

SAMB for “Cs1”

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- Group: No. 6 $C_s^1 Pm$ (b-axis setting) [monoclinic]
 - Associated point group: No. 4 $C_s m$ (b-axis setting) [monoclinic]
 - Generation condition
 - model type: **tight_binding**
 - time-reversal type: **electric**
 - irrep: [A’]
 - **spinful**
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- Unit cell:
 - $a = 1.0, b = 1.0, c = 1.0, \alpha = 90.0, \beta = 90.0, \gamma = 90.0$
- Lattice vectors:
 - $\mathbf{a}_1 = (1.0 \ 0 \ 0)$
 - $\mathbf{a}_2 = (0 \ 1.0 \ 0)$
 - $\mathbf{a}_3 = (0 \ 0 \ 1.0)$

Table 1: High-symmetry line: Γ -X.

symbol	position	symbol	position
Γ	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	X	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$

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- Kets: dimension = 4

Table 2: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket	No.	ket
1	$(p_x, \uparrow)@A_1$	2	$(p_x, \downarrow)@A_1$	3	$(p_y, \uparrow)@A_1$	4	$(p_y, \downarrow)@A_1$

- Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position	mapping
S ₁ [1a: m]	A ₁	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	[1,2]

- Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

bond		tail	head	n	#	$\mathbf{b@c}$	mapping
B ₁ [1b: m]	b ₁	A ₁	A ₁	1	1	$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{2} & 0 \end{pmatrix}$	[1,-2]
B ₂ [1a: m]	b ₂	A ₁	A ₁	1	2	$\begin{pmatrix} -1 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$	[1,2]
B ₃ [1a: m]	b ₃	A ₁	A ₁	1	3	$\begin{pmatrix} 0 & 0 & -1 \end{pmatrix} @ \begin{pmatrix} 0 & 0 & \frac{1}{2} \end{pmatrix}$	[1,2]
B ₄ [1a: m]	b ₄	A ₁	A ₁	2	1	$\begin{pmatrix} -1 & 0 & -1 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$	[1,2]
B ₅ [1b: m]	b ₅	A ₁	A ₁	2	2	$\begin{pmatrix} -1 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	[1]
	b ₆	A ₁	A ₁	2	2	$\begin{pmatrix} -1 & -1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	[2]
B ₆ [1b: m]	b ₇	A ₁	A ₁	2	3	$\begin{pmatrix} 0 & 1 & 1 \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	[1]
	b ₈	A ₁	A ₁	2	3	$\begin{pmatrix} 0 & -1 & 1 \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	[2]
B ₇ [1a: m]	b ₉	A ₁	A ₁	2	4	$\begin{pmatrix} -1 & 0 & 1 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$	[1,2]

- SAMB:

$$\boxed{\text{No. 1}} \quad \hat{\mathbb{Q}}_0^{(A')} \text{ [M}_1, \text{S}_1]$$

$$\hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A')}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A')}]$$

$$\boxed{\text{No. 2}} \quad \hat{\mathbb{Q}}_2^{(A',2)} \text{ [M}_1, \text{S}_1]$$

$$\hat{\mathbb{Z}}_2 = \mathbb{X}_2[\mathbb{Q}_2^{(a,A',2)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A')}]$$

$$\boxed{\text{No. 3}} \quad \hat{\mathbb{Q}}_0^{(A')}(1, 1) \text{ [M}_1, \text{S}_1]$$

$$\hat{\mathbb{Z}}_3 = \mathbb{X}_3[\mathbb{Q}_0^{(a,A')}(1, 1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A')}]$$

$$\boxed{\text{No. 4}} \quad \hat{\mathbb{Q}}_2^{(A',3)}(1, -1) \text{ [M}_1, \text{S}_1]$$

$$\hat{\mathbb{Z}}_4 = \mathbb{X}_4[\mathbb{Q}_2^{(a,A',3)}(1, -1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A')}]$$

$$\boxed{\text{No. 5}} \quad \hat{\mathbb{Q}}_0^{(A')} \text{ [M}_1, \text{B}_1]$$

$$\hat{\mathbb{Z}}_5 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A')}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 6}} \quad \hat{\mathbb{Q}}_2^{(A',2)} \text{ [M}_1, \text{B}_1]$$

$$\hat{\mathbb{Z}}_6 = \mathbb{X}_2[\mathbb{Q}_2^{(a,A',2)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 7}} \quad \hat{\mathbb{Q}}_0^{(A')}(1, 1) \text{ [M}_1, \text{B}_1]$$

$$\hat{\mathbb{Z}}_7 = \mathbb{X}_3[\mathbb{Q}_0^{(a,A')}(1, 1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 8}} \quad \hat{\mathbb{Q}}_2^{(A',3)}(1, -1) \text{ [M}_1, \text{B}_1]$$

$$\hat{\mathbb{Z}}_8 = \mathbb{X}_4[\mathbb{Q}_2^{(a,A',3)}(1, -1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 9}} \quad \hat{\mathbb{Q}}_1^{(A',1)}(1, 1) \text{ [M}_1, \text{B}_1]$$

$$\hat{\mathbb{Z}}_9 = -\mathbb{X}_{12}[\mathbb{M}_1^{(a,A'',2)}(1, 1)] \otimes \mathbb{Y}_3[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 10}} \quad \hat{\mathbb{Q}}_1^{(A',2)}(1,1) \text{ } [M_1, B_1]$$

$$\hat{\mathbb{Z}}_{10} = \mathbb{X}_{11}[\mathbb{M}_1^{(a,A'',1)}(1,1)] \otimes \mathbb{Y}_3[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 11}} \quad \hat{\mathbb{Q}}_1^{(A',2)}(1,-1) \text{ } [M_1, B_1]$$

$$\hat{\mathbb{Z}}_{11} = \mathbb{X}_{13}[\mathbb{M}_1^{(a,A'',1)}(1,-1)] \otimes \mathbb{Y}_3[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 12}} \quad \hat{\mathbb{G}}_2^{(A',1)}(1,-1) \text{ } [M_1, B_1]$$

$$\hat{\mathbb{Z}}_{12} = -\mathbb{X}_{15}[\mathbb{M}_3^{(a,A'',4)}(1,-1)] \otimes \mathbb{Y}_3[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 13}} \quad \hat{\mathbb{G}}_2^{(A',2)}(1,-1) \text{ } [M_1, B_1]$$

$$\hat{\mathbb{Z}}_{13} = -\mathbb{X}_{14}[\mathbb{M}_3^{(a,A'',1)}(1,-1)] \otimes \mathbb{Y}_3[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 14}} \quad \hat{\mathbb{Q}}_1^{(A',1)} \text{ } [M_1, B_1]$$

$$\hat{\mathbb{Z}}_{14} = -\mathbb{X}_{16}[\mathbb{M}_1^{(a,A'',2)}] \otimes \mathbb{Y}_3[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 15}} \quad \hat{\mathbb{Q}}_0^{(A')} \text{ } [M_1, B_2]$$

$$\hat{\mathbb{Z}}_{15} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A')}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 16}} \quad \hat{\mathbb{Q}}_2^{(A',2)} \text{ } [M_1, B_2]$$

$$\hat{\mathbb{Z}}_{16} = \mathbb{X}_2[\mathbb{Q}_2^{(a,A',2)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 17}} \quad \hat{\mathbb{Q}}_0^{(A')}(1,1) \text{ } [M_1, B_2]$$

$$\hat{\mathbb{Z}}_{17} = \mathbb{X}_3[\mathbb{Q}_0^{(a,A')}(1,1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 18}} \quad \hat{\mathbb{Q}}_2^{(A',3)}(1,-1) \text{ } [M_1, B_2]$$

$$\hat{\mathbb{Z}}_{18} = \mathbb{X}_4[\mathbb{Q}_2^{(a,A',3)}(1,-1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 19}} \quad \hat{\mathbb{G}}_1^{(A')}(1, 1) \text{ } [M_1, B_2]$$

$$\hat{\mathbb{Z}}_{19} = \mathbb{X}_7[\mathbb{M}_1^{(a, A')}(1, 1)] \otimes \mathbb{Y}_5[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 20}} \quad \hat{\mathbb{G}}_1^{(A')}(1, -1) \text{ } [M_1, B_2]$$

$$\hat{\mathbb{Z}}_{20} = \mathbb{X}_8[\mathbb{M}_1^{(a, A')}(1, -1)] \otimes \mathbb{Y}_5[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 21}} \quad \hat{\mathbb{G}}_3^{(A', 1)}(1, -1) \text{ } [M_1, B_2]$$

$$\hat{\mathbb{Z}}_{21} = \mathbb{X}_9[\mathbb{M}_3^{(a, A', 1)}(1, -1)] \otimes \mathbb{Y}_5[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 22}} \quad \hat{\mathbb{G}}_3^{(A', 2)}(1, -1) \text{ } [M_1, B_2]$$

$$\hat{\mathbb{Z}}_{22} = \mathbb{X}_{10}[\mathbb{M}_3^{(a, A', 2)}(1, -1)] \otimes \mathbb{Y}_5[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 23}} \quad \hat{\mathbb{Q}}_0^{(A')} \text{ } [M_1, B_3]$$

$$\hat{\mathbb{Z}}_{23} = \mathbb{X}_1[\mathbb{Q}_0^{(a, A')}] \otimes \mathbb{Y}_6[\mathbb{Q}_0^{(b, A')}]$$

$$\boxed{\text{No. 24}} \quad \hat{\mathbb{Q}}_2^{(A', 2)} \text{ } [M_1, B_3]$$

$$\hat{\mathbb{Z}}_{24} = \mathbb{X}_2[\mathbb{Q}_2^{(a, A', 2)}] \otimes \mathbb{Y}_6[\mathbb{Q}_0^{(b, A')}]$$

$$\boxed{\text{No. 25}} \quad \hat{\mathbb{Q}}_0^{(A')}(1, 1) \text{ } [M_1, B_3]$$

$$\hat{\mathbb{Z}}_{25} = \mathbb{X}_3[\mathbb{Q}_0^{(a, A')}(1, 1)] \otimes \mathbb{Y}_6[\mathbb{Q}_0^{(b, A')}]$$

$$\boxed{\text{No. 26}} \quad \hat{\mathbb{Q}}_2^{(A', 3)}(1, -1) \text{ } [M_1, B_3]$$

$$\hat{\mathbb{Z}}_{26} = \mathbb{X}_4[\mathbb{Q}_2^{(a, A', 3)}(1, -1)] \otimes \mathbb{Y}_6[\mathbb{Q}_0^{(b, A')}]$$

$$\boxed{\text{No. 27}} \quad \hat{\mathbb{G}}_1^{(A')}(1, 1) \text{ } [M_1, B_3]$$

$$\hat{\mathbb{Z}}_{27} = \mathbb{X}_7[\mathbb{M}_1^{(a, A')}(1, 1)] \otimes \mathbb{Y}_7[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 28}} \quad \hat{\mathbb{G}}_1^{(A')} (1, -1) \text{ } [M_1, B_3]$$

$$\hat{\mathbb{Z}}_{28} = \mathbb{X}_8[\mathbb{M}_1^{(a, A')} (1, -1)] \otimes \mathbb{Y}_7[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 29}} \quad \hat{\mathbb{G}}_3^{(A', 1)} (1, -1) \text{ } [M_1, B_3]$$

$$\hat{\mathbb{Z}}_{29} = \mathbb{X}_9[\mathbb{M}_3^{(a, A', 1)} (1, -1)] \otimes \mathbb{Y}_7[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 30}} \quad \hat{\mathbb{G}}_3^{(A', 2)} (1, -1) \text{ } [M_1, B_3]$$

$$\hat{\mathbb{Z}}_{30} = \mathbb{X}_{10}[\mathbb{M}_3^{(a, A', 2)} (1, -1)] \otimes \mathbb{Y}_7[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 31}} \quad \hat{\mathbb{Q}}_0^{(A')} \text{ } [M_1, B_4]$$

$$\hat{\mathbb{Z}}_{31} = \mathbb{X}_1[\mathbb{Q}_0^{(a, A')}] \otimes \mathbb{Y}_8[\mathbb{Q}_0^{(b, A')}]$$

$$\boxed{\text{No. 32}} \quad \hat{\mathbb{Q}}_2^{(A', 2)} \text{ } [M_1, B_4]$$

$$\hat{\mathbb{Z}}_{32} = \mathbb{X}_2[\mathbb{Q}_2^{(a, A', 2)}] \otimes \mathbb{Y}_8[\mathbb{Q}_0^{(b, A')}]$$

$$\boxed{\text{No. 33}} \quad \hat{\mathbb{Q}}_0^{(A')} (1, 1) \text{ } [M_1, B_4]$$

$$\hat{\mathbb{Z}}_{33} = \mathbb{X}_3[\mathbb{Q}_0^{(a, A')} (1, 1)] \otimes \mathbb{Y}_8[\mathbb{Q}_0^{(b, A')}]$$

$$\boxed{\text{No. 34}} \quad \hat{\mathbb{Q}}_2^{(A', 3)} (1, -1) \text{ } [M_1, B_4]$$

$$\hat{\mathbb{Z}}_{34} = \mathbb{X}_4[\mathbb{Q}_2^{(a, A', 3)} (1, -1)] \otimes \mathbb{Y}_8[\mathbb{Q}_0^{(b, A')}]$$

$$\boxed{\text{No. 35}} \quad \hat{\mathbb{G}}_1^{(A')} (1, 1) \text{ } [M_1, B_4]$$

$$\hat{\mathbb{Z}}_{35} = \mathbb{X}_7[\mathbb{M}_1^{(a, A')} (1, 1)] \otimes \mathbb{Y}_9[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 36}} \quad \hat{\mathbb{G}}_1^{(A')} (1, -1) \text{ } [M_1, B_4]$$

$$\hat{\mathbb{Z}}_{36} = \mathbb{X}_8[\mathbb{M}_1^{(a, A')} (1, -1)] \otimes \mathbb{Y}_9[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 37}} \quad \hat{\mathbb{G}}_3^{(A',1)}(1, -1) \text{ [M}_1, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{37} = \mathbb{X}_9[\mathbb{M}_3^{(a,A',1)}(1, -1)] \otimes \mathbb{Y}_9[\mathbb{T}_0^{(b,A')}]$$

$$\boxed{\text{No. 38}} \quad \hat{\mathbb{G}}_3^{(A',2)}(1, -1) \text{ [M}_1, \text{B}_4]$$

$$\hat{\mathbb{Z}}_{38} = \mathbb{X}_{10}[\mathbb{M}_3^{(a,A',2)}(1, -1)] \otimes \mathbb{Y}_9[\mathbb{T}_0^{(b,A')}]$$

$$\boxed{\text{No. 39}} \quad \hat{\mathbb{Q}}_0^{(A')} \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{39} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A')}] \otimes \mathbb{Y}_{10}[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 40}} \quad \hat{\mathbb{Q}}_2^{(A',2)} \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{40} = \mathbb{X}_2[\mathbb{Q}_2^{(a,A',2)}] \otimes \mathbb{Y}_{10}[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 41}} \quad \hat{\mathbb{Q}}_1^{(A',1)} \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{41} = \mathbb{X}_5[\mathbb{Q}_2^{(a,A'',2)}] \otimes \mathbb{Y}_{11}[\mathbb{Q}_1^{(b,A'')}]$$

$$\boxed{\text{No. 42}} \quad \hat{\mathbb{Q}}_0^{(A')}(1, 1) \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{42} = \mathbb{X}_3[\mathbb{Q}_0^{(a,A')}(1, 1)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 43}} \quad \hat{\mathbb{Q}}_2^{(A',3)}(1, -1) \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{43} = \mathbb{X}_4[\mathbb{Q}_2^{(a,A',3)}(1, -1)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 44}} \quad \hat{\mathbb{Q}}_1^{(A',2)}(1, -1) \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{44} = \mathbb{X}_6[\mathbb{Q}_2^{(a,A'',1)}(1, -1)] \otimes \mathbb{Y}_{11}[\mathbb{Q}_1^{(b,A'')}]$$

$$\boxed{\text{No. 45}} \quad \hat{\mathbb{G}}_1^{(A')}(1, 1) \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{45} = \mathbb{X}_7[\mathbb{M}_1^{(a,A')}(1, 1)] \otimes \mathbb{Y}_{12}[\mathbb{T}_0^{(b,A')}]$$

$$\boxed{\text{No. 46}} \quad \hat{\mathbb{Q}}_1^{(A',1)}(1,1) \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{46} = -\mathbb{X}_{12}[\mathbb{M}_1^{(a,A'',2)}(1,1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 47}} \quad \hat{\mathbb{Q}}_1^{(A',2)}(1,1) \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{47} = \mathbb{X}_{11}[\mathbb{M}_1^{(a,A'',1)}(1,1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 48}} \quad \hat{\mathbb{G}}_1^{(A')}(1,-1) \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{48} = \mathbb{X}_8[\mathbb{M}_1^{(a,A')}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{T}_0^{(b,A')}]$$

$$\boxed{\text{No. 49}} \quad \hat{\mathbb{Q}}_1^{(A',2)}(1,-1) \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{49} = \mathbb{X}_{13}[\mathbb{M}_1^{(a,A'',1)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 50}} \quad \hat{\mathbb{G}}_3^{(A',1)}(1,-1) \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{50} = \mathbb{X}_9[\mathbb{M}_3^{(a,A',1)}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{T}_0^{(b,A')}]$$

$$\boxed{\text{No. 51}} \quad \hat{\mathbb{G}}_3^{(A',2)}(1,-1) \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{51} = \mathbb{X}_{10}[\mathbb{M}_3^{(a,A',2)}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{T}_0^{(b,A')}]$$

$$\boxed{\text{No. 52}} \quad \hat{\mathbb{G}}_2^{(A',1)}(1,-1) \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{52} = -\mathbb{X}_{15}[\mathbb{M}_3^{(a,A'',4)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 53}} \quad \hat{\mathbb{G}}_2^{(A',2)}(1,-1) \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{53} = -\mathbb{X}_{14}[\mathbb{M}_3^{(a,A'',1)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 54}} \quad \hat{\mathbb{Q}}_1^{(A',1)} \text{ [M}_1, \text{B}_5]$$

$$\hat{\mathbb{Z}}_{54} = -\mathbb{X}_{16}[\mathbb{M}_1^{(a,A'',2)}] \otimes \mathbb{Y}_{13}[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 55}} \quad \hat{\mathbb{Q}}_0^{(A')} \text{ } [M_1, B_6]$$

$$\hat{Z}_{55} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A')}] \otimes \mathbb{Y}_{14}[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 56}} \quad \hat{\mathbb{Q}}_2^{(A',2)} \text{ } [M_1, B_6]$$

$$\hat{Z}_{56} = \mathbb{X}_2[\mathbb{Q}_2^{(a,A',2)}] \otimes \mathbb{Y}_{14}[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 57}} \quad \hat{\mathbb{Q}}_1^{(A',1)} \text{ } [M_1, B_6]$$

$$\hat{Z}_{57} = \mathbb{X}_5[\mathbb{Q}_2^{(a,A'',2)}] \otimes \mathbb{Y}_{15}[\mathbb{Q}_1^{(b,A'')}]$$

$$\boxed{\text{No. 58}} \quad \hat{\mathbb{Q}}_0^{(A')}(1, 1) \text{ } [M_1, B_6]$$

$$\hat{Z}_{58} = \mathbb{X}_3[\mathbb{Q}_0^{(a,A')}(1, 1)] \otimes \mathbb{Y}_{14}[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 59}} \quad \hat{\mathbb{Q}}_2^{(A',3)}(1, -1) \text{ } [M_1, B_6]$$

$$\hat{Z}_{59} = \mathbb{X}_4[\mathbb{Q}_2^{(a,A',3)}(1, -1)] \otimes \mathbb{Y}_{14}[\mathbb{Q}_0^{(b,A')}]$$

$$\boxed{\text{No. 60}} \quad \hat{\mathbb{Q}}_1^{(A',2)}(1, -1) \text{ } [M_1, B_6]$$

$$\hat{Z}_{60} = \mathbb{X}_6[\mathbb{Q}_2^{(a,A'',1)}(1, -1)] \otimes \mathbb{Y}_{15}[\mathbb{Q}_1^{(b,A'')}]$$

$$\boxed{\text{No. 61}} \quad \hat{\mathbb{G}}_1^{(A')}(1, 1) \text{ } [M_1, B_6]$$

$$\hat{Z}_{61} = \mathbb{X}_7[\mathbb{M}_1^{(a,A')}(1, 1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_0^{(b,A')}]$$

$$\boxed{\text{No. 62}} \quad \hat{\mathbb{Q}}_1^{(A',1)}(1, 1) \text{ } [M_1, B_6]$$

$$\hat{Z}_{62} = -\mathbb{X}_{12}[\mathbb{M}_1^{(a,A'',2)}(1, 1)] \otimes \mathbb{Y}_{17}[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 63}} \quad \hat{\mathbb{Q}}_1^{(A',2)}(1, 1) \text{ } [M_1, B_6]$$

$$\hat{Z}_{63} = \mathbb{X}_{11}[\mathbb{M}_1^{(a,A'',1)}(1, 1)] \otimes \mathbb{Y}_{17}[\mathbb{T}_1^{(b,A'')}]$$

$$\boxed{\text{No. 64}} \quad \hat{\mathbb{G}}_1^{(A')}(1, -1) \text{ [M}_1, \text{B}_6]$$

$$\hat{\mathbb{Z}}_{64} = \mathbb{X}_8[\mathbb{M}_1^{(a, A')}(1, -1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 65}} \quad \hat{\mathbb{Q}}_1^{(A', 2)}(1, -1) \text{ [M}_1, \text{B}_6]$$

$$\hat{\mathbb{Z}}_{65} = \mathbb{X}_{13}[\mathbb{M}_1^{(a, A'', 1)}(1, -1)] \otimes \mathbb{Y}_{17}[\mathbb{T}_1^{(b, A'')}]$$

$$\boxed{\text{No. 66}} \quad \hat{\mathbb{G}}_3^{(A', 1)}(1, -1) \text{ [M}_1, \text{B}_6]$$

$$\hat{\mathbb{Z}}_{66} = \mathbb{X}_9[\mathbb{M}_3^{(a, A', 1)}(1, -1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 67}} \quad \hat{\mathbb{G}}_3^{(A', 2)}(1, -1) \text{ [M}_1, \text{B}_6]$$

$$\hat{\mathbb{Z}}_{67} = \mathbb{X}_{10}[\mathbb{M}_3^{(a, A', 2)}(1, -1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 68}} \quad \hat{\mathbb{G}}_2^{(A', 1)}(1, -1) \text{ [M}_1, \text{B}_6]$$

$$\hat{\mathbb{Z}}_{68} = -\mathbb{X}_{15}[\mathbb{M}_3^{(a, A'', 4)}(1, -1)] \otimes \mathbb{Y}_{17}[\mathbb{T}_1^{(b, A'')}]$$

$$\boxed{\text{No. 69}} \quad \hat{\mathbb{G}}_2^{(A', 2)}(1, -1) \text{ [M}_1, \text{B}_6]$$

$$\hat{\mathbb{Z}}_{69} = -\mathbb{X}_{14}[\mathbb{M}_3^{(a, A'', 1)}(1, -1)] \otimes \mathbb{Y}_{17}[\mathbb{T}_1^{(b, A'')}]$$

$$\boxed{\text{No. 70}} \quad \hat{\mathbb{Q}}_1^{(A', 1)} \text{ [M}_1, \text{B}_6]$$

$$\hat{\mathbb{Z}}_{70} = -\mathbb{X}_{16}[\mathbb{M}_1^{(a, A'', 2)}] \otimes \mathbb{Y}_{17}[\mathbb{T}_1^{(b, A'')}]$$

$$\boxed{\text{No. 71}} \quad \hat{\mathbb{Q}}_0^{(A')} \text{ [M}_1, \text{B}_7]$$

$$\hat{\mathbb{Z}}_{71} = \mathbb{X}_1[\mathbb{Q}_0^{(a, A')}] \otimes \mathbb{Y}_{18}[\mathbb{Q}_0^{(b, A')}]$$

$$\boxed{\text{No. 72}} \quad \hat{\mathbb{Q}}_2^{(A', 2)} \text{ [M}_1, \text{B}_7]$$

$$\hat{\mathbb{Z}}_{72} = \mathbb{X}_2[\mathbb{Q}_2^{(a, A', 2)}] \otimes \mathbb{Y}_{18}[\mathbb{Q}_0^{(b, A')}]$$

$$\boxed{\text{No. 73}} \quad \hat{\mathbb{Q}}_0^{(A')}(1, 1) \text{ [M}_1, \text{B}_7]$$

$$\hat{\mathbb{Z}}_{73} = \mathbb{X}_3[\mathbb{Q}_0^{(a, A')}(1, 1)] \otimes \mathbb{Y}_{18}[\mathbb{Q}_0^{(b, A')}]$$

$$\boxed{\text{No. 74}} \quad \hat{\mathbb{Q}}_2^{(A', 3)}(1, -1) \text{ [M}_1, \text{B}_7]$$

$$\hat{\mathbb{Z}}_{74} = \mathbb{X}_4[\mathbb{Q}_2^{(a, A', 3)}(1, -1)] \otimes \mathbb{Y}_{18}[\mathbb{Q}_0^{(b, A')}]$$

$$\boxed{\text{No. 75}} \quad \hat{\mathbb{G}}_1^{(A')}(1, 1) \text{ [M}_1, \text{B}_7]$$

$$\hat{\mathbb{Z}}_{75} = \mathbb{X}_7[\mathbb{M}_1^{(a, A')}(1, 1)] \otimes \mathbb{Y}_{19}[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 76}} \quad \hat{\mathbb{G}}_1^{(A')}(1, -1) \text{ [M}_1, \text{B}_7]$$

$$\hat{\mathbb{Z}}_{76} = \mathbb{X}_8[\mathbb{M}_1^{(a, A')}(1, -1)] \otimes \mathbb{Y}_{19}[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 77}} \quad \hat{\mathbb{G}}_3^{(A', 1)}(1, -1) \text{ [M}_1, \text{B}_7]$$

$$\hat{\mathbb{Z}}_{77} = \mathbb{X}_9[\mathbb{M}_3^{(a, A', 1)}(1, -1)] \otimes \mathbb{Y}_{19}[\mathbb{T}_0^{(b, A')}]$$

$$\boxed{\text{No. 78}} \quad \hat{\mathbb{G}}_3^{(A', 2)}(1, -1) \text{ [M}_1, \text{B}_7]$$

$$\hat{\mathbb{Z}}_{78} = \mathbb{X}_{10}[\mathbb{M}_3^{(a, A', 2)}(1, -1)] \otimes \mathbb{Y}_{19}[\mathbb{T}_0^{(b, A')}]$$

Table 5: Atomic SAMB group.

group	bra	ket
M ₁	$(p_x, \uparrow), (p_x, \downarrow), (p_y, \uparrow), (p_y, \downarrow)$	$(p_x, \uparrow), (p_x, \downarrow), (p_y, \uparrow), (p_y, \downarrow)$

Table 6: Atomic SAMB.

symbol	type	group	form
\mathbb{X}_1	$\mathbb{Q}_0^{(a,A')}$	M_1	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$
\mathbb{X}_2	$\mathbb{Q}_2^{(a,A',2)}$	M_1	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}$
\mathbb{X}_3	$\mathbb{Q}_0^{(a,A')}(1,1)$	M_1	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & \frac{i}{2} \\ \frac{i}{2} & 0 & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & 0 \end{pmatrix}$
\mathbb{X}_4	$\mathbb{Q}_2^{(a,A',3)}(1,-1)$	M_1	$\begin{pmatrix} 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & -\frac{i}{2} & 0 \\ 0 & \frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_5	$\mathbb{Q}_2^{(a,A'',2)}$	M_1	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$
\mathbb{X}_6	$\mathbb{Q}_2^{(a,A'',1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_7	$\mathbb{M}_1^{(a,A')}(1,1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{19}i}{19} & 0 & \frac{3\sqrt{19}}{38} \\ -\frac{\sqrt{19}i}{19} & 0 & \frac{3\sqrt{19}}{38} & 0 \\ 0 & \frac{3\sqrt{19}}{38} & 0 & -\frac{2\sqrt{19}i}{19} \\ \frac{3\sqrt{19}}{38} & 0 & \frac{2\sqrt{19}i}{19} & 0 \end{pmatrix}$
\mathbb{X}_8	$\mathbb{M}_1^{(a,A')}(1,-1)$	M_1	$\begin{pmatrix} 0 & -\frac{7\sqrt{38}i}{76} & 0 & -\frac{\sqrt{38}}{76} \\ \frac{7\sqrt{38}i}{76} & 0 & -\frac{\sqrt{38}}{76} & 0 \\ 0 & -\frac{\sqrt{38}}{76} & 0 & -\frac{5\sqrt{38}i}{76} \\ -\frac{\sqrt{38}}{76} & 0 & \frac{5\sqrt{38}i}{76} & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
\mathbb{X}_9	$\mathbb{M}_3^{(a,A',1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \end{pmatrix}$
\mathbb{X}_{10}	$\mathbb{M}_3^{(a,A',2)}(1,-1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}i}{4} \\ -\frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 \end{pmatrix}$
\mathbb{X}_{11}	$\mathbb{M}_1^{(a,A'',1)}(1,1)$	M_1	$\begin{pmatrix} 0 & \frac{2\sqrt{19}}{19} & 0 & -\frac{3\sqrt{19}i}{38} \\ \frac{2\sqrt{19}}{19} & 0 & \frac{3\sqrt{19}i}{38} & 0 \\ 0 & -\frac{3\sqrt{19}i}{38} & 0 & -\frac{\sqrt{19}}{19} \\ \frac{3\sqrt{19}i}{38} & 0 & -\frac{\sqrt{19}}{19} & 0 \end{pmatrix}$
\mathbb{X}_{12}	$\mathbb{M}_1^{(a,A'',2)}(1,1)$	M_1	$\begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$
\mathbb{X}_{13}	$\mathbb{M}_1^{(a,A'',1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & \frac{5\sqrt{38}}{76} & 0 & \frac{\sqrt{38}i}{76} \\ \frac{5\sqrt{38}}{76} & 0 & -\frac{\sqrt{38}i}{76} & 0 \\ 0 & \frac{\sqrt{38}i}{76} & 0 & \frac{7\sqrt{38}}{76} \\ -\frac{\sqrt{38}i}{76} & 0 & \frac{7\sqrt{38}}{76} & 0 \end{pmatrix}$
\mathbb{X}_{14}	$\mathbb{M}_3^{(a,A'',1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} \\ \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 \end{pmatrix}$
\mathbb{X}_{15}	$\mathbb{M}_3^{(a,A'',4)}(1,-1)$	M_1	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$
\mathbb{X}_{16}	$\mathbb{M}_1^{(a,A'',2)}$	M_1	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & -\frac{i}{2} \\ \frac{i}{2} & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 \end{pmatrix}$

Table 7: Cluster SAMB.

symbol	type	cluster	form
\mathbb{Y}_1	$\mathbb{Q}_0^{(s,A')}$	S_1	$\begin{pmatrix} 1 \end{pmatrix}$
\mathbb{Y}_2	$\mathbb{Q}_0^{(b,A')}$	B_1	$\begin{pmatrix} 1 \end{pmatrix}$
\mathbb{Y}_3	$\mathbb{T}_1^{(b,A'')}$	B_1	$\begin{pmatrix} i \end{pmatrix}$
\mathbb{Y}_4	$\mathbb{Q}_0^{(b,A')}$	B_2	$\begin{pmatrix} 1 \end{pmatrix}$
\mathbb{Y}_5	$\mathbb{T}_0^{(b,A')}$	B_2	$\begin{pmatrix} i \end{pmatrix}$
\mathbb{Y}_6	$\mathbb{Q}_0^{(b,A')}$	B_3	$\begin{pmatrix} 1 \end{pmatrix}$
\mathbb{Y}_7	$\mathbb{T}_0^{(b,A')}$	B_3	$\begin{pmatrix} i \end{pmatrix}$
\mathbb{Y}_8	$\mathbb{Q}_0^{(b,A')}$	B_4	$\begin{pmatrix} 1 \end{pmatrix}$
\mathbb{Y}_9	$\mathbb{T}_0^{(b,A')}$	B_4	$\begin{pmatrix} i \end{pmatrix}$
\mathbb{Y}_{10}	$\mathbb{Q}_0^{(b,A')}$	B_5	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{Y}_{11}	$\mathbb{Q}_1^{(b,A'')}$	B_5	$\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{Y}_{12}	$\mathbb{T}_0^{(b,A')}$	B_5	$\begin{pmatrix} \frac{\sqrt{2}i}{2} & \frac{\sqrt{2}i}{2} \end{pmatrix}$
\mathbb{Y}_{13}	$\mathbb{T}_1^{(b,A'')}$	B_5	$\begin{pmatrix} \frac{\sqrt{2}i}{2} & -\frac{\sqrt{2}i}{2} \end{pmatrix}$
\mathbb{Y}_{14}	$\mathbb{Q}_0^{(b,A')}$	B_6	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{Y}_{15}	$\mathbb{Q}_1^{(b,A'')}$	B_6	$\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{Y}_{16}	$\mathbb{T}_0^{(b,A')}$	B_6	$\begin{pmatrix} \frac{\sqrt{2}i}{2} & \frac{\sqrt{2}i}{2} \end{pmatrix}$
\mathbb{Y}_{17}	$\mathbb{T}_1^{(b,A'')}$	B_6	$\begin{pmatrix} \frac{\sqrt{2}i}{2} & -\frac{\sqrt{2}i}{2} \end{pmatrix}$
\mathbb{Y}_{18}	$\mathbb{Q}_0^{(b,A')}$	B_7	$\begin{pmatrix} 1 \end{pmatrix}$
\mathbb{Y}_{19}	$\mathbb{T}_0^{(b,A')}$	B_7	$\begin{pmatrix} i \end{pmatrix}$

Table 8: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_0^{(A')}$	0	A'	—	—	1
2	$\mathbb{Q}_1^{(A'')}$	1	A''	—	—	y
3	$\mathbb{Q}_2^{(A'',1)}$	2	A''	1	—	$\sqrt{3}yz$
4	$\mathbb{Q}_2^{(A'',2)}$	2	A''	2	—	$\sqrt{3}xy$
5	$\mathbb{Q}_2^{(A',2)}$	2	A'	2	—	$\frac{\sqrt{3}(x-y)(x+y)}{2}$
6	$\mathbb{Q}_2^{(A',3)}$	2	A'	3	—	$\sqrt{3}xz$

Table 9: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_1^{(A'',1)}$	1	A''	1	—	X
2	$\mathbb{G}_1^{(A'',2)}$	1	A''	2	—	Z
3	$\mathbb{G}_1^{(A')}$	1	A'	—	—	Y
4	$\mathbb{G}_3^{(A'',1)}$	3	A''	1	—	$\frac{X(2X^2-3Y^2-3Z^2)}{2}$
5	$\mathbb{G}_3^{(A'',4)}$	3	A''	4	—	$\frac{\sqrt{15}Z(X-Y)(X+Y)}{2}$
6	$\mathbb{G}_3^{(A',1)}$	3	A'	1	—	$\sqrt{15}XYZ$
7	$\mathbb{G}_3^{(A',2)}$	3	A'	2	—	$-\frac{Y(3X^2-2Y^2+3Z^2)}{2}$

-
- Group info.: Generator = $\{m_{010}|0\}$

Table 10: Conjugacy class (point-group part).

rep. SO	symmetry operations
$\{1 0\}$	$\{1 0\}$
$\{m_{010} 0\}$	$\{m_{010} 0\}$

Table 11: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	$\{1 0\}$	2	$\{m_{010} 0\}$						

Table 12: Character table (point-group part).

	1	m_{010}
A'	1	1
A''	1	-1

Table 13: Parity conversion.

\leftrightarrow	\leftrightarrow
$A' (A'')$	$A'' (A')$

Table 14: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

	A'	A''
A'	A'	A''
A''	A'	A'

Table 15: Anti-symmetric product, $[\Gamma \otimes \Gamma']_-$.

A'	A''
$-$	$-$

Table 16: Virtual-cluster sites.

No.	position	No.	position
1	$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$	2	$\begin{pmatrix} 0 & -1 & 0 \end{pmatrix}$

Table 17: Virtual-cluster basis.

symbol	1	2
$Q_0^{(A')}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$Q_1^{(A'')}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$