

MPG No. 16.1.60 3 [ Type I, trigonal ] [G tensor]

\* Rank 0 tensor.

$$[G]$$

$$G = G_s^{(1)}$$

\* Rank 1 tensor.

$$[0 \ 0 \ G_z]$$

$$G_z = G_{pz}^{(1)}$$

\* Rank 2 tensor (s).

$$\begin{bmatrix} G_{xx} & 0 & 0 \\ 0 & G_{xx} & 0 \\ 0 & 0 & G_{zz} \end{bmatrix}$$

$$G_{xx} = -G_{du}^{(1)} + G_s^{(1)}$$

$$G_{zz} = 2G_{du}^{(1)} + G_s^{(1)}$$

\* Rank 2 tensor (a).

$$\begin{bmatrix} 0 & G_{xy} & 0 \\ -G_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{xy} = Q_{pz}^{(1)}$$

\* Rank 3 tensor (s).

$$\begin{bmatrix} G_{xxx} & G_{xxy} & G_{xxz} \\ -G_{xxx} & -G_{xxy} & G_{xxz} \\ 0 & 0 & G_{zzz} \\ G_{yzz} & G_{yzy} & 0 \\ G_{yzy} & -G_{yzx} & 0 \\ G_{xyx} & -G_{xxx} & 0 \end{bmatrix}$$

$$G_{xxx} = G_{f2}^{(1)}$$

$$G_{xxy} = G_{f1}^{(1)}$$

$$G_{xxz} = -G_{faz}^{(1)} + G_{pz}^{(1)}$$

$$G_{zzz} = 2G_{faz}^{(1)} + G_{pz}^{(1)} + 2G_{pz}^{(2)}$$

$$G_{yzx} = -3Q_{du}^{(1)}$$

$$G_{yzy} = -G_{faz}^{(1)} + G_{pz}^{(2)}$$

\* Rank 3 tensor (a).

$$\begin{bmatrix} G_{yzx} & G_{yzy} & 0 \\ -G_{yzy} & G_{yzx} & 0 \\ 0 & 0 & G_{xyz} \end{bmatrix}$$

$$G_{yzx} = -Q_{du}^{(2)} + Q_s^{(1)}$$

$$G_{yzy} = G_{pz}^{(3)}$$

$$G_{xyz} = 2Q_{du}^{(2)} + Q_s^{(1)}$$

\* Rank 4 tensor (sss).

$$\begin{bmatrix} G_{xxxx} & G_{xxyy} & G_{xxzz} & G_{xxyz} & G_{xxzx} & 0 \\ G_{xxyy} & G_{xxxx} & G_{xxzz} & -G_{xxyz} & -G_{xxzx} & 0 \\ G_{xxzz} & G_{xxzz} & G_{zzzz} & 0 & 0 & 0 \\ G_{xxyz} & -G_{xxyz} & 0 & G_{yzyz} & 0 & -G_{xxzx} \\ G_{xxzx} & -G_{xxzx} & 0 & 0 & G_{yzyz} & G_{xxyz} \\ 0 & 0 & 0 & -G_{xxzx} & G_{xxyz} & \frac{G_{xxxx}}{2} - \frac{G_{xxyy}}{2} \end{bmatrix}$$

$$G_{xxxx} = -2G_{du}^{(1)} - 4G_{du}^{(2)} + 3G_{g0}^{(1)} + G_s^{(1)} + 2G_s^{(2)}$$

$$G_{xxyy} = -2G_{du}^{(1)} + G_{g0}^{(1)} + G_s^{(1)}$$

$$G_{xxzz} = G_{du}^{(1)} - 4G_{g0}^{(1)} + G_s^{(1)}$$

$$G_{xxyz} = G_{gb}^{(1)}$$

$$G_{xxzx} = G_{ga}^{(1)}$$

$$G_{zzzz} = 4G_{du}^{(1)} + 8G_{du}^{(2)} + 8G_{g0}^{(1)} + G_s^{(1)} + 2G_s^{(2)}$$

$$G_{yzyz} = G_{du}^{(2)} - 4G_{g0}^{(1)} + G_s^{(2)}$$

\* Rank 4 tensor (ssa).

$$\begin{bmatrix} 0 & 0 & G_{xxzz} & G_{xxyz} & G_{xxzx} & G_{xxxz} \\ 0 & 0 & G_{xxzz} & -G_{xxyz} & -G_{xxzx} & -G_{xxxz} \\ -G_{xxzz} & -G_{xxzz} & 0 & 0 & 0 & 0 \\ -G_{xxyz} & G_{xxyz} & 0 & 0 & G_{yzzx} & G_{xxzx} \\ -G_{xxzx} & G_{xxzx} & 0 & -G_{yzzx} & 0 & -G_{xxyz} \\ -G_{xxxz} & G_{xxxz} & 0 & -G_{xxzx} & G_{xxyz} & 0 \end{bmatrix}$$

$$G_{xxzz} = 3G_{du}^{(3)}$$

$$G_{xxyz} = 2Q_{f2}^{(1)}$$

$$G_{xxzx} = -2Q_{f1}^{(1)}$$

$$G_{xxxz} = -2Q_{faz}^{(1)} + 2Q_{pz}^{(1)}$$

$$G_{yzzx} = -4Q_{faz}^{(1)} - Q_{pz}^{(1)}$$

\* Rank 4 tensor (aas).

$$\begin{bmatrix} G_{yzyz} & 0 & 0 \\ 0 & G_{yzyz} & 0 \\ 0 & 0 & G_{xyxy} \end{bmatrix}$$

$$G_{yzyz} = -2G_{du}^{(4)} + G_s^{(3)}$$

$$G_{xyxy} = 4G_{du}^{(4)} + G_s^{(3)}$$

\* Rank 4 tensor (aaa).

$$\begin{bmatrix} 0 & G_{yzzx} & 0 \\ -G_{yzzx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{yzzx} = -Q_{pz}^{(2)}$$

\* Rank 4 tensor (sa).

$$\begin{bmatrix} G_{xxyz} & G_{xxzx} & G_{xxxz} \\ -G_{xxyz} & -G_{xxzx} & G_{xxxz} \\ 0 & 0 & G_{zzxy} \\ G_{yzyz} & G_{yzzx} & 0 \\ G_{yzzx} & -G_{yzyz} & 0 \\ G_{xxzx} & -G_{xxyz} & 0 \end{bmatrix}$$

$$\begin{aligned}
G_{xxyz} &= Q_{f2}^{(2)} \\
G_{xxzx} &= Q_{f1}^{(2)} \\
G_{xxxy} &= -Q_{faz}^{(2)} + Q_{pz}^{(3)} \\
G_{zzxy} &= 2Q_{faz}^{(2)} + Q_{pz}^{(3)} + 2Q_{pz}^{(4)} \\
G_{yzyz} &= -3G_{du}^{(5)} \\
G_{yzzx} &= -Q_{faz}^{(2)} + Q_{pz}^{(4)}
\end{aligned}$$

\* Rank 4 tensor (as).

$$\begin{bmatrix} G_{yzxx} & -G_{yzxx} & 0 & G_{yzyz} & G_{yzzx} & G_{yzxy} \\ G_{yzxy} & -G_{yzxy} & 0 & G_{yzzx} & -G_{yzyz} & -G_{yzxx} \\ G_{xyxx} & G_{xyxx} & G_{xyzz} & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}
G_{yzzx} &= Q_{f2}^{(3)} \\
G_{yzyz} &= -3G_{du}^{(6)} \\
G_{yzzx} &= -Q_{faz}^{(3)} + Q_{pz}^{(6)} \\
G_{yzxy} &= Q_{f1}^{(3)} \\
G_{xyxx} &= -Q_{faz}^{(3)} + Q_{pz}^{(5)} \\
G_{xyzz} &= 2Q_{faz}^{(3)} + Q_{pz}^{(5)} + 2Q_{pz}^{(6)}
\end{aligned}$$

\* Rank 4 tensor (s).

$$\begin{bmatrix} G_{xxxx} & G_{xxyy} & G_{xxzz} & G_{xxyz} & G_{xxzx} & G_{xxxz} & G_{xxzy} & G_{xxxz} & G_{xxxy} \\ G_{xxyy} & G_{xxxx} & G_{xxzz} & -G_{xxyz} & -G_{xxzx} & -G_{xxxz} & -G_{xxzy} & -G_{xxxz} & -G_{xxxy} \\ G_{zzxx} & G_{zzxx} & G_{zzzz} & 0 & 0 & 0 & 0 & 0 & -G_{zzxy} \\ G_{yzxx} & -G_{yzxx} & 0 & G_{yzyz} & G_{yzzx} & G_{yzxy} & G_{yzyz} & G_{yzzx} & G_{yzxy} \\ -G_{yzxy} & G_{yzxy} & 0 & -G_{yzxx} & G_{yzyz} & G_{yzxx} & -G_{yzzx} & G_{yzyz} & G_{yzxx} \\ -\frac{G_{xxxx}}{2} - \frac{G_{xxyy}}{2} & \frac{G_{xxyy}}{2} + \frac{G_{xxxx}}{2} & 0 & -G_{xxzx} & G_{xxzy} & \frac{G_{xxxx}}{2} - \frac{G_{xxyy}}{2} & -G_{xxzy} & G_{xxxy} & \frac{G_{xxxx}}{2} - \frac{G_{xxyy}}{2} \end{bmatrix}$$

$$\begin{aligned}
G_{xxxx} &= -2G_{du}^{(1)} - 4G_{du}^{(2)} + 3G_{g0}^{(1)} + G_s^{(1)} + 2G_s^{(2)} \\
G_{xxyy} &= -2G_{du}^{(1)} + G_{g0}^{(1)} + G_s^{(1)} \\
G_{xxzz} &= G_{du}^{(1)} + 3G_{du}^{(3)} - 4G_{g0}^{(1)} + G_s^{(1)} \\
G_{xxyz} &= G_{gb}^{(1)} + 2Q_{f2}^{(1)} + Q_{f2}^{(2)} \\
G_{xxzx} &= G_{ga}^{(1)} - 2Q_{f1}^{(1)} + Q_{f1}^{(2)} \\
G_{xxxy} &= -2Q_{faz}^{(1)} - Q_{faz}^{(2)} + 2Q_{pz}^{(1)} + Q_{pz}^{(3)} \\
G_{xxzy} &= G_{gb}^{(1)} + 2Q_{f2}^{(1)} - Q_{f2}^{(2)} \\
G_{xxxz} &= G_{ga}^{(1)} - 2Q_{f1}^{(1)} - Q_{f1}^{(2)} \\
G_{xxyx} &= -2Q_{faz}^{(1)} + Q_{faz}^{(2)} + 2Q_{pz}^{(1)} - Q_{pz}^{(3)} \\
G_{zzxx} &= G_{du}^{(1)} - 3G_{du}^{(3)} - 4G_{g0}^{(1)} + G_s^{(1)} \\
G_{zzzz} &= 4G_{du}^{(1)} + 8G_{du}^{(2)} + 8G_{g0}^{(1)} + G_s^{(1)} + 2G_s^{(2)} \\
G_{zzxy} &= 2Q_{faz}^{(2)} + Q_{pz}^{(3)} + 2Q_{pz}^{(4)} \\
G_{yzzx} &= G_{gb}^{(1)} - 2Q_{f2}^{(1)} \\
G_{yzyz} &= G_{du}^{(2)} - 3G_{du}^{(5)} - 4G_{g0}^{(1)} + G_s^{(2)} \\
G_{yzzx} &= -4Q_{faz}^{(1)} - Q_{faz}^{(2)} - Q_{pz}^{(1)} + Q_{pz}^{(4)} \\
G_{yzxy} &= -G_{ga}^{(1)} - 2Q_{f1}^{(1)} \\
G_{yzzz} &= G_{du}^{(2)} + 3G_{du}^{(5)} - 4G_{g0}^{(1)} + G_s^{(2)} \\
G_{yzzx} &= -4Q_{faz}^{(1)} + Q_{faz}^{(2)} - Q_{pz}^{(1)} - Q_{pz}^{(4)}
\end{aligned}$$

\* Rank 4 tensor (a).

$$\begin{bmatrix} G_{yzxx} & -G_{yzxx} & 0 & G_{yzyz} & G_{yzxx} & G_{yzxy} & G_{yzyz} & G_{yzxz} & G_{yzxy} \\ G_{yzxy} & -G_{yzxy} & 0 & G_{yzxx} & -G_{yzxy} & -G_{yzxx} & G_{yzxx} & -G_{yzyz} & -G_{yzxx} \\ G_{xyxx} & G_{xyxx} & G_{xyzz} & 0 & 0 & G_{xyxy} & 0 & 0 & -G_{xyxy} \end{bmatrix}$$

$$G_{yzxx} = Q_{f2}^{(3)}$$

$$G_{yzyz} = -2G_{du}^{(4)} - 3G_{du}^{(6)} + G_s^{(3)}$$

$$G_{yzxx} = -Q_{faz}^{(3)} - Q_{pz}^{(2)} + Q_{pz}^{(6)}$$

$$G_{yzxy} = Q_{f1}^{(3)}$$

$$G_{yzyy} = 2G_{du}^{(4)} - 3G_{du}^{(6)} - G_s^{(3)}$$

$$G_{yzxz} = -Q_{faz}^{(3)} + Q_{pz}^{(2)} + Q_{pz}^{(6)}$$

$$G_{xyxx} = -Q_{faz}^{(3)} + Q_{pz}^{(5)}$$

$$G_{xyzz} = 2Q_{faz}^{(3)} + Q_{pz}^{(5)} + 2Q_{pz}^{(6)}$$

$$G_{xyxy} = 4G_{du}^{(4)} + G_s^{(3)}$$

\* Rank 4 tensor (t).

$$\begin{bmatrix} G_{xxxx} & 0 & G_{xxxx} & \\ 0 & G_{xxxx} & G_{yyyz} & \\ 0 & 0 & G_{zzzz} & \\ -G_{xxxx} & G_{yyyz} & G_{yyzz} & \\ G_{zzxx} & 0 & 0 & \\ 0 & G_{xxyy} & -G_{yyyz} & \\ 0 & G_{gyzy} & 0 & \\ G_{xxxx} & -G_{yyyz} & -\frac{G_{xxxx}}{2} + \frac{3G_{xxyy}}{2} + G_{yyzz} - G_{yzyy} + G_{zzxx} & \\ \frac{G_{xxxx}}{2} - \frac{G_{xxyy}}{2} & 0 & -G_{xxxx} & \\ -G_{yyyz} & -G_{xxxx} & 0 & \end{bmatrix}$$

$$G_{xxxx} = -2G_{du}^{(1)} - 4G_{du}^{(2)} + 3G_{g0}^{(1)} + 3G_s^{(1)}$$

$$G_{xxxx} = G_{ga}^{(1)}$$

$$G_{yyyz} = -G_{gb}^{(1)}$$

$$G_{zzzz} = 4G_{du}^{(1)} + 8G_{du}^{(2)} + 8G_{g0}^{(1)} + 3G_s^{(1)}$$

$$G_{yyzz} = G_{du}^{(1)} + 3G_{du}^{(3)} - 4G_{g0}^{(1)} + G_s^{(1)}$$

$$G_{zzxx} = G_{du}^{(1)} - 3G_{du}^{(3)} - 4G_{g0}^{(1)} + G_s^{(1)}$$

$$G_{xxyy} = -2G_{du}^{(1)} + G_{g0}^{(1)} + G_s^{(1)}$$

$$G_{yzyy} = G_{du}^{(2)} + 3G_{du}^{(5)} - 4G_{g0}^{(1)} + G_s^{(1)}$$