Turbulent-Viscosity Models – The RNG and Realizable k-ε Models

An interesting development in turbulence modeling occurred in 1986 when Yakhot and Orszag "derived" the k-ε model using renormalization group theory (RNG theory). RNG theory is a technique that has been very successful in solid state physics, but its application to turbulence has been surrounded by controversy. While the mathematics of the technique are sophisticated, it appears that RNG derivations have no more justification than other turbulence theories. RNG k-ε models are very similar to k-ε models presented heretofore, except for "derived" nonstandard model constants and a viscous damping effect near the wall.

Both the standard and RNG k-ε models suffer from certain "realizability" issues. The term "realizable" means that the model satisfies certain mathematical constraints on the Reynolds stresses, consistent with the physics of turbulent flows. Two standard "realizability" constraints are:

(i) $\langle (u')^2\rangle,\ \langle (v')^2\rangle,\ \langle (w')^2\rangle \geq 0$ — Nonnegativity of Normal Stresses

(ii) $\langle (u_i'u_j')^2\rangle \leq \langle (u_i')^2(u_j')^2\rangle$ — Schawarz Inequality for Shear Stresses

The standard k-ε model violates the above constraints when the mean strain rate is large. To see this, note that:

$$\langle (u')^2\rangle = \tfrac{2}{3}k - 2\nu_t\frac{\partial \bar{u}}{\partial x} = \tfrac{2}{3}k - 2C_\mu \frac{k^2}{\varepsilon}\frac{\partial \bar{u}}{\partial x}$$

for the k-ε model. Hence, when:

$$\frac{k}{\varepsilon}\frac{\partial \bar{u}}{\partial x} > \frac{1}{C_\mu} \approx 3.7$$

the modeled normal stress $\langle (u')^2\rangle$ becomes negative. The most straight-forward way to ensure realizability is to make C_μ variable by sensitizing it to mean deformation and the turbulence quantities k and ε. An additional weakness of the k-ε model in its standard form lies with the modeled equation for ε, and indeed the well-known round-jet anomaly is considered to be mainly due the modeled ε equation.

The realizable k-ε model proposed by Shih et al. in 1995 was designed specifically to address the above deficiencies by adopting the following:

(i) A new eddy-viscosity formula involving a variable C_μ (proposed by Reynolds).
(ii) A new model equation for ε based on the dynamic equation for the mean-square vorticity fluctuation.

The details of the realizable k-ε model are somewhat complicated and hence have been omitted here.

The RNG and realizable k-ε models show~~s~~ substantial improvement over the standard k-ε model when the flow features strong streamline curvature, vortices, and rotation. Moreover, ~~it~~ the realizable k-ε model exhibits better performance than both the standard and RNG k-ε models for separated flows and flows with complex secondary features.

The primary limitation of the realizable k-ε model is that it produces non-physical turbulent viscosities when the domain of interest contains both rotating and stationary fluid zones.