

# SAMB for “grapheneAB”

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- Group: No. 187  $D_{3h}^1$   $P-6m2$  [ hexagonal ]
  - Associated point group: No. 26  $D_{3h}$   $-6m2$  (-6m2 setting) [ hexagonal ]
  - Generation condition
    - model type: **tight\_binding**
    - time-reversal type: **electric**
    - irrep: [A1']
    - spinless
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- Unit cell:  
 $a = 2.435$ ,  $b = 2.435$ ,  $c = 10.0$ ,  $\alpha = 90.0$ ,  $\beta = 90.0$ ,  $\gamma = 120.0$
- Lattice vectors:  
 $\mathbf{a}_1 = (2.435 \ 0 \ 0)$   
 $\mathbf{a}_2 = (-1.2175 \ 2.10877185821511 \ 0)$   
 $\mathbf{a}_3 = (0 \ 0 \ 10.0)$

Table 1: High-symmetry line:  $\Gamma$ -X.

symbol	position	symbol	position
$\Gamma$	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	X	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$

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- Kets: dimension = 3

Table 2: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket
1	$s@A_1$	2	$p_x@B_1$	3	$p_y@B_1$

- Sites in (primitive) unit cell:

Table 3: Site-clusters.

site	position	mapping
S <sub>1</sub> A <sub>1</sub>	$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix}$	[1,2,3,4,5,6,7,8,9,10,11,12]
S <sub>2</sub> B <sub>1</sub>	$\begin{pmatrix} \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}$	[1,2,3,4,5,6,7,8,9,10,11,12]

- Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

bond	tail	head	$n$	#	$\mathbf{b@c}$	mapping
B <sub>1</sub>	b <sub>1</sub>	B <sub>1</sub> A <sub>1</sub>	1	1	$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$	[1,2,7,10]
	b <sub>2</sub>	B <sub>1</sub> A <sub>1</sub>	1	1	$\begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	[3,6,8,11]
	b <sub>3</sub>	B <sub>1</sub> A <sub>1</sub>	1	1	$\begin{pmatrix} -\frac{2}{3} & -\frac{1}{3} & 0 \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{2} & 0 \end{pmatrix}$	[4,5,9,12]
B <sub>2</sub>	b <sub>4</sub>	A <sub>1</sub> A <sub>1</sub>	1	1	$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & 0 \end{pmatrix}$	[1,-3,-8,10]
	b <sub>5</sub>	A <sub>1</sub> A <sub>1</sub>	1	1	$\begin{pmatrix} 1 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{5}{6} & \frac{1}{6} & 0 \end{pmatrix}$	[2,-5,7,-12]
	b <sub>6</sub>	A <sub>1</sub> A <sub>1</sub>	1	1	$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{5}{6} & \frac{2}{3} & 0 \end{pmatrix}$	[-4,6,-9,11]
B <sub>3</sub>	b <sub>7</sub>	B <sub>1</sub> B <sub>1</sub>	1	1	$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & 0 \end{pmatrix}$	[1,-2,-7,10]
	b <sub>8</sub>	B <sub>1</sub> B <sub>1</sub>	1	1	$\begin{pmatrix} 1 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{6} & \frac{5}{6} & 0 \end{pmatrix}$	[3,-6,8,-11]
	b <sub>9</sub>	B <sub>1</sub> B <sub>1</sub>	1	1	$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{2}{3} & \frac{5}{6} & 0 \end{pmatrix}$	[-4,5,-9,12]

- SAMB:

$$\boxed{\text{No. 1}} \quad \hat{Q}_0^{(A'_1)} [M_1, S_1]$$

$$\hat{Z}_1 = \mathbb{X}_1[Q_0^{(a, A'_1)}] \otimes \mathbb{Y}_1[Q_0^{(s, A'_1)}]$$

$$\hat{Z}_1(\mathbf{k}) = \mathbb{X}_1[Q_0^{(a, A'_1)}] \otimes \mathbb{U}_1[Q_0^{(s, A'_1)}]$$

$$\boxed{\text{No. 2}} \quad \hat{Q}_0^{(A'_1)} [M_2, S_2]$$

$$\hat{Z}_2 = \mathbb{X}_2[Q_0^{(a, A'_1)}] \otimes \mathbb{Y}_2[Q_0^{(s, A'_1)}]$$

$$\hat{Z}_2(\mathbf{k}) = \mathbb{X}_2[Q_0^{(a, A'_1)}] \otimes \mathbb{U}_2[Q_0^{(s, A'_1)}]$$

$$\boxed{\text{No. 3}} \quad \hat{Q}_0^{(A'_1)} [M_3, B_1]$$

$$\hat{Z}_3 = \frac{\sqrt{2}\mathbb{X}_6[Q_{1,0}^{(a, E')}] \otimes \mathbb{Y}_3[Q_{1,0}^{(b, E')}] }{2} + \frac{\sqrt{2}\mathbb{X}_7[Q_{1,1}^{(a, E')}] \otimes \mathbb{Y}_4[Q_{1,1}^{(b, E')}] }{2}$$

$$\begin{aligned} \hat{Z}_3(\mathbf{k}) = & \frac{\mathbb{X}_6[Q_{1,0}^{(a, E')}] \otimes \mathbb{U}_3[Q_0^{(u, A'_1)}] \otimes \mathbb{F}_1[Q_{1,0}^{(k, E')}] }{2} - \frac{\mathbb{X}_6[Q_{1,0}^{(a, E')}] \otimes \mathbb{U}_4[\mathbb{T}_0^{(u, A'_1)}] \otimes \mathbb{F}_3[\mathbb{T}_{1,0}^{(k, E')}] }{2} \\ & + \frac{\mathbb{X}_7[Q_{1,1}^{(a, E')}] \otimes \mathbb{U}_3[Q_0^{(u, A'_1)}] \otimes \mathbb{F}_2[Q_{1,1}^{(k, E')}] }{2} - \frac{\mathbb{X}_7[Q_{1,1}^{(a, E')}] \otimes \mathbb{U}_4[\mathbb{T}_0^{(u, A'_1)}] \otimes \mathbb{F}_4[\mathbb{T}_{1,1}^{(k, E')}] }{2} \end{aligned}$$

$$\boxed{\text{No. 4}} \quad \hat{Q}_0^{(A'_1)} [M_1, B_2]$$

$$\hat{Z}_4 = \mathbb{X}_1[Q_0^{(a, A'_1)}] \otimes \mathbb{Y}_5[Q_0^{(b, A'_1)}]$$

$$\hat{Z}_4(\mathbf{k}) = \mathbb{X}_1[Q_0^{(a, A'_1)}] \otimes \mathbb{U}_1[Q_0^{(s, A'_1)}] \otimes \mathbb{F}_5[Q_0^{(k, A'_1)}]$$

$$\boxed{\text{No. 5}} \quad \hat{Q}_0^{(A'_1)} [M_2, B_3]$$

$$\hat{Z}_5 = \mathbb{X}_2[Q_0^{(a, A'_1)}] \otimes \mathbb{Y}_6[Q_0^{(b, A'_1)}]$$

$$\hat{\mathbb{Z}}_5(\mathbf{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A'_1)}] \otimes \mathbb{U}_2[\mathbb{Q}_0^{(s,A'_1)}] \otimes \mathbb{F}_6[\mathbb{Q}_0^{(k,A'_1)}]$$

$$\boxed{\text{No. 6}} \quad \hat{\mathbb{Q}}_3^{(A'_1)} [\mathbb{M}_2, \mathbb{B}_3]$$

$$\hat{\mathbb{Z}}_6 = -\frac{\sqrt{2}\mathbb{X}_3[\mathbb{Q}_{2,0}^{(a,E')}] \otimes \mathbb{Y}_7[\mathbb{Q}_{1,0}^{(b,E')}] }{2} - \frac{\sqrt{2}\mathbb{X}_4[\mathbb{Q}_{2,1}^{(a,E')}] \otimes \mathbb{Y}_8[\mathbb{Q}_{1,1}^{(b,E')}] }{2}$$

$$\hat{\mathbb{Z}}_6(\mathbf{k}) = -\frac{\sqrt{2}\mathbb{X}_3[\mathbb{Q}_{2,0}^{(a,E')}] \otimes \mathbb{U}_2[\mathbb{Q}_0^{(s,A'_1)}] \otimes \mathbb{F}_7[\mathbb{Q}_{1,0}^{(k,E')}] }{2} - \frac{\sqrt{2}\mathbb{X}_4[\mathbb{Q}_{2,1}^{(a,E')}] \otimes \mathbb{U}_2[\mathbb{Q}_0^{(s,A'_1)}] \otimes \mathbb{F}_8[\mathbb{Q}_{1,1}^{(k,E')}] }{2}$$

$$\boxed{\text{No. 7}} \quad \hat{\mathbb{Q}}_3^{(A'_1)} [\mathbb{M}_2, \mathbb{B}_3]$$

$$\hat{\mathbb{Z}}_7 = \mathbb{X}_5[\mathbb{M}_1^{(a,A'_2)}] \otimes \mathbb{Y}_9[\mathbb{T}_3^{(b,A'_2)}]$$

$$\hat{\mathbb{Z}}_7(\mathbf{k}) = \mathbb{X}_5[\mathbb{M}_1^{(a,A'_2)}] \otimes \mathbb{U}_2[\mathbb{Q}_0^{(s,A'_1)}] \otimes \mathbb{F}_9[\mathbb{T}_3^{(k,A'_2)}]$$

Table 5: Atomic SAMB group.

group	bra	ket
$\mathbb{M}_1$	$s$	$s$
$\mathbb{M}_2$	$p_x, p_y$	$p_x, p_y$
$\mathbb{M}_3$	$p_x, p_y$	$s$

Table 6: Atomic SAMB.

symbol	type	group	form
$\mathbb{X}_1$	$\mathbb{Q}_0^{(a,A'_1)}$	$\mathbb{M}_1$	$\begin{pmatrix} 1 \end{pmatrix}$
$\mathbb{X}_2$	$\mathbb{Q}_0^{(a,A'_1)}$	$\mathbb{M}_2$	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$

*continued ...*

Table 6

symbol	type	group	form
$\mathbb{X}_3$	$\mathbb{Q}_{2,0}^{(a,E')}$	$M_2$	$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & 0 \end{pmatrix}$
$\mathbb{X}_4$	$\mathbb{Q}_{2,1}^{(a,E')}$	$M_2$	$\begin{pmatrix} -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{X}_5$	$\mathbb{M}_1^{(a,A'_2)}$	$M_2$	$\begin{pmatrix} 0 & -\frac{\sqrt{2}i}{2} \\ \frac{\sqrt{2}i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_6$	$\mathbb{Q}_{1,0}^{(a,E')}$	$M_3$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\mathbb{X}_7$	$\mathbb{Q}_{1,1}^{(a,E')}$	$M_3$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Table 7: Cluster SAMB.

symbol	type	cluster	form
$\mathbb{Y}_1$	$\mathbb{Q}_0^{(s,A'_1)}$	$S_1$	$\begin{pmatrix} 1 \end{pmatrix}$
$\mathbb{Y}_2$	$\mathbb{Q}_0^{(s,A'_1)}$	$S_2$	$\begin{pmatrix} 1 \end{pmatrix}$
$\mathbb{Y}_3$	$\mathbb{Q}_{1,0}^{(b,E')}$	$B_1$	$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_4$	$\mathbb{Q}_{1,1}^{(b,E')}$	$B_1$	$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_5$	$\mathbb{Q}_0^{(b,A'_1)}$	$B_2$	$\begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{Y}_6$	$\mathbb{Q}_0^{(b,A'_1)}$	$B_3$	$\begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{Y}_7$	$\mathbb{Q}_{1,0}^{(b,E')}$	$B_3$	$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_8$	$\mathbb{Q}_{1,1}^{(b,E')}$	$B_3$	$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_9$	$\mathbb{T}_3^{(b,A'_2)}$	$B_3$	$\begin{pmatrix} \frac{\sqrt{3}i}{3} & -\frac{\sqrt{3}i}{3} & \frac{\sqrt{3}i}{3} \end{pmatrix}$

Table 8: Uniform SAMB.

symbol	type	cluster	form
$U_1$	$\mathbb{Q}_0^{(s,A'_1)}$	$S_1$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
$U_2$	$\mathbb{Q}_0^{(s,A'_1)}$	$S_2$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
$U_3$	$\mathbb{Q}_0^{(u,A'_1)}$	$B_1$	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$
$U_4$	$\mathbb{T}_0^{(u,A'_1)}$	$B_1$	$\begin{pmatrix} 0 & -\frac{\sqrt{2}i}{2} \\ \frac{\sqrt{2}i}{2} & 0 \end{pmatrix}$

Table 9: Structure SAMB.

symbol	type	cluster	form
$F_1$	$\mathbb{Q}_{1,0}^{(k,E')}$	$B_1$	$-c_{002} + c_{003}$
$F_2$	$\mathbb{Q}_{1,1}^{(k,E')}$	$B_1$	$-\frac{2\sqrt{3}c_{001}}{3} + \frac{\sqrt{3}c_{002}}{3} + \frac{\sqrt{3}c_{003}}{3}$
$F_3$	$\mathbb{T}_{1,0}^{(k,E')}$	$B_1$	$-s_{002} + s_{003}$
$F_4$	$\mathbb{T}_{1,1}^{(k,E')}$	$B_1$	$-\frac{2\sqrt{3}s_{001}}{3} + \frac{\sqrt{3}s_{002}}{3} + \frac{\sqrt{3}s_{003}}{3}$
$F_5$	$\mathbb{Q}_0^{(k,A'_1)}$	$B_2$	$\frac{\sqrt{6}c_{004}}{3} + \frac{\sqrt{6}c_{005}}{3} + \frac{\sqrt{6}c_{006}}{3}$
$F_6$	$\mathbb{Q}_0^{(k,A'_1)}$	$B_3$	$\frac{\sqrt{6}c_{007}}{3} + \frac{\sqrt{6}c_{008}}{3} + \frac{\sqrt{6}c_{009}}{3}$
$F_7$	$\mathbb{Q}_{1,0}^{(k,E')}$	$B_3$	$-c_{008} + c_{009}$
$F_8$	$\mathbb{Q}_{1,1}^{(k,E')}$	$B_3$	$-\frac{2\sqrt{3}c_{007}}{3} + \frac{\sqrt{3}c_{008}}{3} + \frac{\sqrt{3}c_{009}}{3}$
$F_9$	$\mathbb{T}_3^{(k,A'_2)}$	$B_3$	$\frac{\sqrt{6}s_{007}}{3} - \frac{\sqrt{6}s_{008}}{3} + \frac{\sqrt{6}s_{009}}{3}$

Table 10: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_0^{(A'_1)}$	0	$A'_1$	—	—	1
2	$\mathbb{Q}_{1,0}^{(E')}$	1	$E'$	—	0	$x$
3	$\mathbb{Q}_{1,1}^{(E')}$	1	$E'$	—	1	$y$
4	$\mathbb{Q}_{2,0}^{(E')}$	2	$E'$	—	0	$-\sqrt{3}xy$
5	$\mathbb{Q}_{2,1}^{(E')}$	2	$E'$	—	1	$-\frac{\sqrt{3}(x-y)(x+y)}{2}$
6	$\mathbb{Q}_3^{(A'_2)}$	3	$A'_2$	—	—	$\frac{\sqrt{10}x(x^2-3y^2)}{4}$

Table 11: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_1^{(A'_2)}$	1	$A'_2$	—	—	$Z$

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- Group info.: Generator =  $\{3_{001}^+|0\}$ ,  $\{m_{001}|0\}$ ,  $\{m_{110}|0\}$

Table 12: Conjugacy class (point-group part).

rep. SO	symmetry operations
$\{1 0\}$	$\{1 0\}$
$\{2_{120} 0\}$	$\{2_{120} 0\}$ , $\{2_{210} 0\}$ , $\{2_{1-10} 0\}$
$\{3_{001}^+ 0\}$	$\{3_{001}^+ 0\}$ , $\{3_{001}^- 0\}$
$\{m_{100} 0\}$	$\{m_{100} 0\}$ , $\{m_{010} 0\}$ , $\{m_{110} 0\}$
$\{m_{001} 0\}$	$\{m_{001} 0\}$
$\{-6_{001}^+ 0\}$	$\{-6_{001}^+ 0\}$ , $\{-6_{001}^- 0\}$

Table 13: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	$\{1 0\}$	2	$\{2_{120} 0\}$	3	$\{2_{210} 0\}$	4	$\{2_{1-10} 0\}$	5	$\{3_{001}^+ 0\}$
6	$\{3_{001}^- 0\}$	7	$\{m_{100} 0\}$	8	$\{m_{010} 0\}$	9	$\{m_{110} 0\}$	10	$\{m_{001} 0\}$
11	$\{-6_{001}^+ 0\}$	12	$\{-6_{001}^- 0\}$						

Table 14: Character table (point-group part).

	1	$2_{120}$	$3_{001}^+$	$m_{100}$	$m_{001}$	$-6_{001}^+$
$A_1'$	1	1	1	1	1	1
$A_2'$	1	-1	1	-1	1	1
$A_1''$	1	1	1	-1	-1	-1
$A_2''$	1	-1	1	1	-1	-1
$E'$	2	0	-1	0	2	-1
$E''$	2	0	-1	0	-2	1

Table 15: Parity conversion.

$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$
$A_1' (A_1'')$	$A_2' (A_2'')$	$A_1'' (A_1')$	$A_2'' (A_2')$	$E'' (E')$
$E' (E'')$				

Table 16: Symmetric product,  $[\Gamma \otimes \Gamma']_+$ .

	$A_1'$	$A_2'$	$A_1''$	$A_2''$	$E'$	$E''$
$A_1'$	$A_1'$	$A_2'$	$A_1''$	$A_2''$	$E'$	$E''$
$A_2'$		$A_1'$	$A_2''$	$A_1''$	$E'$	$E''$
$A_1''$			$A_1'$	$A_2'$	$E''$	$E'$

continued ...



Table 16

	$A'_1$	$A'_2$	$A''_1$	$A''_2$	$E'$	$E''$
$A''_2$				$A'_1$	$E''$	$E'$
$E'$					$A'_1 + E'$	$A'_1 + A''_2 + E''$
$E''$						$A'_1 + E'$

Table 17: Anti-symmetric product,  $[\Gamma \otimes \Gamma]_-$ .

$A'_1$	$A'_2$	$A''_1$	$A''_2$	$E'$	$E''$
$-$	$-$	$-$	$-$	$A'_2$	$A'_2$

Table 18: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix} -1 & -1 & 1 \end{pmatrix}$	2	$\begin{pmatrix} 0 & -1 & -1 \end{pmatrix}$	3	$\begin{pmatrix} -1 & 0 & -1 \end{pmatrix}$	4	$\begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$
5	$\begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$	6	$\begin{pmatrix} 0 & 1 & 1 \end{pmatrix}$	7	$\begin{pmatrix} 0 & -1 & 1 \end{pmatrix}$	8	$\begin{pmatrix} -1 & 0 & 1 \end{pmatrix}$
9	$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$	10	$\begin{pmatrix} -1 & -1 & -1 \end{pmatrix}$	11	$\begin{pmatrix} 0 & 1 & -1 \end{pmatrix}$	12	$\begin{pmatrix} 1 & 0 & -1 \end{pmatrix}$

Table 19: Virtual-cluster basis.

symbol	1	2	3	4	5	6	7	8	9	10
$\mathbb{Q}_0^{(A'_1)}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$
	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$								

*continued ...*

Table 19

symbol	1	2	3	4	5	6	7	8	9	10
$\mathbb{Q}_1^{(A_2'')}$	$\frac{\sqrt{3}}{6}$ $-\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$ $-\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$
$\mathbb{Q}_{1,0}^{(E')}$	$-\frac{\sqrt{6}}{12}$ $-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$ $\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$
$\mathbb{Q}_{1,1}^{(E')}$	$-\frac{\sqrt{2}}{4}$ $\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$ 0	0	$\frac{\sqrt{2}}{4}$	0	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$
$\mathbb{Q}_{2,0}^{(E')}$	$-\frac{\sqrt{2}}{4}$ $\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$ 0	0	$-\frac{\sqrt{2}}{4}$	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	0	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$
$\mathbb{Q}_{2,1}^{(E')}$	$\frac{\sqrt{6}}{12}$ $\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$ $-\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
$\mathbb{Q}_{2,0}^{(E'')}$	$-\frac{\sqrt{6}}{12}$ $\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$ $-\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
$\mathbb{Q}_{2,1}^{(E'')}$	$-\frac{\sqrt{2}}{4}$ $-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$ 0	0	$-\frac{\sqrt{2}}{4}$	0	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$
$\mathbb{Q}_3^{(A_2')}$	$\frac{\sqrt{3}}{6}$ $\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$ $\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$
$\mathbb{Q}_{3,0}^{(E'')}$	$-\frac{\sqrt{2}}{4}$ $-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$ 0	0	$\frac{\sqrt{2}}{4}$	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	0	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$
$\mathbb{Q}_{3,1}^{(E'')}$	$\frac{\sqrt{6}}{12}$ $-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$ $\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$
$\mathbb{Q}_4^{(A_1'')}$	$\frac{\sqrt{3}}{6}$ $-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$ $-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$