

MPG No. 9.3.31 4' [Type III, tetragonal] [T tensor]

* Rank 0 tensor. * Rank 1 tensor. * Rank 2 tensor (s).

$$\begin{bmatrix} T_{xx} & T_{xy} & 0 \\ T_{xy} & -T_{xx} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T_{xx} = T_{dv}^{(1)}$$

$$T_{xy} = T_{dx}^{(1)}$$

* Rank 2 tensor (a). * Rank 3 tensor (s).

$$\begin{bmatrix} 0 & 0 & T_{xxz} \\ 0 & 0 & -T_{xxz} \\ 0 & 0 & 0 \\ T_{yzx} & T_{zyy} & 0 \\ -T_{yzy} & T_{yzx} & 0 \\ 0 & 0 & T_{xyz} \end{bmatrix}$$

$$T_{xxz} = -2M_{dxy}^{(1)} + T_{fbz}^{(1)}$$

$$T_{yzx} = -M_{dv}^{(1)} + T_{f3}^{(1)}$$

$$T_{yzy} = -M_{dxy}^{(1)} - T_{fbz}^{(1)}$$

$$T_{xyz} = 2M_{dv}^{(1)} + T_{f3}^{(1)}$$

* Rank 3 tensor (a).

$$\begin{bmatrix} T_{yzx} & T_{yzy} & 0 \\ T_{yzy} & -T_{yzx} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T_{yzx} = M_{dv}^{(2)}$$

$$T_{yzy} = M_{dxy}^{(2)}$$

* Rank 4 tensor (sss).

$$\begin{bmatrix} T_{xxxx} & 0 & T_{xxzz} & 0 & 0 & T_{xxxz} \\ 0 & -T_{xxxx} & -T_{xxzz} & 0 & 0 & T_{xxxy} \\ T_{xxzz} & -T_{xxzz} & 0 & 0 & 0 & T_{zzxy} \\ 0 & 0 & 0 & T_{yzyz} & T_{yzzx} & 0 \\ 0 & 0 & 0 & T_{yzzx} & -T_{yzyz} & 0 \\ T_{xxxz} & T_{xxxy} & T_{zzxy} & 0 & 0 & 0 \end{bmatrix}$$

$$T_{xxxx} = 2T_{dv}^{(1)} + 4T_{dv}^{(2)} + T_{gv}^{(1)}$$

$$T_{xxzz} = T_{dv}^{(1)} - T_{gv}^{(1)}$$

$$T_{xxxy} = T_{dxy}^{(1)} + 2T_{dxy}^{(2)} - T_{gbz}^{(1)}$$

$$T_{zzxy} = T_{dxy}^{(1)} + 2T_{gbz}^{(1)}$$

$$T_{yzyz} = -T_{dv}^{(2)} + T_{gv}^{(1)}$$

$$T_{yzzx} = T_{dxy}^{(2)} + 2T_{gbz}^{(1)}$$

* Rank 4 tensor (ssa).

$$\begin{bmatrix} 0 & T_{xxyy} & T_{xxzz} & 0 & 0 & T_{xxxz} \\ -T_{xxyy} & 0 & -T_{xxzz} & 0 & 0 & T_{xxxy} \\ -T_{xxzz} & T_{xxzz} & 0 & 0 & 0 & T_{zzxy} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -T_{xxxz} & -T_{xxxy} & -T_{zzxy} & 0 & 0 & 0 \end{bmatrix}$$

$$T_{xxyy} = 4M_{f3}^{(1)} - 2T_{dv}^{(3)}$$

$$T_{xxzz} = -4M_{f3}^{(1)} - T_{dv}^{(3)}$$

$$T_{xxxz} = 2M_{fbz}^{(1)} + T_{dxy}^{(3)}$$

$$T_{zzxy} = -4M_{fbz}^{(1)} + T_{dxy}^{(3)}$$

* Rank 4 tensor (aas).

$$\begin{bmatrix} T_{yzyz} & T_{yzzx} & 0 \\ T_{yzzx} & -T_{yzyz} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T_{yzyz} = 2T_{dv}^{(4)}$$

$$T_{yzzx} = 2T_{dxy}^{(4)}$$

* Rank 4 tensor (aaa). * Rank 4 tensor (sa).

$$\begin{bmatrix} 0 & 0 & T_{xxxz} \\ 0 & 0 & -T_{xxxz} \\ 0 & 0 & 0 \\ T_{yzyz} & T_{yzzx} & 0 \\ -T_{yzzx} & T_{yzyz} & 0 \\ 0 & 0 & T_{xyxy} \end{bmatrix}$$

$$T_{xxxz} = M_{fbz}^{(2)} - 2T_{dxy}^{(5)}$$

$$T_{yzyz} = M_{f3}^{(2)} - T_{dv}^{(5)}$$

$$T_{yzzx} = -M_{fbz}^{(2)} - T_{dxy}^{(5)}$$

$$T_{xyxy} = M_{f3}^{(2)} + 2T_{dv}^{(5)}$$

* Rank 4 tensor (as).

$$\begin{bmatrix} 0 & 0 & 0 & T_{yzyz} & T_{yzzx} & 0 \\ 0 & 0 & 0 & -T_{yzzx} & T_{yzyz} & 0 \\ T_{xyxx} & -T_{xyxx} & 0 & 0 & 0 & T_{xyxy} \end{bmatrix}$$

$$T_{yzyz} = M_{f3}^{(3)} - T_{dv}^{(6)}$$

$$T_{yzzx} = M_{fbz}^{(3)} + T_{dxy}^{(6)}$$

$$T_{xyxx} = M_{fbz}^{(3)} - 2T_{dxy}^{(6)}$$

$$T_{xyxy} = M_{f3}^{(3)} + 2T_{dv}^{(6)}$$

* Rank 4 tensor (s).

$$\begin{bmatrix} T_{xxxx} & T_{xxyy} & T_{xxzz} & 0 & 0 & T_{xxxz} & 0 & 0 & T_{xyxy} \\ -T_{xxyy} & -T_{xxxx} & -T_{xxzz} & 0 & 0 & T_{xxxz} & 0 & 0 & T_{xxxz} \\ T_{zzxx} & -T_{zzxx} & 0 & 0 & 0 & T_{zzxy} & 0 & 0 & T_{zzxy} \\ 0 & 0 & 0 & T_{yzyz} & T_{yzzx} & 0 & T_{yzyz} & T_{yzzx} & 0 \\ 0 & 0 & 0 & T_{yzzx} & -T_{yzyz} & 0 & T_{yzyz} & -T_{yzyz} & 0 \\ T_{xyxx} & T_{xyxx} & T_{xyzz} & 0 & 0 & T_{xyxy} & 0 & 0 & -T_{xyxy} \end{bmatrix}$$

$$\begin{aligned}
T_{xxxx} &= 2T_{dv}^{(1)} + 4T_{dv}^{(2)} + T_{gv}^{(1)} \\
T_{xxyy} &= 4M_{f3}^{(1)} - 2T_{dv}^{(3)} \\
T_{xxzz} &= -4M_{f3}^{(1)} + T_{dv}^{(1)} - T_{dv}^{(3)} - T_{gv}^{(1)} \\
T_{xxxy} &= 2M_{fbz}^{(1)} + M_{fbz}^{(2)} + T_{dxy}^{(1)} + 2T_{dxy}^{(2)} + T_{dxy}^{(3)} - 2T_{dxy}^{(5)} - T_{gbz}^{(1)} \\
T_{xxyx} &= 2M_{fbz}^{(1)} - M_{fbz}^{(2)} + T_{dxy}^{(1)} + 2T_{dxy}^{(2)} + T_{dxy}^{(3)} + 2T_{dxy}^{(5)} - T_{gbz}^{(1)} \\
T_{zzxx} &= 4M_{f3}^{(1)} + T_{dv}^{(1)} + T_{dv}^{(3)} - T_{gv}^{(1)} \\
T_{zzxy} &= -4M_{fbz}^{(1)} + T_{dxy}^{(1)} + T_{dxy}^{(3)} + 2T_{gbz}^{(1)} \\
T_{yzyz} &= M_{f3}^{(2)} - T_{dv}^{(2)} - T_{dv}^{(5)} + T_{gv}^{(1)} \\
T_{yzxz} &= -M_{fbz}^{(2)} + T_{dxy}^{(2)} - T_{dxy}^{(5)} + 2T_{gbz}^{(1)} \\
T_{yzyy} &= -M_{f3}^{(2)} - T_{dv}^{(2)} + T_{dv}^{(5)} + T_{gv}^{(1)} \\
T_{yzxz} &= M_{fbz}^{(2)} + T_{dxy}^{(2)} + T_{dxy}^{(5)} + 2T_{gbz}^{(1)} \\
T_{xyxx} &= -2M_{fbz}^{(1)} + T_{dxy}^{(1)} + 2T_{dxy}^{(2)} - T_{dxy}^{(3)} - T_{gbz}^{(1)} \\
T_{xyzz} &= 4M_{fbz}^{(1)} + T_{dxy}^{(1)} - T_{dxy}^{(3)} + 2T_{gbz}^{(1)} \\
T_{xyxy} &= M_{f3}^{(2)} + 2T_{dv}^{(5)}
\end{aligned}$$

* Rank 4 tensor (a).

$$\begin{bmatrix} 0 & 0 & 0 & T_{yzyz} & T_{yzxz} & 0 & T_{yzyy} & T_{yzxz} & 0 \\ 0 & 0 & 0 & -T_{yzxz} & T_{yzyy} & 0 & -T_{yzyx} & T_{yzyz} & 0 \\ T_{xyxx} & -T_{xyxx} & 0 & 0 & 0 & T_{xyxy} & 0 & 0 & T_{xyxy} \end{bmatrix}$$

$$\begin{aligned}
T_{yzyz} &= M_{f3}^{(3)} + 2T_{dv}^{(4)} - T_{dv}^{(6)} \\
T_{yzxz} &= M_{fbz}^{(3)} + 2T_{dxy}^{(4)} + T_{dxy}^{(6)} \\
T_{yzyy} &= M_{f3}^{(3)} - 2T_{dv}^{(4)} - T_{dv}^{(6)} \\
T_{yzxz} &= M_{fbz}^{(3)} - 2T_{dxy}^{(4)} + T_{dxy}^{(6)} \\
T_{xyxx} &= M_{fbz}^{(3)} - 2T_{dxy}^{(6)} \\
T_{xyxy} &= M_{f3}^{(3)} + 2T_{dv}^{(6)}
\end{aligned}$$

* Rank 4 tensor (t).

$$\begin{bmatrix} T_{xxxx} & T_{xxyy} & 0 \\ T_{xxyy} & -T_{xxxx} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & T_{yzyz} \\ T_{zzxx} & T_{zzxy} & 0 \\ T_{xxyx} & -T_{yyzz} - T_{zzxx} & 0 \\ T_{yzxz} & T_{yzyy} & 0 \\ -T_{yzyy} - T_{zzxx} & T_{xyyy} & T_{zzxz} \\ 0 & 0 & -\frac{T_{xxyy}}{2} - \frac{T_{xyxy}}{2} + T_{xyyy} + T_{zzxy} \end{bmatrix}$$

$$T_{xxxx} = 2T_{dv}^{(1)} + 4T_{dv}^{(2)} + T_{gv}^{(1)}$$

$$T_{xxxy} = T_{dxy}^{(1)} + 2T_{dxy}^{(2)} + T_{dxy}^{(3)} - 2T_{dxy}^{(5)} - T_{gbz}^{(1)}$$

$$T_{yyzz} = -T_{dv}^{(1)} + T_{dv}^{(3)} + T_{gv}^{(1)}$$

$$T_{zzxx} = T_{dv}^{(1)} + T_{dv}^{(3)} - T_{gv}^{(1)}$$

$$T_{zzxy} = T_{dxy}^{(1)} + T_{dxy}^{(3)} + 2T_{gbz}^{(1)}$$

$$T_{xxyx} = T_{dxy}^{(1)} + 2T_{dxy}^{(2)} + T_{dxy}^{(3)} + 2T_{dxy}^{(5)} - T_{gbz}^{(1)}$$

$$T_{yzzx} = T_{dxy}^{(2)} - T_{dxy}^{(5)} + 2T_{gbz}^{(1)}$$

$$T_{yzzy} = -T_{dv}^{(2)} + T_{dv}^{(5)} + T_{gv}^{(1)}$$

$$T_{zxxz} = T_{dv}^{(2)} + T_{dv}^{(5)} - T_{gv}^{(1)}$$

$$T_{xyyy} = T_{dxy}^{(1)} + 2T_{dxy}^{(2)} - T_{dxy}^{(3)} - T_{gbz}^{(1)}$$