Response Tensors up to 4th rank in C_{2h}

— polar tensors —

$$C^{(0,Q)} = \begin{pmatrix} C^{(0,Q)} \end{pmatrix}$$

$$C^{(0,Q)} = Q_0$$

$$S^{(2,Q)} = \begin{pmatrix} S^{(2,Q)}_{xx} & 0 & S^{(2,Q)}_{xz} \\ 0 & S^{(2,Q)}_{yy} & 0 \\ S^{(2,Q)}_{xz} & 0 & S^{(2,Q)}_{zz} \end{pmatrix}$$

$$S^{(2,Q)}_{xx} = Q_0 - Q_u + Q_v$$
$$S^{(2,Q)}_{xz} = Q_{zx}$$
$$S^{(2,Q)}_{yy} = Q_0 - Q_u - Q_v$$
$$S^{(2,Q)}_{zz} = Q_0 + 2Q_u$$

$$A^{(2,Q)} = \begin{pmatrix} 0 & 0 & A^{(2,Q)}_{xz} \\ 0 & 0 & 0 \\ -A^{(2,Q)}_{xz} & 0 & 0 \end{pmatrix}$$

$$A^{(2,Q)}_{xz} = -G_y$$

$$S^{(4,Q)} = \begin{pmatrix} S^{(4,Q)}_{11} & S^{(4,Q)}_{12} & S^{(4,Q)}_{13} & 0 & S^{(4,Q)}_{15} & 0 \\ S^{(4,Q)}_{12} & S^{(4,Q)}_{22} & S^{(4,Q)}_{23} & 0 & S^{(4,Q)}_{25} & 0 \\ S^{(4,Q)}_{13} & S^{(4,Q)}_{23} & S^{(4,Q)}_{33} & 0 & S^{(4,Q)}_{35} & 0 \\ 0 & 0 & 0 & S^{(4,Q)}_{44} & 0 & S^{(4,Q)}_{46} \\ S^{(4,Q)}_{15} & S^{(4,Q)}_{25} & S^{(4,Q)}_{35} & 0 & S^{(4,Q)}_{55} & 0 \\ 0 & 0 & 0 & S^{(4,Q)}_{46} & 0 & S^{(4,Q)}_{66} \end{pmatrix}$$

$$S^{(4,Q)}_{11} = Q_0[1] + 2Q_0[2] - Q_{4u} + Q_{4v} + 2Q_4 - 2Q_u[1] - 4Q_u[2] + 2Q_v[1] + 4Q_v[2]$$
$$S^{(4,Q)}_{12} = Q_0[1] + 2Q_{4u} - Q_4 - 2Q_u[1]$$
$$S^{(4,Q)}_{13} = Q_0[1] - Q_{4u} - Q_{4v} - Q_4 + Q_u[1] + Q_v[1]$$
$$S^{(4,Q)}_{15} = -Q^{\alpha}_{4y} - Q^{\beta}_{4y} + Q_{zx}[1] + 2Q_{zx}[2]$$
$$S^{(4,Q)}_{22} = Q_0[1] + 2Q_0[2] - Q_{4u} - Q_{4v} + 2Q_4 - 2Q_u[1] - 4Q_u[2] - 2Q_v[1] - 4Q_v[2]$$
$$S^{(4,Q)}_{23} = Q_0[1] - Q_{4u} + Q_{4v} - Q_4 + Q_u[1] - Q_v[1]$$
$$S^{(4,Q)}_{25} = 2Q^{\beta}_{4y} + Q_{zx}[1]$$
$$S^{(4,Q)}_{33} = Q_0[1] + 2Q_0[2] + 2Q_{4u} + 2Q_4 + 4Q_u[1] + 8Q_u[2]$$
$$S^{(4,Q)}_{35} = Q^{\alpha}_{4y} - Q^{\beta}_{4y} + Q_{zx}[1] + 2Q_{zx}[2]$$
$$S^{(4,Q)}_{44} = Q_0[2] - Q_{4u} + Q_{4v} - Q_4 + Q_u[2] - Q_v[2]$$
$$S^{(4,Q)}_{46} = 2Q^{\beta}_{4y} + Q_{zx}[2]$$
$$S^{(4,Q)}_{55} = Q_0[2] - Q_{4u} - Q_{4v} - Q_4 + Q_u[2] + Q_v[2]$$
$$S^{(4,Q)}_{66} = Q_0[2] + 2Q_{4u} - Q_4 - 2Q_u[2]$$

$$\bar{S}^{(4,Q)} = \begin{pmatrix} 0 & \bar{S}_{12}^{(4,Q)} & \bar{S}_{13}^{(4,Q)} & 0 & \bar{S}_{15}^{(4,Q)} & 0 \\ -\bar{S}_{12}^{(4,Q)} & 0 & \bar{S}_{23}^{(4,Q)} & 0 & \bar{S}_{25}^{(4,Q)} & 0 \\ -\bar{S}_{13}^{(4,Q)} & -\bar{S}_{23}^{(4,Q)} & 0 & 0 & \bar{S}_{35}^{(4,Q)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{S}_{46}^{(4,Q)} \\ -\bar{S}_{15}^{(4,Q)} & -\bar{S}_{25}^{(4,Q)} & -\bar{S}_{35}^{(4,Q)} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\bar{S}_{46}^{(4,Q)} & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}
\bar{S}_{12}^{(4,Q)} &= 4G_{xyz}[1] - 2Q_v[3] \\
\bar{S}_{13}^{(4,Q)} &= -4G_{xyz}[1] + 3Q_u[3] - Q_v[3] \\
\bar{S}_{15}^{(4,Q)} &= -2G_y[1] + 2G_y^{\alpha}[1] + 2G_y^{\beta}[1] + Q_{zx}[3] \\
\bar{S}_{23}^{(4,Q)} &= 4G_{xyz}[1] + 3Q_u[3] + Q_v[3] \\
\bar{S}_{25}^{(4,Q)} &= -4G_y^{\beta}[1] + Q_{zx}[3] \\
\bar{S}_{35}^{(4,Q)} &= 2G_y[1] - 2G_y^{\alpha}[1] + 2G_y^{\beta}[1] + Q_{zx}[3] \\
\bar{S}_{46}^{(4,Q)} &= G_y[1] + 4G_y^{\alpha}[1]
\end{aligned}$$

$$A^{(4,Q)} = \begin{pmatrix} A_{xx}^{(4,Q)} & 0 & A_{xz}^{(4,Q)} \\ 0 & A_{yy}^{(4,Q)} & 0 \\ A_{xz}^{(4,Q)} & 0 & A_{zz}^{(4,Q)} \end{pmatrix}$$

$$\begin{aligned}
A_{xx}^{(4,Q)} &= Q_0[3] - 2Q_u[6] + 2Q_v[6] \\
A_{xz}^{(4,Q)} &= 2Q_{zx}[6] \\
A_{yy}^{(4,Q)} &= Q_0[3] - 2Q_u[6] - 2Q_v[6] \\
A_{zz}^{(4,Q)} &= Q_0[3] + 4Q_u[6]
\end{aligned}$$

$$\bar{A}^{(4,Q)} = \begin{pmatrix} 0 & 0 & \bar{A}_{xz}^{(4,Q)} \\ 0 & 0 & 0 \\ -\bar{A}_{xz}^{(4,Q)} & 0 & 0 \end{pmatrix}$$

$$\bar{A}_{xz}^{(4,Q)} = -G_y[6]$$

$$M^{(4,Q)} = \begin{pmatrix} 0 & M_{1y}^{(4,Q)} & 0 \\ 0 & M_{2y}^{(4,Q)} & 0 \\ 0 & M_{3y}^{(4,Q)} & 0 \\ M_{4x}^{(4,Q)} & 0 & M_{4z}^{(4,Q)} \\ 0 & M_{5y}^{(4,Q)} & 0 \\ M_{6x}^{(4,Q)} & 0 & M_{6z}^{(4,Q)} \end{pmatrix}$$

$$\begin{aligned}
M_{1y}^{(4,Q)} &= G_y[3] - G_y^{\alpha}[2] - G_y^{\beta}[2] + 2Q_{zx}[4] \\
M_{2y}^{(4,Q)} &= 2G_y[2] + G_y[3] + 2G_y^{\alpha}[2] \\
M_{3y}^{(4,Q)} &= G_y[3] - G_y^{\alpha}[2] + G_y^{\beta}[2] - 2Q_{zx}[4] \\
M_{4x}^{(4,Q)} &= G_{xyz}[2] - 3Q_u[4] - Q_v[4] \\
M_{4z}^{(4,Q)} &= G_y[2] - G_y^{\alpha}[2] + G_y^{\beta}[2] + Q_{zx}[4] \\
M_{5y}^{(4,Q)} &= G_{xyz}[2] + 3Q_u[4] - Q_v[4] \\
M_{6x}^{(4,Q)} &= G_y[2] - G_y^{\alpha}[2] - G_y^{\beta}[2] - Q_{zx}[4] \\
M_{6z}^{(4,Q)} &= G_{xyz}[2] + 2Q_v[4]
\end{aligned}$$

$$\bar{M}^{(4,Q)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}_{x4}^{(4,Q)} & 0 & \bar{M}_{x6}^{(4,Q)} \\ \bar{M}_{y1}^{(4,Q)} & \bar{M}_{y2}^{(4,Q)} & \bar{M}_{y3}^{(4,Q)} & 0 & \bar{M}_{y5}^{(4,Q)} & 0 \\ 0 & 0 & 0 & \bar{M}_{z4}^{(4,Q)} & 0 & \bar{M}_{z6}^{(4,Q)} \end{pmatrix}$$

$$\begin{aligned}
\bar{M}_{x4}^{(4,Q)} &= G_{xyz}[3] - 3Q_u[5] - Q_v[5] \\
\bar{M}_{x6}^{(4,Q)} &= G_y[4] - G_y^{\alpha}[3] - G_y^{\beta}[3] - Q_{zx}[5] \\
\bar{M}_{y1}^{(4,Q)} &= G_y[5] - G_y^{\alpha}[3] - G_y^{\beta}[3] + 2Q_{zx}[5] \\
\bar{M}_{y2}^{(4,Q)} &= 2G_y[4] + G_y[5] + 2G_y^{\alpha}[3] \\
\bar{M}_{y3}^{(4,Q)} &= G_y[5] - G_y^{\alpha}[3] + G_y^{\beta}[3] - 2Q_{zx}[5] \\
\bar{M}_{y5}^{(4,Q)} &= G_{xyz}[3] + 3Q_u[5] - Q_v[5] \\
\bar{M}_{z4}^{(4,Q)} &= G_y[4] - G_y^{\alpha}[3] + G_y^{\beta}[3] + Q_{zx}[5] \\
\bar{M}_{z6}^{(4,Q)} &= G_{xyz}[3] + 2Q_v[5]
\end{aligned}$$

— axial tensors —

$$C^{(1,G)} = \begin{pmatrix} 0 & C^{(1,G)}_{y} & 0 \end{pmatrix}$$

$$C^{(1,G)}_{y} = G_{y}$$

$$S^{(3,G)} = \begin{pmatrix} 0 & S^{(3,G)}_{1y} & 0 \\ 0 & S^{(3,G)}_{2y} & 0 \\ 0 & S^{(3,G)}_{3y} & 0 \\ S^{(3,G)}_{4x} & 0 & S^{(3,G)}_{4z} \\ 0 & S^{(3,G)}_{5y} & 0 \\ S^{(3,G)}_{6x} & 0 & S^{(3,G)}_{6z} \end{pmatrix}$$

$$\begin{aligned}
S^{(3,G)}_{1y} &= G_{y}[2] - G^{\alpha}_{y} - G^{\beta}_{y} + 2Q_{zx}[1] \\
S^{(3,G)}_{2y} &= 2G_{y}[1] + G_{y}[2] + 2G^{\alpha}_{y} \\
S^{(3,G)}_{3y} &= G_{y}[2] - G^{\alpha}_{y} + G^{\beta}_{y} - 2Q_{zx}[1] \\
S^{(3,G)}_{4x} &= G_{xyz} - 3Q_{u}[1] - Q_{v}[1] \\
S^{(3,G)}_{4z} &= G_{y}[1] - G^{\alpha}_{y} + G^{\beta}_{y} + Q_{zx}[1] \\
S^{(3,G)}_{5y} &= G_{xyz} + 3Q_{u}[1] - Q_{v}[1] \\
S^{(3,G)}_{6x} &= G_{y}[1] - G^{\alpha}_{y} - G^{\beta}_{y} - Q_{zx}[1] \\
S^{(3,G)}_{6z} &= G_{xyz} + 2Q_{v}[1]
\end{aligned}$$

$$A^{(3,G)} = \begin{pmatrix} A^{(3,G)}_{4x} & 0 & A^{(3,G)}_{4z} \\ 0 & A^{(3,G)}_{5y} & 0 \\ A^{(3,G)}_{6x} & 0 & A^{(3,G)}_{6z} \end{pmatrix}$$

$$\begin{aligned}
A^{(3,G)}_{4x} &= Q_{0} - Q_{u}[2] + Q_{v}[2] \\
A^{(3,G)}_{4z} &= -G_{y}[3] + Q_{zx}[2] \\
A^{(3,G)}_{5y} &= Q_{0} - Q_{u}[2] - Q_{v}[2] \\
A^{(3,G)}_{6x} &= G_{y}[3] + Q_{zx}[2] \\
A^{(3,G)}_{6z} &= Q_{0} + 2Q_{u}[2]
\end{aligned}$$