

PG No. 19 C_{3v} $3m$ (3m1 setting) [trigonal] (polar, internal axial octupole)

* Harmonics for rank 0

* Harmonics for rank 1

$$\vec{\mathbb{Q}}_1^{(3,2)}[g](A_1)$$

** symmetry

z

** expression

$$-\frac{3\sqrt{70}G_1x(x^2 - 3y^2)}{56} + \frac{3\sqrt{70}G_2y(3x^2 - y^2)}{56} - \frac{\sqrt{105}G_3z(x-y)(x+y)}{14} - \frac{\sqrt{42}G_{3xy}(x^2 + y^2 - 4z^2)}{56} + \frac{\sqrt{42}G_{3y}x(x^2 + y^2 - 4z^2)}{56} + \frac{\sqrt{105}G_{bz}xyz}{7}$$

$$\vec{\mathbb{Q}}_{1,1}^{(3,2)}[g](E), \vec{\mathbb{Q}}_{1,2}^{(3,2)}[g](E)$$

** symmetry

x

y

** expression

$$\begin{aligned} & \frac{3\sqrt{70}G_1z(x-y)(x+y)}{56} - \frac{3\sqrt{70}G_2xyz}{28} - \frac{\sqrt{105}G_3x(y-z)(y+z)}{14} - \frac{5\sqrt{42}G_{3x}xyz}{28} \\ & - \frac{\sqrt{42}G_{3y}z(x^2 + 11y^2 - 4z^2)}{56} + \frac{3\sqrt{7}G_{az}y(x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{105}G_{bz}y(x^2 - y^2 + 2z^2)}{28} \end{aligned}$$

$$\begin{aligned} & -\frac{3\sqrt{70}G_1xyz}{28} - \frac{3\sqrt{70}G_2z(x-y)(x+y)}{56} + \frac{\sqrt{105}G_3y(x-z)(x+z)}{14} + \frac{\sqrt{42}G_{3x}z(11x^2 + y^2 - 4z^2)}{56} \\ & + \frac{5\sqrt{42}G_{3y}xyz}{28} - \frac{3\sqrt{7}G_{az}x(x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{105}G_{bz}x(x^2 - y^2 - 2z^2)}{28} \end{aligned}$$

* Harmonics for rank 2

$$\vec{\mathbb{Q}}_2^{(3,0)}[g](A_1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{210}G_3(x-y)(x+y)}{28} + \frac{\sqrt{21}G_{3x}yz}{7} - \frac{\sqrt{21}G_{3y}xz}{7} + \frac{\sqrt{210}G_{bz}xy}{14}$$

$$\vec{\mathbb{Q}}_2^{(3,2)}[g](A_1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\begin{aligned} & -\frac{\sqrt{70}G_1xz(x^2 - 3y^2)}{8} + \frac{\sqrt{70}G_2yz(3x^2 - y^2)}{8} + \frac{\sqrt{105}G_3(x-y)(x+y)(x^2 + y^2 - 6z^2)}{42} \\ & - \frac{5\sqrt{42}G_{3xy}z(3x^2 + 3y^2 - 4z^2)}{168} + \frac{5\sqrt{42}G_{3y}xz(3x^2 + 3y^2 - 4z^2)}{168} - \frac{\sqrt{105}G_{bz}xy(x^2 + y^2 - 6z^2)}{21} \end{aligned}$$

$$\vec{\mathbb{Q}}_{2,1}^{(3,0)}[g](E, 1), \vec{\mathbb{Q}}_{2,2}^{(3,0)}[g](E, 1)$$

** symmetry

$\sqrt{3}xz$

$\sqrt{3}yz$

** expression

$$-\frac{\sqrt{105}G_1(x-y)(x+y)}{28} + \frac{\sqrt{105}G_2xy}{14} - \frac{3\sqrt{7}G_{3x}xy}{14} + \frac{\sqrt{7}G_{3y}(x^2 - 5y^2 + 4z^2)}{28} - \frac{\sqrt{42}G_{az}yz}{7}$$

$$\frac{\sqrt{105}G_1xy}{14} + \frac{\sqrt{105}G_2(x-y)(x+y)}{28} + \frac{\sqrt{7}G_{3x}(5x^2 - y^2 - 4z^2)}{28} + \frac{3\sqrt{7}G_{3y}xy}{14} + \frac{\sqrt{42}G_{az}xz}{7}$$

$$\vec{\mathbb{Q}}_{2,1}^{(3,0)}[g](E, 2), \vec{\mathbb{Q}}_{2,2}^{(3,0)}[g](E, 2)$$

** symmetry

$\sqrt{3}xy$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$-\frac{\sqrt{105}G_1yz}{14} - \frac{\sqrt{105}G_2xz}{14} + \frac{3\sqrt{7}G_{3x}xz}{14} - \frac{3\sqrt{7}G_{3y}yz}{14} - \frac{\sqrt{42}G_{az}(x-y)(x+y)}{28} + \frac{\sqrt{70}G_{bz}(x^2+y^2-2z^2)}{28}$$

$$\frac{\sqrt{105}G_1xz}{14} - \frac{\sqrt{105}G_2yz}{14} - \frac{\sqrt{70}G_3(x^2+y^2-2z^2)}{28} - \frac{3\sqrt{7}G_{3x}yz}{14} - \frac{3\sqrt{7}G_{3y}xz}{14} + \frac{\sqrt{42}G_{az}xy}{14}$$

$$\vec{\mathbb{Q}}_{2,1}^{(3,2)}[g](E,1), \vec{\mathbb{Q}}_{2,2}^{(3,2)}[g](E,1)$$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$\begin{aligned} & -\frac{\sqrt{210}G_1(5x^4-21x^2y^2-9x^2z^2+2y^4+9y^2z^2)}{168} + \frac{\sqrt{210}G_2xy(17x^2-11y^2-18z^2)}{168} - \frac{\sqrt{35}G_3xz(x-z)(x+z)}{6} + \frac{5\sqrt{14}G_{3x}xy(x^2+y^2-6z^2)}{168} \\ & + \frac{5\sqrt{14}G_{3y}(x^4+3x^2y^2-9x^2z^2+2y^4-15y^2z^2+4z^4)}{168} + \frac{5\sqrt{21}G_{az}yz(3x^2+3y^2-4z^2)}{84} + \frac{\sqrt{35}G_{bz}yz(3x^2+y^2-2z^2)}{12} \\ & - \frac{\sqrt{210}G_1xy(11x^2-17y^2+18z^2)}{168} - \frac{\sqrt{210}G_2(2x^4-21x^2y^2+9x^2z^2+5y^4-9y^2z^2)}{168} \\ & + \frac{\sqrt{35}G_3yz(y-z)(y+z)}{6} - \frac{5\sqrt{14}G_{3x}(2x^4+3x^2y^2-15x^2z^2+y^4-9y^2z^2+4z^4)}{168} \\ & - \frac{5\sqrt{14}G_{3y}xy(x^2+y^2-6z^2)}{168} - \frac{5\sqrt{21}G_{az}xz(3x^2+3y^2-4z^2)}{84} + \frac{\sqrt{35}G_{bz}xz(x^2+3y^2-2z^2)}{12} \end{aligned}$$

$$\vec{\mathbb{Q}}_{2,1}^{(3,2)}[g](E,2), \vec{\mathbb{Q}}_{2,2}^{(3,2)}[g](E,2)$$

** symmetry

$$\sqrt{3}xy$$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{210}G_1yz(3x^2+3y^2-4z^2)}{168} - \frac{\sqrt{210}G_2xz(3x^2+3y^2-4z^2)}{168} + \frac{\sqrt{35}G_3xy(x-y)(x+y)}{6} + \frac{5\sqrt{14}G_{3x}xz(13x^2-15y^2-8z^2)}{168} \\ & + \frac{5\sqrt{14}G_{3y}yz(15x^2-13y^2+8z^2)}{168} - \frac{5\sqrt{21}G_{az}(x-y)(x+y)(x^2+y^2-6z^2)}{84} + \frac{\sqrt{35}G_{bz}(5x^4-18x^2y^2-12x^2z^2+5y^4-12y^2z^2+4z^4)}{84} \\ & - \frac{\sqrt{210}G_1xz(3x^2+3y^2-4z^2)}{168} - \frac{\sqrt{210}G_2yz(3x^2+3y^2-4z^2)}{168} + \frac{\sqrt{35}G_3(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4)}{42} \\ & - \frac{5\sqrt{14}G_{3xy}(27x^2-y^2-8z^2)}{168} + \frac{5\sqrt{14}G_{3y}xz(x^2-27y^2+8z^2)}{168} + \frac{5\sqrt{21}G_{az}xy(x^2+y^2-6z^2)}{42} - \frac{\sqrt{35}G_{bz}xy(x-y)(x+y)}{6} \end{aligned}$$

* Harmonics for rank 3

$$\vec{\mathbb{Q}}_3^{(3,-2)}[g](A_1,1)$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$\frac{\sqrt{2}G_{3x}y}{2} - \frac{\sqrt{2}G_{3y}x}{2}$$

$$\vec{\mathbb{Q}}_3^{(3,-2)}[g](A_1,2)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2-y^2)}{4}$$

** expression

$$-\frac{\sqrt{3}G_2z}{2} - \frac{\sqrt{2}G_3y}{4} + \frac{\sqrt{2}G_{bz}x}{4}$$

$$\vec{\mathbb{Q}}_3^{(3,0)}[g](A_1,1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$\frac{\sqrt{15}G_1x(x^2 - 3y^2)}{12} - \frac{\sqrt{15}G_2y(3x^2 - y^2)}{12} - \frac{\sqrt{10}G_3z(x - y)(x + y)}{4} - \frac{G_{3x}y(x^2 + y^2 - 4z^2)}{4} + \frac{G_{3y}x(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{10}G_{bz}xyz}{2}$$

$\tilde{\mathbb{Q}}_3^{(3,0)}[g](A_1, 2)$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$-\frac{\sqrt{6}G_2z(3x^2 + 3y^2 - 2z^2)}{12} - \frac{G_3y(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{10}G_{3x}z(x - y)(x + y)}{4} - \frac{\sqrt{10}G_{3y}xyz}{2} - \frac{\sqrt{15}G_{az}x(x^2 - 3y^2)}{12} + \frac{G_{bz}x(x^2 + y^2 - 4z^2)}{4}$$

$\tilde{\mathbb{Q}}_3^{(3,2)}[g](A_1, 1)$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{7\sqrt{330}G_1x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{528} - \frac{7\sqrt{330}G_2y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{528} + \frac{7\sqrt{55}G_3z(x - y)(x + y)(x^2 + y^2 - 2z^2)}{44} \\ & + \frac{5\sqrt{22}G_{3x}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{176} - \frac{5\sqrt{22}G_{3y}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{176} - \frac{7\sqrt{55}G_{bz}xyz(x^2 + y^2 - 2z^2)}{22} \end{aligned}$$

$\tilde{\mathbb{Q}}_3^{(3,2)}[g](A_1, 2)$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{33}G_2z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{528} + \frac{\sqrt{22}G_3y(25x^4 - 55x^2y^2 + 15x^2z^2 + 4y^4 + 15y^2z^2 - 10z^4)}{88} \\ & + \frac{7\sqrt{55}G_{3x}z(5x^4 - 18x^2y^2 - 4x^2z^2 + y^4 + 4y^2z^2)}{176} + \frac{7\sqrt{55}G_{3y}xyz(x^2 - 2y^2 + z^2)}{22} \\ & - \frac{7\sqrt{330}G_{az}x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{528} + \frac{\sqrt{22}G_{bz}x(13x^4 - 100x^2y^2 - 30x^2z^2 + 55y^4 - 30y^2z^2 + 20z^4)}{176} \end{aligned}$$

$\tilde{\mathbb{Q}}_3^{(3,-2)}[g](A_2)$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{3}G_1z}{2} - \frac{\sqrt{2}G_3x}{4} - \frac{\sqrt{2}G_{bz}y}{4}$$

$\tilde{\mathbb{Q}}_3^{(3,0)}[g](A_2)$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{6}G_1z(3x^2 + 3y^2 - 2z^2)}{12} - \frac{G_3x(x^2 + y^2 - 4z^2)}{4} - \frac{\sqrt{10}G_{3x}xyz}{2} - \frac{\sqrt{10}G_{3y}z(x - y)(x + y)}{4} + \frac{\sqrt{15}G_{az}y(3x^2 - y^2)}{12} - \frac{G_{bz}y(x^2 + y^2 - 4z^2)}{4}$$

$\tilde{\mathbb{Q}}_3^{(3,2)}[g](A_2)$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\begin{aligned} & \frac{\sqrt{33}G_1z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{528} + \frac{\sqrt{22}G_3x(4x^4 - 55x^2y^2 + 15x^2z^2 + 25y^4 + 15y^2z^2 - 10z^4)}{88} \\ & - \frac{7\sqrt{55}G_{3x}xyz(2x^2 - y^2 - z^2)}{22} + \frac{7\sqrt{55}G_{3y}z(x^4 - 18x^2y^2 + 4x^2z^2 + 5y^4 - 4y^2z^2)}{176} \\ & + \frac{7\sqrt{330}G_{az}y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{528} - \frac{\sqrt{22}G_{bz}y(55x^4 - 100x^2y^2 - 30x^2z^2 + 13y^4 - 30y^2z^2 + 20z^4)}{176} \end{aligned}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,-2)}[g](E,1), \vec{\mathbb{Q}}_{3,2}^{(3,-2)}[g](E,1)$
** symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$-\frac{\sqrt{30}G_3x}{12} + \frac{\sqrt{3}G_{3y}z}{6} - \frac{\sqrt{2}G_{az}y}{2} + \frac{\sqrt{30}G_{bz}y}{12}$$

$$\frac{\sqrt{30}G_3y}{12} - \frac{\sqrt{3}G_{3x}z}{6} + \frac{\sqrt{2}G_{az}x}{2} + \frac{\sqrt{30}G_{bz}x}{12}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,-2)}[g](E,2), \vec{\mathbb{Q}}_{3,2}^{(3,-2)}[g](E,2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}G_1y}{4} + \frac{\sqrt{2}G_2x}{4} + \frac{\sqrt{30}G_{3x}x}{12} - \frac{\sqrt{30}G_{3y}y}{12} - \frac{\sqrt{3}G_{bz}z}{3}$$

$$-\frac{\sqrt{2}G_1x}{4} + \frac{\sqrt{2}G_2y}{4} + \frac{\sqrt{3}G_3z}{3} - \frac{\sqrt{30}G_{3x}y}{12} - \frac{\sqrt{30}G_{3y}x}{12}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,0)}[g](E,1), \vec{\mathbb{Q}}_{3,2}^{(3,0)}[g](E,1)$

** symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$-\frac{\sqrt{10}G_1z(x-y)(x+y)}{4} + \frac{\sqrt{10}G_2xyz}{2} - \frac{\sqrt{15}G_3x(x^2 - 3y^2)}{12} - \frac{\sqrt{6}G_{3y}z(3x^2 + 3y^2 - 2z^2)}{12} + \frac{G_{az}y(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{15}G_{bz}y(3x^2 - y^2)}{12}$$

$$\frac{\sqrt{10}G_1xyz}{2} + \frac{\sqrt{10}G_2z(x-y)(x+y)}{4} - \frac{\sqrt{15}G_3y(3x^2 - y^2)}{12} + \frac{\sqrt{6}G_{3x}z(3x^2 + 3y^2 - 2z^2)}{12} - \frac{G_{az}x(x^2 + y^2 - 4z^2)}{4} - \frac{\sqrt{15}G_{bz}x(x^2 - 3y^2)}{12}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,0)}[g](E,2), \vec{\mathbb{Q}}_{3,2}^{(3,0)}[g](E,2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{G_1y(x^2 + y^2 - 4z^2)}{4} + \frac{G_2x(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{15}G_{3x}x(x^2 - 3y^2)}{12} + \frac{\sqrt{15}G_{3y}y(3x^2 - y^2)}{12} + \frac{\sqrt{10}G_{az}z(x-y)(x+y)}{4} + \frac{\sqrt{6}G_{bz}z(3x^2 + 3y^2 - 2z^2)}{12}$$

$$-\frac{G_1x(x^2 + y^2 - 4z^2)}{4} + \frac{G_2y(x^2 + y^2 - 4z^2)}{4} - \frac{\sqrt{6}G_3z(3x^2 + 3y^2 - 2z^2)}{12} - \frac{\sqrt{15}G_{3x}y(3x^2 - y^2)}{12} + \frac{\sqrt{15}G_{3y}x(x^2 - 3y^2)}{12} - \frac{\sqrt{10}G_{az}xyz}{2}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,2)}[g](E,1), \vec{\mathbb{Q}}_{3,2}^{(3,2)}[g](E,1)$

** symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned}
& - \frac{7\sqrt{55}G_1z(5x^4 - 18x^2y^2 - 4x^2z^2 + y^4 + 4y^2z^2)}{176} + \frac{7\sqrt{55}G_2xyz(2x^2 - y^2 - z^2)}{22} \\
& + \frac{\sqrt{330}G_3x(4x^4 + x^2y^2 - 41x^2z^2 - 3y^4 + 15y^2z^2 + 18z^4)}{264} + \frac{5\sqrt{33}G_{3y}z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{528} \\
& - \frac{5\sqrt{22}G_{az}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{176} - \frac{\sqrt{330}G_{bz}y(15x^4 + 16x^2y^2 - 138x^2z^2 + y^4 - 26y^2z^2 + 36z^4)}{528} \\
& - \frac{7\sqrt{55}G_1xyz(x^2 - 2y^2 + z^2)}{22} - \frac{7\sqrt{55}G_2z(x^4 - 18x^2y^2 + 4x^2z^2 + 5y^4 - 4y^2z^2)}{176} \\
& + \frac{\sqrt{330}G_3y(3x^4 - x^2y^2 - 15x^2z^2 - 4y^4 + 41y^2z^2 - 18z^4)}{264} - \frac{5\sqrt{33}G_{3x}z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{528} \\
& + \frac{5\sqrt{22}G_{az}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{176} - \frac{\sqrt{330}G_{bz}x(x^4 + 16x^2y^2 - 26x^2z^2 + 15y^4 - 138y^2z^2 + 36z^4)}{528}
\end{aligned}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,2)}[g](E, 2)$, $\vec{\mathbb{Q}}_{3,2}^{(3,2)}[g](E, 2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned}
& - \frac{\sqrt{22}G_1y(25x^4 - 55x^2y^2 + 15x^2z^2 + 4y^4 + 15y^2z^2 - 10z^4)}{88} - \frac{\sqrt{22}G_2x(4x^4 - 55x^2y^2 + 15x^2z^2 + 25y^4 + 15y^2z^2 - 10z^4)}{88} \\
& - \frac{\sqrt{330}G_{3x}x(4x^4 + x^2y^2 - 41x^2z^2 - 3y^4 + 15y^2z^2 + 18z^4)}{264} - \frac{\sqrt{330}G_{3y}y(3x^4 - x^2y^2 - 15x^2z^2 - 4y^4 + 41y^2z^2 - 18z^4)}{264} \\
& - \frac{7\sqrt{55}G_{az}z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{44} + \frac{\sqrt{33}G_{bz}z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{132} \\
& - \frac{\sqrt{22}G_1x(13x^4 - 100x^2y^2 - 30x^2z^2 + 55y^4 - 30y^2z^2 + 20z^4)}{176} + \frac{\sqrt{22}G_2y(55x^4 - 100x^2y^2 - 30x^2z^2 + 13y^4 - 30y^2z^2 + 20z^4)}{176} \\
& - \frac{\sqrt{33}G_3z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{132} + \frac{\sqrt{330}G_{3x}y(15x^4 + 16x^2y^2 - 138x^2z^2 + y^4 - 26y^2z^2 + 36z^4)}{528} \\
& + \frac{\sqrt{330}G_{3y}x(x^4 + 16x^2y^2 - 26x^2z^2 + 15y^4 - 138y^2z^2 + 36z^4)}{528} + \frac{7\sqrt{55}G_{az}xyz(x^2 + y^2 - 2z^2)}{22}
\end{aligned}$$

* Harmonics for rank 4

$\vec{\mathbb{Q}}_4^{(3,-2)}[g](A_1, 1)$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\frac{\sqrt{21}G_3(x-y)(x+y)}{14} + \frac{\sqrt{210}G_{3x}yz}{14} - \frac{\sqrt{210}G_{3y}xz}{14} - \frac{\sqrt{21}G_{bz}xy}{7}$$

$\vec{\mathbb{Q}}_4^{(3,-2)}[g](A_1, 2)$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\frac{3\sqrt{5}G_2(x^2 + y^2 - 2z^2)}{20} + \frac{\sqrt{30}G_{3y}z}{20} + \frac{\sqrt{3}G_{3x}(x-y)(x+y)}{4} - \frac{\sqrt{3}G_{3y}xy}{2} - \frac{\sqrt{30}G_{bz}xz}{20}$$

$\vec{\mathbb{Q}}_4^{(3,0)}[g](A_1, 1)$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\begin{aligned}
& - \frac{5\sqrt{77}G_1xz(x^2 - 3y^2)}{44} - \frac{5\sqrt{77}G_2yz(3x^2 - y^2)}{44} + \frac{5\sqrt{462}G_3(x-y)(x+y)(x^2 + y^2 - 6z^2)}{616} \\
& - \frac{3\sqrt{1155}G_{3x}yz(3x^2 + 3y^2 - 4z^2)}{308} + \frac{3\sqrt{1155}G_{3y}xz(3x^2 + 3y^2 - 4z^2)}{308} - \frac{5\sqrt{462}G_{bz}xy(x^2 + y^2 - 6z^2)}{308}
\end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,0)}[g](A_1, 2)$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\begin{aligned} & \frac{\sqrt{110}G_2(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{176} - \frac{\sqrt{165}G_3yz(3x^2 + 3y^2 - 4z^2)}{44} + \frac{\sqrt{66}G_{3x}(5x^4 - 42x^2y^2 + 12x^2z^2 + 9y^4 - 12y^2z^2)}{176} \\ & + \frac{\sqrt{66}G_{3y}xy(4x^2 - 3y^2 - 3z^2)}{22} + \frac{7\sqrt{11}G_{az}xz(x^2 - 3y^2)}{44} + \frac{\sqrt{165}G_{bz}xz(3x^2 + 3y^2 - 4z^2)}{44} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,2)}[g](A_1, 1)$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\begin{aligned} & \frac{9\sqrt{10010}G_1xz(x^2 - 3y^2)(3x^2 + 3y^2 - 8z^2)}{2288} - \frac{9\sqrt{10010}G_2yz(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{2288} \\ & - \frac{\sqrt{15015}G_3(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} + \frac{5\sqrt{6006}G_{3x}yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\ & - \frac{5\sqrt{6006}G_{3y}xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} + \frac{\sqrt{15015}G_{bz}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{286} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,2)}[g](A_1, 2)$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\begin{aligned} & - \frac{9\sqrt{143}G_1xy(x^2 - 3y^2)(3x^2 - y^2)}{208} - \frac{3\sqrt{143}G_2(14x^6 - 255x^4y^2 + 45x^4z^2 + 240x^2y^4 + 90x^2y^2z^2 - 60x^2z^4 - 19y^6 + 45y^4z^2 - 60y^2z^4 + 8z^6)}{2288} \\ & + \frac{\sqrt{858}G_3yz(20x^4 - 125x^2y^2 + 85x^2z^2 - 13y^4 + 85y^2z^2 - 34z^4)}{1144} \\ & - \frac{\sqrt{2145}G_{3x}(14x^6 - 31x^4y^2 - 179x^4z^2 - 44x^2y^4 + 450x^2y^2z^2 + 104x^2z^4 + y^6 + 29y^4z^2 - 104y^2z^4)}{2288} \\ & - \frac{\sqrt{2145}G_{3y}xy(17x^4 - 26x^2y^2 - 92x^2z^2 - 43y^4 + 508y^2z^2 - 208z^4)}{2288} - \frac{27\sqrt{1430}G_{az}xz(x^2 - 3y^2)(3x^2 + 3y^2 - 8z^2)}{2288} \\ & + \frac{\sqrt{858}G_{bz}xz(59x^4 - 80x^2y^2 - 170x^2z^2 + 125y^4 - 170y^2z^2 + 68z^4)}{2288} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,-2)}[g](A_2)$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$-\frac{3\sqrt{5}G_1(x^2 + y^2 - 2z^2)}{20} + \frac{\sqrt{30}G_3xz}{20} - \frac{\sqrt{3}G_{3x}xy}{2} - \frac{\sqrt{3}G_{3y}(x-y)(x+y)}{4} + \frac{\sqrt{30}G_{bz}yz}{20}$$

$\tilde{\mathbb{Q}}_4^{(3,0)}[g](A_2)$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\begin{aligned} & - \frac{\sqrt{110}G_1(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{176} - \frac{\sqrt{165}G_3xz(3x^2 + 3y^2 - 4z^2)}{44} - \frac{\sqrt{66}G_{3x}xy(3x^2 - 4y^2 + 3z^2)}{22} \\ & + \frac{\sqrt{66}G_{3y}(9x^4 - 42x^2y^2 - 12x^2z^2 + 5y^4 + 12y^2z^2)}{176} - \frac{7\sqrt{11}G_{az}yz(3x^2 - y^2)}{44} - \frac{\sqrt{165}G_{bz}yz(3x^2 + 3y^2 - 4z^2)}{44} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,2)}[g](A_2)$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\begin{aligned}
& - \frac{3\sqrt{143}G_1(19x^6 - 240x^4y^2 - 45x^4z^2 + 255x^2y^4 - 90x^2y^2z^2 + 60x^2z^4 - 14y^6 - 45y^4z^2 + 60y^2z^4 - 8z^6)}{2288} + \frac{9\sqrt{143}G_2xy(x^2 - 3y^2)(3x^2 - y^2)}{208} \\
& - \frac{\sqrt{858}G_3xz(13x^4 + 125x^2y^2 - 85x^2z^2 - 20y^4 - 85y^2z^2 + 34z^4)}{1144} + \frac{\sqrt{2145}G_{3x}xy(43x^4 + 26x^2y^2 - 508x^2z^2 - 17y^4 + 92y^2z^2 + 208z^4)}{2288} \\
& - \frac{\sqrt{2145}G_{3y}(x^6 - 44x^4y^2 + 29x^4z^2 - 31x^2y^4 + 450x^2y^2z^2 - 104x^2z^4 + 14y^6 - 179y^4z^2 + 104y^2z^4)}{2288} \\
& + \frac{27\sqrt{1430}G_{az}yz(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{2288} - \frac{\sqrt{858}G_{bz}yz(125x^4 - 80x^2y^2 - 170x^2z^2 + 59y^4 - 170y^2z^2 + 68z^4)}{2288}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,-2)}[g](E,1), \vec{\mathbb{Q}}_{4,2}^{(3,-2)}[g](E,1)$

** symmetry

$$-\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{35}G_1(x-y)(x+y)}{140} - \frac{3\sqrt{35}G_2xy}{70} - \frac{\sqrt{210}G_3xz}{20} + \frac{\sqrt{21}G_{3x}xy}{7} - \frac{\sqrt{21}G_{3y}(3x^2 - y^2 - 2z^2)}{28} - \frac{3\sqrt{14}G_{az}yz}{14} + \frac{\sqrt{210}G_{bz}yz}{20} \\
& - \frac{3\sqrt{35}G_1xy}{70} - \frac{3\sqrt{35}G_2(x-y)(x+y)}{140} + \frac{\sqrt{210}G_3yz}{20} - \frac{\sqrt{21}G_{3x}(x^2 - 3y^2 + 2z^2)}{28} - \frac{\sqrt{21}G_{3y}xy}{7} + \frac{3\sqrt{14}G_{az}xz}{14} + \frac{\sqrt{210}G_{bz}xz}{20}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,-2)}[g](E,2), \vec{\mathbb{Q}}_{4,2}^{(3,-2)}[g](E,2)$

** symmetry

$$-\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

** expression

$$\begin{aligned}
& - \frac{3\sqrt{10}G_1yz}{10} + \frac{3\sqrt{10}G_2xz}{10} + \frac{\sqrt{15}G_3xy}{5} - \frac{\sqrt{15}G_{bz}(x-y)(x+y)}{10} \\
& - \frac{3\sqrt{10}G_1xz}{10} + \frac{3\sqrt{10}G_2yz}{10} - \frac{\sqrt{15}G_3(x-y)(x+y)}{10} - \frac{\sqrt{15}G_{bz}xy}{5}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,-2)}[g](E,3), \vec{\mathbb{Q}}_{4,2}^{(3,-2)}[g](E,3)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\begin{aligned}
& \frac{9\sqrt{70}G_1yz}{140} + \frac{9\sqrt{70}G_2xz}{140} + \frac{\sqrt{42}G_{3x}xz}{28} - \frac{\sqrt{42}G_{3y}yz}{28} + \frac{3\sqrt{7}G_{az}(x-y)(x+y)}{14} + \frac{\sqrt{105}G_{bz}(x^2 + y^2 - 2z^2)}{35} \\
& - \frac{9\sqrt{70}G_1xz}{140} + \frac{9\sqrt{70}G_2yz}{140} - \frac{\sqrt{105}G_3(x^2 + y^2 - 2z^2)}{35} - \frac{\sqrt{42}G_{3x}yz}{28} - \frac{\sqrt{42}G_{3y}xz}{28} - \frac{3\sqrt{7}G_{az}xy}{7}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,0)}[g](E,1), \vec{\mathbb{Q}}_{4,2}^{(3,0)}[g](E,1)$

** symmetry

$$-\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned}
& \frac{\sqrt{770}G_1(17x^4 - 42x^2y^2 - 60x^2z^2 - 3y^4 + 60y^2z^2)}{1232} - \frac{\sqrt{770}G_2xy(6x^2 - y^2 - 15z^2)}{154} - \frac{\sqrt{1155}G_3xz(x^2 - 3y^2)}{44} - \frac{\sqrt{462}G_{3x}xy(x^2 + y^2 - 6z^2)}{154} \\
& + \frac{\sqrt{462}G_{3y}(13x^4 + 18x^2y^2 - 96x^2z^2 + 5y^4 - 48y^2z^2 + 24z^4)}{1232} + \frac{9\sqrt{77}G_{az}yz(3x^2 + 3y^2 - 4z^2)}{308} + \frac{\sqrt{1155}G_{bz}yz(3x^2 - y^2)}{44}
\end{aligned}$$

$$\begin{aligned} & \frac{\sqrt{770}G_1xy(x^2 - 6y^2 + 15z^2)}{154} - \frac{\sqrt{770}G_2(3x^4 + 42x^2y^2 - 60x^2z^2 - 17y^4 + 60y^2z^2)}{1232} \\ & - \frac{\sqrt{1155}G_{3yz}(3x^2 - y^2)}{44} - \frac{\sqrt{462}G_{3x}(5x^4 + 18x^2y^2 - 48x^2z^2 + 13y^4 - 96y^2z^2 + 24z^4)}{1232} \\ & + \frac{\sqrt{462}G_{3y}xy(x^2 + y^2 - 6z^2)}{154} - \frac{9\sqrt{77}G_{az}xz(3x^2 + 3y^2 - 4z^2)}{308} - \frac{\sqrt{1155}G_{bz}xz(x^2 - 3y^2)}{44} \end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,0)}[g](E, 2), \vec{\mathbb{Q}}_{4,2}^{(3,0)}[g](E, 2)$

** symmetry

$$-\frac{\sqrt{35}xy(x - y)(x + y)}{2}$$

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{55}G_1yz(3x^2 + 3y^2 - 4z^2)}{44} + \frac{\sqrt{55}G_2xz(3x^2 + 3y^2 - 4z^2)}{44} + \frac{\sqrt{330}G_3xy(x^2 + y^2 - 6z^2)}{44} - \frac{7\sqrt{33}G_{3x}xz(x^2 - 3y^2)}{44} \\ & + \frac{7\sqrt{33}G_{3y}yz(3x^2 - y^2)}{44} + \frac{7\sqrt{22}G_{az}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{88} - \frac{\sqrt{330}G_{bz}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{88} \end{aligned}$$

$$\begin{aligned} & \frac{\sqrt{55}G_1xz(3x^2 + 3y^2 - 4z^2)}{44} + \frac{\sqrt{55}G_2yz(3x^2 + 3y^2 - 4z^2)}{44} - \frac{\sqrt{330}G_3(x - y)(x + y)(x^2 + y^2 - 6z^2)}{88} \\ & - \frac{7\sqrt{33}G_{3x}yz(3x^2 - y^2)}{44} - \frac{7\sqrt{33}G_{3y}xz(x^2 - 3y^2)}{44} + \frac{7\sqrt{22}G_{az}xy(x - y)(x + y)}{22} - \frac{\sqrt{330}G_{bz}xy(x^2 + y^2 - 6z^2)}{44} \end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,0)}[g](E, 3), \vec{\mathbb{Q}}_{4,2}^{(3,0)}[g](E, 3)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

$$-\frac{\sqrt{5}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{5\sqrt{385}G_1yz(3x^2 + 3y^2 - 4z^2)}{308} + \frac{5\sqrt{385}G_2xz(3x^2 + 3y^2 - 4z^2)}{308} - \frac{\sqrt{2310}G_3xy(x - y)(x + y)}{44} + \frac{\sqrt{231}G_{3x}xz(13x^2 - 15y^2 - 8z^2)}{308} \\ & + \frac{\sqrt{231}G_{3y}yz(15x^2 - 13y^2 + 8z^2)}{308} - \frac{13\sqrt{154}G_{az}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{616} - \frac{\sqrt{2310}G_{bz}(5x^4 - 18x^2y^2 - 12x^2z^2 + 5y^4 - 12y^2z^2 + 4z^4)}{616} \\ & - \frac{5\sqrt{385}G_1xz(3x^2 + 3y^2 - 4z^2)}{308} + \frac{5\sqrt{385}G_2yz(3x^2 + 3y^2 - 4z^2)}{308} - \frac{\sqrt{2310}G_3(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{308} \\ & - \frac{\sqrt{231}G_{3x}yz(27x^2 - y^2 - 8z^2)}{308} + \frac{\sqrt{231}G_{3y}xz(x^2 - 27y^2 + 8z^2)}{308} + \frac{13\sqrt{154}G_{az}xy(x^2 + y^2 - 6z^2)}{308} + \frac{\sqrt{2310}G_{bz}xy(x - y)(x + y)}{44} \end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,2)}[g](E, 1), \vec{\mathbb{Q}}_{4,2}^{(3,2)}[g](E, 1)$

** symmetry

$$-\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{1001}G_1(7x^6 - 20x^4y^2 - 85x^4z^2 - 25x^2y^4 + 270x^2y^2z^2 + 40x^2z^4 + 2y^6 - 5y^4z^2 - 40y^2z^4)}{2288} \\ & - \frac{3\sqrt{1001}G_2xy(23x^4 + 10x^2y^2 - 260x^2z^2 - 13y^4 + 100y^2z^2 + 80z^4)}{2288} \\ & + \frac{\sqrt{6006}G_3xz(25x^4 + 5x^2y^2 - 85x^2z^2 - 20y^4 + 35y^2z^2 + 22z^4)}{1144} + \frac{\sqrt{15015}G_{3x}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{2288} \\ & - \frac{\sqrt{15015}G_{3y}(3x^6 + 8x^4y^2 - 53x^4z^2 + 7x^2y^4 - 90x^2y^2z^2 + 68x^2z^4 + 2y^6 - 37y^4z^2 + 52y^2z^4 - 8z^6)}{2288} \\ & - \frac{3\sqrt{10010}G_{az}yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} - \frac{\sqrt{6006}G_{bz}yz(95x^4 + 100x^2y^2 - 290x^2z^2 + 5y^4 - 50y^2z^2 + 44z^4)}{2288} \end{aligned}$$

$$\begin{aligned}
& \frac{3\sqrt{1001}G_1xy(13x^4 - 10x^2y^2 - 100x^2z^2 - 23y^4 + 260y^2z^2 - 80z^4)}{2288} \\
& + \frac{3\sqrt{1001}G_2(2x^6 - 25x^4y^2 - 5x^4z^2 - 20x^2y^4 + 270x^2y^2z^2 - 40x^2z^4 + 7y^6 - 85y^4z^2 + 40y^2z^4)}{2288} \\
& + \frac{\sqrt{6006}G_3yz(20x^4 - 5x^2y^2 - 35x^2z^2 - 25y^4 + 85y^2z^2 - 22z^4)}{1144} \\
& + \frac{\sqrt{15015}G_{3x}(2x^6 + 7x^4y^2 - 37x^4z^2 + 8x^2y^4 - 90x^2y^2z^2 + 52x^2z^4 + 3y^6 - 53y^4z^2 + 68y^2z^4 - 8z^6)}{2288} \\
& - \frac{\sqrt{15015}G_{3y}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{2288} + \frac{3\sqrt{10010}G_{az}xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\
& - \frac{\sqrt{6006}G_{bz}xz(5x^4 + 100x^2y^2 - 50x^2z^2 + 95y^4 - 290y^2z^2 + 44z^4)}{2288}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,2)}[g](E, 2), \vec{\mathbb{Q}}_{4,2}^{(3,2)}[g](E, 2)$

** symmetry

$$-\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{286}G_1yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} + \frac{3\sqrt{286}G_2xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\
& - \frac{\sqrt{429}G_3xy(47x^4 - 170x^2y^2 + 40x^2z^2 + 47y^4 + 40y^2z^2 - 40z^4)}{572} - \frac{3\sqrt{4290}G_{3x}xz(17x^4 - 122x^2y^2 - 16x^2z^2 + 37y^4 + 48y^2z^2)}{2288} \\
& - \frac{3\sqrt{4290}G_{3y}yz(37x^4 - 122x^2y^2 + 48x^2z^2 + 17y^4 - 16y^2z^2)}{2288} + \frac{9\sqrt{715}G_{az}(x^2 + y^2 - 10z^2)(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{1144} \\
& - \frac{\sqrt{429}G_{bz}(x-y)(x+y)(19x^4 - 226x^2y^2 - 40x^2z^2 + 19y^4 - 40y^2z^2 + 40z^4)}{1144}
\end{aligned}$$

$$\begin{aligned}
& \frac{3\sqrt{286}G_1xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} + \frac{3\sqrt{286}G_2yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\
& + \frac{\sqrt{429}G_3(x-y)(x+y)(7x^4 - 118x^2y^2 + 20x^2z^2 + 7y^4 + 20y^2z^2 - 20z^4)}{572} \\
& - \frac{3\sqrt{4290}G_{3x}yz(73x^4 - 98x^2y^2 - 48x^2z^2 + 5y^4 + 16y^2z^2)}{2288} + \frac{3\sqrt{4290}G_{3y}xz(5x^4 - 98x^2y^2 + 16x^2z^2 + 73y^4 - 48y^2z^2)}{2288} \\
& + \frac{9\sqrt{715}G_{az}xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{286} - \frac{\sqrt{429}G_{bz}xy(13x^4 - 40x^2y^2 - 10x^2z^2 + 13y^4 - 10y^2z^2 + 10z^4)}{143}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,2)}[g](E, 3), \vec{\mathbb{Q}}_{4,2}^{(3,2)}[g](E, 3)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\begin{aligned}
& - \frac{9\sqrt{2002}G_1yz(25x^4 - 60x^2y^2 + 10x^2z^2 + 3y^4 + 10y^2z^2 - 4z^4)}{2288} - \frac{9\sqrt{2002}G_2xz(3x^4 - 60x^2y^2 + 10x^2z^2 + 25y^4 + 10y^2z^2 - 4z^4)}{2288} \\
& + \frac{3\sqrt{3003}G_3xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572} - \frac{\sqrt{30030}G_{3x}xz(17x^4 + 16x^2y^2 - 62x^2z^2 - y^4 - 14y^2z^2 + 20z^4)}{2288} \\
& - \frac{\sqrt{30030}G_{3y}yz(x^4 - 16x^2y^2 + 14x^2z^2 - 17y^4 + 62y^2z^2 - 20z^4)}{2288} + \frac{3\sqrt{5005}G_{az}(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144} \\
& - \frac{\sqrt{3003}G_{bz}(x^6 + 15x^4y^2 - 30x^4z^2 + 15x^2y^4 - 180x^2y^2z^2 + 60x^2z^4 + y^6 - 30y^4z^2 + 60y^2z^4 - 8z^6)}{1144}
\end{aligned}$$

$$\begin{aligned}
& - \frac{9\sqrt{2002}G_1xz(4x^4 - 25x^2y^2 - 5x^2z^2 + 15y^4 - 5y^2z^2 + 2z^4)}{1144} + \frac{9\sqrt{2002}G_2yz(15x^4 - 25x^2y^2 - 5x^2z^2 + 4y^4 - 5y^2z^2 + 2z^4)}{1144} \\
& + \frac{\sqrt{3003}G_3(x^6 - 15x^4z^2 + 15x^2z^4 + y^6 - 15y^4z^2 + 15y^2z^4 - 2z^6)}{286} \\
& + \frac{\sqrt{30030}G_{3x}yz(13x^4 + 17x^2y^2 - 43x^2z^2 + 4y^4 - 19y^2z^2 + 10z^4)}{1144} + \frac{\sqrt{30030}G_{3y}xz(4x^4 + 17x^2y^2 - 19x^2z^2 + 13y^4 - 43y^2z^2 + 10z^4)}{1144} \\
& - \frac{3\sqrt{5005}G_{az}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} - \frac{3\sqrt{3003}G_{bz}xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572}
\end{aligned}$$