# No. 8 $D_{2h}$ $mmm$ [ orthorhombic ] (polar)

表 1 rank 0

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 1 | $A_g$ | Ag | – | – | $\mathbb{Q}_0^{(h,A_g)}$ | Qh(0, Ag, , ) | $C_0$ |

表 2 rank 1

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 2 | $B_{1u}$ | B1u | – | – | $\mathbb{Q}_1^{(h,B_{1u})}$ | Qh(1, B1u, , ) | $C_0$ |
| 3 | $B_{2u}$ | B2u | – | – | $\mathbb{Q}_1^{(h,B_{2u})}$ | Qh(1, B2u, , ) | $S_1$ |
| 4 | $B_{3u}$ | B3u | – | – | $\mathbb{Q}_1^{(h,B_{3u})}$ | Qh(1, B3u, , ) | $C_1$ |

表 3 rank 2

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 5 | $A_g$ | Ag | 1 | – | $\mathbb{Q}_2^{(h,A_g,1)}$ | Qh(2, Ag, 1, ) | $C_0$ |
| 6 | $A_g$ | Ag | 2 | – | $\mathbb{Q}_2^{(h,A_g,2)}$ | Qh(2, Ag, 2, ) | $C_2$ |
| 7 | $B_{1g}$ | B1g | – | – | $\mathbb{Q}_2^{(h,B_{1g})}$ | Qh(2, B1g, , ) | $S_2$ |
| 8 | $B_{2g}$ | B2g | – | – | $\mathbb{Q}_2^{(h,B_{2g})}$ | Qh(2, B2g, , ) | $C_1$ |
| 9 | $B_{3g}$ | B3g | – | – | $\mathbb{Q}_2^{(h,B_{3g})}$ | Qh(2, B3g, , ) | $S_1$ |

表 4 rank 3

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 10 | $A_u$ | Au | – | – | $\mathbb{Q}_3^{(h,A_u)}$ | Qh(3, Au, , ) | $S_2$ |
| 11 | $B_{1u}$ | B1u | 1 | – | $\mathbb{Q}_3^{(h,B_{1u},1)}$ | Qh(3, B1u, 1, ) | $C_0$ |
| 12 | $B_{1u}$ | B1u | 2 | – | $\mathbb{Q}_3^{(h,B_{1u},2)}$ | Qh(3, B1u, 2, ) | $C_2$ |
| 13 | $B_{2u}$ | B2u | 1 | – | $\mathbb{Q}_3^{(h,B_{2u},1)}$ | Qh(3, B2u, 1, ) | $-\frac{\sqrt{6}S_1}{4} - \frac{\sqrt{10}S_3}{4}$ |
| 14 | $B_{2u}$ | B2u | 2 | – | $\mathbb{Q}_3^{(h,B_{2u},2)}$ | Qh(3, B2u, 2, ) | $\frac{\sqrt{10}S_1}{4} - \frac{\sqrt{6}S_3}{4}$ |
| 15 | $B_{3u}$ | B3u | 1 | – | $\mathbb{Q}_3^{(h,B_{3u},1)}$ | Qh(3, B3u, 1, ) | $-\frac{\sqrt{6}C_1}{4} + \frac{\sqrt{10}C_3}{4}$ |
| 16 | $B_{3u}$ | B3u | 2 | – | $\mathbb{Q}_3^{(h,B_{3u},2)}$ | Qh(3, B3u, 2, ) | $-\frac{\sqrt{10}C_1}{4} - \frac{\sqrt{6}C_3}{4}$ |

表 5 rank 4

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 17 | $A_g$ | Ag | 1 | – | $\mathbb{Q}_4^{(h,A_g,1)}$ | Qh(4, Ag, 1, ) | $\frac{\sqrt{21}C_0}{6} + \frac{\sqrt{15}C_4}{6}$ |
| 18 | $A_g$ | Ag | 2 | – | $\mathbb{Q}_4^{(h,A_g,2)}$ | Qh(4, Ag, 2, ) | $\frac{\sqrt{15}C_0}{6} - \frac{\sqrt{21}C_4}{6}$ |
| 19 | $A_g$ | Ag | 3 | – | $\mathbb{Q}_4^{(h,A_g,3)}$ | Qh(4, Ag, 3, ) | $-C_2$ |
| 20 | $B_{1g}$ | B1g | 1 | – | $\mathbb{Q}_4^{(h,B_{1g},1)}$ | Qh(4, B1g, 1, ) | $S_4$ |
| 21 | $B_{1g}$ | B1g | 2 | – | $\mathbb{Q}_4^{(h,B_{1g},2)}$ | Qh(4, B1g, 2, ) | $S_2$ |
| 22 | $B_{2g}$ | B2g | 1 | – | $\mathbb{Q}_4^{(h,B_{2g},1)}$ | Qh(4, B2g, 1, ) | $\frac{\sqrt{14}C_1}{4} - \frac{\sqrt{2}C_3}{4}$ |
| 23 | $B_{2g}$ | B2g | 2 | – | $\mathbb{Q}_4^{(h,B_{2g},2)}$ | Qh(4, B2g, 2, ) | $-\frac{\sqrt{2}C_1}{4} - \frac{\sqrt{14}C_3}{4}$ |
| 24 | $B_{3g}$ | B3g | 1 | – | $\mathbb{Q}_4^{(h,B_{3g},1)}$ | Qh(4, B3g, 1, ) | $-\frac{\sqrt{14}S_1}{4} - \frac{\sqrt{2}S_3}{4}$ |
| 25 | $B_{3g}$ | B3g | 2 | – | $\mathbb{Q}_4^{(h,B_{3g},2)}$ | Qh(4, B3g, 2, ) | $-\frac{\sqrt{2}S_1}{4} + \frac{\sqrt{14}S_3}{4}$ |

表 6　rank 5

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 26 | $A_u$ | Au | 1 | – | $\mathbb{Q}_5^{(h,A_u,1)}$ | Qh(5, Au, 1, ) | $S_4$ |
| 27 | $A_u$ | Au | 2 | – | $\mathbb{Q}_5^{(h,A_u,2)}$ | Qh(5, Au, 2, ) | $-S_2$ |
| 28 | $B_{1u}$ | B1u | 1 | – | $\mathbb{Q}_5^{(h,B_{1u},1)}$ | Qh(5, B1u, 1, ) | $C_0$ |
| 29 | $B_{1u}$ | B1u | 2 | – | $\mathbb{Q}_5^{(h,B_{1u},2)}$ | Qh(5, B1u, 2, ) | $C_4$ |
| 30 | $B_{1u}$ | B1u | 3 | – | $\mathbb{Q}_5^{(h,B_{1u},3)}$ | Qh(5, B1u, 3, ) | $C_2$ |
| 31 | $B_{2u}$ | B2u | 1 | – | $\mathbb{Q}_5^{(h,B_{2u},1)}$ | Qh(5, B2u, 1, ) | $\frac{\sqrt{15}S_1}{8} + \frac{\sqrt{70}S_3}{16} + \frac{3\sqrt{14}S_5}{16}$ |
| 32 | $B_{2u}$ | B2u | 2 | – | $\mathbb{Q}_5^{(h,B_{2u},2)}$ | Qh(5, B2u, 2, ) | $\frac{\sqrt{21}S_1}{8} - \frac{9\sqrt{2}S_3}{16} + \frac{\sqrt{10}S_5}{16}$ |
| 33 | $B_{2u}$ | B2u | 3 | – | $\mathbb{Q}_5^{(h,B_{2u},3)}$ | Qh(5, B2u, 3, ) | $-\frac{\sqrt{7}S_1}{4} - \frac{\sqrt{6}S_3}{8} + \frac{\sqrt{30}S_5}{8}$ |
| 34 | $B_{3u}$ | B3u | 1 | – | $\mathbb{Q}_5^{(h,B_{3u},1)}$ | Qh(5, B3u, 1, ) | $\frac{\sqrt{15}C_1}{8} - \frac{\sqrt{70}C_3}{16} + \frac{3\sqrt{14}C_5}{16}$ |
| 35 | $B_{3u}$ | B3u | 2 | – | $\mathbb{Q}_5^{(h,B_{3u},2)}$ | Qh(5, B3u, 2, ) | $\frac{\sqrt{21}C_1}{8} + \frac{9\sqrt{2}C_3}{16} + \frac{\sqrt{10}C_5}{16}$ |
| 36 | $B_{3u}$ | B3u | 3 | – | $\mathbb{Q}_5^{(h,B_{3u},3)}$ | Qh(5, B3u, 3, ) | $\frac{\sqrt{7}C_1}{4} - \frac{\sqrt{6}C_3}{8} - \frac{\sqrt{30}C_5}{8}$ |

表 7　rank 6

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 37 | $A_g$ | Ag | 1 | – | $\mathbb{Q}_6^{(h,A_g,1)}$ | Qh(6, Ag, 1, ) | $\frac{\sqrt{2}C_0}{4} - \frac{\sqrt{14}C_4}{4}$ |
| 38 | $A_g$ | Ag | 2 | – | $\mathbb{Q}_6^{(h,A_g,2)}$ | Qh(6, Ag, 2, ) | $\frac{\sqrt{11}C_2}{4} - \frac{\sqrt{5}C_6}{4}$ |
| 39 | $A_g$ | Ag | 3 | – | $\mathbb{Q}_6^{(h,A_g,3)}$ | Qh(6, Ag, 3, ) | $\frac{\sqrt{14}C_0}{4} + \frac{\sqrt{2}C_4}{4}$ |
| 40 | $A_g$ | Ag | 4 | – | $\mathbb{Q}_6^{(h,A_g,4)}$ | Qh(6, Ag, 4, ) | $\frac{\sqrt{5}C_2}{4} + \frac{\sqrt{11}C_6}{4}$ |
| 41 | $B_{1g}$ | B1g | 1 | – | $\mathbb{Q}_6^{(h,B_{1g},1)}$ | Qh(6, B1g, 1, ) | $S_4$ |
| 42 | $B_{1g}$ | B1g | 2 | – | $\mathbb{Q}_6^{(h,B_{1g},2)}$ | Qh(6, B1g, 2, ) | $S_6$ |
| 43 | $B_{1g}$ | B1g | 3 | – | $\mathbb{Q}_6^{(h,B_{1g},3)}$ | Qh(6, B1g, 3, ) | $S_2$ |
| 44 | $B_{2g}$ | B2g | 1 | – | $\mathbb{Q}_6^{(h,B_{2g},1)}$ | Qh(6, B2g, 1, ) | $-\frac{\sqrt{3}C_1}{4} - \frac{\sqrt{30}C_3}{8} + \frac{\sqrt{22}C_5}{8}$ |
| 45 | $B_{2g}$ | B2g | 2 | – | $\mathbb{Q}_6^{(h,B_{2g},2)}$ | Qh(6, B2g, 2, ) | $\frac{3\sqrt{22}C_1}{16} - \frac{\sqrt{55}C_3}{16} + \frac{\sqrt{3}C_5}{16}$ |
| 46 | $B_{2g}$ | B2g | 3 | – | $\mathbb{Q}_6^{(h,B_{2g},3)}$ | Qh(6, B2g, 3, ) | $\frac{\sqrt{10}C_1}{16} + \frac{9C_3}{16} + \frac{\sqrt{165}C_5}{16}$ |
| 47 | $B_{3g}$ | B3g | 1 | – | $\mathbb{Q}_6^{(h,B_{3g},1)}$ | Qh(6, B3g, 1, ) | $\frac{\sqrt{3}S_1}{4} - \frac{\sqrt{30}S_3}{8} - \frac{\sqrt{22}S_5}{8}$ |
| 48 | $B_{3g}$ | B3g | 2 | – | $\mathbb{Q}_6^{(h,B_{3g},2)}$ | Qh(6, B3g, 2, ) | $\frac{3\sqrt{22}S_1}{16} + \frac{\sqrt{55}S_3}{16} + \frac{\sqrt{3}S_5}{16}$ |
| 49 | $B_{3g}$ | B3g | 3 | – | $\mathbb{Q}_6^{(h,B_{3g},3)}$ | Qh(6, B3g, 3, ) | $\frac{\sqrt{10}S_1}{16} - \frac{9S_3}{16} + \frac{\sqrt{165}S_5}{16}$ |

表 8　rank 7

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 50 | $A_u$ | Au | 1 | – | $\mathbb{Q}_7^{(h,A_u,1)}$ | Qh(7, Au, 1, ) | $\frac{\sqrt{78}S_2}{12} + \frac{\sqrt{66}S_6}{12}$ |
| 51 | $A_u$ | Au | 2 | – | $\mathbb{Q}_7^{(h,A_u,2)}$ | Qh(7, Au, 2, ) | $S_4$ |
| 52 | $A_u$ | Au | 3 | – | $\mathbb{Q}_7^{(h,A_u,3)}$ | Qh(7, Au, 3, ) | $\frac{\sqrt{66}S_2}{12} - \frac{\sqrt{78}S_6}{12}$ |
| 53 | $B_{1u}$ | B1u | 1 | – | $\mathbb{Q}_7^{(h,B_{1u},1)}$ | Qh(7, B1u, 1, ) | $C_0$ |
| 54 | $B_{1u}$ | B1u | 2 | – | $\mathbb{Q}_7^{(h,B_{1u},2)}$ | Qh(7, B1u, 2, ) | $C_4$ |
| 55 | $B_{1u}$ | B1u | 3 | – | $\mathbb{Q}_7^{(h,B_{1u},3)}$ | Qh(7, B1u, 3, ) | $C_6$ |
| 56 | $B_{1u}$ | B1u | 4 | – | $\mathbb{Q}_7^{(h,B_{1u},4)}$ | Qh(7, B1u, 4, ) | $C_2$ |
| 57 | $B_{2u}$ | B2u | 1 | – | $\mathbb{Q}_7^{(h,B_{2u},1)}$ | Qh(7, B2u, 1, ) | $-\frac{5\sqrt{7}S_1}{32} - \frac{3\sqrt{21}S_3}{32} - \frac{\sqrt{231}S_5}{32} - \frac{\sqrt{429}S_7}{32}$ |
| 58 | $B_{2u}$ | B2u | 2 | – | $\mathbb{Q}_7^{(h,B_{2u},2)}$ | Qh(7, B2u, 2, ) | $-\frac{3\sqrt{33}S_1}{32} + \frac{\sqrt{11}S_3}{32} + \frac{25S_5}{32} - \frac{\sqrt{91}S_7}{32}$ |
| 59 | $B_{2u}$ | B2u | 3 | – | $\mathbb{Q}_7^{(h,B_{2u},3)}$ | Qh(7, B2u, 3, ) | $\frac{\sqrt{858}S_1}{64} - \frac{3\sqrt{286}S_3}{64} + \frac{5\sqrt{26}S_5}{64} - \frac{\sqrt{14}S_7}{64}$ |
| 60 | $B_{2u}$ | B2u | 4 | – | $\mathbb{Q}_7^{(h,B_{2u},4)}$ | Qh(7, B2u, 4, ) | $\frac{15\sqrt{6}S_1}{64} + \frac{19\sqrt{2}S_3}{64} + \frac{\sqrt{22}S_5}{64} - \frac{\sqrt{2002}S_7}{64}$ |
| 61 | $B_{3u}$ | B3u | 1 | – | $\mathbb{Q}_7^{(h,B_{3u},1)}$ | Qh(7, B3u, 1, ) | $-\frac{5\sqrt{7}C_1}{32} + \frac{3\sqrt{21}C_3}{32} - \frac{\sqrt{231}C_5}{32} + \frac{\sqrt{429}C_7}{32}$ |
| 62 | $B_{3u}$ | B3u | 2 | – | $\mathbb{Q}_7^{(h,B_{3u},2)}$ | Qh(7, B3u, 2, ) | $-\frac{3\sqrt{33}C_1}{32} - \frac{\sqrt{11}C_3}{32} + \frac{25C_5}{32} + \frac{\sqrt{91}C_7}{32}$ |
| 63 | $B_{3u}$ | B3u | 3 | – | $\mathbb{Q}_7^{(h,B_{3u},3)}$ | Qh(7, B3u, 3, ) | $-\frac{\sqrt{858}C_1}{64} - \frac{3\sqrt{286}C_3}{64} - \frac{5\sqrt{26}C_5}{64} - \frac{\sqrt{14}C_7}{64}$ |
| 64 | $B_{3u}$ | B3u | 4 | – | $\mathbb{Q}_7^{(h,B_{3u},4)}$ | Qh(7, B3u, 4, ) | $-\frac{15\sqrt{6}C_1}{64} + \frac{19\sqrt{2}C_3}{64} - \frac{\sqrt{22}C_5}{64} - \frac{\sqrt{2002}C_7}{64}$ |

表 9　rank 8

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 65 | $A_g$ | Ag | 1 | – | $\mathbb{Q}_8^{(h,A_g,1)}$ | Qh(8, Ag, 1, ) | $\frac{\sqrt{33}C_0}{8} + \frac{\sqrt{21}C_4}{12} + \frac{\sqrt{195}C_8}{24}$ |
| 66 | $A_g$ | Ag | 2 | – | $\mathbb{Q}_8^{(h,A_g,2)}$ | Qh(8, Ag, 2, ) | $-\frac{\sqrt{286}C_0}{32} + \frac{\sqrt{182}C_4}{16} + \frac{\sqrt{10}C_8}{32}$ |
| 67 | $A_g$ | Ag | 3 | – | $\mathbb{Q}_8^{(h,A_g,3)}$ | Qh(8, Ag, 3, ) | $C_6$ |
| 68 | $A_g$ | Ag | 4 | – | $\mathbb{Q}_8^{(h,A_g,4)}$ | Qh(8, Ag, 4, ) | $-\frac{\sqrt{210}C_0}{32} - \frac{\sqrt{330}C_4}{48} + \frac{\sqrt{6006}C_8}{96}$ |
| 69 | $A_g$ | Ag | 5 | – | $\mathbb{Q}_8^{(h,A_g,5)}$ | Qh(8, Ag, 5, ) | $C_2$ |
| 70 | $B_{1g}$ | B1g | 1 | – | $\mathbb{Q}_8^{(h,B_{1g},1)}$ | Qh(8, B1g, 1, ) | $S_8$ |
| 71 | $B_{1g}$ | B1g | 2 | – | $\mathbb{Q}_8^{(h,B_{1g},2)}$ | Qh(8, B1g, 2, ) | $S_4$ |
| 72 | $B_{1g}$ | B1g | 3 | – | $\mathbb{Q}_8^{(h,B_{1g},3)}$ | Qh(8, B1g, 3, ) | $S_6$ |
| 73 | $B_{1g}$ | B1g | 4 | – | $\mathbb{Q}_8^{(h,B_{1g},4)}$ | Qh(8, B1g, 4, ) | $S_2$ |
| 74 | $B_{2g}$ | B2g | 1 | – | $\mathbb{Q}_8^{(h,B_{2g},1)}$ | Qh(8, B2g, 1, ) | $\frac{\sqrt{715}C_1}{32} - \frac{\sqrt{273}C_3}{32} + \frac{\sqrt{35}C_5}{32} - \frac{C_7}{32}$ |
| 75 | $B_{2g}$ | B2g | 2 | – | $\mathbb{Q}_8^{(h,B_{2g},2)}$ | Qh(8, B2g, 2, ) | $\frac{\sqrt{77}C_1}{32} + \frac{5\sqrt{15}C_3}{32} + \frac{3\sqrt{13}C_5}{32} - \frac{\sqrt{455}C_7}{32}$ |
| 76 | $B_{2g}$ | B2g | 3 | – | $\mathbb{Q}_8^{(h,B_{2g},3)}$ | Qh(8, B2g, 3, ) | $-\frac{\sqrt{858}C_1}{64} - \frac{\sqrt{910}C_3}{64} + \frac{7\sqrt{42}C_5}{64} - \frac{3\sqrt{30}C_7}{64}$ |
| 77 | $B_{2g}$ | B2g | 4 | – | $\mathbb{Q}_8^{(h,B_{2g},4)}$ | Qh(8, B2g, 4, ) | $-\frac{\sqrt{70}C_1}{64} - \frac{3\sqrt{66}C_3}{64} - \frac{\sqrt{1430}C_5}{64} - \frac{\sqrt{2002}C_7}{64}$ |
| 78 | $B_{3g}$ | B3g | 1 | – | $\mathbb{Q}_8^{(h,B_{3g},1)}$ | Qh(8, B3g, 1, ) | $-\frac{\sqrt{715}S_1}{32} - \frac{\sqrt{273}S_3}{32} - \frac{\sqrt{35}S_5}{32} - \frac{S_7}{32}$ |
| 79 | $B_{3g}$ | B3g | 2 | – | $\mathbb{Q}_8^{(h,B_{3g},2)}$ | Qh(8, B3g, 2, ) | $-\frac{\sqrt{77}S_1}{32} + \frac{5\sqrt{15}S_3}{32} - \frac{3\sqrt{13}S_5}{32} - \frac{\sqrt{455}S_7}{32}$ |
| 80 | $B_{3g}$ | B3g | 3 | – | $\mathbb{Q}_8^{(h,B_{3g},3)}$ | Qh(8, B3g, 3, ) | $-\frac{\sqrt{858}S_1}{64} + \frac{\sqrt{910}S_3}{64} + \frac{7\sqrt{42}S_5}{64} + \frac{3\sqrt{30}S_7}{64}$ |
| 81 | $B_{3g}$ | B3g | 4 | – | $\mathbb{Q}_8^{(h,B_{3g},4)}$ | Qh(8, B3g, 4, ) | $-\frac{\sqrt{70}S_1}{64} + \frac{3\sqrt{66}S_3}{64} - \frac{\sqrt{1430}S_5}{64} + \frac{\sqrt{2002}S_7}{64}$ |

表 10　rank 9

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 82 | $A_u$ | Au | 1 | – | $\mathbb{Q}_9^{(h,A_u,1)}$ | Qh(9, Au, 1, ) | $\frac{\sqrt{102}S_4}{12} - \frac{\sqrt{42}S_8}{12}$ |
| 83 | $A_u$ | Au | 2 | – | $\mathbb{Q}_9^{(h,A_u,2)}$ | Qh(9, Au, 2, ) | $\frac{\sqrt{3}S_2}{4} - \frac{\sqrt{13}S_6}{4}$ |
| 84 | $A_u$ | Au | 3 | – | $\mathbb{Q}_9^{(h,A_u,3)}$ | Qh(9, Au, 3, ) | $\frac{\sqrt{42}S_4}{12} + \frac{\sqrt{102}S_8}{12}$ |
| 85 | $A_u$ | Au | 4 | – | $\mathbb{Q}_9^{(h,A_u,4)}$ | Qh(9, Au, 4, ) | $-\frac{\sqrt{13}S_2}{4} - \frac{\sqrt{3}S_6}{4}$ |
| 86 | $B_{1u}$ | B1u | 1 | – | $\mathbb{Q}_9^{(h,B_{1u},1)}$ | Qh(9, B1u, 1, ) | $C_0$ |
| 87 | $B_{1u}$ | B1u | 2 | – | $\mathbb{Q}_9^{(h,B_{1u},2)}$ | Qh(9, B1u, 2, ) | $C_8$ |
| 88 | $B_{1u}$ | B1u | 3 | – | $\mathbb{Q}_9^{(h,B_{1u},3)}$ | Qh(9, B1u, 3, ) | $C_4$ |
| 89 | $B_{1u}$ | B1u | 4 | – | $\mathbb{Q}_9^{(h,B_{1u},4)}$ | Qh(9, B1u, 4, ) | $C_6$ |
| 90 | $B_{1u}$ | B1u | 5 | – | $\mathbb{Q}_9^{(h,B_{1u},5)}$ | Qh(9, B1u, 5, ) | $C_2$ |
| 91 | $B_{2u}$ | B2u | 1 | – | $\mathbb{Q}_9^{(h,B_{2u},1)}$ | Qh(9, B2u, 1, ) | $\frac{21\sqrt{5}S_1}{128} + \frac{\sqrt{2310}S_3}{128} + \frac{3\sqrt{286}S_5}{128} + \frac{3\sqrt{1430}S_7}{256} + \frac{\sqrt{24310}S_9}{256}$ |
| 92 | $B_{2u}$ | B2u | 2 | – | $\mathbb{Q}_9^{(h,B_{2u},2)}$ | Qh(9, B2u, 2, ) | $\frac{\sqrt{2431}S_1}{128} - \frac{\sqrt{9282}S_3}{128} + \frac{5\sqrt{170}S_5}{128} - \frac{7\sqrt{34}S_7}{256} + \frac{3\sqrt{2}S_9}{256}$ |
| 93 | $B_{2u}$ | B2u | 3 | – | $\mathbb{Q}_9^{(h,B_{2u},3)}$ | Qh(9, B2u, 3, ) | $\frac{\sqrt{1001}S_1}{64} + \frac{\sqrt{78}S_3}{64} - \frac{3\sqrt{70}S_5}{64} - \frac{23\sqrt{14}S_7}{128} + \frac{3\sqrt{238}S_9}{128}$ |
| 94 | $B_{2u}$ | B2u | 4 | – | $\mathbb{Q}_9^{(h,B_{2u},4)}$ | Qh(9, B2u, 4, ) | $-\frac{\sqrt{858}S_1}{64} + \frac{\sqrt{91}S_3}{32} + \frac{5\sqrt{15}S_5}{32} - \frac{21\sqrt{3}S_7}{64} + \frac{\sqrt{51}S_9}{64}$ |
| 95 | $B_{2u}$ | B2u | 5 | – | $\mathbb{Q}_9^{(h,B_{2u},5)}$ | Qh(9, B2u, 5, ) | $-\frac{7\sqrt{22}S_1}{64} - \frac{3\sqrt{21}S_3}{32} - \frac{\sqrt{65}S_5}{32} + \frac{\sqrt{13}S_7}{64} + \frac{3\sqrt{221}S_9}{64}$ |
| 96 | $B_{3u}$ | B3u | 1 | – | $\mathbb{Q}_9^{(h,B_{3u},1)}$ | Qh(9, B3u, 1, ) | $\frac{21\sqrt{5}C_1}{128} - \frac{\sqrt{2310}C_3}{128} + \frac{3\sqrt{286}C_5}{128} - \frac{3\sqrt{1430}C_7}{256} + \frac{\sqrt{24310}C_9}{256}$ |
| 97 | $B_{3u}$ | B3u | 2 | – | $\mathbb{Q}_9^{(h,B_{3u},2)}$ | Qh(9, B3u, 2, ) | $\frac{\sqrt{2431}C_1}{128} + \frac{\sqrt{9282}C_3}{128} + \frac{5\sqrt{170}C_5}{128} + \frac{7\sqrt{34}C_7}{256} + \frac{3\sqrt{2}C_9}{256}$ |
| 98 | $B_{3u}$ | B3u | 3 | – | $\mathbb{Q}_9^{(h,B_{3u},3)}$ | Qh(9, B3u, 3, ) | $\frac{\sqrt{1001}C_1}{64} - \frac{\sqrt{78}C_3}{64} - \frac{3\sqrt{70}C_5}{64} + \frac{23\sqrt{14}C_7}{128} + \frac{3\sqrt{238}C_9}{128}$ |
| 99 | $B_{3u}$ | B3u | 4 | – | $\mathbb{Q}_9^{(h,B_{3u},4)}$ | Qh(9, B3u, 4, ) | $\frac{\sqrt{858}C_1}{64} + \frac{\sqrt{91}C_3}{32} - \frac{5\sqrt{15}C_5}{32} - \frac{21\sqrt{3}C_7}{64} - \frac{\sqrt{51}C_9}{64}$ |
| 100 | $B_{3u}$ | B3u | 5 | – | $\mathbb{Q}_9^{(h,B_{3u},5)}$ | Qh(9, B3u, 5, ) | $\frac{7\sqrt{22}C_1}{64} - \frac{3\sqrt{21}C_3}{32} + \frac{\sqrt{65}C_5}{32} + \frac{\sqrt{13}C_7}{64} - \frac{3\sqrt{221}C_9}{64}$ |

表 11　rank 10

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 101 | $A_g$ | Ag | 1 | – | $\mathbb{Q}_{10}^{(h,A_g,1)}$ | Qh(10, Ag, 1, ) | $\frac{\sqrt{390}C_0}{48} - \frac{\sqrt{22}C_4}{8} - \frac{\sqrt{1122}C_8}{48}$ |
| 102 | $A_g$ | Ag | 2 | – | $\mathbb{Q}_{10}^{(h,A_g,2)}$ | Qh(10, Ag, 2, ) | $-\frac{\sqrt{85}C_{10}}{16} + \frac{\sqrt{1482}C_2}{48} + \frac{\sqrt{57}C_6}{48}$ |
| 103 | $A_g$ | Ag | 3 | – | $\mathbb{Q}_{10}^{(h,A_g,3)}$ | Qh(10, Ag, 3, ) | $\frac{11\sqrt{420189}C_0}{8988} + \frac{\sqrt{827645}C_4}{1498} - \frac{\sqrt{146055}C_8}{8988}$ |
| 104 | $A_g$ | Ag | 4 | – | $\mathbb{Q}_{10}^{(h,A_g,4)}$ | Qh(10, Ag, 4, ) | $\frac{\sqrt{370006}C_{10}}{749} + \frac{\sqrt{190995}C_2}{749}$ |
| 105 | $A_g$ | Ag | 5 | – | $\mathbb{Q}_{10}^{(h,A_g,5)}$ | Qh(10, Ag, 5, ) | $\frac{3\sqrt{3213210}C_0}{11984} - \frac{83\sqrt{1498}C_4}{5992} + \frac{31\sqrt{76398}C_8}{11984}$ |
| 106 | $A_g$ | Ag | 6 | – | $\mathbb{Q}_{10}^{(h,A_g,6)}$ | Qh(10, Ag, 6, ) | $\frac{\sqrt{1209635}C_{10}}{11984} - \frac{19\sqrt{58422}C_2}{35952} + \frac{\sqrt{2247}C_6}{48}$ |
| 107 | $B_{1g}$ | B1g | 1 | – | $\mathbb{Q}_{10}^{(h,B_{1g},1)}$ | Qh(10, B1g, 1, ) | $S_8$ |
| 108 | $B_{1g}$ | B1g | 2 | – | $\mathbb{Q}_{10}^{(h,B_{1g},2)}$ | Qh(10, B1g, 2, ) | $S_4$ |
| 109 | $B_{1g}$ | B1g | 3 | – | $\mathbb{Q}_{10}^{(h,B_{1g},3)}$ | Qh(10, B1g, 3, ) | $S_{10}$ |
| 110 | $B_{1g}$ | B1g | 4 | – | $\mathbb{Q}_{10}^{(h,B_{1g},4)}$ | Qh(10, B1g, 4, ) | $S_6$ |
| 111 | $B_{1g}$ | B1g | 5 | – | $\mathbb{Q}_{10}^{(h,B_{1g},5)}$ | Qh(10, B1g, 5, ) | $S_2$ |
| 112 | $B_{2g}$ | B2g | 1 | – | $\mathbb{Q}_{10}^{(h,B_{2g},1)}$ | Qh(10, B2g, 1, ) | $-\frac{\sqrt{221}C_1}{32} - \frac{\sqrt{102}C_3}{32} + \frac{\sqrt{510}C_5}{32} - \frac{11\sqrt{6}C_7}{64} + \frac{\sqrt{38}C_9}{64}$ |
| 113 | $B_{2g}$ | B2g | 2 | – | $\mathbb{Q}_{10}^{(h,B_{2g},2)}$ | Qh(10, B2g, 2, ) | $-\frac{\sqrt{39}C_1}{32} - \frac{11\sqrt{2}C_3}{32} - \frac{5\sqrt{10}C_5}{32} - \frac{\sqrt{34}C_7}{64} + \frac{\sqrt{1938}C_9}{64}$ |
| 114 | $B_{2g}$ | B2g | 3 | – | $\mathbb{Q}_{10}^{(h,B_{2g},3)}$ | Qh(10, B2g, 3, ) | $\frac{\sqrt{41990}C_1}{256} - \frac{\sqrt{4845}C_3}{128} + \frac{\sqrt{969}C_5}{128} - \frac{\sqrt{285}C_7}{256} + \frac{\sqrt{5}C_9}{256}$ |
| 115 | $B_{2g}$ | B2g | 4 | – | $\mathbb{Q}_{10}^{(h,B_{2g},4)}$ | Qh(10, B2g, 4, ) | $\frac{9\sqrt{78}C_1}{256} + \frac{69C_3}{128} - \frac{\sqrt{5}C_5}{128} - \frac{43\sqrt{17}C_7}{256} + \frac{3\sqrt{969}C_9}{256}$ |
| 116 | $B_{2g}$ | B2g | 5 | – | $\mathbb{Q}_{10}^{(h,B_{2g},5)}$ | Qh(10, B2g, 5, ) | $\frac{7\sqrt{3}C_1}{128} + \frac{7\sqrt{26}C_3}{128} + \frac{5\sqrt{130}C_5}{128} + \frac{7\sqrt{442}C_7}{256} + \frac{\sqrt{25194}C_9}{256}$ |
| 117 | $B_{3g}$ | B3g | 1 | – | $\mathbb{Q}_{10}^{(h,B_{3g},1)}$ | Qh(10, B3g, 1, ) | $\frac{\sqrt{221}S_1}{32} - \frac{\sqrt{102}S_3}{32} - \frac{\sqrt{510}S_5}{32} - \frac{11\sqrt{6}S_7}{64} - \frac{\sqrt{38}S_9}{64}$ |
| 118 | $B_{3g}$ | B3g | 2 | – | $\mathbb{Q}_{10}^{(h,B_{3g},2)}$ | Qh(10, B3g, 2, ) | $\frac{\sqrt{39}S_1}{32} - \frac{11\sqrt{2}S_3}{32} + \frac{5\sqrt{10}S_5}{32} - \frac{\sqrt{34}S_7}{64} - \frac{\sqrt{1938}S_9}{64}$ |
| 119 | $B_{3g}$ | B3g | 3 | – | $\mathbb{Q}_{10}^{(h,B_{3g},3)}$ | Qh(10, B3g, 3, ) | $\frac{\sqrt{41990}S_1}{256} + \frac{\sqrt{4845}S_3}{128} + \frac{\sqrt{969}S_5}{128} + \frac{\sqrt{285}S_7}{256} + \frac{\sqrt{5}S_9}{256}$ |
| 120 | $B_{3g}$ | B3g | 4 | – | $\mathbb{Q}_{10}^{(h,B_{3g},4)}$ | Qh(10, B3g, 4, ) | $\frac{9\sqrt{78}S_1}{256} - \frac{69S_3}{128} - \frac{\sqrt{5}S_5}{128} + \frac{43\sqrt{17}S_7}{256} + \frac{3\sqrt{969}S_9}{256}$ |
| 121 | $B_{3g}$ | B3g | 5 | – | $\mathbb{Q}_{10}^{(h,B_{3g},5)}$ | Qh(10, B3g, 5, ) | $\frac{7\sqrt{3}S_1}{128} - \frac{7\sqrt{26}S_3}{128} + \frac{5\sqrt{130}S_5}{128} - \frac{7\sqrt{442}S_7}{256} + \frac{\sqrt{25194}S_9}{256}$ |

表 12　rank 11

| No. | irrep. | (tag) | mul. | comp. | harmonics | (tag) | definition |
|---|---|---|---|---|---|---|---|
| 122 | $A_u$ | Au | 1 | – | $\mathbb{Q}_{11}^{(h,A_u,1)}$ | Qh(11, Au, 1, ) | $\frac{\sqrt{798}S_{10}}{48}+\frac{\sqrt{255}S_2}{24}+\frac{3\sqrt{6}S_6}{16}$ |
| 123 | $A_u$ | Au | 2 | – | $\mathbb{Q}_{11}^{(h,A_u,2)}$ | Qh(11, Au, 2, ) | $S_8$ |
| 124 | $A_u$ | Au | 3 | – | $\mathbb{Q}_{11}^{(h,A_u,3)}$ | Qh(11, Au, 3, ) | $-\frac{\sqrt{210}S_{10}}{96}+\frac{\sqrt{969}S_2}{48}-\frac{\sqrt{570}S_6}{32}$ |
| 125 | $A_u$ | Au | 4 | – | $\mathbb{Q}_{11}^{(h,A_u,4)}$ | Qh(11, Au, 4, ) | $S_4$ |
| 126 | $A_u$ | Au | 5 | – | $\mathbb{Q}_{11}^{(h,A_u,5)}$ | Qh(11, Au, 5, ) | $-\frac{\sqrt{646}S_{10}}{32}+\frac{\sqrt{35}S_2}{16}+\frac{\sqrt{238}S_6}{32}$ |
| 127 | $B_{1u}$ | B1u | 1 | – | $\mathbb{Q}_{11}^{(h,B_{1u},1)}$ | Qh(11, B1u, 1, ) | $C_0$ |
| 128 | $B_{1u}$ | B1u | 2 | – | $\mathbb{Q}_{11}^{(h,B_{1u},2)}$ | Qh(11, B1u, 2, ) | $C_8$ |
| 129 | $B_{1u}$ | B1u | 3 | – | $\mathbb{Q}_{11}^{(h,B_{1u},3)}$ | Qh(11, B1u, 3, ) | $C_4$ |
| 130 | $B_{1u}$ | B1u | 4 | – | $\mathbb{Q}_{11}^{(h,B_{1u},4)}$ | Qh(11, B1u, 4, ) | $C_{10}$ |
| 131 | $B_{1u}$ | B1u | 5 | – | $\mathbb{Q}_{11}^{(h,B_{1u},5)}$ | Qh(11, B1u, 5, ) | $C_6$ |
| 132 | $B_{1u}$ | B1u | 6 | – | $\mathbb{Q}_{11}^{(h,B_{1u},6)}$ | Qh(11, B1u, 6, ) | $C_2$ |
| 133 | $B_{2u}$ | B2u | 1 | – | $\mathbb{Q}_{11}^{(h,B_{2u},1)}$ | Qh(11, B2u, 1, ) | $-\frac{21\sqrt{66}S_1}{512}-\frac{\sqrt{88179}S_{11}}{512}-\frac{\sqrt{30030}S_3}{512}-\frac{15\sqrt{143}S_5}{512}-\frac{\sqrt{36465}S_7}{512}-\frac{\sqrt{46189}S_9}{512}$ |
| 134 | $B_{2u}$ | B2u | 2 | – | $\mathbb{Q}_{11}^{(h,B_{2u},2)}$ | Qh(11, B2u, 2, ) | $-\frac{\sqrt{41990}S_1}{512}-\frac{\sqrt{385}S_{11}}{512}+\frac{3\sqrt{4522}S_3}{512}+\frac{3\sqrt{4845}S_5}{512}-\frac{77\sqrt{19}S_7}{512}+\frac{39\sqrt{15}S_9}{512}$ |
| 135 | $B_{2u}$ | B2u | 3 | – | $\mathbb{Q}_{11}^{(h,B_{2u},3)}$ | Qh(11, B2u, 3, ) | $-\frac{5\sqrt{546}S_1}{256}-\frac{\sqrt{10659}S_{11}}{256}-\frac{11\sqrt{30}S_3}{256}+\frac{13\sqrt{7}S_5}{256}+\frac{3\sqrt{1785}S_7}{256}+\frac{3\sqrt{2261}S_9}{256}$ |
| 136 | $B_{2u}$ | B2u | 4 | – | $\mathbb{Q}_{11}^{(h,B_{2u},4)}$ | Qh(11, B2u, 4, ) | $\frac{\sqrt{29393}S_1}{512}-\frac{\sqrt{22}S_{11}}{1024}-\frac{9\sqrt{1615}S_3}{512}+\frac{5\sqrt{13566}S_5}{1024}-\frac{7\sqrt{1330}S_7}{1024}+\frac{9\sqrt{42}S_9}{1024}$ |
| 137 | $B_{2u}$ | B2u | 5 | – | $\mathbb{Q}_{11}^{(h,B_{2u},5)}$ | Qh(11, B2u, 5, ) | $\frac{15\sqrt{221}S_1}{512}-\frac{3\sqrt{2926}S_{11}}{1024}-\frac{\sqrt{595}S_3}{512}-\frac{53\sqrt{102}S_5}{1024}-\frac{105\sqrt{10}S_7}{1024}+\frac{61\sqrt{114}S_9}{1024}$ |
| 138 | $B_{2u}$ | B2u | 6 | – | $\mathbb{Q}_{11}^{(h,B_{2u},6)}$ | Qh(11, B2u, 6, ) | $\frac{21\sqrt{130}S_1}{512}-\frac{\sqrt{124355}S_{11}}{512}+\frac{57\sqrt{14}S_3}{512}+\frac{41\sqrt{15}S_5}{512}+\frac{17\sqrt{17}S_7}{512}-\frac{\sqrt{4845}S_9}{512}$ |
| 139 | $B_{3u}$ | B3u | 1 | – | $\mathbb{Q}_{11}^{(h,B_{3u},1)}$ | Qh(11, B3u, 1, ) | $-\frac{21\sqrt{66}C_1}{512}+\frac{\sqrt{88179}C_{11}}{512}+\frac{\sqrt{30030}C_3}{512}-\frac{15\sqrt{143}C_5}{512}+\frac{\sqrt{36465}C_7}{512}-\frac{\sqrt{46189}C_9}{512}$ |
| 140 | $B_{3u}$ | B3u | 2 | – | $\mathbb{Q}_{11}^{(h,B_{3u},2)}$ | Qh(11, B3u, 2, ) | $-\frac{\sqrt{41990}C_1}{512}+\frac{\sqrt{385}C_{11}}{512}-\frac{3\sqrt{4522}C_3}{512}+\frac{3\sqrt{4845}C_5}{512}+\frac{77\sqrt{19}C_7}{512}+\frac{39\sqrt{15}C_9}{512}$ |
| 141 | $B_{3u}$ | B3u | 3 | – | $\mathbb{Q}_{11}^{(h,B_{3u},3)}$ | Qh(11, B3u, 3, ) | $-\frac{5\sqrt{546}C_1}{256}+\frac{\sqrt{10659}C_{11}}{256}+\frac{11\sqrt{30}C_3}{256}+\frac{13\sqrt{7}C_5}{256}-\frac{3\sqrt{1785}C_7}{256}+\frac{3\sqrt{2261}C_9}{256}$ |
| 142 | $B_{3u}$ | B3u | 4 | – | $\mathbb{Q}_{11}^{(h,B_{3u},4)}$ | Qh(11, B3u, 4, ) | $-\frac{\sqrt{29393}C_1}{512}-\frac{\sqrt{22}C_{11}}{1024}-\frac{9\sqrt{1615}C_3}{512}-\frac{5\sqrt{13566}C_5}{1024}-\frac{7\sqrt{1330}C_7}{1024}-\frac{9\sqrt{42}C_9}{1024}$ |
| 143 | $B_{3u}$ | B3u | 5 | – | $\mathbb{Q}_{11}^{(h,B_{3u},5)}$ | Qh(11, B3u, 5, ) | $-\frac{15\sqrt{221}C_1}{512}-\frac{3\sqrt{2926}C_{11}}{1024}-\frac{\sqrt{595}C_3}{512}+\frac{53\sqrt{102}C_5}{1024}-\frac{105\sqrt{10}C_7}{1024}-\frac{61\sqrt{114}C_9}{1024}$ |
| 144 | $B_{3u}$ | B3u | 6 | – | $\mathbb{Q}_{11}^{(h,B_{3u},6)}$ | Qh(11, B3u, 6, ) | $-\frac{21\sqrt{130}C_1}{512}-\frac{\sqrt{124355}C_{11}}{512}+\frac{57\sqrt{14}C_3}{512}-\frac{41\sqrt{15}C_5}{512}+\frac{17\sqrt{17}C_7}{512}+\frac{\sqrt{4845}C_9}{512}$ |