Turbulent-Viscosity Models - The k-ω Model

Since the turbulent kinetic energy k and dissipation ε naturally define a turbulence time-scale $T = k/\varepsilon$, one might instead combine the model equation for k with a time-scale equation. Unfortunately, the resulting time-scale equation requires a negative diffusion term for physical consistency, resulting in a mathematically ill-posed system. Alternatively, an equation for the frequency $\omega = 1/T = \varepsilon/k$ may be considered. This is the basis of the k-ω model introduced by Wilcox in 1993:

Reynolds Stress Model:	$\langle u_i' u_j' \rangle = -2\nu_T \bar{S}_{ij} + \frac{2}{3} k \delta_{ij}$
Turbulent Viscosity:	$\nu_T = C_\mu k/\omega$
Model Equation for k:	$\bar{D}k/\bar{D}t = \vec{\nabla}\cdot((\nu + \nu_T/\sigma_k)\vec{\nabla}k) + P - k\omega$
Model Equation for ω:	$\bar{D}\omega/\bar{D}t = \vec{\nabla}\cdot((\nu + \nu_T/\sigma_\omega)\vec{\nabla}\omega) + C_{\omega 1}\frac{P\omega}{k} - C_{\omega 2}\omega^2$
Standard Constants:	$C_\mu = 0.09$, $\sigma_k = \sigma_\omega = 2$, $C_{\omega 1} = 5/9$, $C_{\omega 2} = 5/6$

As compared with the standard k-ε model, the k-equation is only altered by changing ε to $k\omega$ and the ω-equation is analogous to the ε-equation. However, as described by Wilcox, the k-ω model is superior in its treatment of the viscous wall region as extra dissipation is produced near walls. To see this, we write the $\varepsilon = k\omega$ evolution equation implied by the k-ω model:

$$\bar{D}\varepsilon/\bar{D}t = \vec{\nabla}\cdot\left(\left(\nu + \frac{\nu_T}{\sigma_\varepsilon}\right)\vec{\nabla}\varepsilon\right) + \frac{C_{\varepsilon 1}P - C_{\varepsilon 2}\varepsilon}{T} + S_\omega$$

$$S_\omega = \frac{2}{T}\left(\nu + \frac{\nu_T}{\sigma_\omega}\right)\left[\frac{|\vec{\nabla}k|^2}{k} - \frac{\vec{\nabla}k\cdot\vec{\nabla}\varepsilon}{\varepsilon}\right]$$

The source term S_ω distinguishes between the k-ε and k-ω models. In the viscous sublayer, k increases while ε decreases so $S_\omega > 0$. Thus, a larger ε is produced with the k-ω model.

In 1994, Menter noted a major failing of the k-ω model: it spurious sensitivity to free-stream conditions. To overcome this shortcoming, Menter proposed the shear stress transport (SST) model which yields the best behavior of the k-ε and k-ω models. It relies on the fact that the $\omega = \varepsilon/k$ evolution equation implied by the k-ε model is:

$$\bar{D}\omega/\bar{D}t = \vec{\nabla}\cdot((\nu + \nu_T/\sigma_\omega)\vec{\nabla}\omega) + C_{\omega 1}\frac{P\omega}{k} - C_{\omega 2}\omega^2 + \frac{2\nu_T}{\sigma_\omega k}\vec{\nabla}\omega\cdot\vec{\nabla}k$$

Consequently, the SST model is written as a nonstandard k-ω model in which the above is used as the ω-equation where the last term is multiplied by a 'blending function'. Close to the walls, the blending function is zero, leading to the standard k-ω model. Far from walls, the blending function is one, leading to the standard k-ε model. In addition, the limiter:

$$\nu_T = \min\left[C_\mu \frac{k}{\omega}, \frac{\sqrt{C_\mu}\,k}{|2\bar{S}|}\right]$$

is used to prevent the k-ω model's (more minor) tendency to overpredict the level of shear stress in adverse pressure gradient boundary layers.