

\* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

| No. | tag  | matrix (polar)   | det | TR |
|-----|--|--|-----|----|
| 1   | $\{1 0\}$  | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                  | 1   | 1  |
| 2   | $\{4_{001}^+ 00\frac{1}{2}\}$                    | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$                       | 1   | 1  |
| 3   | $\{4_{001}^- 00\frac{1}{2}\}$                    | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$                       | 1   | 1  |
| 4   | $\{2_{100} \frac{1}{2}\frac{1}{2}0\}$            | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$            | 1   | 1  |
| 5   | $\{2_{010} \frac{1}{2}\frac{1}{2}0\}$            | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$            | 1   | 1  |
| 6   | $\{2_{001} 0\}$                                  | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                | 1   | 1  |
| 7   | $\{2_{110} \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$   | 1   | 1  |
| 8   | $\{2_{1-10} \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | 1   | 1  |
| 9   | $\{-1 0\}$                                       | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                               | -1  | 1  |
| 10  | $\{-4_{001}^+ 00\frac{1}{2}\}$                   | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$                      | -1  | 1  |
| 11  | $\{-4_{001}^- 00\frac{1}{2}\}$                   | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$                      | -1  | 1  |
| 12  | $\{m_{100} \frac{1}{2}\frac{1}{2}0\}$            | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$             | -1  | 1  |
| 13  | $\{m_{010} \frac{1}{2}\frac{1}{2}0\}$            | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$             | -1  | 1  |

*continued ...*

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| 14  | $\{m_{001} 0\}$                                   | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                                 | -1  | 1  |
| 15  | $\{m_{110} \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$   | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$  | -1  | 1  |
| 16  | $\{m_{1-10} \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$    | -1  | 1  |
| 17  | $\{1' 0\}$  | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                  | 1   | -1 |
| 18  | $\{4_{001}^+' 00\frac{1}{2}\}$                    | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$                       | 1   | -1 |
| 19  | $\{4_{001}^-' 00\frac{1}{2}\}$                    | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$                       | 1   | -1 |
| 20  | $\{2_{100}' \frac{1}{2}\frac{1}{2}0\}$            | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$            | 1   | -1 |
| 21  | $\{2_{010}' \frac{1}{2}\frac{1}{2}0\}$            | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$            | 1   | -1 |
| 22  | $\{2_{001}' 0\}$                                  | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                | 1   | -1 |
| 23  | $\{2_{110}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$   | 1   | -1 |
| 24  | $\{2_{1-10}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 25  | $\{-1' 0\}$                                       | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                               | -1  | -1 |
| 26  | $\{-4_{001}^+' 00\frac{1}{2}\}$                   | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$                      | -1  | -1 |
| 27  | $\{-4_{001}^-' 00\frac{1}{2}\}$                   | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$                      | -1  | -1 |

continued ...

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| No. | tag   | matrix (polar)  | det | TR |
|-----|---|---|-----|----|
| 28  | $\{m_{100}' \frac{1}{2}\frac{1}{2}0\}$            | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$            | -1  | -1 |
| 29  | $\{m_{010}' \frac{1}{2}\frac{1}{2}0\}$            | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$            | -1  | -1 |
| 30  | $\{m_{001}' 0\}$                                  | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                                | -1  | -1 |
| 31  | $\{m_{110}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 32  | $\{m_{1-10}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$   | -1  | -1 |