

\* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

| No. | tag  | matrix (polar)   | det | TR |
|-----|--|--|-----|----|
| 1   | {1 0}  | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                  | 1   | 1  |
| 2   | {4 <sup>+</sup> <sub>100</sub>   $\frac{1}{4}\frac{1}{4}\frac{3}{4}$ } | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 3   | {4 <sup>-</sup> <sub>100</sub>   $\frac{3}{4}\frac{1}{4}\frac{1}{4}$ } | $\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \\ 0 & -1 & 0 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 4   | {4 <sup>+</sup> <sub>010</sub>   $\frac{3}{4}\frac{1}{4}\frac{1}{4}$ } | $\begin{bmatrix} 0 & 0 & 1 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 5   | {4 <sup>-</sup> <sub>010</sub>   $\frac{1}{4}\frac{3}{4}\frac{1}{4}$ } | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 6   | {4 <sup>+</sup> <sub>001</sub>   $\frac{1}{4}\frac{3}{4}\frac{1}{4}$ } | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 7   | {4 <sup>-</sup> <sub>001</sub>   $\frac{1}{4}\frac{1}{4}\frac{3}{4}$ } | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 8   | {2 <sub>100</sub>  00 $\frac{1}{2}$ }                                  | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$                      | 1   | 1  |
| 9   | {2 <sub>010</sub>   $\frac{1}{2}00$ }                                  | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                      | 1   | 1  |
| 10  | {2 <sub>001</sub>  0 $\frac{1}{2}0$ }                                  | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$                      | 1   | 1  |
| 11  | {2 <sub>110</sub>   $\frac{3}{4}\frac{1}{4}\frac{1}{4}$ }              | $\begin{bmatrix} 0 & 1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 12  | {2 <sub>1-10</sub>   $\frac{1}{4}\frac{1}{4}\frac{1}{4}$ }             | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$ | 1   | 1  |
| 13  | {2 <sub>011</sub>   $\frac{1}{4}\frac{3}{4}\frac{1}{4}$ }              | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |

*continued ...*

Table 1

| No. | tag   | matrix (polar)   | det | TR |
|-----|---|--|-----|----|
| 14  | $\{2_{01-1} \frac{1}{4}\frac{1}{4}\frac{1}{4}\}$  | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \\ 0 & -1 & 0 & \frac{1}{4} \end{bmatrix}$ | 1   | 1  |
| 15  | $\{2_{101} \frac{1}{4}\frac{1}{4}\frac{3}{4}\}$   | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{4} \\ 0 & -1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 16  | $\{2_{-101} \frac{1}{4}\frac{1}{4}\frac{1}{4}\}$  | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{4} \\ 0 & -1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{1}{4} \end{bmatrix}$ | 1   | 1  |
| 17  | $\{3^+_{111} 0\}$                                 | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$                                  | 1   | 1  |
| 18  | $\{3^-_{111} 0\}$                                 | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$                                  | 1   | 1  |
| 19  | $\{3^-_{1-1-1} \frac{1}{2}00\}$                   | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$                      | 1   | 1  |
| 20  | $\{3^+_{1-1-1} 0\frac{1}{2}0\}$                   | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \end{bmatrix}$                      | 1   | 1  |
| 21  | $\{3^-_{-11-1} 0\frac{1}{2}0\}$                   | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \\ 1 & 0 & 0 & 0 \end{bmatrix}$                      | 1   | 1  |
| 22  | $\{3^+_{-11-1} 00\frac{1}{2}\}$                   | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \end{bmatrix}$                      | 1   | 1  |
| 23  | $\{3^-_{-1-11} 00\frac{1}{2}\}$                   | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$                      | 1   | 1  |
| 24  | $\{3^+_{-1-11} \frac{1}{2}00\}$                   | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{2} \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$                      | 1   | 1  |
| 25  | $\{1 \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$         | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$    | 1   | 1  |
| 26  | $\{4^+_{100} \frac{3}{4}\frac{3}{4}\frac{1}{4}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 27  | $\{4^-_{100} \frac{1}{4}\frac{3}{4}\frac{3}{4}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \\ 0 & -1 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |

continued ...

Table 1

| No. | tag   | matrix (polar)   | det | TR |
|-----|---|--|-----|----|
| 28  | $\{4_{010}^+   \frac{1}{4} \frac{3}{4} \frac{3}{4}\}$ | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 29  | $\{4_{010}^-   \frac{3}{4} \frac{1}{4} \frac{3}{4}\}$ | $\begin{bmatrix} 0 & 0 & -1 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 30  | $\{4_{001}^+   \frac{3}{4} \frac{1}{4} \frac{3}{4}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 31  | $\{4_{001}^-   \frac{3}{4} \frac{3}{4} \frac{1}{4}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 32  | $\{2_{100}   \frac{1}{2} \frac{1}{2} 0\}$             | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$            | 1   | 1  |
| 33  | $\{2_{010}   0 \frac{1}{2} \frac{1}{2}\}$             | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$            | 1   | 1  |
| 34  | $\{2_{001}   \frac{1}{2} 0 \frac{1}{2}\}$             | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$            | 1   | 1  |
| 35  | $\{2_{110}   \frac{1}{4} \frac{3}{4} \frac{3}{4}\}$   | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 36  | $\{2_{1-10}   \frac{3}{4} \frac{3}{4} \frac{3}{4}\}$  | $\begin{bmatrix} 0 & -1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$ | 1   | 1  |
| 37  | $\{2_{011}   \frac{3}{4} \frac{1}{4} \frac{3}{4}\}$   | $\begin{bmatrix} -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 38  | $\{2_{01-1}   \frac{3}{4} \frac{3}{4} \frac{3}{4}\}$  | $\begin{bmatrix} -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \\ 0 & -1 & 0 & \frac{3}{4} \end{bmatrix}$ | 1   | 1  |
| 39  | $\{2_{101}   \frac{3}{4} \frac{3}{4} \frac{1}{4}\}$   | $\begin{bmatrix} 0 & 0 & 1 & \frac{3}{4} \\ 0 & -1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 40  | $\{2_{-101}   \frac{3}{4} \frac{3}{4} \frac{3}{4}\}$  | $\begin{bmatrix} 0 & 0 & -1 & \frac{3}{4} \\ 0 & -1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{3}{4} \end{bmatrix}$ | 1   | 1  |
| 41  | $\{3_{111}^+   \frac{1}{2} \frac{1}{2} \frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \end{bmatrix}$    | 1   | 1  |

continued ...

Table 1

| No. | tag   | matrix (polar)  | det | TR |
|-----|---|---|-----|----|
| 42  | $\{3^-_{111} \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | 1   | 1  |
| 43  | $\{3^-_{1-1-1} 0\frac{1}{2}\frac{1}{2}\}$         | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$         | 1   | 1  |
| 44  | $\{3^+_{1-1-1} \frac{1}{2}0\frac{1}{2}\}$         | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{2} \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \end{bmatrix}$         | 1   | 1  |
| 45  | $\{3^-_{-11-1} \frac{1}{2}0\frac{1}{2}\}$         | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$         | 1   | 1  |
| 46  | $\{3^+_{-11-1} \frac{1}{2}\frac{1}{2}0\}$         | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \end{bmatrix}$         | 1   | 1  |
| 47  | $\{3^-_{-1-11} \frac{1}{2}\frac{1}{2}0\}$         | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \\ -1 & 0 & 0 & 0 \end{bmatrix}$         | 1   | 1  |
| 48  | $\{3^+_{-1-11} 0\frac{1}{2}\frac{1}{2}\}$         | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \end{bmatrix}$         | 1   | 1  |