ for each } N_n$$

$$\sum_n N_n = N_T$$

$$N_n = \frac{\sqrt{a_n}}{\sqrt{\lambda}}$$

$$\sum_n \frac{\sqrt{a_n}}{\lambda} = N_T$$

$$\frac{\sum_n \sqrt{a_n}}{N_T} = \sqrt{\lambda}$$

$$N_n = \frac{\sqrt{a_n} \cdot N_T}{\sum_n \sqrt{a_n}}$$

$$\text{minimize } \sum_n a_n N_n^{-1} \rightarrow \sum_n a_n \frac{\sum_n \sqrt{a_n}}{\sqrt{a_n} N_T}$$

$$= \frac{\sum_n \sqrt{a_n}}{N_T} = \left(\sum_n \sqrt{a_n} \right)^2 \frac{1}{N_T}$$

~~this~~ it can be seen that this is the same