Structure and Dynamics of Near-Wall Turbulence:

There has been considerable research into the structure and dynamics of the near-wall turbulent boundary layer. There are several reasons for this:

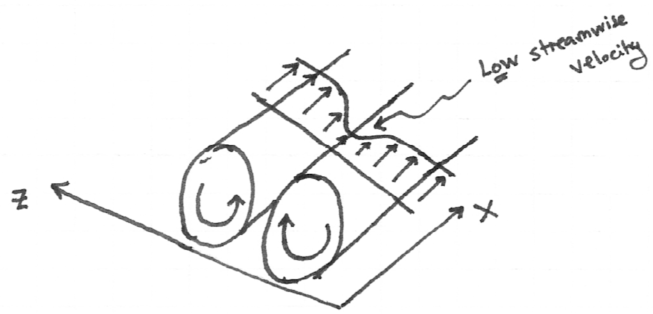
i) The statistics in the near-wall region are generic, as observed by the law of the wall. Thus, what we learn about near-wall turbulence will be applicable to a wide range of flows.

ii) There is much interest in controlling turbulence, and practical considerations demand any actuators or sensors used to effect the contol be located on the walls.

iii) Due to severe inhomogeneity, it is the near-wall region that presents the biggest challenge to modeling of turbulent flows.

In the late 1960s, the world of wall bounded turbulence was revolutionized when Kline and coworkers observed that the near wall region of a turbulent wall bounded flow (subject to zero pressure gradient) was organized into streamwise elongated regions of high- and low-speed flow that alternate in the spanwise direction. These are now known as high- and low-speed streaks. Kline observed the spacing between the streaks was approximately 100 wall-units, and the streaks are coherent in the streamwise direction for more than 1000 wall-units. These streaks are only observed in the viscous wall region for $y^+ < 30$.

Since their discovery, the origin of these streaks has been of great interest. One of the most obvious ways that streaks can be generated is if an array of counter-rotating vortices in the streamwise direction existed near the walls as depicted below:



Between the vortices, there would be alternating regions of fluid moving toward and away from the wall. The fluid moving away from the wall would have low streamwise velocity while the fluid moving toward the wall would have high streamwise velocity, thus giving rise to low- and high-speed streaks.

The above model of streak formation was widely accepted at one time, [redacted] but it suffers a serious flaw. Consider the linearized axial momentum equation:

$$\frac{\partial u}{\partial t} = -v\frac{\partial u}{\partial y} + \nu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \qquad v \text{ is fixed}$$

Scaling in wall-units (viscous scales) and noting that $\partial u^+/\partial y^+ \approx 1$ near the wall yields:

$$\frac{\partial u^+}{\partial t^+} = -v^+ + \frac{\partial^2 u^+}{\partial y^{+2}} + \frac{\partial^2 u^+}{\partial z^{+2}}$$

Assuming the simple form for the vertical velocity $v^+ = \hat{v}\cos(k^+z^+)$ (which in turn implies $u^+ = \hat{u}\cos(k^+z^+)$) where $k^+ = 2\pi/\lambda^+$ and $\lambda^+ = 100$ is the distance between streaks, we obtain the following equation for a steady solution:

$$\frac{\partial^2 \hat{u}}{\partial y^{+2}} - k^{+2}\hat{u} = \hat{v}$$

Since the size of the vortices in the wall-normal direction is about the same as their size in the spanwise direction, $\partial^2\hat{u}/\partial y^{+2}$ is of the same magnitude as $k^{+2}\hat{u}$. Therefore:

$$|\hat{u}| \approx \frac{1}{2k^{+2}}|\hat{v}| \approx 125|\hat{v}|$$

occurs at $\sim y^+ = 20$

This says we should expect that in the middle of the streamwise vortices, the streamwise velocity fluctuations are two orders of magnitude larger than the wall-normal fluctuations. However, this does not agree at all with experimental or computational data. Consequently, if the streamwise vortices do indeed exist, there must be other structures present that produce vertical velocity fluctuations, but such structures would also likely produce analogous streamwise velocity fluctuations.

The solution to the above dilemma comes from observations in direct numerical simulations where it is observed that streamwise vortices in fact do not lie along the wall with long streamwise extents. Instead, they extend along the wall for roughly 100 wall-units before they lift up from the wall on their downstream end. These vortices then propagate in the streamwise direction much further from the wall than the streaks.

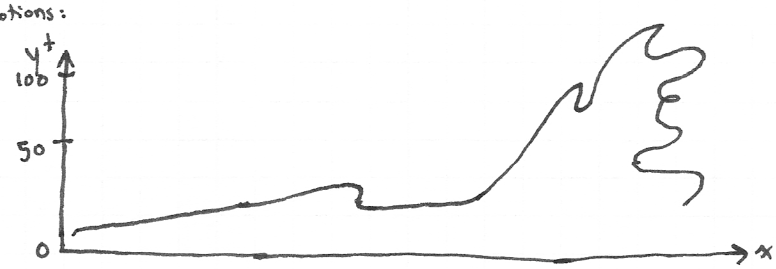
The above discussion brings to light a new question: how can streamwise vortices that are only occasionally present produce the observed streaks? To answer this, we return to our model problem. First, we postulate that $\hat{u}$ and $\hat{v}$ are both order one when the vortex is present. When the vortex is not present, $\hat{v} \approx 0$. Recognizing that $k^+ \ll 1$, we obtain:

$$\frac{\partial u^+}{\partial t^+} \approx -v^+ \qquad \text{vortex present}$$

$$\frac{\partial u^+}{\partial t^+} \approx \frac{\partial^2 u^+}{\partial y^{+2}} + \frac{\partial^2 u^+}{\partial z^{+2}} \approx O(k^{+2}) \qquad \text{no vortex present}$$

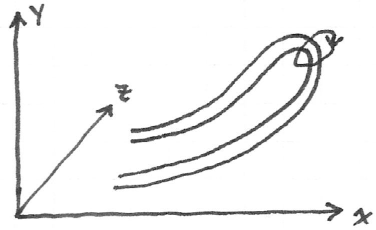
Thus, the time scale for the build up of a streak is $O(1/v^+) \approx 1$ while the time scale for the decay of a streak is of order $1/k^{+2} \approx 250$ which is much longer. Thus, the vortices need only be present for a relatively short period of time for the streaks to be maintained. The question still remains of why are the vortices even there, a question we shall later return to.

Low-speed streaks have a characteristic behavior known as bursting. Namely, with increasing downstream distance, a low-speed streak migrates slowly from the wall and then at some point (usually around $y^+ = 10$) it moves away more rapidly in a process known as streak lifting, or ejection. As it is lifted, the streak rapid oscillates and then experiences a breakdown into finer-scale motions:



As the low-speed fluid moves away from the wall, continuity demands high-speed fluid is brought toward the wall as well. Corino and Brodkey (1969) first identified regions of high-speed fluid moving toward the wall in events called sweeps.

It has long been postulated that horseshoe or hairpin vortices would exist near the wall – Head and Bandyopadhyay suggested that these are the dominant vortical structures further into the boundary layer back in 1981. Hairpin vortices are vortices that extend ~~up~~ along the wall for some distance and then tilt up. At the head, the vorticity turns from being primarily streamwise to spanwise, and then streamwise again in the other direction, much like a hairpin:



The head of the hairpin is pushed up from the wall by self-induction, and the strain associated with the mean shear stretches and strengthens the inclined vortices. The cross-stream dimensions scale with δ_ν, but the overall length may be of order δ, so these structures are extremely elongated at high Reynolds number. They are inclined at approximately 45° to the wall, as predicted by Theodorsen in 1952.

Between the "legs" of a hairpin vortex, the vortices induce a flow up and back relative to the motion of the vortices. Thus this structure produces low-speed flow moving away from the wall, analogous to the bursting phenomena mentioned earlier. High-speed flow coming from behind the hairpin then collides with the low-speed flow, producing an inclined shear layer.

However, hairspin vortices of the kind discussed above are rarely seen in practice. Such two-legged approximately symmetric hairpins are not frequent in numerical simulations, but one-legged asymmetric hairpins are quite common:

Symmetric

Asymmetric

Moreover, asymmetric hairpins are usually associated with an inclined shear layer.

It is often hypothesized that the streamwise vortices observed near the wall are simply the legs of hairpin vortices and current flow visualization technology is simply not sophisticated enough to detect this. This hypothesis is strengthened by a recent theory by Waleffe and Kim (1997) and Hussain and Schoppa (1997) which seeks to answer our earlier question: What is the source of streamwise vortices? This theory notes that streaks are like jets of high- and low-speed fluid, with a vertical shear layer between them. This layer is inviscidly unstable, with the most unstable mode being a sinuous mode. If one draws a straight line about which a streak boundary is oscillating, it is clear that when the line passes (in x) from low-speed streak to high-speed streak, $\partial u/\partial x > 0$. Thus streamwise vorticity in this region will be amplified due to the vortex stretching term $\omega_x \, {\partial u}/{\partial x}$. The opposite situation occurs when the line passes from high-speed streak to low-speed streak. Hussain and Schoppa showed that vortices created in this manner will indeed lift from the wall and turn into the spanwise direction, like a one-legged hairpin. This also explains why one-legged hairpins are observed much more than two-legged hairpins.

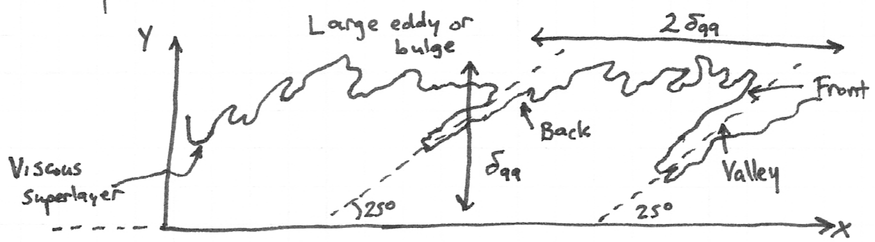
The scenario above suggests a three-step procedure by which streaks and vortices are maintained in the near-wall region:

i) Streaks are created and maintained by streamwise vortices near the wall.
ii) The streaks become unstable to a sinuous mode, producing wavy streaks.
iii) Localized stretching of the streamwise vortices by the wavy streaks strengthens the vortices, allowing them to maintain the streaks.

The above process is self-sustaining – it does not require turbulence away from the wall to be maintained. In fact, Jiminez (1997) demonstrated that if the above features are eliminated from a turbulent boundary layer, the turbulence actually dies. Consequently, streaks and vortices are critical to the maintenance of near-wall turbulence.

Head and Bandyopadhyay (1981) suggested that larger turbulent structures may be composed of an ensemble of hairpin vortices, and measurements from Adrian and co-workers (1997) suggest that for $y^+ > 60$, hairpin vortices organize themselves into long packets in which the hairpin heads line up in a "ramp". Away from the wall, Adrian suggests most of the change of the streamwise ~~vortex~~ velocity occurs discretely across these vortex ramps.

The above discussion is made more clear by recognizing that the outermost part of a turbulent boundary layer is intermittent. There is a thin front – the viscous superlayer, separating the turbulent layer from the irrotational free-stream.



For a given x, the y location of the superlayer is roughly normally distributed with mean $0.8\,\delta_{99}$ and standard deviation $0.15\,\delta_{99}$. There are valleys of non-turbulent fluid that penetrate deep within the layer and separate large eddies or bulges that incline at a characteristic angle of 20-25°. The bulges are roughly of length δ to 3δ in the streamwise direction and $\delta/2$ to $3\delta/2$ in the spanwise direction. These bulges slowly evolve and rotate downstream. They are also composed of finer-scale structures called typically eddies, and Head and Bandyopadhay (1981) hypothesized these were in fact hairpin vortices oriented at 45°. Entrainment of free-stream fluid occurs both via engulfing into the bulges (via the valleys) and mixing by small eddies. This two-fold process advances the viscous superlayer.