

PG No. 10  $S_4 \bar{4}$  [ tetragonal ] (axial, internal polar quadrupole)

\* Harmonics for rank 0

\* Harmonics for rank 1

$$\vec{\mathbb{G}}_{\mathbf{1}}^{(2,1)}[q](A)$$

\*\* symmetry

$z$

\*\* expression

$$\frac{\sqrt{30}Q_vxy}{5} - \frac{\sqrt{30}Q_{xy}(x-y)(x+y)}{10} + \frac{\sqrt{30}Q_{xz}yz}{10} - \frac{\sqrt{30}Q_{yz}xz}{10}$$

$$\vec{\mathbb{G}}_{\mathbf{1},1}^{(2,1)}[q](E), \vec{\mathbb{G}}_{\mathbf{1},2}^{(2,1)}[q](E)$$

\*\* symmetry

$y$

$x$

\*\* expression

$$\frac{3\sqrt{10}Q_{uxz}}{10} - \frac{\sqrt{30}Q_{vzx}}{10} - \frac{\sqrt{30}Q_{xy}yz}{10} + \frac{\sqrt{30}Q_{xz}(x-z)(x+z)}{10} + \frac{\sqrt{30}Q_{yz}xy}{10}$$

$$- \frac{3\sqrt{10}Q_{uyz}}{10} - \frac{\sqrt{30}Q_{vyz}}{10} + \frac{\sqrt{30}Q_{xy}xz}{10} - \frac{\sqrt{30}Q_{xz}xy}{10} - \frac{\sqrt{30}Q_{yz}(y-z)(y+z)}{10}$$

\* Harmonics for rank 2

$$\vec{\mathbb{G}}_{\mathbf{2}}^{(2,-1)}[q](A, 1)$$

\*\* symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

\*\* expression

$$\frac{\sqrt{6}Q_{xy}z}{3} - \frac{\sqrt{6}Q_{xz}y}{6} - \frac{\sqrt{6}Q_{yz}x}{6}$$

$$\vec{\mathbb{G}}_{\mathbf{2}}^{(2,-1)}[q](A, 2)$$

\*\* symmetry

$\sqrt{3}xy$

\*\* expression

$$- \frac{\sqrt{6}Q_vz}{3} + \frac{\sqrt{6}Q_{xz}x}{6} - \frac{\sqrt{6}Q_{yz}y}{6}$$

$$\vec{\mathbb{G}}_{\mathbf{2}}^{(2,1)}[q](A, 1)$$

\*\* symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

\*\* expression

$$- \frac{5\sqrt{42}Q_{uyz}}{14} + \frac{\sqrt{14}Q_{xy}z(3x^2+3y^2-2z^2)}{28} - \frac{\sqrt{14}Q_{xz}y(9x^2-y^2-6z^2)}{28} + \frac{\sqrt{14}Q_{yz}x(x^2-9y^2+6z^2)}{28}$$

$$\vec{\mathbb{G}}_{\mathbf{2}}^{(2,1)}[q](A, 2)$$

\*\* symmetry

$\sqrt{3}xy$

\*\* expression

$$\frac{5\sqrt{42}Q_uz(x-y)(x+y)}{28} - \frac{\sqrt{14}Q_vz(3x^2+3y^2-2z^2)}{28} + \frac{\sqrt{14}Q_{xz}x(2x^2-3y^2-3z^2)}{14} + \frac{\sqrt{14}Q_{yz}y(3x^2-2y^2+3z^2)}{14}$$

$$\vec{\mathbb{G}}_{\mathbf{2}}^{(2,-1)}[q](B)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$\frac{\sqrt{2}Q_{xz}y}{2} - \frac{\sqrt{2}Q_{yz}x}{2}$$

$\vec{\mathbb{G}}_2^{(2,1)}[q](B)$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$\frac{5\sqrt{42}Q_vxyz}{14} - \frac{5\sqrt{42}Q_{xyz}(x-y)(x+y)}{28} - \frac{\sqrt{42}Q_{xz}y(x^2+y^2-4z^2)}{28} + \frac{\sqrt{42}Q_{yz}x(x^2+y^2-4z^2)}{28}$$

$\vec{\mathbb{G}}_{2,1}^{(2,-1)}[q](E), \vec{\mathbb{G}}_{2,2}^{(2,-1)}[q](E)$

\*\* symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

\*\* expression

$$-\frac{\sqrt{2}Q_uy}{2} + \frac{\sqrt{6}Q_vy}{6} - \frac{\sqrt{6}Q_{xy}x}{6} + \frac{\sqrt{6}Q_{yz}z}{6}$$

$$\frac{\sqrt{2}Q_ux}{2} + \frac{\sqrt{6}Q_vx}{6} + \frac{\sqrt{6}Q_{xy}y}{6} - \frac{\sqrt{6}Q_{xz}z}{6}$$

$\vec{\mathbb{G}}_{2,1}^{(2,1)}[q](E), \vec{\mathbb{G}}_{2,2}^{(2,1)}[q](E)$

\*\* symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

\*\* expression

$$\frac{\sqrt{42}Q_uy(x^2+y^2-4z^2)}{28} + \frac{\sqrt{14}Q_vy(9x^2-y^2-6z^2)}{28} - \frac{\sqrt{14}Q_{xy}x(2x^2-3y^2-3z^2)}{14} - \frac{\sqrt{14}Q_{yz}z(3x^2+3y^2-2z^2)}{14}$$

$$-\frac{\sqrt{42}Q_ux(x^2+y^2-4z^2)}{28} - \frac{\sqrt{14}Q_vx(x^2-9y^2+6z^2)}{28} - \frac{\sqrt{14}Q_{xy}y(3x^2-2y^2+3z^2)}{14} + \frac{\sqrt{14}Q_{xz}z(3x^2+3y^2-2z^2)}{14}$$

\* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(2,-1)}[q](A)$

\*\* symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{30}Q_vxy}{10} + \frac{\sqrt{30}Q_{xy}(x-y)(x+y)}{20} + \frac{\sqrt{30}Q_{xz}yz}{5} - \frac{\sqrt{30}Q_{yz}xz}{5}$$

$\vec{\mathbb{G}}_3^{(2,1)}[q](A)$

\*\* symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{15}Q_vxy(x^2+y^2-6z^2)}{6} + \frac{\sqrt{15}Q_{xy}(x-y)(x+y)(x^2+y^2-6z^2)}{12} - \frac{\sqrt{15}Q_{xz}yz(3x^2+3y^2-4z^2)}{12} + \frac{\sqrt{15}Q_{yz}xz(3x^2+3y^2-4z^2)}{12}$$

$\vec{\mathbb{G}}_3^{(2,-1)}[q](B, 1)$

\*\* symmetry

$$\sqrt{15}xyz$$

\*\* expression

$$\frac{\sqrt{6}Q_u(x-y)(x+y)}{4} + \frac{\sqrt{2}Q_v(x^2+y^2-2z^2)}{4}$$

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](B, 2)$$

\*\* symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

\*\* expression

$$-\frac{\sqrt{6}Q_u xy}{2} - \frac{\sqrt{2}Q_{xy}(x^2 + y^2 - 2z^2)}{4}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](B, 1)$$

\*\* symmetry

$$\sqrt{15}xyz$$

\*\* expression

$$-\frac{\sqrt{3}Q_u(x-y)(x+y)(x^2 + y^2 - 6z^2)}{6} - \frac{Q_v(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{6}$$

$$-\frac{7Q_{xy}xy(x-y)(x+y)}{6} + \frac{7Q_{xz}xz(x-z)(x+z)}{6} - \frac{7Q_{yz}yz(y-z)(y+z)}{6}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](B, 2)$$

\*\* symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

\*\* expression

$$\frac{\sqrt{3}Q_u xy(x^2 + y^2 - 6z^2)}{3} + \frac{7Q_v xy(x-y)(x+y)}{6} - \frac{Q_{xy}(5x^4 - 18x^2y^2 - 12x^2z^2 + 5y^4 - 12y^2z^2 + 4z^4)}{12}$$

$$-\frac{7Q_{xz}yz(3x^2 + y^2 - 2z^2)}{12} - \frac{7Q_{yz}xz(x^2 + 3y^2 - 2z^2)}{12}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](E, 1), \vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](E, 1)$$

\*\* symmetry

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

\*\* expression

$$-\frac{3\sqrt{10}Q_uxz}{20} + \frac{\sqrt{30}Q_vxz}{20} - \frac{\sqrt{30}Q_{xy}yz}{5} - \frac{\sqrt{30}Q_{xz}(x-z)(x+z)}{20} + \frac{\sqrt{30}Q_{yz}xy}{5}$$

$$\frac{3\sqrt{10}Q_uyz}{20} + \frac{\sqrt{30}Q_vyz}{20} + \frac{\sqrt{30}Q_{xy}xz}{5} - \frac{\sqrt{30}Q_{xz}xy}{5} + \frac{\sqrt{30}Q_{yz}(y-z)(y+z)}{20}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](E, 2), \vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](E, 2)$$

\*\* symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

\*\* expression

$$\frac{\sqrt{6}Q_uyz}{4} - \frac{3\sqrt{2}Q_vyz}{4} + \frac{\sqrt{2}Q_{yz}(2x^2 - y^2 - z^2)}{4}$$

$$\frac{\sqrt{6}Q_uxz}{4} + \frac{3\sqrt{2}Q_vxz}{4} - \frac{\sqrt{2}Q_{xz}(x^2 - 2y^2 + z^2)}{4}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](E, 1), \vec{\mathbb{G}}_{3,2}^{(2,1)}[q](E, 1)$$

\*\* symmetry

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

\*\* expression

$$\begin{aligned}
& -\frac{\sqrt{5}Q_uxz(x^2-6y^2+z^2)}{4} + \frac{\sqrt{15}Q_vxz(x^2-6y^2+z^2)}{12} + \frac{\sqrt{15}Q_{xy}yz(3x^2-4y^2+3z^2)}{12} \\
& -\frac{\sqrt{15}Q_{xz}(x-z)(x+z)(x^2-6y^2+z^2)}{12} - \frac{\sqrt{15}Q_{yz}xy(3x^2-4y^2+3z^2)}{12} \\
& -\frac{\sqrt{5}Q_uyz(6x^2-y^2-z^2)}{4} - \frac{\sqrt{15}Q_vyz(6x^2-y^2-z^2)}{12} + \frac{\sqrt{15}Q_{xy}xz(4x^2-3y^2-3z^2)}{12} \\
& -\frac{\sqrt{15}Q_{xz}xy(4x^2-3y^2-3z^2)}{12} - \frac{\sqrt{15}Q_{yz}(y-z)(y+z)(6x^2-y^2-z^2)}{12}
\end{aligned}$$

$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](E, 2), \vec{\mathbb{G}}_{3,2}^{(2,1)}[q](E, 2)$

\*\* symmetry

$$-\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

\*\* expression

$$\begin{aligned}
& \frac{\sqrt{3}Q_uyz(12x^2-9y^2+5z^2)}{12} - \frac{Q_vyz(36x^2+y^2-13z^2)}{12} + \frac{7Q_{xy}xz(2x^2-3y^2-z^2)}{12} \\
& + \frac{7Q_{xz}xy(2x^2-y^2-3z^2)}{12} - \frac{Q_{yz}(4x^4-12x^2y^2-12x^2z^2+5y^4-18y^2z^2+5z^4)}{12} \\
& -\frac{\sqrt{3}Q_uxz(9x^2-12y^2-5z^2)}{12} + \frac{Q_vxz(x^2+36y^2-13z^2)}{12} - \frac{7Q_{xy}yz(3x^2-2y^2+z^2)}{12} \\
& -\frac{Q_{xz}(5x^4-12x^2y^2-18x^2z^2+4y^4-12y^2z^2+5z^4)}{12} - \frac{7Q_{yz}xy(x^2-2y^2+3z^2)}{12}
\end{aligned}$$

\* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(2,-1)}[q](A, 1)$

\*\* symmetry

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

\*\* expression

$$-\frac{6\sqrt{7}Q_uxxyz}{7} + \frac{\sqrt{21}Q_{xy}z(3x^2+3y^2-2z^2)}{14} - \frac{\sqrt{21}Q_{xz}y(2x^2-y^2+z^2)}{14} + \frac{\sqrt{21}Q_{yz}x(x^2-2y^2-z^2)}{14}$$

$\vec{\mathbb{G}}_4^{(2,-1)}[q](A, 2)$

\*\* symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

\*\* expression

$$-\frac{3\sqrt{7}Q_uz(x-y)(x+y)}{7} + \frac{\sqrt{21}Q_vz(3x^2+3y^2-2z^2)}{14} - \frac{\sqrt{21}Q_{xz}x(x^2-5y^2+2z^2)}{28} - \frac{\sqrt{21}Q_{yz}y(5x^2-y^2-2z^2)}{28}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](A, 1)$

\*\* symmetry

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

\*\* expression

$$\begin{aligned}
& -\frac{21\sqrt{22}Q_uxxyz(x^2+y^2-2z^2)}{44} - \frac{21\sqrt{66}Q_vxyz(x-y)(x+y)}{44} + \frac{\sqrt{66}Q_{xy}z(9x^4-24x^2y^2-10x^2z^2+9y^4-10y^2z^2+2z^4)}{44} \\
& -\frac{\sqrt{66}Q_{xz}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44} - \frac{\sqrt{66}Q_{yz}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44}
\end{aligned}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](A, 2)$

\*\* symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

\*\* expression

$$-\frac{21\sqrt{22}Q_uz(x-y)(x+y)(x^2+y^2-2z^2)}{88} - \frac{\sqrt{66}Q_vz(3x^4-78x^2y^2+20x^2z^2+3y^4+20y^2z^2-4z^4)}{88} - \frac{21\sqrt{66}Q_{xy}xyz(x-y)(x+y)}{44}$$

$$-\frac{\sqrt{66}Q_{xz}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44} + \frac{\sqrt{66}Q_{yz}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44}$$

$\vec{\mathbb{G}}_4^{(2,-1)}[q](B, 1)$

\*\* symmetry

$$\frac{\sqrt{21}(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4)}{6}$$

\*\* expression

$$\frac{\sqrt{5}Q_{xy}z(x-y)(x+y)}{2} - \frac{\sqrt{5}Q_{xz}y(x-z)(x+z)}{2} + \frac{\sqrt{5}Q_{yz}x(y-z)(y+z)}{2}$$

$\vec{\mathbb{G}}_4^{(2,-1)}[q](B, 2)$

\*\* symmetry

$$-\frac{\sqrt{15}(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4)}{12}$$

\*\* expression

$$-\frac{6\sqrt{7}Q_vxyz}{7} - \frac{\sqrt{7}Q_{xy}z(x-y)(x+y)}{14} + \frac{\sqrt{7}Q_{xz}y(4x^2-3y^2+5z^2)}{14} + \frac{\sqrt{7}Q_{yz}x(3x^2-4y^2-5z^2)}{14}$$

$\vec{\mathbb{G}}_4^{(2,-1)}[q](B, 3)$

\*\* symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

\*\* expression

$$-\frac{\sqrt{3}Q_vz(x-y)(x+y)}{2} + \sqrt{3}Q_{xy}xyz + \frac{\sqrt{3}Q_{xz}x(x^2-3y^2)}{4} - \frac{\sqrt{3}Q_{yz}y(3x^2-y^2)}{4}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](B, 1)$

\*\* symmetry

$$\frac{\sqrt{21}(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4)}{6}$$

\*\* expression

$$-\frac{3\sqrt{2310}Q_uxyz(x-y)(x+y)}{44} - \frac{3\sqrt{770}Q_vxyz(x^2+y^2-2z^2)}{44} + \frac{\sqrt{770}Q_{xy}z(x-y)(x+y)(x^2+y^2-2z^2)}{22}$$

$$-\frac{\sqrt{770}Q_{xz}y(x-z)(x+z)(x^2-2y^2+z^2)}{22} - \frac{\sqrt{770}Q_{yz}x(y-z)(y+z)(2x^2-y^2-z^2)}{22}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](B, 2)$

\*\* symmetry

$$-\frac{\sqrt{15}(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4)}{12}$$

\*\* expression

$$\frac{21\sqrt{66}Q_uxyz(x-y)(x+y)}{44} - \frac{21\sqrt{22}Q_vxyz(x^2+y^2-2z^2)}{44} + \frac{7\sqrt{22}Q_{xy}z(x-y)(x+y)(x^2+y^2-2z^2)}{44}$$

$$+ \frac{\sqrt{22}Q_{xz}y(17x^4-22x^2y^2-36x^2z^2+3y^4-8y^2z^2+10z^4)}{44} - \frac{\sqrt{22}Q_{yz}x(3x^4-22x^2y^2-8x^2z^2+17y^4-36y^2z^2+10z^4)}{44}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](B, 3)$

\*\* symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

\*\* expression

$$\frac{9\sqrt{154}Q_uz(x^2-2xy-y^2)(x^2+2xy-y^2)}{88} - \frac{\sqrt{462}Q_vz(x-y)(x+y)(x^2+y^2-2z^2)}{88} + \frac{\sqrt{462}Q_{xy}xyz(x^2+y^2-2z^2)}{44}$$

$$+ \frac{\sqrt{462}Q_{xz}x(x^4-8x^2y^2-2x^2z^2+3y^4+6y^2z^2)}{44} + \frac{\sqrt{462}Q_{yz}y(3x^4-8x^2y^2+6x^2z^2+y^4-2y^2z^2)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E, 1), \vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E, 1)$

\*\* symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

\*\* expression

$$\frac{3Q_{ux}(y-z)(y+z)}{4} + \frac{\sqrt{3}Q_vx(y-z)(y+z)}{4} + \frac{\sqrt{3}Q_{xy}y(y^2-3z^2)}{4} - \frac{\sqrt{3}Q_{xz}z(3y^2-z^2)}{4} + \sqrt{3}Q_{yz}xyz$$

$$\frac{3Q_{uy}(x-z)(x+z)}{4} - \frac{\sqrt{3}Q_vy(x-z)(x+z)}{4} + \frac{\sqrt{3}Q_{xy}x(x^2-3z^2)}{4} + \sqrt{3}Q_{xz}xyz - \frac{\sqrt{3}Q_{yz}z(3x^2-z^2)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E, 2), \vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E, 2)$

\*\* symmetry

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

\*\* expression

$$\frac{3\sqrt{7}Q_{uy}(5x^2-2y^2+z^2)}{28} + \frac{\sqrt{21}Q_vy(3x^2+2y^2-9z^2)}{28} + \frac{\sqrt{21}Q_{xy}x(x^2+2y^2-5z^2)}{28} + \frac{\sqrt{21}Q_{yz}z(5x^2-2y^2-z^2)}{28}$$

$$\frac{3\sqrt{7}Q_{ux}(2x^2-5y^2-z^2)}{28} + \frac{\sqrt{21}Q_vx(2x^2+3y^2-9z^2)}{28} - \frac{\sqrt{21}Q_{xy}y(2x^2+y^2-5z^2)}{28} + \frac{\sqrt{21}Q_{xz}z(2x^2-5y^2+z^2)}{28}$$

$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E, 1), \vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E, 1)$

\*\* symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

\*\* expression

$$-\frac{3\sqrt{154}Q_{ux}(x^2y^2-x^2z^2+y^4-9y^2z^2+2z^4)}{88} - \frac{\sqrt{462}Q_vx(x^2y^2-x^2z^2-5y^4+27y^2z^2-4z^4)}{88} \\ - \frac{\sqrt{462}Q_{xy}y(2x^2y^2-6x^2z^2-y^4+8y^2z^2-3z^4)}{44} + \frac{\sqrt{462}Q_{xz}z(6x^2y^2-2x^2z^2+3y^4-8y^2z^2+z^4)}{44} - \frac{\sqrt{462}Q_{yz}xyz(2x^2-y^2-z^2)}{44}$$

$$-\frac{3\sqrt{154}Q_{uy}(x^4+x^2y^2-9x^2z^2-y^2z^2+2z^4)}{88} - \frac{\sqrt{462}Q_vy(5x^4-x^2y^2-27x^2z^2+y^2z^2+4z^4)}{88} \\ + \frac{\sqrt{462}Q_{xy}x(x^4-2x^2y^2-8x^2z^2+6y^2z^2+3z^4)}{44} + \frac{\sqrt{462}Q_{xz}xyz(x^2-2y^2+z^2)}{44} + \frac{\sqrt{462}Q_{yz}z(3x^4+6x^2y^2-8x^2z^2-2y^2z^2+z^4)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E, 2), \vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E, 2)$

\*\* symmetry

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

\*\* expression

$$-\frac{3\sqrt{22}Q_{uy}(5x^4+3x^2y^2-39x^2z^2-2y^4+17y^2z^2-2z^4)}{88} - \frac{\sqrt{66}Q_vy(9x^4-31x^2y^2+39x^2z^2+2y^4+11y^2z^2-12z^4)}{88} \\ + \frac{\sqrt{66}Q_{xy}x(x^4-12x^2y^2+2x^2z^2+8y^4-12y^2z^2+z^4)}{44} + \frac{21\sqrt{66}Q_{xz}xyz(x-z)(x+z)}{44} - \frac{\sqrt{66}Q_{yz}z(x^4-12x^2y^2+2x^2z^2+8y^4-12y^2z^2+z^4)}{44}$$

$$-\frac{3\sqrt{22}Q_{ux}(2x^4-3x^2y^2-17x^2z^2-5y^4+39y^2z^2+2z^4)}{88} - \frac{\sqrt{66}Q_vx(2x^4-31x^2y^2+11x^2z^2+9y^4+39y^2z^2-12z^4)}{88} \\ - \frac{\sqrt{66}Q_{xy}y(8x^4-12x^2y^2-12x^2z^2+y^4+2y^2z^2+z^4)}{44} + \frac{\sqrt{66}Q_{xz}z(8x^4-12x^2y^2-12x^2z^2+y^4+2y^2z^2+z^4)}{44} - \frac{21\sqrt{66}Q_{yz}xyz(y-z)(y+z)}{44}$$