

PG No. 35 $C_{3v}(1)$ $3m$ (31m setting) [trigonal] (axial, internal polar quadrupole)

* Harmonics for rank 0

* Harmonics for rank 1

$$\vec{\mathbb{G}}_1^{(2,1)}[q](A_2)$$

** symmetry

z

** expression

$$\frac{\sqrt{30}Q_vxy}{5} - \frac{\sqrt{30}Q_{xy}(x-y)(x+y)}{10} + \frac{\sqrt{30}Q_{xz}yz}{10} - \frac{\sqrt{30}Q_{yz}xz}{10}$$

$$\vec{\mathbb{G}}_{1,1}^{(2,1)}[q](E), \vec{\mathbb{G}}_{1,2}^{(2,1)}[q](E)$$

** symmetry

$-y$

x

** expression

$$-\frac{3\sqrt{10}Q_uxz}{10} + \frac{\sqrt{30}Q_vxz}{10} + \frac{\sqrt{30}Q_{xy}yz}{10} - \frac{\sqrt{30}Q_{xz}(x-z)(x+z)}{10} - \frac{\sqrt{30}Q_{yz}xy}{10}$$

$$-\frac{3\sqrt{10}Q_uyz}{10} - \frac{\sqrt{30}Q_vyz}{10} + \frac{\sqrt{30}Q_{xy}xz}{10} - \frac{\sqrt{30}Q_{xz}xy}{10} - \frac{\sqrt{30}Q_{yz}(y-z)(y+z)}{10}$$

* Harmonics for rank 2

$$\vec{\mathbb{G}}_2^{(2,-1)}[q](A_2)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{2}Q_{xzy}}{2} - \frac{\sqrt{2}Q_{yzx}}{2}$$

$$\vec{\mathbb{G}}_2^{(2,1)}[q](A_2)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{5\sqrt{42}Q_vxyz}{14} - \frac{5\sqrt{42}Q_{xy}z(x-y)(x+y)}{28} - \frac{\sqrt{42}Q_{xzy}(x^2+y^2-4z^2)}{28} + \frac{\sqrt{42}Q_{yzx}(x^2+y^2-4z^2)}{28}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,-1)}[q](E,1), \vec{\mathbb{G}}_{2,2}^{(2,-1)}[q](E,1)$$

** symmetry

$\sqrt{3}yz$

$-\sqrt{3}xz$

** expression

$$\frac{\sqrt{2}Q_ux}{2} + \frac{\sqrt{6}Q_vx}{6} + \frac{\sqrt{6}Q_{xy}y}{6} - \frac{\sqrt{6}Q_{xz}z}{6}$$

$$\frac{\sqrt{2}Q_uy}{2} - \frac{\sqrt{6}Q_vy}{6} + \frac{\sqrt{6}Q_{xy}x}{6} - \frac{\sqrt{6}Q_{yz}z}{6}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,-1)}[q](E,2), \vec{\mathbb{G}}_{2,2}^{(2,-1)}[q](E,2)$$

** symmetry

$\sqrt{3}xy$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$-\frac{\sqrt{6}Q_vz}{3} + \frac{\sqrt{6}Q_{xz}x}{6} - \frac{\sqrt{6}Q_{yz}y}{6}$$

$$\frac{\sqrt{6}Q_{xy}z}{3} - \frac{\sqrt{6}Q_{xz}y}{6} - \frac{\sqrt{6}Q_{yz}x}{6}$$

$\vec{\mathbb{G}}_{2,1}^{(2,1)}[q](E, 1), \vec{\mathbb{G}}_{2,2}^{(2,1)}[q](E, 1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$-\frac{\sqrt{42}Q_ux(x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{14}Q_vx(x^2 - 9y^2 + 6z^2)}{28} - \frac{\sqrt{14}Q_{xy}y(3x^2 - 2y^2 + 3z^2)}{14} + \frac{\sqrt{14}Q_{xz}z(3x^2 + 3y^2 - 2z^2)}{14}$$

$$-\frac{\sqrt{42}Q_uy(x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{14}Q_vy(9x^2 - y^2 - 6z^2)}{28} + \frac{\sqrt{14}Q_{xy}x(2x^2 - 3y^2 - 3z^2)}{14} + \frac{\sqrt{14}Q_{yz}z(3x^2 + 3y^2 - 2z^2)}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(2,1)}[q](E, 2), \vec{\mathbb{G}}_{2,2}^{(2,1)}[q](E, 2)$

** symmetry

$$\sqrt{3}xy$$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{5\sqrt{42}Q_uz(x-y)(x+y)}{28} - \frac{\sqrt{14}Q_vz(3x^2 + 3y^2 - 2z^2)}{28} + \frac{\sqrt{14}Q_{xz}x(2x^2 - 3y^2 - 3z^2)}{14} + \frac{\sqrt{14}Q_{yz}y(3x^2 - 2y^2 + 3z^2)}{14}$$

$$-\frac{5\sqrt{42}Q_uxyz}{14} + \frac{\sqrt{14}Q_{xy}z(3x^2 + 3y^2 - 2z^2)}{28} - \frac{\sqrt{14}Q_{xz}y(9x^2 - y^2 - 6z^2)}{28} + \frac{\sqrt{14}Q_{yz}x(x^2 - 9y^2 + 6z^2)}{28}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(2,-1)}[q](A_1)$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$-\frac{\sqrt{3}Q_vxz}{2} + \frac{\sqrt{3}Q_{xy}yz}{2} + \frac{\sqrt{3}Q_{xz}(x-y)(x+y)}{4} - \frac{\sqrt{3}Q_{yz}xy}{2}$$

$\vec{\mathbb{G}}_3^{(2,1)}[q](A_1)$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$\frac{7\sqrt{2}Q_uxz(x^2 - 3y^2)}{8} - \frac{\sqrt{6}Q_vxz(3x^2 + 3y^2 - 4z^2)}{24} + \frac{\sqrt{6}Q_{xy}yz(3x^2 + 3y^2 - 4z^2)}{24}$$

$$+ \frac{\sqrt{6}Q_{xz}(5x^4 - 21x^2y^2 - 9x^2z^2 + 2y^4 + 9y^2z^2)}{24} + \frac{\sqrt{6}Q_{yz}xy(11x^2 - 17y^2 + 18z^2)}{24}$$

$\vec{\mathbb{G}}_3^{(2,-1)}[q](A_2, 1)$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{\sqrt{30}Q_vxy}{10} + \frac{\sqrt{30}Q_{xy}(x-y)(x+y)}{20} + \frac{\sqrt{30}Q_{xz}yz}{5} - \frac{\sqrt{30}Q_{yz}xz}{5}$$

$\vec{\mathbb{G}}_3^{(2,-1)}[q](A_2, 2)$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{3}Q_vyz}{2} + \frac{\sqrt{3}Q_{xy}xz}{2} - \frac{\sqrt{3}Q_{xz}xy}{2} - \frac{\sqrt{3}Q_{yz}(x-y)(x+y)}{4}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](A_2, 1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_vxy(x^2 + y^2 - 6z^2)}{6} + \frac{\sqrt{15}Q_{xy}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{12} - \frac{\sqrt{15}Q_{xz}yz(3x^2 + 3y^2 - 4z^2)}{12} + \frac{\sqrt{15}Q_{yz}xz(3x^2 + 3y^2 - 4z^2)}{12}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](A_2, 2)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$-\frac{7\sqrt{2}Q_{uyz}(3x^2 - y^2)}{8} + \frac{\sqrt{6}Q_{vyz}(3x^2 + 3y^2 - 4z^2)}{24} + \frac{\sqrt{6}Q_{xyxz}(3x^2 + 3y^2 - 4z^2)}{24} \\ - \frac{\sqrt{6}Q_{xz}xy(17x^2 - 11y^2 - 18z^2)}{24} + \frac{\sqrt{6}Q_{yz}(2x^4 - 21x^2y^2 + 9x^2z^2 + 5y^4 - 9y^2z^2)}{24}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](E, 1), \vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](E, 1)$$

** symmetry

$$\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$-\frac{\sqrt{15}Q_uxz}{5} - \frac{3\sqrt{5}Q_vxz}{10} - \frac{3\sqrt{5}Q_{xy}yz}{10} + \frac{\sqrt{5}Q_{xz}(x^2 - 5y^2 + 4z^2)}{20} + \frac{3\sqrt{5}Q_{yz}xy}{10} \\ - \frac{\sqrt{15}Q_{uyz}}{5} + \frac{3\sqrt{5}Q_{vyz}}{10} - \frac{3\sqrt{5}Q_{xyxz}}{10} + \frac{3\sqrt{5}Q_{xz}xy}{10} - \frac{\sqrt{5}Q_{yz}(5x^2 - y^2 - 4z^2)}{20}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](E, 2), \vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](E, 2)$$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{6}Q_u(x-y)(x+y)}{4} + \frac{\sqrt{2}Q_v(x^2 + y^2 - 2z^2)}{4}$$

$$-\frac{\sqrt{6}Q_uxy}{2} - \frac{\sqrt{2}Q_{xy}(x^2 + y^2 - 2z^2)}{4}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](E, 1), \vec{\mathbb{G}}_{3,2}^{(2,1)}[q](E, 1)$$

** symmetry

$$\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$\frac{\sqrt{30}Q_uxz(3x^2 + 3y^2 - 4z^2)}{24} + \frac{\sqrt{10}Q_vxz(x^2 - 27y^2 + 8z^2)}{24} + \frac{\sqrt{10}Q_{xy}yz(15x^2 - 13y^2 + 8z^2)}{24} \\ + \frac{\sqrt{10}Q_{xz}(x^4 + 3x^2y^2 - 9x^2z^2 + 2y^4 - 15y^2z^2 + 4z^4)}{24} - \frac{\sqrt{10}Q_{yz}xy(x^2 + y^2 - 6z^2)}{24}$$

$$\frac{\sqrt{30}Q_uyz(3x^2 + 3y^2 - 4z^2)}{24} + \frac{\sqrt{10}Q_vyz(27x^2 - y^2 - 8z^2)}{24} - \frac{\sqrt{10}Q_{xy}xz(13x^2 - 15y^2 - 8z^2)}{24} \\ - \frac{\sqrt{10}Q_{xz}xy(x^2 + y^2 - 6z^2)}{24} + \frac{\sqrt{10}Q_{yz}(2x^4 + 3x^2y^2 - 15x^2z^2 + y^4 - 9y^2z^2 + 4z^4)}{24}$$

$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](E, 2), \vec{\mathbb{G}}_{3,2}^{(2,1)}[q](E, 2)$
** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_u(x-y)(x+y)(x^2+y^2-6z^2)}{6} - \frac{Q_v(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4)}{6}$$

$$-\frac{7Q_{xy}xy(x-y)(x+y)}{6} + \frac{7Q_{xz}xz(x-z)(x+z)}{6} - \frac{7Q_{yz}yz(y-z)(y+z)}{6}$$

$$\frac{\sqrt{3}Q_uxy(x^2+y^2-6z^2)}{3} + \frac{7Q_vxy(x-y)(x+y)}{6} - \frac{Q_{xy}(5x^4-18x^2y^2-12x^2z^2+5y^4-12y^2z^2+4z^4)}{12}$$

$$-\frac{7Q_{xz}yz(3x^2+y^2-2z^2)}{12} - \frac{7Q_{yz}xz(x^2+3y^2-2z^2)}{12}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(2,-1)}[q](A_1)$

** symmetry

$$\frac{\sqrt{70}yz(3x^2-y^2)}{4}$$

** expression

$$\frac{3\sqrt{2}Q_ux(x^2-3y^2)}{8} + \frac{\sqrt{6}Q_vx(x^2+y^2-4z^2)}{8} - \frac{\sqrt{6}Q_{xy}y(x^2+y^2-4z^2)}{8} + \frac{\sqrt{6}Q_{xz}z(x-y)(x+y)}{8} - \frac{\sqrt{6}Q_{yz}xyz}{4}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](A_1)$

** symmetry

$$\frac{\sqrt{70}yz(3x^2-y^2)}{4}$$

** expression

$$\frac{3\sqrt{77}Q_ux(x^2-3y^2)(x^2+y^2-8z^2)}{88} - \frac{\sqrt{231}Q_vx(x^4-16x^2y^2+6x^2z^2+7y^4+6y^2z^2-4z^4)}{88}$$

$$-\frac{\sqrt{231}Q_{xy}y(4x^4-7x^2y^2-3x^2z^2+y^4-3y^2z^2+2z^4)}{44} + \frac{\sqrt{231}Q_{xz}z(4x^4-9x^2y^2-5x^2z^2-y^4+5y^2z^2)}{44} + \frac{\sqrt{231}Q_{yz}xyz(x^2-11y^2+10z^2)}{44}$$

$\vec{\mathbb{G}}_4^{(2,-1)}[q](A_2, 1)$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$-\frac{\sqrt{105}Q_vxyz}{7} + \frac{\sqrt{105}Q_{xy}z(x-y)(x+y)}{14} - \frac{\sqrt{105}Q_{xz}y(x^2+y^2-4z^2)}{28} + \frac{\sqrt{105}Q_{yz}x(x^2+y^2-4z^2)}{28}$$

$\vec{\mathbb{G}}_4^{(2,-1)}[q](A_2, 2)$

** symmetry

$$\frac{\sqrt{70}xz(x^2-3y^2)}{4}$$

** expression

$$-\frac{3\sqrt{2}Q_uy(3x^2-y^2)}{8} - \frac{\sqrt{6}Q_vy(x^2+y^2-4z^2)}{8} - \frac{\sqrt{6}Q_{xy}x(x^2+y^2-4z^2)}{8} - \frac{\sqrt{6}Q_{xz}xyz}{4} - \frac{\sqrt{6}Q_{yz}z(x-y)(x+y)}{8}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](A_2, 1)$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$-\frac{7\sqrt{330}Q_vxyz(x^2+y^2-2z^2)}{44} + \frac{7\sqrt{330}Q_{xy}z(x-y)(x+y)(x^2+y^2-2z^2)}{88}$$

$$+\frac{\sqrt{330}Q_{xz}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{88} - \frac{\sqrt{330}Q_{yz}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{88}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](A_2, 2)$
** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{77}Q_{uy}(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{88} + \frac{\sqrt{231}Q_{vy}(7x^4 - 16x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 4z^4)}{88} \\ & - \frac{\sqrt{231}Q_{xy}x(x^4 - 7x^2y^2 - 3x^2z^2 + 4y^4 - 3y^2z^2 + 2z^4)}{44} - \frac{\sqrt{231}Q_{xz}xyz(11x^2 - y^2 - 10z^2)}{44} - \frac{\sqrt{231}Q_{yz}z(x^4 + 9x^2y^2 - 5x^2z^2 - 4y^4 + 5y^2z^2)}{44} \end{aligned}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E, 1), \vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E, 1)$

** symmetry

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned} & -\frac{3\sqrt{14}Q_{ux}(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{42}Q_vx(x^2 + 5y^2 - 8z^2)}{56} + \frac{\sqrt{42}Q_{xy}y(x^2 - 3y^2 + 8z^2)}{56} - \frac{\sqrt{42}Q_{xz}z(x^2 - 13y^2 + 4z^2)}{56} - \frac{\sqrt{42}Q_{yz}xyz}{4} \\ & - \frac{3\sqrt{14}Q_{uy}(x^2 + y^2 - 4z^2)}{56} + \frac{\sqrt{42}Q_vy(5x^2 + y^2 - 8z^2)}{56} - \frac{\sqrt{42}Q_{xy}x(3x^2 - y^2 - 8z^2)}{56} - \frac{\sqrt{42}Q_{xz}xyz}{4} + \frac{\sqrt{42}Q_{yz}z(13x^2 - y^2 - 4z^2)}{56} \end{aligned}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E, 2), \vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E, 2)$

** symmetry

$$-\frac{\sqrt{35}xy(x - y)(x + y)}{2}$$

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

** expression

$$\frac{\sqrt{3}Q_{vz}(x - y)(x + y)}{2} - \sqrt{3}Q_{xy}xyz - \frac{\sqrt{3}Q_{xz}x(x^2 - 3y^2)}{4} + \frac{\sqrt{3}Q_{yz}y(3x^2 - y^2)}{4}$$

$$\sqrt{3}Q_vxyz + \frac{\sqrt{3}Q_{xy}z(x - y)(x + y)}{2} - \frac{\sqrt{3}Q_{xz}y(3x^2 - y^2)}{4} - \frac{\sqrt{3}Q_{yz}x(x^2 - 3y^2)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E, 3), \vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E, 3)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

$$-\frac{\sqrt{5}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\frac{3\sqrt{7}Q_{uz}(x - y)(x + y)}{7} + \frac{\sqrt{21}Q_{vz}(3x^2 + 3y^2 - 2z^2)}{14} - \frac{\sqrt{21}Q_{xz}x(x^2 - 5y^2 + 2z^2)}{28} - \frac{\sqrt{21}Q_{yz}y(5x^2 - y^2 - 2z^2)}{28}$$

$$-\frac{6\sqrt{7}Q_{ux}xyz}{7} - \frac{\sqrt{21}Q_{yz}z(3x^2 + 3y^2 - 2z^2)}{14} + \frac{\sqrt{21}Q_{xz}y(2x^2 - y^2 + z^2)}{14} - \frac{\sqrt{21}Q_{yz}x(x^2 - 2y^2 - z^2)}{14}$$

$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E, 1), \vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E, 1)$

** symmetry

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{11}Q_{ux}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} + \frac{\sqrt{33}Q_vx(x^4 - 12x^2y^2 + 2x^2z^2 - 13y^4 + 114y^2z^2 - 20z^4)}{88} \\
& + \frac{\sqrt{33}Q_{xy}y(4x^4 + x^2y^2 - 27x^2z^2 - 3y^4 + 29y^2z^2 - 10z^4)}{44} \\
& - \frac{\sqrt{33}Q_{xz}z(4x^4 + 15x^2y^2 - 13x^2z^2 + 11y^4 - 27y^2z^2 + 4z^4)}{44} + \frac{7\sqrt{33}Q_{yz}xyz(x^2 + y^2 - 2z^2)}{44} \\
\\
& \frac{3\sqrt{11}Q_{uy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} + \frac{\sqrt{33}Q_vy(13x^4 + 12x^2y^2 - 114x^2z^2 - y^4 - 2y^2z^2 + 20z^4)}{88} \\
& - \frac{\sqrt{33}Q_{xy}x(3x^4 - x^2y^2 - 29x^2z^2 - 4y^4 + 27y^2z^2 + 10z^4)}{44} + \frac{7\sqrt{33}Q_{xz}xyz(x^2 + y^2 - 2z^2)}{44} \\
& - \frac{\sqrt{33}Q_{yz}z(11x^4 + 15x^2y^2 - 27x^2z^2 + 4y^4 - 13y^2z^2 + 4z^4)}{44}
\end{aligned}$$

$\tilde{\mathbb{G}}_{4,1}^{(2,1)}[q](E, 2), \tilde{\mathbb{G}}_{4,2}^{(2,1)}[q](E, 2)$

** symmetry

$$-\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

** expression

$$\begin{aligned}
& -\frac{9\sqrt{154}Q_{uz}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{88} + \frac{\sqrt{462}Q_vz(x-y)(x+y)(x^2 + y^2 - 2z^2)}{88} - \frac{\sqrt{462}Q_{xy}xyz(x^2 + y^2 - 2z^2)}{44} \\
& - \frac{\sqrt{462}Q_{xz}x(x^4 - 8x^2y^2 - 2x^2z^2 + 3y^4 + 6y^2z^2)}{44} - \frac{\sqrt{462}Q_{yz}y(3x^4 - 8x^2y^2 + 6x^2z^2 + y^4 - 2y^2z^2)}{44} \\
\\
& -\frac{9\sqrt{154}Q_{u}xyz(x-y)(x+y)}{22} + \frac{\sqrt{462}Q_vxyz(x^2 + y^2 - 2z^2)}{44} + \frac{\sqrt{462}Q_{xy}z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{88} \\
& - \frac{\sqrt{462}Q_{xz}y(9x^4 - 14x^2y^2 - 12x^2z^2 + y^4 + 4y^2z^2)}{88} + \frac{\sqrt{462}Q_{yz}x(x^4 - 14x^2y^2 + 4x^2z^2 + 9y^4 - 12y^2z^2)}{88}
\end{aligned}$$

$\tilde{\mathbb{G}}_{4,1}^{(2,1)}[q](E, 3), \tilde{\mathbb{G}}_{4,2}^{(2,1)}[q](E, 3)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\begin{aligned}
& -\frac{21\sqrt{22}Q_{uz}(x-y)(x+y)(x^2 + y^2 - 2z^2)}{88} - \frac{\sqrt{66}Q_vz(3x^4 - 78x^2y^2 + 20x^2z^2 + 3y^4 + 20y^2z^2 - 4z^4)}{88} - \frac{21\sqrt{66}Q_{xy}xyz(x-y)(x+y)}{44} \\
& - \frac{\sqrt{66}Q_{xz}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44} + \frac{\sqrt{66}Q_{yz}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44} \\
\\
& \frac{21\sqrt{22}Q_{u}xyz(x^2 + y^2 - 2z^2)}{44} + \frac{21\sqrt{66}Q_vxyz(x-y)(x+y)}{44} - \frac{\sqrt{66}Q_{xy}z(9x^4 - 24x^2y^2 - 10x^2z^2 + 9y^4 - 10y^2z^2 + 2z^4)}{44} \\
& + \frac{\sqrt{66}Q_{xz}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44} + \frac{\sqrt{66}Q_{yz}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44}
\end{aligned}$$