SAMB for "C2h1"

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- Generation condition
 - model type: tight_binding
 - time-reversal type: electric
 - irrep: [Ag]
 - spinful
- Unit cell:

$$a=1.0,\ b=1.2,\ c=1.0,\ \alpha=90.0,\ \beta=90.0,\ \gamma=90.0$$

- Lattice vectors:
 - $\boldsymbol{a}_1 = \begin{pmatrix} 1.0 & 0 & 0 \end{pmatrix}$
 - $\boldsymbol{a}_2 = \begin{pmatrix} 0 & 1.2 & 0 \end{pmatrix}$
 - $\mathbf{a}_3 = \begin{pmatrix} 0 & 0 & 1.0 \end{pmatrix}$

Table 1: High-symmetry line: Γ -X.

symbol	position		symbol	position		on
Γ	$\begin{pmatrix} 0 & 0 \end{pmatrix}$	0)	X	$\left(\frac{1}{2}\right)$	0	0)

• Kets: dimension = 4

Table 2: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket	No.	ket
 1	(s,\uparrow) @A ₁	2	(s,\downarrow) @A ₁	3	(s,\uparrow) @B ₁	4	(s,\downarrow) @B ₁

• Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position			mapping
S_1	A_1	(0	0	0)	[1,2,3,4]
S_2	B_1	$\left(\frac{1}{2}\right)$	$\frac{1}{2}$	0)	[1,2,3,4]

• Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

	bond	tail	head	n	#	b@c	mapping
B_1	b_1	A_1	A_1	1	1	$ \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} @ \begin{pmatrix} 0 & 0 & \frac{1}{2} \end{pmatrix} $	[1,-2,-3,4]
B_2	b_2	A_1	A_1	1	2	$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$	[1,-2,-3,4]
B_3	b_3	B_1	B_1	1	1	$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{2} & 0 \end{pmatrix}$	[1,-2,-3,4]
B_4	b_4	B_1	B_1	1	2	$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	[1,-2,-3,4]
B_{5}	b_5	B_1	A_1	1	1	$ \left(\begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right) @ \left(\begin{array}{ccc} \frac{1}{4} & \frac{1}{4} & 0 \end{array}\right) $	[1]
	b_6	B_1	A_1	1	1	$ \left(\begin{array}{ccc} -\frac{1}{2} & \frac{1}{2} & 0 \end{array} \right) @ \left(\begin{array}{ccc} \frac{3}{4} & \frac{1}{4} & 0 \end{array} \right) $	[2]
	b_7	B_1	A_1	1	1	$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{3}{4} & \frac{3}{4} & 0 \end{pmatrix}$	[3]
	b_8	B_1	A_1	1	1	$\left(\begin{array}{cccc} \frac{1}{2} & -\frac{1}{2} & 0 \end{array}\right) @ \left(\begin{array}{cccc} \frac{1}{4} & \frac{3}{4} & 0 \end{array}\right)$	[4]

• SAMB:

No. 1
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₁, S₁]

$$\hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_g)}]$$

$$\hat{\mathbb{Z}}_1(\mathbf{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}]$$

No. 2
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₁, S₂]

$$\hat{\mathbb{Z}}_2 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(s,A_g)}]$$

$$\hat{\mathbb{Z}}_2(\mathbf{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_2[\mathbb{Q}_0^{(s,A_g)}]$$

No. 3
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₁, B₁]

$$\hat{\mathbb{Z}}_3 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_3(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_g)}]$$

No. 4
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₁, B₂]

$$\hat{\mathbb{Z}}_4 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_4(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_2[\mathbb{Q}_0^{(k,A_g)}]$$

No. 5
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₁, B₃]

$$\hat{\mathbb{Z}}_5 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_5(\mathbf{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_2[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k,A_g)}]$$

No. 6
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₁, B₄]

$$\hat{\mathbb{Z}}_6 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_6[\mathbb{Q}_0^{(b,A_g)}]$$

$$\begin{split} \hat{\mathbb{Z}}_{6}(\boldsymbol{k}) &= \mathbb{X}_{1}[\mathbb{Q}_{0}^{(a,A_{g})}] \otimes \mathbb{U}_{2}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{4}[\mathbb{Q}_{0}^{(k,A_{g})}] \\ & \boxed{No. \ 7} \quad \hat{\mathbb{Q}}_{0}^{(A_{g})} \ [M_{1},B_{5}] \\ \hat{\mathbb{Z}}_{7} &= \mathbb{X}_{1}[\mathbb{Q}_{0}^{(a,A_{g})}] \otimes \mathbb{Y}_{7}[\mathbb{Q}_{0}^{(b,A_{g})}] \\ \hat{\mathbb{Z}}_{7}(\boldsymbol{k}) &= \mathbb{X}_{1}[\mathbb{Q}_{0}^{(a,A_{g})}] \otimes \mathbb{U}_{3}[\mathbb{Q}_{0}^{(u,A_{g})}] \otimes \mathbb{F}_{5}[\mathbb{Q}_{0}^{(k,A_{g})}] \\ \hline \hat{\mathbb{Z}}_{8}(\boldsymbol{k}) &= \mathbb{X}_{2}[\mathbb{M}_{1}^{(a,A_{g})}(1,-1) \ [M_{1},B_{5}] \\ \hat{\mathbb{Z}}_{8} &= \mathbb{X}_{2}[\mathbb{M}_{1}^{(a,A_{g})}(1,-1)] \otimes \mathbb{Y}_{8}[\mathbb{T}_{0}^{(b,A_{g})}] \\ \hline \hat{\mathbb{Z}}_{8}(\boldsymbol{k}) &= \mathbb{X}_{2}[\mathbb{M}_{1}^{(a,A_{g})}(1,-1) \ [M_{1},B_{5}] \\ \hline \hat{\mathbb{Z}}_{9} &= \mathbb{X}_{3}[\mathbb{M}_{1}^{(a,B_{g},1)}(1,-1)] \otimes \mathbb{Y}_{9}[\mathbb{T}_{2}^{(b,B_{g},2)}] \\ \hline \hat{\mathbb{Z}}_{9}(\boldsymbol{k}) &= \mathbb{X}_{3}[\mathbb{M}_{1}^{(a,B_{g},1)}(1,-1)] \otimes \mathbb{U}_{4}[\mathbb{T}_{0}^{(u,A_{g})}] \otimes \mathbb{F}_{6}[\mathbb{Q}_{2}^{(k,B_{g},2)}] \\ \hline \hline \hat{\mathbb{N}}_{0}. \ 10 \quad \hat{\mathbb{Q}}_{2}^{(A_{g},2)}(1,-1) \ [M_{1},B_{5}] \\ \hline \hat{\mathbb{Z}}_{10} &= -\mathbb{X}_{4}[\mathbb{M}_{1}^{(a,B_{g},2)}(1,-1)] \otimes \mathbb{Y}_{9}[\mathbb{T}_{2}^{(b,B_{g},2)}] \\ \hline \end{split}$$

 $\hat{\mathbb{Z}}_{10}(\boldsymbol{k}) = -\mathbb{X}_{4}[\mathbb{M}_{1}^{(a,B_{g},2)}(1,-1)] \otimes \mathbb{U}_{4}[\mathbb{T}_{0}^{(u,A_{g})}] \otimes \mathbb{F}_{6}[\mathbb{Q}_{2}^{(k,B_{g},2)}]$

Table 5: Atomic SAMB group.

group	bra	ket
M_1	$(s,\uparrow),(s,\downarrow)$	$(s,\uparrow),(s,\downarrow)$

Table 6: Atomic SAMB.

symbol	type	group	form
\mathbb{X}_1	$\mathbb{Q}_0^{(a,A_g)}$	M_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0\\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{X}_2	$\mathbb{M}_1^{(a,A_g)}(1,-1)$	M_1	$\begin{pmatrix} 0 & -\frac{\sqrt{2}i}{2} \\ \frac{\sqrt{2}i}{2} & 0 \end{pmatrix}$
\mathbb{X}_3	$\mathbb{M}_{1}^{(a,B_{g},1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$ $\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & 0 \end{pmatrix}$
\mathbb{X}_4	$\mathbb{M}_{1}^{(a,B_{g},2)}(1,-1)$	M_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0\\ 0 & -\frac{\sqrt{2}}{2} \end{pmatrix}$

Table 7: Cluster SAMB.

symbol	type	cluster	form
\mathbb{Y}_1	$\mathbb{Q}_0^{(s,A_g)}$	S_1	(1)
\mathbb{Y}_2	$\mathbb{Q}_0^{(s,A_g)}$	S_2	(1)
\mathbb{Y}_3	$\mathbb{Q}_0^{(b,A_g)}$	B_1	(1)
\mathbb{Y}_4	$\mathbb{Q}_0^{(b,A_g)}$	B_2	(1)
\mathbb{Y}_5	$\mathbb{Q}_0^{(b,A_g)}$	B_3	(1)
\mathbb{Y}_6	$\mathbb{Q}_0^{(b,A_g)}$	B_4	(1)
\mathbb{Y}_7	$\mathbb{Q}_0^{(b,A_g)}$	B_{5}	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$
\mathbb{Y}_8	$\mathbb{T}_0^{(b,A_g)}$ $\mathbb{T}_2^{(b,B_g,2)}$	B_{5}	$\left(\begin{array}{cccc} \frac{i}{2} & \frac{i}{2} & \frac{i}{2} & \frac{i}{2} \end{array}\right)$
\mathbb{Y}_9	$\mathbb{T}_2^{(b,B_g,2)}$	B_{5}	$\left(\begin{array}{cccc} \frac{i}{2} & -\frac{i}{2} & \frac{i}{2} & -\frac{i}{2} \end{array}\right)$

Table 8: Uniform SAMB.

symbol	type	cluster	form
\mathbb{U}_1	$\mathbb{Q}_0^{(s,A_g)}$	S_1	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
\mathbb{U}_2	$\mathbb{Q}_0^{(s,A_g)}$	S_2	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
\mathbb{U}_3	$\mathbb{Q}_0^{(u,A_g)}$	B_5	$ \begin{pmatrix} 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix} $
\mathbb{U}_4	$\mathbb{T}_0^{(u,A_g)}$	B_5	$ \left[\begin{pmatrix} 0 & -\frac{\sqrt{2}i}{2} \\ \frac{\sqrt{2}i}{2} & 0 \end{pmatrix} \right] $

Table 9: Structure SAMB.

symbol	type	cluster	form
\mathbb{F}_1	$\mathbb{Q}_0^{(k,A_g)}$	B_1	$\sqrt{2}c_{001}$
\mathbb{F}_2	$\mathbb{Q}_0^{(k,A_g)}$	B_2	$\sqrt{2}c_{002}$
\mathbb{F}_3	$\mathbb{Q}_0^{(k,A_g)}$	B_3	$\sqrt{2}c_{003}$
\mathbb{F}_4	$\mathbb{Q}_0^{(k,A_g)}$	B_4	$\sqrt{2}c_{004}$
\mathbb{F}_5	$\mathbb{Q}_0^{(k,A_g)}$ $\mathbb{Q}_2^{(k,B_g,2)}$	B_5	$c_{005} + c_{006}$
\mathbb{F}_6	$\mathbb{Q}_2^{(k,B_g,2)}$	B_5	$c_{005} - c_{006}$

Table 10: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	$_{ m form}$
1	$\mathbb{Q}_0^{(A_g)}$	0	A_g	_	_	1

 $continued\ \dots$

Table 10

No.	symbol	rank	irrep.	mul.	comp.	form
2	$\mathbb{Q}_2^{(B_g,2)}$	2	B_g	2	_	$\sqrt{3}xy$

Table 11: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_1^{(A_g)}$	1	A_g	_	_	Y
2	$\mathbb{G}_1^{(B_g,1)}$	1	B_g	1	_	X
3	$\mathbb{G}_1^{(B_g,2)}$	1	B_g	2	_	Z

• Group info.: Generator = $\{2_{010}|0\}$, $\{-1|0\}$

Table 12: Conjugacy class (point-group part).

rep. SO	symmetry operations
{1 0}	{1 0}
${\{2_{010} 0\}}$	${2_{010} 0}$
{-1 0}	{-1 0}
$\{m_{010} 0\}$	$\{m_{010} 0\}$

Table 13: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	$\{1 0\}$	2	$\{2_{010} 0\}$	3	$\{-1 0\}$	4	$\{m_{010} 0\}$		

Table 14: Character table (point-group part).

	1	2_{010}	-1	m_{010}
A_g	1	1	1	1
B_g	1	-1	1	-1
A_u	1	1	-1	-1
B_u	1	-1	-1	1

Table 15: Parity conversion.

\leftrightarrow	\leftrightarrow	\leftrightarrow	\leftrightarrow
$A_g (A_u)$	$B_g (B_u)$	$A_u (A_g)$	$B_u (B_g)$

Table 16: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

	A_g	B_g	A_u	B_u
$\overline{A_g}$	A_g	B_g	A_u	B_u
B_g		A_g	B_u	A_u
A_u			A_g	B_g
B_u				A_g

Table 17: Anti-symmetric product, $[\Gamma \otimes \Gamma]_-$.

A_g	B_g	A_u	B_u
_	_	_	_

Table 18: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}$	2	$\begin{pmatrix} -1 & 1 & 0 \end{pmatrix}$	3	$\begin{pmatrix} -1 & -1 & 0 \end{pmatrix}$	4	$\begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$

Table 19: Virtual-cluster basis.

symbol	1	2	3	4
$\mathbb{Q}_0^{(A_g)}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\mathbb{Q}_1^{(A_u)}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$\mathbb{Q}_1^{(B_u,1)}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
$\mathbb{Q}_2^{(B_g,2)}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$