

PG No. 35  $C_{3v}(1)$   $3m$  (31m setting) [ trigonal ] (polar, internal axial quadrupole)

\* Harmonics for rank 0

\* Harmonics for rank 1

$$\vec{\mathbb{Q}}_1^{(2,1)}[g](A_1)$$

\*\* symmetry

$z$

\*\* expression

$$\frac{\sqrt{30}G_vxy}{5} - \frac{\sqrt{30}G_{xy}(x-y)(x+y)}{10} + \frac{\sqrt{30}G_{xz}yz}{10} - \frac{\sqrt{30}G_{yz}xz}{10}$$

$$\vec{\mathbb{Q}}_{1,1}^{(2,1)}[g](E), \vec{\mathbb{Q}}_{1,2}^{(2,1)}[g](E)$$

\*\* symmetry

$x$

$y$

\*\* expression

$$-\frac{3\sqrt{10}G_uyz}{10} - \frac{\sqrt{30}G_vyz}{10} + \frac{\sqrt{30}G_{xy}xz}{10} - \frac{\sqrt{30}G_{xz}xy}{10} - \frac{\sqrt{30}G_{yz}(y-z)(y+z)}{10}$$

$$\frac{3\sqrt{10}G_uxz}{10} - \frac{\sqrt{30}G_vxz}{10} - \frac{\sqrt{30}G_{xy}yz}{10} + \frac{\sqrt{30}G_{xz}(x-z)(x+z)}{10} + \frac{\sqrt{30}G_{yz}xy}{10}$$

\* Harmonics for rank 2

$$\vec{\mathbb{Q}}_2^{(2,-1)}[g](A_1)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$\frac{\sqrt{2}G_{xz}y}{2} - \frac{\sqrt{2}G_{yz}x}{2}$$

$$\vec{\mathbb{Q}}_2^{(2,1)}[g](A_1)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$\frac{5\sqrt{42}G_vxyz}{14} - \frac{5\sqrt{42}G_{xy}z(x-y)(x+y)}{28} - \frac{\sqrt{42}G_{xz}y(x^2+y^2-4z^2)}{28} + \frac{\sqrt{42}G_{yz}x(x^2+y^2-4z^2)}{28}$$

$$\vec{\mathbb{Q}}_{2,1}^{(2,-1)}[g](E,1), \vec{\mathbb{Q}}_{2,2}^{(2,-1)}[g](E,1)$$

\*\* symmetry

$\sqrt{3}xz$

$\sqrt{3}yz$

\*\* expression

$$-\frac{\sqrt{2}G_uy}{2} + \frac{\sqrt{6}G_vy}{6} - \frac{\sqrt{6}G_{xy}x}{6} + \frac{\sqrt{6}G_{yz}z}{6}$$

$$\frac{\sqrt{2}G_ux}{2} + \frac{\sqrt{6}G_vx}{6} + \frac{\sqrt{6}G_{xy}y}{6} - \frac{\sqrt{6}G_{xz}z}{6}$$

$$\vec{\mathbb{Q}}_{2,1}^{(2,-1)}[g](E,2), \vec{\mathbb{Q}}_{2,2}^{(2,-1)}[g](E,2)$$

\*\* symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$-\sqrt{3}xy$

\*\* expression

$$\frac{\sqrt{6}G_{xy}z}{3} - \frac{\sqrt{6}G_{xz}y}{6} - \frac{\sqrt{6}G_{yz}x}{6}$$

$$\frac{\sqrt{6}G_vz}{3} - \frac{\sqrt{6}G_{xz}x}{6} + \frac{\sqrt{6}G_{yz}y}{6}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,1)}[g](E, 1), \vec{\mathbb{Q}}_{2,2}^{(2,1)}[g](E, 1)$

\*\* symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{42}G_uy(x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{14}G_vy(9x^2 - y^2 - 6z^2)}{28} - \frac{\sqrt{14}G_{xy}x(2x^2 - 3y^2 - 3z^2)}{14} - \frac{\sqrt{14}G_{yz}z(3x^2 + 3y^2 - 2z^2)}{14} \\ & - \frac{\sqrt{42}G_ux(x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{14}G_vx(x^2 - 9y^2 + 6z^2)}{28} - \frac{\sqrt{14}G_{xy}y(3x^2 - 2y^2 + 3z^2)}{14} + \frac{\sqrt{14}G_{xz}z(3x^2 + 3y^2 - 2z^2)}{14} \end{aligned}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,1)}[g](E, 2), \vec{\mathbb{Q}}_{2,2}^{(2,1)}[g](E, 2)$

\*\* symmetry

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

$$-\sqrt{3}xy$$

\*\* expression

$$\begin{aligned} & -\frac{5\sqrt{42}G_uxyz}{14} + \frac{\sqrt{14}G_{xy}z(3x^2 + 3y^2 - 2z^2)}{28} - \frac{\sqrt{14}G_{xz}y(9x^2 - y^2 - 6z^2)}{28} + \frac{\sqrt{14}G_{yz}x(x^2 - 9y^2 + 6z^2)}{28} \\ & - \frac{5\sqrt{42}G_uz(x - y)(x + y)}{28} + \frac{\sqrt{14}G_vz(3x^2 + 3y^2 - 2z^2)}{28} - \frac{\sqrt{14}G_{xz}x(2x^2 - 3y^2 - 3z^2)}{14} - \frac{\sqrt{14}G_{yz}y(3x^2 - 2y^2 + 3z^2)}{14} \end{aligned}$$

\* Harmonics for rank 3

$\vec{\mathbb{Q}}_3^{(2,-1)}[g](A_1, 1)$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{30}G_vxy}{10} + \frac{\sqrt{30}G_{xy}(x - y)(x + y)}{20} + \frac{\sqrt{30}G_{xz}yz}{5} - \frac{\sqrt{30}G_{yz}xz}{5}$$

$\vec{\mathbb{Q}}_3^{(2,-1)}[g](A_1, 2)$

\*\* symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

\*\* expression

$$\frac{\sqrt{3}G_vyz}{2} + \frac{\sqrt{3}G_{xy}xz}{2} - \frac{\sqrt{3}G_{xz}xy}{2} - \frac{\sqrt{3}G_{yz}(x - y)(x + y)}{4}$$

$\vec{\mathbb{Q}}_3^{(2,1)}[g](A_1, 1)$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{15}G_vxy(x^2 + y^2 - 6z^2)}{6} + \frac{\sqrt{15}G_{xy}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{12} - \frac{\sqrt{15}G_{xz}yz(3x^2 + 3y^2 - 4z^2)}{12} + \frac{\sqrt{15}G_{yz}xz(3x^2 + 3y^2 - 4z^2)}{12}$$

$\vec{\mathbb{Q}}_3^{(2,1)}[g](A_1, 2)$

\*\* symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

\*\* expression

$$\begin{aligned} & -\frac{7\sqrt{2}G_uyz(3x^2 - y^2)}{8} + \frac{\sqrt{6}G_vyz(3x^2 + 3y^2 - 4z^2)}{24} + \frac{\sqrt{6}G_{xy}xz(3x^2 + 3y^2 - 4z^2)}{24} \\ & - \frac{\sqrt{6}G_{xz}xy(17x^2 - 11y^2 - 18z^2)}{24} + \frac{\sqrt{6}G_{yz}(2x^4 - 21x^2y^2 + 9x^2z^2 + 5y^4 - 9y^2z^2)}{24} \end{aligned}$$

$$\vec{\mathbb{Q}}_3^{(2,-1)}[g](A_2)$$

\*\* symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

\*\* expression

$$-\frac{\sqrt{3}G_vxz}{2} + \frac{\sqrt{3}G_{xy}yz}{2} + \frac{\sqrt{3}G_{xz}(x-y)(x+y)}{4} - \frac{\sqrt{3}G_{yz}xy}{2}$$

$$\vec{\mathbb{Q}}_3^{(2,1)}[g](A_2)$$

\*\* symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{7\sqrt{2}G_uxz(x^2 - 3y^2)}{8} - \frac{\sqrt{6}G_vxz(3x^2 + 3y^2 - 4z^2)}{24} + \frac{\sqrt{6}G_{xy}yz(3x^2 + 3y^2 - 4z^2)}{24} \\ & + \frac{\sqrt{6}G_{xz}(5x^4 - 21x^2y^2 - 9x^2z^2 + 2y^4 + 9y^2z^2)}{24} + \frac{\sqrt{6}G_{yz}xy(11x^2 - 17y^2 + 18z^2)}{24} \end{aligned}$$

$$\vec{\mathbb{Q}}_{3,1}^{(2,-1)}[g](E, 1), \vec{\mathbb{Q}}_{3,2}^{(2,-1)}[g](E, 1)$$

\*\* symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

\*\* expression

$$-\frac{\sqrt{15}G_uyz}{5} + \frac{3\sqrt{5}G_vyz}{10} - \frac{3\sqrt{5}G_{xy}xz}{10} + \frac{3\sqrt{5}G_{xz}xy}{10} - \frac{\sqrt{5}G_{yz}(5x^2 - y^2 - 4z^2)}{20}$$

$$\frac{\sqrt{15}G_uxz}{5} + \frac{3\sqrt{5}G_vxz}{10} + \frac{3\sqrt{5}G_{xy}yz}{10} - \frac{\sqrt{5}G_{xz}(x^2 - 5y^2 + 4z^2)}{20} - \frac{3\sqrt{5}G_{yz}xy}{10}$$

$$\vec{\mathbb{Q}}_{3,1}^{(2,-1)}[g](E, 2), \vec{\mathbb{Q}}_{3,2}^{(2,-1)}[g](E, 2)$$

\*\* symmetry

$$-\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

$$\sqrt{15}xyz$$

\*\* expression

$$\frac{\sqrt{6}G_uxy}{2} + \frac{\sqrt{2}G_{xy}(x^2 + y^2 - 2z^2)}{4}$$

$$\frac{\sqrt{6}G_u(x-y)(x+y)}{4} + \frac{\sqrt{2}G_v(x^2 + y^2 - 2z^2)}{4}$$

$$\vec{\mathbb{Q}}_{3,1}^{(2,1)}[g](E, 1), \vec{\mathbb{Q}}_{3,2}^{(2,1)}[g](E, 1)$$

\*\* symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{30}G_uyz(3x^2 + 3y^2 - 4z^2)}{24} + \frac{\sqrt{10}G_vyz(27x^2 - y^2 - 8z^2)}{24} - \frac{\sqrt{10}G_{xy}xz(13x^2 - 15y^2 - 8z^2)}{24} \\ & - \frac{\sqrt{10}G_{xz}xy(x^2 + y^2 - 6z^2)}{24} + \frac{\sqrt{10}G_{yz}(2x^4 + 3x^2y^2 - 15x^2z^2 + y^4 - 9y^2z^2 + 4z^4)}{24} \end{aligned}$$

$$\begin{aligned} & - \frac{\sqrt{30}G_uxz(3x^2 + 3y^2 - 4z^2)}{24} - \frac{\sqrt{10}G_vxz(x^2 - 27y^2 + 8z^2)}{24} - \frac{\sqrt{10}G_{xy}yz(15x^2 - 13y^2 + 8z^2)}{24} \\ & - \frac{\sqrt{10}G_{xz}(x^4 + 3x^2y^2 - 9x^2z^2 + 2y^4 - 15y^2z^2 + 4z^4)}{24} + \frac{\sqrt{10}G_{yz}xy(x^2 + y^2 - 6z^2)}{24} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(2,1)}[g](E,2), \tilde{\mathbb{Q}}_{3,2}^{(2,1)}[g](E,2)$$

\*\* symmetry

$$-\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

$$\sqrt{15}xyz$$

\*\* expression

$$\begin{aligned} & -\frac{\sqrt{3}G_u xy(x^2 + y^2 - 6z^2)}{3} - \frac{7G_v xy(x-y)(x+y)}{6} + \frac{G_{xy}(5x^4 - 18x^2y^2 - 12x^2z^2 + 5y^4 - 12y^2z^2 + 4z^4)}{12} \\ & + \frac{7G_{xz}yz(3x^2 + y^2 - 2z^2)}{12} + \frac{7G_{yz}xz(x^2 + 3y^2 - 2z^2)}{12} \\ & - \frac{\sqrt{3}G_u(x-y)(x+y)(x^2 + y^2 - 6z^2)}{6} - \frac{G_v(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{6} \\ & - \frac{7G_{xy}xy(x-y)(x+y)}{6} + \frac{7G_{xz}xz(x-z)(x+z)}{6} - \frac{7G_{yz}yz(y-z)(y+z)}{6} \end{aligned}$$

\* Harmonics for rank 4

$$\tilde{\mathbb{Q}}_4^{(2,-1)}[g](A_1, 1)$$

\*\* symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

\*\* expression

$$-\frac{\sqrt{105}G_vxyz}{7} + \frac{\sqrt{105}G_{xy}z(x-y)(x+y)}{14} - \frac{\sqrt{105}G_{xz}y(x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{105}G_{yz}x(x^2 + y^2 - 4z^2)}{28}$$

$$\tilde{\mathbb{Q}}_4^{(2,-1)}[g](A_1, 2)$$

\*\* symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

\*\* expression

$$-\frac{3\sqrt{2}G_{uy}(3x^2 - y^2)}{8} - \frac{\sqrt{6}G_vy(x^2 + y^2 - 4z^2)}{8} - \frac{\sqrt{6}G_{xy}x(x^2 + y^2 - 4z^2)}{8} - \frac{\sqrt{6}G_{xz}xyz}{4} - \frac{\sqrt{6}G_{yz}z(x-y)(x+y)}{8}$$

$$\tilde{\mathbb{Q}}_4^{(2,1)}[g](A_1, 1)$$

\*\* symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

\*\* expression

$$\begin{aligned} & -\frac{7\sqrt{330}G_vxyz(x^2 + y^2 - 2z^2)}{44} + \frac{7\sqrt{330}G_{xy}z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{88} \\ & + \frac{\sqrt{330}G_{xz}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} - \frac{\sqrt{330}G_{yz}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} \end{aligned}$$

$$\tilde{\mathbb{Q}}_4^{(2,1)}[g](A_1, 2)$$

\*\* symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{77}G_{uy}(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{88} + \frac{\sqrt{231}G_vy(7x^4 - 16x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 4z^4)}{88} \\ & - \frac{\sqrt{231}G_{xy}x(x^4 - 7x^2y^2 - 3x^2z^2 + 4y^4 - 3y^2z^2 + 2z^4)}{44} - \frac{\sqrt{231}G_{xz}xyz(11x^2 - y^2 - 10z^2)}{44} - \frac{\sqrt{231}G_{yz}z(x^4 + 9x^2y^2 - 5x^2z^2 - 4y^4 + 5y^2z^2)}{44} \end{aligned}$$

$$\tilde{\mathbb{Q}}_4^{(2,-1)}[g](A_2)$$

\*\* symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

\*\* expression

$$\frac{3\sqrt{2}G_{ux}(x^2 - 3y^2)}{8} + \frac{\sqrt{6}G_vx(x^2 + y^2 - 4z^2)}{8} - \frac{\sqrt{6}G_{xy}y(x^2 + y^2 - 4z^2)}{8} + \frac{\sqrt{6}G_{xz}z(x-y)(x+y)}{8} - \frac{\sqrt{6}G_{yz}xyz}{4}$$

$$\vec{\mathbb{Q}}_4^{(2,1)}[g](A_2)$$

\*\* symmetry

$$-\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

\*\* expression

$$-\frac{3\sqrt{77}G_{ux}(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{88} - \frac{\sqrt{231}G_{vx}(x^4 - 16x^2y^2 + 6x^2z^2 + 7y^4 + 6y^2z^2 - 4z^4)}{88} \\ - \frac{\sqrt{231}G_{xy}y(4x^4 - 7x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + 2z^4)}{44} + \frac{\sqrt{231}G_{xz}z(4x^4 - 9x^2y^2 - 5x^2z^2 - y^4 + 5y^2z^2)}{44} + \frac{\sqrt{231}G_{yz}xyz(x^2 - 11y^2 + 10z^2)}{44}$$

$$\vec{\mathbb{Q}}_{4,1}^{(2,-1)}[g](E, 1), \vec{\mathbb{Q}}_{4,2}^{(2,-1)}[g](E, 1)$$

\*\* symmetry

$$-\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

\*\* expression

$$\frac{3\sqrt{14}G_{uy}(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{42}G_{vy}(5x^2 + y^2 - 8z^2)}{56} + \frac{\sqrt{42}G_{xy}x(3x^2 - y^2 - 8z^2)}{56} + \frac{\sqrt{42}G_{xz}xyz}{4} - \frac{\sqrt{42}G_{yz}z(13x^2 - y^2 - 4z^2)}{56}$$

$$-\frac{3\sqrt{14}G_{ux}(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{42}G_vx(x^2 + 5y^2 - 8z^2)}{56} + \frac{\sqrt{42}G_{xy}y(x^2 - 3y^2 + 8z^2)}{56} - \frac{\sqrt{42}G_{xz}z(x^2 - 13y^2 + 4z^2)}{56} - \frac{\sqrt{42}G_{yz}xyz}{4}$$

$$\vec{\mathbb{Q}}_{4,1}^{(2,-1)}[g](E, 2), \vec{\mathbb{Q}}_{4,2}^{(2,-1)}[g](E, 2)$$

\*\* symmetry

$$-\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x - y)(x + y)}{2}$$

\*\* expression

$$\sqrt{3}G_vxyz + \frac{\sqrt{3}G_{xy}z(x - y)(x + y)}{2} - \frac{\sqrt{3}G_{xz}y(3x^2 - y^2)}{4} - \frac{\sqrt{3}G_{yz}x(x^2 - 3y^2)}{4}$$

$$-\frac{\sqrt{3}G_vz(x - y)(x + y)}{2} + \sqrt{3}G_{xy}xyz + \frac{\sqrt{3}G_{xz}x(x^2 - 3y^2)}{4} - \frac{\sqrt{3}G_{yz}y(3x^2 - y^2)}{4}$$

$$\vec{\mathbb{Q}}_{4,1}^{(2,-1)}[g](E, 3), \vec{\mathbb{Q}}_{4,2}^{(2,-1)}[g](E, 3)$$

\*\* symmetry

$$-\frac{\sqrt{5}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

\*\* expression

$$-\frac{6\sqrt{7}G_uxyz}{7} - \frac{\sqrt{21}G_{xy}z(3x^2 + 3y^2 - 2z^2)}{14} + \frac{\sqrt{21}G_{xz}y(2x^2 - y^2 + z^2)}{14} - \frac{\sqrt{21}G_{yz}x(x^2 - 2y^2 - z^2)}{14}$$

$$-\frac{3\sqrt{7}G_uz(x - y)(x + y)}{7} - \frac{\sqrt{21}G_vz(3x^2 + 3y^2 - 2z^2)}{14} + \frac{\sqrt{21}G_{xz}x(x^2 - 5y^2 + 2z^2)}{28} + \frac{\sqrt{21}G_{yz}y(5x^2 - y^2 - 2z^2)}{28}$$

$$\vec{\mathbb{Q}}_{4,1}^{(2,1)}[g](E, 1), \vec{\mathbb{Q}}_{4,2}^{(2,1)}[g](E, 1)$$

\*\* symmetry

$$-\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

\*\* expression

$$\begin{aligned}
& -\frac{3\sqrt{11}G_u y (x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} - \frac{\sqrt{33}G_v y (13x^4 + 12x^2y^2 - 114x^2z^2 - y^4 - 2y^2z^2 + 20z^4)}{88} \\
& + \frac{\sqrt{33}G_{xy} x (3x^4 - x^2y^2 - 29x^2z^2 - 4y^4 + 27y^2z^2 + 10z^4)}{44} - \frac{7\sqrt{33}G_{xz}xyz (x^2 + y^2 - 2z^2)}{44} \\
& + \frac{\sqrt{33}G_{yz} z (11x^4 + 15x^2y^2 - 27x^2z^2 + 4y^4 - 13y^2z^2 + 4z^4)}{44}
\end{aligned}$$
  

$$\begin{aligned}
& 3\sqrt{11}G_u x (x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4) + \frac{\sqrt{33}G_v x (x^4 - 12x^2y^2 + 2x^2z^2 - 13y^4 + 114y^2z^2 - 20z^4)}{88} \\
& + \frac{\sqrt{33}G_{xy} y (4x^4 + x^2y^2 - 27x^2z^2 - 3y^4 + 29y^2z^2 - 10z^4)}{44} \\
& - \frac{\sqrt{33}G_{xz} z (4x^4 + 15x^2y^2 - 13x^2z^2 + 11y^4 - 27y^2z^2 + 4z^4)}{44} + \frac{7\sqrt{33}G_{yz}xyz (x^2 + y^2 - 2z^2)}{44}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,1)}[g](E, 2), \vec{\mathbb{Q}}_{4,2}^{(2,1)}[g](E, 2)$

\*\* symmetry

$$-\frac{\sqrt{35} (x^2 - 2xy - y^2) (x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy (x - y)(x + y)}{2}$$

\*\* expression

$$\begin{aligned}
& -\frac{9\sqrt{154}G_u xyz (x - y)(x + y)}{22} + \frac{\sqrt{462}G_v xyz (x^2 + y^2 - 2z^2)}{44} + \frac{\sqrt{462}G_{xy} z (x - y)(x + y)(x^2 + y^2 - 2z^2)}{88} \\
& - \frac{\sqrt{462}G_{xz} y (9x^4 - 14x^2y^2 - 12x^2z^2 + y^4 + 4y^2z^2)}{88} + \frac{\sqrt{462}G_{yz} x (x^4 - 14x^2y^2 + 4x^2z^2 + 9y^4 - 12y^2z^2)}{88}
\end{aligned}$$

$$\begin{aligned}
& 9\sqrt{154}G_u z (x^2 - 2xy - y^2)(x^2 + 2xy - y^2) - \frac{\sqrt{462}G_v z (x - y)(x + y)(x^2 + y^2 - 2z^2)}{88} + \frac{\sqrt{462}G_{xy} xyz (x^2 + y^2 - 2z^2)}{44} \\
& + \frac{\sqrt{462}G_{xz} x (x^4 - 8x^2y^2 - 2x^2z^2 + 3y^4 + 6y^2z^2)}{44} + \frac{\sqrt{462}G_{yz} y (3x^4 - 8x^2y^2 + 6x^2z^2 + y^4 - 2y^2z^2)}{44}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,1)}[g](E, 3), \vec{\mathbb{Q}}_{4,2}^{(2,1)}[g](E, 3)$

\*\* symmetry

$$-\frac{\sqrt{5} (x - y)(x + y)(x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy (x^2 + y^2 - 6z^2)}{2}$$

\*\* expression

$$\begin{aligned}
& 21\sqrt{22}G_u xyz (x^2 + y^2 - 2z^2) + \frac{21\sqrt{66}G_v xyz (x - y)(x + y)}{44} - \frac{\sqrt{66}G_{xy} z (9x^4 - 24x^2y^2 - 10x^2z^2 + 9y^4 - 10y^2z^2 + 2z^4)}{44} \\
& + \frac{\sqrt{66}G_{xz} y (x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44} + \frac{\sqrt{66}G_{yz} x (x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44}
\end{aligned}$$

$$\begin{aligned}
& 21\sqrt{22}G_u z (x - y)(x + y)(x^2 + y^2 - 2z^2) + \frac{\sqrt{66}G_v z (3x^4 - 78x^2y^2 + 20x^2z^2 + 3y^4 + 20y^2z^2 - 4z^4)}{88} + \frac{21\sqrt{66}G_{xy} xyz (x - y)(x + y)}{44} \\
& + \frac{\sqrt{66}G_{xz} x (x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44} - \frac{\sqrt{66}G_{yz} y (x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44}
\end{aligned}$$