Stats_with_Scipy

October 4, 2019

1 Basic stats using Scipy

In this example we will go over how to draw samples from various built in probability distributions and define your own custom distributions.

1.1 Packages being used

- scipy: has all the stats stuff
- numpy: has all the array stuff

1.2 Relevant documentation

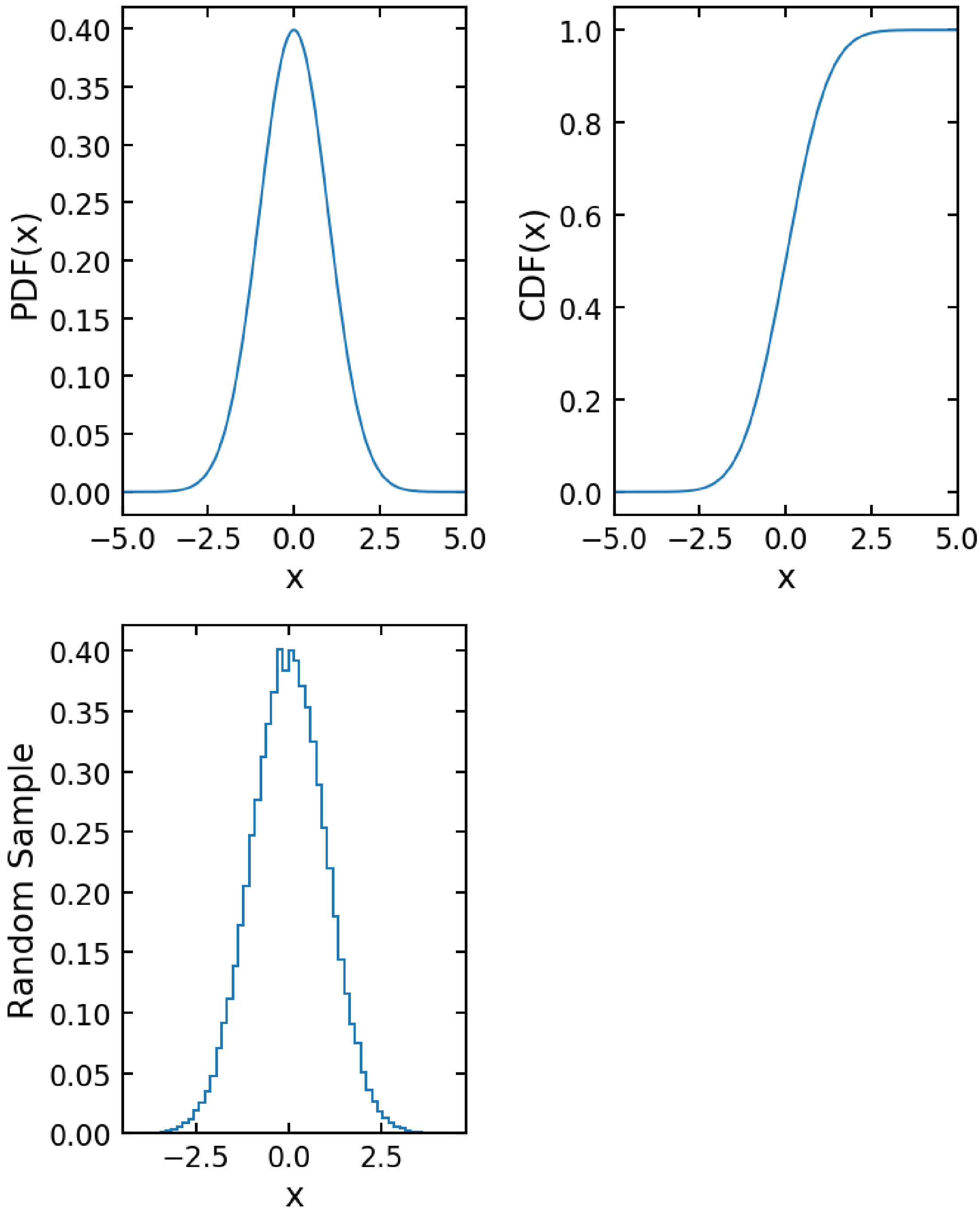
- scipy.stats: http://docs.scipy.org/doc/scipy/reference/tutorial/stats.html, http://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.rv_continuous.html#scipy.stats.rv_conti http://docs.scipy.org/doc/scipy/reference/stats.html#module-scipy.stats

[2]:
```
import numpy as np
import scipy.stats as st
# some special functions we will make use of later on
from scipy.special import erfc
from matplotlib import pyplot as plt
from astropy.visualization import hist
import mpl_style
%matplotlib inline
plt.style.use('default')
plt.style.use(mpl_style.style1)
```

There are many probability distributions that are already available in scipy: http://docs.scipy.org/doc/scipy/reference/stats.html#module-scipy.stats. These classes allow for the evaluations of PDFs, CDFs, PPFs, moments, random draws, and fitting. As an example lets take a look at the normal distribution.

[3]:
```
norm = st.norm(loc=0, scale=1)
x = np.linspace(-5, 5, 1000)
plt.figure(1, figsize=(8, 10))
plt.subplot2grid((2, 2), (0, 0))
plt.plot(x, norm.pdf(x))
plt.xlabel('x')
```

```python
plt.ylabel('PDF(x)')
plt.xlim(-5, 5)
plt.subplot2grid((2, 2), (0, 1))
plt.plot(x, norm.cdf(x))
plt.xlabel('x')
plt.ylabel('CDF(x)')
plt.xlim(-5, 5)
plt.subplot2grid((2, 2), (1, 0))
sample_norm = norm.rvs(size=100000)
hist(sample_norm, bins='knuth', histtype='step', lw=1.5, density=True)
plt.xlabel('x')
plt.ylabel('Random Sample')
plt.tight_layout()
```



You can calculate moments and fit data:

[4]:
```
for i in range(4):
    print('moment {0}: {1}'.format(i+1, norm.moment(i+1)))

print('best fit: {0}'.format(st.norm.fit(sample_norm)))
```

```
moment 1: 0.0
moment 2: 1.0
moment 3: 0.0
moment 4: 3.0
best fit: (-0.0012332358901274972, 0.9975667586725817)
```

2 Custom probability distributions

Sometimes you need to use obscure PDFs that are not already in `scipy` or `astropy`. When this is the case you can make your own subclass of `st.rv_continuous` and overwrite the `_pdf` or `_cdf` methods. This new sub class will act exactly like the built in distributions.

The methods you can override in the subclass are:

- _rvs: create a random sample drawn from the distribution
- _pdf: calculate the PDF at any point
- _cdf: calculate the CDF at any point
- _sf: survival function, a.k.a. 1-CDF(x)
- _ppf: percent point function, a.k.a. inverse CDF
- _isf: inverse survival function
- _stats: function that calculates the first 4 moments
- _munp: function that calculates the nth moment
- _entropy: differential entropy
- _argcheck: function to check the input arguments are valid (e.g. var>0)

You should override any method you have analytic functions for, otherwise (typically slow) numerical integration, differentiation, and function inversion are used to transform the ones that are specified.

2.1 The exponentially modified Gaussian distribution

As and example lets create a class for the EMG distribution (https://en.wikipedia.org/wiki/Exponentially_modified_Gaussian_distribution). This is the distributions resulting from the sum of a Gaussian random variable and an exponential random variable. The PDF and CDF are:

$$f(x;\mu,\sigma,\lambda) = \frac{\lambda}{2}\exp\left(\frac{\lambda}{2}\left[2\mu+\lambda\sigma^2-2x\right]\right)\operatorname{erfc}\left(\frac{\mu+\lambda\sigma^2-x}{\sigma\sqrt{2}}\right) \tag{1}$$

$$F(x;\mu,\sigma,\lambda) = \Phi(u,0,v) - \Phi(u,v^2,v)\exp\left(-u+\frac{v^2}{2}\right) \tag{2}$$

$$\Phi(x,a,b) = \frac{1}{2}\left[1+\operatorname{erf}\left(\frac{x-a}{b\sqrt{2}}\right)\right] \tag{3}$$

$$u = \lambda(x-\mu) \tag{4}$$

$$v = \lambda\sigma \tag{5}$$

[5]:
```python
# create a generating class
class EMG_gen1(st.rv_continuous):
```

```
    def _pdf(self, x, mu, sig, lam):
        u = 0.5 * lam * (2 * mu + lam * sig**2 - 2 * x)
        v = (mu + lam * sig**2 - x)/(sig * np.sqrt(2))
        return 0.5 * lam * np.exp(u) * erfc(v)
    def _cdf(self, x, mu, sig, lam):
        u = lam * (x - mu)
        v = lam * sig
        phi1 = st.norm.cdf(u, loc=0, scale=v)
        phi2 = st.norm.cdf(u, loc=v**2, scale=v)
        return phi1 - phi2 * np.exp(-u + 0.5 * v**2)
    def _stats(self, mu, sig, lam):
        # reutrn the mean, variance, skewness, and kurtosis
        mean = mu + 1 / lam
        var = sig**2 + 1 / lam**2
        sl = sig * lam
        u = 1 + 1 / sl**2
        skew = (2 / sl**3) * u**(-3 / 2)
        v = 3 * (1 + 2 / sl**2 + 3 / sl**4) / u**2
        kurt = v - 3
        return mean, var, skew, kurt
    def _argcheck(self, mu, sig, lam):
        return np.isfinite(mu) and (sig > 0) and (lam > 0)

class EMG_gen2(EMG_gen1):
    def _ppf(self, q, mu, sig, lam):
        # use linear interpolation to solve this faster (not exact, but much␣
 ↪faster than the built in method)
        # pick range large enough to fit the full cdf
        var = sig**2 + 1 / lam**2
        x = np.arange(mu - 50 * np.sqrt(var), mu + 50 * np.sqrt(var), 0.01)
        y = self.cdf(x, mu, sig, lam)
        return np.interp(q, y, x)

class EMG_gen3(EMG_gen1):
    def _rvs(self, mu, sig, lam):
        # redefine the random sampler to sample based on a normal and exp dist
        return st.norm.rvs(loc=mu, scale=sig, size=self._size) + st.expon.
 ↪rvs(loc=0, scale=1/lam, size=self._size)

# use generator to make the new class
EMG1 = EMG_gen1(name='EMG1')
EMG2 = EMG_gen2(name='EMG2')
EMG3 = EMG_gen3(name='EMG3')
```

Lets look at how long it takes to create readom samples for each of these version of the EMG:

[6]:
```
%time EMG1.rvs(0, 1, 0.5, size=1000)
print('=========')
```

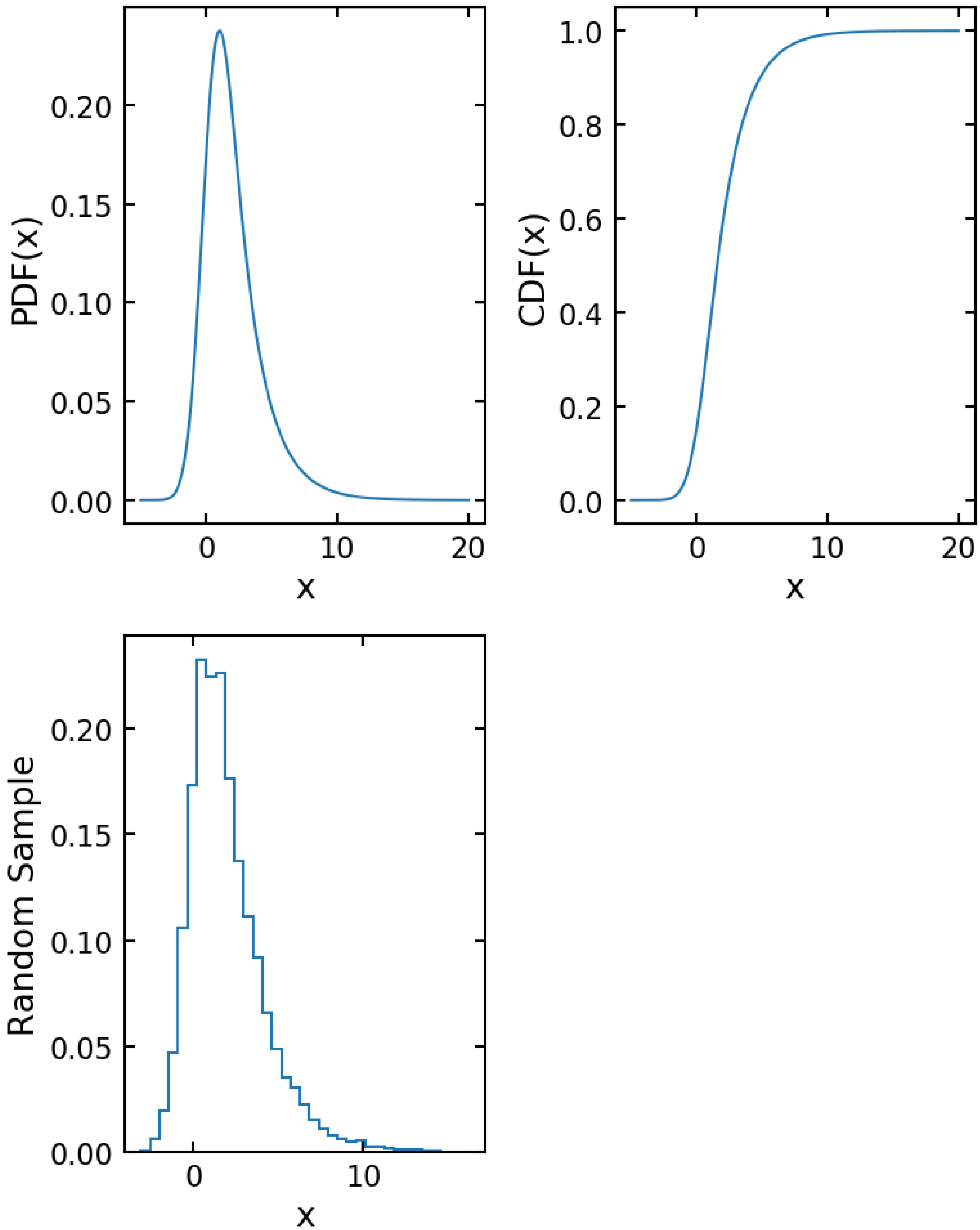
```
%time EMG2.rvs(0, 1, 0.5, size=1000)
print('=========')
%time EMG3.rvs(0, 1, 0.5, size=1000)
print('=========')
```

```
CPU times: user 8.49 s, sys: 61.2 ms, total: 8.55 s
Wall time: 9.54 s
=========
CPU times: user 7.22 ms, sys: 3.25 ms, total: 10.5 ms
Wall time: 11.7 ms
=========
CPU times: user 1.1 ms, sys: 130 ţs, total: 1.23 ms
Wall time: 1.34 ms
=========
```

As you can see, the numerical inversion of the CDF is very slow, the approximation to the inversion is much faster, and defining `_rvs` in terms of the `normal` and `exp` distributions is the fastest.

Lets take a look at the results for `EMG3`:

[7]:
```
dist = EMG3(0, 1, 0.5)
x = np.linspace(-5, 20, 1000)
plt.figure(2, figsize=(8, 10))
plt.subplot2grid((2, 2), (0, 0))
plt.plot(x, dist.pdf(x))
plt.xlabel('x')
plt.ylabel('PDF(x)')
plt.subplot2grid((2, 2), (0, 1))
plt.plot(x, dist.cdf(x))
plt.xlabel('x')
plt.ylabel('CDF(x)')
plt.subplot2grid((2, 2), (1, 0))
sample_emg = dist.rvs(size=10000)
hist(sample_emg, bins='knuth', histtype='step', lw=1.5, density=True)
plt.xlabel('x')
plt.ylabel('Random Sample')
plt.tight_layout()
```



As with the built in functions we can calculate moments and do fits to data. **Note** Since we are not using the built in `loc` and `scale` params they are fixed to 0 and 1 in the fit below.

[8]:
```
for i in range(4):
    print('moment {0}: {1}'.format(i+1, dist.moment(i+1)))
```

```
print('best fit: {0}'.format(EMG3.fit(sample_emg, floc=0, fscale=1)))
```

```
moment 1: 2.0
moment 2: 9.0
moment 3: 54.0
moment 4: 435.0
best fit: (-0.025441047775655215, 0.9875431617248385, 0.5018381039808805, 0, 1)
```

For reference here is how scipy defines this distribution (found under the name exponnorm):

[9]:
```
import scipy.stats._continuous_distns as cd
np.source(cd.exponnorm_gen)
```

```
In file: /Users/coleman/anaconda3/lib/python3.7/site-
packages/scipy/stats/_continuous_distns.py

class exponnorm_gen(rv_continuous):
    r"""An exponentially modified Normal continuous random variable.

    %(before_notes)s

    Notes
    -----
    The probability density function for `exponnorm` is:

    .. math::

        f(x, K) = \frac{1}{2K} \exp\left(\frac{1}{2 K^2} - x / K \right)
                  \text{erfc}\left(-\frac{x - 1/K}{\sqrt{2}}\right)

    where :math:`x` is a real number and :math:`K > 0`.

    It can be thought of as the sum of a standard normal random variable
    and an independent exponentially distributed random variable with rate
    ``1/K``.

    %(after_notes)s

    An alternative parameterization of this distribution (for example, in
    `Wikipedia
<https://en.wikipedia.org/wiki/Exponentially_modified_Gaussian_distribution>`_)
    involves three parameters, :math:`\mu`, :math:`\lambda` and
    :math:`\sigma`.
    In the present parameterization this corresponds to having ``loc`` and
    ``scale`` equal to :math:`\mu` and :math:`\sigma`, respectively, and
    shape parameter :math:`K = 1/(\sigma\lambda)`.

    .. versionadded:: 0.16.0
```

```
    %(example)s

    """
    def _rvs(self, K):
        expval = self._random_state.standard_exponential(self._size) * K
        gval = self._random_state.standard_normal(self._size)
        return expval + gval

    def _pdf(self, x, K):
        # exponnorm.pdf(x, K) =
        #     1/(2*K) exp(1/(2 * K**2)) exp(-x / K) * erfc-(x - 1/K) / sqrt(2))
        invK = 1.0 / K
        exparg = 0.5 * invK**2 - invK * x
        # Avoid overflows; setting np.exp(exparg) to the max float works
        #  all right here
        expval = _lazywhere(exparg < _LOGXMAX, (exparg,), np.exp, _XMAX)
        return 0.5 * invK * (expval * sc.erfc(-(x - invK) / np.sqrt(2)))

    def _logpdf(self, x, K):
        invK = 1.0 / K
        exparg = 0.5 * invK**2 - invK * x
        return exparg + np.log(0.5 * invK * sc.erfc(-(x - invK) / np.sqrt(2)))

    def _cdf(self, x, K):
        invK = 1.0 / K
        expval = invK * (0.5 * invK - x)
        return _norm_cdf(x) - np.exp(expval) * _norm_cdf(x - invK)

    def _sf(self, x, K):
        invK = 1.0 / K
        expval = invK * (0.5 * invK - x)
        return _norm_cdf(-x) + np.exp(expval) * _norm_cdf(x - invK)

    def _stats(self, K):
        K2 = K * K
        opK2 = 1.0 + K2
        skw = 2 * K**3 * opK2**(-1.5)
        krt = 6.0 * K2 * K2 * opK2**(-2)
        return K, opK2, skw, krt
```

[12]:
```
%time st.exponnorm.rvs(0.5, size=1000)
print('=========')
```

```
CPU times: user 765 ţs, sys: 326 ţs, total: 1.09 ms
Wall time: 966 ţs
=========
```

[ ]:
```

```