Turbulent-Viscosity Models - Turbulent Viscosity Transport Models

The turbulent viscosity models considered so far construct a turbulent viscosity from velocity and length scales. It might seem prudent to instead formulate a transport equation directly for the turbulent viscosity, resulting in a complete one-equation model. This idea has been proposed many times in the past, and the most recent incarnation, initiated by Baldwin and Barth in 1990, has proven quite effective. Flaws to the formulation were rectified by Spalart and Allmaras in 1992, and the resulting model, the so-called Spalart-Allmaras model, has enjoyed great success in predicting aerodynamic flows such as transonic flows over airfoils.

The Spalart-Allmaras model assumes that an effective viscosity $\tilde{\nu}$ satisfies a prototype transport equation consisting of production, destruction, and transport terms. These terms are then modeled to obtain good behavior for flows of interest. An effective viscosity is employed rather than the turbulent viscosity to ease model construction. Namely, $\tilde{\nu}$ may be designed to respect the log-layer dependence $\tilde{\nu} = \kappa u_\tau y$ all the way to the wall. The turbulent viscosity is then obtained via a nonlinear transformation. Construction of the Spalart-Allmaras model is a complicated affair, so we simply present the final model for the sake of brevity.

Transport Equation for $\tilde{\nu}$:

$$\frac{\bar{D}\tilde{\nu}}{\bar{D}t} = \frac{1}{\sigma_\nu}\left[\vec{\nabla}\cdot\left((\nu+\tilde{\nu})\vec{\nabla}\tilde{\nu}\right) + c_{b2}|\vec{\nabla}\tilde{\nu}|^2\right] + c_{b1}\tilde{S}\tilde{\nu} - c_{w1} f_w \left(\frac{\tilde{\nu}}{d}\right)^2$$

where:

$$\tilde{S} \equiv |\vec{\nabla}\times\vec{u}| - \frac{\tilde{\nu}}{(\nu+\nu_T)(\kappa y)^2} + \frac{\tilde{\nu}}{(\kappa y)^2} = \text{Effective Vorticity}$$

$$d \equiv \text{Minimum Distance to the Wall}$$

$$f_w(r) \equiv g(r)\left[\frac{65}{g(r)^6+64}\right]^{1/6} \quad \text{with} \quad g(r) = r + 0.3\,(r^6 - r)$$

$$\sigma_\nu = 2/3, \quad c_{b1} = 0.1355, \quad c_{b2} = 0.622, \quad \kappa = 0.41,$$

$$c_{w1} = c_{b1}\kappa^{-2} + (1+c_{b2})/\sigma_\nu = 3.2391$$

Nonlinear Transformation for ν_T:

$$\nu_T = \tilde{\nu}\, f_\nu(\tilde{\nu}/\nu) \quad \text{with} \quad f_\nu(r) = \frac{r^3}{r^3 + 7.1^3}$$

In applications to the aerodynamic flows for which it is intended, the Spalart-Allmaras model has proven quite successful. On D2L, a plot comparing Stanton number (non-dimensional heat transfer coefficient) contours obtained from the Spalart-Allmaras model with experimental data may be found. The application is heat transfer in a transonic turbine passage. The performance of the Spalart-Allmaras model is quite good, especially given the complexity of the model.

Despite all of its successes, the Spalart-Allmaras model has clear limitations. It is incapable of accounting for the decay of ν_T in isotropic turbulence and overpredicts the rate of spreading of the plane jet by almost 40%. Nonetheless, it is a simple and particularly powerful model for aerodynamic flows of interest.