

MPG No. 7.1.20 $mm2$ [Type I, orthorhombic] [M tensor]

* Rank 0 tensor. * Rank 1 tensor. * Rank 2 tensor (s).

$$\begin{bmatrix} 0 & M_{xy} & 0 \\ M_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{xy} = M_{dxy}^{(1)}$$

* Rank 2 tensor (a).

$$\begin{bmatrix} 0 & M_{xy} & 0 \\ -M_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{xy} = T_{pz}^{(1)}$$

* Rank 3 tensor (s).

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ M_{yzz} & 0 & 0 \\ 0 & M_{zxy} & 0 \\ 0 & 0 & M_{xyz} \end{bmatrix}$$

$$M_{yzz} = M_{f3}^{(1)} - 3T_{du}^{(1)} - T_{dv}^{(1)}$$

$$M_{zxy} = M_{f3}^{(1)} + 3T_{du}^{(1)} - T_{dv}^{(1)}$$

$$M_{xyz} = M_{f3}^{(1)} + 2T_{dv}^{(1)}$$

* Rank 3 tensor (a).

$$\begin{bmatrix} M_{yzz} & 0 & 0 \\ 0 & M_{zxy} & 0 \\ 0 & 0 & M_{xyz} \end{bmatrix}$$

$$M_{yzz} = -T_{du}^{(2)} + T_{dv}^{(2)} + T_s^{(1)}$$

$$M_{zxy} = -T_{du}^{(2)} - T_{dv}^{(2)} + T_s^{(1)}$$

$$M_{xyz} = 2T_{du}^{(2)} + T_s^{(1)}$$

* Rank 4 tensor (sss).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & M_{xxxx} \\ 0 & 0 & 0 & 0 & 0 & M_{yyxy} \\ 0 & 0 & 0 & 0 & 0 & M_{zzxy} \\ 0 & 0 & 0 & 0 & M_{yzzx} & 0 \\ 0 & 0 & 0 & M_{yzzx} & 0 & 0 \\ M_{xxxx} & M_{yyxy} & M_{zzxy} & 0 & 0 & 0 \end{bmatrix}$$

$$M_{xxxx} = M_{dxy}^{(1)} + 2M_{dxy}^{(2)} + M_{gaz}^{(1)} - M_{gbz}^{(1)}$$

$$M_{yyxy} = M_{dxy}^{(1)} + 2M_{dxy}^{(2)} - M_{gaz}^{(1)} - M_{gbz}^{(1)}$$

$$M_{zzxy} = M_{dxy}^{(1)} + 2M_{gbz}^{(1)}$$

$$M_{yzzx} = M_{dxy}^{(2)} + 2M_{gbz}^{(1)}$$

* Rank 4 tensor (ssa).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & M_{xxx} \\ 0 & 0 & 0 & 0 & 0 & M_{yyxy} \\ 0 & 0 & 0 & 0 & 0 & M_{zzxy} \\ 0 & 0 & 0 & 0 & 0 & M_{yzzx} \\ 0 & 0 & 0 & -M_{yzzx} & 0 & 0 \\ -M_{xxx} & -M_{yyxy} & -M_{zzxy} & 0 & 0 & 0 \end{bmatrix}$$

$$M_{xxx} = M_{dxy}^{(3)} - 2T_{faz}^{(1)} + 2T_{fbz}^{(1)} + 2T_{pz}^{(1)}$$

$$M_{yyxy} = M_{dxy}^{(3)} + 2T_{faz}^{(1)} + 2T_{fbz}^{(1)} - 2T_{pz}^{(1)}$$

$$M_{zzxy} = M_{dxy}^{(3)} - 4T_{fbz}^{(1)}$$

$$M_{yzzx} = -4T_{faz}^{(1)} - T_{pz}^{(1)}$$

* Rank 4 tensor (aas).

$$\begin{bmatrix} 0 & M_{yzzx} & 0 \\ M_{yzzx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{yzzx} = 2M_{dxy}^{(4)}$$

* Rank 4 tensor (aaa).

$$\begin{bmatrix} 0 & M_{yzzx} & 0 \\ -M_{yzzx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{yzzx} = -T_{pz}^{(2)}$$

* Rank 4 tensor (sa).

$$\begin{bmatrix} 0 & 0 & M_{xxx} \\ 0 & 0 & M_{yyxy} \\ 0 & 0 & M_{zzxy} \\ 0 & M_{yzzx} & 0 \\ M_{zxyz} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{xxx} = -2M_{dxy}^{(5)} - T_{faz}^{(2)} + T_{fbz}^{(2)} + T_{pz}^{(3)}$$

$$M_{yyxy} = 2M_{dxy}^{(5)} - T_{faz}^{(2)} - T_{fbz}^{(2)} + T_{pz}^{(3)}$$

$$M_{zzxy} = 2T_{faz}^{(2)} + T_{pz}^{(3)} + 2T_{pz}^{(4)}$$

$$M_{yzzx} = -M_{dxy}^{(5)} - T_{faz}^{(2)} - T_{fbz}^{(2)} + T_{pz}^{(4)}$$

$$M_{zxyz} = M_{dxy}^{(5)} - T_{faz}^{(2)} + T_{fbz}^{(2)} + T_{pz}^{(4)}$$

* Rank 4 tensor (as).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & M_{yzzx} & 0 \\ 0 & 0 & 0 & M_{zxyz} & 0 & 0 \\ M_{xyxx} & M_{xyyy} & M_{xyzz} & 0 & 0 & 0 \end{bmatrix}$$

$$M_{yzzx} = M_{dxy}^{(6)} - T_{faz}^{(3)} + T_{fbz}^{(3)} + T_{pz}^{(6)}$$

$$M_{zxyz} = -M_{dxy}^{(6)} - T_{faz}^{(3)} - T_{fbz}^{(3)} + T_{pz}^{(6)}$$

$$M_{xyxx} = -2M_{dxy}^{(6)} - T_{faz}^{(3)} + T_{fbz}^{(3)} + T_{pz}^{(5)}$$

$$M_{xyyy} = 2M_{dxy}^{(6)} - T_{faz}^{(3)} - T_{fbz}^{(3)} + T_{pz}^{(5)}$$

$$M_{xyzz} = 2T_{faz}^{(3)} + T_{pz}^{(5)} + 2T_{pz}^{(6)}$$

* Rank 4 tensor (s).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & M_{xxyy} & 0 & 0 & M_{xxyx} \\ 0 & 0 & 0 & 0 & 0 & M_{yyxy} & 0 & 0 & M_{yyyx} \\ 0 & 0 & 0 & 0 & 0 & M_{zzxy} & 0 & 0 & M_{zzyx} \\ 0 & 0 & 0 & 0 & M_{yzzx} & 0 & 0 & M_{yzxz} & 0 \\ 0 & 0 & 0 & M_{zxyz} & 0 & 0 & M_{zxzy} & 0 & 0 \\ M_{xyxx} & M_{xyyy} & M_{xyzz} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} M_{xxyy} &= M_{dxy}^{(1)} + 2M_{dxy}^{(2)} + M_{dxy}^{(3)} - 2M_{dxy}^{(5)} + M_{gaz}^{(1)} - M_{gbz}^{(1)} - 2T_{faz}^{(1)} - T_{fbz}^{(2)} + 2T_{fbz}^{(1)} + T_{pz}^{(2)} + T_{pz}^{(3)} \\ M_{xxyx} &= M_{dxy}^{(1)} + 2M_{dxy}^{(2)} + M_{dxy}^{(3)} + 2M_{dxy}^{(5)} + M_{gaz}^{(1)} - M_{gbz}^{(1)} - 2T_{faz}^{(1)} + T_{faz}^{(2)} + 2T_{fbz}^{(1)} - T_{fbz}^{(2)} + 2T_{pz}^{(1)} - T_{pz}^{(3)} \\ M_{yyxy} &= M_{dxy}^{(1)} + 2M_{dxy}^{(2)} + M_{dxy}^{(3)} + 2M_{dxy}^{(5)} - M_{gaz}^{(1)} - M_{gbz}^{(1)} + 2T_{faz}^{(1)} - T_{faz}^{(2)} + 2T_{fbz}^{(1)} - T_{fbz}^{(2)} - 2T_{pz}^{(1)} + T_{pz}^{(3)} \\ M_{yyyx} &= M_{dxy}^{(1)} + 2M_{dxy}^{(2)} + M_{dxy}^{(3)} - 2M_{dxy}^{(5)} - M_{gaz}^{(1)} - M_{gbz}^{(1)} + 2T_{faz}^{(1)} + T_{faz}^{(2)} + 2T_{fbz}^{(1)} + T_{fbz}^{(2)} - 2T_{pz}^{(1)} - T_{pz}^{(3)} \\ M_{zzxy} &= M_{dxy}^{(1)} + M_{dxy}^{(3)} + 2M_{gbz}^{(1)} + 2T_{faz}^{(2)} - 4T_{fbz}^{(1)} + T_{pz}^{(3)} + 2T_{pz}^{(4)} \\ M_{zzyx} &= M_{dxy}^{(1)} + M_{dxy}^{(3)} + 2M_{gbz}^{(1)} - 2T_{faz}^{(2)} - 4T_{fbz}^{(1)} - T_{pz}^{(3)} - 2T_{pz}^{(4)} \\ M_{yzzx} &= M_{dxy}^{(2)} - M_{dxy}^{(5)} + 2M_{gbz}^{(1)} - 4T_{faz}^{(1)} - T_{fbz}^{(2)} - T_{pz}^{(1)} + T_{pz}^{(4)} \\ M_{yzxz} &= M_{dxy}^{(2)} + M_{dxy}^{(5)} + 2M_{gbz}^{(1)} - 4T_{faz}^{(1)} + T_{faz}^{(2)} + T_{fbz}^{(1)} - T_{pz}^{(1)} - T_{pz}^{(4)} \\ M_{zxyz} &= M_{dxy}^{(2)} + M_{dxy}^{(5)} + 2M_{gbz}^{(1)} + 4T_{faz}^{(1)} - T_{fbz}^{(2)} + T_{fbz}^{(1)} + T_{pz}^{(1)} + T_{pz}^{(4)} \\ M_{zxzy} &= M_{dxy}^{(2)} - M_{dxy}^{(5)} + 2M_{gbz}^{(1)} + 4T_{faz}^{(1)} + T_{faz}^{(2)} - T_{fbz}^{(1)} + T_{pz}^{(1)} - T_{pz}^{(4)} \\ M_{xyxx} &= M_{dxy}^{(1)} + 2M_{dxy}^{(2)} - M_{dxy}^{(3)} + M_{gaz}^{(1)} - M_{gbz}^{(1)} + 2T_{faz}^{(1)} - 2T_{fbz}^{(1)} - 2T_{pz}^{(1)} \\ M_{xyyy} &= M_{dxy}^{(1)} + 2M_{dxy}^{(2)} - M_{dxy}^{(3)} - M_{gaz}^{(1)} - M_{gbz}^{(1)} - 2T_{faz}^{(1)} - 2T_{fbz}^{(1)} + 2T_{pz}^{(1)} \\ M_{xyzz} &= M_{dxy}^{(1)} - M_{dxy}^{(3)} + 2M_{gbz}^{(1)} + 4T_{fbz}^{(1)} \end{aligned}$$

* Rank 4 tensor (a).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & M_{yzzx} & 0 & 0 & M_{yzxz} & 0 \\ 0 & 0 & 0 & M_{zxyz} & 0 & 0 & M_{zxzy} & 0 & 0 \\ M_{xyxx} & M_{xyyy} & M_{xyzz} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} M_{yzzx} &= 2M_{dxy}^{(4)} + M_{dxy}^{(6)} - T_{faz}^{(3)} + T_{fbz}^{(3)} - T_{pz}^{(2)} + T_{pz}^{(6)} \\ M_{yzxz} &= -2M_{dxy}^{(4)} + M_{dxy}^{(6)} - T_{faz}^{(3)} + T_{fbz}^{(3)} + T_{pz}^{(2)} + T_{pz}^{(6)} \\ M_{zxyz} &= 2M_{dxy}^{(4)} - M_{dxy}^{(6)} - T_{faz}^{(3)} - T_{fbz}^{(3)} + T_{pz}^{(2)} + T_{pz}^{(6)} \\ M_{zxzy} &= -2M_{dxy}^{(4)} - M_{dxy}^{(6)} - T_{faz}^{(3)} - T_{fbz}^{(3)} - T_{pz}^{(2)} + T_{pz}^{(6)} \\ M_{xyxx} &= -2M_{dxy}^{(6)} - T_{faz}^{(3)} + T_{fbz}^{(3)} + T_{pz}^{(5)} \\ M_{xyyy} &= 2M_{dxy}^{(6)} - T_{faz}^{(3)} - T_{fbz}^{(3)} + T_{pz}^{(5)} \\ M_{xyzz} &= 2T_{faz}^{(3)} + T_{pz}^{(5)} + 2T_{pz}^{(6)} \end{aligned}$$

* Rank 4 tensor (t).

$$\begin{bmatrix} 0 & M_{xxyy} & 0 \\ M_{yyyx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & M_{zzxy} & 0 \\ M_{xxyx} & 0 & 0 \\ M_{yzzx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & M_{xyyy} & 0 \\ 0 & 0 & \frac{M_{xxyy}}{2} - \frac{M_{yyyx}}{2} + M_{xyyy} - M_{yyyx} + M_{zzxy} \end{bmatrix}$$

$$M_{xxyy} = M_{dxy}^{(1)} + 2M_{dxy}^{(2)} + M_{dxy}^{(3)} - 2M_{dxy}^{(5)} + M_{gaz}^{(1)} - M_{gbz}^{(1)}$$

$$M_{yyxx} = M_{dxy}^{(1)} + 2M_{dxy}^{(2)} + M_{dxy}^{(3)} - 2M_{dxy}^{(5)} - M_{gaz}^{(1)} - M_{gbz}^{(1)}$$

$$M_{zzxy} = M_{dxy}^{(1)} + M_{dxy}^{(3)} + 2M_{gbz}^{(1)}$$

$$M_{xxyx} = M_{dxy}^{(1)} + 2M_{dxy}^{(2)} + M_{dxy}^{(3)} + 2M_{dxy}^{(5)} + M_{gaz}^{(1)} - M_{gbz}^{(1)}$$

$$M_{yzzx} = M_{dxy}^{(2)} - M_{dxy}^{(5)} + 2M_{gbz}^{(1)}$$

$$M_{xyyy} = M_{dxy}^{(1)} + 2M_{dxy}^{(2)} - M_{dxy}^{(3)} - M_{gaz}^{(1)} - M_{gbz}^{(1)}$$