1. Gsolve

Gsolve is a Python computer program designed to make the routine data processing of relative gravity surveys user friendly through the use of an intuitive graphical user interface. The program corrects relative gravity meter readings for gravimeter drift and tidal gravitational effects and can account for tares in the gravimeter base line and an additional gravimeter calibration factor.

Gsolve also has the ability to calculate free air gravity anomalies relative to the GRS80 and GRS67 reference ellipsoids and compute terrain corrections with a user supplied digital elevation model. Details of the program are given in the following subsections.

1.1. Relative gravity data processing

Importing survey data. The relative gravity measurements, location identities, the time and date (in UTC) and location of each observation must first be imported or manually entered into the program. These input data will be used to construct the system of linear equations of equation **??**. Figure 1 shows the graphical user interface where this operation is performed, this is the first page the user is shown upon running the program.

The user can either import the data from an excel spread sheet (by press-

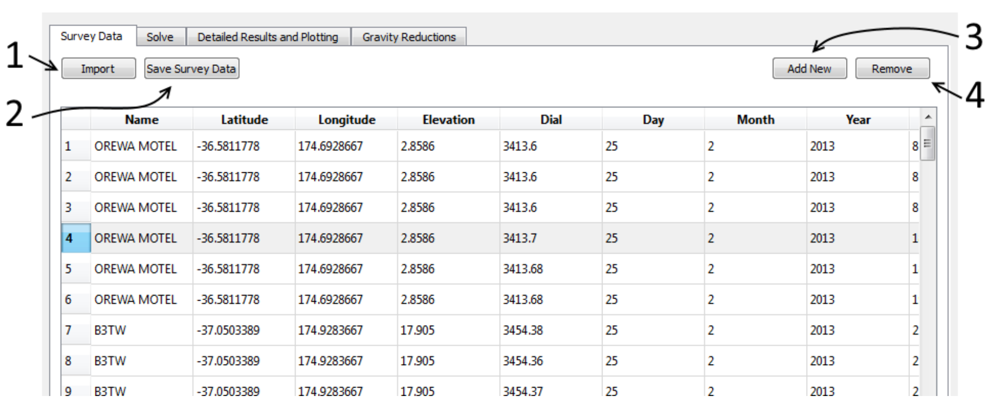


Figure 1: Graphical user interface to import relative gravity survey data into Gsolve.

ing (1)) with the specified format given by the example survey data file example_survey_data.xls or enter the observations manually one by one (by pressing (3)). Rows of data can be removed using button (4) and the survey data can be exported from Gsolve by clicking (2).

Here the user can also assign individual loop numbers to portions of the observation data. When solving the system of linear equations, given by equations **??** and **??**, Gsolve computes separate gravimeter baseline values a and drift coefficients b for each loop. This is advantageous since it can be used to account for tares in the gravimeter base line value and variable drift rates between separate days of data.

Absolute gravity sites and gravimeter calibration tables stored in Gsolve. Gsolve stores 2 databases, the first contains the absolute gravity reference sites to be used for equation **??** and the second contains gravimeter calibration tables to transform the gravimeter dial readings into gravity units $x_{i,j}$.

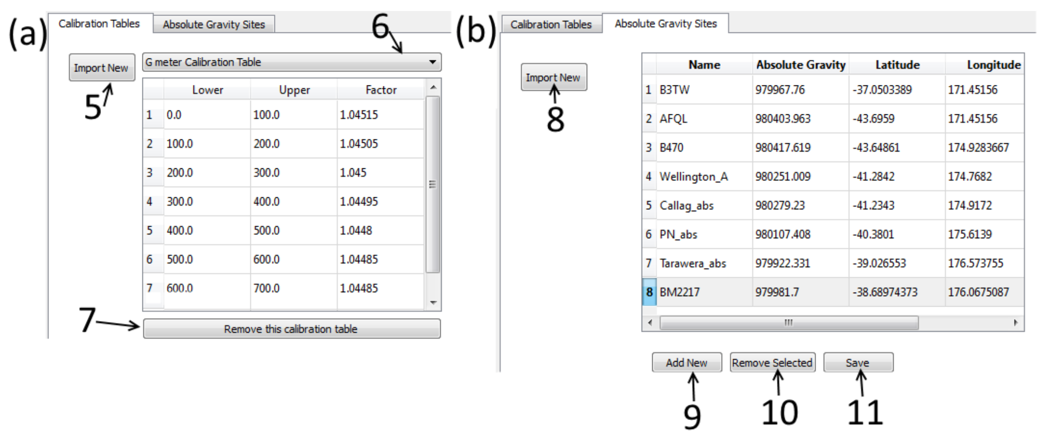


Figure 2: (a) shows the graphical user interface to view, import and delete gravimeter calibration tables from the Gsolve database. (b) shows the graphical user interface to view, import, modify, save and delete absolute gravity reference sites from the Gsolve database.

Figure 2 shows the graphical user interface which can be used to interact with the databases. New gravimeter calibration tables can be imported and saved in the Gsolve database by clicking button (5) and following the on screen instructions (the file format is given by calibration_table_example.xls), tables in the database can be viewed by selecting one of the drop down options from (6) and tables can be deleted from the database by clicking (7). New absolute gravity values can be added to the database by clicking "Import" (8) and importing a files with the same format as example_absolute_gravity.xls, or by clicking "Add New" (9) and entering the reference site details manually.

Determining least squares estimate of the surveyed sites absolute gravity value. The graphical user interface to solve the systems of linear equations by least squares is given by figure 3.

First the gravimeter calibration table (which must be already stored in the Gsolve database) must be specified using using the drop menu (12). Next the user must specify the reference absolute gravity site, to construct equations of the form of equation **??**, by clicking (13), the program will prompt the user to find them in the database by matching the surveyed site names with those stored or to enter them manually, any falsely added reference site can be removed by selecting the row and clicking (14).

The user must then select whether or not to calculate separate gravimeter baselines $\tilde{a}$ and drift rates $\tilde{b}$ for each loop identified in the survey data by clicking (15). Typically, processing the data with multiple loops results in smaller $\tilde{g}_j$ parameter standard error estimates since tares in the gravimeter base line between separate loops can be accounted for adequately.
A residual confidence interval can then be chosen (16), the default is 100%. When this value is set to be less than 100%, it causes Gsolve to produce an initial solution, identify outliers outside of the specified confidence interval then compute a new solution with the outliers removed. This is useful to remove the influence of erroneous measurements on the determined parameters $\tilde{g}_j$, $\tilde{a}$ and $\tilde{b}$.

Reilly (1970) identified a method to determine a further calibration scaling factor β for the relative gravity readings to account for inaccurate or non existent calibration tables. To compute this value by least squares, equation

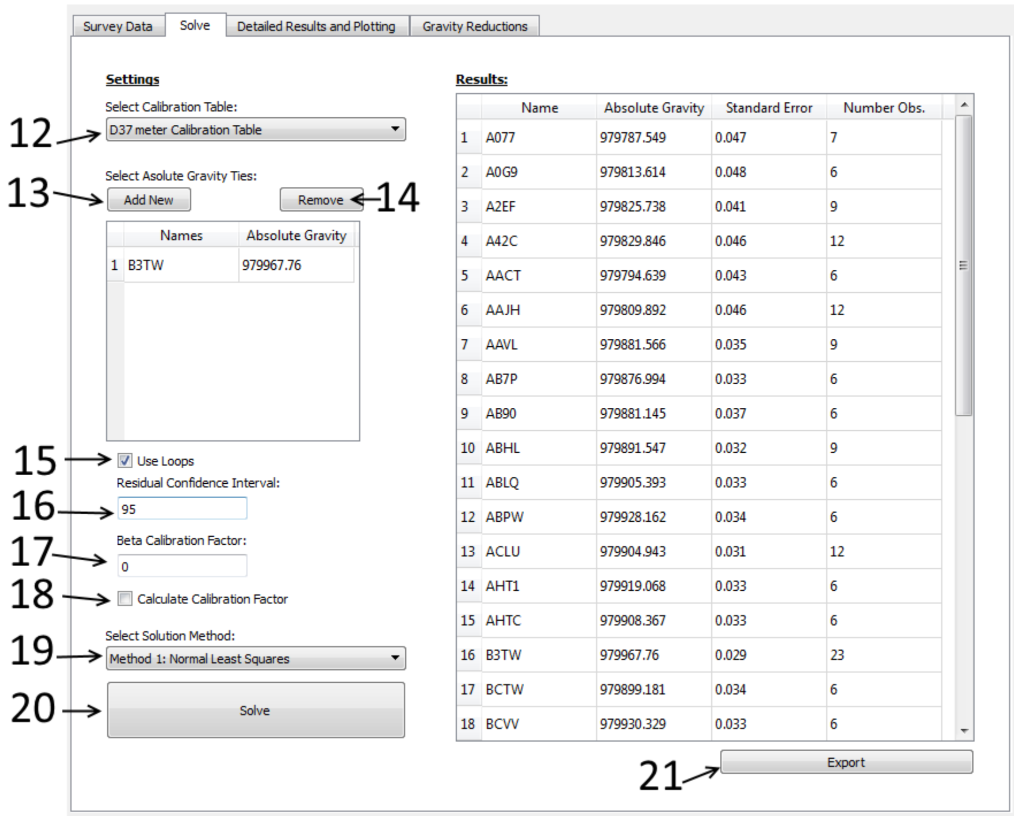


Figure 3: Specify the gravimeter calibration table to be used, the absolute gravity reference sites, the settings of the least squares solution and calculate the absolute gravity values.

?? must be modified to give equation 1,

$$\begin{vmatrix} P & -\mathbf{1} & -\mathbf{T} & \mathbf{x} \\ Q & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} \mathbf{g} \\ a \\ b \\ \beta \end{vmatrix} = \begin{vmatrix} \mathbf{x}-\mathbf{z} \\ \mathbf{h} \end{vmatrix} + \begin{vmatrix} \mathbf{e} \\ \epsilon \end{vmatrix} \tag{1}$$

To determine the value of β the survey data must be reference to at least two absolute gravity sites.

The β value can be determined by Gsolve by clicking (18). Alternatively it can be specified by the user in (17), using a previously derived value, and used to calibrate the gravimeter readings prior to determining the least squares squares solution. The default option is to not calculate β and to set it equal to zero, i.e. it is excluded from the calculation.

The users can then specify the least squares method to be used, (i.e. normal least squares, decoupled least squares or constrained least squares) to find the absolute gravity at each surveyed location.

The least squares calculation is finally initiated initiated by pressing "Solve" (20). The gravimeter readings are first calibrated using the specified calibration table (and β factor if supplied), tidal gravity effects are then calculated and subtracted from the observations using the Longman (1959) formula, then the system of linear equations is arranged and the least squares solution to parameters $\tilde{\mathbf{g}}$, $\tilde{a}$, $\tilde{b}$ and β (if required) are determined.

The least squares determined absolute gravity at each site is shown in the table along side standard errors of the determined value and the number of observations used. The data in the table can be exported to an xls file by clicking (21) and following the on screen instructions.

Detailed Output. A detailed output of the least squares solution is given in the "Detailed Results and Plotting" tab which can be seen in figure 4.

For each observation in the survey data the site name, position, dial and calibrated dial reading, date and time, tidal effect, loop identity and residuals are given in the table. The drift rate per hour for each loop is given by (22) and the β value (whether specified or calculated) is given by (23).
The drift function and observation residuals around the drift function of any individual loop can be plotted using (25) and similarly a cumulative distribution function of the observation residuals can be plotted using (24) . These plots are useful to help identify outliers or whether the use of a confidence interval confidence interval would be appropriate.

The detailed output data can be exported to an xls file by clicking (26).

1.2. Computing gravity anomalies

Free air anomalies. Once absolute gravity values have been obtained at each observation site, Gsolve can also be used to determine free air gravity anomalies with respect to the GRS80 or GRS67 reference ellipsoid. This can be

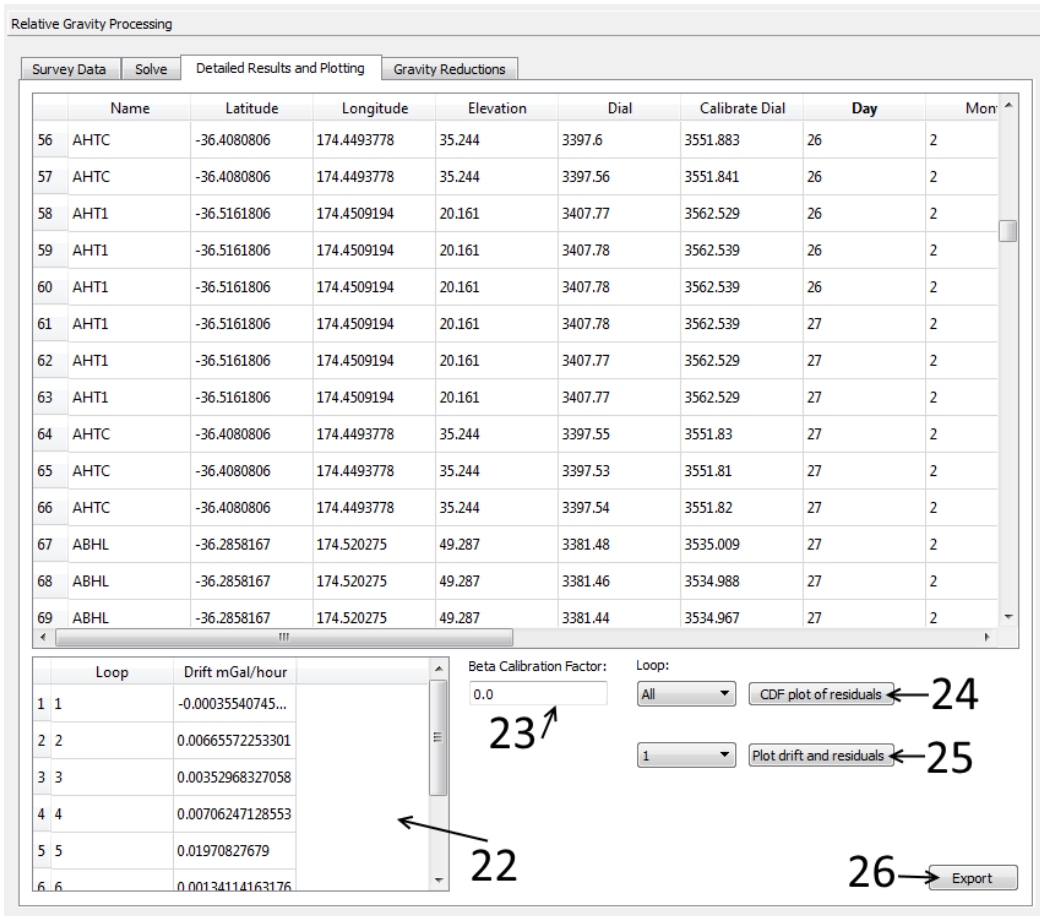
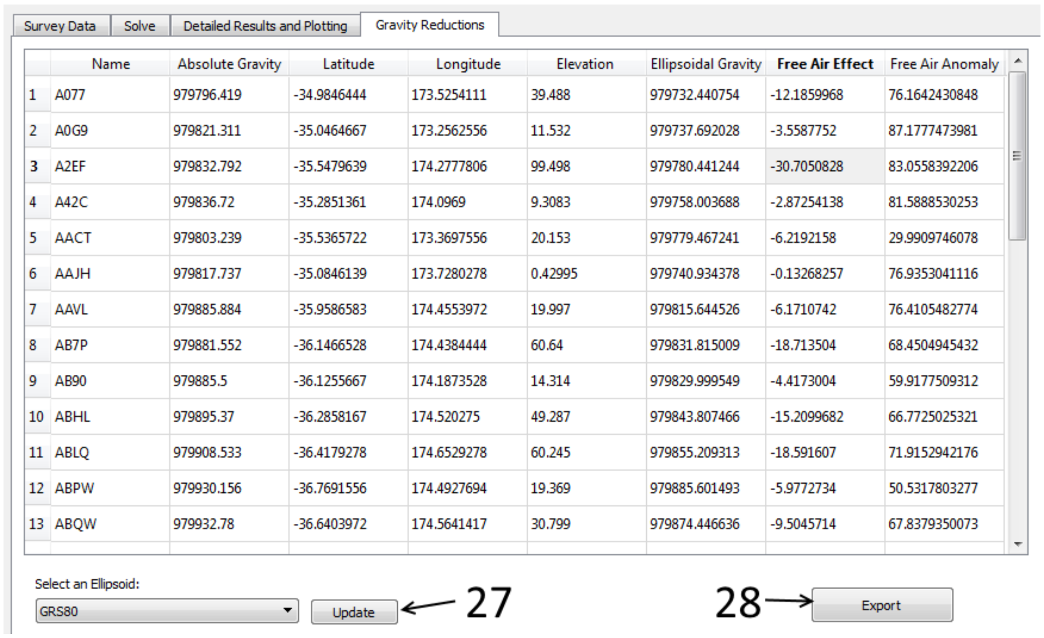


Figure 4: Detailed results output graphical user interface. It gives a table with the site name, position, dial and calibrated dial reading, date and time, tidal effect, loop identity and residuals for each observation, drfit rates for each loop, β calibration factor and plotting options of the least squares residuals and drift functions.

done by selecting an ellipsoid from the drop down menu and clicking the update button (27). The graphical user interface to view and export (using button (28)) the gravity correction values and gravity anomalies is shown in figure 5.

The ellipsoidal gravity is calculated using the Moritz (1980) formula given



	Name	Absolute Gravity	Latitude	Longitude	Elevation	Ellipsoidal Gravity	**Free Air Effect**	Free Air Anomaly
1	A077	979796.419	-34.9846444	173.5254111	39.488	979732.440754	-12.1859968	76.1642430848
2	A0G9	979821.311	-35.0464667	173.2562556	11.532	979737.692028	-3.5587752	87.1777473981
3	A2EF	979832.792	-35.5479639	174.2777806	99.498	979780.441244	-30.7050828	83.0558392206
4	A42C	979836.72	-35.2851361	174.0969	9.3083	979758.003688	-2.87254138	81.5888530253
5	AACT	979803.239	-35.5365722	173.3697556	20.153	979779.467241	-6.2192158	29.9909746078
6	AAJH	979817.737	-35.0846139	173.7280278	0.42995	979740.934378	-0.13268257	76.9353041116
7	AAVL	979885.884	-35.9586583	174.4553972	19.997	979815.644526	-6.1710742	76.4105482774
8	AB7P	979881.552	-36.1466528	174.4384444	60.64	979831.815009	-18.713504	68.4504945432
9	AB90	979885.5	-36.1255667	174.1873528	14.314	979829.999549	-4.4173004	59.9177509312
10	ABHL	979895.37	-36.2858167	174.520275	49.287	979843.807466	-15.2099682	66.7725025321
11	ABLQ	979908.533	-36.4179278	174.6529278	60.245	979855.209313	-18.591607	71.9152942176
12	ABPW	979930.156	-36.7691556	174.4927694	19.369	979885.601493	-5.9772734	50.5317803277
13	ABQW	979932.78	-36.6403972	174.5641417	30.799	979874.446636	-9.5045714	67.8379350073

Figure 5: Graphical user interface of surveyed site gravity anomaly derivation. Here a choice of the GRS80 or GRS67 reference ellipsoids can be made and the data can be exported to an xls file.

by equation 2,

$$\gamma(\phi) = \gamma_a \frac{1 + ksin^2(\phi)}{\sqrt{1 - e^2 sin^2(\phi)}} \tag{2}$$

where ϕ is the ellipsoidal latitude and remaining the parameters are,

- γ_a=978032.67715
- k=0.001931851353
- e^2=0.00669438002290

for GRS80 and

- γ_a=978031.84558
- k=0.001931663383
- e^2=0.00669460532856

for GRS67. The free air effect is estimated using the standard formula $-0.3086 H mGal/m$ where H is the height in meters.

Terrain corrections. Gsolve is also able to determine terrain corrections using the the Naggy (1966) prism formula. The user must first read in a digital elevation model (DEM) in an xyz (Eastings, Northing, Height in metres) data format (an example is given by the file NLDEM.txt). This file format can easily be changed within the code to accommodate other data types if required.

The user interface to import the DEM is shown in figure 6. Button (29) allows the user to select their DEM file which is plotted and the resolution is specified in text box (30).

Next, the user must again import the location data file which contains the Eastings and Northings of the gravity sites. The graphical user interface to do this is shown in figure 7.

The file read in is the same file type as read in for the relative gravity data processing but must have two additional channels, Eastings and Northings, on the locations sheet of the xls file. The Eastings and Northings must be in terms of the same projection as the DEM. These additional columns are shown in the example file example_survey_data.xls.

When the gravity sites are read in (by clicking (31)), the program prompts the user to specify which type of elevation to use for the correction. Here there are three options,

1. Gsolve can determine the heights from the DEM by clicking "yes" to the first option,
2. extract priorly determined DEM heights from the survey data file (i.e. stored in the 7th column of the locations sheet) by clicking "No" tot he first option and "Yes" to the second option,
3. or extract field observed heights from the survey files (i.e. stored in the 4th column of the locations sheet) by clicking "No" to the first and second options.

If the gravity sites are located within the bounds of the DEM, option 1 is typical used. If the gravity sites are located outside the bounds of the DEM

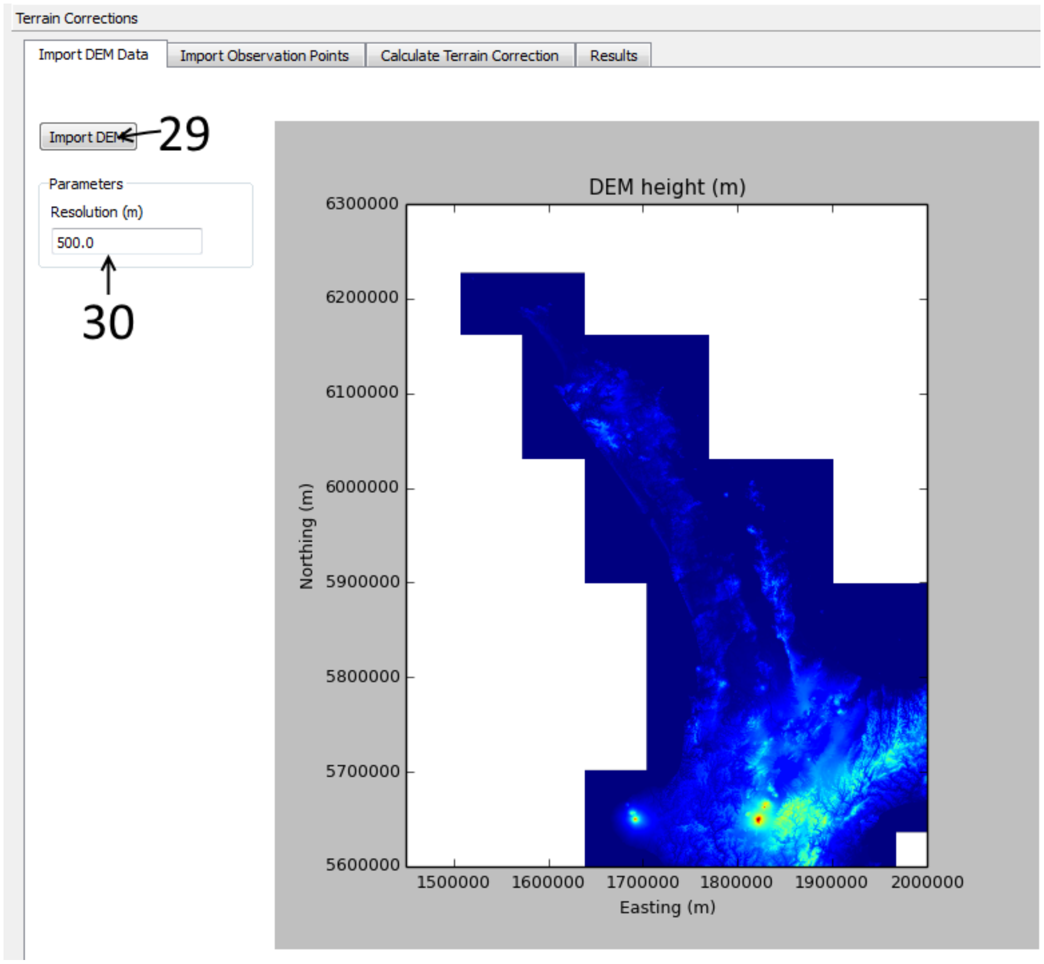


Figure 6: Graphical user interface to import a digital elevation model file.

then but DEM heights have been computed previously these can be placed in column 7 of the survey data file (e.g. example_survey_data.xls) of the "locations" sheet and option 2 should be used. The user may, in some circumstances, find it preferable to use heights established in the field or some other height for the terrain correction, in this instance option 3 can be used.

The user must then select the "Calculate Terrain Correction" tab to specify the inner (32) and outer (33) radius of the terrain correction (i.e. the correction only accounts for the topography in side an annulus with the specified

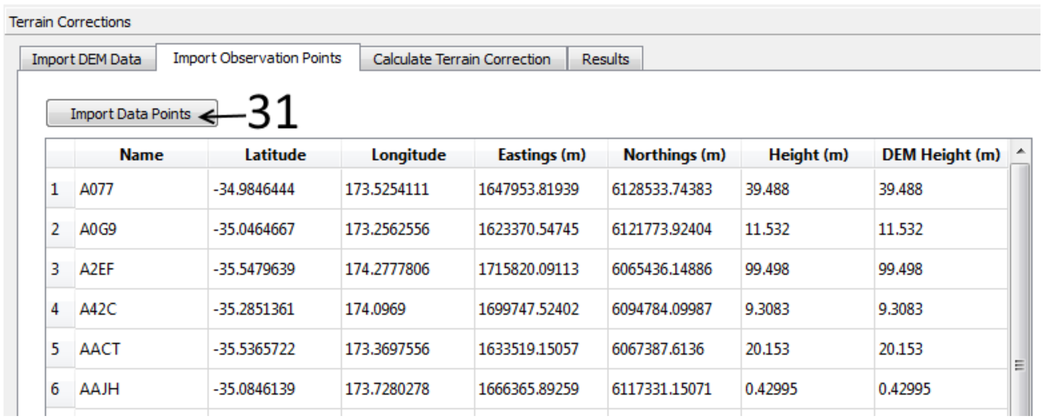


Figure 7: Graphical user interface to import a gravity sites data for terrain corrections.

inner radius and outer radius) and the rock density (34) in kg/m^3 using the graphical user interface shown by figure 8.

To initiate the calculation the user needs to click the button (35). When the calculation is complete the 'x' marks on the plot turn white and the terrain correction at each site is given on the plot.

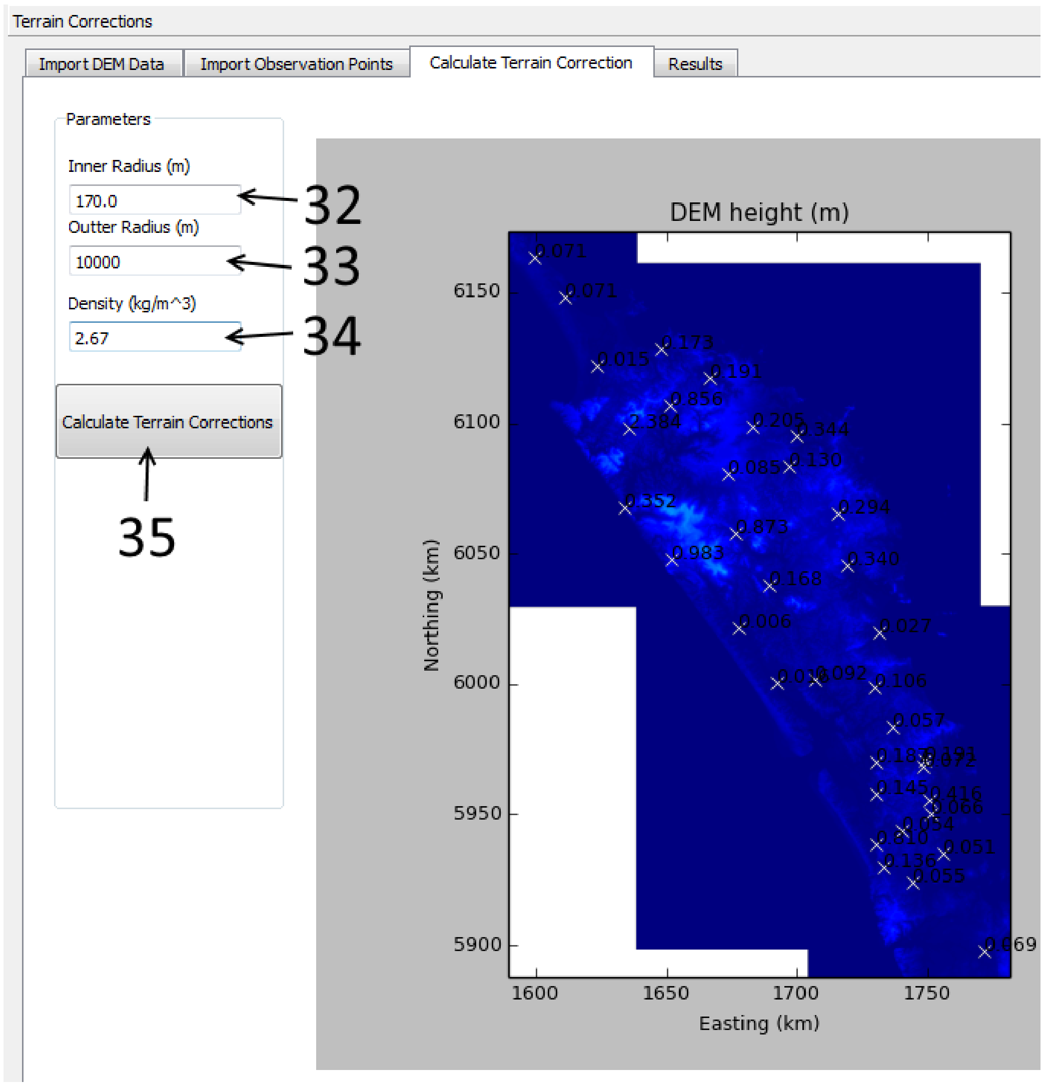
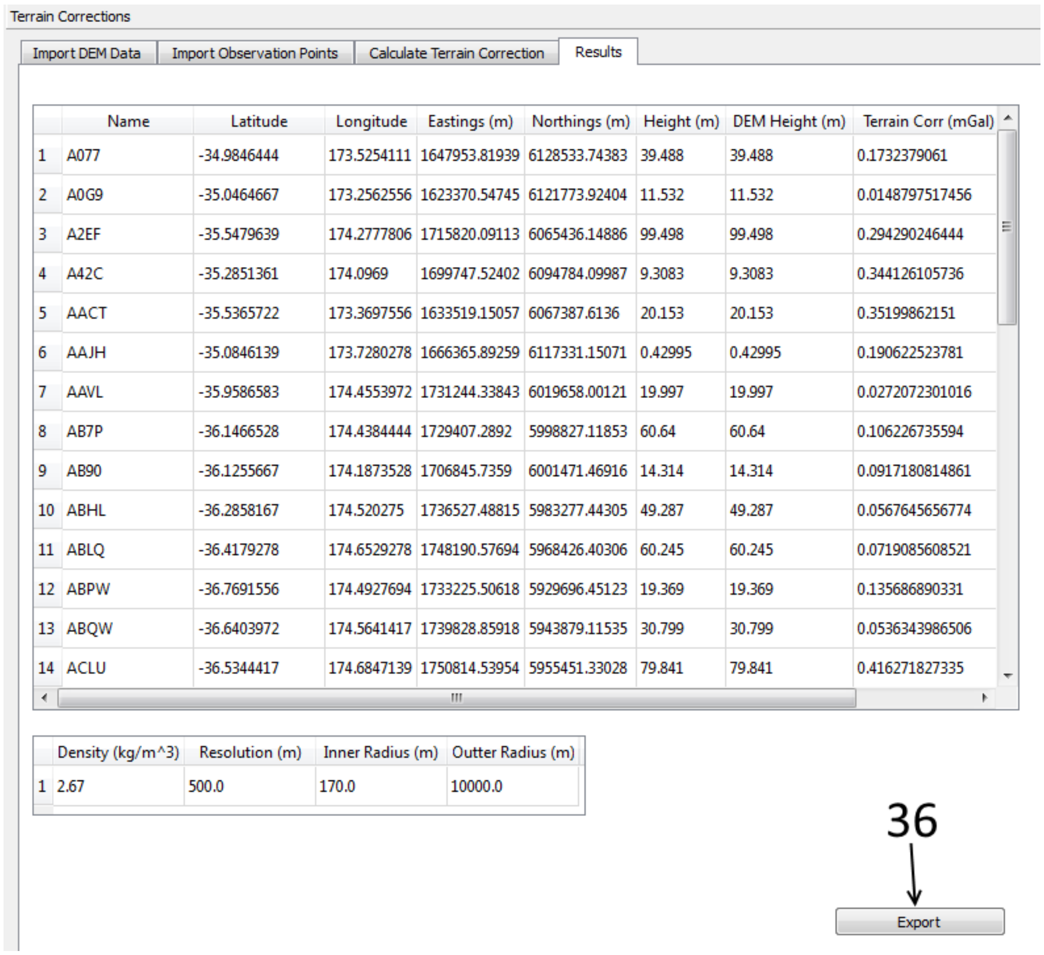


Figure 8: Graphical user interface to set parameters and calculate terrain corrections.

The final results of the terrain correction are given in the graphical user interface under the “Results” tab and this can be seen in figure 9. The results table contains the gravity site name, position data and the terrain correction. The parameters of the computed terrain correction are also given and the data can be exported using button (36).



	Name	Latitude	Longitude	Eastings (m)	Northings (m)	Height (m)	DEM Height (m)	Terrain Corr (mGal)
1	A077	-34.9846444	173.5254111	1647953.81939	6128533.74383	39.488	39.488	0.1732379061
2	A0G9	-35.0464667	173.2562556	1623370.54745	6121773.92404	11.532	11.532	0.0148797517456
3	A2EF	-35.5479639	174.2777806	1715820.09113	6065436.14886	99.498	99.498	0.294290246444
4	A42C	-35.2851361	174.0969	1699747.52402	6094784.09987	9.3083	9.3083	0.344126105736
5	AACT	-35.5365722	173.3697556	1633519.15057	6067387.6136	20.153	20.153	0.35199862151
6	AAJH	-35.0846139	173.7280278	1666365.89259	6117331.15071	0.42995	0.42995	0.190622523781
7	AAVL	-35.9586583	174.4553972	1731244.33843	6019658.00121	19.997	19.997	0.0272072301016
8	AB7P	-36.1466528	174.4384444	1729407.2892	5998827.11853	60.64	60.64	0.106226735594
9	AB90	-36.1255667	174.1873528	1706845.7359	6001471.46916	14.314	14.314	0.0917180814861
10	ABHL	-36.2858167	174.520275	1736527.48815	5983277.44305	49.287	49.287	0.0567645656774
11	ABLQ	-36.4179278	174.6529278	1748190.57694	5968426.40306	60.245	60.245	0.0719085608521
12	ABPW	-36.7691556	174.4927694	1733225.50618	5929696.45123	19.369	19.369	0.135686890331
13	ABQW	-36.6403972	174.5641417	1739828.85918	5943879.11535	30.799	30.799	0.0536343986506
14	ACLU	-36.5344417	174.6847139	1750814.53954	5955451.33028	79.841	79.841	0.416271827335

	Density (kg/m^3)	Resolution (m)	Inner Radius (m)	Outter Radius (m)
1	2.67	500.0	170.0	10000.0

Figure 9: Graphical user interface to set parameters and calculate terrain corrections.

2. Conclusions

Reilly's (1970) least squares method to reduce relative gravity readings to absolute gravity values has been reviewed. Two alternative least squares formulations of the problem are given which relate the relative gravity measurements to the reference absolute gravity values with different degrees of flexibility compared to Reilly's orgirinal method.

A new Python computer program, called Gsolve, which has a graphical user interface has been presented. The purpose of the program is to assist in the routine reduction of relative gravity survey data to absolute gravity values by least squares. The software is capable of accounting for tidal gravitational effects of the sun and moon, gravimeter drift and tares in the survey data. It is also possible to use the software to determine further calibration factors for the gravimeter.

It has further been shown the Gsolve is able to compute free air gravity anomalies from the least squares derived absolute gravity data relative to the GRS80 and GRS67 reference ellipsoids and moreover be used to calculate terrain corrections from a user supplied digital elevation model and specified density parameter.

3. References

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[5] Woodward, D.J and Carman, A.F 1984:*Computer program to reduce precise gravity observations*, Geophysics Division, Technical Note No. 93