Response Tensors up to 4th rank in C_{3h}

— polar tensors —

$$C^{(0,Q)} = \begin{pmatrix} C^{(0,Q)} \end{pmatrix}$$

$$C^{(0,Q)} = Q_0$$

$$S^{(2,Q)} = \begin{pmatrix} S^{(2,Q)}_{xx} & 0 & 0 \\ 0 & S^{(2,Q)}_{xx} & 0 \\ 0 & 0 & S^{(2,Q)}_{zz} \end{pmatrix}$$

$$S^{(2,Q)}_{xx} = Q_0 - Q_u$$
$$S^{(2,Q)}_{zz} = Q_0 + 2Q_u$$

$$A^{(2,Q)} = \begin{pmatrix} 0 & A^{(2,Q)}_{xy} & 0 \\ -A^{(2,Q)}_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A^{(2,Q)}_{xy} = G_z$$

$$S^{(3,Q)} = \begin{pmatrix} S^{(3,Q)}_{1x} & S^{(3,Q)}_{1y} & 0 \\ -S^{(3,Q)}_{1x} & -S^{(3,Q)}_{1y} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ S^{(3,Q)}_{1y} & -S^{(3,Q)}_{1x} & 0 \end{pmatrix}$$

$$S^{(3,Q)}_{1x} = Q_3^{\gamma}$$
$$S^{(3,Q)}_{1y} = Q_3^{\beta}$$

$$S^{(4,Q)} = \begin{pmatrix} S^{(4,Q)}_{11} & S^{(4,Q)}_{12} & S^{(4,Q)}_{13} & 0 & 0 & 0 \\ S^{(4,Q)}_{12} & S^{(4,Q)}_{11} & S^{(4,Q)}_{13} & 0 & 0 & 0 \\ S^{(4,Q)}_{13} & S^{(4,Q)}_{13} & S^{(4,Q)}_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S^{(4,Q)}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S^{(4,Q)}_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{S^{(4,Q)}_{11}}{2} - \frac{S^{(4,Q)}_{12}}{2} \end{pmatrix}$$

$$S^{(4,Q)}_{11} = Q_0[1] + 2Q_0[2] + 3Q_4 - 2Q_u[1] - 4Q_u[2]$$
$$S^{(4,Q)}_{12} = Q_0[1] + Q_4 - 2Q_u[1]$$
$$S^{(4,Q)}_{13} = Q_0[1] - 4Q_4 + Q_u[1]$$
$$S^{(4,Q)}_{33} = Q_0[1] + 2Q_0[2] + 8Q_4 + 4Q_u[1] + 8Q_u[2]$$
$$S^{(4,Q)}_{44} = Q_0[2] - 4Q_4 + Q_u[2]$$

$$\bar{S}^{(4,Q)} = \begin{pmatrix} 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & 0 & \bar{S}_{16}^{(4,Q)} \\ 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & 0 & \bar{S}_{26}^{(4,Q)} \\ -\bar{S}_{13}^{(4,Q)} & -\bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 & -\bar{S}_{16}^{(4,Q)} - \bar{S}_{26}^{(4,Q)} \\ 0 & 0 & 0 & 0 & \frac{7\bar{S}_{16}^{(4,Q)}}{4} + \frac{9\bar{S}_{26}^{(4,Q)}}{4} & 0 \\ 0 & 0 & 0 & -\frac{7\bar{S}_{16}^{(4,Q)}}{4} - \frac{9\bar{S}_{26}^{(4,Q)}}{4} & 0 & 0 \\ -\bar{S}_{16}^{(4,Q)} & -\bar{S}_{26}^{(4,Q)} & \bar{S}_{16}^{(4,Q)} + \bar{S}_{26}^{(4,Q)} & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \bar{S}_{13}^{(4,Q)} &= 3Q_u[3] \\ \bar{S}_{16}^{(4,Q)} &= -6G_3^{\alpha}[1] + 2G_z[1] \\ \bar{S}_{26}^{(4,Q)} &= 2G_3^{\alpha}[1] - 2G_z[1] \end{aligned}$$

$$A^{(4,Q)} = \begin{pmatrix} A_{xx}^{(4,Q)} & 0 & 0 \\ 0 & A_{xx}^{(4,Q)} & 0 \\ 0 & 0 & A_{zz}^{(4,Q)} \end{pmatrix}$$

$$\begin{aligned} A_{xx}^{(4,Q)} &= Q_0[3] - 2Q_u[6] \\ A_{zz}^{(4,Q)} &= Q_0[3] + 4Q_u[6] \end{aligned}$$

$$\bar{A}^{(4,Q)} = \begin{pmatrix} 0 & \bar{A}_{xy}^{(4,Q)} & 0 \\ -\bar{A}_{xy}^{(4,Q)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\bar{A}_{xy}^{(4,Q)} = G_z[6]$$

$$M^{(4,Q)} = \begin{pmatrix} 0 & 0 & M_{1z}^{(4,Q)} \\ 0 & 0 & M_{2z}^{(4,Q)} \\ 0 & 0 & M_{3z}^{(4,Q)} \\ M_{4x}^{(4,Q)} & \frac{5M_{1z}^{(4,Q)}}{4} - \frac{7M_{2z}^{(4,Q)}}{4} + \frac{M_{3z}^{(4,Q)}}{2} & 0 \\ \frac{9M_{1z}^{(4,Q)}}{4} - \frac{11M_{2z}^{(4,Q)}}{4} + \frac{M_{3z}^{(4,Q)}}{2} & -M_{4x}^{(4,Q)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} M_{1z}^{(4,Q)} &= -3G_3^{\alpha}[2] + G_z[3] \\ M_{2z}^{(4,Q)} &= -G_3^{\alpha}[2] + G_z[3] \\ M_{3z}^{(4,Q)} &= 2G_3^{\alpha}[2] + 2G_z[2] + G_z[3] \\ M_{4x}^{(4,Q)} &= -3Q_u[4] \end{aligned}$$

$$\bar{M}^{(4,Q)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}_{x4}^{(4,Q)} & \bar{M}_{x5}^{(4,Q)} & 0 \\ 0 & 0 & 0 & \bar{M}_{y4}^{(4,Q)} & -\bar{M}_{x4}^{(4,Q)} & 0 \\ \bar{M}_{z1}^{(4,Q)} & -\bar{M}_{x5}^{(4,Q)} + \bar{M}_{y4}^{(4,Q)} + \bar{M}_{z1}^{(4,Q)} & -\frac{7\bar{M}_{x5}^{(4,Q)}}{2} + \frac{11\bar{M}_{y4}^{(4,Q)}}{2} + \bar{M}_{z1}^{(4,Q)} & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \bar{M}_{x4}^{(4,Q)} &= -3Q_u[5] \\ \bar{M}_{x5}^{(4,Q)} &= -3G_3^{\alpha}[3] + G_z[4] \\ \bar{M}_{y4}^{(4,Q)} &= -G_3^{\alpha}[3] + G_z[4] \\ \bar{M}_{z1}^{(4,Q)} &= -3G_3^{\alpha}[3] + G_z[5] \end{aligned}$$

— axial tensors —

$$C^{(1,G)} = \begin{pmatrix} 0 & 0 & C_z^{(1,G)} \end{pmatrix}$$

$$C_z^{(1,G)} = G_z$$

$$S^{(3,G)} = \begin{pmatrix} 0 & 0 & S_{1z}^{(3,G)} \\ 0 & 0 & S_{2z}^{(3,G)} \\ 0 & 0 & S_{3z}^{(3,G)} \\ S_{4x}^{(3,G)} & \frac{5S_{1z}^{(3,G)}}{4} - \frac{7S_{2z}^{(3,G)}}{4} + \frac{S_{3z}^{(3,G)}}{2} & 0 \\ \frac{9S_{1z}^{(3,G)}}{4} - \frac{11S_{2z}^{(3,G)}}{4} + \frac{S_{3z}^{(3,G)}}{2} & -S_{4x}^{(3,G)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_{1z}^{(3,G)} = -3G_3^\alpha + G_z[2]$$
$$S_{2z}^{(3,G)} = -G_3^\alpha + G_z[2]$$
$$S_{3z}^{(3,G)} = 2G_3^\alpha + 2G_z[1] + G_z[2]$$
$$S_{4x}^{(3,G)} = -3Q_u[1]$$

$$A^{(3,G)} = \begin{pmatrix} A_{4x}^{(3,G)} & A_{4y}^{(3,G)} & 0 \\ -A_{4y}^{(3,G)} & A_{4x}^{(3,G)} & 0 \\ 0 & 0 & A_{6z}^{(3,G)} \end{pmatrix}$$

$$A_{4x}^{(3,G)} = Q_0 - Q_u[2]$$
$$A_{4y}^{(3,G)} = G_z[3]$$
$$A_{6z}^{(3,G)} = Q_0 + 2Q_u[2]$$

$$S^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & S_{14}^{(4,G)} & S_{15}^{(4,G)} & 0 \\ 0 & 0 & 0 & -S_{14}^{(4,G)} & -S_{15}^{(4,G)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ S_{14}^{(4,G)} & -S_{14}^{(4,G)} & 0 & 0 & 0 & -S_{15}^{(4,G)} \\ S_{15}^{(4,G)} & -S_{15}^{(4,G)} & 0 & 0 & 0 & S_{14}^{(4,G)} \\ 0 & 0 & 0 & -S_{15}^{(4,G)} & S_{14}^{(4,G)} & 0 \end{pmatrix}$$

$$S_{14}^{(4,G)} = G_4^\beta$$
$$S_{15}^{(4,G)} = G_4^\alpha$$

$$\bar{S}^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & \bar{S}_{14}^{(4,G)} & \bar{S}_{15}^{(4,G)} & 0 \\ 0 & 0 & 0 & -\bar{S}_{14}^{(4,G)} & -\bar{S}_{15}^{(4,G)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\bar{S}_{14}^{(4,G)} & \bar{S}_{14}^{(4,G)} & 0 & 0 & 0 & \bar{S}_{15}^{(4,G)} \\ -\bar{S}_{15}^{(4,G)} & \bar{S}_{15}^{(4,G)} & 0 & 0 & 0 & -\bar{S}_{14}^{(4,G)} \\ 0 & 0 & 0 & -\bar{S}_{15}^{(4,G)} & \bar{S}_{14}^{(4,G)} & 0 \end{pmatrix}$$

$$
\bar{S}_{14}^{(4,G)} = 2Q_3^{\gamma}[1]
$$
$$
\bar{S}_{15}^{(4,G)} = -2Q_3^{\beta}[1]
$$

$$
M^{(4,G)} = \begin{pmatrix} M_{1x}^{(4,G)} & M_{1y}^{(4,G)} & 0 \\ -M_{1x}^{(4,G)} & -M_{1y}^{(4,G)} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ M_{1y}^{(4,G)} & -M_{1x}^{(4,G)} & 0 \end{pmatrix}
$$

$$
M_{1x}^{(4,G)} = Q_3^{\gamma}[2]
$$
$$
M_{1y}^{(4,G)} = Q_3^{\beta}[2]
$$

$$
\bar{M}^{(4,G)} = \begin{pmatrix} \bar{M}_{x1}^{(4,G)} & -\bar{M}_{x1}^{(4,G)} & 0 & 0 & 0 & \bar{M}_{x6}^{(4,G)} \\ \bar{M}_{x6}^{(4,G)} & -\bar{M}_{x6}^{(4,G)} & 0 & 0 & 0 & -\bar{M}_{x1}^{(4,G)} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}
$$

$$
\bar{M}_{x1}^{(4,G)} = Q_3^{\gamma}[3]
$$
$$
\bar{M}_{x6}^{(4,G)} = Q_3^{\beta}[3]
$$