

# SAMB for “kagome”

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- Group: No. 147  $C_{3i}^1$   $P-3$  [ trigonal ]
  - Associated point group: No. 17  $C_{3i}$   $-3$  [ trigonal ]
  - Generation condition
    - model type: **tight\_binding**
    - time-reversal type: **electric**
    - irrep: [Ag]
    - spinful
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- Unit cell:  
 $a = 1.0$ ,  $b = 1.0$ ,  $c = 1.0$ ,  $\alpha = 90.0$ ,  $\beta = 90.0$ ,  $\gamma = 120.0$
- Lattice vectors:  
 $\mathbf{a}_1 = (1.0 \ 0 \ 0)$   
 $\mathbf{a}_2 = (-0.5 \ 0.86602540378444 \ 0)$   
 $\mathbf{a}_3 = (0 \ 0 \ 1.0)$

Table 1: High-symmetry line:  $\Gamma$ -X.

symbol	position	symbol	position
$\Gamma$	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	X	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$

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- Kets: dimension = 24

Table 2: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket	No.	ket	No.	ket
1	$(s, \uparrow)@A_1$	2	$(s, \downarrow)@A_1$	3	$(p_x, \uparrow)@A_1$	4	$(p_x, \downarrow)@A_1$	5	$(p_y, \uparrow)@A_1$
6	$(p_y, \downarrow)@A_1$	7	$(p_z, \uparrow)@A_1$	8	$(p_z, \downarrow)@A_1$	9	$(s, \uparrow)@A_2$	10	$(s, \downarrow)@A_2$
11	$(p_x, \uparrow)@A_2$	12	$(p_x, \downarrow)@A_2$	13	$(p_y, \uparrow)@A_2$	14	$(p_y, \downarrow)@A_2$	15	$(p_z, \uparrow)@A_2$
16	$(p_z, \downarrow)@A_2$	17	$(s, \uparrow)@A_3$	18	$(s, \downarrow)@A_3$	19	$(p_x, \uparrow)@A_3$	20	$(p_x, \downarrow)@A_3$
21	$(p_y, \uparrow)@A_3$	22	$(p_y, \downarrow)@A_3$	23	$(p_z, \uparrow)@A_3$	24	$(p_z, \downarrow)@A_3$		

- Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position	mapping
S <sub>1</sub> [3e: -1]	A <sub>1</sub>	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$	[1,4]
	A <sub>2</sub>	$\begin{pmatrix} 0 & \frac{1}{2} & 0 \end{pmatrix}$	[2,5]
	A <sub>3</sub>	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	[3,6]

- Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

	bond	tail	head	$n$	#	$\mathbf{b@c}$	mapping
B <sub>1</sub> [6g: 1]	b <sub>1</sub>	A <sub>2</sub>	A <sub>1</sub>	1	1	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \end{pmatrix}$	[1]
	b <sub>2</sub>	A <sub>3</sub>	A <sub>2</sub>	1	1	$\begin{pmatrix} -\frac{1}{2} & 0 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{3}{4} & \frac{1}{2} & 0 \end{pmatrix}$	[2]
	b <sub>3</sub>	A <sub>3</sub>	A <sub>1</sub>	1	1	$\begin{pmatrix} 0 & \frac{1}{2} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 \end{pmatrix}$	[-3]
	b <sub>4</sub>	A <sub>2</sub>	A <sub>1</sub>	1	1	$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{4} & \frac{3}{4} & 0 \end{pmatrix}$	[4]
	b <sub>5</sub>	A <sub>3</sub>	A <sub>2</sub>	1	1	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{4} & \frac{1}{2} & 0 \end{pmatrix}$	[5]
	b <sub>6</sub>	A <sub>3</sub>	A <sub>1</sub>	1	1	$\begin{pmatrix} 0 & -\frac{1}{2} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{3}{4} & 0 \end{pmatrix}$	[-6]

- SAMB:

$$\boxed{\text{No. 1}} \quad \hat{Q}_0^{(A_g)} [M_1, S_1]$$

$$\hat{Z}_1 = \mathbb{X}_1[Q_0^{(a, A_g)}] \otimes \mathbb{Y}_1[Q_0^{(s, A_g)}]$$

$$\boxed{\text{No. 2}} \quad \hat{Q}_0^{(A_g)} [M_3, S_1]$$

$$\hat{Z}_2 = \mathbb{X}_{29}[Q_0^{(a, A_g)}] \otimes \mathbb{Y}_1[Q_0^{(s, A_g)}]$$

$$\boxed{\text{No. 3}} \quad \hat{Q}_2^{(A_g)} [M_3, S_1]$$

$$\hat{Z}_3 = \mathbb{X}_{30}[Q_2^{(a, A_g)}] \otimes \mathbb{Y}_1[Q_0^{(s, A_g)}]$$

$$\boxed{\text{No. 4}} \quad \hat{Q}_0^{(A_g)} [M_3, S_1]$$

$$\hat{Z}_4 = \frac{\sqrt{2}\mathbb{X}_{36}[Q_{2,0}^{(a, E_g, 2)}] \otimes \mathbb{Y}_2[Q_{2,0}^{(s, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{37}[Q_{2,1}^{(a, E_g, 2)}] \otimes \mathbb{Y}_3[Q_{2,1}^{(s, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 5}} \quad \hat{G}_1^{(A_g)} [M_3, S_1]$$

$$\hat{Z}_5 = -\frac{\sqrt{2}\mathbb{X}_{36}[Q_{2,0}^{(a, E_g, 2)}] \otimes \mathbb{Y}_3[Q_{2,1}^{(s, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{37}[Q_{2,1}^{(a, E_g, 2)}] \otimes \mathbb{Y}_2[Q_{2,0}^{(s, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 6}} \quad \hat{G}_3^{(A_g, 2)} [M_3, S_1]$$

$$\hat{Z}_6 = \frac{\sqrt{2}\mathbb{X}_{34}[Q_{2,0}^{(a, E_g, 1)}] \otimes \mathbb{Y}_2[Q_{2,0}^{(s, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{35}[Q_{2,1}^{(a, E_g, 1)}] \otimes \mathbb{Y}_3[Q_{2,1}^{(s, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 7}} \quad \hat{G}_3^{(A_g, 3)} [M_3, S_1]$$

$$\hat{Z}_7 = \frac{\sqrt{2}\mathbb{X}_{34}[Q_{2,0}^{(a, E_g, 1)}] \otimes \mathbb{Y}_3[Q_{2,1}^{(s, E_g, 2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{35}[Q_{2,1}^{(a, E_g, 1)}] \otimes \mathbb{Y}_2[Q_{2,0}^{(s, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 8}} \quad \hat{Q}_0^{(A_g)}(1, 1) [M_3, S_1]$$

$$\hat{Z}_8 = \mathbb{X}_{31}[Q_0^{(a, A_g)}(1, 1)] \otimes \mathbb{Y}_1[Q_0^{(s, A_g)}]$$

$$\boxed{\text{No. 9}} \quad \hat{\mathbb{Q}}_2^{(A_g)}(1, -1) \text{ [M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_9 = \mathbb{X}_{32}[\mathbb{Q}_2^{(a, A_g)}(1, -1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\boxed{\text{No. 10}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, -1) \text{ [M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_{10} = \frac{\sqrt{2}\mathbb{X}_{40}[\mathbb{Q}_{2,0}^{(a, E_g, 2)}(1, -1)] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{41}[\mathbb{Q}_{2,1}^{(a, E_g, 2)}(1, -1)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 11}} \quad \hat{\mathbb{G}}_1^{(A_g)}(1, -1) \text{ [M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_{11} = -\frac{\sqrt{2}\mathbb{X}_{40}[\mathbb{Q}_{2,0}^{(a, E_g, 2)}(1, -1)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{41}[\mathbb{Q}_{2,1}^{(a, E_g, 2)}(1, -1)] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 12}} \quad \hat{\mathbb{G}}_3^{(A_g, 2)}(1, -1) \text{ [M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_{12} = \frac{\sqrt{2}\mathbb{X}_{38}[\mathbb{Q}_{2,0}^{(a, E_g, 1)}(1, -1)] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{39}[\mathbb{Q}_{2,1}^{(a, E_g, 1)}(1, -1)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 13}} \quad \hat{\mathbb{G}}_3^{(A_g, 3)}(1, -1) \text{ [M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_{13} = \frac{\sqrt{2}\mathbb{X}_{38}[\mathbb{Q}_{2,0}^{(a, E_g, 1)}(1, -1)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s, E_g, 2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{39}[\mathbb{Q}_{2,1}^{(a, E_g, 1)}(1, -1)] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 14}} \quad \hat{\mathbb{G}}_1^{(A_g)}(1, 0) \text{ [M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_{14} = \mathbb{X}_{33}[\mathbb{G}_1^{(a, A_g)}(1, 0)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\boxed{\text{No. 15}} \quad \hat{\mathbb{G}}_3^{(A_g, 2)}(1, 0) \text{ [M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_{15} = -\frac{\sqrt{2}\mathbb{X}_{42}[\mathbb{G}_{1,0}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{43}[\mathbb{G}_{1,1}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 16}} \quad \hat{\mathbb{G}}_3^{(A_g, 3)}(1, 0) \text{ [M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_{16} = \frac{\sqrt{2}\mathbb{X}_{42}[\mathbb{G}_{1,0}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_2[\mathbb{Q}_{2,0}^{(s, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{43}[\mathbb{G}_{1,1}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,1}^{(s, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 17}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_1, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{17} = \mathbb{X}_1[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b, A_g)}]$$

$$\boxed{\text{No. 18}} \quad \hat{\mathbb{G}}_1^{(A_g)} (1, -1) [\text{M}_1, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{18} = \mathbb{X}_2[\text{M}_1^{(a, A_g)} (1, -1)] \otimes \mathbb{Y}_{10}[\mathbb{T}_0^{(b, A_g)}]$$

$$\boxed{\text{No. 19}} \quad \hat{\mathbb{G}}_3^{(A_g, 2)} (1, -1) [\text{M}_1, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{19} = -\frac{\sqrt{2}\mathbb{X}_3[\text{M}_{1,0}^{(a, E_g)} (1, -1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_4[\text{M}_{1,1}^{(a, E_g)} (1, -1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 20}} \quad \hat{\mathbb{G}}_3^{(A_g, 3)} (1, -1) [\text{M}_1, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{20} = \frac{\sqrt{2}\mathbb{X}_3[\text{M}_{1,0}^{(a, E_g)} (1, -1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_4[\text{M}_{1,1}^{(a, E_g)} (1, -1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 21}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{21} = \frac{\sqrt{2}\mathbb{X}_6[\mathbb{Q}_{1,0}^{(a, E_u)}] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(b, E_u)}]}{2} + \frac{\sqrt{2}\mathbb{X}_7[\mathbb{Q}_{1,1}^{(a, E_u)}] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(b, E_u)}]}{2}$$

$$\boxed{\text{No. 22}} \quad \hat{\mathbb{G}}_1^{(A_g)} [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{22} = \frac{\sqrt{2}\mathbb{X}_6[\mathbb{Q}_{1,0}^{(a, E_u)}] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(b, E_u)}]}{2} - \frac{\sqrt{2}\mathbb{X}_7[\mathbb{Q}_{1,1}^{(a, E_u)}] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(b, E_u)}]}{2}$$

$$\boxed{\text{No. 23}} \quad \hat{\mathbb{G}}_3^{(A_g, 2)} [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{23} = \mathbb{X}_5[\mathbb{Q}_1^{(a, A_u)}] \otimes \mathbb{Y}_9[\mathbb{Q}_3^{(b, A_u, 3)}]$$

$$\boxed{\text{No. 24}} \quad \hat{\mathbb{Q}}_0^{(A_g)} (1, 0) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{24} = \frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{Q}_{1,0}^{(a, E_u)} (1, 0)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(b, E_u)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{Q}_{1,1}^{(a, E_u)} (1, 0)] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(b, E_u)}]}{2}$$

$$\boxed{\text{No. 25}} \quad \hat{\mathbb{G}}_1^{(Ag)}(1, 0) \text{ [M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{25} = \frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{Q}_{1,0}^{(a,E_u)}(1, 0)] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(b,E_u)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{Q}_{1,1}^{(a,E_u)}(1, 0)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(b,E_u)}]}{2}$$

$$\boxed{\text{No. 26}} \quad \hat{\mathbb{G}}_3^{(Ag,2)}(1, 0) \text{ [M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{26} = \mathbb{X}_8[\mathbb{Q}_1^{(a,A_u)}(1, 0)] \otimes \mathbb{Y}_9[\mathbb{Q}_3^{(b,A_u,3)}]$$

$$\boxed{\text{No. 27}} \quad \hat{\mathbb{G}}_3^{(Ag,3)}(1, 1) \text{ [M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{27} = \mathbb{X}_9[\mathbb{G}_0^{(a,A_u)}(1, 1)] \otimes \mathbb{Y}_9[\mathbb{Q}_3^{(b,A_u,3)}]$$

$$\boxed{\text{No. 28}} \quad \hat{\mathbb{G}}_1^{(Ag)}(1, -1) \text{ [M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{28} = \frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{G}_{2,0}^{(a,E_u,1)}(1, -1)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(b,E_u)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{2,1}^{(a,E_u,1)}(1, -1)] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(b,E_u)}]}{2}$$

$$\boxed{\text{No. 29}} \quad \hat{\mathbb{Q}}_2^{(Ag)}(1, -1) \text{ [M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{29} = \frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{G}_{2,0}^{(a,E_u,1)}(1, -1)] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(b,E_u)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{2,1}^{(a,E_u,1)}(1, -1)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(b,E_u)}]}{2}$$

$$\boxed{\text{No. 30}} \quad \hat{\mathbb{G}}_3^{(Ag,2)}(1, -1) \text{ [M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{30} = \frac{\sqrt{2}\mathbb{X}_{15}[\mathbb{G}_{2,0}^{(a,E_u,2)}(1, -1)] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(b,E_u)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{16}[\mathbb{G}_{2,1}^{(a,E_u,2)}(1, -1)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(b,E_u)}]}{2}$$

$$\boxed{\text{No. 31}} \quad \hat{\mathbb{G}}_3^{(Ag,3)}(1, -1) \text{ [M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{31} = \frac{\sqrt{2}\mathbb{X}_{15}[\mathbb{G}_{2,0}^{(a,E_u,2)}(1, -1)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,0}^{(b,E_u)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{16}[\mathbb{G}_{2,1}^{(a,E_u,2)}(1, -1)] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,1}^{(b,E_u)}]}{2}$$

$$\boxed{\text{No. 32}} \quad \hat{\mathbb{G}}_3^{(Ag,3)}(1, -1) \text{ [M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{32} = -\mathbb{X}_{10}[\mathbb{G}_2^{(a,A_u)}(1, -1)] \otimes \mathbb{Y}_9[\mathbb{Q}_3^{(b,A_u,3)}]$$

$$\boxed{\text{No. 33}} \quad \hat{\mathbb{G}}_3^{(Ag,3)}(1,1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{33} = \mathbb{X}_{17}[\mathbb{M}_0^{(a,A_u)}(1,1)] \otimes \mathbb{Y}_{15}[\mathbb{T}_3^{(b,A_u,3)}]$$

$$\boxed{\text{No. 34}} \quad \hat{\mathbb{G}}_1^{(Ag)}(1,-1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{34} = \frac{\sqrt{2}\mathbb{X}_{20}[\mathbb{M}_{2,0}^{(a,E_u,1)}(1,-1)] \otimes \mathbb{Y}_{11}[\mathbb{T}_{1,0}^{(b,E_u)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{21}[\mathbb{M}_{2,1}^{(a,E_u,1)}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{T}_{1,1}^{(b,E_u)}]}{2}$$

$$\boxed{\text{No. 35}} \quad \hat{\mathbb{Q}}_2^{(Ag)}(1,-1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{35} = \frac{\sqrt{2}\mathbb{X}_{20}[\mathbb{M}_{2,0}^{(a,E_u,1)}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{T}_{1,1}^{(b,E_u)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{21}[\mathbb{M}_{2,1}^{(a,E_u,1)}(1,-1)] \otimes \mathbb{Y}_{11}[\mathbb{T}_{1,0}^{(b,E_u)}]}{2}$$

$$\boxed{\text{No. 36}} \quad \hat{\mathbb{G}}_3^{(Ag,2)}(1,-1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{36} = \frac{\sqrt{2}\mathbb{X}_{22}[\mathbb{M}_{2,0}^{(a,E_u,2)}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{T}_{1,1}^{(b,E_u)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{23}[\mathbb{M}_{2,1}^{(a,E_u,2)}(1,-1)] \otimes \mathbb{Y}_{11}[\mathbb{T}_{1,0}^{(b,E_u)}]}{2}$$

$$\boxed{\text{No. 37}} \quad \hat{\mathbb{G}}_3^{(Ag,3)}(1,-1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{37} = \frac{\sqrt{2}\mathbb{X}_{22}[\mathbb{M}_{2,0}^{(a,E_u,2)}(1,-1)] \otimes \mathbb{Y}_{11}[\mathbb{T}_{1,0}^{(b,E_u)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{23}[\mathbb{M}_{2,1}^{(a,E_u,2)}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{T}_{1,1}^{(b,E_u)}]}{2}$$

$$\boxed{\text{No. 38}} \quad \hat{\mathbb{G}}_3^{(Ag,3)}(1,-1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{38} = -\mathbb{X}_{18}[\mathbb{M}_2^{(a,A_u)}(1,-1)] \otimes \mathbb{Y}_{15}[\mathbb{T}_3^{(b,A_u,3)}]$$

$$\boxed{\text{No. 39}} \quad \hat{\mathbb{Q}}_0^{(Ag)}(1,0) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{39} = \frac{\sqrt{2}\mathbb{X}_{24}[\mathbb{T}_{1,0}^{(a,E_u)}(1,0)] \otimes \mathbb{Y}_{11}[\mathbb{T}_{1,0}^{(b,E_u)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{T}_{1,1}^{(a,E_u)}(1,0)] \otimes \mathbb{Y}_{12}[\mathbb{T}_{1,1}^{(b,E_u)}]}{2}$$

$$\boxed{\text{No. 40}} \quad \hat{\mathbb{G}}_1^{(Ag)}(1,0) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{40} = \frac{\sqrt{2}\mathbb{X}_{24}[\mathbb{T}_{1,0}^{(a,E_u)}(1,0)] \otimes \mathbb{Y}_{12}[\mathbb{T}_{1,1}^{(b,E_u)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{T}_{1,1}^{(a,E_u)}(1,0)] \otimes \mathbb{Y}_{11}[\mathbb{T}_{1,0}^{(b,E_u)}]}{2}$$

$$\boxed{\text{No. 41}} \quad \hat{\mathbb{G}}_3^{(A_g, 2)}(1, 0) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{41} = \mathbb{X}_{19}[\mathbb{T}_1^{(a, A_u)}(1, 0)] \otimes \mathbb{Y}_{15}[\mathbb{T}_3^{(b, A_u, 3)}]$$

$$\boxed{\text{No. 42}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{42} = \frac{\sqrt{2}\mathbb{X}_{27}[\mathbb{T}_{1,0}^{(a, E_u)}] \otimes \mathbb{Y}_{11}[\mathbb{T}_{1,0}^{(b, E_u)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{28}[\mathbb{T}_{1,1}^{(a, E_u)}] \otimes \mathbb{Y}_{12}[\mathbb{T}_{1,1}^{(b, E_u)}]}{2}$$

$$\boxed{\text{No. 43}} \quad \hat{\mathbb{G}}_1^{(A_g)} [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{43} = \frac{\sqrt{2}\mathbb{X}_{27}[\mathbb{T}_{1,0}^{(a, E_u)}] \otimes \mathbb{Y}_{12}[\mathbb{T}_{1,1}^{(b, E_u)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{28}[\mathbb{T}_{1,1}^{(a, E_u)}] \otimes \mathbb{Y}_{11}[\mathbb{T}_{1,0}^{(b, E_u)}]}{2}$$

$$\boxed{\text{No. 44}} \quad \hat{\mathbb{G}}_3^{(A_g, 2)} [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{44} = \mathbb{X}_{26}[\mathbb{T}_1^{(a, A_u)}] \otimes \mathbb{Y}_{15}[\mathbb{T}_3^{(b, A_u, 3)}]$$

$$\boxed{\text{No. 45}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{45} = \mathbb{X}_{29}[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b, A_g)}]$$

$$\boxed{\text{No. 46}} \quad \hat{\mathbb{Q}}_2^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{46} = \mathbb{X}_{30}[\mathbb{Q}_2^{(a, A_g)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b, A_g)}]$$

$$\boxed{\text{No. 47}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{47} = \frac{\sqrt{2}\mathbb{X}_{36}[\mathbb{Q}_{2,0}^{(a, E_g, 2)}] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(b, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{37}[\mathbb{Q}_{2,1}^{(a, E_g, 2)}] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(b, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 48}} \quad \hat{\mathbb{G}}_1^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{48} = -\frac{\sqrt{2}\mathbb{X}_{36}[\mathbb{Q}_{2,0}^{(a, E_g, 2)}] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(b, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{37}[\mathbb{Q}_{2,1}^{(a, E_g, 2)}] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(b, E_g, 2)}]}{2}$$



$$\boxed{\text{No. 49}} \quad \hat{\mathbb{G}}_3^{(A_g, 2)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{49} = \frac{\sqrt{2}\mathbb{X}_{34}[\mathbb{Q}_{2,0}^{(a,E_g,1)}] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(b,E_g,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{35}[\mathbb{Q}_{2,1}^{(a,E_g,1)}] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 50}} \quad \hat{\mathbb{G}}_3^{(A_g, 3)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{50} = \frac{\sqrt{2}\mathbb{X}_{34}[\mathbb{Q}_{2,0}^{(a,E_g,1)}] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(b,E_g,2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{35}[\mathbb{Q}_{2,1}^{(a,E_g,1)}] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 51}} \quad \hat{\mathbb{Q}}_0^{(A_g)} (1, 1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{51} = \mathbb{X}_{31}[\mathbb{Q}_0^{(a,A_g)} (1, 1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 52}} \quad \hat{\mathbb{Q}}_2^{(A_g)} (1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{52} = \mathbb{X}_{32}[\mathbb{Q}_2^{(a,A_g)} (1, -1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 53}} \quad \hat{\mathbb{Q}}_0^{(A_g)} (1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{53} = \frac{\sqrt{2}\mathbb{X}_{40}[\mathbb{Q}_{2,0}^{(a,E_g,2)} (1, -1)] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(b,E_g,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{41}[\mathbb{Q}_{2,1}^{(a,E_g,2)} (1, -1)] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 54}} \quad \hat{\mathbb{G}}_1^{(A_g)} (1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{54} = -\frac{\sqrt{2}\mathbb{X}_{40}[\mathbb{Q}_{2,0}^{(a,E_g,2)} (1, -1)] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(b,E_g,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{41}[\mathbb{Q}_{2,1}^{(a,E_g,2)} (1, -1)] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 55}} \quad \hat{\mathbb{G}}_3^{(A_g, 2)} (1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{55} = \frac{\sqrt{2}\mathbb{X}_{38}[\mathbb{Q}_{2,0}^{(a,E_g,1)} (1, -1)] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(b,E_g,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{39}[\mathbb{Q}_{2,1}^{(a,E_g,1)} (1, -1)] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 56}} \quad \hat{\mathbb{G}}_3^{(A_g, 3)} (1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{56} = \frac{\sqrt{2}\mathbb{X}_{38}[\mathbb{Q}_{2,0}^{(a,E_g,1)} (1, -1)] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(b,E_g,2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{39}[\mathbb{Q}_{2,1}^{(a,E_g,1)} (1, -1)] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 57}} \quad \hat{\mathbb{G}}_1^{(A_g)}(1, 0) \text{ [M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{57} = \mathbb{X}_{33}[\mathbb{G}_1^{(a, A_g)}(1, 0)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b, A_g)}]$$

$$\boxed{\text{No. 58}} \quad \hat{\mathbb{G}}_3^{(A_g, 2)}(1, 0) \text{ [M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{58} = -\frac{\sqrt{2}\mathbb{X}_{42}[\mathbb{G}_{1,0}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(b, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{43}[\mathbb{G}_{1,1}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(b, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 59}} \quad \hat{\mathbb{G}}_3^{(A_g, 3)}(1, 0) \text{ [M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{59} = \frac{\sqrt{2}\mathbb{X}_{42}[\mathbb{G}_{1,0}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_7[\mathbb{Q}_{2,0}^{(b, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{43}[\mathbb{G}_{1,1}^{(a, E_g)}(1, 0)] \otimes \mathbb{Y}_8[\mathbb{Q}_{2,1}^{(b, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 60}} \quad \hat{\mathbb{G}}_1^{(A_g)} \text{ [M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{60} = \mathbb{X}_{44}[\mathbb{M}_1^{(a, A_g)}] \otimes \mathbb{Y}_{10}[\mathbb{T}_0^{(b, A_g)}]$$

$$\boxed{\text{No. 61}} \quad \hat{\mathbb{G}}_3^{(A_g, 2)} \text{ [M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{61} = -\frac{\sqrt{2}\mathbb{X}_{45}[\mathbb{M}_{1,0}^{(a, E_g)}] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{46}[\mathbb{M}_{1,1}^{(a, E_g)}] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 62}} \quad \hat{\mathbb{G}}_3^{(A_g, 3)} \text{ [M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{62} = \frac{\sqrt{2}\mathbb{X}_{45}[\mathbb{M}_{1,0}^{(a, E_g)}] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{46}[\mathbb{M}_{1,1}^{(a, E_g)}] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 63}} \quad \hat{\mathbb{G}}_1^{(A_g)}(1, 1) \text{ [M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{63} = \mathbb{X}_{47}[\mathbb{M}_1^{(a, A_g)}(1, 1)] \otimes \mathbb{Y}_{10}[\mathbb{T}_0^{(b, A_g)}]$$

$$\boxed{\text{No. 64}} \quad \hat{\mathbb{G}}_3^{(A_g, 2)}(1, 1) \text{ [M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{64} = -\frac{\sqrt{2}\mathbb{X}_{53}[\mathbb{M}_{1,0}^{(a, E_g)}(1, 1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b, E_g, 2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{54}[\mathbb{M}_{1,1}^{(a, E_g)}(1, 1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b, E_g, 2)}]}{2}$$

$$\boxed{\text{No. 65}} \quad \hat{\mathbb{G}}_3^{(A_g,3)}(1,1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{65} = \frac{\sqrt{2}\mathbb{X}_{53}[\mathbb{M}_{1,0}^{(a,E_g)}(1,1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b,E_g,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{54}[\mathbb{M}_{1,1}^{(a,E_g)}(1,1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 66}} \quad \hat{\mathbb{G}}_1^{(A_g)}(1,-1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{66} = \mathbb{X}_{48}[\mathbb{M}_1^{(a,A_g)}(1,-1)] \otimes \mathbb{Y}_{10}[\mathbb{T}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 67}} \quad \hat{\mathbb{G}}_3^{(A_g,2)}(1,-1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{67} = -\frac{\sqrt{2}\mathbb{X}_{55}[\mathbb{M}_{1,0}^{(a,E_g)}(1,-1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b,E_g,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{56}[\mathbb{M}_{1,1}^{(a,E_g)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 68}} \quad \hat{\mathbb{G}}_3^{(A_g,3)}(1,-1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{68} = \frac{\sqrt{2}\mathbb{X}_{55}[\mathbb{M}_{1,0}^{(a,E_g)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b,E_g,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{56}[\mathbb{M}_{1,1}^{(a,E_g)}(1,-1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 69}} \quad \hat{\mathbb{G}}_3^{(A_g,1)}(1,-1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{69} = \mathbb{X}_{49}[\mathbb{M}_3^{(a,A_g,1)}(1,-1)] \otimes \mathbb{Y}_{10}[\mathbb{T}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 70}} \quad \hat{\mathbb{G}}_3^{(A_g,2)}(1,-1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{70} = \mathbb{X}_{50}[\mathbb{M}_3^{(a,A_g,2)}(1,-1)] \otimes \mathbb{Y}_{10}[\mathbb{T}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 71}} \quad \hat{\mathbb{G}}_3^{(A_g,3)}(1,-1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{71} = \mathbb{X}_{51}[\mathbb{M}_3^{(a,A_g,3)}(1,-1)] \otimes \mathbb{Y}_{10}[\mathbb{T}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 72}} \quad \hat{\mathbb{G}}_1^{(A_g)}(1,-1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{72} = \frac{\sqrt{2}\mathbb{X}_{59}[\mathbb{M}_{3,0}^{(a,E_g,2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b,E_g,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{60}[\mathbb{M}_{3,1}^{(a,E_g,2)}(1,-1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 73}} \quad \hat{\mathbb{Q}}_2^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{73} = -\frac{\sqrt{2}\mathbb{X}_{59}[\mathbb{M}_{3,0}^{(a,E_g,2)}(1, -1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b,E_g,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{60}[\mathbb{M}_{3,1}^{(a,E_g,2)}(1, -1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 74}} \quad \hat{\mathbb{G}}_3^{(A_g,2)}(1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{74} = \frac{\sqrt{2}\mathbb{X}_{57}[\mathbb{M}_{3,0}^{(a,E_g,1)}(1, -1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b,E_g,2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{58}[\mathbb{M}_{3,1}^{(a,E_g,1)}(1, -1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 75}} \quad \hat{\mathbb{G}}_3^{(A_g,3)}(1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{75} = -\frac{\sqrt{2}\mathbb{X}_{57}[\mathbb{M}_{3,0}^{(a,E_g,1)}(1, -1)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b,E_g,2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{58}[\mathbb{M}_{3,1}^{(a,E_g,1)}(1, -1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 76}} \quad \hat{\mathbb{Q}}_2^{(A_g)}(1, 0) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{76} = \mathbb{X}_{52}[\mathbb{T}_2^{(a,A_g)}(1, 0)] \otimes \mathbb{Y}_{10}[\mathbb{T}_0^{(b,A_g)}]$$

$$\boxed{\text{No. 77}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 0) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{77} = \frac{\sqrt{2}\mathbb{X}_{63}[\mathbb{T}_{2,0}^{(a,E_g,2)}(1, 0)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b,E_g,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{64}[\mathbb{T}_{2,1}^{(a,E_g,2)}(1, 0)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 78}} \quad \hat{\mathbb{G}}_1^{(A_g)}(1, 0) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{78} = -\frac{\sqrt{2}\mathbb{X}_{63}[\mathbb{T}_{2,0}^{(a,E_g,2)}(1, 0)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b,E_g,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{64}[\mathbb{T}_{2,1}^{(a,E_g,2)}(1, 0)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 79}} \quad \hat{\mathbb{G}}_3^{(A_g,2)}(1, 0) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{79} = \frac{\sqrt{2}\mathbb{X}_{61}[\mathbb{T}_{2,0}^{(a,E_g,1)}(1, 0)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b,E_g,2)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{62}[\mathbb{T}_{2,1}^{(a,E_g,1)}(1, 0)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b,E_g,2)}]}{2}$$

$$\boxed{\text{No. 80}} \quad \hat{\mathbb{G}}_3^{(A_g,3)}(1, 0) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{80} = \frac{\sqrt{2}\mathbb{X}_{61}[\mathbb{T}_{2,0}^{(a,E_g,1)}(1, 0)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{2,1}^{(b,E_g,2)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{62}[\mathbb{T}_{2,1}^{(a,E_g,1)}(1, 0)] \otimes \mathbb{Y}_{13}[\mathbb{T}_{2,0}^{(b,E_g,2)}]}{2}$$

Table 5: Atomic SAMB group.

group	bra	ket
M <sub>1</sub>	$(s, \uparrow), (s, \downarrow)$	$(s, \uparrow), (s, \downarrow)$
M <sub>2</sub>	$(s, \uparrow), (s, \downarrow)$	$(p_x, \uparrow), (p_x, \downarrow), (p_y, \uparrow), (p_y, \downarrow), (p_z, \uparrow), (p_z, \downarrow)$
M <sub>3</sub>	$(p_x, \uparrow), (p_x, \downarrow), (p_y, \uparrow), (p_y, \downarrow), (p_z, \uparrow), (p_z, \downarrow)$	$(p_x, \uparrow), (p_x, \downarrow), (p_y, \uparrow), (p_y, \downarrow), (p_z, \uparrow), (p_z, \downarrow)$

Table 6: Atomic SAMB.

symbol	type	group	form
$\mathbb{X}_1$	$\mathbb{Q}_0^{(a, A_g)}$	M <sub>1</sub>	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{X}_2$	$\mathbb{M}_1^{(a, A_g)}(1, -1)$	M <sub>1</sub>	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{X}_3$	$\mathbb{M}_{1,0}^{(a, E_g)}(1, -1)$	M <sub>1</sub>	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$
$\mathbb{X}_4$	$\mathbb{M}_{1,1}^{(a, E_g)}(1, -1)$	M <sub>1</sub>	$\begin{pmatrix} 0 & -\frac{\sqrt{2}i}{2} \\ \frac{\sqrt{2}i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_5$	$\mathbb{Q}_1^{(a, A_u)}$	M <sub>2</sub>	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{X}_6$	$\mathbb{Q}_{1,0}^{(a, E_u)}$	M <sub>2</sub>	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_7$	$\mathbb{Q}_{1,1}^{(a, E_u)}$	M <sub>2</sub>	$\begin{pmatrix} 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_8$	$\mathbb{Q}_1^{(a, A_u)}(1, 0)$	M <sub>2</sub>	$\begin{pmatrix} 0 & -\frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_9$	$\mathbb{G}_0^{(a, A_u)}(1, 1)$	M <sub>2</sub>	$\begin{pmatrix} 0 & \frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}i}{6} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}i}{6} \end{pmatrix}$
$\mathbb{X}_{10}$	$\mathbb{G}_2^{(a, A_u)}(1, -1)$	M <sub>2</sub>	$\begin{pmatrix} 0 & -\frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}i}{3} & 0 \\ -\frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}i}{3} \end{pmatrix}$
$\mathbb{X}_{11}$	$\mathbb{Q}_{1,0}^{(a, E_u)}(1, 0)$	M <sub>2</sub>	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{i}{2} & -\frac{1}{2} & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{12}$	$\mathbb{Q}_{1,1}^{(a,E_u)}(1,0)$	$M_2$	$\begin{pmatrix} \frac{i}{2} & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & -\frac{i}{2} & 0 & 0 & -\frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{13}$	$\mathbb{G}_{2,0}^{(a,E_u,1)}(1,-1)$	$M_2$	$\begin{pmatrix} \frac{i}{2} & 0 & 0 & 0 & 0 & \frac{i}{2} \\ 0 & -\frac{i}{2} & 0 & 0 & \frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{14}$	$\mathbb{G}_{2,1}^{(a,E_u,1)}(1,-1)$	$M_2$	$\begin{pmatrix} 0 & 0 & \frac{i}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & -\frac{i}{2} & -\frac{1}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{15}$	$\mathbb{G}_{2,0}^{(a,E_u,2)}(1,-1)$	$M_2$	$\begin{pmatrix} 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{16}$	$\mathbb{G}_{2,1}^{(a,E_u,2)}(1,-1)$	$M_2$	$\begin{pmatrix} 0 & -\frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 \\ \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{17}$	$\mathbb{M}_0^{(a,A_u)}(1,1)$	$M_2$	$\begin{pmatrix} 0 & \frac{\sqrt{6}}{6} & 0 & -\frac{\sqrt{6}i}{6} & \frac{\sqrt{6}}{6} & 0 \\ \frac{\sqrt{6}}{6} & 0 & \frac{\sqrt{6}i}{6} & 0 & 0 & -\frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_{18}$	$\mathbb{M}_2^{(a,A_u)}(1,-1)$	$M_2$	$\begin{pmatrix} 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{3}}{6} & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{X}_{19}$	$\mathbb{T}_1^{(a,A_u)}(1,0)$	$M_2$	$\begin{pmatrix} 0 & \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 \\ -\frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{20}$	$\mathbb{M}_{2,0}^{(a,E_u,1)}(1,-1)$	$M_2$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{21}$	$\mathbb{M}_{2,1}^{(a,E_u,1)}(1,-1)$	$M_2$	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{22}$	$\mathbb{M}_{2,0}^{(a,E_u,2)}(1,-1)$	$M_2$	$\begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 \\ \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{23}$	$\mathbb{M}_{2,1}^{(a,E_u,2)}(1,-1)$	$M_2$	$\begin{pmatrix} 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{24}$	$\mathbb{T}_{1,0}^{(a,E_u)}(1,0)$	$M_2$	$\begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{25}$	$\mathbb{T}_{1,1}^{(a,E_u)}(1,0)$	$M_2$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{26}$	$\mathbb{T}_1^{(a,A_u)}$	$M_2$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{2} \end{pmatrix}$
$\mathbb{X}_{27}$	$\mathbb{T}_{1,0}^{(a,E_u)}$	$M_2$	$\begin{pmatrix} \frac{\sqrt{2}i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{28}$	$\mathbb{T}_{1,1}^{(a, Eu)}$	$M_2$	$\begin{pmatrix} 0 & 0 & \frac{\sqrt{2}i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}i}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{29}$	$\mathbb{Q}_0^{(a, Ag)}$	$M_3$	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_{30}$	$\mathbb{Q}_2^{(a, Ag)}$	$M_3$	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{X}_{31}$	$\mathbb{Q}_0^{(a, Ag)}(1, 1)$	$M_3$	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ \frac{\sqrt{3}i}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{32}$	$\mathbb{Q}_2^{(a, Ag)}(1, -1)$	$M_3$	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{33}$	$\mathbb{G}_1^{(a, Ag)}(1, 0)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{34}$	$\mathbb{Q}_{2,0}^{(a,E_g,1)}$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{35}$	$\mathbb{Q}_{2,1}^{(a,E_g,1)}$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{36}$	$\mathbb{Q}_{2,0}^{(a,E_g,2)}$	$M_3$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{37}$	$\mathbb{Q}_{2,1}^{(a,E_g,2)}$	$M_3$	$\begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{38}$	$\mathbb{Q}_{2,0}^{(a,E_g,1)}(1, -1)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$

*continued ...*



Table 6

symbol	type	group	form
$\mathbb{X}_{39}$	$\mathbb{Q}_{2,1}^{(a,E_g,1)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{40}$	$\mathbb{Q}_{2,0}^{(a,E_g,2)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{41}$	$\mathbb{Q}_{2,1}^{(a,E_g,2)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{42}$	$\mathbb{G}_{1,0}^{(a,E_g)}(1,0)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{43}$	$\mathbb{G}_{1,1}^{(a,E_g)}(1,0)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{44}$	$\mathbb{M}_1^{(a, A_g)}$	$M_3$	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{45}$	$\mathbb{M}_{1,0}^{(a, E_g)}$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{46}$	$\mathbb{M}_{1,1}^{(a, E_g)}$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{47}$	$\mathbb{M}_1^{(a, A_g)}(1, 1)$	$M_3$	$\begin{pmatrix} -\frac{\sqrt{30}}{30} & 0 & 0 & 0 & 0 & \frac{\sqrt{30}}{20} \\ 0 & \frac{\sqrt{30}}{30} & 0 & 0 & \frac{\sqrt{30}}{20} & 0 \\ 0 & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & -\frac{\sqrt{30}i}{20} \\ 0 & 0 & 0 & \frac{\sqrt{30}}{30} & \frac{\sqrt{30}i}{20} & 0 \\ 0 & \frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{15} & 0 \\ \frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{15} \end{pmatrix}$
$\mathbb{X}_{48}$	$\mathbb{M}_1^{(a, A_g)}(1, -1)$	$M_3$	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{49}$	$\mathbb{M}_3^{(a, A_g, 1)}(1, -1)$	$M_3$	$\begin{pmatrix} -\frac{\sqrt{5}}{10} & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}}{10} & 0 & 0 & -\frac{\sqrt{5}}{10} & 0 \\ 0 & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & \frac{\sqrt{5}i}{10} \\ 0 & 0 & 0 & \frac{\sqrt{5}}{10} & -\frac{\sqrt{5}i}{10} & 0 \\ 0 & -\frac{\sqrt{5}}{10} & 0 & \frac{\sqrt{5}i}{10} & \frac{\sqrt{5}}{5} & 0 \\ -\frac{\sqrt{5}}{10} & 0 & -\frac{\sqrt{5}i}{10} & 0 & 0 & -\frac{\sqrt{5}}{5} \end{pmatrix}$
$\mathbb{X}_{50}$	$\mathbb{M}_3^{(a, A_g, 2)}(1, -1)$	$M_3$	$\begin{pmatrix} 0 & -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{51}$	$\mathbb{M}_3^{(a, A_g, 3)}(1, -1)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{52}$	$\mathbb{T}_2^{(a, A_g)}(1, 0)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{53}$	$\mathbb{M}_{1,0}^{(a, E_g)}(1, 1)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{30}}{15} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{20} & 0 \\ \frac{\sqrt{30}}{15} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{20} \\ 0 & -\frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 \\ \frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & 0 \\ \frac{\sqrt{30}}{20} & 0 & 0 & 0 & 0 & -\frac{\sqrt{30}}{30} \\ 0 & -\frac{\sqrt{30}}{20} & 0 & 0 & -\frac{\sqrt{30}}{30} & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{54}$	$\mathbb{M}_{1,1}^{(a,E_g)}(1,1)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{30}i}{30} & 0 & \frac{\sqrt{30}}{20} & 0 & 0 \\ -\frac{\sqrt{30}i}{30} & 0 & \frac{\sqrt{30}}{20} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{15} & \frac{\sqrt{30}}{20} & 0 \\ \frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{15} & 0 & 0 & -\frac{\sqrt{30}}{20} \\ 0 & 0 & \frac{\sqrt{30}}{20} & 0 & 0 & \frac{\sqrt{30}i}{30} \\ 0 & 0 & 0 & -\frac{\sqrt{30}}{20} & -\frac{\sqrt{30}i}{30} & 0 \end{pmatrix}$
$\mathbb{X}_{55}$	$\mathbb{M}_{1,0}^{(a,E_g)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{56}$	$\mathbb{M}_{1,1}^{(a,E_g)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}i}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{57}$	$\mathbb{M}_{3,0}^{(a,E_g,1)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & -\frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{60} & \frac{\sqrt{30}}{15} & 0 \\ -\frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{60} & 0 & 0 & -\frac{\sqrt{30}}{15} \\ 0 & \frac{\sqrt{30}i}{60} & 0 & -\frac{\sqrt{30}}{60} & 0 & 0 \\ -\frac{\sqrt{30}i}{60} & 0 & -\frac{\sqrt{30}}{60} & 0 & 0 & 0 \\ \frac{\sqrt{30}}{15} & 0 & 0 & 0 & 0 & \frac{\sqrt{30}}{15} \\ 0 & -\frac{\sqrt{30}}{15} & 0 & 0 & \frac{\sqrt{30}}{15} & 0 \end{pmatrix}$
$\mathbb{X}_{58}$	$\mathbb{M}_{3,1}^{(a,E_g,1)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{30}i}{60} & 0 & -\frac{\sqrt{30}}{60} & 0 & 0 \\ -\frac{\sqrt{30}i}{60} & 0 & -\frac{\sqrt{30}}{60} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{30}}{60} & 0 & \frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{15} & 0 \\ -\frac{\sqrt{30}}{60} & 0 & -\frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{15} \\ 0 & 0 & \frac{\sqrt{30}}{15} & 0 & 0 & -\frac{\sqrt{30}i}{15} \\ 0 & 0 & 0 & -\frac{\sqrt{30}}{15} & \frac{\sqrt{30}i}{15} & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{59}$	$\mathbb{M}_{3,0}^{(a,E_g,2)}(1,-1)$	$M_3$	$\begin{pmatrix} \frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{60}$	$\mathbb{M}_{3,1}^{(a,E_g,2)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}i}{6} & 0 \\ -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \\ 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 \\ 0 & \frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ -\frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{61}$	$\mathbb{T}_{2,0}^{(a,E_g,1)}(1,0)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 \\ -\frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{12} \\ 0 & 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{12} & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{62}$	$\mathbb{T}_{2,1}^{(a,E_g,1)}(1,0)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ -\frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \\ 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{63}$	$\mathbb{T}_{2,0}^{(a,E_g,2)}(1,0)$	$M_3$	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}i}{12} & 0 \\ -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{12} \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & \frac{\sqrt{6}}{12} & 0 \\ 0 & -\frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 \\ \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{64}$	$\mathbb{T}_{2,1}^{(a,E_g,2)}(1,0)$	$M_3$	$\begin{pmatrix} -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{12} \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & \frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}i}{12} & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & \frac{\sqrt{6}i}{12} & 0 & 0 \\ \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$

Table 7: Cluster SAMB.

symbol	type	cluster	form
$\mathbb{Y}_1$	$\mathbb{Q}_0^{(s,A_g)}$	$S_1$	$\begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{Y}_2$	$\mathbb{Q}_{2,0}^{(s,E_g,2)}$	$S_1$	$\begin{pmatrix} -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} & -\frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_3$	$\mathbb{Q}_{2,1}^{(s,E_g,2)}$	$S_1$	$\begin{pmatrix} -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{Y}_4$	$\mathbb{Q}_0^{(b,A_g)}$	$B_1$	$\begin{pmatrix} \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_5$	$\mathbb{Q}_{1,0}^{(b,E_u)}$	$B_1$	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{6} \end{pmatrix}$
$\mathbb{Y}_6$	$\mathbb{Q}_{1,1}^{(b,E_u)}$	$B_1$	$\begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$
$\mathbb{Y}_7$	$\mathbb{Q}_{2,0}^{(b,E_g,2)}$	$B_1$	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{6} \end{pmatrix}$
$\mathbb{Y}_8$	$\mathbb{Q}_{2,1}^{(b,E_g,2)}$	$B_1$	$\begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$
$\mathbb{Y}_9$	$\mathbb{Q}_3^{(b,A_u,3)}$	$B_1$	$\begin{pmatrix} \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_{10}$	$\mathbb{T}_0^{(b,A_g)}$	$B_1$	$\begin{pmatrix} \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} & -\frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} & -\frac{\sqrt{6}i}{6} \end{pmatrix}$
$\mathbb{Y}_{11}$	$\mathbb{T}_{1,0}^{(b,E_u)}$	$B_1$	$\begin{pmatrix} -\frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{3} & \frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}i}{3} & -\frac{\sqrt{3}i}{6} \end{pmatrix}$
$\mathbb{Y}_{12}$	$\mathbb{T}_{1,1}^{(b,E_u)}$	$B_1$	$\begin{pmatrix} -\frac{i}{2} & 0 & -\frac{i}{2} & \frac{i}{2} & 0 & \frac{i}{2} \end{pmatrix}$
$\mathbb{Y}_{13}$	$\mathbb{T}_{2,0}^{(b,E_g,2)}$	$B_1$	$\begin{pmatrix} -\frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{3} & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}i}{6} & \frac{\sqrt{3}i}{3} & \frac{\sqrt{3}i}{6} \end{pmatrix}$
$\mathbb{Y}_{14}$	$\mathbb{T}_{2,1}^{(b,E_g,2)}$	$B_1$	$\begin{pmatrix} -\frac{i}{2} & 0 & -\frac{i}{2} & -\frac{i}{2} & 0 & -\frac{i}{2} \end{pmatrix}$
$\mathbb{Y}_{15}$	$\mathbb{T}_3^{(b,A_u,3)}$	$B_1$	$\begin{pmatrix} \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} & -\frac{\sqrt{6}i}{6} & -\frac{\sqrt{6}i}{6} & -\frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} \end{pmatrix}$

Table 8: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_0^{(A_g)}$	0	$A_g$	—	—	1
2	$\mathbb{Q}_1^{(A_u)}$	1	$A_u$	—	—	$z$
3	$\mathbb{Q}_{1,0}^{(E_u)}$	1	$E_u$	—	0	$x$
4	$\mathbb{Q}_{1,1}^{(E_u)}$	1	$E_u$	—	1	$y$
5	$\mathbb{Q}_2^{(A_g)}$	2	$A_g$	—	—	$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$
6	$\mathbb{Q}_{2,0}^{(E_g,1)}$	2	$E_g$	1	0	$\sqrt{3}xz$
7	$\mathbb{Q}_{2,1}^{(E_g,1)}$	2	$E_g$	1	1	$\sqrt{3}yz$
8	$\mathbb{Q}_{2,0}^{(E_g,2)}$	2	$E_g$	2	0	$\frac{\sqrt{3}(x^2-y^2)}{2}$
9	$\mathbb{Q}_{2,1}^{(E_g,2)}$	2	$E_g$	2	1	$-\sqrt{3}xy$
10	$\mathbb{Q}_3^{(A_u,3)}$	3	$A_u$	3	—	$\frac{\sqrt{10}x(x^2-3y^2)}{4}$

Table 9: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_0^{(A_u)}$	0	$A_u$	—	—	1
2	$\mathbb{G}_1^{(A_g)}$	1	$A_g$	—	—	$Z$
3	$\mathbb{G}_{1,0}^{(E_g)}$	1	$E_g$	—	0	$X$
4	$\mathbb{G}_{1,1}^{(E_g)}$	1	$E_g$	—	1	$Y$
5	$\mathbb{G}_2^{(A_u)}$	2	$A_u$	—	—	$-\frac{X^2}{2} - \frac{Y^2}{2} + Z^2$
6	$\mathbb{G}_{2,0}^{(E_u,1)}$	2	$E_u$	1	0	$\sqrt{3}XZ$
7	$\mathbb{G}_{2,1}^{(E_u,1)}$	2	$E_u$	1	1	$\sqrt{3}YZ$
8	$\mathbb{G}_{2,0}^{(E_u,2)}$	2	$E_u$	2	0	$\frac{\sqrt{3}(X^2-Y^2)}{2}$
9	$\mathbb{G}_{2,1}^{(E_u,2)}$	2	$E_u$	2	1	$-\sqrt{3}XY$
10	$\mathbb{G}_3^{(A_g,1)}$	3	$A_g$	1	—	$-\frac{Z(3X^2+3Y^2-2Z^2)}{2}$
11	$\mathbb{G}_3^{(A_g,2)}$	3	$A_g$	2	—	$\frac{\sqrt{10}Y(3X^2-Y^2)}{4}$

continued ...

Table 9

No.	symbol	rank	irrep.	mul.	comp.	form
12	$\mathbb{G}_3^{(A_g,3)}$	3	$A_g$	3	—	$\sqrt{10}X(X^2-3Y^2)$
13	$\mathbb{G}_{3,0}^{(E_g,1)}$	3	$E_g$	1	0	$\sqrt{6}X(-X^2-Y^2+4Z^2)$
14	$\mathbb{G}_{3,1}^{(E_g,1)}$	3	$E_g$	1	1	$\sqrt{6}Y(-X^2-Y^2+4Z^2)$
15	$\mathbb{G}_{3,0}^{(E_g,2)}$	3	$E_g$	2	0	$\frac{\sqrt{15}Z}{2}(X^2-Y^2)$
16	$\mathbb{G}_{3,1}^{(E_g,2)}$	3	$E_g$	2	1	$-\sqrt{15}XYZ$

- 
- Group info.: Generator =  $\{3_{001}^+|0\rangle, \{-1|0\rangle\}$

Table 10: Conjugacy class (point-group part).

rep. SO	symmetry operations
$\{1 0\rangle$	$\{1 0\rangle$
$\{3_{001}^+ 0\rangle$	$\{3_{001}^+ 0\rangle$
$\{3_{001}^- 0\rangle$	$\{3_{001}^- 0\rangle$
$\{-1 0\rangle$	$\{-1 0\rangle$
$\{-3_{001}^+ 0\rangle$	$\{-3_{001}^+ 0\rangle$
$\{-3_{001}^- 0\rangle$	$\{-3_{001}^- 0\rangle$

Table 11: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	$\{1 0\rangle$	2	$\{3_{001}^+ 0\rangle$	3	$\{3_{001}^- 0\rangle$	4	$\{-1 0\rangle$	5	$\{-3_{001}^+ 0\rangle$
6	$\{-3_{001}^- 0\rangle$								



Table 12: Character table (point-group part).

	1	$3_{001}^+$	$3_{001}^-$	-1	$-3_{001}^+$	$-3_{001}^-$
$A_g$	1	1	1	1	1	1
$E_g^{(a)}$	1	$\omega^*$	$\omega$	1	$\omega^*$	$\omega$
$E_g^{(b)}$	1	$\omega$	$\omega^*$	1	$\omega$	$\omega^*$
$A_u$	1	1	1	-1	-1	-1
$E_u^{(a)}$	1	$\omega^*$	$\omega$	-1	$-\omega^*$	$-\omega$
$E_u^{(b)}$	1	$\omega$	$\omega^*$	-1	$-\omega$	$-\omega^*$

Table 13: Parity conversion.

$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$
$A_g$ ( $A_u$ )	$E_g^{(a)}$ ( $E_u^{(a)}$ )	$E_g^{(b)}$ ( $E_u^{(b)}$ )	$A_u$ ( $A_g$ )	$E_u^{(a)}$ ( $E_g^{(a)}$ )
$E_u^{(b)}$ ( $E_g^{(b)}$ )				

Table 14: Symmetric product,  $[\Gamma \otimes \Gamma']_+$ .

	$A_g$	$E_g^{(a)}$	$E_g^{(b)}$	$A_u$	$E_u^{(a)}$	$E_u^{(b)}$
$A_g$	$A_g$	$E_g^{(a)}$	$E_g^{(b)}$	$A_u$	$E_u^{(a)}$	$E_u^{(b)}$
$E_g^{(a)}$		$E_g^{(b)}$	$A_g$	$E_u^{(a)}$	$E_u^{(b)}$	$A_u$
$E_g^{(b)}$			$E_g^{(a)}$	$E_u^{(b)}$	$A_u$	$E_u^{(a)}$
$A_u$				$A_g$	$E_g^{(a)}$	$E_g^{(b)}$
$E_u^{(a)}$					$E_g^{(b)}$	$A_g$
$E_u^{(b)}$						$E_g^{(a)}$

Table 15: Anti-symmetric product,  $[\Gamma \otimes \Gamma]_-$ .

$A_g$	$E_g^{(a)}$	$E_g^{(b)}$	$A_u$	$E_u^{(a)}$	$E_u^{(b)}$
—	—	—	—	—	—

Table 16: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix} -1 & -1 & 0 \end{pmatrix}$	2	$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$	3	$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$	4	$\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}$
5	$\begin{pmatrix} -1 & 0 & 0 \end{pmatrix}$	6	$\begin{pmatrix} 0 & -1 & 0 \end{pmatrix}$				

Table 17: Virtual-cluster basis.

symbol	1	2	3	4	5	6
$\mathbb{Q}_0^{(A_g)}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$
$\mathbb{Q}_{1,0}^{(E_u)}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{6}$
$\mathbb{Q}_{1,1}^{(E_u)}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$
$\mathbb{Q}_{2,0}^{(E_g, 2)}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{6}$
$\mathbb{Q}_{2,1}^{(E_g, 2)}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$
$\mathbb{Q}_3^{(A_u, 3)}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$