CDL Quantum Hackathon 2021

CosmiQ Team Presentation

Qutriscendence

Example : 27-qubit IBM Q (NISQ device)

Qubits: 2^27 ~ 135M

Qutrits: 3^27 ~ 7.5T

Qutrit >> Qubit

Vision and Mission Statement

Vision: Empower the world with advanced quantum algorithms using qudits.

Mission: Expand the capabilities of quantum algorithms by leveraging the multi-level quantum states.

Added Value: Qudit-based algorithms can process more information and solve more complex problems on existing NISQ hardware.

Use Case: Using Qutrits for Financial Portfolio Optimization, QUTO formulation.

Our Results

- We explored similarities between different quantum **hardware** and quantum **software** implementations of **multi-level quantum states** (qudits). We demonstrated that qudit based quantum algorithms offer an advantage compared to qubit based algorithms.

- We used **Qiskit Pulse** open source library to access the higher energy states of the **IBMQ** quantum devices. We designed custom pulse schedules and executed them on quantum hardware to calibrate qutrit states and to construct **single-qutrit quantum gates.**

- We developed a novel quantum algorithm to solve a **financial portfolio optimization problem** using qutrit states. We implemented the **QAOA algorithm** for qutrits on a photonic quantum circuit simulator using **Xanadu's Pennylane library**.

- We reformulated a famous partitioning QUBO into a **QUTO** (quadratic unconstrained binary/ternary optimisation) and showed the complexity reduction associated. It was solved using the **D-Wave libraries**, and can therefore be run on HW.

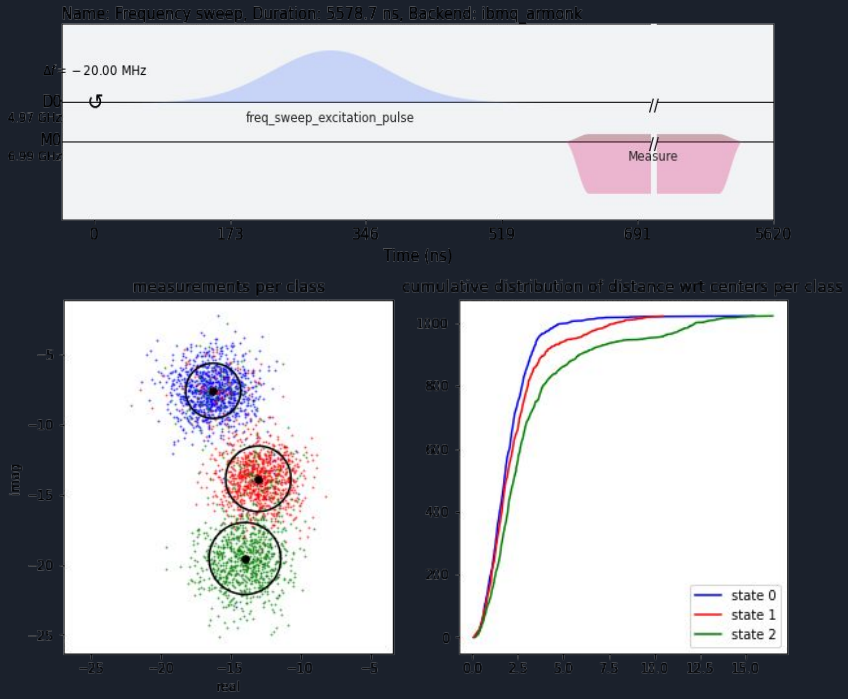
Portfolio Optimization Market

- Top 10 Asset management companies account for more than **$30 trillion** worth of Assets Under Management.
- It costs **30 cents** to run a single task on a quantum computer using existing NISQ.
- **Large mutual funds** like VTSAX have include the entirety of the US equity market, which amounts to more than **3,600 stocks** ranging from small to large cap growth and value stocks.
- Rating agencies like Standard and Poor's, Moody's track thousands of financial assets over large period of time to determine their financial health.
- International Financial Regulatory agencies like the **World Bank** look at the historical data of the bonds of the sovereign nations, to determine the investment climate in the country.

Calibrating Qutrits with Qiskit Pulse

Qiskit Pulse provides a formalism for specifying pulse level control of a general quantum device.

- We **calibrated** first and second excited states **pulses** of an IBMQ quantum device on *ibmq_armonk* backend.
- We manipulated qutrit, managing to create for example an **equal superposition** of the 3 lowest states (0,1,2)

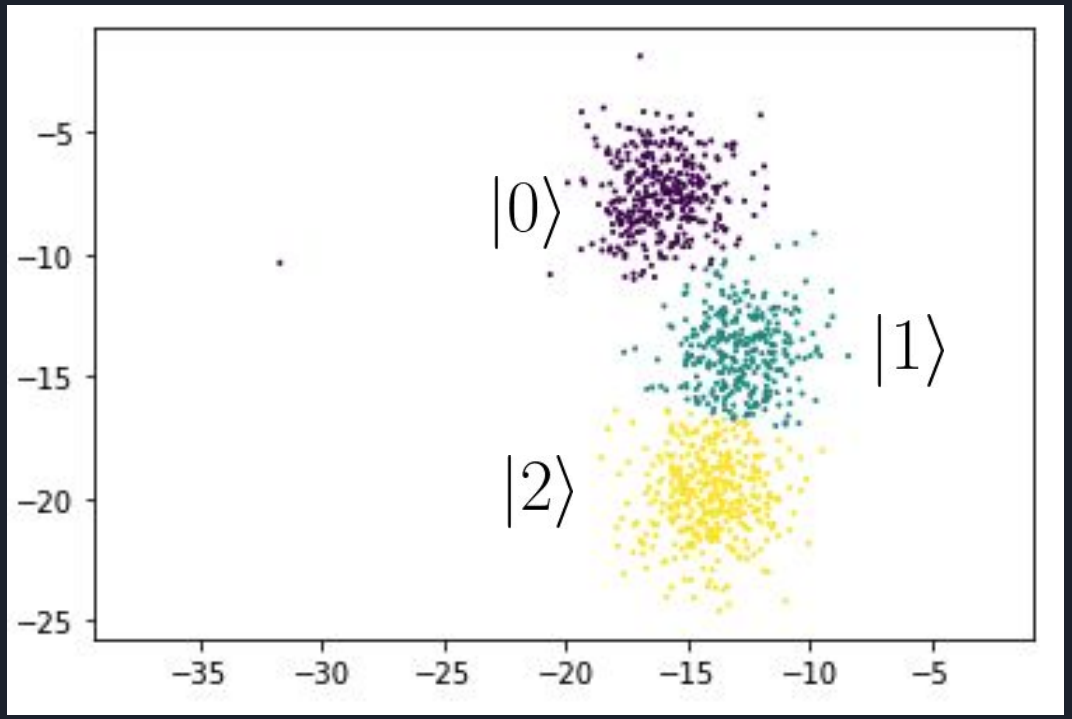


Qiskit pulse: programming quantum computers through the cloud with pulses, Quantum Sci. Technol. **5**, 044006 (2020)

Constructing Qutrit States

$$|i\rangle = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle$$

$$c_0 = c_1 = c_2$$



Financial Portfolio Optimization Using Qutrits

$$\mathcal{H} = \sum_{t_0}^{t_f}\sum_{i}\sum_{q}(-\mu(t,i) - 2\rho)\frac{d^q}{K}\hat{n}(t,i,q) + \sum_{t_0}^{t_f}\sum_{i}\sum_{q}(\rho)\frac{d^{2q}}{K^2}\hat{n}(t,i,q)\hat{n}(t,i,q)$$
$$+ \sum_{t_0}^{t_f}\sum_{i}\sum_{q<p} 2(\rho)\frac{d^{p+q}}{K^2}\hat{n}(t,i,q)\hat{n}(t,i,p)$$
$$+ \sum_{t_0}^{t_f}\sum_{i<j}\sum_{q} 2(\rho)\frac{d^{2q}}{K^2}\hat{n}(t,i,q)\hat{n}(t,j,q)$$
$$+ \sum_{t_0}^{t_f}\sum_{i<j}\sum_{q<p} 4(\rho)\frac{d^{p+q}}{K^2}\hat{n}(t,i,q)\hat{n}(t,j,p)$$

$\omega \rightarrow$ binary representation of the assets
$\Sigma \rightarrow$ risk matrix
$R^* \rightarrow$ minimum return value

The World Bank: there are approximately 43,000 listed companies, if we try monitoring all of them for a year we need to track roughly ***16 million data points***

For a general discrete probability distribution of n points:

- classical computer: 2 ^ (16 * n)
- quantum computer (qubits): 16 * n
- quantum computer (qutrits): 16 * n / 1.585

n qutrits could represent 1.5 ^ n times more information than n qubits

Financial Portfolio Optimization Using Qutrits

Hamiltonian Cost Function

Risk aversion (controls volatility) — Transaction costs — Lagrange multiplier — Vector of ones

$$\mathcal{H} = \sum_{t=t_i}^{t_f} -\mu_t^T \omega_t + \frac{\gamma}{2}\omega_t^T \Sigma_t \omega_t + \lambda(\Delta\omega_t)^2 + \rho\left(u^T\omega_t - 1\right)^2$$

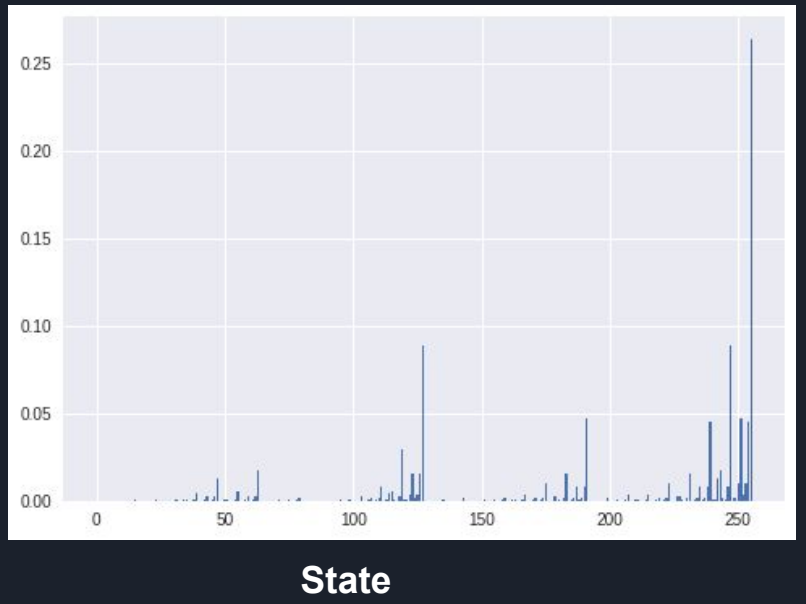
TO MINIMIZE

→ QUBO (D-WAVE)

→ QAOA (**Xanadu**) arXiv:2012.01121

```
+ (0.15625) [Z0 Z2]
+ (0.15625) [Z4 Z6]
+ (0.15625) [I0 I2]
+ (0.15625) [I4 I6]
+ (0.3125) [Z0 Z1]
+ (0.3125) [Z2 Z3]
+ (0.3125) [Z4 Z5]
+ (0.3125) [Z6 Z7]
+ (0.3125) [I0 I1]
+ (0.3125) [I2 I3]
+ (0.3125) [I4 I5]
+ (0.3125) [I6 I7]
+ (0.625) [Z0 Z3]
+ (0.625) [Z1 Z3]
+ (0.625) [Z4 Z7]
+ (0.625) [Z5 Z7]
+ (0.625) [I0 I3]
+ (0.625) [I1 I3]
+ (0.625) [I4 I7]
+ (0.625) [I5 I7]
```

```
  (-2.1880812564998986) [I1]
+ (-2.1880736479337526) [I5]
+ (-2.187871854002614) [I3]
+ (-2.18784328290288) [I7]
+ (-1.2496567170971198) [Z7]
+ (-1.2496281459973861) [Z3]
+ (-1.1721656282499493) [I0]
+ (-1.1721618239668763) [I4]
+ (-1.172060927001307) [I2]
+ (-1.17204664145144) [I6]
+ (-1.0934631760331237) [Z4]
+ (-1.0934593717500507) [Z0]
+ (0.0005736479337525857) [Z5]
+ (0.000581256499898597) [Z1]
+ (0.1564216414514401) [Z6]
+ (0.15643592700130693) [Z2]
```



QUTO vs QUBO : Robust Equal Sum Partitioning

RQUTO :
3 N binary $x\epsilon\{0,1\}$
$x\_(i,0)=i\epsilon O, x\_(i,-1)=i\epsilon A, x\_(i,1)=i\epsilon B$
One constraint : $\sum x\_i =0$
Minimise : $-\sum x\_i$ ^2

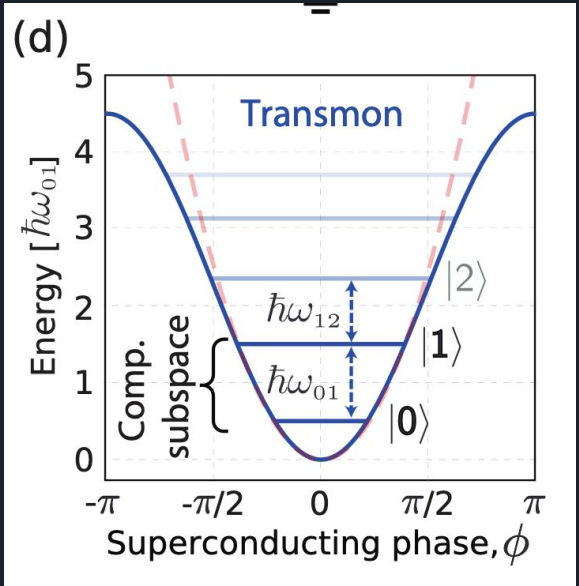
RQUTO :
N Ising trinary $x\epsilon\{-1,0,1\}$
One constraint : $\sum x\_i =0$
Minimise : $-\sum x\_i$ ^2

Thank you!

Backup Slides

Qutrits

$$|i\rangle = |0\rangle, |1\rangle, |2\rangle$$



Applied Physics Reviews 6, 021318 (2019)

Other advantages of qutrits

- ".. qudit provides a larger state space to store and process information, and thus can provide reduction of the circuit complexity, simplification of the experimental setup and enhancement of the algorithm efficiency." Front. Phys., 8, 479 (2020)
- Demonstration of quantum volume 64 on a superconducting quantum computing system arXiv:2008.08571
- It can be realized using existing quantum hardware: IBMQ