

Hackathon Docs

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Introduction

Task 1

Task 2

$$\begin{aligned}\hat{D} = & \sum_{t_0}^{t_f-1} \sum_i -\mu(t, i) \left[\frac{1}{K} \sum_{q=0}^{N_q-1} d^q \hat{n}(i, t, q) \right] + \frac{\gamma}{2} \sum_{t=t_0}^{t_f-1} \sum_{ij} \Sigma_{ij}^t \left[\frac{1}{K} \sum_{q=0}^{N_q-1} d^q \hat{n}(i, t, q) \right] \left[\frac{1}{K} \sum_{\bar{q}=0}^{N_q-1} d^q \hat{n}(j, t, \bar{q}) \right] \\ & + \sum_{t_0}^{t_f-1} \sum_{ij} \lambda(t, i) \lambda(t, j) \left\{ \left[\frac{1}{K} \sum_{q=0}^{N_q-1} d^q \hat{n}(j, t+1, q) \right] \left[\frac{1}{K} \sum_{q=0}^{N_q-1} d^q \hat{n}(i, t+1, q) \right] \right. \\ & - 2 \left[\frac{1}{K} \sum_{q=0}^{N_q-1} d^q \hat{n}(i, t+1, q) \right] \left[\frac{1}{K} \sum_{q=0}^{N_q-1} d^q \hat{n}(j, t, q) \right] + \left[\frac{1}{K} \sum_{q=0}^{N_q-1} d^q \hat{n}(i, t, q) \right] \left[\frac{1}{K} \sum_{q=0}^{N_q-1} d^q \hat{n}(j, t, q) \right] \left. \vphantom{\sum_{q=0}^{N_q-1}} \right\} \\ & + \rho \sum_{t_0}^{t_f} \sum_{ij} \left[\frac{1}{K} \sum_{q=0}^{N_q-1} d^q \hat{n}(i, t, q) \right] \left[\frac{1}{K} \sum_{\bar{q}=0}^{N_q-1} d^q \hat{n}(j, t, \bar{q}) \right] - 2\rho \sum_{t_0}^{t_f} \sum_i \left[\frac{1}{K} \sum_{q=0}^{N_q-1} d^q \hat{n}(i, t, q) \right] + N_t \cdot N^2\end{aligned}\tag{1}$$

Which we may further simplify by collecting terms:

$$\begin{aligned}K\hat{D} = & \sum_{t_0}^{t_f-1} \sum_i \sum_{q=0}^{N_q-1} (-\mu(t, i) - 2\rho) d^q \hat{n}(i, t, q) \\ & + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_{ij} \sum_{q=0}^{N_q-1} \sum_{\bar{q}=0}^{N_q-1} \left[\frac{\gamma}{2} \Sigma_{ij}^t + \lambda(t, i) \lambda(t, j) + \rho \right] d^q \hat{n}(i, t, q) d^q \hat{n}(j, t, \bar{q}) \\ & + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_{ij} \sum_{q=0}^{N_q-1} \sum_{\bar{q}=0}^{N_q-1} \lambda(t, i) \lambda(t, j) [d^q \hat{n}(i, t+1, q) d^q \hat{n}(j, t+1, \bar{q}) - 2d^q \hat{n}(i, t+1, q) d^q \hat{n}(j, t, \bar{q})] \\ & + \frac{\rho}{K} \sum_{ij} \sum_{q=0}^{N_q-1} \sum_{\bar{q}=0}^{N_q-1} d^q \hat{n}(i, t_f, q) d^q \hat{n}(j, t_f, \bar{q}) - 2\rho \sum_i \sum_{q=0}^{N_q-1} d^q \hat{n}(i, t_f, q) \\ & + K \cdot N_t \cdot N^2\end{aligned}\tag{2}$$

Note that while the boundary term contributes to the inner Hamiltonian, it only appears as a separate term due to the constraint. We have no information available about relative profits ($\mu(i, t_f)$), risk ($\Sigma_{ij}^{t_f}$), and transaction cost ($\lambda(t, i)$) at the final time $t = t_f$.

Using $\mu(t_f, i) = 0$ and $\Sigma^{t_f} = 0$, a more compressed notation gives us:

$$\begin{aligned}
K\hat{D} = & \sum_{t_0}^{t_f} \sum_i \sum_{q=0}^{N_q-1} (-\mu(t, i) - 2\rho) d^q \hat{n}(i, t, q) \\
& + \frac{1}{K} \sum_{t_0}^{t_f} \sum_{i < j} \sum_{q=0}^{N_q-1} \sum_{\bar{q}=0}^{N_q-1} 2 \left[\frac{\gamma}{2} \Sigma_{ij}^t + \lambda(t, i) \lambda(t, j) + \rho \right] d^q \hat{n}(i, t, q) d^{\bar{q}} \hat{n}(j, t, \bar{q}) \\
& + \frac{1}{K} \sum_{t_0}^{t_f} \sum_i \sum_{q=0}^{N_q-1} \sum_{\bar{q}=0}^{N_q-1} \left[\frac{\gamma}{2} \Sigma_{ii}^t + \lambda(t, i) \lambda(t, i) + \rho \right] d^q \hat{n}(i, t, q) d^{\bar{q}} \hat{n}(i, t, \bar{q}) \\
& + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_{i < j} \sum_{q=0}^{N_q-1} \sum_{\bar{q}=0}^{N_q-1} 2 \lambda(t, i) \lambda(t, j) [d^q \hat{n}(i, t+1, q) d^{\bar{q}} \hat{n}(j, t+1, \bar{q}) - 2 d^q \hat{n}(i, t+1, q) d^{\bar{q}} \hat{n}(j, t, \bar{q})] \\
& + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_i \sum_{q=0}^{N_q-1} \sum_{\bar{q}=0}^{N_q-1} 2 \lambda(t, i) \lambda(t, i) [d^q \hat{n}(i, t+1, q) d^{\bar{q}} \hat{n}(i, t+1, \bar{q}) - 2 d^q \hat{n}(i, t+1, q) d^{\bar{q}} \hat{n}(i, t, \bar{q})] \\
& + K \cdot N_t \cdot N^2
\end{aligned} \tag{3}$$

For MPS we need to collect terms a little differently:

$$\begin{aligned}
K\hat{D} = & \sum_{t_0}^{t_f} \sum_i \sum_{q=0}^{N_q-1} (-\mu(t, i) - 2\rho) d^q \hat{n}(i, t, q) \\
& + \frac{1}{K} \sum_{t_0}^{t_f} \sum_{i < j} \sum_{q < \bar{q}}^{N_q-1} 4 \left[\frac{\gamma}{2} \Sigma_{ij}^t + \lambda(t, i) \lambda(t, j) + \rho \right] d^q \hat{n}(i, t, q) d^q \hat{n}(j, t, \bar{q}) \\
& + \frac{1}{K} \sum_{t_0}^{t_f} \sum_{i < j} \sum_q^{N_q-1} 2 \left[\frac{\gamma}{2} \Sigma_{ij}^t + \lambda(t, i) \lambda(t, j) + \rho \right] d^q \hat{n}(i, t, q) d^q \hat{n}(j, t, q) \\
& + \frac{1}{K} \sum_{t_0}^{t_f} \sum_i \sum_{q < \bar{q}}^{N_q-1} 2 \left[\frac{\gamma}{2} \Sigma_{ii}^t + \lambda(t, i) \lambda(t, i) + \rho \right] d^q \hat{n}(i, t, q) d^q \hat{n}(i, t, \bar{q}) \\
& + \frac{1}{K} \sum_{t_0}^{t_f} \sum_i \sum_{q=0}^{N_q-1} \left[\frac{\gamma}{2} \Sigma_{ii}^t + \lambda(t, i) \lambda(t, i) + \rho \right] d^q \hat{n}(i, t, q) d^q \hat{n}(i, t, q) \\
& + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_{i < j} \sum_{q < \bar{q}}^{N_q-1} 4 \lambda(t, i) \lambda(t, j) [d^q \hat{n}(i, t+1, q) d^q \hat{n}(j, t+1, \bar{q}) - 2 d^q \hat{n}(i, t+1, q) d^q \hat{n}(j, t, \bar{q})] \\
& + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_{i < j} \sum_{q=0}^{N_q-1} 2 \lambda(t, i) \lambda(t, j) [d^q \hat{n}(i, t+1, q) d^q \hat{n}(j, t+1, q) - 2 d^q \hat{n}(i, t+1, q) d^q \hat{n}(j, t, q)] \\
& + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_i \sum_{q < \bar{q}}^{N_q-1} 4 \lambda(t, i) \lambda(t, i) [d^q \hat{n}(i, t+1, q) d^q \hat{n}(i, t+1, \bar{q}) - 2 d^q \hat{n}(i, t+1, q) d^q \hat{n}(i, t, \bar{q})] \\
& + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_i \sum_{q=0}^{N_q-1} 2 \lambda(t, i) \lambda(t, i) [d^q \hat{n}(i, t+1, q) d^q \hat{n}(i, t+1, q) - 2 d^q \hat{n}(i, t+1, q) d^q \hat{n}(i, t, q)] \\
& + K \cdot N_t \cdot N^2
\end{aligned} \tag{4}$$

And we rearrange to have all the on-site pieces together:

$$\begin{aligned}
K\hat{D} = & \sum_{t_0}^{t_f} \sum_i \sum_{q=0}^{N_q-1} (-\mu(t, i) - 2\rho) d^q \hat{n}(i, t, q) \\
& + \frac{1}{K} \sum_{t_0}^{t_f} \sum_i \sum_{q=0}^{N_q-1} \left[\frac{\gamma}{2} \Sigma_{ii}^t + \lambda(t, i) \lambda(t, i) + \rho \right] d^q \hat{n}(i, t, q) d^q \hat{n}(i, t, q) \\
& + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_i \sum_{q=0}^{N_q-1} 2\lambda(t, i) \lambda(t, i) d^q \hat{n}(i, t+1, q) d^q \hat{n}(i, t+1, q) \\
& + \frac{1}{K} \sum_{t_0}^{t_f} \sum_{i < j} \sum_{q < \bar{q}}^{N_q-1} 4 \left[\frac{\gamma}{2} \Sigma_{ij}^t + \lambda(t, i) \lambda(t, j) + \rho \right] d^q \hat{n}(i, t, q) d^q \hat{n}(j, t, \bar{q}) \\
& + \frac{1}{K} \sum_{t_0}^{t_f} \sum_{i < j} \sum_q^{N_q-1} 2 \left[\frac{\gamma}{2} \Sigma_{ij}^t + \lambda(t, i) \lambda(t, j) + \rho \right] d^q \hat{n}(i, t, q) d^q \hat{n}(j, t, q) \\
& + \frac{1}{K} \sum_{t_0}^{t_f} \sum_i \sum_{q < \bar{q}}^{N_q-1} 2 \left[\frac{\gamma}{2} \Sigma_{ii}^t + \lambda(t, i) \lambda(t, i) + \rho \right] d^q \hat{n}(i, t, q) d^q \hat{n}(i, t, \bar{q}) \\
& + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_{i < j} \sum_{q < \bar{q}}^{N_q-1} 4\lambda(t, i) \lambda(t, j) [d^q \hat{n}(i, t+1, q) d^q \hat{n}(j, t+1, \bar{q}) - 2d^q \hat{n}(i, t+1, q) d^q \hat{n}(j, t, \bar{q})] \\
& + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_{i < j} \sum_{q=0}^{N_q-1} 2\lambda(t, i) \lambda(t, j) [d^q \hat{n}(i, t+1, q) d^q \hat{n}(j, t+1, q) - 2d^q \hat{n}(i, t+1, q) d^q \hat{n}(j, t, q)] \\
& + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_i \sum_{q < \bar{q}}^{N_q-1} 4\lambda(t, i) \lambda(t, i) [d^q \hat{n}(i, t+1, q) d^q \hat{n}(i, t+1, \bar{q}) - 2d^q \hat{n}(i, t+1, q) d^q \hat{n}(i, t, \bar{q})] \\
& + \frac{1}{K} \sum_{t_0}^{t_f-1} \sum_i \sum_{q=0}^{N_q-1} 2\lambda(t, i) \lambda(t, i) [-2d^q \hat{n}(i, t+1, q) d^q \hat{n}(i, t, q)] \\
& + K \cdot N_t \cdot N^2
\end{aligned} \tag{5}$$

References