

\* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

| No. | tag   | matrix (polar)  | det | TR |
|-----|---|---|-----|----|
| 1   | {1 0}   | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                 | 1   | 1  |
| 2   | {2 <sub>001</sub>   $\frac{3}{4}\frac{3}{4}0$ }           | $\begin{bmatrix} -1 & 0 & 0 & \frac{3}{4} \\ 0 & -1 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & 0 \end{bmatrix}$           | 1   | 1  |
| 3   | {-1 0}  | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                              | -1  | 1  |
| 4   | {m <sub>001</sub>   $\frac{1}{4}\frac{1}{4}0$ }           | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & 0 \end{bmatrix}$            | -1  | 1  |
| 5   | {2 <sub>100</sub> '  $0\frac{3}{4}\frac{3}{4}$ }          | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$           | 1   | -1 |
| 6   | {2 <sub>010</sub> '  $\frac{3}{4}0\frac{3}{4}$ }          | $\begin{bmatrix} -1 & 0 & 0 & \frac{3}{4} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$           | 1   | -1 |
| 7   | {m <sub>100</sub> '  $0\frac{1}{4}\frac{1}{4}$ }          | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$            | -1  | -1 |
| 8   | {m <sub>010</sub> '  $\frac{1}{4}0\frac{1}{4}$ }          | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$            | -1  | -1 |
| 9   | {1 0 $\frac{1}{2}\frac{1}{2}$ }                           | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$             | 1   | 1  |
| 10  | {2 <sub>001</sub>   $\frac{3}{4}\frac{1}{4}\frac{1}{2}$ } | $\begin{bmatrix} -1 & 0 & 0 & \frac{3}{4} \\ 0 & -1 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | 1   | 1  |
| 11  | {-1 0 $\frac{1}{2}\frac{1}{2}$ }                          | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$          | -1  | 1  |
| 12  | {m <sub>001</sub>   $\frac{1}{4}\frac{3}{4}\frac{1}{2}$ } | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$  | -1  | 1  |
| 13  | {2 <sub>100</sub> '  $0\frac{1}{4}\frac{1}{4}$ }          | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$           | 1   | -1 |

continued ...

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| No. | tag  | matrix (polar)  | det | TR |
|-----|--|---|-----|----|
| 14  | $\{2_{010}' \frac{3}{4}\frac{1}{2}\frac{1}{4}\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$ | 1   | -1 |
| 15  | $\{m_{100}' 0\frac{3}{4}\frac{3}{4}\}$           | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$            | -1  | -1 |
| 16  | $\{m_{010}' \frac{1}{4}\frac{1}{2}\frac{3}{4}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$  | -1  | -1 |
| 17  | $\{1 \frac{1}{2}0\frac{1}{2}\}$                  | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$             | 1   | 1  |
| 18  | $\{2_{001} \frac{1}{4}\frac{3}{4}\frac{1}{2}\}$  | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{4} \\ 0 & -1 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | 1   | 1  |
| 19  | $\{-1 \frac{1}{2}0\frac{1}{2}\}$                 | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$          | -1  | 1  |
| 20  | $\{m_{001} \frac{3}{4}\frac{1}{4}\frac{1}{2}\}$  | $\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$  | -1  | 1  |
| 21  | $\{2_{100}' \frac{1}{2}\frac{3}{4}\frac{1}{4}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$ | 1   | -1 |
| 22  | $\{2_{010}' \frac{1}{4}0\frac{1}{4}\}$           | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$           | 1   | -1 |
| 23  | $\{m_{100}' \frac{1}{2}\frac{1}{4}\frac{3}{4}\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$  | -1  | -1 |
| 24  | $\{m_{010}' \frac{3}{4}0\frac{3}{4}\}$           | $\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$            | -1  | -1 |
| 25  | $\{1 \frac{1}{2}\frac{1}{2}0\}$                  | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$             | 1   | 1  |
| 26  | $\{2_{001} \frac{1}{4}\frac{1}{4}0\}$            | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{4} \\ 0 & -1 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & 0 \end{bmatrix}$           | 1   | 1  |
| 27  | $\{-1 \frac{1}{2}\frac{1}{2}0\}$                 | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$          | -1  | 1  |

continued ...

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| No. | tag  | matrix (polar)  | det | TR |
|-----|--|---|-----|----|
| 28  | $\{m_{001}   \frac{3}{4} \frac{3}{4} 0\}$            | $\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & 0 \end{bmatrix}$            | -1  | 1  |
| 29  | $\{2_{100}'   \frac{1}{2} \frac{1}{4} \frac{3}{4}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$ | 1   | -1 |
| 30  | $\{2_{010}'   \frac{1}{4} \frac{1}{2} \frac{3}{4}\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$ | 1   | -1 |
| 31  | $\{m_{100}'   \frac{1}{2} \frac{3}{4} \frac{1}{4}\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$  | -1  | -1 |
| 32  | $\{m_{010}'   \frac{3}{4} \frac{1}{2} \frac{1}{4}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$  | -1  | -1 |