

PG No. 35 $C_{3v}(1)$ 3m (31m setting) [trigonal] (axial, internal axial octupole)

* Harmonics for rank 0

$$\vec{\mathbb{G}}_0^{(3,3)}[g](A_2)$$

** symmetry

1

** expression

$$\begin{aligned} & \frac{\sqrt{70}G_1y(3x^2-y^2)}{28} + \frac{\sqrt{70}G_2x(x^2-3y^2)}{28} + \frac{\sqrt{105}G_3xyz}{7} - \frac{\sqrt{42}G_{3x}x(x^2+y^2-4z^2)}{28} \\ & - \frac{\sqrt{42}G_{3y}y(x^2+y^2-4z^2)}{28} - \frac{\sqrt{7}G_{az}z(3x^2+3y^2-2z^2)}{14} + \frac{\sqrt{105}G_{bz}z(x-y)(x+y)}{14} \end{aligned}$$

* Harmonics for rank 1

$$\vec{\mathbb{G}}_1^{(3,1)}[g](A_2)$$

** symmetry

z

** expression

$$\begin{aligned} & \frac{\sqrt{21}G_{3xy}}{7} + \frac{2\sqrt{210}G_{3x}xz}{35} + \frac{2\sqrt{210}G_{3y}yz}{35} - \frac{3\sqrt{35}G_{az}(x^2+y^2-2z^2)}{70} + \frac{\sqrt{21}G_{bz}(x-y)(x+y)}{14} \end{aligned}$$

$$\vec{\mathbb{G}}_1^{(3,3)}[g](A_2)$$

** symmetry

z

** expression

$$\begin{aligned} & \frac{\sqrt{210}G_1yz(3x^2-y^2)}{24} + \frac{\sqrt{210}G_2xz(x^2-3y^2)}{24} - \frac{\sqrt{35}G_3xy(x^2+y^2-6z^2)}{14} - \frac{5\sqrt{14}G_{3x}xz(3x^2+3y^2-4z^2)}{56} \\ & - \frac{5\sqrt{14}G_{3y}yz(3x^2+3y^2-4z^2)}{56} + \frac{\sqrt{21}G_{az}(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4)}{84} - \frac{\sqrt{35}G_{bz}(x-y)(x+y)(x^2+y^2-6z^2)}{28} \end{aligned}$$

$$\vec{\mathbb{G}}_{1,1}^{(3,1)}[g](E), \vec{\mathbb{G}}_{1,2}^{(3,1)}[g](E)$$

** symmetry

$-y$

x

** expression

$$\begin{aligned} & -\frac{3\sqrt{14}G_1(x-y)(x+y)}{28} + \frac{3\sqrt{14}G_2xy}{14} - \frac{\sqrt{21}G_3xz}{7} + \frac{\sqrt{210}G_{3x}xy}{70} + \frac{\sqrt{210}G_{3y}(x^2+3y^2-4z^2)}{140} + \frac{3\sqrt{35}G_{az}yz}{35} + \frac{\sqrt{21}G_{bz}yz}{7} \\ & - \frac{3\sqrt{14}G_1xy}{14} + \frac{3\sqrt{14}G_2(x-y)(x+y)}{28} + \frac{\sqrt{21}G_3yz}{7} - \frac{\sqrt{210}G_{3x}(3x^2+y^2-4z^2)}{140} - \frac{\sqrt{210}G_{3y}xy}{70} - \frac{3\sqrt{35}G_{az}xz}{35} + \frac{\sqrt{21}G_{bz}xz}{7} \end{aligned}$$

$$\vec{\mathbb{G}}_{1,1}^{(3,3)}[g](E), \vec{\mathbb{G}}_{1,2}^{(3,3)}[g](E)$$

** symmetry

$-y$

x

** expression

$$\begin{aligned} & \frac{\sqrt{210}G_1(3x^4-21x^2y^2+3x^2z^2+4y^4-3y^2z^2)}{168} - \frac{\sqrt{210}G_2xy(13x^2-15y^2+6z^2)}{168} + \frac{\sqrt{35}G_3xz(x^2-6y^2+z^2)}{14} + \frac{5\sqrt{14}G_{3x}xy(x^2+y^2-6z^2)}{56} \\ & - \frac{\sqrt{14}G_{3y}(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4)}{56} + \frac{5\sqrt{21}G_{az}yz(3x^2+3y^2-4z^2)}{84} - \frac{\sqrt{35}G_{bz}yz(9x^2-5y^2+2z^2)}{28} \\ & - \frac{\sqrt{210}G_1xy(15x^2-13y^2-6z^2)}{168} + \frac{\sqrt{210}G_2(4x^4-21x^2y^2-3x^2z^2+3y^4+3y^2z^2)}{168} \\ & + \frac{\sqrt{35}G_3yz(6x^2-y^2-z^2)}{14} - \frac{\sqrt{14}G_{3x}(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4)}{56} \\ & - \frac{5\sqrt{14}G_{3y}xy(x^2+y^2-6z^2)}{56} - \frac{5\sqrt{21}G_{az}xz(3x^2+3y^2-4z^2)}{84} + \frac{\sqrt{35}G_{bz}xz(5x^2-9y^2-2z^2)}{28} \end{aligned}$$

* Harmonics for rank 2

$$\vec{\mathbb{G}}_2^{(3,-1)}[g](A_2)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{14}G_{3xx}}{7} + \frac{\sqrt{14}G_{3yy}}{7} + \frac{\sqrt{21}G_{azz}}{7}$$

$\vec{\mathbb{G}}_2^{(3,1)}[g](A_2)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{5\sqrt{210}G_1y(3x^2 - y^2)}{168} - \frac{5\sqrt{210}G_2x(x^2 - 3y^2)}{168} - \frac{3\sqrt{14}G_{3xx}(x^2 + y^2 - 4z^2)}{56} - \frac{3\sqrt{14}G_{3yy}(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{21}G_{azz}(3x^2 + 3y^2 - 2z^2)}{21}$$

$\vec{\mathbb{G}}_2^{(3,3)}[g](A_2)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{1155}G_1y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{264} - \frac{\sqrt{1155}G_2x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{264} - \frac{3\sqrt{770}G_{3xyz}(x^2 + y^2 - 2z^2)}{44} \\ + \frac{15\sqrt{77}G_{3xx}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{616} + \frac{15\sqrt{77}G_{3yy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{616} \\ + \frac{5\sqrt{462}G_{azz}(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{1848} - \frac{3\sqrt{770}G_{bz}(x-y)(x+y)(x^2 + y^2 - 2z^2)}{88}$$

$\vec{\mathbb{G}}_{2,1}^{(3,-1)}[g](E,1), \vec{\mathbb{G}}_{2,2}^{(3,-1)}[g](E,1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{105}G_{3xx}}{21} + \frac{2\sqrt{42}G_{3yy}}{21} - \frac{\sqrt{7}G_{azz}}{7} - \frac{\sqrt{105}G_{bz}}{21} \\ - \frac{\sqrt{105}G_{3xy}}{21} - \frac{2\sqrt{42}G_{3xz}}{21} + \frac{\sqrt{7}G_{azx}}{7} - \frac{\sqrt{105}G_{bxz}}{21}$$

$\vec{\mathbb{G}}_{2,1}^{(3,-1)}[g](E,2), \vec{\mathbb{G}}_{2,2}^{(3,-1)}[g](E,2)$

** symmetry

$$\sqrt{3}xy$$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{70}G_1x}{14} - \frac{\sqrt{70}G_2y}{14} + \frac{\sqrt{105}G_3z}{21} - \frac{\sqrt{42}G_{3xy}}{42} - \frac{\sqrt{42}G_{3yy}}{42} \\ \frac{\sqrt{70}G_1y}{14} + \frac{\sqrt{70}G_2x}{14} - \frac{\sqrt{42}G_{3xx}}{42} + \frac{\sqrt{42}G_{3yy}}{42} + \frac{\sqrt{105}G_{bz}}{21}$$

$\vec{\mathbb{G}}_{2,1}^{(3,1)}[g](E,1), \vec{\mathbb{G}}_{2,2}^{(3,1)}[g](E,1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{5\sqrt{70}G_1z(x-y)(x+y)}{56} - \frac{5\sqrt{70}G_2xyz}{28} - \frac{\sqrt{105}G_3x(2x^2 - 3y^2 - 3z^2)}{42} + \frac{5\sqrt{42}G_{3xyz}}{28} \\ - \frac{\sqrt{42}G_{3yz}(21x^2 - 9y^2 - 4z^2)}{168} - \frac{\sqrt{7}G_{azy}(x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{105}G_{bz}(9x^2 - y^2 - 6z^2)}{84}$$

$$-\frac{5\sqrt{70}G_1xyz}{28} - \frac{5\sqrt{70}G_2z(x-y)(x+y)}{56} - \frac{\sqrt{105}G_3y(3x^2-2y^2+3z^2)}{42} - \frac{\sqrt{42}G_{3x}z(9x^2-21y^2+4z^2)}{168}$$

$$-\frac{5\sqrt{42}G_{3y}xyz}{28} + \frac{\sqrt{7}G_{az}x(x^2+y^2-4z^2)}{28} - \frac{\sqrt{105}G_{bz}x(x^2-9y^2+6z^2)}{84}$$

$\vec{\mathbb{G}}_{2,1}^{(3,1)}[g](E, 2), \vec{\mathbb{G}}_{2,2}^{(3,1)}[g](E, 2)$

** symmetry

$\sqrt{3}xy$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$-\frac{\sqrt{70}G_1x(x^2+y^2-4z^2)}{56} - \frac{\sqrt{70}G_2y(x^2+y^2-4z^2)}{56} + \frac{\sqrt{105}G_3z(3x^2+3y^2-2z^2)}{42}$$

$$-\frac{\sqrt{42}G_{3x}y(21x^2+y^2-24z^2)}{168} - \frac{\sqrt{42}G_{3y}x(x^2+21y^2-24z^2)}{168} - \frac{5\sqrt{7}G_{az}xyz}{7}$$

$$-\frac{\sqrt{70}G_1y(x^2+y^2-4z^2)}{56} + \frac{\sqrt{70}G_2x(x^2+y^2-4z^2)}{56} - \frac{\sqrt{42}G_{3x}x(11x^2-9y^2-24z^2)}{168}$$

$$-\frac{\sqrt{42}G_{3y}y(9x^2-11y^2+24z^2)}{168} - \frac{5\sqrt{7}G_{az}z(x-y)(x+y)}{14} + \frac{\sqrt{105}G_{bz}z(3x^2+3y^2-2z^2)}{42}$$

$\vec{\mathbb{G}}_{2,1}^{(3,3)}[g](E, 1), \vec{\mathbb{G}}_{2,2}^{(3,3)}[g](E, 1)$

** symmetry

$\sqrt{3}yz$

$$-\sqrt{3}xz$$

** expression

$$-\frac{\sqrt{385}G_1z(x^4-9x^2y^2+x^2z^2+2y^4-y^2z^2)}{44} + \frac{\sqrt{385}G_2xyz(5x^2-7y^2+2z^2)}{44} + \frac{\sqrt{2310}G_3x(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{462}$$

$$-\frac{5\sqrt{231}G_{3x}xyz(x^2+y^2-2z^2)}{44} + \frac{5\sqrt{231}G_{3y}z(3x^4-15x^2y^2-x^2z^2-18y^4+41y^2z^2-4z^4)}{924}$$

$$+\frac{5\sqrt{154}G_{az}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{308} - \frac{\sqrt{2310}G_{bz}y(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4)}{924}$$

$$-\frac{\sqrt{385}G_1xyz(7x^2-5y^2-2z^2)}{44} - \frac{\sqrt{385}G_2z(2x^4-9x^2y^2-x^2z^2+y^4+y^2z^2)}{44} + \frac{\sqrt{2310}G_3y(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{462}$$

$$+\frac{5\sqrt{231}G_{3x}z(18x^4+15x^2y^2-41x^2z^2-3y^4+y^2z^2+4z^4)}{924} + \frac{5\sqrt{231}G_{3y}xyz(x^2+y^2-2z^2)}{44}$$

$$-\frac{5\sqrt{154}G_{az}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{308} + \frac{\sqrt{2310}G_{bz}x(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4)}{924}$$

$\vec{\mathbb{G}}_{2,1}^{(3,3)}[g](E, 2), \vec{\mathbb{G}}_{2,2}^{(3,3)}[g](E, 2)$

** symmetry

$\sqrt{3}xy$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$-\frac{\sqrt{385}G_1x(5x^4-53x^2y^2+3x^2z^2+26y^4+3y^2z^2-2z^4)}{308} + \frac{\sqrt{385}G_2y(26x^4-53x^2y^2+3x^2z^2+5y^4+3y^2z^2-2z^4)}{308}$$

$$-\frac{\sqrt{2310}G_3z(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4)}{462} - \frac{5\sqrt{231}G_{3x}y(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{924}$$

$$+\frac{5\sqrt{231}G_{3y}x(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{924} - \frac{5\sqrt{154}G_{az}xyz(x^2+y^2-2z^2)}{44} + \frac{3\sqrt{2310}G_{bz}xyz(x-y)(x+y)}{44}$$

$$-\frac{\sqrt{385}G_1y(53x^4-104x^2y^2-6x^2z^2+11y^4-6y^2z^2+4z^4)}{616} + \frac{\sqrt{385}G_2x(11x^4-104x^2y^2-6x^2z^2+53y^4-6y^2z^2+4z^4)}{616}$$

$$+\frac{3\sqrt{2310}G_3xyz(x-y)(x+y)}{44} - \frac{5\sqrt{231}G_{3x}x(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4)}{1848}$$

$$-\frac{5\sqrt{231}G_{3y}y(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4)}{1848} - \frac{5\sqrt{154}G_{az}z(x-y)(x+y)(x^2+y^2-2z^2)}{88}$$

$$+\frac{\sqrt{2310}G_{bz}z(39x^4-174x^2y^2-20x^2z^2+39y^4-20y^2z^2+4z^4)}{1848}$$

* Harmonics for rank 3

$$\vec{\mathbb{G}}_3^{(3,-3)}[g](A_1)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$G_1$$

$$\vec{\mathbb{G}}_3^{(3,-1)}[g](A_1)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$\frac{\sqrt{15}G_1(x^2 + y^2 - 2z^2)}{12} + \frac{\sqrt{10}G_3xz}{4} - \frac{G_{3x}xy}{2} - \frac{G_{3y}(x-y)(x+y)}{4} + \frac{\sqrt{10}G_{bz}yz}{4}$$

$$\vec{\mathbb{G}}_3^{(3,1)}[g](A_1)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$\begin{aligned} & \frac{\sqrt{22}G_1(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{176} + \frac{5\sqrt{33}G_3xz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{\sqrt{330}G_{3x}xy(5x^2 - 2y^2 - 9z^2)}{66} \\ & + \frac{\sqrt{330}G_{3y}(x^4 - 42x^2y^2 + 36x^2z^2 + 13y^4 - 36y^2z^2)}{528} - \frac{7\sqrt{55}G_{az}yz(3x^2 - y^2)}{44} + \frac{5\sqrt{33}G_{bz}yz(3x^2 + 3y^2 - 4z^2)}{132} \end{aligned}$$

$$\vec{\mathbb{G}}_3^{(3,3)}[g](A_1)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$\begin{aligned} & \frac{\sqrt{429}G_1(113x^6 - 1740x^4y^2 + 45x^4z^2 + 1725x^2y^4 + 90x^2y^2z^2 - 60x^2z^4 - 118y^6 + 45y^4z^2 - 60y^2z^4 + 8z^6)}{6864} + \frac{7\sqrt{429}G_2xy(x^2 - 3y^2)(3x^2 - y^2)}{208} \\ & - \frac{7\sqrt{286}G_3xz(7x^4 - 85x^2y^2 + 5x^2z^2 + 40y^4 + 5y^2z^2 - 2z^4)}{1144} - \frac{7\sqrt{715}G_{3x}xy(7x^4 + 2x^2y^2 - 76x^2z^2 - 5y^4 + 44y^2z^2 + 16z^4)}{2288} \\ & + \frac{7\sqrt{715}G_{3y}(x^6 - 8x^4y^2 - 7x^4z^2 - 7x^2y^4 + 90x^2y^2z^2 - 8x^2z^4 + 2y^6 - 23y^4z^2 + 8y^2z^4)}{2288} \\ & - \frac{7\sqrt{4290}G_{az}yz(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{2288} + \frac{7\sqrt{286}G_{bz}yz(85x^4 - 160x^2y^2 - 10x^2z^2 + 19y^4 - 10y^2z^2 + 4z^4)}{2288} \end{aligned}$$

$$\vec{\mathbb{G}}_3^{(3,-3)}[g](A_2, 1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$G_{az}$$

$$\vec{\mathbb{G}}_3^{(3,-3)}[g](A_2, 2)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$G_2$$

$$\vec{\mathbb{G}}_3^{(3,-1)}[g](A_2, 1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-G_3xy + \frac{\sqrt{10}G_{3x}xz}{10} + \frac{\sqrt{10}G_{3y}yz}{10} - \frac{\sqrt{15}G_{az}(x^2 + y^2 - 2z^2)}{15} - \frac{G_{bz}(x-y)(x+y)}{2}$$

$$\vec{\mathbb{G}}_3^{(3,-1)}[g](A_2, 2)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{15}G_2(x^2 + y^2 - 2z^2)}{12} - \frac{\sqrt{10}G_3yz}{4} - \frac{G_{3x}(x-y)(x+y)}{4} + \frac{G_{3y}xy}{2} + \frac{\sqrt{10}G_{bz}xz}{4}$$

$$\vec{\mathbb{G}}_3^{(3,1)}[g](A_2, 1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{7\sqrt{55}G_1yz(3x^2 - y^2)}{44} - \frac{7\sqrt{55}G_2xz(x^2 - 3y^2)}{44} + \frac{\sqrt{330}G_3xy(x^2 + y^2 - 6z^2)}{132} - \frac{5\sqrt{33}G_{3x}xz(3x^2 + 3y^2 - 4z^2)}{132} \\ - \frac{5\sqrt{33}G_{3y}yz(3x^2 + 3y^2 - 4z^2)}{132} + \frac{\sqrt{22}G_{az}(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{88} + \frac{\sqrt{330}G_{bz}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{264}$$

$$\vec{\mathbb{G}}_3^{(3,1)}[g](A_2, 2)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{22}G_2(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{176} - \frac{5\sqrt{33}G_3yz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{\sqrt{330}G_{3x}(13x^4 - 42x^2y^2 - 36x^2z^2 + y^4 + 36y^2z^2)}{528} \\ - \frac{\sqrt{330}G_{3y}xy(2x^2 - 5y^2 + 9z^2)}{66} - \frac{7\sqrt{55}G_{az}xz(x^2 - 3y^2)}{44} + \frac{5\sqrt{33}G_{bz}xz(3x^2 + 3y^2 - 4z^2)}{132}$$

$$\vec{\mathbb{G}}_3^{(3,3)}[g](A_2, 1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{7\sqrt{4290}G_1yz(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{2288} - \frac{7\sqrt{4290}G_2xz(x^2 - 3y^2)(3x^2 + 3y^2 - 8z^2)}{2288} \\ + \frac{7\sqrt{715}G_3xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} + \frac{35\sqrt{286}G_{3x}xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\ + \frac{35\sqrt{286}G_{3y}yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\ - \frac{5\sqrt{429}G_{az}(5x^6 + 15x^4y^2 - 90x^4z^2 + 15x^2y^4 - 180x^2y^2z^2 + 120x^2z^4 + 5y^6 - 90y^4z^2 + 120y^2z^4 - 16z^6)}{3432} \\ + \frac{7\sqrt{715}G_{bz}(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144}$$

$$\vec{\mathbb{G}}_3^{(3,3)}[g](A_2, 2)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{7\sqrt{429}G_1xy(x^2 - 3y^2)(3x^2 - y^2)}{208} + \frac{\sqrt{429}G_2(118x^6 - 1725x^4y^2 - 45x^4z^2 + 1740x^2y^4 - 90x^2y^2z^2 + 60x^2z^4 - 113y^6 - 45y^4z^2 + 60y^2z^4 - 8z^6)}{6864} \\ + \frac{7\sqrt{286}G_3yz(40x^4 - 85x^2y^2 + 5x^2z^2 + 7y^4 + 5y^2z^2 - 2z^4)}{1144} \\ - \frac{7\sqrt{715}G_{3x}(2x^6 - 7x^4y^2 - 23x^4z^2 - 8x^2y^4 + 90x^2y^2z^2 + 8x^2z^4 + y^6 - 7y^4z^2 - 8y^2z^4)}{2288} \\ - \frac{7\sqrt{715}G_{3y}xy(5x^4 - 2x^2y^2 - 44x^2z^2 - 7y^4 + 76y^2z^2 - 16z^4)}{2288} - \frac{7\sqrt{4290}G_{az}xz(x^2 - 3y^2)(3x^2 + 3y^2 - 8z^2)}{2288} \\ + \frac{7\sqrt{286}G_{bz}xz(19x^4 - 160x^2y^2 - 10x^2z^2 + 85y^4 - 10y^2z^2 + 4z^4)}{2288}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,-3)}[g](E, 1), \vec{\mathbb{G}}_{3,2}^{(3,-3)}[g](E, 1)$$

** symmetry

$$\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$-G_{3y}$$

$$G_{3x}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,-3)}[g](E,2), \vec{\mathbb{G}}_{3,2}^{(3,-3)}[g](E,2)$$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$G_3$$

$$G_{bz}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,-1)}[g](E,1), \vec{\mathbb{G}}_{3,2}^{(3,-1)}[g](E,1)$$

** symmetry

$$\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$\frac{G_1(x-y)(x+y)}{4} - \frac{G_2xy}{2} - \frac{\sqrt{6}G_3xz}{4} - \frac{\sqrt{15}G_{3x}xy}{5} + \frac{\sqrt{15}G_{3y}(3x^2 - y^2 - 2z^2)}{20} - \frac{\sqrt{10}G_{az}yz}{10} + \frac{\sqrt{6}G_{bz}yz}{4}$$

$$-\frac{G_1xy}{2} - \frac{G_2(x-y)(x+y)}{4} + \frac{\sqrt{6}G_3yz}{4} + \frac{\sqrt{15}G_{3x}(x^2 - 3y^2 + 2z^2)}{20} + \frac{\sqrt{15}G_{3y}xy}{5} + \frac{\sqrt{10}G_{az}xz}{10} + \frac{\sqrt{6}G_{bz}xz}{4}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,-1)}[g](E,2), \vec{\mathbb{G}}_{3,2}^{(3,-1)}[g](E,2)$$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{10}G_1xz}{4} - \frac{\sqrt{10}G_2yz}{4} + \frac{\sqrt{6}G_{3x}yz}{4} + \frac{\sqrt{6}G_{3y}xz}{4} - G_{az}xy$$

$$\frac{\sqrt{10}G_1yz}{4} + \frac{\sqrt{10}G_2xz}{4} + \frac{\sqrt{6}G_{3x}xz}{4} - \frac{\sqrt{6}G_{3y}yz}{4} - \frac{G_{az}(x-y)(x+y)}{2}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,1)}[g](E,1), \vec{\mathbb{G}}_{3,2}^{(3,1)}[g](E,1)$$

** symmetry

$$\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{330}G_1(x^4 - 42x^2y^2 + 36x^2z^2 + 13y^4 - 36y^2z^2)}{528} + \frac{\sqrt{330}G_2xy(2x^2 - 5y^2 + 9z^2)}{66} + \frac{\sqrt{55}G_{3x}xz(19x^2 - 9y^2 - 16z^2)}{132} + \frac{5\sqrt{22}G_{3x}xy(x^2 + y^2 - 6z^2)}{66} \\ & - \frac{\sqrt{22}G_{3y}(23x^4 + 6x^2y^2 - 144x^2z^2 - 17y^4 + 96y^2z^2 + 8z^4)}{528} + \frac{5\sqrt{33}G_{az}yz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{\sqrt{55}G_{bz}yz(33x^2 + 5y^2 - 16z^2)}{132} \end{aligned}$$

$$\begin{aligned}
& -\frac{\sqrt{330}G_1xy(5x^2 - 2y^2 - 9z^2)}{66} - \frac{\sqrt{330}G_2(13x^4 - 42x^2y^2 - 36x^2z^2 + y^4 + 36y^2z^2)}{528} \\
& + \frac{\sqrt{55}G_3yz(9x^2 - 19y^2 + 16z^2)}{132} - \frac{\sqrt{22}G_{3x}(17x^4 - 6x^2y^2 - 96x^2z^2 - 23y^4 + 144y^2z^2 - 8z^4)}{528} \\
& - \frac{5\sqrt{22}G_{3y}xy(x^2 + y^2 - 6z^2)}{66} - \frac{5\sqrt{33}G_{az}xz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{\sqrt{55}G_{bz}xz(5x^2 + 33y^2 - 16z^2)}{132}
\end{aligned}$$

$\vec{\mathbb{G}}_{3,1}^{(3,1)}[g](E, 2), \vec{\mathbb{G}}_{3,2}^{(3,1)}[g](E, 2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

** expression

$$\begin{aligned}
& \frac{5\sqrt{33}G_1xz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{5\sqrt{33}G_2yz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{7\sqrt{22}G_3(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{66} \\
& + \frac{\sqrt{55}G_{3x}yz(9x^2 - 19y^2 + 16z^2)}{132} - \frac{\sqrt{55}G_{3y}xz(19x^2 - 9y^2 - 16z^2)}{132} + \frac{\sqrt{330}G_{az}xy(x^2 + y^2 - 6z^2)}{132} + \frac{35\sqrt{22}G_{bz}xy(x - y)(x + y)}{132} \\
& \frac{5\sqrt{33}G_1yz(3x^2 + 3y^2 - 4z^2)}{132} + \frac{5\sqrt{33}G_2xz(3x^2 + 3y^2 - 4z^2)}{132} + \frac{35\sqrt{22}G_3xy(x - y)(x + y)}{132} - \frac{\sqrt{55}G_{3x}xz(5x^2 + 33y^2 - 16z^2)}{132} \\
& + \frac{\sqrt{55}G_{3y}yz(33x^2 + 5y^2 - 16z^2)}{132} + \frac{\sqrt{330}G_{az}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{264} + \frac{7\sqrt{22}G_{bz}(x^4 - 18x^2y^2 + 12x^2z^2 + y^4 + 12y^2z^2 - 4z^4)}{264}
\end{aligned}$$

$\vec{\mathbb{G}}_{3,1}^{(3,3)}[g](E, 1), \vec{\mathbb{G}}_{3,2}^{(3,3)}[g](E, 1)$

** symmetry

$$\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned}
& -\frac{7\sqrt{715}G_1(x^6 - 8x^4y^2 - 7x^4z^2 - 7x^2y^4 + 90x^2y^2z^2 - 8x^2z^4 + 2y^6 - 23y^4z^2 + 8y^2z^4)}{2288} \\
& + \frac{7\sqrt{715}G_2xy(5x^4 - 2x^2y^2 - 44x^2z^2 - 7y^4 + 76y^2z^2 - 16z^4)}{2288} \\
& - \frac{7\sqrt{4290}G_3xz(x^4 - 7x^2y^2 - x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4)}{1144} - \frac{35\sqrt{429}G_{3x}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{2288} \\
& + \frac{5\sqrt{429}G_{3y}(x^6 - 4x^4y^2 - 11x^4z^2 - 11x^2y^4 + 90x^2y^2z^2 - 4x^2z^4 - 6y^6 + 101y^4z^2 - 116y^2z^4 + 8z^6)}{2288} \\
& - \frac{35\sqrt{286}G_{az}yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} + \frac{7\sqrt{4290}G_{bz}yz(11x^4 + 4x^2y^2 - 26x^2z^2 - 7y^4 + 22y^2z^2 - 4z^4)}{2288} \\
& \frac{7\sqrt{715}G_1xy(7x^4 + 2x^2y^2 - 76x^2z^2 - 5y^4 + 44y^2z^2 + 16z^4)}{2288} \\
& - \frac{7\sqrt{715}G_2(2x^6 - 7x^4y^2 - 23x^4z^2 - 8x^2y^4 + 90x^2y^2z^2 + 8x^2z^4 + y^6 - 7y^4z^2 - 8y^2z^4)}{2288} - \frac{7\sqrt{4290}G_3yz(8x^4 + 7x^2y^2 - 23x^2z^2 - y^4 + y^2z^2 + 2z^4)}{1144} \\
& + \frac{5\sqrt{429}G_{3x}(6x^6 + 11x^4y^2 - 101x^4z^2 + 4x^2y^4 - 90x^2y^2z^2 + 116x^2z^4 - y^6 + 11y^4z^2 + 4y^2z^4 - 8z^6)}{2288} \\
& + \frac{35\sqrt{429}G_{3y}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{2288} + \frac{35\sqrt{286}G_{az}xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\
& - \frac{7\sqrt{4290}G_{bz}xz(7x^4 - 4x^2y^2 - 22x^2z^2 - 11y^4 + 26y^2z^2 + 4z^4)}{2288}
\end{aligned}$$

$\vec{\mathbb{G}}_{3,1}^{(3,3)}[g](E, 2), \vec{\mathbb{G}}_{3,2}^{(3,3)}[g](E, 2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

** expression

$$\begin{aligned}
& -\frac{7\sqrt{286}G_1xz(7x^4 - 85x^2y^2 + 5x^2z^2 + 40y^4 + 5y^2z^2 - 2z^4)}{1144} + \frac{7\sqrt{286}G_2yz(40x^4 - 85x^2y^2 + 5x^2z^2 + 7y^4 + 5y^2z^2 - 2z^4)}{1144} \\
& + \frac{\sqrt{429}G_3(2x^6 - 15x^4y^2 - 15x^4z^2 - 15x^2y^4 + 180x^2y^2z^2 - 15x^2z^4 + 2y^6 - 15y^4z^2 - 15y^2z^4 + 2z^6)}{286} \\
& - \frac{7\sqrt{4290}G_{3x}yz(8x^4 + 7x^2y^2 - 23x^2z^2 - y^4 + y^2z^2 + 2z^4)}{1144} + \frac{7\sqrt{4290}G_{3y}xz(x^4 - 7x^2y^2 - x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4)}{1144} \\
& + \frac{7\sqrt{715}G_{az}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} - \frac{21\sqrt{429}G_{bz}xy(x - y)(x + y)(x^2 + y^2 - 10z^2)}{572}
\end{aligned}$$

$$\begin{aligned}
& \frac{7\sqrt{286}G_1yz(85x^4 - 160x^2y^2 - 10x^2z^2 + 19y^4 - 10y^2z^2 + 4z^4)}{2288} + \frac{7\sqrt{286}G_2xz(19x^4 - 160x^2y^2 - 10x^2z^2 + 85y^4 - 10y^2z^2 + 4z^4)}{2288} \\
& - \frac{21\sqrt{429}G_3xy(x-y)(x+y)(x^2+y^2-10z^2)}{572} - \frac{7\sqrt{4290}G_{3x}xz(7x^4 - 4x^2y^2 - 22x^2z^2 - 11y^4 + 26y^2z^2 + 4z^4)}{2288} \\
& - \frac{7\sqrt{4290}G_{3y}yz(11x^4 + 4x^2y^2 - 26x^2z^2 - 7y^4 + 22y^2z^2 - 4z^4)}{2288} + \frac{7\sqrt{715}G_{az}(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144} \\
& - \frac{\sqrt{429}G_{bz}(13x^6 - 45x^4y^2 - 150x^4z^2 - 45x^2y^4 + 540x^2y^2z^2 + 60x^2z^4 + 13y^6 - 150y^4z^2 + 60y^2z^4 - 8z^6)}{1144}
\end{aligned}$$

* Harmonics for rank 4

$$\tilde{\mathbb{G}}_4^{(3,-3)}[g](A_1)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\frac{G_1z}{2} + \frac{\sqrt{6}G_3x}{4} + \frac{\sqrt{6}G_{bz}y}{4}$$

$$\tilde{\mathbb{G}}_4^{(3,-1)}[g](A_1)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{22}G_1z(3x^2 + 3y^2 - 2z^2)}{44} - \frac{\sqrt{33}G_3x(x^2 + y^2 - 4z^2)}{44} + \frac{\sqrt{330}G_{3x}xyz}{22} \\
& + \frac{\sqrt{330}G_{3y}z(x-y)(x+y)}{44} - \frac{3\sqrt{55}G_{az}y(3x^2 - y^2)}{44} - \frac{\sqrt{33}G_{bz}y(x^2 + y^2 - 4z^2)}{44}
\end{aligned}$$

$$\tilde{\mathbb{G}}_4^{(3,1)}[g](A_1)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{715}G_1z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{2288} - \frac{\sqrt{4290}G_3x(8x^4 - 47x^2y^2 - 33x^2z^2 + 29y^4 - 33y^2z^2 + 22z^4)}{1144} \\
& - \frac{7\sqrt{429}G_{3x}xyz(2x^2 + 5y^2 - 7z^2)}{286} - \frac{7\sqrt{429}G_{3y}z(17x^4 - 18x^2y^2 - 28x^2z^2 - 11y^4 + 28y^2z^2)}{2288} \\
& + \frac{21\sqrt{286}G_{az}y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{2288} + \frac{\sqrt{4290}G_{bz}y(47x^4 - 116x^2y^2 + 66x^2z^2 + 5y^4 + 66y^2z^2 - 44z^4)}{2288}
\end{aligned}$$

$$\tilde{\mathbb{G}}_4^{(3,3)}[g](A_1)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\begin{aligned}
& - \frac{\sqrt{715}G_1z(197x^6 - 3270x^4y^2 + 105x^4z^2 + 3165x^2y^4 + 210x^2y^2z^2 - 84x^2z^4 - 232y^6 + 105y^4z^2 - 84y^2z^4 + 8z^6)}{5720} \\
& + \frac{3\sqrt{715}G_2xyz(x^2 - 3y^2)(3x^2 - y^2)}{40} \\
& + \frac{\sqrt{4290}G_3x(7x^6 - 78x^4y^2 - 69x^4z^2 - 45x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 40y^6 - 465y^4z^2 - 60y^2z^4 + 16z^6)}{2860} \\
& - \frac{3\sqrt{429}G_{3x}xyz(81x^4 + 30x^2y^2 - 300x^2z^2 - 51y^4 + 140y^2z^2 + 48z^4)}{1144} \\
& + \frac{3\sqrt{429}G_{3y}z(9x^6 - 90x^4y^2 - 15x^4z^2 - 75x^2y^4 + 330x^2y^2z^2 - 24x^2z^4 + 24y^6 - 95y^4z^2 + 24y^2z^4)}{1144} \\
& + \frac{3\sqrt{286}G_{az}y(3x^2 - y^2)(3x^4 + 6x^2y^2 - 60x^2z^2 + 3y^4 - 60y^2z^2 + 80z^4)}{1144} \\
& - \frac{\sqrt{4290}G_{bz}y(85x^6 - 75x^4y^2 - 1050x^4z^2 - 141x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 19y^6 - 258y^4z^2 + 120y^2z^4 - 32z^6)}{5720}
\end{aligned}$$

$$\tilde{\mathbb{G}}_4^{(3,-3)}[g](A_2, 1)$$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$-\frac{\sqrt{42}G_{3x}x}{14} - \frac{\sqrt{42}G_{3y}y}{14} + \frac{2\sqrt{7}G_{az}z}{7}$$

$\tilde{\mathbb{G}}_4^{(3,-3)}[g](A_2, 2)$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\frac{G_2z}{2} - \frac{\sqrt{6}G_3y}{4} + \frac{\sqrt{6}G_{bz}x}{4}$$

$\tilde{\mathbb{G}}_4^{(3,-1)}[g](A_2, 1)$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\begin{aligned} & \frac{3\sqrt{385}G_1y(3x^2 - y^2)}{308} + \frac{3\sqrt{385}G_2x(x^2 - 3y^2)}{308} - \frac{\sqrt{2310}G_3xyz}{22} - \frac{\sqrt{231}G_{3x}x(x^2 + y^2 - 4z^2)}{308} \\ & - \frac{\sqrt{231}G_{3y}y(x^2 + y^2 - 4z^2)}{308} - \frac{3\sqrt{154}G_{az}z(3x^2 + 3y^2 - 2z^2)}{154} - \frac{\sqrt{2310}G_{bz}z(x - y)(x + y)}{44} \end{aligned}$$

$\tilde{\mathbb{G}}_4^{(3,-1)}[g](A_2, 2)$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{22}G_2z(3x^2 + 3y^2 - 2z^2)}{44} + \frac{\sqrt{33}G_3y(x^2 + y^2 - 4z^2)}{44} + \frac{\sqrt{330}G_{3x}z(x - y)(x + y)}{44} \\ & - \frac{\sqrt{330}G_{3y}xyz}{22} - \frac{3\sqrt{55}G_{az}x(x^2 - 3y^2)}{44} - \frac{\sqrt{33}G_{bz}x(x^2 + y^2 - 4z^2)}{44} \end{aligned}$$

$\tilde{\mathbb{G}}_4^{(3,1)}[g](A_2, 1)$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\begin{aligned} & \frac{15\sqrt{2002}G_1y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{2288} + \frac{15\sqrt{2002}G_2x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{2288} + \frac{5\sqrt{3003}G_3xyz(x^2 + y^2 - 2z^2)}{286} \\ & + \frac{\sqrt{3003}G_{3x}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{1456} + \frac{\sqrt{3003}G_{3y}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{1456} \\ & + \frac{3\sqrt{5005}G_{az}z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{4004} + \frac{5\sqrt{3003}G_{bz}z(x - y)(x + y)(x^2 + y^2 - 2z^2)}{572} \end{aligned}$$

$\tilde{\mathbb{G}}_4^{(3,1)}[g](A_2, 2)$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{715}G_2z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{2288} + \frac{\sqrt{4290}G_3y(29x^4 - 47x^2y^2 - 33x^2z^2 + 8y^4 - 33y^2z^2 + 22z^4)}{1144} \\ & - \frac{7\sqrt{429}G_{3x}z(11x^4 + 18x^2y^2 - 28x^2z^2 - 17y^4 + 28y^2z^2)}{2288} + \frac{7\sqrt{429}G_{3y}xyz(5x^2 + 2y^2 - 7z^2)}{286} \\ & + \frac{21\sqrt{286}G_{az}x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{2288} + \frac{\sqrt{4290}G_{bz}x(5x^4 - 116x^2y^2 + 66x^2z^2 + 47y^4 + 66y^2z^2 - 44z^4)}{2288} \end{aligned}$$

$\tilde{\mathbb{G}}_4^{(3,3)}[g](A_2, 1)$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{2002}G_1y(3x^2 - y^2)(3x^4 + 6x^2y^2 - 60x^2z^2 + 3y^4 - 60y^2z^2 + 80z^4)}{4576} \\
& + \frac{3\sqrt{2002}G_2x(x^2 - 3y^2)(3x^4 + 6x^2y^2 - 60x^2z^2 + 3y^4 - 60y^2z^2 + 80z^4)}{4576} + \frac{3\sqrt{3003}G_3xyz(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{1144} \\
& - \frac{\sqrt{3003}G_{3xx}(5x^6 + 15x^4y^2 - 120x^4z^2 + 15x^2y^4 - 240x^2y^2z^2 + 240x^2z^4 + 5y^6 - 120y^4z^2 + 240y^2z^4 - 64z^6)}{4576} \\
& - \frac{\sqrt{3003}G_{3yy}(5x^6 + 15x^4y^2 - 120x^4z^2 + 15x^2y^4 - 240x^2y^2z^2 + 240x^2z^4 + 5y^6 - 120y^4z^2 + 240y^2z^4 - 64z^6)}{4576} \\
& - \frac{\sqrt{5005}G_{az}z(35x^6 + 105x^4y^2 - 210x^4z^2 + 105x^2y^4 - 420x^2y^2z^2 + 168x^2z^4 + 35y^6 - 210y^4z^2 + 168y^2z^4 - 16z^6)}{2288} \\
& + \frac{3\sqrt{3003}G_{bz}z(x - y)(x + y)(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{2288}
\end{aligned}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](A_2, 2)$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{715}G_1xyz(x^2 - 3y^2)(3x^2 - y^2)}{40} \\
& + \frac{\sqrt{715}G_2z(232x^6 - 3165x^4y^2 - 105x^4z^2 + 3270x^2y^4 - 210x^2y^2z^2 + 84x^2z^4 - 197y^6 - 105y^4z^2 + 84y^2z^4 - 8z^6)}{5720} \\
& - \frac{\sqrt{4290}G_3y(40x^6 - 45x^4y^2 - 465x^4z^2 - 78x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 7y^6 - 69y^4z^2 - 60y^2z^4 + 16z^6)}{2860} \\
& - \frac{3\sqrt{429}G_{3xz}(24x^6 - 75x^4y^2 - 95x^4z^2 - 90x^2y^4 + 330x^2y^2z^2 + 24x^2z^4 + 9y^6 - 15y^4z^2 - 24y^2z^4)}{1144} \\
& - \frac{3\sqrt{429}G_{3yz}(51x^4 - 30x^2y^2 - 140x^2z^2 - 81y^4 + 300y^2z^2 - 48z^4)}{1144} + \frac{3\sqrt{286}G_{az}x(x^2 - 3y^2)(3x^4 + 6x^2y^2 - 60x^2z^2 + 3y^4 - 60y^2z^2 + 80z^4)}{1144} \\
& - \frac{\sqrt{4290}G_{bz}x(19x^6 - 141x^4y^2 - 258x^4z^2 - 75x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 85y^6 - 1050y^4z^2 + 120y^2z^4 - 32z^6)}{5720}
\end{aligned}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-3)}[g](E, 1), \vec{\mathbb{G}}_{4,2}^{(3,-3)}[g](E, 1)$

** symmetry

$$\frac{-\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$-\frac{\sqrt{42}G_3x}{28} + \frac{\sqrt{105}G_{3y}z}{14} + \frac{\sqrt{70}G_{az}y}{14} + \frac{\sqrt{42}G_{bz}y}{28}$$

$$\frac{\sqrt{42}G_3y}{28} - \frac{\sqrt{105}G_{3x}z}{14} - \frac{\sqrt{70}G_{az}x}{14} + \frac{\sqrt{42}G_{bx}x}{28}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-3)}[g](E, 2), \vec{\mathbb{G}}_{4,2}^{(3,-3)}[g](E, 2)$

** symmetry

$$\frac{-\sqrt{35}xy(x - y)(x + y)}{2}$$

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

** expression

$$-\frac{\sqrt{2}G_1x}{2} - \frac{\sqrt{2}G_2y}{2}$$

$$-\frac{\sqrt{2}G_1y}{2} + \frac{\sqrt{2}G_2x}{2}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-3)}[g](E, 3), \vec{\mathbb{G}}_{4,2}^{(3,-3)}[g](E, 3)$

** symmetry

$$\frac{-\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

$$-\frac{\sqrt{5} (x-y) (x+y) (x^2+y^2-6 z^2)}{4}$$

** expression

$$-\frac{\sqrt{14} G_1 x}{28} + \frac{\sqrt{14} G_2 y}{28} + \frac{\sqrt{21} G_3 z}{7} + \frac{\sqrt{210} G_{3x} y}{28} + \frac{\sqrt{210} G_{3y} x}{28}$$

$$-\frac{\sqrt{14} G_1 y}{28} - \frac{\sqrt{14} G_2 x}{28} + \frac{\sqrt{210} G_{3x} x}{28} - \frac{\sqrt{210} G_{3y} y}{28} + \frac{\sqrt{21} G_{bz} z}{7}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,-1)}[g](E,1), \vec{\mathbb{G}}_{4,2}^{(3,-1)}[g](E,1)$$

** symmetry

$$-\frac{\sqrt{10} y z (3 x^2 + 3 y^2 - 4 z^2)}{4}$$

$$\frac{\sqrt{10} x z (3 x^2 + 3 y^2 - 4 z^2)}{4}$$

** expression

$$-\frac{15 \sqrt{154} G_1 z (x-y) (x+y)}{308} + \frac{15 \sqrt{154} G_2 x y z}{154} + \frac{\sqrt{231} G_3 x (x^2 - 19 y^2 + 16 z^2)}{308} + \frac{2 \sqrt{2310} G_{3x} x y z}{77}$$

$$-\frac{\sqrt{2310} G_{3y} z (7 x^2 - y^2 - 2 z^2)}{308} - \frac{3 \sqrt{385} G_{az} y (x^2 + y^2 - 4 z^2)}{308} - \frac{\sqrt{231} G_{bz} y (11 x^2 - 9 y^2 + 16 z^2)}{308}$$

$$\frac{15 \sqrt{154} G_1 x y z}{154} + \frac{15 \sqrt{154} G_2 z (x-y) (x+y)}{308} + \frac{\sqrt{231} G_3 y (19 x^2 - y^2 - 16 z^2)}{308} - \frac{\sqrt{2310} G_{3x} z (x^2 - 7 y^2 + 2 z^2)}{308}$$

$$-\frac{2 \sqrt{2310} G_{3y} x y z}{77} + \frac{3 \sqrt{385} G_{az} x (x^2 + y^2 - 4 z^2)}{308} + \frac{\sqrt{231} G_{bz} x (9 x^2 - 11 y^2 - 16 z^2)}{308}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,-1)}[g](E,2), \vec{\mathbb{G}}_{4,2}^{(3,-1)}[g](E,2)$$

** symmetry

$$-\frac{\sqrt{35} x y (x-y) (x+y)}{2}$$

$$-\frac{\sqrt{35} (x^2 - 2 x y - y^2) (x^2 + 2 x y - y^2)}{8}$$

** expression

$$-\frac{3 \sqrt{11} G_1 x (x^2 + y^2 - 4 z^2)}{44} - \frac{3 \sqrt{11} G_2 y (x^2 + y^2 - 4 z^2)}{44} - \frac{5 \sqrt{66} G_3 z (x-y) (x+y)}{44} + \frac{\sqrt{165} G_{3x} y (3 x^2 - y^2)}{44} + \frac{\sqrt{165} G_{3y} x (x^2 - 3 y^2)}{44} - \frac{5 \sqrt{66} G_{bz} x y z}{22}$$

$$-\frac{3 \sqrt{11} G_1 y (x^2 + y^2 - 4 z^2)}{44} + \frac{3 \sqrt{11} G_2 x (x^2 + y^2 - 4 z^2)}{44} - \frac{5 \sqrt{66} G_3 x y z}{22} - \frac{\sqrt{165} G_{3x} x (x^2 - 3 y^2)}{44} + \frac{\sqrt{165} G_{3y} y (3 x^2 - y^2)}{44} + \frac{5 \sqrt{66} G_{bz} z (x-y) (x+y)}{44}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,-1)}[g](E,3), \vec{\mathbb{G}}_{4,2}^{(3,-1)}[g](E,3)$$

** symmetry

$$-\frac{\sqrt{5} x y (x^2 + y^2 - 6 z^2)}{2}$$

$$-\frac{\sqrt{5} (x-y) (x+y) (x^2 + y^2 - 6 z^2)}{4}$$

** expression

$$-\frac{9 \sqrt{77} G_1 x (x^2 + y^2 - 4 z^2)}{308} + \frac{9 \sqrt{77} G_2 y (x^2 + y^2 - 4 z^2)}{308} + \frac{\sqrt{462} G_3 z (3 x^2 + 3 y^2 - 2 z^2)}{308}$$

$$+ \frac{\sqrt{1155} G_{3x} y (7 x^2 - 5 y^2 + 8 z^2)}{308} - \frac{\sqrt{1155} G_{3y} x (5 x^2 - 7 y^2 - 8 z^2)}{308} - \frac{3 \sqrt{770} G_{az} x y z}{154}$$

$$-\frac{9 \sqrt{77} G_1 y (x^2 + y^2 - 4 z^2)}{308} - \frac{9 \sqrt{77} G_2 x (x^2 + y^2 - 4 z^2)}{308} + \frac{\sqrt{1155} G_{3x} x (x^2 - 11 y^2 + 8 z^2)}{308}$$

$$+ \frac{\sqrt{1155} G_{3y} y (11 x^2 - y^2 - 8 z^2)}{308} - \frac{3 \sqrt{770} G_{az} z (x-y) (x+y)}{308} + \frac{\sqrt{462} G_{bz} z (3 x^2 + 3 y^2 - 2 z^2)}{308}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,1)}[g](E,1), \vec{\mathbb{G}}_{4,2}^{(3,1)}[g](E,1)$$

** symmetry

$$-\frac{\sqrt{10} y z (3 x^2 + 3 y^2 - 4 z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned} & -\frac{3\sqrt{5005}G_1z(x^4 + 54x^2y^2 - 20x^2z^2 - 19y^4 + 20y^2z^2)}{2288} - \frac{3\sqrt{5005}G_2xyz(2x^2 - 7y^2 + 5z^2)}{286} \\ & + \frac{\sqrt{30030}G_3x(8x^4 + 9x^2y^2 - 89x^2z^2 + y^4 - 33y^2z^2 + 50z^4)}{8008} - \frac{4\sqrt{3003}G_{3x}xyz(x^2 + y^2 - 2z^2)}{143} \\ & + \frac{\sqrt{3003}G_{3y}z(239x^4 + 30x^2y^2 - 488x^2z^2 - 209y^4 + 408y^2z^2 + 8z^4)}{16016} \\ & + \frac{57\sqrt{2002}G_{az}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{16016} - \frac{\sqrt{30030}G_{bz}y(23x^4 + 32x^2y^2 - 234x^2z^2 + 9y^4 - 122y^2z^2 + 100z^4)}{16016} \end{aligned}$$

$$\begin{aligned} & \frac{3\sqrt{5005}G_1xyz(7x^2 - 2y^2 - 5z^2)}{286} + \frac{3\sqrt{5005}G_2z(19x^4 - 54x^2y^2 - 20x^2z^2 - y^4 + 20y^2z^2)}{2288} \\ & - \frac{\sqrt{30030}G_3y(x^4 + 9x^2y^2 - 33x^2z^2 + 8y^4 - 89y^2z^2 + 50z^4)}{8008} + \frac{\sqrt{3003}G_{3x}z(209x^4 - 30x^2y^2 - 408x^2z^2 - 239y^4 + 488y^2z^2 - 8z^4)}{16016} \\ & + \frac{4\sqrt{3003}G_{3y}xyz(x^2 + y^2 - 2z^2)}{143} - \frac{57\sqrt{2002}G_{az}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{16016} \\ & - \frac{\sqrt{30030}G_{bz}x(9x^4 + 32x^2y^2 - 122x^2z^2 + 23y^4 - 234y^2z^2 + 100z^4)}{16016} \end{aligned}$$

$\vec{\mathbb{G}}_{4,1}^{(3,1)}[g](E, 2), \vec{\mathbb{G}}_{4,2}^{(3,1)}[g](E, 2)$

** symmetry

$$-\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

** expression

$$\begin{aligned} & -\frac{3\sqrt{1430}G_1x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} - \frac{3\sqrt{1430}G_2y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} \\ & - \frac{7\sqrt{2145}G_3z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{572} + \frac{7\sqrt{858}G_{3x}y(21x^4 - 26x^2y^2 - 48x^2z^2 + y^4 + 16y^2z^2)}{2288} \\ & - \frac{7\sqrt{858}G_{3y}x(x^4 - 26x^2y^2 + 16x^2z^2 + 21y^4 - 48y^2z^2)}{2288} + \frac{63\sqrt{143}G_{az}xyz(x-y)(x+y)}{143} - \frac{7\sqrt{2145}G_{bz}xyz(x^2 + y^2 - 2z^2)}{286} \\ & - \frac{3\sqrt{1430}G_1y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} + \frac{3\sqrt{1430}G_2x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} \\ & - \frac{7\sqrt{2145}G_3xyz(x^2 + y^2 - 2z^2)}{286} - \frac{7\sqrt{858}G_{3x}x(5x^4 - 34x^2y^2 - 16x^2z^2 + 9y^4 + 48y^2z^2)}{2288} - \frac{7\sqrt{858}G_{3y}y(9x^4 - 34x^2y^2 + 48x^2z^2 + 5y^4 - 16y^2z^2)}{2288} \\ & - \frac{63\sqrt{143}G_{az}z(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{572} + \frac{7\sqrt{2145}G_{bz}z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{572} \end{aligned}$$

$\vec{\mathbb{G}}_{4,1}^{(3,1)}[g](E, 3), \vec{\mathbb{G}}_{4,2}^{(3,1)}[g](E, 3)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{9\sqrt{10010}G_1x(x^4 - 40x^2y^2 + 30x^2z^2 + 15y^4 + 30y^2z^2 - 20z^4)}{16016} - \frac{9\sqrt{10010}G_2y(15x^4 - 40x^2y^2 + 30x^2z^2 + y^4 + 30y^2z^2 - 20z^4)}{16016} \\ & - \frac{\sqrt{15015}G_3z(51x^4 - 66x^2y^2 - 80x^2z^2 + 51y^4 - 80y^2z^2 + 16z^4)}{4004} - \frac{\sqrt{6006}G_{3x}y(73x^4 + 20x^2y^2 - 498x^2z^2 - 53y^4 + 510y^2z^2 - 172z^4)}{16016} \\ & + \frac{\sqrt{6006}G_{3y}x(53x^4 - 20x^2y^2 - 510x^2z^2 - 73y^4 + 498y^2z^2 + 172z^4)}{16016} - \frac{3\sqrt{1001}G_{az}xyz(x^2 + y^2 - 2z^2)}{286} + \frac{3\sqrt{15015}G_{bz}xyz(x-y)(x+y)}{143} \\ & - \frac{9\sqrt{10010}G_1y(10x^4 - 15x^2y^2 - 15x^2z^2 + 3y^4 - 15y^2z^2 + 10z^4)}{8008} - \frac{9\sqrt{10010}G_2x(3x^4 - 15x^2y^2 - 15x^2z^2 + 10y^4 - 15y^2z^2 + 10z^4)}{8008} \\ & + \frac{3\sqrt{15015}G_3xyz(x-y)(x+y)}{143} - \frac{\sqrt{6006}G_{3x}x(5x^4 - 53x^2y^2 + 3x^2z^2 - 58y^4 + 507y^2z^2 - 86z^4)}{8008} \\ & - \frac{\sqrt{6006}G_{3y}y(58x^4 + 53x^2y^2 - 507x^2z^2 - 5y^4 - 3y^2z^2 + 86z^4)}{8008} - \frac{3\sqrt{1001}G_{az}z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{572} \\ & - \frac{\sqrt{15015}G_{bz}z(9x^4 + 186x^2y^2 - 80x^2z^2 + 9y^4 - 80y^2z^2 + 16z^4)}{4004} \end{aligned}$$

$$\tilde{\mathbb{G}}_{4,1}^{(3,3)}[g](E, 1), \tilde{\mathbb{G}}_{4,2}^{(3,3)}[g](E, 1)$$

** symmetry

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{5005}G_1z(9x^6 - 90x^4y^2 - 15x^4z^2 - 75x^2y^4 + 330x^2y^2z^2 - 24x^2z^4 + 24y^6 - 95y^4z^2 + 24y^2z^4)}{5720} \\ & - \frac{3\sqrt{5005}G_2xyz(51x^4 - 30x^2y^2 - 140x^2z^2 - 81y^4 + 300y^2z^2 - 48z^4)}{5720} \\ & - \frac{\sqrt{30030}G_3x(x^6 - 6x^4y^2 - 15x^4z^2 - 15x^2y^4 + 150x^2y^2z^2 - 8y^6 + 165y^4z^2 - 240y^2z^4 + 16z^6)}{2860} \\ & + \frac{3\sqrt{3003}G_{3x}xyz(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{1144} \\ & - \frac{\sqrt{3003}G_{3y}z(5x^6 - 30x^4y^2 - 15x^4z^2 - 75x^2y^4 + 210x^2y^2z^2 - 12x^2z^4 - 40y^6 + 225y^4z^2 - 156y^2z^4 + 8z^6)}{1144} \\ & - \frac{\sqrt{2002}G_{az}y(5x^6 + 15x^4y^2 - 120x^4z^2 + 15x^2y^4 - 240x^2y^2z^2 + 240x^2z^4 + 5y^6 - 120y^4z^2 + 240y^2z^4 - 64z^6)}{1144} \\ & + \frac{\sqrt{30030}G_{bz}y(11x^6 + 15x^4y^2 - 210x^4z^2 - 3x^2y^4 - 60x^2y^2z^2 + 240x^2z^4 - 7y^6 + 150y^4z^2 - 240y^2z^4 + 32z^6)}{5720} \end{aligned}$$

$$\begin{aligned} & \frac{3\sqrt{5005}G_1xyz(81x^4 + 30x^2y^2 - 300x^2z^2 - 51y^4 + 140y^2z^2 + 48z^4)}{5720} \\ & + \frac{3\sqrt{5005}G_2z(24x^6 - 75x^4y^2 - 95x^4z^2 - 90x^2y^4 + 330x^2y^2z^2 + 24x^2z^4 + 9y^6 - 15y^4z^2 - 24y^2z^4)}{5720} \\ & - \frac{\sqrt{30030}G_3y(8x^6 + 15x^4y^2 - 165x^4z^2 + 6x^2y^4 - 150x^2y^2z^2 + 240x^2z^4 - y^6 + 15y^4z^2 - 16z^6)}{2860} \\ & - \frac{\sqrt{3003}G_{3x}z(40x^6 + 75x^4y^2 - 225x^4z^2 + 30x^2y^4 - 210x^2y^2z^2 + 156x^2z^4 - 5y^6 + 15y^4z^2 + 12y^2z^4 - 8z^6)}{1144} \\ & - \frac{3\sqrt{3003}G_{3y}xyz(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{1144} \\ & + \frac{\sqrt{2002}G_{az}x(5x^6 + 15x^4y^2 - 120x^4z^2 + 15x^2y^4 - 240x^2y^2z^2 + 240x^2z^4 + 5y^6 - 120y^4z^2 + 240y^2z^4 - 64z^6)}{1144} \\ & - \frac{\sqrt{30030}G_{bz}x(7x^6 + 3x^4y^2 - 150x^4z^2 - 15x^2y^4 + 60x^2y^2z^2 + 240x^2z^4 - 11y^6 + 210y^4z^2 - 240y^2z^4 - 32z^6)}{5720} \end{aligned}$$

$$\tilde{\mathbb{G}}_{4,1}^{(3,3)}[g](E, 2), \tilde{\mathbb{G}}_{4,2}^{(3,3)}[g](E, 2)$$

** symmetry

$$-\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

** expression

$$\begin{aligned} & \frac{\sqrt{1430}G_1x(53x^6 - 1128x^4y^2 + 15x^4z^2 + 1875x^2y^4 + 30x^2y^2z^2 - 30x^2z^4 - 376y^6 + 15y^4z^2 - 30y^2z^4 + 8z^6)}{5720} \\ & - \frac{\sqrt{1430}G_2y(376x^6 - 1875x^4y^2 - 15x^4z^2 + 1128x^2y^4 - 30x^2y^2z^2 + 30x^2z^4 - 53y^6 - 15y^4z^2 + 30y^2z^4 - 8z^6)}{5720} \\ & + \frac{3\sqrt{2145}G_3z(x-y)(x+y)(8x^4 - 127x^2y^2 + 5x^2z^2 + 8y^4 + 5y^2z^2 - 3z^4)}{1430} \\ & + \frac{3\sqrt{858}G_{3x}y(8x^6 - 5x^4y^2 - 105x^4z^2 - 12x^2y^4 + 150x^2y^2z^2 + 30x^2z^4 + y^6 - 9y^4z^2 - 10y^2z^4)}{1144} \\ & - \frac{3\sqrt{858}G_{3y}x(x^6 - 12x^4y^2 - 9x^4z^2 - 5x^2y^4 + 150x^2y^2z^2 - 10x^2z^4 + 8y^6 - 105y^4z^2 + 30y^2z^4)}{1144} \\ & + \frac{3\sqrt{143}G_{az}xyz(x-y)(x+y)(3x^2 + 3y^2 - 10z^2)}{52} - \frac{3\sqrt{2145}G_{bz}xyz(111x^4 - 350x^2y^2 - 20x^2z^2 + 111y^4 - 20y^2z^2 + 12z^4)}{2860} \end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{1430}G_1y(1499x^6 - 7515x^4y^2 + 60x^4z^2 + 4497x^2y^4 + 120x^2y^2z^2 - 120x^2z^4 - 217y^6 + 60y^4z^2 - 120y^2z^4 + 32z^6)}{22880} \\
& + \frac{\sqrt{1430}G_2x(217x^6 - 4497x^4y^2 - 60x^4z^2 + 7515x^2y^4 - 120x^2y^2z^2 + 120x^2z^4 - 1499y^6 - 60y^4z^2 + 120y^2z^4 - 32z^6)}{22880} \\
& + \frac{3\sqrt{2145}G_3xyz(207x^4 - 730x^2y^2 + 40x^2z^2 + 207y^4 + 40y^2z^2 - 24z^4)}{5720} \\
& - \frac{3\sqrt{858}G_{3x}x(7x^6 - 51x^4y^2 - 96x^4z^2 - 35x^2y^4 + 720x^2y^2z^2 + 40x^2z^4 + 23y^6 - 240y^4z^2 - 120y^2z^4)}{4576} \\
& - \frac{3\sqrt{858}G_{3y}y(23x^6 - 35x^4y^2 - 240x^4z^2 - 51x^2y^4 + 720x^2y^2z^2 - 120x^2z^4 + 7y^6 - 96y^4z^2 + 40y^2z^4)}{4576} \\
& - \frac{3\sqrt{143}G_{az}z(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)(3x^2 + 3y^2 - 10z^2)}{208} \\
& + \frac{3\sqrt{2145}G_{bz}z(x - y)(x + y)(79x^4 - 986x^2y^2 - 40x^2z^2 + 79y^4 - 40y^2z^2 + 24z^4)}{11440}
\end{aligned}$$

$\vec{\mathbb{G}}_{4,1}^{(3,3)}[g](E, 3), \vec{\mathbb{G}}_{4,2}^{(3,3)}[g](E, 3)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

$$-\frac{\sqrt{5}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\begin{aligned}
& \frac{\sqrt{10010}G_1x(7x^6 - 78x^4y^2 - 69x^4z^2 - 45x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 40y^6 - 465y^4z^2 - 60y^2z^4 + 16z^6)}{5720} \\
& - \frac{\sqrt{10010}G_2y(40x^6 - 45x^4y^2 - 465x^4z^2 - 78x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 7y^6 - 69y^4z^2 - 60y^2z^4 + 16z^6)}{5720} \\
& + \frac{\sqrt{15015}G_3z(8x^6 - 75x^4y^2 - 15x^4z^2 - 75x^2y^4 + 300x^2y^2z^2 - 21x^2z^4 + 8y^6 - 15y^4z^2 - 21y^2z^4 + 2z^6)}{1430} \\
& + \frac{\sqrt{6006}G_{3x}y(8x^6 + 15x^4y^2 - 165x^4z^2 + 6x^2y^4 - 150x^2y^2z^2 + 240x^2z^4 - y^6 + 15y^4z^2 - 16z^6)}{1144} \\
& - \frac{\sqrt{6006}G_{3y}x(x^6 - 6x^4y^2 - 15x^4z^2 - 15x^2y^4 + 150x^2y^2z^2 - 8y^6 + 165y^4z^2 - 240y^2z^4 + 16z^6)}{1144} \\
& + \frac{3\sqrt{1001}G_{az}xyz(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{572} - \frac{3\sqrt{15015}G_{bz}xyz(x - y)(x + y)(3x^2 + 3y^2 - 10z^2)}{260}
\end{aligned}$$

$$\begin{aligned}
& - \frac{\sqrt{10010}G_1y(85x^6 - 75x^4y^2 - 1050x^4z^2 - 141x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 19y^6 - 258y^4z^2 + 120y^2z^4 - 32z^6)}{11440} \\
& - \frac{\sqrt{10010}G_2x(19x^6 - 141x^4y^2 - 258x^4z^2 - 75x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 85y^6 - 1050y^4z^2 + 120y^2z^4 - 32z^6)}{11440} \\
& - \frac{3\sqrt{15015}G_3xyz(x - y)(x + y)(3x^2 + 3y^2 - 10z^2)}{260} \\
& + \frac{\sqrt{6006}G_{3x}x(7x^6 + 3x^4y^2 - 150x^4z^2 - 15x^2y^4 + 60x^2y^2z^2 + 240x^2z^4 - 11y^6 + 210y^4z^2 - 240y^2z^4 - 32z^6)}{2288} \\
& + \frac{\sqrt{6006}G_{3y}y(11x^6 + 15x^4y^2 - 210x^4z^2 - 3x^2y^4 - 60x^2y^2z^2 + 240x^2z^4 - 7y^6 + 150y^4z^2 - 240y^2z^4 + 32z^6)}{2288} \\
& + \frac{3\sqrt{1001}G_{az}z(x - y)(x + y)(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{1144} \\
& - \frac{\sqrt{15015}G_{bz}z(67x^6 - 195x^4y^2 - 270x^4z^2 - 195x^2y^4 + 780x^2y^2z^2 + 84x^2z^4 + 67y^6 - 270y^4z^2 + 84y^2z^4 - 8z^6)}{5720}
\end{aligned}$$