

# SAMB for “Th”

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- Group: No. 29  $T_h$   $m - 3$  [ cubic ]
- Generation condition
  - model type: **tight\_binding**
  - time-reversal type: **electric**
  - irrep: [Ag]
  - **spinful**

- Kets: dimension = 64

Table 1: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket	No.	ket	No.	ket
1	$(s, \uparrow)@A_1$	2	$(s, \downarrow)@A_1$	3	$(p_x, \uparrow)@A_1$	4	$(p_x, \downarrow)@A_1$	5	$(p_y, \uparrow)@A_1$
6	$(p_y, \downarrow)@A_1$	7	$(p_z, \uparrow)@A_1$	8	$(p_z, \downarrow)@A_1$				

- Sites in (primitive) unit cell:

Table 2: Site-clusters.

site	position	mapping
S <sub>1</sub> A <sub>1</sub>	$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$	[1,5,9]
A <sub>2</sub>	$\begin{pmatrix} -1 & -1 & 1 \end{pmatrix}$	[2,6,11]
A <sub>3</sub>	$\begin{pmatrix} 1 & -1 & -1 \end{pmatrix}$	[3,7,12]
A <sub>4</sub>	$\begin{pmatrix} -1 & 1 & -1 \end{pmatrix}$	[4,8,10]

*continued ...*

Table 2

site	position	mapping
A <sub>5</sub>	$\begin{pmatrix} -1 & -1 & -1 \end{pmatrix}$	[13,17,21]
A <sub>6</sub>	$\begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$	[14,18,23]
A <sub>7</sub>	$\begin{pmatrix} -1 & 1 & 1 \end{pmatrix}$	[15,19,24]
A <sub>8</sub>	$\begin{pmatrix} 1 & -1 & 1 \end{pmatrix}$	[16,20,22]

- Bonds in (primitive) unit cell:

Table 3: Bond-clusters.

bond	tail	head	$n$	#	$\mathbf{b@c}$	mapping	
B <sub>1</sub>	b <sub>1</sub>	A <sub>1</sub>	A <sub>6</sub>	1	1	$\begin{pmatrix} 0 & 0 & -2 \end{pmatrix} @ \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}$	[1,-14]
	b <sub>2</sub>	A <sub>2</sub>	A <sub>5</sub>	1	1	$\begin{pmatrix} 0 & 0 & -2 \end{pmatrix} @ \begin{pmatrix} -1 & -1 & 0 \end{pmatrix}$	[2,-13]
	b <sub>3</sub>	A <sub>3</sub>	A <sub>8</sub>	1	1	$\begin{pmatrix} 0 & 0 & 2 \end{pmatrix} @ \begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$	[3,-16]
	b <sub>4</sub>	A <sub>4</sub>	A <sub>7</sub>	1	1	$\begin{pmatrix} 0 & 0 & 2 \end{pmatrix} @ \begin{pmatrix} -1 & 1 & 0 \end{pmatrix}$	[4,-15]
	b <sub>5</sub>	A <sub>1</sub>	A <sub>7</sub>	1	1	$\begin{pmatrix} -2 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} 0 & 1 & 1 \end{pmatrix}$	[5,-19]
	b <sub>6</sub>	A <sub>2</sub>	A <sub>8</sub>	1	1	$\begin{pmatrix} 2 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} 0 & -1 & 1 \end{pmatrix}$	[6,-20]
	b <sub>7</sub>	A <sub>3</sub>	A <sub>5</sub>	1	1	$\begin{pmatrix} -2 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} 0 & -1 & -1 \end{pmatrix}$	[7,-17]
	b <sub>8</sub>	A <sub>4</sub>	A <sub>6</sub>	1	1	$\begin{pmatrix} 2 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} 0 & 1 & -1 \end{pmatrix}$	[8,-18]
	b <sub>9</sub>	A <sub>1</sub>	A <sub>8</sub>	1	1	$\begin{pmatrix} 0 & -2 & 0 \end{pmatrix} @ \begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$	[9,-22]
	b <sub>10</sub>	A <sub>4</sub>	A <sub>5</sub>	1	1	$\begin{pmatrix} 0 & -2 & 0 \end{pmatrix} @ \begin{pmatrix} -1 & 0 & -1 \end{pmatrix}$	[10,-21]
	b <sub>11</sub>	A <sub>2</sub>	A <sub>7</sub>	1	1	$\begin{pmatrix} 0 & 2 & 0 \end{pmatrix} @ \begin{pmatrix} -1 & 0 & 1 \end{pmatrix}$	[11,-24]
	b <sub>12</sub>	A <sub>3</sub>	A <sub>6</sub>	1	1	$\begin{pmatrix} 0 & 2 & 0 \end{pmatrix} @ \begin{pmatrix} 1 & 0 & -1 \end{pmatrix}$	[12,-23]

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- SAMB:

$$\boxed{\text{No. 1}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_1, \text{S}_1]$$

$$\hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\boxed{\text{No. 2}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_2, \text{S}_1]$$

$$\hat{\mathbb{Z}}_2 = \frac{\sqrt{3}\mathbb{X}_5[\mathbb{Q}_{1,0}^{(a, T_u)}] \otimes \mathbb{U}_2[\mathbb{Q}_{1,0}^{(s, T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_6[\mathbb{Q}_{1,1}^{(a, T_u)}] \otimes \mathbb{U}_3[\mathbb{Q}_{1,1}^{(s, T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_7[\mathbb{Q}_{1,2}^{(a, T_u)}] \otimes \mathbb{U}_4[\mathbb{Q}_{1,2}^{(s, T_u)}]}{3}$$

$$\boxed{\text{No. 3}} \quad \hat{\mathbb{Q}}_0^{(A_g)} (1, 0) [\text{M}_2, \text{S}_1]$$

$$\hat{\mathbb{Z}}_3 = \frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{Q}_{1,2}^{(a, T_u)}(1, 0)] \otimes \mathbb{U}_4[\mathbb{Q}_{1,2}^{(s, T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_8[\mathbb{Q}_{1,0}^{(a, T_u)}(1, 0)] \otimes \mathbb{U}_2[\mathbb{Q}_{1,0}^{(s, T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_9[\mathbb{Q}_{1,1}^{(a, T_u)}(1, 0)] \otimes \mathbb{U}_3[\mathbb{Q}_{1,1}^{(s, T_u)}]}{3}$$

$$\boxed{\text{No. 4}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_4 = \mathbb{X}_{17}[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\boxed{\text{No. 5}} \quad \hat{\mathbb{Q}}_0^{(A_g)} (1, 1) [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_5 = \mathbb{X}_{18}[\mathbb{Q}_0^{(a, A_g)}(1, 1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s, A_g)}]$$

$$\boxed{\text{No. 6}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_6 = \frac{\sqrt{3}\mathbb{X}_{21}[\mathbb{Q}_{2,0}^{(a, T_g)}] \otimes \mathbb{U}_5[\mathbb{Q}_{2,0}^{(s, T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{22}[\mathbb{Q}_{2,1}^{(a, T_g)}] \otimes \mathbb{U}_6[\mathbb{Q}_{2,1}^{(s, T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{23}[\mathbb{Q}_{2,2}^{(a, T_g)}] \otimes \mathbb{U}_7[\mathbb{Q}_{2,2}^{(s, T_g)}]}{3}$$

$$\boxed{\text{No. 7}} \quad \hat{\mathbb{Q}}_0^{(A_g)} (1, -1) [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_7 = \frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{Q}_{2,0}^{(a, T_g)}(1, -1)] \otimes \mathbb{U}_5[\mathbb{Q}_{2,0}^{(s, T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{27}[\mathbb{Q}_{2,1}^{(a, T_g)}(1, -1)] \otimes \mathbb{U}_6[\mathbb{Q}_{2,1}^{(s, T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{28}[\mathbb{Q}_{2,2}^{(a, T_g)}(1, -1)] \otimes \mathbb{U}_7[\mathbb{Q}_{2,2}^{(s, T_g)}]}{3}$$

$$\boxed{\text{No. 8}} \quad \hat{\mathbb{G}}_3^{(A_g)} (1, 0) [\text{M}_3, \text{S}_1]$$

$$\hat{\mathbb{Z}}_8 = \frac{\sqrt{3}\mathbb{X}_{29}[\mathbb{G}_{1,0}^{(a, T_g)}(1, 0)] \otimes \mathbb{U}_5[\mathbb{Q}_{2,0}^{(s, T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{30}[\mathbb{G}_{1,1}^{(a, T_g)}(1, 0)] \otimes \mathbb{U}_6[\mathbb{Q}_{2,1}^{(s, T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{G}_{1,2}^{(a, T_g)}(1, 0)] \otimes \mathbb{U}_7[\mathbb{Q}_{2,2}^{(s, T_g)}]}{3}$$

$$\boxed{\text{No. 9}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\mathbb{M}_1, \mathbb{B}_1]$$

$$\hat{\mathbb{Z}}_9 = \mathbb{X}_1[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{U}_8[\mathbb{Q}_0^{(u, A_g)}]$$

$$\boxed{\text{No. 10}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\mathbb{M}_1, \mathbb{B}_1]$$

$$\hat{\mathbb{Z}}_{10} = \frac{\sqrt{3}\mathbb{X}_2[\mathbb{M}_{1,0}^{(a, T_g)}(1, -1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{2,0}^{(u, T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_3[\mathbb{M}_{1,1}^{(a, T_g)}(1, -1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{2,1}^{(u, T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,2}^{(a, T_g)}(1, -1)] \otimes \mathbb{U}_{25}[\mathbb{T}_{2,2}^{(u, T_g)}]}{3}$$

$$\boxed{\text{No. 11}} \quad \hat{\mathbb{Q}}_4^{(A_g)}(1, -1) [\mathbb{M}_1, \mathbb{B}_1]$$

$$\hat{\mathbb{Z}}_{11} = -\frac{\sqrt{3}\mathbb{X}_2[\mathbb{M}_{1,0}^{(a, T_g)}(1, -1)] \otimes \mathbb{U}_{27}[\mathbb{T}_{4,0}^{(u, T_g, 1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_3[\mathbb{M}_{1,1}^{(a, T_g)}(1, -1)] \otimes \mathbb{U}_{28}[\mathbb{T}_{4,1}^{(u, T_g, 1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,2}^{(a, T_g)}(1, -1)] \otimes \mathbb{U}_{29}[\mathbb{T}_{4,2}^{(u, T_g, 1)}]}{3}$$

$$\boxed{\text{No. 12}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\mathbb{M}_2, \mathbb{B}_1]$$

$$\hat{\mathbb{Z}}_{12} = \frac{\sqrt{3}\mathbb{X}_5[\mathbb{Q}_{1,0}^{(a, T_u)}] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u, T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_6[\mathbb{Q}_{1,1}^{(a, T_u)}] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u, T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_7[\mathbb{Q}_{1,2}^{(a, T_u)}] \otimes \mathbb{U}_{11}[\mathbb{Q}_{1,2}^{(u, T_u)}]}{3}$$

$$\boxed{\text{No. 13}} \quad \hat{\mathbb{Q}}_4^{(A_g)} [\mathbb{M}_2, \mathbb{B}_1]$$

$$\hat{\mathbb{Z}}_{13} = \frac{\sqrt{3}\mathbb{X}_5[\mathbb{Q}_{1,0}^{(a, T_u)}] \otimes \mathbb{U}_{17}[\mathbb{Q}_{3,0}^{(u, T_u, 1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_6[\mathbb{Q}_{1,1}^{(a, T_u)}] \otimes \mathbb{U}_{18}[\mathbb{Q}_{3,1}^{(u, T_u, 1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_7[\mathbb{Q}_{1,2}^{(a, T_u)}] \otimes \mathbb{U}_{19}[\mathbb{Q}_{3,2}^{(u, T_u, 1)}]}{3}$$

$$\boxed{\text{No. 14}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 0) [\mathbb{M}_2, \mathbb{B}_1]$$

$$\hat{\mathbb{Z}}_{14} = \frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{Q}_{1,2}^{(a, T_u)}(1, 0)] \otimes \mathbb{U}_{11}[\mathbb{Q}_{1,2}^{(u, T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_8[\mathbb{Q}_{1,0}^{(a, T_u)}(1, 0)] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u, T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_9[\mathbb{Q}_{1,1}^{(a, T_u)}(1, 0)] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u, T_u)}]}{3}$$

$$\boxed{\text{No. 15}} \quad \hat{\mathbb{Q}}_4^{(A_g)}(1, 0) [\mathbb{M}_2, \mathbb{B}_1]$$

$$\hat{\mathbb{Z}}_{15} = \frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{Q}_{1,2}^{(a, T_u)}(1, 0)] \otimes \mathbb{U}_{19}[\mathbb{Q}_{3,2}^{(u, T_u, 1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_8[\mathbb{Q}_{1,0}^{(a, T_u)}(1, 0)] \otimes \mathbb{U}_{17}[\mathbb{Q}_{3,0}^{(u, T_u, 1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_9[\mathbb{Q}_{1,1}^{(a, T_u)}(1, 0)] \otimes \mathbb{U}_{18}[\mathbb{Q}_{3,1}^{(u, T_u, 1)}]}{3}$$

$$\boxed{\text{No. 16}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\mathbb{M}_2, \mathbb{B}_1]$$

$$\hat{\mathbb{Z}}_{16} = \frac{\sqrt{3}\mathbb{X}_{11}[\mathbb{M}_{2,0}^{(a, T_u)}(1, -1)] \otimes \mathbb{U}_{20}[\mathbb{T}_{1,0}^{(u, T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{12}[\mathbb{M}_{2,1}^{(a, T_u)}(1, -1)] \otimes \mathbb{U}_{21}[\mathbb{T}_{1,1}^{(u, T_u)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{13}[\mathbb{M}_{2,2}^{(a, T_u)}(1, -1)] \otimes \mathbb{U}_{22}[\mathbb{T}_{1,2}^{(u, T_u)}]}{3}$$

$$\boxed{\text{No. 17}} \quad \hat{\mathbb{G}}_3^{(Ag)}(1, -1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{17} = -\frac{\sqrt{2}\mathbb{X}_{15}[\mathbb{M}_{2,0}^{(a,Eu)}(1, -1)] \otimes \mathbb{U}_{31}[\mathbb{T}_{5,1}^{(u,Eu)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{16}[\mathbb{M}_{2,1}^{(a,Eu)}(1, -1)] \otimes \mathbb{U}_{30}[\mathbb{T}_{5,0}^{(u,Eu)}]}{2}$$

$$\boxed{\text{No. 18}} \quad \hat{\mathbb{Q}}_4^{(Ag)}(1, -1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{18} = -\frac{\sqrt{2}\mathbb{X}_{15}[\mathbb{M}_{2,0}^{(a,Eu)}(1, -1)] \otimes \mathbb{U}_{30}[\mathbb{T}_{5,0}^{(u,Eu)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{16}[\mathbb{M}_{2,1}^{(a,Eu)}(1, -1)] \otimes \mathbb{U}_{31}[\mathbb{T}_{5,1}^{(u,Eu)}]}{2}$$

$$\boxed{\text{No. 19}} \quad \hat{\mathbb{G}}_3^{(Ag)}(1, 1) [\text{M}_2, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{19} = \mathbb{X}_{14}[\mathbb{M}_0^{(a,Au)}(1, 1)] \otimes \mathbb{U}_{26}[\mathbb{T}_3^{(u,Au)}]$$

$$\boxed{\text{No. 20}} \quad \hat{\mathbb{Q}}_0^{(Ag)} [\text{M}_4, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{20} = \frac{\sqrt{3}\mathbb{X}_{50}[\mathbb{Q}_{1,0}^{(a,Tu)}] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,Tu)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{51}[\mathbb{Q}_{1,1}^{(a,Tu)}] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,Tu)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{52}[\mathbb{Q}_{1,2}^{(a,Tu)}] \otimes \mathbb{U}_{11}[\mathbb{Q}_{1,2}^{(u,Tu)}]}{3}$$

$$\boxed{\text{No. 21}} \quad \hat{\mathbb{Q}}_4^{(Ag)} [\text{M}_4, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{21} = \frac{\sqrt{3}\mathbb{X}_{50}[\mathbb{Q}_{1,0}^{(a,Tu)}] \otimes \mathbb{U}_{17}[\mathbb{Q}_{3,0}^{(u,Tu,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{51}[\mathbb{Q}_{1,1}^{(a,Tu)}] \otimes \mathbb{U}_{18}[\mathbb{Q}_{3,1}^{(u,Tu,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{52}[\mathbb{Q}_{1,2}^{(a,Tu)}] \otimes \mathbb{U}_{19}[\mathbb{Q}_{3,2}^{(u,Tu,1)}]}{3}$$

$$\boxed{\text{No. 22}} \quad \hat{\mathbb{Q}}_0^{(Ag)}(1, 0) [\text{M}_4, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{22} = \frac{\sqrt{3}\mathbb{X}_{53}[\mathbb{Q}_{1,0}^{(a,Tu)}(1, 0)] \otimes \mathbb{U}_9[\mathbb{Q}_{1,0}^{(u,Tu)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{54}[\mathbb{Q}_{1,1}^{(a,Tu)}(1, 0)] \otimes \mathbb{U}_{10}[\mathbb{Q}_{1,1}^{(u,Tu)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{55}[\mathbb{Q}_{1,2}^{(a,Tu)}(1, 0)] \otimes \mathbb{U}_{11}[\mathbb{Q}_{1,2}^{(u,Tu)}]}{3}$$

$$\boxed{\text{No. 23}} \quad \hat{\mathbb{Q}}_4^{(Ag)}(1, 0) [\text{M}_4, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{23} = \frac{\sqrt{3}\mathbb{X}_{53}[\mathbb{Q}_{1,0}^{(a,Tu)}(1, 0)] \otimes \mathbb{U}_{17}[\mathbb{Q}_{3,0}^{(u,Tu,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{54}[\mathbb{Q}_{1,1}^{(a,Tu)}(1, 0)] \otimes \mathbb{U}_{18}[\mathbb{Q}_{3,1}^{(u,Tu,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{55}[\mathbb{Q}_{1,2}^{(a,Tu)}(1, 0)] \otimes \mathbb{U}_{19}[\mathbb{Q}_{3,2}^{(u,Tu,1)}]}{3}$$

$$\boxed{\text{No. 24}} \quad \hat{\mathbb{G}}_3^{(Ag)}(1, -1) [\text{M}_4, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{24} = \frac{\sqrt{3}\mathbb{X}_{56}[\mathbb{M}_{2,0}^{(a,Tu)}(1, -1)] \otimes \mathbb{U}_{20}[\mathbb{T}_{1,0}^{(u,Tu)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{57}[\mathbb{M}_{2,1}^{(a,Tu)}(1, -1)] \otimes \mathbb{U}_{21}[\mathbb{T}_{1,1}^{(u,Tu)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{58}[\mathbb{M}_{2,2}^{(a,Tu)}(1, -1)] \otimes \mathbb{U}_{22}[\mathbb{T}_{1,2}^{(u,Tu)}]}{3}$$

$$\boxed{\text{No. 25}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_4, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{25} = -\frac{\sqrt{2}\mathbb{X}_{60}[\mathbb{M}_{2,0}^{(a,E_u)}(1, -1)] \otimes \mathbb{U}_{31}[\mathbb{T}_{5,1}^{(u,E_u)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{61}[\mathbb{M}_{2,1}^{(a,E_u)}(1, -1)] \otimes \mathbb{U}_{30}[\mathbb{T}_{5,0}^{(u,E_u)}]}{2}$$

$$\boxed{\text{No. 26}} \quad \hat{\mathbb{Q}}_4^{(A_g)}(1, -1) [\text{M}_4, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{26} = -\frac{\sqrt{2}\mathbb{X}_{60}[\mathbb{M}_{2,0}^{(a,E_u)}(1, -1)] \otimes \mathbb{U}_{30}[\mathbb{T}_{5,0}^{(u,E_u)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{61}[\mathbb{M}_{2,1}^{(a,E_u)}(1, -1)] \otimes \mathbb{U}_{31}[\mathbb{T}_{5,1}^{(u,E_u)}]}{2}$$

$$\boxed{\text{No. 27}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, 1) [\text{M}_4, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{27} = \mathbb{X}_{59}[\mathbb{M}_0^{(a,A_u)}(1, 1)] \otimes \mathbb{U}_{26}[\mathbb{T}_3^{(u,A_u)}]$$

$$\boxed{\text{No. 28}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{28} = \mathbb{X}_{17}[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_8[\mathbb{Q}_0^{(u,A_g)}]$$

$$\boxed{\text{No. 29}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{29} = \mathbb{X}_{18}[\mathbb{Q}_0^{(a,A_g)}(1, 1)] \otimes \mathbb{U}_8[\mathbb{Q}_0^{(u,A_g)}]$$

$$\boxed{\text{No. 30}} \quad \hat{\mathbb{Q}}_0^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{30} = \frac{\sqrt{5}\mathbb{X}_{19}[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,0}^{(u,E_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{20}[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{U}_{13}[\mathbb{Q}_{2,1}^{(u,E_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{21}[\mathbb{Q}_{2,0}^{(a,T_g)}] \otimes \mathbb{U}_{14}[\mathbb{Q}_{2,0}^{(u,T_g)}]}{5}$$

$$+ \frac{\sqrt{5}\mathbb{X}_{22}[\mathbb{Q}_{2,1}^{(a,T_g)}] \otimes \mathbb{U}_{15}[\mathbb{Q}_{2,1}^{(u,T_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{23}[\mathbb{Q}_{2,2}^{(a,T_g)}] \otimes \mathbb{U}_{16}[\mathbb{Q}_{2,2}^{(u,T_g)}]}{5}$$

$$\boxed{\text{No. 31}} \quad \hat{\mathbb{G}}_3^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{31} = \frac{\sqrt{2}\mathbb{X}_{19}[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{U}_{13}[\mathbb{Q}_{2,1}^{(u,E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{20}[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,0}^{(u,E_g)}]}{2}$$

$$\boxed{\text{No. 32}} \quad \hat{\mathbb{Q}}_4^{(A_g)} [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{32} = \frac{\sqrt{30}\mathbb{X}_{19}[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,0}^{(u,E_g)}]}{10} + \frac{\sqrt{30}\mathbb{X}_{20}[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{U}_{13}[\mathbb{Q}_{2,1}^{(u,E_g)}]}{10} - \frac{\sqrt{30}\mathbb{X}_{21}[\mathbb{Q}_{2,0}^{(a,T_g)}] \otimes \mathbb{U}_{14}[\mathbb{Q}_{2,0}^{(u,T_g)}]}{15}$$

$$- \frac{\sqrt{30}\mathbb{X}_{22}[\mathbb{Q}_{2,1}^{(a,T_g)}] \otimes \mathbb{U}_{15}[\mathbb{Q}_{2,1}^{(u,T_g)}]}{15} - \frac{\sqrt{30}\mathbb{X}_{23}[\mathbb{Q}_{2,2}^{(a,T_g)}] \otimes \mathbb{U}_{16}[\mathbb{Q}_{2,2}^{(u,T_g)}]}{15}$$

$$\begin{aligned}
\text{No. 33} \quad & \hat{\mathbb{Q}}_0^{(A_g)}(1, -1) [\mathbb{M}_3, \mathbb{B}_1] \\
\hat{\mathbb{Z}}_{33} = & \frac{\sqrt{5}\mathbb{X}_{24}[\mathbb{Q}_{2,0}^{(a,E_g)}(1, -1)] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,0}^{(u,E_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{25}[\mathbb{Q}_{2,1}^{(a,E_g)}(1, -1)] \otimes \mathbb{U}_{13}[\mathbb{Q}_{2,1}^{(u,E_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{26}[\mathbb{Q}_{2,0}^{(a,T_g)}(1, -1)] \otimes \mathbb{U}_{14}[\mathbb{Q}_{2,0}^{(u,T_g)}]}{5} \\
& + \frac{\sqrt{5}\mathbb{X}_{27}[\mathbb{Q}_{2,1}^{(a,T_g)}(1, -1)] \otimes \mathbb{U}_{15}[\mathbb{Q}_{2,1}^{(u,T_g)}]}{5} + \frac{\sqrt{5}\mathbb{X}_{28}[\mathbb{Q}_{2,2}^{(a,T_g)}(1, -1)] \otimes \mathbb{U}_{16}[\mathbb{Q}_{2,2}^{(u,T_g)}]}{5} \\
\text{No. 34} \quad & \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\mathbb{M}_3, \mathbb{B}_1] \\
\hat{\mathbb{Z}}_{34} = & \frac{\sqrt{2}\mathbb{X}_{24}[\mathbb{Q}_{2,0}^{(a,E_g)}(1, -1)] \otimes \mathbb{U}_{13}[\mathbb{Q}_{2,1}^{(u,E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{25}[\mathbb{Q}_{2,1}^{(a,E_g)}(1, -1)] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,0}^{(u,E_g)}]}{2} \\
\text{No. 35} \quad & \hat{\mathbb{Q}}_4^{(A_g)}(1, -1) [\mathbb{M}_3, \mathbb{B}_1] \\
\hat{\mathbb{Z}}_{35} = & \frac{\sqrt{30}\mathbb{X}_{24}[\mathbb{Q}_{2,0}^{(a,E_g)}(1, -1)] \otimes \mathbb{U}_{12}[\mathbb{Q}_{2,0}^{(u,E_g)}]}{10} + \frac{\sqrt{30}\mathbb{X}_{25}[\mathbb{Q}_{2,1}^{(a,E_g)}(1, -1)] \otimes \mathbb{U}_{13}[\mathbb{Q}_{2,1}^{(u,E_g)}]}{10} - \frac{\sqrt{30}\mathbb{X}_{26}[\mathbb{Q}_{2,0}^{(a,T_g)}(1, -1)] \otimes \mathbb{U}_{14}[\mathbb{Q}_{2,0}^{(u,T_g)}]}{15} \\
& - \frac{\sqrt{30}\mathbb{X}_{27}[\mathbb{Q}_{2,1}^{(a,T_g)}(1, -1)] \otimes \mathbb{U}_{15}[\mathbb{Q}_{2,1}^{(u,T_g)}]}{15} - \frac{\sqrt{30}\mathbb{X}_{28}[\mathbb{Q}_{2,2}^{(a,T_g)}(1, -1)] \otimes \mathbb{U}_{16}[\mathbb{Q}_{2,2}^{(u,T_g)}]}{15} \\
\text{No. 36} \quad & \hat{\mathbb{G}}_3^{(A_g)}(1, 0) [\mathbb{M}_3, \mathbb{B}_1] \\
\hat{\mathbb{Z}}_{36} = & \frac{\sqrt{3}\mathbb{X}_{29}[\mathbb{G}_{1,0}^{(a,T_g)}(1, 0)] \otimes \mathbb{U}_{14}[\mathbb{Q}_{2,0}^{(u,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{30}[\mathbb{G}_{1,1}^{(a,T_g)}(1, 0)] \otimes \mathbb{U}_{15}[\mathbb{Q}_{2,1}^{(u,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{G}_{1,2}^{(a,T_g)}(1, 0)] \otimes \mathbb{U}_{16}[\mathbb{Q}_{2,2}^{(u,T_g)}]}{3} \\
\text{No. 37} \quad & \hat{\mathbb{G}}_3^{(A_g)} [\mathbb{M}_3, \mathbb{B}_1] \\
\hat{\mathbb{Z}}_{37} = & \frac{\sqrt{3}\mathbb{X}_{32}[\mathbb{M}_{1,0}^{(a,T_g)}] \otimes \mathbb{U}_{23}[\mathbb{T}_{2,0}^{(u,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{33}[\mathbb{M}_{1,1}^{(a,T_g)}] \otimes \mathbb{U}_{24}[\mathbb{T}_{2,1}^{(u,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{34}[\mathbb{M}_{1,2}^{(a,T_g)}] \otimes \mathbb{U}_{25}[\mathbb{T}_{2,2}^{(u,T_g)}]}{3} \\
\text{No. 38} \quad & \hat{\mathbb{Q}}_4^{(A_g)} [\mathbb{M}_3, \mathbb{B}_1] \\
\hat{\mathbb{Z}}_{38} = & -\frac{\sqrt{3}\mathbb{X}_{32}[\mathbb{M}_{1,0}^{(a,T_g)}] \otimes \mathbb{U}_{27}[\mathbb{T}_{4,0}^{(u,T_g,1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{33}[\mathbb{M}_{1,1}^{(a,T_g)}] \otimes \mathbb{U}_{28}[\mathbb{T}_{4,1}^{(u,T_g,1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{34}[\mathbb{M}_{1,2}^{(a,T_g)}] \otimes \mathbb{U}_{29}[\mathbb{T}_{4,2}^{(u,T_g,1)}]}{3} \\
\text{No. 39} \quad & \hat{\mathbb{G}}_3^{(A_g)}(1, 1) [\mathbb{M}_3, \mathbb{B}_1] \\
\hat{\mathbb{Z}}_{39} = & \frac{\sqrt{3}\mathbb{X}_{35}[\mathbb{M}_{1,0}^{(a,T_g)}(1, 1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{2,0}^{(u,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{36}[\mathbb{M}_{1,1}^{(a,T_g)}(1, 1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{2,1}^{(u,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{37}[\mathbb{M}_{1,2}^{(a,T_g)}(1, 1)] \otimes \mathbb{U}_{25}[\mathbb{T}_{2,2}^{(u,T_g)}]}{3}
\end{aligned}$$

$$\boxed{\text{No. 40}} \quad \hat{\mathbb{Q}}_4^{(A_g)}(1, 1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{40} = -\frac{\sqrt{3}\mathbb{X}_{35}[\text{M}_{1,0}^{(a,T_g)}(1, 1)] \otimes \mathbb{U}_{27}[\mathbb{T}_{4,0}^{(u,T_g,1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{36}[\text{M}_{1,1}^{(a,T_g)}(1, 1)] \otimes \mathbb{U}_{28}[\mathbb{T}_{4,1}^{(u,T_g,1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{37}[\text{M}_{1,2}^{(a,T_g)}(1, 1)] \otimes \mathbb{U}_{29}[\mathbb{T}_{4,2}^{(u,T_g,1)}]}{3}$$

$$\boxed{\text{No. 41}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{41} = \frac{\sqrt{3}\mathbb{X}_{38}[\text{M}_{1,0}^{(a,T_g)}(1, -1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{2,0}^{(u,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{39}[\text{M}_{1,1}^{(a,T_g)}(1, -1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{2,1}^{(u,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{40}[\text{M}_{1,2}^{(a,T_g)}(1, -1)] \otimes \mathbb{U}_{25}[\mathbb{T}_{2,2}^{(u,T_g)}]}{3}$$

$$\boxed{\text{No. 42}} \quad \hat{\mathbb{Q}}_4^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{42} = -\frac{\sqrt{3}\mathbb{X}_{38}[\text{M}_{1,0}^{(a,T_g)}(1, -1)] \otimes \mathbb{U}_{27}[\mathbb{T}_{4,0}^{(u,T_g,1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{39}[\text{M}_{1,1}^{(a,T_g)}(1, -1)] \otimes \mathbb{U}_{28}[\mathbb{T}_{4,1}^{(u,T_g,1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{40}[\text{M}_{1,2}^{(a,T_g)}(1, -1)] \otimes \mathbb{U}_{29}[\mathbb{T}_{4,2}^{(u,T_g,1)}]}{3}$$

$$\boxed{\text{No. 43}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{43} = -\frac{\sqrt{3}\mathbb{X}_{41}[\text{M}_{3,0}^{(a,T_g,1)}(1, -1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{2,0}^{(u,T_g)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{42}[\text{M}_{3,1}^{(a,T_g,1)}(1, -1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{2,1}^{(u,T_g)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{43}[\text{M}_{3,2}^{(a,T_g,1)}(1, -1)] \otimes \mathbb{U}_{25}[\mathbb{T}_{2,2}^{(u,T_g)}]}{3}$$

$$\boxed{\text{No. 44}} \quad \hat{\mathbb{Q}}_4^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{44} = -\frac{\sqrt{3}\mathbb{X}_{44}[\text{M}_{3,0}^{(a,T_g,2)}(1, -1)] \otimes \mathbb{U}_{23}[\mathbb{T}_{2,0}^{(u,T_g)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{45}[\text{M}_{3,1}^{(a,T_g,2)}(1, -1)] \otimes \mathbb{U}_{24}[\mathbb{T}_{2,1}^{(u,T_g)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{46}[\text{M}_{3,2}^{(a,T_g,2)}(1, -1)] \otimes \mathbb{U}_{25}[\mathbb{T}_{2,2}^{(u,T_g)}]}{3}$$

$$\boxed{\text{No. 45}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{45} = \frac{\sqrt{3}\mathbb{X}_{44}[\text{M}_{3,0}^{(a,T_g,2)}(1, -1)] \otimes \mathbb{U}_{27}[\mathbb{T}_{4,0}^{(u,T_g,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{45}[\text{M}_{3,1}^{(a,T_g,2)}(1, -1)] \otimes \mathbb{U}_{28}[\mathbb{T}_{4,1}^{(u,T_g,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{46}[\text{M}_{3,2}^{(a,T_g,2)}(1, -1)] \otimes \mathbb{U}_{29}[\mathbb{T}_{4,2}^{(u,T_g,1)}]}{3}$$

$$\boxed{\text{No. 46}} \quad \hat{\mathbb{Q}}_4^{(A_g)}(1, -1) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{46} = \frac{\sqrt{3}\mathbb{X}_{41}[\text{M}_{3,0}^{(a,T_g,1)}(1, -1)] \otimes \mathbb{U}_{27}[\mathbb{T}_{4,0}^{(u,T_g,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{42}[\text{M}_{3,1}^{(a,T_g,1)}(1, -1)] \otimes \mathbb{U}_{28}[\mathbb{T}_{4,1}^{(u,T_g,1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\text{M}_{3,2}^{(a,T_g,1)}(1, -1)] \otimes \mathbb{U}_{29}[\mathbb{T}_{4,2}^{(u,T_g,1)}]}{3}$$

$$\boxed{\text{No. 47}} \quad \hat{\mathbb{Q}}_0^{(A_g)}(1, 0) [\text{M}_3, \text{B}_1]$$

$$\hat{\mathbb{Z}}_{47} = \frac{\sqrt{3}\mathbb{X}_{47}[\mathbb{T}_{2,0}^{(a,T_g)}(1, 0)] \otimes \mathbb{U}_{23}[\mathbb{T}_{2,0}^{(u,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{48}[\mathbb{T}_{2,1}^{(a,T_g)}(1, 0)] \otimes \mathbb{U}_{24}[\mathbb{T}_{2,1}^{(u,T_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{49}[\mathbb{T}_{2,2}^{(a,T_g)}(1, 0)] \otimes \mathbb{U}_{25}[\mathbb{T}_{2,2}^{(u,T_g)}]}{3}$$



$$\boxed{\text{No. 48}} \quad \hat{\mathbb{G}}_3^{(A_g)}(1, 0) [M_3, B_1]$$

$$\hat{Z}_{48} = -\frac{\sqrt{3}\mathbb{X}_{47}[\mathbb{T}_{2,0}^{(a,T_g)}(1, 0)] \otimes \mathbb{U}_{27}[\mathbb{T}_{4,0}^{(u,T_g,1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{48}[\mathbb{T}_{2,1}^{(a,T_g)}(1, 0)] \otimes \mathbb{U}_{28}[\mathbb{T}_{4,1}^{(u,T_g,1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{49}[\mathbb{T}_{2,2}^{(a,T_g)}(1, 0)] \otimes \mathbb{U}_{29}[\mathbb{T}_{4,2}^{(u,T_g,1)}]}{3}$$

Table 4: Atomic SAMB group.

group	bra	ket
M <sub>1</sub>	(s, ↑), (s, ↓)	(s, ↑), (s, ↓)
M <sub>2</sub>	(s, ↑), (s, ↓)	(p <sub>x</sub> , ↑), (p <sub>x</sub> , ↓), (p <sub>y</sub> , ↑), (p <sub>y</sub> , ↓), (p <sub>z</sub> , ↑), (p <sub>z</sub> , ↓)
M <sub>3</sub>	(p <sub>x</sub> , ↑), (p <sub>x</sub> , ↓), (p <sub>y</sub> , ↑), (p <sub>y</sub> , ↓), (p <sub>z</sub> , ↑), (p <sub>z</sub> , ↓)	(p <sub>x</sub> , ↑), (p <sub>x</sub> , ↓), (p <sub>y</sub> , ↑), (p <sub>y</sub> , ↓), (p <sub>z</sub> , ↑), (p <sub>z</sub> , ↓)
M <sub>4</sub>	(p <sub>x</sub> , ↑), (p <sub>x</sub> , ↓), (p <sub>y</sub> , ↑), (p <sub>y</sub> , ↓), (p <sub>z</sub> , ↑), (p <sub>z</sub> , ↓)	(s, ↑), (s, ↓)

Table 5: Atomic SAMB.

symbol	type	group	form
X <sub>1</sub>	$\mathbb{Q}_0^{(a,A_g)}$	M <sub>1</sub>	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
X <sub>2</sub>	$\mathbb{M}_{1,0}^{(a,T_g)}(1, -1)$	M <sub>1</sub>	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$
X <sub>3</sub>	$\mathbb{M}_{1,1}^{(a,T_g)}(1, -1)$	M <sub>1</sub>	$\begin{pmatrix} 0 & -\frac{\sqrt{2}i}{2} \\ \frac{\sqrt{2}i}{2} & 0 \end{pmatrix}$
X <sub>4</sub>	$\mathbb{M}_{1,2}^{(a,T_g)}(1, -1)$	M <sub>1</sub>	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} \end{pmatrix}$
X <sub>5</sub>	$\mathbb{Q}_{1,0}^{(a,T_u)}$	M <sub>2</sub>	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
X <sub>6</sub>	$\mathbb{Q}_{1,1}^{(a,T_u)}$	M <sub>2</sub>	$\begin{pmatrix} 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 \end{pmatrix}$
X <sub>7</sub>	$\mathbb{Q}_{1,2}^{(a,T_u)}$	M <sub>2</sub>	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
X <sub>8</sub>	$\mathbb{Q}_{1,0}^{(a,T_u)}(1, 0)$	M <sub>2</sub>	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{i}{2} & -\frac{1}{2} & 0 \end{pmatrix}$

*continued ...*

Table 5

symbol	type	group	form
$\mathbb{X}_9$	$\mathbb{Q}_{1,1}^{(a,T_u)}(1,0)$	$M_2$	$\begin{pmatrix} \frac{i}{2} & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & -\frac{i}{2} & 0 & 0 & -\frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{10}$	$\mathbb{Q}_{1,2}^{(a,T_u)}(1,0)$	$M_2$	$\begin{pmatrix} 0 & -\frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{11}$	$\mathbb{M}_{2,0}^{(a,T_u)}(1,-1)$	$M_2$	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{12}$	$\mathbb{M}_{2,1}^{(a,T_u)}(1,-1)$	$M_2$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{13}$	$\mathbb{M}_{2,2}^{(a,T_u)}(1,-1)$	$M_2$	$\begin{pmatrix} 0 & -\frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{14}$	$\mathbb{M}_0^{(a,A_u)}(1,1)$	$M_2$	$\begin{pmatrix} 0 & \frac{\sqrt{6}}{6} & 0 & -\frac{\sqrt{6}i}{6} & \frac{\sqrt{6}}{6} & 0 \\ \frac{\sqrt{6}}{6} & 0 & \frac{\sqrt{6}i}{6} & 0 & 0 & -\frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_{15}$	$\mathbb{M}_{2,0}^{(a,E_u)}(1,-1)$	$M_2$	$\begin{pmatrix} 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{3}}{6} & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{X}_{16}$	$\mathbb{M}_{2,1}^{(a,E_u)}(1,-1)$	$M_2$	$\begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 \\ \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{17}$	$\mathbb{Q}_0^{(a,A_g)}$	$M_3$	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_{18}$	$\mathbb{Q}_0^{(a,A_g)}(1,1)$	$M_3$	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ \frac{\sqrt{3}i}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 5

symbol	type	group	form
$\mathbb{X}_{19}$	$\mathbb{Q}_{2,0}^{(a,E_g)}$	$M_3$	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{X}_{20}$	$\mathbb{Q}_{2,1}^{(a,E_g)}$	$M_3$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{21}$	$\mathbb{Q}_{2,0}^{(a,T_g)}$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{22}$	$\mathbb{Q}_{2,1}^{(a,T_g)}$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{23}$	$\mathbb{Q}_{2,2}^{(a,T_g)}$	$M_3$	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 5

symbol	type	group	form
$\mathbb{X}_{24}$	$\mathbb{Q}_{2,0}^{(a,E_g)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{25}$	$\mathbb{Q}_{2,1}^{(a,E_g)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{26}$	$\mathbb{Q}_{2,0}^{(a,T_g)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{27}$	$\mathbb{Q}_{2,1}^{(a,T_g)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{28}$	$\mathbb{Q}_{2,2}^{(a,T_g)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 5

symbol	type	group	form
$\mathbb{X}_{29}$	$\mathbb{G}_{1,0}^{(a,T_g)}(1,0)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{30}$	$\mathbb{G}_{1,1}^{(a,T_g)}(1,0)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{31}$	$\mathbb{G}_{1,2}^{(a,T_g)}(1,0)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{32}$	$\mathbb{M}_{1,0}^{(a,T_g)}$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{33}$	$\mathbb{M}_{1,1}^{(a,T_g)}$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 5

symbol	type	group	form
$\mathbb{X}_{34}$	$\mathbb{M}_{1,2}^{(a,T_g)}$	$M_3$	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{35}$	$\mathbb{M}_{1,0}^{(a,T_g)}(1,1)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{30}}{15} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{20} & 0 \\ \frac{\sqrt{30}}{15} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{20} \\ 0 & -\frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 \\ \frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & 0 \\ \frac{\sqrt{30}}{20} & 0 & 0 & 0 & 0 & -\frac{\sqrt{30}}{30} \\ 0 & -\frac{\sqrt{30}}{20} & 0 & 0 & -\frac{\sqrt{30}}{30} & 0 \end{pmatrix}$
$\mathbb{X}_{36}$	$\mathbb{M}_{1,1}^{(a,T_g)}(1,1)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{30}i}{30} & 0 & \frac{\sqrt{30}}{20} & 0 & 0 \\ -\frac{\sqrt{30}i}{30} & 0 & \frac{\sqrt{30}}{20} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{15} & \frac{\sqrt{30}}{20} & 0 \\ \frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{15} & 0 & 0 & -\frac{\sqrt{30}}{20} \\ 0 & 0 & \frac{\sqrt{30}}{20} & 0 & 0 & \frac{\sqrt{30}i}{30} \\ 0 & 0 & 0 & -\frac{\sqrt{30}}{20} & -\frac{\sqrt{30}i}{30} & 0 \end{pmatrix}$
$\mathbb{X}_{37}$	$\mathbb{M}_{1,2}^{(a,T_g)}(1,1)$	$M_3$	$\begin{pmatrix} -\frac{\sqrt{30}}{30} & 0 & 0 & 0 & 0 & \frac{\sqrt{30}}{20} \\ 0 & \frac{\sqrt{30}}{30} & 0 & 0 & \frac{\sqrt{30}}{20} & 0 \\ 0 & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & -\frac{\sqrt{30}i}{20} \\ 0 & 0 & 0 & \frac{\sqrt{30}}{30} & \frac{\sqrt{30}i}{20} & 0 \\ 0 & \frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{15} & 0 \\ \frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{15} \end{pmatrix}$
$\mathbb{X}_{38}$	$\mathbb{M}_{1,0}^{(a,T_g)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \end{pmatrix}$

continued ...

Table 5

symbol	type	group	form
$\mathbb{X}_{39}$	$\mathbb{M}_{1,1}^{(a,T_g)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}i}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{40}$	$\mathbb{M}_{1,2}^{(a,T_g)}(1,-1)$	$M_3$	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_{41}$	$\mathbb{M}_{3,0}^{(a,T_g,1)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{5}}{5} & 0 & \frac{\sqrt{5}i}{10} & -\frac{\sqrt{5}}{10} & 0 \\ \frac{\sqrt{5}}{5} & 0 & -\frac{\sqrt{5}i}{10} & 0 & 0 & \frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 \\ -\frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & 0 \\ -\frac{\sqrt{5}}{10} & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}}{10} & 0 & 0 & -\frac{\sqrt{5}}{10} & 0 \end{pmatrix}$
$\mathbb{X}_{42}$	$\mathbb{M}_{3,1}^{(a,T_g,1)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 \\ -\frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{5}}{10} & 0 & -\frac{\sqrt{5}i}{5} & -\frac{\sqrt{5}}{10} & 0 \\ -\frac{\sqrt{5}}{10} & 0 & \frac{\sqrt{5}i}{5} & 0 & 0 & \frac{\sqrt{5}}{10} \\ 0 & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & \frac{\sqrt{5}i}{10} \\ 0 & 0 & 0 & \frac{\sqrt{5}}{10} & -\frac{\sqrt{5}i}{10} & 0 \end{pmatrix}$
$\mathbb{X}_{43}$	$\mathbb{M}_{3,2}^{(a,T_g,1)}(1,-1)$	$M_3$	$\begin{pmatrix} -\frac{\sqrt{5}}{10} & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}}{10} & 0 & 0 & -\frac{\sqrt{5}}{10} & 0 \\ 0 & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & \frac{\sqrt{5}i}{10} \\ 0 & 0 & 0 & \frac{\sqrt{5}}{10} & -\frac{\sqrt{5}i}{10} & 0 \\ 0 & -\frac{\sqrt{5}}{10} & 0 & \frac{\sqrt{5}i}{10} & \frac{\sqrt{5}}{5} & 0 \\ -\frac{\sqrt{5}}{10} & 0 & -\frac{\sqrt{5}i}{10} & 0 & 0 & -\frac{\sqrt{5}}{5} \end{pmatrix}$

continued ...

Table 5

symbol	type	group	form
$\mathbb{X}_{44}$	$\mathbb{M}_{3,0}^{(a,T_g,2)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ 0 & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 \\ \frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \\ 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{45}$	$\mathbb{M}_{3,1}^{(a,T_g,2)}(1,-1)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ -\frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \\ -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \\ 0 & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}i}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{46}$	$\mathbb{M}_{3,2}^{(a,T_g,2)}(1,-1)$	$M_3$	$\begin{pmatrix} \frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{47}$	$\mathbb{T}_{2,0}^{(a,T_g)}(1,0)$	$M_3$	$\begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ -\frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \\ 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{48}$	$\mathbb{T}_{2,1}^{(a,T_g)}(1,0)$	$M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 \\ -\frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{12} \\ 0 & 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{12} & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$

continued ...



Table 5

symbol	type	group	form
$\mathbb{X}_{49}$	$\mathbb{T}_{2,2}^{(a,T_g)}(1,0)$	$M_3$	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}i}{12} & 0 \\ 0 & -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & \frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{50}$	$\mathbb{Q}_{1,0}^{(a,T_u)}$	$M_4$	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$
$\mathbb{X}_{51}$	$\mathbb{Q}_{1,1}^{(a,T_u)}$	$M_4$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$
$\mathbb{X}_{52}$	$\mathbb{Q}_{1,2}^{(a,T_u)}$	$M_4$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{X}_{53}$	$\mathbb{Q}_{1,0}^{(a,T_u)}(1,0)$	$M_4$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{i}{2} & 0 \\ 0 & -\frac{i}{2} \\ 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$

continued ...

Table 5

symbol	type	group	form
$\mathbb{X}_{54}$	$\mathbb{Q}_{1,1}^{(a,T_u)}(1,0)$	$M_4$	$\begin{pmatrix} -\frac{i}{2} & 0 \\ 0 & \frac{i}{2} \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{i}{2} \\ \frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{55}$	$\mathbb{Q}_{1,2}^{(a,T_u)}(1,0)$	$M_4$	$\begin{pmatrix} 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 \\ 0 & -\frac{i}{2} \\ -\frac{i}{2} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$
$\mathbb{X}_{56}$	$\mathbb{M}_{2,0}^{(a,T_u)}(1,-1)$	$M_4$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \\ 0 & -\frac{i}{2} \\ \frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{57}$	$\mathbb{M}_{2,1}^{(a,T_u)}(1,-1)$	$M_4$	$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{58}$	$\mathbb{M}_{2,2}^{(a,T_u)}(1,-1)$	$M_4$	$\begin{pmatrix} 0 & -\frac{i}{2} \\ \frac{i}{2} & 0 \\ 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

*continued ...*

Table 5

symbol	type	group	form
$\mathbb{X}_{59}$	$\mathbb{M}_0^{(a, A_u)}(1, 1)$	$M_4$	$\begin{pmatrix} 0 & \frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{6} & 0 \\ 0 & -\frac{\sqrt{6}i}{6} \\ \frac{\sqrt{6}i}{6} & 0 \\ \frac{\sqrt{6}}{6} & 0 \\ 0 & -\frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_{60}$	$\mathbb{M}_{2,0}^{(a, E_u)}(1, -1)$	$M_4$	$\begin{pmatrix} 0 & -\frac{\sqrt{3}}{6} \\ -\frac{\sqrt{3}}{6} & 0 \\ 0 & \frac{\sqrt{3}i}{6} \\ -\frac{\sqrt{3}i}{6} & 0 \\ \frac{\sqrt{3}}{3} & 0 \\ 0 & -\frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{X}_{61}$	$\mathbb{M}_{2,1}^{(a, E_u)}(1, -1)$	$M_4$	$\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \\ 0 & \frac{i}{2} \\ -\frac{i}{2} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

Table 6: Uniform SAMB.

symbol	type	cluster	form
$\mathbb{U}_1$	$\mathbb{Q}_0^{(s, A_g)}$	$S_1$	$\begin{pmatrix} \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \end{pmatrix}$
$\mathbb{U}_2$	$\mathbb{Q}_{1,0}^{(s, T_u)}$	$S_1$	$\begin{pmatrix} \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \end{pmatrix}$
$\mathbb{U}_3$	$\mathbb{Q}_{1,1}^{(s, T_u)}$	$S_1$	$\begin{pmatrix} \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \end{pmatrix}$
$\mathbb{U}_4$	$\mathbb{Q}_{1,2}^{(s, T_u)}$	$S_1$	$\begin{pmatrix} \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \end{pmatrix}$

continued ...

[illegible]

Table 6

symbol	type	cluster	form
$\mathbb{U}_9$	$\mathbb{Q}_{1,0}^{(u,T_u)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{3\sqrt{26}}{52} & 0 & \frac{\sqrt{26}}{26} \\ 0 & 0 & 0 & 0 & -\frac{3\sqrt{26}}{52} & 0 & -\frac{\sqrt{26}}{26} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{26}}{26} & 0 & \frac{3\sqrt{26}}{52} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{26}}{26} & 0 & -\frac{3\sqrt{26}}{52} & 0 \\ 0 & -\frac{3\sqrt{26}}{52} & 0 & -\frac{\sqrt{26}}{26} & 0 & 0 & 0 & 0 \\ \frac{3\sqrt{26}}{52} & 0 & \frac{\sqrt{26}}{26} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{26}}{26} & 0 & -\frac{3\sqrt{26}}{52} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{26}}{26} & 0 & \frac{3\sqrt{26}}{52} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{10}$	$\mathbb{Q}_{1,1}^{(u,T_u)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{26}}{26} & \frac{3\sqrt{26}}{52} & 0 \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{26}}{26} & 0 & 0 & -\frac{3\sqrt{26}}{52} \\ 0 & 0 & 0 & 0 & -\frac{3\sqrt{26}}{52} & 0 & 0 & -\frac{\sqrt{26}}{26} \\ 0 & 0 & 0 & 0 & 0 & \frac{3\sqrt{26}}{52} & \frac{\sqrt{26}}{26} & 0 \\ 0 & -\frac{\sqrt{26}}{26} & -\frac{3\sqrt{26}}{52} & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{26}}{26} & 0 & 0 & \frac{3\sqrt{26}}{52} & 0 & 0 & 0 & 0 \\ \frac{3\sqrt{26}}{52} & 0 & 0 & \frac{\sqrt{26}}{26} & 0 & 0 & 0 & 0 \\ 0 & -\frac{3\sqrt{26}}{52} & -\frac{\sqrt{26}}{26} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{11}$	$\mathbb{Q}_{1,2}^{(u,T_u)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{26}}{26} & \frac{3\sqrt{26}}{52} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{3\sqrt{26}}{52} & \frac{\sqrt{26}}{26} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{26}}{26} & -\frac{3\sqrt{26}}{52} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{3\sqrt{26}}{52} & -\frac{\sqrt{26}}{26} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{26}}{26} & -\frac{3\sqrt{26}}{52} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{3\sqrt{26}}{52} & -\frac{\sqrt{26}}{26} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{26}}{26} & \frac{3\sqrt{26}}{52} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3\sqrt{26}}{52} & \frac{\sqrt{26}}{26} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{12}$	$\mathbb{Q}_{2,0}^{(u,E_g)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{11\sqrt{3}}{84} & -\frac{\sqrt{3}}{42} & \frac{13\sqrt{3}}{84} \\ 0 & 0 & 0 & 0 & -\frac{11\sqrt{3}}{84} & 0 & \frac{13\sqrt{3}}{84} & -\frac{\sqrt{3}}{42} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{42} & \frac{13\sqrt{3}}{84} & 0 & -\frac{11\sqrt{3}}{84} \\ 0 & 0 & 0 & 0 & \frac{13\sqrt{3}}{84} & -\frac{\sqrt{3}}{42} & -\frac{11\sqrt{3}}{84} & 0 \\ 0 & -\frac{11\sqrt{3}}{84} & -\frac{\sqrt{3}}{42} & \frac{13\sqrt{3}}{84} & 0 & 0 & 0 & 0 \\ -\frac{11\sqrt{3}}{84} & 0 & \frac{13\sqrt{3}}{84} & -\frac{\sqrt{3}}{42} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{3}}{42} & \frac{13\sqrt{3}}{84} & 0 & -\frac{11\sqrt{3}}{84} & 0 & 0 & 0 & 0 \\ \frac{13\sqrt{3}}{84} & -\frac{\sqrt{3}}{42} & -\frac{11\sqrt{3}}{84} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	cluster	form
$\mathbb{U}_{13}$	$\mathbb{Q}_{2,1}^{(u,E_g)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{5}{28} & -\frac{2}{7} & \frac{3}{28} \\ 0 & 0 & 0 & 0 & \frac{5}{28} & 0 & \frac{3}{28} & -\frac{2}{7} \\ 0 & 0 & 0 & 0 & -\frac{2}{7} & \frac{3}{28} & 0 & \frac{5}{28} \\ 0 & 0 & 0 & 0 & \frac{3}{28} & -\frac{2}{7} & \frac{5}{28} & 0 \\ 0 & \frac{5}{28} & -\frac{2}{7} & \frac{3}{28} & 0 & 0 & 0 & 0 \\ \frac{5}{28} & 0 & \frac{3}{28} & -\frac{2}{7} & 0 & 0 & 0 & 0 \\ -\frac{2}{7} & \frac{3}{28} & 0 & \frac{5}{28} & 0 & 0 & 0 & 0 \\ \frac{3}{28} & -\frac{2}{7} & \frac{5}{28} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{14}$	$\mathbb{Q}_{2,0}^{(u,T_g)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{15}$	$\mathbb{Q}_{2,1}^{(u,T_g)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{16}$	$\mathbb{Q}_{2,2}^{(u,T_g)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	cluster	form
$\mathbb{U}_{17}$	$\mathbb{Q}_{3,0}^{(u,T_u,1)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{26}}{26} & 0 & -\frac{3\sqrt{26}}{52} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{26}}{26} & 0 & \frac{3\sqrt{26}}{52} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{3\sqrt{26}}{52} & 0 & \frac{\sqrt{26}}{26} \\ 0 & 0 & 0 & 0 & \frac{3\sqrt{26}}{52} & 0 & -\frac{\sqrt{26}}{26} & 0 \\ 0 & -\frac{\sqrt{26}}{26} & 0 & \frac{3\sqrt{26}}{52} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{26}}{26} & 0 & -\frac{3\sqrt{26}}{52} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3\sqrt{26}}{52} & 0 & -\frac{\sqrt{26}}{26} & 0 & 0 & 0 & 0 \\ -\frac{3\sqrt{26}}{52} & 0 & \frac{\sqrt{26}}{26} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{18}$	$\mathbb{Q}_{3,1}^{(u,T_u,1)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{3\sqrt{26}}{52} & \frac{\sqrt{26}}{26} & 0 \\ 0 & 0 & 0 & 0 & \frac{3\sqrt{26}}{52} & 0 & 0 & -\frac{\sqrt{26}}{26} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{26}}{26} & 0 & 0 & \frac{3\sqrt{26}}{52} \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{26}}{26} & -\frac{3\sqrt{26}}{52} & 0 \\ 0 & \frac{3\sqrt{26}}{52} & -\frac{\sqrt{26}}{26} & 0 & 0 & 0 & 0 & 0 \\ -\frac{3\sqrt{26}}{52} & 0 & 0 & \frac{\sqrt{26}}{26} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{26}}{26} & 0 & 0 & -\frac{3\sqrt{26}}{52} & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{26}}{26} & \frac{3\sqrt{26}}{52} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{19}$	$\mathbb{Q}_{3,2}^{(u,T_u,1)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3\sqrt{26}}{52} & \frac{\sqrt{26}}{26} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{26}}{26} & -\frac{3\sqrt{26}}{52} \\ 0 & 0 & 0 & 0 & \frac{3\sqrt{26}}{52} & -\frac{\sqrt{26}}{26} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{26}}{26} & \frac{3\sqrt{26}}{52} & 0 & 0 \\ 0 & 0 & \frac{3\sqrt{26}}{52} & -\frac{\sqrt{26}}{26} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{26}}{26} & \frac{3\sqrt{26}}{52} & 0 & 0 & 0 & 0 \\ -\frac{3\sqrt{26}}{52} & \frac{\sqrt{26}}{26} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{26}}{26} & -\frac{3\sqrt{26}}{52} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{20}$	$\mathbb{T}_{1,0}^{(u,T_u)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...



Table 6

symbol	type	cluster	form
$\mathbb{U}_{21}$	$\mathbb{T}_{1,1}^{(u,T_u)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{22}$	$\mathbb{T}_{1,2}^{(u,T_u)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{23}$	$\mathbb{T}_{2,0}^{(u,T_g)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{26}i}{26} & 0 & \frac{3\sqrt{26}i}{52} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{26}i}{26} & 0 & -\frac{3\sqrt{26}i}{52} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{3\sqrt{26}i}{52} & 0 & \frac{\sqrt{26}i}{26} \\ 0 & 0 & 0 & 0 & -\frac{3\sqrt{26}i}{52} & 0 & -\frac{\sqrt{26}i}{26} & 0 \\ 0 & \frac{\sqrt{26}i}{26} & 0 & \frac{3\sqrt{26}i}{52} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{26}i}{26} & 0 & -\frac{3\sqrt{26}i}{52} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3\sqrt{26}i}{52} & 0 & \frac{\sqrt{26}i}{26} & 0 & 0 & 0 & 0 \\ -\frac{3\sqrt{26}i}{52} & 0 & -\frac{\sqrt{26}i}{26} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{24}$	$\mathbb{T}_{2,1}^{(u,T_g)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{3\sqrt{26}i}{52} & \frac{\sqrt{26}i}{26} & 0 \\ 0 & 0 & 0 & 0 & -\frac{3\sqrt{26}i}{52} & 0 & 0 & -\frac{\sqrt{26}i}{26} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{26}i}{26} & 0 & 0 & -\frac{3\sqrt{26}i}{52} \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{26}i}{26} & \frac{3\sqrt{26}i}{52} & 0 \\ 0 & \frac{3\sqrt{26}i}{52} & \frac{\sqrt{26}i}{26} & 0 & 0 & 0 & 0 & 0 \\ -\frac{3\sqrt{26}i}{52} & 0 & 0 & -\frac{\sqrt{26}i}{26} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{26}i}{26} & 0 & 0 & -\frac{3\sqrt{26}i}{52} & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{26}i}{26} & \frac{3\sqrt{26}i}{52} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	cluster	form
$\mathbb{U}_{25}$	$\mathbb{T}_{2,2}^{(u,T_g)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3\sqrt{26}i}{52} & \frac{\sqrt{26}i}{26} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{26}i}{26} & \frac{3\sqrt{26}i}{52} & 0 \\ 0 & 0 & 0 & 0 & -\frac{3\sqrt{26}i}{52} & -\frac{\sqrt{26}i}{26} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{26}i}{26} & -\frac{3\sqrt{26}i}{52} & 0 & 0 & 0 \\ 0 & 0 & \frac{3\sqrt{26}i}{52} & \frac{\sqrt{26}i}{26} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{26}i}{26} & \frac{3\sqrt{26}i}{52} & 0 & 0 & 0 & 0 & 0 \\ -\frac{3\sqrt{26}i}{52} & -\frac{\sqrt{26}i}{26} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{26}i}{26} & -\frac{3\sqrt{26}i}{52} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{26}$	$\mathbb{T}_3^{(u,A_u)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}i}{12} & 0 \\ 0 & -\frac{\sqrt{6}i}{12} & -\frac{\sqrt{6}i}{12} & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{6}i}{12} & 0 & -\frac{\sqrt{6}i}{12} & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{6}i}{12} & -\frac{\sqrt{6}i}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{6}i}{12} & -\frac{\sqrt{6}i}{12} & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{27}$	$\mathbb{T}_{4,0}^{(u,T_g,1)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{3\sqrt{26}i}{52} & 0 & -\frac{\sqrt{26}i}{26} \\ 0 & 0 & 0 & 0 & -\frac{3\sqrt{26}i}{52} & 0 & \frac{\sqrt{26}i}{26} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{26}i}{26} & 0 & \frac{3\sqrt{26}i}{52} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{26}i}{26} & 0 & -\frac{3\sqrt{26}i}{52} & 0 \\ 0 & \frac{3\sqrt{26}i}{52} & 0 & -\frac{\sqrt{26}i}{26} & 0 & 0 & 0 & 0 \\ -\frac{3\sqrt{26}i}{52} & 0 & \frac{\sqrt{26}i}{26} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{26}i}{26} & 0 & \frac{3\sqrt{26}i}{52} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{26}i}{26} & 0 & -\frac{3\sqrt{26}i}{52} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{28}$	$\mathbb{T}_{4,1}^{(u,T_g,1)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{26}i}{26} & \frac{3\sqrt{26}i}{52} & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{26}i}{26} & 0 & 0 & -\frac{3\sqrt{26}i}{52} \\ 0 & 0 & 0 & 0 & -\frac{3\sqrt{26}i}{52} & 0 & 0 & \frac{\sqrt{26}i}{26} \\ 0 & 0 & 0 & 0 & 0 & \frac{3\sqrt{26}i}{52} & -\frac{\sqrt{26}i}{26} & 0 \\ 0 & -\frac{\sqrt{26}i}{26} & \frac{3\sqrt{26}i}{52} & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{26}i}{26} & 0 & 0 & -\frac{3\sqrt{26}i}{52} & 0 & 0 & 0 & 0 \\ -\frac{3\sqrt{26}i}{52} & 0 & 0 & \frac{\sqrt{26}i}{26} & 0 & 0 & 0 & 0 \\ 0 & \frac{3\sqrt{26}i}{52} & -\frac{\sqrt{26}i}{26} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	cluster	form
$\mathbb{U}_{29}$	$\mathbb{T}_{4,2}^{(u,T_g,1)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{26}i}{26} & \frac{3\sqrt{26}i}{52} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{3\sqrt{26}i}{52} & -\frac{\sqrt{26}i}{26} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{26}i}{26} & -\frac{3\sqrt{26}i}{52} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{3\sqrt{26}i}{52} & \frac{\sqrt{26}i}{26} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{26}i}{26} & \frac{3\sqrt{26}i}{52} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3\sqrt{26}i}{52} & -\frac{\sqrt{26}i}{26} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{26}i}{26} & -\frac{3\sqrt{26}i}{52} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{3\sqrt{26}i}{52} & \frac{\sqrt{26}i}{26} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{30}$	$\mathbb{T}_{5,0}^{(u,E_u)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{5i}{28} & -\frac{2i}{7} & \frac{3i}{28} \\ 0 & 0 & 0 & 0 & \frac{5i}{28} & 0 & \frac{3i}{28} & -\frac{2i}{7} \\ 0 & 0 & 0 & 0 & -\frac{2i}{7} & \frac{3i}{28} & 0 & \frac{5i}{28} \\ 0 & 0 & 0 & 0 & \frac{3i}{28} & -\frac{2i}{7} & \frac{5i}{28} & 0 \\ 0 & -\frac{5i}{28} & \frac{2i}{7} & -\frac{3i}{28} & 0 & 0 & 0 & 0 \\ -\frac{5i}{28} & 0 & -\frac{3i}{28} & \frac{2i}{7} & 0 & 0 & 0 & 0 \\ \frac{2i}{7} & -\frac{3i}{28} & 0 & -\frac{5i}{28} & 0 & 0 & 0 & 0 \\ -\frac{3i}{28} & \frac{2i}{7} & -\frac{5i}{28} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{U}_{31}$	$\mathbb{T}_{5,1}^{(u,E_u)}$	$B_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{11\sqrt{3}i}{84} & \frac{\sqrt{3}i}{42} & -\frac{13\sqrt{3}i}{84} \\ 0 & 0 & 0 & 0 & \frac{11\sqrt{3}i}{84} & 0 & -\frac{13\sqrt{3}i}{84} & \frac{\sqrt{3}i}{42} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}i}{42} & -\frac{13\sqrt{3}i}{84} & 0 & \frac{11\sqrt{3}i}{84} \\ 0 & 0 & 0 & 0 & -\frac{13\sqrt{3}i}{84} & \frac{\sqrt{3}i}{42} & \frac{11\sqrt{3}i}{84} & 0 \\ 0 & -\frac{11\sqrt{3}i}{84} & -\frac{\sqrt{3}i}{42} & \frac{13\sqrt{3}i}{84} & 0 & 0 & 0 & 0 \\ -\frac{11\sqrt{3}i}{84} & 0 & \frac{13\sqrt{3}i}{84} & -\frac{\sqrt{3}i}{42} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{3}i}{42} & \frac{13\sqrt{3}i}{84} & 0 & -\frac{11\sqrt{3}i}{84} & 0 & 0 & 0 & 0 \\ \frac{13\sqrt{3}i}{84} & -\frac{\sqrt{3}i}{42} & -\frac{11\sqrt{3}i}{84} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

Table 7: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_0^{(A_g)}$	0	$A_g$	—	—	1
2	$\mathbb{Q}_{1,0}^{(T_u)}$	1	$T_u$	—	0	$x$
3	$\mathbb{Q}_{1,1}^{(T_u)}$	1	$T_u$	—	1	$y$
4	$\mathbb{Q}_{1,2}^{(T_u)}$	1	$T_u$	—	2	$z$
5	$\mathbb{Q}_{2,0}^{(E_g)}$	2	$E_g$	—	0	$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$
6	$\mathbb{Q}_{2,1}^{(E_g)}$	2	$E_g$	—	1	$\frac{\sqrt{3}(x^2 - y^2)}{2}$
7	$\mathbb{Q}_{2,0}^{(T_g)}$	2	$T_g$	—	0	$\sqrt{3}yz$
8	$\mathbb{Q}_{2,1}^{(T_g)}$	2	$T_g$	—	1	$\sqrt{3}xz$
9	$\mathbb{Q}_{2,2}^{(T_g)}$	2	$T_g$	—	2	$\sqrt{3}xy$
10	$\mathbb{Q}_3^{(A_u)}$	3	$A_u$	—	—	$\sqrt{15}xyz$
11	$\mathbb{Q}_{3,0}^{(T_u,1)}$	3	$T_u$	1	0	$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$
12	$\mathbb{Q}_{3,1}^{(T_u,1)}$	3	$T_u$	1	1	$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$
13	$\mathbb{Q}_{3,2}^{(T_u,1)}$	3	$T_u$	1	2	$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$
14	$\mathbb{Q}_{4,0}^{(T_g,1)}$	4	$T_g$	1	0	$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$
15	$\mathbb{Q}_{4,1}^{(T_g,1)}$	4	$T_g$	1	1	$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$
16	$\mathbb{Q}_{4,2}^{(T_g,1)}$	4	$T_g$	1	2	$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$
17	$\mathbb{Q}_{5,0}^{(E_u)}$	5	$E_u$	—	0	$\frac{3\sqrt{35}xyz(x^2 - y^2)}{2}$
18	$\mathbb{Q}_{5,1}^{(E_u)}$	5	$E_u$	—	1	$\frac{\sqrt{105}xyz(x^2 + y^2 - 2z^2)}{2}$

Table 8: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_0^{(A_u)}$	0	$A_u$	—	—	1
2	$\mathbb{G}_{1,0}^{(T_g)}$	1	$T_g$	—	0	$X$
3	$\mathbb{G}_{1,1}^{(T_g)}$	1	$T_g$	—	1	$Y$
4	$\mathbb{G}_{1,2}^{(T_g)}$	1	$T_g$	—	2	$Z$
5	$\mathbb{G}_{2,0}^{(E_u)}$	2	$E_u$	—	0	$-\frac{X^2}{2} - \frac{Y^2}{2} + Z^2$

continued ...

Table 8

No.	symbol	rank	irrep.	mul.	comp.	form
6	$\mathbb{G}_{2,1}^{(E_u)}$	2	$E_u$	—	1	$\frac{\sqrt{3}(X^2-Y^2)}{2}$
7	$\mathbb{G}_{2,0}^{(T_u)}$	2	$T_u$	—	0	$\sqrt{3}YZ$
8	$\mathbb{G}_{2,1}^{(T_u)}$	2	$T_u$	—	1	$\sqrt{3}XZ$
9	$\mathbb{G}_{2,2}^{(T_u)}$	2	$T_u$	—	2	$\sqrt{3}XY$
10	$\mathbb{G}_{3,0}^{(T_g,1)}$	3	$T_g$	1	0	$\frac{X(2X^2-3Y^2-3Z^2)}{2}$
11	$\mathbb{G}_{3,1}^{(T_g,1)}$	3	$T_g$	1	1	$-\frac{Y(3X^2-2Y^2+3Z^2)}{2}$
12	$\mathbb{G}_{3,2}^{(T_g,1)}$	3	$T_g$	1	2	$-\frac{Z(3X^2+3Y^2-2Z^2)}{2}$
13	$\mathbb{G}_{3,0}^{(T_g,2)}$	3	$T_g$	2	0	$\frac{\sqrt{15}X(Y-Z)(Y+Z)}{2}$
14	$\mathbb{G}_{3,1}^{(T_g,2)}$	3	$T_g$	2	1	$-\frac{\sqrt{15}Y(X-Z)(X+Z)}{2}$
15	$\mathbb{G}_{3,2}^{(T_g,2)}$	3	$T_g$	2	2	$\frac{\sqrt{15}Z(X-Y)(X+Y)}{2}$

- 
- Group info.: Generator =  $2_{001}$ ,  $2_{010}$ ,  $3_{111}^+$ ,  $-1$

Table 9: Conjugacy class.

rep. SO	symmetry operations
1	1
$2_{001}$	$2_{001}$ , $2_{100}$ , $2_{010}$
$3_{111}^+$	$3_{111}^+$ , $3_{1-1-1}^+$ , $3_{-11-1}^+$ , $3_{-1-11}^+$
$3_{111}^-$	$3_{111}^-$ , $3_{1-1-1}^-$ , $3_{-11-1}^-$ , $3_{-1-11}^-$
$-1$	$-1$
$m_{001}$	$m_{001}$ , $m_{100}$ , $m_{010}$
$-3_{111}^+$	$-3_{111}^+$ , $-3_{1-1-1}^+$ , $-3_{-11-1}^+$ , $-3_{-1-11}^+$
$-3_{111}^-$	$-3_{111}^-$ , $-3_{1-1-1}^-$ , $-3_{-11-1}^-$ , $-3_{-1-11}^-$

Table 10: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	1	2	$2_{001}$	3	$2_{100}$	4	$2_{010}$	5	$3_{111}^+$
6	$3_{1-1-1}^+$	7	$3_{-11-1}^+$	8	$3_{-1-11}^+$	9	$3_{111}^-$	10	$3_{1-1-1}^-$
11	$3_{-11-1}^-$	12	$3_{-1-11}^-$	13	-1	14	$m_{001}$	15	$m_{100}$
16	$m_{010}$	17	$-3_{111}^+$	18	$-3_{1-1-1}^+$	19	$-3_{-11-1}^+$	20	$-3_{-1-11}^+$
21	$-3_{111}^-$	22	$-3_{1-1-1}^-$	23	$-3_{-11-1}^-$	24	$-3_{-1-11}^-$		

Table 11: Character table.

	1	$2_{001}$	$3_{111}^+$	$3_{111}^-$	-1	$m_{001}$	$-3_{111}^+$	$-3_{111}^-$
$A_g$	1	1	1	1	1	1	1	1
$E_g^{(a)}$	1	1	$\omega^*$	$\omega$	1	1	$\omega^*$	$\omega$
$E_g^{(b)}$	1	1	$\omega$	$\omega^*$	1	1	$\omega$	$\omega^*$
$T_g$	3	-1	0	0	3	-1	0	0
$A_u$	1	1	1	1	-1	-1	-1	-1
$E_u^{(a)}$	1	1	$\omega^*$	$\omega$	-1	-1	$-\omega^*$	$-\omega$
$E_u^{(b)}$	1	1	$\omega$	$\omega^*$	-1	-1	$-\omega$	$-\omega^*$
$T_u$	3	-1	0	0	-3	1	0	0

Table 12: Parity conversion.

$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$
$A_g$ ( $A_u$ )	$E_g^{(a)}$ ( $E_u^{(a)}$ )	$E_g^{(b)}$ ( $E_u^{(b)}$ )	$T_g$ ( $T_u$ )	$A_u$ ( $A_g$ )
$E_u^{(a)}$ ( $E_g^{(a)}$ )	$E_u^{(b)}$ ( $E_g^{(b)}$ )	$T_u$ ( $T_g$ )		

Table 13: Symmetric product,  $[\Gamma \otimes \Gamma']_+$ .

	$A_g$	$E_g^{(a)}$	$E_g^{(b)}$	$T_g$	$A_u$	$E_u^{(a)}$	$E_u^{(b)}$	$T_u$
$A_g$	$A_g$	$E_g^{(a)}$	$E_g^{(b)}$	$T_g$	$A_u$	$E_u^{(a)}$	$E_u^{(b)}$	$T_u$
$E_g^{(a)}$		$E_g^{(a)}$	$A_g$	$T_g$	$E_u^{(a)}$	$E_u^{(b)}$	$A_u$	$T_u$
$E_g^{(b)}$			$E_g^{(a)}$	$T_g$	$E_u^{(b)}$	$A_u$	$E_u^{(a)}$	$T_u$
$T_g$				$A_g + E_g^{(a)} + E_g^{(b)} + T_g$	$T_u$	$T_u$	$T_u$	$A_u + E_u^{(a)} + E_u^{(b)} + 2T_u$
$A_u$					$A_g$	$E_g^{(a)}$	$E_g^{(b)}$	$T_g$
$E_u^{(a)}$						$E_g^{(b)}$	$A_g$	$T_g$
$E_u^{(b)}$							$E_g^{(a)}$	$T_g$
$T_u$								$A_g + E_g^{(a)} + E_g^{(b)} + T_g$

Table 14: Anti-symmetric product,  $[\Gamma \otimes \Gamma]_-$ .

$A_g$	$E_g^{(a)}$	$E_g^{(b)}$	$T_g$	$A_u$	$E_u^{(a)}$	$E_u^{(b)}$	$T_u$
$-$	$-$	$-$	$T_g$	$-$	$-$	$-$	$T_g$

Table 15: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix} 3 & 2 & 1 \end{pmatrix}$	2	$\begin{pmatrix} -3 & -2 & 1 \end{pmatrix}$	3	$\begin{pmatrix} 3 & -2 & -1 \end{pmatrix}$	4	$\begin{pmatrix} -3 & 2 & -1 \end{pmatrix}$
5	$\begin{pmatrix} 1 & 3 & 2 \end{pmatrix}$	6	$\begin{pmatrix} -1 & -3 & 2 \end{pmatrix}$	7	$\begin{pmatrix} 1 & -3 & -2 \end{pmatrix}$	8	$\begin{pmatrix} -1 & 3 & -2 \end{pmatrix}$
9	$\begin{pmatrix} 2 & 1 & 3 \end{pmatrix}$	10	$\begin{pmatrix} -2 & 1 & -3 \end{pmatrix}$	11	$\begin{pmatrix} -2 & -1 & 3 \end{pmatrix}$	12	$\begin{pmatrix} 2 & -1 & -3 \end{pmatrix}$
13	$\begin{pmatrix} -3 & -2 & -1 \end{pmatrix}$	14	$\begin{pmatrix} 3 & 2 & -1 \end{pmatrix}$	15	$\begin{pmatrix} -3 & 2 & 1 \end{pmatrix}$	16	$\begin{pmatrix} 3 & -2 & 1 \end{pmatrix}$
17	$\begin{pmatrix} -1 & -3 & -2 \end{pmatrix}$	18	$\begin{pmatrix} 1 & 3 & -2 \end{pmatrix}$	19	$\begin{pmatrix} -1 & 3 & 2 \end{pmatrix}$	20	$\begin{pmatrix} 1 & -3 & 2 \end{pmatrix}$
21	$\begin{pmatrix} -2 & -1 & -3 \end{pmatrix}$	22	$\begin{pmatrix} 2 & -1 & 3 \end{pmatrix}$	23	$\begin{pmatrix} 2 & 1 & -3 \end{pmatrix}$	24	$\begin{pmatrix} -2 & 1 & 3 \end{pmatrix}$

Table 16: Virtual-cluster basis.

[illegible]

*continued ...*



Table 16

[illegible]

*continued ...*

Table 16

symbol	1	2	3	4	5	6	7	8	9	10
	$\frac{\sqrt{829}}{829}$	$-\frac{\sqrt{829}}{829}$	$\frac{39\sqrt{829}}{3316}$	$-\frac{39\sqrt{829}}{3316}$	$\frac{39\sqrt{829}}{3316}$	$-\frac{39\sqrt{829}}{3316}$	$-\frac{11\sqrt{829}}{3316}$	$\frac{11\sqrt{829}}{3316}$	$-\frac{11\sqrt{829}}{3316}$	$\frac{11\sqrt{829}}{3316}$
	$-\frac{\sqrt{829}}{829}$	$\frac{\sqrt{829}}{829}$	$\frac{\sqrt{829}}{829}$	$-\frac{\sqrt{829}}{829}$						
$\mathbb{Q}_{4,1}^{(Tg,2)}$	$-\frac{\sqrt{829}}{829}$	$\frac{\sqrt{829}}{829}$	$\frac{\sqrt{829}}{829}$	$-\frac{\sqrt{829}}{829}$	$\frac{39\sqrt{829}}{3316}$	$-\frac{39\sqrt{829}}{3316}$	$-\frac{39\sqrt{829}}{3316}$	$\frac{39\sqrt{829}}{3316}$	$-\frac{11\sqrt{829}}{3316}$	$-\frac{11\sqrt{829}}{3316}$
	$\frac{11\sqrt{829}}{3316}$	$\frac{11\sqrt{829}}{3316}$	$-\frac{\sqrt{829}}{829}$	$\frac{\sqrt{829}}{829}$	$\frac{\sqrt{829}}{829}$	$-\frac{\sqrt{829}}{829}$	$\frac{39\sqrt{829}}{3316}$	$-\frac{39\sqrt{829}}{3316}$	$-\frac{39\sqrt{829}}{3316}$	$\frac{39\sqrt{829}}{3316}$
	$-\frac{11\sqrt{829}}{3316}$	$-\frac{11\sqrt{829}}{3316}$	$\frac{11\sqrt{829}}{3316}$	$\frac{11\sqrt{829}}{3316}$						
$\mathbb{Q}_{4,2}^{(Tg,2)}$	$-\frac{11\sqrt{829}}{3316}$	$-\frac{11\sqrt{829}}{3316}$	$\frac{11\sqrt{829}}{3316}$	$\frac{11\sqrt{829}}{3316}$	$-\frac{\sqrt{829}}{829}$	$-\frac{\sqrt{829}}{829}$	$\frac{\sqrt{829}}{829}$	$\frac{\sqrt{829}}{829}$	$\frac{39\sqrt{829}}{3316}$	$-\frac{39\sqrt{829}}{3316}$
	$\frac{39\sqrt{829}}{3316}$	$-\frac{39\sqrt{829}}{3316}$	$-\frac{11\sqrt{829}}{3316}$	$-\frac{11\sqrt{829}}{3316}$	$\frac{11\sqrt{829}}{3316}$	$\frac{11\sqrt{829}}{3316}$	$-\frac{\sqrt{829}}{829}$	$-\frac{\sqrt{829}}{829}$	$\frac{\sqrt{829}}{829}$	$\frac{\sqrt{829}}{829}$
	$\frac{39\sqrt{829}}{3316}$	$-\frac{39\sqrt{829}}{3316}$	$\frac{39\sqrt{829}}{3316}$	$-\frac{39\sqrt{829}}{3316}$						
$\mathbb{Q}_{5,0}^{(Eu)}$	$\frac{5}{28}$	$\frac{5}{28}$	$\frac{5}{28}$	$\frac{5}{28}$	$-\frac{2}{7}$	$-\frac{2}{7}$	$-\frac{2}{7}$	$-\frac{2}{7}$	$\frac{3}{28}$	$\frac{3}{28}$
	$\frac{3}{28}$	$\frac{3}{28}$	$-\frac{5}{28}$	$-\frac{5}{28}$	$-\frac{5}{28}$	$-\frac{5}{28}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{2}{7}$
	$-\frac{3}{28}$	$-\frac{3}{28}$	$-\frac{3}{28}$	$-\frac{3}{28}$						
$\mathbb{Q}_{5,1}^{(Eu)}$	$\frac{11\sqrt{3}}{84}$	$\frac{11\sqrt{3}}{84}$	$\frac{11\sqrt{3}}{84}$	$\frac{11\sqrt{3}}{84}$	$\frac{\sqrt{3}}{42}$	$\frac{\sqrt{3}}{42}$	$\frac{\sqrt{3}}{42}$	$\frac{\sqrt{3}}{42}$	$-\frac{13\sqrt{3}}{84}$	$-\frac{13\sqrt{3}}{84}$
	$-\frac{13\sqrt{3}}{84}$	$-\frac{13\sqrt{3}}{84}$	$-\frac{11\sqrt{3}}{84}$	$-\frac{11\sqrt{3}}{84}$	$-\frac{11\sqrt{3}}{84}$	$-\frac{11\sqrt{3}}{84}$	$-\frac{\sqrt{3}}{42}$	$-\frac{\sqrt{3}}{42}$	$-\frac{\sqrt{3}}{42}$	$-\frac{\sqrt{3}}{42}$
	$\frac{13\sqrt{3}}{84}$	$\frac{13\sqrt{3}}{84}$	$\frac{13\sqrt{3}}{84}$	$\frac{13\sqrt{3}}{84}$						