

MPG No. 2.1.3  $\bar{1}$  [ Type I, triclinic ] [Q tensor]

\* Rank 0 tensor.

$$[Q]$$

$$Q = Q_s^{(1)}$$

\* Rank 1 tensor. \* Rank 2 tensor (s).

$$\begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xz} \\ Q_{xy} & Q_{yy} & Q_{yz} \\ Q_{xz} & Q_{yz} & Q_{zz} \end{bmatrix}$$

$$Q_{xx} = -Q_{du}^{(1)} + Q_{dv}^{(1)} + Q_s^{(1)}$$

$$Q_{xy} = Q_{dxy}^{(1)}$$

$$Q_{xz} = Q_{dxz}^{(1)}$$

$$Q_{yy} = -Q_{du}^{(1)} - Q_{dv}^{(1)} + Q_s^{(1)}$$

$$Q_{yz} = Q_{dyz}^{(1)}$$

$$Q_{zz} = 2Q_{du}^{(1)} + Q_s^{(1)}$$

\* Rank 2 tensor (a).

$$\begin{bmatrix} 0 & Q_{xy} & Q_{xz} \\ -Q_{xy} & 0 & Q_{yz} \\ -Q_{xz} & -Q_{yz} & 0 \end{bmatrix}$$

$$Q_{xy} = G_{pz}^{(1)}$$

$$Q_{xz} = -G_{py}^{(1)}$$

$$Q_{yz} = G_{px}^{(1)}$$

\* Rank 3 tensor (s). \* Rank 3 tensor (a). \* Rank 4 tensor (sss).

$$\begin{bmatrix} Q_{xxxx} & Q_{xxyy} & Q_{xxzz} & Q_{xxyz} & Q_{xxzx} & Q_{xxxz} \\ Q_{xxyy} & Q_{yyyy} & Q_{yyzz} & Q_{yyyz} & Q_{yyzx} & Q_{yyxy} \\ Q_{xxzz} & Q_{yyzz} & Q_{zzzz} & Q_{zzyz} & Q_{zzzx} & Q_{zzxy} \\ Q_{xxyz} & Q_{yyyz} & Q_{zzyz} & Q_{yzyz} & Q_{yzzx} & Q_{yzxy} \\ Q_{xxzx} & Q_{yyzx} & Q_{zzzx} & Q_{yzzx} & Q_{zxzx} & Q_{zxxy} \\ Q_{xxxz} & Q_{yyxy} & Q_{zzxy} & Q_{yzxy} & Q_{zxxy} & Q_{xyxy} \end{bmatrix}$$

$$Q_{xxxx} = -2Q_{du}^{(1)} - 4Q_{du}^{(2)} + 2Q_{dv}^{(1)} + 4Q_{dv}^{(2)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)}$$

$$Q_{xxyy} = -2Q_{du}^{(1)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{xxzz} = Q_{du}^{(1)} + Q_{dv}^{(1)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{xxyz} = Q_{dyz}^{(1)} + 2Q_{gby}^{(1)}$$

$$Q_{xxzx} = Q_{dxz}^{(1)} + 2Q_{dxz}^{(2)} - Q_{gay}^{(1)} - Q_{gby}^{(1)}$$

$$Q_{xxxz} = Q_{dxy}^{(1)} + 2Q_{dxy}^{(2)} + Q_{gaz}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{yyyy} = -2Q_{du}^{(1)} - 4Q_{du}^{(2)} - 2Q_{dv}^{(1)} - 4Q_{dv}^{(2)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)}$$

$$Q_{yyzz} = Q_{du}^{(1)} - Q_{dv}^{(1)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{yyyz} = Q_{dyz}^{(1)} + 2Q_{dyz}^{(2)} + Q_{gax}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{yyzx} = Q_{dxz}^{(1)} + 2Q_{gby}^{(1)}$$

$$Q_{yyxy} = Q_{dxy}^{(1)} + 2Q_{dxy}^{(2)} - Q_{gaz}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{zzzz} = 4Q_{du}^{(1)} + 8Q_{du}^{(2)} + 2Q_{gu}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)}$$

$$Q_{zzyz} = Q_{dyz}^{(1)} + 2Q_{dyz}^{(2)} - Q_{gax}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{zzzx} = Q_{dxz}^{(1)} + 2Q_{dxz}^{(2)} + Q_{gay}^{(1)} - Q_{gby}^{(1)}$$

$$Q_{zzxy} = Q_{dxy}^{(1)} + 2Q_{gby}^{(1)}$$

$$Q_{yzyz} = Q_{du}^{(2)} - Q_{dv}^{(2)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(2)}$$

$$Q_{yzzx} = Q_{dxy}^{(2)} + 2Q_{gbz}^{(1)}$$

$$Q_{yxzy} = Q_{dxz}^{(2)} + 2Q_{gby}^{(1)}$$

$$Q_{zxxz} = Q_{du}^{(2)} + Q_{dv}^{(2)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(2)}$$

$$Q_{zxxz} = Q_{dyz}^{(2)} + 2Q_{gbx}^{(1)}$$

$$Q_{xyxy} = -2Q_{du}^{(2)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(2)}$$

\* Rank 4 tensor (ssa).

$$\begin{bmatrix} 0 & Q_{xxyy} & Q_{xxzz} & Q_{xxyz} & Q_{xxzx} & Q_{xxxz} \\ -Q_{xxyy} & 0 & Q_{yyzz} & Q_{yyyz} & Q_{yyzx} & Q_{yyxy} \\ -Q_{xxzz} & -Q_{yyzz} & 0 & Q_{zzyz} & Q_{zzzx} & Q_{zzxy} \\ -Q_{xxyz} & -Q_{yyyz} & -Q_{zzyz} & 0 & Q_{yzzx} & Q_{yzxy} \\ -Q_{xxzx} & -Q_{yyzx} & -Q_{zzzx} & -Q_{yzzx} & 0 & Q_{zxxy} \\ -Q_{xxxz} & -Q_{yyxy} & -Q_{zzxy} & -Q_{yzxy} & -Q_{zxxy} & 0 \end{bmatrix}$$

$$\begin{aligned}
Q_{xxyy} &= 4G_{f3}^{(1)} - 2Q_{dv}^{(3)} \\
Q_{xxzz} &= -4G_{f3}^{(1)} + 3Q_{du}^{(3)} - Q_{dv}^{(3)} \\
Q_{xxyz} &= -4G_{fbx}^{(1)} + Q_{dyz}^{(3)} \\
Q_{xxzx} &= 2G_{fay}^{(1)} + 2G_{fbz}^{(1)} - 2G_{py}^{(1)} + Q_{dxz}^{(3)} \\
Q_{xxxz} &= -2G_{faz}^{(1)} + 2G_{fbz}^{(1)} + 2G_{pz}^{(1)} + Q_{dxy}^{(3)} \\
Q_{yyzz} &= 4G_{f3}^{(1)} + 3Q_{du}^{(3)} + Q_{dv}^{(3)} \\
Q_{yyyy} &= -2G_{fax}^{(1)} + 2G_{fbx}^{(1)} + 2G_{px}^{(1)} + Q_{dyz}^{(3)} \\
Q_{yyzx} &= -4G_{fbz}^{(1)} + Q_{dxz}^{(3)} \\
Q_{yyxy} &= 2G_{faz}^{(1)} + 2G_{fbz}^{(1)} - 2G_{pz}^{(1)} + Q_{dxy}^{(3)} \\
Q_{zzyz} &= 2G_{fax}^{(1)} + 2G_{fbx}^{(1)} - 2G_{px}^{(1)} + Q_{dyz}^{(3)} \\
Q_{zzzx} &= -2G_{fay}^{(1)} + 2G_{fbz}^{(1)} + 2G_{py}^{(1)} + Q_{dxz}^{(3)} \\
Q_{zzxy} &= -4G_{fbz}^{(1)} + Q_{dxy}^{(3)} \\
Q_{yzzx} &= -4G_{faz}^{(1)} - G_{pz}^{(1)} \\
Q_{yzxy} &= 4G_{fay}^{(1)} + G_{py}^{(1)} \\
Q_{zxxz} &= -4G_{fax}^{(1)} - G_{px}^{(1)}
\end{aligned}$$

\* Rank 4 tensor (aas).

$$\begin{bmatrix} Q_{yzyz} & Q_{yzzx} & Q_{yzxy} \\ Q_{yzzx} & Q_{zxxz} & Q_{zxxy} \\ Q_{yzxy} & Q_{zxxy} & Q_{xyxy} \end{bmatrix}$$

$$\begin{aligned}
Q_{yzyz} &= -2Q_{du}^{(4)} + 2Q_{dv}^{(4)} + Q_s^{(3)} \\
Q_{yzzx} &= 2Q_{dxy}^{(4)} \\
Q_{yzxy} &= 2Q_{dxz}^{(4)} \\
Q_{zxxz} &= -2Q_{du}^{(4)} - 2Q_{dv}^{(4)} + Q_s^{(3)} \\
Q_{zxxy} &= 2Q_{dyz}^{(4)} \\
Q_{xyxy} &= 4Q_{du}^{(4)} + Q_s^{(3)}
\end{aligned}$$

\* Rank 4 tensor (aaa).

$$\begin{bmatrix} 0 & Q_{yzzx} & Q_{yzxy} \\ -Q_{yzzx} & 0 & Q_{zxxy} \\ -Q_{yzxy} & -Q_{zxxy} & 0 \end{bmatrix}$$

$$\begin{aligned}
Q_{yzzx} &= -G_{pz}^{(2)} \\
Q_{yzxy} &= G_{py}^{(2)} \\
Q_{zxxy} &= -G_{px}^{(2)}
\end{aligned}$$

\* Rank 4 tensor (sa).

$$\begin{bmatrix} Q_{xxyz} & Q_{xxzx} & Q_{xxxz} \\ Q_{yyyz} & Q_{yyzx} & Q_{yyxy} \\ Q_{zzyz} & Q_{zzzx} & Q_{zzxy} \\ Q_{yzyz} & Q_{yzzx} & Q_{yzxy} \\ Q_{zxyz} & Q_{zxzx} & Q_{zxxy} \\ Q_{xyyz} & Q_{xyzx} & Q_{xyxy} \end{bmatrix}$$

$$\begin{aligned}
Q_{xxyz} &= 2G_{fax}^{(2)} + G_{px}^{(3)} + 2G_{px}^{(4)} \\
Q_{xxzx} &= -G_{fay}^{(2)} - G_{fbz}^{(2)} + G_{py}^{(3)} + 2Q_{dxz}^{(5)} \\
Q_{xxxz} &= -G_{faz}^{(2)} + G_{fbz}^{(2)} + G_{pz}^{(3)} - 2Q_{dxy}^{(5)} \\
Q_{yyyz} &= -G_{fax}^{(2)} + G_{fbx}^{(2)} + G_{px}^{(3)} - 2Q_{dyz}^{(5)} \\
Q_{yyzx} &= 2G_{fay}^{(2)} + G_{py}^{(3)} + 2G_{py}^{(4)} \\
Q_{yyxy} &= -G_{faz}^{(2)} - G_{fbz}^{(2)} + G_{pz}^{(3)} + 2Q_{dxy}^{(5)} \\
Q_{zzyz} &= -G_{fax}^{(2)} - G_{fbx}^{(2)} + G_{px}^{(3)} + 2Q_{dyz}^{(5)} \\
Q_{zzzx} &= -G_{fay}^{(2)} + G_{fbz}^{(2)} + G_{py}^{(3)} - 2Q_{dxz}^{(5)} \\
Q_{zzxy} &= 2G_{faz}^{(2)} + G_{pz}^{(3)} + 2G_{pz}^{(4)} \\
Q_{yzyz} &= G_{f3}^{(2)} - 3Q_{du}^{(5)} - Q_{dv}^{(5)} \\
Q_{yzzx} &= -G_{faz}^{(2)} - G_{fbz}^{(2)} + G_{pz}^{(4)} - Q_{dxy}^{(5)} \\
Q_{yzxy} &= -G_{fay}^{(2)} + G_{fbz}^{(2)} + G_{py}^{(4)} + Q_{dxz}^{(5)} \\
Q_{zxyz} &= -G_{faz}^{(2)} + G_{fbz}^{(2)} + G_{pz}^{(4)} + Q_{dxy}^{(5)} \\
Q_{zxzx} &= G_{f3}^{(2)} + 3Q_{du}^{(5)} - Q_{dv}^{(5)} \\
Q_{zxxz} &= -G_{fax}^{(2)} - G_{fbx}^{(2)} + G_{px}^{(4)} - Q_{dyz}^{(5)} \\
Q_{xyyz} &= -G_{fay}^{(2)} - G_{fbz}^{(2)} + G_{py}^{(4)} - Q_{dxz}^{(5)} \\
Q_{xyzx} &= -G_{fax}^{(2)} + G_{fbx}^{(2)} + G_{px}^{(4)} + Q_{dyz}^{(5)} \\
Q_{xyxy} &= G_{f3}^{(2)} + 2Q_{dv}^{(5)}
\end{aligned}$$

\* Rank 4 tensor (as).

$$\begin{bmatrix}
Q_{yzxx} & Q_{yzyy} & Q_{yzzz} & Q_{yzyz} & Q_{yzzx} & Q_{yzxy} \\
Q_{zxxx} & Q_{zxyy} & Q_{zxzz} & Q_{zxzy} & Q_{zxzx} & Q_{zxxy} \\
Q_{xyxx} & Q_{xyyy} & Q_{xyzz} & Q_{xyyz} & Q_{xyzx} & Q_{xyxy}
\end{bmatrix}$$

$$\begin{aligned}
Q_{yzxx} &= 2G_{fax}^{(3)} + G_{px}^{(5)} + 2G_{px}^{(6)} \\
Q_{yzyy} &= -G_{fax}^{(3)} + G_{fbx}^{(3)} + G_{px}^{(5)} - 2Q_{dyz}^{(6)} \\
Q_{yzzz} &= -G_{fax}^{(3)} - G_{fbx}^{(3)} + G_{px}^{(5)} + 2Q_{dyz}^{(6)} \\
Q_{yzyz} &= G_{f3}^{(3)} - 3Q_{du}^{(6)} - Q_{dv}^{(6)} \\
Q_{yzzx} &= -G_{faz}^{(3)} + G_{fbz}^{(3)} + G_{pz}^{(6)} + Q_{dxy}^{(6)} \\
Q_{yzxy} &= -G_{fay}^{(3)} - G_{fbz}^{(3)} + G_{py}^{(6)} - Q_{dxz}^{(6)} \\
Q_{zxzx} &= -G_{fay}^{(3)} - G_{fbz}^{(3)} + G_{py}^{(6)} + 2Q_{dxz}^{(6)} \\
Q_{zxyy} &= 2G_{fay}^{(3)} + G_{py}^{(5)} + 2G_{py}^{(6)} \\
Q_{zxzz} &= -G_{fay}^{(3)} + G_{fbz}^{(3)} + G_{py}^{(5)} - 2Q_{dxz}^{(6)} \\
Q_{zxyz} &= -G_{faz}^{(3)} - G_{fbz}^{(3)} + G_{pz}^{(6)} - Q_{dxy}^{(6)} \\
Q_{zxzx} &= G_{f3}^{(3)} + 3Q_{du}^{(6)} - Q_{dv}^{(6)} \\
Q_{zxxz} &= -G_{fax}^{(3)} + G_{fbx}^{(3)} + G_{px}^{(6)} + Q_{dyz}^{(6)} \\
Q_{xyxx} &= -G_{faz}^{(3)} + G_{fbz}^{(3)} + G_{pz}^{(5)} - 2Q_{dxy}^{(6)} \\
Q_{xyyy} &= -G_{faz}^{(3)} - G_{fbz}^{(3)} + G_{pz}^{(5)} + 2Q_{dxy}^{(6)} \\
Q_{xyzz} &= 2G_{faz}^{(3)} + G_{pz}^{(5)} + 2G_{pz}^{(6)} \\
Q_{xyyz} &= -G_{fay}^{(3)} + G_{fbz}^{(3)} + G_{py}^{(6)} + Q_{dxz}^{(6)} \\
Q_{xyzx} &= -G_{fax}^{(3)} - G_{fbx}^{(3)} + G_{px}^{(6)} - Q_{dyz}^{(6)} \\
Q_{xyxy} &= G_{f3}^{(3)} + 2Q_{dv}^{(6)}
\end{aligned}$$

\* Rank 4 tensor (s).

$$\begin{bmatrix} Q_{xxxx} & Q_{xxyy} & Q_{xxzz} & Q_{xxyz} & Q_{xxzx} & Q_{xxxy} & Q_{xxzy} & Q_{xxxz} & Q_{xxyx} \\ Q_{yyxx} & Q_{yyyy} & Q_{yyzz} & Q_{yyyz} & Q_{yyzx} & Q_{yyxy} & Q_{yyzy} & Q_{yyxz} & Q_{yyyx} \\ Q_{zzxx} & Q_{zzyy} & Q_{zzzz} & Q_{zzyz} & Q_{zzzx} & Q_{zzxy} & Q_{zzzy} & Q_{zzxz} & Q_{zzyx} \\ Q_{yzxx} & Q_{yzyy} & Q_{yzzz} & Q_{yzyz} & Q_{yzzx} & Q_{yzxy} & Q_{yzzy} & Q_{yzxz} & Q_{zyyx} \\ Q_{zxxx} & Q_{zxyy} & Q_{zxzz} & Q_{zxyz} & Q_{zxzx} & Q_{zxxy} & Q_{zxzy} & Q_{zxxz} & Q_{zxyx} \\ Q_{xyxx} & Q_{xyyy} & Q_{xyzz} & Q_{xyyz} & Q_{xyzx} & Q_{xyxy} & Q_{xyzy} & Q_{xyxz} & Q_{xyyx} \end{bmatrix}$$

$$Q_{xxxx} = -2Q_{du}^{(1)} - 4Q_{du}^{(2)} + 2Q_{dv}^{(1)} + 4Q_{dv}^{(2)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)}$$

$$Q_{xxyy} = 4G_{f3}^{(1)} - 2Q_{du}^{(1)} - 2Q_{dv}^{(3)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{xxzz} = -4G_{f3}^{(1)} + Q_{du}^{(1)} + 3Q_{du}^{(3)} + Q_{dv}^{(1)} - Q_{dv}^{(3)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{xxyz} = 2G_{fax}^{(2)} - 4G_{fbx}^{(1)} + G_{px}^{(3)} + 2G_{px}^{(4)} + Q_{dyz}^{(1)} + Q_{dyz}^{(3)} + 2Q_{gbx}^{(1)}$$

$$Q_{xxzx} = 2G_{fay}^{(1)} - G_{fay}^{(2)} + 2G_{fby}^{(1)} - G_{fby}^{(2)} - 2G_{py}^{(1)} + G_{py}^{(3)} + Q_{dxz}^{(1)} + 2Q_{dxz}^{(2)} + Q_{dxz}^{(3)} + 2Q_{dxz}^{(5)} - Q_{gaz}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{xxxy} = -2G_{faz}^{(1)} - G_{faz}^{(2)} + 2G_{fbz}^{(1)} + G_{fbz}^{(2)} + 2G_{pz}^{(1)} + G_{pz}^{(3)} + Q_{dxy}^{(1)} + 2Q_{dxy}^{(2)} + Q_{dxy}^{(3)} - 2Q_{dxy}^{(5)} + Q_{gaz}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{xxzy} = -2G_{fax}^{(2)} - 4G_{fbx}^{(1)} - G_{px}^{(3)} - 2G_{px}^{(4)} + Q_{dyz}^{(1)} + Q_{dyz}^{(3)} + 2Q_{gbx}^{(1)}$$

$$Q_{xxxz} = 2G_{fay}^{(1)} + G_{fay}^{(2)} + 2G_{fby}^{(1)} + G_{fby}^{(2)} - 2G_{py}^{(1)} - G_{py}^{(3)} + Q_{dxz}^{(1)} + 2Q_{dxz}^{(2)} + Q_{dxz}^{(3)} - 2Q_{dxz}^{(5)} - Q_{gaz}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{xxyx} = -2G_{faz}^{(1)} + G_{faz}^{(2)} + 2G_{fbz}^{(1)} - G_{fbz}^{(2)} + 2G_{pz}^{(1)} - G_{pz}^{(3)} + Q_{dxy}^{(1)} + 2Q_{dxy}^{(2)} + Q_{dxy}^{(3)} + 2Q_{dxy}^{(5)} + Q_{gaz}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{yyxx} = -4G_{f3}^{(1)} - 2Q_{du}^{(1)} + 2Q_{dv}^{(3)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{yyyy} = -2Q_{du}^{(1)} - 4Q_{du}^{(2)} - 2Q_{dv}^{(1)} - 4Q_{dv}^{(2)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)}$$

$$Q_{yyzz} = 4G_{f3}^{(1)} + Q_{du}^{(1)} + 3Q_{du}^{(3)} - Q_{dv}^{(1)} + Q_{dv}^{(3)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{yyyz} = -2G_{fax}^{(1)} - G_{fax}^{(2)} + 2G_{fbx}^{(1)} + G_{fbx}^{(2)} + 2G_{px}^{(1)} + G_{px}^{(3)} + Q_{dyz}^{(1)} + 2Q_{dyz}^{(2)} + Q_{dyz}^{(3)} - 2Q_{dyz}^{(5)} + Q_{gax}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{yyzx} = 2G_{fay}^{(2)} - 4G_{fbx}^{(1)} + G_{py}^{(3)} + 2G_{py}^{(4)} + Q_{dxz}^{(1)} + Q_{dxz}^{(3)} + 2Q_{gbx}^{(1)}$$

$$Q_{yyxy} = 2G_{faz}^{(1)} - G_{faz}^{(2)} + 2G_{fbz}^{(1)} - G_{fbz}^{(2)} - 2G_{pz}^{(1)} + G_{pz}^{(3)} + Q_{dxy}^{(1)} + 2Q_{dxy}^{(2)} + Q_{dxy}^{(3)} + 2Q_{dxy}^{(5)} - Q_{gaz}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{yyzy} = -2G_{fax}^{(1)} + G_{fax}^{(2)} + 2G_{fbx}^{(1)} - G_{fbx}^{(2)} + 2G_{px}^{(1)} - G_{px}^{(3)} + Q_{dyz}^{(1)} + 2Q_{dyz}^{(2)} + Q_{dyz}^{(3)} + 2Q_{dyz}^{(5)} + Q_{gax}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{yyxz} = -2G_{fay}^{(2)} - 4G_{fbx}^{(1)} - G_{py}^{(3)} - 2G_{py}^{(4)} + Q_{dxz}^{(1)} + Q_{dxz}^{(3)} + 2Q_{gbx}^{(1)}$$

$$Q_{yyyx} = 2G_{faz}^{(1)} + G_{faz}^{(2)} + 2G_{fbx}^{(1)} + G_{fbx}^{(2)} - 2G_{pz}^{(1)} - G_{pz}^{(3)} + Q_{dxy}^{(1)} + 2Q_{dxy}^{(2)} + Q_{dxy}^{(3)} - 2Q_{dxy}^{(5)} - Q_{gaz}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{zzxx} = 4G_{f3}^{(1)} + Q_{du}^{(1)} - 3Q_{du}^{(3)} + Q_{dv}^{(1)} + Q_{dv}^{(3)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{zzyy} = -4G_{f3}^{(1)} + Q_{du}^{(1)} - 3Q_{du}^{(3)} - Q_{dv}^{(1)} - Q_{dv}^{(3)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{zzzz} = 4Q_{du}^{(1)} + 8Q_{du}^{(2)} + 2Q_{gu}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)}$$

$$Q_{zzyz} = 2G_{fax}^{(1)} - G_{fax}^{(2)} + 2G_{fbx}^{(1)} - G_{fbx}^{(2)} - 2G_{px}^{(1)} + G_{px}^{(3)} + Q_{dyz}^{(1)} + 2Q_{dyz}^{(2)} + Q_{dyz}^{(3)} + 2Q_{dyz}^{(5)} - Q_{gax}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{zzzx} = -2G_{fay}^{(1)} - G_{fay}^{(2)} + 2G_{fbx}^{(1)} + G_{fbx}^{(2)} + 2G_{py}^{(1)} + G_{py}^{(3)} + Q_{dxz}^{(1)} + 2Q_{dxz}^{(2)} + Q_{dxz}^{(3)} - 2Q_{dxz}^{(5)} + Q_{gaz}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{zzxy} = 2G_{fax}^{(2)} - 4G_{fbx}^{(1)} + G_{pz}^{(3)} + 2G_{pz}^{(4)} + Q_{dxy}^{(1)} + Q_{dxy}^{(3)} + 2Q_{gbx}^{(1)}$$

$$Q_{zzzy} = 2G_{faz}^{(1)} + G_{faz}^{(2)} + 2G_{fbx}^{(1)} + G_{fbx}^{(2)} - 2G_{px}^{(1)} - G_{px}^{(3)} + Q_{dyz}^{(1)} + 2Q_{dyz}^{(2)} + Q_{dyz}^{(3)} - 2Q_{dyz}^{(5)} - Q_{gax}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{zzxz} = -2G_{fay}^{(1)} + G_{fay}^{(2)} + 2G_{fbx}^{(1)} - G_{fbx}^{(2)} + 2G_{py}^{(1)} - G_{py}^{(3)} + Q_{dxz}^{(1)} + 2Q_{dxz}^{(2)} + Q_{dxz}^{(3)} + 2Q_{dxz}^{(5)} + Q_{gaz}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{zzyx} = -2G_{faz}^{(2)} - 4G_{fbx}^{(1)} - G_{pz}^{(3)} - 2G_{pz}^{(4)} + Q_{dxy}^{(1)} + Q_{dxy}^{(3)} + 2Q_{gbx}^{(1)}$$

$$Q_{yzxx} = 4G_{fbx}^{(1)} + Q_{dyz}^{(1)} - Q_{dyz}^{(3)} + 2Q_{gbx}^{(1)}$$

$$Q_{yzyy} = 2G_{fax}^{(1)} - 2G_{fbx}^{(1)} - 2G_{px}^{(1)} + Q_{dyz}^{(1)} + 2Q_{dyz}^{(2)} - Q_{dyz}^{(3)} + Q_{gax}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{yzzz} = -2G_{fax}^{(1)} - 2G_{fbx}^{(1)} + 2G_{px}^{(1)} + Q_{dyz}^{(1)} + 2Q_{dyz}^{(2)} - Q_{dyz}^{(3)} - Q_{gax}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{yzyz} = G_{f3}^{(2)} + Q_{du}^{(2)} - 3Q_{du}^{(5)} - Q_{dv}^{(2)} - Q_{dv}^{(5)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(2)}$$

$$Q_{yzzz} = -4G_{faz}^{(1)} - G_{faz}^{(2)} - G_{fbz}^{(1)} - G_{pbz}^{(1)} + G_{px}^{(4)} + Q_{dxy}^{(2)} - Q_{dxy}^{(5)} + 2Q_{gbx}^{(1)}$$

$$Q_{yzxy} = 4G_{fay}^{(1)} - G_{fay}^{(2)} + G_{fbx}^{(1)} + G_{py}^{(1)} + G_{py}^{(4)} + Q_{dxz}^{(2)} + Q_{dxz}^{(5)} + 2Q_{gbx}^{(1)}$$

$$Q_{yzyy} = -G_{f3}^{(2)} + Q_{du}^{(2)} + 3Q_{du}^{(5)} - Q_{dv}^{(2)} + Q_{dv}^{(5)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(2)}$$

$$Q_{yzxz} = -4G_{faz}^{(1)} + G_{faz}^{(2)} + G_{fbz}^{(1)} - G_{pbz}^{(1)} - G_{px}^{(4)} + Q_{dxy}^{(2)} + Q_{dxy}^{(5)} + 2Q_{gbx}^{(1)}$$

$$Q_{yzyx} = 4G_{fay}^{(1)} + G_{fay}^{(2)} - G_{fbx}^{(1)} + G_{py}^{(1)} - G_{py}^{(4)} + Q_{dxz}^{(2)} - Q_{dxz}^{(5)} + 2Q_{gbx}^{(1)}$$

$$Q_{zzxx} = -2G_{fay}^{(1)} - 2G_{fbx}^{(1)} + 2G_{py}^{(1)} + Q_{dxz}^{(1)} + 2Q_{dxz}^{(2)} - Q_{dxz}^{(3)} - Q_{gaz}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{zzyy} = 4G_{fbx}^{(1)} + Q_{dxz}^{(1)} - Q_{dxz}^{(3)} + 2Q_{gbx}^{(1)}$$

$$Q_{zzzz} = 2G_{fay}^{(1)} - 2G_{fbx}^{(1)} - 2G_{py}^{(1)} + Q_{dxz}^{(1)} + 2Q_{dxz}^{(2)} - Q_{dxz}^{(3)} + Q_{gaz}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{zzxy} = 4G_{faz}^{(1)} + G_{faz}^{(2)} - G_{fbz}^{(1)} + G_{pbz}^{(1)} + G_{px}^{(4)} + Q_{dxy}^{(2)} - Q_{dxy}^{(5)} + 2Q_{gbx}^{(1)}$$

$$Q_{zzxz} = 4G_{fay}^{(1)} + G_{fay}^{(2)} + G_{fbz}^{(1)} + G_{pbz}^{(1)} + G_{px}^{(4)} + Q_{dxy}^{(2)} + Q_{dxy}^{(5)} + 2Q_{gbx}^{(1)}$$

$$Q_{zzzx} = G_{f3}^{(2)} + Q_{du}^{(2)} + 3Q_{du}^{(5)} - Q_{dv}^{(2)} - Q_{dv}^{(5)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(2)}$$

$$Q_{zzxy} = -4G_{faz}^{(1)} - G_{faz}^{(2)} - G_{fbz}^{(1)} - G_{pbz}^{(1)} + G_{px}^{(4)} + Q_{dxy}^{(2)} - Q_{dxy}^{(5)} + 2Q_{gbx}^{(1)}$$

$$Q_{zzzy} = 4G_{faz}^{(1)} + G_{faz}^{(2)} - G_{fbz}^{(1)} + G_{pbz}^{(1)} + G_{px}^{(4)} + Q_{dxy}^{(2)} - Q_{dxy}^{(5)} + 2Q_{gbx}^{(1)}$$

$$Q_{zzzz} = -G_{f3}^{(2)} + Q_{du}^{(2)} - 3Q_{du}^{(5)} + Q_{dv}^{(2)} + Q_{dv}^{(5)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(2)}$$

\* Rank 4 tensor (a).

$$\begin{bmatrix} Q_{yzxx} & Q_{yzyy} & Q_{yzzz} & Q_{yzyz} & Q_{yzxx} & Q_{yzxy} & Q_{yzyy} & Q_{yzxz} & Q_{yzyx} \\ Q_{zxxx} & Q_{zxyy} & Q_{zxzz} & Q_{zxyz} & Q_{zxzz} & Q_{zxyy} & Q_{zxzy} & Q_{zxxz} & Q_{zxyx} \\ Q_{xyxx} & Q_{xyyy} & Q_{xyzz} & Q_{xyyz} & Q_{xyzx} & Q_{xyxy} & Q_{xyzy} & Q_{xyxz} & Q_{xyyx} \end{bmatrix}$$

$$\begin{aligned} Q_{yzxx} &= 2G_{fax}^{(3)} + G_{px}^{(5)} + 2G_{px}^{(6)} \\ Q_{yzyy} &= -G_{fax}^{(3)} + G_{fbx}^{(3)} + G_{px}^{(5)} - 2Q_{dyz}^{(6)} \\ Q_{yzzz} &= -G_{fax}^{(3)} - G_{fbx}^{(3)} + G_{px}^{(5)} + 2Q_{dyz}^{(6)} \\ Q_{yzyz} &= G_{f3}^{(3)} - 2Q_{du}^{(4)} - 3Q_{du}^{(6)} + 2Q_{dv}^{(4)} - Q_{dv}^{(6)} + Q_s^{(3)} \\ Q_{yzzx} &= -G_{faz}^{(3)} + G_{fbz}^{(3)} - G_{pz}^{(2)} + G_{pz}^{(6)} + 2Q_{dxy}^{(4)} + Q_{dxy}^{(6)} \\ Q_{yzxy} &= -G_{fay}^{(3)} - G_{fyb}^{(3)} + G_{py}^{(2)} + G_{py}^{(6)} + 2Q_{dxz}^{(4)} - Q_{dxz}^{(6)} \\ Q_{yzyy} &= G_{f3}^{(3)} + 2Q_{du}^{(4)} - 3Q_{du}^{(6)} - 2Q_{dv}^{(4)} - Q_{dv}^{(6)} - Q_s^{(3)} \\ Q_{yzxz} &= -G_{faz}^{(3)} + G_{fbz}^{(3)} + G_{pz}^{(2)} + G_{pz}^{(6)} - 2Q_{dxy}^{(4)} + Q_{dxy}^{(6)} \\ Q_{yzyx} &= -G_{fay}^{(3)} - G_{fyb}^{(3)} - G_{py}^{(2)} + G_{py}^{(6)} - 2Q_{dxz}^{(4)} - Q_{dxz}^{(6)} \\ Q_{zxxx} &= -G_{fay}^{(3)} - G_{fbx}^{(3)} + G_{py}^{(5)} + 2Q_{daz}^{(6)} \\ Q_{zxyy} &= 2G_{fay}^{(3)} + G_{py}^{(5)} + 2G_{py}^{(6)} \\ Q_{zxzz} &= -G_{fay}^{(3)} + G_{fbx}^{(3)} + G_{py}^{(5)} - 2Q_{daz}^{(6)} \\ Q_{zxyz} &= -G_{faz}^{(3)} - G_{fbz}^{(3)} + G_{pz}^{(2)} + G_{pz}^{(6)} + 2Q_{dxy}^{(4)} - Q_{dxy}^{(6)} \\ Q_{zzxx} &= G_{f3}^{(3)} - 2Q_{du}^{(4)} + 3Q_{du}^{(6)} - 2Q_{dv}^{(4)} - Q_{dv}^{(6)} + Q_s^{(3)} \\ Q_{zxxz} &= -G_{fax}^{(3)} + G_{fbx}^{(3)} - G_{px}^{(2)} + G_{px}^{(6)} + 2Q_{dyz}^{(4)} + Q_{dyz}^{(6)} \\ Q_{zxxz} &= -G_{faz}^{(3)} - G_{fbz}^{(3)} - G_{pz}^{(2)} + G_{pz}^{(6)} - 2Q_{dxy}^{(4)} - Q_{dxy}^{(6)} \\ Q_{zzxz} &= G_{f3}^{(3)} + 2Q_{du}^{(4)} + 3Q_{du}^{(6)} + 2Q_{dv}^{(4)} - Q_{dv}^{(6)} - Q_s^{(3)} \\ Q_{zxyx} &= -G_{fax}^{(3)} + G_{fbx}^{(3)} + G_{px}^{(2)} + G_{px}^{(6)} - 2Q_{dyz}^{(4)} + Q_{dyz}^{(6)} \\ Q_{xyxx} &= -G_{faz}^{(3)} + G_{fbz}^{(3)} + G_{pz}^{(5)} - 2Q_{dxy}^{(6)} \\ Q_{xyyy} &= -G_{faz}^{(3)} - G_{fbz}^{(3)} + G_{pz}^{(5)} + 2Q_{dxy}^{(6)} \\ Q_{xyzz} &= 2G_{faz}^{(3)} + G_{pz}^{(5)} + 2G_{pz}^{(6)} \\ Q_{xyyz} &= -G_{fay}^{(3)} + G_{fyb}^{(3)} - G_{py}^{(2)} + G_{py}^{(6)} + 2Q_{dxz}^{(4)} + Q_{dxz}^{(6)} \\ Q_{xyzx} &= -G_{fax}^{(3)} - G_{fbx}^{(3)} + G_{px}^{(2)} + G_{px}^{(6)} + 2Q_{dyz}^{(4)} - Q_{dyz}^{(6)} \\ Q_{xyxy} &= G_{f3}^{(3)} + 4Q_{du}^{(4)} + 2Q_{dv}^{(6)} + Q_s^{(3)} \\ Q_{xyzy} &= -G_{fay}^{(3)} + G_{fyb}^{(3)} + G_{py}^{(2)} + G_{py}^{(6)} - 2Q_{dxz}^{(4)} + Q_{dxz}^{(6)} \\ Q_{xyxz} &= -G_{fax}^{(3)} - G_{fbx}^{(3)} - G_{px}^{(2)} + G_{px}^{(6)} - 2Q_{dyz}^{(4)} - Q_{dyz}^{(6)} \\ Q_{xyyx} &= G_{f3}^{(3)} - 4Q_{du}^{(4)} + 2Q_{dv}^{(6)} - Q_s^{(3)} \end{aligned}$$

\* Rank 4 tensor (t).

$$\left[ \begin{array}{ccc} Q_{xxxx} & Q_{xxxy} & Q_{xxxx} \\ Q_{yyyy} & Q_{yyyy} & Q_{yyyz} \\ Q_{zzzz} & Q_{zzzz} & Q_{zzzz} \\ Q_{yyzz} & Q_{yyzz} & Q_{yyzz} \\ Q_{zzxx} & Q_{zzxy} & Q_{zzxx} \\ Q_{xxyx} & Q_{xxyy} & Q_{xxyz} \\ Q_{yzzx} & Q_{yzyy} & Q_{yzzz} \\ Q_{zxxx} & Q_{zxyy} & Q_{zxxz} \\ Q_{xxyy} + Q_{yyzz} - Q_{yzyy} - Q_{zzxx} + Q_{zzxz} & Q_{xxyy} + \frac{Q_{zzzz}}{2} - \frac{Q_{zzxx}}{2} - \frac{Q_{xxyx}}{2} + Q_{xxyy} - Q_{yyyz} + Q_{zzyy} & Q_{xxyy} - \frac{Q_{xxyx}}{2} + Q_{xxyy} - Q_{yyyz} + Q_{zzyy} \end{array} \right]$$

$$Q_{xxxx} = -2Q_{du}^{(1)} - 4Q_{du}^{(2)} + 2Q_{dv}^{(1)} + 4Q_{dv}^{(2)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} + 2Q_g^{(1)} + 3Q_s^{(1)}$$

$$Q_{xxyy} = Q_{dxy}^{(1)} + 2Q_{dxy}^{(2)} + Q_{dxy}^{(3)} - 2Q_{dxy}^{(5)} + Q_{gaz}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{xxxz} = Q_{dxz}^{(1)} + 2Q_{dxz}^{(2)} + Q_{dxz}^{(3)} - 2Q_{dxz}^{(5)} - Q_{gay}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{yyyx} = Q_{dxy}^{(1)} + 2Q_{dxy}^{(2)} + Q_{dxy}^{(3)} - 2Q_{dxy}^{(5)} - Q_{gaz}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{yyyy} = -2Q_{du}^{(1)} - 4Q_{du}^{(2)} - 2Q_{dv}^{(1)} - 4Q_{dv}^{(2)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} + 2Q_g^{(1)} + 3Q_s^{(1)}$$

$$Q_{yyyz} = Q_{dyz}^{(1)} + 2Q_{dyz}^{(2)} + Q_{dyz}^{(3)} - 2Q_{dyz}^{(5)} + Q_{gax}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{zzzx} = Q_{dxz}^{(1)} + 2Q_{dxz}^{(2)} + Q_{dxz}^{(3)} - 2Q_{dxz}^{(5)} + Q_{gay}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{zzzy} = Q_{dyz}^{(1)} + 2Q_{dyz}^{(2)} + Q_{dyz}^{(3)} - 2Q_{dyz}^{(5)} - Q_{gax}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{zzzz} = 4Q_{du}^{(1)} + 8Q_{du}^{(2)} + 2Q_{gu}^{(1)} + 2Q_g^{(1)} + 3Q_s^{(1)}$$

$$Q_{yyzx} = Q_{dxz}^{(1)} + Q_{dxz}^{(3)} + 2Q_{gbz}^{(1)}$$

$$Q_{yyzy} = Q_{dyz}^{(1)} + 2Q_{dyz}^{(2)} + Q_{dyz}^{(3)} + 2Q_{dyz}^{(5)} + Q_{gax}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{yyzz} = Q_{du}^{(1)} + 3Q_{du}^{(3)} - Q_{du}^{(5)} + Q_{dv}^{(1)} + Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{zzxx} = Q_{du}^{(1)} - 3Q_{du}^{(3)} + Q_{du}^{(5)} + Q_{dv}^{(1)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{zzxy} = Q_{dxy}^{(1)} + Q_{dxy}^{(3)} + 2Q_{gbz}^{(1)}$$

$$Q_{zzxz} = Q_{dxz}^{(1)} + 2Q_{dxz}^{(2)} + Q_{dxz}^{(3)} + 2Q_{dxz}^{(5)} + Q_{gay}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{xxxy} = Q_{dxy}^{(1)} + 2Q_{dxy}^{(2)} + Q_{dxy}^{(3)} + 2Q_{dxy}^{(5)} + Q_{gaz}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{xxyy} = -2Q_{du}^{(1)} - 2Q_{dv}^{(1)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{xxyz} = Q_{dyz}^{(1)} + Q_{dyz}^{(3)} + 2Q_{gbx}^{(1)}$$

$$Q_{yzzx} = Q_{dxy}^{(2)} - Q_{dxy}^{(5)} + 2Q_{gbz}^{(1)}$$

$$Q_{yzzz} = Q_{du}^{(2)} + 3Q_{du}^{(5)} - Q_{dv}^{(2)} + Q_{dv}^{(5)} - Q_{gu}^{(1)} + Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{yzzz} = Q_{dyz}^{(1)} + 2Q_{dyz}^{(2)} - Q_{dyz}^{(3)} - Q_{gax}^{(1)} - Q_{gbx}^{(1)}$$

$$Q_{zxxx} = Q_{dxz}^{(1)} + 2Q_{dxz}^{(2)} - Q_{dxz}^{(3)} - Q_{gax}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{zxxz} = Q_{dyz}^{(2)} - Q_{dyz}^{(5)} + 2Q_{gbx}^{(1)}$$

$$Q_{zxxz} = Q_{du}^{(2)} - 3Q_{du}^{(5)} + Q_{dv}^{(2)} + Q_{dv}^{(5)} - Q_{gu}^{(1)} - Q_{gv}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{xyyy} = Q_{dxy}^{(1)} + 2Q_{dxy}^{(2)} - Q_{dxy}^{(3)} - Q_{gaz}^{(1)} - Q_{gbz}^{(1)}$$

$$Q_{xyyz} = Q_{dxz}^{(2)} - Q_{dxz}^{(5)} + 2Q_{gbz}^{(1)}$$