

PG No. 7 C_{2v} $mm2$ [orthorhombic] (axial, internal axial dipole)

* Harmonics for rank 0

$$\tilde{\mathbb{G}}_0^{(1,1)}[g](A_2)$$

** symmetry

1

** expression

$$\frac{\sqrt{3}G_x x}{3} + \frac{\sqrt{3}G_y y}{3} + \frac{\sqrt{3}G_z z}{3}$$

* Harmonics for rank 1

$$\tilde{\mathbb{G}}_1^{(1,-1)}[g](A_2)$$

** symmetry

z

** expression

G_z

$$\tilde{\mathbb{G}}_1^{(1,1)}[g](A_2)$$

** symmetry

z

** expression

$$\frac{3\sqrt{10}G_x x z}{10} + \frac{3\sqrt{10}G_y y z}{10} - \frac{\sqrt{10}G_z (x^2 + y^2 - 2z^2)}{10}$$

$$\tilde{\mathbb{G}}_1^{(1,-1)}[g](B_1)$$

** symmetry

y

** expression

G_y

$$\tilde{\mathbb{G}}_1^{(1,1)}[g](B_1)$$

** symmetry

y

** expression

$$\frac{3\sqrt{10}G_x x y}{10} - \frac{\sqrt{10}G_y (x^2 - 2y^2 + z^2)}{10} + \frac{3\sqrt{10}G_z y z}{10}$$

$$\tilde{\mathbb{G}}_1^{(1,-1)}[g](B_2)$$

** symmetry

x

** expression

G_x

$$\tilde{\mathbb{G}}_1^{(1,1)}[g](B_2)$$

** symmetry

x

** expression

$$\frac{\sqrt{10}G_x (2x^2 - y^2 - z^2)}{10} + \frac{3\sqrt{10}G_y x y}{10} + \frac{3\sqrt{10}G_z x z}{10}$$

* Harmonics for rank 2

$$\tilde{\mathbb{G}}_2^{(1,-1)}[g](A_1)$$

** symmetry

$\sqrt{3}xy$

** expression

$$\frac{\sqrt{2}G_x y}{2} + \frac{\sqrt{2}G_y x}{2}$$

$$\vec{\mathbb{G}}_2^{(1,1)}[g](A_1)$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{7}G_xy(4x^2-y^2-z^2)}{7}-\frac{\sqrt{7}G_yx(x^2-4y^2+z^2)}{7}+\frac{5\sqrt{7}G_zxyz}{7}$$

$$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_2, 1)$$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$-\frac{\sqrt{6}G_xx}{6}-\frac{\sqrt{6}G_yy}{6}+\frac{\sqrt{6}G_zz}{3}$$

$$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_2, 2)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}G_xx}{2}-\frac{\sqrt{2}G_yy}{2}$$

$$\vec{\mathbb{G}}_2^{(1,1)}[g](A_2, 1)$$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$-\frac{\sqrt{21}G_xx(x^2+y^2-4z^2)}{14}-\frac{\sqrt{21}G_yy(x^2+y^2-4z^2)}{14}-\frac{\sqrt{21}G_zz(3x^2+3y^2-2z^2)}{14}$$

$$\vec{\mathbb{G}}_2^{(1,1)}[g](A_2, 2)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{7}G_xx(3x^2-7y^2-2z^2)}{14}+\frac{\sqrt{7}G_yy(7x^2-3y^2+2z^2)}{14}+\frac{5\sqrt{7}G_zz(x-y)(x+y)}{14}$$

$$\vec{\mathbb{G}}_2^{(1,-1)}[g](B_1)$$

** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{2}G_yz}{2}+\frac{\sqrt{2}G_zy}{2}$$

$$\vec{\mathbb{G}}_2^{(1,1)}[g](B_1)$$

** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{5\sqrt{7}G_xxyz}{7}-\frac{\sqrt{7}G_yz(x^2-4y^2+z^2)}{7}-\frac{\sqrt{7}G_zy(x^2+y^2-4z^2)}{7}$$

$$\vec{\mathbb{G}}_2^{(1,-1)}[g](B_2)$$

** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}G_xz}{2} + \frac{\sqrt{2}G_zx}{2}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](B_2)$

** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{7}G_xz(4x^2 - y^2 - z^2)}{7} + \frac{5\sqrt{7}G_yxyz}{7} - \frac{\sqrt{7}G_zx(x^2 + y^2 - 4z^2)}{7}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$G_xyz + G_yxz + G_zxy$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{15}G_xyz(6x^2 - y^2 - z^2)}{6} - \frac{\sqrt{15}G_yxz(x^2 - 6y^2 + z^2)}{6} - \frac{\sqrt{15}G_zxy(x^2 + y^2 - 6z^2)}{6}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_2, 1)$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{\sqrt{15}G_xxz}{5} - \frac{\sqrt{15}G_yyz}{5} - \frac{\sqrt{15}G_z(x^2 + y^2 - 2z^2)}{10}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_2, 2)$

** symmetry

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

** expression

$$G_xxz - G_yyz + \frac{G_z(x - y)(x + y)}{2}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_2, 1)$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{5G_xxz(3x^2 + 3y^2 - 4z^2)}{12} - \frac{5G_yyz(3x^2 + 3y^2 - 4z^2)}{12} + \frac{G_z(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_2, 2)$

** symmetry

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

** expression

$$\frac{\sqrt{15}G_xxz(5x^2 - 9y^2 - 2z^2)}{12} + \frac{\sqrt{15}G_yyz(9x^2 - 5y^2 + 2z^2)}{12} - \frac{\sqrt{15}G_z(x - y)(x + y)(x^2 + y^2 - 6z^2)}{12}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B_1, 1)$

** symmetry

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$-\frac{\sqrt{15}G_xxy}{5} - \frac{\sqrt{15}G_y(x^2 - 2y^2 + z^2)}{10} - \frac{\sqrt{15}G_zyz}{5}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B_1, 2)$

** symmetry

$$-\frac{\sqrt{15}y(x - z)(x + z)}{2}$$

** expression

$$-G_xxy - \frac{G_y(x - z)(x + z)}{2} + G_zyz$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B_1, 1)$

** symmetry

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$-\frac{5G_xxy(3x^2 - 4y^2 + 3z^2)}{12} + \frac{G_y(3x^4 - 24x^2y^2 + 6x^2z^2 + 8y^4 - 24y^2z^2 + 3z^4)}{12} - \frac{5G_zyz(3x^2 - 4y^2 + 3z^2)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B_1, 2)$

** symmetry

$$-\frac{\sqrt{15}y(x - z)(x + z)}{2}$$

** expression

$$-\frac{\sqrt{15}G_xxy(5x^2 - 2y^2 - 9z^2)}{12} + \frac{\sqrt{15}G_y(x - z)(x + z)(x^2 - 6y^2 + z^2)}{12} - \frac{\sqrt{15}G_zyz(9x^2 + 2y^2 - 5z^2)}{12}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B_2, 1)$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

** expression

$$\frac{\sqrt{15}G_x(2x^2 - y^2 - z^2)}{10} - \frac{\sqrt{15}G_yxy}{5} - \frac{\sqrt{15}G_zxz}{5}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B_2, 2)$

** symmetry

$$\frac{\sqrt{15}x(y - z)(y + z)}{2}$$

** expression

$$\frac{G_x(y - z)(y + z)}{2} + G_yxy - G_zxz$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B_2, 1)$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

** expression

$$\frac{G_x(8x^4 - 24x^2y^2 - 24x^2z^2 + 3y^4 + 6y^2z^2 + 3z^4)}{12} + \frac{5G_yxy(4x^2 - 3y^2 - 3z^2)}{12} + \frac{5G_zxz(4x^2 - 3y^2 - 3z^2)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B_2, 2)$

** symmetry

$$\frac{\sqrt{15}x(y - z)(y + z)}{2}$$

** expression

$$\frac{\sqrt{15}G_x(y-z)(y+z)(6x^2-y^2-z^2)}{12}-\frac{\sqrt{15}G_yxy(2x^2-5y^2+9z^2)}{12}+\frac{\sqrt{15}G_zxz(2x^2+9y^2-5z^2)}{12}$$

* Harmonics for rank 4

$$\tilde{\mathbb{G}}_4^{(1,-1)}[g](A_1, 1)$$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{5}G_xy(3x^2-y^2)}{4}+\frac{\sqrt{5}G_yx(x^2-3y^2)}{4}$$

$$\tilde{\mathbb{G}}_4^{(1,-1)}[g](A_1, 2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$-\frac{\sqrt{35}G_xy(3x^2+y^2-6z^2)}{28}-\frac{\sqrt{35}G_yx(x^2+3y^2-6z^2)}{28}+\frac{3\sqrt{35}G_zxyz}{7}$$

$$\tilde{\mathbb{G}}_4^{(1,1)}[g](A_1, 1)$$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{77}G_xy(6x^4-11x^2y^2-3x^2z^2+y^4+y^2z^2)}{22}-\frac{\sqrt{77}G_yx(x^4-11x^2y^2+x^2z^2+6y^4-3y^2z^2)}{22}+\frac{9\sqrt{77}G_zxyz(x-y)(x+y)}{22}$$

$$\tilde{\mathbb{G}}_4^{(1,1)}[g](A_1, 2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$-\frac{\sqrt{11}G_xy(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{22}+\frac{\sqrt{11}G_yx(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{22}-\frac{21\sqrt{11}G_zxyz(x^2+y^2-2z^2)}{22}$$

$$\tilde{\mathbb{G}}_4^{(1,-1)}[g](A_2, 1)$$

** symmetry

$$\frac{\sqrt{21}(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4)}{6}$$

** expression

$$\frac{\sqrt{3}G_xx(2x^2-3y^2-3z^2)}{6}-\frac{\sqrt{3}G_yy(3x^2-2y^2+3z^2)}{6}-\frac{\sqrt{3}G_zz(3x^2+3y^2-2z^2)}{6}$$

$$\tilde{\mathbb{G}}_4^{(1,-1)}[g](A_2, 2)$$

** symmetry

$$-\frac{\sqrt{15}(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4)}{12}$$

** expression

$$-\frac{\sqrt{105}G_xx(x^2-6y^2+3z^2)}{42}+\frac{\sqrt{105}G_yy(6x^2-y^2-3z^2)}{42}-\frac{\sqrt{105}G_zz(3x^2+3y^2-2z^2)}{42}$$

$$\tilde{\mathbb{G}}_4^{(1,-1)}[g](A_2, 3)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\frac{\sqrt{35}G_xx(x^2-3z^2)}{14}-\frac{\sqrt{35}G_yy(y^2-3z^2)}{14}-\frac{3\sqrt{35}G_zz(x-y)(x+y)}{14}$$

$$\vec{\mathbb{G}}_4^{(1,1)}[g](A_2, 1)$$

** symmetry

$$\frac{\sqrt{21} (x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\begin{aligned} & \frac{\sqrt{1155}G_x x (x^4 - 5x^2y^2 - 5x^2z^2 + 3y^4 - 3y^2z^2 + 3z^4)}{66} + \frac{\sqrt{1155}G_y y (3x^4 - 5x^2y^2 - 3x^2z^2 + y^4 - 5y^2z^2 + 3z^4)}{66} \\ & + \frac{\sqrt{1155}G_z z (3x^4 - 3x^2y^2 - 5x^2z^2 + 3y^4 - 5y^2z^2 + z^4)}{66} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(1,1)}[g](A_2, 2)$$

** symmetry

$$-\frac{\sqrt{15} (x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{33}G_x x (5x^4 - 88x^2y^2 + 38x^2z^2 + 33y^4 + 66y^2z^2 - 30z^4)}{132} - \frac{\sqrt{33}G_y y (33x^4 - 88x^2y^2 + 66x^2z^2 + 5y^4 + 38y^2z^2 - 30z^4)}{132} \\ & + \frac{\sqrt{33}G_z z (3x^4 + 132x^2y^2 - 50x^2z^2 + 3y^4 - 50y^2z^2 + 10z^4)}{132} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(1,1)}[g](A_2, 3)$$

** symmetry

$$\frac{\sqrt{5} (x - y) (x + y) (x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{\sqrt{11}G_x x (5x^4 - 4x^2y^2 - 46x^2z^2 - 9y^4 + 66y^2z^2 + 12z^4)}{44} \\ & + \frac{\sqrt{11}G_y y (9x^4 + 4x^2y^2 - 66x^2z^2 - 5y^4 + 46y^2z^2 - 12z^4)}{44} + \frac{21\sqrt{11}G_z z (x - y) (x + y) (x^2 + y^2 - 2z^2)}{44} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](B_1, 1)$$

** symmetry

$$\frac{\sqrt{35}yz (y - z) (y + z)}{2}$$

** expression

$$\frac{\sqrt{5}G_y z (3y^2 - z^2)}{4} + \frac{\sqrt{5}G_z y (y^2 - 3z^2)}{4}$$

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](B_1, 2)$$

** symmetry

$$\frac{\sqrt{5}yz (6x^2 - y^2 - z^2)}{2}$$

** expression

$$\frac{3\sqrt{35}G_x xyz}{7} + \frac{\sqrt{35}G_y z (6x^2 - 3y^2 - z^2)}{28} + \frac{\sqrt{35}G_z y (6x^2 - y^2 - 3z^2)}{28}$$

$$\vec{\mathbb{G}}_4^{(1,1)}[g](B_1, 1)$$

** symmetry

$$\frac{\sqrt{35}yz (y - z) (y + z)}{2}$$

** expression

$$\frac{9\sqrt{77}G_x xyz (y - z) (y + z)}{22} - \frac{\sqrt{77}G_y z (3x^2y^2 - x^2z^2 - 6y^4 + 11y^2z^2 - z^4)}{22} - \frac{\sqrt{77}G_z y (x^2y^2 - 3x^2z^2 + y^4 - 11y^2z^2 + 6z^4)}{22}$$

$$\vec{\mathbb{G}}_4^{(1,1)}[g](B_1, 2)$$

** symmetry

$$\frac{\sqrt{5}yz (6x^2 - y^2 - z^2)}{2}$$

** expression

$$\frac{21\sqrt{11}G_x xyz (2x^2 - y^2 - z^2)}{22} - \frac{\sqrt{11}G_y z (6x^4 - 51x^2y^2 + 5x^2z^2 + 6y^4 + 5y^2z^2 - z^4)}{22} - \frac{\sqrt{11}G_z y (6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4)}{22}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](B_2, 1)$
** symmetry

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

** expression

$$-\frac{\sqrt{5}G_xz(3x^2-z^2)}{4} - \frac{\sqrt{5}G_zx(x^2-3z^2)}{4}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](B_2, 2)$
** symmetry

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

** expression

$$-\frac{\sqrt{35}G_xz(3x^2-6y^2+z^2)}{28} + \frac{3\sqrt{35}G_yxyz}{7} - \frac{\sqrt{35}G_zx(x^2-6y^2+3z^2)}{28}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](B_2, 1)$
** symmetry

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

** expression

$$-\frac{\sqrt{77}G_xz(6x^4-3x^2y^2-11x^2z^2+y^2z^2+z^4)}{22} - \frac{9\sqrt{77}G_yxyz(x-z)(x+z)}{22} + \frac{\sqrt{77}G_zx(x^4+x^2y^2-11x^2z^2-3y^2z^2+6z^4)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](B_2, 2)$
** symmetry

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

** expression

$$-\frac{\sqrt{11}G_xz(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4)}{22} - \frac{21\sqrt{11}G_yxyz(x^2-2y^2+z^2)}{22} + \frac{\sqrt{11}G_zx(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{22}$$