

MPG No. 9.3.31 4' [Type III, tetragonal] [G tensor]

* Rank 0 tensor.

$$[G]$$

$$G = G_s^{(1)}$$

* Rank 1 tensor.

$$[0 \ 0 \ G_z]$$

$$G_z = G_{pz}^{(1)}$$

* Rank 2 tensor (s).

$$\begin{bmatrix} G_{xx} & 0 & 0 \\ 0 & G_{xx} & 0 \\ 0 & 0 & G_{zz} \end{bmatrix}$$

$$G_{xx} = -G_{du}^{(1)} + G_s^{(1)}$$

$$G_{zz} = 2G_{du}^{(1)} + G_s^{(1)}$$

* Rank 2 tensor (a).

$$\begin{bmatrix} 0 & G_{xy} & 0 \\ -G_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{xy} = Q_{pz}^{(1)}$$

* Rank 3 tensor (s).

$$\begin{bmatrix} 0 & 0 & G_{xxz} \\ 0 & 0 & G_{xxz} \\ 0 & 0 & G_{zzz} \\ G_{yzx} & G_{yzy} & 0 \\ G_{yzy} & -G_{yzx} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{xxz} = -G_{faz}^{(1)} + G_{pz}^{(1)}$$

$$G_{zzz} = 2G_{faz}^{(1)} + G_{pz}^{(1)} + 2G_{pz}^{(2)}$$

$$G_{yzx} = -3Q_{du}^{(1)}$$

$$G_{yzy} = -G_{faz}^{(1)} + G_{pz}^{(2)}$$

* Rank 3 tensor (a).

$$\begin{bmatrix} G_{yzx} & G_{yzy} & 0 \\ -G_{yzy} & G_{yzx} & 0 \\ 0 & 0 & G_{xyz} \end{bmatrix}$$

$$G_{yzx} = -Q_{du}^{(2)} + Q_s^{(1)}$$

$$G_{yzy} = G_{pz}^{(3)}$$

$$G_{xyz} = 2Q_{du}^{(2)} + Q_s^{(1)}$$

* Rank 4 tensor (sss).

$$\begin{bmatrix} G_{xxxx} & G_{xxyy} & G_{xxzz} & 0 & 0 & G_{xxxz} \\ G_{xxyy} & G_{xxxx} & G_{xxzz} & 0 & 0 & -G_{xxxy} \\ G_{xxzz} & G_{xxzz} & G_{zzzz} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{yzyz} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{yzyz} & 0 \\ G_{xxxy} & -G_{xxxy} & 0 & 0 & 0 & G_{xyxy} \end{bmatrix}$$

$$G_{xxxx} = -2G_{du}^{(1)} - 4G_{du}^{(2)} - G_{gu}^{(1)} + 2G_g^{(1)} + G_s^{(1)} + 2G_s^{(2)}$$

$$G_{xxyy} = -2G_{du}^{(1)} + 2G_{gu}^{(1)} - G_g^{(1)} + G_s^{(1)}$$

$$G_{xxzz} = G_{du}^{(1)} - G_{gu}^{(1)} - G_g^{(1)} + G_s^{(1)}$$

$$G_{xxxy} = G_{gaz}^{(1)}$$

$$G_{zzzz} = 4G_{du}^{(1)} + 8G_{du}^{(2)} + 2G_{gu}^{(1)} + 2G_g^{(1)} + G_s^{(1)} + 2G_s^{(2)}$$

$$G_{yzyz} = G_{du}^{(2)} - G_{gu}^{(1)} - G_g^{(1)} + G_s^{(2)}$$

$$G_{xyxy} = -2G_{du}^{(2)} + 2G_{gu}^{(1)} - G_g^{(1)} + G_s^{(2)}$$

* Rank 4 tensor (ssa).

$$\begin{bmatrix} 0 & 0 & G_{xxzz} & 0 & 0 & G_{xxxy} \\ 0 & 0 & G_{xxzz} & 0 & 0 & -G_{xxxy} \\ -G_{xxzz} & -G_{xxzz} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{yzzx} & 0 \\ 0 & 0 & 0 & -G_{yzzx} & 0 & 0 \\ -G_{xxxy} & G_{xxxy} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G_{xxzz} = 3G_{du}^{(3)}$$

$$G_{xxxy} = -2Q_{faz}^{(1)} + 2Q_{pz}^{(1)}$$

$$G_{yzzx} = -4Q_{faz}^{(1)} - Q_{pz}^{(1)}$$

* Rank 4 tensor (aas).

$$\begin{bmatrix} G_{yzyz} & 0 & 0 \\ 0 & G_{yzyz} & 0 \\ 0 & 0 & G_{xyxy} \end{bmatrix}$$

$$G_{yzyz} = -2G_{du}^{(4)} + G_s^{(3)}$$

$$G_{xyxy} = 4G_{du}^{(4)} + G_s^{(3)}$$

* Rank 4 tensor (aaa).

$$\begin{bmatrix} 0 & G_{yzzx} & 0 \\ -G_{yzzx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{yzzx} = -Q_{pz}^{(2)}$$

* Rank 4 tensor (sa).

$$\begin{bmatrix} 0 & 0 & G_{xxxy} \\ 0 & 0 & G_{xxyy} \\ 0 & 0 & G_{zzxy} \\ G_{yzyz} & G_{yzzx} & 0 \\ G_{yzzx} & -G_{yzyz} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}
G_{xxyy} &= -Q_{faz}^{(2)} + Q_{pz}^{(3)} \\
G_{zzxy} &= 2Q_{faz}^{(2)} + Q_{pz}^{(3)} + 2Q_{pz}^{(4)} \\
G_{yzyz} &= -3G_{du}^{(5)} \\
G_{yzzx} &= -Q_{faz}^{(2)} + Q_{pz}^{(4)}
\end{aligned}$$

* Rank 4 tensor (as).

$$\begin{bmatrix}
0 & 0 & 0 & G_{yzyz} & G_{yzzx} & 0 \\
0 & 0 & 0 & G_{yzzx} & -G_{yzyz} & 0 \\
G_{xyxx} & G_{xyxx} & G_{xyzz} & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{aligned}
G_{yzyz} &= -3G_{du}^{(6)} \\
G_{yzzx} &= -Q_{faz}^{(3)} + Q_{pz}^{(6)} \\
G_{xyxx} &= -Q_{faz}^{(3)} + Q_{pz}^{(5)} \\
G_{xyzz} &= 2Q_{faz}^{(3)} + Q_{pz}^{(5)} + 2Q_{pz}^{(6)}
\end{aligned}$$

* Rank 4 tensor (s).

$$\begin{bmatrix}
G_{xxxx} & G_{xxyy} & G_{xxzz} & 0 & 0 & G_{xxyy} & 0 & 0 & G_{xxyx} \\
G_{xxyy} & G_{xxxx} & G_{xxzz} & 0 & 0 & -G_{xxyx} & 0 & 0 & -G_{xxyy} \\
G_{zzxx} & G_{zzxx} & G_{zzzz} & 0 & 0 & G_{zzxy} & 0 & 0 & -G_{zzxy} \\
0 & 0 & 0 & G_{yzyz} & G_{yzzx} & 0 & G_{yzyy} & G_{yzzx} & 0 \\
0 & 0 & 0 & -G_{yzxx} & G_{yzyy} & 0 & -G_{yzxx} & G_{yzyz} & 0 \\
G_{xyxx} & -G_{xyxx} & 0 & 0 & 0 & G_{xyxy} & 0 & 0 & G_{xyxy}
\end{bmatrix}$$

$$\begin{aligned}
G_{xxxx} &= -2G_{du}^{(1)} - 4G_{du}^{(2)} - G_{gu}^{(1)} + 2G_g^{(1)} + G_s^{(1)} + 2G_s^{(2)} \\
G_{xxyy} &= -2G_{du}^{(1)} + 2G_{gu}^{(1)} - G_g^{(1)} + G_s^{(1)} \\
G_{xxzz} &= G_{du}^{(1)} + 3G_{du}^{(3)} - G_{gu}^{(1)} - G_g^{(1)} + G_s^{(1)} \\
G_{xxyy} &= G_{gaz}^{(1)} - 2Q_{faz}^{(1)} - Q_{faz}^{(2)} + 2Q_{pz}^{(1)} + Q_{pz}^{(3)} \\
G_{xxyx} &= G_{gaz}^{(1)} - 2Q_{faz}^{(1)} + Q_{faz}^{(2)} + 2Q_{pz}^{(1)} - Q_{pz}^{(3)} \\
G_{zzxx} &= G_{du}^{(1)} - 3G_{du}^{(3)} - G_{gu}^{(1)} - G_g^{(1)} + G_s^{(1)} \\
G_{zzzz} &= 4G_{du}^{(1)} + 8G_{du}^{(2)} + 2G_{gu}^{(1)} + 2G_g^{(1)} + G_s^{(1)} + 2G_s^{(2)} \\
G_{zzxy} &= 2Q_{faz}^{(2)} + Q_{pz}^{(3)} + 2Q_{pz}^{(4)} \\
G_{yzyz} &= G_{du}^{(2)} - 3G_{du}^{(5)} - G_{gu}^{(1)} - G_g^{(1)} + G_s^{(2)} \\
G_{yzzx} &= -4Q_{faz}^{(1)} - Q_{faz}^{(2)} - Q_{pz}^{(1)} + Q_{pz}^{(4)} \\
G_{yzyy} &= G_{du}^{(2)} + 3G_{du}^{(5)} - G_{gu}^{(1)} - G_g^{(1)} + G_s^{(2)} \\
G_{yzxx} &= -4Q_{faz}^{(1)} + Q_{faz}^{(2)} - Q_{pz}^{(1)} - Q_{pz}^{(4)} \\
G_{xyxx} &= G_{gaz}^{(1)} + 2Q_{faz}^{(1)} - 2Q_{pz}^{(1)} \\
G_{xyxy} &= -2G_{du}^{(2)} + 2G_{gu}^{(1)} - G_g^{(1)} + G_s^{(2)}
\end{aligned}$$

* Rank 4 tensor (a).

$$\begin{bmatrix}
0 & 0 & 0 & G_{yzyz} & G_{yzzx} & 0 & G_{yzyy} & G_{yzzx} & 0 \\
0 & 0 & 0 & G_{yzzx} & -G_{yzyy} & 0 & G_{yzyy} & -G_{yzyz} & 0 \\
G_{xyxx} & G_{xyxx} & G_{xyzz} & 0 & 0 & G_{xyxy} & 0 & 0 & -G_{xyxy}
\end{bmatrix}$$

$$G_{yzyz} = -2G_{du}^{(4)} - 3G_{du}^{(6)} + G_s^{(3)}$$

$$G_{yzzx} = -Q_{faz}^{(3)} - Q_{pz}^{(2)} + Q_{pz}^{(6)}$$

$$G_{yzzy} = 2G_{du}^{(4)} - 3G_{du}^{(6)} - G_s^{(3)}$$

$$G_{yzxz} = -Q_{faz}^{(3)} + Q_{pz}^{(2)} + Q_{pz}^{(6)}$$

$$G_{xyxx} = -Q_{faz}^{(3)} + Q_{pz}^{(5)}$$

$$G_{xyzz} = 2Q_{faz}^{(3)} + Q_{pz}^{(5)} + 2Q_{pz}^{(6)}$$

$$G_{xyxy} = 4G_{du}^{(4)} + G_s^{(3)}$$

* Rank 4 tensor (t).

$$\begin{bmatrix} G_{xxxx} & G_{xxxz} & 0 \\ -G_{xxxy} & G_{xxxx} & 0 \\ 0 & 0 & G_{zzzz} \\ 0 & 0 & G_{yyzz} \\ G_{zzxx} & 0 & 0 \\ G_{xxxy} & G_{xxyy} & 0 \\ 0 & G_{yzyy} & 0 \\ 0 & 0 & G_{zxzx} \\ G_{xxyy} + G_{yyzz} - G_{yzyy} - G_{zxzx} + G_{zzxx} & -G_{xxxy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{xxxx} = -2G_{du}^{(1)} - 4G_{du}^{(2)} - G_{gu}^{(1)} + 2G_g^{(1)} + 3G_s^{(1)}$$

$$G_{xxxy} = G_{gaz}^{(1)}$$

$$G_{zzzz} = 4G_{du}^{(1)} + 8G_{du}^{(2)} + 2G_{gu}^{(1)} + 2G_g^{(1)} + 3G_s^{(1)}$$

$$G_{yyzz} = G_{du}^{(1)} + 3G_{du}^{(3)} - G_{gu}^{(1)} - G_g^{(1)} + G_s^{(1)}$$

$$G_{zzxx} = G_{du}^{(1)} - 3G_{du}^{(3)} - G_{gu}^{(1)} - G_g^{(1)} + G_s^{(1)}$$

$$G_{xxyy} = -2G_{du}^{(1)} + 2G_{gu}^{(1)} - G_g^{(1)} + G_s^{(1)}$$

$$G_{yzyy} = G_{du}^{(2)} + 3G_{du}^{(5)} - G_{gu}^{(1)} - G_g^{(1)} + G_s^{(1)}$$

$$G_{zxzx} = G_{du}^{(2)} - 3G_{du}^{(5)} - G_{gu}^{(1)} - G_g^{(1)} + G_s^{(1)}$$