Characterization and Modeling of the Pressure-Rate-of-Strain Tensor for Homogeneous Turbulence

As mentioned previously, the pressure-rate-of-strain tensor is the most difficult term to model in Reynolds Stress Transport Models. The tensor acts to redistribute energy among the Reynolds stresses via the interaction of the fluctuating pressure field p' and the rate-of-strain S_{ij}'. Some insight into the pressure-rate-of-strain tensor may be gained by splitting the pressure field into three components: a rapid term, a slow term, and a harmonic term. This decomposition is motivated by examining the Poisson problem for the fluctuating pressure field. Begin by taking the divergence of the Navier-Stokes momentum to obtain the following equation for the total pressure field:

$$\frac{1}{\rho} \nabla^2 p = - \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i}$$

Then, we take the fluctuating part of the above equation:

$$\frac{1}{\rho} \nabla^2 p' = -2 \frac{\partial \bar{u}_i}{\partial x_j} \frac{\partial u_j'}{\partial x_i} - \frac{\partial^2}{\partial x_i x_j} \left((u_i' u_j')' \right)$$

We now decompose the pressure field as follows:

$$p' = \underbrace{p^{(r)}}_{\text{Rapid part}} + \underbrace{p^{(s)}}_{\text{Slow part}} + \underbrace{p^{(h)}}_{\text{Harmonic part}}$$

where:

$$\frac{1}{\rho} \nabla^2 p^{(r)} = -2 \frac{\partial \bar{u}_i}{\partial x_j} \frac{\partial u_j'}{\partial x_i}$$

$$\frac{1}{\rho} \nabla^2 p^{(s)} = - \frac{\partial^2}{\partial x_i \partial x_j} \left((u_i' u_j')' \right)$$

$$\frac{1}{\rho} \nabla^2 p^{(h)} = 0$$

$$\frac{\partial p^{(r)}}{\partial n} = \frac{\partial p^{(s)}}{\partial n} = 0 \quad \text{and} \quad \frac{\partial p^{(h)}}{\partial n} = \nu \frac{\partial^2 v'}{\partial n^2} \quad \text{at walls}$$

vertical fluctuating velocity

The harmonic component exists primarily to ensure the fluctuating pressure field satisfies the correct boundary condition at walls. Indeed, it is important only near walls and is zero in homogeneous turbulence.

The rapid component is so called because it responds immediately to a change in the mean velocity gradient. The slow component, on the other hand, responds to changes in mean velocity gradients through the gradual response in the turbulence.

Corresponding to $p^{(r)}$, $p^{(s)}$, and $p^{(h)}$ the pressure-rate-of-strain tensor may be decomposed into three components $R_{ij}^{(r)}$, $R_{ij}^{(s)}$, and $R_{ij}^{(h)}$, with obvious definitions.

It helps now to consider the setting of <u>homogeneous</u> <u>turbulence</u>. In this setting, the components of R_{ij} are uniform in space, and it is natural to assume that:

$$R_{ij} = R_{ij}\left(b_{ij}, \frac{\partial \bar{u}_i}{\partial x_j}, k, \varepsilon \right)$$

where b_{ij} is the <u>normalized</u> anisotropy tensor:

$$b_{ij} \equiv \frac{a_{ij}}{2k} = \frac{\langle u_i' u_j' \rangle}{2k} - \frac{1}{3}\delta_{ij}$$

Here, we have split the dependence of R_{ij} on the Reynolds stresses into a dependence on b_{ij} and k separately. We have implicitly assumed locality in time. However, it should be mentioned that history effects are present through the evolution equations for the Reynolds stresses.

In what follows, we will consider two distinct limits:

(i) $Sk/\varepsilon \to 0$ Zero-<u>distortion</u> limit

(ii) $Sk/\varepsilon \to \infty$ Rapid-<u>distortion</u> limit

Zero-<u>distortion limit</u>:

We first consider the setting of zero distortion. This corresponds to the case of decaying homogeneous isotropic turbulence, and hence there is <u>no</u> <u>production</u>, <u>transport</u>, <u>rapid</u> <u>pressure</u>, or homogen<u>eous</u> <u>pressure</u>. Our resulting evolution equation for the Reynolds stress tensor is then:

$$\frac{d}{dt}\langle u_i' u_j' \rangle = R_{ij}^{(s)} - \varepsilon_{ij}$$

Since we expect the turbulence to become <u>less</u> anisotropic as it decays away, then it is precisely the sl<u>ow</u> redistribution term which drives the turbulence towards isotropy. To better understand this process, we rewrite the above as an evolution equation for b_{ij}:

$$\frac{d b_{ij}}{dt} = \frac{\varepsilon}{k}\left(b_{ij} + \frac{R_{ij}^{(s)}}{2\varepsilon} \right)$$

In order for $b_{ij} \to 0$ as $t \to \infty$, the right hand side must be negative. This inspires Rotta's model:

$$R_{ij}^{(s)} = -2 C_R \varepsilon b_{ij} \qquad \text{(Rotta 1951)}$$

where necessarily $C_R > 1$ to guarantee decay of b_{ij}. Typical empirical values of C_1 are in the range 1.5 to 2.0.

The Rotta model is usually quite effective. However, the representation assumption:

$$R_{ij} = R_{ij}\left(b_{ij}, \frac{\partial \bar{u}_i}{\partial x_j}, k, \varepsilon \right)$$

suggests the most general functional dependence of the sl<u>ow</u> redistribution term is:

$$R_{ij}^{(s)} = -2 C_R \varepsilon b_{ij} + \underset{4}{\cancel{2}}\,\varepsilon C_R^{n} \left(b_{ij}^2 - \tfrac{1}{3} b_{kk}^2 \delta_{ij} \right)$$

where C_R and C_R^n can be functions of the invariants of b_{ij}. With this form, the evolution equation for the normalized anisotropy becomes:

$$\frac{db_{ij}}{dt} = (1-C_R)\frac{b_{ij}}{T} + 2C_R^n \frac{(b_{ij}^2 - \frac{1}{3} b_{kk}^2 \delta_{ij})}{T}$$

where $T = \varepsilon/k$ and $b_{kk}^2 = b_{11}^2 + b_{22}^2 + b_{33}^2$. An important property of the redistribution model is whether or not it ensures the Reynolds normal stresses are nonnegative. This realizability constraint holds if:

$$b_{11} = \frac{\langle (u_1')^2 \rangle}{2k} - \frac{1}{3} \geq -\frac{1}{3}$$

and similarly, $b_{22} \geq -1/3$ and $b_{33} \geq -1/3$. Without loss of generality, we may consider that the normalized anisotropy is expressed in principal coordinates, in which case:

$$[b_{ij}] = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & -(b_{11}+b_{22}) \end{bmatrix} \qquad \text{since } b_{ii} \equiv 0!$$

Consequently, $b_{11}, b_{22}, b_{33} \in [-1/3, 2/3]$. Suppose that $b_{11} = -1/3$. Then it is required that $db_{11}/dt \geq 0$. We have:

$$\frac{db_{11}}{dt} = (1-C_R)\frac{b_{11}}{T} + 2C_R^n \frac{(\frac{1}{3} b_{11}^2 - \frac{2}{3}(b_{22}^2 + b_{11}b_{22}))}{T}$$

$$= \frac{1}{3T}\left(-(1-C_R) + \left(\left(\frac{2}{9} + \frac{4}{3} b_{22} - 4 b_{22}^2\right)\right) C_R^n\right)$$

The right hand side is minimized when $b_{22} = 2/3$. Therefore:

$$\frac{db_{11}}{dt} \geq \frac{1}{3T}\left(-(1-C_R) - \frac{2}{3} C_R^n\right)$$

In order to ensure db_{11}/dt, we must then enforce:

$$C_R^n \leq \frac{3}{2}(C_R - 1)$$

Note that if $C_R^n \equiv 0$ as in the Rotta model, the above returns $C_R \geq 1$ as expected.

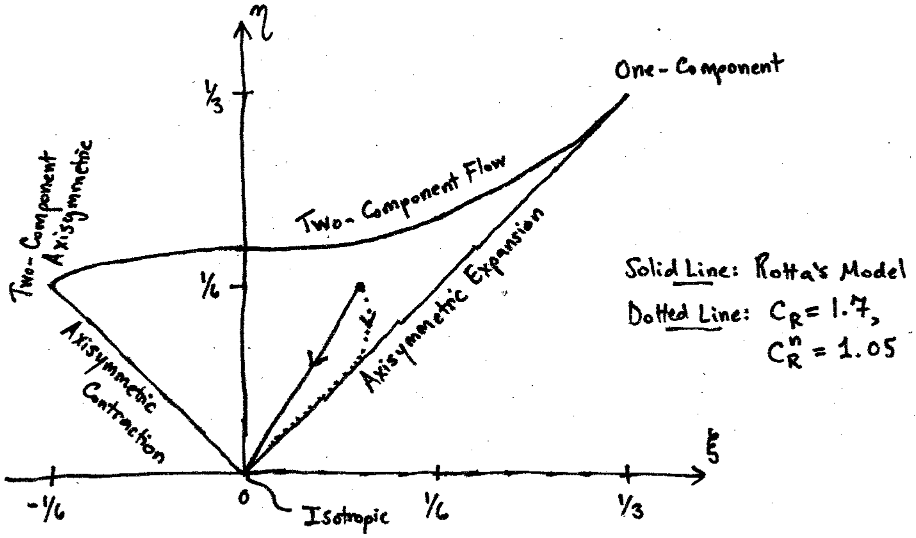
Note that the normalized anisotropy is fully characterized by b_{11} and b_{22}. Alternately, we may characterize b_{ij} in terms of the variables:

$$\eta = \sqrt{-\frac{1}{3} II_b} = \sqrt{\frac{1}{6} b_{kk}^2}$$

$$\xi = \sqrt[3]{\frac{1}{2} III_b} = \sqrt[3]{\frac{1}{6} b_{kk}^3}$$

where II_b and III_b are the second and third invariants of b_{ij}.

At any point and time in a turbulent flow, ξ and η can be determined by the Reynolds stresses and the result plotted on the ξ-η plane:



The realizable states of the Reynolds stresses correspond to points inside the triangle drawn above, known as the Lumley triangle. The triangle edges correspond to special states of the Reynolds stress tensor, as do the nodes of the triangle. These states are summarized in the below table:

State of Turbulence	Invariants	Eigenvalues of Normalized Anisotropy
Isotropic	$\xi = \eta = 0$	$\lambda_1 = \lambda_2 = \lambda_3 = 0$
Two-Component Axisymmetric	$\xi = -\frac{1}{6}, \eta = \frac{1}{6}$	$\lambda_1 = \lambda_2 = \frac{1}{6}, \lambda_3 = -\frac{1}{3}$
One-Component	$\xi = \frac{1}{3}, \eta = \frac{1}{3}$	$\lambda_1 = 2/3, \lambda_2 = \lambda_3 = -\frac{1}{3}$
Axisymmetric Expansion	$\eta = \xi$	$-\frac{1}{3} \le \lambda_1 = \lambda_2 \le 0, \lambda_3 \ge 0$
Axisymmetric Contraction	$\eta = -\xi$	$0 \le \lambda_1 = \lambda_2 \le \frac{1}{6}, \lambda_3 \le 0$
Two-Component	$\det\left(\frac{\langle u_i' u_j' \rangle}{\frac{1}{3}\langle u_l' u_l' \rangle}\right) = 0$ $\eta = \left(\frac{1}{27} + 2\xi^3\right)^{1/2}$	$\lambda_1 + \lambda_2 = \frac{1}{3}, \lambda_3 = -\frac{1}{3}$

The above states are perhaps best characterized by the shape of the corresponding Reynolds-stress ellipsoid.

State of Turbulence	Shape of Reynolds-stress Ellipsoid
Isotropic	Sphere
Two-Comp. Axi.	Disk
One-Comp.	Line
Axi. Expansion	Prolate Spheroid
Axi. Contraction	Oblate Spheroid
Two-Comp.	Ellipse

The one-component, two-component, and axisymmetric states are signposts for characterizing anisotropy. When the normalized anisotropy and Reynolds stress tensors are written in terms of principal coordinates, then:

$$\langle (u_i')^2 \rangle = 2k\lambda_i + \tfrac{2}{3}k$$

Thus, when $\lambda_i = -\tfrac{1}{3}$, $\langle (u_i')^2 \rangle = 0$. This immediately suggests the one-component ($\lambda_2 = \lambda_3 = -\tfrac{1}{3}$) and two-component ($\lambda_3 = -\tfrac{1}{3}$) states. The axisymmetric states correspond to $\lambda_1 = \lambda_2$ and hence $\langle (u_1')^2 \rangle = \langle (u_2')^2 \rangle$. If $\lambda_3 \leq 0$, then $\lambda_1 = \lambda_2 \geq 0$ and $\langle (u_1')^2 \rangle = \langle (u_2')^2 \rangle$ are greater than $\tfrac{2}{3}k$ while $\langle (u_3')^2 \rangle$ is lesser. This is called the case of axisymmetric contraction. Such anisotropy could be produced by contraction in the plane normal to the x_3-axis and expansion along that axis. On the other hand, if $\lambda_3 \geq 0$, then $\langle (u_1')^2 \rangle = \langle (u_2')^2 \rangle$ are lesser than $\tfrac{2}{3}k$ while $\langle (u_3')^2 \rangle$ is greater, resulting in axisymmetric expansion.

For the Rotta model, it can be shown that:

$$\frac{d\xi}{dt} = -(C_R - 1)\frac{\varepsilon}{k}\xi, \quad \frac{d\eta}{dt} = -(C_R - 1)\frac{\varepsilon}{k}\eta$$

Hence, the trajectories in the ξ-η plane generated by the Rotta model are straight lines directed toward the origin for $C_R > 1$. Consequently, the trajectories stay within the Lumley triangle. However, in reality the trajectories swerve toward axisymmetric expansion (Choi and Lumley, 1984) during return to isotropy. The nonlinear model is equipped to handle this swerving. For illustrative purpose, trajectories associated with Rotta's model and the nonlinear model with $C_R = 1.7$, $C_R^n = 1.05$ are plotted on the Lumley triangle included on the previous page. Indeed, the nonlinear model's trajectories swerve toward axisymmetric expansion during return to isotropy.

Rapid-distortion limit:

We next consider the setting of rapid distortion. Homogeneous turbulence can be subjected to a time-dependent uniform mean velocity gradient, the magnitude of which can be characterized as:

$$\mathcal{S}(t) \equiv (2\bar{S}_{ij}\bar{S}_{ij})^{1/2}$$

The setting of rapid distortion is when the turbulence-to-mean-shear time-scale ratio $\tau\mathcal{S} = \mathcal{S}k/\varepsilon$ is arbitrarily large. In this limiting case, the evolution of the turbulence is exactly described by the rapid-distortion equations:

$$\frac{\bar{D}u_j'}{\bar{D}t} = -u_i'\frac{\partial \bar{u}_j}{\partial x_i} - \frac{1}{\rho}\frac{\partial p^{(r)}}{\partial x_j}$$

$$\frac{1}{\rho}\nabla^2 p^{(r)} = -2\frac{\partial \bar{u}_i}{\partial x_j}\frac{\partial u_j'}{\partial x_i}$$

The deformation caused by the mean velocity gradients can be considered in terms of the rate $\mathcal{S}(t)$, the amount:

$$s(t) \equiv \int_0^{t'} \mathcal{S}(t')\,dt',$$

and the geometry of the deformation:

$$G_{ij}(t) \equiv \frac{1}{\mathcal{S}(t)}\frac{\partial \bar{u}_i}{\partial x_j}$$

Using s in place of t and defining:

$$\hat{u}_i(\vec{x},s) \equiv u_i'(\vec{x},t),\quad \hat{G}_{ij}(s) \equiv G_{ij}(t),\quad \check{p}(\vec{x},s) \equiv \frac{p^{(r)}(\vec{x},t)}{\rho\, S(t)}$$

we find:

$$\frac{\bar{D}\hat{u}_j}{\bar{D}t} = -\hat{u}_i\,\hat{G}_{ij} - \frac{\partial \check{p}}{\partial x_j}$$

$$\nabla^2 \check{p} = -2\,\hat{G}_{ij}\,\frac{\partial \hat{u}_j}{\partial x_i}$$

Hence, in the limit of rapid distortion, the turbulence depends on the <u>geometry</u> and the <u>amount</u> of distortion but <u>not</u> its <u>rate</u>.

To make progress with the rapid distortion equations, it is necessary to circumvent or to solve the Poisson equation for $p^{(r)}$. We note the Poisson equation admits the formal solution:

$$p^{(r)}(\vec{x}) = \iiint_{-\infty}^{\infty} G(\vec{x},\vec{x}')\left(-2\rho\,\frac{\partial \bar{u}_i}{\partial x_j'}\,\frac{\partial u_j'}{\partial x_i'}\right)(\vec{x}')\,d\vec{x}'$$

where $G(\vec{x},\vec{x}')$ is the free-space Green's function:

$$G(\vec{x},\vec{x}') = -\frac{1}{4\pi}\,\frac{1}{|\vec{x}-\vec{x}'|}$$

We express the above as:

$$\begin{aligned} p^{(r)}(\vec{x}) &= \left(\nabla_\xi^2\right)^{-1}\left(\left(-2\,\frac{\partial \bar{u}_i}{\partial \xi_j}\,\frac{\partial u_j'}{\partial \xi_i}\right)(\vec{\xi})\right)(\vec{x}) \\ &= -2\rho\,\frac{\partial \bar{u}_i}{\partial x_j}\left(\nabla_\xi^2\right)^{-1}\left(\frac{\partial u_j'}{\partial \xi_i}(\vec{\xi})\right)(\vec{x}) \end{aligned}$$

By exploiting the properties of homogeneous turbulence, we can then show:

$$\begin{aligned} \left\langle \frac{p^{(r)}(\vec{x})}{\rho}\,\frac{\partial u_i'}{\partial x_j}(\vec{x})\right\rangle &= -2\,\frac{\partial \bar{u}_k}{\partial x_\ell}\left\langle \left(\nabla_\xi^2\right)^{-1}\left(\frac{\partial u_\ell'}{\partial \xi_k}(\vec{\xi})\right)(\vec{x})\,\frac{\partial u_i'}{\partial x_j}(\vec{x})\right\rangle \\ &= -2\,\frac{\partial \bar{u}_k}{\partial x_\ell}\left\langle \left(\nabla_\xi^2\right)^{-1}\left(\frac{\partial u_\ell'}{\partial \xi_k}(\vec{\xi})\,\frac{\partial u_i'}{\partial x_j}(\vec{x})\right)(\vec{x})\right\rangle \\ &= -2\,\frac{\partial \bar{u}_k}{\partial x_\ell}\left\langle \left(\nabla_\xi^2\right)^{-1}\left(\frac{\partial^2}{\partial x_j\,\partial \xi_k}\left(u_i'(\vec{x})\,u_\ell'(\vec{\xi})\right)(\vec{x})\right)\right\rangle \\ &= -2\,\frac{\partial \bar{u}_k}{\partial x_\ell}\left\{-\frac{1}{4\pi}\iiint_{-\infty}^{\infty}\frac{1}{|\vec{x}-\vec{\xi}|}\,\frac{\partial^2}{\partial x_j\,\partial \xi_k}\left(u_i'(\vec{x})\,u_\ell'(\vec{\xi})\right)d\vec{\xi}\right\} \\ \underset{\text{Change of Variables}}{} &= 2\,\frac{\partial \bar{u}_k}{\partial x_\ell}\left\{-\frac{1}{4\pi}\iiint_{-\infty}^{\infty}\frac{1}{|\vec{r}|}\,\frac{\partial^2}{\partial r_j\,\partial r_k}\left(u_i'(\vec{x})\,u_\ell'(\vec{x}-\vec{r})\right)d\vec{r}\right\} \\ \underset{\text{Homogeneity}}{} &= 2\,\frac{\partial \bar{u}_k}{\partial x_\ell}\left\{-\frac{1}{4\pi}\iiint_{-\infty}^{\infty}\frac{1}{|\vec{r}|}\,\frac{\partial^2 R_{i\ell}(\vec{r})}{\partial r_j\,\partial r_k}\,d\vec{r}\right\} \end{aligned}$$

where $R_{ij}(\vec{r}) = \langle u_i'(\vec{x}) u_j'(\vec{x}+\vec{r}) \rangle$ is the two-point velocity correlation. Defining:

$$M_{i\ell jk} \equiv -\frac{1}{4\pi} \iiint_{-\infty}^{\infty} \frac{1}{|\vec{r}|} \frac{\partial^2 R_{i\ell}(\vec{r})}{\partial r_j \partial r_k} d\vec{r} = (\nabla_r^2)^{-1} \left(\frac{\partial^2 R_{i\ell}}{\partial r_j \partial r_k}(\vec{r}) \right)(0)$$

we have the following expression for the rapid redistribution tensor:

$$R_{ij}^{(r)} = 2 \frac{\partial \bar{u}_k}{\partial x_\ell} \left(M_{i\ell jk} + M_{j\ell ik} \right)$$

The fact that the rapid redistribution tensor depends on two-point correlations speaks to its nonlocal nature and has motivated the development of two-point closure models. Nonetheless, insight into its form can be obtained by examing its behavior for specific flow regimes, as is done in Section 11.4 in Pope. The fourth-order tensor $M_{ijk\ell}$ satisfies several constraints, including:

$$M_{ijk\ell} = M_{jik\ell} = M_{ij\ell k}, \quad M_{ijj\ell} = 0, \quad M_{ijkk} = \langle u_i' u_j' \rangle$$

For isotropic turbulence, when $M_{ijk\ell}$ is necessarily an isotropic tensor, we then have:

$$M_{ijk\ell} = \frac{1}{15} k \left(4 \delta_{ij} \delta_{k\ell} - \delta_{ik} \delta_{j\ell} - \delta_{i\ell} \delta_{jk} \right)$$

and hence the rapid pressure-rate-of-strain tensor is:

$$R_{ij}^{(r)} = \frac{4}{5} k \bar{S}_{ij} = -\frac{3}{5} P_{ij}$$

Thus, at the initial instant when distortion is applied, the effect of the rapid pressure on the Reynold's stress is to counteract 60% of the production. After this initial instant, the tensor $M_{ijk\ell}$ is no longer isotropic. To proceed, we assume:

$$M_{ijk\ell} = M_{ijk\ell}(b_{ij}, k, \varepsilon)$$

Dimensional analysis and the Cayley-Hamilton theorem then dictate that $M_{ijk\ell}$ can be written as a quadratic polynomial of b_{ij}. However, this form is quite complicated, so we instead elect to only include terms which are linear in the normalized anisotropy, leading to:

$$\begin{aligned} R_{ij}^{(r)} = \; & C_1 k \bar{S}_{ij} \\ & + 2k (C_2 + C_3) \left(b_{ik} \bar{S}_{kj} + b_{jk} \bar{S}_{ki} - \tfrac{2}{3} \delta_{ij} b_{k\ell} \bar{S}_{\ell k} \right) \\ & + 2k (C_2 - C_3) \left(b_{ik} \bar{\Omega}_{kj} + b_{jk} \bar{\Omega}_{ki} \right) \end{aligned}$$

The coefficients C_1, C_2, and C_3 are in general functions of the invariants II_b and III_b, resulting in a General Quasi-Linear Model, but they are most often taken as constants. A number of authors have proposed models based on the above, based on a variety of modeling considerations.

1973: Launder, Reece, and Rodi (LRR) Model: $C_1 = 4/5$, $C_2 = \frac{c+8}{11}$, $C_3 = \frac{8c-2}{11}$ $\quad c \approx 0.4$

1989: Isotropization of Production (IP) Model: $C_1 = 4/5$, $C_2 = 3/5$, $C_3 = 0$

1989: Speziale, Sarkhar, and Gatski (SSG) Model: $C_1 = 4/5 - 0.65 \sqrt{|II_b|}$, $C_2 = 0.41125$, $C_3 = 0.2125$

After choosing among the rapid models above, the Reynolds stress model is complete for homogeneous turbulence. Analytic and/or numerical solutions can then be found and compared to empirical information. For example, in homogeneous shear flow, experimental data yields:

$$[b_{ij}] = \begin{bmatrix} 0.36 \pm 0.08 & -0.32 \pm 0.02 & 0 \\ -0.32 \pm 0.02 & -0.22 \pm 0.05 & 0 \\ 0 & 0 & -0.14 \pm 0.06 \end{bmatrix}$$

whereas the IP model coupled with the Rotta model yields:

$$[b_{ij}] = \begin{bmatrix} 0.356 & -0.361 & 0 \\ -0.361 & -0.178 & 0 \\ 0 & 0 & -0.173 \end{bmatrix}$$

and the SSG model coupled with the Rotta model yields:

$$[b_{ij}] = \begin{bmatrix} 0.433 & -0.328 & 0 \\ -0.328 & -0.282 & 0 \\ 0 & 0 & -0.151 \end{bmatrix}$$

The Basic Model:

Now that we have discussed the zero-distortion and rapid-distortion limits at length, we return to the problem of modeling the complete pressure-rate-of-strain tensor. The most basic, and likely most utilized, model is the basic model put forth by Naot, Shavit, and Wolfstein in 1970:

$$R_{ij} = \underbrace{-C_R \frac{\varepsilon}{k}\left(\langle u_i' u_j' \rangle - \tfrac{2}{3} k \delta_{ij}\right)}_{\text{Return to Isotropy}} \underbrace{- C_2 \left(P_{ij} - \tfrac{2}{3} P \delta_{ij} \right)}_{\text{Isotropization of Production}}$$

The above model supposes that the rapid pressure partially counteracts the effect of production to increase the Reynolds-stress anisotropy. This is indeed the effect observed for rapid distortion axisymmetric expansion. If C_R is taken to be 1.8, Rotta's original return to isotropy model is obtained, and if C_2 is taken to be $3/5$, the simple basic model yields the correct initial response of isotropic turbulence to all rapid distortions.

On D2L, a plot comparing calculations of the basic model to DNS data for homogeneous shear flow may be found. From this plot, it may be seen that the evolution of b_{12} and b_{11} is reasonably represented by the basic model, but the model predicts that b_{22} and b_{33} are equal while the DNS data demonstrates that $b_{22} \ll b_{33}$. Nonetheless, the basic model is much simpler than most other pressure-rate-of-strain models and often forms the basis of more sophisticated models (e.g., elliptic relaxation models).