Response Tensors up to 4th rank in D_{2h}

— polar tensors —

$$C^{(0,Q)} = (C^{(0,Q)})$$

$$C^{(0,Q)} = Q_0$$

$$S^{(2,Q)} = \begin{pmatrix} S_{xx}^{(2,Q)} & 0 & 0\\ 0 & S_{yy}^{(2,Q)} & 0\\ 0 & 0 & S_{zz}^{(2,Q)} \end{pmatrix}$$

$$S_{xx}^{(2,Q)} = Q_0 - Q_u + Q_v$$

$$S_{yy}^{(2,Q)} = Q_0 - Q_u - Q_v$$

$$S_{zz}^{(2,Q)} = Q_0 + 2Q_u$$

$$S^{(4,Q)} = \begin{pmatrix} S_{11}^{(4,Q)} & S_{12}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & 0 & 0 \\ S_{12}^{(4,Q)} & S_{22}^{(4,Q)} & S_{23}^{(4,Q)} & 0 & 0 & 0 \\ S_{13}^{(4,Q)} & S_{23}^{(4,Q)} & S_{33}^{(4,Q)} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55}^{(4,Q)} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66}^{(4,Q)} \end{pmatrix}$$

$$\begin{split} S_{11}^{(4,Q)} &= Q_0[1] + 2Q_0[2] - Q_{4u} + Q_{4v} + 2Q_4 - 2Q_u[1] - 4Q_u[2] + 2Q_v[1] + 4Q_v[2] \\ S_{12}^{(4,Q)} &= Q_0[1] + 2Q_{4u} - Q_4 - 2Q_u[1] \\ S_{13}^{(4,Q)} &= Q_0[1] - Q_{4u} - Q_{4v} - Q_4 + Q_u[1] + Q_v[1] \\ S_{22}^{(4,Q)} &= Q_0[1] + 2Q_0[2] - Q_{4u} - Q_{4v} + 2Q_4 - 2Q_u[1] - 4Q_u[2] - 2Q_v[1] - 4Q_v[2] \\ S_{23}^{(4,Q)} &= Q_0[1] - Q_{4u} + Q_{4v} - Q_4 + Q_u[1] - Q_v[1] \\ S_{33}^{(4,Q)} &= Q_0[1] + 2Q_0[2] + 2Q_{4u} + 2Q_4 + 4Q_u[1] + 8Q_u[2] \\ S_{44}^{(4,Q)} &= Q_0[2] - Q_{4u} + Q_{4v} - Q_4 + Q_u[2] - Q_v[2] \\ S_{56}^{(4,Q)} &= Q_0[2] + 2Q_{4u} - Q_4v - Q_4 + Q_u[2] + Q_v[2] \\ S_{66}^{(4,Q)} &= Q_0[2] + 2Q_{4u} - Q_4 - 2Q_u[2] \end{split}$$

$$\bar{S}_{12}^{(4,Q)} = 4G_{xyz}[1] - 2Q_v[3]$$

$$\bar{S}_{13}^{(4,Q)} = -4G_{xyz}[1] + 3Q_u[3] - Q_v[3]$$

$$\bar{S}_{23}^{(4,Q)} = 4G_{xyz}[1] + 3Q_u[3] + Q_v[3]$$

$$A^{(4,Q)} = \begin{pmatrix} A_{xx}^{(4,Q)} & 0 & 0\\ 0 & A_{yy}^{(4,Q)} & 0\\ 0 & 0 & A_{zz}^{(4,Q)} \end{pmatrix}$$

$$\begin{split} A_{xx}^{(4,Q)} &= Q_0[3] - 2Q_u[6] + 2Q_v[6] \\ A_{yy}^{(4,Q)} &= Q_0[3] - 2Q_u[6] - 2Q_v[6] \\ A_{zz}^{(4,Q)} &= Q_0[3] + 4Q_u[6] \end{split}$$

$$\begin{split} M_{4x}^{(4,Q)} &= G_{xyz}[2] - 3Q_u[4] - Q_v[4] \\ M_{5y}^{(4,Q)} &= G_{xyz}[2] + 3Q_u[4] - Q_v[4] \\ M_{6z}^{(4,Q)} &= G_{xyz}[2] + 2Q_v[4] \end{split}$$

$$\bar{M}^{(4,Q)} = \begin{pmatrix} 0 & 0 & \bar{M}_{x4}^{(4,Q)} & 0 & 0\\ 0 & 0 & 0 & \bar{M}_{y5}^{(4,Q)} & 0\\ 0 & 0 & 0 & 0 & \bar{M}_{z6}^{(4,Q)} \end{pmatrix}$$

$$\begin{split} \bar{M}_{x4}^{(4,Q)} &= G_{xyz}[3] - 3Q_u[5] - Q_v[5] \\ \bar{M}_{y5}^{(4,Q)} &= G_{xyz}[3] + 3Q_u[5] - Q_v[5] \\ \bar{M}_{z6}^{(4,Q)} &= G_{xyz}[3] + 2Q_v[5] \end{split}$$

$$S^{(3,G)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ S_{4x}^{(3,G)} & 0 & 0 \\ 0 & S_{5y}^{(3,G)} & 0 \\ 0 & 0 & S_{6z}^{(3,G)} \end{pmatrix}$$

$$\begin{split} S_{4x}^{(3,G)} &= G_{xyz} - 3Q_u[1] - Q_v[1] \\ S_{5y}^{(3,G)} &= G_{xyz} + 3Q_u[1] - Q_v[1] \\ S_{6z}^{(3,G)} &= G_{xyz} + 2Q_v[1] \end{split}$$

$$A^{(3,G)} = \begin{pmatrix} A_{4x}^{(3,G)} & 0 & 0\\ 0 & A_{5y}^{(3,G)} & 0\\ 0 & 0 & A_{6z}^{(3,G)} \end{pmatrix}$$

$$A_{4x}^{(3,G)} = Q_0 - Q_u[2] + Q_v[2]$$

$$A_{5y}^{(3,G)} = Q_0 - Q_u[2] - Q_v[2]$$

$$A_{6z}^{(3,G)} = Q_0 + 2Q_u[2]$$