Reynolds Averaged Navier-Stokes Models - An Introduction

Reynolds Averaged Navier-Stokes (RANS) models are by far the most commonly used in engineering turbulence prediction. They are implemented as components of CFD codes and have been applied with varying degrees of success to a wide range of turbulent fluid flow problems. The basic principle lies in solving the Reynolds equations:

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(2\nu \bar{S}_{ij} - \langle u_i' u_j' \rangle\right)$$

$$\bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$$

for the mean velocity field. The primary challenge lies in the fact that the Reynolds stresses:

$$r_{ij} = \langle u_i' u_j' \rangle$$

are unknown. Hence, a model is needed for the Reynolds stresses. There are several restrictions we can place on our models for r_{ij}, and these constraints are generally helpful in reducing the range of possible models. The constraints are:

(i) The Reynolds stress approximation should be tensorially consistent. That is, it should map as a tensor under a change of coordinates.

(ii) The Reynolds stress approximation should maintain the realizability constraint:

$$r_{ii} = \langle u_i' u_i' \rangle \geq 0$$

(iii) The Reynolds stress approximation should be invariant under Galilean transformations.

The two primary classes of RANS models are turbulent-viscosity models and Reynolds-stress models, both of which will be presented in detail.