

SAMB for “D2h1”

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- Group: No. 47 D_{2h}^1 $Pmmm$ [orthorhombic]
 - Associated point group: No. 8 D_{2h} mmm [orthorhombic]
 - Generation condition
 - model type: **tight_binding**
 - time-reversal type: **electric**
 - irrep: [Ag]
 - spinful
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- Unit cell:
 $a = 1.0$, $b = 1.2$, $c = 1.5$, $\alpha = 90.0$, $\beta = 90.0$, $\gamma = 90.0$
- Lattice vectors:
 $\mathbf{a}_1 = (1.0 \ 0 \ 0)$
 $\mathbf{a}_2 = (0 \ 1.2 \ 0)$
 $\mathbf{a}_3 = (0 \ 0 \ 1.5)$

Table 1: High-symmetry line: Γ -X.

symbol	position	symbol	position
Γ	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	X	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$

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- Kets: dimension = 8

Table 2: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket	No.	ket	No.	ket
1	$(s, \uparrow)@A_1$	2	$(s, \downarrow)@A_1$	3	$(p_x, \uparrow)@A_1$	4	$(p_x, \downarrow)@A_1$	5	$(p_y, \uparrow)@A_1$
6	$(p_y, \downarrow)@A_1$	7	$(p_z, \uparrow)@A_1$	8	$(p_z, \downarrow)@A_1$				

- Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position	mapping
S ₁ [1a: mmm]	A ₁	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	[1,2,3,4,5,6,7,8]

- Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

	bond	tail	head	n	#	$\mathbf{b}@c$	mapping
B ₁ [1b: mmm]	b ₁	A ₁	A ₁	1	1	$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$	[1,-2,-3,4,-5,6,7,-8]
B ₂ [1e: mmm]	b ₂	A ₁	A ₁	2	1	$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{2} & 0 \end{pmatrix}$	[1,-2,3,-4,-5,6,-7,8]
B ₃ [1c: mmm]	b ₃	A ₁	A ₁	3	1	$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} @ \begin{pmatrix} 0 & 0 & \frac{1}{2} \end{pmatrix}$	[1,2,-3,-4,-5,-6,7,8]
B ₄ [1f: mmm]	b ₄	A ₁	A ₁	4	1	$\begin{pmatrix} 1 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	[1,-2,-5,6]
	b ₅	A ₁	A ₁	4	1	$\begin{pmatrix} 1 & -1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	[-3,4,7,-8]

- SAMB:

$$\boxed{\text{No. 1}} \quad \hat{Q}_0^{(A_g)} [M_1, S_1]$$

$$\hat{Z}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\hat{\mathbb{Z}}_1(\mathbf{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\boxed{\text{No. 2}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_2 = \mathbb{X}_2[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\hat{\mathbb{Z}}_2(\mathbf{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\boxed{\text{No. 3}} \quad \hat{\mathbb{Q}}_2^{(A_g, 1)} [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_3 = \mathbb{X}_3[\mathbb{Q}_2^{(a, A_g, 1)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\hat{\mathbb{Z}}_3(\mathbf{k}) = \mathbb{X}_3[\mathbb{Q}_2^{(a, A_g, 1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\boxed{\text{No. 4}} \quad \hat{\mathbb{Q}}_2^{(A_g, 2)} [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_4 = \mathbb{X}_4[\mathbb{Q}_2^{(a, A_g, 2)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\hat{\mathbb{Z}}_4(\mathbf{k}) = \mathbb{X}_4[\mathbb{Q}_2^{(a, A_g, 2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\boxed{\text{No. 5}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 1) [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_5 = \mathbb{X}_5[\mathbb{Q}_0^{(a, A_g)}(1, 1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\hat{\mathbb{Z}}_5(\mathbf{k}) = \mathbb{X}_5[\mathbb{Q}_0^{(a, A_g)}(1, 1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\boxed{\text{No. 6}} \quad \hat{\mathbb{Q}}_2^{(A_g, 1)}(1, -1) [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_6 = \mathbb{X}_6[\mathbb{Q}_2^{(a, A_g, 1)}(1, -1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\hat{\mathbb{Z}}_6(\mathbf{k}) = \mathbb{X}_6[\mathbb{Q}_2^{(a, A_g, 1)}(1, -1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\boxed{\text{No. 7}} \quad \hat{\mathbb{Q}}_2^{(A_g, 2)}(1, -1) [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_7 = \mathbb{X}_7[\mathbb{Q}_2^{(a, A_g, 2)}(1, -1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\hat{Z}_7(\mathbf{k}) = \mathbb{X}_7[\mathbb{Q}_2^{(a,A_g,2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}]$$

$$\boxed{\text{No. 8}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_1, \text{B}_1]$$

$$\hat{Z}_8 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{Z}_8(\mathbf{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 9}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\hat{Z}_9 = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{Z}_9(\mathbf{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 10}} \quad \hat{\mathbb{Q}}_2^{(A_g,1)} [\text{M}_3, \text{B}_1]$$

$$\hat{Z}_{10} = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_g,1)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{Z}_{10}(\mathbf{k}) = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_g,1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 11}} \quad \hat{\mathbb{Q}}_2^{(A_g,2)} [\text{M}_3, \text{B}_1]$$

$$\hat{Z}_{11} = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{Z}_{11}(\mathbf{k}) = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 12}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1,1) [\text{M}_3, \text{B}_1]$$

$$\hat{Z}_{12} = \mathbb{X}_5[\mathbb{Q}_0^{(a,A_g)}(1,1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{Z}_{12}(\mathbf{k}) = \mathbb{X}_5[\mathbb{Q}_0^{(a,A_g)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 13}} \quad \hat{\mathbb{Q}}_2^{(A_g,1)}(1,-1) [\text{M}_3, \text{B}_1]$$

$$\hat{Z}_{13} = \mathbb{X}_6[\mathbb{Q}_2^{(a,A_g,1)}(1,-1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{Z}_{13}(\mathbf{k}) = \mathbb{X}_6[\mathbb{Q}_2^{(a,A_g,1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 14}} \quad \hat{\mathbb{Q}}_2^{(A_g,2)}(1,-1) [\text{M}_3, \text{B}_1]$$

$$\hat{Z}_{14} = \mathbb{X}_7[\mathbb{Q}_2^{(a,A_g,2)}(1,-1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{Z}_{14}(\mathbf{k}) = \mathbb{X}_7[\mathbb{Q}_2^{(a,A_g,2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 15}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_1, \text{B}_2]$$

$$\hat{Z}_{15} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{Z}_{15}(\mathbf{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_2[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 16}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_2]$$

$$\hat{Z}_{16} = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{Z}_{16}(\mathbf{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_2[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 17}} \quad \hat{\mathbb{Q}}_2^{(A_g,1)} [\text{M}_3, \text{B}_2]$$

$$\hat{Z}_{17} = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_g,1)}] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{Z}_{17}(\mathbf{k}) = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_g,1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_2[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 18}} \quad \hat{\mathbb{Q}}_2^{(A_g,2)} [\text{M}_3, \text{B}_2]$$

$$\hat{Z}_{18} = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{Z}_{18}(\mathbf{k}) = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_2[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 19}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1,1) [\text{M}_3, \text{B}_2]$$

$$\hat{Z}_{19} = \mathbb{X}_5[\mathbb{Q}_0^{(a,A_g)}(1,1)] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{19}(\mathbf{k}) = \mathbb{X}_5[\mathbb{Q}_0^{(a, A_g)}(1, 1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_2[\mathbb{Q}_0^{(k, A_g)}]$$

$$\boxed{\text{No. 20}} \quad \hat{\mathbb{Q}}_2^{(A_g, 1)}(1, -1) [\text{M}_3, \text{B}_2]$$

$$\hat{\mathbb{Z}}_{20} = \mathbb{X}_6[\mathbb{Q}_2^{(a, A_g, 1)}(1, -1)] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{\mathbb{Z}}_{20}(\mathbf{k}) = \mathbb{X}_6[\mathbb{Q}_2^{(a, A_g, 1)}(1, -1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_2[\mathbb{Q}_0^{(k, A_g)}]$$

$$\boxed{\text{No. 21}} \quad \hat{\mathbb{Q}}_2^{(A_g, 2)}(1, -1) [\text{M}_3, \text{B}_2]$$

$$\hat{\mathbb{Z}}_{21} = \mathbb{X}_7[\mathbb{Q}_2^{(a, A_g, 2)}(1, -1)] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{\mathbb{Z}}_{21}(\mathbf{k}) = \mathbb{X}_7[\mathbb{Q}_2^{(a, A_g, 2)}(1, -1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_2[\mathbb{Q}_0^{(k, A_g)}]$$

$$\boxed{\text{No. 22}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_1, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{22} = \mathbb{X}_1[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{\mathbb{Z}}_{22}(\mathbf{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k, A_g)}]$$

$$\boxed{\text{No. 23}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{23} = \mathbb{X}_2[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{\mathbb{Z}}_{23}(\mathbf{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k, A_g)}]$$

$$\boxed{\text{No. 24}} \quad \hat{\mathbb{Q}}_2^{(A_g, 1)} [\text{M}_3, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{24} = \mathbb{X}_3[\mathbb{Q}_2^{(a, A_g, 1)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{\mathbb{Z}}_{24}(\mathbf{k}) = \mathbb{X}_3[\mathbb{Q}_2^{(a, A_g, 1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k, A_g)}]$$

$$\boxed{\text{No. 25}} \quad \hat{\mathbb{Q}}_2^{(A_g, 2)} [\text{M}_3, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{25} = \mathbb{X}_4[\mathbb{Q}_2^{(a, A_g, 2)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{Z}_{25}(\mathbf{k}) = \mathbb{X}_4[\mathbb{Q}_2^{(a, A_g, 2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k, A_g)}]$$

$$\boxed{\text{No. 26}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 1) \text{ [M}_3, \text{B}_3]$$

$$\hat{Z}_{26} = \mathbb{X}_5[\mathbb{Q}_0^{(a, A_g)}(1, 1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{Z}_{26}(\mathbf{k}) = \mathbb{X}_5[\mathbb{Q}_0^{(a, A_g)}(1, 1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k, A_g)}]$$

$$\boxed{\text{No. 27}} \quad \hat{\mathbb{Q}}_2^{(A_g, 1)}(1, -1) \text{ [M}_3, \text{B}_3]$$

$$\hat{Z}_{27} = \mathbb{X}_6[\mathbb{Q}_2^{(a, A_g, 1)}(1, -1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{Z}_{27}(\mathbf{k}) = \mathbb{X}_6[\mathbb{Q}_2^{(a, A_g, 1)}(1, -1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k, A_g)}]$$

$$\boxed{\text{No. 28}} \quad \hat{\mathbb{Q}}_2^{(A_g, 2)}(1, -1) \text{ [M}_3, \text{B}_3]$$

$$\hat{Z}_{28} = \mathbb{X}_7[\mathbb{Q}_2^{(a, A_g, 2)}(1, -1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{Z}_{28}(\mathbf{k}) = \mathbb{X}_7[\mathbb{Q}_2^{(a, A_g, 2)}(1, -1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k, A_g)}]$$

$$\boxed{\text{No. 29}} \quad \hat{\mathbb{Q}}_0^{(A_g)} \text{ [M}_1, \text{B}_4]$$

$$\hat{Z}_{29} = \mathbb{X}_1[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{Z}_{29}(\mathbf{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_4[\mathbb{Q}_0^{(k, A_g)}]$$

$$\boxed{\text{No. 30}} \quad \hat{\mathbb{Q}}_0^{(A_g)} \text{ [M}_3, \text{B}_4]$$

$$\hat{Z}_{30} = \mathbb{X}_2[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{Z}_{30}(\mathbf{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_4[\mathbb{Q}_0^{(k, A_g)}]$$

$$\boxed{\text{No. 31}} \quad \hat{\mathbb{Q}}_2^{(A_g, 1)} \text{ [M}_3, \text{B}_4]$$

$$\hat{Z}_{31} = \mathbb{X}_3[\mathbb{Q}_2^{(a, A_g, 1)}] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{\mathbb{Z}}_{31}(\mathbf{k}) = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_g,1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_4[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 32}} \quad \hat{\mathbb{Q}}_2^{(A_g,2)} [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{32} = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{32}(\mathbf{k}) = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_4[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 33}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{33} = \mathbb{X}_8[\mathbb{Q}_2^{(a,B_{1g})}] \otimes \mathbb{Y}_6[\mathbb{Q}_2^{(b,B_{1g})}]$$

$$\hat{\mathbb{Z}}_{33}(\mathbf{k}) = \mathbb{X}_8[\mathbb{Q}_2^{(a,B_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_5[\mathbb{Q}_2^{(k,B_{1g})}]$$

$$\boxed{\text{No. 34}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1,1) [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{34} = \mathbb{X}_5[\mathbb{Q}_0^{(a,A_g)}(1,1)] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{34}(\mathbf{k}) = \mathbb{X}_5[\mathbb{Q}_0^{(a,A_g)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_4[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 35}} \quad \hat{\mathbb{Q}}_2^{(A_g,1)}(1,-1) [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{35} = \mathbb{X}_6[\mathbb{Q}_2^{(a,A_g,1)}(1,-1)] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{35}(\mathbf{k}) = \mathbb{X}_6[\mathbb{Q}_2^{(a,A_g,1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_4[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 36}} \quad \hat{\mathbb{Q}}_2^{(A_g,2)}(1,-1) [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{36} = \mathbb{X}_7[\mathbb{Q}_2^{(a,A_g,2)}(1,-1)] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{36}(\mathbf{k}) = \mathbb{X}_7[\mathbb{Q}_2^{(a,A_g,2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_4[\mathbb{Q}_0^{(k,A_g)}]$$

$$\boxed{\text{No. 37}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1,-1) [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{37} = \mathbb{X}_9[\mathbb{Q}_2^{(a,B_{1g})}(1,-1)] \otimes \mathbb{Y}_6[\mathbb{Q}_2^{(b,B_{1g})}]$$

$$\hat{\mathbb{Z}}_{37}(\mathbf{k}) = \mathbb{X}_9[\mathbb{Q}_2^{(a, B_{1g})}(1, -1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_5[\mathbb{Q}_2^{(k, B_{1g})}]$$

$$\boxed{\text{No. 38}} \quad \hat{\mathbb{Q}}_2^{(A_g, 2)}(1, 0) [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{38} = -\mathbb{X}_{10}[\mathbb{G}_1^{(a, B_{1g})}(1, 0)] \otimes \mathbb{Y}_6[\mathbb{Q}_2^{(b, B_{1g})}]$$

$$\hat{\mathbb{Z}}_{38}(\mathbf{k}) = -\mathbb{X}_{10}[\mathbb{G}_1^{(a, B_{1g})}(1, 0)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}] \otimes \mathbb{F}_5[\mathbb{Q}_2^{(k, B_{1g})}]$$

Table 5: Atomic SAMB group.

group	bra	ket
M ₁	(s, ↑), (s, ↓)	(s, ↑), (s, ↓)
M ₂	(s, ↑), (s, ↓)	(p _x , ↑), (p _x , ↓), (p _y , ↑), (p _y , ↓), (p _z , ↑), (p _z , ↓)
M ₃	(p _x , ↑), (p _x , ↓), (p _y , ↑), (p _y , ↓), (p _z , ↑), (p _z , ↓)	(p _x , ↑), (p _x , ↓), (p _y , ↑), (p _y , ↓), (p _z , ↑), (p _z , ↓)

Table 6: Atomic SAMB.

symbol	type	group	form
\mathbb{X}_1	$\mathbb{Q}_0^{(a, A_g)}$	M ₁	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{X}_2	$\mathbb{Q}_0^{(a, A_g)}$	M ₃	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
\mathbb{X}_3	$\mathbb{Q}_2^{(a, A_g, 1)}$	M_3	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} \end{pmatrix}$
\mathbb{X}_4	$\mathbb{Q}_2^{(a, A_g, 2)}$	M_3	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_5	$\mathbb{Q}_0^{(a, A_g)}(1, 1)$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ \frac{\sqrt{3}i}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_6	$\mathbb{Q}_2^{(a, A_g, 1)}(1, -1)$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_7	$\mathbb{Q}_2^{(a, A_g, 2)}(1, -1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
\mathbb{X}_8	$\mathbb{Q}_2^{(a, B_{1g})}$	M_3	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_9	$\mathbb{Q}_2^{(a, B_{1g})}(1, -1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{10}	$\mathbb{G}_1^{(a, B_{1g})}(1, 0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$

Table 7: Cluster SAMB.

symbol	type	cluster	form
\mathbb{Y}_1	$\mathbb{Q}_0^{(s, A_g)}$	S_1	$\begin{pmatrix} 1 \end{pmatrix}$
\mathbb{Y}_2	$\mathbb{Q}_0^{(b, A_g)}$	B_1	$\begin{pmatrix} 1 \end{pmatrix}$
\mathbb{Y}_3	$\mathbb{Q}_0^{(b, A_g)}$	B_2	$\begin{pmatrix} 1 \end{pmatrix}$
\mathbb{Y}_4	$\mathbb{Q}_0^{(b, A_g)}$	B_3	$\begin{pmatrix} 1 \end{pmatrix}$
\mathbb{Y}_5	$\mathbb{Q}_0^{(b, A_g)}$	B_4	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{Y}_6	$\mathbb{Q}_2^{(b, B_{1g})}$	B_4	$\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$

Table 8: Uniform SAMB.

symbol	type	cluster	form
\mathbb{U}_1	$\mathbb{Q}_0^{(s, A_g)}$	S_1	$\binom{1}{1}$

Table 9: Structure SAMB.

symbol	type	cluster	form
\mathbb{F}_1	$\mathbb{Q}_0^{(k, A_g)}$	B_1	$\sqrt{2}c_{001}$
\mathbb{F}_2	$\mathbb{Q}_0^{(k, A_g)}$	B_2	$\sqrt{2}c_{002}$
\mathbb{F}_3	$\mathbb{Q}_0^{(k, A_g)}$	B_3	$\sqrt{2}c_{003}$
\mathbb{F}_4	$\mathbb{Q}_0^{(k, A_g)}$	B_4	$c_{004} + c_{005}$
\mathbb{F}_5	$\mathbb{Q}_2^{(k, B_{1g})}$	B_4	$c_{004} - c_{005}$

Table 10: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_0^{(A_g)}$	0	A_g	—	—	1
2	$\mathbb{Q}_2^{(A_g, 1)}$	2	A_g	1	—	$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$
3	$\mathbb{Q}_2^{(A_g, 2)}$	2	A_g	2	—	$\frac{\sqrt{3}(x-y)(x+y)}{2}$
4	$\mathbb{Q}_2^{(B_{1g})}$	2	B_{1g}	—	—	$\sqrt{3}xy$

Table 11: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_1^{(B_{1g})}$	1	B_{1g}	—	—	Z

-
- Group info.: Generator = $\{2_{001}|0\}$, $\{2_{010}|0\}$, $\{-1|0\}$

Table 12: Conjugacy class (point-group part).

rep. SO	symmetry operations
$\{1 0\}$	$\{1 0\}$
$\{2_{001} 0\}$	$\{2_{001} 0\}$
$\{2_{010} 0\}$	$\{2_{010} 0\}$
$\{2_{100} 0\}$	$\{2_{100} 0\}$
$\{-1 0\}$	$\{-1 0\}$
$\{m_{001} 0\}$	$\{m_{001} 0\}$
$\{m_{010} 0\}$	$\{m_{010} 0\}$
$\{m_{100} 0\}$	$\{m_{100} 0\}$

Table 13: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	$\{1 0\}$	2	$\{2_{001} 0\}$	3	$\{2_{010} 0\}$	4	$\{2_{100} 0\}$	5	$\{-1 0\}$
6	$\{m_{001} 0\}$	7	$\{m_{010} 0\}$	8	$\{m_{100} 0\}$				

Table 14: Character table (point-group part).

	1	2 ₀₀₁	2 ₀₁₀	2 ₁₀₀	-1	m ₀₀₁	m ₀₁₀	m ₁₀₀
A_g	1	1	1	1	1	1	1	1
B_{1g}	1	1	-1	-1	1	1	-1	-1
B_{2g}	1	-1	1	-1	1	-1	1	-1
B_{3g}	1	-1	-1	1	1	-1	-1	1
A_u	1	1	1	1	-1	-1	-1	-1
B_{1u}	1	1	-1	-1	-1	-1	1	1
B_{2u}	1	-1	1	-1	-1	1	-1	1
B_{3u}	1	-1	-1	1	-1	1	1	-1

Table 15: Parity conversion.

\leftrightarrow	\leftrightarrow	\leftrightarrow	\leftrightarrow	\leftrightarrow
A_g (A_u)	B_{3g} (B_{3u})	B_{2g} (B_{2u})	B_{1g} (B_{1u})	A_u (A_g)
B_{3u} (B_{3g})	B_{2u} (B_{2g})	B_{1u} (B_{1g})		

Table 16: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

	A_g	B_{1g}	B_{2g}	B_{3g}	A_u	B_{1u}	B_{2u}	B_{3u}
A_g	A_g	B_{1g}	B_{2g}	B_{3g}	A_u	B_{1u}	B_{2u}	B_{3u}
B_{1g}		A_g	B_{3g}	B_{2g}	B_{1u}	A_u	B_{3u}	B_{2u}
B_{2g}			A_g	B_{1g}	B_{2u}	B_{3u}	A_u	B_{1u}
B_{3g}				A_g	B_{3u}	B_{2u}	B_{1u}	A_u
A_u					A_g	B_{1g}	B_{2g}	B_{3g}
B_{1u}						A_g	B_{3g}	B_{2g}
B_{2u}							A_g	B_{1g}
B_{3u}								A_g

Table 17: Anti-symmetric product, $[\Gamma \otimes \Gamma]_-$.

A_g	B_{1g}	B_{2g}	B_{3g}	A_u	B_{1u}	B_{2u}	B_{3u}
—	—	—	—	—	—	—	—

Table 18: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$	2	$\begin{pmatrix} -1 & -1 & 1 \end{pmatrix}$	3	$\begin{pmatrix} -1 & 1 & -1 \end{pmatrix}$	4	$\begin{pmatrix} 1 & -1 & -1 \end{pmatrix}$
5	$\begin{pmatrix} -1 & -1 & -1 \end{pmatrix}$	6	$\begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$	7	$\begin{pmatrix} 1 & -1 & 1 \end{pmatrix}$	8	$\begin{pmatrix} -1 & 1 & 1 \end{pmatrix}$

Table 19: Virtual-cluster basis.

symbol	1	2	3	4	5	6	7	8
$Q_0^{(A_g)}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$
$Q_1^{(B_{1u})}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$
$Q_1^{(B_{2u})}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$
$Q_1^{(B_{3u})}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$
$Q_2^{(B_{1g})}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$
$Q_2^{(B_{2g})}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$
$Q_2^{(B_{3g})}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$
$Q_3^{(A_u)}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$