Turbulent Viscosity Models – Algebraic Models

If the turbulent-viscosity hypothesis is accepted as valid, all that remains is to determine an appropriate specification for the turbulent viscosity $\nu_T(\vec{x},t)$. The simplest models are predicated on the fact that $\nu_T(\vec{x},t)$ can be written as the product of a velocity $u^*(\vec{x},t)$ and a length $\ell^*(\vec{x},t)$:

$$\nu_T = u^* \ell^*$$

and the task of specifying ν_T is generally approached through specifications of u^* and ℓ^*. This immediately inspires the simplest type of turbulent viscosity model, the so-called uniform turbulent viscosity model. For planar two-dimensional free shear flow, the uniform-turbulent-viscosity model is:

$$\nu_T(x,y) = \frac{U_0(x)\,\delta(x)}{R_T} \qquad \text{uniform-turbulent-viscosity model}$$

where $U_0(x)$ and $\delta(x)$ are the characteristic velocity and length scales of the mean flow and R_T, the turbulent Reynolds number, is a flow-dependent constant. Thus the turbulent viscosity is taken to be constant in the cross-stream direction, but it varies in the mean-flow direction.

The range of the uniform-turbulent-viscosity model is extremely limited as it is necessary to define unambiguously the direction of flow, x; the characteristic flow width, $\delta(x)$; and the characteristic velocity $U_0(x)$. This only is possible for the simplest flows, such as planar jets, wakes, and mixing layers.

An alternative and historically important algebraic model was introduced by Prandtl by appealing to the notion of a mixing length. In this model, one writes the time-scale $t^* = \ell^*/u^*$ as:

$$t^* \approx \left| \frac{\partial \bar{u}}{\partial y} \right|$$

for a boundary layer or other nearly parallel shear flow. Assuming the length-scale $\ell^* = \ell_m(x,y)$ is known, the turbulent viscosity is then written as:

$$\nu_T(x,y) = \ell_m^2(x,y) \left| \frac{\partial \bar{u}}{\partial y}(x,y) \right| \qquad \text{mixing length model}$$

The length-scale ℓ_m is called the mixing length and must be specified. In wall-bounded flows, the mixing length is taken to be $\ell_m = \kappa y$ in the log-law region where κ is the von Karman constant and y is the distance from the wall.

Several generalizations of the mixing length model have been proposed for general flows. The first of note is due to Smagorinsky (1963) and is based on the mean rate of strain:

$$\nu_T = \ell_m^2 \left(2 \bar{S}_{ij} \bar{S}_{ij}\right)^{1/2} = \ell_m^2 S \qquad \text{Smagorinsky (1963)}$$

and this model is also commonly used in large eddy simulation. The second generalization of note is due to Baldwin and Lomax (1978) and is based on the mean rate of rotation:

$$\nu_T = \ell_m^2 \left(2 \bar{\Omega}_{ij} \bar{\Omega}_{ij}\right)^{1/2} = \ell_m^2 \Omega \qquad \text{Baldwin \& Lomax (1978)}$$

In its generalized form, the mixing length model is applicable to all turbulent flows and is arguably the simplest turbulence model. Its major drawback, however, is that the mixing length $\ell_m(x)$ has to be specified. Nonetheless, there are classes of technologically important flows that have been studied extensively, so that specifications of $\ell_m(\vec{x})$ are well-established. The prime example is boundary-layer flows in aeronautical applications.