

MPG No. 8.1.24 *mmm* [Type I, orthorhombic] [Q tensor]

* Rank 0 tensor.

$$[Q]$$

$$Q = Q_s^{(1)}$$

* Rank 1 tensor. * Rank 2 tensor (s).

$$\begin{bmatrix} Q_{xx} & 0 & 0 \\ 0 & Q_{yy} & 0 \\ 0 & 0 & Q_{zz} \end{bmatrix}$$

$$Q_{xx} = -Q_{du}^{(1)} + Q_{dv}^{(1)} + Q_s^{(1)}$$

$$Q_{yy} = -Q_{du}^{(1)} - Q_{dv}^{(1)} + Q_s^{(1)}$$

$$Q_{zz} = 2Q_{du}^{(1)} + Q_s^{(1)}$$

* Rank 2 tensor (a). * Rank 3 tensor (s). * Rank 3 tensor (a). * Rank 4 tensor (sss).

$$\begin{bmatrix} Q_{xxxx} & Q_{xxyy} & Q_{xxzz} & 0 & 0 & 0 \\ Q_{xxyy} & Q_{yyyy} & Q_{yyzz} & 0 & 0 & 0 \\ Q_{xxzz} & Q_{yyzz} & Q_{zzzz} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{yzyz} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{zxzx} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{xyxy} \end{bmatrix}$$

$$Q_{xxxx} = -2Q_{du}^{(1)} - 4Q_{du}^{(2)} + 2Q_{dv}^{(1)} + 4Q_{dv}^{(2)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)}$$

$$Q_{xxyy} = -2Q_{du}^{(1)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{xxzz} = Q_{du}^{(1)} + Q_{dv}^{(1)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{yyyy} = -2Q_{du}^{(1)} - 4Q_{du}^{(2)} - 2Q_{dv}^{(1)} - 4Q_{dv}^{(2)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)}$$

$$Q_{yyzz} = Q_{du}^{(1)} - Q_{dv}^{(1)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{zzzz} = 4Q_{du}^{(1)} + 8Q_{du}^{(2)} + 2Q_{gu}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)}$$

$$Q_{yzyz} = Q_{du}^{(2)} - Q_{dv}^{(1)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(2)}$$

$$Q_{zxzx} = Q_{du}^{(2)} + Q_{dv}^{(2)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(2)}$$

$$Q_{xyxy} = -2Q_{du}^{(2)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(2)}$$

* Rank 4 tensor (ssa).

$$\begin{bmatrix} 0 & Q_{xxyy} & Q_{xxzz} & 0 & 0 & 0 \\ -Q_{xxyy} & 0 & Q_{yyzz} & 0 & 0 & 0 \\ -Q_{xxzz} & -Q_{yyzz} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Q_{xxyy} = 4G_{f3}^{(1)} - 2Q_{dv}^{(3)}$$

$$Q_{xxzz} = -4G_{f3}^{(1)} + 3Q_{du}^{(3)} - Q_{dv}^{(3)}$$

$$Q_{yyzz} = 4G_{f3}^{(1)} + 3Q_{du}^{(3)} + Q_{dv}^{(3)}$$

* Rank 4 tensor (aas).

$$\begin{bmatrix} Q_{yzyz} & 0 & 0 \\ 0 & Q_{zxzx} & 0 \\ 0 & 0 & Q_{xyxy} \end{bmatrix}$$

$$\begin{aligned}
Q_{yzyz} &= -2Q_{du}^{(4)} + 2Q_{dv}^{(4)} + Q_s^{(3)} \\
Q_{zxzx} &= -2Q_{du}^{(4)} - 2Q_{dv}^{(4)} + Q_s^{(3)} \\
Q_{xyxy} &= 4Q_{du}^{(4)} + Q_s^{(3)}
\end{aligned}$$

* Rank 4 tensor (aaa). * Rank 4 tensor (sa).

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ Q_{yzyz} & 0 & 0 \\ 0 & Q_{zxzx} & 0 \\ 0 & 0 & Q_{xyxy} \end{bmatrix}$$

$$\begin{aligned}
Q_{yzyz} &= G_{f3}^{(2)} - 3Q_{du}^{(5)} - Q_{dv}^{(5)} \\
Q_{zxzx} &= G_{f3}^{(2)} + 3Q_{du}^{(5)} - Q_{dv}^{(5)} \\
Q_{xyxy} &= G_{f3}^{(2)} + 2Q_{dv}^{(5)}
\end{aligned}$$

* Rank 4 tensor (as).

$$\begin{bmatrix} 0 & 0 & 0 & Q_{yzyz} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{zxzx} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{xyxy} \end{bmatrix}$$

$$\begin{aligned}
Q_{yzyz} &= G_{f3}^{(3)} - 3Q_{du}^{(6)} - Q_{dv}^{(6)} \\
Q_{zxzx} &= G_{f3}^{(3)} + 3Q_{du}^{(6)} - Q_{dv}^{(6)} \\
Q_{xyxy} &= G_{f3}^{(3)} + 2Q_{dv}^{(6)}
\end{aligned}$$

* Rank 4 tensor (s).

$$\begin{bmatrix} Q_{xxxx} & Q_{xxyy} & Q_{xxzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{uyxx} & Q_{uyyy} & Q_{yyzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{zzxx} & Q_{zzyy} & Q_{zzzz} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{yzyz} & 0 & 0 & Q_{yzyy} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{zxzx} & 0 & 0 & Q_{zxzx} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{xyxy} & 0 & 0 & Q_{xyyx} \end{bmatrix}$$

$$\begin{aligned}
Q_{xxxx} &= -2Q_{du}^{(1)} - 4Q_{du}^{(2)} + 2Q_{dv}^{(1)} + 4Q_{dv}^{(2)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)} \\
Q_{xxyy} &= 4G_{f3}^{(1)} - 2Q_{du}^{(1)} - 2Q_{dv}^{(1)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)} \\
Q_{xxzz} &= -4G_{f3}^{(1)} + Q_{du}^{(1)} + 3Q_{du}^{(3)} + Q_{dv}^{(1)} - Q_{dv}^{(3)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)} \\
Q_{yyxx} &= -4G_{f3}^{(1)} - 2Q_{du}^{(1)} + 2Q_{dv}^{(3)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)} \\
Q_{yyyy} &= -2Q_{du}^{(1)} - 4Q_{du}^{(2)} - 2Q_{dv}^{(1)} - 4Q_{dv}^{(2)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)} \\
Q_{yyzz} &= 4G_{f3}^{(1)} + Q_{du}^{(1)} + 3Q_{du}^{(3)} - Q_{dv}^{(1)} + Q_{dv}^{(3)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)} \\
Q_{zzxx} &= 4G_{f3}^{(1)} + Q_{du}^{(1)} - 3Q_{du}^{(3)} + Q_{dv}^{(1)} + Q_{dv}^{(3)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)} \\
Q_{zzyy} &= -4G_{f3}^{(1)} + Q_{du}^{(1)} - 3Q_{du}^{(3)} - Q_{dv}^{(1)} - Q_{dv}^{(3)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)} \\
Q_{zzzz} &= 4Q_{du}^{(1)} + 8Q_{du}^{(2)} + 2Q_{gu}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)} \\
Q_{yzyz} &= G_{f3}^{(2)} + Q_{du}^{(2)} - 3Q_{du}^{(5)} - Q_{dv}^{(2)} - Q_{dv}^{(5)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(2)} \\
Q_{yzyy} &= -G_{f3}^{(2)} + Q_{du}^{(2)} + 3Q_{du}^{(5)} - Q_{dv}^{(2)} + Q_{dv}^{(5)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(2)} \\
Q_{zxzx} &= G_{f3}^{(2)} + Q_{du}^{(2)} + 3Q_{du}^{(5)} + Q_{dv}^{(2)} - Q_{dv}^{(5)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(2)} \\
Q_{zxzx} &= -G_{f3}^{(2)} + Q_{du}^{(2)} - 3Q_{du}^{(5)} + Q_{dv}^{(2)} + Q_{dv}^{(5)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(2)} \\
Q_{xyxy} &= G_{f3}^{(2)} - 2Q_{du}^{(2)} + 2Q_{dv}^{(5)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(2)} \\
Q_{xyyx} &= -G_{f3}^{(2)} - 2Q_{du}^{(2)} - 2Q_{dv}^{(5)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(2)}
\end{aligned}$$

* Rank 4 tensor (a).

$$\begin{bmatrix} 0 & 0 & 0 & Q_{yzyz} & 0 & 0 & Q_{yzyy} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{zxzx} & 0 & 0 & Q_{zxzx} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{xyxy} & 0 & 0 & Q_{xyyx} \end{bmatrix}$$

$$Q_{yzyz} = G_{f3}^{(3)} - 2Q_{du}^{(4)} - 3Q_{du}^{(6)} + 2Q_{dv}^{(4)} - Q_{dv}^{(6)} + Q_s^{(3)}$$

$$Q_{yzyy} = G_{f3}^{(3)} + 2Q_{du}^{(4)} - 3Q_{du}^{(6)} - 2Q_{dv}^{(4)} - Q_{dv}^{(6)} - Q_s^{(3)}$$

$$Q_{zxzx} = G_{f3}^{(3)} - 2Q_{du}^{(4)} + 3Q_{du}^{(6)} - 2Q_{dv}^{(4)} - Q_{dv}^{(6)} + Q_s^{(3)}$$

$$Q_{zxzx} = G_{f3}^{(3)} + 2Q_{du}^{(4)} + 3Q_{du}^{(6)} + 2Q_{dv}^{(4)} - Q_{dv}^{(6)} - Q_s^{(3)}$$

$$Q_{xyxy} = G_{f3}^{(3)} + 4Q_{du}^{(4)} + 2Q_{dv}^{(6)} + Q_s^{(3)}$$

$$Q_{xyyx} = G_{f3}^{(3)} - 4Q_{du}^{(4)} + 2Q_{dv}^{(6)} - Q_s^{(3)}$$

* Rank 4 tensor (t).

$$\begin{bmatrix} Q_{xxxx} & 0 & 0 \\ 0 & Q_{yyyy} & 0 \\ 0 & 0 & Q_{zzzz} \\ 0 & 0 & Q_{yyzz} \\ Q_{zxzx} & 0 & 0 \\ 0 & Q_{xxyy} & 0 \\ 0 & Q_{yzzy} & 0 \\ 0 & 0 & Q_{zxzx} \\ Q_{xxyy} + Q_{yyzz} - Q_{yzzy} - Q_{zxzx} + Q_{zzxx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Q_{xxxx} = -2Q_{du}^{(1)} - 4Q_{du}^{(2)} + 2Q_{dv}^{(1)} + 4Q_{dv}^{(2)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} + 2Q_g^{(1)} + 3Q_s^{(1)}$$

$$Q_{yyyy} = -2Q_{du}^{(1)} - 4Q_{du}^{(2)} - 2Q_{dv}^{(1)} - 4Q_{dv}^{(2)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} + 2Q_g^{(1)} + 3Q_s^{(1)}$$

$$Q_{zzzz} = 4Q_{du}^{(1)} + 8Q_{du}^{(2)} + 2Q_{gu}^{(1)} + 2Q_g^{(1)} + 3Q_s^{(1)}$$

$$Q_{yyzz} = Q_{du}^{(1)} + 3Q_{du}^{(3)} - Q_{dv}^{(1)} + Q_{dv}^{(3)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{zzxx} = Q_{du}^{(1)} - 3Q_{du}^{(3)} + Q_{dv}^{(1)} + Q_{dv}^{(3)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{xxyy} = -2Q_{du}^{(1)} - 2Q_{dv}^{(3)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{yzzy} = Q_{du}^{(2)} + 3Q_{du}^{(5)} - Q_{dv}^{(2)} + Q_{dv}^{(5)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{zxzx} = Q_{du}^{(2)} - 3Q_{du}^{(5)} + Q_{dv}^{(2)} + Q_{dv}^{(5)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$