

# Model for “C4v1”

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## General Condition

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- Basis type: 1gs
- SAMB selection:
  - Type: [Q, G]
  - Rank: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]
  - Irrep.: [A<sub>1</sub>, A<sub>2</sub>, B<sub>1</sub>, B<sub>2</sub>, E]
  - Spin (s): [0, 1]
- Atomic selection:
  - Type: [Q, G, M, T]
  - Rank: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]
  - Irrep.: [A<sub>1</sub>, A<sub>2</sub>, B<sub>1</sub>, B<sub>2</sub>, E]
  - Spin (s): [0, 1]
- Site-cluster selection:
  - Rank: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]
  - Irrep.: [A<sub>1</sub>, A<sub>2</sub>, B<sub>1</sub>, B<sub>2</sub>, E]
- Bond-cluster selection:
  - Type: [Q, G, M, T]
  - Rank: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]
  - Irrep.: [A<sub>1</sub>, A<sub>2</sub>, B<sub>1</sub>, B<sub>2</sub>, E]
- Max. neighbor: 10
- Search cell range: (-2, 3), (-2, 3), (-2, 3)
- Toroidal priority: false

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## Group and Unit Cell

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- Group: SG No. 99 C<sub>4v</sub><sup>1</sup> P4mm [ tetragonal ]
- Associated point group: PG No. 99 C<sub>4v</sub> 4mm [ tetragonal ]
- Unit cell:

$a = 1.00000, b = 1.00000, c = 1.00000, \alpha = 90.0, \beta = 90.0, \gamma = 90.0$
- Lattice vectors (conventional cell):

$\mathbf{a}_1 = [1.00000, 0.00000, 0.00000]$   
 $\mathbf{a}_2 = [0.00000, 1.00000, 0.00000]$   
 $\mathbf{a}_3 = [0.00000, 0.00000, 1.00000]$

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 — Symmetry Operation —

Table 1: Symmetry operation

#	SO	#	SO	#	SO	#	SO	#	SO
1	{1 0}	2	{2 <sub>001</sub>  0}	3	{4 <sub>001</sub> <sup>+</sup>  0}	4	{4 <sub>001</sub> <sup>-</sup>  0}	5	{m <sub>010</sub>  0}
6	{m <sub>100</sub>  0}	7	{m <sub>110</sub>  0}	8	{m <sub>1-10</sub>  0}				

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 — Harmonics —

Table 2: Harmonics

#	symbol	irrep.	rank	X	multiplicity	component	symmetry
1	$\mathbb{Q}_0(A_1)$	$A_1$	0	$Q, T$	-	-	1
2	$\mathbb{Q}_1(A_1)$	$A_1$	1	$Q, T$	-	-	$z$
3	$\mathbb{G}_0(A_2)$	$A_2$	0	$G, M$	-	-	1
4	$\mathbb{G}_1(A_2)$	$A_2$	1	$G, M$	-	-	$z$
5	$\mathbb{G}_2(B_1)$	$B_1$	2	$G, M$	-	-	$\sqrt{3}xy$
6	$\mathbb{Q}_2(B_1)$	$B_1$	2	$Q, T$	-	-	$\frac{\sqrt{3}(x-y)(x+y)}{2}$
7	$\mathbb{G}_2(B_2)$	$B_2$	2	$G, M$	-	-	$\frac{\sqrt{3}(x-y)(x+y)}{2}$
8	$\mathbb{G}_{1,1}(E)$	$E$	1	$G, M$	-	1	$-y$

*continued ...*

Table 2

#	symbol	irrep.	rank	X	multiplicity	component	symmetry
9	$\mathbb{G}_{1,2}(E)$					2	$x$
10	$\mathbb{Q}_{1,1}(E)$	$E$	1	$Q, T$	-	1	$x$
11	$\mathbb{Q}_{1,2}(E)$					2	$y$

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Basis in full matrix

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Table 3: dimension = 2

#	orbital@atom(SL)	#	orbital@atom(SL)
0	$ s, \uparrow\rangle @A(1)$	1	$ s, \downarrow\rangle @A(1)$

Table 4: Atomic basis (orbital part only)

orbital	definition
$ s\rangle$	1

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**SAMB**


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13 (all 13) SAMBs

- 'A' site-cluster : A
  - \* bra:  $\langle s, \uparrow |$ ,  $\langle s, \downarrow |$
  - \* ket:  $|s, \uparrow \rangle$ ,  $|s, \downarrow \rangle$
  - \* wyckoff: 1a

$$\boxed{z1} \quad \mathbb{Q}_0^{(c)}(A_1) = \mathbb{Q}_0^{(a)}(A_1)\mathbb{Q}_0^{(s)}(A_1)$$

- 'A'-'A' bond-cluster : A;A\_001\_1

- \* bra:  $\langle s, \uparrow |$ ,  $\langle s, \downarrow |$
- \* ket:  $|s, \uparrow \rangle$ ,  $|s, \downarrow \rangle$
- \* wyckoff: 2a@2c

$$\boxed{z2} \quad \mathbb{Q}_0^{(c)}(A_1) = \mathbb{Q}_0^{(a)}(A_1)\mathbb{Q}_0^{(b)}(A_1)$$

$$\boxed{z3} \quad \mathbb{Q}_1^{(1,-1;c)}(A_1) = \frac{\sqrt{2}\mathbb{M}_{1,1}^{(1,-1;a)}(E)\mathbb{T}_{1,1}^{(b)}(E)}{2} + \frac{\sqrt{2}\mathbb{M}_{1,2}^{(1,-1;a)}(E)\mathbb{T}_{1,2}^{(b)}(E)}{2}$$

$$\boxed{z5} \quad \mathbb{G}_0^{(1,-1;c)}(A_2) = -\frac{\sqrt{2}\mathbb{M}_{1,1}^{(1,-1;a)}(E)\mathbb{T}_{1,2}^{(b)}(E)}{2} + \frac{\sqrt{2}\mathbb{M}_{1,2}^{(1,-1;a)}(E)\mathbb{T}_{1,1}^{(b)}(E)}{2}$$

$$\boxed{z7} \quad \mathbb{Q}_2^{(c)}(B_1) = \mathbb{Q}_0^{(a)}(A_1)\mathbb{Q}_2^{(b)}(B_1)$$

$$\boxed{z8} \quad \mathbb{G}_2^{(1,-1;c)}(B_1) = -\frac{\sqrt{2}\mathbb{M}_{1,1}^{(1,-1;a)}(E)\mathbb{T}_{1,1}^{(b)}(E)}{2} + \frac{\sqrt{2}\mathbb{M}_{1,2}^{(1,-1;a)}(E)\mathbb{T}_{1,2}^{(b)}(E)}{2}$$

$$\boxed{z9} \quad \mathbb{G}_2^{(1,-1;c)}(B_2) = \frac{\sqrt{2}\mathbb{M}_{1,1}^{(1,-1;a)}(E)\mathbb{T}_{1,2}^{(b)}(E)}{2} + \frac{\sqrt{2}\mathbb{M}_{1,2}^{(1,-1;a)}(E)\mathbb{T}_{1,1}^{(b)}(E)}{2}$$

$$\boxed{z10} \quad \mathbb{Q}_{1,1}^{(1,-1;c)}(E) = -\frac{\sqrt{2}\mathbb{M}_1^{(1,-1;a)}(A_2)\mathbb{T}_{1,2}^{(b)}(E)}{2}$$

$$\boxed{z11} \quad \mathbb{Q}_{1,2}^{(1,-1;c)}(E) = \frac{\sqrt{2}\mathbb{M}_1^{(1,-1;a)}(A_2)\mathbb{T}_{1,1}^{(b)}(E)}{2}$$

- 'A'-'A' bond-cluster : A;A\_001\_2

- \* bra:  $\langle s, \uparrow |$ ,  $\langle s, \downarrow |$
- \* ket:  $|s, \uparrow \rangle$ ,  $|s, \downarrow \rangle$
- \* wyckoff: **1a@1a**

$$\boxed{z4} \quad \mathbb{Q}_0^{(c)}(A_1) = \mathbb{Q}_0^{(a)}(A_1) \mathbb{Q}_0^{(b)}(A_1)$$

$$\boxed{z6} \quad \mathbb{G}_1^{(1,-1;c)}(A_2) = \mathbb{M}_1^{(1,-1;a)}(A_2) \mathbb{T}_0^{(b)}(A_1)$$

$$\boxed{z12} \quad \mathbb{G}_{1,1}^{(1,-1;c)}(E) = \frac{\sqrt{2} \mathbb{M}_{1,1}^{(1,-1;a)}(E) \mathbb{T}_0^{(b)}(A_1)}{2}$$

$$\boxed{z13} \quad \mathbb{G}_{1,2}^{(1,-1;c)}(E) = \frac{\sqrt{2} \mathbb{M}_{1,2}^{(1,-1;a)}(E) \mathbb{T}_0^{(b)}(A_1)}{2}$$

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### — Atomic SAMB —

- bra:  $\langle s, \uparrow |$ ,  $\langle s, \downarrow |$
- ket:  $|s, \uparrow \rangle$ ,  $|s, \downarrow \rangle$

$$\boxed{x1} \quad \mathbb{Q}_0^{(a)}(A_1) = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\boxed{x2} \quad \mathbb{M}_1^{(1,-1;a)}(A_2) = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\boxed{x3} \quad \mathbb{M}_{1,1}^{(1,-1;a)}(E) = \begin{bmatrix} 0 & \frac{\sqrt{2}i}{2} \\ -\frac{\sqrt{2}i}{2} & 0 \end{bmatrix}$$

$$\boxed{x4} \quad \mathbb{M}_{1,2}^{(1,-1;a)}(E) = \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{bmatrix}$$

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### — Cluster SAMB —

- Site cluster

\*\* Wyckoff: **1a**

[y1]  $\mathbb{Q}_0^{(s)}(A_1) = [1]$

- Bond cluster

\*\* Wyckoff: 1a@1a

[y2]  $\mathbb{Q}_0^{(s)}(A_1) = [1]$

[y3]  $\mathbb{T}_0^{(s)}(A_1) = [i]$

\*\* Wyckoff: 2a@2c

[y4]  $\mathbb{Q}_0^{(s)}(A_1) = \left[ \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right]$

[y5]  $\mathbb{Q}_2^{(s)}(B_1) = \left[ \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right]$

[y6]  $\mathbb{T}_{1,1}^{(s)}(E) = [i, 0]$

[y7]  $\mathbb{T}_{1,2}^{(s)}(E) = [0, i]$

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— Site and Bond —

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Table 5: Orbital of each site

#	site	orbital
1	A	$ s, \uparrow\rangle,  s, \downarrow\rangle$

Table 6: Neighbor and bra-ket of each bond

#	head	tail	neighbor	head (bra)	tail (ket)
1	A	A	[1]	[s]	[s]

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### — Site in Unit Cell —

Sites in (conventional) cell (no plus set), SL = sublattice

Table 7: 'A' (#1) site cluster (1a), 4mm

SL	position ( $s$ )	mapping
1	[ 0.00000, 0.00000, 0.00000 ]	[1,2,3,4,5,6,7,8]

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### — Bond in Unit Cell —

Bonds in (conventional) cell (no plus set): tail, head = (SL, plus set), (N)D = (non)directional (listed up to 5th neighbor at most)

Table 8: 1-th 'A'-'A' [1] (#1) bond cluster ( $2\text{a}@\text{2c}$ ), ND,  $|\mathbf{v}|=1.0$  (cartesian)

SL	vector ( $\mathbf{v}$ )	center ( $\mathbf{c}$ )	mapping	head	tail	$\mathbf{R}$ (primitive)
1	[ -1.00000, 0.00000, 0.00000 ]	[ 0.50000, 0.00000, 0.00000 ]	[ 1,-2,5,-6 ]	(1,1)	(1,1)	[ 1,0,0 ]
2	[ 0.00000,-1.00000, 0.00000 ]	[ 0.00000, 0.50000, 0.00000 ]	[ 3,-4,-7,8 ]	(1,1)	(1,1)	[ 0,1,0 ]

Table 9: 1-th 'A'-'A' [2] (#2) bond cluster ( $1\text{a}@\text{1a}$ ), D,  $|\mathbf{v}|=1.0$  (cartesian)

SL	vector ( $\mathbf{v}$ )	center ( $\mathbf{c}$ )	mapping	head	tail	$\mathbf{R}$ (primitive)
1	[ 0.00000, 0.00000,-1.00000 ]	[ 0.00000, 0.00000, 0.50000 ]	[ 1,2,3,4,5,6,7,8 ]	(1,1)	(1,1)	[ 0,0,1 ]