CDL Quantum Hackathon 2021

CosmiQ Team Presentation

Qutriscendence

Example : 27-qubit IBM Q (NISQ device)

Qubits: 2^{27} ~ 135M

Qutrits: 3^{27} ~ 7.5T

Qutrit >> Qubit

Vision and Mission Statement

Vision: Empower the world with advanced quantum algorithms using qudits.

Mission: Expand the capabilities of quantum algorithms by leveraging the multi-level quantum states.

Added Value: Qudit-based algorithms can process more information and solve more complex problems on existing NISQ hardware.

Use Case: Using Qutrits for Financial Portfolio Optimization, QUTO formulation.

Our Results

- We explored similarities between different quantum **hardware** and quantum **software** implementations of **multi-level quantum states** (qudits). We demonstrated that qudit based quantum algorithms offer an advantage compared to qubit based algorithms.

- We used **Qiskit Pulse** open source library to access the higher energy states of the **IBMQ** quantum devices. We designed custom pulse schedules and executed them on quantum hardware to calibrate qutrit states and to construct **single-qutrit quantum gates.**

- We developed a novel quantum algorithm to solve a **financial portfolio optimization problem** using qutrit states. We implemented the **QAOA algorithm** for qutrits on a photonic quantum circuit simulator using **Xanadu's Pennylane library**.

- We reformulated a famous partitioning QUBO into a **QUTO** (quadratic unconstrained binary/ternary optimisation) and showed the complexity reduction associated. It was solved using the **D-Wave libraries**, and can therefore be run on HW.

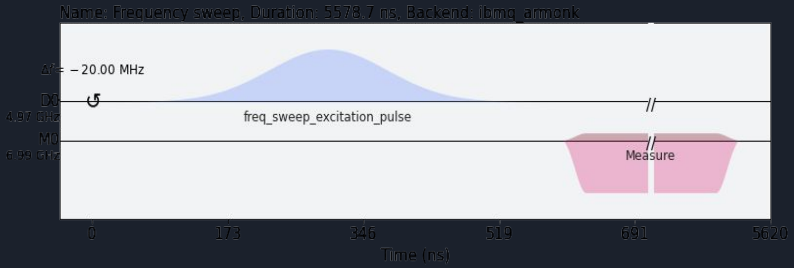
Portfolio Optimization Market

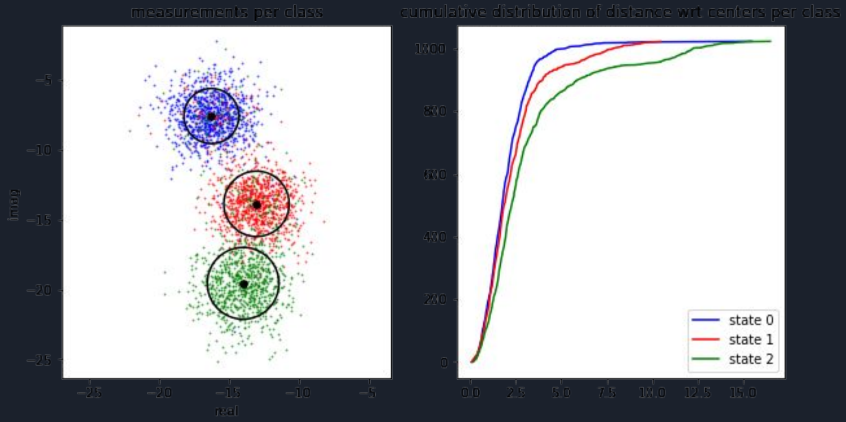
- Top 10 Asset management companies account for more than **$30 trillion** worth of Assets Under Management.
- It costs **30 cents** to run a single task on a quantum computer using existing NISQ.
- **Large mutual funds** like VTSAX have include the entirety of the US equity market, which amounts to more than **3,600 stocks** ranging from small to large cap growth and value stocks.
- Rating agencies like Standard and Poor's, Moody's track thousands of financial assets over large period of time to determine their financial health.
- International Financial Regulatory agencies like the **World Bank** look at the historical data of the bonds of the sovereign nations, to determine the investment climate in the country.

Calibrating Qutrits with Qiskit Pulse

Qiskit Pulse provides a formalism for specifying pulse level control of a general quantum device.

- We **calibrated** first and second excited states **pulses** of an IBMQ quantum device on *ibmq_armonk* backend.
- We manipulated qutrit, managing to create for example an **equal superposition** of the 3 lowest states (0,1,2)



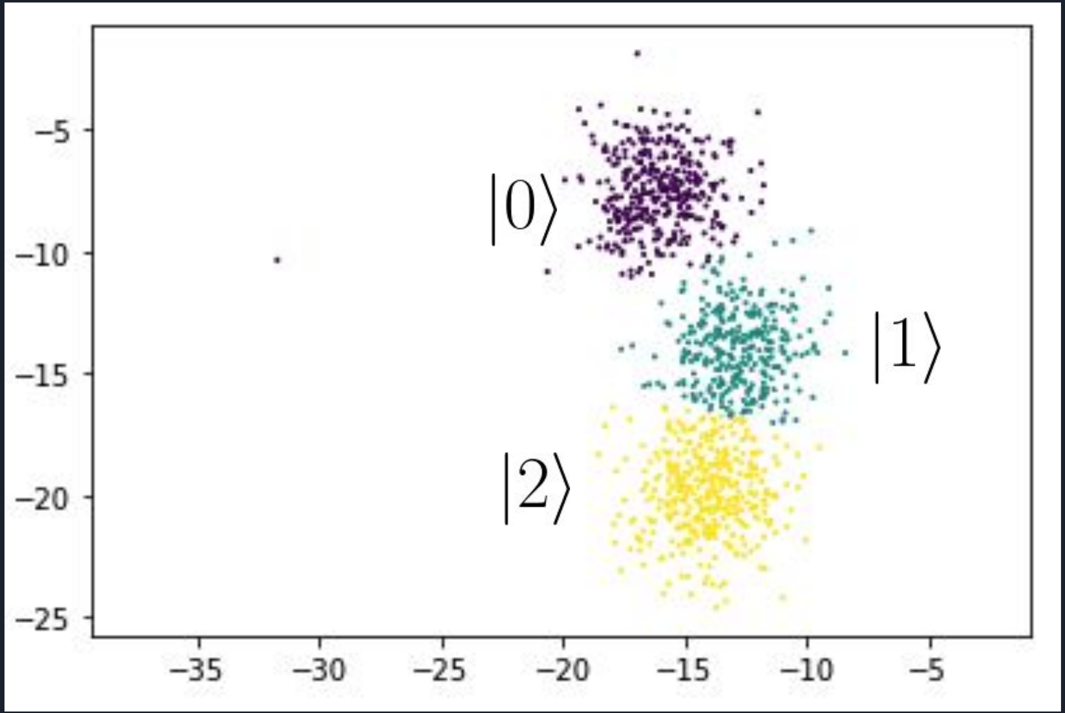


Qiskit pulse: programming quantum computers through the cloud with pulses, Quantum Sci. Technol. **5**, 044006 (2020)

Constructing Qutrit States

$$|i\rangle = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle$$

$$c_0 = c_1 = c_2$$



Financial Portfolio Optimization Using Qutrits

$$
\begin{aligned}
\mathcal{H} = &\sum_{t_0}^{t_f}\sum_{i}\sum_{q}(-\mu(t,i) - 2\rho)\frac{d^q}{K}\hat{n}(t,i,q) + \sum_{t_0}^{t_f}\sum_{i}\sum_{q}(\rho)\frac{d^{2q}}{K^2}\hat{n}(t,i,q)\hat{n}(t,i,q) \\
&+ \sum_{t_0}^{t_f}\sum_{i}\sum_{q<p}2(\rho)\frac{d^{p+q}}{K^2}\hat{n}(t,i,q)\hat{n}(t,i,p) \\
&+ \sum_{t_0}^{t_f}\sum_{i<j}\sum_{q}2(\rho)\frac{d^{2q}}{K^2}\hat{n}(t,i,q)\hat{n}(t,j,q) \\
&+ \sum_{t_0}^{t_f}\sum_{i<j}\sum_{q<p}4(\rho)\frac{d^{p+q}}{K^2}\hat{n}(t,i,q)\hat{n}(t,j,p)
\end{aligned}
$$

n(i,t,q) $\rightarrow$ binary representation of the assets
Σ $\rightarrow$ risk matrix
ρ $\rightarrow$ lagrange multiplier

The World Bank: there are approximately 43,000 listed companies, if we try monitoring all of them for a year we need to track roughly ***16 million data points***

For a general discrete probability distribution of n points:

- classical computer: 2 ^ (16 * n)
- quantum computer (qubits): 16 * n
- quantum computer (qutrits): 16 * n / 1.585

n qutrits could represent 1.5 ^ n times more information than n qubits

Financial Portfolio Optimization Using Qutrits

Hamiltonian Cost Function

Risk aversion (controls volatility) | Transaction costs | Lagrange multiplier | Vector of ones

$$\mathcal{H} = \sum_{t=t_i}^{t_f} -\mu_t^T \omega_t + \frac{\gamma}{2} \omega_t^T \Sigma_t \omega_t + \lambda (\Delta \omega_t)^2 + \rho \left(u^T \omega_t - 1 \right)^2$$

TO MINIMIZE

QUBO (D-WAVE)

QAOA (**Xanadu**)

arXiv:2012.01121

```
+ (0.15625) [Z0 Z2]
+ (0.15625) [Z4 Z6]
+ (0.15625) [I0 I2]
+ (0.15625) [I4 I6]
+ (0.3125) [Z0 Z1]
+ (0.3125) [Z2 Z3]
+ (0.3125) [Z4 Z5]
+ (0.3125) [Z6 Z7]
+ (0.3125) [I0 I1]
+ (0.3125) [I2 I3]
+ (0.3125) [I4 I5]
+ (0.3125) [I6 I7]
+ (0.625) [Z0 Z3]
+ (0.625) [Z1 Z3]
+ (0.625) [Z4 Z7]
+ (0.625) [Z5 Z7]
+ (0.625) [I0 I3]
+ (0.625) [I1 I3]
+ (0.625) [I4 I7]
+ (0.625) [I5 I7]
```

```
  (-2.1880812564998986) [I1]
+ (-2.1880736479337526) [I5]
+ (-2.187871854002614) [I3]
+ (-2.18784328290288) [I7]
+ (-1.2496567170971198) [Z7]
+ (-1.2496281459973861) [Z3]
+ (-1.1721656282499493) [I0]
+ (-1.1721618239668763) [I4]
+ (-1.172060927001307) [I2]
+ (-1.17204664145144) [I6]
+ (-1.0934631760331237) [Z4]
+ (-1.0934593717500507) [Z0]
+ (0.0005736479337525857) [Z5]
+ (0.000581256499898597) [Z1]
+ (0.1564216414514401) [Z6]
+ (0.1564359270130693) [Z2]
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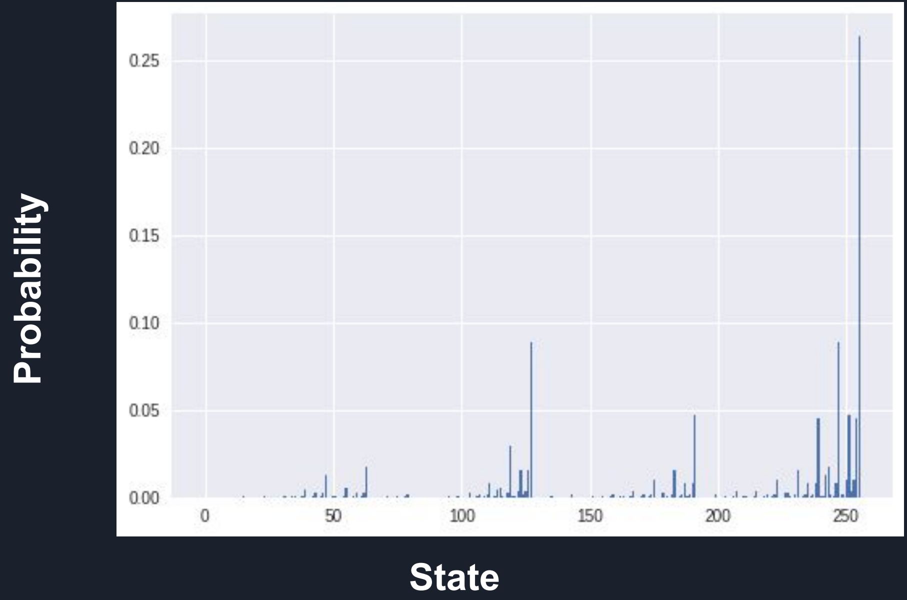

QUTO vs QUBO : Robust Equal Sum Partitioning

QUBO
3 N binary $x\epsilon\{0,1\}$
$x$io = $i \epsilon O$, $x$ia = $i \epsilon A$, $x$ib = $i \epsilon B$
One constraint $\sum ai\ x$ia+ $-\sum ai\ x$ib =0
N constraints $x$ia + $x$ib + $x$io=1
Minimise : $\sum x$io

QUTO :
N Ising trinary $x\epsilon\{-1,0,1\}$
$xi = 0 \rightarrow i\epsilon O,\ xi = -1 \rightarrow i\epsilon A,\ xi = +1 \rightarrow i\epsilon B$
One constraint : $\sum xi$ =0
Minimise : $-\sum xi$ ^2

	QUBO	QUTO
Variables	3 N	N
Constraints	N + 1	1
Cost	1	1

Thank you!