

PG No. 10 S_4 $\bar{4}$ [tetragonal] (axial, internal polar octupole)

* Harmonics for rank 0

* Harmonics for rank 1

$$\vec{G}_1^{(3,2)}[q](A)$$

** symmetry

$$z$$

** expression

$$-\frac{3\sqrt{70}Q_1x(x^2-3y^2)}{56} + \frac{3\sqrt{70}Q_2y(3x^2-y^2)}{56} - \frac{\sqrt{105}Q_3z(x-y)(x+y)}{14} - \frac{\sqrt{42}Q_{3x}y(x^2+y^2-4z^2)}{56} + \frac{\sqrt{42}Q_{3y}x(x^2+y^2-4z^2)}{56} + \frac{\sqrt{105}Q_{bz}xyz}{7}$$

$$\vec{G}_{1,1}^{(3,2)}[q](E), \vec{G}_{1,2}^{(3,2)}[q](E)$$

** symmetry

$$y$$

$$x$$

** expression

$$\begin{aligned} & -\frac{3\sqrt{70}Q_1xyz}{28} - \frac{3\sqrt{70}Q_2z(x-y)(x+y)}{56} + \frac{\sqrt{105}Q_3y(x-z)(x+z)}{14} + \frac{\sqrt{42}Q_{3x}z(11x^2+y^2-4z^2)}{56} \\ & + \frac{5\sqrt{42}Q_{3y}xyz}{28} - \frac{3\sqrt{7}Q_{az}x(x^2+y^2-4z^2)}{28} + \frac{\sqrt{105}Q_{bz}x(x^2-y^2-2z^2)}{28} \\ & \frac{3\sqrt{70}Q_1z(x-y)(x+y)}{56} - \frac{3\sqrt{70}Q_2xyz}{28} - \frac{\sqrt{105}Q_3x(y-z)(y+z)}{14} - \frac{5\sqrt{42}Q_{3x}xyz}{28} \\ & - \frac{\sqrt{42}Q_{3y}z(x^2+11y^2-4z^2)}{56} + \frac{3\sqrt{7}Q_{az}y(x^2+y^2-4z^2)}{28} - \frac{\sqrt{105}Q_{bz}y(x^2-y^2+2z^2)}{28} \end{aligned}$$

* Harmonics for rank 2

$$\vec{G}_2^{(3,0)}[q](A,1)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{105}Q_1xz}{14} - \frac{\sqrt{105}Q_2yz}{14} - \frac{\sqrt{70}Q_3(x^2+y^2-2z^2)}{28} - \frac{3\sqrt{7}Q_{3x}yz}{14} - \frac{3\sqrt{7}Q_{3y}xz}{14} + \frac{\sqrt{42}Q_{az}xy}{14}$$

$$\vec{G}_2^{(3,0)}[q](A,2)$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{105}Q_1yz}{14} - \frac{\sqrt{105}Q_2xz}{14} + \frac{3\sqrt{7}Q_{3x}xz}{14} - \frac{3\sqrt{7}Q_{3y}yz}{14} - \frac{\sqrt{42}Q_{az}(x-y)(x+y)}{28} + \frac{\sqrt{70}Q_{bz}(x^2+y^2-2z^2)}{28}$$

$$\vec{G}_2^{(3,2)}[q](A,1)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{210}Q_1xz(3x^2+3y^2-4z^2)}{168} - \frac{\sqrt{210}Q_2yz(3x^2+3y^2-4z^2)}{168} + \frac{\sqrt{35}Q_3(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4)}{42} \\ & - \frac{5\sqrt{14}Q_{3x}yz(27x^2-y^2-8z^2)}{168} + \frac{5\sqrt{14}Q_{3y}xz(x^2-27y^2+8z^2)}{168} + \frac{5\sqrt{21}Q_{az}xy(x^2+y^2-6z^2)}{42} - \frac{\sqrt{35}Q_{bz}xy(x-y)(x+y)}{6} \end{aligned}$$

$$\vec{G}_2^{(3,2)}[q](A,2)$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\begin{aligned} & -\frac{\sqrt{210}Q_1yz(3x^2+3y^2-4z^2)}{168} - \frac{\sqrt{210}Q_2xz(3x^2+3y^2-4z^2)}{168} + \frac{\sqrt{35}Q_3xy(x-y)(x+y)}{6} + \frac{5\sqrt{14}Q_{3x}xz(13x^2-15y^2-8z^2)}{168} \\ & + \frac{5\sqrt{14}Q_{3y}yz(15x^2-13y^2+8z^2)}{168} - \frac{5\sqrt{21}Q_{az}(x-y)(x+y)(x^2+y^2-6z^2)}{84} + \frac{\sqrt{35}Q_{bz}(5x^4-18x^2y^2-12x^2z^2+5y^4-12y^2z^2+4z^4)}{84} \end{aligned}$$

$$\vec{\mathbb{G}}_2^{(3,0)}[q](B)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{210}Q_3(x-y)(x+y)}{28} + \frac{\sqrt{21}Q_{3x}yz}{7} - \frac{\sqrt{21}Q_{3y}xz}{7} + \frac{\sqrt{210}Q_{bz}xy}{14}$$

$$\vec{\mathbb{G}}_2^{(3,2)}[q](B)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{70}Q_{1xz}(x^2-3y^2)}{8} + \frac{\sqrt{70}Q_{2yz}(3x^2-y^2)}{8} + \frac{\sqrt{105}Q_3(x-y)(x+y)(x^2+y^2-6z^2)}{42} \\ - \frac{5\sqrt{42}Q_{3x}yz(3x^2+3y^2-4z^2)}{168} + \frac{5\sqrt{42}Q_{3y}xz(3x^2+3y^2-4z^2)}{168} - \frac{\sqrt{105}Q_{bz}xy(x^2+y^2-6z^2)}{21}$$

$$\vec{\mathbb{G}}_{2,1}^{(3,0)}[q](E), \vec{\mathbb{G}}_{2,2}^{(3,0)}[q](E)$$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$-\frac{\sqrt{105}Q_1(x-y)(x+y)}{28} + \frac{\sqrt{105}Q_2xy}{14} - \frac{3\sqrt{7}Q_{3x}xy}{14} + \frac{\sqrt{7}Q_{3y}(x^2-5y^2+4z^2)}{28} - \frac{\sqrt{42}Q_{az}yz}{7} \\ \frac{\sqrt{105}Q_1xy}{14} + \frac{\sqrt{105}Q_2(x-y)(x+y)}{28} + \frac{\sqrt{7}Q_{3x}(5x^2-y^2-4z^2)}{28} + \frac{3\sqrt{7}Q_{3y}xy}{14} + \frac{\sqrt{42}Q_{az}xz}{7}$$

$$\vec{\mathbb{G}}_{2,1}^{(3,2)}[q](E), \vec{\mathbb{G}}_{2,2}^{(3,2)}[q](E)$$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$-\frac{\sqrt{210}Q_1(5x^4-21x^2y^2-9x^2z^2+2y^4+9y^2z^2)}{168} + \frac{\sqrt{210}Q_2xy(17x^2-11y^2-18z^2)}{168} - \frac{\sqrt{35}Q_3xz(x-z)(x+z)}{6} + \frac{5\sqrt{14}Q_{3x}xy(x^2+y^2-6z^2)}{168} \\ + \frac{5\sqrt{14}Q_{3y}(x^4+3x^2y^2-9x^2z^2+2y^4-15y^2z^2+4z^4)}{168} + \frac{5\sqrt{21}Q_{az}yz(3x^2+3y^2-4z^2)}{84} + \frac{\sqrt{35}Q_{bz}yz(3x^2+y^2-2z^2)}{12} \\ - \frac{\sqrt{210}Q_{1xy}(11x^2-17y^2+18z^2)}{168} - \frac{\sqrt{210}Q_2(2x^4-21x^2y^2+9x^2z^2+5y^4-9y^2z^2)}{168} \\ + \frac{\sqrt{35}Q_3yz(y-z)(y+z)}{6} - \frac{5\sqrt{14}Q_{3x}(2x^4+3x^2y^2-15x^2z^2+y^4-9y^2z^2+4z^4)}{168} \\ - \frac{5\sqrt{14}Q_{3y}xy(x^2+y^2-6z^2)}{168} - \frac{5\sqrt{21}Q_{az}xz(3x^2+3y^2-4z^2)}{84} + \frac{\sqrt{35}Q_{bz}xz(x^2+3y^2-2z^2)}{12}$$

* Harmonics for rank 3

$$\vec{\mathbb{G}}_3^{(3,-2)}[q](A)$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$\frac{\sqrt{2}Q_{3xy}}{2} - \frac{\sqrt{2}Q_{3yx}}{2}$$

$$\vec{\mathbb{G}}_3^{(3,0)}[q](A)$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$\frac{\sqrt{15}Q_1x(x^2-3y^2)}{12} - \frac{\sqrt{15}Q_2y(3x^2-y^2)}{12} - \frac{\sqrt{10}Q_3z(x-y)(x+y)}{4} - \frac{Q_{3xy}(x^2+y^2-4z^2)}{4} + \frac{Q_{3yx}(x^2+y^2-4z^2)}{4} + \frac{\sqrt{10}Q_{bxyz}}{2}$$

$\vec{\mathbb{G}}_3^{(3,2)}[q](A)$

** symmetry

$$- \frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$\frac{7\sqrt{330}Q_1x(x^2-3y^2)(x^2+y^2-8z^2)}{528} - \frac{7\sqrt{330}Q_2y(3x^2-y^2)(x^2+y^2-8z^2)}{528} + \frac{7\sqrt{55}Q_3z(x-y)(x+y)(x^2+y^2-2z^2)}{44} \\ + \frac{5\sqrt{22}Q_{3xy}(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{176} - \frac{5\sqrt{22}Q_{3yx}(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{176} - \frac{7\sqrt{55}Q_{bxyz}(x^2+y^2-2z^2)}{22}$$

$\vec{\mathbb{G}}_3^{(3,-2)}[q](B,1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{2}Q_1y}{4} + \frac{\sqrt{2}Q_2x}{4} + \frac{\sqrt{30}Q_{3xx}}{12} - \frac{\sqrt{30}Q_{3yy}}{12} - \frac{\sqrt{3}Q_{bzz}}{3}$$

$\vec{\mathbb{G}}_3^{(3,-2)}[q](B,2)$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$- \frac{\sqrt{2}Q_1x}{4} + \frac{\sqrt{2}Q_2y}{4} + \frac{\sqrt{3}Q_3z}{3} - \frac{\sqrt{30}Q_{3xy}}{12} - \frac{\sqrt{30}Q_{3yx}}{12}$$

$\vec{\mathbb{G}}_3^{(3,0)}[q](B,1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{Q_1y(x^2+y^2-4z^2)}{4} + \frac{Q_2x(x^2+y^2-4z^2)}{4} + \frac{\sqrt{15}Q_{3xx}(x^2-3y^2)}{12} + \frac{\sqrt{15}Q_{3yy}(3x^2-y^2)}{12} + \frac{\sqrt{10}Q_{azz}(x-y)(x+y)}{4} + \frac{\sqrt{6}Q_{bzz}(3x^2+3y^2-2z^2)}{12}$$

$\vec{\mathbb{G}}_3^{(3,0)}[q](B,2)$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$- \frac{Q_1x(x^2+y^2-4z^2)}{4} + \frac{Q_2y(x^2+y^2-4z^2)}{4} - \frac{\sqrt{6}Q_3z(3x^2+3y^2-2z^2)}{12} - \frac{\sqrt{15}Q_{3xy}(3x^2-y^2)}{12} + \frac{\sqrt{15}Q_{3yx}(x^2-3y^2)}{12} - \frac{\sqrt{10}Q_{azxyz}}{2}$$

$\vec{\mathbb{G}}_3^{(3,2)}[q](B,1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$- \frac{\sqrt{22}Q_1y(25x^4-55x^2y^2+15x^2z^2+4y^4+15y^2z^2-10z^4)}{88} - \frac{\sqrt{22}Q_2x(4x^4-55x^2y^2+15x^2z^2+25y^4+15y^2z^2-10z^4)}{88} \\ - \frac{\sqrt{330}Q_{3xx}(4x^4+x^2y^2-41x^2z^2-3y^4+15y^2z^2+18z^4)}{264} - \frac{\sqrt{330}Q_{3yy}(3x^4-x^2y^2-15x^2z^2-4y^4+41y^2z^2-18z^4)}{264} \\ - \frac{7\sqrt{55}Q_{azz}(x-y)(x+y)(x^2+y^2-2z^2)}{44} + \frac{\sqrt{33}Q_{bzz}(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4)}{132}$$

$\vec{\mathbb{G}}_3^{(3,2)}[q](B,2)$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{22}Q_1x(13x^4-100x^2y^2-30x^2z^2+55y^4-30y^2z^2+20z^4)}{176} + \frac{\sqrt{22}Q_2y(55x^4-100x^2y^2-30x^2z^2+13y^4-30y^2z^2+20z^4)}{176} \\ & -\frac{\sqrt{33}Q_3z(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4)}{132} + \frac{\sqrt{330}Q_{3xy}(15x^4+16x^2y^2-138x^2z^2+y^4-26y^2z^2+36z^4)}{528} \\ & +\frac{\sqrt{330}Q_{3yz}(x^4+16x^2y^2-26x^2z^2+15y^4-138y^2z^2+36z^4)}{528} + \frac{7\sqrt{55}Q_{az}xyz(x^2+y^2-2z^2)}{22} \end{aligned}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,-2)}[q](E,1), \vec{\mathbb{G}}_{3,2}^{(3,-2)}[q](E,1)$$

** symmetry

$$-\frac{y(3x^2-2y^2+3z^2)}{2}$$

$$\frac{x(2x^2-3y^2-3z^2)}{2}$$

** expression

$$\frac{\sqrt{30}Q_2z}{8} + \frac{\sqrt{2}Q_{3xz}}{8} - \frac{\sqrt{3}Q_{azx}}{4} - \frac{\sqrt{5}Q_{bzx}}{4}$$

$$\frac{\sqrt{30}Q_1z}{8} - \frac{\sqrt{2}Q_{3yz}}{8} + \frac{\sqrt{3}Q_{azy}}{4} - \frac{\sqrt{5}Q_{bzy}}{4}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,-2)}[q](E,2), \vec{\mathbb{G}}_{3,2}^{(3,-2)}[q](E,2)$$

** symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

** expression

$$-\frac{3\sqrt{2}Q_1z}{8} + \frac{\sqrt{3}Q_{3x}}{3} - \frac{\sqrt{30}Q_{3yz}}{24} + \frac{\sqrt{5}Q_{azy}}{4} - \frac{\sqrt{3}Q_{bzy}}{12}$$

$$\frac{3\sqrt{2}Q_2z}{8} + \frac{\sqrt{3}Q_{3y}}{3} - \frac{\sqrt{30}Q_{3xz}}{24} + \frac{\sqrt{5}Q_{azx}}{4} + \frac{\sqrt{3}Q_{bzx}}{12}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,0)}[q](E,1), \vec{\mathbb{G}}_{3,2}^{(3,0)}[q](E,1)$$

** symmetry

$$-\frac{y(3x^2-2y^2+3z^2)}{2}$$

$$\frac{x(2x^2-3y^2-3z^2)}{2}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{15}Q_1xyz}{4} + \frac{\sqrt{15}Q_2z(3y^2-z^2)}{12} + \frac{\sqrt{10}Q_3y(x-z)(x+z)}{4} - \frac{Q_{3xz}(4x^2-y^2-z^2)}{4} \\ & + \frac{5Q_{3y}xyz}{4} + \frac{\sqrt{6}Q_{azx}(2x^2-3y^2-3z^2)}{12} - \frac{\sqrt{10}Q_{bzx}(y-z)(y+z)}{4} \end{aligned}$$

$$\begin{aligned} & \frac{\sqrt{15}Q_1z(3x^2-z^2)}{12} - \frac{\sqrt{15}Q_2xyz}{4} - \frac{\sqrt{10}Q_3x(y-z)(y+z)}{4} - \frac{5Q_{3x}xyz}{4} \\ & - \frac{Q_{3yz}(x^2-4y^2+z^2)}{4} + \frac{\sqrt{6}Q_{azx}(3x^2-2y^2+3z^2)}{12} - \frac{\sqrt{10}Q_{bzy}(x-z)(x+z)}{4} \end{aligned}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,0)}[q](E,2), \vec{\mathbb{G}}_{3,2}^{(3,0)}[q](E,2)$$

** symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

** expression

$$\begin{aligned} & \frac{Q_1 z (x^2 - 4y^2 + z^2)}{4} - \frac{5Q_2 xyz}{4} + \frac{\sqrt{6}Q_3 x (2x^2 - 3y^2 - 3z^2)}{12} + \frac{\sqrt{15}Q_{3x}xyz}{4} \\ & + \frac{\sqrt{15}Q_{3y}z (3x^2 - z^2)}{12} - \frac{\sqrt{10}Q_{az}y (x - z) (x + z)}{4} - \frac{\sqrt{6}Q_{bz}y (3x^2 - 2y^2 + 3z^2)}{12} \\ & \frac{5Q_1 xyz}{4} + \frac{Q_2 z (4x^2 - y^2 - z^2)}{4} - \frac{\sqrt{6}Q_3 y (3x^2 - 2y^2 + 3z^2)}{12} + \frac{\sqrt{15}Q_{3x}z (3y^2 - z^2)}{12} \\ & + \frac{\sqrt{15}Q_{3y}xyz}{4} - \frac{\sqrt{10}Q_{az}x (y - z) (y + z)}{4} - \frac{\sqrt{6}Q_{bz}x (2x^2 - 3y^2 - 3z^2)}{12} \end{aligned}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,2)}[q](E, 1), \vec{\mathbb{G}}_{3,2}^{(3,2)}[q](E, 1)$$

** symmetry

$$-\frac{y (3x^2 - 2y^2 + 3z^2)}{2}$$

$$\frac{x (2x^2 - 3y^2 - 3z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{7\sqrt{330}Q_1xyz (x^2 - 2y^2 + z^2)}{88} + \frac{\sqrt{330}Q_2z (9x^4 - 87x^2y^2 + 11x^2z^2 + 30y^4 - 31y^2z^2 + 2z^4)}{528} - \frac{7\sqrt{55}Q_3y (x - z) (x + z) (x^2 - 2y^2 + z^2)}{44} \\ & - \frac{5\sqrt{22}Q_{3x}z (5x^4 - 39x^2y^2 + 3x^2z^2 - 2y^4 + 17y^2z^2 - 2z^4)}{176} - \frac{35\sqrt{22}Q_{3y}xyz (x^2 - 2y^2 + z^2)}{88} \\ & + \frac{5\sqrt{33}Q_{az}x (x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{264} - \frac{\sqrt{55}Q_{bz}x (3x^4 - 29x^2y^2 - x^2z^2 + 10y^4 + 27y^2z^2 - 4z^4)}{88} \\ & \frac{\sqrt{330}Q_1z (30x^4 - 87x^2y^2 - 31x^2z^2 + 9y^4 + 11y^2z^2 + 2z^4)}{528} - \frac{7\sqrt{330}Q_2xyz (2x^2 - y^2 - z^2)}{88} - \frac{7\sqrt{55}Q_3x (y - z) (y + z) (2x^2 - y^2 - z^2)}{44} \\ & - \frac{35\sqrt{22}Q_{3x}xyz (2x^2 - y^2 - z^2)}{88} - \frac{5\sqrt{22}Q_{3y}z (2x^4 + 39x^2y^2 - 17x^2z^2 - 5y^4 - 3y^2z^2 + 2z^4)}{176} \\ & + \frac{5\sqrt{33}Q_{az}y (6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4)}{264} - \frac{\sqrt{55}Q_{bz}y (10x^4 - 29x^2y^2 + 27x^2z^2 + 3y^4 - y^2z^2 - 4z^4)}{88} \end{aligned}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,2)}[q](E, 2), \vec{\mathbb{G}}_{3,2}^{(3,2)}[q](E, 2)$$

** symmetry

$$\frac{\sqrt{15}x (y - z) (y + z)}{2}$$

$$-\frac{\sqrt{15}y (x - z) (x + z)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{22}Q_1z (40x^4 - 165x^2y^2 - 25x^2z^2 + 5y^4 + 45y^2z^2 - 2z^4)}{176} - \frac{35\sqrt{22}Q_2xyz (2x^2 - y^2 - z^2)}{88} \\ & - \frac{\sqrt{33}Q_3x (8x^4 - 40x^2y^2 - 40x^2z^2 + 15y^4 + 30y^2z^2 + 15z^4)}{132} + \frac{7\sqrt{330}Q_{3x}xyz (2x^2 - y^2 - z^2)}{88} \\ & - \frac{\sqrt{330}Q_{3y}z (24x^4 - 57x^2y^2 - 29x^2z^2 + 45y^4 - 71y^2z^2 + 10z^4)}{528} \\ & - \frac{\sqrt{55}Q_{az}y (4x^4 + x^2y^2 - 27x^2z^2 - 3y^4 + 29y^2z^2 - 10z^4)}{88} + \frac{\sqrt{33}Q_{bz}y (60x^4 - 55x^2y^2 - 195x^2z^2 + 11y^4 - 55y^2z^2 + 60z^4)}{264} \\ & \frac{35\sqrt{22}Q_1xyz (x^2 - 2y^2 + z^2)}{88} - \frac{\sqrt{22}Q_2z (5x^4 - 165x^2y^2 + 45x^2z^2 + 40y^4 - 25y^2z^2 - 2z^4)}{176} \\ & - \frac{\sqrt{33}Q_3y (15x^4 - 40x^2y^2 + 30x^2z^2 + 8y^4 - 40y^2z^2 + 15z^4)}{132} - \frac{\sqrt{330}Q_{3x}z (45x^4 - 57x^2y^2 - 71x^2z^2 + 24y^4 - 29y^2z^2 + 10z^4)}{528} \\ & - \frac{7\sqrt{330}Q_{3y}xyz (x^2 - 2y^2 + z^2)}{88} + \frac{\sqrt{55}Q_{az}x (3x^4 - x^2y^2 - 29x^2z^2 - 4y^4 + 27y^2z^2 + 10z^4)}{88} \\ & - \frac{\sqrt{33}Q_{bz}x (11x^4 - 55x^2y^2 - 55x^2z^2 + 60y^4 - 195y^2z^2 + 60z^4)}{264} \end{aligned}$$

* Harmonics for rank 4

$$\vec{\mathbb{G}}_4^{(3,-2)}[q](A, 1)$$

** symmetry

$$\frac{\sqrt{5}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\frac{9\sqrt{70}Q_1xz}{140} - \frac{9\sqrt{70}Q_2yz}{140} + \frac{\sqrt{105}Q_3(x^2 + y^2 - 2z^2)}{35} + \frac{\sqrt{42}Q_{3x}yz}{28} + \frac{\sqrt{42}Q_{3y}xz}{28} + \frac{3\sqrt{7}Q_{az}xy}{7}$$

$$\bar{\mathbb{G}}_4^{(3,-2)}[q](A, 2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\frac{9\sqrt{70}Q_1yz}{140} + \frac{9\sqrt{70}Q_2xz}{140} + \frac{\sqrt{42}Q_{3x}xz}{28} - \frac{\sqrt{42}Q_{3y}yz}{28} + \frac{3\sqrt{7}Q_{az}(x-y)(x+y)}{14} + \frac{\sqrt{105}Q_{bz}(x^2 + y^2 - 2z^2)}{35}$$

$$\bar{\mathbb{G}}_4^{(3,0)}[q](A, 1)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{5\sqrt{385}Q_1xz(3x^2 + 3y^2 - 4z^2)}{308} - \frac{5\sqrt{385}Q_2yz(3x^2 + 3y^2 - 4z^2)}{308} + \frac{\sqrt{2310}Q_3(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{308} \\ & + \frac{\sqrt{231}Q_{3x}yz(27x^2 - y^2 - 8z^2)}{308} - \frac{\sqrt{231}Q_{3y}xz(x^2 - 27y^2 + 8z^2)}{308} - \frac{13\sqrt{154}Q_{az}xy(x^2 + y^2 - 6z^2)}{308} - \frac{\sqrt{2310}Q_{bz}xy(x-y)(x+y)}{44} \end{aligned}$$

$$\bar{\mathbb{G}}_4^{(3,0)}[q](A, 2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{5\sqrt{385}Q_1yz(3x^2 + 3y^2 - 4z^2)}{308} + \frac{5\sqrt{385}Q_2xz(3x^2 + 3y^2 - 4z^2)}{308} - \frac{\sqrt{2310}Q_{3x}xy(x-y)(x+y)}{44} + \frac{\sqrt{231}Q_{3x}xz(13x^2 - 15y^2 - 8z^2)}{308} \\ & + \frac{\sqrt{231}Q_{3y}yz(15x^2 - 13y^2 + 8z^2)}{308} - \frac{13\sqrt{154}Q_{az}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{616} - \frac{\sqrt{2310}Q_{bz}(5x^4 - 18x^2y^2 - 12x^2z^2 + 5y^4 - 12y^2z^2 + 4z^4)}{616} \end{aligned}$$

$$\bar{\mathbb{G}}_4^{(3,2)}[q](A, 1)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{9\sqrt{2002}Q_1xz(4x^4 - 25x^2y^2 - 5x^2z^2 + 15y^4 - 5y^2z^2 + 2z^4)}{1144} - \frac{9\sqrt{2002}Q_2yz(15x^4 - 25x^2y^2 - 5x^2z^2 + 4y^4 - 5y^2z^2 + 2z^4)}{1144} \\ & - \frac{\sqrt{3003}Q_3(x^6 - 15x^4z^2 + 15x^2z^4 + y^6 - 15y^4z^2 + 15y^2z^4 - 2z^6)}{286} \\ & - \frac{\sqrt{30030}Q_{3x}yz(13x^4 + 17x^2y^2 - 43x^2z^2 + 4y^4 - 19y^2z^2 + 10z^4)}{1144} - \frac{\sqrt{30030}Q_{3y}xz(4x^4 + 17x^2y^2 - 19x^2z^2 + 13y^4 - 43y^2z^2 + 10z^4)}{1144} \\ & + \frac{3\sqrt{5005}Q_{az}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} + \frac{3\sqrt{3003}Q_{bz}xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572} \end{aligned}$$

$$\bar{\mathbb{G}}_4^{(3,2)}[q](A, 2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\begin{aligned} & -\frac{9\sqrt{2002}Q_1yz(25x^4 - 60x^2y^2 + 10x^2z^2 + 3y^4 + 10y^2z^2 - 4z^4)}{2288} - \frac{9\sqrt{2002}Q_2xz(3x^4 - 60x^2y^2 + 10x^2z^2 + 25y^4 + 10y^2z^2 - 4z^4)}{2288} \\ & + \frac{3\sqrt{3003}Q_{3x}xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572} - \frac{\sqrt{30030}Q_{3x}xz(17x^4 + 16x^2y^2 - 62x^2z^2 - y^4 - 14y^2z^2 + 20z^4)}{2288} \\ & - \frac{\sqrt{30030}Q_{3y}yz(x^4 - 16x^2y^2 + 14x^2z^2 - 17y^4 + 62y^2z^2 - 20z^4)}{2288} + \frac{3\sqrt{5005}Q_{az}(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144} \\ & - \frac{\sqrt{3003}Q_{bz}(x^6 + 15x^4y^2 - 30x^4z^2 + 15x^2y^4 - 180x^2y^2z^2 + 60x^2z^4 + y^6 - 30y^4z^2 + 60y^2z^4 - 8z^6)}{1144} \end{aligned}$$

$$\bar{\mathbb{G}}_4^{(3,-2)}[q](B, 1)$$

** symmetry

$$\frac{\sqrt{21} (x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\frac{\sqrt{6}Q_1xz}{4} + \frac{\sqrt{6}Q_2yz}{4} + \frac{\sqrt{10}Q_3xyz}{4} - \frac{\sqrt{10}Q_3yxz}{4} - Q_{bzx}y$$

$$\vec{\mathbb{G}}_4^{(3,-2)}[q](B, 2)$$

** symmetry

$$-\frac{\sqrt{15} (x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$-\frac{\sqrt{210}Q_1xz}{20} - \frac{\sqrt{210}Q_2yz}{20} + \frac{3\sqrt{35}Q_3(x-y)(x+y)}{35} + \frac{5\sqrt{14}Q_3xyz}{28} - \frac{5\sqrt{14}Q_3yxz}{28} + \frac{\sqrt{35}Q_{bzx}y}{35}$$

$$\vec{\mathbb{G}}_4^{(3,-2)}[q](B, 3)$$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{3\sqrt{10}Q_1yz}{10} - \frac{3\sqrt{10}Q_2xz}{10} - \frac{\sqrt{15}Q_3xy}{5} + \frac{\sqrt{15}Q_{bzx}(x-y)(x+y)}{10}$$

$$\vec{\mathbb{G}}_4^{(3,0)}[q](B, 1)$$

** symmetry

$$\frac{\sqrt{21} (x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\frac{5\sqrt{33}Q_1xz(5x^2 - 9y^2 - 2z^2)}{132} - \frac{5\sqrt{33}Q_2yz(9x^2 - 5y^2 + 2z^2)}{132} - \frac{\sqrt{55}Q_3xyz(15x^2 + y^2 - 6z^2)}{44} \\ + \frac{\sqrt{55}Q_3yxz(x^2 + 15y^2 - 6z^2)}{44} + \frac{7\sqrt{330}Q_{az}xy(x-y)(x+y)}{132} - \frac{5\sqrt{22}Q_{bzx}y(x^2 + y^2 - 6z^2)}{44}$$

$$\vec{\mathbb{G}}_4^{(3,0)}[q](B, 2)$$

** symmetry

$$-\frac{\sqrt{15} (x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\frac{\sqrt{1155}Q_1xz(x^2 - 9y^2 + 2z^2)}{132} - \frac{\sqrt{1155}Q_2yz(9x^2 - y^2 - 2z^2)}{132} + \frac{3\sqrt{770}Q_3(x-y)(x+y)(x^2 + y^2 - 6z^2)}{308} \\ + \frac{\sqrt{77}Q_3xyz(51x^2 - 47y^2 + 30z^2)}{308} + \frac{\sqrt{77}Q_3yxz(47x^2 - 51y^2 - 30z^2)}{308} - \frac{7\sqrt{462}Q_{az}xy(x-y)(x+y)}{132} + \frac{\sqrt{770}Q_{bzx}y(x^2 + y^2 - 6z^2)}{308}$$

$$\vec{\mathbb{G}}_4^{(3,0)}[q](B, 3)$$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{55}Q_1yz(3x^2 + 3y^2 - 4z^2)}{44} - \frac{\sqrt{55}Q_2xz(3x^2 + 3y^2 - 4z^2)}{44} - \frac{\sqrt{330}Q_3xy(x^2 + y^2 - 6z^2)}{44} + \frac{7\sqrt{33}Q_{3x}xz(x^2 - 3y^2)}{44} \\ - \frac{7\sqrt{33}Q_{3y}yz(3x^2 - y^2)}{44} - \frac{7\sqrt{22}Q_{az}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{88} + \frac{\sqrt{330}Q_{bzx}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{88}$$

$$\vec{\mathbb{G}}_4^{(3,2)}[q](B, 1)$$

** symmetry

$$\frac{\sqrt{21} (x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\begin{aligned} & \frac{\sqrt{4290}Q_1xz(17x^4 - 29x^2y^2 - 47x^2z^2 - 46y^4 + 121y^2z^2 + 2z^4)}{1144} - \frac{\sqrt{4290}Q_2yz(46x^4 + 29x^2y^2 - 121x^2z^2 - 17y^4 + 47y^2z^2 - 2z^4)}{1144} \\ & - \frac{3\sqrt{715}Q_3(x-y)(x+y)(x-z)(x+z)(y-z)(y+z)}{26} - \frac{5\sqrt{286}Q_{3x}yz(46x^4 - 91x^2y^2 - x^2z^2 - 5y^4 + 47y^2z^2 - 14z^4)}{1144} \\ & - \frac{5\sqrt{286}Q_{3y}xz(5x^4 + 91x^2y^2 - 47x^2z^2 - 46y^4 + y^2z^2 + 14z^4)}{1144} + \frac{15\sqrt{429}Q_{az}xy(x-y)(x+y)(x^2+y^2-10z^2)}{572} \\ & - \frac{\sqrt{715}Q_{bz}xy(19x^4 - 94x^2y^2 + 92x^2z^2 + 19y^4 + 92y^2z^2 - 92z^4)}{572} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(3,2)}[q](B, 2)$

** symmetry

$$- \frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\begin{aligned} & \frac{\sqrt{6006}Q_1xz(10x^4 - 25x^2y^2 - 25x^2z^2 - 35y^4 + 95y^2z^2 - 2z^4)}{1144} - \frac{\sqrt{6006}Q_2yz(35x^4 + 25x^2y^2 - 95x^2z^2 - 10y^4 + 25y^2z^2 + 2z^4)}{1144} \\ & - \frac{3\sqrt{1001}Q_3(x-y)(x+y)(x^4 - 9x^2y^2 - 5x^2z^2 + y^4 - 5y^2z^2 + 5z^4)}{286} \\ & + \frac{\sqrt{10010}Q_{3x}yz(61x^4 - 61x^2y^2 - 61x^2z^2 + 10y^4 - 13y^2z^2 + 10z^4)}{1144} - \frac{\sqrt{10010}Q_{3y}xz(10x^4 - 61x^2y^2 - 13x^2z^2 + 61y^4 - 61y^2z^2 + 10z^4)}{1144} \\ & - \frac{3\sqrt{15015}Q_{az}xy(x-y)(x+y)(x^2+y^2-10z^2)}{572} + \frac{\sqrt{1001}Q_{bz}xy(31x^4 - 70x^2y^2 - 100x^2z^2 + 31y^4 - 100y^2z^2 + 100z^4)}{572} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(3,2)}[q](B, 3)$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{286}Q_1yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} - \frac{3\sqrt{286}Q_2xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\ & + \frac{\sqrt{429}Q_3xy(47x^4 - 170x^2y^2 + 40x^2z^2 + 47y^4 + 40y^2z^2 - 40z^4)}{572} + \frac{3\sqrt{4290}Q_{3x}xz(17x^4 - 122x^2y^2 - 16x^2z^2 + 37y^4 + 48y^2z^2)}{2288} \\ & + \frac{3\sqrt{4290}Q_{3y}yz(37x^4 - 122x^2y^2 + 48x^2z^2 + 17y^4 - 16y^2z^2)}{2288} - \frac{9\sqrt{715}Q_{az}(x^2+y^2-10z^2)(x^2-2xy-y^2)(x^2+2xy-y^2)}{1144} \\ & + \frac{\sqrt{429}Q_{bz}(x-y)(x+y)(19x^4 - 226x^2y^2 - 40x^2z^2 + 19y^4 - 40y^2z^2 + 40z^4)}{1144} \end{aligned}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-2)}[q](E, 1), \vec{\mathbb{G}}_{4,2}^{(3,-2)}[q](E, 1)$

** symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$- \frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{10}Q_1xy}{40} - \frac{3\sqrt{10}Q_2(y-z)(y+z)}{40} - \frac{\sqrt{15}Q_3yz}{5} - \frac{\sqrt{6}Q_{3x}(y-z)(y+z)}{8} + \frac{3\sqrt{6}Q_{3y}xy}{8} - \frac{3Q_{az}xz}{4} - \frac{3\sqrt{15}Q_{bz}xz}{20} \\ & \frac{3\sqrt{10}Q_1(x-z)(x+z)}{40} - \frac{3\sqrt{10}Q_2xy}{40} - \frac{\sqrt{15}Q_3xz}{5} + \frac{3\sqrt{6}Q_{3x}xy}{8} - \frac{\sqrt{6}Q_{3y}(x-z)(x+z)}{8} - \frac{3Q_{az}yz}{4} + \frac{3\sqrt{15}Q_{bz}yz}{20} \end{aligned}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-2)}[q](E, 2), \vec{\mathbb{G}}_{4,2}^{(3,-2)}[q](E, 2)$

** symmetry

$$- \frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

** expression

$$\frac{3\sqrt{70}Q_1(3x^2 + 4y^2 - 7z^2)}{280} + \frac{3\sqrt{70}Q_2xy}{280} + \frac{5\sqrt{42}Q_{3x}xy}{56} + \frac{\sqrt{42}Q_{3y}(5x^2 - 4y^2 - z^2)}{56} + \frac{3\sqrt{7}Q_{az}yz}{28} - \frac{\sqrt{105}Q_{bz}yz}{20}$$

$$\frac{3\sqrt{70}Q_1xy}{280} + \frac{3\sqrt{70}Q_2(4x^2+3y^2-7z^2)}{280} + \frac{\sqrt{42}Q_{3x}(4x^2-5y^2+z^2)}{56} - \frac{5\sqrt{42}Q_{3y}xy}{56} - \frac{3\sqrt{7}Q_{az}xz}{28} - \frac{\sqrt{105}Q_{bz}xz}{20}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,0)}[q](E, 1), \vec{\mathbb{G}}_{4,2}^{(3,0)}[q](E, 1)$$

** symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{55}Q_1xy(x^2-6y^2+15z^2)}{44} + \frac{\sqrt{55}Q_2(9x^2y^2-9x^2z^2-5y^4+21y^2z^2-2z^4)}{88} + \frac{\sqrt{330}Q_{3yz}(6x^2-y^2-z^2)}{44} \\ & + \frac{\sqrt{33}Q_{3x}(15x^2y^2-15x^2z^2+y^4-21y^2z^2+6z^4)}{88} - \frac{\sqrt{33}Q_{3y}xy(5x^2-2y^2-9z^2)}{44} + \frac{\sqrt{22}Q_{az}xz(5x^2+12y^2-9z^2)}{44} + \frac{\sqrt{330}Q_{bz}xz(x^2-6y^2+z^2)}{44} \\ & \frac{\sqrt{55}Q_1(5x^4-9x^2y^2-21x^2z^2+9y^2z^2+2z^4)}{88} - \frac{\sqrt{55}Q_2xy(6x^2-y^2-15z^2)}{44} - \frac{\sqrt{330}Q_{3xz}(x^2-6y^2+z^2)}{44} + \frac{\sqrt{33}Q_{3xy}(2x^2-5y^2+9z^2)}{44} \\ & + \frac{\sqrt{33}Q_{3y}(x^4+15x^2y^2-21x^2z^2-15y^2z^2+6z^4)}{88} + \frac{\sqrt{22}Q_{az}yz(12x^2+5y^2-9z^2)}{44} + \frac{\sqrt{330}Q_{bzyz}(6x^2-y^2-z^2)}{44} \end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,0)}[q](E, 2), \vec{\mathbb{G}}_{4,2}^{(3,0)}[q](E, 2)$$

** symmetry

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{385}Q_1(x^4+21x^2y^2-27x^2z^2+6y^4-57y^2z^2+14z^4)}{616} + \frac{\sqrt{385}Q_2xy(6x^2-y^2-15z^2)}{308} + \frac{\sqrt{2310}Q_{3xz}(x-z)(x+z)}{44} \\ & + \frac{\sqrt{231}Q_{3xy}(22x^2-27y^2+15z^2)}{308} - \frac{\sqrt{231}Q_{3y}(19x^4-69x^2y^2-45x^2z^2+10y^4+9y^2z^2+6z^4)}{616} \\ & + \frac{\sqrt{154}Q_{az}yz(30x^2-19y^2+9z^2)}{308} + \frac{\sqrt{2310}Q_{bzyz}(y-z)(y+z)}{44} \\ & - \frac{\sqrt{385}Q_1xy(x^2-6y^2+15z^2)}{308} + \frac{\sqrt{385}Q_2(6x^4+21x^2y^2-57x^2z^2+y^4-27y^2z^2+14z^4)}{616} \\ & - \frac{\sqrt{2310}Q_{3yz}(y-z)(y+z)}{44} + \frac{\sqrt{231}Q_{3x}(10x^4-69x^2y^2+9x^2z^2+19y^4-45y^2z^2+6z^4)}{616} \\ & + \frac{\sqrt{231}Q_{3xy}(27x^2-22y^2-15z^2)}{308} + \frac{\sqrt{154}Q_{az}xz(19x^2-30y^2-9z^2)}{308} + \frac{\sqrt{2310}Q_{bzxz}(x-z)(x+z)}{44} \end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,2)}[q](E, 1), \vec{\mathbb{G}}_{4,2}^{(3,2)}[q](E, 1)$$

** symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{286}Q_1xy(2x^4-65x^2y^2+175x^2z^2+65y^4-455y^2z^2+140z^4)}{2288} \\ & - \frac{3\sqrt{286}Q_2(20x^4y^2-20x^4z^2-95x^2y^4+450x^2y^2z^2-55x^2z^4+17y^6-160y^4z^2+85y^2z^4-2z^6)}{2288} \\ & - \frac{\sqrt{429}Q_{3yz}(40x^4-40x^2y^2-40x^2z^2-47y^4+170y^2z^2-47z^4)}{572} \\ & - \frac{\sqrt{4290}Q_{3x}(20x^4y^2-20x^4z^2+25x^2y^4-270x^2y^2z^2+65x^2z^4+5y^6-100y^4z^2+145y^2z^4-14z^6)}{2288} \\ & + \frac{3\sqrt{4290}Q_{3xy}(2x^4-x^2y^2-17x^2z^2-3y^4+33y^2z^2-8z^4)}{2288} - \frac{3\sqrt{715}Q_{az}xz(2x^4+31x^2y^2-17x^2z^2+29y^4-89y^2z^2+14z^4)}{1144} \\ & - \frac{3\sqrt{429}Q_{bz}xz(2x^4-65x^2y^2+15x^2z^2-45y^4+155y^2z^2-20z^4)}{1144} \end{aligned}$$

$$\begin{aligned}
& \frac{3\sqrt{286}Q_1 (17x^6 - 95x^4y^2 - 160x^4z^2 + 20x^2y^4 + 450x^2y^2z^2 + 85x^2z^4 - 20y^4z^2 - 55y^2z^4 - 2z^6)}{2288} \\
& - \frac{3\sqrt{286}Q_2xy (65x^4 - 65x^2y^2 - 455x^2z^2 + 2y^4 + 175y^2z^2 + 140z^4)}{2288} \\
& + \frac{\sqrt{429}Q_3xz (47x^4 + 40x^2y^2 - 170x^2z^2 - 40y^4 + 40y^2z^2 + 47z^4)}{572} - \frac{3\sqrt{4290}Q_{3x}xy (3x^4 + x^2y^2 - 33x^2z^2 - 2y^4 + 17y^2z^2 + 8z^4)}{2288} \\
& - \frac{\sqrt{4290}Q_{3y} (5x^6 + 25x^4y^2 - 100x^4z^2 + 20x^2y^4 - 270x^2y^2z^2 + 145x^2z^4 - 20y^4z^2 + 65y^2z^4 - 14z^6)}{2288} \\
& - \frac{3\sqrt{715}Q_{az}yz (29x^4 + 31x^2y^2 - 89x^2z^2 + 2y^4 - 17y^2z^2 + 14z^4)}{1144} - \frac{3\sqrt{429}Q_{bz}yz (45x^4 + 65x^2y^2 - 155x^2z^2 - 2y^4 - 15y^2z^2 + 20z^4)}{1144}
\end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,2)}[q](E, 2), \vec{\mathbb{G}}_{4,2}^{(3,2)}[q](E, 2)$$

** symmetry

$$- \frac{\sqrt{5}xz (x^2 - 6y^2 + z^2)}{2}$$

$$\frac{\sqrt{5}yz (6x^2 - y^2 - z^2)}{2}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{2002}Q_1 (3x^6 - 55x^4y^2 + 10x^4z^2 + 70x^2y^4 - 90x^2y^2z^2 + 5x^2z^4 - 4y^6 - 10y^4z^2 + 25y^2z^4 - 2z^6)}{2288} \\
& - \frac{3\sqrt{2002}Q_2xy (19x^4 - 85x^2y^2 + 65x^2z^2 + 28y^4 - 25y^2z^2 - 20z^4)}{2288} \\
& - \frac{3\sqrt{3003}Q_3xz (x - z) (x + z) (x^2 - 10y^2 + z^2)}{572} - \frac{\sqrt{30030}Q_{3x}xy (11x^4 + 7x^2y^2 - 131x^2z^2 - 4y^4 + 19y^2z^2 + 56z^4)}{2288} \\
& + \frac{\sqrt{30030}Q_{3y} (x^6 - 9x^4y^2 - 6x^4z^2 - 6x^2y^4 + 90x^2y^2z^2 - 9x^2z^4 + 4y^6 - 54y^4z^2 + 39y^2z^4 - 2z^6)}{2288} \\
& - \frac{3\sqrt{5005}Q_{az}yz (19x^4 + 11x^2y^2 - 49x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4)}{1144} + \frac{\sqrt{3003}Q_{bz}yz (55x^4 + 5x^2y^2 - 115x^2z^2 + 16y^4 - 55y^2z^2 + 28z^4)}{1144} \\
& - \frac{3\sqrt{2002}Q_1xy (28x^4 - 85x^2y^2 - 25x^2z^2 + 19y^4 + 65y^2z^2 - 20z^4)}{2288} \\
& - \frac{3\sqrt{2002}Q_2 (4x^6 - 70x^4y^2 + 10x^4z^2 + 55x^2y^4 + 90x^2y^2z^2 - 25x^2z^4 - 3y^6 - 10y^4z^2 - 5y^2z^4 + 2z^6)}{2288} \\
& - \frac{3\sqrt{3003}Q_3yz (y - z) (y + z) (10x^2 - y^2 - z^2)}{572} - \frac{\sqrt{30030}Q_{3x} (4x^6 - 6x^4y^2 - 54x^4z^2 - 9x^2y^4 + 90x^2y^2z^2 + 39x^2z^4 + y^6 - 6y^4z^2 - 9y^2z^4 - 2z^6)}{2288} \\
& - \frac{\sqrt{30030}Q_{3y}xy (4x^4 - 7x^2y^2 - 19x^2z^2 - 11y^4 + 131y^2z^2 - 56z^4)}{2288} - \frac{3\sqrt{5005}Q_{az}xz (8x^4 - 11x^2y^2 - 23x^2z^2 - 19y^4 + 49y^2z^2 + 2z^4)}{1144} \\
& + \frac{\sqrt{3003}Q_{bz}xz (16x^4 + 5x^2y^2 - 55x^2z^2 + 55y^4 - 115y^2z^2 + 28z^4)}{1144}
\end{aligned}$$