

PG No. 31  $T_d$   $\bar{4}3m$  [ cubic ] (polar, internal axial quadrupole)

\* Harmonics for rank 0

\* Harmonics for rank 1

$$\vec{\mathbb{Q}}_{1,1}^{(2,1)}[g](T_2), \vec{\mathbb{Q}}_{1,2}^{(2,1)}[g](T_2), \vec{\mathbb{Q}}_{1,3}^{(2,1)}[g](T_2)$$

\*\* symmetry

$x$

$y$

$z$

\*\* expression

$$-\frac{3\sqrt{10}G_uyz}{10} - \frac{\sqrt{30}G_vyz}{10} + \frac{\sqrt{30}G_{xy}xz}{10} - \frac{\sqrt{30}G_{xz}xy}{10} - \frac{\sqrt{30}G_{yz}(y-z)(y+z)}{10}$$

$$\frac{3\sqrt{10}G_{uxz}}{10} - \frac{\sqrt{30}G_{vxz}}{10} - \frac{\sqrt{30}G_{xy}yz}{10} + \frac{\sqrt{30}G_{xz}(x-z)(x+z)}{10} + \frac{\sqrt{30}G_{yz}xy}{10}$$

$$\frac{\sqrt{30}G_vxy}{5} - \frac{\sqrt{30}G_{xy}(x-y)(x+y)}{10} + \frac{\sqrt{30}G_{xz}yz}{10} - \frac{\sqrt{30}G_{yz}xz}{10}$$

\* Harmonics for rank 2

$$\vec{\mathbb{Q}}_{2,1}^{(2,-1)}[g](E), \vec{\mathbb{Q}}_{2,2}^{(2,-1)}[g](E)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

\*\* expression

$$\frac{\sqrt{2}G_{xz}y}{2} - \frac{\sqrt{2}G_{yz}x}{2}$$

$$\frac{\sqrt{6}G_{xy}z}{3} - \frac{\sqrt{6}G_{xz}y}{6} - \frac{\sqrt{6}G_{yz}x}{6}$$

$$\vec{\mathbb{Q}}_{2,1}^{(2,1)}[g](E), \vec{\mathbb{Q}}_{2,2}^{(2,1)}[g](E)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

\*\* expression

$$\frac{5\sqrt{42}G_vxyz}{14} - \frac{5\sqrt{42}G_{xy}z(x-y)(x+y)}{28} - \frac{\sqrt{42}G_{xz}y(x^2+y^2-4z^2)}{28} + \frac{\sqrt{42}G_{yz}x(x^2+y^2-4z^2)}{28}$$

$$-\frac{5\sqrt{42}G_uxyz}{14} + \frac{\sqrt{14}G_{xy}z(3x^2+3y^2-2z^2)}{28} - \frac{\sqrt{14}G_{xz}y(9x^2-y^2-6z^2)}{28} + \frac{\sqrt{14}G_{yz}x(x^2-9y^2+6z^2)}{28}$$

$$\vec{\mathbb{Q}}_{2,1}^{(2,-1)}[g](T_2), \vec{\mathbb{Q}}_{2,2}^{(2,-1)}[g](T_2), \vec{\mathbb{Q}}_{2,3}^{(2,-1)}[g](T_2)$$

\*\* symmetry

$\sqrt{3}yz$

$\sqrt{3}xz$

$\sqrt{3}xy$

\*\* expression

$$\frac{\sqrt{2}G_ux}{2} + \frac{\sqrt{6}G_vx}{6} + \frac{\sqrt{6}G_{xy}y}{6} - \frac{\sqrt{6}G_{xz}z}{6}$$

$$-\frac{\sqrt{2}G_uy}{2} + \frac{\sqrt{6}G_vy}{6} - \frac{\sqrt{6}G_{xy}x}{6} + \frac{\sqrt{6}G_{yz}z}{6}$$

$$-\frac{\sqrt{6}G_vz}{3} + \frac{\sqrt{6}G_{xz}x}{6} - \frac{\sqrt{6}G_{yz}y}{6}$$

$$\vec{\mathbb{Q}}_{2,1}^{(2,1)}[g](T_2), \vec{\mathbb{Q}}_{2,2}^{(2,1)}[g](T_2), \vec{\mathbb{Q}}_{2,3}^{(2,1)}[g](T_2)$$

\*\* symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

\*\* expression

$$-\frac{\sqrt{42}G_u x (x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{14}G_v x (x^2 - 9y^2 + 6z^2)}{28} - \frac{\sqrt{14}G_{xy}y (3x^2 - 2y^2 + 3z^2)}{14} + \frac{\sqrt{14}G_{xz}z (3x^2 + 3y^2 - 2z^2)}{14}$$

$$\frac{\sqrt{42}G_u y (x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{14}G_v y (9x^2 - y^2 - 6z^2)}{28} - \frac{\sqrt{14}G_{xy}x (2x^2 - 3y^2 - 3z^2)}{14} - \frac{\sqrt{14}G_{yz}z (3x^2 + 3y^2 - 2z^2)}{14}$$

$$\frac{5\sqrt{42}G_u z (x - y) (x + y)}{28} - \frac{\sqrt{14}G_v z (3x^2 + 3y^2 - 2z^2)}{28} + \frac{\sqrt{14}G_{xz}x (2x^2 - 3y^2 - 3z^2)}{14} + \frac{\sqrt{14}G_{yz}y (3x^2 - 2y^2 + 3z^2)}{14}$$

\* Harmonics for rank 3

$$\vec{\mathbb{Q}}_3^{(2,-1)}[g](A_1)$$

\*\* symmetry

$$\sqrt{15}xyz$$

\*\* expression

$$\frac{\sqrt{6}G_u (x - y) (x + y)}{4} + \frac{\sqrt{2}G_v (x^2 + y^2 - 2z^2)}{4}$$

$$\vec{\mathbb{Q}}_3^{(2,1)}[g](A_1)$$

\*\* symmetry

$$\sqrt{15}xyz$$

\*\* expression

$$-\frac{\sqrt{3}G_u (x - y) (x + y) (x^2 + y^2 - 6z^2)}{6} - \frac{G_v (x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{6}$$

$$- \frac{7G_{xy}xy (x - y) (x + y)}{6} + \frac{7G_{xz}xz (x - z) (x + z)}{6} - \frac{7G_{yz}yz (y - z) (y + z)}{6}$$

$$\vec{\mathbb{Q}}_{3,1}^{(2,-1)}[g](T_1), \vec{\mathbb{Q}}_{3,2}^{(2,-1)}[g](T_1), \vec{\mathbb{Q}}_{3,3}^{(2,-1)}[g](T_1)$$

\*\* symmetry

$$\frac{\sqrt{15}x (y - z) (y + z)}{2}$$

$$- \frac{\sqrt{15}y (x - z) (x + z)}{2}$$

$$\frac{\sqrt{15}z (x - y) (x + y)}{2}$$

\*\* expression

$$\frac{\sqrt{6}G_uyz}{4} - \frac{3\sqrt{2}G_vyz}{4} + \frac{\sqrt{2}G_{yz}(2x^2 - y^2 - z^2)}{4}$$

$$\frac{\sqrt{6}G_uxz}{4} + \frac{3\sqrt{2}G_vxz}{4} - \frac{\sqrt{2}G_{xz}(x^2 - 2y^2 + z^2)}{4}$$

$$- \frac{\sqrt{6}G_uxy}{2} - \frac{\sqrt{2}G_{xy}(x^2 + y^2 - 2z^2)}{4}$$

$$\vec{\mathbb{Q}}_{3,1}^{(2,1)}[g](T_1), \vec{\mathbb{Q}}_{3,2}^{(2,1)}[g](T_1), \vec{\mathbb{Q}}_{3,3}^{(2,1)}[g](T_1)$$

\*\* symmetry

$$\frac{\sqrt{15}x (y - z) (y + z)}{2}$$

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{3}G_{uyz}(12x^2 - 9y^2 + 5z^2)}{12} - \frac{G_vyz(36x^2 + y^2 - 13z^2)}{12} + \frac{7G_{xy}xz(2x^2 - 3y^2 - z^2)}{12} \\ & + \frac{7G_{xz}xy(2x^2 - y^2 - 3z^2)}{12} - \frac{G_{yz}(4x^4 - 12x^2y^2 - 12x^2z^2 + 5y^4 - 18y^2z^2 + 5z^4)}{12} \end{aligned}$$

$$\begin{aligned} & -\frac{\sqrt{3}G_{uxz}(9x^2 - 12y^2 - 5z^2)}{12} + \frac{G_vxz(x^2 + 36y^2 - 13z^2)}{12} - \frac{7G_{xy}yz(3x^2 - 2y^2 + z^2)}{12} \\ & - \frac{G_{xz}(5x^4 - 12x^2y^2 - 18x^2z^2 + 4y^4 - 12y^2z^2 + 5z^4)}{12} - \frac{7G_{yz}xy(x^2 - 2y^2 + 3z^2)}{12} \end{aligned}$$

$$\begin{aligned} & \frac{\sqrt{3}G_{u}xy(x^2 + y^2 - 6z^2)}{3} + \frac{7G_vxy(x-y)(x+y)}{6} - \frac{G_{xy}(5x^4 - 18x^2y^2 - 12x^2z^2 + 5y^4 - 12y^2z^2 + 4z^4)}{12} \\ & - \frac{7G_{xyz}(3x^2 + y^2 - 2z^2)}{12} - \frac{7G_{yz}xz(x^2 + 3y^2 - 2z^2)}{12} \end{aligned}$$

$$\vec{\mathbb{Q}}_{3,1}^{(2,-1)}[g](T_2), \vec{\mathbb{Q}}_{3,2}^{(2,-1)}[g](T_2), \vec{\mathbb{Q}}_{3,3}^{(2,-1)}[g](T_2)$$

\*\* symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{10}G_{uyz}}{20} + \frac{\sqrt{30}G_vyz}{20} + \frac{\sqrt{30}G_{xy}xz}{5} - \frac{\sqrt{30}G_{xz}xy}{5} + \frac{\sqrt{30}G_{yz}(y-z)(y+z)}{20} \\ & - \frac{3\sqrt{10}G_{uxz}}{20} + \frac{\sqrt{30}G_vxz}{20} - \frac{\sqrt{30}G_{xy}yz}{5} - \frac{\sqrt{30}G_{xz}(x-z)(x+z)}{20} + \frac{\sqrt{30}G_{yz}xy}{5} \\ & - \frac{\sqrt{30}G_vxy}{10} + \frac{\sqrt{30}G_{xy}(x-y)(x+y)}{20} + \frac{\sqrt{30}G_{xz}yz}{5} - \frac{\sqrt{30}G_{yz}xz}{5} \end{aligned}$$

$$\vec{\mathbb{Q}}_{3,1}^{(2,1)}[g](T_2), \vec{\mathbb{Q}}_{3,2}^{(2,1)}[g](T_2), \vec{\mathbb{Q}}_{3,3}^{(2,1)}[g](T_2)$$

\*\* symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$\begin{aligned} & -\frac{\sqrt{5}G_{uyz}(6x^2 - y^2 - z^2)}{4} - \frac{\sqrt{15}G_vyz(6x^2 - y^2 - z^2)}{12} + \frac{\sqrt{15}G_{xy}xz(4x^2 - 3y^2 - 3z^2)}{12} \\ & - \frac{\sqrt{15}G_{xz}xy(4x^2 - 3y^2 - 3z^2)}{12} - \frac{\sqrt{15}G_{yz}(y-z)(y+z)(6x^2 - y^2 - z^2)}{12} \\ & - \frac{\sqrt{5}G_{uxz}(x^2 - 6y^2 + z^2)}{4} + \frac{\sqrt{15}G_vxz(x^2 - 6y^2 + z^2)}{12} + \frac{\sqrt{15}G_{xy}yz(3x^2 - 4y^2 + 3z^2)}{12} \\ & - \frac{\sqrt{15}G_{xz}(x-z)(x+z)(x^2 - 6y^2 + z^2)}{12} - \frac{\sqrt{15}G_{yz}xy(3x^2 - 4y^2 + 3z^2)}{12} \\ & - \frac{\sqrt{15}G_vxy(x^2 + y^2 - 6z^2)}{6} + \frac{\sqrt{15}G_{xy}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{12} - \frac{\sqrt{15}G_{xz}yz(3x^2 + 3y^2 - 4z^2)}{12} + \frac{\sqrt{15}G_{yz}xz(3x^2 + 3y^2 - 4z^2)}{12} \end{aligned}$$

\* Harmonics for rank 4

$$\vec{\mathbb{Q}}_4^{(2,-1)}[g](A_1)$$

\*\* symmetry

$$\frac{\sqrt{21} (x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

\*\* expression

$$\frac{\sqrt{5}G_{xy}z(x-y)(x+y)}{2} - \frac{\sqrt{5}G_{xz}y(x-z)(x+z)}{2} + \frac{\sqrt{5}G_{yz}x(y-z)(y+z)}{2}$$

$$\vec{\mathbb{Q}}_4^{(2,1)}[g](A_1)$$

\*\* symmetry

$$\frac{\sqrt{21} (x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

\*\* expression

$$-\frac{3\sqrt{2310}G_uxyz(x-y)(x+y)}{44} - \frac{3\sqrt{770}G_vxyz(x^2+y^2-2z^2)}{44} + \frac{\sqrt{770}G_{xy}z(x-y)(x+y)(x^2+y^2-2z^2)}{22} \\ - \frac{\sqrt{770}G_{xz}y(x-z)(x+z)(x^2-2y^2+z^2)}{22} - \frac{\sqrt{770}G_{yz}x(y-z)(y+z)(2x^2-y^2-z^2)}{22}$$

$$\vec{\mathbb{Q}}_{4,1}^{(2,-1)}[g](E), \vec{\mathbb{Q}}_{4,2}^{(2,-1)}[g](E)$$

\*\* symmetry

$$-\frac{\sqrt{15} (x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

$$\frac{\sqrt{5} (x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

\*\* expression

$$-\frac{6\sqrt{7}G_vxyz}{7} - \frac{\sqrt{7}G_{xy}z(x-y)(x+y)}{14} + \frac{\sqrt{7}G_{xz}y(4x^2-3y^2+5z^2)}{14} + \frac{\sqrt{7}G_{yz}x(3x^2-4y^2-5z^2)}{14}$$

$$\frac{6\sqrt{7}G_uxyz}{7} + \frac{\sqrt{21}G_{xy}z(3x^2+3y^2-2z^2)}{14} - \frac{\sqrt{21}G_{xz}y(2x^2-y^2+z^2)}{14} + \frac{\sqrt{21}G_{yz}x(x^2-2y^2-z^2)}{14}$$

$$\vec{\mathbb{Q}}_{4,1}^{(2,1)}[g](E), \vec{\mathbb{Q}}_{4,2}^{(2,1)}[g](E)$$

\*\* symmetry

$$-\frac{\sqrt{15} (x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

$$\frac{\sqrt{5} (x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

\*\* expression

$$\frac{21\sqrt{66}G_uxyz(x-y)(x+y)}{44} - \frac{21\sqrt{22}G_vxyz(x^2+y^2-2z^2)}{44} + \frac{7\sqrt{22}G_{xy}z(x-y)(x+y)(x^2+y^2-2z^2)}{44} \\ + \frac{\sqrt{22}G_{xz}y(17x^4-22x^2y^2-36x^2z^2+3y^4-8y^2z^2+10z^4)}{44} - \frac{\sqrt{22}G_{yz}x(3x^4-22x^2y^2-8x^2z^2+17y^4-36y^2z^2+10z^4)}{44}$$

$$-\frac{21\sqrt{22}G_uxyz(x^2+y^2-2z^2)}{44} - \frac{21\sqrt{66}G_vxyz(x-y)(x+y)}{44} + \frac{\sqrt{66}G_{xy}z(9x^4-24x^2y^2-10x^2z^2+9y^4-10y^2z^2+2z^4)}{44} \\ - \frac{\sqrt{66}G_{xz}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44} - \frac{\sqrt{66}G_{yz}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44}$$

$$\vec{\mathbb{Q}}_{4,1}^{(2,-1)}[g](T_1), \vec{\mathbb{Q}}_{4,2}^{(2,-1)}[g](T_1), \vec{\mathbb{Q}}_{4,3}^{(2,-1)}[g](T_1)$$

\*\* symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

\*\* expression

$$\frac{3G_{ux}(y-z)(y+z)}{4} + \frac{\sqrt{3}G_vx(y-z)(y+z)}{4} + \frac{\sqrt{3}G_{xy}y(y^2-3z^2)}{4} - \frac{\sqrt{3}G_{xz}z(3y^2-z^2)}{4} + \sqrt{3}G_{yz}xyz$$

$$\frac{3G_{uy}(x-z)(x+z)}{4} - \frac{\sqrt{3}G_vy(x-z)(x+z)}{4} + \frac{\sqrt{3}G_{xy}x(x^2-3z^2)}{4} + \sqrt{3}G_{xz}xyz - \frac{\sqrt{3}G_{yz}z(3x^2-z^2)}{4}$$

$$-\frac{\sqrt{3}G_vz(x-y)(x+y)}{2} + \sqrt{3}G_{xy}xyz + \frac{\sqrt{3}G_{xz}x(x^2-3y^2)}{4} - \frac{\sqrt{3}G_{yz}y(3x^2-y^2)}{4}$$

$$\vec{\mathbb{Q}}_{4,1}^{(2,1)}[g](T_1), \vec{\mathbb{Q}}_{4,2}^{(2,1)}[g](T_1), \vec{\mathbb{Q}}_{4,3}^{(2,1)}[g](T_1)$$

\*\* symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

\*\* expression

$$-\frac{3\sqrt{154}G_{ux}(x^2y^2-x^2z^2+y^4-9y^2z^2+2z^4)}{88} - \frac{\sqrt{462}G_vx(x^2y^2-x^2z^2-5y^4+27y^2z^2-4z^4)}{88} \\ - \frac{\sqrt{462}G_{xy}y(2x^2y^2-6x^2z^2-y^4+8y^2z^2-3z^4)}{44} + \frac{\sqrt{462}G_{xz}z(6x^2y^2-2x^2z^2+3y^4-8y^2z^2+z^4)}{44} - \frac{\sqrt{462}G_{yz}xyz(2x^2-y^2-z^2)}{44}$$

$$-\frac{3\sqrt{154}G_{uy}(x^4+x^2y^2-9x^2z^2-y^2z^2+2z^4)}{88} - \frac{\sqrt{462}G_vy(5x^4-x^2y^2-27x^2z^2+y^2z^2+4z^4)}{88} \\ + \frac{\sqrt{462}G_{xy}x(x^4-2x^2y^2-8x^2z^2+6y^2z^2+3z^4)}{44} + \frac{\sqrt{462}G_{xz}xyz(x^2-2y^2+z^2)}{44} + \frac{\sqrt{462}G_{yz}z(3x^4+6x^2y^2-8x^2z^2-2y^2z^2+z^4)}{44}$$

$$\frac{9\sqrt{154}G_uz(x^2-2xy-y^2)(x^2+2xy-y^2)}{88} - \frac{\sqrt{462}G_vz(x-y)(x+y)(x^2+y^2-2z^2)}{88} + \frac{\sqrt{462}G_{xy}xyz(x^2+y^2-2z^2)}{44} \\ + \frac{\sqrt{462}G_{xz}x(x^4-8x^2y^2-2x^2z^2+3y^4+6y^2z^2)}{44} + \frac{\sqrt{462}G_{yz}y(3x^4-8x^2y^2+6x^2z^2+y^4-2y^2z^2)}{44}$$

$$\vec{\mathbb{Q}}_{4,1}^{(2,-1)}[g](T_2), \vec{\mathbb{Q}}_{4,2}^{(2,-1)}[g](T_2), \vec{\mathbb{Q}}_{4,3}^{(2,-1)}[g](T_2)$$

\*\* symmetry

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

\*\* expression

$$\frac{3\sqrt{7}G_{ux}(2x^2-5y^2-z^2)}{28} + \frac{\sqrt{21}G_vx(2x^2+3y^2-9z^2)}{28} - \frac{\sqrt{21}G_{xy}y(2x^2+y^2-5z^2)}{28} + \frac{\sqrt{21}G_{xz}z(2x^2-5y^2+z^2)}{28}$$

$$\frac{3\sqrt{7}G_{uy}(5x^2-2y^2+z^2)}{28} + \frac{\sqrt{21}G_vy(3x^2+2y^2-9z^2)}{28} + \frac{\sqrt{21}G_{xy}x(x^2+2y^2-5z^2)}{28} + \frac{\sqrt{21}G_{yz}z(5x^2-2y^2-z^2)}{28}$$

$$\frac{3\sqrt{7}G_uz(x-y)(x+y)}{7} + \frac{\sqrt{21}G_vz(3x^2+3y^2-2z^2)}{14} - \frac{\sqrt{21}G_{xz}x(x^2-5y^2+2z^2)}{28} - \frac{\sqrt{21}G_{yz}y(5x^2-y^2-2z^2)}{28}$$

$$\vec{\mathbb{Q}}_{4,1}^{(2,1)}[g](T_2), \vec{\mathbb{Q}}_{4,2}^{(2,1)}[g](T_2), \vec{\mathbb{Q}}_{4,3}^{(2,1)}[g](T_2)$$

\*\* symmetry

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

\*\* expression

$$\begin{aligned}
& -\frac{3\sqrt{22}G_u x (2x^4 - 3x^2y^2 - 17x^2z^2 - 5y^4 + 39y^2z^2 + 2z^4)}{88} - \frac{\sqrt{66}G_v x (2x^4 - 31x^2y^2 + 11x^2z^2 + 9y^4 + 39y^2z^2 - 12z^4)}{88} \\
& - \frac{\sqrt{66}G_{xy}y (8x^4 - 12x^2y^2 - 12x^2z^2 + y^4 + 2y^2z^2 + z^4)}{44} + \frac{\sqrt{66}G_{xz}z (8x^4 - 12x^2y^2 - 12x^2z^2 + y^4 + 2y^2z^2 + z^4)}{44} - \frac{21\sqrt{66}G_{yz}xyz(y - z)(y + z)}{44} \\
& - \frac{3\sqrt{22}G_u y (5x^4 + 3x^2y^2 - 39x^2z^2 - 2y^4 + 17y^2z^2 - 2z^4)}{88} - \frac{\sqrt{66}G_v y (9x^4 - 31x^2y^2 + 39x^2z^2 + 2y^4 + 11y^2z^2 - 12z^4)}{88} \\
& + \frac{\sqrt{66}G_{xy}x (x^4 - 12x^2y^2 + 2x^2z^2 + 8y^4 - 12y^2z^2 + z^4)}{44} + \frac{21\sqrt{66}G_{xz}xyz(x - z)(x + z)}{44} - \frac{\sqrt{66}G_{yz}z (x^4 - 12x^2y^2 + 2x^2z^2 + 8y^4 - 12y^2z^2 + z^4)}{44} \\
& - \frac{21\sqrt{22}G_u z (x - y)(x + y)(x^2 + y^2 - 2z^2)}{88} - \frac{\sqrt{66}G_v z (3x^4 - 78x^2y^2 + 20x^2z^2 + 3y^4 + 20y^2z^2 - 4z^4)}{88} - \frac{21\sqrt{66}G_{xy}xyz(x - y)(x + y)}{44} \\
& - \frac{\sqrt{66}G_{xz}x (x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44} + \frac{\sqrt{66}G_{yz}y (x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44}
\end{aligned}$$