Turbulent Boundary Layer – Mean Velocity Profiles:

The profiles of the mean axial velocity and shear stress for laminar and turbulent boundary layers subject to zero pressure gradient are included in a PDF on D2L at the link:

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The turbulent layer corresponds to $Re_\theta = 8{,}000$ and the profile associated with the layer was obtained from Klebanoff (1954). Immediately observe that the turbulent mean velocity profile rises much more steeply from the wall than the laminar profile, and it is flatter away from the wall. The "flatness" of the profile is quantified by the shape factor:

$$H \equiv \delta^*/\theta$$

The flatness factor is $H \cong 2.6$ for the laminar profile and $H \cong 1.3$ for the $Re_\theta = 8{,}000$ profile. The shape factor decreases with increasing Re_θ.

Near the wall (i.e., plate), the law of the wall is satisfied:

$$u^+ = f_w(y^+)$$

In the viscous sublayer ($y^+ < 5$), $u^+ \approx y^+$, and in the log law region ($y^+ > 30$, $y/\delta \ll 1$), $u^+ = 1/\kappa \ln(y^+) + B$. The form of the law of the wall in the buffer layer ($5 < y^+ < 30$) may be approximated using the mixing-length hypothesis:

$$\langle u'v' \rangle = \underset{\text{Turbulent Viscosity}}{\nu_T} \frac{\partial \bar{u}}{\partial y} = \underset{\text{Mixing Length}}{l_m^2} \left(\frac{\partial \bar{u}}{\partial y}\right)^2$$

Setting $l_m^+ \equiv l_m/\delta_\nu$:

$$\tau/\tau_w = \frac{\partial u^+}{\partial y^+} + \left(l_m^+ \frac{\partial u^+}{\partial y^+}\right)^2$$

Solving the above for the velocity gradient gives:

$$\frac{\partial u^+}{\partial y^+} = \frac{2\tau/\tau_w}{1 + \left[1 + (4\tau/\tau_m)(l_m^+)^2\right]^{1/2}}$$

In the inner layer where the law of the wall holds, $\tau/\tau_w \approx 1$, giving:

$$u^+ = \int_0^{y^+} \frac{2\,dy'}{1 + \left[1 + 4(l_m^+(y'))^2\right]^{1/2}}$$

In the log law region, $l_m = \kappa y$, while the mixing length should be damped near the wall in order to ensure $u^+ \approx y^+$. Thus, van Driest proposed the following specification in 1956:

$$l_m^+ = \kappa y^+ \underbrace{\left[1 - \exp(-y^+/A^+)\right]}_{\text{van Driest damping function}}$$

where A^+ is commonly taken to be 26. For large y^+, $l_m^+ \approx \kappa y^+$, while for small y^+, $l_m^+ \approx 0$. On the third page of Plots.pdf, van Driest's law of the wall is compared with DNS data. A very good fit is observed.

Outside of the inner layer, the mean velocity profile begins to deviate from the channel flow profile. In particular, in the defect layer ($y/\delta > 0.2$), the mean velocity deviates from the log law. This is seen from the figure on the fourth page of Plots.pdf. In 1956, Coles hypothesized that the mean velocity profile throughout the boundary layer may be well represented by the sum of two functions:

$$\frac{\bar{u}}{u_\tau} = \underbrace{f_w\left(\frac{y}{\delta_\nu}\right)}_{\text{Law of the Wall}} + \frac{\Pi}{\kappa} \underbrace{w\left(\frac{y}{\delta}\right)}_{\text{Wake Function}}$$

where Π is the so-called wake-strength parameter. The wake function is assumed universal, while the value of Π is flow-dependent. Coles originally tabulated $w(y/\delta)$ based on experimental data, but a more convenient approximation is:

$$w\left(\frac{y}{\delta}\right) = 2 \sin^2\left(\frac{\pi}{2} \frac{y}{\delta}\right)$$

The approximate profile of Cole is compared with experimental data in the figure on the fourth page of Plots.pdf, and good agreement between the profiles is observed.

If one is to apply the mixing-length turbulence model to a boundary layer flow, one must modify the formula $\ell_m = \kappa y$ in the defect layer. For example, Escudier proposed to set

$$\ell_m = \min\{\kappa y, 0.09\delta\}$$

in 1966.

It should be mentioned that one may obtain Reynolds number dependencies of statistical quantities in much the same manner as for a turbulent channel flow. An explicit approximation of the skin-friction coefficient due to Schlichting (1979), for example, is:

$$c_f = 0.370 \left(\log_{10} Re_x\right)^{-2.584}$$