# No. 11　$C_{4h}$　$4/m$　[ tetragonal ] (polar)

表 1　rank 0

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 1 | $A_g$ | Ag | – | – | $\mathbb{Q}_0^{(h,A_g)}$ | Qh(0, Ag, , ) | $C_0$ |

表 2　rank 1

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 2 | $A_u$ | Au | – | – | $\mathbb{Q}_1^{(h,A_u)}$ | Qh(1, Au, , ) | $C_0$ |
| 3 | $E_u$ | Eu | – | 0 | $\mathbb{Q}_{1,0}^{(h,E_u)}$ | Qh(1, Eu, , 0) | $C_1$ |
| 4 | $E_u$ | Eu | – | 1 | $\mathbb{Q}_{1,1}^{(h,E_u)}$ | Qh(1, Eu, , 1) | $S_1$ |

表 3　rank 2

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 5 | $A_g$ | Ag | – | – | $\mathbb{Q}_2^{(h,A_g)}$ | Qh(2, Ag, , ) | $C_0$ |
| 6 | $B_g$ | Bg | 1 | – | $\mathbb{Q}_2^{(h,B_g,1)}$ | Qh(2, Bg, 1, ) | $C_2$ |
| 7 | $B_g$ | Bg | 2 | – | $\mathbb{Q}_2^{(h,B_g,2)}$ | Qh(2, Bg, 2, ) | $S_2$ |
| 8 | $E_g$ | Eg | – | 0 | $\mathbb{Q}_{2,0}^{(h,E_g)}$ | Qh(2, Eg, , 0) | $S_1$ |
| 9 | $E_g$ | Eg | – | 1 | $\mathbb{Q}_{2,1}^{(h,E_g)}$ | Qh(2, Eg, , 1) | $-C_1$ |

表 4　rank 3

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 10 | $A_u$ | Au | – | – | $\mathbb{Q}_3^{(h,A_u)}$ | Qh(3, Au, , ) | $C_0$ |
| 11 | $B_u$ | Bu | 1 | – | $\mathbb{Q}_3^{(h,B_u,1)}$ | Qh(3, Bu, 1, ) | $S_2$ |
| 12 | $B_u$ | Bu | 2 | – | $\mathbb{Q}_3^{(h,B_u,2)}$ | Qh(3, Bu, 2, ) | $C_2$ |
| 13 | $E_u$ | Eu | 1 | 0 | $\mathbb{Q}_{3,0}^{(h,E_u,1)}$ | Qh(3, Eu, 1, 0) | $-\frac{\sqrt{6}C_1}{4}+\frac{\sqrt{10}C_3}{4}$ |
| 14 | $E_u$ | Eu | 1 | 1 | $\mathbb{Q}_{3,1}^{(h,E_u,1)}$ | Qh(3, Eu, 1, 1) | $-\frac{\sqrt{6}S_1}{4}-\frac{\sqrt{10}S_3}{4}$ |
| 15 | $E_u$ | Eu | 2 | 0 | $\mathbb{Q}_{3,0}^{(h,E_u,2)}$ | Qh(3, Eu, 2, 0) | $-\frac{\sqrt{10}C_1}{4}-\frac{\sqrt{6}C_3}{4}$ |
| 16 | $E_u$ | Eu | 2 | 1 | $\mathbb{Q}_{3,1}^{(h,E_u,2)}$ | Qh(3, Eu, 2, 1) | $-\frac{\sqrt{10}S_1}{4}+\frac{\sqrt{6}S_3}{4}$ |

表 5　rank 4

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 17 | $A_g$ | Ag | 1 | – | $\mathbb{Q}_4^{(h,A_g,1)}$ | Qh(4, Ag, 1, ) | $\frac{\sqrt{21}C_0}{6}+\frac{\sqrt{15}C_4}{6}$ |
| 18 | $A_g$ | Ag | 2 | – | $\mathbb{Q}_4^{(h,A_g,2)}$ | Qh(4, Ag, 2, ) | $\frac{\sqrt{15}C_0}{6}-\frac{\sqrt{21}C_4}{6}$ |
| 19 | $A_g$ | Ag | 3 | – | $\mathbb{Q}_4^{(h,A_g,3)}$ | Qh(4, Ag, 3, ) | $S_4$ |
| 20 | $B_g$ | Bg | 1 | – | $\mathbb{Q}_4^{(h,B_g,1)}$ | Qh(4, Bg, 1, ) | $-C_2$ |
| 21 | $B_g$ | Bg | 2 | – | $\mathbb{Q}_4^{(h,B_g,2)}$ | Qh(4, Bg, 2, ) | $S_2$ |
| 22 | $E_g$ | Eg | 1 | 0 | $\mathbb{Q}_{4,0}^{(h,E_g,1)}$ | Qh(4, Eg, 1, 0) | $\frac{\sqrt{2}\left(-\sqrt{7}S_1-S_3\right)}{4}$ |
| 23 | $E_g$ | Eg | 1 | 1 | $\mathbb{Q}_{4,1}^{(h,E_g,1)}$ | Qh(4, Eg, 1, 1) | $\frac{\sqrt{14}C_1}{4}-\frac{\sqrt{2}C_3}{4}$ |
| 24 | $E_g$ | Eg | 2 | 0 | $\mathbb{Q}_{4,0}^{(h,E_g,2)}$ | Qh(4, Eg, 2, 0) | $\frac{\sqrt{2}\left(-S_1+\sqrt{7}S_3\right)}{4}$ |
| 25 | $E_g$ | Eg | 2 | 1 | $\mathbb{Q}_{4,1}^{(h,E_g,2)}$ | Qh(4, Eg, 2, 1) | $\frac{\sqrt{2}C_1}{4}+\frac{\sqrt{14}C_3}{4}$ |

表 6　rank 5

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 26 | $A_u$ | Au | 1 | – | $\mathbb{Q}_5^{(h,A_u,1)}$ | Qh(5, Au, 1, ) | $S_4$ |
| 27 | $A_u$ | Au | 2 | – | $\mathbb{Q}_5^{(h,A_u,2)}$ | Qh(5, Au, 2, ) | $C_0$ |
| 28 | $A_u$ | Au | 3 | – | $\mathbb{Q}_5^{(h,A_u,3)}$ | Qh(5, Au, 3, ) | $C_4$ |
| 29 | $B_u$ | Bu | 1 | – | $\mathbb{Q}_5^{(h,B_u,1)}$ | Qh(5, Bu, 1, ) | $-S_2$ |
| 30 | $B_u$ | Bu | 2 | – | $\mathbb{Q}_5^{(h,B_u,2)}$ | Qh(5, Bu, 2, ) | $C_2$ |
| 31 | $E_u$ | Eu | 1 | 0 | $\mathbb{Q}_{5,0}^{(h,E_u,1)}$ | Qh(5, Eu, 1, 0) | $\frac{\sqrt{15}C_1}{8} - \frac{\sqrt{70}C_3}{16} + \frac{3\sqrt{14}C_5}{16}$ |
| 32 | $E_u$ | Eu | 1 | 1 | $\mathbb{Q}_{5,1}^{(h,E_u,1)}$ | Qh(5, Eu, 1, 1) | $\frac{\sqrt{15}S_1}{8} + \frac{\sqrt{70}S_3}{16} + \frac{3\sqrt{14}S_5}{16}$ |
| 33 | $E_u$ | Eu | 2 | 0 | $\mathbb{Q}_{5,0}^{(h,E_u,2)}$ | Qh(5, Eu, 2, 0) | $\frac{\sqrt{2}\left(\sqrt{42}C_1+9C_3+\sqrt{5}C_5\right)}{16}$ |
| 34 | $E_u$ | Eu | 2 | 1 | $\mathbb{Q}_{5,1}^{(h,E_u,2)}$ | Qh(5, Eu, 2, 1) | $\frac{\sqrt{2}\left(\sqrt{42}S_1-9S_3+\sqrt{5}S_5\right)}{16}$ |
| 35 | $E_u$ | Eu | 3 | 0 | $\mathbb{Q}_{5,0}^{(h,E_u,3)}$ | Qh(5, Eu, 3, 0) | $\frac{\sqrt{7}C_1}{4} - \frac{\sqrt{6}C_3}{8} - \frac{\sqrt{30}C_5}{8}$ |
| 36 | $E_u$ | Eu | 3 | 1 | $\mathbb{Q}_{5,1}^{(h,E_u,3)}$ | Qh(5, Eu, 3, 1) | $\frac{\sqrt{7}S_1}{4} + \frac{\sqrt{6}S_3}{8} - \frac{\sqrt{30}S_5}{8}$ |

表 7　rank 6

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 37 | $A_g$ | Ag | 1 | – | $\mathbb{Q}_6^{(h,A_g,1)}$ | Qh(6, Ag, 1, ) | $\frac{\sqrt{2}C_0}{4} - \frac{\sqrt{14}C_4}{4}$ |
| 38 | $A_g$ | Ag | 2 | – | $\mathbb{Q}_6^{(h,A_g,2)}$ | Qh(6, Ag, 2, ) | $\frac{\sqrt{14}C_0}{4} + \frac{\sqrt{2}C_4}{4}$ |
| 39 | $A_g$ | Ag | 3 | – | $\mathbb{Q}_6^{(h,A_g,3)}$ | Qh(6, Ag, 3, ) | $S_4$ |
| 40 | $B_g$ | Bg | 1 | – | $\mathbb{Q}_6^{(h,B_g,1)}$ | Qh(6, Bg, 1, ) | $\frac{\sqrt{11}C_2}{4} - \frac{\sqrt{5}C_6}{4}$ |
| 41 | $B_g$ | Bg | 2 | – | $\mathbb{Q}_6^{(h,B_g,2)}$ | Qh(6, Bg, 2, ) | $\frac{\sqrt{5}C_2}{4} + \frac{\sqrt{11}C_6}{4}$ |
| 42 | $B_g$ | Bg | 3 | – | $\mathbb{Q}_6^{(h,B_g,3)}$ | Qh(6, Bg, 3, ) | $S_6$ |
| 43 | $B_g$ | Bg | 4 | – | $\mathbb{Q}_6^{(h,B_g,4)}$ | Qh(6, Bg, 4, ) | $S_2$ |
| 44 | $E_g$ | Eg | 1 | 0 | $\mathbb{Q}_{6,0}^{(h,E_g,1)}$ | Qh(6, Eg, 1, 0) | $\frac{\sqrt{3}S_1}{4} - \frac{\sqrt{30}S_3}{8} - \frac{\sqrt{22}S_5}{8}$ |
| 45 | $E_g$ | Eg | 1 | 1 | $\mathbb{Q}_{6,1}^{(h,E_g,1)}$ | Qh(6, Eg, 1, 1) | $-\frac{\sqrt{3}C_1}{4} - \frac{\sqrt{30}C_3}{8} + \frac{\sqrt{22}C_5}{8}$ |
| 46 | $E_g$ | Eg | 2 | 0 | $\mathbb{Q}_{6,0}^{(h,E_g,2)}$ | Qh(6, Eg, 2, 0) | $\frac{3\sqrt{22}S_1}{16} + \frac{\sqrt{55}S_3}{16} + \frac{\sqrt{3}S_5}{16}$ |
| 47 | $E_g$ | Eg | 2 | 1 | $\mathbb{Q}_{6,1}^{(h,E_g,2)}$ | Qh(6, Eg, 2, 1) | $-\frac{3\sqrt{22}C_1}{16} + \frac{\sqrt{55}C_3}{16} - \frac{\sqrt{3}C_5}{16}$ |
| 48 | $E_g$ | Eg | 3 | 0 | $\mathbb{Q}_{6,0}^{(h,E_g,3)}$ | Qh(6, Eg, 3, 0) | $\frac{\sqrt{10}S_1}{16} - \frac{9S_3}{16} + \frac{\sqrt{165}S_5}{16}$ |
| 49 | $E_g$ | Eg | 3 | 1 | $\mathbb{Q}_{6,1}^{(h,E_g,3)}$ | Qh(6, Eg, 3, 1) | $-\frac{\sqrt{10}C_1}{16} - \frac{9C_3}{16} - \frac{\sqrt{165}C_5}{16}$ |

表 8　rank 7

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 50 | $A_u$ | Au | 1 | – | $\mathbb{Q}_7^{(h,A_u,1)}$ | Qh(7, Au, 1, ) | $S_4$ |
| 51 | $A_u$ | Au | 2 | – | $\mathbb{Q}_7^{(h,A_u,2)}$ | Qh(7, Au, 2, ) | $C_0$ |
| 52 | $A_u$ | Au | 3 | – | $\mathbb{Q}_7^{(h,A_u,3)}$ | Qh(7, Au, 3, ) | $C_4$ |
| 53 | $B_u$ | Bu | 1 | – | $\mathbb{Q}_7^{(h,B_u,1)}$ | Qh(7, Bu, 1, ) | $\frac{\sqrt{78}S_2}{12} + \frac{\sqrt{66}S_6}{12}$ |
| 54 | $B_u$ | Bu | 2 | – | $\mathbb{Q}_7^{(h,B_u,2)}$ | Qh(7, Bu, 2, ) | $\frac{\sqrt{66}S_2}{12} - \frac{\sqrt{78}S_6}{12}$ |
| 55 | $B_u$ | Bu | 3 | – | $\mathbb{Q}_7^{(h,B_u,3)}$ | Qh(7, Bu, 3, ) | $C_6$ |
| 56 | $B_u$ | Bu | 4 | – | $\mathbb{Q}_7^{(h,B_u,4)}$ | Qh(7, Bu, 4, ) | $C_2$ |
| 57 | $E_u$ | Eu | 1 | 0 | $\mathbb{Q}_{7,0}^{(h,E_u,1)}$ | Qh(7, Eu, 1, 0) | $-\frac{5\sqrt{7}C_1}{32} + \frac{3\sqrt{21}C_3}{32} - \frac{\sqrt{231}C_5}{32} + \frac{\sqrt{429}C_7}{32}$ |
| 58 | $E_u$ | Eu | 1 | 1 | $\mathbb{Q}_{7,1}^{(h,E_u,1)}$ | Qh(7, Eu, 1, 1) | $-\frac{5\sqrt{7}S_1}{32} - \frac{3\sqrt{21}S_3}{32} - \frac{\sqrt{231}S_5}{32} - \frac{\sqrt{429}S_7}{32}$ |
| 59 | $E_u$ | Eu | 2 | 0 | $\mathbb{Q}_{7,0}^{(h,E_u,2)}$ | Qh(7, Eu, 2, 0) | $-\frac{3\sqrt{33}C_1}{32} - \frac{\sqrt{11}C_3}{32} + \frac{25C_5}{32} + \frac{\sqrt{91}C_7}{32}$ |
| 60 | $E_u$ | Eu | 2 | 1 | $\mathbb{Q}_{7,1}^{(h,E_u,2)}$ | Qh(7, Eu, 2, 1) | $-\frac{3\sqrt{33}S_1}{32} + \frac{\sqrt{11}S_3}{32} + \frac{25S_5}{32} - \frac{\sqrt{91}S_7}{32}$ |
| 61 | $E_u$ | Eu | 3 | 0 | $\mathbb{Q}_{7,0}^{(h,E_u,3)}$ | Qh(7, Eu, 3, 0) | $-\frac{\sqrt{858}C_1}{64} - \frac{3\sqrt{286}C_3}{64} - \frac{5\sqrt{26}C_5}{64} - \frac{\sqrt{14}C_7}{64}$ |
| 62 | $E_u$ | Eu | 3 | 1 | $\mathbb{Q}_{7,1}^{(h,E_u,3)}$ | Qh(7, Eu, 3, 1) | $-\frac{\sqrt{858}S_1}{64} + \frac{3\sqrt{286}S_3}{64} - \frac{5\sqrt{26}S_5}{64} + \frac{\sqrt{14}S_7}{64}$ |
| 63 | $E_u$ | Eu | 4 | 0 | $\mathbb{Q}_{7,0}^{(h,E_u,4)}$ | Qh(7, Eu, 4, 0) | $\frac{\sqrt{2}\left(-15\sqrt{3}C_1+19C_3-\sqrt{11}C_5-\sqrt{1001}C_7\right)}{64}$ |
| 64 | $E_u$ | Eu | 4 | 1 | $\mathbb{Q}_{7,1}^{(h,E_u,4)}$ | Qh(7, Eu, 4, 1) | $-\frac{15\sqrt{6}S_1}{64} - \frac{19\sqrt{2}S_3}{64} - \frac{\sqrt{22}S_5}{64} + \frac{\sqrt{2002}S_7}{64}$ |

表 9　rank 8

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 65 | $A_g$ | Ag | 1 | – | $\mathbb{Q}_8^{(h,A_g,1)}$ | Qh(8, Ag, 1, ) | $\frac{\sqrt{33}C_0}{8} + \frac{\sqrt{21}C_4}{12} + \frac{\sqrt{195}C_8}{24}$ |
| 66 | $A_g$ | Ag | 2 | – | $\mathbb{Q}_8^{(h,A_g,2)}$ | Qh(8, Ag, 2, ) | $-\frac{\sqrt{286}C_0}{32} + \frac{\sqrt{182}C_4}{16} + \frac{\sqrt{10}C_8}{32}$ |
| 67 | $A_g$ | Ag | 3 | – | $\mathbb{Q}_8^{(h,A_g,3)}$ | Qh(8, Ag, 3, ) | $-\frac{\sqrt{210}C_0}{32} - \frac{\sqrt{330}C_4}{48} + \frac{\sqrt{6006}C_8}{96}$ |
| 68 | $A_g$ | Ag | 4 | – | $\mathbb{Q}_8^{(h,A_g,4)}$ | Qh(8, Ag, 4, ) | $S_8$ |
| 69 | $A_g$ | Ag | 5 | – | $\mathbb{Q}_8^{(h,A_g,5)}$ | Qh(8, Ag, 5, ) | $S_4$ |
| 70 | $B_g$ | Bg | 1 | – | $\mathbb{Q}_8^{(h,B_g,1)}$ | Qh(8, Bg, 1, ) | $C_6$ |
| 71 | $B_g$ | Bg | 2 | – | $\mathbb{Q}_8^{(h,B_g,2)}$ | Qh(8, Bg, 2, ) | $C_2$ |
| 72 | $B_g$ | Bg | 3 | – | $\mathbb{Q}_8^{(h,B_g,3)}$ | Qh(8, Bg, 3, ) | $S_6$ |
| 73 | $B_g$ | Bg | 4 | – | $\mathbb{Q}_8^{(h,B_g,4)}$ | Qh(8, Bg, 4, ) | $S_2$ |
| 74 | $E_g$ | Eg | 1 | 0 | $\mathbb{Q}_{8,0}^{(h,E_g,1)}$ | Qh(8, Eg, 1, 0) | $-\frac{\sqrt{715}S_1}{32} - \frac{\sqrt{273}S_3}{32} - \frac{\sqrt{35}S_5}{32} - \frac{S_7}{32}$ |
| 75 | $E_g$ | Eg | 1 | 1 | $\mathbb{Q}_{8,1}^{(h,E_g,1)}$ | Qh(8, Eg, 1, 1) | $\frac{\sqrt{715}C_1}{32} - \frac{\sqrt{273}C_3}{32} + \frac{\sqrt{35}C_5}{32} - \frac{C_7}{32}$ |
| 76 | $E_g$ | Eg | 2 | 0 | $\mathbb{Q}_{8,0}^{(h,E_g,2)}$ | Qh(8, Eg, 2, 0) | $-\frac{\sqrt{77}S_1}{32} + \frac{5\sqrt{15}S_3}{32} - \frac{3\sqrt{13}S_5}{32} - \frac{\sqrt{455}S_7}{32}$ |
| 77 | $E_g$ | Eg | 2 | 1 | $\mathbb{Q}_{8,1}^{(h,E_g,2)}$ | Qh(8, Eg, 2, 1) | $\frac{\sqrt{77}C_1}{32} + \frac{5\sqrt{15}C_3}{32} + \frac{3\sqrt{13}C_5}{32} - \frac{\sqrt{455}C_7}{32}$ |
| 78 | $E_g$ | Eg | 3 | 0 | $\mathbb{Q}_{8,0}^{(h,E_g,3)}$ | Qh(8, Eg, 3, 0) | $-\frac{\sqrt{858}S_1}{64} + \frac{\sqrt{910}S_3}{64} + \frac{7\sqrt{42}S_5}{64} + \frac{3\sqrt{30}S_7}{64}$ |
| 79 | $E_g$ | Eg | 3 | 1 | $\mathbb{Q}_{8,1}^{(h,E_g,3)}$ | Qh(8, Eg, 3, 1) | $\frac{\sqrt{858}C_1}{64} + \frac{\sqrt{910}C_3}{64} - \frac{7\sqrt{42}C_5}{64} + \frac{3\sqrt{30}C_7}{64}$ |
| 80 | $E_g$ | Eg | 4 | 0 | $\mathbb{Q}_{8,0}^{(h,E_g,4)}$ | Qh(8, Eg, 4, 0) | $-\frac{\sqrt{70}S_1}{64} + \frac{3\sqrt{66}S_3}{64} - \frac{\sqrt{1430}S_5}{64} + \frac{\sqrt{2002}S_7}{64}$ |
| 81 | $E_g$ | Eg | 4 | 1 | $\mathbb{Q}_{8,1}^{(h,E_g,4)}$ | Qh(8, Eg, 4, 1) | $\frac{\sqrt{70}C_1}{64} + \frac{3\sqrt{66}C_3}{64} + \frac{\sqrt{1430}C_5}{64} + \frac{\sqrt{2002}C_7}{64}$ |

表 10　rank 9

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 82 | $A_u$ | Au | 1 | – | $\mathbb{Q}_9^{(h,A_u,1)}$ | Qh(9, Au, 1, ) | $\frac{\sqrt{102}S_4}{12} - \frac{\sqrt{42}S_8}{12}$ |
| 83 | $A_u$ | Au | 2 | – | $\mathbb{Q}_9^{(h,A_u,2)}$ | Qh(9, Au, 2, ) | $\frac{\sqrt{42}S_4}{12} + \frac{\sqrt{102}S_8}{12}$ |
| 84 | $A_u$ | Au | 3 | – | $\mathbb{Q}_9^{(h,A_u,3)}$ | Qh(9, Au, 3, ) | $C_0$ |
| 85 | $A_u$ | Au | 4 | – | $\mathbb{Q}_9^{(h,A_u,4)}$ | Qh(9, Au, 4, ) | $C_8$ |
| 86 | $A_u$ | Au | 5 | – | $\mathbb{Q}_9^{(h,A_u,5)}$ | Qh(9, Au, 5, ) | $C_4$ |
| 87 | $B_u$ | Bu | 1 | – | $\mathbb{Q}_9^{(h,B_u,1)}$ | Qh(9, Bu, 1, ) | $\frac{\sqrt{3}S_2}{4} - \frac{\sqrt{13}S_6}{4}$ |
| 88 | $B_u$ | Bu | 2 | – | $\mathbb{Q}_9^{(h,B_u,2)}$ | Qh(9, Bu, 2, ) | $-\frac{\sqrt{13}S_2}{4} - \frac{\sqrt{3}S_6}{4}$ |
| 89 | $B_u$ | Bu | 3 | – | $\mathbb{Q}_9^{(h,B_u,3)}$ | Qh(9, Bu, 3, ) | $C_6$ |
| 90 | $B_u$ | Bu | 4 | – | $\mathbb{Q}_9^{(h,B_u,4)}$ | Qh(9, Bu, 4, ) | $C_2$ |
| 91 | $E_u$ | Eu | 1 | 0 | $\mathbb{Q}_{9,0}^{(h,E_u,1)}$ | Qh(9, Eu, 1, 0) | $\frac{21\sqrt{5}C_1}{128} - \frac{\sqrt{2310}C_3}{128} + \frac{3\sqrt{286}C_5}{128} - \frac{3\sqrt{1430}C_7}{256} + \frac{\sqrt{24310}C_9}{256}$ |
| 92 | $E_u$ | Eu | 1 | 1 | $\mathbb{Q}_{9,1}^{(h,E_u,1)}$ | Qh(9, Eu, 1, 1) | $\frac{21\sqrt{5}S_1}{128} + \frac{\sqrt{2310}S_3}{128} + \frac{3\sqrt{286}S_5}{128} + \frac{3\sqrt{1430}S_7}{256} + \frac{\sqrt{24310}S_9}{256}$ |
| 93 | $E_u$ | Eu | 2 | 0 | $\mathbb{Q}_{9,0}^{(h,E_u,2)}$ | Qh(9, Eu, 2, 0) | $\frac{\sqrt{2}\left(\sqrt{4862}C_1+2\sqrt{4641}C_3+10\sqrt{85}C_5+7\sqrt{17}C_7+3C_9\right)}{256}$ |
| 94 | $E_u$ | Eu | 2 | 1 | $\mathbb{Q}_{9,1}^{(h,E_u,2)}$ | Qh(9, Eu, 2, 1) | $\frac{\sqrt{2}\left(\sqrt{4862}S_1-2\sqrt{4641}S_3+10\sqrt{85}S_5-7\sqrt{17}S_7+3S_9\right)}{256}$ |
| 95 | $E_u$ | Eu | 3 | 0 | $\mathbb{Q}_{9,0}^{(h,E_u,3)}$ | Qh(9, Eu, 3, 0) | $\frac{\sqrt{1001}C_1}{64} - \frac{\sqrt{78}C_3}{64} - \frac{3\sqrt{70}C_5}{64} + \frac{23\sqrt{14}C_7}{128} + \frac{3\sqrt{238}C_9}{128}$ |
| 96 | $E_u$ | Eu | 3 | 1 | $\mathbb{Q}_{9,1}^{(h,E_u,3)}$ | Qh(9, Eu, 3, 1) | $\frac{\sqrt{1001}S_1}{64} + \frac{\sqrt{78}S_3}{64} - \frac{3\sqrt{70}S_5}{64} - \frac{23\sqrt{14}S_7}{128} + \frac{3\sqrt{238}S_9}{128}$ |
| 97 | $E_u$ | Eu | 4 | 0 | $\mathbb{Q}_{9,0}^{(h,E_u,4)}$ | Qh(9, Eu, 4, 0) | $\frac{\sqrt{858}C_1}{64} + \frac{\sqrt{91}C_3}{32} - \frac{5\sqrt{15}C_5}{32} - \frac{21\sqrt{3}C_7}{64} - \frac{\sqrt{51}C_9}{64}$ |
| 98 | $E_u$ | Eu | 4 | 1 | $\mathbb{Q}_{9,1}^{(h,E_u,4)}$ | Qh(9, Eu, 4, 1) | $\frac{\sqrt{858}S_1}{64} - \frac{\sqrt{91}S_3}{32} - \frac{5\sqrt{15}S_5}{32} + \frac{21\sqrt{3}S_7}{64} - \frac{\sqrt{51}S_9}{64}$ |
| 99 | $E_u$ | Eu | 5 | 0 | $\mathbb{Q}_{9,0}^{(h,E_u,5)}$ | Qh(9, Eu, 5, 0) | $\frac{7\sqrt{22}C_1}{64} - \frac{3\sqrt{21}C_3}{32} + \frac{\sqrt{65}C_5}{32} + \frac{\sqrt{13}C_7}{64} - \frac{3\sqrt{221}C_9}{64}$ |
| 100 | $E_u$ | Eu | 5 | 1 | $\mathbb{Q}_{9,1}^{(h,E_u,5)}$ | Qh(9, Eu, 5, 1) | $\frac{7\sqrt{22}S_1}{64} + \frac{3\sqrt{21}S_3}{32} + \frac{\sqrt{65}S_5}{32} - \frac{\sqrt{13}S_7}{64} - \frac{3\sqrt{221}S_9}{64}$ |

表 11　rank 10

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 101 | $A_g$ | Ag | 1 | – | $\mathbb{Q}_{10}^{(h,A_g,1)}$ | Qh(10, Ag, 1, ) | $\frac{\sqrt{390}C_0}{48} - \frac{\sqrt{22}C_4}{8} - \frac{\sqrt{1122}C_8}{48}$ |
| 102 | $A_g$ | Ag | 2 | – | $\mathbb{Q}_{10}^{(h,A_g,2)}$ | Qh(10, Ag, 2, ) | $\frac{11\sqrt{420189}C_0}{8988} + \frac{\sqrt{827645}C_4}{1498} - \frac{\sqrt{146055}C_8}{8988}$ |
| 103 | $A_g$ | Ag | 3 | – | $\mathbb{Q}_{10}^{(h,A_g,3)}$ | Qh(10, Ag, 3, ) | $\frac{3\sqrt{3213210}C_0}{11984} - \frac{83\sqrt{1498}C_4}{5992} + \frac{31\sqrt{76398}C_8}{11984}$ |
| 104 | $A_g$ | Ag | 4 | – | $\mathbb{Q}_{10}^{(h,A_g,4)}$ | Qh(10, Ag, 4, ) | $S_8$ |
| 105 | $A_g$ | Ag | 5 | – | $\mathbb{Q}_{10}^{(h,A_g,5)}$ | Qh(10, Ag, 5, ) | $S_4$ |
| 106 | $B_g$ | Bg | 1 | – | $\mathbb{Q}_{10}^{(h,B_g,1)}$ | Qh(10, Bg, 1, ) | $-\frac{\sqrt{85}C_{10}}{16} + \frac{\sqrt{1482}C_2}{48} + \frac{\sqrt{57}C_6}{48}$ |
| 107 | $B_g$ | Bg | 2 | – | $\mathbb{Q}_{10}^{(h,B_g,2)}$ | Qh(10, Bg, 2, ) | $\frac{\sqrt{370006}C_{10}}{749} + \frac{\sqrt{190995}C_2}{749}$ |
| 108 | $B_g$ | Bg | 3 | – | $\mathbb{Q}_{10}^{(h,B_g,3)}$ | Qh(10, Bg, 3, ) | $\frac{\sqrt{1209635}C_{10}}{11984} - \frac{19\sqrt{58422}C_2}{35952} + \frac{\sqrt{2247}C_6}{48}$ |
| 109 | $B_g$ | Bg | 4 | – | $\mathbb{Q}_{10}^{(h,B_g,4)}$ | Qh(10, Bg, 4, ) | $S_{10}$ |
| 110 | $B_g$ | Bg | 5 | – | $\mathbb{Q}_{10}^{(h,B_g,5)}$ | Qh(10, Bg, 5, ) | $S_6$ |
| 111 | $B_g$ | Bg | 6 | – | $\mathbb{Q}_{10}^{(h,B_g,6)}$ | Qh(10, Bg, 6, ) | $S_2$ |
| 112 | $E_g$ | Eg | 1 | 0 | $\mathbb{Q}_{10,0}^{(h,E_g,1)}$ | Qh(10, Eg, 1, 0) | $\frac{\sqrt{221}S_1}{32} - \frac{\sqrt{102}S_3}{32} - \frac{\sqrt{510}S_5}{32} - \frac{11\sqrt{6}S_7}{64} - \frac{\sqrt{38}S_9}{64}$ |
| 113 | $E_g$ | Eg | 1 | 1 | $\mathbb{Q}_{10,1}^{(h,E_g,1)}$ | Qh(10, Eg, 1, 1) | $-\frac{\sqrt{221}C_1}{32} - \frac{\sqrt{102}C_3}{32} + \frac{\sqrt{510}C_5}{32} - \frac{11\sqrt{6}C_7}{64} + \frac{\sqrt{38}C_9}{64}$ |
| 114 | $E_g$ | Eg | 2 | 0 | $\mathbb{Q}_{10,0}^{(h,E_g,2)}$ | Qh(10, Eg, 2, 0) | $\frac{\sqrt{2}\left(\sqrt{78}S_1-22S_3+10\sqrt{5}S_5-\sqrt{17}S_7-\sqrt{969}S_9\right)}{64}$ |
| 115 | $E_g$ | Eg | 2 | 1 | $\mathbb{Q}_{10,1}^{(h,E_g,2)}$ | Qh(10, Eg, 2, 1) | $-\frac{\sqrt{2}\left(\sqrt{78}C_1+22C_3+10\sqrt{5}C_5+\sqrt{17}C_7-\sqrt{969}C_9\right)}{64}$ |
| 116 | $E_g$ | Eg | 3 | 0 | $\mathbb{Q}_{10,0}^{(h,E_g,3)}$ | Qh(10, Eg, 3, 0) | $\frac{\sqrt{41990}S_1}{256} + \frac{\sqrt{4845}S_3}{128} + \frac{\sqrt{969}S_5}{128} + \frac{\sqrt{285}S_7}{256} + \frac{\sqrt{5}S_9}{256}$ |
| 117 | $E_g$ | Eg | 3 | 1 | $\mathbb{Q}_{10,1}^{(h,E_g,3)}$ | Qh(10, Eg, 3, 1) | $-\frac{\sqrt{41990}C_1}{256} + \frac{\sqrt{4845}C_3}{128} - \frac{\sqrt{969}C_5}{128} + \frac{\sqrt{285}C_7}{256} - \frac{\sqrt{5}C_9}{256}$ |
| 118 | $E_g$ | Eg | 4 | 0 | $\mathbb{Q}_{10,0}^{(h,E_g,4)}$ | Qh(10, Eg, 4, 0) | $\frac{9\sqrt{78}S_1}{256} - \frac{69S_3}{128} - \frac{\sqrt{5}S_5}{128} + \frac{43\sqrt{17}S_7}{256} + \frac{3\sqrt{969}S_9}{256}$ |
| 119 | $E_g$ | Eg | 4 | 1 | $\mathbb{Q}_{10,1}^{(h,E_g,4)}$ | Qh(10, Eg, 4, 1) | $-\frac{9\sqrt{78}C_1}{256} - \frac{69C_3}{128} + \frac{\sqrt{5}C_5}{128} + \frac{43\sqrt{17}C_7}{256} - \frac{3\sqrt{969}C_9}{256}$ |
| 120 | $E_g$ | Eg | 5 | 0 | $\mathbb{Q}_{10,0}^{(h,E_g,5)}$ | Qh(10, Eg, 5, 0) | $\frac{7\sqrt{3}S_1}{128} - \frac{7\sqrt{26}S_3}{128} + \frac{5\sqrt{130}S_5}{128} - \frac{7\sqrt{442}S_7}{256} + \frac{\sqrt{25194}S_9}{256}$ |
| 121 | $E_g$ | Eg | 5 | 1 | $\mathbb{Q}_{10,1}^{(h,E_g,5)}$ | Qh(10, Eg, 5, 1) | $-\frac{7\sqrt{3}C_1}{128} - \frac{7\sqrt{26}C_3}{128} - \frac{5\sqrt{130}C_5}{128} - \frac{7\sqrt{442}C_7}{256} - \frac{\sqrt{25194}C_9}{256}$ |

表 12　rank 11

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 122 | $A_u$ | Au | 1 | – | $\mathbb{Q}_{11}^{(h,A_u,1)}$ | Qh(11, Au, 1, ) | $S_8$ |
| 123 | $A_u$ | Au | 2 | – | $\mathbb{Q}_{11}^{(h,A_u,2)}$ | Qh(11, Au, 2, ) | $S_4$ |
| 124 | $A_u$ | Au | 3 | – | $\mathbb{Q}_{11}^{(h,A_u,3)}$ | Qh(11, Au, 3, ) | $C_0$ |
| 125 | $A_u$ | Au | 4 | – | $\mathbb{Q}_{11}^{(h,A_u,4)}$ | Qh(11, Au, 4, ) | $C_8$ |
| 126 | $A_u$ | Au | 5 | – | $\mathbb{Q}_{11}^{(h,A_u,5)}$ | Qh(11, Au, 5, ) | $C_4$ |
| 127 | $B_u$ | Bu | 1 | – | $\mathbb{Q}_{11}^{(h,B_u,1)}$ | Qh(11, Bu, 1, ) | $\frac{\sqrt{798}S_{10}}{48} + \frac{\sqrt{255}S_2}{24} + \frac{3\sqrt{6}S_6}{16}$ |
| 128 | $B_u$ | Bu | 2 | – | $\mathbb{Q}_{11}^{(h,B_u,2)}$ | Qh(11, Bu, 2, ) | $-\frac{\sqrt{210}S_{10}}{96} + \frac{\sqrt{969}S_2}{48} - \frac{\sqrt{570}S_6}{32}$ |
| 129 | $B_u$ | Bu | 3 | – | $\mathbb{Q}_{11}^{(h,B_u,3)}$ | Qh(11, Bu, 3, ) | $-\frac{\sqrt{646}S_{10}}{32} + \frac{\sqrt{35}S_2}{16} + \frac{\sqrt{238}S_6}{32}$ |
| 130 | $B_u$ | Bu | 4 | – | $\mathbb{Q}_{11}^{(h,B_u,4)}$ | Qh(11, Bu, 4, ) | $C_{10}$ |
| 131 | $B_u$ | Bu | 5 | – | $\mathbb{Q}_{11}^{(h,B_u,5)}$ | Qh(11, Bu, 5, ) | $C_6$ |
| 132 | $B_u$ | Bu | 6 | – | $\mathbb{Q}_{11}^{(h,B_u,6)}$ | Qh(11, Bu, 6, ) | $C_2$ |
| 133 | $E_u$ | Eu | 1 | 0 | $\mathbb{Q}_{11,0}^{(h,E_u,1)}$ | Qh(11, Eu, 1, 0) | $-\frac{21\sqrt{66}C_1}{512} + \frac{\sqrt{88179}C_{11}}{512} + \frac{\sqrt{30030}C_3}{512} - \frac{15\sqrt{143}C_5}{512} + \frac{\sqrt{36465}C_7}{512} - \frac{\sqrt{46189}C_9}{512}$ |
| 134 | $E_u$ | Eu | 1 | 1 | $\mathbb{Q}_{11,1}^{(h,E_u,1)}$ | Qh(11, Eu, 1, 1) | $-\frac{21\sqrt{66}S_1}{512} - \frac{\sqrt{88179}S_{11}}{512} - \frac{\sqrt{30030}S_3}{512} - \frac{15\sqrt{143}S_5}{512} - \frac{\sqrt{36465}S_7}{512} - \frac{\sqrt{46189}S_9}{512}$ |
| 135 | $E_u$ | Eu | 2 | 0 | $\mathbb{Q}_{11,0}^{(h,E_u,2)}$ | Qh(11, Eu, 2, 0) | $-\frac{\sqrt{41990}C_1}{512} + \frac{\sqrt{385}C_{11}}{512} - \frac{3\sqrt{4522}C_3}{512} + \frac{3\sqrt{4845}C_5}{512} + \frac{77\sqrt{19}C_7}{512} + \frac{39\sqrt{15}C_9}{512}$ |
| 136 | $E_u$ | Eu | 2 | 1 | $\mathbb{Q}_{11,1}^{(h,E_u,2)}$ | Qh(11, Eu, 2, 1) | $-\frac{\sqrt{41990}S_1}{512} - \frac{\sqrt{385}S_{11}}{512} + \frac{3\sqrt{4522}S_3}{512} + \frac{3\sqrt{4845}S_5}{512} - \frac{77\sqrt{19}S_7}{512} + \frac{39\sqrt{15}S_9}{512}$ |
| 137 | $E_u$ | Eu | 3 | 0 | $\mathbb{Q}_{11,0}^{(h,E_u,3)}$ | Qh(11, Eu, 3, 0) | $-\frac{5\sqrt{546}C_1}{256} + \frac{\sqrt{10659}C_{11}}{256} + \frac{11\sqrt{30}C_3}{256} + \frac{13\sqrt{7}C_5}{256} - \frac{3\sqrt{1785}C_7}{256} + \frac{3\sqrt{2261}C_9}{256}$ |
| 138 | $E_u$ | Eu | 3 | 1 | $\mathbb{Q}_{11,1}^{(h,E_u,3)}$ | Qh(11, Eu, 3, 1) | $-\frac{5\sqrt{546}S_1}{256} - \frac{\sqrt{10659}S_{11}}{256} - \frac{11\sqrt{30}S_3}{256} + \frac{13\sqrt{7}S_5}{256} + \frac{3\sqrt{1785}S_7}{256} + \frac{3\sqrt{2261}S_9}{256}$ |
| 139 | $E_u$ | Eu | 4 | 0 | $\mathbb{Q}_{11,0}^{(h,E_u,4)}$ | Qh(11, Eu, 4, 0) | $-\frac{\sqrt{29393}C_1}{512} - \frac{\sqrt{22}C_{11}}{1024} - \frac{9\sqrt{1615}C_3}{512} - \frac{5\sqrt{13566}C_5}{1024} - \frac{7\sqrt{1330}C_7}{1024} - \frac{9\sqrt{42}C_9}{1024}$ |
| 140 | $E_u$ | Eu | 4 | 1 | $\mathbb{Q}_{11,1}^{(h,E_u,4)}$ | Qh(11, Eu, 4, 1) | $-\frac{\sqrt{29393}S_1}{512} + \frac{\sqrt{22}S_{11}}{1024} + \frac{9\sqrt{1615}S_3}{512} - \frac{5\sqrt{13566}S_5}{1024} + \frac{7\sqrt{1330}S_7}{1024} - \frac{9\sqrt{42}S_9}{1024}$ |
| 141 | $E_u$ | Eu | 5 | 0 | $\mathbb{Q}_{11,0}^{(h,E_u,5)}$ | Qh(11, Eu, 5, 0) | $-\frac{15\sqrt{221}C_1}{512} - \frac{3\sqrt{2926}C_{11}}{1024} - \frac{\sqrt{595}C_3}{512} + \frac{53\sqrt{102}C_5}{1024} - \frac{105\sqrt{10}C_7}{1024} - \frac{61\sqrt{114}C_9}{1024}$ |
| 142 | $E_u$ | Eu | 5 | 1 | $\mathbb{Q}_{11,1}^{(h,E_u,5)}$ | Qh(11, Eu, 5, 1) | $-\frac{15\sqrt{221}S_1}{512} + \frac{3\sqrt{2926}S_{11}}{1024} + \frac{\sqrt{595}S_3}{512} + \frac{53\sqrt{102}S_5}{1024} + \frac{105\sqrt{10}S_7}{1024} - \frac{61\sqrt{114}S_9}{1024}$ |
| 143 | $E_u$ | Eu | 6 | 0 | $\mathbb{Q}_{11,0}^{(h,E_u,6)}$ | Qh(11, Eu, 6, 0) | $-\frac{21\sqrt{130}C_1}{512} - \frac{\sqrt{124355}C_{11}}{512} + \frac{57\sqrt{14}C_3}{512} - \frac{41\sqrt{15}C_5}{512} + \frac{17\sqrt{17}C_7}{512} + \frac{\sqrt{4845}C_9}{512}$ |
| 144 | $E_u$ | Eu | 6 | 1 | $\mathbb{Q}_{11,1}^{(h,E_u,6)}$ | Qh(11, Eu, 6, 1) | $-\frac{21\sqrt{130}S_1}{512} + \frac{\sqrt{124355}S_{11}}{512} - \frac{57\sqrt{14}S_3}{512} - \frac{41\sqrt{15}S_5}{512} - \frac{17\sqrt{17}S_7}{512} + \frac{\sqrt{4845}S_9}{512}$ |