Dynamics of Turbulent Kinetic Energy in Free-Shear Flows:

The nonlinear equation for the evolution of turbulent kinetic energy $k = \frac{1}{2}\langle \vec{u}' \cdot \vec{u}' \rangle$ is:

$$\frac{\partial k}{\partial t} + \underbrace{\bar{u} \cdot \vec{\nabla} k}_{\text{Mean-flow Convection}} + \underbrace{\vec{\nabla} \cdot \vec{T}'}_{\text{Turbulent Transport}} = \underbrace{P}_{\text{Production}} - \underbrace{\varepsilon}_{\text{Dissipation}}.$$

where:

$$T_i' \equiv \frac{1}{2}\langle u_i' u_j' u_j' \rangle + \langle u_i' p' \rangle / \rho - 2\nu \langle u_j' S_{ij}' \rangle$$

$$P \equiv -\langle u_i' u_j' \rangle \frac{\partial \bar{u}_i}{\partial x_j}$$

$$\varepsilon \equiv 2\nu \langle S_{ij}' S_{ij}' \rangle$$

$$S_{ij}' \equiv \frac{1}{2}\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i} \right)$$ ← *Fluctuating rate of strain!*

The production term is generally positive while the dissipation term is negative. Generally speaking, the action of mean velocity gradients working against the Reynolds stresses removes kinetic energy from the mean flow and transfers it to the fluctuating velocity field, resulting in production of turbulent kinetic energy, while the fluctuating velocity gradients working against the fluctuating deviatoric stresses transform kinetic energy into internal energy, resulting in dissipation.

Plots of the budget for the turbulent kinetic energy for a round jet and an axisymmetric wake are included in a PDF on D2L at the link:

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Note from the plots that the budget is very different between the two cases. Dissipation is a dominant term throughout the round jet, while it is only half the size of the mean-flow convection in the axisymmetric wake. The production peaks at $r/r_{1/2} \approx 0.6$ for the round jet where the ratio P/ε is about 0.8. In contrast, the production P is just 20% of ε in the axisymmetric wake and 15% of convection. The dominance of convection, and the relatively small amount of production, suggest that the turbulence in an axisymmetric wake is strongly influenced by conditions upstream. This makes sense since the spreading rate for a wake depends very significantly on the geometry of the body that generates the wake. On going from streamlined bodies to bluff bodies, S increases by a factor of 10.