Flow Rates of Mass, Momentum, and Energy:

Using the conservative form of the shear layer equation:

$$\frac{\partial}{\partial x}\left((\bar{u})^2\right) + \frac{\partial}{\partial y}\left(\bar{u}\,\bar{v}\right) = -\frac{\partial}{\partial y}\langle u'v'\rangle \qquad \text{(ignoring viscous terms)}$$

we can make some fundamental observations about flow rates. Integrate the above from $y=-\infty$ to $y=+\infty$ to obtain:

$$\frac{d}{dx}\int_{-\infty}^{\infty}(\bar{u})^2\,dy = -\Big[\bar{u}\,\bar{v} + \langle u'v'\rangle\Big]_{-\infty}^{\infty}$$

For a planar jet or wake, both $\bar{u}\bar{v}\Big|_{-\infty}^{\infty}$ and $\langle u'v'\rangle\Big|_{-\infty}^{\infty}$ go to zero (as $y \to \pm\infty$). Thus:

$$\frac{d}{dx}\int_{-\infty}^{\infty}(\bar{u})^2\,dy = 0 \qquad \text{jet or wake}$$

This implies that the momentum flow rate of the mean flow:

$$\dot{M}(x) \equiv \int_{-\infty}^{\infty}\rho\left(\bar{u}(x,y)\right)^2 dy$$

is conserved in that the rate is independent of x for jets and wakes. For a mixing layer, $\bar{u}\bar{v}\Big|_{-\infty}^{\infty} \neq 0$, so the rate is not conserved.

We will show later that we expect:

$$\bar{u}(x,y) = u_{offset} + u_s(x)\,\mathcal{F}(\eta) \qquad \eta = \frac{y}{\delta(x)}$$

for a fully developed free shear flow that is planar. If $u_{offset} = 0$ (as for a non-coflowing jet), then the flow rates of mass and energy are, in the mean:

Flow Rate of Mass of the Mean Flow:

$$\dot{m}(x) \equiv \int_{-\infty}^{\infty}\rho\,u_s(x)\,\mathcal{F}\!\left(\frac{y}{\delta(x)}\right)dy$$

$$= \rho\,u_s(x)\,\delta(x)\int_{-\infty}^{\infty}\mathcal{F}(\eta)\,d\eta$$

Linearly proportional to $u_s(x)\,\delta(x)$!

Flow Rate of Energy of the Mean Flow:

$$\dot{E}(x) \equiv \int_{-\infty}^{\infty}\frac{1}{2}\rho\,u_s^3(x)\,\mathcal{F}^3\!\left(\frac{y}{\delta(x)}\right)dy$$

$$= \int_{-\infty}^{\infty}\frac{1}{2}\rho\,u_s^3(x)\,\mathcal{F}^3(\eta)\,\delta(x)\,d\eta$$

$$= \frac{1}{2}\rho\,u_s^3(x)\,\delta(x)\int_{-\infty}^{\infty}\mathcal{F}^3(\eta)\,d\eta$$

Linearly proportional to $u_s^3(x)\,\delta(x)$!

For a non-coflowing jet, we will later show that $\delta \sim x$ and $U_s \sim x^{-1/2}$, so:

$$\left.\begin{aligned} \dot{m}(x) &\sim x^{1/2} \\ \dot{E}(x) &\sim x^{-1/2} \end{aligned}\right\} \text{For a non-conflowing jet!}$$

Therefore, the mass flow rate increases and the energy flow rate decreases as x increases.