No. 12 D_4 422 [tetragonal] (polar)

表 1 rank 0

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
1	A_1	A1	_	_	$\mathbb{Q}_0^{(h,A_1)}$	$\mathtt{Qh}(\mathtt{0},\mathtt{A1},,)$	C_0

表 2 rank 1

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
2	A_2	A2	-	-	$\mathbb{Q}_1^{(h,A_2)}$	$\mathtt{Qh}(\mathtt{1},\mathtt{A2},,)$	C_0
3	E	E	_	0	$\mathbb{Q}_{1,0}^{(h,E)}$	$\mathtt{Qh}(\mathtt{1},\mathtt{E},,\mathtt{0})$	C_1
4	E	E	_	1	$\mathbb{Q}_{1,1}^{(h,E)}$	$\mathtt{Qh}(\mathtt{1},\mathtt{E},,\mathtt{1})$	S_1

表 3 rank 2

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
5	A_1	A1	-	-	$\mathbb{Q}_2^{(h,A_1)}$	$\mathtt{Qh}(\mathtt{2},\mathtt{A1},,)$	C_0
6	B_1	B1	_	_	$\mathbb{Q}_2^{(h,B_1)}$	$\mathtt{Qh}(\mathtt{2},\mathtt{B1},,)$	C_2
7	B_2	B2	_	_	$\mathbb{Q}_2^{(h,B_2)}$	$\mathtt{Qh}(\mathtt{2},\mathtt{B2},,)$	S_2
8	E	E	_	0	$\mathbb{Q}_{2,0}^{(h,E)}$	$\mathtt{Qh}(2,\mathtt{E},,\mathtt{0})$	S_1
9	E	E	-	1	$\mathbb{Q}_{2,1}^{(h,E)}$	$\mathtt{Qh}(2,\mathtt{E},,1)$	$-C_1$

表 4 rank 3

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
10	A_2	A2	-	-	$\mathbb{Q}_3^{(h,A_2)}$	$\mathtt{Qh}(\mathtt{3},\mathtt{A2},,)$	C_0
11	B_1	B1	_	_	$\mathbb{Q}_3^{(h,B_1)}$	$\mathtt{Qh}(\mathtt{3},\mathtt{B1},,)$	S_2
12	B_2	B2	_	_	$\mathbb{Q}_3^{(h,B_2)}$	$\mathtt{Qh}(\mathtt{3},\mathtt{B2},,)$	C_2
13	E	E	1	0	$\mathbb{Q}_{3,0}^{(h,E,1)}$	$\mathtt{Qh}(3,\mathtt{E},1,0)$	$-\frac{\sqrt{6}C_1}{4} + \frac{\sqrt{10}C_3}{4}$
14	E	E	1	1	$\mathbb{Q}_{3,1}^{(h,E,1)}$	$\mathtt{Qh}(3,\mathtt{E},1,1)$	$-\frac{\sqrt{6}S_1}{4} - \frac{\sqrt{10}S_3}{4}$
15	E	E	2	0	$\mathbb{Q}_{3,0}^{(h,E,2)}$	$\mathtt{Qh}(3,\mathtt{E},2,0)$	$-\frac{\sqrt{10}C_1}{4} - \frac{\sqrt{6}C_3}{4}$
16	E	E	2	1	$\mathbb{Q}_{3,1}^{(h,E,2)}$	$\mathtt{Qh}(3,\mathtt{E},2,1)$	$-\frac{\sqrt{10}S_1}{4} + \frac{\sqrt{6}S_3}{4}$

表 5 rank 4

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
17	A_1	A1	1	_	$\mathbb{Q}_4^{(h,A_1,1)}$	$\mathtt{Qh}(\mathtt{4},\mathtt{A1},\mathtt{1},)$	$\frac{\sqrt{21}C_0}{6} + \frac{\sqrt{15}C_4}{6}$
18	A_1	A1	2	_	$\mathbb{Q}_4^{(h,A_1,2)}$	$\mathtt{Qh}(\mathtt{4},\mathtt{A1},\mathtt{2},)$	$\frac{\sqrt{15}C_0}{6} - \frac{\sqrt{21}C_4}{6}$
19	A_2	A2	_	_	$\mathbb{Q}_4^{(h,A_2)}$	$\mathtt{Qh}(\mathtt{4},\mathtt{A2},,)$	S_4
20	B_1	B1	_	_	$\mathbb{Q}_4^{(h,B_1)}$	$\mathtt{Qh}(\mathtt{4},\mathtt{B1},,)$	$-C_2$
21	B_2	B2	_	_	$\mathbb{Q}_4^{(h,B_2)}$	$\mathtt{Qh}(\mathtt{4},\mathtt{B2},,)$	S_2
22	E	E	1	0	$\mathbb{Q}_{4,0}^{(h,E,1)}$	$\mathtt{Qh}(\mathtt{4},\mathtt{E},\mathtt{1},\mathtt{0})$	$-\frac{\sqrt{14}S_1}{4} - \frac{\sqrt{2}S_3}{4}$
23	E	E	1	1	$\mathbb{Q}_{4,1}^{(h,E,1)}$	$\mathtt{Qh}(\mathtt{4},\mathtt{E},\mathtt{1},\mathtt{1})$	$\frac{\sqrt{14}C_1}{4} - \frac{\sqrt{2}C_3}{4}$
24	E	E	2	0	$\mathbb{Q}_{4,0}^{(h,E,2)}$	$\mathtt{Qh}(4,\mathtt{E},2,0)$	$-\frac{\sqrt{2}S_1}{4} + \frac{\sqrt{14}S_3}{4}$
25	E	E	2	1	$\mathbb{Q}_{4,1}^{(h,E,2)}$	$\mathtt{Qh}(4,\mathtt{E},2,1)$	$\frac{\sqrt{2}C_1}{4} + \frac{\sqrt{14}C_3}{4}$

表 6 rank 5

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
26	A_1	A1	-	-	$\mathbb{Q}_5^{(h,A_1)}$	$\mathtt{Qh}(\mathtt{5},\mathtt{A1},,)$	S_4
27	A_2	A2	1	_	$\mathbb{Q}_5^{(h,A_2,1)}$	$\mathtt{Qh}(\mathtt{5},\mathtt{A2},\mathtt{1},)$	C_0
28	A_2	A2	2	_	$\mathbb{Q}_5^{(h,A_2,2)}$	$\mathtt{Qh}(\mathtt{5},\mathtt{A2},\mathtt{2},\mathtt{)}$	C_4
29	B_1	B1	_	-	$\mathbb{Q}_5^{(h,B_1)}$	$\mathtt{Qh}(\mathtt{5},\mathtt{B1},,)$	$-S_2$
30	B_2	B2	_	-	$\mathbb{Q}_5^{(h,B_2)}$	$\mathtt{Qh}(5,\mathtt{B2},,)$	C_2
31	E	E	1	0	$\mathbb{Q}_{5,0}^{(h,E,1)}$	$\mathtt{Qh}(\mathtt{5},\mathtt{E},\mathtt{1},\mathtt{0})$	$\frac{\sqrt{15}C_1}{8} - \frac{\sqrt{70}C_3}{16} + \frac{3\sqrt{14}C_5}{16}$
32	E	E	1	1	$\mathbb{Q}_{5,1}^{(h,E,1)}$	$\mathtt{Qh}(\mathtt{5},\mathtt{E},\mathtt{1},\mathtt{1})$	$\frac{\sqrt{15}S_1}{8} + \frac{\sqrt{70}S_3}{16} + \frac{3\sqrt{14}S_5}{16}$
33	E	E	2	0	$\mathbb{Q}_{5,0}^{(h,E,2)}$	$\mathtt{Qh}(5,\mathtt{E},2,0)$	$\frac{\sqrt{21}C_1}{8} + \frac{9\sqrt{2}C_3}{16} + \frac{\sqrt{10}C_5}{16}$
34	E	E	2	1	$\mathbb{Q}_{5,1}^{(h,E,2)}$	$\mathtt{Qh}(\mathtt{5},\mathtt{E},\mathtt{2},\mathtt{1})$	$\frac{\sqrt{21}S_1}{8} - \frac{9\sqrt{2}S_3}{16} + \frac{\sqrt{10}S_5}{16}$
35	E	E	3	0	$\mathbb{Q}_{5,0}^{(h,E,3)}$	$\mathtt{Qh}(\mathtt{5},\mathtt{E},\mathtt{3},\mathtt{0})$	$\frac{\sqrt{7}C_1}{4} - \frac{\sqrt{6}C_3}{8} - \frac{\sqrt{30}C_5}{8}$
36	E	E	3	1	$\mathbb{Q}_{5,1}^{(h,E,3)}$	$\mathtt{Qh}(\mathtt{5},\mathtt{E},\mathtt{3},\mathtt{1})$	$\frac{\sqrt{7}S_1}{4} + \frac{\sqrt{6}S_3}{8} - \frac{\sqrt{30}S_5}{8}$

表 7 rank 6

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
37	A_1	A1	1	=	$\mathbb{Q}_6^{(h,A_1,1)}$	Qh(6, A1, 1,)	$\frac{\sqrt{2}C_0}{4} - \frac{\sqrt{14}C_4}{4}$
38	A_1	A1	2	_	$\mathbb{Q}_6^{(h,A_1,2)}$	$\mathtt{Qh}(6,\mathtt{A1},\mathtt{2},)$	$\frac{\sqrt{14}C_0}{4} + \frac{\sqrt{2}C_4}{4}$
39	A_2	A2	_	_	$\mathbb{Q}_6^{(h,A_2)}$	$\mathtt{Qh}(6,\mathtt{A2},,)$	S_4
40	B_1	B1	1	_	$\mathbb{Q}_6^{(h,B_1,1)}$	$\mathtt{Qh}(6,\mathtt{B1},\mathtt{1},)$	$\frac{\sqrt{11}C_2}{4} - \frac{\sqrt{5}C_6}{4}$
41	B_1	B1	2	_	$\mathbb{Q}_6^{(h,B_1,2)}$	$\mathtt{Qh}(6,\mathtt{B1},\mathtt{2},)$	$\frac{\sqrt{5}C_2}{4} + \frac{\sqrt{11}C_6}{4}$
42	B_2	B2	1	_	$\mathbb{Q}_6^{(h,B_2,1)}$	$\mathtt{Qh}(6,\mathtt{B2},1,)$	S_6
43	B_2	B2	2	_	$\mathbb{Q}_6^{(h,B_2,2)}$	$\mathtt{Qh}(6,\mathtt{B2},\mathtt{2},)$	S_2
44	E	E	1	0	$\mathbb{Q}_{6,0}^{(h,E,1)}$	$\mathtt{Qh}(6,\mathtt{E},\mathtt{1},\mathtt{0})$	$\frac{\sqrt{3}S_1}{4} - \frac{\sqrt{30}S_3}{8} - \frac{\sqrt{22}S_5}{8}$
45	E	E	1	1	$\mathbb{Q}_{6,1}^{(h,E,1)}$	$\mathtt{Qh}(6,\mathtt{E},1,1)$	$-\frac{\sqrt{3}C_1}{4} - \frac{\sqrt{30}C_3}{8} + \frac{\sqrt{22}C_5}{8}$
46	E	E	2	0	$\mathbb{Q}_{6,0}^{(h,E,2)}$	$\mathtt{Qh}(6,\mathtt{E},2,0)$	$\frac{3\sqrt{22}S_1}{16} + \frac{\sqrt{55}S_3}{16} + \frac{\sqrt{3}S_5}{16}$
47	E	E	2	1	$\mathbb{Q}_{6,1}^{(h,E,2)}$	$\mathtt{Qh}(6,\mathtt{E},2,1)$	$-\frac{3\sqrt{22}C_1}{16} + \frac{\sqrt{55}C_3}{16} - \frac{\sqrt{3}C_5}{16}$
48	E	E	3	0	$\mathbb{Q}_{6,0}^{(h,E,3)}$	$\mathtt{Qh}(6,\mathtt{E},3,0)$	$\frac{\sqrt{10}S_1}{16} - \frac{9S_3}{16} + \frac{\sqrt{165}S_5}{16}$
49	E	E	3	1	$\mathbb{Q}_{6,1}^{(h,E,3)}$	$\mathtt{Qh}(6,\mathtt{E},3,1)$	$-\frac{\sqrt{10}C_1}{16} - \frac{9C_3}{16} - \frac{\sqrt{165}C_5}{16}$

表 8 $\operatorname{rank} 7$

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
50	A_1	A1	_	_	$\mathbb{Q}_7^{(h,A_1)}$	$\mathtt{Qh}(\mathtt{7},\mathtt{A1},,)$	S_4
51	A_2	A2	1	_	$\mathbb{Q}_7^{(h,A_2,1)}$	$\mathtt{Qh}(7,\mathtt{A2},\mathtt{1},)$	C_0
52	A_2	A2	2	_	$\mathbb{Q}_7^{(h,A_2,2)}$	$\mathtt{Qh}(7,\mathtt{A2},2,)$	C_4
53	B_1	B1	1	_	$\mathbb{Q}_7^{(h,B_1,1)}$	$\mathtt{Qh}(7,\mathtt{B1},1,)$	$\frac{\sqrt{78}S_2}{12} + \frac{\sqrt{66}S_6}{12}$
54	B_1	B1	2	_	$\mathbb{Q}_7^{(h,B_1,2)}$	$\mathtt{Qh}(7,\mathtt{B1},2,)$	$\frac{\sqrt{66}S_2}{12} - \frac{\sqrt{78}S_6}{12}$
55	B_2	B2	1	_	$\mathbb{Q}_7^{(h,B_2,1)}$	$\mathtt{Qh}(7,\mathtt{B2},1,)$	C_6
56	B_2	B2	2	_	$\mathbb{Q}_7^{(h,B_2,2)}$	$\mathtt{Qh}(7,\mathtt{B2},\mathtt{2},)$	C_2
57	E	E	1	0	$\mathbb{Q}_{7,0}^{(h,E,1)}$	$\mathtt{Qh}(7,\mathtt{E},\mathtt{1},\mathtt{0})$	$-\frac{5\sqrt{7}C_1}{32} + \frac{3\sqrt{21}C_3}{32} - \frac{\sqrt{231}C_5}{32} + \frac{\sqrt{429}C_7}{32}$
58	E	E	1	1	$\mathbb{Q}_{7,1}^{(h,E,1)}$	$\mathtt{Qh}(7,\mathtt{E},1,1)$	$-\frac{5\sqrt{7}S_1}{32} - \frac{3\sqrt{21}S_3}{32} - \frac{\sqrt{231}S_5}{32} - \frac{\sqrt{429}S_7}{32}$
59	E	E	2	0	$\mathbb{Q}_{7,0}^{(h,E,2)}$	$\mathtt{Qh}(7,\mathtt{E},2,\mathtt{0})$	$-\frac{3\sqrt{33}C_1}{32} - \frac{\sqrt{11}C_3}{32} + \frac{25C_5}{32} + \frac{\sqrt{91}C_7}{32}$
60	E	E	2	1	$\mathbb{Q}_{7,1}^{(h,E,2)}$	$\mathtt{Qh}(7,\mathtt{E},2,1)$	$-\frac{3\sqrt{33}S_1}{32} + \frac{\sqrt{11}S_3}{32} + \frac{25S_5}{32} - \frac{\sqrt{91}S_7}{32}$
61	E	E	3	0	$\mathbb{Q}_{7,0}^{(h,E,3)}$	$\mathtt{Qh}(7,\mathtt{E},3,0)$	$-\frac{\sqrt{858}C_1}{64} - \frac{3\sqrt{286}C_3}{64} - \frac{5\sqrt{26}C_5}{64} - \frac{\sqrt{14}C_7}{64}$
62	E	E	3	1	$\mathbb{Q}_{7,1}^{(h,E,3)}$	$\mathtt{Qh}(7,\mathtt{E},3,1)$	$-\frac{\sqrt{858}S_1}{64} + \frac{3\sqrt{286}S_3}{64} - \frac{5\sqrt{26}S_5}{64} + \frac{\sqrt{14}S_7}{64}$
63	E	E	4	0	$\mathbb{Q}_{7,0}^{(h,E,4)}$	$\mathtt{Qh}(7,\mathtt{E},4,\mathtt{0})$	$-\frac{15\sqrt{6}C_1}{64} + \frac{19\sqrt{2}C_3}{64} - \frac{\sqrt{22}C_5}{64} - \frac{\sqrt{2002}C_7}{64}$
64	E	E	4	1	$\mathbb{Q}_{7,1}^{(h,E,4)}$	$\mathtt{Qh}(7,\mathtt{E},4,1)$	$-\frac{15\sqrt{6}S_1}{64} - \frac{19\sqrt{2}S_3}{64} - \frac{\sqrt{22}S_5}{64} + \frac{\sqrt{2002}S_7}{64}$

表 9 rank 8

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
65	A_1	A1	1	_	$\mathbb{Q}_8^{(h,A_1,1)}$	Qh(8, A1, 1,)	$\frac{\sqrt{33}C_0}{8} + \frac{\sqrt{21}C_4}{12} + \frac{\sqrt{195}C_8}{24}$
66	A_1	A1	2	_	$\mathbb{Q}_8^{(h,A_1,2)}$	$\mathtt{Qh}(8,\mathtt{A1},2,)$	$-\frac{\sqrt{286}C_0}{32} + \frac{\sqrt{182}C_4}{16} + \frac{\sqrt{10}C_8}{32}$
67	A_1	A1	3	_	$\mathbb{Q}_8^{(h,A_1,3)}$	$\mathtt{Qh}(8,\mathtt{A1},\mathtt{3},)$	$-\frac{\sqrt{210}C_0}{32} - \frac{\sqrt{330}C_4}{48} + \frac{\sqrt{6006}C_8}{96}$
68	A_2	A2	1	_	$\mathbb{Q}_8^{(h,A_2,1)}$	$\mathtt{Qh}(8,\mathtt{A2},\mathtt{1},)$	S_8
69	A_2	A2	2	_	$\mathbb{Q}_8^{(h,A_2,2)}$	$\mathtt{Qh}(8,\mathtt{A2},2,)$	S_4
70	B_1	B1	1	_	$\mathbb{Q}_8^{(h,B_1,1)}$	$\mathtt{Qh}(8,\mathtt{B1},\mathtt{1},)$	C_6
71	B_1	B1	2	_	$\mathbb{Q}_8^{(h,B_1,2)}$	$\mathtt{Qh}(8,\mathtt{B1},2,)$	C_2
72	B_2	B2	1	_	$\mathbb{Q}_8^{(h,B_2,1)}$	$\mathtt{Qh}(8,\mathtt{B2},\mathtt{1},)$	S_6
73	B_2	B2	2	_	$\mathbb{Q}_8^{(h,B_2,2)}$	$\mathtt{Qh}(8,\mathtt{B2},\mathtt{2},)$	S_2
74	E	E	1	0	$\mathbb{Q}_{8,0}^{(h,E,1)}$	$\mathtt{Qh}(8,\mathtt{E},\mathtt{1},\mathtt{0})$	$-\frac{\sqrt{715}S_1}{32} - \frac{\sqrt{273}S_3}{32} - \frac{\sqrt{35}S_5}{32} - \frac{S_7}{32}$
75	E	E	1	1	$\mathbb{Q}_{8,1}^{(h,E,1)}$	$\mathtt{Qh}(8,\mathtt{E},1,1)$	$\frac{\sqrt{715}C_1}{32} - \frac{\sqrt{273}C_3}{32} + \frac{\sqrt{35}C_5}{32} - \frac{C_7}{32}$
76	E	E	2	0	$\mathbb{Q}_{8,0}^{(h,E,2)}$	$\mathtt{Qh}(8,\mathtt{E},2,\mathtt{0})$	$-\frac{\sqrt{77}S_1}{32} + \frac{5\sqrt{15}S_3}{32} - \frac{3\sqrt{13}S_5}{32} - \frac{\sqrt{455}S_7}{32}$
77	E	E	2	1	$\mathbb{Q}_{8,1}^{(h,E,2)}$	$\mathtt{Qh}(8,\mathtt{E},2,1)$	$\frac{\sqrt{77}C_1}{32} + \frac{5\sqrt{15}C_3}{32} + \frac{3\sqrt{13}C_5}{32} - \frac{\sqrt{455}C_7}{32}$
78	E	E	3	0	$\mathbb{Q}_{8,0}^{(h,E,3)}$	$\mathtt{Qh}(8,\mathtt{E},3,0)$	$-\frac{\sqrt{858}S_1}{64} + \frac{\sqrt{910}S_3}{64} + \frac{7\sqrt{42}S_5}{64} + \frac{3\sqrt{30}S_7}{64}$
79	E	E	3	1	$\mathbb{Q}_{8,1}^{(h,E,3)}$	$\mathtt{Qh}(8,\mathtt{E},3,1)$	$\frac{\sqrt{858}C_1}{64} + \frac{\sqrt{910}C_3}{64} - \frac{7\sqrt{42}C_5}{64} + \frac{3\sqrt{30}C_7}{64}$
80	E	E	4	0	$\mathbb{Q}_{8,0}^{(h,E,4)}$	$\mathtt{Qh}(8,\mathtt{E},4,0)$	$-\frac{\sqrt{70}S_1}{64} + \frac{3\sqrt{66}S_3}{64} - \frac{\sqrt{1430}S_5}{64} + \frac{\sqrt{2002}S_7}{64}$
81	E	E	4	1	$\mathbb{Q}_{8,1}^{(h,E,4)}$	$\mathtt{Qh}(8,\mathtt{E},4,1)$	$\frac{\sqrt{70}C_1}{64} + \frac{3\sqrt{66}C_3}{64} + \frac{\sqrt{1430}C_5}{64} + \frac{\sqrt{2002}C_7}{64}$

表 10 rank 9

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
82	A_1	A1	1	-	$\mathbb{Q}_9^{(h,A_1,1)}$	Qh(9, A1, 1,)	$rac{\sqrt{102}S_4}{12} - rac{\sqrt{42}S_8}{12}$
83	A_1	A1	2	_	$\mathbb{Q}_9^{(h,A_1,2)}$	$\mathtt{Qh}(9,\mathtt{A1},2,)$	$rac{\sqrt{42}S_4}{12} + rac{\sqrt{102}S_8}{12}$
84	A_2	A2	1	-	$\mathbb{Q}_9^{(h,A_2,1)}$	$\mathtt{Qh}(9,\mathtt{A2},\mathtt{1},)$	C_0
85	A_2	A2	2	_	$\mathbb{Q}_9^{(h,A_2,2)}$	$\mathtt{Qh}(9,\mathtt{A2},2,)$	C_8
86	A_2	A2	3	_	$\mathbb{Q}_9^{(h,A_2,3)}$	$\mathtt{Qh}(9,\mathtt{A2},\mathtt{3},)$	C_4
87	B_1	B1	1	_	$\mathbb{Q}_9^{(h,B_1,1)}$	$\mathtt{Qh}(9,\mathtt{B1},1,)$	$\frac{\sqrt{3}S_2}{4} - \frac{\sqrt{13}S_6}{4}$
88	B_1	B1	2	_	$\mathbb{Q}_9^{(h,B_1,2)}$	$\mathtt{Qh}(9,\mathtt{B1},2,)$	$-rac{\sqrt{13}S_2}{4} - rac{\sqrt{3}S_6}{4}$
89	B_2	B2	1	-	$\mathbb{Q}_9^{(h,B_2,1)}$	$\mathtt{Qh}(9,\mathtt{B2},1,)$	C_6
90	B_2	B2	2	-	$\mathbb{Q}_9^{(h,B_2,2)}$	$\mathtt{Qh}(9,\mathtt{B2},2,)$	C_2
91	E	E	1	0	$\mathbb{Q}_{9,0}^{(h,E,1)}$	$\mathtt{Qh}(9,\mathtt{E},1,0)$	$\frac{21\sqrt{5}C_1}{128} - \frac{\sqrt{2310}C_3}{128} + \frac{3\sqrt{286}C_5}{128} - \frac{3\sqrt{1430}C_7}{256} + \frac{\sqrt{24310}C_9}{256}$
92	E	E	1	1	$\mathbb{Q}_{9,1}^{(h,E,1)}$	$\mathtt{Qh}(9,\mathtt{E},\mathtt{1},\mathtt{1})$	$\frac{21\sqrt{5}S_1}{128} + \frac{\sqrt{2310}S_3}{128} + \frac{3\sqrt{286}S_5}{128} + \frac{3\sqrt{1430}S_7}{256} + \frac{\sqrt{24310}S_9}{256}$
93	E	E	2	0	$\mathbb{Q}_{9,0}^{(h,E,2)}$	$\mathtt{Qh}(9,\mathtt{E},2,0)$	$\frac{\sqrt{2431}C_1}{128} + \frac{\sqrt{9282}C_3}{128} + \frac{5\sqrt{170}C_5}{128} + \frac{7\sqrt{34}C_7}{256} + \frac{3\sqrt{2}C_9}{256}$
94	E	E	2	1	$\mathbb{Q}_{9,1}^{(h,E,2)}$	$\mathtt{Qh}(9,\mathtt{E},2,1)$	$\frac{\sqrt{2431}S_1}{128} - \frac{\sqrt{9282}S_3}{128} + \frac{5\sqrt{170}S_5}{128} - \frac{7\sqrt{34}S_7}{256} + \frac{3\sqrt{2}S_9}{256}$
95	E	E	3	0	$\mathbb{Q}_{9,0}^{(h,E,3)}$	$\mathtt{Qh}(9,\mathtt{E},3,0)$	$\frac{\sqrt{1001}C_1}{64} - \frac{\sqrt{78}C_3}{64} - \frac{3\sqrt{70}C_5}{64} + \frac{23\sqrt{14}C_7}{128} + \frac{3\sqrt{238}C_9}{128}$
96	E	E	3	1	$\mathbb{Q}_{9,1}^{(h,E,3)}$	$\mathtt{Qh}(9,\mathtt{E},3,1)$	$\frac{\sqrt{1001}S_1}{64} + \frac{\sqrt{78}S_3}{64} - \frac{3\sqrt{70}S_5}{64} - \frac{23\sqrt{14}S_7}{128} + \frac{3\sqrt{238}S_9}{128}$
97	E	E	4	0	$\mathbb{Q}_{9,0}^{(h,E,4)}$	$\mathtt{Qh}(9,\mathtt{E},4,0)$	$\frac{\sqrt{858}C_1}{64} + \frac{\sqrt{91}C_3}{32} - \frac{5\sqrt{15}C_5}{32} - \frac{21\sqrt{3}C_7}{64} - \frac{\sqrt{51}C_9}{64}$
98	E	E	4	1	$\mathbb{Q}_{9,1}^{(h,E,4)}$	$\mathtt{Qh}(9,\mathtt{E},4,1)$	$\frac{\sqrt{858}S_1}{64} - \frac{\sqrt{91}S_3}{32} - \frac{5\sqrt{15}S_5}{32} + \frac{21\sqrt{3}S_7}{64} - \frac{\sqrt{51}S_9}{64}$
99	E	E	5	0	$\mathbb{Q}_{9,0}^{(h,E,5)}$	$\mathtt{Qh}(9,\mathtt{E},5,0)$	$\frac{7\sqrt{22}C_1}{64} - \frac{3\sqrt{21}C_3}{32} + \frac{\sqrt{65}C_5}{32} + \frac{\sqrt{13}C_7}{64} - \frac{3\sqrt{221}C_9}{64}$
100	E	E	5	1	$\mathbb{Q}_{9,1}^{(h,E,5)}$	$\mathtt{Qh}(9,\mathtt{E},5,1)$	$\frac{7\sqrt{22}S_1}{64} + \frac{3\sqrt{21}S_3}{32} + \frac{\sqrt{65}S_5}{32} - \frac{\sqrt{13}S_7}{64} - \frac{3\sqrt{221}S_9}{64}$

表 11 rank 10

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
101	A_1	A1	1	_	$\mathbb{Q}_{10}^{(h,A_1,1)}$	Qh(10, A1, 1,)	$\frac{\sqrt{390}C_0}{48} - \frac{\sqrt{22}C_4}{8} - \frac{\sqrt{1122}C_8}{48}$
102	A_1	A1	2	_	$\mathbb{Q}_{10}^{(h,A_1,2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{A1},\mathtt{2},)$	$\frac{11\sqrt{420189}C_0}{8988} + \frac{\sqrt{827645}C_4}{1498} - \frac{\sqrt{146055}C_8}{8988}$
103	A_1	A1	3	_	$\mathbb{Q}_{10}^{(h,A_1,3)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{A1},\mathtt{3},)$	$\frac{3\sqrt{3213210}C_0}{11984} - \frac{83\sqrt{1498}C_4}{5992} + \frac{31\sqrt{76398}C_8}{11984}$
104	A_2	A2	1	_	$\mathbb{Q}_{10}^{(h,A_2,1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{A2},\mathtt{1},)$	S_8
105	A_2	A2	2	_	$\mathbb{Q}_{10}^{(h,A_2,2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{A2},\mathtt{2},)$	S_4
106	B_1	B1	1	_	$\mathbb{Q}_{10}^{(h,B_1,1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{B1},\mathtt{1},)$	$-\frac{\sqrt{85}C_{10}}{16} + \frac{\sqrt{1482}C_2}{48} + \frac{\sqrt{57}C_6}{48}$
107	B_1	B1	2	_	$\mathbb{Q}_{10}^{(h,B_1,2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{B1},\mathtt{2},)$	$\frac{\sqrt{370006}C_{10}}{749} + \frac{\sqrt{190995}C_2}{749}$
108	B_1	B1	3	_	$\mathbb{Q}_{10}^{(h,B_1,3)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{B1},\mathtt{3},)$	$\frac{\sqrt{1209635}C_{10}}{11984} - \frac{19\sqrt{58422}C_2}{35952} + \frac{\sqrt{2247}C_6}{48}$
109	B_2	B2	1	_	$\mathbb{Q}_{10}^{(h,B_2,1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{B2},\mathtt{1},)$	S_{10}
110	B_2	B2	2	_	$\mathbb{Q}_{10}^{(h,B_2,2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{B2},\mathtt{2},)$	S_6
111	B_2	B2	3	_	$\mathbb{Q}_{10}^{(h,B_2,3)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{B2},\mathtt{3},)$	S_2
112	E	E	1	0	$\mathbb{Q}_{10,0}^{(h,E,1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{E},\mathtt{1},\mathtt{0})$	$\frac{\sqrt{221}S_1}{32} - \frac{\sqrt{102}S_3}{32} - \frac{\sqrt{510}S_5}{32} - \frac{11\sqrt{6}S_7}{64} - \frac{\sqrt{38}S_9}{64}$
113	E	E	1	1	$\mathbb{Q}_{10,1}^{(h,E,1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{E},\mathtt{1},\mathtt{1})$	$-\frac{\sqrt{221}C_1}{32} - \frac{\sqrt{102}C_3}{32} + \frac{\sqrt{510}C_5}{32} - \frac{11\sqrt{6}C_7}{64} + \frac{\sqrt{38}C_9}{64}$
114	E	E	2	0	$\mathbb{Q}_{10,0}^{(h,E,2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{E},\mathtt{2},\mathtt{0})$	$\frac{\sqrt{39}S_1}{32} - \frac{11\sqrt{2}S_3}{32} + \frac{5\sqrt{10}S_5}{32} - \frac{\sqrt{34}S_7}{64} - \frac{\sqrt{1938}S_9}{64}$
115	E	E	2	1	$\mathbb{Q}_{10,1}^{(h,E,2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{E},\mathtt{2},\mathtt{1})$	$-\frac{\sqrt{39}C_1}{32} - \frac{11\sqrt{2}C_3}{32} - \frac{5\sqrt{10}C_5}{32} - \frac{\sqrt{34}C_7}{64} + \frac{\sqrt{1938}C_9}{64}$
116	E	E	3	0	$\mathbb{Q}_{10,0}^{(h,E,3)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{E},\mathtt{3},\mathtt{0})$	$\frac{\sqrt{41990}S_1}{256} + \frac{\sqrt{4845}S_3}{128} + \frac{\sqrt{969}S_5}{128} + \frac{\sqrt{285}S_7}{256} + \frac{\sqrt{5}S_9}{256}$
117	E	E	3	1	$\mathbb{Q}_{10,1}^{(h,E,3)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{E},\mathtt{3},\mathtt{1})$	$-\frac{\sqrt{41990}C_1}{256} + \frac{\sqrt{4845}C_3}{128} - \frac{\sqrt{969}C_5}{128} + \frac{\sqrt{285}C_7}{256} - \frac{\sqrt{5}C_9}{256}$
118	E	E	4	0	$\mathbb{Q}_{10,0}^{(h,E,4)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{E},\mathtt{4},\mathtt{0})$	$\frac{9\sqrt{78}S_1}{256} - \frac{69S_3}{128} - \frac{\sqrt{5}S_5}{128} + \frac{43\sqrt{17}S_7}{256} + \frac{3\sqrt{969}S_9}{256}$
119	E	E	4	1	$\mathbb{Q}_{10,1}^{(h,E,4)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{E},\mathtt{4},\mathtt{1})$	$-\frac{9\sqrt{78}C_1}{256} - \frac{69C_3}{128} + \frac{\sqrt{5}C_5}{128} + \frac{43\sqrt{17}C_7}{256} - \frac{3\sqrt{969}C_9}{256}$
120	E	E	5	0	$\mathbb{Q}_{10,0}^{(h,E,5)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{E},\mathtt{5},\mathtt{0})$	$\frac{7\sqrt{3}S_1}{128} - \frac{7\sqrt{26}S_3}{128} + \frac{5\sqrt{130}S_5}{128} - \frac{7\sqrt{442}S_7}{256} + \frac{\sqrt{25194}S_9}{256}$
121	E	Е	5	1	$\mathbb{Q}_{10,1}^{(h,E,5)}$	Qh(10, E, 5, 1)	$-\frac{7\sqrt{3}C_1}{128} - \frac{7\sqrt{26}C_3}{128} - \frac{5\sqrt{130}C_5}{128} - \frac{7\sqrt{442}C_7}{256} - \frac{\sqrt{25194}C_9}{256}$

表 12 rank 11

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
122	A_1	A1	1	-	$\mathbb{Q}_{11}^{(h,A_1,1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{A1},\mathtt{1},)$	S_8
123	A_1	A1	2	_	$\mathbb{Q}_{11}^{(h,A_1,2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{A1},\mathtt{2},)$	S_4
124	A_2	A2	1	_	$\mathbb{Q}_{11}^{(h,A_2,1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{A2},\mathtt{1},)$	C_0
125	A_2	A2	2	-	$\mathbb{Q}_{11}^{(h,A_2,2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{A2},\mathtt{2},)$	C_8
126	A_2	A2	3	-	$\mathbb{Q}_{11}^{(h,A_2,3)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{A2},\mathtt{3},)$	C_4
127	B_1	B1	1	-	$\mathbb{Q}_{11}^{(h,B_1,1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{B1},\mathtt{1},)$	$\frac{\sqrt{798}S_{10}}{48} + \frac{\sqrt{255}S_2}{24} + \frac{3\sqrt{6}S_6}{16}$
128	B_1	B1	2	-	$\mathbb{Q}_{11}^{(h,B_1,2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{B1},\mathtt{2},)$	$-\frac{\sqrt{210}S_{10}}{96} + \frac{\sqrt{969}S_2}{48} - \frac{\sqrt{570}S_6}{32}$
129	B_1	B1	3	-	$\mathbb{Q}_{11}^{(h,B_1,3)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{B1},\mathtt{3},)$	$-\frac{\sqrt{646}S_{10}}{32} + \frac{\sqrt{35}S_2}{16} + \frac{\sqrt{238}S_6}{32}$
130	B_2	B2	1	-	$\mathbb{Q}_{11}^{(h,B_2,1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{B2},\mathtt{1},)$	C_{10}
131	B_2	B2	2	-	$\mathbb{Q}_{11}^{(h,B_2,2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{B2},\mathtt{2},)$	C_6
132	B_2	B2	3	_	$\mathbb{Q}_{11}^{(h,B_2,3)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{B2},\mathtt{3},)$	C_2
133	E	E	1	0	$\mathbb{Q}_{11,0}^{(h,E,1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{E},\mathtt{1},\mathtt{0})$	$-\frac{21\sqrt{66}C_1}{512}+\frac{\sqrt{88179}C_{11}}{512}+\frac{\sqrt{30030}C_3}{512}-\frac{15\sqrt{143}C_5}{512}+\frac{\sqrt{36465}C_7}{512}-\frac{\sqrt{46189}C_9}{512}$
134	E	E	1	1	$\mathbb{Q}_{11,1}^{(h,E,1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{E},\mathtt{1},\mathtt{1})$	$-\frac{21\sqrt{66}S_1}{512}-\frac{\sqrt{88179}S_{11}}{512}-\frac{\sqrt{30030}S_3}{512}-\frac{15\sqrt{143}S_5}{512}-\frac{\sqrt{36465}S_7}{512}-\frac{\sqrt{46189}S_9}{512}$
135	E	E	2	0	$\mathbb{Q}_{11,0}^{(h,E,2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{E},\mathtt{2},\mathtt{0})$	$-\frac{\sqrt{41990}C_1}{512} + \frac{\sqrt{385}C_{11}}{512} - \frac{3\sqrt{4522}C_3}{512} + \frac{3\sqrt{4845}C_5}{512} + \frac{77\sqrt{19}C_7}{512} + \frac{39\sqrt{15}C_9}{512}$
136	E	E	2	1	$\mathbb{Q}_{11,1}^{(h,E,2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{E},\mathtt{2},\mathtt{1})$	$-\frac{\sqrt{41990}S_1}{512} - \frac{\sqrt{385}S_{11}}{512} + \frac{3\sqrt{4522}S_3}{512} + \frac{3\sqrt{4845}S_5}{512} - \frac{77\sqrt{19}S_7}{512} + \frac{39\sqrt{15}S_9}{512}$
137	E	E	3	0	$\mathbb{Q}_{11,0}^{(h,E,3)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{E},\mathtt{3},\mathtt{0})$	$-\frac{5\sqrt{546}C_1}{256}+\frac{\sqrt{10659}C_{11}}{256}+\frac{11\sqrt{30}C_3}{256}+\frac{13\sqrt{7}C_5}{256}-\frac{3\sqrt{1785}C_7}{256}+\frac{3\sqrt{2261}C_9}{256}$
138	E	E	3	1	$\mathbb{Q}_{11,1}^{(h,E,3)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{E},\mathtt{3},\mathtt{1})$	$-\frac{5\sqrt{546}S_1}{256} - \frac{\sqrt{10659}S_{11}}{256} - \frac{11\sqrt{30}S_3}{256} + \frac{13\sqrt{7}S_5}{256} + \frac{3\sqrt{1785}S_7}{256} + \frac{3\sqrt{2261}S_9}{256}$
139	E	E	4	0	$\mathbb{Q}_{11,0}^{(h,E,4)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{E},\mathtt{4},\mathtt{0})$	$-\frac{\sqrt{29393}C_1}{512}-\frac{\sqrt{22}C_{11}}{1024}-\frac{9\sqrt{1615}C_3}{512}-\frac{5\sqrt{13566}C_5}{1024}-\frac{7\sqrt{1330}C_7}{1024}-\frac{9\sqrt{42}C_9}{1024}$
140	E	E	4	1	$\mathbb{Q}_{11,1}^{(h,E,4)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{E},\mathtt{4},\mathtt{1})$	$-\frac{\sqrt{29393}S_1}{512} + \frac{\sqrt{22}S_{11}}{1024} + \frac{9\sqrt{1615}S_3}{512} - \frac{5\sqrt{13566}S_5}{1024} + \frac{7\sqrt{1330}S_7}{1024} - \frac{9\sqrt{42}S_9}{1024}$
141	E	E	5	0	$\mathbb{Q}_{11,0}^{(h,E,5)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{E},\mathtt{5},\mathtt{0})$	$-\frac{15\sqrt{221}C_1}{512}-\frac{3\sqrt{2926}C_{11}}{1024}-\frac{\sqrt{595}C_3}{512}+\frac{53\sqrt{102}C_5}{1024}-\frac{105\sqrt{10}C_7}{1024}-\frac{61\sqrt{114}C_9}{1024}$
142	E	E	5	1	$\mathbb{Q}_{11,1}^{(h,E,5)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{E},\mathtt{5},\mathtt{1})$	$-\frac{15\sqrt{221}S_1}{512}+\frac{3\sqrt{2926}S_{11}}{1024}+\frac{\sqrt{595}S_3}{512}+\frac{53\sqrt{102}S_5}{1024}+\frac{105\sqrt{10}S_7}{1024}-\frac{61\sqrt{114}S_9}{1024}$
143	E	E	6	0	$\mathbb{Q}_{11,0}^{(h,E,6)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{E},6,\mathtt{0})$	$-\frac{21\sqrt{130}C_1}{512}-\frac{\sqrt{124355}C_{11}}{512}+\frac{57\sqrt{14}C_3}{512}-\frac{41\sqrt{15}C_5}{512}+\frac{17\sqrt{17}C_7}{512}+\frac{\sqrt{4845}C_9}{512}$
144	E	E	6	1	$\mathbb{Q}_{11,1}^{(h,E,6)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{E},6,\mathtt{1})$	$-\frac{21\sqrt{130}S_1}{512} + \frac{\sqrt{124355}S_{11}}{512} - \frac{57\sqrt{14}S_3}{512} - \frac{41\sqrt{15}S_5}{512} - \frac{17\sqrt{17}S_7}{512} + \frac{\sqrt{4845}S_9}{512}$