

Model for “Cs1”

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General Condition

- Basis type: **lgs**
- SAMB selection:
 - Type: **[Q, G]**
 - Rank: **[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]**
 - Irrep.: **[A', A'']**
 - Spin (s): **[0, 1]**
- Atomic selection:
 - Type: **[Q, G, M, T]**
 - Rank: **[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]**
 - Irrep.: **[A', A'']**
 - Spin (s): **[0, 1]**
- Site-cluster selection:
 - Rank: **[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]**
 - Irrep.: **[A', A'']**
- Bond-cluster selection:
 - Type: **[Q, G, M, T]**
 - Rank: **[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]**
 - Irrep.: **[A', A'']**
- Max. neighbor: **10**
- Search cell range: **(-2, 3), (-2, 3), (-2, 3)**
- Toroidal priority: **false**

Group and Unit Cell

- Group: SG No. 6 C_s^1 Pm (b-axis setting) [monoclinic]
- Associated point group: PG No. 6 C_s m (b-axis setting) [monoclinic]
- Unit cell:
 - $a = 1.00000$, $b = 1.00000$, $c = 1.00000$, $\alpha = 90.0$, $\beta = 90.0$, $\gamma = 90.0$
- Lattice vectors (conventional cell):
 - $\mathbf{a}_1 = [1.00000, 0.00000, 0.00000]$
 - $\mathbf{a}_2 = [0.00000, 1.00000, 0.00000]$
 - $\mathbf{a}_3 = [0.00000, 0.00000, 1.00000]$

Symmetry Operation

Table 1: Symmetry operation

| # | SO | # | SO | # | SO | # | SO |
|---|-----------|---|-----------------|---|----|---|----|
| 1 | $\{1 0\}$ | 2 | $\{m_{010} 0\}$ | | | | |

Harmonics

Table 2: Harmonics

| # | symbol | irrep. | rank | X | multiplicity | component | symmetry |
|----|-----------------------|--------|------|--------|--------------|-----------|--|
| 1 | $\mathbb{Q}_0(A')$ | A' | 0 | Q, T | - | - | 1 |
| 2 | $\mathbb{G}_1(A')$ | A' | 1 | G, M | - | - | y |
| 3 | $\mathbb{Q}_1(A', 1)$ | A' | 1 | Q, T | 1 | - | x |
| 4 | $\mathbb{Q}_1(A', 2)$ | A' | 1 | Q, T | 2 | - | z |
| 5 | $\mathbb{Q}_2(A', 1)$ | A' | 2 | Q, T | 1 | - | $-\frac{x^2}{2} - \frac{y^2}{2} + z^2$ |
| 6 | $\mathbb{Q}_2(A', 2)$ | A' | 2 | Q, T | 2 | - | $\frac{\sqrt{3}(x-y)(x+y)}{2}$ |
| 7 | $\mathbb{Q}_2(A', 3)$ | A' | 2 | Q, T | 3 | - | $\sqrt{3}xz$ |
| 8 | $\mathbb{G}_3(A', 1)$ | A' | 3 | G, M | 1 | - | $\sqrt{15}xyz$ |
| 9 | $\mathbb{G}_3(A', 2)$ | A' | 3 | G, M | 2 | - | $-\frac{y(3x^2-2y^2+3z^2)}{2}$ |
| 10 | $\mathbb{G}_3(A', 3)$ | A' | 3 | G, M | 3 | - | $-\frac{\sqrt{15}y(x-z)(x+z)}{2}$ |

continued ...

Table 2

| # | symbol | irrep. | rank | X | multiplicity | component | symmetry |
|----|------------------------|--------|------|--------|--------------|-----------|--|
| 11 | $\mathbb{Q}_3(A', 1)$ | A' | 3 | Q, T | 1 | - | $\frac{x(2x^2-3y^2-3z^2)}{2}$ |
| 12 | $\mathbb{Q}_3(A', 2)$ | A' | 3 | Q, T | 2 | - | $-\frac{z(3x^2+3y^2-2z^2)}{2}$ |
| 13 | $\mathbb{Q}_3(A', 4)$ | A' | 3 | Q, T | 4 | - | $\frac{\sqrt{15}z(x-y)(x+y)}{2}$ |
| 14 | $\mathbb{G}_0(A'')$ | A'' | 0 | G, M | - | - | 1 |
| 15 | $\mathbb{G}_1(A'', 1)$ | A'' | 1 | G, M | 1 | - | x |
| 16 | $\mathbb{G}_1(A'', 2)$ | A'' | 1 | G, M | 2 | - | z |
| 17 | $\mathbb{Q}_1(A'')$ | A'' | 1 | Q, T | - | - | y |
| 18 | $\mathbb{G}_2(A'', 1)$ | A'' | 2 | G, M | 1 | - | $-\frac{x^2}{2} - \frac{y^2}{2} + z^2$ |
| 19 | $\mathbb{Q}_2(A'', 1)$ | A'' | 2 | Q, T | 1 | - | $\sqrt{3}yz$ |
| 20 | $\mathbb{Q}_2(A'', 2)$ | A'' | 2 | Q, T | 2 | - | $\sqrt{3}xy$ |
| 21 | $\mathbb{G}_3(A'', 1)$ | A'' | 3 | G, M | 1 | - | $\frac{x(2x^2-3y^2-3z^2)}{2}$ |
| 22 | $\mathbb{G}_3(A'', 3)$ | A'' | 3 | G, M | 3 | - | $\frac{\sqrt{15}x(y-z)(y+z)}{2}$ |
| 23 | $\mathbb{G}_3(A'', 4)$ | A'' | 3 | G, M | 4 | - | $\frac{\sqrt{15}z(x-y)(x+y)}{2}$ |
| 24 | $\mathbb{Q}_3(A'', 1)$ | A'' | 3 | Q, T | 1 | - | $\sqrt{15}xyz$ |
| 25 | $\mathbb{Q}_3(A'', 3)$ | A'' | 3 | Q, T | 3 | - | $-\frac{\sqrt{15}y(x-z)(x+z)}{2}$ |

— Basis in full matrix —

Table 3: dimension = 4

| # | orbital@atom(SL) | # | orbital@atom(SL) | # | orbital@atom(SL) | # | orbital@atom(SL) |
|---|-------------------------------|---|---------------------------------|---|-------------------------------|---|---------------------------------|
| 0 | $ p_x, \uparrow\rangle @A(1)$ | 1 | $ p_x, \downarrow\rangle @A(1)$ | 2 | $ p_y, \uparrow\rangle @A(1)$ | 3 | $ p_y, \downarrow\rangle @A(1)$ |

Table 4: Atomic basis (orbital part only)

| orbital | definition |
|---------------|------------|
| $ p_x\rangle$ | x |
| $ p_y\rangle$ | y |
| $ p_z\rangle$ | z |

— **SAMB: 70 (all 150)** —

• **A : 'A' site-cluster**

* bra: $\langle p_x, \uparrow |, \langle p_x, \downarrow |, \langle p_y, \uparrow |, \langle p_y, \downarrow |$

* ket: $|p_x, \uparrow\rangle, |p_x, \downarrow\rangle, |p_y, \uparrow\rangle, |p_y, \downarrow\rangle$

* wyckoff: **1a**

$$\boxed{\text{z1}} \quad \mathbb{Q}_0^{(c)}(A') = \mathbb{Q}_0^{(a)}(A')\mathbb{Q}_0^{(s)}(A')$$

$$\boxed{\text{z2}} \quad \mathbb{Q}_2^{(c)}(A', 2) = \mathbb{Q}_2^{(a)}(A', 2)\mathbb{Q}_0^{(s)}(A')$$

$$\boxed{\text{z3}} \quad \mathbb{Q}_2^{(1, -1; c)}(A', 1) = \mathbb{Q}_2^{(1, -1; a)}(A', 1)\mathbb{Q}_0^{(s)}(A')$$

$$\boxed{\text{z4}} \quad \mathbb{Q}_2^{(1, -1; c)}(A', 3) = \mathbb{Q}_2^{(1, -1; a)}(A', 3)\mathbb{Q}_0^{(s)}(A')$$

$$\boxed{\text{z79}} \quad \mathbb{Q}_2^{(c)}(A'', 2) = \mathbb{Q}_2^{(a)}(A'', 2) \mathbb{Q}_0^{(s)}(A')$$

$$\boxed{\text{z80}} \quad \mathbb{Q}_2^{(1, -1; c)}(A'', 1) = \mathbb{Q}_2^{(1, -1; a)}(A'', 1) \mathbb{Q}_0^{(s)}(A')$$

• **A;A_001_1** : 'A'-'A' bond-cluster

* bra: $\langle p_x, \uparrow |, \langle p_x, \downarrow |, \langle p_y, \uparrow |, \langle p_y, \downarrow |$

* ket: $|p_x, \uparrow \rangle, |p_x, \downarrow \rangle, |p_y, \uparrow \rangle, |p_y, \downarrow \rangle$

* wyckoff: **1a@1a**

$$\boxed{\text{z5}} \quad \mathbb{Q}_0^{(c)}(A') = \mathbb{Q}_0^{(a)}(A') \mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z6}} \quad \mathbb{Q}_2^{(c)}(A', 2) = \mathbb{Q}_2^{(a)}(A', 2) \mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z7}} \quad \mathbb{Q}_2^{(1, -1; c)}(A', 1) = \mathbb{Q}_2^{(1, -1; a)}(A', 1) \mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z8}} \quad \mathbb{Q}_2^{(1, -1; c)}(A', 3) = \mathbb{Q}_2^{(1, -1; a)}(A', 3) \mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z9}} \quad \mathbb{G}_1^{(1, -1; c)}(A') = \mathbb{M}_1^{(1, -1; a)}(A') \mathbb{T}_0^{(b)}(A')$$

$$\boxed{\text{z10}} \quad \mathbb{G}_3^{(1, -1; c)}(A', 1) = \mathbb{M}_3^{(1, -1; a)}(A', 1) \mathbb{T}_0^{(b)}(A')$$

$$\boxed{\text{z11}} \quad \mathbb{G}_3^{(1, -1; c)}(A', 2) = \mathbb{M}_3^{(1, -1; a)}(A', 2) \mathbb{T}_0^{(b)}(A')$$

$$\boxed{\text{z12}} \quad \mathbb{G}_3^{(1, -1; c)}(A', 3) = \mathbb{M}_3^{(1, -1; a)}(A', 3) \mathbb{T}_0^{(b)}(A')$$

$$\boxed{\text{z81}} \quad \mathbb{Q}_2^{(c)}(A'', 2) = \mathbb{Q}_2^{(a)}(A'', 2) \mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z82}} \quad \mathbb{Q}_2^{(1, -1; c)}(A'', 1) = \mathbb{Q}_2^{(1, -1; a)}(A'', 1) \mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z83}} \quad \mathbb{G}_1^{(c)}(A'', 2) = \mathbb{M}_1^{(a)}(A'', 2) \mathbb{T}_0^{(b)}(A')$$

$$\boxed{\text{z84}} \quad \mathbb{G}_1^{(1, -1; c)}(A'', 1) = \mathbb{M}_1^{(1, -1; a)}(A'', 1) \mathbb{T}_0^{(b)}(A')$$

$$\boxed{\text{z85}} \quad \mathbb{G}_1^{(1, -1; c)}(A'', 2) = \mathbb{M}_1^{(1, -1; a)}(A'', 2) \mathbb{T}_0^{(b)}(A')$$

$$\boxed{\text{z86}} \quad \mathbb{G}_3^{(1, -1; c)}(A'', 1) = \mathbb{M}_3^{(1, -1; a)}(A'', 1) \mathbb{T}_0^{(b)}(A')$$

$$\boxed{\text{z87}} \quad \mathbb{G}_3^{(1,-1;c)}(A'', 3) = \mathbb{M}_3^{(1,-1;a)}(A'', 3) \mathbb{T}_0^{(b)}(A')$$

$$\boxed{\text{z88}} \quad \mathbb{G}_3^{(1,-1;c)}(A'', 4) = \mathbb{M}_3^{(1,-1;a)}(A'', 4) \mathbb{T}_0^{(b)}(A')$$

* common SAMBs

(A;A_001_1, A;A_001_3, A;A_002_2, A;A_002_3), (z5, z23, z47, z55), (z6, z24, z48, z56), (z7, z25, z49, z57), (z8, z26, z50, z58), (z9, z27, z51, z59), (z10, z28, z52, z60), (z11, z29, z53, z61), (z12, z30, z54, z62), (z81, z95, z119, z127), (z82, z96, z120, z128), (z83, z97, z121, z129), (z84, z98, z122, z130), (z85, z99, z123, z131), (z86, z100, z124, z132), (z87, z101, z125, z133), (z88, z102, z126, z134)

• A;A_001_2 : 'A'-'A' bond-cluster

* bra: $\langle p_x, \uparrow |, \langle p_x, \downarrow |, \langle p_y, \uparrow |, \langle p_y, \downarrow |$

* ket: $|p_x, \uparrow \rangle, |p_x, \downarrow \rangle, |p_y, \uparrow \rangle, |p_y, \downarrow \rangle$

* wyckoff: 1b@1b

$$\boxed{\text{z13}} \quad \mathbb{Q}_0^{(c)}(A') = \mathbb{Q}_0^{(a)}(A') \mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z14}} \quad \mathbb{Q}_1^{(c)}(A', 1) = -\mathbb{M}_1^{(a)}(A'', 2) \mathbb{T}_1^{(b)}(A'')$$

$$\boxed{\text{z15}} \quad \mathbb{Q}_2^{(c)}(A', 2) = \mathbb{Q}_2^{(a)}(A', 2) \mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z16}} \quad \mathbb{Q}_1^{(1,-1;c)}(A', 1) = -\mathbb{M}_1^{(1,-1;a)}(A'', 2) \mathbb{T}_1^{(b)}(A'')$$

$$\boxed{\text{z17}} \quad \mathbb{Q}_1^{(1,-1;c)}(A', 2) = \mathbb{M}_1^{(1,-1;a)}(A'', 1) \mathbb{T}_1^{(b)}(A'')$$

$$\boxed{\text{z18}} \quad \mathbb{Q}_2^{(1,-1;c)}(A', 1) = \mathbb{Q}_2^{(1,-1;a)}(A', 1) \mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z19}} \quad \mathbb{Q}_2^{(1,-1;c)}(A', 3) = \mathbb{Q}_2^{(1,-1;a)}(A', 3) \mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z20}} \quad \mathbb{Q}_3^{(1,-1;c)}(A', 1) = -\mathbb{M}_3^{(1,-1;a)}(A'', 4) \mathbb{T}_1^{(b)}(A'')$$

$$\boxed{\text{z21}} \quad \mathbb{Q}_3^{(1,-1;c)}(A', 2) = -\frac{\sqrt{6}\mathbb{M}_3^{(1,-1;a)}(A'', 1) \mathbb{T}_1^{(b)}(A'')}{4} - \frac{\sqrt{10}\mathbb{M}_3^{(1,-1;a)}(A'', 3) \mathbb{T}_1^{(b)}(A'')}{4}$$

$$\boxed{\text{z22}} \quad \mathbb{Q}_3^{(1,-1;c)}(A', 4) = \frac{\sqrt{10}\mathbb{M}_3^{(1,-1;a)}(A'', 1) \mathbb{T}_1^{(b)}(A'')}{4} - \frac{\sqrt{6}\mathbb{M}_3^{(1,-1;a)}(A'', 3) \mathbb{T}_1^{(b)}(A'')}{4}$$

$$\boxed{\text{z89}} \quad \mathbb{Q}_2^{(c)}(A'', 2) = \mathbb{Q}_2^{(a)}(A'', 2) \mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z90}} \quad \mathbb{Q}_2^{(1,-1;c)}(A'', 1) = \mathbb{Q}_2^{(1,-1;a)}(A'', 1) \mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z91}} \quad \mathbb{Q}_3^{(1,-1;c)}(A'', 1) = -\mathbb{M}_3^{(1,-1;a)}(A', 3)\mathbb{T}_1^{(b)}(A'')$$

$$\boxed{\text{z92}} \quad \mathbb{Q}_3^{(1,-1;c)}(A'', 3) = \mathbb{M}_3^{(1,-1;a)}(A', 1)\mathbb{T}_1^{(b)}(A'')$$

$$\boxed{\text{z93}} \quad \mathbb{G}_0^{(1,-1;c)}(A'') = \mathbb{M}_1^{(1,-1;a)}(A')\mathbb{T}_1^{(b)}(A'')$$

$$\boxed{\text{z94}} \quad \mathbb{G}_2^{(1,-1;c)}(A'', 1) = -\mathbb{M}_3^{(1,-1;a)}(A', 2)\mathbb{T}_1^{(b)}(A'')$$

• **A;A_002_1** : 'A'-'A' bond-cluster

* bra: $\langle p_x, \uparrow |, \langle p_x, \downarrow |, \langle p_y, \uparrow |, \langle p_y, \downarrow |$

* ket: $|p_x, \uparrow \rangle, |p_x, \downarrow \rangle, |p_y, \uparrow \rangle, |p_y, \downarrow \rangle$

* wyckoff: **2c@1b**

$$\boxed{\text{z31}} \quad \mathbb{Q}_0^{(c)}(A') = \mathbb{Q}_0^{(a)}(A')\mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z32}} \quad \mathbb{Q}_1^{(c)}(A', 1a) = \mathbb{Q}_2^{(a)}(A'', 2)\mathbb{Q}_1^{(b)}(A'')$$

$$\boxed{\text{z33}} \quad \mathbb{Q}_1^{(c)}(A', 1b) = -\mathbb{M}_1^{(a)}(A'', 2)\mathbb{T}_1^{(b)}(A'')$$

$$\boxed{\text{z34}} \quad \mathbb{Q}_2^{(c)}(A', 2) = \mathbb{Q}_2^{(a)}(A', 2)\mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z35}} \quad \mathbb{Q}_1^{(1,-1;c)}(A', 1) = -\mathbb{M}_1^{(1,-1;a)}(A'', 2)\mathbb{T}_1^{(b)}(A'')$$

$$\boxed{\text{z36}} \quad \mathbb{Q}_1^{(1,-1;c)}(A', 2a) = \mathbb{Q}_2^{(1,-1;a)}(A'', 1)\mathbb{Q}_1^{(b)}(A'')$$

$$\boxed{\text{z37}} \quad \mathbb{Q}_1^{(1,-1;c)}(A', 2b) = \mathbb{M}_1^{(1,-1;a)}(A'', 1)\mathbb{T}_1^{(b)}(A'')$$

$$\boxed{\text{z38}} \quad \mathbb{Q}_2^{(1,-1;c)}(A', 1) = \mathbb{Q}_2^{(1,-1;a)}(A', 1)\mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z39}} \quad \mathbb{Q}_2^{(1,-1;c)}(A', 3) = \mathbb{Q}_2^{(1,-1;a)}(A', 3)\mathbb{Q}_0^{(b)}(A')$$

$$\boxed{\text{z40}} \quad \mathbb{Q}_3^{(1,-1;c)}(A', 1) = -\mathbb{M}_3^{(1,-1;a)}(A'', 4)\mathbb{T}_1^{(b)}(A'')$$

$$\boxed{\text{z41}} \quad \mathbb{Q}_3^{(1,-1;c)}(A', 2) = -\frac{\sqrt{6}\mathbb{M}_3^{(1,-1;a)}(A'', 1)\mathbb{T}_1^{(b)}(A'')}{4} - \frac{\sqrt{10}\mathbb{M}_3^{(1,-1;a)}(A'', 3)\mathbb{T}_1^{(b)}(A'')}{4}$$

$$\boxed{\text{z42}} \quad \mathbb{Q}_3^{(1,-1;c)}(A', 4) = \frac{\sqrt{10}\mathbb{M}_3^{(1,-1;a)}(A'', 1)\mathbb{T}_1^{(b)}(A'')}{4} - \frac{\sqrt{6}\mathbb{M}_3^{(1,-1;a)}(A'', 3)\mathbb{T}_1^{(b)}(A'')}{4}$$

$$\begin{aligned}
\boxed{\text{z43}} \quad \mathbb{G}_1^{(1,-1;c)}(A') &= \mathbb{M}_1^{(1,-1;a)}(A')\mathbb{T}_0^{(b)}(A') \\
\boxed{\text{z44}} \quad \mathbb{G}_3^{(1,-1;c)}(A', 1) &= \mathbb{M}_3^{(1,-1;a)}(A', 1)\mathbb{T}_0^{(b)}(A') \\
\boxed{\text{z45}} \quad \mathbb{G}_3^{(1,-1;c)}(A', 2) &= \mathbb{M}_3^{(1,-1;a)}(A', 2)\mathbb{T}_0^{(b)}(A') \\
\boxed{\text{z46}} \quad \mathbb{G}_3^{(1,-1;c)}(A', 3) &= \mathbb{M}_3^{(1,-1;a)}(A', 3)\mathbb{T}_0^{(b)}(A') \\
\boxed{\text{z103}} \quad \mathbb{Q}_1^{(c)}(A'', a) &= \mathbb{Q}_0^{(a)}(A')\mathbb{Q}_1^{(b)}(A'') \\
\boxed{\text{z104}} \quad \mathbb{Q}_1^{(c)}(A'', b) &= -\mathbb{Q}_2^{(a)}(A', 2)\mathbb{Q}_1^{(b)}(A'') \\
\boxed{\text{z105}} \quad \mathbb{Q}_2^{(c)}(A'', 2) &= \mathbb{Q}_2^{(a)}(A'', 2)\mathbb{Q}_0^{(b)}(A') \\
\boxed{\text{z106}} \quad \mathbb{Q}_1^{(1,-1;c)}(A'') &= -\mathbb{Q}_2^{(1,-1;a)}(A', 1)\mathbb{Q}_1^{(b)}(A'') \\
\boxed{\text{z107}} \quad \mathbb{Q}_2^{(1,-1;c)}(A'', 1) &= \mathbb{Q}_2^{(1,-1;a)}(A'', 1)\mathbb{Q}_0^{(b)}(A') \\
\boxed{\text{z108}} \quad \mathbb{Q}_3^{(1,-1;c)}(A'', 1a) &= \mathbb{Q}_2^{(1,-1;a)}(A', 3)\mathbb{Q}_1^{(b)}(A'') \\
\boxed{\text{z109}} \quad \mathbb{Q}_3^{(1,-1;c)}(A'', 1b) &= -\mathbb{M}_3^{(1,-1;a)}(A', 3)\mathbb{T}_1^{(b)}(A'') \\
\boxed{\text{z110}} \quad \mathbb{Q}_3^{(1,-1;c)}(A'', 3) &= \mathbb{M}_3^{(1,-1;a)}(A', 1)\mathbb{T}_1^{(b)}(A'') \\
\boxed{\text{z111}} \quad \mathbb{G}_1^{(c)}(A'', 2) &= \mathbb{M}_1^{(a)}(A'', 2)\mathbb{T}_0^{(b)}(A') \\
\boxed{\text{z112}} \quad \mathbb{G}_0^{(1,-1;c)}(A'') &= \mathbb{M}_1^{(1,-1;a)}(A')\mathbb{T}_1^{(b)}(A'') \\
\boxed{\text{z113}} \quad \mathbb{G}_1^{(1,-1;c)}(A'', 1) &= \mathbb{M}_1^{(1,-1;a)}(A'', 1)\mathbb{T}_0^{(b)}(A') \\
\boxed{\text{z114}} \quad \mathbb{G}_1^{(1,-1;c)}(A'', 2) &= \mathbb{M}_1^{(1,-1;a)}(A'', 2)\mathbb{T}_0^{(b)}(A') \\
\boxed{\text{z115}} \quad \mathbb{G}_2^{(1,-1;c)}(A'', 1) &= -\mathbb{M}_3^{(1,-1;a)}(A', 2)\mathbb{T}_1^{(b)}(A'') \\
\boxed{\text{z116}} \quad \mathbb{G}_3^{(1,-1;c)}(A'', 1) &= \mathbb{M}_3^{(1,-1;a)}(A'', 1)\mathbb{T}_0^{(b)}(A') \\
\boxed{\text{z117}} \quad \mathbb{G}_3^{(1,-1;c)}(A'', 3) &= \mathbb{M}_3^{(1,-1;a)}(A'', 3)\mathbb{T}_0^{(b)}(A')
\end{aligned}$$

$$\boxed{\text{z118}} \quad \mathbb{G}_3^{(1,-1;c)}(A'',4) = \mathbb{M}_3^{(1,-1;a)}(A'',4)\mathbb{T}_0^{(b)}(A')$$

* common SAMBs

(A;A_002_1, A;A_002_4), (z31, z63), (z32, z64), (z33, z65), (z34, z66), (z35, z67), (z36, z68), (z37, z69), (z38, z70), (z39, z71), (z40, z72), (z41, z73), (z42, z74), (z43, z75), (z44, z76), (z45, z77), (z46, z78), (z103, z135), (z104, z136), (z105, z137), (z106, z138), (z107, z139), (z108, z140), (z109, z141), (z110, z142), (z111, z143), (z112, z144), (z113, z145), (z114, z146), (z115, z147), (z116, z148), (z117, z149), (z118, z150)

Atomic SAMB

- bra: $\langle p_x, \uparrow |, \langle p_x, \downarrow |, \langle p_y, \uparrow |, \langle p_y, \downarrow |$
- ket: $|p_x, \uparrow \rangle, |p_x, \downarrow \rangle, |p_y, \uparrow \rangle, |p_y, \downarrow \rangle$

$$\boxed{\text{x1}} \quad \mathbb{Q}_0^{(a)}(A') = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$\boxed{\text{x2}} \quad \mathbb{Q}_2^{(a)}(A',2) = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

$$\boxed{\text{x3}} \quad \mathbb{Q}_2^{(a)}(A'',2) = \begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

$$\boxed{\text{x4}} \quad \mathbb{Q}_2^{(1,-1;a)}(A',1) = \begin{bmatrix} 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & \frac{i}{2} \\ \frac{i}{2} & 0 & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & 0 \end{bmatrix}$$

$$\boxed{\text{x5}} \quad \mathbb{Q}_2^{(1,-1;a)}(A', 3) = \begin{bmatrix} 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & -\frac{i}{2} & 0 \\ 0 & \frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\text{x6}} \quad \mathbb{Q}_2^{(1,-1;a)}(A'', 1) = \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\text{x7}} \quad \mathbb{M}_1^{(a)}(A'', 2) = \begin{bmatrix} 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & -\frac{i}{2} \\ \frac{i}{2} & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 \end{bmatrix}$$

$$\boxed{\text{x8}} \quad \mathbb{M}_1^{(1,-1;a)}(A') = \begin{bmatrix} 0 & -\frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & \frac{i}{2} & 0 \end{bmatrix}$$

$$\boxed{\text{x9}} \quad \mathbb{M}_3^{(1,-1;a)}(A', 1) = \begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \end{bmatrix}$$

$$\boxed{\text{x10}} \quad \mathbb{M}_3^{(1,-1;a)}(A', 2) = \begin{bmatrix} 0 & \frac{3\sqrt{13}i}{26} & 0 & -\frac{\sqrt{13}}{13} \\ -\frac{3\sqrt{13}i}{26} & 0 & -\frac{\sqrt{13}}{13} & 0 \\ 0 & -\frac{\sqrt{13}}{13} & 0 & -\frac{3\sqrt{13}i}{26} \\ -\frac{\sqrt{13}}{13} & 0 & \frac{3\sqrt{13}i}{26} & 0 \end{bmatrix}$$

$$\boxed{\text{x11}} \quad \mathbb{M}_3^{(1,-1;a)}(A', 3) = \begin{bmatrix} 0 & -\frac{\sqrt{13}i}{13} & 0 & -\frac{3\sqrt{13}}{26} \\ \frac{\sqrt{13}i}{13} & 0 & -\frac{3\sqrt{13}}{26} & 0 \\ 0 & -\frac{3\sqrt{13}}{26} & 0 & \frac{\sqrt{13}i}{13} \\ -\frac{3\sqrt{13}}{26} & 0 & -\frac{\sqrt{13}i}{13} & 0 \end{bmatrix}$$

$$\boxed{\text{x12}} \quad \mathbb{M}_1^{(1,-1;a)}(A'', 1) = \begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

$$\boxed{\text{x13}} \quad \mathbb{M}_1^{(1,-1;a)}(A'', 2) = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

$$\boxed{\text{x14}} \quad \mathbb{M}_3^{(1,-1;a)}(A'', 1) = \begin{bmatrix} 0 & \frac{3\sqrt{13}}{26} & 0 & \frac{\sqrt{13}i}{13} \\ \frac{3\sqrt{13}}{26} & 0 & -\frac{\sqrt{13}i}{13} & 0 \\ 0 & \frac{\sqrt{13}i}{13} & 0 & -\frac{3\sqrt{13}}{26} \\ -\frac{\sqrt{13}i}{13} & 0 & -\frac{3\sqrt{13}}{26} & 0 \end{bmatrix}$$

$$\boxed{\text{x15}} \quad \mathbb{M}_3^{(1,-1;a)}(A'', 3) = \begin{bmatrix} 0 & \frac{\sqrt{13}}{13} & 0 & -\frac{3\sqrt{13}i}{26} \\ \frac{\sqrt{13}}{13} & 0 & \frac{3\sqrt{13}i}{26} & 0 \\ 0 & -\frac{3\sqrt{13}i}{26} & 0 & -\frac{\sqrt{13}}{13} \\ \frac{3\sqrt{13}i}{26} & 0 & -\frac{\sqrt{13}}{13} & 0 \end{bmatrix}$$

$$\boxed{\text{x16}} \quad \mathbb{M}_3^{(1,-1;a)}(A'', 4) = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

- Site cluster

** Wyckoff: **1a**

$$\boxed{\text{y1}} \quad \mathbb{Q}_0^{(s)}(A') = [1]$$

- Bond cluster

** Wyckoff: **1b@1b**

$$\boxed{\text{y2}} \quad \mathbb{Q}_0^{(s)}(A') = [1]$$

$$\boxed{\text{y3}} \quad \mathbb{T}_1^{(s)}(A'') = [i]$$

** Wyckoff: **1a@1a**

$$\boxed{\text{y4}} \quad \mathbb{Q}_0^{(s)}(A') = [1]$$

$$\boxed{\text{y5}} \quad \mathbb{T}_0^{(s)}(A') = [i]$$

** Wyckoff: **2c@1b**

$$\boxed{\text{y6}} \quad \mathbb{Q}_0^{(s)}(A') = \left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right]$$

$$\boxed{\text{y7}} \quad \mathbb{T}_0^{(s)}(A') = \left[\frac{\sqrt{2}i}{2}, \frac{\sqrt{2}i}{2} \right]$$

$$\boxed{\text{y8}} \quad \mathbb{Q}_1^{(s)}(A'') = \left[\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right]$$

$$\boxed{\text{y9}} \quad \mathbb{T}_1^{(s)}(A'') = \left[\frac{\sqrt{2}i}{2}, -\frac{\sqrt{2}i}{2} \right]$$

— Site and Bond —

Table 5: Orbital of each site

| # | site | orbital |
|---|------|--|
| 1 | A | $ p_x, \uparrow\rangle, p_x, \downarrow\rangle, p_y, \uparrow\rangle, p_y, \downarrow\rangle$ |

Table 6: Neighbor and bra-ket of each bond

| # | head | tail | neighbor | head (bra) | tail (ket) |
|---|------|------|----------|------------|------------|
| 1 | A | A | [1,2] | [p] | [p] |

— Site in Unit Cell —

Sites in (conventional) cell (no plus set), SL = sublattice

Table 7: 'A' (#1) site cluster (1a), m

| SL | position (s) | mapping |
|----|------------------------------|---------|
| 1 | [0.00000, 0.00000, 0.00000] | [1,2] |

— Bond in Unit Cell —

Bonds in (conventional) cell (no plus set): tail, head = (SL, plus set), (N)D = (non)directional (listed up to 5th neighbor at most)

Table 8: 1-th 'A'-'A' [1] (#1) bond cluster (1a@1a), D, $|\mathbf{v}|=1.0$ (cartesian)

| SL | vector (\mathbf{v}) | center (\mathbf{c}) | mapping | head | tail | \mathbf{R} (primitive) |
|----|------------------------------|------------------------------|---------|-------|-------|--------------------------|
| 1 | [-1.00000, 0.00000, 0.00000] | [0.50000, 0.00000, 0.00000] | [1,2] | (1,1) | (1,1) | [1,0,0] |

Table 9: 1-th 'A'-'A' [2] (#2) bond cluster (1b@1b), ND, $|\mathbf{v}|=1.0$ (cartesian)

| SL | vector (\mathbf{v}) | center (\mathbf{c}) | mapping | head | tail | \mathbf{R} (primitive) |
|----|------------------------------|------------------------------|---------|-------|-------|--------------------------|
| 1 | [0.00000,-1.00000, 0.00000] | [0.00000, 0.50000, 0.00000] | [1,-2] | (1,1) | (1,1) | [0,1,0] |

Table 10: 1-th 'A'-'A' [3] (#3) bond cluster (1a@1a), D, $|\mathbf{v}|=1.0$ (cartesian)

| SL | vector (\mathbf{v}) | center (\mathbf{c}) | mapping | head | tail | \mathbf{R} (primitive) |
|----|------------------------------|------------------------------|---------|-------|-------|--------------------------|
| 1 | [0.00000, 0.00000,-1.00000] | [0.00000, 0.00000, 0.50000] | [1,2] | (1,1) | (1,1) | [0,0,1] |

Table 11: 2-th 'A'-'A' [1] (#4) bond cluster (2c@1b), D, $|\mathbf{v}|=1.41421$ (cartesian)

| SL | vector (\mathbf{v}) | center (\mathbf{c}) | mapping | head | tail | \mathbf{R} (primitive) |
|----|------------------------------|------------------------------|---------|-------|-------|--------------------------|
| 1 | [-1.00000,-1.00000, 0.00000] | [0.50000, 0.50000, 0.00000] | [1] | (1,1) | (1,1) | [1,1,0] |
| 2 | [-1.00000, 1.00000, 0.00000] | [0.50000, 0.50000, 0.00000] | [2] | (1,1) | (1,1) | [1,-1,0] |

Table 12: 2-th 'A'-'A' [2] (#5) bond cluster (1a@1a), D, $|\mathbf{v}|=1.41421$ (cartesian)

| SL | vector (\mathbf{v}) | center (\mathbf{c}) | mapping | head | tail | \mathbf{R} (primitive) |
|----|-------------------------------|------------------------------|---------|-------|-------|--------------------------|
| 1 | [-1.00000, 0.00000, -1.00000] | [0.50000, 0.00000, 0.50000] | [1,2] | (1,1) | (1,1) | [1,0,1] |

Table 13: 2-th 'A'-'A' [3] (#6) bond cluster (1a@1a), D, $|\mathbf{v}|=1.41421$ (cartesian)

| SL | vector (\mathbf{v}) | center (\mathbf{c}) | mapping | head | tail | \mathbf{R} (primitive) |
|----|------------------------------|------------------------------|---------|-------|-------|--------------------------|
| 1 | [-1.00000, 0.00000, 1.00000] | [0.50000, 0.00000, 0.50000] | [1,2] | (1,1) | (1,1) | [1,0,-1] |

Table 14: 2-th 'A'-'A' [4] (#7) bond cluster (2c@1b), D, $|\mathbf{v}|=1.41421$ (cartesian)

| SL | vector (\mathbf{v}) | center (\mathbf{c}) | mapping | head | tail | \mathbf{R} (primitive) |
|----|--------------------------------|------------------------------|---------|-------|-------|--------------------------|
| 1 | [0.00000, -1.00000, -1.00000] | [0.00000, 0.50000, 0.50000] | [1] | (1,1) | (1,1) | [0,1,1] |
| 2 | [0.00000, 1.00000, -1.00000] | [0.00000, 0.50000, 0.50000] | [2] | (1,1) | (1,1) | [0,-1,1] |