Modeling and Simulation of Turbulence

Until this point, we have employed analytical tools to study the impact of turbulence on mean flow features for several simple problems, including jets, mixing layers, wakes, channels, boundary layers, and homogeneous isotropic turbulence. However, for complex flows of engineering interest, analytical tools are unable to be employed successfully. Consequently, turbulence modeling and simulation must be employed to predict the effects of turbulence in flows of technological relevance.

There are three primary approaches to turbulence modeling and simulation: direct numerical simulation (DNS), large eddy simulation (LES), and Reynolds-averaged Navier Stokes (RANS). These three approaches comprise the spectrum of turbulence modeling, and each approach exhibits a tradeoff between resolution and computational cost.

Increasing Resolution and Computational Cost →

Steady RANS — Unsteady RANS — Hybrid LES/RANS, LES w/ Wall Modeling — LES — DNS

Broadly speaking, DNS and LES are turbulent-flow simulation approaches in which equations for a time-dependent velocity field are solved for one realization of the turbulent flow. In this sense, DNS and LES are deterministic approaches. DNS directly solves the Navier-Stokes equations, resolving all the scales of motion, with initial and boundary conditions appropriate to the flow considered. While DNS is very expensive, it provides unparalleled resolution of flow details. In LES, equations are solved for a 'filtered' velocity field which is representative of the larger-scale turbulent motions. The equations solved include a model for the influence of the smaller-scale motions which are not directly represented.

RANS is a turbulence modeling approach in which equations are solved for mean quantities. In this sense, RANS is a statistical approach. In RANS, the Reynolds equations are solved for the mean velocity field, and appropriate models must be postulated for the Reynolds stresses.

There are several criteria that can be used to assess different models. Principal among these are the following:

(i) Level of Description: In RANS, the description of the flow is at the mean-flow level. No information is provided about higher-order moments of velocity such as two-point correlations. In DNS, the flow is described by the instantaneous velocity from which all other information can be determined.

(ii) Completeness: A model is deemed complete if the constituent equations are free from flow-dependent specifications. DNS is complete, while most RANS and LES approaches are not.

(iii) Cost and Ease of Use: In DNS, the computational cost rapidly increases with Re, while in many RANS approaches, the increase in cost is insignificant or nonexistent.

(iv) Applicability: Not all models are applicable to all flows. For example, mixing-length RANS models typically make assumptions about the flow geometry in the specification of the mixing length. Computational requirements place another limitation on applicabilty. For DNS, such requirements rise so steeply with Re that DNS is applicable to only low and medium Reynolds number flows. It is emphasized that applicability is a separate notion from accuracy. A model is applicable to a flow if the model equations are well-posed and can be solved, irrespective of whether the solutions are accurate.

(v) Accuracy: It goes without saying that accuracy is a desirable attribute in any model. That being said, there are many potential sources of error, including:

* Inaccuracies of the model (e.g. turbulence closures)
* Numerical error (e.g., spatial truncation error)
* Measurement error
* Discrepancies in the boundary conditions

DNS is free from model inaccuracies, but it may still suffer from the other sources of error listed above. Hence, there is a danger of drawing false conclusions about the accuracy of a model no matter its level of resolution.

The suitability of a particular model for a particular turbulent-flow problem depends on a weighted combination of the above criteria, and the relative weighting further depends on the problem. Hence, there is no one 'best' model, but simply a range of models that can be usefully applied to a broad range of problems.