# SAMB for "C3v1"

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- Group: No. 156 $C_{3v}^{1}$ $P3m1$ [ trigonal ]
- Associated point group: No. 19 $C_{3v}$ $3m1$ (3m1 setting) [ trigonal ]

- Generation condition
  - model type: `tight_binding`
  - time-reversal type: `electric`
  - irrep: `[A1]`
  - `spinful`

- Unit cell:
  $a = 1.0,\ b = 1.0,\ c = 1.0,\ \alpha = 90.0,\ \beta = 90.0,\ \gamma = 120.0$
- Lattice vectors:
  $\boldsymbol{a}_1 = \begin{pmatrix} 1.0 & 0 & 0 \end{pmatrix}$
  $\boldsymbol{a}_2 = \begin{pmatrix} -0.5 & 0.86602540378444 & 0 \end{pmatrix}$
  $\boldsymbol{a}_3 = \begin{pmatrix} 0 & 0 & 1.0 \end{pmatrix}$

Table 1: High-symmetry line: Γ-X.

| | symbol | position | symbol | position |
|---|---|---|---|---|
| | Γ | $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$ | X | $\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$ |

- Kets: dimension = 8

Table 2: Hilbert space for full matrix.

| | No. | ket | No. | ket | No. | ket | No. | ket | No. | ket |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | $(p_x,\uparrow)@\mathrm{A}_1$ | 2 | $(p_x,\downarrow)@\mathrm{A}_1$ | 3 | $(p_y,\uparrow)@\mathrm{A}_1$ | 4 | $(p_y,\downarrow)@\mathrm{A}_1$ | 5 | $(p_x,\uparrow)@\mathrm{B}_1$ |
| | 6 | $(p_x,\downarrow)@\mathrm{B}_1$ | 7 | $(p_y,\uparrow)@\mathrm{B}_1$ | 8 | $(p_y,\downarrow)@\mathrm{B}_1$ | | | | |

- Sites in (primitive) unit cell:

Table 3: Site-clusters.

| | site | position | mapping |
|---|---|---|---|
| $\mathrm{S}_1$ [1b: 3m.] | $\mathrm{A}_1$ | $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix}$ | [1,2,3,4,5,6] |
| $\mathrm{S}_2$ [1c: 3m.] | $\mathrm{B}_1$ | $\begin{pmatrix} \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}$ | [1,2,3,4,5,6] |

- Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

| | bond | tail | head | $n$ | # | $\boldsymbol{b}@\boldsymbol{c}$ | mapping |
|---|---|---|---|---|---|---|---|
| $\mathrm{B}_1$ [3d: .m.] | $\mathrm{b}_1$ | $\mathrm{B}_1$ | $\mathrm{A}_1$ | 1 | 1 | $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$ | [1,4] |
| | $\mathrm{b}_2$ | $\mathrm{B}_1$ | $\mathrm{A}_1$ | 1 | 1 | $\begin{pmatrix} -\frac{2}{3} & -\frac{1}{3} & 0 \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{2} & 0 \end{pmatrix}$ | [2,6] |
| | $\mathrm{b}_3$ | $\mathrm{B}_1$ | $\mathrm{A}_1$ | 1 | 1 | $\begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$ | [3,5] |

- SAMB:

$\boxed{\text{No. 1}}$ $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1, \mathrm{S}_1]$

$\hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$

$\boxed{\text{No. 2}}$ $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_1, \mathrm{S}_1]$

$\hat{\mathbb{Z}}_2 = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$

$\boxed{\text{No. 3}}$ $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1, \mathrm{S}_2]$

$\hat{\mathbb{Z}}_3 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(s,A_1)}]$

$\boxed{\text{No. 4}}$ $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_1, \mathrm{S}_2]$

$\hat{\mathbb{Z}}_4 = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(s,A_1)}]$

$\boxed{\text{No. 5}}$ $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1, \mathrm{B}_1]$

$\hat{\mathbb{Z}}_5 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_1)}]$

$\boxed{\text{No. 6}}$ $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$\hat{\mathbb{Z}}_6 = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_1)}]$

$\boxed{\text{No. 7}}$ $\hat{\mathbb{Q}}_3^{(A_1,2)}$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_7 = -\frac{\sqrt{2}\mathbb{X}_3[\mathbb{Q}_{2,0}^{(a,E,2)}] \otimes \mathbb{Y}_4[\mathbb{Q}_{1,0}^{(b,E)}]}{2} - \frac{\sqrt{2}\mathbb{X}_4[\mathbb{Q}_{2,1}^{(a,E,2)}] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,1}^{(b,E)}]}{2}$$

$\boxed{\text{No. 8}}$ $\hat{\mathbb{Q}}_1^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_8 = \frac{\sqrt{2}\mathbb{X}_5[\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_4[\mathbb{Q}_{1,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_6[\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,1}^{(b,E)}]}{2}$$

$\boxed{\text{No. 9}}$ $\hat{\mathbb{Q}}_1^{(A_1)}(1,1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_9 = \frac{\sqrt{2}\mathbb{X}_7[\mathbb{M}_{1,0}^{(a,E)}(1,1)] \otimes \mathbb{Y}_7[\mathbb{T}_{1,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_8[\mathbb{M}_{1,1}^{(a,E)}(1,1)] \otimes \mathbb{Y}_8[\mathbb{T}_{1,1}^{(b,E)}]}{2}$$

$\boxed{\text{No. 10}}$ $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$\hat{\mathbb{Z}}_{10} = \mathbb{X}_{13}[\mathbb{M}_3^{(a,A_1)}(1,-1)] \otimes \mathbb{Y}_6[\mathbb{T}_0^{(b,A_1)}]$

$\boxed{\text{No. 11}}$ $\hat{\mathbb{Q}}_3^{(A_1,2)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{11} = \frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_7[\mathbb{T}_{1,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_8[\mathbb{T}_{1,1}^{(b,E)}]}{2}$$

$\boxed{\text{No. 12}}$ $\hat{\mathbb{Q}}_1^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{12} = \frac{\sqrt{2}\mathbb{X}_{10}[\mathbb{M}_{1,1}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_8[\mathbb{T}_{1,1}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{9}[\mathbb{M}_{1,0}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_7[\mathbb{T}_{1,0}^{(b,E)}]}{2}$$

Table 5: Atomic SAMB group.

| group | bra | ket |
|---|---|---|
| $\mathrm{M}_1$ | $(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow)$ | $(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow)$ |

Table 6: Atomic SAMB.

| symbol | type | group | form |
|---|---|---|---|
| $\mathbb{X}_1$ | $\mathbb{Q}_0^{(a,A_1)}$ | $\mathrm{M}_1$ | $\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ |
| $\mathbb{X}_2$ | $\mathbb{Q}_0^{(a,A_1)}(1,1)$ | $\mathrm{M}_1$ | $\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & \frac{i}{2} \\ \frac{i}{2} & 0 & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & 0 \end{pmatrix}$ |
| $\mathbb{X}_3$ | $\mathbb{Q}_{2,0}^{(a,E,2)}$ | $\mathrm{M}_1$ | $\begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \end{pmatrix}$ |

*continued ...*

Table 6

| symbol | type | group | form |
|---|---|---|---|
| $\mathbb{X}_4$ | $\mathbb{Q}_{2,1}^{(a,E,2)}$ | $\mathrm{M}_1$ | $\begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ |
| $\mathbb{X}_5$ | $\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)$ | $\mathrm{M}_1$ | $\begin{pmatrix} 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & -\frac{i}{2} & 0 \\ 0 & \frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$ |
| $\mathbb{X}_6$ | $\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)$ | $\mathrm{M}_1$ | $\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$ |
| $\mathbb{X}_7$ | $\mathbb{M}_{1,0}^{(a,E)}(1,1)$ | $\mathrm{M}_1$ | $\begin{pmatrix} 0 & -\frac{\sqrt{19}i}{19} & 0 & -\frac{3\sqrt{19}}{38} \\ \frac{\sqrt{19}i}{19} & 0 & -\frac{3\sqrt{19}}{38} & 0 \\ 0 & -\frac{3\sqrt{19}}{38} & 0 & \frac{2\sqrt{19}i}{19} \\ -\frac{3\sqrt{19}}{38} & 0 & -\frac{2\sqrt{19}i}{19} & 0 \end{pmatrix}$ |
| $\mathbb{X}_8$ | $\mathbb{M}_{1,1}^{(a,E)}(1,1)$ | $\mathrm{M}_1$ | $\begin{pmatrix} 0 & \frac{2\sqrt{19}}{19} & 0 & -\frac{3\sqrt{19}i}{38} \\ \frac{2\sqrt{19}}{19} & 0 & \frac{3\sqrt{19}i}{38} & 0 \\ 0 & -\frac{3\sqrt{19}i}{38} & 0 & -\frac{\sqrt{19}}{19} \\ \frac{3\sqrt{19}i}{38} & 0 & -\frac{\sqrt{19}}{19} & 0 \end{pmatrix}$ |
| $\mathbb{X}_9$ | $\mathbb{M}_{1,0}^{(a,E)}(1,-1)$ | $\mathrm{M}_1$ | $\begin{pmatrix} 0 & \frac{7\sqrt{38}i}{76} & 0 & \frac{\sqrt{38}}{76} \\ -\frac{7\sqrt{38}i}{76} & 0 & \frac{\sqrt{38}}{76} & 0 \\ 0 & \frac{\sqrt{38}}{76} & 0 & \frac{5\sqrt{38}i}{76} \\ \frac{\sqrt{38}}{76} & 0 & -\frac{5\sqrt{38}i}{76} & 0 \end{pmatrix}$ |
| $\mathbb{X}_{10}$ | $\mathbb{M}_{1,1}^{(a,E)}(1,-1)$ | $\mathrm{M}_1$ | $\begin{pmatrix} 0 & \frac{5\sqrt{38}}{76} & 0 & \frac{\sqrt{38}i}{76} \\ \frac{5\sqrt{38}}{76} & 0 & -\frac{\sqrt{38}i}{76} & 0 \\ 0 & \frac{\sqrt{38}i}{76} & 0 & \frac{7\sqrt{38}}{76} \\ -\frac{\sqrt{38}i}{76} & 0 & \frac{7\sqrt{38}}{76} & 0 \end{pmatrix}$ |
| $\mathbb{X}_{11}$ | $\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)$ | $\mathrm{M}_1$ | $\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ |

*continued ...*

Table 6

| symbol | type | group | form |
|---|---|---|---|
| $\mathbb{X}_{12}$ | $\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)$ | $\mathrm{M}_1$ | $\begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ |
| $\mathbb{X}_{13}$ | $\mathbb{M}_{3}^{(a,A_1)}(1,-1)$ | $\mathrm{M}_1$ | $\begin{pmatrix} 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} \\ \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 \end{pmatrix}$ |

Table 7: Cluster SAMB.

| symbol | type | cluster | form |
|---|---|---|---|
| $\mathbb{Y}_1$ | $\mathbb{Q}_0^{(s,A_1)}$ | $\mathrm{S}_1$ | $\begin{pmatrix} 1 \end{pmatrix}$ |
| $\mathbb{Y}_2$ | $\mathbb{Q}_0^{(s,A_1)}$ | $\mathrm{S}_2$ | $\begin{pmatrix} 1 \end{pmatrix}$ |
| $\mathbb{Y}_3$ | $\mathbb{Q}_0^{(b,A_1)}$ | $\mathrm{B}_1$ | $\begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$ |
| $\mathbb{Y}_4$ | $\mathbb{Q}_{1,0}^{(b,E)}$ | $\mathrm{B}_1$ | $\begin{pmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$ |
| $\mathbb{Y}_5$ | $\mathbb{Q}_{1,1}^{(b,E)}$ | $\mathrm{B}_1$ | $\begin{pmatrix} -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix}$ |
| $\mathbb{Y}_6$ | $\mathbb{T}_0^{(b,A_1)}$ | $\mathrm{B}_1$ | $\begin{pmatrix} \frac{\sqrt{3}i}{3} & \frac{\sqrt{3}i}{3} & \frac{\sqrt{3}i}{3} \end{pmatrix}$ |
| $\mathbb{Y}_7$ | $\mathbb{T}_{1,0}^{(b,E)}$ | $\mathrm{B}_1$ | $\begin{pmatrix} 0 & \frac{\sqrt{2}i}{2} & -\frac{\sqrt{2}i}{2} \end{pmatrix}$ |
| $\mathbb{Y}_8$ | $\mathbb{T}_{1,1}^{(b,E)}$ | $\mathrm{B}_1$ | $\begin{pmatrix} -\frac{\sqrt{6}i}{3} & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} \end{pmatrix}$ |

Table 8: Polar harmonics.

| No. | symbol | rank | irrep. | mul. | comp. | form |
|---|---|---|---|---|---|---|
| 1 | $\mathbb{Q}_{0}^{(A_1)}$ | 0 | $A_1$ | – | – | 1 |
| 2 | $\mathbb{Q}_{1,0}^{(E)}$ | 1 | $E$ | – | 0 | $x$ |
| 3 | $\mathbb{Q}_{1,1}^{(E)}$ | 1 | $E$ | – | 1 | $y$ |
| 4 | $\mathbb{Q}_{2,0}^{(E,1)}$ | 2 | $E$ | 1 | 0 | $\sqrt{3}xz$ |
| 5 | $\mathbb{Q}_{2,1}^{(E,1)}$ | 2 | $E$ | 1 | 1 | $\sqrt{3}yz$ |
| 6 | $\mathbb{Q}_{2,0}^{(E,2)}$ | 2 | $E$ | 2 | 0 | $-\sqrt{3}xy$ |
| 7 | $\mathbb{Q}_{2,1}^{(E,2)}$ | 2 | $E$ | 2 | 1 | $-\frac{\sqrt{3}(x-y)(x+y)}{2}$ |

Table 9: Axial harmonics.

| No. | symbol | rank | irrep. | mul. | comp. | form |
|---|---|---|---|---|---|---|
| 1 | $\mathbb{G}_{1,0}^{(E)}$ | 1 | $E$ | – | 0 | $-Y$ |
| 2 | $\mathbb{G}_{1,1}^{(E)}$ | 1 | $E$ | – | 1 | $X$ |
| 3 | $\mathbb{G}_{3}^{(A_1)}$ | 3 | $A_1$ | – | – | $\frac{\sqrt{10}X\left(X^2-3Y^2\right)}{4}$ |
| 4 | $\mathbb{G}_{3,0}^{(E,2)}$ | 3 | $E$ | 2 | 0 | $\frac{\sqrt{15}Z(X-Y)(X+Y)}{2}$ |
| 5 | $\mathbb{G}_{3,1}^{(E,2)}$ | 3 | $E$ | 2 | 1 | $-\sqrt{15}XYZ$ |

- Group info.: Generator = $\{3^{+}_{001}|0\}$, $\{\mathrm{m}_{110}|0\}$

Table 10: Conjugacy class (point-group part).

| rep. SO | symmetry operations |
|---|---|
| $\{1\|0\}$ | $\{1\|0\}$ |
| $\{3^{+}_{001}\|0\}$ | $\{3^{+}_{001}\|0\}$, $\{3^{-}_{001}\|0\}$ |

*continued ...*

Table 10

| rep. SO | symmetry operations |
|---|---|
| $\{m_{100}\|0\}$ | $\{m_{100}\|0\}$, $\{m_{010}\|0\}$, $\{m_{110}\|0\}$ |

Table 11: Symmetry operations.

| No. | SO | No. | SO | No. | SO | No. | SO | No. | SO |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $\{1\|0\}$ | 2 | $\{3^{+}_{001}\|0\}$ | 3 | $\{3^{-}_{001}\|0\}$ | 4 | $\{m_{100}\|0\}$ | 5 | $\{m_{010}\|0\}$ |
| 6 | $\{m_{110}\|0\}$ | | | | | | | | |

Table 12: Character table (point-group part).

| | 1 | $3^{+}_{001}$ | $m_{100}$ |
|---|---|---|---|
| $A_1$ | 1 | 1 | 1 |
| $A_2$ | 1 | 1 | $-1$ |
| $E$ | 2 | $-1$ | 0 |

Table 13: Parity conversion.

| $\leftrightarrow$ | $\leftrightarrow$ | $\leftrightarrow$ |
|---|---|---|
| $A_1$ ($A_2$) | $A_2$ ($A_1$) | $E$ ($E$) |

Table 14: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

| | $A_1$ | $A_2$ | $E$ |
|---|---|---|---|
| $A_1$ | $A_1$ | $A_2$ | $E$ |
| $A_2$ | | $A_1$ | $E$ |
| $E$ | | | $A_1 + E$ |

Table 15: Anti-symmetric product, $[\Gamma \otimes \Gamma]_-$.

| $A_1$ | $A_2$ | $E$ |
|---|---|---|
| $-$ | $-$ | $A_2$ |

Table 16: Virtual-cluster sites.

| No. | position | No. | position | No. | position | No. | position |
|---|---|---|---|---|---|---|---|
| 1 | $\begin{pmatrix} -1 & -1 & 0 \end{pmatrix}$ | 2 | $\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$ | 3 | $\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$ | 4 | $\begin{pmatrix} 0 & -1 & 0 \end{pmatrix}$ |
| 5 | $\begin{pmatrix} -1 & 0 & 0 \end{pmatrix}$ | 6 | $\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}$ | | | | |

Table 17: Virtual-cluster basis.

| symbol | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $\mathbb{Q}_0^{(A_1)}$ | $\frac{\sqrt{6}}{6}$ | $\frac{\sqrt{6}}{6}$ | $\frac{\sqrt{6}}{6}$ | $\frac{\sqrt{6}}{6}$ | $\frac{\sqrt{6}}{6}$ | $\frac{\sqrt{6}}{6}$ |
| $\mathbb{Q}_{1,0}^{(E)}$ | $-\frac{\sqrt{3}}{6}$ | $\frac{\sqrt{3}}{3}$ | $-\frac{\sqrt{3}}{6}$ | $\frac{\sqrt{3}}{6}$ | $-\frac{\sqrt{3}}{3}$ | $\frac{\sqrt{3}}{6}$ |
| $\mathbb{Q}_{1,1}^{(E)}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ |
| $\mathbb{Q}_{2,0}^{(E,2)}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ |

*continued ...*

Table 17

| symbol | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $\mathbb{Q}_{2,1}^{(E,2)}$ | $\frac{\sqrt{3}}{6}$ | $-\frac{\sqrt{3}}{3}$ | $\frac{\sqrt{3}}{6}$ | $\frac{\sqrt{3}}{6}$ | $-\frac{\sqrt{3}}{3}$ | $\frac{\sqrt{3}}{6}$ |
| $\mathbb{Q}_{3}^{(A_2)}$ | $\frac{\sqrt{6}}{6}$ | $\frac{\sqrt{6}}{6}$ | $\frac{\sqrt{6}}{6}$ | $-\frac{\sqrt{6}}{6}$ | $-\frac{\sqrt{6}}{6}$ | $-\frac{\sqrt{6}}{6}$ |