

PG No. 19  $C_{3v}$  3m (3m1 setting) [ trigonal ] (polar, internal polar octupole)

\* Harmonics for rank 0

$$\tilde{\mathbb{Q}}_0^{(3,3)}[q](A_1)$$

\*\* symmetry

1

\*\* expression

$$\begin{aligned} & \frac{\sqrt{70}Q_1y(3x^2-y^2)}{28} + \frac{\sqrt{70}Q_2x(x^2-3y^2)}{28} + \frac{\sqrt{105}Q_3xyz}{7} - \frac{\sqrt{42}Q_{3x}x(x^2+y^2-4z^2)}{28} \\ & - \frac{\sqrt{42}Q_{3y}y(x^2+y^2-4z^2)}{28} - \frac{\sqrt{7}Q_{az}z(3x^2+3y^2-2z^2)}{14} + \frac{\sqrt{105}Q_{bz}z(x-y)(x+y)}{14} \end{aligned}$$

\* Harmonics for rank 1

$$\tilde{\mathbb{Q}}_1^{(3,1)}[q](A_1)$$

\*\* symmetry

$z$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{21}Q_{3xy}}{7} + \frac{2\sqrt{210}Q_{3xxz}}{35} + \frac{2\sqrt{210}Q_{3yyz}}{35} - \frac{3\sqrt{35}Q_{az}(x^2+y^2-2z^2)}{70} + \frac{\sqrt{21}Q_{bz}(x-y)(x+y)}{14} \end{aligned}$$

$$\tilde{\mathbb{Q}}_1^{(3,3)}[q](A_1)$$

\*\* symmetry

$z$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{210}Q_1yz(3x^2-y^2)}{24} + \frac{\sqrt{210}Q_2xz(x^2-3y^2)}{24} - \frac{\sqrt{35}Q_3xy(x^2+y^2-6z^2)}{14} - \frac{5\sqrt{14}Q_{3x}xz(3x^2+3y^2-4z^2)}{56} \\ & - \frac{5\sqrt{14}Q_{3yyz}(3x^2+3y^2-4z^2)}{56} + \frac{\sqrt{21}Q_{az}(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4)}{84} - \frac{\sqrt{35}Q_{bz}(x-y)(x+y)(x^2+y^2-6z^2)}{28} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{1,1}^{(3,1)}[q](E), \tilde{\mathbb{Q}}_{1,2}^{(3,1)}[q](E)$$

\*\* symmetry

$x$

$y$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{14}Q_1xy}{14} + \frac{3\sqrt{14}Q_2(x-y)(x+y)}{28} + \frac{\sqrt{21}Q_3yz}{7} - \frac{\sqrt{210}Q_{3x}(3x^2+y^2-4z^2)}{140} - \frac{\sqrt{210}Q_{3y}xy}{70} - \frac{3\sqrt{35}Q_{az}xz}{35} + \frac{\sqrt{21}Q_{bz}xz}{7} \\ & \frac{3\sqrt{14}Q_1(x-y)(x+y)}{28} - \frac{3\sqrt{14}Q_2xy}{14} + \frac{\sqrt{21}Q_3xz}{7} - \frac{\sqrt{210}Q_{3x}xy}{70} - \frac{\sqrt{210}Q_{3y}(x^2+3y^2-4z^2)}{140} - \frac{3\sqrt{35}Q_{az}yz}{35} - \frac{\sqrt{21}Q_{bz}yz}{7} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{1,1}^{(3,3)}[q](E), \tilde{\mathbb{Q}}_{1,2}^{(3,3)}[q](E)$$

\*\* symmetry

$x$

$y$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{210}Q_1xy(15x^2-13y^2-6z^2)}{168} + \frac{\sqrt{210}Q_2(4x^4-21x^2y^2-3x^2z^2+3y^4+3y^2z^2)}{168} \\ & + \frac{\sqrt{35}Q_3yz(6x^2-y^2-z^2)}{14} - \frac{\sqrt{14}Q_{3x}(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4)}{56} \\ & - \frac{5\sqrt{14}Q_{3y}xy(x^2+y^2-6z^2)}{56} - \frac{5\sqrt{21}Q_{az}xz(3x^2+3y^2-4z^2)}{84} + \frac{\sqrt{35}Q_{bz}xz(5x^2-9y^2-2z^2)}{28} \\ & - \frac{\sqrt{210}Q_1(3x^4-21x^2y^2+3x^2z^2+4y^4-3y^2z^2)}{168} + \frac{\sqrt{210}Q_2xy(13x^2-15y^2+6z^2)}{168} - \frac{\sqrt{35}Q_{3xz}(x^2-6y^2+z^2)}{14} - \frac{5\sqrt{14}Q_{3x}xy(x^2+y^2-6z^2)}{56} \\ & + \frac{\sqrt{14}Q_{3y}(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4)}{56} - \frac{5\sqrt{21}Q_{az}yz(3x^2+3y^2-4z^2)}{84} + \frac{\sqrt{35}Q_{bz}yz(9x^2-5y^2+2z^2)}{28} \end{aligned}$$

\* Harmonics for rank 2

$$\tilde{\mathbb{Q}}_2^{(3,-1)}[q](A_1)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$\frac{\sqrt{14}Q_{3x}x}{7} + \frac{\sqrt{14}Q_{3y}y}{7} + \frac{\sqrt{21}Q_{az}z}{7}$$

$\vec{\mathbb{Q}}_2^{(3,1)}[q](A_1)$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$-\frac{5\sqrt{210}Q_1y(3x^2 - y^2)}{168} - \frac{5\sqrt{210}Q_2x(x^2 - 3y^2)}{168} - \frac{3\sqrt{14}Q_{3x}x(x^2 + y^2 - 4z^2)}{56} - \frac{3\sqrt{14}Q_{3y}y(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{21}Q_{az}z(3x^2 + 3y^2 - 2z^2)}{21}$$

$\vec{\mathbb{Q}}_2^{(3,3)}[q](A_1)$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$\begin{aligned} & -\frac{\sqrt{1155}Q_1y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{264} - \frac{\sqrt{1155}Q_2x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{264} - \frac{3\sqrt{770}Q_3xyz(x^2 + y^2 - 2z^2)}{44} \\ & + \frac{15\sqrt{77}Q_{3x}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{616} + \frac{15\sqrt{77}Q_{3y}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{616} \\ & + \frac{5\sqrt{462}Q_{az}z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{1848} - \frac{3\sqrt{770}Q_{bz}z(x - y)(x + y)(x^2 + y^2 - 2z^2)}{88} \end{aligned}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,-1)}[q](E, 1), \vec{\mathbb{Q}}_{2,2}^{(3,-1)}[q](E, 1)$

\*\* symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

\*\* expression

$$\frac{\sqrt{105}Q_3y}{21} + \frac{2\sqrt{42}Q_{3x}z}{21} - \frac{\sqrt{7}Q_{az}x}{7} + \frac{\sqrt{105}Q_{bz}x}{21}$$

$$\frac{\sqrt{105}Q_3x}{21} + \frac{2\sqrt{42}Q_{3y}z}{21} - \frac{\sqrt{7}Q_{az}y}{7} - \frac{\sqrt{105}Q_{bz}y}{21}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,-1)}[q](E, 2), \vec{\mathbb{Q}}_{2,2}^{(3,-1)}[q](E, 2)$

\*\* symmetry

$$\sqrt{3}xy$$

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

\*\* expression

$$\frac{\sqrt{70}Q_1x}{14} - \frac{\sqrt{70}Q_2y}{14} + \frac{\sqrt{105}Q_3z}{21} - \frac{\sqrt{42}Q_{3x}y}{42} - \frac{\sqrt{42}Q_{3y}x}{42}$$

$$\frac{\sqrt{70}Q_1y}{14} + \frac{\sqrt{70}Q_2x}{14} - \frac{\sqrt{42}Q_{3x}x}{42} + \frac{\sqrt{42}Q_{3y}y}{42} + \frac{\sqrt{105}Q_{bz}z}{21}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,1)}[q](E, 1), \vec{\mathbb{Q}}_{2,2}^{(3,1)}[q](E, 1)$

\*\* symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

\*\* expression

$$\begin{aligned} & \frac{5\sqrt{70}Q_1xyz}{28} + \frac{5\sqrt{70}Q_2z(x - y)(x + y)}{56} + \frac{\sqrt{105}Q_3y(3x^2 - 2y^2 + 3z^2)}{42} + \frac{\sqrt{42}Q_{3x}z(9x^2 - 21y^2 + 4z^2)}{168} \\ & + \frac{5\sqrt{42}Q_{3y}xyz}{28} - \frac{\sqrt{7}Q_{az}x(x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{105}Q_{bz}x(x^2 - 9y^2 + 6z^2)}{84} \end{aligned}$$

$$\begin{aligned} & \frac{5\sqrt{70}Q_1z(x-y)(x+y)}{56} - \frac{5\sqrt{70}Q_2xyz}{28} - \frac{\sqrt{105}Q_3x(2x^2-3y^2-3z^2)}{42} + \frac{5\sqrt{42}Q_{3x}xyz}{28} \\ & - \frac{\sqrt{42}Q_{3y}z(21x^2-9y^2-4z^2)}{168} - \frac{\sqrt{7}Q_{az}y(x^2+y^2-4z^2)}{28} + \frac{\sqrt{105}Q_{bz}y(9x^2-y^2-6z^2)}{84} \end{aligned}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,1)}[q](E, 2), \vec{\mathbb{Q}}_{2,2}^{(3,1)}[q](E, 2)$

\*\* symmetry

$$\sqrt{3}xy$$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{70}Q_1x(x^2+y^2-4z^2)}{56} - \frac{\sqrt{70}Q_2y(x^2+y^2-4z^2)}{56} + \frac{\sqrt{105}Q_3z(3x^2+3y^2-2z^2)}{42} \\ & - \frac{\sqrt{42}Q_{3x}y(21x^2+y^2-24z^2)}{168} - \frac{\sqrt{42}Q_{3y}x(x^2+21y^2-24z^2)}{168} - \frac{5\sqrt{7}Q_{az}xyz}{7} \\ & \frac{\sqrt{70}Q_1y(x^2+y^2-4z^2)}{56} + \frac{\sqrt{70}Q_2x(x^2+y^2-4z^2)}{56} - \frac{\sqrt{42}Q_{3x}x(11x^2-9y^2-24z^2)}{168} \\ & - \frac{\sqrt{42}Q_{3y}y(9x^2-11y^2+24z^2)}{168} - \frac{5\sqrt{7}Q_{az}z(x-y)(x+y)}{14} + \frac{\sqrt{105}Q_{bz}z(3x^2+3y^2-2z^2)}{42} \end{aligned}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,3)}[q](E, 1), \vec{\mathbb{Q}}_{2,2}^{(3,3)}[q](E, 1)$

\*\* symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{385}Q_1xyz(7x^2-5y^2-2z^2)}{44} + \frac{\sqrt{385}Q_2z(2x^4-9x^2y^2-x^2z^2+y^4+y^2z^2)}{44} - \frac{\sqrt{2310}Q_3y(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{462} \\ & - \frac{5\sqrt{231}Q_{3x}z(18x^4+15x^2y^2-41x^2z^2-3y^4+y^2z^2+4z^4)}{924} - \frac{5\sqrt{231}Q_{3y}xyz(x^2+y^2-2z^2)}{44} \\ & + \frac{5\sqrt{154}Q_{az}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{308} - \frac{\sqrt{2310}Q_{bz}x(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4)}{924} \\ & - \frac{\sqrt{385}Q_1z(x^4-9x^2y^2+x^2z^2+2y^4-y^2z^2)}{44} + \frac{\sqrt{385}Q_2xyz(5x^2-7y^2+2z^2)}{44} + \frac{\sqrt{2310}Q_3x(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{462} \\ & - \frac{5\sqrt{231}Q_{3x}xyz(x^2+y^2-2z^2)}{44} + \frac{5\sqrt{231}Q_{3y}z(3x^4-15x^2y^2-x^2z^2-18y^4+41y^2z^2-4z^4)}{924} \\ & + \frac{5\sqrt{154}Q_{az}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{308} - \frac{\sqrt{2310}Q_{bz}y(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4)}{924} \end{aligned}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,3)}[q](E, 2), \vec{\mathbb{Q}}_{2,2}^{(3,3)}[q](E, 2)$

\*\* symmetry

$$\sqrt{3}xy$$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

\*\* expression

$$\begin{aligned} & - \frac{\sqrt{385}Q_1x(5x^4-53x^2y^2+3x^2z^2+26y^4+3y^2z^2-2z^4)}{308} + \frac{\sqrt{385}Q_2y(26x^4-53x^2y^2+3x^2z^2+5y^4+3y^2z^2-2z^4)}{308} \\ & - \frac{\sqrt{2310}Q_3z(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4)}{462} - \frac{5\sqrt{231}Q_{3x}y(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{924} \\ & + \frac{5\sqrt{231}Q_{3y}x(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{924} - \frac{5\sqrt{154}Q_{az}xyz(x^2+y^2-2z^2)}{44} + \frac{3\sqrt{2310}Q_{bz}xyz(x-y)(x+y)}{44} \\ & - \frac{\sqrt{385}Q_1y(53x^4-104x^2y^2-6x^2z^2+11y^4-6y^2z^2+4z^4)}{616} + \frac{\sqrt{385}Q_2x(11x^4-104x^2y^2-6x^2z^2+53y^4-6y^2z^2+4z^4)}{616} \\ & + \frac{3\sqrt{2310}Q_3xyz(x-y)(x+y)}{44} - \frac{5\sqrt{231}Q_{3x}x(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4)}{1848} \\ & - \frac{5\sqrt{231}Q_{3y}y(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4)}{1848} - \frac{5\sqrt{154}Q_{az}z(x-y)(x+y)(x^2+y^2-2z^2)}{88} \\ & + \frac{\sqrt{2310}Q_{bz}z(39x^4-174x^2y^2-20x^2z^2+39y^4-20y^2z^2+4z^4)}{1848} \end{aligned}$$

\* Harmonics for rank 3

$$\tilde{\mathbb{Q}}_3^{(3,-3)}[q](A_1, 1)$$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$Q_{az}$$

$$\tilde{\mathbb{Q}}_3^{(3,-3)}[q](A_1, 2)$$

\*\* symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

\*\* expression

$$Q_1$$

$$\tilde{\mathbb{Q}}_3^{(3,-1)}[q](A_1, 1)$$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$-Q_3xy + \frac{\sqrt{10}Q_{3x}xz}{10} + \frac{\sqrt{10}Q_{3y}yz}{10} - \frac{\sqrt{15}Q_{az}(x^2 + y^2 - 2z^2)}{15} - \frac{Q_{bz}(x - y)(x + y)}{2}$$

$$\tilde{\mathbb{Q}}_3^{(3,-1)}[q](A_1, 2)$$

\*\* symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

\*\* expression

$$\frac{\sqrt{15}Q_1(x^2 + y^2 - 2z^2)}{12} + \frac{\sqrt{10}Q_{3x}xz}{4} - \frac{Q_{3x}xy}{2} - \frac{Q_{3y}(x - y)(x + y)}{4} + \frac{\sqrt{10}Q_{bz}yz}{4}$$

$$\tilde{\mathbb{Q}}_3^{(3,1)}[q](A_1, 1)$$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$\begin{aligned} & -\frac{7\sqrt{55}Q_1yz(3x^2 - y^2)}{44} - \frac{7\sqrt{55}Q_2xz(x^2 - 3y^2)}{44} + \frac{\sqrt{330}Q_3xy(x^2 + y^2 - 6z^2)}{132} - \frac{5\sqrt{33}Q_{3x}xz(3x^2 + 3y^2 - 4z^2)}{132} \\ & - \frac{5\sqrt{33}Q_{3y}yz(3x^2 + 3y^2 - 4z^2)}{132} + \frac{\sqrt{22}Q_{az}(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{88} + \frac{\sqrt{330}Q_{bz}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{264} \end{aligned}$$

$$\tilde{\mathbb{Q}}_3^{(3,1)}[q](A_1, 2)$$

\*\* symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{22}Q_1(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{176} + \frac{5\sqrt{33}Q_3xz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{\sqrt{330}Q_{3x}xy(5x^2 - 2y^2 - 9z^2)}{66} \\ & + \frac{\sqrt{330}Q_{3y}(x^4 - 42x^2y^2 + 36x^2z^2 + 13y^4 - 36y^2z^2)}{528} - \frac{7\sqrt{55}Q_{az}yz(3x^2 - y^2)}{44} + \frac{5\sqrt{33}Q_{bz}yz(3x^2 + 3y^2 - 4z^2)}{132} \end{aligned}$$

$$\tilde{\mathbb{Q}}_3^{(3,3)}[q](A_1, 1)$$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$\begin{aligned} & -\frac{7\sqrt{4290}Q_1yz(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{2288} - \frac{7\sqrt{4290}Q_2xz(x^2 - 3y^2)(3x^2 + 3y^2 - 8z^2)}{2288} \\ & + \frac{7\sqrt{715}Q_3xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} + \frac{35\sqrt{286}Q_{3x}xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\ & + \frac{35\sqrt{286}Q_{3y}yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\ & - \frac{5\sqrt{429}Q_{az}(5x^6 + 15x^4y^2 - 90x^4z^2 + 15x^2y^4 - 180x^2y^2z^2 + 120x^2z^4 + 5y^6 - 90y^4z^2 + 120y^2z^4 - 16z^6)}{3432} \\ & + \frac{7\sqrt{715}Q_{bz}(x - y)(x + y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144} \end{aligned}$$

$$\vec{\mathbb{Q}}_3^{(3,3)}[q](A_1, 2)$$

\*\* symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{429}Q_1(113x^6 - 1740x^4y^2 + 45x^4z^2 + 1725x^2y^4 + 90x^2y^2z^2 - 60x^2z^4 - 118y^6 + 45y^4z^2 - 60y^2z^4 + 8z^6)}{6864} + \frac{7\sqrt{429}Q_2xy(x^2 - 3y^2)(3x^2 - y^2)}{208} \\ & - \frac{7\sqrt{286}Q_3xz(7x^4 - 85x^2y^2 + 5x^2z^2 + 40y^4 + 5y^2z^2 - 2z^4)}{1144} - \frac{7\sqrt{715}Q_{3x}xy(7x^4 + 2x^2y^2 - 76x^2z^2 - 5y^4 + 44y^2z^2 + 16z^4)}{2288} \\ & + \frac{7\sqrt{715}Q_{3y}(x^6 - 8x^4y^2 - 7x^4z^2 - 7x^2y^4 + 90x^2y^2z^2 - 8x^2z^4 + 2y^6 - 23y^4z^2 + 8y^2z^4)}{2288} \\ & - \frac{7\sqrt{4290}Q_{az}yz(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{2288} + \frac{7\sqrt{286}Q_{bz}yz(85x^4 - 160x^2y^2 - 10x^2z^2 + 19y^4 - 10y^2z^2 + 4z^4)}{2288} \end{aligned}$$

$$\vec{\mathbb{Q}}_3^{(3,-3)}[q](A_2)$$

\*\* symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

\*\* expression

$$Q_2$$

$$\vec{\mathbb{Q}}_3^{(3,-1)}[q](A_2)$$

\*\* symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

\*\* expression

$$\frac{\sqrt{15}Q_2(x^2 + y^2 - 2z^2)}{12} - \frac{\sqrt{10}Q_3yz}{4} - \frac{Q_{3x}(x - y)(x + y)}{4} + \frac{Q_{3y}xy}{2} + \frac{\sqrt{10}Q_{bz}xz}{4}$$

$$\vec{\mathbb{Q}}_3^{(3,1)}[q](A_2)$$

\*\* symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{22}Q_2(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{176} - \frac{5\sqrt{33}Q_3yz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{\sqrt{330}Q_{3x}(13x^4 - 42x^2y^2 - 36x^2z^2 + y^4 + 36y^2z^2)}{528} \\ & - \frac{\sqrt{330}Q_{3y}xy(2x^2 - 5y^2 + 9z^2)}{66} - \frac{7\sqrt{55}Q_{az}xz(x^2 - 3y^2)}{44} + \frac{5\sqrt{33}Q_{bz}xz(3x^2 + 3y^2 - 4z^2)}{132} \end{aligned}$$

$$\vec{\mathbb{Q}}_3^{(3,3)}[q](A_2)$$

\*\* symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{7\sqrt{429}Q_1xy(x^2 - 3y^2)(3x^2 - y^2)}{208} + \frac{\sqrt{429}Q_2(118x^6 - 1725x^4y^2 - 45x^4z^2 + 1740x^2y^4 - 90x^2y^2z^2 + 60x^2z^4 - 113y^6 - 45y^4z^2 + 60y^2z^4 - 8z^6)}{6864} \\ & + \frac{7\sqrt{286}Q_3yz(40x^4 - 85x^2y^2 + 5x^2z^2 + 7y^4 + 5y^2z^2 - 2z^4)}{1144} \\ & - \frac{7\sqrt{715}Q_{3x}(2x^6 - 7x^4y^2 - 23x^4z^2 - 8x^2y^4 + 90x^2y^2z^2 + 8x^2z^4 + y^6 - 7y^4z^2 - 8y^2z^4)}{2288} \\ & - \frac{7\sqrt{715}Q_{3y}xy(5x^4 - 2x^2y^2 - 44x^2z^2 - 7y^4 + 76y^2z^2 - 16z^4)}{2288} - \frac{7\sqrt{4290}Q_{az}xz(x^2 - 3y^2)(3x^2 + 3y^2 - 8z^2)}{2288} \\ & + \frac{7\sqrt{286}Q_{bz}xz(19x^4 - 160x^2y^2 - 10x^2z^2 + 85y^4 - 10y^2z^2 + 4z^4)}{2288} \end{aligned}$$

$$\vec{\mathbb{Q}}_{3,1}^{(3,-3)}[q](E, 1), \vec{\mathbb{Q}}_{3,2}^{(3,-3)}[q](E, 1)$$

\*\* symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

\*\* expression

$$Q_{3x}$$

$$Q_{3y}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,-3)}[q](E,2), \vec{\mathbb{Q}}_{3,2}^{(3,-3)}[q](E,2)$

\*\* symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

\*\* expression

$$Q_3$$

$$Q_{bz}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,-1)}[q](E,1), \vec{\mathbb{Q}}_{3,2}^{(3,-1)}[q](E,1)$

\*\* symmetry

$$-\frac{\sqrt{6}x(x^2+y^2-4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2+y^2-4z^2)}{4}$$

\*\* expression

$$-\frac{Q_1xy}{2} - \frac{Q_2(x-y)(x+y)}{4} + \frac{\sqrt{6}Q_3yz}{4} + \frac{\sqrt{15}Q_{3x}(x^2-3y^2+2z^2)}{20} + \frac{\sqrt{15}Q_{3y}xy}{5} + \frac{\sqrt{10}Q_{az}xz}{10} + \frac{\sqrt{6}Q_{bz}xz}{4}$$

$$-\frac{Q_1(x-y)(x+y)}{4} + \frac{Q_2xy}{2} + \frac{\sqrt{6}Q_{3xz}}{4} + \frac{\sqrt{15}Q_{3x}xy}{5} - \frac{\sqrt{15}Q_{3y}(3x^2-y^2-2z^2)}{20} + \frac{\sqrt{10}Q_{az}yz}{10} - \frac{\sqrt{6}Q_{bz}yz}{4}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,-1)}[q](E,2), \vec{\mathbb{Q}}_{3,2}^{(3,-1)}[q](E,2)$

\*\* symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

\*\* expression

$$\frac{\sqrt{10}Q_1xz}{4} - \frac{\sqrt{10}Q_2yz}{4} + \frac{\sqrt{6}Q_{3x}yz}{4} + \frac{\sqrt{6}Q_{3y}xz}{4} - Q_{az}xy$$

$$\frac{\sqrt{10}Q_1yz}{4} + \frac{\sqrt{10}Q_2xz}{4} + \frac{\sqrt{6}Q_{3x}xz}{4} - \frac{\sqrt{6}Q_{3y}yz}{4} - \frac{Q_{az}(x-y)(x+y)}{2}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,1)}[q](E,1), \vec{\mathbb{Q}}_{3,2}^{(3,1)}[q](E,1)$

\*\* symmetry

$$-\frac{\sqrt{6}x(x^2+y^2-4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2+y^2-4z^2)}{4}$$

\*\* expression

$$\begin{aligned} & -\frac{\sqrt{330}Q_1xy(5x^2-2y^2-9z^2)}{66} - \frac{\sqrt{330}Q_2(13x^4-42x^2y^2-36x^2z^2+y^4+36y^2z^2)}{528} \\ & + \frac{\sqrt{55}Q_3yz(9x^2-19y^2+16z^2)}{132} - \frac{\sqrt{22}Q_{3x}(17x^4-6x^2y^2-96x^2z^2-23y^4+144y^2z^2-8z^4)}{528} \\ & - \frac{5\sqrt{22}Q_{3y}xy(x^2+y^2-6z^2)}{66} - \frac{5\sqrt{33}Q_{az}xz(3x^2+3y^2-4z^2)}{132} - \frac{\sqrt{55}Q_{bz}xz(5x^2+33y^2-16z^2)}{132} \end{aligned}$$

$$\begin{aligned} & \frac{\sqrt{330}Q_1(x^4-42x^2y^2+36x^2z^2+13y^4-36y^2z^2)}{528} - \frac{\sqrt{330}Q_2xy(2x^2-5y^2+9z^2)}{66} - \frac{\sqrt{55}Q_{3xz}(19x^2-9y^2-16z^2)}{132} - \frac{5\sqrt{22}Q_{3x}xy(x^2+y^2-6z^2)}{66} \\ & + \frac{\sqrt{22}Q_{3y}(23x^4+6x^2y^2-144x^2z^2-17y^4+96y^2z^2+8z^4)}{528} - \frac{5\sqrt{33}Q_{az}yz(3x^2+3y^2-4z^2)}{132} + \frac{\sqrt{55}Q_{bz}yz(33x^2+5y^2-16z^2)}{132} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(3,1)}[q](E, 2), \tilde{\mathbb{Q}}_{3,2}^{(3,1)}[q](E, 2)$$

\*\* symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{5\sqrt{33}Q_1xz(3x^2+3y^2-4z^2)}{132} - \frac{5\sqrt{33}Q_2yz(3x^2+3y^2-4z^2)}{132} - \frac{7\sqrt{22}Q_3(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4)}{66} \\ & + \frac{\sqrt{55}Q_{3x}yz(9x^2-19y^2+16z^2)}{132} - \frac{\sqrt{55}Q_{3y}xz(19x^2-9y^2-16z^2)}{132} + \frac{\sqrt{330}Q_{az}xy(x^2+y^2-6z^2)}{132} + \frac{35\sqrt{22}Q_{bz}xy(x-y)(x+y)}{132} \\ \\ & \frac{5\sqrt{33}Q_1yz(3x^2+3y^2-4z^2)}{132} + \frac{5\sqrt{33}Q_2xz(3x^2+3y^2-4z^2)}{132} + \frac{35\sqrt{22}Q_3xy(x-y)(x+y)}{132} - \frac{\sqrt{55}Q_{3x}xz(5x^2+33y^2-16z^2)}{132} \\ & + \frac{\sqrt{55}Q_{3y}yz(33x^2+5y^2-16z^2)}{132} + \frac{\sqrt{330}Q_{az}(x-y)(x+y)(x^2+y^2-6z^2)}{264} + \frac{7\sqrt{22}Q_{bz}(x^4-18x^2y^2+12x^2z^2+y^4+12y^2z^2-4z^4)}{264} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(3,3)}[q](E, 1), \tilde{\mathbb{Q}}_{3,2}^{(3,3)}[q](E, 1)$$

\*\* symmetry

$$\frac{\sqrt{6}x(x^2+y^2-4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2+y^2-4z^2)}{4}$$

\*\* expression

$$\begin{aligned} & -\frac{7\sqrt{715}Q_1xy(7x^4+2x^2y^2-76x^2z^2-5y^4+44y^2z^2+16z^4)}{2288} \\ & -\frac{7\sqrt{715}Q_2(2x^6-7x^4y^2-23x^4z^2-8x^2y^4+90x^2y^2z^2+8x^2z^4+y^6-7y^4z^2-8y^2z^4)}{2288} - \frac{7\sqrt{4290}Q_3yz(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4)}{1144} \\ & + \frac{5\sqrt{429}Q_{3x}(6x^6+11x^4y^2-101x^4z^2+4x^2y^4-90x^2y^2z^2+116x^2z^4-y^6+11y^4z^2+4y^2z^4-8z^6)}{2288} \\ & + \frac{35\sqrt{429}Q_{3y}xy(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{2288} + \frac{35\sqrt{286}Q_{az}xz(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4)}{2288} \\ & - \frac{7\sqrt{4290}Q_{bz}xz(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4)}{2288} \\ \\ & \frac{7\sqrt{715}Q_1(x^6-8x^4y^2-7x^4z^2-7x^2y^4+90x^2y^2z^2-8x^2z^4+2y^6-23y^4z^2+8y^2z^4)}{2288} \\ & - \frac{7\sqrt{715}Q_2xy(5x^4-2x^2y^2-44x^2z^2-7y^4+76y^2z^2-16z^4)}{2288} \\ & + \frac{7\sqrt{4290}Q_3xz(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4)}{1144} + \frac{35\sqrt{429}Q_{3x}xy(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{2288} \\ & - \frac{5\sqrt{429}Q_{3y}(x^6-4x^4y^2-11x^4z^2-11x^2y^4+90x^2y^2z^2-4x^2z^4-6y^6+101y^4z^2-116y^2z^4+8z^6)}{2288} \\ & + \frac{35\sqrt{286}Q_{az}yz(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4)}{2288} - \frac{7\sqrt{4290}Q_{bz}yz(11x^4+4x^2y^2-26x^2z^2-7y^4+22y^2z^2-4z^4)}{2288} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(3,3)}[q](E, 2), \tilde{\mathbb{Q}}_{3,2}^{(3,3)}[q](E, 2)$$

\*\* symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

\*\* expression

$$\begin{aligned} & -\frac{7\sqrt{286}Q_1xz(7x^4-85x^2y^2+5x^2z^2+40y^4+5y^2z^2-2z^4)}{1144} + \frac{7\sqrt{286}Q_2yz(40x^4-85x^2y^2+5x^2z^2+7y^4+5y^2z^2-2z^4)}{1144} \\ & + \frac{\sqrt{429}Q_3(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6)}{286} \\ & - \frac{7\sqrt{4290}Q_{3x}yz(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4)}{1144} + \frac{7\sqrt{4290}Q_{3y}xz(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4)}{1144} \\ & + \frac{7\sqrt{715}Q_{az}xy(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{572} - \frac{21\sqrt{429}Q_{bz}xy(x-y)(x+y)(x^2+y^2-10z^2)}{572} \end{aligned}$$

$$\begin{aligned}
& \frac{7\sqrt{286}Q_1yz(85x^4 - 160x^2y^2 - 10x^2z^2 + 19y^4 - 10y^2z^2 + 4z^4)}{2288} + \frac{7\sqrt{286}Q_2xz(19x^4 - 160x^2y^2 - 10x^2z^2 + 85y^4 - 10y^2z^2 + 4z^4)}{2288} \\
& - \frac{21\sqrt{429}Q_3xy(x-y)(x+y)(x^2+y^2-10z^2)}{572} - \frac{7\sqrt{4290}Q_{3x}xz(7x^4 - 4x^2y^2 - 22x^2z^2 - 11y^4 + 26y^2z^2 + 4z^4)}{2288} \\
& - \frac{7\sqrt{4290}Q_{3y}yz(11x^4 + 4x^2y^2 - 26x^2z^2 - 7y^4 + 22y^2z^2 - 4z^4)}{2288} + \frac{7\sqrt{715}Q_{az}(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144} \\
& - \frac{\sqrt{429}Q_{bz}(13x^6 - 45x^4y^2 - 150x^4z^2 - 45x^2y^4 + 540x^2y^2z^2 + 60x^2z^4 + 13y^6 - 150y^4z^2 + 60y^2z^4 - 8z^6)}{1144}
\end{aligned}$$

\* Harmonics for rank 4

$\vec{\mathbb{Q}}_4^{(3,-3)}[q](A_1, 1)$

\*\* symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

\*\* expression

$$-\frac{\sqrt{42}Q_{3x}x}{14} - \frac{\sqrt{42}Q_{3y}y}{14} + \frac{2\sqrt{7}Q_{az}z}{7}$$

$\vec{\mathbb{Q}}_4^{(3,-3)}[q](A_1, 2)$

\*\* symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

\*\* expression

$$\frac{Q_1z}{2} + \frac{\sqrt{6}Q_{3x}}{4} + \frac{\sqrt{6}Q_{bz}y}{4}$$

$\vec{\mathbb{Q}}_4^{(3,-1)}[q](A_1, 1)$

\*\* symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

\*\* expression

$$\begin{aligned}
& \frac{3\sqrt{385}Q_1y(3x^2 - y^2)}{308} + \frac{3\sqrt{385}Q_2x(x^2 - 3y^2)}{308} - \frac{\sqrt{2310}Q_{3xy}z}{22} - \frac{\sqrt{231}Q_{3x}x(x^2 + y^2 - 4z^2)}{308} \\
& - \frac{\sqrt{231}Q_{3y}y(x^2 + y^2 - 4z^2)}{308} - \frac{3\sqrt{154}Q_{az}z(3x^2 + 3y^2 - 2z^2)}{154} - \frac{\sqrt{2310}Q_{bz}z(x-y)(x+y)}{44}
\end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,-1)}[q](A_1, 2)$

\*\* symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

\*\* expression

$$\begin{aligned}
& \frac{3\sqrt{22}Q_1z(3x^2 + 3y^2 - 2z^2)}{44} - \frac{\sqrt{33}Q_3x(x^2 + y^2 - 4z^2)}{44} + \frac{\sqrt{330}Q_{3x}xyz}{22} \\
& + \frac{\sqrt{330}Q_{3y}z(x-y)(x+y)}{44} - \frac{3\sqrt{55}Q_{az}y(3x^2 - y^2)}{44} - \frac{\sqrt{33}Q_{bz}y(x^2 + y^2 - 4z^2)}{44}
\end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,1)}[q](A_1, 1)$

\*\* symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

\*\* expression

$$\begin{aligned}
& \frac{15\sqrt{2002}Q_1y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{2288} + \frac{15\sqrt{2002}Q_2x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{2288} + \frac{5\sqrt{3003}Q_{3xy}z(x^2 + y^2 - 2z^2)}{286} \\
& + \frac{\sqrt{30030}Q_{3x}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{1456} + \frac{\sqrt{30030}Q_{3y}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{1456} \\
& + \frac{3\sqrt{5005}Q_{az}z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{4004} + \frac{5\sqrt{3003}Q_{bz}z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{572}
\end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,1)}[q](A_1, 2)$

\*\* symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{715}Q_1z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{2288} - \frac{\sqrt{4290}Q_3x(8x^4 - 47x^2y^2 - 33x^2z^2 + 29y^4 - 33y^2z^2 + 22z^4)}{1144} \\ & - \frac{7\sqrt{429}Q_{3x}xyz(2x^2 + 5y^2 - 7z^2)}{286} - \frac{7\sqrt{429}Q_{3y}z(17x^4 - 18x^2y^2 - 28x^2z^2 - 11y^4 + 28y^2z^2)}{2288} \\ & + \frac{21\sqrt{286}Q_{az}y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{2288} + \frac{\sqrt{4290}Q_{bz}y(47x^4 - 116x^2y^2 + 66x^2z^2 + 5y^4 + 66y^2z^2 - 44z^4)}{2288} \end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,3)}[q](A_1, 1)$

\*\* symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{2002}Q_1y(3x^2 - y^2)(3x^4 + 6x^2y^2 - 60x^2z^2 + 3y^4 - 60y^2z^2 + 80z^4)}{4576} \\ & + \frac{3\sqrt{2002}Q_2x(x^2 - 3y^2)(3x^4 + 6x^2y^2 - 60x^2z^2 + 3y^4 - 60y^2z^2 + 80z^4)}{4576} + \frac{3\sqrt{3003}Q_3xyz(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{1144} \\ & - \frac{\sqrt{3003}Q_{3x}x(5x^6 + 15x^4y^2 - 120x^4z^2 + 15x^2y^4 - 240x^2y^2z^2 + 240x^2z^4 + 5y^6 - 120y^4z^2 + 240y^2z^4 - 64z^6)}{4576} \\ & - \frac{\sqrt{3003}Q_{3y}y(5x^6 + 15x^4y^2 - 120x^4z^2 + 15x^2y^4 - 240x^2y^2z^2 + 240x^2z^4 + 5y^6 - 120y^4z^2 + 240y^2z^4 - 64z^6)}{4576} \\ & - \frac{\sqrt{5005}Q_{az}z(35x^6 + 105x^4y^2 - 210x^4z^2 + 105x^2y^4 - 420x^2y^2z^2 + 168x^2z^4 + 35y^6 - 210y^4z^2 + 168y^2z^4 - 16z^6)}{2288} \\ & + \frac{3\sqrt{3003}Q_{bz}z(x - y)(x + y)(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{2288} \end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,3)}[q](A_1, 2)$

\*\* symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

\*\* expression

$$\begin{aligned} & - \frac{\sqrt{715}Q_1z(197x^6 - 3270x^4y^2 + 105x^4z^2 + 3165x^2y^4 + 210x^2y^2z^2 - 84x^2z^4 - 232y^6 + 105y^4z^2 - 84y^2z^4 + 8z^6)}{5720} \\ & + \frac{3\sqrt{715}Q_2xyz(x^2 - 3y^2)(3x^2 - y^2)}{40} \\ & + \frac{\sqrt{4290}Q_3x(7x^6 - 78x^4y^2 - 69x^4z^2 - 45x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 40y^6 - 465y^4z^2 - 60y^2z^4 + 16z^6)}{2860} \\ & - \frac{3\sqrt{429}Q_{3x}xyz(81x^4 + 30x^2y^2 - 300x^2z^2 - 51y^4 + 140y^2z^2 + 48z^4)}{1144} \\ & + \frac{3\sqrt{429}Q_{3y}z(9x^6 - 90x^4y^2 - 15x^4z^2 - 75x^2y^4 + 330x^2y^2z^2 - 24x^2z^4 + 24y^6 - 95y^4z^2 + 24y^2z^4)}{1144} \\ & + \frac{3\sqrt{286}Q_{az}y(3x^2 - y^2)(3x^4 + 6x^2y^2 - 60x^2z^2 + 3y^4 - 60y^2z^2 + 80z^4)}{1144} \\ & - \frac{\sqrt{4290}Q_{bz}y(85x^6 - 75x^4y^2 - 1050x^4z^2 - 141x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 19y^6 - 258y^4z^2 + 120y^2z^4 - 32z^6)}{5720} \end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,-3)}[q](A_2)$

\*\* symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

\*\* expression

$$\frac{Q_2z}{2} - \frac{\sqrt{6}Q_3y}{4} + \frac{\sqrt{6}Q_{bz}x}{4}$$

$\vec{\mathbb{Q}}_4^{(3,-1)}[q](A_2)$

\*\* symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{22}Q_2z(3x^2 + 3y^2 - 2z^2)}{44} + \frac{\sqrt{33}Q_3y(x^2 + y^2 - 4z^2)}{44} + \frac{\sqrt{330}Q_{3x}z(x - y)(x + y)}{44} \\ & - \frac{\sqrt{330}Q_{3y}xyz}{22} - \frac{3\sqrt{55}Q_{az}x(x^2 - 3y^2)}{44} - \frac{\sqrt{33}Q_{bz}x(x^2 + y^2 - 4z^2)}{44} \end{aligned}$$

$$\vec{\mathbb{Q}}_4^{(3,1)}[q](A_2)$$

\*\* symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{715}Q_2z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{2288} + \frac{\sqrt{4290}Q_3y(29x^4 - 47x^2y^2 - 33x^2z^2 + 8y^4 - 33y^2z^2 + 22z^4)}{1144} \\ & - \frac{7\sqrt{429}Q_{3x}z(11x^4 + 18x^2y^2 - 28x^2z^2 - 17y^4 + 28y^2z^2)}{2288} + \frac{7\sqrt{429}Q_{3y}xyz(5x^2 + 2y^2 - 7z^2)}{286} \\ & + \frac{21\sqrt{286}Q_{az}x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{2288} + \frac{\sqrt{4290}Q_{bz}x(5x^4 - 116x^2y^2 + 66x^2z^2 + 47y^4 + 66y^2z^2 - 44z^4)}{2288} \end{aligned}$$

$$\vec{\mathbb{Q}}_4^{(3,3)}[q](A_2)$$

\*\* symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{715}Q_1xyz(x^2 - 3y^2)(3x^2 - y^2)}{40} \\ & + \frac{\sqrt{715}Q_2z(232x^6 - 3165x^4y^2 - 105x^4z^2 + 3270x^2y^4 - 210x^2y^2z^2 + 84x^2z^4 - 197y^6 - 105y^4z^2 + 84y^2z^4 - 8z^6)}{5720} \\ & - \frac{\sqrt{4290}Q_3y(40x^6 - 45x^4y^2 - 465x^4z^2 - 78x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 7y^6 - 69y^4z^2 - 60y^2z^4 + 16z^6)}{2860} \\ & - \frac{3\sqrt{429}Q_{3x}z(24x^6 - 75x^4y^2 - 95x^4z^2 - 90x^2y^4 + 330x^2y^2z^2 + 24x^2z^4 + 9y^6 - 15y^4z^2 - 24y^2z^4)}{1144} \\ & - \frac{3\sqrt{429}Q_{3y}xyz(51x^4 - 30x^2y^2 - 140x^2z^2 - 81y^4 + 300y^2z^2 - 48z^4)}{1144} + \frac{3\sqrt{286}Q_{az}x(x^2 - 3y^2)(3x^4 + 6x^2y^2 - 60x^2z^2 + 3y^4 - 60y^2z^2 + 80z^4)}{1144} \\ & - \frac{\sqrt{4290}Q_{bz}x(19x^6 - 141x^4y^2 - 258x^4z^2 - 75x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 85y^6 - 1050y^4z^2 + 120y^2z^4 - 32z^6)}{5720} \end{aligned}$$

$$\vec{\mathbb{Q}}_{4,1}^{(3,-3)}[q](E, 1), \vec{\mathbb{Q}}_{4,2}^{(3,-3)}[q](E, 1)$$

\*\* symmetry

$$-\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

\*\* expression

$$\begin{aligned} & -\frac{\sqrt{42}Q_3y}{28} + \frac{\sqrt{105}Q_{3x}z}{14} + \frac{\sqrt{70}Q_{az}x}{14} - \frac{\sqrt{42}Q_{bz}x}{28} \\ & - \frac{\sqrt{42}Q_{3x}}{28} + \frac{\sqrt{105}Q_{3y}z}{14} + \frac{\sqrt{70}Q_{az}y}{14} + \frac{\sqrt{42}Q_{bz}y}{28} \end{aligned}$$

$$\vec{\mathbb{Q}}_{4,1}^{(3,-3)}[q](E, 2), \vec{\mathbb{Q}}_{4,2}^{(3,-3)}[q](E, 2)$$

\*\* symmetry

$$-\frac{\sqrt{35}xy(x - y)(x + y)}{2}$$

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

\*\* expression

$$-\frac{\sqrt{2}Q_1x}{2} - \frac{\sqrt{2}Q_2y}{2}$$

$$-\frac{\sqrt{2}Q_1y}{2} + \frac{\sqrt{2}Q_2x}{2}$$

$$\vec{\mathbb{Q}}_{4,1}^{(3,-3)}[q](E, 3), \vec{\mathbb{Q}}_{4,2}^{(3,-3)}[q](E, 3)$$

\*\* symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

$$-\frac{\sqrt{5} (x-y) (x+y) (x^2+y^2-6 z^2)}{4}$$

\*\* expression

$$-\frac{\sqrt{14} Q_1 x}{28} + \frac{\sqrt{14} Q_2 y}{28} + \frac{\sqrt{21} Q_3 z}{7} + \frac{\sqrt{210} Q_{3x} y}{28} + \frac{\sqrt{210} Q_{3y} x}{28}$$

$$-\frac{\sqrt{14} Q_1 y}{28} - \frac{\sqrt{14} Q_2 x}{28} + \frac{\sqrt{210} Q_{3x} x}{28} - \frac{\sqrt{210} Q_{3y} y}{28} + \frac{\sqrt{21} Q_{bz} z}{7}$$

$$\vec{\mathbb{Q}}_{4,1}^{(3,-1)}[q](E,1), \vec{\mathbb{Q}}_{4,2}^{(3,-1)}[q](E,1)$$

\*\* symmetry

$$-\frac{\sqrt{10} x z (3 x^2 + 3 y^2 - 4 z^2)}{4}$$

$$-\frac{\sqrt{10} y z (3 x^2 + 3 y^2 - 4 z^2)}{4}$$

\*\* expression

$$-\frac{15 \sqrt{154} Q_1 x y z}{154} - \frac{15 \sqrt{154} Q_2 z (x-y) (x+y)}{308} - \frac{\sqrt{231} Q_3 y (19 x^2 - y^2 - 16 z^2)}{308} + \frac{\sqrt{2310} Q_{3x} z (x^2 - 7 y^2 + 2 z^2)}{308} \\ + \frac{2 \sqrt{2310} Q_{3y} x y z}{77} - \frac{3 \sqrt{385} Q_{az} x (x^2 + y^2 - 4 z^2)}{308} - \frac{\sqrt{231} Q_{bz} x (9 x^2 - 11 y^2 - 16 z^2)}{308}$$

$$-\frac{15 \sqrt{154} Q_1 z (x-y) (x+y)}{308} + \frac{15 \sqrt{154} Q_2 x y z}{154} + \frac{\sqrt{231} Q_3 x (x^2 - 19 y^2 + 16 z^2)}{308} + \frac{2 \sqrt{2310} Q_{3x} x y z}{77} \\ - \frac{\sqrt{2310} Q_{3y} z (7 x^2 - y^2 - 2 z^2)}{308} - \frac{3 \sqrt{385} Q_{az} y (x^2 + y^2 - 4 z^2)}{308} - \frac{\sqrt{231} Q_{bz} y (11 x^2 - 9 y^2 + 16 z^2)}{308}$$

$$\vec{\mathbb{Q}}_{4,1}^{(3,-1)}[q](E,2), \vec{\mathbb{Q}}_{4,2}^{(3,-1)}[q](E,2)$$

\*\* symmetry

$$-\frac{\sqrt{35} x y (x-y) (x+y)}{2}$$

$$-\frac{\sqrt{35} (x^2 - 2 x y - y^2) (x^2 + 2 x y - y^2)}{8}$$

\*\* expression

$$-\frac{3 \sqrt{11} Q_1 x (x^2 + y^2 - 4 z^2)}{44} - \frac{3 \sqrt{11} Q_2 y (x^2 + y^2 - 4 z^2)}{44} - \frac{5 \sqrt{66} Q_3 z (x-y) (x+y)}{44} + \frac{\sqrt{165} Q_{3x} y (3 x^2 - y^2)}{44} + \frac{\sqrt{165} Q_{3y} x (x^2 - 3 y^2)}{44} - \frac{5 \sqrt{66} Q_{bz} x y z}{22} \\ - \frac{3 \sqrt{11} Q_1 y (x^2 + y^2 - 4 z^2)}{44} + \frac{3 \sqrt{11} Q_2 x (x^2 + y^2 - 4 z^2)}{44} - \frac{5 \sqrt{66} Q_{3x} y z}{22} - \frac{\sqrt{165} Q_{3x} x (x^2 - 3 y^2)}{44} + \frac{\sqrt{165} Q_{3y} y (3 x^2 - y^2)}{44} + \frac{5 \sqrt{66} Q_{bz} z (x-y) (x+y)}{44}$$

$$\vec{\mathbb{Q}}_{4,1}^{(3,-1)}[q](E,3), \vec{\mathbb{Q}}_{4,2}^{(3,-1)}[q](E,3)$$

\*\* symmetry

$$-\frac{\sqrt{5} x y (x^2 + y^2 - 6 z^2)}{2}$$

$$-\frac{\sqrt{5} (x-y) (x+y) (x^2 + y^2 - 6 z^2)}{4}$$

\*\* expression

$$-\frac{9 \sqrt{77} Q_1 x (x^2 + y^2 - 4 z^2)}{308} + \frac{9 \sqrt{77} Q_2 y (x^2 + y^2 - 4 z^2)}{308} + \frac{\sqrt{462} Q_3 z (3 x^2 + 3 y^2 - 2 z^2)}{308} \\ + \frac{\sqrt{1155} Q_{3x} y (7 x^2 - 5 y^2 + 8 z^2)}{308} - \frac{\sqrt{1155} Q_{3y} x (5 x^2 - 7 y^2 - 8 z^2)}{308} - \frac{3 \sqrt{770} Q_{az} x y z}{154}$$

$$-\frac{9 \sqrt{77} Q_1 y (x^2 + y^2 - 4 z^2)}{308} - \frac{9 \sqrt{77} Q_2 x (x^2 + y^2 - 4 z^2)}{308} + \frac{\sqrt{1155} Q_{3x} x (x^2 - 11 y^2 + 8 z^2)}{308} \\ + \frac{\sqrt{1155} Q_{3y} y (11 x^2 - y^2 - 8 z^2)}{308} - \frac{3 \sqrt{770} Q_{az} z (x-y) (x+y)}{308} + \frac{\sqrt{462} Q_{bz} z (3 x^2 + 3 y^2 - 2 z^2)}{308}$$

$$\vec{\mathbb{Q}}_{4,1}^{(3,1)}[q](E,1), \vec{\mathbb{Q}}_{4,2}^{(3,1)}[q](E,1)$$

\*\* symmetry

$$-\frac{\sqrt{10} x z (3 x^2 + 3 y^2 - 4 z^2)}{4}$$

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

\*\* expression

$$\begin{aligned} & -\frac{3\sqrt{5005}Q_1xyz(7x^2 - 2y^2 - 5z^2)}{286} - \frac{3\sqrt{5005}Q_2z(19x^4 - 54x^2y^2 - 20x^2z^2 - y^4 + 20y^2z^2)}{2288} \\ & + \frac{\sqrt{30030}Q_3y(x^4 + 9x^2y^2 - 33x^2z^2 + 8y^4 - 89y^2z^2 + 50z^4)}{8008} - \frac{\sqrt{3003}Q_{3x}z(209x^4 - 30x^2y^2 - 408x^2z^2 - 239y^4 + 488y^2z^2 - 8z^4)}{16016} \\ & - \frac{4\sqrt{3003}Q_{3y}xyz(x^2 + y^2 - 2z^2)}{143} + \frac{57\sqrt{2002}Q_{az}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{16016} \\ & + \frac{\sqrt{30030}Q_{bz}x(9x^4 + 32x^2y^2 - 122x^2z^2 + 23y^4 - 234y^2z^2 + 100z^4)}{16016} \\ & - \frac{3\sqrt{5005}Q_1z(x^4 + 54x^2y^2 - 20x^2z^2 - 19y^4 + 20y^2z^2)}{2288} - \frac{3\sqrt{5005}Q_2xyz(2x^2 - 7y^2 + 5z^2)}{286} \\ & + \frac{\sqrt{30030}Q_{3x}(8x^4 + 9x^2y^2 - 89x^2z^2 + y^4 - 33y^2z^2 + 50z^4)}{8008} - \frac{4\sqrt{3003}Q_{3x}xyz(x^2 + y^2 - 2z^2)}{143} \\ & + \frac{\sqrt{3003}Q_{3y}z(239x^4 + 30x^2y^2 - 488x^2z^2 - 209y^4 + 408y^2z^2 + 8z^4)}{16016} \\ & + \frac{57\sqrt{2002}Q_{az}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{16016} - \frac{\sqrt{30030}Q_{bz}y(23x^4 + 32x^2y^2 - 234x^2z^2 + 9y^4 - 122y^2z^2 + 100z^4)}{16016} \end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,1)}[q](E, 2), \vec{\mathbb{Q}}_{4,2}^{(3,1)}[q](E, 2)$

\*\* symmetry

$$-\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

\*\* expression

$$\begin{aligned} & -\frac{3\sqrt{1430}Q_1x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} - \frac{3\sqrt{1430}Q_2y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} \\ & - \frac{7\sqrt{2145}Q_3z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{572} + \frac{7\sqrt{858}Q_{3x}y(21x^4 - 26x^2y^2 - 48x^2z^2 + y^4 + 16y^2z^2)}{2288} \\ & - \frac{7\sqrt{858}Q_{3y}x(x^4 - 26x^2y^2 + 16x^2z^2 + 21y^4 - 48y^2z^2)}{2288} + \frac{63\sqrt{143}Q_{az}xyz(x-y)(x+y)}{143} - \frac{7\sqrt{2145}Q_{bz}xyz(x^2 + y^2 - 2z^2)}{286} \\ & - \frac{3\sqrt{1430}Q_1y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} + \frac{3\sqrt{1430}Q_2x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} \\ & - \frac{7\sqrt{2145}Q_3xyz(x^2 + y^2 - 2z^2)}{286} - \frac{7\sqrt{858}Q_{3x}x(5x^4 - 34x^2y^2 - 16x^2z^2 + 9y^4 + 48y^2z^2)}{2288} - \frac{7\sqrt{858}Q_{3y}y(9x^4 - 34x^2y^2 + 48x^2z^2 + 5y^4 - 16y^2z^2)}{2288} \\ & - \frac{63\sqrt{143}Q_{az}z(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{572} + \frac{7\sqrt{2145}Q_{bz}z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{572} \end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,1)}[q](E, 3), \vec{\mathbb{Q}}_{4,2}^{(3,1)}[q](E, 3)$

\*\* symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{9\sqrt{10010}Q_1x(x^4 - 40x^2y^2 + 30x^2z^2 + 15y^4 + 30y^2z^2 - 20z^4)}{16016} - \frac{9\sqrt{10010}Q_2y(15x^4 - 40x^2y^2 + 30x^2z^2 + y^4 + 30y^2z^2 - 20z^4)}{16016} \\ & - \frac{\sqrt{15015}Q_3z(51x^4 - 66x^2y^2 - 80x^2z^2 + 51y^4 - 80y^2z^2 + 16z^4)}{4004} - \frac{\sqrt{6006}Q_{3x}y(73x^4 + 20x^2y^2 - 498x^2z^2 - 53y^4 + 510y^2z^2 - 172z^4)}{16016} \\ & + \frac{\sqrt{6006}Q_{3y}x(53x^4 - 20x^2y^2 - 510x^2z^2 - 73y^4 + 498y^2z^2 + 172z^4)}{16016} - \frac{3\sqrt{1001}Q_{az}xyz(x^2 + y^2 - 2z^2)}{286} + \frac{3\sqrt{15015}Q_{bz}xyz(x-y)(x+y)}{143} \\ & - \frac{9\sqrt{10010}Q_1y(10x^4 - 15x^2y^2 - 15x^2z^2 + 3y^4 - 15y^2z^2 + 10z^4)}{8008} - \frac{9\sqrt{10010}Q_2x(3x^4 - 15x^2y^2 - 15x^2z^2 + 10y^4 - 15y^2z^2 + 10z^4)}{8008} \\ & + \frac{3\sqrt{15015}Q_3xyz(x-y)(x+y)}{143} - \frac{\sqrt{6006}Q_{3x}x(5x^4 - 53x^2y^2 + 3x^2z^2 - 58y^4 + 507y^2z^2 - 86z^4)}{8008} \\ & - \frac{\sqrt{6006}Q_{3y}y(58x^4 + 53x^2y^2 - 507x^2z^2 - 5y^4 - 3y^2z^2 + 86z^4)}{8008} - \frac{3\sqrt{1001}Q_{az}z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{572} \\ & - \frac{\sqrt{15015}Q_{bz}z(9x^4 + 186x^2y^2 - 80x^2z^2 + 9y^4 - 80y^2z^2 + 16z^4)}{4004} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{4,1}^{(3,3)}[q](E, 1), \tilde{\mathbb{Q}}_{4,2}^{(3,3)}[q](E, 1)$$

\*\* symmetry

$$-\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

\*\* expression

$$\begin{aligned} & -\frac{3\sqrt{5005}Q_1xyz(81x^4 + 30x^2y^2 - 300x^2z^2 - 51y^4 + 140y^2z^2 + 48z^4)}{5720} \\ & -\frac{3\sqrt{5005}Q_2z(24x^6 - 75x^4y^2 - 95x^4z^2 - 90x^2y^4 + 330x^2y^2z^2 + 24x^2z^4 + 9y^6 - 15y^4z^2 - 24y^2z^4)}{5720} \\ & +\frac{\sqrt{30030}Q_3y(8x^6 + 15x^4y^2 - 165x^4z^2 + 6x^2y^4 - 150x^2y^2z^2 + 240x^2z^4 - y^6 + 15y^4z^2 - 16z^6)}{2860} \\ & +\frac{\sqrt{3003}Q_3xz(40x^6 + 75x^4y^2 - 225x^4z^2 + 30x^2y^4 - 210x^2y^2z^2 + 156x^2z^4 - 5y^6 + 15y^4z^2 + 12y^2z^4 - 8z^6)}{1144} \\ & +\frac{3\sqrt{3003}Q_{3y}xyz(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{1144} \\ & -\frac{\sqrt{2002}Q_{az}x(5x^6 + 15x^4y^2 - 120x^4z^2 + 15x^2y^4 - 240x^2y^2z^2 + 240x^2z^4 + 5y^6 - 120y^4z^2 + 240y^2z^4 - 64z^6)}{1144} \\ & +\frac{\sqrt{30030}Q_{bz}x(7x^6 + 3x^4y^2 - 150x^4z^2 - 15x^2y^4 + 60x^2y^2z^2 + 240x^2z^4 - 11y^6 + 210y^4z^2 - 240y^2z^4 - 32z^6)}{5720} \end{aligned}$$

$$\begin{aligned} & \frac{3\sqrt{5005}Q_1z(9x^6 - 90x^4y^2 - 15x^4z^2 - 75x^2y^4 + 330x^2y^2z^2 - 24x^2z^4 + 24y^6 - 95y^4z^2 + 24y^2z^4)}{5720} \\ & -\frac{3\sqrt{5005}Q_2xyz(51x^4 - 30x^2y^2 - 140x^2z^2 - 81y^4 + 300y^2z^2 - 48z^4)}{5720} \\ & -\frac{\sqrt{30030}Q_3x(x^6 - 6x^4y^2 - 15x^4z^2 - 15x^2y^4 + 150x^2y^2z^2 - 8y^6 + 165y^4z^2 - 240y^2z^4 + 16z^6)}{2860} \\ & +\frac{3\sqrt{3003}Q_{3x}xyz(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{1144} \\ & -\frac{\sqrt{3003}Q_{3y}z(5x^6 - 30x^4y^2 - 15x^4z^2 - 75x^2y^4 + 210x^2y^2z^2 - 12x^2z^4 - 40y^6 + 225y^4z^2 - 156y^2z^4 + 8z^6)}{1144} \\ & -\frac{\sqrt{2002}Q_{az}y(5x^6 + 15x^4y^2 - 120x^4z^2 + 15x^2y^4 - 240x^2y^2z^2 + 240x^2z^4 + 5y^6 - 120y^4z^2 + 240y^2z^4 - 64z^6)}{1144} \\ & +\frac{\sqrt{30030}Q_{bz}y(11x^6 + 15x^4y^2 - 210x^4z^2 - 3x^2y^4 - 60x^2y^2z^2 + 240x^2z^4 - 7y^6 + 150y^4z^2 - 240y^2z^4 + 32z^6)}{5720} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{4,1}^{(3,3)}[q](E, 2), \tilde{\mathbb{Q}}_{4,2}^{(3,3)}[q](E, 2)$$

\*\* symmetry

$$-\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{1430}Q_1x(53x^6 - 1128x^4y^2 + 15x^4z^2 + 1875x^2y^4 + 30x^2y^2z^2 - 30x^2z^4 - 376y^6 + 15y^4z^2 - 30y^2z^4 + 8z^6)}{5720} \\ & -\frac{\sqrt{1430}Q_2y(376x^6 - 1875x^4y^2 - 15x^4z^2 + 1128x^2y^4 - 30x^2y^2z^2 + 30x^2z^4 - 53y^6 - 15y^4z^2 + 30y^2z^4 - 8z^6)}{5720} \\ & +\frac{3\sqrt{2145}Q_3z(x-y)(x+y)(8x^4 - 127x^2y^2 + 5x^2z^2 + 8y^4 + 5y^2z^2 - 3z^4)}{1430} \\ & +\frac{3\sqrt{858}Q_{3x}y(8x^6 - 5x^4y^2 - 105x^4z^2 - 12x^2y^4 + 150x^2y^2z^2 + 30x^2z^4 + y^6 - 9y^4z^2 - 10y^2z^4)}{1144} \\ & -\frac{3\sqrt{858}Q_{3y}x(x^6 - 12x^4y^2 - 9x^4z^2 - 5x^2y^4 + 150x^2y^2z^2 - 10x^2z^4 + 8y^6 - 105y^4z^2 + 30y^2z^4)}{1144} \\ & +\frac{3\sqrt{143}Q_{az}xyz(x-y)(x+y)(3x^2 + 3y^2 - 10z^2)}{52} -\frac{3\sqrt{2145}Q_{bz}xyz(111x^4 - 350x^2y^2 - 20x^2z^2 + 111y^4 - 20y^2z^2 + 12z^4)}{2860} \end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{1430}Q_1y(1499x^6 - 7515x^4y^2 + 60x^4z^2 + 4497x^2y^4 + 120x^2y^2z^2 - 120x^2z^4 - 217y^6 + 60y^4z^2 - 120y^2z^4 + 32z^6)}{22880} \\
& + \frac{\sqrt{1430}Q_2x(217x^6 - 4497x^4y^2 - 60x^4z^2 + 7515x^2y^4 - 120x^2y^2z^2 + 120x^2z^4 - 1499y^6 - 60y^4z^2 + 120y^2z^4 - 32z^6)}{22880} \\
& + \frac{3\sqrt{2145}Q_3xyz(207x^4 - 730x^2y^2 + 40x^2z^2 + 207y^4 + 40y^2z^2 - 24z^4)}{5720} \\
& - \frac{3\sqrt{858}Q_{3x}x(7x^6 - 51x^4y^2 - 96x^4z^2 - 35x^2y^4 + 720x^2y^2z^2 + 40x^2z^4 + 23y^6 - 240y^4z^2 - 120y^2z^4)}{4576} \\
& - \frac{3\sqrt{858}Q_{3y}y(23x^6 - 35x^4y^2 - 240x^4z^2 - 51x^2y^4 + 720x^2y^2z^2 - 120x^2z^4 + 7y^6 - 96y^4z^2 + 40y^2z^4)}{4576} \\
& - \frac{3\sqrt{143}Q_{az}z(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)(3x^2 + 3y^2 - 10z^2)}{208} \\
& + \frac{3\sqrt{2145}Q_{bz}z(x - y)(x + y)(79x^4 - 986x^2y^2 - 40x^2z^2 + 79y^4 - 40y^2z^2 + 24z^4)}{11440}
\end{aligned}$$

$\tilde{\mathbb{Q}}_{4,1}^{(3,3)}[q](E, 3), \tilde{\mathbb{Q}}_{4,2}^{(3,3)}[q](E, 3)$

\*\* symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

$$-\frac{\sqrt{5}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{4}$$

\*\* expression

$$\begin{aligned}
& \frac{\sqrt{10010}Q_1x(7x^6 - 78x^4y^2 - 69x^4z^2 - 45x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 40y^6 - 465y^4z^2 - 60y^2z^4 + 16z^6)}{5720} \\
& - \frac{\sqrt{10010}Q_2y(40x^6 - 45x^4y^2 - 465x^4z^2 - 78x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 7y^6 - 69y^4z^2 - 60y^2z^4 + 16z^6)}{5720} \\
& + \frac{\sqrt{15015}Q_3z(8x^6 - 75x^4y^2 - 15x^4z^2 - 75x^2y^4 + 300x^2y^2z^2 - 21x^2z^4 + 8y^6 - 15y^4z^2 - 21y^2z^4 + 2z^6)}{1430} \\
& + \frac{\sqrt{6006}Q_{3x}y(8x^6 + 15x^4y^2 - 165x^4z^2 + 6x^2y^4 - 150x^2y^2z^2 + 240x^2z^4 - y^6 + 15y^4z^2 - 16z^6)}{1144} \\
& - \frac{\sqrt{6006}Q_{3y}x(x^6 - 6x^4y^2 - 15x^4z^2 - 15x^2y^4 + 150x^2y^2z^2 - 8y^6 + 165y^4z^2 - 240y^2z^4 + 16z^6)}{1144} \\
& + \frac{3\sqrt{1001}Q_{az}xyz(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{572} - \frac{3\sqrt{15015}Q_{bz}xyz(x - y)(x + y)(3x^2 + 3y^2 - 10z^2)}{260}
\end{aligned}$$

$$\begin{aligned}
& - \frac{\sqrt{10010}Q_1y(85x^6 - 75x^4y^2 - 1050x^4z^2 - 141x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 19y^6 - 258y^4z^2 + 120y^2z^4 - 32z^6)}{11440} \\
& - \frac{\sqrt{10010}Q_2x(19x^6 - 141x^4y^2 - 258x^4z^2 - 75x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 85y^6 - 1050y^4z^2 + 120y^2z^4 - 32z^6)}{11440} \\
& - \frac{3\sqrt{15015}Q_3xyz(x - y)(x + y)(3x^2 + 3y^2 - 10z^2)}{260} \\
& + \frac{\sqrt{6006}Q_{3x}x(7x^6 + 3x^4y^2 - 150x^4z^2 - 15x^2y^4 + 60x^2y^2z^2 + 240x^2z^4 - 11y^6 + 210y^4z^2 - 240y^2z^4 - 32z^6)}{2288} \\
& + \frac{\sqrt{6006}Q_{3y}y(11x^6 + 15x^4y^2 - 210x^4z^2 - 3x^2y^4 - 60x^2y^2z^2 + 240x^2z^4 - 7y^6 + 150y^4z^2 - 240y^2z^4 + 32z^6)}{2288} \\
& + \frac{3\sqrt{1001}Q_{az}z(x - y)(x + y)(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{1144} \\
& - \frac{\sqrt{15015}Q_{bz}z(67x^6 - 195x^4y^2 - 270x^4z^2 - 195x^2y^4 + 780x^2y^2z^2 + 84x^2z^4 + 67y^6 - 270y^4z^2 + 84y^2z^4 - 8z^6)}{5720}
\end{aligned}$$