## Linear difference equations

## wontdo: nonlinear

## wontdo: Stochastic difference equations

Let $u_t$ be a random variable. Then so will be x:

$$x_t = ax_{t-1} + u_t$$

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}\mathbf{u}_t$$

with x is K by 1, A K by K, u is M by 1, B K by M.

> TODO: clean up theoretical results

# Break until 14:38

## Markov chains

## Dynamic programming with Markov chains

## ... and difference equations

## Application 1: Diamond-Mortensen-Pissarides search model

## Application 2: A GE technology adoption problem (Julia vs Matlab)

# Markov chains

Suppose $x_t \in \{1, 2, \ldots, K\}$. discrete or categorical variables, with K different "states".

To make it stochastic, assume random variable, give it a **probability mass function**:

$$\pi_{tk} = \Pr(x_t = k)$$

Suppose K=2 and 1=employment, 2=unemployment. Then $\pi_{t1}$ is the prob of being employed in period t, $\pi_{t2} = 1 - \pi_{t1}$.

### PMFs

Because pmfs have

$$1 \geq \pi_k \geq 0$$

and

$$\sum_{k=1}^{K} \pi_k = 1.$$

How do we transition from employment to unemployment? Suppose $x_1 = 1$. What is the probability that $x_2 = 1$? This will be characterized by a PMF.

## Markov property

My forecast, that is, my PMF can only depend on the current value of $x$. Then I will have K PMFs, each of length K.

For every $i$, collect PMF into a row vector of probs:

$$\Pr(x_2 = k | x_1 = i) = \pi_{ik}$$

So let's arrange them in a square matrix, KxK:

$$\mathbf{P} = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}$$

But we know that

$$1 \geq \mathbf{P} \geq 0 \tag{*}$$

and

$$\sum_k \pi_{ik} = 1 \text{ for all } i$$

in matrix notation:

$$\mathbf{P1} = \mathbf{1} \tag{**}$$

P is called a **transition matrix**. Square matrices satisfying ($*$) and ($**$) are **stohastic matrices**.

$$\mathbf{P} = \begin{bmatrix} 0.95 & 0.05 \\ 0.12 & 0.88 \end{bmatrix}$$