SAMB for "C3v5"

Generated on 2023-09-27 07:01 by MultiPie 1.1.14

- Group: No. 160 C_{3v}^5 $R3m$ [trigonal]
- Associated point group: No. 19 C_{3v} $3m1$ (3m1 setting) [trigonal]

- Generation condition
  - model type: `tight_binding`
  - time-reversal type: `electric`
  - irrep: `[A1]`
  - `spinful`

- Unit cell:
  $a = 1.0,\ b = 1.0,\ c = 1.0,\ \alpha = 90.0,\ \beta = 90.0,\ \gamma = 120.0$
- Lattice vectors:
  $\boldsymbol{a}_1 = \begin{pmatrix} 1.0 & 0 & 0 \end{pmatrix}$
  $\boldsymbol{a}_2 = \begin{pmatrix} -0.5 & 0.86602540378444 & 0 \end{pmatrix}$
  $\boldsymbol{a}_3 = \begin{pmatrix} 0 & 0 & 1.0 \end{pmatrix}$
- Plus sets:
  $+\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$
  $+\begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$
  $+\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix}$

Table 1: High-symmetry line: Γ-X.

	symbol	position	symbol	position
	Γ	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	X	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$

- Kets: dimension = 54

Table 2: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket	No.	ket	No.	ket
1	$(p_x,\uparrow)@A_1$	2	$(p_x,\downarrow)@A_1$	3	$(p_y,\uparrow)@A_1$	4	$(p_y,\downarrow)@A_1$	5	$(p_z,\uparrow)@A_1$
6	$(p_z,\downarrow)@A_1$	7	$(p_x,\uparrow)@A_2$	8	$(p_x,\downarrow)@A_2$	9	$(p_y,\uparrow)@A_2$	10	$(p_y,\downarrow)@A_2$
11	$(p_z,\uparrow)@A_2$	12	$(p_z,\downarrow)@A_2$	13	$(p_x,\uparrow)@A_3$	14	$(p_x,\downarrow)@A_3$	15	$(p_y,\uparrow)@A_3$
16	$(p_y,\downarrow)@A_3$	17	$(p_z,\uparrow)@A_3$	18	$(p_z,\downarrow)@A_3$	19	$(p_x,\uparrow)@B_1$	20	$(p_x,\downarrow)@B_1$
21	$(p_y,\uparrow)@B_1$	22	$(p_y,\downarrow)@B_1$	23	$(p_z,\uparrow)@B_1$	24	$(p_z,\downarrow)@B_1$	25	$(p_x,\uparrow)@B_2$
26	$(p_x,\downarrow)@B_2$	27	$(p_y,\uparrow)@B_2$	28	$(p_y,\downarrow)@B_2$	29	$(p_z,\uparrow)@B_2$	30	$(p_z,\downarrow)@B_2$
31	$(p_x,\uparrow)@B_3$	32	$(p_x,\downarrow)@B_3$	33	$(p_y,\uparrow)@B_3$	34	$(p_y,\downarrow)@B_3$	35	$(p_z,\uparrow)@B_3$
36	$(p_z,\downarrow)@B_3$	37	$(p_x,\uparrow)@B_4$	38	$(p_x,\downarrow)@B_4$	39	$(p_y,\uparrow)@B_4$	40	$(p_y,\downarrow)@B_4$
41	$(p_z,\uparrow)@B_4$	42	$(p_z,\downarrow)@B_4$	43	$(p_x,\uparrow)@B_5$	44	$(p_x,\downarrow)@B_5$	45	$(p_y,\uparrow)@B_5$
46	$(p_y,\downarrow)@B_5$	47	$(p_z,\uparrow)@B_5$	48	$(p_z,\downarrow)@B_5$	49	$(p_x,\uparrow)@B_6$	50	$(p_x,\downarrow)@B_6$
51	$(p_y,\uparrow)@B_6$	52	$(p_y,\downarrow)@B_6$	53	$(p_z,\uparrow)@B_6$	54	$(p_z,\downarrow)@B_6$		

- Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position	mapping
S_1 [9b: .m]	A_1	$\left(\frac{1}{6}\ \frac{5}{6}\ \frac{1}{3}\right)$	[1,6]
	A_2	$\left(\frac{1}{6}\ \frac{1}{3}\ \frac{1}{3}\right)$	[2,5]
	A_3	$\left(\frac{2}{3}\ \frac{5}{6}\ \frac{1}{3}\right)$	[3,4]
S_2 [18c: 1]	B_1	$\left(\frac{5}{6}\ 0\ \frac{2}{3}\right)$	[1]
	B_2	$\left(0\ \frac{5}{6}\ \frac{2}{3}\right)$	[2]
	B_3	$\left(\frac{1}{6}\ \frac{1}{6}\ \frac{2}{3}\right)$	[3]
	B_4	$\left(\frac{1}{6}\ 0\ \frac{2}{3}\right)$	[4]
	B_5	$\left(\frac{5}{6}\ \frac{5}{6}\ \frac{2}{3}\right)$	[5]
	B_6	$\left(0\ \frac{1}{6}\ \frac{2}{3}\right)$	[6]

- Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

	bond	tail	head	n	#	$\boldsymbol{b}@\boldsymbol{c}$	mapping
B_1 [18c: 1]	b_1	B_1	A_1	1	1	$\begin{pmatrix}0 & -\frac{1}{6} & 0\end{pmatrix}@\begin{pmatrix}\frac{5}{6} & \frac{1}{12} & \frac{2}{3}\end{pmatrix}$	[1]
	b_2	B_2	A_2	1	1	$\begin{pmatrix}\frac{1}{6} & \frac{1}{6} & 0\end{pmatrix}@\begin{pmatrix}\frac{11}{12} & \frac{3}{4} & \frac{2}{3}\end{pmatrix}$	[2]
	b_3	B_3	A_3	1	1	$\begin{pmatrix}-\frac{1}{6} & 0 & 0\end{pmatrix}@\begin{pmatrix}\frac{1}{4} & \frac{1}{6} & \frac{2}{3}\end{pmatrix}$	[3]
	b_4	B_4	A_3	1	1	$\begin{pmatrix}-\frac{1}{6} & -\frac{1}{6} & 0\end{pmatrix}@\begin{pmatrix}\frac{1}{4} & \frac{1}{12} & \frac{2}{3}\end{pmatrix}$	[4]
	b_5	B_5	A_2	1	1	$\begin{pmatrix}0 & \frac{1}{6} & 0\end{pmatrix}@\begin{pmatrix}\frac{5}{6} & \frac{3}{4} & \frac{2}{3}\end{pmatrix}$	[5]
	b_6	B_6	A_1	1	1	$\begin{pmatrix}\frac{1}{6} & 0 & 0\end{pmatrix}@\begin{pmatrix}\frac{11}{12} & \frac{1}{6} & \frac{2}{3}\end{pmatrix}$	[6]

- SAMB:

No. 1 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1, \mathrm{S}_1]$

$\hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$

No. 2 $\hat{\mathbb{Q}}_2^{(A_1)}$ $[\mathrm{M}_1, \mathrm{S}_1]$

$\hat{\mathbb{Z}}_2 = \mathbb{X}_2[\mathbb{Q}_2^{(a,A_1)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$

No. 3 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_3 = \frac{\sqrt{2}\mathbb{X}_7[\mathbb{Q}_{2,0}^{(a,E,2)}] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_8[\mathbb{Q}_{2,1}^{(a,E,2)}] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s,E,2)}]}{2}$$

No. 4 $\hat{\mathbb{G}}_3^{(A_1)}$ $[\mathrm{M}_1, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_4 = \frac{\sqrt{2}\mathbb{X}_5[\mathbb{Q}_{2,0}^{(a,E,1)}] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_6[\mathbb{Q}_{2,1}^{(a,E,1)}] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s,E,2)}]}{2}$$

No. 5 $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_1, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_5 = \mathbb{X}_3[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 6 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_6 = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_1)}(1,-1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 7 $\hat{\mathbb{Q}}_0^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_7 = \frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{Q}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{Q}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s,E,2)}]}{2}$$

No. 8 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_8 = \frac{\sqrt{2}\mathbb{X}_{10}[\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_9[\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s,E,2)}]}{2}$$

No. 9 $\hat{\mathbb{G}}_3^{(A_1)}(1,0)$ $[\mathrm{M}_1, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_9 = -\frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{G}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s,E,2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s,E,2)}]}{2}$$

No. 10 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1, \mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{10} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(s,A_1)}]$$

No. 11 $\hat{\mathbb{Q}}_2^{(A_1)}$ $[\mathrm{M}_1, \mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{11} = \mathbb{X}_2[\mathbb{Q}_2^{(a,A_1)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(s,A_1)}]$$

No. 12 $\hat{\mathbb{Q}}_1^{(A_1)}$ $[\mathrm{M}_1, \mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{12} = \frac{\sqrt{2}\mathbb{X}_5[\mathbb{Q}_{2,0}^{(a,E,1)}] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(s,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_6[\mathbb{Q}_{2,1}^{(a,E,1)}] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(s,E)}]}{2}$$

No. 13 $\hat{\mathbb{Q}}_3^{(A_1,2)}$ $[\mathrm{M}_1, \mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{13} = -\frac{\sqrt{2}\mathbb{X}_7[\mathbb{Q}_{2,0}^{(a,E,2)}] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(s,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_8[\mathbb{Q}_{2,1}^{(a,E,2)}] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(s,E)}]}{2}$$

No. 14 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1, \mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{14} = \frac{\sqrt{2}\mathbb{X}_7[\mathbb{Q}_{2,0}^{(a,E,2)}] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(s,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_8[\mathbb{Q}_{2,1}^{(a,E,2)}] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(s,E,2)}]}{2}$$

No. 15 $\hat{\mathbb{G}}_3^{(A_1)}$ $[\mathrm{M}_1, \mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{15} = \frac{\sqrt{2}\mathbb{X}_5[\mathbb{Q}_{2,0}^{(a,E,1)}] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(s,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_6[\mathbb{Q}_{2,1}^{(a,E,1)}] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(s,E,2)}]}{2}$$

No. 16 $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_1, \mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{16} = \mathbb{X}_3[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(s,A_1)}]$$

No. 17 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{17} = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_1)}(1,-1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(s,A_1)}]$$

No. 18 $\hat{\mathbb{Q}}_1^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{18} = \frac{\sqrt{2}\mathbb{X}_{10}[\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(s,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_9[\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(s,E)}]}{2}$$

No. 19 $\hat{\mathbb{Q}}_3^{(A_1,2)}(1,-1)$ $[\mathrm{M}_1, \mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{19} = -\frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{Q}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(s,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{Q}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(s,E)}]}{2}$$

No. 20 $\hat{\mathbb{Q}}_0^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{20} = \frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{Q}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(s,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{Q}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(s,E,2)}]}{2}$$

No. 21 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_1,\mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{21} = \frac{\sqrt{2}\mathbb{X}_{10}[\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(s,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_9[\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(s,E,2)}]}{2}$$

No. 22 $\hat{\mathbb{Q}}_1^{(A_1)}(1,0)$ $[\mathrm{M}_1,\mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{22} = \frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{G}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(s,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(s,E)}]}{2}$$

No. 23 $\hat{\mathbb{G}}_3^{(A_1)}(1,0)$ $[\mathrm{M}_1,\mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{23} = -\frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{G}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(s,E,2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(s,E,2)}]}{2}$$

No. 24 $\hat{\mathbb{Q}}_3^{(A_1,2)}(1,0)$ $[\mathrm{M}_1,\mathrm{S}_2]$

$$\hat{\mathbb{Z}}_{24} = \mathbb{X}_{36}[\mathbb{G}_1^{(a,A_2)}(1,0)] \otimes \mathbb{Y}_9[\mathbb{Q}_3^{(s,A_2)}]$$

No. 25 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{25} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_{10}[\mathbb{Q}_0^{(b,A_1)}]$$

No. 26 $\hat{\mathbb{Q}}_2^{(A_1)}$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{26} = \mathbb{X}_2[\mathbb{Q}_2^{(a,A_1)}] \otimes \mathbb{Y}_{10}[\mathbb{Q}_0^{(b,A_1)}]$$

No. 27 $\hat{\mathbb{Q}}_1^{(A_1)}$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{27} = \frac{\sqrt{2}\mathbb{X}_5[\mathbb{Q}_{2,0}^{(a,E,1)}] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{1,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_6[\mathbb{Q}_{2,1}^{(a,E,1)}] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{1,1}^{(b,E)}]}{2}$$

No. 28 $\hat{\mathbb{Q}}_3^{(A_1,2)}$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{28} = -\frac{\sqrt{2}\mathbb{X}_7[\mathbb{Q}_{2,0}^{(a,E,2)}] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{1,0}^{(b,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_8[\mathbb{Q}_{2,1}^{(a,E,2)}] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{1,1}^{(b,E)}]}{2}$$

No. 29 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{29} = \frac{\sqrt{2}\mathbb{X}_7[\mathbb{Q}_{2,0}^{(a,E,2)}] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{2,0}^{(b,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_8[\mathbb{Q}_{2,1}^{(a,E,2)}] \otimes \mathbb{Y}_{14}[\mathbb{Q}_{2,1}^{(b,E,2)}]}{2}$$

No. 30 $\hat{\mathbb{G}}_3^{(A_1)}$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{30} = \frac{\sqrt{2}\mathbb{X}_5[\mathbb{Q}_{2,0}^{(a,E,1)}] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{2,0}^{(b,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_6[\mathbb{Q}_{2,1}^{(a,E,1)}] \otimes \mathbb{Y}_{14}[\mathbb{Q}_{2,1}^{(b,E,2)}]}{2}$$

No. 31 $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{31} = \mathbb{X}_3[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_0^{(b,A_1)}]$$

No. 32 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{32} = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_1)}(1,-1)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_0^{(b,A_1)}]$$

No. 33 $\hat{\mathbb{Q}}_1^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{33} = \frac{\sqrt{2}\mathbb{X}_{10}[\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{1,1}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_9[\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{1,0}^{(b,E)}]}{2}$$

No. 34 $\hat{\mathbb{Q}}_3^{(A_1,2)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{34} = -\frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{Q}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{1,0}^{(b,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{Q}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{1,1}^{(b,E)}]}{2}$$

No. 35 $\hat{\mathbb{Q}}_0^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{35} = \frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{Q}_{2,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{2,0}^{(b,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{Q}_{2,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_{14}[\mathbb{Q}_{2,1}^{(b,E,2)}]}{2}$$

No. 36 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{36} = \frac{\sqrt{2}\mathbb{X}_{10}[\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_{14}[\mathbb{Q}_{2,1}^{(b,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_9[\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{2,0}^{(b,E,2)}]}{2}$$

No. 37 $\hat{\mathbb{Q}}_1^{(A_1)}(1,0)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{37} = \frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{G}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{1,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{1,1}^{(b,E)}]}{2}$$

No. 38 $\hat{\mathbb{G}}_3^{(A_1)}(1,0)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{38} = -\frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{G}_{1,0}^{(a,E)}(1,0)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{2,0}^{(b,E,2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{1,1}^{(a,E)}(1,0)] \otimes \mathbb{Y}_{14}[\mathbb{Q}_{2,1}^{(b,E,2)}]}{2}$$

No. 39 $\hat{\mathbb{Q}}_3^{(A_1,2)}(1,0)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{39} = \mathbb{X}_{36}[\mathbb{G}_1^{(a,A_2)}(1,0)] \otimes \mathbb{Y}_{15}[\mathbb{Q}_3^{(b,A_2)}]$$

No. 40 $\hat{\mathbb{Q}}_1^{(A_1)}$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{40} = \frac{\sqrt{2}\mathbb{X}_{16}[\mathbb{M}_{1,0}^{(a,E)}] \otimes \mathbb{Y}_{17}[\mathbb{T}_{1,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{17}[\mathbb{M}_{1,1}^{(a,E)}] \otimes \mathbb{Y}_{18}[\mathbb{T}_{1,1}^{(b,E)}]}{2}$$

No. 41 $\hat{\mathbb{G}}_3^{(A_1)}$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{41} = -\frac{\sqrt{2}\mathbb{X}_{16}[\mathbb{M}_{1,0}^{(a,E)}] \otimes \mathbb{Y}_{19}[\mathbb{T}_{2,0}^{(b,E,2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{17}[\mathbb{M}_{1,1}^{(a,E)}] \otimes \mathbb{Y}_{20}[\mathbb{T}_{2,1}^{(b,E,2)}]}{2}$$

No. 42 $\hat{\mathbb{Q}}_3^{(A_1,2)}$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{42} = \mathbb{X}_{15}[\mathbb{M}_1^{(a,A_2)}] \otimes \mathbb{Y}_{21}[\mathbb{T}_3^{(b,A_2)}]$$

No. 43 $\hat{\mathbb{Q}}_1^{(A_1)}(1,1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{43} = \frac{\sqrt{2}\mathbb{X}_{22}[\mathbb{M}_{1,0}^{(a,E)}(1,1)] \otimes \mathbb{Y}_{17}[\mathbb{T}_{1,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{23}[\mathbb{M}_{1,1}^{(a,E)}(1,1)] \otimes \mathbb{Y}_{18}[\mathbb{T}_{1,1}^{(b,E)}]}{2}$$

No. 44 $\hat{\mathbb{G}}_3^{(A_1)}(1,1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{44} = -\frac{\sqrt{2}\mathbb{X}_{22}[\mathbb{M}_{1,0}^{(a,E)}(1,1)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{2,0}^{(b,E,2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{23}[\mathbb{M}_{1,1}^{(a,E)}(1,1)] \otimes \mathbb{Y}_{20}[\mathbb{T}_{2,1}^{(b,E,2)}]}{2}$$

No. 45 $\hat{\mathbb{Q}}_3^{(A_1,2)}(1,1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{45} = \mathbb{X}_{18}[\mathbb{M}_1^{(a,A_2)}(1,1)] \otimes \mathbb{Y}_{21}[\mathbb{T}_3^{(b,A_2)}]$$

No. 46 $\hat{\mathbb{Q}}_1^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{46} = \frac{\sqrt{2}\mathbb{X}_{24}[\mathbb{M}_{1,0}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{17}[\mathbb{T}_{1,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{M}_{1,1}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{18}[\mathbb{T}_{1,1}^{(b,E)}]}{2}$$

No. 47 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{47} = -\frac{\sqrt{2}\mathbb{X}_{24}[\mathbb{M}_{1,0}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{2,0}^{(b,E,2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{M}_{1,1}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{20}[\mathbb{T}_{2,1}^{(b,E,2)}]}{2}$$

No. 48 $\hat{\mathbb{Q}}_3^{(A_1,2)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{48} = \mathbb{X}_{19}[\mathbb{M}_1^{(a,A_2)}(1,-1)] \otimes \mathbb{Y}_{21}[\mathbb{T}_3^{(b,A_2)}]$$

No. 49 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{49} = \mathbb{X}_{34}[\mathbb{M}_3^{(a,A_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_0^{(b,A_1)}]$$

No. 50 $\hat{\mathbb{Q}}_3^{(A_1,1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{50} = \frac{\sqrt{2}\mathbb{X}_{26}[\mathbb{M}_{3,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_{17}[\mathbb{T}_{1,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{27}[\mathbb{M}_{3,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_{18}[\mathbb{T}_{1,1}^{(b,E)}]}{2}$$

No. 51 $\hat{\mathbb{Q}}_3^{(A_1,2)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{51} = \frac{\sqrt{2}\mathbb{X}_{28}[\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_{17}[\mathbb{T}_{1,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{29}[\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_{18}[\mathbb{T}_{1,1}^{(b,E)}]}{2}$$

No. 52 $\hat{\mathbb{Q}}_2^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{52} = -\frac{\sqrt{2}\mathbb{X}_{28}[\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{2,0}^{(b,E,2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{29}[\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_{20}[\mathbb{T}_{2,1}^{(b,E,2)}]}{2}$$

No. 53 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{53} = \frac{\sqrt{2}\mathbb{X}_{26}[\mathbb{M}_{3,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{2,0}^{(b,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{27}[\mathbb{M}_{3,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_{20}[\mathbb{T}_{2,1}^{(b,E,2)}]}{2}$$

No. 54 $\hat{\mathbb{Q}}_1^{(A_1)}(1,-1)$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{54} = -\mathbb{X}_{21}[\mathbb{M}_3^{(a,A_2,2)}(1,-1)] \otimes \mathbb{Y}_{21}[\mathbb{T}_3^{(b,A_2)}]$$

No. 55 $\hat{\mathbb{Q}}_3^{(A_1,2)}(1,-1)$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{55} = -\mathbb{X}_{20}[\mathbb{M}_3^{(a,A_2,1)}(1,-1)] \otimes \mathbb{Y}_{21}[\mathbb{T}_3^{(b,A_2)}]$$

No. 56 $\hat{\mathbb{Q}}_2^{(A_1)}(1,0)$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{56} = \mathbb{X}_{35}[\mathbb{T}_2^{(a,A_1)}(1,0)] \otimes \mathbb{Y}_{16}[\mathbb{T}_0^{(b,A_1)}]$$

No. 57 $\hat{\mathbb{Q}}_1^{(A_1)}(1,0)$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{57} = \frac{\sqrt{2}\mathbb{X}_{30}[\mathbb{T}_{2,0}^{(a,E,1)}(1,0)] \otimes \mathbb{Y}_{17}[\mathbb{T}_{1,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{31}[\mathbb{T}_{2,1}^{(a,E,1)}(1,0)] \otimes \mathbb{Y}_{18}[\mathbb{T}_{1,1}^{(b,E)}]}{2}$$

No. 58 $\hat{\mathbb{Q}}_3^{(A_1,2)}(1,0)$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{58} = -\frac{\sqrt{2}\mathbb{X}_{32}[\mathbb{T}_{2,0}^{(a,E,2)}(1,0)] \otimes \mathbb{Y}_{17}[\mathbb{T}_{1,0}^{(b,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{33}[\mathbb{T}_{2,1}^{(a,E,2)}(1,0)] \otimes \mathbb{Y}_{18}[\mathbb{T}_{1,1}^{(b,E)}]}{2}$$

No. 59 $\hat{\mathbb{Q}}_0^{(A_1)}(1,0)$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{59} = \frac{\sqrt{2}\mathbb{X}_{32}[\mathbb{T}_{2,0}^{(a,E,2)}(1,0)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{2,0}^{(b,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{33}[\mathbb{T}_{2,1}^{(a,E,2)}(1,0)] \otimes \mathbb{Y}_{20}[\mathbb{T}_{2,1}^{(b,E,2)}]}{2}$$

No. 60 $\hat{\mathbb{G}}_3^{(A_1)}(1,0)$ $[\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{60} = \frac{\sqrt{2}\mathbb{X}_{30}[\mathbb{T}_{2,0}^{(a,E,1)}(1,0)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{2,0}^{(b,E,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{31}[\mathbb{T}_{2,1}^{(a,E,1)}(1,0)] \otimes \mathbb{Y}_{20}[\mathbb{T}_{2,1}^{(b,E,2)}]}{2}$$

Table 5: Atomic SAMB group.

group	bra	ket
M_1	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$

Table 6: Atomic SAMB.

symbol	type	group	form
$\mathbb{X}_1$	$\mathbb{Q}_0^{(a,A_1)}$	M_1	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_2$	$\mathbb{Q}_2^{(a,A_1)}$	M_1	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{X}_3$	$\mathbb{Q}_0^{(a,A_1)}(1,1)$	M_1	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ \frac{\sqrt{3}i}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_4$	$\mathbb{Q}_2^{(a,A_1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_5$	$\mathbb{Q}_{2,0}^{(a,E,1)}$	M_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_6$	$\mathbb{Q}_{2,1}^{(a,E,1)}$	M_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_7$	$\mathbb{Q}_{2,0}^{(a,E,2)}$	M_1	$\begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_8$	$\mathbb{Q}_{2,1}^{(a,E,2)}$	M_1	$\begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_9$	$\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{10}$	$\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{11}$	$\mathbb{Q}_{2,0}^{(a,E,2)}(1,-1)$	M_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{12}$	$\mathbb{Q}_{2,1}^{(a,E,2)}(1,-1)$	M_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & -\frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{13}$	$\mathbb{G}_{1,0}^{(a,E)}(1,0)$	M_1	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{14}$	$\mathbb{G}_{1,1}^{(a,E)}(1,0)$	M_1	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{15}$	$\mathbb{M}_1^{(a,A_2)}$	M_1	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{16}$	$\mathbb{M}_{1,0}^{(a,E)}$	M_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{17}$	$\mathbb{M}_{1,1}^{(a,E)}$	M_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{18}$	$\mathbb{M}_1^{(a,A_2)}(1,1)$	M_1	$\begin{pmatrix} -\frac{\sqrt{30}}{30} & 0 & 0 & 0 & 0 & \frac{\sqrt{30}}{20} \\ 0 & \frac{\sqrt{30}}{30} & 0 & 0 & \frac{\sqrt{30}}{20} & 0 \\ 0 & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & -\frac{\sqrt{30}i}{20} \\ 0 & 0 & 0 & \frac{\sqrt{30}}{30} & \frac{\sqrt{30}i}{20} & 0 \\ 0 & \frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{15} & 0 \\ \frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{15} \end{pmatrix}$
$\mathbb{X}_{19}$	$\mathbb{M}_1^{(a,A_2)}(1,-1)$	M_1	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{20}$	$\mathbb{M}_3^{(a,A_2,1)}(1,-1)$	M_1	$\begin{pmatrix} -\frac{\sqrt{5}}{10} & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}}{10} & 0 & 0 & -\frac{\sqrt{5}}{10} & 0 \\ 0 & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & \frac{\sqrt{5}i}{10} \\ 0 & 0 & 0 & \frac{\sqrt{5}}{10} & -\frac{\sqrt{5}i}{10} & 0 \\ 0 & -\frac{\sqrt{5}}{10} & 0 & \frac{\sqrt{5}i}{10} & \frac{\sqrt{5}}{5} & 0 \\ -\frac{\sqrt{5}}{10} & 0 & -\frac{\sqrt{5}i}{10} & 0 & 0 & -\frac{\sqrt{5}}{5} \end{pmatrix}$
$\mathbb{X}_{21}$	$\mathbb{M}_3^{(a,A_2,2)}(1,-1)$	M_1	$\begin{pmatrix} 0 & -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{22}$	$\mathbb{M}_{1,0}^{(a,E)}(1,1)$	M_1	$\begin{pmatrix} 0 & -\frac{\sqrt{30}i}{30} & 0 & -\frac{\sqrt{30}}{20} & 0 & 0 \\ \frac{\sqrt{30}i}{30} & 0 & -\frac{\sqrt{30}}{20} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{15} & -\frac{\sqrt{30}}{20} & 0 \\ -\frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{15} & 0 & 0 & \frac{\sqrt{30}}{20} \\ 0 & 0 & -\frac{\sqrt{30}}{20} & 0 & 0 & -\frac{\sqrt{30}i}{30} \\ 0 & 0 & 0 & \frac{\sqrt{30}}{20} & \frac{\sqrt{30}i}{30} & 0 \end{pmatrix}$
$\mathbb{X}_{23}$	$\mathbb{M}_{1,1}^{(a,E)}(1,1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{30}}{15} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{20} & 0 \\ \frac{\sqrt{30}}{15} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{20} \\ 0 & -\frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 \\ \frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & 0 \\ \frac{\sqrt{30}}{20} & 0 & 0 & 0 & 0 & -\frac{\sqrt{30}}{30} \\ 0 & -\frac{\sqrt{30}}{20} & 0 & 0 & -\frac{\sqrt{30}}{30} & 0 \end{pmatrix}$
$\mathbb{X}_{24}$	$\mathbb{M}_{1,0}^{(a,E)}(1,-1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{25}$	$\mathbb{M}_{1,1}^{(a,E)}(1,-1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{26}$	$\mathbb{M}_{3,0}^{(a,E,1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & -\frac{\sqrt{30}i}{60} & 0 & \frac{\sqrt{30}}{60} & 0 & 0 \\ \frac{\sqrt{30}i}{60} & 0 & \frac{\sqrt{30}}{60} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{30}}{60} & 0 & -\frac{\sqrt{30}i}{20} & -\frac{\sqrt{30}}{15} & 0 \\ \frac{\sqrt{30}}{60} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & \frac{\sqrt{30}}{15} \\ 0 & 0 & -\frac{\sqrt{30}}{15} & 0 & 0 & \frac{\sqrt{30}i}{15} \\ 0 & 0 & 0 & \frac{\sqrt{30}}{15} & -\frac{\sqrt{30}i}{15} & 0 \end{pmatrix}$
$\mathbb{X}_{27}$	$\mathbb{M}_{3,1}^{(a,E,1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & -\frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{60} & \frac{\sqrt{30}}{15} & 0 \\ -\frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{60} & 0 & 0 & -\frac{\sqrt{30}}{15} \\ 0 & \frac{\sqrt{30}i}{60} & 0 & -\frac{\sqrt{30}}{60} & 0 & 0 \\ -\frac{\sqrt{30}i}{60} & 0 & -\frac{\sqrt{30}}{60} & 0 & 0 & 0 \\ \frac{\sqrt{30}}{15} & 0 & 0 & 0 & 0 & \frac{\sqrt{30}}{15} \\ 0 & -\frac{\sqrt{30}}{15} & 0 & 0 & \frac{\sqrt{30}}{15} & 0 \end{pmatrix}$
$\mathbb{X}_{28}$	$\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)$	M_1	$\begin{pmatrix} \frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{29}$	$\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)$	M_1	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}i}{6} & 0 \\ -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \\ 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 \\ 0 & \frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ -\frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{30}$	$\mathbb{T}_{2,0}^{(a,E,1)}(1,0)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 \\ -\frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{12} \\ 0 & 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{12} & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{31}$	$\mathbb{T}_{2,1}^{(a,E,1)}(1,0)$	M_1	$\begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ -\frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \\ 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{32}$	$\mathbb{T}_{2,0}^{(a,E,2)}(1,0)$	M_1	$\begin{pmatrix} -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{12} \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & \frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}i}{12} & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & \frac{\sqrt{6}i}{12} & 0 & 0 \\ \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{33}$	$\mathbb{T}_{2,1}^{(a,E,2)}(1,0)$	M_1	$\begin{pmatrix} 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}i}{12} & 0 \\ \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} & 0 \\ 0 & \frac{\sqrt{6}i}{12} & 0 & -\frac{\sqrt{6}}{12} & 0 & 0 \\ -\frac{\sqrt{6}i}{12} & 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{34}$	$\mathbb{M}_3^{(a,A_1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{35}$	$\mathbb{T}_2^{(a,A_1)}(1,0)$	M_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{36}$	$\mathbb{G}_1^{(a,A_2)}(1,0)$	M_1	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$

Table 7: Cluster SAMB.

symbol	type	cluster	form
$\mathbb{Y}_1$	$\mathbb{Q}_0^{(s,A_1)}$	S_1	$\begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{Y}_2$	$\mathbb{Q}_{2,0}^{(s,E,2)}$	S_1	$\begin{pmatrix} -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{Y}_3$	$\mathbb{Q}_{2,1}^{(s,E,2)}$	S_1	$\begin{pmatrix} \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_4$	$\mathbb{Q}_0^{(s,A_1)}$	S_2	$\begin{pmatrix} \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_5$	$\mathbb{Q}_{1,0}^{(s,E)}$	S_2	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{6} \end{pmatrix}$
$\mathbb{Y}_6$	$\mathbb{Q}_{1,1}^{(s,E)}$	S_2	$\begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$
$\mathbb{Y}_7$	$\mathbb{Q}_{2,0}^{(s,E,2)}$	S_2	$\begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$
$\mathbb{Y}_8$	$\mathbb{Q}_{2,1}^{(s,E,2)}$	S_2	$\begin{pmatrix} \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{6} \end{pmatrix}$
$\mathbb{Y}_9$	$\mathbb{Q}_3^{(s,A_2)}$	S_2	$\begin{pmatrix} \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_{10}$	$\mathbb{Q}_0^{(b,A_1)}$	B_1	$\begin{pmatrix} \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_{11}$	$\mathbb{Q}_{1,0}^{(b,E)}$	B_1	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{6} \end{pmatrix}$
$\mathbb{Y}_{12}$	$\mathbb{Q}_{1,1}^{(b,E)}$	B_1	$\begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$

continued ...

Table 7

symbol	type	cluster	form
$\mathbb{Y}_{13}$	$\mathbb{Q}_{2,0}^{(b,E,2)}$	B_1	$\begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$
$\mathbb{Y}_{14}$	$\mathbb{Q}_{2,1}^{(b,E,2)}$	B_1	$\begin{pmatrix} \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{6} \end{pmatrix}$
$\mathbb{Y}_{15}$	$\mathbb{Q}_{3}^{(b,A_2)}$	B_1	$\begin{pmatrix} \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_{16}$	$\mathbb{T}_{0}^{(b,A_1)}$	B_1	$\begin{pmatrix} \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} \end{pmatrix}$
$\mathbb{Y}_{17}$	$\mathbb{T}_{1,0}^{(b,E)}$	B_1	$\begin{pmatrix} -\frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{3} & -\frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}i}{3} & \frac{\sqrt{3}i}{6} \end{pmatrix}$
$\mathbb{Y}_{18}$	$\mathbb{T}_{1,1}^{(b,E)}$	B_1	$\begin{pmatrix} -\frac{i}{2} & 0 & \frac{i}{2} & -\frac{i}{2} & 0 & \frac{i}{2} \end{pmatrix}$
$\mathbb{Y}_{19}$	$\mathbb{T}_{2,0}^{(b,E,2)}$	B_1	$\begin{pmatrix} -\frac{i}{2} & 0 & \frac{i}{2} & \frac{i}{2} & 0 & -\frac{i}{2} \end{pmatrix}$
$\mathbb{Y}_{20}$	$\mathbb{T}_{2,1}^{(b,E,2)}$	B_1	$\begin{pmatrix} \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}i}{3} & \frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}i}{3} & \frac{\sqrt{3}i}{6} \end{pmatrix}$
$\mathbb{Y}_{21}$	$\mathbb{T}_{3}^{(b,A_2)}$	B_1	$\begin{pmatrix} \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} & -\frac{\sqrt{6}i}{6} & -\frac{\sqrt{6}i}{6} & -\frac{\sqrt{6}i}{6} \end{pmatrix}$

Table 8: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_{0}^{(A_1)}$	0	A_1	–	–	1
2	$\mathbb{Q}_{1,0}^{(E)}$	1	E	–	0	x
3	$\mathbb{Q}_{1,1}^{(E)}$	1	E	–	1	y
4	$\mathbb{Q}_{2}^{(A_1)}$	2	A_1	–	–	$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$
5	$\mathbb{Q}_{2,0}^{(E,1)}$	2	E	1	0	$\sqrt{3}xz$
6	$\mathbb{Q}_{2,1}^{(E,1)}$	2	E	1	1	$\sqrt{3}yz$
7	$\mathbb{Q}_{2,0}^{(E,2)}$	2	E	2	0	$-\sqrt{3}xy$
8	$\mathbb{Q}_{2,1}^{(E,2)}$	2	E	2	1	$-\frac{\sqrt{3}(x-y)(x+y)}{2}$
9	$\mathbb{Q}_{3}^{(A_2)}$	3	A_2	–	–	$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$

Table 9: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_1^{(A_2)}$	1	A_2	–	–	Z
2	$\mathbb{G}_{1,0}^{(E)}$	1	E	–	0	$-Y$
3	$\mathbb{G}_{1,1}^{(E)}$	1	E	–	1	X
4	$\mathbb{G}_3^{(A_1)}$	3	A_1	–	–	$\frac{\sqrt{10}X\left(X^2-3Y^2\right)}{4}$
5	$\mathbb{G}_3^{(A_2,1)}$	3	A_2	1	–	$-\frac{Z\left(3X^2+3Y^2-2Z^2\right)}{2}$
6	$\mathbb{G}_3^{(A_2,2)}$	3	A_2	2	–	$\frac{\sqrt{10}Y\left(3X^2-Y^2\right)}{4}$
7	$\mathbb{G}_{3,0}^{(E,1)}$	3	E	1	0	$\frac{\sqrt{6}Y\left(X^2+Y^2-4Z^2\right)}{4}$
8	$\mathbb{G}_{3,1}^{(E,1)}$	3	E	1	1	$-\frac{\sqrt{6}X\left(X^2+Y^2-4Z^2\right)}{4}$
9	$\mathbb{G}_{3,0}^{(E,2)}$	3	E	2	0	$\frac{\sqrt{15}Z(X-Y)(X+Y)}{2}$
10	$\mathbb{G}_{3,1}^{(E,2)}$	3	E	2	1	$-\sqrt{15}XYZ$

- Group info.: Generator = $\{3^+_{001}|0\},\ \{\mathrm{m}_{110}|0\}$

Table 10: Conjugacy class (point-group part).

rep. SO	symmetry operations
$\{1\|0\}$	$\{1\|0\}$
$\{3^+_{001}\|0\}$	$\{3^+_{001}\|0\},\ \{3^-_{001}\|0\}$
$\{\mathrm{m}_{100}\|0\}$	$\{\mathrm{m}_{100}\|0\},\ \{\mathrm{m}_{010}\|0\},\ \{\mathrm{m}_{110}\|0\}$

Table 11: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	$\{1\|0\}$	2	$\{3^{+}_{001}\|0\}$	3	$\{3^{-}_{001}\|0\}$	4	$\{\mathrm{m}_{100}\|0\}$	5	$\{\mathrm{m}_{010}\|0\}$
6	$\{\mathrm{m}_{110}\|0\}$								

Table 12: Character table (point-group part).

	1	3^{+}_{001}	m_{100}
A_1	1	1	1
A_2	1	1	-1
E	2	-1	0

Table 13: Parity conversion.

$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$
A_1 (A_2)	A_2 (A_1)	E (E)

Table 14: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

	A_1	A_2	E
A_1	A_1	A_2	E
A_2		A_1	E
E			$A_1 + E$

Table 15: Anti-symmetric product, $[\Gamma \otimes \Gamma]_-$.

	A_1	A_2	E
	$-$	$-$	A_2

Table 16: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix}-1 & -1 & 0\end{pmatrix}$	2	$\begin{pmatrix}1 & 0 & 0\end{pmatrix}$	3	$\begin{pmatrix}0 & 1 & 0\end{pmatrix}$	4	$\begin{pmatrix}0 & -1 & 0\end{pmatrix}$
5	$\begin{pmatrix}-1 & 0 & 0\end{pmatrix}$	6	$\begin{pmatrix}1 & 1 & 0\end{pmatrix}$				

Table 17: Virtual-cluster basis.

symbol	1	2	3	4	5	6
$\mathbb{Q}_0^{(A_1)}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$
$\mathbb{Q}_{1,0}^{(E)}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{6}$
$\mathbb{Q}_{1,1}^{(E)}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$
$\mathbb{Q}_{2,0}^{(E,2)}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$
$\mathbb{Q}_{2,1}^{(E,2)}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{6}$
$\mathbb{Q}_3^{(A_2)}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$