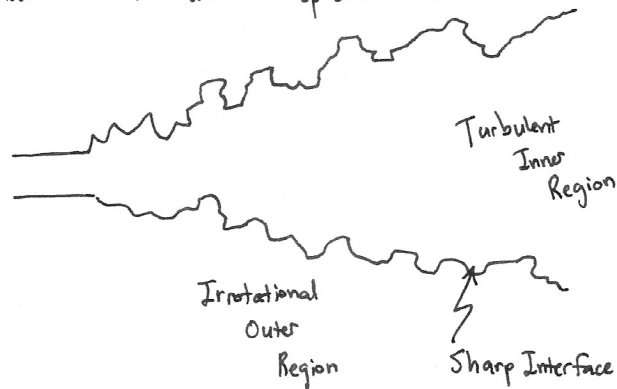


Intermittency, the Viscous Superlayer, and Large-Scale Turbulent Structures in Free Shear Flows:

Visual observations of free-shear flows at a particular instance in time suggests there is a sharp (but highly irregular) interface between the inner turbulent flow and the outer irrotational ambient fluid as depicted below.



At a fixed position near the edge of the turbulent inner region, the motion is sometimes turbulent and sometimes irrotational. Hence, it is said to be intermittent, and as a consequence of intermittency, the sharp interface between the inner and outer regions is constantly changing.

We describe intermittency at a point in space and time using the intermittency function:

$$I(\vec{x}, t) = H(|\vec{\omega}(\vec{x}, t)| - \omega_{\text{thresh.}})$$

\nwarrow \nwarrow \nwarrow
 Heaviside function vorticity threshold value

The intermittency factor is the probability that the flow at (\vec{x}, t) is turbulent:

$$\gamma(\vec{x}, t) = \text{Prob}(|\vec{\omega}(\vec{x}, t)| > \omega_{\text{thresh.}}) = \langle I(\vec{x}, t) \rangle$$

Outside of the turbulent inner region, $\gamma < 1$, and inside the irrotational outer region, $\gamma \approx 0$. The intermittency function may be utilized to obtain conditional statistics of a flow - see, for example, the discussion on pg. 168-170 in Pope. Moreover, it may be used to enhance a turbulent viscosity model as:

$$\nu_T = \nu_s(x) f(x) \hat{\gamma}_T(\eta) \gamma(\eta)$$

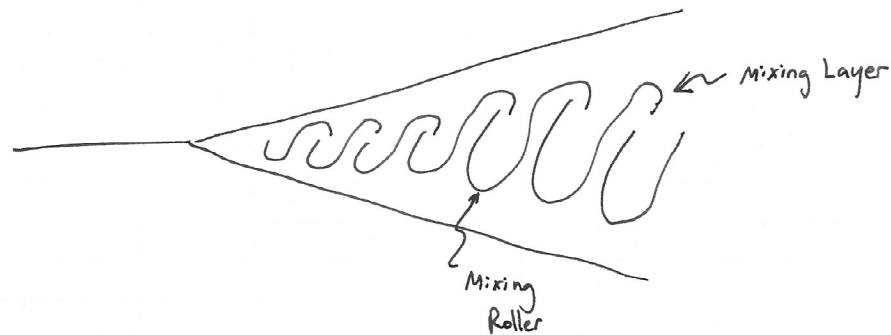
\nwarrow intermittency factor

Thickness: $O(\delta Re \delta^{-3/4})$

The sharp layer between the inner and outer regions is referred to as the viscous superlayer because viscous effects dominate turbulent effects within the interface region. This is because shear stresses are greatly enhanced within the viscous superlayer.

In addition to intermittency, free shear flows may exhibit large-scale turbulent motion. This motion may not be characterized by one-point statistics, and instead two-point (or more) velocity correlations must be employed.

The canonical large-scale turbulent structure associated with a free shear flow is the mixing roller:



Mixing rollers are regions of concentrated spanwise vorticity that are coherent over substantial spanwise distances. As rollers move downstream, they merge and tear themselves apart. When two rollers merge, this is referred to as a "pairing" process. When a roller is torn apart, its vorticity is absorbed by adjacent rollers.

Mixing layer rollers resemble the motion induced by the Kelvin-Helmholtz instability, an inviscid instability that occurs when there is a velocity shear in a single continuous fluid. To further investigate this, consider the Rayleigh equation, the Orr-Sommerfeld equation with no viscosity:

$$(U - c) \left(\frac{d^2 \xi_2}{dx_2^2} - \alpha^2 \xi_2 \right) - \frac{d^2 U}{dx_2^2} \xi_2 = 0 \quad -h \leq x_2 \leq h$$

$\xi_2 = \text{amplitude}$
 $\alpha = \text{angular frequency}$
 $c = \text{wave speed}$

$\xi_2(\pm h) = 0$

See "Stability and Transition to Turbulence"

If there exists a solution to the Rayleigh equation with complex wave speed with positive imaginary part, the background flow is unstable. Fjortoft's theorem states that any flow whose vorticity magnitude is maximized within the region of flow away from boundaries is unstable. Thus, the shear profile:

$$U(x_2) = \tanh\left(\frac{x_2}{h}\right)$$

which has maximum vorticity magnitude at $x_2 = 0$ is unstable, and as this instability evolves, the shear profile "rolls" up just like a sequence of mixing rollers.

In a mixing layer, rollers make a large contribution to entrainment into the boundary layer. In fully turbulent flow, the gap between rollers, the so-called "braid region", contains a good deal of small scale turbulence.

In the near field of wakes behind bluff bodies (e.g., cylinders and spheres), there are large-scale motions with preferred frequencies, the canonical example being vortex shedding. These motions, like mixing rollers, are linked to basic fluid dynamic instabilities, and they are also responsible for differences among observed spreading rates (e.g., for a cylinder, plate, or airfoil).

Large-scale turbulent structures are important for a number of reasons. Perhaps most importantly, they act as a mechanism for mixing in turbulent flows. Generally speaking, there are two different ways in which a turbulent flow mixes a passive scalar in a mixing layer:

Entrainment by Nibbling: Diffusion processes draw free stream fluid into the viscous superlayer, and turbulent eddies at the edge of the turbulent region act to engulf small bits of fluid and mix that fluid into the turbulent region.

Entrainment by Engulfment: Regions of free stream fluid are engulfed into the mixing layer via large-scale spanwise vortices undergoing pair interactions.

Entrainment by nibbling results in the mixed value of a passive scalar having continuous variation as a function of y . The peak of the PDF (probability density function) for the passive scalar will march from one free-stream value to the other, leading to the name "marching PDF."

Entrainment by engulfment, on the other hand, results in the mixed value of a passive scalar having the same value at any y location within the mixing layer. The only difference is the magnitude of the "spike" at the two free stream values, as the mixed value jumps from the constant "mixed value" to the free stream values. In this case, the peak of the PDF does not depend on y within the layer and jumps at the two free stream values, leading to the name "non-marching PDF."

In general, mixing will occur due to both entrainment by nibbling and entrainment by engulfment. Consequently, the PDF for a passive scalar will have both "marching" and "non-marching" characteristics for most flow settings.