Hackathon Docs

cosmiQ

July 27, 2021

Introduction

Task 1

Task 2

$$
\begin{aligned}
\hat{D} = &\sum_{t_0}^{t_f-1}\sum_{i} -\mu(t,i)\left[\frac{1}{K}\sum_{q=0}^{N_q-1} d^q\hat{n}(i,t,q)\right] + \frac{\gamma}{2}\sum_{t=t_0}^{t_f-1}\sum_{ij}\Sigma_{ij}^{t}\left[\frac{1}{K}\sum_{q=0}^{N_q-1} d^q\hat{n}(i,t,q)\right]\left[\frac{1}{K}\sum_{\bar{q}=0}^{N_q-1} d^q\hat{n}(j,t,\bar{q})\right] \\
&+ \sum_{t_0}^{t_f-1}\sum_{ij}\lambda(t,i)\lambda(t,j)\Bigg\{\left[\frac{1}{K}\sum_{q=0}^{N_q-1} d^q\hat{n}(j,t+1,q)\right]\left[\frac{1}{K}\sum_{q=0}^{N_q-1} d^q\hat{n}(i,t+1,q)\right] \\
&- 2\left[\frac{1}{K}\sum_{q=0}^{N_q-1} d^q\hat{n}(i,t+1,q)\right]\left[\frac{1}{K}\sum_{q=0}^{N_q-1} d^q\hat{n}(j,t,q)\right] + \left[\frac{1}{K}\sum_{q=0}^{N_q-1} d^q\hat{n}(i,t,q)\right]\left[\frac{1}{K}\sum_{q=0}^{N_q-1} d^q\hat{n}(j,t,q)\right]\Bigg\} \\
&+ \rho\sum_{t_0}^{t_f}\sum_{ij}\left[\frac{1}{K}\sum_{q=0}^{N_q-1} d^q\hat{n}(i,t,q)\right]\left[\frac{1}{K}\sum_{\bar{q}=0}^{N_q-1} d^q\hat{n}(j,t,\bar{q})\right] - 2\rho\sum_{t_0}^{t_f}\sum_{i}\left[\frac{1}{K}\sum_{q=0}^{N_q-1} d^q\hat{n}(i,t,q)\right] + N_t\cdot N^2
\end{aligned}
\tag{1}
$$

Which we may further simplify by collecting terms:

$$
\begin{aligned}
K\hat{D} = &\sum_{t_0}^{t_f-1}\sum_{i}\sum_{q=0}^{N_q-1}(-\mu(t,i)-2\rho)d^q\hat{n}(i,t,q) \\
&+ \frac{1}{K}\sum_{t_0}^{t_f-1}\sum_{ij}\sum_{q=0}^{N_q-1}\sum_{\bar{q}=0}^{N_q-1}\left[\frac{\gamma}{2}\Sigma_{ij}^{t} + \lambda(t,i)\lambda(t,j) + \rho\right] d^q\hat{n}(i,t,q)d^q\hat{n}(j,t,\bar{q}) \\
&+ \frac{1}{K}\sum_{t_0}^{t_f-1}\sum_{ij}\sum_{q=0}^{N_q-1}\sum_{\bar{q}=0}^{N_q-1}\lambda(t,i)\lambda(t,j)\left[d^q\hat{n}(i,t+1,q)d^q\hat{n}(j,t+1,\bar{q}) - 2d^q\hat{n}(i,t+1,q)d^q\hat{n}(j,t,\bar{q})\right] \\
&+ \frac{\rho}{K}\sum_{ij}\sum_{q=0}^{N_q-1}\sum_{\bar{q}=0}^{N_q-1} d^q\hat{n}(i,t_f,q)d^q\hat{n}(j,t_f,\bar{q}) - 2\rho\sum_{i}\sum_{q=0}^{N_q-1} d^q\hat{n}(i,t_f,q) \\
&+ K\cdot N_t\cdot N^2
\end{aligned}
\tag{2}
$$

Note that while the boundary term contributes to the inner Hamiltonian, it only appears as a separate term due to the constraint. We have no information available about relative profits ($\mu(i, t_f)$), risk ($\Sigma_{ij}^{t_f}$), and transaction cost ($\lambda(t, i)$) at the final time $t = t_f$.

Using $\mu(t_f, i) = 0$ and $\Sigma^{t_f} = 0$, a more compressed notation gives us:

$$
\begin{aligned}
K\hat{D} = &\sum_{t_0}^{t_f} \sum_{i} \sum_{q=0}^{N_q-1} (-\mu(t,i) - 2\rho) d^q \hat{n}(i,t,q) \\
&+ \frac{1}{K} \sum_{t_0}^{t_f} \sum_{i<j} \sum_{q=0}^{N_q-1} \sum_{\bar{q}=0}^{N_q-1} 2\left[\frac{\gamma}{2}\Sigma_{ij}^t + \lambda(t,i)\lambda(t,j) + \rho\right] d^q \hat{n}(i,t,q) d^q \hat{n}(j,t,\bar{q}) \\
&+ \frac{1}{K} \sum_{t_0}^{t_f} \sum_{i} \sum_{q=0}^{N_q-1} \sum_{\bar{q}=0}^{N_q-1} \left[\frac{\gamma}{2}\Sigma_{ii}^t + \lambda(t,i)\lambda(t,i) + \rho\right] d^q \hat{n}(i,t,q) d^q \hat{n}(i,t,\bar{q}) \\
&+ \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_{i<j} \sum_{q=0}^{N_q-1} \sum_{\bar{q}=0}^{N_q-1} 2\lambda(t,i)\lambda(t,j) \left[d^q \hat{n}(i,t+1,q) d^q \hat{n}(j,t+1,\bar{q}) - 2 d^q \hat{n}(i,t+1,q) d^q \hat{n}(j,t,\bar{q})\right] \\
&+ \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_{i} \sum_{q=0}^{N_q-1} \sum_{\bar{q}=0}^{N_q-1} 2\lambda(t,i)\lambda(t,i) \left[d^q \hat{n}(i,t+1,q) d^q \hat{n}(i,t+1,\bar{q}) - 2 d^q \hat{n}(i,t+1,q) d^q \hat{n}(i,t,\bar{q})\right] \\
&+ K \cdot N_t \cdot N^2
\end{aligned} \tag{3}
$$

For MPS we need to collect terms a little differently:

$$
\begin{aligned}
K\hat{D} = &\sum_{t_0}^{t_f}\sum_{i}\sum_{q=0}^{N_q-1}(-\mu(t,i)-2\rho)d^q\hat{n}(i,t,q)\\
&+\frac{1}{K}\sum_{t_0}^{t_f}\sum_{i<j}\sum_{q<\bar{q}}^{N_q-1}4\left[\frac{\gamma}{2}\Sigma^t_{ij}+\lambda(t,i)\lambda(t,j)+\rho\right]d^q\hat{n}(i,t,q)d^q\hat{n}(j,t,\bar{q})\\
&+\frac{1}{K}\sum_{t_0}^{t_f}\sum_{i<j}\sum_{q}^{N_q-1}2\left[\frac{\gamma}{2}\Sigma^t_{ij}+\lambda(t,i)\lambda(t,j)+\rho\right]d^q\hat{n}(i,t,q)d^q\hat{n}(j,t,q)\\
&+\frac{1}{K}\sum_{t_0}^{t_f}\sum_{i}\sum_{q<\bar{q}}^{N_q-1}2\left[\frac{\gamma}{2}\Sigma^t_{ii}+\lambda(t,i)\lambda(t,i)+\rho\right]d^q\hat{n}(i,t,q)d^q\hat{n}(i,t,\bar{q})\\
&+\frac{1}{K}\sum_{t_0}^{t_f}\sum_{i}\sum_{q=0}^{N_q-1}\left[\frac{\gamma}{2}\Sigma^t_{ii}+\lambda(t,i)\lambda(t,i)+\rho\right]d^q\hat{n}(i,t,q)d^q\hat{n}(i,t,q)\\
&+\frac{1}{K}\sum_{t_0}^{t_f-1}\sum_{i<j}\sum_{q<\bar{q}}^{N_q-1}4\lambda(t,i)\lambda(t,j)\left[d^q\hat{n}(i,t+1,q)d^q\hat{n}(j,t+1,\bar{q})-2d^q\hat{n}(i,t+1,q)d^q\hat{n}(j,t,\bar{q})\right]\\
&+\frac{1}{K}\sum_{t_0}^{t_f-1}\sum_{i<j}\sum_{q=0}^{N_q-1}2\lambda(t,i)\lambda(t,j)\left[d^q\hat{n}(i,t+1,q)d^q\hat{n}(j,t+1,q)-2d^q\hat{n}(i,t+1,q)d^q\hat{n}(j,t,q)\right]\\
&+\frac{1}{K}\sum_{t_0}^{t_f-1}\sum_{i}\sum_{q<\bar{q}}^{N_q-1}4\lambda(t,i)\lambda(t,i)\left[d^q\hat{n}(i,t+1,q)d^q\hat{n}(i,t+1,\bar{q})-2d^q\hat{n}(i,t+1,q)d^q\hat{n}(i,t,\bar{q})\right]\\
&+\frac{1}{K}\sum_{t_0}^{t_f-1}\sum_{i}\sum_{q=0}^{N_q-1}2\lambda(t,i)\lambda(t,i)\left[d^q\hat{n}(i,t+1,q)d^q\hat{n}(i,t+1,q)-2d^q\hat{n}(i,t+1,q)d^q\hat{n}(i,t,q)\right]\\
&+K\cdot N_t\cdot N^2
\end{aligned}
\tag{4}
$$

And we rearrange to have all the on-site pieces together:

$$
\begin{aligned}
K\hat{D} = &\sum_{t_0}^{t_f}\sum_{i}\sum_{q=0}^{N_q-1}(-\mu(t,i)-2\rho)d^q\hat{n}(i,t,q) \\
&+\frac{1}{K}\sum_{t_0}^{t_f}\sum_{i}\sum_{q=0}^{N_q-1}\left[\frac{\gamma}{2}\Sigma_{ii}^t+\lambda(t,i)\lambda(t,i)+\rho\right]d^q\hat{n}(i,t,q)d^q\hat{n}(i,t,q) \\
&+\frac{1}{K}\sum_{t_0=1}^{t_f}\sum_{i}\sum_{q=0}^{N_q-1}2\lambda(t-1,i)\lambda(t-1,i)d^q\hat{n}(i,t,q)d^q\hat{n}(i,t,q) \\
&+\frac{1}{K}\sum_{t_0}^{t_f}\sum_{i}\sum_{q<\bar{q}}^{N_q-1}2\left[\frac{\gamma}{2}\Sigma_{ii}^t+\lambda(t,i)\lambda(t,i)+\rho\right]d^q\hat{n}(i,t,q)d^q\hat{n}(i,t,\bar{q}) \\
&+\frac{1}{K}\sum_{t_0}^{t_f-1}\sum_{i}\sum_{q<\bar{q}}^{N_q-1}4\lambda(t,i)\lambda(t,i)\left[d^q\hat{n}(i,t+1,q)d^q\hat{n}(i,t+1,\bar{q})-2d^q\hat{n}(i,t+1,q)d^q\hat{n}(i,t,\bar{q})\right] \\
&+\frac{1}{K}\sum_{t_0}^{t_f}\sum_{i<j}\sum_{q<\bar{q}}^{N_q-1}4\left[\frac{\gamma}{2}\Sigma_{ij}^t+\lambda(t,i)\lambda(t,j)+\rho\right]d^q\hat{n}(i,t,q)d^q\hat{n}(j,t,\bar{q}) \\
&+\frac{1}{K}\sum_{t_0}^{t_f}\sum_{i<j}\sum_{q}^{N_q-1}2\left[\frac{\gamma}{2}\Sigma_{ij}^t+\lambda(t,i)\lambda(t,j)+\rho\right]d^q\hat{n}(i,t,q)d^q\hat{n}(j,t,q) \\
&+\frac{1}{K}\sum_{t_0}^{t_f-1}\sum_{i<j}\sum_{q<\bar{q}}^{N_q-1}4\lambda(t,i)\lambda(t,j)\left[d^q\hat{n}(i,t+1,q)d^q\hat{n}(j,t+1,\bar{q})-2d^q\hat{n}(i,t+1,q)d^q\hat{n}(j,t,\bar{q})\right] \\
&+\frac{1}{K}\sum_{t_0}^{t_f-1}\sum_{i<j}\sum_{q=0}^{N_q-1}2\lambda(t,i)\lambda(t,j)\left[d^q\hat{n}(i,t+1,q)d^q\hat{n}(j,t+1,q)-2d^q\hat{n}(i,t+1,q)d^q\hat{n}(j,t,q)\right] \\
&+\frac{1}{K}\sum_{t_0}^{t_f-1}\sum_{i}\sum_{q=0}^{N_q-1}2\lambda(t,i)\lambda(t,i)\left[-2d^q\hat{n}(i,t+1,q)d^q\hat{n}(i,t,q)\right] \\
&+K\cdot N_t\cdot N^2
\end{aligned}
\tag{5}
$$

References