Response Tensors up to 4th rank in C_3

— polar tensors —

$$C^{(0,Q)} = \left(C^{(0,Q)}\right)$$

$$C^{(0,Q)} = Q_0$$

$$C^{(1,Q)} = \begin{pmatrix} 0 & 0 & C_z^{(1,Q)} \end{pmatrix}$$

$$C_z^{(1,Q)} = Q_z$$

$$S^{(2,Q)} = \begin{pmatrix} S_{xx}^{(2,Q)} & 0 & 0 \\ 0 & S_{xx}^{(2,Q)} & 0 \\ 0 & 0 & S_{zz}^{(2,Q)} \end{pmatrix}$$

$$S_{xx}^{(2,Q)} = Q_0 - Q_u$$
$$S_{zz}^{(2,Q)} = Q_0 + 2Q_u$$

$$A^{(2,Q)} = \begin{pmatrix} 0 & A_{xy}^{(2,Q)} & 0 \\ -A_{xy}^{(2,Q)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{xy}^{(2,Q)} = G_z$$

$$S^{(3,Q)} = \begin{pmatrix} S_{1x}^{(3,Q)} & S_{1y}^{(3,Q)} & S_{1z}^{(3,Q)} \\ -S_{1x}^{(3,Q)} & -S_{1y}^{(3,Q)} & S_{2z}^{(3,Q)} \\ 0 & 0 & S_{3z}^{(3,Q)} \\ S_{4x}^{(3,Q)} & \frac{5S_{1z}^{(3,Q)}}{4} - \frac{7S_{2z}^{(3,Q)}}{4} + \frac{S_{3z}^{(3,Q)}}{2} & 0 \\ \frac{9S_{1z}^{(3,Q)}}{4} - \frac{11S_{2z}^{(3,Q)}}{4} + \frac{S_{3z}^{(3,Q)}}{2} & -S_{4x}^{(3,Q)} & 0 \\ S_{1y}^{(3,Q)} & -S_{1x}^{(3,Q)} & 0 \end{pmatrix}$$

$$S_{1x}^{(3,Q)} = Q_3^{\gamma}$$
$$S_{1y}^{(3,Q)} = Q_3^{\beta}$$
$$S_{1z}^{(3,Q)} = -3Q_3^{\alpha} + Q_z[2]$$
$$S_{2z}^{(3,Q)} = -Q_3^{\alpha} + Q_z[2]$$
$$S_{3z}^{(3,Q)} = 2Q_3^{\alpha} + 2Q_z[1] + Q_z[2]$$
$$S_{4x}^{(3,Q)} = -3G_u[1]$$

$$A^{(3,Q)} = \begin{pmatrix} A_{4x}^{(3,Q)} & A_{4y}^{(3,Q)} & 0 \\ -A_{4y}^{(3,Q)} & A_{4x}^{(3,Q)} & 0 \\ 0 & 0 & A_{6z}^{(3,Q)} \end{pmatrix}$$

$$
\begin{aligned}
A_{4x}^{(3,Q)} &= G_0 - G_u[2] \\
A_{4y}^{(3,Q)} &= Q_z[3] \\
A_{6z}^{(3,Q)} &= G_0 + 2G_u[2]
\end{aligned}
$$

$$
S^{(4,Q)} = \begin{pmatrix}
S_{11}^{(4,Q)} & S_{12}^{(4,Q)} & S_{13}^{(4,Q)} & S_{14}^{(4,Q)} & S_{15}^{(4,Q)} & 0 \\
S_{12}^{(4,Q)} & S_{11}^{(4,Q)} & S_{13}^{(4,Q)} & -S_{14}^{(4,Q)} & -S_{15}^{(4,Q)} & 0 \\
S_{13}^{(4,Q)} & S_{13}^{(4,Q)} & S_{33}^{(4,Q)} & 0 & 0 & 0 \\
S_{14}^{(4,Q)} & -S_{14}^{(4,Q)} & 0 & S_{44}^{(4,Q)} & 0 & -S_{15}^{(4,Q)} \\
S_{15}^{(4,Q)} & -S_{15}^{(4,Q)} & 0 & 0 & S_{44}^{(4,Q)} & S_{14}^{(4,Q)} \\
0 & 0 & 0 & -S_{15}^{(4,Q)} & S_{14}^{(4,Q)} & \frac{S_{11}^{(4,Q)}}{2} - \frac{S_{12}^{(4,Q)}}{2}
\end{pmatrix}
$$

$$
\begin{aligned}
S_{11}^{(4,Q)} &= Q_0[1] + 2Q_0[2] + 3Q_4 - 2Q_u[1] - 4Q_u[2] \\
S_{12}^{(4,Q)} &= Q_0[1] + Q_4 - 2Q_u[1] \\
S_{13}^{(4,Q)} &= Q_0[1] - 4Q_4 + Q_u[1] \\
S_{14}^{(4,Q)} &= Q_4^{\beta} \\
S_{15}^{(4,Q)} &= Q_4^{\alpha} \\
S_{33}^{(4,Q)} &= Q_0[1] + 2Q_0[2] + 8Q_4 + 4Q_u[1] + 8Q_u[2] \\
S_{44}^{(4,Q)} &= Q_0[2] - 4Q_4 + Q_u[2]
\end{aligned}
$$

$$
\bar{S}^{(4,Q)} = \begin{pmatrix}
0 & 0 & \bar{S}_{13}^{(4,Q)} & \bar{S}_{14}^{(4,Q)} & \bar{S}_{15}^{(4,Q)} & \bar{S}_{16}^{(4,Q)} \\
0 & 0 & \bar{S}_{13}^{(4,Q)} & -\bar{S}_{14}^{(4,Q)} & -\bar{S}_{15}^{(4,Q)} & \bar{S}_{26}^{(4,Q)} \\
-\bar{S}_{13}^{(4,Q)} & -\bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 & -\bar{S}_{16}^{(4,Q)} - \bar{S}_{26}^{(4,Q)} \\
-\bar{S}_{14}^{(4,Q)} & \bar{S}_{14}^{(4,Q)} & 0 & 0 & \frac{7\bar{S}_{16}^{(4,Q)}}{4} + \frac{9\bar{S}_{26}^{(4,Q)}}{4} & \bar{S}_{15}^{(4,Q)} \\
-\bar{S}_{15}^{(4,Q)} & \bar{S}_{15}^{(4,Q)} & 0 & -\frac{7\bar{S}_{16}^{(4,Q)}}{4} - \frac{9\bar{S}_{26}^{(4,Q)}}{4} & 0 & -\bar{S}_{14}^{(4,Q)} \\
-\bar{S}_{16}^{(4,Q)} & -\bar{S}_{26}^{(4,Q)} & \bar{S}_{16}^{(4,Q)} + \bar{S}_{26}^{(4,Q)} & -\bar{S}_{15}^{(4,Q)} & \bar{S}_{14}^{(4,Q)} & 0
\end{pmatrix}
$$

$$
\begin{aligned}
\bar{S}_{13}^{(4,Q)} &= 3Q_u[3] \\
\bar{S}_{14}^{(4,Q)} &= 2G_3^{\gamma}[1] \\
\bar{S}_{15}^{(4,Q)} &= -2G_3^{\beta}[1] \\
\bar{S}_{16}^{(4,Q)} &= -6G_3^{\alpha}[1] + 2G_z[1] \\
\bar{S}_{26}^{(4,Q)} &= 2G_3^{\alpha}[1] - 2G_z[1]
\end{aligned}
$$

$$
A^{(4,Q)} = \begin{pmatrix}
A_{xx}^{(4,Q)} & 0 & 0 \\
0 & A_{xx}^{(4,Q)} & 0 \\
0 & 0 & A_{zz}^{(4,Q)}
\end{pmatrix}
$$

$$
\begin{aligned}
A_{xx}^{(4,Q)} &= Q_0[3] - 2Q_u[6] \\
A_{zz}^{(4,Q)} &= Q_0[3] + 4Q_u[6]
\end{aligned}
$$

$$
\bar{A}^{(4,Q)} = \begin{pmatrix}
0 & \bar{A}_{xy}^{(4,Q)} & 0 \\
-\bar{A}_{xy}^{(4,Q)} & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
$$

$$\bar{A}_{xy}^{(4,Q)} = G_z[6]$$

$$M^{(4,Q)} = \begin{pmatrix} M_{1x}^{(4,Q)} & M_{1y}^{(4,Q)} & M_{1z}^{(4,Q)} \\ -M_{1x}^{(4,Q)} & -M_{1y}^{(4,Q)} & M_{2z}^{(4,Q)} \\ 0 & 0 & M_{3z}^{(4,Q)} \\ M_{4x}^{(4,Q)} & \frac{5M_{1z}^{(4,Q)}}{4} - \frac{7M_{2z}^{(4,Q)}}{4} + \frac{M_{3z}^{(4,Q)}}{2} & 0 \\ \frac{9M_{1z}^{(4,Q)}}{4} - \frac{11M_{2z}^{(4,Q)}}{4} + \frac{M_{3z}^{(4,Q)}}{2} & -M_{4x}^{(4,Q)} & 0 \\ M_{1y}^{(4,Q)} & -M_{1x}^{(4,Q)} & 0 \end{pmatrix}$$

$$\begin{aligned} M_{1x}^{(4,Q)} &= G_3^{\gamma}[2] \\ M_{1y}^{(4,Q)} &= G_3^{\beta}[2] \\ M_{1z}^{(4,Q)} &= -3G_3^{\alpha}[2] + G_z[3] \\ M_{2z}^{(4,Q)} &= -G_3^{\alpha}[2] + G_z[3] \\ M_{3z}^{(4,Q)} &= 2G_3^{\alpha}[2] + 2G_z[2] + G_z[3] \\ M_{4x}^{(4,Q)} &= -3Q_u[4] \end{aligned}$$

$$\bar{M}^{(4,Q)} = \begin{pmatrix} \bar{M}_{x1}^{(4,Q)} & -\bar{M}_{x1}^{(4,Q)} & 0 & \bar{M}_{x4}^{(4,Q)} & \bar{M}_{x5}^{(4,Q)} & \bar{M}_{x6}^{(4,Q)} \\ \bar{M}_{x6}^{(4,Q)} & -\bar{M}_{x6}^{(4,Q)} & 0 & \bar{M}_{y4}^{(4,Q)} & -\bar{M}_{x4}^{(4,Q)} & -\bar{M}_{x1}^{(4,Q)} \\ \bar{M}_{z1}^{(4,Q)} & -\bar{M}_{x5}^{(4,Q)} + \bar{M}_{y4}^{(4,Q)} + \bar{M}_{z1}^{(4,Q)} & -\frac{7\bar{M}_{x5}^{(4,Q)}}{2} + \frac{11\bar{M}_{y4}^{(4,Q)}}{2} + \bar{M}_{z1}^{(4,Q)} & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \bar{M}_{x1}^{(4,Q)} &= G_3^{\gamma}[3] \\ \bar{M}_{x4}^{(4,Q)} &= -3Q_u[5] \\ \bar{M}_{x5}^{(4,Q)} &= -3G_3^{\alpha}[3] + G_z[4] \\ \bar{M}_{x6}^{(4,Q)} &= G_3^{\beta}[3] \\ \bar{M}_{y4}^{(4,Q)} &= -G_3^{\alpha}[3] + G_z[4] \\ \bar{M}_{z1}^{(4,Q)} &= -3G_3^{\alpha}[3] + G_z[5] \end{aligned}$$

— axial tensors —

$$C^{(0,G)} = \left(C^{(0,G)}\right)$$

$$C^{(0,G)} = G_0$$

$$C^{(1,G)} = \begin{pmatrix} 0 & 0 & C_z^{(1,G)} \end{pmatrix}$$

$$C_z^{(1,G)} = G_z$$

$$S^{(2,G)} = \begin{pmatrix} S_{xx}^{(2,G)} & 0 & 0 \\ 0 & S_{xx}^{(2,G)} & 0 \\ 0 & 0 & S_{zz}^{(2,G)} \end{pmatrix}$$

$$S_{xx}^{(2,G)} = G_0 - G_u$$
$$S_{zz}^{(2,G)} = G_0 + 2G_u$$

$$A^{(2,G)} = \begin{pmatrix} 0 & A_{xy}^{(2,G)} & 0 \\ -A_{xy}^{(2,G)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{xy}^{(2,G)} = Q_z$$

$$S^{(3,G)} = \begin{pmatrix} S_{1x}^{(3,G)} & S_{1y}^{(3,G)} & S_{1z}^{(3,G)} \\ -S_{1x}^{(3,G)} & -S_{1y}^{(3,G)} & S_{2z}^{(3,G)} \\ 0 & 0 & S_{3z}^{(3,G)} \\ S_{4x}^{(3,G)} & \frac{5S_{1z}^{(3,G)}}{4} - \frac{7S_{2z}^{(3,G)}}{4} + \frac{S_{3z}^{(3,G)}}{2} & 0 \\ \frac{9S_{1z}^{(3,G)}}{4} - \frac{11S_{2z}^{(3,G)}}{4} + \frac{S_{3z}^{(3,G)}}{2} & -S_{4x}^{(3,G)} & 0 \\ S_{1y}^{(3,G)} & -S_{1x}^{(3,G)} & 0 \end{pmatrix}$$

$$S_{1x}^{(3,G)} = G_3^{\gamma}$$
$$S_{1y}^{(3,G)} = G_3^{\beta}$$
$$S_{1z}^{(3,G)} = -3G_3^{\alpha} + G_z[2]$$
$$S_{2z}^{(3,G)} = -G_3^{\alpha} + G_z[2]$$
$$S_{3z}^{(3,G)} = 2G_3^{\alpha} + 2G_z[1] + G_z[2]$$
$$S_{4x}^{(3,G)} = -3Q_u[1]$$

$$A^{(3,G)} = \begin{pmatrix} A_{4x}^{(3,G)} & A_{4y}^{(3,G)} & 0 \\ -A_{4y}^{(3,G)} & A_{4x}^{(3,G)} & 0 \\ 0 & 0 & A_{6z}^{(3,G)} \end{pmatrix}$$

$$
\begin{aligned}
A_{4x}^{(3,G)} &= Q_0 - Q_u[2] \\
A_{4y}^{(3,G)} &= G_z[3] \\
A_{6z}^{(3,G)} &= Q_0 + 2Q_u[2]
\end{aligned}
$$

$$
S^{(4,G)} = \begin{pmatrix}
S_{11}^{(4,G)} & S_{12}^{(4,G)} & S_{13}^{(4,G)} & S_{14}^{(4,G)} & S_{15}^{(4,G)} & 0 \\
S_{12}^{(4,G)} & S_{11}^{(4,G)} & S_{13}^{(4,G)} & -S_{14}^{(4,G)} & -S_{15}^{(4,G)} & 0 \\
S_{13}^{(4,G)} & S_{13}^{(4,G)} & S_{33}^{(4,G)} & 0 & 0 & 0 \\
S_{14}^{(4,G)} & -S_{14}^{(4,G)} & 0 & S_{44}^{(4,G)} & 0 & -S_{15}^{(4,G)} \\
S_{15}^{(4,G)} & -S_{15}^{(4,G)} & 0 & 0 & S_{44}^{(4,G)} & S_{14}^{(4,G)} \\
0 & 0 & 0 & -S_{15}^{(4,G)} & S_{14}^{(4,G)} & \frac{S_{11}^{(4,G)}}{2} - \frac{S_{12}^{(4,G)}}{2}
\end{pmatrix}
$$

$$
\begin{aligned}
S_{11}^{(4,G)} &= G_0[1] + 2G_0[2] + 3G_4 - 2G_u[1] - 4G_u[2] \\
S_{12}^{(4,G)} &= G_0[1] + G_4 - 2G_u[1] \\
S_{13}^{(4,G)} &= G_0[1] - 4G_4 + G_u[1] \\
S_{14}^{(4,G)} &= G_4^{\beta} \\
S_{15}^{(4,G)} &= G_4^{\alpha} \\
S_{33}^{(4,G)} &= G_0[1] + 2G_0[2] + 8G_4 + 4G_u[1] + 8G_u[2] \\
S_{44}^{(4,G)} &= G_0[2] - 4G_4 + G_u[2]
\end{aligned}
$$

$$
\bar{S}^{(4,G)} = \begin{pmatrix}
0 & 0 & \bar{S}_{13}^{(4,G)} & \bar{S}_{14}^{(4,G)} & \bar{S}_{15}^{(4,G)} & \bar{S}_{16}^{(4,G)} \\
0 & 0 & \bar{S}_{13}^{(4,G)} & -\bar{S}_{14}^{(4,G)} & -\bar{S}_{15}^{(4,G)} & \bar{S}_{26}^{(4,G)} \\
-\bar{S}_{13}^{(4,G)} & -\bar{S}_{13}^{(4,G)} & 0 & 0 & 0 & -\bar{S}_{16}^{(4,G)} - \bar{S}_{26}^{(4,G)} \\
-\bar{S}_{14}^{(4,G)} & \bar{S}_{14}^{(4,G)} & 0 & 0 & \frac{7\bar{S}_{16}^{(4,G)}}{4} + \frac{9\bar{S}_{26}^{(4,G)}}{4} & \bar{S}_{15}^{(4,G)} \\
-\bar{S}_{15}^{(4,G)} & \bar{S}_{15}^{(4,G)} & 0 & -\frac{7\bar{S}_{16}^{(4,G)}}{4} - \frac{9\bar{S}_{26}^{(4,G)}}{4} & 0 & -\bar{S}_{14}^{(4,G)} \\
-\bar{S}_{16}^{(4,G)} & -\bar{S}_{26}^{(4,G)} & \bar{S}_{16}^{(4,G)} + \bar{S}_{26}^{(4,G)} & -\bar{S}_{15}^{(4,G)} & \bar{S}_{14}^{(4,G)} & 0
\end{pmatrix}
$$

$$
\begin{aligned}
\bar{S}_{13}^{(4,G)} &= 3G_u[3] \\
\bar{S}_{14}^{(4,G)} &= 2Q_3^{\gamma}[1] \\
\bar{S}_{15}^{(4,G)} &= -2Q_3^{\beta}[1] \\
\bar{S}_{16}^{(4,G)} &= -6Q_3^{\alpha}[1] + 2Q_z[1] \\
\bar{S}_{26}^{(4,G)} &= 2Q_3^{\alpha}[1] - 2Q_z[1]
\end{aligned}
$$

$$
A^{(4,G)} = \begin{pmatrix}
A_{xx}^{(4,G)} & 0 & 0 \\
0 & A_{xx}^{(4,G)} & 0 \\
0 & 0 & A_{zz}^{(4,G)}
\end{pmatrix}
$$

$$
\begin{aligned}
A_{xx}^{(4,G)} &= G_0[3] - 2G_u[6] \\
A_{zz}^{(4,G)} &= G_0[3] + 4G_u[6]
\end{aligned}
$$

$$
\bar{A}^{(4,G)} = \begin{pmatrix}
0 & \bar{A}_{xy}^{(4,G)} & 0 \\
-\bar{A}_{xy}^{(4,G)} & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
$$

$$\bar{A}_{xy}^{(4,G)} = Q_z[6]$$

$$M^{(4,G)} = \begin{pmatrix} M_{1x}^{(4,G)} & M_{1y}^{(4,G)} & M_{1z}^{(4,G)} \\ -M_{1x}^{(4,G)} & -M_{1y}^{(4,G)} & M_{2z}^{(4,G)} \\ 0 & 0 & M_{3z}^{(4,G)} \\ M_{4x}^{(4,G)} & \frac{5M_{1z}^{(4,G)}}{4} - \frac{7M_{2z}^{(4,G)}}{4} + \frac{M_{3z}^{(4,G)}}{2} & 0 \\ \frac{9M_{1z}^{(4,G)}}{4} - \frac{11M_{2z}^{(4,G)}}{4} + \frac{M_{3z}^{(4,G)}}{2} & -M_{4x}^{(4,G)} & 0 \\ M_{1y}^{(4,G)} & -M_{1x}^{(4,G)} & 0 \end{pmatrix}$$

$$\begin{aligned} M_{1x}^{(4,G)} &= Q_3^{\gamma}[2] \\ M_{1y}^{(4,G)} &= Q_3^{\beta}[2] \\ M_{1z}^{(4,G)} &= -3Q_3^{\alpha}[2] + Q_z[3] \\ M_{2z}^{(4,G)} &= -Q_3^{\alpha}[2] + Q_z[3] \\ M_{3z}^{(4,G)} &= 2Q_3^{\alpha}[2] + 2Q_z[2] + Q_z[3] \\ M_{4x}^{(4,G)} &= -3G_u[4] \end{aligned}$$

$$\bar{M}^{(4,G)} = \begin{pmatrix} \bar{M}_{x1}^{(4,G)} & -\bar{M}_{x1}^{(4,G)} & 0 & \bar{M}_{x4}^{(4,G)} & \bar{M}_{x5}^{(4,G)} & \bar{M}_{x6}^{(4,G)} \\ \bar{M}_{x6}^{(4,G)} & -\bar{M}_{x6}^{(4,G)} & 0 & \bar{M}_{y4}^{(4,G)} & -\bar{M}_{x4}^{(4,G)} & -\bar{M}_{x1}^{(4,G)} \\ \bar{M}_{z1}^{(4,G)} & -\bar{M}_{x5}^{(4,G)} + \bar{M}_{y4}^{(4,G)} + \bar{M}_{z1}^{(4,G)} & -\frac{7\bar{M}_{x5}^{(4,G)}}{2} + \frac{11\bar{M}_{y4}^{(4,G)}}{2} + \bar{M}_{z1}^{(4,G)} & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \bar{M}_{x1}^{(4,G)} &= Q_3^{\gamma}[3] \\ \bar{M}_{x4}^{(4,G)} &= -3G_u[5] \\ \bar{M}_{x5}^{(4,G)} &= -3Q_3^{\alpha}[3] + Q_z[4] \\ \bar{M}_{x6}^{(4,G)} &= Q_3^{\beta}[3] \\ \bar{M}_{y4}^{(4,G)} &= -Q_3^{\alpha}[3] + Q_z[4] \\ \bar{M}_{z1}^{(4,G)} &= -3Q_3^{\alpha}[3] + Q_z[5] \end{aligned}$$