

# Model for “graphene”

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## General Condition

- Basis type: **1g**
- SAMB selection:
  - Type: **[Q, G]**
  - Rank: **[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]**
  - Irrep.: **[ $A_{1g}$ ,  $A_{2g}$ ,  $B_{1g}$ ,  $B_{2g}$ ,  $E_{1g}$ ,  $E_{2g}$ ,  $A_{1u}$ ,  $A_{2u}$ ,  $B_{1u}$ ,  $B_{2u}$ ,  $E_{1u}$ ,  $E_{2u}$ ]**
  - Spin (s): **[0, 1]**
- Atomic selection:
  - Type: **[Q, G, M, T]**
  - Rank: **[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]**
  - Irrep.: **[ $A_{1g}$ ,  $A_{2g}$ ,  $B_{1g}$ ,  $B_{2g}$ ,  $E_{1g}$ ,  $E_{2g}$ ,  $A_{1u}$ ,  $A_{2u}$ ,  $B_{1u}$ ,  $B_{2u}$ ,  $E_{1u}$ ,  $E_{2u}$ ]**
  - Spin (s): **[0, 1]**
- Site-cluster selection:
  - Rank: **[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]**
  - Irrep.: **[ $A_{1g}$ ,  $A_{2g}$ ,  $B_{1g}$ ,  $B_{2g}$ ,  $E_{1g}$ ,  $E_{2g}$ ,  $A_{1u}$ ,  $A_{2u}$ ,  $B_{1u}$ ,  $B_{2u}$ ,  $E_{1u}$ ,  $E_{2u}$ ]**
- Bond-cluster selection:
  - Type: **[Q, G, M, T]**
  - Rank: **[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]**
  - Irrep.: **[ $A_{1g}$ ,  $A_{2g}$ ,  $B_{1g}$ ,  $B_{2g}$ ,  $E_{1g}$ ,  $E_{2g}$ ,  $A_{1u}$ ,  $A_{2u}$ ,  $B_{1u}$ ,  $B_{2u}$ ,  $E_{1u}$ ,  $E_{2u}$ ]**
- Max. neighbor: **10**
- Search cell range: **(-2, 3), (-2, 3), (-2, 3)**
- Toroidal priority: **false**

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## Group and Unit Cell

- Group: SG No. 191  $D_{6h}^1$   $P6/mmm$  [ hexagonal ]
- Associated point group: PG No. 191  $D_{6h}$   $6/mmm$  [ hexagonal ]
- Unit cell:
  - $a = 1.00000$ ,  $b = 1.00000$ ,  $c = 4.00000$ ,  $\alpha = 90.0$ ,  $\beta = 90.0$ ,  $\gamma = 120.0$
- Lattice vectors (conventional cell):
  - $\mathbf{a}_1 = [ 1.00000, 0.00000, 0.00000 ]$
  - $\mathbf{a}_2 = [ -0.50000, 0.86603, 0.00000 ]$
  - $\mathbf{a}_3 = [ 0.00000, 0.00000, 4.00000 ]$

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**Symmetry Operation**


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Table 1: Symmetry operation

#	SO	#	SO	#	SO	#	SO	#	SO
1	$\{1 0\}$	2	$\{3_{001}^+ 0\}$	3	$\{3_{001}^- 0\}$	4	$\{2_{001} 0\}$	5	$\{6_{001}^- 0\}$
6	$\{6_{001}^+ 0\}$	7	$\{2_{110} 0\}$	8	$\{2_{100} 0\}$	9	$\{2_{010} 0\}$	10	$\{2_{1-10} 0\}$
11	$\{2_{120} 0\}$	12	$\{2_{210} 0\}$	13	$\{-1 0\}$	14	$\{-3_{001}^+ 0\}$	15	$\{-3_{001}^- 0\}$
16	$\{m_{001} 0\}$	17	$\{-6_{001}^- 0\}$	18	$\{-6_{001}^+ 0\}$	19	$\{m_{110} 0\}$	20	$\{m_{100} 0\}$
21	$\{m_{010} 0\}$	22	$\{m_{1-10} 0\}$	23	$\{m_{120} 0\}$	24	$\{m_{210} 0\}$		

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**Harmonics**


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Table 2: Harmonics

#	symbol	irrep.	rank	X	multiplicity	component	symmetry
1	$\mathbb{Q}_0(A_{1g})$	$A_{1g}$	0	$Q, T$	-	-	1
2	$\mathbb{G}_1(A_{2g})$	$A_{2g}$	1	$G, M$	-	-	$z$
3	$\mathbb{Q}_6(A_{2g})$	$A_{2g}$	6	$Q, T$	-	-	$\frac{\sqrt{462}xy(x^2-3y^2)(3x^2-y^2)}{16}$
4	$\mathbb{Q}_3(B_{1u})$	$B_{1u}$	3	$Q, T$	-	-	$\frac{\sqrt{10}y(3x^2-y^2)}{4}$
5	$\mathbb{Q}_3(B_{2u})$	$B_{2u}$	3	$Q, T$	-	-	$\frac{\sqrt{10}x(x^2-3y^2)}{4}$
6	$\mathbb{Q}_{1,1}(E_{1u})$	$E_{1u}$	1	$Q, T$	-	1	$x$

*continued ...*

Table 2

#	symbol	irrep.	rank	X	multiplicity	component	symmetry
7	$\mathbb{Q}_{1,2}(E_{1u})$					2	$y$
8	$\mathbb{Q}_{2,1}(E_{2g})$	$E_{2g}$	2	$Q, T$	-	1	$\frac{\sqrt{3}(x-y)(x+y)}{2}$
9	$\mathbb{Q}_{2,2}(E_{2g})$					2	$-\sqrt{3}xy$
10	$\mathbb{Q}_{4,1}(E_{2g}, 1)$	$E_{2g}$	4	$Q, T$	1	1	$\frac{\sqrt{35}(x^2-2xy-y^2)(x^2+2xy-y^2)}{8}$
11	$\mathbb{Q}_{4,2}(E_{2g}, 1)$					2	$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$

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**Basis in full matrix**


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Table 3: dimension = 2

#	orbital@atom(SL)	#	orbital@atom(SL)
0	$ p_z\rangle @C(1)$	1	$ p_z\rangle @C(2)$

Table 4: Atomic basis (orbital part only)

orbital	definition
$ p_x\rangle$	$x$
$ p_y\rangle$	$y$
$ p_z\rangle$	$z$

• **C : 'C' site-cluster**

- \* bra:  $\langle p_z |$
- \* ket:  $|p_z\rangle$
- \* wyckoff: **2c**

$$\boxed{\text{z1}} \quad \mathbb{Q}_0^{(c)}(A_{1g}) = \mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_0^{(s)}(A_{1g})$$

$$\boxed{\text{z23}} \quad \mathbb{Q}_3^{(c)}(B_{1u}) = \mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_3^{(s)}(B_{1u})$$

• **C;C\_001\_1 : 'C'-'C' bond-cluster**

- \* bra:  $\langle p_z |$
- \* ket:  $|p_z\rangle$
- \* wyckoff: **3a03f**

$$\boxed{\text{z2}} \quad \mathbb{Q}_0^{(c)}(A_{1g}) = \mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_0^{(b)}(A_{1g})$$

$$\boxed{\text{z9}} \quad \mathbb{Q}_{2,1}^{(c)}(E_{2g}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{2,1}^{(b)}(E_{2g})}{2}$$

$$\boxed{\text{z10}} \quad \mathbb{Q}_{2,2}^{(c)}(E_{2g}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{2,2}^{(b)}(E_{2g})}{2}$$

• **C;C\_002\_1 : 'C'-'C' bond-cluster**

- \* bra:  $\langle p_z |$
- \* ket:  $|p_z\rangle$
- \* wyckoff: **6b061**

$$\boxed{\text{z3}} \quad \mathbb{Q}_0^{(c)}(A_{1g}) = \mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_0^{(b)}(A_{1g})$$

$$\boxed{\text{z11}} \quad \mathbb{Q}_3^{(c)}(B_{1u}) = \mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_3^{(b)}(B_{1u})$$

$$\boxed{\text{z12}} \quad \mathbb{Q}_{1,1}^{(c)}(E_{1u}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{1,1}^{(b)}(E_{1u})}{2}$$

$$\boxed{\text{z24}} \quad \mathbb{Q}_{1,2}^{(c)}(E_{1u}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{1,2}^{(b)}(E_{1u})}{2}$$

$$\boxed{\text{z27}} \quad \mathbb{Q}_{2,1}^{(c)}(E_{2g}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{2,1}^{(b)}(E_{2g})}{2}$$

$$\boxed{\text{z28}} \quad \mathbb{Q}_{2,2}^{(c)}(E_{2g}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{2,2}^{(b)}(E_{2g})}{2}$$

- **C;C\_003\_1** : 'C'-'C' bond-cluster

\* bra:  $\langle p_z |$

\* ket:  $|p_z\rangle$

\* wyckoff: **3b@1a**

$$\boxed{\text{z4}} \quad \mathbb{Q}_0^{(c)}(A_{1g}) = \mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_0^{(b)}(A_{1g})$$

$$\boxed{\text{z13}} \quad \mathbb{Q}_{2,1}^{(c)}(E_{2g}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{2,1}^{(b)}(E_{2g})}{2}$$

$$\boxed{\text{z14}} \quad \mathbb{Q}_{2,2}^{(c)}(E_{2g}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{2,2}^{(b)}(E_{2g})}{2}$$

- **C;C\_004\_1** : 'C'-'C' bond-cluster

\* bra:  $\langle p_z |$

\* ket:  $|p_z\rangle$

\* wyckoff: **6d@3f**

$$\boxed{\text{z5}} \quad \mathbb{Q}_0^{(c)}(A_{1g}) = \mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_0^{(b)}(A_{1g})$$

$$\boxed{\text{z8}} \quad \mathbb{Q}_6^{(c)}(A_{2g}) = \mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_6^{(b)}(A_{2g})$$

$$\boxed{\text{z15}} \quad \mathbb{Q}_{2,1}^{(c)}(E_{2g}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{2,1}^{(b)}(E_{2g})}{2}$$

$$\boxed{\text{z16}} \quad \mathbb{Q}_{2,2}^{(c)}(E_{2g}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{2,2}^{(b)}(E_{2g})}{2}$$

$$\boxed{\text{z17}} \quad \mathbb{Q}_{4,1}^{(c)}(E_{2g}, 1) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{4,1}^{(b)}(E_{2g}, 1)}{2}$$

$$\boxed{\text{z18}} \quad \mathbb{Q}_{4,2}^{(c)}(E_{2g}, 1) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{4,2}^{(b)}(E_{2g}, 1)}{2}$$

• **C;C\_005\_1** : 'C'-'C' bond-cluster

\* bra:  $\langle p_z |$

\* ket:  $|p_z\rangle$

\* wyckoff: **6a@61**

$$\boxed{\text{z6}} \quad \mathbb{Q}_0^{(c)}(A_{1g}) = \mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_0^{(b)}(A_{1g})$$

$$\boxed{\text{z19}} \quad \mathbb{Q}_3^{(c)}(B_{1u}) = \mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_3^{(b)}(B_{1u})$$

$$\boxed{\text{z20}} \quad \mathbb{Q}_{1,1}^{(c)}(E_{1u}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{1,1}^{(b)}(E_{1u})}{2}$$

$$\boxed{\text{z25}} \quad \mathbb{Q}_{1,2}^{(c)}(E_{1u}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{1,2}^{(b)}(E_{1u})}{2}$$

$$\boxed{\text{z29}} \quad \mathbb{Q}_{2,1}^{(c)}(E_{2g}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{2,1}^{(b)}(E_{2g})}{2}$$

$$\boxed{\text{z30}} \quad \mathbb{Q}_{2,2}^{(c)}(E_{2g}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{2,2}^{(b)}(E_{2g})}{2}$$

• **C;C\_006\_1** : 'C'-'C' bond-cluster

\* bra:  $\langle p_z |$

\* ket:  $|p_z\rangle$

\* wyckoff: **6c@2c**

$$\boxed{\text{z7}} \quad \mathbb{Q}_0^{(c)}(A_{1g}) = \mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_0^{(b)}(A_{1g})$$

$$\boxed{\text{z21}} \quad \mathbb{Q}_3^{(c)}(B_{1u}) = \mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_3^{(b)}(B_{1u})$$

$$\boxed{\text{z22}} \quad \mathbb{Q}_{1,1}^{(c)}(E_{1u}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{1,1}^{(b)}(E_{1u})}{2}$$

$$\boxed{\text{z26}} \quad \mathbb{Q}_{1,2}^{(c)}(E_{1u}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{1,2}^{(b)}(E_{1u})}{2}$$

$$\boxed{\text{z31}} \quad \mathbb{Q}_{2,1}^{(c)}(E_{2g}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{2,1}^{(b)}(E_{2g})}{2}$$

$$\boxed{\text{z32}} \quad \mathbb{Q}_{2,2}^{(c)}(E_{2g}) = \frac{\sqrt{2}\mathbb{Q}_0^{(a)}(A_{1g})\mathbb{Q}_{2,2}^{(b)}(E_{2g})}{2}$$

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### Atomic SAMB

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- bra:  $\langle p_z |$
- ket:  $|p_z\rangle$

$$\boxed{\text{x1}} \quad \mathbb{Q}_0^{(a)}(A_{1g}) = [1]$$

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### Cluster SAMB

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- Site cluster

\*\* Wyckoff: 2c

$$\boxed{\text{y1}} \quad \mathbb{Q}_0^{(s)}(A_{1g}) = \left[ \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right]$$

$$\boxed{\text{y2}} \quad \mathbb{Q}_3^{(s)}(B_{1u}) = \left[ \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right]$$

- Bond cluster

\*\* Wyckoff: 3b@1a

$$\boxed{\text{y3}} \quad \mathbb{Q}_0^{(s)}(A_{1g}) = \left[ \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3} \right]$$

$$\boxed{\text{y4}} \quad \mathbb{T}_3^{(s)}(B_{1u}) = \left[ \frac{\sqrt{3}i}{3}, \frac{\sqrt{3}i}{3}, \frac{\sqrt{3}i}{3} \right]$$

$$\boxed{\text{y5}} \quad \mathbb{T}_{1,1}^{(s)}(E_{1u}) = \left[ 0, -\frac{\sqrt{2}i}{2}, \frac{\sqrt{2}i}{2} \right]$$

$$\boxed{\text{y6}} \quad \mathbb{T}_{1,2}^{(s)}(E_{1u}) = \left[ \frac{\sqrt{6}i}{3}, -\frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6} \right]$$

$$\boxed{\text{y7}} \quad \mathbb{Q}_{2,1}^{(s)}(E_{2g}) = \left[ \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6} \right]$$

$$\boxed{\text{y8}} \quad \mathbb{Q}_{2,2}^{(s)}(E_{2g}) = \left[ 0, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right]$$

\*\* Wyckoff: 3a@3f

$$\boxed{\text{y9}} \quad \mathbb{Q}_0^{(s)}(A_{1g}) = \left[ \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3} \right]$$

$$\boxed{\text{y10}} \quad \mathbb{T}_3^{(s)}(B_{1u}) = \left[ \frac{\sqrt{3}i}{3}, \frac{\sqrt{3}i}{3}, \frac{\sqrt{3}i}{3} \right]$$

$$\boxed{\text{y11}} \quad \mathbb{T}_{1,1}^{(s)}(E_{1u}) = \left[ 0, -\frac{\sqrt{2}i}{2}, \frac{\sqrt{2}i}{2} \right]$$

$$\boxed{\text{y12}} \quad \mathbb{T}_{1,2}^{(s)}(E_{1u}) = \left[ \frac{\sqrt{6}i}{3}, -\frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6} \right]$$

$$\boxed{\text{y13}} \quad \mathbb{Q}_{2,1}^{(s)}(E_{2g}) = \left[ \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6} \right]$$

$$\boxed{\text{y14}} \quad \mathbb{Q}_{2,2}^{(s)}(E_{2g}) = \left[ 0, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right]$$

\*\* Wyckoff: 6b@6l

$$\boxed{\text{y15}} \quad \mathbb{Q}_0^{(s)}(A_{1g}) = \left[ \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6} \right]$$

$$\boxed{\text{y16}} \quad \mathbb{M}_1^{(s)}(A_{2g}) = \left[ \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6} \right]$$

$$\boxed{\text{y17}} \quad \mathbb{Q}_3^{(s)}(B_{1u}) = \left[ \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6} \right]$$



$$\boxed{\text{y18}} \quad \mathbb{T}_3^{(s)}(B_{2u}) = \left[ \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6} \right]$$

$$\boxed{\text{y19}} \quad \mathbb{Q}_{1,1}^{(s)}(E_{1u}) = \left[ 0, -\frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2}, -\frac{1}{2} \right]$$

$$\boxed{\text{y20}} \quad \mathbb{Q}_{1,2}^{(s)}(E_{1u}) = \left[ \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{6}, -\frac{\sqrt{3}}{6}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6} \right]$$

$$\boxed{\text{y21}} \quad \mathbb{T}_{1,1}^{(s)}(E_{1u}) = \left[ \frac{\sqrt{3}i}{3}, -\frac{\sqrt{3}i}{6}, -\frac{\sqrt{3}i}{6}, -\frac{\sqrt{3}i}{3}, \frac{\sqrt{3}i}{6}, \frac{\sqrt{3}i}{6} \right]$$

$$\boxed{\text{y22}} \quad \mathbb{T}_{1,2}^{(s)}(E_{1u}) = \left[ 0, \frac{i}{2}, -\frac{i}{2}, 0, -\frac{i}{2}, \frac{i}{2} \right]$$

$$\boxed{\text{y23}} \quad \mathbb{Q}_{2,1}^{(s)}(E_{2g}) = \left[ \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{6}, -\frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{6}, -\frac{\sqrt{3}}{6} \right]$$

$$\boxed{\text{y24}} \quad \mathbb{Q}_{2,2}^{(s)}(E_{2g}) = \left[ 0, \frac{1}{2}, -\frac{1}{2}, 0, \frac{1}{2}, -\frac{1}{2} \right]$$

$$\boxed{\text{y25}} \quad \mathbb{T}_{2,1}^{(s)}(E_{2g}) = \left[ 0, \frac{i}{2}, -\frac{i}{2}, 0, \frac{i}{2}, -\frac{i}{2} \right]$$

$$\boxed{\text{y26}} \quad \mathbb{T}_{2,2}^{(s)}(E_{2g}) = \left[ -\frac{\sqrt{3}i}{3}, \frac{\sqrt{3}i}{6}, \frac{\sqrt{3}i}{6}, -\frac{\sqrt{3}i}{3}, \frac{\sqrt{3}i}{6}, \frac{\sqrt{3}i}{6} \right]$$

\*\* Wyckoff: 6c@2c

$$\boxed{\text{y27}} \quad \mathbb{Q}_0^{(s)}(A_{1g}) = \left[ \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6} \right]$$

$$\boxed{\text{y28}} \quad \mathbb{M}_1^{(s)}(A_{2g}) = \left[ \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6} \right]$$

$$\boxed{\text{y29}} \quad \mathbb{Q}_3^{(s)}(B_{1u}) = \left[ \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6} \right]$$

$$\boxed{\text{y30}} \quad \mathbb{T}_3^{(s)}(B_{2u}) = \left[ \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6} \right]$$

$$\boxed{\text{y31}} \quad \mathbb{Q}_{1,1}^{(s)}(E_{1u}) = \left[ 0, -\frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2}, -\frac{1}{2} \right]$$

$$\boxed{\text{y32}} \quad \mathbb{Q}_{1,2}^{(s)}(E_{1u}) = \left[ \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{6}, -\frac{\sqrt{3}}{6}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6} \right]$$

$$\boxed{\text{y33}} \quad \mathbb{T}_{1,1}^{(s)}(E_{1u}) = \left[ \frac{\sqrt{3}i}{3}, -\frac{\sqrt{3}i}{6}, -\frac{\sqrt{3}i}{6}, -\frac{\sqrt{3}i}{3}, \frac{\sqrt{3}i}{6}, \frac{\sqrt{3}i}{6} \right]$$

$$\boxed{\text{y34}} \quad \mathbb{T}_{1,2}^{(s)}(E_{1u}) = \left[ 0, \frac{i}{2}, -\frac{i}{2}, 0, -\frac{i}{2}, \frac{i}{2} \right]$$

$$\boxed{\text{y35}} \quad \mathbb{Q}_{2,1}^{(s)}(E_{2g}) = \left[ \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{6}, -\frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{6}, -\frac{\sqrt{3}}{6} \right]$$

$$\boxed{\text{y36}} \quad \mathbb{Q}_{2,2}^{(s)}(E_{2g}) = \left[ 0, \frac{1}{2}, -\frac{1}{2}, 0, \frac{1}{2}, -\frac{1}{2} \right]$$

$$\boxed{\text{y37}} \quad \mathbb{T}_{2,1}^{(s)}(E_{2g}) = \left[ 0, \frac{i}{2}, -\frac{i}{2}, 0, \frac{i}{2}, -\frac{i}{2} \right]$$

$$\boxed{\text{y38}} \quad \mathbb{T}_{2,2}^{(s)}(E_{2g}) = \left[ -\frac{\sqrt{3}i}{3}, \frac{\sqrt{3}i}{6}, \frac{\sqrt{3}i}{6}, -\frac{\sqrt{3}i}{3}, \frac{\sqrt{3}i}{6}, \frac{\sqrt{3}i}{6} \right]$$

\*\* Wyckoff: **6d@3f**

$$\boxed{\text{y39}} \quad \mathbb{Q}_0^{(s)}(A_{1g}) = \left[ \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6} \right]$$

$$\boxed{\text{y40}} \quad \mathbb{Q}_6^{(s)}(A_{2g}) = \left[ \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6} \right]$$

$$\boxed{\text{y41}} \quad \mathbb{T}_3^{(s)}(B_{1u}) = \left[ \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6} \right]$$

$$\boxed{\text{y42}} \quad \mathbb{T}_3^{(s)}(B_{2u}) = \left[ \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6} \right]$$

$$\boxed{\text{y43}} \quad \mathbb{T}_{1,1}^{(s)}(E_{1u}, a) = \left[ \frac{5\sqrt{21}i}{42}, -\frac{2\sqrt{21}i}{21}, -\frac{\sqrt{21}i}{42}, -\frac{\sqrt{21}i}{42}, \frac{5\sqrt{21}i}{42}, -\frac{2\sqrt{21}i}{21} \right]$$

$$\boxed{\text{y44}} \quad \mathbb{T}_{1,2}^{(s)}(E_{1u}, a) = \left[ \frac{\sqrt{7}i}{14}, \frac{\sqrt{7}i}{7}, -\frac{3\sqrt{7}i}{14}, \frac{3\sqrt{7}i}{14}, -\frac{\sqrt{7}i}{14}, -\frac{\sqrt{7}i}{7} \right]$$

$$\boxed{\text{y45}} \quad \mathbb{T}_{1,1}^{(s)}(E_{1u}, b) = \left[ \frac{\sqrt{7}i}{14}, \frac{\sqrt{7}i}{7}, -\frac{3\sqrt{7}i}{14}, -\frac{3\sqrt{7}i}{14}, \frac{\sqrt{7}i}{14}, \frac{\sqrt{7}i}{7} \right]$$

$$\boxed{\text{y46}} \quad \mathbb{T}_{1,2}^{(s)}(E_{1u}, b) = \left[ -\frac{5\sqrt{21}i}{42}, \frac{2\sqrt{21}i}{21}, \frac{\sqrt{21}i}{42}, -\frac{\sqrt{21}i}{42}, \frac{5\sqrt{21}i}{42}, -\frac{2\sqrt{21}i}{21} \right]$$

$$\boxed{\text{y47}} \quad \mathbb{Q}_{2,1}^{(s)}(E_{2g}) = \left[ \frac{11\sqrt{3}}{42}, \frac{\sqrt{3}}{21}, -\frac{13\sqrt{3}}{42}, -\frac{13\sqrt{3}}{42}, \frac{11\sqrt{3}}{42}, \frac{\sqrt{3}}{21} \right]$$

$$\boxed{\text{y48}} \quad \mathbb{Q}_{2,2}^{(s)}(E_{2g}) = \left[ -\frac{5}{14}, \frac{4}{7}, -\frac{3}{14}, \frac{3}{14}, \frac{5}{14}, -\frac{4}{7} \right]$$

$$\boxed{\text{y49}} \quad \mathbb{Q}_{4,1}^{(s)}(E_{2g}, 1) = \left[ \frac{5}{14}, -\frac{4}{7}, \frac{3}{14}, \frac{3}{14}, \frac{5}{14}, -\frac{4}{7} \right]$$

$$\boxed{\text{y50}} \quad \mathbb{Q}_{4,2}^{(s)}(E_{2g}, 1) = \left[ \frac{11\sqrt{3}}{42}, \frac{\sqrt{3}}{21}, -\frac{13\sqrt{3}}{42}, \frac{13\sqrt{3}}{42}, -\frac{11\sqrt{3}}{42}, -\frac{\sqrt{3}}{21} \right]$$

\*\* Wyckoff: **6a@61**

$$\boxed{\text{y51}} \quad \mathbb{Q}_0^{(s)}(A_{1g}) = \left[ \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6} \right]$$

$$\boxed{\text{y52}} \quad \mathbb{T}_0^{(s)}(A_{1g}) = \left[ \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6} \right]$$

$$\boxed{\text{y53}} \quad \mathbb{Q}_3^{(s)}(B_{1u}) = \left[ \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6} \right]$$

$$\boxed{\text{y54}} \quad \mathbb{T}_3^{(s)}(B_{1u}) = \left[ \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, \frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6}, -\frac{\sqrt{6}i}{6} \right]$$

$$\boxed{\text{y55}} \quad \mathbb{Q}_{1,1}^{(s)}(E_{1u}) = \left[ 0, -\frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2}, -\frac{1}{2} \right]$$

$$\boxed{\text{y56}} \quad \mathbb{Q}_{1,2}^{(s)}(E_{1u}) = \left[ \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{6}, -\frac{\sqrt{3}}{6}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6} \right]$$

$$\boxed{\text{y57}} \quad \mathbb{T}_{1,1}^{(s)}(E_{1u}) = \left[ 0, -\frac{i}{2}, \frac{i}{2}, 0, \frac{i}{2}, -\frac{i}{2} \right]$$

$$\boxed{\text{y58}} \quad \mathbb{T}_{1,2}^{(s)}(E_{1u}) = \left[ \frac{\sqrt{3}i}{3}, -\frac{\sqrt{3}i}{6}, -\frac{\sqrt{3}i}{6}, -\frac{\sqrt{3}i}{3}, \frac{\sqrt{3}i}{6}, \frac{\sqrt{3}i}{6} \right]$$

$$\boxed{\text{y59}} \quad \mathbb{Q}_{2,1}^{(s)}(E_{2g}) = \left[ \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{6}, -\frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{6}, -\frac{\sqrt{3}}{6} \right]$$

$$\boxed{\text{y60}} \quad \mathbb{Q}_{2,2}^{(s)}(E_{2g}) = \left[ 0, \frac{1}{2}, -\frac{1}{2}, 0, \frac{1}{2}, -\frac{1}{2} \right]$$

$$\boxed{\text{y61}} \quad \mathbb{T}_{2,1}^{(s)}(E_{2g}) = \left[ \frac{\sqrt{3}i}{3}, -\frac{\sqrt{3}i}{6}, -\frac{\sqrt{3}i}{6}, \frac{\sqrt{3}i}{3}, -\frac{\sqrt{3}i}{6}, -\frac{\sqrt{3}i}{6} \right]$$

$$\boxed{\text{y62}} \quad \mathbb{T}_{2,2}^{(s)}(E_{2g}) = \left[ 0, \frac{i}{2}, -\frac{i}{2}, 0, \frac{i}{2}, -\frac{i}{2} \right]$$

— Site and Bond —

Table 5: Orbital of each site

#	site	orbital
1	<b>c</b>	$ p_z\rangle$

Table 6: Neighbor and bra-ket of each bond

#	head	tail	neighbor	head (bra)	tail (ket)
1	C	C	[1,2,3,4,5,6]	[p]	[p]

#### Site in Unit Cell

Sites in (conventional) cell (no plus set), SL = sublattice

Table 7: 'C' (#1) site cluster (2c),  $-6m2$

SL	position ( <i>s</i> )	mapping
1	[ 0.33333, 0.66667, 0.00000]	[1,2,3,10,11,12,16,17,18,19,20,21]
2	[ 0.66667, 0.33333, 0.00000]	[4,5,6,7,8,9,13,14,15,22,23,24]

#### Bond in Unit Cell

Bonds in (conventional) cell (no plus set): tail, head = (SL, plus set), (N)D = (non)directional (listed up to 5th neighbor at most)

Table 8: 1-th 'C'-'C' [1] (#1) bond cluster (3a@3f), ND,  $|v| = 0.57735$  (cartesian)

SL	vector ( <i>v</i> )	center ( <i>c</i> )	mapping	head	tail	<i>R</i> (primitive)
1	[ 0.33333, 0.66667, 0.00000]	[ 0.50000, 0.00000, 0.00000]	[1,-4,-8,11,-13,16,20,-23]	(2,1)	(1,1)	[0,-1,0]
2	[-0.66667,-0.33333, 0.00000]	[ 0.00000, 0.50000, 0.00000]	[2,-5,-7,10,-14,17,19,-22]	(2,1)	(1,1)	[1,0,0]

*continued ...*

Table 8

SL	vector ( $\boldsymbol{v}$ )	center ( $\boldsymbol{c}$ )	mapping	head	tail	$\boldsymbol{R}$ (primitive)
3	[ 0.33333, -0.33333, 0.00000]	[ 0.50000, 0.50000, 0.00000]	[3, -6, -9, 12, -15, 18, 21, -24]	(2, 1)	(1, 1)	[0, 0, 0]

Table 9: 2-th 'C'-'C' [1] (#2) bond cluster (6b@6l), ND,  $|\boldsymbol{v}|=1.0$  (cartesian)

SL	vector ( $\boldsymbol{v}$ )	center ( $\boldsymbol{c}$ )	mapping	head	tail	$\boldsymbol{R}$ (primitive)
1	[ 1.00000, 0.00000, 0.00000]	[ 0.83333, 0.66667, 0.00000]	[1, -11, 16, -20]	(1, 1)	(1, 1)	[-1, 0, 0]
2	[ 0.00000, 1.00000, 0.00000]	[ 0.33333, 0.16667, 0.00000]	[2, -10, 17, -19]	(1, 1)	(1, 1)	[0, -1, 0]
3	[-1.00000, -1.00000, 0.00000]	[ 0.83333, 0.16667, 0.00000]	[3, -12, 18, -21]	(1, 1)	(1, 1)	[1, 1, 0]
4	[-1.00000, 0.00000, 0.00000]	[ 0.16667, 0.33333, 0.00000]	[4, -8, 13, -23]	(2, 1)	(2, 1)	[1, 0, 0]
5	[ 0.00000, -1.00000, 0.00000]	[ 0.66667, 0.83333, 0.00000]	[5, -7, 14, -22]	(2, 1)	(2, 1)	[0, 1, 0]
6	[ 1.00000, 1.00000, 0.00000]	[ 0.16667, 0.83333, 0.00000]	[6, -9, 15, -24]	(2, 1)	(2, 1)	[-1, -1, 0]

Table 10: 3-th 'C'-'C' [1] (#3) bond cluster (3b@1a), ND,  $|\boldsymbol{v}|=1.1547$  (cartesian)

SL	vector ( $\boldsymbol{v}$ )	center ( $\boldsymbol{c}$ )	mapping	head	tail	$\boldsymbol{R}$ (primitive)
1	[-0.66667, -1.33333, 0.00000]	[ 0.00000, 0.00000, 0.00000]	[1, -4, -8, 11, -13, 16, 20, -23]	(2, 1)	(1, 1)	[1, 1, 0]
2	[ 1.33333, 0.66667, 0.00000]	[ 0.00000, 0.00000, 0.00000]	[2, -5, -7, 10, -14, 17, 19, -22]	(2, 1)	(1, 1)	[-1, -1, 0]
3	[-0.66667, 0.66667, 0.00000]	[ 0.00000, 0.00000, 0.00000]	[3, -6, -9, 12, -15, 18, 21, -24]	(2, 1)	(1, 1)	[1, -1, 0]

Table 11: 4-th 'C'-'C' [1] (#4) bond cluster (6d@3f), ND,  $|\mathbf{v}|=1.52753$  (cartesian)

SL	vector ( $\mathbf{v}$ )	center ( $\mathbf{c}$ )	mapping	head	tail	$\mathbf{R}$ (primitive)
1	[-1.66667, -1.33333, 0.00000]	[ 0.50000, 0.00000, 0.00000]	[1, -4, -13, 16]	(2, 1)	(1, 1)	[2, 1, 0]
2	[ 1.33333, -0.33333, 0.00000]	[ 0.00000, 0.50000, 0.00000]	[2, -5, -14, 17]	(2, 1)	(1, 1)	[-1, 0, 0]
3	[ 0.33333, 1.66667, 0.00000]	[ 0.50000, 0.50000, 0.00000]	[3, -6, -15, 18]	(2, 1)	(1, 1)	[0, -2, 0]
4	[-1.33333, -1.66667, 0.00000]	[ 0.00000, 0.50000, 0.00000]	[7, -10, -19, 22]	(1, 1)	(2, 1)	[1, 2, 0]
5	[-0.33333, 1.33333, 0.00000]	[ 0.50000, 0.00000, 0.00000]	[8, -11, -20, 23]	(1, 1)	(2, 1)	[0, -1, 0]
6	[ 1.66667, 0.33333, 0.00000]	[ 0.50000, 0.50000, 0.00000]	[9, -12, -21, 24]	(1, 1)	(2, 1)	[-2, 0, 0]

Table 12: 5-th 'C'-'C' [1] (#5) bond cluster (6a@6l), D,  $|\mathbf{v}|=1.73205$  (cartesian)

SL	vector ( $\mathbf{v}$ )	center ( $\mathbf{c}$ )	mapping	head	tail	$\mathbf{R}$ (primitive)
1	[ 1.00000, 2.00000, 0.00000]	[ 0.83333, 0.66667, 0.00000]	[1, 11, 16, 20]	(1, 1)	(1, 1)	[-1, -2, 0]
2	[-2.00000, -1.00000, 0.00000]	[ 0.33333, 0.16667, 0.00000]	[2, 10, 17, 19]	(1, 1)	(1, 1)	[2, 1, 0]
3	[ 1.00000, -1.00000, 0.00000]	[ 0.83333, 0.16667, 0.00000]	[3, 12, 18, 21]	(1, 1)	(1, 1)	[-1, 1, 0]
4	[-1.00000, -2.00000, 0.00000]	[ 0.16667, 0.33333, 0.00000]	[4, 8, 13, 23]	(2, 1)	(2, 1)	[1, 2, 0]
5	[ 2.00000, 1.00000, 0.00000]	[ 0.66667, 0.83333, 0.00000]	[5, 7, 14, 22]	(2, 1)	(2, 1)	[-2, -1, 0]
6	[-1.00000, 1.00000, 0.00000]	[ 0.16667, 0.83333, 0.00000]	[6, 9, 15, 24]	(2, 1)	(2, 1)	[1, -1, 0]