

SAMB for “Th1”

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- Group: No. 200 T_h^1 $Pm-3$ [cubic]
 - Associated point group: No. 29 T_h $m-3$ [cubic]
 - Generation condition
 - model type: **tight_binding**
 - time-reversal type: **electric**
 - irrep: [Ag]
 - spinful
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- Unit cell:
 - $a = 1.0$, $b = 1.0$, $c = 1.0$, $\alpha = 90.0$, $\beta = 90.0$, $\gamma = 90.0$
- Lattice vectors:
 - $\mathbf{a}_1 = (1.0 \ 0 \ 0)$
 - $\mathbf{a}_2 = (0 \ 1.0 \ 0)$
 - $\mathbf{a}_3 = (0 \ 0 \ 1.0)$

Table 1: High-symmetry line: Γ -X.

	symbol	position		symbol	position
	Γ	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$		X	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$

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- Kets: dimension = 24

Table 2: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket	No.	ket	No.	ket
1	$(s, \uparrow)@A_1$	2	$(s, \downarrow)@A_1$	3	$(p_x, \uparrow)@A_1$	4	$(p_x, \downarrow)@A_1$	5	$(p_y, \uparrow)@A_1$
6	$(p_y, \downarrow)@A_1$	7	$(p_z, \uparrow)@A_1$	8	$(p_z, \downarrow)@A_1$	9	$(s, \uparrow)@A_2$	10	$(s, \downarrow)@A_2$
11	$(p_x, \uparrow)@A_2$	12	$(p_x, \downarrow)@A_2$	13	$(p_y, \uparrow)@A_2$	14	$(p_y, \downarrow)@A_2$	15	$(p_z, \uparrow)@A_2$
16	$(p_z, \downarrow)@A_2$	17	$(s, \uparrow)@A_3$	18	$(s, \downarrow)@A_3$	19	$(p_x, \uparrow)@A_3$	20	$(p_x, \downarrow)@A_3$
21	$(p_y, \uparrow)@A_3$	22	$(p_y, \downarrow)@A_3$	23	$(p_z, \uparrow)@A_3$	24	$(p_z, \downarrow)@A_3$		

- Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position	mapping
S ₁ [3d: mmm..]	A ₁	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$	[1,2,3,4,13,14,15,16]
	A ₂	$\begin{pmatrix} 0 & \frac{1}{2} & 0 \end{pmatrix}$	[5,6,7,8,17,18,19,20]
	A ₃	$\begin{pmatrix} 0 & 0 & \frac{1}{2} \end{pmatrix}$	[9,10,11,12,21,22,23,24]

- Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

	bond	tail	head	n	#	$\mathbf{b@c}$	mapping
B ₁ [12j: m..]	b ₁	A ₂	A ₁	1	1	$\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$	[1,14]
	b ₂	A ₂	A ₁	1	1	$\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{3}{4} & \frac{3}{4} & 0 \end{pmatrix}$	[2,13]
	b ₃	A ₂	A ₁	1	1	$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{4} & \frac{3}{4} & 0 \end{pmatrix}$	[3,16]
	b ₄	A ₂	A ₁	1	1	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \end{pmatrix}$	[4,15]
	b ₅	A ₃	A ₂	1	1	$\begin{pmatrix} 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$	[5,19]
	b ₆	A ₃	A ₂	1	1	$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} @ \begin{pmatrix} 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$	[6,20]

continued ...

Table 4

	bond	tail	head	n	#	$\mathbf{b}@\mathbf{c}$	mapping
	b ₇	A ₃	A ₂	1	1	$\begin{pmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} @ \begin{pmatrix} 0 & \frac{3}{4} & \frac{3}{4} \end{pmatrix}$	[7,17]
	b ₈	A ₃	A ₂	1	1	$\begin{pmatrix} 0 & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{4} & \frac{3}{4} \end{pmatrix}$	[8,18]
	b ₉	A ₃	A ₁	1	1	$\begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} @ \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$	[-9,-22]
	b ₁₀	A ₃	A ₁	1	1	$\begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} @ \begin{pmatrix} \frac{3}{4} & 0 & \frac{3}{4} \end{pmatrix}$	[-10,-21]
	b ₁₁	A ₃	A ₁	1	1	$\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} @ \begin{pmatrix} \frac{3}{4} & 0 & \frac{1}{4} \end{pmatrix}$	[-11,-24]
	b ₁₂	A ₃	A ₁	1	1	$\begin{pmatrix} -\frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} @ \begin{pmatrix} \frac{1}{4} & 0 & \frac{3}{4} \end{pmatrix}$	[-12,-23]
B ₂ [3c: mmm..]	b ₁₃	A ₁	A ₁	2	1	$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$	[1,2,-3,-4,-13,-14,15,16]
	b ₁₄	A ₂	A ₂	2	1	$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	[5,-6,7,-8,-17,18,-19,20]
	b ₁₅	A ₃	A ₃	2	1	$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	[9,10,-11,-12,-21,-22,23,24]
B ₃ [1a: m-3.]	b ₁₆	A ₁	A ₁	2	2	$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	[1,-2,3,-4,-13,14,-15,16]
	b ₁₇	A ₂	A ₂	2	2	$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	[5,-6,-7,8,-17,18,19,-20]
	b ₁₈	A ₃	A ₃	2	2	$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} @ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	[9,-10,11,-12,-21,22,-23,24]
B ₄ [3c: mmm..]	b ₁₉	A ₁	A ₁	2	3	$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	[1,-2,-3,4,-13,14,15,-16]
	b ₂₀	A ₂	A ₂	2	3	$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	[5,6,-7,-8,-17,-18,19,20]
	b ₂₁	A ₃	A ₃	2	3	$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$	[9,-10,-11,12,-21,22,23,-24]

- SAMB:

$$\boxed{\text{No. 1}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\mathbb{M}_1, \mathbb{S}_1]$$

$$\hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_g)}]$$

$$\boxed{\text{No. 2}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\mathbb{M}_3, \mathbb{S}_1]$$

$$\hat{\mathbb{Z}}_2 = \mathbb{X}_{23}[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_g)}]$$

$$\boxed{\text{No. 3}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 1) [\mathbb{M}_3, \mathbb{S}_1]$$

$$\hat{\mathbb{Z}}_3 = \mathbb{X}_{24}[\mathbb{Q}_0^{(a,A_g)}(1, 1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_g)}]$$

$$\boxed{\text{No. 4}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_4 = \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s,E_g)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{26}[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s,E_g)}]}{2}$$

$$\boxed{\text{No. 5}} \quad \hat{\mathbb{G}}_3^{(A_g)} [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_5 = \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s,E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{26}[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s,E_g)}]}{2}$$

$$\boxed{\text{No. 6}} \quad \hat{\mathbb{Q}}_0^{(A_g)} (1, -1) [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_6 = \frac{\sqrt{2}\mathbb{X}_{30}[\mathbb{Q}_{2,0}^{(a,E_g)} (1, -1)] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s,E_g)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{31}[\mathbb{Q}_{2,1}^{(a,E_g)} (1, -1)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s,E_g)}]}{2}$$

$$\boxed{\text{No. 7}} \quad \hat{\mathbb{G}}_3^{(A_g)} (1, -1) [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_7 = \frac{\sqrt{2}\mathbb{X}_{30}[\mathbb{Q}_{2,0}^{(a,E_g)} (1, -1)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s,E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{31}[\mathbb{Q}_{2,1}^{(a,E_g)} (1, -1)] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s,E_g)}]}{2}$$

$$\boxed{\text{No. 8}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_1, \text{B}_1]$$

$$\hat{\mathbb{Z}}_8 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 9}} \quad \hat{\mathbb{G}}_3^{(A_g)} (1, -1) [\text{M}_1, \text{B}_1]$$

$$\hat{\mathbb{Z}}_9 = \frac{\sqrt{3}\mathbb{X}_2[\mathbb{M}_{1,0}^{(a,T_g)} (1, -1)] \otimes \mathbb{Y}_{22}[\mathbb{T}_{2,0}^{(b,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_3[\mathbb{M}_{1,1}^{(a,T_g)} (1, -1)] \otimes \mathbb{Y}_{23}[\mathbb{T}_{2,1}^{(b,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_g)} (1, -1)] \otimes \mathbb{Y}_{24}[\mathbb{T}_{2,2}^{(b,T_g)}]}{3}$$

$$\boxed{\text{No. 10}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{10} = \frac{\sqrt{3}\mathbb{X}_5[\mathbb{Q}_{1,0}^{(a,T_u)}] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_6[\mathbb{Q}_{1,1}^{(a,T_u)}] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_7[\mathbb{Q}_{1,2}^{(a,T_u)}] \otimes \mathbb{Y}_7[\mathbb{Q}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 11}} \quad \hat{\mathbb{Q}}_4^{(A_g)} [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{11} = \frac{\sqrt{3}\mathbb{X}_5[\mathbb{Q}_{1,0}^{(a,T_u)}] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,0}^{(b,T_u,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_6[\mathbb{Q}_{1,1}^{(a,T_u)}] \otimes \mathbb{Y}_{14}[\mathbb{Q}_{3,1}^{(b,T_u,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_7[\mathbb{Q}_{1,2}^{(a,T_u)}] \otimes \mathbb{Y}_{15}[\mathbb{Q}_{3,2}^{(b,T_u,1)}]}{3}$$

$$\boxed{\text{No. 12}} \quad \hat{\mathbb{Q}}_0^{(Ag)}(1, 0) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{12} = \frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{Q}_{1,2}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_7[\mathbb{Q}_{1,2}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_8[\mathbb{Q}_{1,0}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_9[\mathbb{Q}_{1,1}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 13}} \quad \hat{\mathbb{Q}}_4^{(Ag)}(1, 0) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{13} = \frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{Q}_{1,2}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{15}[\mathbb{Q}_{3,2}^{(b,T_u,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_8[\mathbb{Q}_{1,0}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,0}^{(b,T_u,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_9[\mathbb{Q}_{1,1}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{14}[\mathbb{Q}_{3,1}^{(b,T_u,1)}]}{3}$$

$$\boxed{\text{No. 14}} \quad \hat{\mathbb{G}}_3^{(Ag)}(1, -1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{14} = \frac{\sqrt{3}\mathbb{X}_{11}[\mathbb{G}_{2,0}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{12}[\mathbb{G}_{2,1}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{13}[\mathbb{G}_{2,2}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_7[\mathbb{Q}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 15}} \quad \hat{\mathbb{G}}_3^{(Ag)}(1, -1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{15} = -\frac{\sqrt{3}\mathbb{X}_{11}[\mathbb{G}_{2,0}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,0}^{(b,T_u,1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{12}[\mathbb{G}_{2,1}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{14}[\mathbb{Q}_{3,1}^{(b,T_u,1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{13}[\mathbb{G}_{2,2}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{15}[\mathbb{Q}_{3,2}^{(b,T_u,1)}]}{3}$$

$$\boxed{\text{No. 16}} \quad \hat{\mathbb{G}}_3^{(Ag)}(1, -1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{16} = \frac{\sqrt{3}\mathbb{X}_{14}[\mathbb{M}_{2,0}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{17}[\mathbb{T}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{15}[\mathbb{M}_{2,1}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{18}[\mathbb{T}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{16}[\mathbb{M}_{2,2}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 17}} \quad \hat{\mathbb{G}}_3^{(Ag)}(1, -1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{17} = -\frac{\sqrt{3}\mathbb{X}_{14}[\mathbb{M}_{2,0}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{25}[\mathbb{T}_{3,0}^{(b,T_u,1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{15}[\mathbb{M}_{2,1}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{26}[\mathbb{T}_{3,1}^{(b,T_u,1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{16}[\mathbb{M}_{2,2}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{27}[\mathbb{T}_{3,2}^{(b,T_u,1)}]}{3}$$

$$\boxed{\text{No. 18}} \quad \hat{\mathbb{Q}}_0^{(Ag)}(1, 0) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{18} = \frac{\sqrt{3}\mathbb{X}_{17}[\mathbb{T}_{1,0}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{17}[\mathbb{T}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{18}[\mathbb{T}_{1,1}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{18}[\mathbb{T}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{19}[\mathbb{T}_{1,2}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 19}} \quad \hat{\mathbb{Q}}_4^{(Ag)}(1, 0) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{19} = \frac{\sqrt{3}\mathbb{X}_{17}[\mathbb{T}_{1,0}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{25}[\mathbb{T}_{3,0}^{(b,T_u,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{18}[\mathbb{T}_{1,1}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{26}[\mathbb{T}_{3,1}^{(b,T_u,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{19}[\mathbb{T}_{1,2}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{27}[\mathbb{T}_{3,2}^{(b,T_u,1)}]}{3}$$

$$\boxed{\text{No. 20}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{20} = \frac{\sqrt{3}\mathbb{X}_{20}[\mathbb{T}_{1,0}^{(a,T_u)}] \otimes \mathbb{Y}_{17}[\mathbb{T}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{21}[\mathbb{T}_{1,1}^{(a,T_u)}] \otimes \mathbb{Y}_{18}[\mathbb{T}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{22}[\mathbb{T}_{1,2}^{(a,T_u)}] \otimes \mathbb{Y}_{19}[\mathbb{T}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 21}} \quad \hat{\mathbb{Q}}_4^{(A_g)} [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{21} = \frac{\sqrt{3}\mathbb{X}_{20}[\mathbb{T}_{1,0}^{(a,T_u)}] \otimes \mathbb{Y}_{25}[\mathbb{T}_{3,0}^{(b,T_u,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{21}[\mathbb{T}_{1,1}^{(a,T_u)}] \otimes \mathbb{Y}_{26}[\mathbb{T}_{3,1}^{(b,T_u,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{22}[\mathbb{T}_{1,2}^{(a,T_u)}] \otimes \mathbb{Y}_{27}[\mathbb{T}_{3,2}^{(b,T_u,1)}]}{3}$$

$$\boxed{\text{No. 22}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{22} = \mathbb{X}_{23}[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 23}} \quad \hat{\mathbb{Q}}_0^{(A_g)} (1, 1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{23} = \mathbb{X}_{24}[\mathbb{Q}_0^{(a,A_g)} (1, 1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 24}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{24} = \frac{\sqrt{5}\mathbb{X}_{25}[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,0}^{(b,E_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{26}[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,1}^{(b,E_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{27}[\mathbb{Q}_{2,0}^{(a,T_g)}] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,0}^{(b,T_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{28}[\mathbb{Q}_{2,1}^{(a,T_g)}] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{2,1}^{(b,T_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{29}[\mathbb{Q}_{2,2}^{(a,T_g)}] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{2,2}^{(b,T_g)}]}{5}$$

$$\boxed{\text{No. 25}} \quad \hat{\mathbb{G}}_3^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{25} = \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,1}^{(b,E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{26}[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,0}^{(b,E_g)}]}{2}$$

$$\boxed{\text{No. 26}} \quad \hat{\mathbb{Q}}_4^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\begin{aligned} \hat{\mathbb{Z}}_{26} = & \frac{\sqrt{30}\mathbb{X}_{25}[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,0}^{(b,E_g)}]}{10} + \frac{\sqrt{30}\mathbb{X}_{26}[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,1}^{(b,E_g)}]}{10} - \frac{\sqrt{30}\mathbb{X}_{27}[\mathbb{Q}_{2,0}^{(a,T_g)}] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,0}^{(b,T_g)}]}{15} \\ & - \frac{\sqrt{30}\mathbb{X}_{28}[\mathbb{Q}_{2,1}^{(a,T_g)}] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{2,1}^{(b,T_g)}]}{15} - \frac{\sqrt{30}\mathbb{X}_{29}[\mathbb{Q}_{2,2}^{(a,T_g)}] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{2,2}^{(b,T_g)}]}{15} \end{aligned}$$

$$\begin{aligned}
\text{No. 27} \quad & \hat{\mathbb{Q}}_0^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1] \\
\hat{\mathbb{Z}}_{27} = & \frac{\sqrt{5}\mathbb{X}_{30}[\mathbb{Q}_{2,0}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,0}^{(b,E_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{31}[\mathbb{Q}_{2,1}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,1}^{(b,E_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{32}[\mathbb{Q}_{2,0}^{(a,T_g)}(1, -1)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,0}^{(b,T_g)}]}{5} \\
& + \frac{\sqrt{5}\mathbb{X}_{33}[\mathbb{Q}_{2,1}^{(a,T_g)}(1, -1)] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{2,1}^{(b,T_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{34}[\mathbb{Q}_{2,2}^{(a,T_g)}(1, -1)] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{2,2}^{(b,T_g)}]}{5} \\
\text{No. 28} \quad & \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1] \\
\hat{\mathbb{Z}}_{28} = & \frac{\sqrt{2}\mathbb{X}_{30}[\mathbb{Q}_{2,0}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,1}^{(b,E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{31}[\mathbb{Q}_{2,1}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,0}^{(b,E_g)}]}{2} \\
\text{No. 29} \quad & \hat{\mathbb{Q}}_4^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1] \\
\hat{\mathbb{Z}}_{29} = & \frac{\sqrt{30}\mathbb{X}_{30}[\mathbb{Q}_{2,0}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,0}^{(b,E_g)}]}{10} + \frac{\sqrt{30}\mathbb{X}_{31}[\mathbb{Q}_{2,1}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,1}^{(b,E_g)}]}{10} - \frac{\sqrt{30}\mathbb{X}_{32}[\mathbb{Q}_{2,0}^{(a,T_g)}(1, -1)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,0}^{(b,T_g)}]}{15} \\
& - \frac{\sqrt{30}\mathbb{X}_{33}[\mathbb{Q}_{2,1}^{(a,T_g)}(1, -1)] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{2,1}^{(b,T_g)}]}{15} - \frac{\sqrt{30}\mathbb{X}_{34}[\mathbb{Q}_{2,2}^{(a,T_g)}(1, -1)] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{2,2}^{(b,T_g)}]}{15} \\
\text{No. 30} \quad & \hat{\mathbb{G}}_3^{(A_g)}(1, 0) [\text{M}_3, \text{B}_1] \\
\hat{\mathbb{Z}}_{30} = & \frac{\sqrt{3}\mathbb{X}_{35}[\mathbb{G}_{1,0}^{(a,T_g)}(1, 0)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,0}^{(b,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{36}[\mathbb{G}_{1,1}^{(a,T_g)}(1, 0)] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{2,1}^{(b,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{37}[\mathbb{G}_{1,2}^{(a,T_g)}(1, 0)] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{2,2}^{(b,T_g)}]}{3} \\
\text{No. 31} \quad & \hat{\mathbb{G}}_3^{(A_g)} [\text{M}_3, \text{B}_1] \\
\hat{\mathbb{Z}}_{31} = & \frac{\sqrt{3}\mathbb{X}_{38}[\mathbb{M}_{1,0}^{(a,T_g)}] \otimes \mathbb{Y}_{22}[\mathbb{T}_{2,0}^{(b,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{39}[\mathbb{M}_{1,1}^{(a,T_g)}] \otimes \mathbb{Y}_{23}[\mathbb{T}_{2,1}^{(b,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_g)}] \otimes \mathbb{Y}_{24}[\mathbb{T}_{2,2}^{(b,T_g)}]}{3} \\
\text{No. 32} \quad & \hat{\mathbb{G}}_3^{(A_g)}(1, 1) [\text{M}_3, \text{B}_1] \\
\hat{\mathbb{Z}}_{32} = & \frac{\sqrt{3}\mathbb{X}_{41}[\mathbb{M}_{1,0}^{(a,T_g)}(1, 1)] \otimes \mathbb{Y}_{22}[\mathbb{T}_{2,0}^{(b,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{42}[\mathbb{M}_{1,1}^{(a,T_g)}(1, 1)] \otimes \mathbb{Y}_{23}[\mathbb{T}_{2,1}^{(b,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{1,2}^{(a,T_g)}(1, 1)] \otimes \mathbb{Y}_{24}[\mathbb{T}_{2,2}^{(b,T_g)}]}{3} \\
\text{No. 33} \quad & \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1] \\
\hat{\mathbb{Z}}_{33} = & \frac{\sqrt{3}\mathbb{X}_{44}[\mathbb{M}_{1,0}^{(a,T_g)}(1, -1)] \otimes \mathbb{Y}_{22}[\mathbb{T}_{2,0}^{(b,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{45}[\mathbb{M}_{1,1}^{(a,T_g)}(1, -1)] \otimes \mathbb{Y}_{23}[\mathbb{T}_{2,1}^{(b,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{46}[\mathbb{M}_{1,2}^{(a,T_g)}(1, -1)] \otimes \mathbb{Y}_{24}[\mathbb{T}_{2,2}^{(b,T_g)}]}{3}
\end{aligned}$$

$$\boxed{\text{No. 34}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{34} = \mathbb{X}_{56}[\mathbb{M}_3^{(a, A_g)}(1, -1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_0^{(b, A_g)}]$$

$$\boxed{\text{No. 35}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{35} = -\frac{\sqrt{3}\mathbb{X}_{47}[\mathbb{M}_{3,0}^{(a, T_g, 1)}(1, -1)] \otimes \mathbb{Y}_{22}[\mathbb{T}_{2,0}^{(b, T_g)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{48}[\mathbb{M}_{3,1}^{(a, T_g, 1)}(1, -1)] \otimes \mathbb{Y}_{23}[\mathbb{T}_{2,1}^{(b, T_g)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{49}[\mathbb{M}_{3,2}^{(a, T_g, 1)}(1, -1)] \otimes \mathbb{Y}_{24}[\mathbb{T}_{2,2}^{(b, T_g)}]}{3}$$

$$\boxed{\text{No. 36}} \quad \hat{\mathbb{Q}}_4^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{36} = -\frac{\sqrt{3}\mathbb{X}_{50}[\mathbb{M}_{3,0}^{(a, T_g, 2)}(1, -1)] \otimes \mathbb{Y}_{22}[\mathbb{T}_{2,0}^{(b, T_g)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{51}[\mathbb{M}_{3,1}^{(a, T_g, 2)}(1, -1)] \otimes \mathbb{Y}_{23}[\mathbb{T}_{2,1}^{(b, T_g)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{52}[\mathbb{M}_{3,2}^{(a, T_g, 2)}(1, -1)] \otimes \mathbb{Y}_{24}[\mathbb{T}_{2,2}^{(b, T_g)}]}{3}$$

$$\boxed{\text{No. 37}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 0) [\text{M}_3, \text{B}_1]$$

$$\begin{aligned} \hat{\mathbb{Z}}_{37} = & \frac{\sqrt{5}\mathbb{X}_{53}[\mathbb{T}_{2,0}^{(a, T_g)}(1, 0)] \otimes \mathbb{Y}_{22}[\mathbb{T}_{2,0}^{(b, T_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{54}[\mathbb{T}_{2,1}^{(a, T_g)}(1, 0)] \otimes \mathbb{Y}_{23}[\mathbb{T}_{2,1}^{(b, T_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{55}[\mathbb{T}_{2,2}^{(a, T_g)}(1, 0)] \otimes \mathbb{Y}_{24}[\mathbb{T}_{2,2}^{(b, T_g)}]}{5} \\ & + \frac{\sqrt{5}\mathbb{X}_{57}[\mathbb{T}_{2,0}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_{20}[\mathbb{T}_{2,0}^{(b, E_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{58}[\mathbb{T}_{2,1}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_{21}[\mathbb{T}_{2,1}^{(b, E_g)}]}{5} \end{aligned}$$

$$\boxed{\text{No. 38}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, 0) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{38} = \frac{\sqrt{2}\mathbb{X}_{57}[\mathbb{T}_{2,0}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_{21}[\mathbb{T}_{2,1}^{(b, E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{58}[\mathbb{T}_{2,1}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_{20}[\mathbb{T}_{2,0}^{(b, E_g)}]}{2}$$

$$\boxed{\text{No. 39}} \quad \hat{\mathbb{Q}}_4^{(A_g)}(1, 0) [\text{M}_3, \text{B}_1]$$

$$\begin{aligned} \hat{\mathbb{Z}}_{39} = & -\frac{\sqrt{30}\mathbb{X}_{53}[\mathbb{T}_{2,0}^{(a, T_g)}(1, 0)] \otimes \mathbb{Y}_{22}[\mathbb{T}_{2,0}^{(b, T_g)}]}{15} - \frac{\sqrt{30}\mathbb{X}_{54}[\mathbb{T}_{2,1}^{(a, T_g)}(1, 0)] \otimes \mathbb{Y}_{23}[\mathbb{T}_{2,1}^{(b, T_g)}]}{15} - \frac{\sqrt{30}\mathbb{X}_{55}[\mathbb{T}_{2,2}^{(a, T_g)}(1, 0)] \otimes \mathbb{Y}_{24}[\mathbb{T}_{2,2}^{(b, T_g)}]}{15} \\ & + \frac{\sqrt{30}\mathbb{X}_{57}[\mathbb{T}_{2,0}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_{20}[\mathbb{T}_{2,0}^{(b, E_g)}]}{10} + \frac{\sqrt{30}\mathbb{X}_{58}[\mathbb{T}_{2,1}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_{21}[\mathbb{T}_{2,1}^{(b, E_g)}]}{10} \end{aligned}$$

$$\boxed{\text{No. 40}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_1, \text{B}_2]$$

$$\hat{\mathbb{Z}}_{40} = \mathbb{X}_1[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{Y}_{28}[\mathbb{Q}_0^{(b, A_g)}]$$

$$\boxed{\text{No. 41}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_2, \text{B}_2]$$

$$\hat{\mathbb{Z}}_{41} = \frac{\sqrt{3}\mathbb{X}_{14}[\mathbb{M}_{2,0}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{31}[\mathbb{T}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{15}[\mathbb{M}_{2,1}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{32}[\mathbb{T}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{16}[\mathbb{M}_{2,2}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{33}[\mathbb{T}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 42}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 0) [\text{M}_2, \text{B}_2]$$

$$\hat{\mathbb{Z}}_{42} = \frac{\sqrt{3}\mathbb{X}_{17}[\mathbb{T}_{1,0}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{31}[\mathbb{T}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{18}[\mathbb{T}_{1,1}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{32}[\mathbb{T}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{19}[\mathbb{T}_{1,2}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{33}[\mathbb{T}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 43}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_2, \text{B}_2]$$

$$\hat{\mathbb{Z}}_{43} = \frac{\sqrt{3}\mathbb{X}_{20}[\mathbb{T}_{1,0}^{(a,T_u)}] \otimes \mathbb{Y}_{31}[\mathbb{T}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{21}[\mathbb{T}_{1,1}^{(a,T_u)}] \otimes \mathbb{Y}_{32}[\mathbb{T}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{22}[\mathbb{T}_{1,2}^{(a,T_u)}] \otimes \mathbb{Y}_{33}[\mathbb{T}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 44}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_2]$$

$$\hat{\mathbb{Z}}_{44} = \mathbb{X}_{23}[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_{28}[\mathbb{Q}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 45}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 1) [\text{M}_3, \text{B}_2]$$

$$\hat{\mathbb{Z}}_{45} = \mathbb{X}_{24}[\mathbb{Q}_0^{(a,A_g)}(1, 1)] \otimes \mathbb{Y}_{28}[\mathbb{Q}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 46}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_2]$$

$$\hat{\mathbb{Z}}_{46} = \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{Y}_{29}[\mathbb{Q}_{2,0}^{(b,E_g)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{26}[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{Y}_{30}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{2}$$

$$\boxed{\text{No. 47}} \quad \hat{\mathbb{G}}_3^{(A_g)} [\text{M}_3, \text{B}_2]$$

$$\hat{\mathbb{Z}}_{47} = \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{Y}_{30}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{26}[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{Y}_{29}[\mathbb{Q}_{2,0}^{(b,E_g)}]}{2}$$

$$\boxed{\text{No. 48}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, -1) [\text{M}_3, \text{B}_2]$$

$$\hat{\mathbb{Z}}_{48} = \frac{\sqrt{2}\mathbb{X}_{30}[\mathbb{Q}_{2,0}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_{29}[\mathbb{Q}_{2,0}^{(b,E_g)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{31}[\mathbb{Q}_{2,1}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_{30}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{2}$$

$$\boxed{\text{No. 49}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_3, \text{B}_2]$$

$$\hat{\mathbb{Z}}_{49} = \frac{\sqrt{2}\mathbb{X}_{30}[\mathbb{Q}_{2,0}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_{30}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{31}[\mathbb{Q}_{2,1}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_{29}[\mathbb{Q}_{2,0}^{(b,E_g)}]}{2}$$

$$\boxed{\text{No. 50}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_1, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{50} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_{34}[\mathbb{Q}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 51}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_2, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{51} = \frac{\sqrt{3}\mathbb{X}_{14}[\mathbb{M}_{2,0}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{37}[\mathbb{T}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{15}[\mathbb{M}_{2,1}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{38}[\mathbb{T}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{16}[\mathbb{M}_{2,2}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{39}[\mathbb{T}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 52}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 0) [\text{M}_2, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{52} = \frac{\sqrt{3}\mathbb{X}_{17}[\mathbb{T}_{1,0}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{37}[\mathbb{T}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{18}[\mathbb{T}_{1,1}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{38}[\mathbb{T}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{19}[\mathbb{T}_{1,2}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{39}[\mathbb{T}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 53}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_2, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{53} = \frac{\sqrt{3}\mathbb{X}_{20}[\mathbb{T}_{1,0}^{(a,T_u)}] \otimes \mathbb{Y}_{37}[\mathbb{T}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{21}[\mathbb{T}_{1,1}^{(a,T_u)}] \otimes \mathbb{Y}_{38}[\mathbb{T}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{22}[\mathbb{T}_{1,2}^{(a,T_u)}] \otimes \mathbb{Y}_{39}[\mathbb{T}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 54}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{54} = \mathbb{X}_{23}[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_{34}[\mathbb{Q}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 55}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 1) [\text{M}_3, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{55} = \mathbb{X}_{24}[\mathbb{Q}_0^{(a,A_g)}(1, 1)] \otimes \mathbb{Y}_{34}[\mathbb{Q}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 56}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{56} = \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{Y}_{35}[\mathbb{Q}_{2,0}^{(b,E_g)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{26}[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{Y}_{36}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{2}$$

$$\boxed{\text{No. 57}} \quad \hat{\mathbb{G}}_3^{(A_g)} [\text{M}_3, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{57} = \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{Y}_{36}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{26}[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{Y}_{35}[\mathbb{Q}_{2,0}^{(b,E_g)}]}{2}$$

$$\boxed{\text{No. 58}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, -1) [\text{M}_3, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{58} = \frac{\sqrt{2}\mathbb{X}_{30}[\mathbb{Q}_{2,0}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_{35}[\mathbb{Q}_{2,0}^{(b,E_g)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{31}[\mathbb{Q}_{2,1}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_{36}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{2}$$

$$\boxed{\text{No. 59}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_3, \text{B}_3]$$

$$\hat{\mathbb{Z}}_{59} = \frac{\sqrt{2}\mathbb{X}_{30}[\mathbb{Q}_{2,0}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_{36}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{31}[\mathbb{Q}_{2,1}^{(a,E_g)}(1, -1)] \otimes \mathbb{Y}_{35}[\mathbb{Q}_{2,0}^{(b,E_g)}]}{2}$$

$$\boxed{\text{No. 60}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_1, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{60} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_{40}[\mathbb{Q}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 61}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_2, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{61} = \frac{\sqrt{3}\mathbb{X}_{14}[\mathbb{M}_{2,0}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{43}[\mathbb{T}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{15}[\mathbb{M}_{2,1}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{44}[\mathbb{T}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{16}[\mathbb{M}_{2,2}^{(a,T_u)}(1, -1)] \otimes \mathbb{Y}_{45}[\mathbb{T}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 62}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 0) [\text{M}_2, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{62} = \frac{\sqrt{3}\mathbb{X}_{17}[\mathbb{T}_{1,0}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{43}[\mathbb{T}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{18}[\mathbb{T}_{1,1}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{44}[\mathbb{T}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{19}[\mathbb{T}_{1,2}^{(a,T_u)}(1, 0)] \otimes \mathbb{Y}_{45}[\mathbb{T}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 63}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_2, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{63} = \frac{\sqrt{3}\mathbb{X}_{20}[\mathbb{T}_{1,0}^{(a,T_u)}] \otimes \mathbb{Y}_{43}[\mathbb{T}_{1,0}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{21}[\mathbb{T}_{1,1}^{(a,T_u)}] \otimes \mathbb{Y}_{44}[\mathbb{T}_{1,1}^{(b,T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{22}[\mathbb{T}_{1,2}^{(a,T_u)}] \otimes \mathbb{Y}_{45}[\mathbb{T}_{1,2}^{(b,T_u)}]}{3}$$

$$\boxed{\text{No. 64}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{64} = \mathbb{X}_{23}[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_{40}[\mathbb{Q}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 65}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 1) [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{65} = \mathbb{X}_{24}[\mathbb{Q}_0^{(a, A_g)}(1, 1)] \otimes \mathbb{Y}_{40}[\mathbb{Q}_0^{(b, A_g)}]$$

$$\boxed{\text{No. 66}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{66} = \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{Q}_{2,0}^{(a, E_g)}] \otimes \mathbb{Y}_{41}[\mathbb{Q}_{2,0}^{(b, E_g)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{26}[\mathbb{Q}_{2,1}^{(a, E_g)}] \otimes \mathbb{Y}_{42}[\mathbb{Q}_{2,1}^{(b, E_g)}]}{2}$$

$$\boxed{\text{No. 67}} \quad \hat{\mathbb{G}}_3^{(A_g)} [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{67} = \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{Q}_{2,0}^{(a, E_g)}] \otimes \mathbb{Y}_{42}[\mathbb{Q}_{2,1}^{(b, E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{26}[\mathbb{Q}_{2,1}^{(a, E_g)}] \otimes \mathbb{Y}_{41}[\mathbb{Q}_{2,0}^{(b, E_g)}]}{2}$$

$$\boxed{\text{No. 68}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, -1) [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{68} = \frac{\sqrt{2}\mathbb{X}_{30}[\mathbb{Q}_{2,0}^{(a, E_g)}(1, -1)] \otimes \mathbb{Y}_{41}[\mathbb{Q}_{2,0}^{(b, E_g)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{31}[\mathbb{Q}_{2,1}^{(a, E_g)}(1, -1)] \otimes \mathbb{Y}_{42}[\mathbb{Q}_{2,1}^{(b, E_g)}]}{2}$$

$$\boxed{\text{No. 69}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_3, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{69} = \frac{\sqrt{2}\mathbb{X}_{30}[\mathbb{Q}_{2,0}^{(a, E_g)}(1, -1)] \otimes \mathbb{Y}_{42}[\mathbb{Q}_{2,1}^{(b, E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{31}[\mathbb{Q}_{2,1}^{(a, E_g)}(1, -1)] \otimes \mathbb{Y}_{41}[\mathbb{Q}_{2,0}^{(b, E_g)}]}{2}$$

Table 5: Atomic SAMB group.

group	bra	ket
M ₁	$(s, \uparrow), (s, \downarrow)$	$(s, \uparrow), (s, \downarrow)$
M ₂	$(s, \uparrow), (s, \downarrow)$	$(p_x, \uparrow), (p_x, \downarrow), (p_y, \uparrow), (p_y, \downarrow), (p_z, \uparrow), (p_z, \downarrow)$
M ₃	$(p_x, \uparrow), (p_x, \downarrow), (p_y, \uparrow), (p_y, \downarrow), (p_z, \uparrow), (p_z, \downarrow)$	$(p_x, \uparrow), (p_x, \downarrow), (p_y, \uparrow), (p_y, \downarrow), (p_z, \uparrow), (p_z, \downarrow)$

Table 6: Atomic SAMB.

symbol	type	group	form
\mathbb{X}_1	$\mathbb{Q}_0^{(a, Ag)}$	M_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{X}_2	$\mathbb{M}_{1,0}^{(a, Tg)}(1, -1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$
\mathbb{X}_3	$\mathbb{M}_{1,1}^{(a, Tg)}(1, -1)$	M_1	$\begin{pmatrix} 0 & -\frac{\sqrt{2}i}{2} \\ \frac{\sqrt{2}i}{2} & 0 \end{pmatrix}$
\mathbb{X}_4	$\mathbb{M}_{1,2}^{(a, Tg)}(1, -1)$	M_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{X}_5	$\mathbb{Q}_{1,0}^{(a, Tu)}$	M_2	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_6	$\mathbb{Q}_{1,1}^{(a, Tu)}$	M_2	$\begin{pmatrix} 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 \end{pmatrix}$
\mathbb{X}_7	$\mathbb{Q}_{1,2}^{(a, Tu)}$	M_2	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{X}_8	$\mathbb{Q}_{1,0}^{(a, Tu)}(1, 0)$	M_2	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{i}{2} & -\frac{1}{2} & 0 \end{pmatrix}$
\mathbb{X}_9	$\mathbb{Q}_{1,1}^{(a, Tu)}(1, 0)$	M_2	$\begin{pmatrix} \frac{i}{2} & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & -\frac{i}{2} & 0 & 0 & -\frac{i}{2} & 0 \end{pmatrix}$
\mathbb{X}_{10}	$\mathbb{Q}_{1,2}^{(a, Tu)}(1, 0)$	M_2	$\begin{pmatrix} 0 & -\frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{11}	$\mathbb{G}_{2,0}^{(a, Tu)}(1, -1)$	M_2	$\begin{pmatrix} 0 & 0 & \frac{i}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & -\frac{i}{2} & -\frac{1}{2} & 0 \end{pmatrix}$
\mathbb{X}_{12}	$\mathbb{G}_{2,1}^{(a, Tu)}(1, -1)$	M_2	$\begin{pmatrix} \frac{i}{2} & 0 & 0 & 0 & 0 & \frac{i}{2} \\ 0 & -\frac{i}{2} & 0 & 0 & \frac{i}{2} & 0 \end{pmatrix}$
\mathbb{X}_{13}	$\mathbb{G}_{2,2}^{(a, Tu)}(1, -1)$	M_2	$\begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{14}	$\mathbb{M}_{2,0}^{(a, Tu)}(1, -1)$	M_2	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{i}{2} & 0 \end{pmatrix}$
\mathbb{X}_{15}	$\mathbb{M}_{2,1}^{(a, Tu)}(1, -1)$	M_2	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$
\mathbb{X}_{16}	$\mathbb{M}_{2,2}^{(a, Tu)}(1, -1)$	M_2	$\begin{pmatrix} 0 & -\frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
\mathbb{X}_{17}	$\mathbb{T}_{1,0}^{(a,T_u)}(1,0)$	M_2	$\begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & 0 \end{pmatrix}$
\mathbb{X}_{18}	$\mathbb{T}_{1,1}^{(a,T_u)}(1,0)$	M_2	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 0 \end{pmatrix}$
\mathbb{X}_{19}	$\mathbb{T}_{1,2}^{(a,T_u)}(1,0)$	M_2	$\begin{pmatrix} 0 & \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 \\ -\frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{20}	$\mathbb{T}_{1,0}^{(a,T_u)}$	M_2	$\begin{pmatrix} \frac{\sqrt{2}i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{21}	$\mathbb{T}_{1,1}^{(a,T_u)}$	M_2	$\begin{pmatrix} 0 & 0 & \frac{\sqrt{2}i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}i}{2} & 0 & 0 \end{pmatrix}$
\mathbb{X}_{22}	$\mathbb{T}_{1,2}^{(a,T_u)}$	M_2	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{2} \end{pmatrix}$
\mathbb{X}_{23}	$\mathbb{Q}_0^{(a,A_g)}$	M_3	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \end{pmatrix}$
\mathbb{X}_{24}	$\mathbb{Q}_0^{(a,A_g)}(1,1)$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ \frac{\sqrt{3}i}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{25}	$\mathbb{Q}_{2,0}^{(a,E_g)}$	M_3	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
\mathbb{X}_{26}	$\mathbb{Q}_{2,1}^{(a,E_g)}$	M_3	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{27}	$\mathbb{Q}_{2,0}^{(a,T_g)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$
\mathbb{X}_{28}	$\mathbb{Q}_{2,1}^{(a,T_g)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{29}	$\mathbb{Q}_{2,2}^{(a,T_g)}$	M_3	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{30}	$\mathbb{Q}_{2,0}^{(a,E_g)}(1, -1)$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
\mathbb{X}_{31}	$\mathbb{Q}_{2,1}^{(a,E_g)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{32}	$\mathbb{Q}_{2,0}^{(a,T_g)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{33}	$\mathbb{Q}_{2,1}^{(a,T_g)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$
\mathbb{X}_{34}	$\mathbb{Q}_{2,2}^{(a,T_g)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{35}	$\mathbb{G}_{1,0}^{(a,T_g)}(1,0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
\mathbb{X}_{36}	$\mathbb{G}_{1,1}^{(a,T_g)}(1,0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$
\mathbb{X}_{37}	$\mathbb{G}_{1,2}^{(a,T_g)}(1,0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{38}	$\mathbb{M}_{1,0}^{(a,T_g)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \end{pmatrix}$
\mathbb{X}_{39}	$\mathbb{M}_{1,1}^{(a,T_g)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{40}	$\mathbb{M}_{1,2}^{(a,T_g)}$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
\mathbb{X}_{41}	$\mathbb{M}_{1,0}^{(a,T_g)}(1,1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{30}}{15} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{20} & 0 \\ \frac{\sqrt{30}}{15} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{20} \\ 0 & -\frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 \\ \frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & 0 \\ \frac{\sqrt{30}}{20} & 0 & 0 & 0 & 0 & -\frac{\sqrt{30}}{30} \\ 0 & -\frac{\sqrt{30}}{20} & 0 & 0 & -\frac{\sqrt{30}}{30} & 0 \end{pmatrix}$
\mathbb{X}_{42}	$\mathbb{M}_{1,1}^{(a,T_g)}(1,1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{30}i}{30} & 0 & \frac{\sqrt{30}}{20} & 0 & 0 \\ -\frac{\sqrt{30}i}{30} & 0 & \frac{\sqrt{30}}{20} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{15} & \frac{\sqrt{30}}{20} & 0 \\ \frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{15} & 0 & 0 & -\frac{\sqrt{30}}{20} \\ 0 & 0 & \frac{\sqrt{30}}{20} & 0 & 0 & \frac{\sqrt{30}i}{30} \\ 0 & 0 & 0 & -\frac{\sqrt{30}}{20} & -\frac{\sqrt{30}i}{30} & 0 \end{pmatrix}$
\mathbb{X}_{43}	$\mathbb{M}_{1,2}^{(a,T_g)}(1,1)$	M_3	$\begin{pmatrix} -\frac{\sqrt{30}}{30} & 0 & 0 & 0 & 0 & \frac{\sqrt{30}}{20} \\ 0 & \frac{\sqrt{30}}{30} & 0 & 0 & \frac{\sqrt{30}}{20} & 0 \\ 0 & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & -\frac{\sqrt{30}i}{20} \\ 0 & 0 & 0 & \frac{\sqrt{30}}{30} & \frac{\sqrt{30}i}{20} & 0 \\ 0 & \frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{15} & 0 \\ \frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{15} \end{pmatrix}$
\mathbb{X}_{44}	$\mathbb{M}_{1,0}^{(a,T_g)}(1,-1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \end{pmatrix}$
\mathbb{X}_{45}	$\mathbb{M}_{1,1}^{(a,T_g)}(1,-1)$	M_3	$\begin{pmatrix} 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}i}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
\mathbb{X}_{46}	$\mathbb{M}_{1,2}^{(a,T_g)}(1, -1)$	M_3	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \end{pmatrix}$
\mathbb{X}_{47}	$\mathbb{M}_{3,0}^{(a,T_g,1)}(1, -1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{5}}{5} & 0 & \frac{\sqrt{5}i}{10} & -\frac{\sqrt{5}}{10} & 0 \\ \frac{\sqrt{5}}{5} & 0 & -\frac{\sqrt{5}i}{10} & 0 & 0 & \frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 \\ -\frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & 0 \\ -\frac{\sqrt{5}}{10} & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}}{10} & 0 & 0 & -\frac{\sqrt{5}}{10} & 0 \end{pmatrix}$
\mathbb{X}_{48}	$\mathbb{M}_{3,1}^{(a,T_g,1)}(1, -1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 \\ -\frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{5}}{10} & 0 & -\frac{\sqrt{5}i}{5} & -\frac{\sqrt{5}}{10} & 0 \\ -\frac{\sqrt{5}}{10} & 0 & \frac{\sqrt{5}i}{5} & 0 & 0 & \frac{\sqrt{5}}{10} \\ 0 & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & \frac{\sqrt{5}i}{10} \\ 0 & 0 & 0 & \frac{\sqrt{5}}{10} & -\frac{\sqrt{5}i}{10} & 0 \end{pmatrix}$
\mathbb{X}_{49}	$\mathbb{M}_{3,2}^{(a,T_g,1)}(1, -1)$	M_3	$\begin{pmatrix} -\frac{\sqrt{5}}{10} & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}}{10} & 0 & 0 & -\frac{\sqrt{5}}{10} & 0 \\ 0 & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & \frac{\sqrt{5}i}{10} \\ 0 & 0 & 0 & \frac{\sqrt{5}}{10} & -\frac{\sqrt{5}i}{10} & 0 \\ 0 & -\frac{\sqrt{5}}{10} & 0 & \frac{\sqrt{5}i}{10} & \frac{\sqrt{5}}{5} & 0 \\ -\frac{\sqrt{5}}{10} & 0 & -\frac{\sqrt{5}i}{10} & 0 & 0 & -\frac{\sqrt{5}}{5} \end{pmatrix}$
\mathbb{X}_{50}	$\mathbb{M}_{3,0}^{(a,T_g,2)}(1, -1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ 0 & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 \\ \frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \\ 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
\mathbb{X}_{51}	$\mathbb{M}_{3,1}^{(a,T_g,2)}(1,-1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ -\frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \\ -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \\ 0 & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}i}{6} & 0 \end{pmatrix}$
\mathbb{X}_{52}	$\mathbb{M}_{3,2}^{(a,T_g,2)}(1,-1)$	M_3	$\begin{pmatrix} \frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{53}	$\mathbb{T}_{2,0}^{(a,T_g)}(1,0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ -\frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \\ 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 \end{pmatrix}$
\mathbb{X}_{54}	$\mathbb{T}_{2,1}^{(a,T_g)}(1,0)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 \\ -\frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{12} \\ 0 & 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{12} & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
\mathbb{X}_{55}	$\mathbb{T}_{2,2}^{(a,T_g)}(1,0)$	M_3	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}i}{12} & 0 \\ 0 & -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & \frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
\mathbb{X}_{56}	$\mathbb{M}_3^{(a, A_g)}(1, -1)$	M_3	$\begin{pmatrix} 0 & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}i}{6} & 0 \\ \frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 \\ \frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{57}	$\mathbb{T}_{2,0}^{(a, E_g)}(1, 0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{58}	$\mathbb{T}_{2,1}^{(a, E_g)}(1, 0)$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}i}{12} & 0 \\ -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{12} \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & \frac{\sqrt{6}}{12} & 0 \\ 0 & -\frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 \\ \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 & 0 \end{pmatrix}$

Table 7: Cluster SAMB.

symbol	type	cluster	form
\mathbb{Y}_1	$\mathbb{Q}_0^{(s, A_g)}$	S_1	$\begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$
\mathbb{Y}_2	$\mathbb{Q}_{2,0}^{(s, E_g)}$	S_1	$\begin{pmatrix} -\frac{11\sqrt{6}}{42} & -\frac{\sqrt{6}}{21} & \frac{13\sqrt{6}}{42} \end{pmatrix}$
\mathbb{Y}_3	$\mathbb{Q}_{2,1}^{(s, E_g)}$	S_1	$\begin{pmatrix} \frac{5\sqrt{2}}{14} & -\frac{4\sqrt{2}}{7} & \frac{3\sqrt{2}}{14} \end{pmatrix}$
\mathbb{Y}_4	$\mathbb{Q}_0^{(b, A_g)}$	B_1	$\begin{pmatrix} \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} \end{pmatrix}$
\mathbb{Y}_5	$\mathbb{Q}_{1,0}^{(b, T_u)}$	B_1	$\begin{pmatrix} \frac{3\sqrt{13}}{26} & -\frac{3\sqrt{13}}{26} & \frac{3\sqrt{13}}{26} & -\frac{3\sqrt{13}}{26} & 0 & 0 & 0 & \frac{\sqrt{13}}{13} & -\frac{\sqrt{13}}{13} & -\frac{\sqrt{13}}{13} & \frac{\sqrt{13}}{13} \end{pmatrix}$
\mathbb{Y}_6	$\mathbb{Q}_{1,1}^{(b, T_u)}$	B_1	$\begin{pmatrix} \frac{\sqrt{13}}{13} & -\frac{\sqrt{13}}{13} & -\frac{\sqrt{13}}{13} & \frac{\sqrt{13}}{13} & \frac{3\sqrt{13}}{26} & -\frac{3\sqrt{13}}{26} & -\frac{3\sqrt{13}}{26} & \frac{3\sqrt{13}}{26} & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

symbol	type	cluster	form
Y_7	$Q_{1,2}^{(b,T_u)}$	B_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{13}}{13} & \frac{\sqrt{13}}{13} & -\frac{\sqrt{13}}{13} & -\frac{\sqrt{13}}{13} & \frac{3\sqrt{13}}{26} & -\frac{3\sqrt{13}}{26} & \frac{3\sqrt{13}}{26} & -\frac{3\sqrt{13}}{26} \end{pmatrix}$
Y_8	$Q_{2,0}^{(b,E_g)}$	B_1	$\begin{pmatrix} -\frac{11\sqrt{6}}{84} & -\frac{11\sqrt{6}}{84} & -\frac{11\sqrt{6}}{84} & -\frac{11\sqrt{6}}{84} & -\frac{\sqrt{6}}{42} & -\frac{\sqrt{6}}{42} & -\frac{\sqrt{6}}{42} & -\frac{\sqrt{6}}{42} & \frac{13\sqrt{6}}{84} & \frac{13\sqrt{6}}{84} & \frac{13\sqrt{6}}{84} & \frac{13\sqrt{6}}{84} \end{pmatrix}$
Y_9	$Q_{2,1}^{(b,E_g)}$	B_1	$\begin{pmatrix} \frac{5\sqrt{2}}{28} & \frac{5\sqrt{2}}{28} & \frac{5\sqrt{2}}{28} & \frac{5\sqrt{2}}{28} & -\frac{2\sqrt{2}}{7} & -\frac{2\sqrt{2}}{7} & -\frac{2\sqrt{2}}{7} & -\frac{2\sqrt{2}}{7} & \frac{3\sqrt{2}}{28} & \frac{3\sqrt{2}}{28} & \frac{3\sqrt{2}}{28} & \frac{3\sqrt{2}}{28} \end{pmatrix}$
Y_{10}	$Q_{2,0}^{(b,T_g)}$	B_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
Y_{11}	$Q_{2,1}^{(b,T_g)}$	B_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$
Y_{12}	$Q_{2,2}^{(b,T_g)}$	B_1	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
Y_{13}	$Q_{3,0}^{(b,T_u,1)}$	B_1	$\begin{pmatrix} \frac{\sqrt{13}}{13} & -\frac{\sqrt{13}}{13} & \frac{\sqrt{13}}{13} & -\frac{\sqrt{13}}{13} & 0 & 0 & 0 & 0 & -\frac{3\sqrt{13}}{26} & \frac{3\sqrt{13}}{26} & \frac{3\sqrt{13}}{26} & -\frac{3\sqrt{13}}{26} \end{pmatrix}$
Y_{14}	$Q_{3,1}^{(b,T_u,1)}$	B_1	$\begin{pmatrix} -\frac{3\sqrt{13}}{26} & \frac{3\sqrt{13}}{26} & \frac{3\sqrt{13}}{26} & -\frac{3\sqrt{13}}{26} & \frac{\sqrt{13}}{13} & -\frac{\sqrt{13}}{13} & -\frac{\sqrt{13}}{13} & \frac{\sqrt{13}}{13} & 0 & 0 & 0 & 0 \end{pmatrix}$
Y_{15}	$Q_{3,2}^{(b,T_u,1)}$	B_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{3\sqrt{13}}{26} & -\frac{3\sqrt{13}}{26} & \frac{3\sqrt{13}}{26} & \frac{3\sqrt{13}}{26} & \frac{\sqrt{13}}{13} & -\frac{\sqrt{13}}{13} & \frac{\sqrt{13}}{13} & -\frac{\sqrt{13}}{13} \end{pmatrix}$
Y_{16}	$T_0^{(b,A_g)}$	B_1	$\begin{pmatrix} \frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}i}{6} \end{pmatrix}$
Y_{17}	$T_{1,0}^{(b,T_u)}$	B_1	$\begin{pmatrix} \frac{3\sqrt{13}i}{26} & -\frac{3\sqrt{13}i}{26} & \frac{3\sqrt{13}i}{26} & -\frac{3\sqrt{13}i}{26} & 0 & 0 & 0 & 0 & -\frac{\sqrt{13}i}{13} & \frac{\sqrt{13}i}{13} & \frac{\sqrt{13}i}{13} & -\frac{\sqrt{13}i}{13} \end{pmatrix}$
Y_{18}	$T_{1,1}^{(b,T_u)}$	B_1	$\begin{pmatrix} \frac{\sqrt{13}i}{13} & -\frac{\sqrt{13}i}{13} & -\frac{\sqrt{13}i}{13} & \frac{\sqrt{13}i}{13} & \frac{3\sqrt{13}i}{26} & -\frac{3\sqrt{13}i}{26} & -\frac{3\sqrt{13}i}{26} & \frac{3\sqrt{13}i}{26} & 0 & 0 & 0 & 0 \end{pmatrix}$
Y_{19}	$T_{1,2}^{(b,T_u)}$	B_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{13}i}{13} & \frac{\sqrt{13}i}{13} & -\frac{\sqrt{13}i}{13} & -\frac{\sqrt{13}i}{13} & -\frac{3\sqrt{13}i}{26} & \frac{3\sqrt{13}i}{26} & -\frac{3\sqrt{13}i}{26} & \frac{3\sqrt{13}i}{26} \end{pmatrix}$
Y_{20}	$T_{2,0}^{(b,E_g)}$	B_1	$\begin{pmatrix} -\frac{11\sqrt{6}i}{84} & -\frac{11\sqrt{6}i}{84} & -\frac{11\sqrt{6}i}{84} & -\frac{11\sqrt{6}i}{84} & -\frac{\sqrt{6}i}{42} & -\frac{\sqrt{6}i}{42} & -\frac{\sqrt{6}i}{42} & -\frac{\sqrt{6}i}{42} & -\frac{13\sqrt{6}i}{84} & -\frac{13\sqrt{6}i}{84} & -\frac{13\sqrt{6}i}{84} & -\frac{13\sqrt{6}i}{84} \end{pmatrix}$
Y_{21}	$T_{2,1}^{(b,E_g)}$	B_1	$\begin{pmatrix} \frac{5\sqrt{2}i}{28} & \frac{5\sqrt{2}i}{28} & \frac{5\sqrt{2}i}{28} & \frac{5\sqrt{2}i}{28} & -\frac{2\sqrt{2}i}{7} & -\frac{2\sqrt{2}i}{7} & -\frac{2\sqrt{2}i}{7} & -\frac{2\sqrt{2}i}{7} & -\frac{3\sqrt{2}i}{28} & -\frac{3\sqrt{2}i}{28} & -\frac{3\sqrt{2}i}{28} & -\frac{3\sqrt{2}i}{28} \end{pmatrix}$
Y_{22}	$T_{2,0}^{(b,T_g)}$	B_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{i}{2} & -\frac{i}{2} & \frac{i}{2} & -\frac{i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
Y_{23}	$T_{2,1}^{(b,T_g)}$	B_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & -\frac{i}{2} & \frac{i}{2} & \frac{i}{2} \end{pmatrix}$
Y_{24}	$T_{2,2}^{(b,T_g)}$	B_1	$\begin{pmatrix} \frac{i}{2} & \frac{i}{2} & -\frac{i}{2} & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
Y_{25}	$T_{3,0}^{(b,T_u,1)}$	B_1	$\begin{pmatrix} \frac{\sqrt{13}i}{13} & -\frac{\sqrt{13}i}{13} & \frac{\sqrt{13}i}{13} & -\frac{\sqrt{13}i}{13} & 0 & 0 & 0 & 0 & \frac{3\sqrt{13}i}{26} & -\frac{3\sqrt{13}i}{26} & -\frac{3\sqrt{13}i}{26} & \frac{3\sqrt{13}i}{26} \end{pmatrix}$
Y_{26}	$T_{3,1}^{(b,T_u,1)}$	B_1	$\begin{pmatrix} -\frac{3\sqrt{13}i}{26} & \frac{3\sqrt{13}i}{26} & \frac{3\sqrt{$

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Table 7

symbol	type	cluster	form
Y ₃₅	$\mathbb{Q}_{2,0}^{(b,E_g)}$	B ₃	$\begin{pmatrix} -\frac{11\sqrt{6}}{42} & -\frac{\sqrt{6}}{21} & \frac{13\sqrt{6}}{42} \end{pmatrix}$
Y ₃₆	$\mathbb{Q}_{2,1}^{(b,E_g)}$	B ₃	$\begin{pmatrix} \frac{5\sqrt{2}}{14} & -\frac{4\sqrt{2}}{7} & \frac{3\sqrt{2}}{14} \end{pmatrix}$
Y ₃₇	$\mathbb{T}_{1,0}^{(b,T_u)}$	B ₃	$\begin{pmatrix} i & 0 & 0 \end{pmatrix}$
Y ₃₈	$\mathbb{T}_{1,1}^{(b,T_u)}$	B ₃	$\begin{pmatrix} 0 & i & 0 \end{pmatrix}$
Y ₃₉	$\mathbb{T}_{1,2}^{(b,T_u)}$	B ₃	$\begin{pmatrix} 0 & 0 & i \end{pmatrix}$
Y ₄₀	$\mathbb{Q}_0^{(b,A_g)}$	B ₄	$\begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$
Y ₄₁	$\mathbb{Q}_{2,0}^{(b,E_g)}$	B ₄	$\begin{pmatrix} -\frac{11\sqrt{6}}{42} & -\frac{\sqrt{6}}{21} & \frac{13\sqrt{6}}{42} \end{pmatrix}$
Y ₄₂	$\mathbb{Q}_{2,1}^{(b,E_g)}$	B ₄	$\begin{pmatrix} \frac{5\sqrt{2}}{14} & -\frac{4\sqrt{2}}{7} & \frac{3\sqrt{2}}{14} \end{pmatrix}$
Y ₄₃	$\mathbb{T}_{1,0}^{(b,T_u)}$	B ₄	$\begin{pmatrix} 0 & 0 & i \end{pmatrix}$
Y ₄₄	$\mathbb{T}_{1,1}^{(b,T_u)}$	B ₄	$\begin{pmatrix} i & 0 & 0 \end{pmatrix}$
Y ₄₅	$\mathbb{T}_{1,2}^{(b,T_u)}$	B ₄	$\begin{pmatrix} 0 & i & 0 \end{pmatrix}$

Table 8: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_0^{(A_g)}$	0	A_g	—	—	1
2	$\mathbb{Q}_{1,0}^{(T_u)}$	1	T_u	—	0	x
3	$\mathbb{Q}_{1,1}^{(T_u)}$	1	T_u	—	1	y
4	$\mathbb{Q}_{1,2}^{(T_u)}$	1	T_u	—	2	z
5	$\mathbb{Q}_{2,0}^{(E_g)}$	2	E_g	—	0	$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$
6	$\mathbb{Q}_{2,1}^{(E_g)}$	2	E_g	—	1	$\frac{\sqrt{3}(x^2 - y^2)}{2}$
7	$\mathbb{Q}_{2,0}^{(T_g)}$	2	T_g	—	0	$\sqrt{3}yz$
8	$\mathbb{Q}_{2,1}^{(T_g)}$	2	T_g	—	1	$\sqrt{3}xz$
9	$\mathbb{Q}_{2,2}^{(T_g)}$	2	T_g	—	2	$\sqrt{3}xy$
10	$\mathbb{Q}_{3,0}^{(T_u,1)}$	3	T_u	1	0	$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$

continued ...

Table 8

No.	symbol	rank	irrep.	mul.	comp.	form
11	$\mathbb{Q}_{3,1}^{(T_u,1)}$	3	T_u	1	1	$-\frac{y(3x^2-2y^2+3z^2)}{2}$
12	$\mathbb{Q}_{3,2}^{(T_u,1)}$	3	T_u	1	2	$-\frac{z(3x^2+3y^2-2z^2)}{2}$

Table 9: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_{1,0}^{(T_g)}$	1	T_g	—	0	X
2	$\mathbb{G}_{1,1}^{(T_g)}$	1	T_g	—	1	Y
3	$\mathbb{G}_{1,2}^{(T_g)}$	1	T_g	—	2	Z
4	$\mathbb{G}_{2,0}^{(T_u)}$	2	T_u	—	0	$\sqrt{3}YZ$
5	$\mathbb{G}_{2,1}^{(T_u)}$	2	T_u	—	1	$\sqrt{3}XZ$
6	$\mathbb{G}_{2,2}^{(T_u)}$	2	T_u	—	2	$\sqrt{3}XY$
7	$\mathbb{G}_3^{(A_g)}$	3	A_g	—	—	$\sqrt{15}XYZ$
8	$\mathbb{G}_{3,0}^{(T_g,1)}$	3	T_g	1	0	$\frac{X(2X^2-3Y^2-3Z^2)}{2}$
9	$\mathbb{G}_{3,1}^{(T_g,1)}$	3	T_g	1	1	$-\frac{Y(3X^2-2Y^2+3Z^2)}{2}$
10	$\mathbb{G}_{3,2}^{(T_g,1)}$	3	T_g	1	2	$-\frac{Z(3X^2+3Y^2-2Z^2)}{2}$
11	$\mathbb{G}_{3,0}^{(T_g,2)}$	3	T_g	2	0	$\frac{\sqrt{15}X(Y-Z)(Y+Z)}{2}$
12	$\mathbb{G}_{3,1}^{(T_g,2)}$	3	T_g	2	1	$-\frac{\sqrt{15}Y(X-Z)(X+Z)}{2}$
13	$\mathbb{G}_{3,2}^{(T_g,2)}$	3	T_g	2	2	$\frac{\sqrt{15}Z(X-Y)(X+Y)}{2}$

-
- Group info.: Generator = $\{2_{001}|0\}$, $\{2_{010}|0\}$, $\{3_{111}^+|0\}$, $\{-1|0\}$

Table 10: Conjugacy class (point-group part).

rep. SO	symmetry operations
$\{1 0\}$	$\{1 0\}$
$\{2_{001} 0\}$	$\{2_{001} 0\}, \{2_{100} 0\}, \{2_{010} 0\}$
$\{3_{111}^+ 0\}$	$\{3_{111}^+ 0\}, \{3_{1-1-1}^+ 0\}, \{3_{-11-1}^+ 0\}, \{3_{-1-11}^+ 0\}$
$\{3_{111}^- 0\}$	$\{3_{111}^- 0\}, \{3_{1-1-1}^- 0\}, \{3_{-11-1}^- 0\}, \{3_{-1-11}^- 0\}$
$\{-1 0\}$	$\{-1 0\}$
$\{m_{001} 0\}$	$\{m_{001} 0\}, \{m_{100} 0\}, \{m_{010} 0\}$
$\{-3_{111}^+ 0\}$	$\{-3_{111}^+ 0\}, \{-3_{1-1-1}^+ 0\}, \{-3_{-11-1}^+ 0\}, \{-3_{-1-11}^+ 0\}$
$\{-3_{111}^- 0\}$	$\{-3_{111}^- 0\}, \{-3_{1-1-1}^- 0\}, \{-3_{-11-1}^- 0\}, \{-3_{-1-11}^- 0\}$

Table 11: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	$\{1 0\}$	2	$\{2_{001} 0\}$	3	$\{2_{100} 0\}$	4	$\{2_{010} 0\}$	5	$\{3_{111}^+ 0\}$
6	$\{3_{1-1-1}^+ 0\}$	7	$\{3_{-11-1}^+ 0\}$	8	$\{3_{-1-11}^+ 0\}$	9	$\{3_{111}^- 0\}$	10	$\{3_{1-1-1}^- 0\}$
11	$\{3_{-11-1}^- 0\}$	12	$\{3_{-1-11}^- 0\}$	13	$\{-1 0\}$	14	$\{m_{001} 0\}$	15	$\{m_{100} 0\}$
16	$\{m_{010} 0\}$	17	$\{-3_{111}^+ 0\}$	18	$\{-3_{1-1-1}^+ 0\}$	19	$\{-3_{-11-1}^+ 0\}$	20	$\{-3_{-1-11}^+ 0\}$
21	$\{-3_{111}^- 0\}$	22	$\{-3_{1-1-1}^- 0\}$	23	$\{-3_{-11-1}^- 0\}$	24	$\{-3_{-1-11}^- 0\}$		

Table 12: Character table (point-group part).

	1	2_{001}	3_{111}^+	3_{111}^-	-1	m_{001}	-3_{111}^+	-3_{111}^-
A_g	1	1	1	1	1	1	1	1
$E_g^{(a)}$	1	1	ω^*	ω	1	1	ω^*	ω
$E_g^{(b)}$	1	1	ω	ω^*	1	1	ω	ω^*
T_g	3	-1	0	0	3	-1	0	0
A_u	1	1	1	1	-1	-1	-1	-1

continued ...

Table 12

	1	2 ₀₀₁	3 ₁₁₁ ⁺	3 ₁₁₁ ⁻	-1	m ₀₀₁	-3 ₁₁₁ ⁺	-3 ₁₁₁ ⁻
$E_u^{(a)}$	1	1	ω^*	ω	-1	-1	$-\omega^*$	$-\omega$
$E_u^{(b)}$	1	1	ω	ω^*	-1	-1	$-\omega$	$-\omega^*$
T_u	3	-1	0	0	-3	1	0	0

Table 13: Parity conversion.

\leftrightarrow	\leftrightarrow	\leftrightarrow	\leftrightarrow	\leftrightarrow
$A_g (A_u)$	$E_g^{(a)} (E_u^{(a)})$	$E_g^{(b)} (E_u^{(b)})$	$T_g (T_u)$	$A_u (A_g)$
$E_u^{(a)} (E_g^{(a)})$	$E_u^{(b)} (E_g^{(b)})$	$T_u (T_g)$		

Table 14: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

	A_g	$E_g^{(a)}$	$E_g^{(b)}$	T_g	A_u	$E_u^{(a)}$	$E_u^{(b)}$	T_u
A_g	A_g	$E_g^{(a)}$	$E_g^{(b)}$	T_g	A_u	$E_u^{(a)}$	$E_u^{(b)}$	T_u
$E_g^{(a)}$		$E_g^{(b)}$	A_g	T_g	$E_u^{(a)}$	$E_u^{(b)}$	A_u	T_u
$E_g^{(b)}$			$E_g^{(a)}$	T_g	$E_u^{(b)}$	A_u	$E_u^{(a)}$	T_u
T_g				$A_g + E_g^{(a)} + E_g^{(b)} + T_g$	T_u	T_u	T_u	$A_u + E_u^{(a)} + E_u^{(b)} + 2T_u$
A_u					A_g	$E_g^{(a)}$	$E_g^{(b)}$	T_g
$E_u^{(a)}$						$E_g^{(b)}$	A_g	T_g
$E_u^{(b)}$							$E_g^{(a)}$	T_g
T_u								$A_g + E_g^{(a)} + E_g^{(b)} + T_g$

Table 15: Anti-symmetric product, $[\Gamma \otimes \Gamma]_-$.

A_g	$E_g^{(a)}$	$E_g^{(b)}$	T_g	A_u	$E_u^{(a)}$	$E_u^{(b)}$	T_u
-	-	-	T_g	-	-	-	T_g

Table 16: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix} 3 & 2 & 1 \end{pmatrix}$	2	$\begin{pmatrix} -3 & -2 & 1 \end{pmatrix}$	3	$\begin{pmatrix} 3 & -2 & -1 \end{pmatrix}$	4	$\begin{pmatrix} -3 & 2 & -1 \end{pmatrix}$
5	$\begin{pmatrix} 1 & 3 & 2 \end{pmatrix}$	6	$\begin{pmatrix} -1 & -3 & 2 \end{pmatrix}$	7	$\begin{pmatrix} 1 & -3 & -2 \end{pmatrix}$	8	$\begin{pmatrix} -1 & 3 & -2 \end{pmatrix}$
9	$\begin{pmatrix} 2 & 1 & 3 \end{pmatrix}$	10	$\begin{pmatrix} -2 & 1 & -3 \end{pmatrix}$	11	$\begin{pmatrix} -2 & -1 & 3 \end{pmatrix}$	12	$\begin{pmatrix} 2 & -1 & -3 \end{pmatrix}$
13	$\begin{pmatrix} -3 & -2 & -1 \end{pmatrix}$	14	$\begin{pmatrix} 3 & 2 & -1 \end{pmatrix}$	15	$\begin{pmatrix} -3 & 2 & 1 \end{pmatrix}$	16	$\begin{pmatrix} 3 & -2 & 1 \end{pmatrix}$
17	$\begin{pmatrix} -1 & -3 & -2 \end{pmatrix}$	18	$\begin{pmatrix} 1 & 3 & -2 \end{pmatrix}$	19	$\begin{pmatrix} -1 & 3 & 2 \end{pmatrix}$	20	$\begin{pmatrix} 1 & -3 & 2 \end{pmatrix}$
21	$\begin{pmatrix} -2 & -1 & -3 \end{pmatrix}$	22	$\begin{pmatrix} 2 & -1 & 3 \end{pmatrix}$	23	$\begin{pmatrix} 2 & 1 & -3 \end{pmatrix}$	24	$\begin{pmatrix} -2 & 1 & 3 \end{pmatrix}$

Table 17: Virtual-cluster basis.

symbol	1	2	3	4	5	6	7	8	9	10
$Q_0^{(A_g)}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$						
$Q_{1,0}^{(T_u)}$	$\frac{3\sqrt{7}}{28}$	$-\frac{3\sqrt{7}}{28}$	$\frac{3\sqrt{7}}{28}$	$-\frac{3\sqrt{7}}{28}$	$\frac{\sqrt{7}}{28}$	$-\frac{\sqrt{7}}{28}$	$\frac{\sqrt{7}}{28}$	$-\frac{\sqrt{7}}{28}$	$\frac{\sqrt{7}}{14}$	$-\frac{\sqrt{7}}{14}$
	$-\frac{\sqrt{7}}{14}$	$\frac{\sqrt{7}}{14}$	$-\frac{3\sqrt{7}}{28}$	$\frac{3\sqrt{7}}{28}$	$-\frac{3\sqrt{7}}{28}$	$\frac{3\sqrt{7}}{28}$	$-\frac{\sqrt{7}}{28}$	$\frac{\sqrt{7}}{28}$	$-\frac{\sqrt{7}}{28}$	$\frac{\sqrt{7}}{28}$
	$-\frac{\sqrt{7}}{14}$	$\frac{\sqrt{7}}{14}$	$\frac{\sqrt{7}}{14}$	$-\frac{\sqrt{7}}{14}$						
$Q_{1,1}^{(T_u)}$	$\frac{\sqrt{7}}{14}$	$-\frac{\sqrt{7}}{14}$	$-\frac{\sqrt{7}}{14}$	$\frac{\sqrt{7}}{14}$	$\frac{3\sqrt{7}}{28}$	$-\frac{3\sqrt{7}}{28}$	$-\frac{3\sqrt{7}}{28}$	$\frac{3\sqrt{7}}{28}$	$\frac{\sqrt{7}}{28}$	$\frac{\sqrt{7}}{28}$
	$-\frac{\sqrt{7}}{28}$	$-\frac{\sqrt{7}}{28}$	$-\frac{\sqrt{7}}{14}$	$\frac{\sqrt{7}}{14}$	$\frac{\sqrt{7}}{14}$	$-\frac{\sqrt{7}}{14}$	$-\frac{3\sqrt{7}}{28}$	$\frac{3\sqrt{7}}{28}$	$\frac{3\sqrt{7}}{28}$	$-\frac{3\sqrt{7}}{28}$
	$-\frac{\sqrt{7}}{28}$	$-\frac{\sqrt{7}}{28}$	$\frac{\sqrt{7}}{28}$	$\frac{\sqrt{7}}{28}$						
$Q_{1,2}^{(T_u)}$	$\frac{\sqrt{7}}{28}$	$\frac{\sqrt{7}}{28}$	$-\frac{\sqrt{7}}{28}$	$-\frac{\sqrt{7}}{28}$	$\frac{\sqrt{7}}{14}$	$\frac{\sqrt{7}}{14}$	$-\frac{\sqrt{7}}{14}$	$-\frac{\sqrt{7}}{14}$	$\frac{3\sqrt{7}}{28}$	$-\frac{3\sqrt{7}}{28}$
	$\frac{3\sqrt{7}}{28}$	$-\frac{3\sqrt{7}}{28}$	$-\frac{\sqrt{7}}{28}$	$-\frac{\sqrt{7}}{28}$	$\frac{\sqrt{7}}{28}$	$\frac{\sqrt{7}}{28}$	$-\frac{\sqrt{7}}{14}$	$-\frac{\sqrt{7}}{14}$	$\frac{\sqrt{7}}{14}$	$\frac{\sqrt{7}}{14}$
	$-\frac{3\sqrt{7}}{28}$	$\frac{3\sqrt{7}}{28}$	$-\frac{3\sqrt{7}}{28}$	$\frac{3\sqrt{7}}{28}$						
$Q_{2,0}^{(E_g)}$	$-\frac{11\sqrt{3}}{84}$	$-\frac{11\sqrt{3}}{84}$	$-\frac{11\sqrt{3}}{84}$	$-\frac{11\sqrt{3}}{84}$	$-\frac{\sqrt{3}}{42}$	$-\frac{\sqrt{3}}{42}$	$-\frac{\sqrt{3}}{42}$	$-\frac{\sqrt{3}}{42}$	$\frac{13\sqrt{3}}{84}$	$\frac{13\sqrt{3}}{84}$
	$\frac{13\sqrt{3}}{84}$	$\frac{13\sqrt{3}}{84}$	$-\frac{11\sqrt{3}}{84}$	$-\frac{11\sqrt{3}}{84}$	$-\frac{11\sqrt{3}}{84}$	$-\frac{11\sqrt{3}}{84}$	$-\frac{\sqrt{3}}{42}$	$-\frac{\sqrt{3}}{42}$	$-\frac{\sqrt{3}}{42}$	$-\frac{\sqrt{3}}{42}$

continued ...

[illegible]

continued ...

Table 17

[illegible]

