

PG No. 34 $D_3(1)$ 32 (312 setting) [trigonal] (axial, internal polar dipole)

* Harmonics for rank 0

* Harmonics for rank 1

$$\vec{\mathbb{G}}_1^{(1,0)}[q](A_2)$$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}Q_{xy}}{2} - \frac{\sqrt{2}Q_{yx}}{2}$$

$$\vec{\mathbb{G}}_{1,1}^{(1,0)}[q](E), \vec{\mathbb{G}}_{1,2}^{(1,0)}[q](E)$$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{\sqrt{2}Q_{yz}}{2} - \frac{\sqrt{2}Q_{zy}}{2}$$

$$-\frac{\sqrt{2}Q_{xz}}{2} + \frac{\sqrt{2}Q_{zx}}{2}$$

* Harmonics for rank 2

$$\vec{\mathbb{G}}_2^{(1,0)}[q](A_1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{6}Q_{xyz}}{2} - \frac{\sqrt{6}Q_{yxz}}{2}$$

$$\vec{\mathbb{G}}_{2,1}^{(1,0)}[q](E, 1), \vec{\mathbb{G}}_{2,2}^{(1,0)}[q](E, 1)$$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_x(y-z)(y+z)}{2} - \frac{\sqrt{2}Q_yxy}{2} + \frac{\sqrt{2}Q_zxz}{2}$$

$$-\frac{\sqrt{2}Q_xxy}{2} + \frac{\sqrt{2}Q_y(x-z)(x+z)}{2} + \frac{\sqrt{2}Q_zyz}{2}$$

$$\vec{\mathbb{G}}_{2,1}^{(1,0)}[q](E, 2), \vec{\mathbb{G}}_{2,2}^{(1,0)}[q](E, 2)$$

** symmetry

$$\sqrt{3}xy$$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$-\frac{\sqrt{2}Q_xxz}{2} + \frac{\sqrt{2}Q_yyz}{2} + \frac{\sqrt{2}Q_z(x-y)(x+y)}{2}$$

$$\frac{\sqrt{2}Q_xyz}{2} + \frac{\sqrt{2}Q_yxz}{2} - \sqrt{2}Q_zxy$$

* Harmonics for rank 3

$$\vec{\mathbb{G}}_3^{(1,0)}[q](A_1)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$-\frac{\sqrt{30}Q_x z (x-y)(x+y)}{8} + \frac{\sqrt{30}Q_y x y z}{4} + \frac{\sqrt{30}Q_z x (x^2 - 3y^2)}{8}$$

$$\vec{\mathbb{G}}_3^{(1,0)}[q](A_2, 1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_x y (x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{3}Q_y x (x^2 + y^2 - 4z^2)}{4}$$

$$\vec{\mathbb{G}}_3^{(1,0)}[q](A_2, 2)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{30}Q_x x y z}{4} + \frac{\sqrt{30}Q_y z (x-y)(x+y)}{8} - \frac{\sqrt{30}Q_z y (3x^2 - y^2)}{8}$$

$$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E, 1), \vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E, 1)$$

** symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$\frac{5\sqrt{2}Q_x x y z}{4} - \frac{\sqrt{2}Q_y z (11x^2 + y^2 - 4z^2)}{8} + \frac{\sqrt{2}Q_z y (x^2 + y^2 - 4z^2)}{8}$$

$$\frac{\sqrt{2}Q_x z (x^2 + 11y^2 - 4z^2)}{8} - \frac{5\sqrt{2}Q_y x y z}{4} - \frac{\sqrt{2}Q_z x (x^2 + y^2 - 4z^2)}{8}$$

$$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E, 2), \vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E, 2)$$

** symmetry

$$-\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

$$\sqrt{15}x y z$$

** expression

$$-\frac{\sqrt{5}Q_x y (x^2 - y^2 + 2z^2)}{4} + \frac{\sqrt{5}Q_y x (x^2 - y^2 - 2z^2)}{4} + \sqrt{5}Q_z x y z$$

$$\frac{\sqrt{5}Q_x x (y-z)(y+z)}{2} - \frac{\sqrt{5}Q_y y (x-z)(x+z)}{2} + \frac{\sqrt{5}Q_z z (x-y)(x+y)}{2}$$

* Harmonics for rank 4

$$\vec{\mathbb{G}}_4^{(1,0)}[q](A_1, 1)$$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2 y^2}{4} - 3x^2 z^2 + \frac{3y^4}{8} - 3y^2 z^2 + z^4$$

** expression

$$-\frac{\sqrt{5}Q_x y z (3x^2 + 3y^2 - 4z^2)}{4} + \frac{\sqrt{5}Q_y x z (3x^2 + 3y^2 - 4z^2)}{4}$$

$$\vec{\mathbb{G}}_4^{(1,0)}[q](A_1, 2)$$

** symmetry

$$\frac{\sqrt{70}x z (x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{14}Q_xxy(x^2-3y^2+6z^2)}{8} - \frac{\sqrt{14}Q_y(x^4-3x^2y^2-3x^2z^2+3y^2z^2)}{8} - \frac{3\sqrt{14}Q_zyz(3x^2-y^2)}{8}$$

$$\tilde{\mathbb{G}}_4^{(1,0)}[q](A_2)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2-y^2)}{4}$$

** expression

$$\frac{\sqrt{14}Q_x(3x^2y^2-3x^2z^2-y^4+3y^2z^2)}{8} - \frac{\sqrt{14}Q_yxy(3x^2-y^2-6z^2)}{8} + \frac{3\sqrt{14}Q_zxz(x^2-3y^2)}{8}$$

$$\tilde{\mathbb{G}}_{4,1}^{(1,0)}[q](E,1), \tilde{\mathbb{G}}_{4,2}^{(1,0)}[q](E,1)$$

** symmetry

$$-\frac{\sqrt{10}yz(3x^2+3y^2-4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2+3y^2-4z^2)}{4}$$

** expression

$$-\frac{\sqrt{2}Q_x(3x^2y^2-3x^2z^2+3y^4-21y^2z^2+4z^4)}{8} + \frac{3\sqrt{2}Q_yxy(x^2+y^2-6z^2)}{8} - \frac{\sqrt{2}Q_zxz(3x^2+3y^2-4z^2)}{8}$$

$$\frac{3\sqrt{2}Q_xxy(x^2+y^2-6z^2)}{8} - \frac{\sqrt{2}Q_y(3x^4+3x^2y^2-21x^2z^2-3y^2z^2+4z^4)}{8} - \frac{\sqrt{2}Q_zyz(3x^2+3y^2-4z^2)}{8}$$

$$\tilde{\mathbb{G}}_{4,1}^{(1,0)}[q](E,2), \tilde{\mathbb{G}}_{4,2}^{(1,0)}[q](E,2)$$

** symmetry

$$-\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

$$\frac{\sqrt{35}(x^2-2xy-y^2)(x^2+2xy-y^2)}{8}$$

** expression

$$\frac{\sqrt{7}Q_xxz(x^2-3y^2)}{4} - \frac{\sqrt{7}Q_yyz(3x^2-y^2)}{4} - \frac{\sqrt{7}Q_z(x^2-2xy-y^2)(x^2+2xy-y^2)}{4}$$

$$\frac{\sqrt{7}Q_xyz(3x^2-y^2)}{4} + \frac{\sqrt{7}Q_yxz(x^2-3y^2)}{4} - \sqrt{7}Q_zxy(x-y)(x+y)$$

$$\tilde{\mathbb{G}}_{4,1}^{(1,0)}[q](E,3), \tilde{\mathbb{G}}_{4,2}^{(1,0)}[q](E,3)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\frac{Q_xxz(x^2+15y^2-6z^2)}{4} - \frac{Q_yyz(15x^2+y^2-6z^2)}{4} - \frac{Q_z(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

$$\frac{Q_xyz(3x^2-4y^2+3z^2)}{2} - \frac{Q_yxz(4x^2-3y^2-3z^2)}{2} + \frac{Q_zxy(x^2+y^2-6z^2)}{2}$$