SAMB for “D4h1”

Generated on 2023-06-12 22:53 by MultiPie 1.1.7

- Group: No. 123 D_{4h}^{1} $P4/mmm$ [tetragonal]
- Associated point group: No. 15 D_{4h} $4/mmm$ [tetragonal]

- Generation condition
  - model type: `tight_binding`
  - time-reversal type: `electric`
  - irrep: `[A1g]`
  - `spinful`

- Unit cell:
  $a = 1.0,\ b = 1.0,\ c = 1.5,\ \alpha = 90.0,\ \beta = 90.0,\ \gamma = 90.0$
- Lattice vectors:
  $\boldsymbol{a}_1 = \begin{pmatrix} 1.0 & 0 & 0 \end{pmatrix}$
  $\boldsymbol{a}_2 = \begin{pmatrix} 0 & 1.0 & 0 \end{pmatrix}$
  $\boldsymbol{a}_3 = \begin{pmatrix} 0 & 0 & 1.5 \end{pmatrix}$

Table 1: High-symmetry line: Γ-X.

	symbol	position	symbol	position
	Γ	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	X	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$

- Kets: dimension = 8

Table 2: Hilbert space for full matrix.

	No.	ket	No.	ket	No.	ket	No.	ket	No.	ket
	1	$(s,\uparrow)$@A_1	2	$(s,\downarrow)$@A_1	3	$(p_x,\uparrow)$@A_1	4	$(p_x,\downarrow)$@A_1	5	$(p_y,\uparrow)$@A_1
	6	$(p_y,\downarrow)$@A_1	7	$(p_z,\uparrow)$@A_1	8	$(p_z,\downarrow)$@A_1				

- Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position	mapping
S_1	A_1	$\begin{pmatrix}0 & 0 & 0\end{pmatrix}$	[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16]

- Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

	bond	tail	head	n	#	$\boldsymbol{b}$@$\boldsymbol{c}$	mapping
B_1	b_1	A_1	A_1	1	1	$\begin{pmatrix}0 & 1 & 0\end{pmatrix}$@$\begin{pmatrix}0 & \frac{1}{2} & 0\end{pmatrix}$	[1,-2,-3,4,-9,10,11,-12]
	b_2	A_1	A_1	1	1	$\begin{pmatrix}1 & 0 & 0\end{pmatrix}$@$\begin{pmatrix}\frac{1}{2} & 0 & 0\end{pmatrix}$	[5,-6,-7,8,-13,14,15,-16]
B_2	b_3	A_1	A_1	2	1	$\begin{pmatrix}1 & 1 & 0\end{pmatrix}$@$\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & 0\end{pmatrix}$	[1,-2,5,-6,-9,10,-13,14]
	b_4	A_1	A_1	2	1	$\begin{pmatrix}1 & -1 & 0\end{pmatrix}$@$\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & 0\end{pmatrix}$	[3,-4,-7,8,-11,12,15,-16]
B_3	b_5	A_1	A_1	3	1	$\begin{pmatrix}0 & 0 & 1\end{pmatrix}$@$\begin{pmatrix}0 & 0 & \frac{1}{2}\end{pmatrix}$	[1,2,-3,-4,-5,-6,7,8,-9,-10,11,12,13,14,-15,-16]
B_4	b_6	A_1	A_1	4	1	$\begin{pmatrix}0 & 1 & 1\end{pmatrix}$@$\begin{pmatrix}0 & \frac{1}{2} & \frac{1}{2}\end{pmatrix}$	[1,-3,-9,11]
	b_7	A_1	A_1	4	1	$\begin{pmatrix}0 & 1 & -1\end{pmatrix}$@$\begin{pmatrix}0 & \frac{1}{2} & \frac{1}{2}\end{pmatrix}$	[-2,4,10,-12]
	b_8	A_1	A_1	4	1	$\begin{pmatrix}1 & 0 & -1\end{pmatrix}$@$\begin{pmatrix}\frac{1}{2} & 0 & \frac{1}{2}\end{pmatrix}$	[5,-7,-13,15]
	b_9	A_1	A_1	4	1	$\begin{pmatrix}1 & 0 & 1\end{pmatrix}$@$\begin{pmatrix}\frac{1}{2} & 0 & \frac{1}{2}\end{pmatrix}$	[-6,8,14,-16]

- SAMB:

$\boxed{\text{No. 1}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_1, \mathrm{S}_1]$

$\hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_{1g})}]$

$\hat{\mathbb{Z}}_1(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}]$

$\boxed{\text{No. 2}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{S}_1]$

$\hat{\mathbb{Z}}_2 = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_{1g})}]$

$\hat{\mathbb{Z}}_2(\boldsymbol{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}]$

$\boxed{\text{No. 3}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{S}_1]$

$\hat{\mathbb{Z}}_3 = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_{1g})}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_{1g})}]$

$\hat{\mathbb{Z}}_3(\boldsymbol{k}) = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}]$

$\boxed{\text{No. 4}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}(1,1)$ $[\mathrm{M}_3, \mathrm{S}_1]$

$\hat{\mathbb{Z}}_4 = \mathbb{X}_4[\mathbb{Q}_0^{(a,A_{1g})}(1,1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_{1g})}]$

$\hat{\mathbb{Z}}_4(\boldsymbol{k}) = \mathbb{X}_4[\mathbb{Q}_0^{(a,A_{1g})}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}]$

$\boxed{\text{No. 5}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}(1,-1)$ $[\mathrm{M}_3, \mathrm{S}_1]$

$\hat{\mathbb{Z}}_5 = \mathbb{X}_5[\mathbb{Q}_2^{(a,A_{1g})}(1,-1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_{1g})}]$

$\hat{\mathbb{Z}}_5(\boldsymbol{k}) = \mathbb{X}_5[\mathbb{Q}_2^{(a,A_{1g})}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}]$

$\boxed{\text{No. 6}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_1, \mathrm{B}_1]$

$\hat{\mathbb{Z}}_6 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_{1g})}]$

$\hat{\mathbb{Z}}_6(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_{1g})}]$

$\boxed{\text{No. 7}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{B}_1]$

$\hat{\mathbb{Z}}_7 = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_{1g})}]$

$\hat{\mathbb{Z}}_7(\boldsymbol{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_{1g})}]$

$\boxed{\text{No. 8}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{B}_1]$

$\hat{\mathbb{Z}}_8 = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_{1g})}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_{1g})}]$

$\hat{\mathbb{Z}}_8(\boldsymbol{k}) = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_{1g})}]$

$\boxed{\text{No. 9}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{B}_1]$

$\hat{\mathbb{Z}}_9 = \mathbb{X}_6[\mathbb{Q}_2^{(a,B_{1g})}] \otimes \mathbb{Y}_3[\mathbb{Q}_2^{(b,B_{1g})}]$

$\hat{\mathbb{Z}}_9(\boldsymbol{k}) = \mathbb{X}_6[\mathbb{Q}_2^{(a,B_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_2[\mathbb{Q}_2^{(k,B_{1g})}]$

$\boxed{\text{No. 10}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}(1,1)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$\hat{\mathbb{Z}}_{10} = \mathbb{X}_4[\mathbb{Q}_0^{(a,A_{1g})}(1,1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_{1g})}]$

$\hat{\mathbb{Z}}_{10}(\boldsymbol{k}) = \mathbb{X}_4[\mathbb{Q}_0^{(a,A_{1g})}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_{1g})}]$

$\boxed{\text{No. 11}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$\hat{\mathbb{Z}}_{11} = \mathbb{X}_5[\mathbb{Q}_2^{(a,A_{1g})}(1,-1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_{1g})}]$

$\hat{\mathbb{Z}}_{11}(\boldsymbol{k}) = \mathbb{X}_5[\mathbb{Q}_2^{(a,A_{1g})}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_{1g})}]$

$\boxed{\text{No. 12}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$\hat{\mathbb{Z}}_{12} = \mathbb{X}_{10}[\mathbb{Q}_2^{(a,B_{1g})}(1,-1)] \otimes \mathbb{Y}_3[\mathbb{Q}_2^{(b,B_{1g})}]$

$$\hat{\mathbb{Z}}_{12}(\boldsymbol{k}) = \mathbb{X}_{10}[\mathbb{Q}_2^{(a,B_{1g})}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_2[\mathbb{Q}_2^{(k,B_{1g})}]$$

$\boxed{\text{No. 13}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_1, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{13} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{13}(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 14}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{14} = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{14}(\boldsymbol{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 15}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{15} = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_{1g})}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{15}(\boldsymbol{k}) = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 16}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{16} = \mathbb{X}_7[\mathbb{Q}_2^{(a,B_{2g})}] \otimes \mathbb{Y}_5[\mathbb{Q}_2^{(b,B_{2g})}]$$

$$\hat{\mathbb{Z}}_{16}(\boldsymbol{k}) = \mathbb{X}_7[\mathbb{Q}_2^{(a,B_{2g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_4[\mathbb{Q}_2^{(k,B_{2g})}]$$

$\boxed{\text{No. 17}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}(1,1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{17} = \mathbb{X}_4[\mathbb{Q}_0^{(a,A_{1g})}(1,1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{17}(\boldsymbol{k}) = \mathbb{X}_4[\mathbb{Q}_0^{(a,A_{1g})}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 18}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{18} = \mathbb{X}_5[\mathbb{Q}_2^{(a,A_{1g})}(1,-1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{18}(\boldsymbol{k}) = \mathbb{X}_5[\mathbb{Q}_2^{(a,A_{1g})}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 19}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{19} = \mathbb{X}_{11}[\mathbb{Q}_2^{(a,B_{2g})}(1,-1)] \otimes \mathbb{Y}_5[\mathbb{Q}_2^{(b,B_{2g})}]$$

$$\hat{\mathbb{Z}}_{19}(\boldsymbol{k}) = \mathbb{X}_{11}[\mathbb{Q}_2^{(a,B_{2g})}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_4[\mathbb{Q}_2^{(k,B_{2g})}]$$

$\boxed{\text{No. 20}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_1, \mathrm{B}_3]$

$$\hat{\mathbb{Z}}_{20} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{Y}_6[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{20}(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_5[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 21}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{B}_3]$

$$\hat{\mathbb{Z}}_{21} = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{Y}_6[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{21}(\boldsymbol{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_5[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 22}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{B}_3]$

$$\hat{\mathbb{Z}}_{22} = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_{1g})}] \otimes \mathbb{Y}_6[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{22}(\boldsymbol{k}) = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_5[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 23}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}(1,1)$ $[\mathrm{M}_3, \mathrm{B}_3]$

$$\hat{\mathbb{Z}}_{23} = \mathbb{X}_4[\mathbb{Q}_0^{(a,A_{1g})}(1,1)] \otimes \mathbb{Y}_6[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{23}(\boldsymbol{k}) = \mathbb{X}_4[\mathbb{Q}_0^{(a,A_{1g})}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_5[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 24}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_3]$

$$\hat{\mathbb{Z}}_{24} = \mathbb{X}_5[\mathbb{Q}_2^{(a,A_{1g})}(1,-1)] \otimes \mathbb{Y}_6[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{24}(\boldsymbol{k}) = \mathbb{X}_5[\mathbb{Q}_2^{(a,A_{1g})}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_5[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 25}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_1, \mathrm{B}_4]$

$$\hat{\mathbb{Z}}_{25} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{Y}_7[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{25}(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_6[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 26}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{B}_4]$

$$\hat{\mathbb{Z}}_{26} = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{Y}_7[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{26}(\boldsymbol{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_6[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 27}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{B}_4]$

$$\hat{\mathbb{Z}}_{27} = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_{1g})}] \otimes \mathbb{Y}_7[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{27}(\boldsymbol{k}) = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_6[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 28}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{B}_4]$

$$\hat{\mathbb{Z}}_{28} = \frac{\sqrt{3}\mathbb{X}_6[\mathbb{Q}_2^{(a,B_{1g})}] \otimes \mathbb{Y}_8[\mathbb{Q}_2^{(b,B_{1g})}]}{3} + \frac{\sqrt{3}\mathbb{X}_8[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,0}^{(b,E_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_9[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{3}$$

$$\hat{\mathbb{Z}}_{28}(\boldsymbol{k}) = \frac{\sqrt{3}\mathbb{X}_6[\mathbb{Q}_2^{(a,B_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_7[\mathbb{Q}_2^{(k,B_{1g})}]}{3} + \frac{\sqrt{3}\mathbb{X}_8[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_8[\mathbb{Q}_{2,0}^{(k,E_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_9[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_9[\mathbb{Q}_{2,1}^{(k,E_g)}]}{3}$$

$\boxed{\text{No. 29}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}$ $[\mathrm{M}_3, \mathrm{B}_4]$

$$\hat{\mathbb{Z}}_{29} = -\frac{\sqrt{6}\mathbb{X}_6[\mathbb{Q}_2^{(a,B_{1g})}] \otimes \mathbb{Y}_8[\mathbb{Q}_2^{(b,B_{1g})}]}{3} + \frac{\sqrt{6}\mathbb{X}_8[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,0}^{(b,E_g)}]}{6} + \frac{\sqrt{6}\mathbb{X}_9[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{6}$$

$$\hat{\mathbb{Z}}_{29}(\boldsymbol{k}) = -\frac{\sqrt{6}\mathbb{X}_6[\mathbb{Q}_2^{(a,B_{1g})}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_7[\mathbb{Q}_2^{(k,B_{1g})}]}{3} + \frac{\sqrt{6}\mathbb{X}_8[\mathbb{Q}_{2,0}^{(a,E_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_8[\mathbb{Q}_{2,0}^{(k,E_g)}]}{6} + \frac{\sqrt{6}\mathbb{X}_9[\mathbb{Q}_{2,1}^{(a,E_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_9[\mathbb{Q}_{2,1}^{(k,E_g)}]}{6}$$

$\boxed{\text{No. 30}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}(1,1)$ $[\mathrm{M}_3, \mathrm{B}_4]$

$$\hat{\mathbb{Z}}_{30} = \mathbb{X}_4[\mathbb{Q}_0^{(a,A_{1g})}(1,1)] \otimes \mathbb{Y}_7[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{30}(\boldsymbol{k}) = \mathbb{X}_4[\mathbb{Q}_0^{(a,A_{1g})}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_6[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 31}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_4]$

$$\hat{\mathbb{Z}}_{31} = \mathbb{X}_5[\mathbb{Q}_2^{(a,A_{1g})}(1,-1)] \otimes \mathbb{Y}_7[\mathbb{Q}_0^{(b,A_{1g})}]$$

$$\hat{\mathbb{Z}}_{31}(\boldsymbol{k}) = \mathbb{X}_5[\mathbb{Q}_2^{(a,A_{1g})}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_6[\mathbb{Q}_0^{(k,A_{1g})}]$$

$\boxed{\text{No. 32}}$ $\hat{\mathbb{Q}}_0^{(A_{1g})}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_4]$

$$\hat{\mathbb{Z}}_{32} = \frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{Q}_2^{(a,B_{1g})}(1,-1)] \otimes \mathbb{Y}_8[\mathbb{Q}_2^{(b,B_{1g})}]}{3} + \frac{\sqrt{3}\mathbb{X}_{12}[\mathbb{Q}_{2,0}^{(a,E_g)}(1,-1)] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,0}^{(b,E_g)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{13}[\mathbb{Q}_{2,1}^{(a,E_g)}(1,-1)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{3}$$

$$\begin{aligned}\hat{\mathbb{Z}}_{32}(\boldsymbol{k}) = &\frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{Q}_2^{(a,B_{1g})}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_7[\mathbb{Q}_2^{(k,B_{1g})}]}{3} + \frac{\sqrt{3}\mathbb{X}_{12}[\mathbb{Q}_{2,0}^{(a,E_g)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_8[\mathbb{Q}_{2,0}^{(k,E_g)}]}{3}\\ &+ \frac{\sqrt{3}\mathbb{X}_{13}[\mathbb{Q}_{2,1}^{(a,E_g)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_9[\mathbb{Q}_{2,1}^{(k,E_g)}]}{3}\end{aligned}$$

$\boxed{\text{No. 33}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_4]$

$$\hat{\mathbb{Z}}_{33} = -\frac{\sqrt{6}\mathbb{X}_{10}[\mathbb{Q}_2^{(a,B_{1g})}(1,-1)] \otimes \mathbb{Y}_8[\mathbb{Q}_2^{(b,B_{1g})}]}{3} + \frac{\sqrt{6}\mathbb{X}_{12}[\mathbb{Q}_{2,0}^{(a,E_g)}(1,-1)] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,0}^{(b,E_g)}]}{6} + \frac{\sqrt{6}\mathbb{X}_{13}[\mathbb{Q}_{2,1}^{(a,E_g)}(1,-1)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{6}$$

$$\begin{aligned}\hat{\mathbb{Z}}_{33}(\boldsymbol{k}) = &-\frac{\sqrt{6}\mathbb{X}_{10}[\mathbb{Q}_2^{(a,B_{1g})}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_7[\mathbb{Q}_2^{(k,B_{1g})}]}{3} + \frac{\sqrt{6}\mathbb{X}_{12}[\mathbb{Q}_{2,0}^{(a,E_g)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_8[\mathbb{Q}_{2,0}^{(k,E_g)}]}{6}\\ &+ \frac{\sqrt{6}\mathbb{X}_{13}[\mathbb{Q}_{2,1}^{(a,E_g)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_9[\mathbb{Q}_{2,1}^{(k,E_g)}]}{6}\end{aligned}$$

$\boxed{\text{No. 34}}$ $\hat{\mathbb{Q}}_2^{(A_{1g})}(1,0)$ $[\mathrm{M}_3, \mathrm{B}_4]$

$$\hat{\mathbb{Z}}_{34} = \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{1,0}^{(a,E_g)}(1,0)] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,0}^{(b,E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{15}[\mathbb{G}_{1,1}^{(a,E_g)}(1,0)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,1}^{(b,E_g)}]}{2}$$

$$\hat{\mathbb{Z}}_{34}(\boldsymbol{k}) = \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{G}_{1,0}^{(a,E_g)}(1,0)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_8[\mathbb{Q}_{2,0}^{(k,E_g)}]}{2} - \frac{\sqrt{2}\mathbb{X}_{15}[\mathbb{G}_{1,1}^{(a,E_g)}(1,0)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_{1g})}] \otimes \mathbb{F}_9[\mathbb{Q}_{2,1}^{(k,E_g)}]}{2}$$

Table 5: Atomic SAMB group.

group	bra	ket
M_1	$(s,\uparrow),(s,\downarrow)$	$(s,\uparrow),(s,\downarrow)$
M_2	$(s,\uparrow),(s,\downarrow)$	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$
M_3	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$

Table 6: Atomic SAMB.

symbol	type	group	form
$\mathbb{X}_1$	$\mathbb{Q}_0^{(a,A_{1g})}$	M_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{X}_2$	$\mathbb{Q}_0^{(a,A_{1g})}$	M_3	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_3$	$\mathbb{Q}_2^{(a,A_{1g})}$	M_3	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{X}_4$	$\mathbb{Q}_0^{(a,A_{1g})}(1,1)$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ \frac{\sqrt{3}i}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_5$	$\mathbb{Q}_2^{(a,A_{1g})}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_6$	$\mathbb{Q}_2^{(a,B_{1g})}$	M_3	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_7$	$\mathbb{Q}_2^{(a,B_{2g})}$	M_3	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_8$	$\mathbb{Q}_{2,0}^{(a,E_g)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_9$	$\mathbb{Q}_{2,1}^{(a,E_g)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{10}$	$\mathbb{Q}_{2}^{(a,B_{1g})}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{11}$	$\mathbb{Q}_{2}^{(a,B_{2g})}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{12}$	$\mathbb{Q}_{2,0}^{(a,E_{g})}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{13}$	$\mathbb{Q}_{2,1}^{(a,E_{g})}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{14}$	$\mathbb{G}_{1,0}^{(a,E_{g})}(1,0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{15}$	$\mathbb{G}_{1,1}^{(a,E_g)}(1,0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$

Table 7: Cluster SAMB.

symbol	type	cluster	form
$\mathbb{Y}_1$	$\mathbb{Q}_0^{(s,A_{1g})}$	S_1	$\begin{pmatrix} 1 \end{pmatrix}$
$\mathbb{Y}_2$	$\mathbb{Q}_0^{(b,A_{1g})}$	B_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{Y}_3$	$\mathbb{Q}_2^{(b,B_{1g})}$	B_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{Y}_4$	$\mathbb{Q}_0^{(b,A_{1g})}$	B_2	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{Y}_5$	$\mathbb{Q}_2^{(b,B_{2g})}$	B_2	$\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{Y}_6$	$\mathbb{Q}_0^{(b,A_{1g})}$	B_3	$\begin{pmatrix} 1 \end{pmatrix}$
$\mathbb{Y}_7$	$\mathbb{Q}_0^{(b,A_{1g})}$	B_4	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$
$\mathbb{Y}_8$	$\mathbb{Q}_2^{(b,B_{1g})}$	B_4	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$
$\mathbb{Y}_9$	$\mathbb{Q}_{2,0}^{(b,E_g)}$	B_4	$\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{Y}_{10}$	$\mathbb{Q}_{2,1}^{(b,E_g)}$	B_4	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$

Table 8: Uniform SAMB.

symbol	type	cluster	form
$\mathbb{U}_1$	$\mathbb{Q}_0^{(s,A_{1g})}$	S_1	$\begin{pmatrix} 1 \end{pmatrix}$

Table 9: Structure SAMB.

symbol	type	cluster	form
$\mathbb{F}_1$	$\mathbb{Q}_0^{(k,A_{1g})}$	B_1	$c_{001} + c_{002}$
$\mathbb{F}_2$	$\mathbb{Q}_2^{(k,B_{1g})}$	B_1	$c_{001} - c_{002}$
$\mathbb{F}_3$	$\mathbb{Q}_0^{(k,A_{1g})}$	B_2	$c_{003} + c_{004}$
$\mathbb{F}_4$	$\mathbb{Q}_2^{(k,B_{2g})}$	B_2	$c_{003} - c_{004}$
$\mathbb{F}_5$	$\mathbb{Q}_0^{(k,A_{1g})}$	B_3	$\sqrt{2}c_{005}$
$\mathbb{F}_6$	$\mathbb{Q}_0^{(k,A_{1g})}$	B_4	$\frac{\sqrt{2}c_{006}}{2} + \frac{\sqrt{2}c_{007}}{2} + \frac{\sqrt{2}c_{008}}{2} + \frac{\sqrt{2}c_{009}}{2}$
$\mathbb{F}_7$	$\mathbb{Q}_2^{(k,B_{1g})}$	B_4	$\frac{\sqrt{2}c_{006}}{2} + \frac{\sqrt{2}c_{007}}{2} - \frac{\sqrt{2}c_{008}}{2} - \frac{\sqrt{2}c_{009}}{2}$
$\mathbb{F}_8$	$\mathbb{Q}_{2,0}^{(k,E_g)}$	B_4	$c_{006} - c_{007}$
$\mathbb{F}_9$	$\mathbb{Q}_{2,1}^{(k,E_g)}$	B_4	$-c_{008} + c_{009}$

Table 10: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_0^{(A_{1g})}$	0	A_{1g}	–	–	1
2	$\mathbb{Q}_2^{(A_{1g})}$	2	A_{1g}	–	–	$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$
3	$\mathbb{Q}_2^{(B_{1g})}$	2	B_{1g}	–	–	$\frac{\sqrt{3}(x-y)(x+y)}{2}$
4	$\mathbb{Q}_2^{(B_{2g})}$	2	B_{2g}	–	–	$\sqrt{3}xy$
5	$\mathbb{Q}_{2,0}^{(E_g)}$	2	E_g	–	0	$\sqrt{3}yz$

continued ...

Table 10

No.	symbol	rank	irrep.	mul.	comp.	form
6	$\mathbb{Q}_{2,1}^{(E_g)}$	2	E_g	–	1	$\sqrt{3}xz$

Table 11: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_{1,0}^{(E_g)}$	1	E_g	–	0	X
2	$\mathbb{G}_{1,1}^{(E_g)}$	1	E_g	–	1	Y

- Group info.: Generator = $\{2_{001}|0\}$, $\{4^+_{001}|0\}$, $\{2_{010}|0\}$, $\{-1|0\}$

Table 12: Conjugacy class (point-group part).

rep. SO	symmetry operations
$\{1|0\}$	$\{1|0\}$
$\{2_{001}|0\}$	$\{2_{001}|0\}$
$\{2_{100}|0\}$	$\{2_{100}|0\}$, $\{2_{010}|0\}$
$\{2_{110}|0\}$	$\{2_{110}|0\}$, $\{2_{1-10}|0\}$
$\{4^+_{001}|0\}$	$\{4^+_{001}|0\}$, $\{4^-_{001}|0\}$
$\{-1|0\}$	$\{-1|0\}$
$\{\mathrm{m}_{001}|0\}$	$\{\mathrm{m}_{001}|0\}$
$\{\mathrm{m}_{100}|0\}$	$\{\mathrm{m}_{100}|0\}$, $\{\mathrm{m}_{010}|0\}$
$\{\mathrm{m}_{110}|0\}$	$\{\mathrm{m}_{110}|0\}$, $\{\mathrm{m}_{1-10}|0\}$
$\{-4^+_{001}|0\}$	$\{-4^+_{001}|0\}$, $\{-4^-_{001}|0\}$

Table 13: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	$\{1\|0\}$	2	$\{2_{001}\|0\}$	3	$\{2_{100}\|0\}$	4	$\{2_{010}\|0\}$	5	$\{2_{110}\|0\}$
6	$\{2_{1-10}\|0\}$	7	$\{4^+_{001}\|0\}$	8	$\{4^-_{001}\|0\}$	9	$\{-1\|0\}$	10	$\{m_{001}\|0\}$
11	$\{m_{100}\|0\}$	12	$\{m_{010}\|0\}$	13	$\{m_{110}\|0\}$	14	$\{m_{1-10}\|0\}$	15	$\{-4^+_{001}\|0\}$
16	$\{-4^-_{001}\|0\}$								

Table 14: Character table (point-group part).

	1	2_{001}	2_{100}	2_{110}	4^+_{001}	-1	m_{001}	m_{100}	m_{110}	-4^+_{001}
A_{1g}	1	1	1	1	1	1	1	1	1	1
A_{2g}	1	1	−1	−1	1	1	1	−1	−1	1
B_{1g}	1	1	1	−1	−1	1	1	1	−1	−1
B_{2g}	1	1	−1	1	−1	1	1	−1	1	−1
E_g	2	−2	0	0	0	2	−2	0	0	0
A_{1u}	1	1	1	1	1	−1	−1	−1	−1	−1
A_{2u}	1	1	−1	−1	1	−1	−1	1	1	−1
B_{1u}	1	1	1	−1	−1	−1	−1	−1	1	1
B_{2u}	1	1	−1	1	−1	−1	−1	1	−1	1
E_u	2	−2	0	0	0	−2	2	0	0	0

Table 15: Parity conversion.

$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$
A_{1g} (A_{1u})	B_{1g} (B_{1u})	E_g (E_u)	A_{2g} (A_{2u})	B_{2g} (B_{2u})
A_{1u} (A_{1g})	B_{1u} (B_{1g})	E_u (E_g)	A_{2u} (A_{2g})	B_{2u} (B_{2g})

Table 16: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_g	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_u
A_{1g}	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_g	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_u
A_{2g}		A_{1g}	B_{2g}	B_{1g}	E_g	A_{2u}	A_{1u}	B_{2u}	B_{1u}	E_u
B_{1g}			A_{1g}	A_{2g}	E_g	B_{1u}	B_{2u}	A_{1u}	A_{2u}	E_u
B_{2g}				A_{1g}	E_g	B_{2u}	B_{1u}	A_{2u}	A_{1u}	E_u
E_g					$A_{1g}+B_{1g}+B_{2g}$	E_u	E_u	E_u	E_u	$A_{1u}+A_{2u}+B_{1u}+B_{2u}$
A_{1u}						A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_g
A_{2u}							A_{1g}	B_{2g}	B_{1g}	E_g
B_{1u}								A_{1g}	A_{2g}	E_g
B_{2u}									A_{1g}	E_g
E_u										$A_{1g}+B_{1g}+B_{2g}$

Table 17: Anti-symmetric product, $[\Gamma \otimes \Gamma]_-$.

A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_g	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_u
–	–	–	–	A_{2g}	–	–	–	–	A_{2g}

Table 18: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix}2 & 1 & 1\end{pmatrix}$	2	$\begin{pmatrix}-2 & -1 & 1\end{pmatrix}$	3	$\begin{pmatrix}2 & -1 & -1\end{pmatrix}$	4	$\begin{pmatrix}-2 & 1 & -1\end{pmatrix}$
5	$\begin{pmatrix}1 & 2 & -1\end{pmatrix}$	6	$\begin{pmatrix}-1 & -2 & -1\end{pmatrix}$	7	$\begin{pmatrix}-1 & 2 & 1\end{pmatrix}$	8	$\begin{pmatrix}1 & -2 & 1\end{pmatrix}$
9	$\begin{pmatrix}-2 & -1 & -1\end{pmatrix}$	10	$\begin{pmatrix}2 & 1 & -1\end{pmatrix}$	11	$\begin{pmatrix}-2 & 1 & 1\end{pmatrix}$	12	$\begin{pmatrix}2 & -1 & 1\end{pmatrix}$
13	$\begin{pmatrix}-1 & -2 & 1\end{pmatrix}$	14	$\begin{pmatrix}1 & 2 & 1\end{pmatrix}$	15	$\begin{pmatrix}1 & -2 & -1\end{pmatrix}$	16	$\begin{pmatrix}-1 & 2 & -1\end{pmatrix}$

Table 19: Virtual-cluster basis.

symbol	1	2	3	4	5	6	7	8	9	10
$\mathbb{Q}_{0}^{(A_{1g})}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$				
$\mathbb{Q}_{1}^{(A_{2u})}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$
	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$				
$\mathbb{Q}_{1,0}^{(E_{u})}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$
	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$				
$\mathbb{Q}_{1,1}^{(E_{u})}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$
	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$				
$\mathbb{Q}_{2}^{(B_{1g})}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$				
$\mathbb{Q}_{2}^{(B_{2g})}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$				
$\mathbb{Q}_{2,0}^{(E_{g})}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$
	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$				
$\mathbb{Q}_{2,1}^{(E_{g})}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$
	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$				
$\mathbb{Q}_{3}^{(B_{1u})}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$
	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$				
$\mathbb{Q}_{3}^{(B_{2u})}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$
	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$				
$\mathbb{Q}_{3,0}^{(E_{u},1)}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$
	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$				
$\mathbb{Q}_{3,1}^{(E_{u},1)}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$
	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$				
$\mathbb{Q}_{4}^{(A_{2g})}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$				
$\mathbb{Q}_{4,0}^{(E_{g},1)}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$
	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$				
$\mathbb{Q}_{4,1}^{(E_{g},1)}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$

continued ...

Table 19

symbol	1	2	3	4	5	6	7	8	9	10
	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$				
$\mathbb{Q}_5^{(A_{1u})}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$
	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$				