Turbulent Channel Flow – Reynolds Stresses:

The total shear stress is made up of the viscous shear stress and the Reynolds shear stress. Here, we examine the Reynolds stresses in more detail. A plot of the various Reynolds stresses versus y^+ and normalized by the square of u_τ is included on the first page of:

31 – Turbulent Channel Flow – Reynolds Stresses → Plots.pdf
(from a DNS at $Re = 13{,}750$)

From this plot, we observe that the Reynolds stresses vary wildly over the height of the layer. Moreover, the stress $\langle (u')^2 \rangle$ is qualitatively similar to k, the turbulent kinetic energy. A better plot of the Reynolds stresses is thus obtained by rescaling with k, and such a plot is provided on the second page of Plots.pdf. The Reynolds stresses, when scaled by k, behave differently over three distinct regions:

1. The Viscous Wall Region $(y^+ < 50)$
2. The Log-law Region $(50 < y^+ < 120$ for $Re = 13{,}750)$
3. The Core $(y^+ > 120$ for $Re = 13{,}750)$

In the log-law region, there is approximate self-similarity. The normalized Reynolds stresses are nearly uniform. This may be explained by considering the turbulent kinetic energy budget for a channel:

$$0 = \frac{\partial k}{\partial t} = \underbrace{-(\langle \vec{u} \rangle \cdot \vec{\nabla})k}_{\text{Mean Convection}} + \underbrace{\vec{\nabla} \cdot \vec{T}}_{\text{Flux}} + \underbrace{\mathcal{P}}_{\text{Production}} - \underbrace{\varepsilon}_{\text{Dissipation}}$$

For a turbulent channel at steady state, the Reynolds stresses are closely related to the ratio $\mathcal{P}/\varepsilon$. This statistic is plotted on the third page of Plots.pdf where it is seen that $\mathcal{P}/\varepsilon \approx 1$ in the log-law region, indicating production and dissipation are in balance. Moreover, in this region, viscous and turbulent transport contributions are small by comparison, resulting in approximate self-similarity.

On the centerline, the mean velocity gradient and the shear stress vanish, giving zero production. There is an analogous drop in $\langle (u')^2 \rangle$ in the core. The Reynolds stresses are anisotropic in the core.

The viscous wall region contains the most vigorous turbulent activity. The production peaks in this region, as does the ratio of production to dissipation, which reaches 1.81 at $y^+ \approx 11.8$. At this value of y^+, the Reynolds stress $\langle (u')^2 \rangle$ also sharply peaks, but the other Reynolds stresses do not. This is because the production term is nonzero only for the dynamical component equation for $\langle (u')^2 \rangle$:

$$0 = \frac{\partial \langle (u')^2 \rangle}{\partial t} = -2 \langle u'v' \rangle \frac{\partial \bar{u}}{\partial y} + \dots$$

$$0 = \frac{\partial \langle (v')^2 \rangle}{\partial t} = 0 + \dots$$

$$0 = \frac{\partial \langle (w')^2 \rangle}{\partial t} = 0 + \dots$$

Production terms

since $\bar{u} = \bar{u}(y)$ and $\bar{v} = \bar{w} = 0$. Finally, a plot of the production, dissipation, viscous diffusion, turbulent convection, and pressure transport terms ~~are~~ is included on the fourth page of Plots.pdf.