SAMB for "D3"

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- Group: No. 18 $D_3 - 1$ 321 (321 setting) [trigonal]

- Generation condition
    - model type: `tight_binding`
    - time-reversal type: `electric`
    - irrep: `[A1]`
    - `spinful`

- Kets: dimension = 48

Table 1: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket	No.	ket	No.	ket
1	$(s,\uparrow)$@A$_1$	2	$(s,\downarrow)$@A$_1$	3	$(p_x,\uparrow)$@A$_1$	4	$(p_x,\downarrow)$@A$_1$	5	$(p_y,\uparrow)$@A$_1$
6	$(p_y,\downarrow)$@A$_1$	7	$(p_z,\uparrow)$@A$_1$	8	$(p_z,\downarrow)$@A$_1$	9	$(s,\uparrow)$@A$_2$	10	$(s,\downarrow)$@A$_2$
11	$(p_x,\uparrow)$@A$_2$	12	$(p_x,\downarrow)$@A$_2$	13	$(p_y,\uparrow)$@A$_2$	14	$(p_y,\downarrow)$@A$_2$	15	$(p_z,\uparrow)$@A$_2$
16	$(p_z,\downarrow)$@A$_2$	17	$(s,\uparrow)$@A$_3$	18	$(s,\downarrow)$@A$_3$	19	$(p_x,\uparrow)$@A$_3$	20	$(p_x,\downarrow)$@A$_3$
21	$(p_y,\uparrow)$@A$_3$	22	$(p_y,\downarrow)$@A$_3$	23	$(p_z,\uparrow)$@A$_3$	24	$(p_z,\downarrow)$@A$_3$	25	$(s,\uparrow)$@A$_4$
26	$(s,\downarrow)$@A$_4$	27	$(p_x,\uparrow)$@A$_4$	28	$(p_x,\downarrow)$@A$_4$	29	$(p_y,\uparrow)$@A$_4$	30	$(p_y,\downarrow)$@A$_4$
31	$(p_z,\uparrow)$@A$_4$	32	$(p_z,\downarrow)$@A$_4$	33	$(s,\uparrow)$@A$_5$	34	$(s,\downarrow)$@A$_5$	35	$(p_x,\uparrow)$@A$_5$
36	$(p_x,\downarrow)$@A$_5$	37	$(p_y,\uparrow)$@A$_5$	38	$(p_y,\downarrow)$@A$_5$	39	$(p_z,\uparrow)$@A$_5$	40	$(p_z,\downarrow)$@A$_5$
41	$(s,\uparrow)$@A$_6$	42	$(s,\downarrow)$@A$_6$	43	$(p_x,\uparrow)$@A$_6$	44	$(p_x,\downarrow)$@A$_6$	45	$(p_y,\uparrow)$@A$_6$
46	$(p_y,\downarrow)$@A$_6$	47	$(p_z,\uparrow)$@A$_6$	48	$(p_z,\downarrow)$@A$_6$				

- Sites in (primitive) unit cell:

1

Table 2: Site-clusters.

	site	position	mapping
S_1 [6c: 1]	A_1	$\begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$	[1]
	A_2	$\begin{pmatrix} 1 & 0 & -1 \end{pmatrix}$	[2]
	A_3	$\begin{pmatrix} -1 & -1 & -1 \end{pmatrix}$	[3]
	A_4	$\begin{pmatrix} 0 & 1 & -1 \end{pmatrix}$	[4]
	A_5	$\begin{pmatrix} 0 & 1 & 1 \end{pmatrix}$	[5]
	A_6	$\begin{pmatrix} -1 & -1 & 1 \end{pmatrix}$	[6]

- Bonds in (primitive) unit cell:

Table 3: Bond-clusters.

	bond	tail	head	n	#	$b@c$	mapping
B_1 [6c: 1]	b_1	A_5	A_1	1	1	$\begin{pmatrix} -1 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$	[1]
	b_2	A_3	A_2	1	1	$\begin{pmatrix} -2 & -1 & 0 \end{pmatrix} @ \begin{pmatrix} 0 & -\frac{1}{2} & -1 \end{pmatrix}$	[2]
	b_3	A_4	A_3	1	1	$\begin{pmatrix} 1 & 2 & 0 \end{pmatrix} @ \begin{pmatrix} -\frac{1}{2} & 0 & -1 \end{pmatrix}$	[3]
	b_4	A_4	A_2	1	1	$\begin{pmatrix} -1 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -1 \end{pmatrix}$	[-4]
	b_5	A_6	A_5	1	1	$\begin{pmatrix} -1 & -2 & 0 \end{pmatrix} @ \begin{pmatrix} -\frac{1}{2} & 0 & 1 \end{pmatrix}$	[5]
	b_6	A_6	A_1	1	1	$\begin{pmatrix} -2 & -1 & 0 \end{pmatrix} @ \begin{pmatrix} 0 & -\frac{1}{2} & 1 \end{pmatrix}$	[-6]
B_2 [3b: .2.]	b_7	A_2	A_1	2	1	$\begin{pmatrix} 0 & 0 & -2 \end{pmatrix} @ \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$	[1,-2]
	b_8	A_6	A_3	2	1	$\begin{pmatrix} 0 & 0 & 2 \end{pmatrix} @ \begin{pmatrix} -1 & -1 & 0 \end{pmatrix}$	[3,-6]
	b_9	A_5	A_4	2	1	$\begin{pmatrix} 0 & 0 & 2 \end{pmatrix} @ \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$	[4,-5]

- SAMB:

No. 1 $\hat{\mathbb{Q}}_0^{(A_1)}$ [M_1, S_1]

$\hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}]$

No. 2 | $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_2 = \frac{\sqrt{3}\mathbb{X}_5[\mathbb{Q}_1^{(a,A_2)}] \otimes \mathbb{U}_2[\mathbb{Q}_1^{(s,A_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_6[\mathbb{Q}_{1,0}^{(a,E)}] \otimes \mathbb{U}_3[\mathbb{Q}_{1,0}^{(s,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_7[\mathbb{Q}_{1,1}^{(a,E)}] \otimes \mathbb{U}_4[\mathbb{Q}_{1,1}^{(s,E)}]}{3}$$

No. 3 | $\hat{\mathbb{Q}}_2^{(A_1)}$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_3 = \frac{\sqrt{6}\mathbb{X}_5[\mathbb{Q}_1^{(a,A_2)}] \otimes \mathbb{U}_2[\mathbb{Q}_1^{(s,A_2)}]}{3} - \frac{\sqrt{6}\mathbb{X}_6[\mathbb{Q}_{1,0}^{(a,E)}] \otimes \mathbb{U}_3[\mathbb{Q}_{1,0}^{(s,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_7[\mathbb{Q}_{1,1}^{(a,E)}] \otimes \mathbb{U}_4[\mathbb{Q}_{1,1}^{(s,E)}]}{6}$$

No. 4 | $\hat{\mathbb{G}}_2^{(A_1)}$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_4 = \frac{\sqrt{2}\mathbb{X}_6[\mathbb{Q}_{1,0}^{(a,E)}] \otimes \mathbb{U}_5[\mathbb{Q}_{2,0}^{(s,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_7[\mathbb{Q}_{1,1}^{(a,E)}] \otimes \mathbb{U}_6[\mathbb{Q}_{2,1}^{(s,E,1)}]}{2}$$

No. 5 | $\hat{\mathbb{Q}}_0^{(A_1)}(1,0)$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_5 = \frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{Q}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_4[\mathbb{Q}_{1,1}^{(s,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_8[\mathbb{Q}_1^{(a,A_2)}(1,0)] \otimes \mathbb{U}_2[\mathbb{Q}_1^{(s,A_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_9[\mathbb{Q}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_3[\mathbb{Q}_{1,0}^{(s,E)}]}{3}$$

No. 6 | $\hat{\mathbb{Q}}_2^{(A_1)}(1,0)$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_6 = -\frac{\sqrt{6}\mathbb{X}_{10}[\mathbb{Q}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_4[\mathbb{Q}_{1,1}^{(s,E)}]}{6} + \frac{\sqrt{6}\mathbb{X}_8[\mathbb{Q}_1^{(a,A_2)}(1,0)] \otimes \mathbb{U}_2[\mathbb{Q}_1^{(s,A_2)}]}{3} - \frac{\sqrt{6}\mathbb{X}_9[\mathbb{Q}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_3[\mathbb{Q}_{1,0}^{(s,E)}]}{6}$$

No. 7 | $\hat{\mathbb{G}}_2^{(A_1)}(1,0)$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_7 = \frac{\sqrt{2}\mathbb{X}_{10}[\mathbb{Q}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_6[\mathbb{Q}_{2,1}^{(s,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_9[\mathbb{Q}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_5[\mathbb{Q}_{2,0}^{(s,E,1)}]}{2}$$

No. 8 | $\hat{\mathbb{G}}_2^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_8 = \mathbb{X}_{28}[\mathbb{G}_2^{(a,A_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 9 | $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_9 = \frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{G}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_4[\mathbb{Q}_{1,1}^{(s,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{G}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_3[\mathbb{Q}_{1,0}^{(s,E)}]}{2}$$

3

No. 10 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_{10} = -\frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{G}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_4[\mathbb{Q}_{1,1}^{(s,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_3[\mathbb{Q}_{1,0}^{(s,E)}]}{2}$$

No. 11 $\hat{\mathbb{G}}_0^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_{11} = -\frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{G}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_6[\mathbb{Q}_{2,1}^{(s,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{G}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_5[\mathbb{Q}_{2,0}^{(s,E,1)}]}{2}$$

No. 12 $\hat{\mathbb{Q}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_{12} = -\frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{G}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_6[\mathbb{Q}_{2,1}^{(s,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_5[\mathbb{Q}_{2,0}^{(s,E,1)}]}{2}$$

No. 13 $\hat{\mathbb{G}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_{13} = \mathbb{X}_{27}[\mathbb{G}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 14 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_3, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_{14} = \mathbb{X}_{29}[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 15 $\hat{\mathbb{Q}}_2^{(A_1)}$ $[\mathrm{M}_3, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_{15} = \mathbb{X}_{30}[\mathbb{Q}_2^{(a,A_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 16 $\hat{\mathbb{G}}_2^{(A_1)}$ $[\mathrm{M}_3, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_{16} = -\frac{\sqrt{2}\mathbb{X}_{33}[\mathbb{Q}_{2,0}^{(a,E,1)}] \otimes \mathbb{U}_3[\mathbb{Q}_{1,0}^{(s,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{34}[\mathbb{Q}_{2,1}^{(a,E,1)}] \otimes \mathbb{U}_4[\mathbb{Q}_{1,1}^{(s,E)}]}{2}$$

No. 17 $\hat{\mathbb{Q}}_3^{(A_1)}$ $[\mathrm{M}_3, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_{17} = \frac{\sqrt{2}\mathbb{X}_{35}[\mathbb{Q}_{2,0}^{(a,E,2)}] \otimes \mathbb{U}_3[\mathbb{Q}_{1,0}^{(s,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{36}[\mathbb{Q}_{2,1}^{(a,E,2)}] \otimes \mathbb{U}_4[\mathbb{Q}_{1,1}^{(s,E)}]}{2}$$

4

No. 18 $\hat{\mathbb{Q}}_0^{(A_1)}$ [M$_3$, S$_1$]

$$\hat{\mathbb{Z}}_{18} = \frac{\sqrt{2}\mathbb{X}_{33}[\mathbb{Q}_{2,0}^{(a,E,1)}] \otimes \mathbb{U}_5[\mathbb{Q}_{2,0}^{(s,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{34}[\mathbb{Q}_{2,1}^{(a,E,1)}] \otimes \mathbb{U}_6[\mathbb{Q}_{2,1}^{(s,E,1)}]}{2}$$

No. 19 $\hat{\mathbb{G}}_3^{(A_1)}$ [M$_3$, S$_1$]

$$\hat{\mathbb{Z}}_{19} = \frac{\sqrt{2}\mathbb{X}_{35}[\mathbb{Q}_{2,0}^{(a,E,2)}] \otimes \mathbb{U}_5[\mathbb{Q}_{2,0}^{(s,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{36}[\mathbb{Q}_{2,1}^{(a,E,2)}] \otimes \mathbb{U}_6[\mathbb{Q}_{2,1}^{(s,E,1)}]}{2}$$

No. 20 $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ [M$_3$, S$_1$]

$$\hat{\mathbb{Z}}_{20} = \mathbb{X}_{31}[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 21 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)$ [M$_3$, S$_1$]

$$\hat{\mathbb{Z}}_{21} = \mathbb{X}_{32}[\mathbb{Q}_2^{(a,A_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 22 $\hat{\mathbb{G}}_2^{(A_1)}(1,-1)$ [M$_3$, S$_1$]

$$\hat{\mathbb{Z}}_{22} = -\frac{\sqrt{2}\mathbb{X}_{37}[\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_3[\mathbb{Q}_{1,0}^{(s,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{38}[\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_4[\mathbb{Q}_{1,1}^{(s,E)}]}{2}$$

No. 23 $\hat{\mathbb{Q}}_3^{(A_1)}(1,-1)$ [M$_3$, S$_1$]

$$\hat{\mathbb{Z}}_{23} = \frac{\sqrt{2}\mathbb{X}_{39}[\mathbb{Q}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_3[\mathbb{Q}_{1,0}^{(s,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{40}[\mathbb{Q}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_4[\mathbb{Q}_{1,1}^{(s,E)}]}{2}$$

No. 24 $\hat{\mathbb{Q}}_0^{(A_1)}(1,-1)$ [M$_3$, S$_1$]

$$\hat{\mathbb{Z}}_{24} = \frac{\sqrt{2}\mathbb{X}_{37}[\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_5[\mathbb{Q}_{2,0}^{(s,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{38}[\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_6[\mathbb{Q}_{2,1}^{(s,E,1)}]}{2}$$

No. 25 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ [M$_3$, S$_1$]

$$\hat{\mathbb{Z}}_{25} = \frac{\sqrt{2}\mathbb{X}_{39}[\mathbb{Q}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_5[\mathbb{Q}_{2,0}^{(s,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{40}[\mathbb{Q}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_6[\mathbb{Q}_{2,1}^{(s,E,1)}]}{2}$$

No. 26 $\hat{\mathbb{G}}_0^{(A_1)}(1,0)$ $[\mathrm{M}_3,\mathrm{S}_1]$

$$\hat{\mathbb{Z}}_{26} = -\frac{\sqrt{3}\mathbb{X}_{41}[\mathbb{G}_{1,0}^{(a,E)}(1,0)]\otimes\mathbb{U}_4[\mathbb{Q}_{1,1}^{(s,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{42}[\mathbb{G}_{1,1}^{(a,E)}(1,0)]\otimes\mathbb{U}_3[\mathbb{Q}_{1,0}^{(s,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{64}[\mathbb{G}_1^{(a,A_2)}(1,0)]\otimes\mathbb{U}_2[\mathbb{Q}_1^{(s,A_2)}]}{3}$$

No. 27 $\hat{\mathbb{G}}_2^{(A_1)}(1,0)$ $[\mathrm{M}_3,\mathrm{S}_1]$

$$\hat{\mathbb{Z}}_{27} = \frac{\sqrt{6}\mathbb{X}_{41}[\mathbb{G}_{1,0}^{(a,E)}(1,0)]\otimes\mathbb{U}_4[\mathbb{Q}_{1,1}^{(s,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_{42}[\mathbb{G}_{1,1}^{(a,E)}(1,0)]\otimes\mathbb{U}_3[\mathbb{Q}_{1,0}^{(s,E)}]}{6} + \frac{\sqrt{6}\mathbb{X}_{64}[\mathbb{G}_1^{(a,A_2)}(1,0)]\otimes\mathbb{U}_2[\mathbb{Q}_1^{(s,A_2)}]}{3}$$

No. 28 $\hat{\mathbb{Q}}_2^{(A_1)}(1,0)$ $[\mathrm{M}_3,\mathrm{S}_1]$

$$\hat{\mathbb{Z}}_{28} = -\frac{\sqrt{2}\mathbb{X}_{41}[\mathbb{G}_{1,0}^{(a,E)}(1,0)]\otimes\mathbb{U}_6[\mathbb{Q}_{2,1}^{(s,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{42}[\mathbb{G}_{1,1}^{(a,E)}(1,0)]\otimes\mathbb{U}_5[\mathbb{Q}_{2,0}^{(s,E,1)}]}{2}$$

No. 29 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{29} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}]\otimes\mathbb{U}_7[\mathbb{Q}_0^{(u,A_1)}]$$

No. 30 $\hat{\mathbb{G}}_0^{(A_1)}(1,-1)$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{30} = \frac{\sqrt{3}\mathbb{X}_2[\mathbb{M}_1^{(a,A_2)}(1,-1)]\otimes\mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_3[\mathbb{M}_{1,0}^{(a,E)}(1,-1)]\otimes\mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,1}^{(a,E)}(1,-1)]\otimes\mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{3}$$

No. 31 $\hat{\mathbb{G}}_2^{(A_1)}(1,-1)$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{31} = \frac{\sqrt{6}\mathbb{X}_2[\mathbb{M}_1^{(a,A_2)}(1,-1)]\otimes\mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{3} + \frac{\sqrt{6}\mathbb{X}_3[\mathbb{M}_{1,0}^{(a,E)}(1,-1)]\otimes\mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_4[\mathbb{M}_{1,1}^{(a,E)}(1,-1)]\otimes\mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{6}$$

No. 32 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{32} = -\frac{\sqrt{2}\mathbb{X}_3[\mathbb{M}_{1,0}^{(a,E)}(1,-1)]\otimes\mathbb{U}_{18}[\mathbb{T}_{2,1}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_4[\mathbb{M}_{1,1}^{(a,E)}(1,-1)]\otimes\mathbb{U}_{17}[\mathbb{T}_{2,0}^{(u,E,1)}]}{2}$$

No. 33 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_2,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{33} = \frac{\sqrt{3}\mathbb{X}_5[\mathbb{Q}_1^{(a,A_2)}]\otimes\mathbb{U}_8[\mathbb{Q}_1^{(u,A_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_6[\mathbb{Q}_{1,0}^{(a,E)}]\otimes\mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_7[\mathbb{Q}_{1,1}^{(a,E)}]\otimes\mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,E)}]}{3}$$

No. 34 $\hat{\mathbb{Q}}_2^{(A_1)}$ [M$_2$, B$_1$]

$$\hat{\mathbb{Z}}_{34} = \frac{\sqrt{6}\mathbb{X}_5[\mathbb{Q}_1^{(a,A_2)}] \otimes \mathbb{U}_8[\mathbb{Q}_1^{(u,A_2)}]}{3} - \frac{\sqrt{6}\mathbb{X}_6[\mathbb{Q}_{1,0}^{(a,E)}] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_7[\mathbb{Q}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,E)}]}{6}$$

No. 35 $\hat{\mathbb{G}}_2^{(A_1)}$ [M$_2$, B$_1$]

$$\hat{\mathbb{Z}}_{35} = \frac{\sqrt{2}\mathbb{X}_6[\mathbb{Q}_{1,0}^{(a,E)}] \otimes \mathbb{U}_{11}[\mathbb{Q}_{2,0}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_7[\mathbb{Q}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,1}^{(u,E,1)}]}{2}$$

No. 36 $\hat{\mathbb{Q}}_0^{(A_1)}(1,0)$ [M$_2$, B$_1$]

$$\hat{\mathbb{Z}}_{36} = \frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{Q}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_8[\mathbb{Q}_1^{(a,A_2)}(1,0)] \otimes \mathbb{U}_8[\mathbb{Q}_1^{(u,A_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_9[\mathbb{Q}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,E)}]}{3}$$

No. 37 $\hat{\mathbb{Q}}_2^{(A_1)}(1,0)$ [M$_2$, B$_1$]

$$\hat{\mathbb{Z}}_{37} = -\frac{\sqrt{6}\mathbb{X}_{10}[\mathbb{Q}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,E)}]}{6} + \frac{\sqrt{6}\mathbb{X}_8[\mathbb{Q}_1^{(a,A_2)}(1,0)] \otimes \mathbb{U}_8[\mathbb{Q}_1^{(u,A_2)}]}{3} - \frac{\sqrt{6}\mathbb{X}_9[\mathbb{Q}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,E)}]}{6}$$

No. 38 $\hat{\mathbb{G}}_2^{(A_1)}(1,0)$ [M$_2$, B$_1$]

$$\hat{\mathbb{Z}}_{38} = \frac{\sqrt{2}\mathbb{X}_{10}[\mathbb{Q}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,1}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_9[\mathbb{Q}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_{11}[\mathbb{Q}_{2,0}^{(u,E,1)}]}{2}$$

No. 39 $\hat{\mathbb{G}}_2^{(A_1)}(1,-1)$ [M$_2$, B$_1$]

$$\hat{\mathbb{Z}}_{39} = \mathbb{X}_{28}[\mathbb{G}_2^{(a,A_1)}(1,-1)] \otimes \mathbb{U}_7[\mathbb{Q}_0^{(u,A_1)}]$$

No. 40 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)$ [M$_2$, B$_1$]

$$\hat{\mathbb{Z}}_{40} = \frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{G}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{G}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,E)}]}{2}$$

No. 41 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ [M$_2$, B$_1$]

$$\hat{\mathbb{Z}}_{41} = -\frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{G}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,E)}]}{2}$$

No. 42 $\hat{\mathbb{G}}_0^{(A_1)}(1,-1)\ [\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{42} = -\frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{G}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,1}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{G}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{11}[\mathbb{Q}_{2,0}^{(u,E,1)}]}{2}$$

No. 43 $\hat{\mathbb{Q}}_3^{(A_1)}(1,-1)\ [\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{43} = -\frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{G}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,1}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{11}[\mathbb{Q}_{2,0}^{(u,E,1)}]}{2}$$

No. 44 $\hat{\mathbb{G}}_0^{(A_1)}(1,1)\ [\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{44} = \mathbb{X}_{15}[\mathbb{M}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_{13}[\mathbb{T}_0^{(u,A_1)}]$$

No. 45 $\hat{\mathbb{G}}_2^{(A_1)}(1,-1)\ [\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{45} = \mathbb{X}_{16}[\mathbb{M}_2^{(a,A_1)}(1,-1)] \otimes \mathbb{U}_{13}[\mathbb{T}_0^{(u,A_1)}]$$

No. 46 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)\ [\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{46} = \frac{\sqrt{2}\mathbb{X}_{17}[\mathbb{M}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{18}[\mathbb{M}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{2}$$

No. 47 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)\ [\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{47} = -\frac{\sqrt{2}\mathbb{X}_{19}[\mathbb{M}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{20}[\mathbb{M}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{2}$$

No. 48 $\hat{\mathbb{G}}_0^{(A_1)}(1,-1)\ [\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{48} = -\frac{\sqrt{2}\mathbb{X}_{17}[\mathbb{M}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{18}[\mathbb{T}_{2,1}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{18}[\mathbb{M}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{17}[\mathbb{T}_{2,0}^{(u,E,1)}]}{2}$$

No. 49 $\hat{\mathbb{Q}}_3^{(A_1)}(1,-1)\ [\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{49} = -\frac{\sqrt{2}\mathbb{X}_{19}[\mathbb{M}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{18}[\mathbb{T}_{2,1}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{20}[\mathbb{M}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{17}[\mathbb{T}_{2,0}^{(u,E,1)}]}{2}$$

No. 50 $\hat{\mathbb{Q}}_0^{(A_1)}(1,0)$ $[\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{50} = \frac{\sqrt{3}\mathbb{X}_{21}[\mathbb{T}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{22}[\mathbb{T}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{T}_1^{(a,A_2)}(1,0)] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{3}$$

No. 51 $\hat{\mathbb{Q}}_2^{(A_1)}(1,0)$ $[\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{51} = -\frac{\sqrt{6}\mathbb{X}_{21}[\mathbb{T}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_{22}[\mathbb{T}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{6} + \frac{\sqrt{6}\mathbb{X}_{26}[\mathbb{T}_1^{(a,A_2)}(1,0)] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{3}$$

No. 52 $\hat{\mathbb{G}}_2^{(A_1)}(1,0)$ $[\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{52} = \frac{\sqrt{2}\mathbb{X}_{21}[\mathbb{T}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_{17}[\mathbb{T}_{2,0}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{22}[\mathbb{T}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_{18}[\mathbb{T}_{2,1}^{(u,E,1)}]}{2}$$

No. 53 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{53} = \frac{\sqrt{3}\mathbb{X}_{23}[\mathbb{T}_1^{(a,A_2)}] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{24}[\mathbb{T}_{1,0}^{(a,E)}] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{25}[\mathbb{T}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{3}$$

No. 54 $\hat{\mathbb{Q}}_2^{(A_1)}$ $[\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{54} = \frac{\sqrt{6}\mathbb{X}_{23}[\mathbb{T}_1^{(a,A_2)}] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{3} - \frac{\sqrt{6}\mathbb{X}_{24}[\mathbb{T}_{1,0}^{(a,E)}] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_{25}[\mathbb{T}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{6}$$

No. 55 $\hat{\mathbb{G}}_2^{(A_1)}$ $[\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{55} = \frac{\sqrt{2}\mathbb{X}_{24}[\mathbb{T}_{1,0}^{(a,E)}] \otimes \mathbb{U}_{17}[\mathbb{T}_{2,0}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{T}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{18}[\mathbb{T}_{2,1}^{(u,E,1)}]}{2}$$

No. 56 $\hat{\mathbb{G}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{56} = \mathbb{X}_{27}[\mathbb{G}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_7[\mathbb{Q}_0^{(u,A_1)}]$$

No. 57 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{57} = \mathbb{X}_{29}[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_7[\mathbb{Q}_0^{(u,A_1)}]$$

No. 58 $\hat{\mathbb{Q}}_2^{(A_1)}$ [M$_3$, B$_1$]

$$\hat{\mathbb{Z}}_{58} = \mathbb{X}_{30}[\mathbb{Q}_2^{(a,A_1)}] \otimes \mathbb{U}_7[\mathbb{Q}_0^{(u,A_1)}]$$

No. 59 $\hat{\mathbb{G}}_2^{(A_1)}$ [M$_3$, B$_1$]

$$\hat{\mathbb{Z}}_{59} = -\frac{\sqrt{2}\mathbb{X}_{33}[\mathbb{Q}_{2,0}^{(a,E,1)}] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{34}[\mathbb{Q}_{2,1}^{(a,E,1)}] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,E)}]}{2}$$

No. 60 $\hat{\mathbb{Q}}_3^{(A_1)}$ [M$_3$, B$_1$]

$$\hat{\mathbb{Z}}_{60} = \frac{\sqrt{2}\mathbb{X}_{35}[\mathbb{Q}_{2,0}^{(a,E,2)}] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{36}[\mathbb{Q}_{2,1}^{(a,E,2)}] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,E)}]}{2}$$

No. 61 $\hat{\mathbb{Q}}_0^{(A_1)}$ [M$_3$, B$_1$]

$$\hat{\mathbb{Z}}_{61} = \frac{\sqrt{2}\mathbb{X}_{33}[\mathbb{Q}_{2,0}^{(a,E,1)}] \otimes \mathbb{U}_{11}[\mathbb{Q}_{2,0}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{34}[\mathbb{Q}_{2,1}^{(a,E,1)}] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,1}^{(u,E,1)}]}{2}$$

No. 62 $\hat{\mathbb{G}}_3^{(A_1)}$ [M$_3$, B$_1$]

$$\hat{\mathbb{Z}}_{62} = \frac{\sqrt{2}\mathbb{X}_{35}[\mathbb{Q}_{2,0}^{(a,E,2)}] \otimes \mathbb{U}_{11}[\mathbb{Q}_{2,0}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{36}[\mathbb{Q}_{2,1}^{(a,E,2)}] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,1}^{(u,E,1)}]}{2}$$

No. 63 $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ [M$_3$, B$_1$]

$$\hat{\mathbb{Z}}_{63} = \mathbb{X}_{31}[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_7[\mathbb{Q}_0^{(u,A_1)}]$$

No. 64 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)$ [M$_3$, B$_1$]

$$\hat{\mathbb{Z}}_{64} = \mathbb{X}_{32}[\mathbb{Q}_2^{(a,A_1)}(1,-1)] \otimes \mathbb{U}_7[\mathbb{Q}_0^{(u,A_1)}]$$

No. 65 $\hat{\mathbb{G}}_2^{(A_1)}(1,-1)$ [M$_3$, B$_1$]

$$\hat{\mathbb{Z}}_{65} = -\frac{\sqrt{2}\mathbb{X}_{37}[\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{38}[\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,E)}]}{2}$$

10

No. 66 $\hat{\mathbb{Q}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{66} = \frac{\sqrt{2}\mathbb{X}_{39}[\mathbb{Q}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{40}[\mathbb{Q}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,E)}]}{2}$$

No. 67 $\hat{\mathbb{Q}}_0^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{67} = \frac{\sqrt{2}\mathbb{X}_{37}[\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{11}[\mathbb{Q}_{2,0}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{38}[\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,1}^{(u,E,1)}]}{2}$$

No. 68 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{68} = \frac{\sqrt{2}\mathbb{X}_{39}[\mathbb{Q}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{11}[\mathbb{Q}_{2,0}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{40}[\mathbb{Q}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,1}^{(u,E,1)}]}{2}$$

No. 69 $\hat{\mathbb{G}}_0^{(A_1)}(1,0)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{69} = -\frac{\sqrt{3}\mathbb{X}_{41}[\mathbb{G}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{42}[\mathbb{G}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{64}[\mathbb{G}_1^{(a,A_2)}(1,0)] \otimes \mathbb{U}_8[\mathbb{Q}_1^{(u,A_2)}]}{3}$$

No. 70 $\hat{\mathbb{G}}_2^{(A_1)}(1,0)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{70} = \frac{\sqrt{6}\mathbb{X}_{41}[\mathbb{G}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_{42}[\mathbb{G}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,E)}]}{6} + \frac{\sqrt{6}\mathbb{X}_{64}[\mathbb{G}_1^{(a,A_2)}(1,0)] \otimes \mathbb{U}_8[\mathbb{Q}_1^{(u,A_2)}]}{3}$$

No. 71 $\hat{\mathbb{Q}}_2^{(A_1)}(1,0)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{71} = -\frac{\sqrt{2}\mathbb{X}_{41}[\mathbb{G}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,1}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{42}[\mathbb{G}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_{11}[\mathbb{Q}_{2,0}^{(u,E,1)}]}{2}$$

No. 72 $\hat{\mathbb{G}}_0^{(A_1)}$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{72} = \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_1^{(a,A_2)}] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{44}[\mathbb{M}_{1,0}^{(a,E)}] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{45}[\mathbb{M}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{3}$$

No. 73 $\hat{\mathbb{G}}_2^{(A_1)}$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{73} = \frac{\sqrt{6}\mathbb{X}_{43}[\mathbb{M}_1^{(a,A_2)}] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{3} + \frac{\sqrt{6}\mathbb{X}_{44}[\mathbb{M}_{1,0}^{(a,E)}] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_{45}[\mathbb{M}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{6}$$

No. 74 $\hat{\mathbb{Q}}_2^{(A_1)}\,[\mathrm{M_3, B_1}]$

$$\hat{\mathbb{Z}}_{74} = -\frac{\sqrt{2}\mathbb{X}_{44}[\mathbb{M}_{1,0}^{(a,E)}] \otimes \mathbb{U}_{18}[\mathbb{T}_{2,1}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{45}[\mathbb{M}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{17}[\mathbb{T}_{2,0}^{(u,E,1)}]}{2}$$

No. 75 $\hat{\mathbb{G}}_0^{(A_1)}(1,1)\,[\mathrm{M_3, B_1}]$

$$\hat{\mathbb{Z}}_{75} = \frac{\sqrt{3}\mathbb{X}_{46}[\mathbb{M}_1^{(a,A_2)}(1,1)] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{50}[\mathbb{M}_{1,0}^{(a,E)}(1,1)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{51}[\mathbb{M}_{1,1}^{(a,E)}(1,1)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{3}$$

No. 76 $\hat{\mathbb{G}}_2^{(A_1)}(1,1)\,[\mathrm{M_3, B_1}]$

$$\hat{\mathbb{Z}}_{76} = \frac{\sqrt{6}\mathbb{X}_{46}[\mathbb{M}_1^{(a,A_2)}(1,1)] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{3} + \frac{\sqrt{6}\mathbb{X}_{50}[\mathbb{M}_{1,0}^{(a,E)}(1,1)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_{51}[\mathbb{M}_{1,1}^{(a,E)}(1,1)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{6}$$

No. 77 $\hat{\mathbb{Q}}_2^{(A_1)}(1,1)\,[\mathrm{M_3, B_1}]$

$$\hat{\mathbb{Z}}_{77} = -\frac{\sqrt{2}\mathbb{X}_{50}[\mathbb{M}_{1,0}^{(a,E)}(1,1)] \otimes \mathbb{U}_{18}[\mathbb{T}_{2,1}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{51}[\mathbb{M}_{1,1}^{(a,E)}(1,1)] \otimes \mathbb{U}_{17}[\mathbb{T}_{2,0}^{(u,E,1)}]}{2}$$

No. 78 $\hat{\mathbb{G}}_0^{(A_1)}(1,-1)\,[\mathrm{M_3, B_1}]$

$$\hat{\mathbb{Z}}_{78} = \frac{\sqrt{3}\mathbb{X}_{47}[\mathbb{M}_1^{(a,A_2)}(1,-1)] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{52}[\mathbb{M}_{1,0}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{53}[\mathbb{M}_{1,1}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{3}$$

No. 79 $\hat{\mathbb{G}}_2^{(A_1)}(1,-1)\,[\mathrm{M_3, B_1}]$

$$\hat{\mathbb{Z}}_{79} = \frac{\sqrt{6}\mathbb{X}_{47}[\mathbb{M}_1^{(a,A_2)}(1,-1)] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{3} + \frac{\sqrt{6}\mathbb{X}_{52}[\mathbb{M}_{1,0}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_{53}[\mathbb{M}_{1,1}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{6}$$

No. 80 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)\,[\mathrm{M_3, B_1}]$

$$\hat{\mathbb{Z}}_{80} = -\frac{\sqrt{2}\mathbb{X}_{52}[\mathbb{M}_{1,0}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{18}[\mathbb{T}_{2,1}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{53}[\mathbb{M}_{1,1}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{17}[\mathbb{T}_{2,0}^{(u,E,1)}]}{2}$$

No. 81 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)\,[\mathrm{M_3, B_1}]$

$$\hat{\mathbb{Z}}_{81} = \mathbb{X}_{62}[\mathbb{M}_3^{(a,A_1)}(1,-1)] \otimes \mathbb{U}_{13}[\mathbb{T}_0^{(u,A_1)}]$$

No. 82 $\hat{\mathbb{G}}_2^{(A_1)}(1,-1)\,[\mathrm{M}_3,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{82} = \frac{\sqrt{21}\mathbb{X}_{48}[\mathbb{M}_3^{(a,A_2,1)}(1,-1)] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{7} - \frac{\sqrt{14}\mathbb{X}_{54}[\mathbb{M}_{3,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{7} + \frac{\sqrt{14}\mathbb{X}_{55}[\mathbb{M}_{3,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{7}$$

No. 83 $\hat{\mathbb{Q}}_3^{(A_1)}(1,-1)\,[\mathrm{M}_3,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{83} = \frac{\sqrt{3}\mathbb{X}_{49}[\mathbb{M}_3^{(a,A_2,2)}(1,-1)] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{56}[\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{4} + \frac{\sqrt{2}\mathbb{X}_{57}[\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{4}$$

No. 84 $\hat{\mathbb{G}}_4^{(A_1,1)}(1,-1)\,[\mathrm{M}_3,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{84} = \frac{2\sqrt{7}\mathbb{X}_{48}[\mathbb{M}_3^{(a,A_2,1)}(1,-1)] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{7} + \frac{\sqrt{42}\mathbb{X}_{54}[\mathbb{M}_{3,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{14} - \frac{\sqrt{42}\mathbb{X}_{55}[\mathbb{M}_{3,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{14}$$

No. 85 $\hat{\mathbb{G}}_4^{(A_1,2)}(1,-1)\,[\mathrm{M}_3,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{85} = \frac{\mathbb{X}_{49}[\mathbb{M}_3^{(a,A_2,2)}(1,-1)] \otimes \mathbb{U}_{14}[\mathbb{T}_1^{(u,A_2)}]}{2} + \frac{\sqrt{6}\mathbb{X}_{56}[\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{4} - \frac{\sqrt{6}\mathbb{X}_{57}[\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{4}$$

No. 86 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)\,[\mathrm{M}_3,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{86} = -\frac{\sqrt{2}\mathbb{X}_{54}[\mathbb{M}_{3,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{18}[\mathbb{T}_{2,1}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{55}[\mathbb{M}_{3,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{17}[\mathbb{T}_{2,0}^{(u,E,1)}]}{2}$$

No. 87 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)\,[\mathrm{M}_3,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{87} = -\frac{\sqrt{2}\mathbb{X}_{56}[\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{18}[\mathbb{T}_{2,1}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{57}[\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{17}[\mathbb{T}_{2,0}^{(u,E,1)}]}{2}$$

No. 88 $\hat{\mathbb{Q}}_2^{(A_1)}(1,0)\,[\mathrm{M}_3,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{88} = \mathbb{X}_{63}[\mathbb{T}_2^{(a,A_1)}(1,0)] \otimes \mathbb{U}_{13}[\mathbb{T}_0^{(u,A_1)}]$$

No. 89 $\hat{\mathbb{G}}_2^{(A_1)}(1,0)\,[\mathrm{M}_3,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{89} = -\frac{\sqrt{2}\mathbb{X}_{58}[\mathbb{T}_{2,0}^{(a,E,1)}(1,0)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{59}[\mathbb{T}_{2,1}^{(a,E,1)}(1,0)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{2}$$

$\boxed{\text{No. 90}}$ $\hat{\mathbb{Q}}_3^{(A_1)}(1,0)$ $[\text{M}_3, \text{B}_1]$

$$\hat{\mathbb{Z}}_{90} = \frac{\sqrt{2}\mathbb{X}_{60}[\mathbb{T}_{2,0}^{(a,E,2)}(1,0)] \otimes \mathbb{U}_{15}[\mathbb{T}_{1,0}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{61}[\mathbb{T}_{2,1}^{(a,E,2)}(1,0)] \otimes \mathbb{U}_{16}[\mathbb{T}_{1,1}^{(u,E)}]}{2}$$

$\boxed{\text{No. 91}}$ $\hat{\mathbb{Q}}_0^{(A_1)}(1,0)$ $[\text{M}_3, \text{B}_1]$

$$\hat{\mathbb{Z}}_{91} = \frac{\sqrt{2}\mathbb{X}_{58}[\mathbb{T}_{2,0}^{(a,E,1)}(1,0)] \otimes \mathbb{U}_{17}[\mathbb{T}_{2,0}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{59}[\mathbb{T}_{2,1}^{(a,E,1)}(1,0)] \otimes \mathbb{U}_{18}[\mathbb{T}_{2,1}^{(u,E,1)}]}{2}$$

$\boxed{\text{No. 92}}$ $\hat{\mathbb{G}}_3^{(A_1)}(1,0)$ $[\text{M}_3, \text{B}_1]$

$$\hat{\mathbb{Z}}_{92} = \frac{\sqrt{2}\mathbb{X}_{60}[\mathbb{T}_{2,0}^{(a,E,2)}(1,0)] \otimes \mathbb{U}_{17}[\mathbb{T}_{2,0}^{(u,E,1)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{61}[\mathbb{T}_{2,1}^{(a,E,2)}(1,0)] \otimes \mathbb{U}_{18}[\mathbb{T}_{2,1}^{(u,E,1)}]}{2}$$

$\boxed{\text{No. 93}}$ $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\text{M}_1, \text{B}_2]$

$$\hat{\mathbb{Z}}_{93} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_{19}[\mathbb{Q}_0^{(u,A_1)}]$$

$\boxed{\text{No. 94}}$ $\hat{\mathbb{G}}_0^{(A_1)}(1,-1)$ $[\text{M}_1, \text{B}_2]$

$$\hat{\mathbb{Z}}_{94} = \frac{\sqrt{3}\mathbb{X}_2[\mathbb{M}_1^{(a,A_2)}(1,-1)] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_3[\mathbb{M}_{1,0}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,1}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{3}$$

$\boxed{\text{No. 95}}$ $\hat{\mathbb{G}}_2^{(A_1)}(1,-1)$ $[\text{M}_1, \text{B}_2]$

$$\hat{\mathbb{Z}}_{95} = \frac{\sqrt{6}\mathbb{X}_2[\mathbb{M}_1^{(a,A_2)}(1,-1)] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{3} + \frac{\sqrt{6}\mathbb{X}_3[\mathbb{M}_{1,0}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_4[\mathbb{M}_{1,1}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{6}$$

$\boxed{\text{No. 96}}$ $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\text{M}_2, \text{B}_2]$

$$\hat{\mathbb{Z}}_{96} = \frac{\sqrt{2}\mathbb{X}_6[\mathbb{Q}_{1,0}^{(a,E)}] \otimes \mathbb{U}_{20}[\mathbb{Q}_{1,0}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_7[\mathbb{Q}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{21}[\mathbb{Q}_{1,1}^{(u,E)}]}{2}$$

$\boxed{\text{No. 97}}$ $\hat{\mathbb{Q}}_0^{(A_1)}(1,0)$ $[\text{M}_2, \text{B}_2]$

$$\hat{\mathbb{Z}}_{97} = \frac{\sqrt{2}\mathbb{X}_{10}[\mathbb{Q}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_{21}[\mathbb{Q}_{1,1}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_9[\mathbb{Q}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_{20}[\mathbb{Q}_{1,0}^{(u,E)}]}{2}$$

14

No. 98 $\hat{\mathbb{G}}_2^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{98} = \mathbb{X}_{28}[\mathbb{G}_2^{(a,A_1)}(1,-1)] \otimes \mathbb{U}_{19}[\mathbb{Q}_0^{(u,A_1)}]$$

No. 99 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{99} = \frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{G}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{21}[\mathbb{Q}_{1,1}^{(u,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{G}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{20}[\mathbb{Q}_{1,0}^{(u,E)}]}{2}$$

No. 100 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{100} = -\frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{G}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{21}[\mathbb{Q}_{1,1}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{20}[\mathbb{Q}_{1,0}^{(u,E)}]}{2}$$

No. 101 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{101} = \frac{\sqrt{2}\mathbb{X}_{17}[\mathbb{M}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{18}[\mathbb{M}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{2}$$

No. 102 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{102} = -\frac{\sqrt{2}\mathbb{X}_{19}[\mathbb{M}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{20}[\mathbb{M}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{2}$$

No. 103 $\hat{\mathbb{Q}}_0^{(A_1)}(1,0)$ $[\mathrm{M}_2, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{103} = \frac{\sqrt{3}\mathbb{X}_{21}[\mathbb{T}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{22}[\mathbb{T}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{T}_1^{(a,A_2)}(1,0)] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{3}$$

No. 104 $\hat{\mathbb{Q}}_2^{(A_1)}(1,0)$ $[\mathrm{M}_2, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{104} = -\frac{\sqrt{6}\mathbb{X}_{21}[\mathbb{T}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_{22}[\mathbb{T}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{6} + \frac{\sqrt{6}\mathbb{X}_{26}[\mathbb{T}_1^{(a,A_2)}(1,0)] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{3}$$

No. 105 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_2, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{105} = \frac{\sqrt{3}\mathbb{X}_{23}[\mathbb{T}_1^{(a,A_2)}] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{24}[\mathbb{T}_{1,0}^{(a,E)}] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{25}[\mathbb{T}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{3}$$

No. 106 $\quad \hat{\mathbb{Q}}_2^{(A_1)} \, [M_2, B_2]$

$$\hat{\mathbb{Z}}_{106} = \frac{\sqrt{6}\mathbb{X}_{23}[\mathbb{T}_1^{(a,A_2)}] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{3} - \frac{\sqrt{6}\mathbb{X}_{24}[\mathbb{T}_{1,0}^{(a,E)}] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_{25}[\mathbb{T}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{6}$$

No. 107 $\quad \hat{\mathbb{G}}_0^{(A_1)}(1,1) \, [M_2, B_2]$

$$\hat{\mathbb{Z}}_{107} = \mathbb{X}_{27}[\mathbb{G}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_{19}[\mathbb{Q}_0^{(u,A_1)}]$$

No. 108 $\quad \hat{\mathbb{Q}}_0^{(A_1)} \, [M_3, B_2]$

$$\hat{\mathbb{Z}}_{108} = \mathbb{X}_{29}[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_{19}[\mathbb{Q}_0^{(u,A_1)}]$$

No. 109 $\quad \hat{\mathbb{Q}}_2^{(A_1)} \, [M_3, B_2]$

$$\hat{\mathbb{Z}}_{109} = \mathbb{X}_{30}[\mathbb{Q}_2^{(a,A_1)}] \otimes \mathbb{U}_{19}[\mathbb{Q}_0^{(u,A_1)}]$$

No. 110 $\quad \hat{\mathbb{G}}_2^{(A_1)} \, [M_3, B_2]$

$$\hat{\mathbb{Z}}_{110} = -\frac{\sqrt{2}\mathbb{X}_{33}[\mathbb{Q}_{2,0}^{(a,E,1)}] \otimes \mathbb{U}_{20}[\mathbb{Q}_{1,0}^{(u,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{34}[\mathbb{Q}_{2,1}^{(a,E,1)}] \otimes \mathbb{U}_{21}[\mathbb{Q}_{1,1}^{(u,E)}]}{2}$$

No. 111 $\quad \hat{\mathbb{Q}}_3^{(A_1)} \, [M_3, B_2]$

$$\hat{\mathbb{Z}}_{111} = \frac{\sqrt{2}\mathbb{X}_{35}[\mathbb{Q}_{2,0}^{(a,E,2)}] \otimes \mathbb{U}_{20}[\mathbb{Q}_{1,0}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{36}[\mathbb{Q}_{2,1}^{(a,E,2)}] \otimes \mathbb{U}_{21}[\mathbb{Q}_{1,1}^{(u,E)}]}{2}$$

No. 112 $\quad \hat{\mathbb{Q}}_0^{(A_1)}(1,1) \, [M_3, B_2]$

$$\hat{\mathbb{Z}}_{112} = \mathbb{X}_{31}[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_{19}[\mathbb{Q}_0^{(u,A_1)}]$$

No. 113 $\quad \hat{\mathbb{Q}}_2^{(A_1)}(1,-1) \, [M_3, B_2]$

$$\hat{\mathbb{Z}}_{113} = \mathbb{X}_{32}[\mathbb{Q}_2^{(a,A_1)}(1,-1)] \otimes \mathbb{U}_{19}[\mathbb{Q}_0^{(u,A_1)}]$$

No. 114 $\quad \hat{\mathbb{G}}_2^{(A_1)}(1,-1) \, [M_3, B_2]$

$$\hat{\mathbb{Z}}_{114} = -\frac{\sqrt{2}\mathbb{X}_{37}[\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{20}[\mathbb{Q}_{1,0}^{(u,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{38}[\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{21}[\mathbb{Q}_{1,1}^{(u,E)}]}{2}$$

16

No. 115 $\quad \hat{\mathbb{Q}}_3^{(A_1)}(1,-1) \ [\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{115} = \frac{\sqrt{2}\mathbb{X}_{39}[\mathbb{Q}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{20}[\mathbb{Q}_{1,0}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{40}[\mathbb{Q}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{21}[\mathbb{Q}_{1,1}^{(u,E)}]}{2}$$

No. 116 $\quad \hat{\mathbb{G}}_0^{(A_1)}(1,0) \ [\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{116} = -\frac{\sqrt{2}\mathbb{X}_{41}[\mathbb{G}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{U}_{21}[\mathbb{Q}_{1,1}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{42}[\mathbb{G}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{U}_{20}[\mathbb{Q}_{1,0}^{(u,E)}]}{2}$$

No. 117 $\quad \hat{\mathbb{G}}_0^{(A_1)} \ [\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{117} = \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_1^{(a,A_2)}] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{44}[\mathbb{M}_{1,0}^{(a,E)}] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{45}[\mathbb{M}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{3}$$

No. 118 $\quad \hat{\mathbb{G}}_2^{(A_1)} \ [\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{118} = \frac{\sqrt{6}\mathbb{X}_{43}[\mathbb{M}_1^{(a,A_2)}] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{3} + \frac{\sqrt{6}\mathbb{X}_{44}[\mathbb{M}_{1,0}^{(a,E)}] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_{45}[\mathbb{M}_{1,1}^{(a,E)}] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{6}$$

No. 119 $\quad \hat{\mathbb{G}}_0^{(A_1)}(1,1) \ [\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{119} = \frac{\sqrt{3}\mathbb{X}_{46}[\mathbb{M}_1^{(a,A_2)}(1,1)] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{50}[\mathbb{M}_{1,0}^{(a,E)}(1,1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{51}[\mathbb{M}_{1,1}^{(a,E)}(1,1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{3}$$

No. 120 $\quad \hat{\mathbb{G}}_2^{(A_1)}(1,1) \ [\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{120} = \frac{\sqrt{6}\mathbb{X}_{46}[\mathbb{M}_1^{(a,A_2)}(1,1)] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{3} + \frac{\sqrt{6}\mathbb{X}_{50}[\mathbb{M}_{1,0}^{(a,E)}(1,1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_{51}[\mathbb{M}_{1,1}^{(a,E)}(1,1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{6}$$

No. 121 $\quad \hat{\mathbb{G}}_0^{(A_1)}(1,-1) \ [\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{121} = \frac{\sqrt{3}\mathbb{X}_{47}[\mathbb{M}_1^{(a,A_2)}(1,-1)] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{52}[\mathbb{M}_{1,0}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{53}[\mathbb{M}_{1,1}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{3}$$

No. 122 $\quad \hat{\mathbb{G}}_2^{(A_1)}(1,-1) \ [\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{122} = \frac{\sqrt{6}\mathbb{X}_{47}[\mathbb{M}_1^{(a,A_2)}(1,-1)] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{3} + \frac{\sqrt{6}\mathbb{X}_{52}[\mathbb{M}_{1,0}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{6} - \frac{\sqrt{6}\mathbb{X}_{53}[\mathbb{M}_{1,1}^{(a,E)}(1,-1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{6}$$

No. 123 $\hat{\mathbb{G}}_2^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{123} = \frac{\sqrt{21}\mathbb{X}_{48}[\mathbb{M}_3^{(a,A_2,1)}(1,-1)] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{7} - \frac{\sqrt{14}\mathbb{X}_{54}[\mathbb{M}_{3,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{7} + \frac{\sqrt{14}\mathbb{X}_{55}[\mathbb{M}_{3,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{7}$$

No. 124 $\hat{\mathbb{Q}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{124} = \frac{\sqrt{3}\mathbb{X}_{49}[\mathbb{M}_3^{(a,A_2,2)}(1,-1)] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{56}[\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{4} + \frac{\sqrt{2}\mathbb{X}_{57}[\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{4}$$

No. 125 $\hat{\mathbb{G}}_4^{(A_1,1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{125} = \frac{2\sqrt{7}\mathbb{X}_{48}[\mathbb{M}_3^{(a,A_2,1)}(1,-1)] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{7} + \frac{\sqrt{42}\mathbb{X}_{54}[\mathbb{M}_{3,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{14} - \frac{\sqrt{42}\mathbb{X}_{55}[\mathbb{M}_{3,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{14}$$

No. 126 $\hat{\mathbb{G}}_4^{(A_1,2)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{126} = \frac{\mathbb{X}_{49}[\mathbb{M}_3^{(a,A_2,2)}(1,-1)] \otimes \mathbb{U}_{22}[\mathbb{T}_1^{(u,A_2)}]}{2} + \frac{\sqrt{6}\mathbb{X}_{56}[\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{4} - \frac{\sqrt{6}\mathbb{X}_{57}[\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{4}$$

No. 127 $\hat{\mathbb{G}}_2^{(A_1)}(1,0)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{127} = -\frac{\sqrt{2}\mathbb{X}_{58}[\mathbb{T}_{2,0}^{(a,E,1)}(1,0)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{59}[\mathbb{T}_{2,1}^{(a,E,1)}(1,0)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{2}$$

No. 128 $\hat{\mathbb{Q}}_3^{(A_1)}(1,0)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{128} = \frac{\sqrt{2}\mathbb{X}_{60}[\mathbb{T}_{2,0}^{(a,E,2)}(1,0)] \otimes \mathbb{U}_{23}[\mathbb{T}_{1,0}^{(u,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{61}[\mathbb{T}_{2,1}^{(a,E,2)}(1,0)] \otimes \mathbb{U}_{24}[\mathbb{T}_{1,1}^{(u,E)}]}{2}$$

Table 4: Atomic SAMB group.

group	bra	ket
M_1	$(s,\uparrow),(s,\downarrow)$	$(s,\uparrow),(s,\downarrow)$
M_2	$(s,\uparrow),(s,\downarrow)$	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$

continued …

18

Table 4

group	bra	ket
M_3	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$

Table 5: Atomic SAMB.

symbol	type	group	form
$\mathbb{X}_1$	$\mathbb{Q}_0^{(a,A_1)}$	M_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{X}_2$	$\mathbb{M}_1^{(a,A_2)}(1,-1)$	M_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{X}_3$	$\mathbb{M}_{1,0}^{(a,E)}(1,-1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{2}i}{2} \\ -\frac{\sqrt{2}i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_4$	$\mathbb{M}_{1,1}^{(a,E)}(1,-1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$
$\mathbb{X}_5$	$\mathbb{Q}_1^{(a,A_2)}$	M_2	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{X}_6$	$\mathbb{Q}_{1,0}^{(a,E)}$	M_2	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_7$	$\mathbb{Q}_{1,1}^{(a,E)}$	M_2	$\begin{pmatrix} 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_8$	$\mathbb{Q}_1^{(a,A_2)}(1,0)$	M_2	$\begin{pmatrix} 0 & -\frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_9$	$\mathbb{Q}_{1,0}^{(a,E)}(1,0)$	M_2	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{i}{2} & -\frac{1}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{10}$	$\mathbb{Q}_{1,1}^{(a,E)}(1,0)$	M_2	$\begin{pmatrix} \frac{i}{2} & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & -\frac{i}{2} & 0 & 0 & -\frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{11}$	$\mathbb{G}_{2,0}^{(a,E,1)}(1,-1)$	M_2	$\begin{pmatrix} \frac{i}{2} & 0 & 0 & 0 & 0 & \frac{i}{2} \\ 0 & -\frac{i}{2} & 0 & 0 & \frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{12}$	$\mathbb{G}_{2,1}^{(a,E,1)}(1,-1)$	M_2	$\begin{pmatrix} 0 & 0 & \frac{i}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & -\frac{i}{2} & -\frac{1}{2} & 0 \end{pmatrix}$

continued ...

19

Table 5

symbol	type	group	form
$\mathbb{X}_{13}$	$\mathbb{G}_{2,0}^{(a,E,2)}(1,-1)$	M$_2$	$\begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{14}$	$\mathbb{G}_{2,1}^{(a,E,2)}(1,-1)$	M$_2$	$\begin{pmatrix} 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{15}$	$\mathbb{M}_0^{(a,A_1)}(1,1)$	M$_2$	$\begin{pmatrix} 0 & \frac{\sqrt{6}}{6} & 0 & -\frac{\sqrt{6}i}{6} & \frac{\sqrt{6}}{6} & 0 \\ \frac{\sqrt{6}}{6} & 0 & \frac{\sqrt{6}i}{6} & 0 & 0 & -\frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_{16}$	$\mathbb{M}_2^{(a,A_1)}(1,-1)$	M$_2$	$\begin{pmatrix} 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{3}}{6} & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{X}_{17}$	$\mathbb{M}_{2,0}^{(a,E,1)}(1,-1)$	M$_2$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{18}$	$\mathbb{M}_{2,1}^{(a,E,1)}(1,-1)$	M$_2$	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{19}$	$\mathbb{M}_{2,0}^{(a,E,2)}(1,-1)$	M$_2$	$\begin{pmatrix} 0 & -\frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{20}$	$\mathbb{M}_{2,1}^{(a,E,2)}(1,-1)$	M$_2$	$\begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 \\ \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{21}$	$\mathbb{T}_{1,0}^{(a,E)}(1,0)$	M$_2$	$\begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{22}$	$\mathbb{T}_{1,1}^{(a,E)}(1,0)$	M$_2$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{23}$	$\mathbb{T}_1^{(a,A_2)}$	M$_2$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{2} \end{pmatrix}$
$\mathbb{X}_{24}$	$\mathbb{T}_{1,0}^{(a,E)}$	M$_2$	$\begin{pmatrix} \frac{\sqrt{2}i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{25}$	$\mathbb{T}_{1,1}^{(a,E)}$	M$_2$	$\begin{pmatrix} 0 & 0 & \frac{\sqrt{2}i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}i}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{26}$	$\mathbb{T}_1^{(a,A_2)}(1,0)$	M$_2$	$\begin{pmatrix} 0 & \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 \\ -\frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{27}$	$\mathbb{G}_0^{(a,A_1)}(1,1)$	M$_2$	$\begin{pmatrix} 0 & \frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}i}{6} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}i}{6} \end{pmatrix}$
$\mathbb{X}_{28}$	$\mathbb{G}_2^{(a,A_1)}(1,-1)$	M$_2$	$\begin{pmatrix} 0 & -\frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}i}{3} & 0 \\ -\frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}i}{3} \end{pmatrix}$

Table 5

symbol	type	group	form
$\mathbb{X}_{29}$	$\mathbb{Q}_0^{(a,A_1)}$	M_3	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_{30}$	$\mathbb{Q}_2^{(a,A_1)}$	M_3	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{X}_{31}$	$\mathbb{Q}_0^{(a,A_1)}(1,1)$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ \frac{\sqrt{3}i}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{32}$	$\mathbb{Q}_2^{(a,A_1)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{33}$	$\mathbb{Q}_{2,0}^{(a,E,1)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$

21

Table 5

symbol	type	group	form
$\mathbb{X}_{34}$	$\mathbb{Q}_{2,1}^{(a,E,1)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{35}$	$\mathbb{Q}_{2,0}^{(a,E,2)}$	M_3	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{36}$	$\mathbb{Q}_{2,1}^{(a,E,2)}$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{37}$	$\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{38}$	$\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$

Table 5

symbol	type	group	form
$\mathbb{X}_{39}$	$\mathbb{Q}_{2,0}^{(a,E,2)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{40}$	$\mathbb{Q}_{2,1}^{(a,E,2)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{41}$	$\mathbb{G}_{1,0}^{(a,E)}(1,0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{42}$	$\mathbb{G}_{1,1}^{(a,E)}(1,0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{43}$	$\mathbb{M}_{1}^{(a,A_2)}$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

Table 5

symbol	type	group	form
$\mathbb{X}_{44}$	$\mathbb{M}_{1,0}^{(a,E)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{45}$	$\mathbb{M}_{1,1}^{(a,E)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{46}$	$\mathbb{M}_{1}^{(a,A_2)}(1,1)$	M_3	$\begin{pmatrix} -\frac{\sqrt{30}}{30} & 0 & 0 & 0 & 0 & \frac{\sqrt{30}}{20} \\ 0 & \frac{\sqrt{30}}{30} & 0 & 0 & \frac{\sqrt{30}}{20} & 0 \\ 0 & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & -\frac{\sqrt{30}i}{20} \\ 0 & 0 & 0 & \frac{\sqrt{30}}{30} & \frac{\sqrt{30}i}{20} & 0 \\ 0 & \frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{15} & 0 \\ \frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{15} \end{pmatrix}$
$\mathbb{X}_{47}$	$\mathbb{M}_{1}^{(a,A_2)}(1,-1)$	M_3	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_{48}$	$\mathbb{M}_{3}^{(a,A_2,1)}(1,-1)$	M_3	$\begin{pmatrix} -\frac{\sqrt{5}}{10} & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}}{10} & 0 & 0 & -\frac{\sqrt{5}}{10} & 0 \\ 0 & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & \frac{\sqrt{5}i}{10} \\ 0 & 0 & 0 & \frac{\sqrt{5}}{10} & -\frac{\sqrt{5}i}{10} & 0 \\ 0 & -\frac{\sqrt{5}}{10} & 0 & \frac{\sqrt{5}i}{10} & \frac{\sqrt{5}}{5} & 0 \\ -\frac{\sqrt{5}}{10} & 0 & -\frac{\sqrt{5}i}{10} & 0 & 0 & -\frac{\sqrt{5}}{5} \end{pmatrix}$

Table 5

symbol	type	group	form
$\mathbb{X}_{49}$	$\mathbb{M}_3^{(a,A_2,2)}(1,-1)$	M_3	$\begin{pmatrix} 0 & -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{50}$	$\mathbb{M}_{1,0}^{(a,E)}(1,1)$	M_3	$\begin{pmatrix} 0 & -\frac{\sqrt{30}i}{30} & 0 & -\frac{\sqrt{30}}{20} & 0 & 0 \\ \frac{\sqrt{30}i}{30} & 0 & -\frac{\sqrt{30}}{20} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{15} & -\frac{\sqrt{30}}{20} & 0 \\ -\frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{15} & 0 & 0 & \frac{\sqrt{30}}{20} \\ 0 & 0 & -\frac{\sqrt{30}}{20} & 0 & 0 & -\frac{\sqrt{30}i}{30} \\ 0 & 0 & 0 & \frac{\sqrt{30}}{20} & \frac{\sqrt{30}i}{30} & 0 \end{pmatrix}$
$\mathbb{X}_{51}$	$\mathbb{M}_{1,1}^{(a,E)}(1,1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{30}}{15} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{20} & 0 \\ \frac{\sqrt{30}}{15} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{20} \\ 0 & -\frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 \\ \frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & 0 \\ \frac{\sqrt{30}}{20} & 0 & 0 & 0 & 0 & -\frac{\sqrt{30}}{30} \\ 0 & -\frac{\sqrt{30}}{20} & 0 & 0 & -\frac{\sqrt{30}}{30} & 0 \end{pmatrix}$
$\mathbb{X}_{52}$	$\mathbb{M}_{1,0}^{(a,E)}(1,-1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{53}$	$\mathbb{M}_{1,1}^{(a,E)}(1,-1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \end{pmatrix}$

continued ...

Table 5

symbol	type	group	form
$\mathbb{X}_{54}$	$\mathbb{M}_{3,0}^{(a,E,1)}(1,-1)$	M_3	$\begin{pmatrix} 0 & -\frac{\sqrt{30}i}{60} & 0 & \frac{\sqrt{30}}{60} & 0 & 0 \\ \frac{\sqrt{30}i}{60} & 0 & \frac{\sqrt{30}}{60} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{30}}{60} & 0 & -\frac{\sqrt{30}i}{20} & -\frac{\sqrt{30}}{15} & 0 \\ \frac{\sqrt{30}}{60} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & \frac{\sqrt{30}}{15} \\ 0 & 0 & -\frac{\sqrt{30}}{15} & 0 & 0 & \frac{\sqrt{30}i}{15} \\ 0 & 0 & 0 & \frac{\sqrt{30}}{15} & -\frac{\sqrt{30}i}{15} & 0 \end{pmatrix}$
$\mathbb{X}_{55}$	$\mathbb{M}_{3,1}^{(a,E,1)}(1,-1)$	M_3	$\begin{pmatrix} 0 & -\frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{60} & \frac{\sqrt{30}}{15} & 0 \\ -\frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{60} & 0 & 0 & -\frac{\sqrt{30}}{15} \\ 0 & \frac{\sqrt{30}i}{60} & 0 & -\frac{\sqrt{30}}{60} & 0 & 0 \\ -\frac{\sqrt{30}i}{60} & 0 & -\frac{\sqrt{30}}{60} & 0 & 0 & 0 \\ \frac{\sqrt{30}}{15} & 0 & 0 & 0 & 0 & \frac{\sqrt{30}}{15} \\ 0 & -\frac{\sqrt{30}}{15} & 0 & 0 & \frac{\sqrt{30}}{15} & 0 \end{pmatrix}$
$\mathbb{X}_{56}$	$\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)$	M_3	$\begin{pmatrix} \frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{57}$	$\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}i}{6} & 0 \\ -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \\ 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 \\ 0 & \frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ -\frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{58}$	$\mathbb{T}_{2,0}^{(a,E,1)}(1,0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ -\frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \\ 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 \end{pmatrix}$

Table 5

symbol	type	group	form
$\mathbb{X}_{59}$	$\mathbb{T}_{2,1}^{(a,E,1)}(1,0)$	M$_3$	$\begin{pmatrix} 0 & -\frac{\sqrt{6}i}{6} & 0 & -\frac{\sqrt{6}}{12} & 0 & 0 \\ \frac{\sqrt{6}i}{6} & 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & \frac{\sqrt{6}}{12} & 0 \\ -\frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & 0 & \frac{\sqrt{6}}{12} & 0 & 0 & \frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{12} & -\frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{60}$	$\mathbb{T}_{2,0}^{(a,E,2)}(1,0)$	M$_3$	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}i}{12} & 0 \\ -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{12} \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & \frac{\sqrt{6}}{12} & 0 \\ 0 & -\frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 \\ \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{61}$	$\mathbb{T}_{2,1}^{(a,E,2)}(1,0)$	M$_3$	$\begin{pmatrix} -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{12} \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & \frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}i}{12} & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & \frac{\sqrt{6}i}{12} & 0 & 0 \\ \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{62}$	$\mathbb{M}_3^{(a,A_1)}(1,-1)$	M$_3$	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{63}$	$\mathbb{T}_2^{(a,A_1)}(1,0)$	M$_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$

continued ...

27

Table 5

symbol	type	group	form
$\mathbb{X}_{64}$	$\mathbb{G}_1^{(a,A_2)}(1,0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$

Table 6: Uniform SAMB.

symbol	type	cluster	form
$\mathbb{U}_1$	$\mathbb{Q}_0^{(s,A_1)}$	S_1	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{U}_2$	$\mathbb{Q}_1^{(s,A_2)}$	S_1	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{U}_3$	$\mathbb{Q}_{1,0}^{(s,E)}$	S_1	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}$

Table 6

symbol	type	cluster	form
$\mathbb{U}_4$	$\mathbb{Q}_{1,1}^{(s,E)}$	S_1	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \end{pmatrix}$
$\mathbb{U}_5$	$\mathbb{Q}_{2,0}^{(s,E,1)}$	S_1	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{3}}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \end{pmatrix}$
$\mathbb{U}_6$	$\mathbb{Q}_{2,1}^{(s,E,1)}$	S_1	$\begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$
$\mathbb{U}_7$	$\mathbb{Q}_{0}^{(u,A_1)}$	B_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} \\ 0 & 0 & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{6} \\ \frac{\sqrt{3}}{6} & 0 & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \end{pmatrix}$
$\mathbb{U}_8$	$\mathbb{Q}_{1}^{(u,A_2)}$	B_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} \\ 0 & 0 & -\frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{6} \\ \frac{\sqrt{3}}{6} & 0 & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \end{pmatrix}$

continued ...

29

Table 6

symbol	type	cluster	form
$\mathbb{U}_9$	$\mathbb{Q}_{1,0}^{(u,E)}$	B$_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} \\ 0 & 0 & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & 0 & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{10}$	$\mathbb{Q}_{1,1}^{(u,E)}$	B$_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{12} & -\frac{\sqrt{6}}{12} \\ 0 & 0 & \frac{\sqrt{6}}{12} & \frac{\sqrt{6}}{12} & 0 & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & \frac{\sqrt{6}}{12} & -\frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{12} & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \end{pmatrix}$
$\mathbb{U}_{11}$	$\mathbb{Q}_{2,0}^{(u,E,1)}$	B$_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{12} & -\frac{\sqrt{6}}{12} \\ 0 & 0 & -\frac{\sqrt{6}}{12} & -\frac{\sqrt{6}}{12} & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{12} & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{12} & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \end{pmatrix}$
$\mathbb{U}_{12}$	$\mathbb{Q}_{2,1}^{(u,E,1)}$	B$_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} \\ 0 & 0 & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & 0 & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{13}$	$\mathbb{T}_{0}^{(u,A_1)}$	B$_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} \\ 0 & 0 & -\frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} & 0 & 0 \\ 0 & \frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 \\ 0 & -\frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} & 0 & 0 & 0 \\ \frac{\sqrt{3}i}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ -\frac{\sqrt{3}i}{6} & 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & 0 \end{pmatrix}$

continued …

30

Table 6

symbol	type	cluster	form
$\mathbb{U}_{14}$	$\mathbb{T}_1^{(u,A_2)}$	B$_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} \\ 0 & 0 & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}i}{6} & 0 & 0 \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ 0 & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}i}{6} & 0 & 0 & 0 \\ \frac{\sqrt{3}i}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ -\frac{\sqrt{3}i}{6} & 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & 0 \end{pmatrix}$
$\mathbb{U}_{15}$	$\mathbb{T}_{1,0}^{(u,E)}$	B$_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{16}$	$\mathbb{T}_{1,1}^{(u,E)}$	B$_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & -\frac{\sqrt{6}i}{12} \\ 0 & 0 & -\frac{\sqrt{6}i}{12} & \frac{\sqrt{6}i}{12} & 0 & 0 \\ 0 & \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}i}{6} & 0 & 0 \\ 0 & -\frac{\sqrt{6}i}{12} & -\frac{\sqrt{6}i}{6} & 0 & 0 & 0 \\ -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} \\ \frac{\sqrt{6}i}{12} & 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
$\mathbb{U}_{17}$	$\mathbb{T}_{2,0}^{(u,E,1)}$	B$_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & -\frac{\sqrt{6}i}{12} \\ 0 & 0 & \frac{\sqrt{6}i}{12} & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ 0 & -\frac{\sqrt{6}i}{12} & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 \\ 0 & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}i}{6} & 0 & 0 & 0 \\ -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} \\ \frac{\sqrt{6}i}{12} & 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
$\mathbb{U}_{18}$	$\mathbb{T}_{2,1}^{(u,E,1)}$	B$_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & \frac{\sqrt{2}i}{4} \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

31

Table 6

symbol	type	cluster	form
$\mathbb{U}_{19}$	$\mathbb{Q}_0^{(u,A_1)}$	B$_2$	$\begin{pmatrix} 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{20}$	$\mathbb{Q}_{1,0}^{(u,E)}$	B$_2$	$\begin{pmatrix} 0 & \frac{\sqrt{3}}{3} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{3}}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{21}$	$\mathbb{Q}_{1,1}^{(u,E)}$	B$_2$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{22}$	$\mathbb{T}_1^{(u,A_2)}$	B$_2$	$\begin{pmatrix} 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{23}$	$\mathbb{T}_{1,0}^{(u,E)}$	B$_2$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\ 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$

continued …

Table 6

symbol	type	cluster	form
$\mathbb{U}_{24}$	$\mathbb{T}_{1,1}^{(u,E)}$	B$_2$	$\begin{pmatrix} 0 & \frac{\sqrt{3}i}{3} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{3}i}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$

Table 7: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_0^{(A_1)}$	0	A_1	$-$	$-$	1
2	$\mathbb{Q}_1^{(A_2)}$	1	A_2	$-$	$-$	z
3	$\mathbb{Q}_{1,0}^{(E)}$	1	E	$-$	0	x
4	$\mathbb{Q}_{1,1}^{(E)}$	1	E	$-$	1	y
5	$\mathbb{Q}_2^{(A_1)}$	2	A_1	$-$	$-$	$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$
6	$\mathbb{Q}_{2,0}^{(E,1)}$	2	E	1	0	$\sqrt{3}yz$
7	$\mathbb{Q}_{2,1}^{(E,1)}$	2	E	1	1	$-\sqrt{3}xz$
8	$\mathbb{Q}_{2,0}^{(E,2)}$	2	E	2	0	$\frac{\sqrt{3}(x-y)(x+y)}{2}$
9	$\mathbb{Q}_{2,1}^{(E,2)}$	2	E	2	1	$-\sqrt{3}xy$

Table 8: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_0^{(A_1)}$	0	A_1	$-$	$-$	1

continued …

Table 8

No.	symbol	rank	irrep.	mul.	comp.	form
2	$\mathbb{G}_1^{(A_2)}$	1	A_2	$-$	$-$	Z
3	$\mathbb{G}_{1,0}^{(E)}$	1	E	$-$	0	$-Y$
4	$\mathbb{G}_{1,1}^{(E)}$	1	E	$-$	1	X
5	$\mathbb{G}_2^{(A_1)}$	2	A_1	$-$	$-$	$-\frac{X^2}{2}-\frac{Y^2}{2}+Z^2$
6	$\mathbb{G}_{2,0}^{(E,1)}$	2	E	1	0	$\sqrt{3}XZ$
7	$\mathbb{G}_{2,1}^{(E,1)}$	2	E	1	1	$\sqrt{3}YZ$
8	$\mathbb{G}_{2,0}^{(E,2)}$	2	E	2	0	$\sqrt{3}XY$
9	$\mathbb{G}_{2,1}^{(E,2)}$	2	E	2	1	$\frac{\sqrt{3}(X-Y)(X+Y)}{2}$
10	$\mathbb{G}_3^{(A_1)}$	3	A_1	$-$	$-$	$\frac{\sqrt{10}X(X^2-3Y^2)}{4}$
11	$\mathbb{G}_3^{(A_2,1)}$	3	A_2	1	$-$	$-\frac{Z(3X^2+3Y^2-2Z^2)}{2}$
12	$\mathbb{G}_3^{(A_2,2)}$	3	A_2	2	$-$	$\frac{\sqrt{10}Y(3X^2-Y^2)}{4}$
13	$\mathbb{G}_{3,0}^{(E,1)}$	3	E	1	0	$\frac{\sqrt{6}Y(X^2+Y^2-4Z^2)}{4}$
14	$\mathbb{G}_{3,1}^{(E,1)}$	3	E	1	1	$-\frac{\sqrt{6}X(X^2+Y^2-4Z^2)}{4}$
15	$\mathbb{G}_{3,0}^{(E,2)}$	3	E	2	0	$\frac{\sqrt{15}Z(X-Y)(X+Y)}{2}$
16	$\mathbb{G}_{3,1}^{(E,2)}$	3	E	2	1	$-\sqrt{15}XYZ$

- Group info.: Generator = 3_{001}^+, 2_{110}

Table 9: Conjugacy class.

rep. SO	symmetry operations
1	1
2_{100}	2_{100}, 2_{010}, 2_{110}
3_{001}^+	3_{001}^+, 3_{001}^-

Table 10: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	1	2	2_{100}	3	2_{010}	4	2_{110}	5	3^{+}_{001}
6	3^{-}_{001}								

Table 11: Character table.

	1	2_{100}	3^{+}_{001}
A_1	1	1	1
A_2	1	-1	1
E	2	0	-1

Table 12: Parity conversion.

$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$
A_1 (A_1)	A_2 (A_2)	E (E)

Table 13: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

	A_1	A_2	E
A_1	A_1	A_2	E
A_2		A_1	E
E			$A_1 + E$

Table 14: Anti-symmetric product, $[\Gamma \otimes \Gamma]_-$.

A_1	A_2	E
−	−	A_2

Table 15: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix} 1 & -1 & 1 \end{pmatrix}$	2	$\begin{pmatrix} 2 & 1 & -1 \end{pmatrix}$	3	$\begin{pmatrix} -1 & -2 & -1 \end{pmatrix}$	4	$\begin{pmatrix} -1 & 1 & -1 \end{pmatrix}$
5	$\begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$	6	$\begin{pmatrix} -2 & -1 & 1 \end{pmatrix}$				

Table 16: Virtual-cluster basis.

symbol	1	2	3	4	5	6
$\mathbb{Q}_0^{(A_1)}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$
$\mathbb{Q}_1^{(A_2)}$	$\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$
$\mathbb{Q}_{1,0}^{(E)}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
$\mathbb{Q}_{1,1}^{(E)}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{6}$
$\mathbb{Q}_{2,0}^{(E,1)}$	$-\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{6}$
$\mathbb{Q}_{2,1}^{(E,1)}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$