Response Tensors up to 4th rank in $C_{3v} - 1$

— polar tensors —

$$C^{(0,Q)} = \left(C^{(0,Q)}\right)$$

$$C^{(0,Q)} = Q_0$$

$$C^{(1,Q)} = \begin{pmatrix} 0 & 0 & C_z^{(1,Q)} \end{pmatrix}$$

$$C_z^{(1,Q)} = Q_z$$

$$S^{(2,Q)} = \begin{pmatrix} S_{xx}^{(2,Q)} & 0 & 0 \\ 0 & S_{xx}^{(2,Q)} & 0 \\ 0 & 0 & S_{zz}^{(2,Q)} \end{pmatrix}$$

$$S_{xx}^{(2,Q)} = Q_0 - Q_u$$
$$S_{zz}^{(2,Q)} = Q_0 + 2Q_u$$

$$S^{(3,Q)} = \begin{pmatrix} S_{1x}^{(3,Q)} & 0 & S_{1z}^{(3,Q)} \\ -S_{1x}^{(3,Q)} & 0 & S_{2z}^{(3,Q)} \\ 0 & 0 & S_{3z}^{(3,Q)} \\ 0 & \frac{5S_{1z}^{(3,Q)}}{4} - \frac{7S_{2z}^{(3,Q)}}{4} + \frac{S_{3z}^{(3,Q)}}{2} & 0 \\ \frac{9S_{1z}^{(3,Q)}}{4} - \frac{11S_{2z}^{(3,Q)}}{4} + \frac{S_{3z}^{(3,Q)}}{2} & 0 & 0 \\ 0 & -S_{1x}^{(3,Q)} & 0 \end{pmatrix}$$

$$S_{1x}^{(3,Q)} = Q_3^\gamma$$
$$S_{1z}^{(3,Q)} = -3Q_3^\alpha + Q_z[2]$$
$$S_{2z}^{(3,Q)} = -Q_3^\alpha + Q_z[2]$$
$$S_{3z}^{(3,Q)} = 2Q_3^\alpha + 2Q_z[1] + Q_z[2]$$

$$A^{(3,Q)} = \begin{pmatrix} 0 & A_{4y}^{(3,Q)} & 0 \\ -A_{4y}^{(3,Q)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{4y}^{(3,Q)} = Q_z[3]$$

$$S^{(4,Q)} = \begin{pmatrix} S_{11}^{(4,Q)} & S_{12}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & S_{15}^{(4,Q)} & 0 \\ S_{12}^{(4,Q)} & S_{11}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & -S_{15}^{(4,Q)} & 0 \\ S_{13}^{(4,Q)} & S_{13}^{(4,Q)} & S_{33}^{(4,Q)} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 & -S_{15}^{(4,Q)} \\ S_{15}^{(4,Q)} & -S_{15}^{(4,Q)} & 0 & 0 & S_{44}^{(4,Q)} & 0 \\ 0 & 0 & 0 & -S_{15}^{(4,Q)} & 0 & \frac{S_{11}^{(4,Q)}}{2} - \frac{S_{12}^{(4,Q)}}{2} \end{pmatrix}$$

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$$S_{11}^{(4,Q)} = Q_0[1] + 2Q_0[2] + 3Q_4 - 2Q_u[1] - 4Q_u[2]$$

$$S_{12}^{(4,Q)} = Q_0[1] + Q_4 - 2Q_u[1]$$

$$S_{13}^{(4,Q)} = Q_0[1] - 4Q_4 + Q_u[1]$$

$$S_{15}^{(4,Q)} = Q_4^\alpha$$

$$S_{33}^{(4,Q)} = Q_0[1] + 2Q_0[2] + 8Q_4 + 4Q_u[1] + 8Q_u[2]$$

$$S_{44}^{(4,Q)} = Q_0[2] - 4Q_4 + Q_u[2]$$

$$\bar{S}^{(4,Q)} = \begin{pmatrix} 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & \bar{S}_{15}^{(4,Q)} & 0 \\ 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & -\bar{S}_{15}^{(4,Q)} & 0 \\ -\bar{S}_{13}^{(4,Q)} & -\bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{S}_{15}^{(4,Q)} \\ -\bar{S}_{15}^{(4,Q)} & \bar{S}_{15}^{(4,Q)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\bar{S}_{15}^{(4,Q)} & 0 & 0 \end{pmatrix}$$

$$\bar{S}_{13}^{(4,Q)} = 3Q_u[3]$$

$$\bar{S}_{15}^{(4,Q)} = -2G_3^\beta[1]$$

$$A^{(4,Q)} = \begin{pmatrix} A_{xx}^{(4,Q)} & 0 & 0 \\ 0 & A_{xx}^{(4,Q)} & 0 \\ 0 & 0 & A_{zz}^{(4,Q)} \end{pmatrix}$$

$$A_{xx}^{(4,Q)} = Q_0[3] - 2Q_u[6]$$

$$A_{zz}^{(4,Q)} = Q_0[3] + 4Q_u[6]$$

$$M^{(4,Q)} = \begin{pmatrix} 0 & M_{1y}^{(4,Q)} & 0 \\ 0 & -M_{1y}^{(4,Q)} & 0 \\ 0 & 0 & 0 \\ M_{4x}^{(4,Q)} & 0 & 0 \\ 0 & -M_{4x}^{(4,Q)} & 0 \\ M_{1y}^{(4,Q)} & 0 & 0 \end{pmatrix}$$

$$M_{1y}^{(4,Q)} = G_3^\beta[2]$$

$$M_{4x}^{(4,Q)} = -3Q_u[4]$$

$$\bar{M}^{(4,Q)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}_{x4}^{(4,Q)} & 0 & \bar{M}_{x6}^{(4,Q)} \\ \bar{M}_{x6}^{(4,Q)} & -\bar{M}_{x6}^{(4,Q)} & 0 & 0 & -\bar{M}_{x4}^{(4,Q)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{M}_{x4}^{(4,Q)} = -3Q_u[5]$$

$$\bar{M}_{x6}^{(4,Q)} = G_3^\beta[3]$$

— axial tensors —

$$A^{(2,G)} = \begin{pmatrix} 0 & A_{xy}^{(2,G)} & 0 \\ -A_{xy}^{(2,G)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{xy}^{(2,G)} = Q_z$$

$$S^{(3,G)} = \begin{pmatrix} 0 & S_{1y}^{(3,G)} & 0 \\ 0 & -S_{1y}^{(3,G)} & 0 \\ 0 & 0 & 0 \\ S_{4x}^{(3,G)} & 0 & 0 \\ 0 & -S_{4x}^{(3,G)} & 0 \\ S_{1y}^{(3,G)} & 0 & 0 \end{pmatrix}$$

$$S_{1y}^{(3,G)} = G_3^\beta$$
$$S_{4x}^{(3,G)} = -3Q_u[1]$$

$$A^{(3,G)} = \begin{pmatrix} A_{4x}^{(3,G)} & 0 & 0 \\ 0 & A_{4x}^{(3,G)} & 0 \\ 0 & 0 & A_{6z}^{(3,G)} \end{pmatrix}$$

$$A_{4x}^{(3,G)} = Q_0 - Q_u[2]$$
$$A_{6z}^{(3,G)} = Q_0 + 2Q_u[2]$$

$$S^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & S_{14}^{(4,G)} & 0 & 0 \\ 0 & 0 & 0 & -S_{14}^{(4,G)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ S_{14}^{(4,G)} & -S_{14}^{(4,G)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{14}^{(4,G)} \\ 0 & 0 & 0 & 0 & S_{14}^{(4,G)} & 0 \end{pmatrix}$$

$$S_{14}^{(4,G)} = G_4^\beta$$

$$\bar{S}^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & \bar{S}_{14}^{(4,G)} & 0 & \bar{S}_{16}^{(4,G)} \\ 0 & 0 & 0 & -\bar{S}_{14}^{(4,G)} & 0 & \bar{S}_{26}^{(4,G)} \\ 0 & 0 & 0 & 0 & 0 & -\bar{S}_{16}^{(4,G)} - \bar{S}_{26}^{(4,G)} \\ -\bar{S}_{14}^{(4,G)} & \bar{S}_{14}^{(4,G)} & 0 & 0 & \frac{7\bar{S}_{16}^{(4,G)}}{4} + \frac{9\bar{S}_{26}^{(4,G)}}{4} & 0 \\ 0 & 0 & 0 & -\frac{7\bar{S}_{16}^{(4,G)}}{4} - \frac{9\bar{S}_{26}^{(4,G)}}{4} & 0 & -\bar{S}_{14}^{(4,G)} \\ -\bar{S}_{16}^{(4,G)} & -\bar{S}_{26}^{(4,G)} & \bar{S}_{16}^{(4,G)} + \bar{S}_{26}^{(4,G)} & 0 & \bar{S}_{14}^{(4,G)} & 0 \end{pmatrix}$$

$$\bar{S}_{14}^{(4,G)} = 2Q_3^\gamma[1]$$
$$\bar{S}_{16}^{(4,G)} = -6Q_3^\alpha[1] + 2Q_z[1]$$
$$\bar{S}_{26}^{(4,G)} = 2Q_3^\alpha[1] - 2Q_z[1]$$

$$\bar{A}^{(4,G)} = \begin{pmatrix} 0 & \bar{A}_{xy}^{(4,G)} & 0 \\ -\bar{A}_{xy}^{(4,G)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\bar{A}_{xy}^{(4,G)} = Q_z[6]$$

$$M^{(4,G)} = \begin{pmatrix} M_{1x}^{(4,G)} & 0 & M_{1z}^{(4,G)} \\ -M_{1x}^{(4,G)} & 0 & M_{2z}^{(4,G)} \\ 0 & 0 & M_{3z}^{(4,G)} \\ 0 & \frac{5M_{1z}^{(4,G)}}{4} - \frac{7M_{2z}^{(4,G)}}{4} + \frac{M_{3z}^{(4,G)}}{2} & 0 \\ \frac{9M_{1z}^{(4,G)}}{4} - \frac{11M_{2z}^{(4,G)}}{4} + \frac{M_{3z}^{(4,G)}}{2} & 0 & 0 \\ 0 & -M_{1x}^{(4,G)} & 0 \end{pmatrix}$$

$$M_{1x}^{(4,G)} = Q_3^\gamma[2]$$
$$M_{1z}^{(4,G)} = -3Q_3^\alpha[2] + Q_z[3]$$
$$M_{2z}^{(4,G)} = -Q_3^\alpha[2] + Q_z[3]$$
$$M_{3z}^{(4,G)} = 2Q_3^\alpha[2] + 2Q_z[2] + Q_z[3]$$

$$\bar{M}^{(4,G)} = \begin{pmatrix} \bar{M}_{x1}^{(4,G)} & -\bar{M}_{x1}^{(4,G)} & 0 & 0 & \bar{M}_{x5}^{(4,G)} & 0 \\ 0 & 0 & 0 & \bar{M}_{y4}^{(4,G)} & 0 & -\bar{M}_{x1}^{(4,G)} \\ \bar{M}_{z1}^{(4,G)} & -\bar{M}_{x5}^{(4,G)} + \bar{M}_{y4}^{(4,G)} + \bar{M}_{z1}^{(4,G)} & -\frac{7\bar{M}_{x5}^{(4,G)}}{2} + \frac{11\bar{M}_{y4}^{(4,G)}}{2} + \bar{M}_{z1}^{(4,G)} & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{M}_{x1}^{(4,G)} = Q_3^\gamma[3]$$
$$\bar{M}_{x5}^{(4,G)} = -3Q_3^\alpha[3] + Q_z[4]$$
$$\bar{M}_{y4}^{(4,G)} = -Q_3^\alpha[3] + Q_z[4]$$
$$\bar{M}_{z1}^{(4,G)} = -3Q_3^\alpha[3] + Q_z[5]$$