

PG No. 15  $D_{4h}$  4/mmm [ tetragonal ] (axial, internal axial octupole)

\* Harmonics for rank 0

$$\vec{\mathbb{G}}_0^{(3,3)}[g](A_{1u})$$

\*\* symmetry

1

\*\* expression

$$\begin{aligned} & \frac{\sqrt{70}G_1y(3x^2 - y^2)}{28} + \frac{\sqrt{70}G_2x(x^2 - 3y^2)}{28} + \frac{\sqrt{105}G_3xyz}{7} - \frac{\sqrt{42}G_{3x}x(x^2 + y^2 - 4z^2)}{28} \\ & - \frac{\sqrt{42}G_{3y}y(x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{7}G_{az}z(3x^2 + 3y^2 - 2z^2)}{14} + \frac{\sqrt{105}G_{bz}z(x - y)(x + y)}{14} \end{aligned}$$

\* Harmonics for rank 1

$$\vec{\mathbb{G}}_1^{(3,1)}[g](A_{2g})$$

\*\* symmetry

$z$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{21}G_{3xy}}{7} + \frac{2\sqrt{210}G_{3x}xz}{35} + \frac{2\sqrt{210}G_{3y}yz}{35} - \frac{3\sqrt{35}G_{az}(x^2 + y^2 - 2z^2)}{70} + \frac{\sqrt{21}G_{bz}(x - y)(x + y)}{14} \end{aligned}$$

$$\vec{\mathbb{G}}_1^{(3,3)}[g](A_{2g})$$

\*\* symmetry

$z$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{210}G_1yz(3x^2 - y^2)}{24} + \frac{\sqrt{210}G_2xz(x^2 - 3y^2)}{24} - \frac{\sqrt{35}G_3xy(x^2 + y^2 - 6z^2)}{14} - \frac{5\sqrt{14}G_{3x}xz(3x^2 + 3y^2 - 4z^2)}{56} \\ & - \frac{5\sqrt{14}G_{3y}yz(3x^2 + 3y^2 - 4z^2)}{56} + \frac{\sqrt{21}G_{az}(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{84} - \frac{\sqrt{35}G_{bz}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{28} \end{aligned}$$

$$\vec{\mathbb{G}}_{1,1}^{(3,1)}[g](E_g), \vec{\mathbb{G}}_{1,2}^{(3,1)}[g](E_g)$$

\*\* symmetry

$x$

$-y$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{14}G_1xy}{14} + \frac{3\sqrt{14}G_2(x - y)(x + y)}{28} + \frac{\sqrt{21}G_3yz}{7} - \frac{\sqrt{210}G_{3x}(3x^2 + y^2 - 4z^2)}{140} - \frac{\sqrt{210}G_{3y}xy}{70} - \frac{3\sqrt{35}G_{az}xz}{35} + \frac{\sqrt{21}G_{bz}xz}{7} \\ & - \frac{3\sqrt{14}G_1(x - y)(x + y)}{28} + \frac{3\sqrt{14}G_2xy}{14} - \frac{\sqrt{21}G_3xz}{7} + \frac{\sqrt{210}G_{3x}xy}{70} + \frac{\sqrt{210}G_{3y}(x^2 + 3y^2 - 4z^2)}{140} + \frac{3\sqrt{35}G_{az}yz}{35} + \frac{\sqrt{21}G_{bz}yz}{7} \end{aligned}$$

$$\vec{\mathbb{G}}_{1,1}^{(3,3)}[g](E_g), \vec{\mathbb{G}}_{1,2}^{(3,3)}[g](E_g)$$

\*\* symmetry

$x$

$-y$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{210}G_1xy(15x^2 - 13y^2 - 6z^2)}{168} + \frac{\sqrt{210}G_2(4x^4 - 21x^2y^2 - 3x^2z^2 + 3y^4 + 3y^2z^2)}{168} \\ & + \frac{\sqrt{35}G_3yz(6x^2 - y^2 - z^2)}{14} - \frac{\sqrt{14}G_{3x}(4x^4 + 3x^2y^2 - 27x^2z^2 - y^4 + 3y^2z^2 + 4z^4)}{56} \\ & - \frac{5\sqrt{14}G_{3y}xy(x^2 + y^2 - 6z^2)}{56} - \frac{5\sqrt{21}G_{az}xz(3x^2 + 3y^2 - 4z^2)}{84} + \frac{\sqrt{35}G_{bz}xz(5x^2 - 9y^2 - 2z^2)}{28} \end{aligned}$$

$$\begin{aligned} & \frac{\sqrt{210}G_1(3x^4 - 21x^2y^2 + 3x^2z^2 + 4y^4 - 3y^2z^2)}{168} - \frac{\sqrt{210}G_2xy(13x^2 - 15y^2 + 6z^2)}{168} + \frac{\sqrt{35}G_{3xz}(x^2 - 6y^2 + z^2)}{14} + \frac{5\sqrt{14}G_{3x}xy(x^2 + y^2 - 6z^2)}{56} \\ & - \frac{\sqrt{14}G_{3y}(x^4 - 3x^2y^2 - 3x^2z^2 - 4y^4 + 27y^2z^2 - 4z^4)}{56} + \frac{5\sqrt{21}G_{az}yz(3x^2 + 3y^2 - 4z^2)}{84} - \frac{\sqrt{35}G_{bz}yz(9x^2 - 5y^2 + 2z^2)}{28} \end{aligned}$$

\* Harmonics for rank 2

$$\vec{\mathbb{G}}_2^{(3,-1)}[g](A_{1u})$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$\frac{\sqrt{14}G_{3x}x}{7} + \frac{\sqrt{14}G_{3y}y}{7} + \frac{\sqrt{21}G_{az}z}{7}$$

$\vec{\mathbb{G}}_2^{(3,1)}[g](A_{1u})$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$-\frac{5\sqrt{210}G_1y(3x^2 - y^2)}{168} - \frac{5\sqrt{210}G_2x(x^2 - 3y^2)}{168} - \frac{3\sqrt{14}G_{3x}x(x^2 + y^2 - 4z^2)}{56} - \frac{3\sqrt{14}G_{3y}y(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{21}G_{az}z(3x^2 + 3y^2 - 2z^2)}{21}$$

$\vec{\mathbb{G}}_2^{(3,3)}[g](A_{1u})$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$\begin{aligned} & -\frac{\sqrt{1155}G_1y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{264} - \frac{\sqrt{1155}G_2x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{264} - \frac{3\sqrt{770}G_3xyz(x^2 + y^2 - 2z^2)}{44} \\ & + \frac{15\sqrt{77}G_{3x}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{616} + \frac{15\sqrt{77}G_{3y}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{616} \\ & + \frac{5\sqrt{462}G_{az}z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{1848} - \frac{3\sqrt{770}G_{bz}z(x - y)(x + y)(x^2 + y^2 - 2z^2)}{88} \end{aligned}$$

$\vec{\mathbb{G}}_2^{(3,-1)}[g](B_{1u})$

\*\* symmetry

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

\*\* expression

$$\frac{\sqrt{70}G_1y}{14} + \frac{\sqrt{70}G_2x}{14} - \frac{\sqrt{42}G_{3x}x}{42} + \frac{\sqrt{42}G_{3y}y}{42} + \frac{\sqrt{105}G_{bz}z}{21}$$

$\vec{\mathbb{G}}_2^{(3,1)}[g](B_{1u})$

\*\* symmetry

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{70}G_1y(x^2 + y^2 - 4z^2)}{56} + \frac{\sqrt{70}G_2x(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{42}G_{3x}x(11x^2 - 9y^2 - 24z^2)}{168} \\ & - \frac{\sqrt{42}G_{3y}y(9x^2 - 11y^2 + 24z^2)}{168} - \frac{5\sqrt{7}G_{az}z(x - y)(x + y)}{14} + \frac{\sqrt{105}G_{bz}z(3x^2 + 3y^2 - 2z^2)}{42} \end{aligned}$$

$\vec{\mathbb{G}}_2^{(3,3)}[g](B_{1u})$

\*\* symmetry

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{385}G_1y(53x^4 - 104x^2y^2 - 6x^2z^2 + 11y^4 - 6y^2z^2 + 4z^4)}{616} + \frac{\sqrt{385}G_2x(11x^4 - 104x^2y^2 - 6x^2z^2 + 53y^4 - 6y^2z^2 + 4z^4)}{616} \\ & + \frac{3\sqrt{2310}G_3xyz(x - y)(x + y)}{44} - \frac{5\sqrt{231}G_{3x}x(5x^4 - 4x^2y^2 - 46x^2z^2 - 9y^4 + 66y^2z^2 + 12z^4)}{1848} \\ & - \frac{5\sqrt{231}G_{3y}y(9x^4 + 4x^2y^2 - 66x^2z^2 - 5y^4 + 46y^2z^2 - 12z^4)}{1848} - \frac{5\sqrt{154}G_{az}z(x - y)(x + y)(x^2 + y^2 - 2z^2)}{88} \\ & + \frac{\sqrt{2310}G_{bz}z(39x^4 - 174x^2y^2 - 20x^2z^2 + 39y^4 - 20y^2z^2 + 4z^4)}{1848} \end{aligned}$$

$\vec{\mathbb{G}}_2^{(3,-1)}[g](B_{2u})$

\*\* symmetry

$$\sqrt{3}xy$$

\*\* expression

$$\frac{\sqrt{70}G_1x}{14} - \frac{\sqrt{70}G_2y}{14} + \frac{\sqrt{105}G_3z}{21} - \frac{\sqrt{42}G_{3x}y}{42} - \frac{\sqrt{42}G_{3y}x}{42}$$

$\vec{\mathbb{G}}_2^{(3,1)}[g](B_{2u})$

\*\* symmetry

$$\sqrt{3}xy$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{70}G_1x(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{70}G_2y(x^2 + y^2 - 4z^2)}{56} + \frac{\sqrt{105}G_3z(3x^2 + 3y^2 - 2z^2)}{42} \\ & - \frac{\sqrt{42}G_{3x}y(21x^2 + y^2 - 24z^2)}{168} - \frac{\sqrt{42}G_{3y}x(x^2 + 21y^2 - 24z^2)}{168} - \frac{5\sqrt{7}G_{az}xyz}{7} \end{aligned}$$

$\vec{\mathbb{G}}_2^{(3,3)}[g](B_{2u})$

\*\* symmetry

$$\sqrt{3}xy$$

\*\* expression

$$\begin{aligned} & -\frac{\sqrt{385}G_1x(5x^4 - 53x^2y^2 + 3x^2z^2 + 26y^4 + 3y^2z^2 - 2z^4)}{308} + \frac{\sqrt{385}G_2y(26x^4 - 53x^2y^2 + 3x^2z^2 + 5y^4 + 3y^2z^2 - 2z^4)}{308} \\ & - \frac{\sqrt{2310}G_3z(6x^4 - 51x^2y^2 + 5x^2z^2 + 6y^4 + 5y^2z^2 - z^4)}{462} - \frac{5\sqrt{231}G_{3x}y(6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4)}{924} \\ & + \frac{5\sqrt{231}G_{3y}x(x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{924} - \frac{5\sqrt{154}G_{az}xyz(x^2 + y^2 - 2z^2)}{44} + \frac{3\sqrt{2310}G_{bz}xyz(x - y)(x + y)}{44} \end{aligned}$$

$\vec{\mathbb{G}}_{2,1}^{(3,-1)}[g](E_u), \vec{\mathbb{G}}_{2,2}^{(3,-1)}[g](E_u)$

\*\* symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

\*\* expression

$$\frac{\sqrt{105}G_3x}{21} + \frac{2\sqrt{42}G_{3y}z}{21} - \frac{\sqrt{7}G_{az}y}{7} - \frac{\sqrt{105}G_{bz}y}{21}$$

$$-\frac{\sqrt{105}G_3y}{21} - \frac{2\sqrt{42}G_{3x}z}{21} + \frac{\sqrt{7}G_{az}x}{7} - \frac{\sqrt{105}G_{bz}x}{21}$$

$\vec{\mathbb{G}}_{2,1}^{(3,1)}[g](E_u), \vec{\mathbb{G}}_{2,2}^{(3,1)}[g](E_u)$

\*\* symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

\*\* expression

$$\begin{aligned} & \frac{5\sqrt{70}G_1z(x - y)(x + y)}{56} - \frac{5\sqrt{70}G_2xyz}{28} - \frac{\sqrt{105}G_3x(2x^2 - 3y^2 - 3z^2)}{42} + \frac{5\sqrt{42}G_{3x}xyz}{28} \\ & - \frac{\sqrt{42}G_{3y}z(21x^2 - 9y^2 - 4z^2)}{168} - \frac{\sqrt{7}G_{az}y(x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{105}G_{bz}y(9x^2 - y^2 - 6z^2)}{84} \end{aligned}$$

$$\begin{aligned} & -\frac{5\sqrt{70}G_1xyz}{28} - \frac{5\sqrt{70}G_2z(x - y)(x + y)}{56} - \frac{\sqrt{105}G_3y(3x^2 - 2y^2 + 3z^2)}{42} - \frac{\sqrt{42}G_{3x}z(9x^2 - 21y^2 + 4z^2)}{168} \\ & - \frac{5\sqrt{42}G_{3y}xyz}{28} + \frac{\sqrt{7}G_{az}x(x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{105}G_{bz}x(x^2 - 9y^2 + 6z^2)}{84} \end{aligned}$$

$\vec{\mathbb{G}}_{2,1}^{(3,3)}[g](E_u), \vec{\mathbb{G}}_{2,2}^{(3,3)}[g](E_u)$

\*\* symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

\*\* expression

$$\begin{aligned}
& - \frac{\sqrt{385}G_1z(x^4 - 9x^2y^2 + x^2z^2 + 2y^4 - y^2z^2)}{44} + \frac{\sqrt{385}G_2xyz(5x^2 - 7y^2 + 2z^2)}{44} + \frac{\sqrt{2310}G_3x(x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{462} \\
& - \frac{5\sqrt{231}G_{3x}xyz(x^2 + y^2 - 2z^2)}{44} + \frac{5\sqrt{231}G_{3y}z(3x^4 - 15x^2y^2 - x^2z^2 - 18y^4 + 41y^2z^2 - 4z^4)}{924} \\
& + \frac{5\sqrt{154}G_{az}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{308} - \frac{\sqrt{2310}G_{bz}y(9x^4 + 4x^2y^2 - 66x^2z^2 - 5y^4 + 46y^2z^2 - 12z^4)}{924} \\
& - \frac{\sqrt{385}G_1xyz(7x^2 - 5y^2 - 2z^2)}{44} - \frac{\sqrt{385}G_2z(2x^4 - 9x^2y^2 - x^2z^2 + y^4 + y^2z^2)}{44} + \frac{\sqrt{2310}G_3y(6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4)}{462} \\
& + \frac{5\sqrt{231}G_{3x}z(18x^4 + 15x^2y^2 - 41x^2z^2 - 3y^4 + y^2z^2 + 4z^4)}{924} + \frac{5\sqrt{231}G_{3y}xyz(x^2 + y^2 - 2z^2)}{44} \\
& - \frac{5\sqrt{154}G_{az}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{308} + \frac{\sqrt{2310}G_{bz}x(5x^4 - 4x^2y^2 - 46x^2z^2 - 9y^4 + 66y^2z^2 + 12z^4)}{924}
\end{aligned}$$

\* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(3,-3)}[g](A_{2g})$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$G_{az}$

$\vec{\mathbb{G}}_3^{(3,-1)}[g](A_{2g})$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$-G_3xy + \frac{\sqrt{10}G_{3x}xz}{10} + \frac{\sqrt{10}G_{3y}yz}{10} - \frac{\sqrt{15}G_{az}(x^2 + y^2 - 2z^2)}{15} - \frac{G_{bz}(x - y)(x + y)}{2}$$

$\vec{\mathbb{G}}_3^{(3,1)}[g](A_{2g})$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$\begin{aligned}
& - \frac{7\sqrt{55}G_1yz(3x^2 - y^2)}{44} - \frac{7\sqrt{55}G_2xz(x^2 - 3y^2)}{44} + \frac{\sqrt{330}G_3xy(x^2 + y^2 - 6z^2)}{132} - \frac{5\sqrt{33}G_{3x}xz(3x^2 + 3y^2 - 4z^2)}{132} \\
& - \frac{5\sqrt{33}G_{3y}yz(3x^2 + 3y^2 - 4z^2)}{132} + \frac{\sqrt{22}G_{az}(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{88} + \frac{\sqrt{330}G_{bz}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{264}
\end{aligned}$$

$\vec{\mathbb{G}}_3^{(3,3)}[g](A_{2g})$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$\begin{aligned}
& - \frac{7\sqrt{4290}G_1yz(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{2288} - \frac{7\sqrt{4290}G_2xz(x^2 - 3y^2)(3x^2 + 3y^2 - 8z^2)}{2288} \\
& + \frac{7\sqrt{715}G_3xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} + \frac{35\sqrt{286}G_{3x}xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\
& + \frac{35\sqrt{286}G_{3y}yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\
& - \frac{5\sqrt{429}G_{az}(5x^6 + 15x^4y^2 - 90x^4z^2 + 15x^2y^4 - 180x^2y^2z^2 + 120x^2z^4 + 5y^6 - 90y^4z^2 + 120y^2z^4 - 16z^6)}{3432} \\
& + \frac{7\sqrt{715}G_{bz}(x - y)(x + y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144}
\end{aligned}$$

$\vec{\mathbb{G}}_3^{(3,-3)}[g](B_{1g})$

\*\* symmetry

$\sqrt{15}xyz$

\*\* expression

$G_3$

$\tilde{\mathbb{G}}_3^{(3,-1)}[g](B_{1g})$   
\*\* symmetry

$$\sqrt{15}xyz$$

\*\* expression

$$\frac{\sqrt{10}G_1xz}{4} - \frac{\sqrt{10}G_2yz}{4} + \frac{\sqrt{6}G_{3x}yz}{4} + \frac{\sqrt{6}G_{3y}xz}{4} - G_{azxy}$$

$\tilde{\mathbb{G}}_3^{(3,1)}[g](B_{1g})$   
\*\* symmetry

$$\sqrt{15}xyz$$

\*\* expression

$$\begin{aligned} & \frac{5\sqrt{33}G_1xz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{5\sqrt{33}G_2yz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{7\sqrt{22}G_3(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{66} \\ & + \frac{\sqrt{55}G_{3x}yz(9x^2 - 19y^2 + 16z^2)}{132} - \frac{\sqrt{55}G_{3y}xz(19x^2 - 9y^2 - 16z^2)}{132} + \frac{\sqrt{330}G_{az}xy(x^2 + y^2 - 6z^2)}{132} + \frac{35\sqrt{22}G_{bz}xy(x - y)(x + y)}{132} \end{aligned}$$

$\tilde{\mathbb{G}}_3^{(3,3)}[g](B_{1g})$   
\*\* symmetry

$$\sqrt{15}xyz$$

\*\* expression

$$\begin{aligned} & \frac{7\sqrt{286}G_1xz(7x^4 - 85x^2y^2 + 5x^2z^2 + 40y^4 + 5y^2z^2 - 2z^4)}{1144} + \frac{7\sqrt{286}G_2yz(40x^4 - 85x^2y^2 + 5x^2z^2 + 7y^4 + 5y^2z^2 - 2z^4)}{1144} \\ & + \frac{\sqrt{429}G_3(2x^6 - 15x^4y^2 - 15x^4z^2 - 15x^2y^4 + 180x^2y^2z^2 - 15x^2z^4 + 2y^6 - 15y^4z^2 - 15y^2z^4 + 2z^6)}{286} \\ & - \frac{7\sqrt{4290}G_{3x}yz(8x^4 + 7x^2y^2 - 23x^2z^2 - y^4 + y^2z^2 + 2z^4)}{1144} + \frac{7\sqrt{4290}G_{3y}xz(x^4 - 7x^2y^2 - x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4)}{1144} \\ & + \frac{7\sqrt{715}G_{az}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} - \frac{21\sqrt{429}G_{bz}xy(x - y)(x + y)(x^2 + y^2 - 10z^2)}{572} \end{aligned}$$

$\tilde{\mathbb{G}}_3^{(3,-3)}[g](B_{2g})$   
\*\* symmetry

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

\*\* expression

$$G_{bz}$$

$\tilde{\mathbb{G}}_3^{(3,-1)}[g](B_{2g})$   
\*\* symmetry

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

\*\* expression

$$\frac{\sqrt{10}G_1yz}{4} + \frac{\sqrt{10}G_2xz}{4} + \frac{\sqrt{6}G_{3x}xz}{4} - \frac{\sqrt{6}G_{3y}yz}{4} - \frac{G_{az}(x - y)(x + y)}{2}$$

$\tilde{\mathbb{G}}_3^{(3,1)}[g](B_{2g})$   
\*\* symmetry

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{5\sqrt{33}G_1yz(3x^2 + 3y^2 - 4z^2)}{132} + \frac{5\sqrt{33}G_2xz(3x^2 + 3y^2 - 4z^2)}{132} + \frac{35\sqrt{22}G_3xy(x - y)(x + y)}{132} - \frac{\sqrt{55}G_{3x}xz(5x^2 + 33y^2 - 16z^2)}{132} \\ & + \frac{\sqrt{55}G_{3y}yz(33x^2 + 5y^2 - 16z^2)}{132} + \frac{\sqrt{330}G_{az}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{264} + \frac{7\sqrt{22}G_{bz}(x^4 - 18x^2y^2 + 12x^2z^2 + y^4 + 12y^2z^2 - 4z^4)}{264} \end{aligned}$$

$\tilde{\mathbb{G}}_3^{(3,3)}[g](B_{2g})$   
\*\* symmetry

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{7\sqrt{286}G_1yz(85x^4 - 160x^2y^2 - 10x^2z^2 + 19y^4 - 10y^2z^2 + 4z^4)}{2288} + \frac{7\sqrt{286}G_2xz(19x^4 - 160x^2y^2 - 10x^2z^2 + 85y^4 - 10y^2z^2 + 4z^4)}{2288} \\ & - \frac{21\sqrt{429}G_3xy(x-y)(x+y)(x^2+y^2-10z^2)}{572} - \frac{7\sqrt{4290}G_{3x}xz(7x^4 - 4x^2y^2 - 22x^2z^2 - 11y^4 + 26y^2z^2 + 4z^4)}{2288} \\ & - \frac{7\sqrt{4290}G_{3y}yz(11x^4 + 4x^2y^2 - 26x^2z^2 - 7y^4 + 22y^2z^2 - 4z^4)}{2288} + \frac{7\sqrt{715}G_{az}(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144} \\ & - \frac{\sqrt{429}G_{bz}(13x^6 - 45x^4y^2 - 150x^4z^2 - 45x^2y^4 + 540x^2y^2z^2 + 60x^2z^4 + 13y^6 - 150y^4z^2 + 60y^2z^4 - 8z^6)}{1144} \end{aligned}$$

$\vec{\mathbb{G}}_{3,1}^{(3,-3)}[g](E_g, 1), \vec{\mathbb{G}}_{3,2}^{(3,-3)}[g](E_g, 1)$

\*\* symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

\*\* expression

$$\frac{\sqrt{10}G_2}{4} - \frac{\sqrt{6}G_{3x}}{4}$$

$$\frac{\sqrt{10}G_1}{4} + \frac{\sqrt{6}G_{3y}}{4}$$

$\vec{\mathbb{G}}_{3,1}^{(3,-3)}[g](E_g, 2), \vec{\mathbb{G}}_{3,2}^{(3,-3)}[g](E_g, 2)$

\*\* symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

\*\* expression

$$-\frac{\sqrt{6}G_2}{4} - \frac{\sqrt{10}G_{3x}}{4}$$

$$-\frac{\sqrt{6}G_1}{4} + \frac{\sqrt{10}G_{3y}}{4}$$

$\vec{\mathbb{G}}_{3,1}^{(3,-1)}[g](E_g, 1), \vec{\mathbb{G}}_{3,2}^{(3,-1)}[g](E_g, 1)$

\*\* symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

\*\* expression

$$\frac{\sqrt{6}G_1xy}{8} + \frac{\sqrt{6}G_2(4x^2 + y^2 - 5z^2)}{24} - G_3yz - \frac{\sqrt{10}G_{3x}(4x^2 - 7y^2 + 3z^2)}{40} - \frac{\sqrt{10}G_{3y}xy}{40} - \frac{\sqrt{15}G_{az}xz}{20} + \frac{G_{bz}xz}{4}$$

$$\frac{\sqrt{6}G_1(x^2 + 4y^2 - 5z^2)}{24} + \frac{\sqrt{6}G_2xy}{8} + G_3xz + \frac{\sqrt{10}G_{3x}xy}{40} - \frac{\sqrt{10}G_{3y}(7x^2 - 4y^2 - 3z^2)}{40} + \frac{\sqrt{15}G_{az}yz}{20} + \frac{G_{bz}yz}{4}$$

$\vec{\mathbb{G}}_{3,1}^{(3,-1)}[g](E_g, 2), \vec{\mathbb{G}}_{3,2}^{(3,-1)}[g](E_g, 2)$

\*\* symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

\*\* expression

$$\frac{\sqrt{10}G_1xy}{8} - \frac{\sqrt{10}G_2(y-z)(y+z)}{8} + \frac{\sqrt{6}G_{3x}(y-z)(y+z)}{8} - \frac{3\sqrt{6}G_{3y}xy}{8} - \frac{G_{az}xz}{4} - \frac{\sqrt{15}G_{bz}xz}{4}$$

$$-\frac{\sqrt{10}G_1(x-z)(x+z)}{8} + \frac{\sqrt{10}G_2xy}{8} + \frac{3\sqrt{6}G_{3x}xy}{8} - \frac{\sqrt{6}G_{3y}(x-z)(x+z)}{8} + \frac{G_{az}yz}{4} - \frac{\sqrt{15}G_{bz}yz}{4}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,1)}[g](E_g, 1), \vec{\mathbb{G}}_{3,2}^{(3,1)}[g](E_g, 1)$$

\*\* symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{55}G_1xy(5x^2 - 2y^2 - 9z^2)}{44} + \frac{\sqrt{55}G_2(4x^4 - 9x^2y^2 - 15x^2z^2 + y^4 + 3y^2z^2 + 2z^4)}{88} \\ & - \frac{\sqrt{330}G_3yz(6x^2 - y^2 - z^2)}{132} - \frac{\sqrt{33}G_{3x}(12x^4 - 51x^2y^2 - 21x^2z^2 + 7y^4 + 9y^2z^2 + 2z^4)}{264} \\ & - \frac{5\sqrt{33}G_{3y}xy(x^2 - 6y^2 + 15z^2)}{132} - \frac{5\sqrt{22}G_{az}xz(x^2 - 6y^2 + z^2)}{44} + \frac{\sqrt{330}G_{bz}xz(5x^2 + 12y^2 - 9z^2)}{132} \\ & \frac{\sqrt{55}G_1(x^4 - 9x^2y^2 + 3x^2z^2 + 4y^4 - 15y^2z^2 + 2z^4)}{88} - \frac{\sqrt{55}G_2xy(2x^2 - 5y^2 + 9z^2)}{44} - \frac{\sqrt{330}G_3xz(x^2 - 6y^2 + z^2)}{132} - \frac{5\sqrt{33}G_{3x}xy(6x^2 - y^2 - 15z^2)}{132} \\ & + \frac{\sqrt{33}G_{3y}(7x^4 - 51x^2y^2 + 9x^2z^2 + 12y^4 - 21y^2z^2 + 2z^4)}{264} - \frac{5\sqrt{22}G_{az}yz(6x^2 - y^2 - z^2)}{44} + \frac{\sqrt{330}G_{bz}yz(12x^2 + 5y^2 - 9z^2)}{132} \end{aligned}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,1)}[g](E_g, 2), \vec{\mathbb{G}}_{3,2}^{(3,1)}[g](E_g, 2)$$

\*\* symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{5\sqrt{33}G_1xy(5x^2 - 2y^2 - 9z^2)}{132} + \frac{\sqrt{33}G_2(14x^4 - 57x^2y^2 - 27x^2z^2 - y^4 + 63y^2z^2 - 6z^4)}{264} \\ & + \frac{35\sqrt{22}G_3yz(y-z)(y+z)}{132} + \frac{\sqrt{55}G_{3x}(14x^4 - 33x^2y^2 - 51x^2z^2 - 5y^4 + 63y^2z^2 - 2z^4)}{264} \\ & + \frac{\sqrt{55}G_{3y}xy(11x^2 - 10y^2 - 3z^2)}{132} + \frac{\sqrt{330}G_{az}xz(9x^2 - 12y^2 - 5z^2)}{132} - \frac{5\sqrt{22}G_{bz}xz(x^2 - 6y^2 + z^2)}{132} \\ & - \frac{\sqrt{33}G_1(x^4 + 57x^2y^2 - 63x^2z^2 - 14y^4 + 27y^2z^2 + 6z^4)}{264} - \frac{5\sqrt{33}G_2xy(2x^2 - 5y^2 + 9z^2)}{132} - \frac{35\sqrt{22}G_3xz(x-z)(x+z)}{132} \\ & + \frac{\sqrt{55}G_{3x}xy(10x^2 - 11y^2 + 3z^2)}{132} + \frac{\sqrt{55}G_{3y}(5x^4 + 33x^2y^2 - 63x^2z^2 - 14y^4 + 51y^2z^2 + 2z^4)}{264} \\ & + \frac{\sqrt{330}G_{az}yz(12x^2 - 9y^2 + 5z^2)}{132} + \frac{5\sqrt{22}G_{bz}yz(6x^2 - y^2 - z^2)}{132} \end{aligned}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,3)}[g](E_g, 1), \vec{\mathbb{G}}_{3,2}^{(3,3)}[g](E_g, 1)$$

\*\* symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{7\sqrt{4290}G_1xy(10x^4 - 27x^2y^2 - 19x^2z^2 + 7y^4 + 11y^2z^2 + 4z^4)}{2288} \\ & + \frac{\sqrt{4290}G_2(40x^6 - 468x^4y^2 - 132x^4z^2 + 393x^2y^4 + 450x^2y^2z^2 + 57x^2z^4 - 23y^6 - 48y^4z^2 - 27y^2z^4 - 2z^6)}{6864} \\ & + \frac{7\sqrt{715}G_3yz(16x^4 - 16x^2y^2 - 16x^2z^2 + y^4 + 2y^2z^2 + z^4)}{572} \\ & - \frac{5\sqrt{286}G_{3x}(8x^6 - 4x^4y^2 - 116x^4z^2 - 11x^2y^4 + 90x^2y^2z^2 + 101x^2z^4 + y^6 - 4y^4z^2 - 11y^2z^4 - 6z^6)}{2288} \\ & - \frac{35\sqrt{286}G_{3y}xy(2x^4 + x^2y^2 - 23x^2z^2 - y^4 + 7y^2z^2 + 8z^4)}{2288} - \frac{35\sqrt{429}G_{az}xz(2x^4 + x^2y^2 - 7x^2z^2 - y^4 + y^2z^2 + 2z^4)}{1144} \\ & + \frac{7\sqrt{715}G_{bz}xz(10x^4 - 43x^2y^2 - 19x^2z^2 + 13y^4 + 17y^2z^2 + 4z^4)}{1144} \end{aligned}$$

$$\begin{aligned}
& -\frac{\sqrt{4290}G_1(23x^6 - 393x^4y^2 + 48x^4z^2 + 468x^2y^4 - 450x^2y^2z^2 + 27x^2z^4 - 40y^6 + 132y^4z^2 - 57y^2z^4 + 2z^6)}{6864} \\
& + \frac{7\sqrt{4290}G_2xy(7x^4 - 27x^2y^2 + 11x^2z^2 + 10y^4 - 19y^2z^2 + 4z^4)}{2288} \\
& - \frac{7\sqrt{715}G_3xz(x^4 - 16x^2y^2 + 2x^2z^2 + 16y^4 - 16y^2z^2 + z^4)}{572} - \frac{35\sqrt{286}G_{3x}xy(x^4 - x^2y^2 - 7x^2z^2 - 2y^4 + 23y^2z^2 - 8z^4)}{2288} \\
& + \frac{5\sqrt{286}G_{3y}(x^6 - 11x^4y^2 - 4x^4z^2 - 4x^2y^4 + 90x^2y^2z^2 - 11x^2z^4 + 8y^6 - 116y^4z^2 + 101y^2z^4 - 6z^6)}{2288} \\
& - \frac{35\sqrt{429}G_{az}yz(x^4 - x^2y^2 - x^2z^2 - 2y^4 + 7y^2z^2 - 2z^4)}{1144} + \frac{7\sqrt{715}G_{bz}yz(13x^4 - 43x^2y^2 + 17x^2z^2 + 10y^4 - 19y^2z^2 + 4z^4)}{1144}
\end{aligned}$$

$\vec{\mathbb{G}}_{3,1}^{(3,3)}[g](E_g, 2), \vec{\mathbb{G}}_{3,2}^{(3,3)}[g](E_g, 2)$

\*\* symmetry

$$-\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

\*\* expression

$$\begin{aligned}
& -\frac{7\sqrt{286}G_1xy(16x^4 - 85x^2y^2 + 95x^2z^2 + 31y^4 - 55y^2z^2 - 20z^4)}{2288} \\
& - \frac{\sqrt{286}G_2(12x^6 - 370x^4y^2 + 190x^4z^2 + 505x^2y^4 - 810x^2y^2z^2 - 55x^2z^4 - 37y^6 + 50y^4z^2 + 85y^2z^4 - 2z^6)}{2288} \\
& + \frac{21\sqrt{429}G_3yz(y-z)(y+z)(10x^2 - y^2 - z^2)}{572} \\
& - \frac{\sqrt{4290}G_{3x}(4x^6 + 26x^4y^2 - 86x^4z^2 + 19x^2y^4 - 270x^2y^2z^2 + 131x^2z^4 - 3y^6 + 26y^4z^2 + 19y^2z^4 - 10z^6)}{2288} \\
& - \frac{21\sqrt{4290}G_{3y}xy(x^2y^2 - 3x^2z^2 + y^4 - 13y^2z^2 + 8z^4)}{2288} - \frac{7\sqrt{715}G_{az}xz(4x^4 + 17x^2y^2 - 19x^2z^2 + 13y^4 - 43y^2z^2 + 10z^4)}{1144} \\
& + \frac{7\sqrt{429}G_{bz}xz(4x^4 + 35x^2y^2 - 25x^2z^2 - 35y^4 + 35y^2z^2 + 4z^4)}{1144}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{286}G_1(37x^6 - 505x^4y^2 - 50x^4z^2 + 370x^2y^4 + 810x^2y^2z^2 - 85x^2z^4 - 12y^6 - 190y^4z^2 + 55y^2z^4 + 2z^6) \\
& - \frac{7\sqrt{286}G_2xy(31x^4 - 85x^2y^2 - 55x^2z^2 + 16y^4 + 95y^2z^2 - 20z^4)}{2288} \\
& + \frac{21\sqrt{429}G_3xz(x-z)(x+z)(x^2 - 10y^2 + z^2)}{572} + \frac{21\sqrt{4290}G_{3x}xy(x^4 + x^2y^2 - 13x^2z^2 - 3y^2z^2 + 8z^4)}{2288} \\
& - \frac{\sqrt{4290}G_{3y}(3x^6 - 19x^4y^2 - 26x^4z^2 - 26x^2y^4 + 270x^2y^2z^2 - 19x^2z^4 - 4y^6 + 86y^4z^2 - 131y^2z^4 + 10z^6)}{2288} \\
& + \frac{7\sqrt{715}G_{az}yz(13x^4 + 17x^2y^2 - 43x^2z^2 + 4y^4 - 19y^2z^2 + 10z^4)}{1144} - \frac{7\sqrt{429}G_{bz}yz(35x^4 - 35x^2y^2 - 35x^2z^2 - 4y^4 + 25y^2z^2 - 4z^4)}{1144}
\end{aligned}$$

\* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(3,-3)}[g](A_{1u}, 1)$

\*\* symmetry

$$-\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

\*\* expression

$$-\frac{\sqrt{30}G_1y}{12} + \frac{\sqrt{30}G_2x}{12} - \frac{\sqrt{2}G_{3x}x}{4} - \frac{\sqrt{2}G_{3y}y}{4} + \frac{\sqrt{3}G_{az}z}{3}$$

$\vec{\mathbb{G}}_4^{(3,-3)}[g](A_{1u}, 2)$

\*\* symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

\*\* expression

$$-\frac{\sqrt{42}G_1y}{12} - \frac{\sqrt{42}G_2x}{12} - \frac{\sqrt{70}G_{3x}x}{28} - \frac{\sqrt{70}G_{3y}y}{28} + \frac{\sqrt{105}G_{az}z}{21}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](A_{1u}, 1)$

\*\* symmetry

$$-\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{165}G_{1y}(x^2 - y^2 + 2z^2)}{44} + \frac{\sqrt{165}G_{2x}(x^2 - y^2 - 2z^2)}{44} - \frac{3\sqrt{110}G_3xyz}{11} - \frac{\sqrt{11}G_{3x}x(3x^2 - 7y^2 - 2z^2)}{44} \\ & + \frac{\sqrt{11}G_{3y}y(7x^2 - 3y^2 + 2z^2)}{44} - \frac{\sqrt{66}G_{az}z(3x^2 + 3y^2 - 2z^2)}{44} - \frac{\sqrt{110}G_{bz}z(x - y)(x + y)}{44} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](A_{1u}, 2)$

\*\* symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{231}G_{1y}(11x^2 + y^2 - 14z^2)}{308} - \frac{\sqrt{231}G_{2x}(x^2 + 11y^2 - 14z^2)}{308} + \frac{\sqrt{385}G_{3x}x(3x^2 - 11y^2 + 2z^2)}{308} \\ & - \frac{\sqrt{385}G_{3y}y(11x^2 - 3y^2 - 2z^2)}{308} - \frac{\sqrt{2310}G_{az}z(3x^2 + 3y^2 - 2z^2)}{308} - \frac{5\sqrt{154}G_{bz}z(x - y)(x + y)}{44} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](A_{1u}, 1)$

\*\* symmetry

$$-\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

\*\* expression

$$\begin{aligned} & \frac{5\sqrt{858}G_{1y}(5x^4 + 3x^2y^2 - 39x^2z^2 - 2y^4 + 17y^2z^2 - 2z^4)}{1144} + \frac{5\sqrt{858}G_{2x}(2x^4 - 3x^2y^2 - 17x^2z^2 - 5y^4 + 39y^2z^2 + 2z^4)}{1144} \\ & - \frac{\sqrt{1430}G_{3x}x(6x^4 - 65x^2y^2 + 5x^2z^2 + 13y^4 + 117y^2z^2 - 22z^4)}{1144} - \frac{\sqrt{1430}G_{3y}y(13x^4 - 65x^2y^2 + 117x^2z^2 + 6y^4 + 5y^2z^2 - 22z^4)}{1144} \\ & - \frac{\sqrt{2145}G_{az}z(3x^4 - 78x^2y^2 + 20x^2z^2 + 3y^4 + 20y^2z^2 - 4z^4)}{572} + \frac{35\sqrt{143}G_{bz}z(x - y)(x + y)(x^2 + y^2 - 2z^2)}{572} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](A_{1u}, 2)$

\*\* symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{30030}G_{1y}(4x^4 + 3x^2y^2 - 33x^2z^2 - y^4 + 7y^2z^2 + 2z^4)}{1144} + \frac{\sqrt{30030}G_{2x}(x^4 - 3x^2y^2 - 7x^2z^2 - 4y^4 + 33y^2z^2 - 2z^4)}{1144} + \frac{3\sqrt{5005}G_3xyz(x^2 + y^2 - 2z^2)}{143} \\ & + \frac{\sqrt{2002}G_{3x}x(75x^4 - 389x^2y^2 - 361x^2z^2 + 124y^4 + 423y^2z^2 + 110z^4)}{8008} + \frac{\sqrt{2002}G_{3y}y(124x^4 - 389x^2y^2 + 423x^2z^2 + 75y^4 - 361y^2z^2 + 110z^4)}{8008} \\ & + \frac{\sqrt{3003}G_{az}z(111x^4 - 366x^2y^2 - 100x^2z^2 + 111y^4 - 100y^2z^2 + 20z^4)}{4004} - \frac{\sqrt{5005}G_{bz}z(x - y)(x + y)(x^2 + y^2 - 2z^2)}{572} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](A_{1u}, 1)$

\*\* symmetry

$$-\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{858}G_{1y}(211x^6 - 900x^4y^2 - 465x^4z^2 + 570x^2y^4 - 300x^2y^2z^2 + 615x^2z^4 - 35y^6 + 165y^4z^2 - 225y^2z^4 + 4z^6)}{3432} \\ & + \frac{\sqrt{858}G_{2x}(35x^6 - 570x^4y^2 - 165x^4z^2 + 900x^2y^4 + 300x^2y^2z^2 + 225x^2z^4 - 211y^6 + 465y^4z^2 - 615y^2z^4 - 4z^6)}{3432} \\ & + \frac{3\sqrt{143}G_3xyz(3x^4 - 5x^2y^2 - 5x^2z^2 + 3y^4 - 5y^2z^2 + 3z^4)}{22} \\ & - \frac{\sqrt{1430}G_{3x}x(7x^6 - 6x^4y^2 - 141x^4z^2 + 60x^2y^2z^2 + 225x^2z^4 + 13y^6 - 195y^4z^2 + 165y^2z^4 - 56z^6)}{1144} \\ & - \frac{\sqrt{1430}G_{3y}y(13x^6 - 195x^4z^2 - 6x^2y^4 + 60x^2y^2z^2 + 165x^2z^4 + 7y^6 - 141y^4z^2 + 225y^2z^4 - 56z^6)}{1144} \\ & - \frac{\sqrt{2145}G_{az}z(43x^6 + 30x^4y^2 - 225x^4z^2 + 30x^2y^4 - 120x^2y^2z^2 + 147x^2z^4 + 43y^6 - 225y^4z^2 + 147y^2z^4 - 14z^6)}{1716} \\ & + \frac{3\sqrt{143}G_{bz}z(x - y)(x + y)(23x^4 - 97x^2y^2 - 75x^2z^2 + 23y^4 - 75y^2z^2 + 45z^4)}{572} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](A_{1u}, 2)$

\*\* symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

\*\* expression

$$\begin{aligned}
& - \frac{\sqrt{30030}G_1y(341x^6 - 1935x^4y^2 + 690x^4z^2 + 1113x^2y^4 + 480x^2y^2z^2 - 930x^2z^4 - 43y^6 - 210y^4z^2 + 270y^2z^4 + 8z^6)}{34320} \\
& - \frac{\sqrt{30030}G_2x(43x^6 - 1113x^4y^2 + 210x^4z^2 + 1935x^2y^4 - 480x^2y^2z^2 - 270x^2z^4 - 341y^6 - 690y^4z^2 + 930y^2z^4 - 8z^6)}{34320} \\
& - \frac{3\sqrt{5005}G_3xyz(3x^4 - 20x^2y^2 + 10x^2z^2 + 3y^4 + 10y^2z^2 - 6z^4)}{260} \\
& - \frac{\sqrt{2002}G_{3xx}(x^6 + 57x^4y^2 - 78x^4z^2 + 45x^2y^4 - 840x^2y^2z^2 + 270x^2z^4 - 11y^6 + 30y^4z^2 + 390y^2z^4 - 80z^6)}{2288} \\
& + \frac{\sqrt{2002}G_{3yy}(11x^6 - 45x^4y^2 - 30x^4z^2 - 57x^2y^4 + 840x^2y^2z^2 - 390x^2z^4 - y^6 + 78y^4z^2 - 270y^2z^4 + 80z^6)}{2288} \\
& - \frac{\sqrt{3003}G_{azz}(19x^6 + 255x^4y^2 - 180x^4z^2 + 255x^2y^4 - 1020x^2y^2z^2 + 210x^2z^4 + 19y^6 - 180y^4z^2 + 210y^2z^4 - 20z^6)}{3432} \\
& - \frac{3\sqrt{5005}G_{bzz}(x-y)(x+y)(x^4 - 284x^2y^2 + 90x^2z^2 + y^4 + 90y^2z^2 - 54z^4)}{5720}
\end{aligned}$$

$\tilde{\mathbb{G}}_4^{(3,-3)}[g](A_{2u})$

\*\* symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

\*\* expression

$$\frac{\sqrt{2}G_1x}{2} + \frac{\sqrt{2}G_2y}{2}$$

$\tilde{\mathbb{G}}_4^{(3,-1)}[g](A_{2u})$

\*\* symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

\*\* expression

$$\frac{3\sqrt{11}G_1x(x^2 + y^2 - 4z^2)}{44} + \frac{3\sqrt{11}G_2y(x^2 + y^2 - 4z^2)}{44} + \frac{5\sqrt{66}G_3z(x-y)(x+y)}{44} - \frac{\sqrt{165}G_{3xy}(3x^2 - y^2)}{44} - \frac{\sqrt{165}G_{3yx}(x^2 - 3y^2)}{44} + \frac{5\sqrt{66}G_{bzx}xyz}{22}$$

$\tilde{\mathbb{G}}_4^{(3,1)}[g](A_{2u})$

\*\* symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

\*\* expression

$$\begin{aligned}
& \frac{3\sqrt{1430}G_1x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} + \frac{3\sqrt{1430}G_2y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} \\
& + \frac{7\sqrt{2145}G_3z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{572} - \frac{7\sqrt{858}G_{3xy}(21x^4 - 26x^2y^2 - 48x^2z^2 + y^4 + 16y^2z^2)}{2288} \\
& + \frac{7\sqrt{858}G_{3yx}(x^4 - 26x^2y^2 + 16x^2z^2 + 21y^4 - 48y^2z^2)}{2288} - \frac{63\sqrt{143}G_{azxyz}(x-y)(x+y)}{143} + \frac{7\sqrt{2145}G_{bzx}xyz(x^2 + y^2 - 2z^2)}{286}
\end{aligned}$$

$\tilde{\mathbb{G}}_4^{(3,3)}[g](A_{2u})$

\*\* symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

\*\* expression

$$\begin{aligned}
& - \frac{\sqrt{1430}G_1x(53x^6 - 1128x^4y^2 + 15x^4z^2 + 1875x^2y^4 + 30x^2y^2z^2 - 30x^2z^4 - 376y^6 + 15y^4z^2 - 30y^2z^4 + 8z^6)}{5720} \\
& + \frac{\sqrt{1430}G_2y(376x^6 - 1875x^4y^2 - 15x^4z^2 + 1128x^2y^4 - 30x^2y^2z^2 + 30x^2z^4 - 53y^6 - 15y^4z^2 + 30y^2z^4 - 8z^6)}{5720} \\
& - \frac{3\sqrt{2145}G_3z(x-y)(x+y)(8x^4 - 127x^2y^2 + 5x^2z^2 + 8y^4 + 5y^2z^2 - 3z^4)}{1430} \\
& - \frac{3\sqrt{858}G_{3xy}(8x^6 - 5x^4y^2 - 105x^4z^2 - 12x^2y^4 + 150x^2y^2z^2 + 30x^2z^4 + y^6 - 9y^4z^2 - 10y^2z^4)}{1144} \\
& + \frac{3\sqrt{858}G_{3yx}(x^6 - 12x^4y^2 - 9x^4z^2 - 5x^2y^4 + 150x^2y^2z^2 - 10x^2z^4 + 8y^6 - 105y^4z^2 + 30y^2z^4)}{1144} \\
& - \frac{3\sqrt{143}G_{azxyz}(x-y)(x+y)(3x^2 + 3y^2 - 10z^2)}{52} + \frac{3\sqrt{2145}G_{bzx}xyz(111x^4 - 350x^2y^2 - 20x^2z^2 + 111y^4 - 20y^2z^2 + 12z^4)}{2860}
\end{aligned}$$

$\tilde{\mathbb{G}}_4^{(3,-3)}[g](B_{1u})$

\*\* symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

\*\* expression

$$\frac{\sqrt{14}G_1y}{28} + \frac{\sqrt{14}G_2x}{28} - \frac{\sqrt{210}G_{3x}x}{28} + \frac{\sqrt{210}G_{3y}y}{28} - \frac{\sqrt{21}G_{bz}z}{7}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](B_{1u})$

\*\* symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{9\sqrt{77}G_1y(x^2+y^2-4z^2)}{308} + \frac{9\sqrt{77}G_2x(x^2+y^2-4z^2)}{308} - \frac{\sqrt{1155}G_{3x}x(x^2-11y^2+8z^2)}{308} \\ & - \frac{\sqrt{1155}G_{3y}y(11x^2-y^2-8z^2)}{308} + \frac{3\sqrt{770}G_{az}z(x-y)(x+y)}{308} - \frac{\sqrt{462}G_{bz}z(3x^2+3y^2-2z^2)}{308} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](B_{1u})$

\*\* symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{9\sqrt{10010}G_1y(10x^4-15x^2y^2-15x^2z^2+3y^4-15y^2z^2+10z^4)}{8008} + \frac{9\sqrt{10010}G_2x(3x^4-15x^2y^2-15x^2z^2+10y^4-15y^2z^2+10z^4)}{8008} \\ & - \frac{3\sqrt{15015}G_3xyz(x-y)(x+y)}{143} + \frac{\sqrt{6006}G_{3x}x(5x^4-53x^2y^2+3x^2z^2-58y^4+507y^2z^2-86z^4)}{8008} \\ & + \frac{\sqrt{6006}G_{3y}y(58x^4+53x^2y^2-507x^2z^2-5y^4-3y^2z^2+86z^4)}{8008} + \frac{3\sqrt{1001}G_{az}z(x-y)(x+y)(x^2+y^2-2z^2)}{572} \\ & + \frac{\sqrt{15015}G_{bz}z(9x^4+186x^2y^2-80x^2z^2+9y^4-80y^2z^2+16z^4)}{4004} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](B_{1u})$

\*\* symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{10010}G_1y(85x^6-75x^4y^2-1050x^4z^2-141x^2y^4+1860x^2y^2z^2+120x^2z^4+19y^6-258y^4z^2+120y^2z^4-32z^6)}{11440} \\ & + \frac{\sqrt{10010}G_2x(19x^6-141x^4y^2-258x^4z^2-75x^2y^4+1860x^2y^2z^2+120x^2z^4+85y^6-1050y^4z^2+120y^2z^4-32z^6)}{11440} \\ & + \frac{3\sqrt{15015}G_3xyz(x-y)(x+y)(3x^2+3y^2-10z^2)}{260} \\ & - \frac{\sqrt{6006}G_{3x}x(7x^6+3x^4y^2-150x^4z^2-15x^2y^4+60x^2y^2z^2+240x^2z^4-11y^6+210y^4z^2-240y^2z^4-32z^6)}{2288} \\ & - \frac{\sqrt{6006}G_{3y}y(11x^6+15x^4y^2-210x^4z^2-3x^2y^4-60x^2y^2z^2+240x^2z^4-7y^6+150y^4z^2-240y^2z^4+32z^6)}{2288} \\ & - \frac{3\sqrt{1001}G_{az}z(x-y)(x+y)(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4)}{1144} \\ & + \frac{\sqrt{15015}G_{bz}z(67x^6-195x^4y^2-270x^4z^2-195x^2y^4+780x^2y^2z^2+84x^2z^4+67y^6-270y^4z^2+84y^2z^4-8z^6)}{5720} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(3,-3)}[g](B_{2u})$

\*\* symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{14}G_1x}{28} + \frac{\sqrt{14}G_2y}{28} + \frac{\sqrt{21}G_3z}{7} + \frac{\sqrt{210}G_{3x}y}{28} + \frac{\sqrt{210}G_{3y}x}{28}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](B_{2u})$

\*\* symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

\*\* expression

$$-\frac{9\sqrt{77}G_1x(x^2 + y^2 - 4z^2)}{308} + \frac{9\sqrt{77}G_2y(x^2 + y^2 - 4z^2)}{308} + \frac{\sqrt{462}G_3z(3x^2 + 3y^2 - 2z^2)}{308} \\ + \frac{\sqrt{1155}G_{3xy}(7x^2 - 5y^2 + 8z^2)}{308} - \frac{\sqrt{1155}G_{3yx}(5x^2 - 7y^2 - 8z^2)}{308} - \frac{3\sqrt{770}G_{azxyz}}{154}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](B_{2u})$

\*\* symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

\*\* expression

$$\frac{9\sqrt{10010}G_1x(x^4 - 40x^2y^2 + 30x^2z^2 + 15y^4 + 30y^2z^2 - 20z^4)}{16016} - \frac{9\sqrt{10010}G_2y(15x^4 - 40x^2y^2 + 30x^2z^2 + y^4 + 30y^2z^2 - 20z^4)}{16016} \\ - \frac{\sqrt{15015}G_3z(51x^4 - 66x^2y^2 - 80x^2z^2 + 51y^4 - 80y^2z^2 + 16z^4)}{4004} - \frac{\sqrt{6006}G_{3xy}(73x^4 + 20x^2y^2 - 498x^2z^2 - 53y^4 + 510y^2z^2 - 172z^4)}{16016} \\ + \frac{\sqrt{6006}G_{3yx}(53x^4 - 20x^2y^2 - 510x^2z^2 - 73y^4 + 498y^2z^2 + 172z^4)}{16016} - \frac{3\sqrt{1001}G_{azxyz}(x^2 + y^2 - 2z^2)}{286} + \frac{3\sqrt{15015}G_{bzxyz}(x - y)(x + y)}{143}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](B_{2u})$

\*\* symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

\*\* expression

$$\frac{\sqrt{10010}G_1x(7x^6 - 78x^4y^2 - 69x^4z^2 - 45x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 40y^6 - 465y^4z^2 - 60y^2z^4 + 16z^6)}{5720} \\ - \frac{\sqrt{10010}G_2y(40x^6 - 45x^4y^2 - 465x^4z^2 - 78x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 7y^6 - 69y^4z^2 - 60y^2z^4 + 16z^6)}{5720} \\ + \frac{\sqrt{15015}G_3z(8x^6 - 75x^4y^2 - 15x^4z^2 - 75x^2y^4 + 300x^2y^2z^2 - 21x^2z^4 + 8y^6 - 15y^4z^2 - 21y^2z^4 + 2z^6)}{1430} \\ + \frac{\sqrt{6006}G_{3xy}(8x^6 + 15x^4y^2 - 165x^4z^2 + 6x^2y^4 - 150x^2y^2z^2 + 240x^2z^4 - y^6 + 15y^4z^2 - 16z^6)}{1144} \\ - \frac{\sqrt{6006}G_{3yx}(x^6 - 6x^4y^2 - 15x^4z^2 - 15x^2y^4 + 150x^2y^2z^2 - 8y^6 + 165y^4z^2 - 240y^2z^4 + 16z^6)}{1144} \\ + \frac{3\sqrt{1001}G_{azxyz}(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{572} - \frac{3\sqrt{15015}G_{bzxyz}(x - y)(x + y)(3x^2 + 3y^2 - 10z^2)}{260}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-3)}[g](E_u, 1), \vec{\mathbb{G}}_{4,2}^{(3,-3)}[g](E_u, 1)$

\*\* symmetry

$$-\frac{\sqrt{35}yz(y - z)(y + z)}{2}$$

$$-\frac{\sqrt{35}xz(x - z)(x + z)}{2}$$

\*\* expression

$$-\frac{\sqrt{2}G_1z}{8} - \frac{\sqrt{30}G_{3yz}}{8} - \frac{\sqrt{5}G_{azy}}{4} - \frac{\sqrt{3}G_{bzy}}{4} \\ - \frac{\sqrt{2}G_2z}{8} + \frac{\sqrt{30}G_{3xz}}{8} + \frac{\sqrt{5}G_{azx}}{4} - \frac{\sqrt{3}G_{bxz}}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-3)}[g](E_u, 2), \vec{\mathbb{G}}_{4,2}^{(3,-3)}[g](E_u, 2)$

\*\* symmetry

$$-\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

$$-\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{14}G_1z}{8} + \frac{\sqrt{21}G_3x}{7} - \frac{\sqrt{210}G_{3yz}}{56} - \frac{\sqrt{35}G_{azy}}{28} + \frac{3\sqrt{21}G_{bzy}}{28} \\ - \frac{\sqrt{14}G_2z}{8} - \frac{\sqrt{21}G_{3y}}{7} + \frac{\sqrt{210}G_{3xz}}{56} + \frac{\sqrt{35}G_{azx}}{28} + \frac{3\sqrt{21}G_{bxz}}{28}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,-1)}[g](E_u, 1), \vec{\mathbb{G}}_{4,2}^{(3,-1)}[g](E_u, 1)$$

\*\* symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{11}G_1z(x^2 - 4y^2 + z^2)}{44} - \frac{15\sqrt{11}G_2xyz}{44} + \frac{5\sqrt{66}G_3x(y-z)(y+z)}{44} - \frac{5\sqrt{165}G_{3x}xyz}{44} \\ & + \frac{\sqrt{165}G_{3y}z(3x^2 - z^2)}{44} + \frac{3\sqrt{110}G_{az}y(x-z)(x+z)}{44} + \frac{\sqrt{66}G_{bz}y(3x^2 - 2y^2 + 3z^2)}{44} \\ & - \frac{15\sqrt{11}G_1xyz}{44} - \frac{3\sqrt{11}G_2z(4x^2 - y^2 - z^2)}{44} - \frac{5\sqrt{66}G_3y(x-z)(x+z)}{44} - \frac{\sqrt{165}G_{3x}z(3y^2 - z^2)}{44} \\ & + \frac{5\sqrt{165}G_{3y}xyz}{44} - \frac{3\sqrt{110}G_{az}x(y-z)(y+z)}{44} - \frac{\sqrt{66}G_{bz}x(2x^2 - 3y^2 - 3z^2)}{44} \end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,-1)}[g](E_u, 2), \vec{\mathbb{G}}_{4,2}^{(3,-1)}[g](E_u, 2)$$

\*\* symmetry

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

$$\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{77}G_1z(13x^2 + 8y^2 - 7z^2)}{308} - \frac{15\sqrt{77}G_2xyz}{308} - \frac{\sqrt{462}G_3x(2x^2 - 3y^2 - 3z^2)}{308} + \frac{3\sqrt{1155}G_{3x}xyz}{308} \\ & + \frac{\sqrt{1155}G_{3y}z(7x^2 - 4y^2 - z^2)}{308} - \frac{3\sqrt{770}G_{az}y(5x^2 - 2y^2 + z^2)}{308} + \frac{\sqrt{462}G_{bz}y(x^2 - 4y^2 + 11z^2)}{308} \\ & - \frac{15\sqrt{77}G_1xyz}{308} + \frac{3\sqrt{77}G_2z(8x^2 + 13y^2 - 7z^2)}{308} - \frac{\sqrt{462}G_3y(3x^2 - 2y^2 + 3z^2)}{308} + \frac{\sqrt{1155}G_{3x}z(4x^2 - 7y^2 + z^2)}{308} \\ & - \frac{3\sqrt{1155}G_{3y}xyz}{308} - \frac{3\sqrt{770}G_{az}x(2x^2 - 5y^2 - z^2)}{308} - \frac{\sqrt{462}G_{bz}x(4x^2 - y^2 - 11z^2)}{308} \end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,1)}[g](E_u, 1), \vec{\mathbb{G}}_{4,2}^{(3,1)}[g](E_u, 1)$$

\*\* symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

\*\* expression

$$\begin{aligned} & -\frac{3\sqrt{1430}G_1z(2x^4 - 87x^2y^2 + 25x^2z^2 + 37y^4 - 45y^2z^2 + 2z^4)}{2288} + \frac{21\sqrt{1430}G_2xyz(2x^2 - 7y^2 + 5z^2)}{1144} - \frac{7\sqrt{2145}G_3x(y-z)(y+z)(2x^2 - y^2 - z^2)}{572} \\ & + \frac{7\sqrt{858}G_{3x}xyz(10x^2 + 13y^2 - 23z^2)}{1144} - \frac{\sqrt{858}G_{3y}z(30x^4 + 39x^2y^2 - 73x^2z^2 - 33y^4 + 53y^2z^2 + 2z^4)}{2288} \\ & - \frac{3\sqrt{143}G_{az}y(10x^4 + 13x^2y^2 - 99x^2z^2 + 3y^4 - 43y^2z^2 + 38z^4)}{1144} - \frac{\sqrt{2145}G_{bz}y(6x^4 - 37x^2y^2 + 75x^2z^2 - y^4 + 47y^2z^2 - 36z^4)}{1144} \\ & - \frac{21\sqrt{1430}G_1xyz(7x^2 - 2y^2 - 5z^2)}{1144} - \frac{3\sqrt{1430}G_2z(37x^4 - 87x^2y^2 - 45x^2z^2 + 2y^4 + 25y^2z^2 + 2z^4)}{2288} - \frac{7\sqrt{2145}G_3y(x-z)(x+z)(x^2 - 2y^2 + z^2)}{572} \\ & - \frac{\sqrt{858}G_{3x}z(33x^4 - 39x^2y^2 - 53x^2z^2 - 30y^4 + 73y^2z^2 - 2z^4)}{2288} - \frac{7\sqrt{858}G_{3y}xyz(13x^2 + 10y^2 - 23z^2)}{1144} \\ & + \frac{3\sqrt{143}G_{az}x(3x^4 + 13x^2y^2 - 43x^2z^2 + 10y^4 - 99y^2z^2 + 38z^4)}{1144} + \frac{\sqrt{2145}G_{bz}x(x^4 + 37x^2y^2 - 47x^2z^2 - 6y^4 - 75y^2z^2 + 36z^4)}{1144} \end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,1)}[g](E_u, 2), \vec{\mathbb{G}}_{4,2}^{(3,1)}[g](E_u, 2)$$

\*\* symmetry

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

$$\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

\*\* expression

$$\begin{aligned}
& \frac{3\sqrt{10010}G_1z(4x^4 + 21x^2y^2 - 15x^2z^2 - y^4 - 5y^2z^2 + 2z^4)}{2288} + \frac{3\sqrt{10010}G_2xyz(2x^2 - 7y^2 + 5z^2)}{1144} \\
& - \frac{\sqrt{15015}G_3x(16x^4 - 80x^2y^2 - 80x^2z^2 + 51y^4 - 66y^2z^2 + 51z^4)}{4004} - \frac{3\sqrt{6006}G_{3x}xyz(2x^2 + 9y^2 - 11z^2)}{1144} \\
& - \frac{\sqrt{6006}G_{3y}z(268x^4 - 213x^2y^2 - 465x^2z^2 - 187y^4 + 445y^2z^2 + 2z^4)}{16016} \\
& + \frac{3\sqrt{1001}G_{az}y(32x^4 + 15x^2y^2 - 237x^2z^2 - 17y^4 + 155y^2z^2 - 38z^4)}{8008} + \frac{\sqrt{15015}G_{bz}y(88x^4 - 195x^2y^2 + 57x^2z^2 + 11y^4 + 85y^2z^2 - 52z^4)}{8008} \\
& - \frac{3\sqrt{10010}G_1xyz(7x^2 - 2y^2 - 5z^2)}{1144} - \frac{3\sqrt{10010}G_2z(x^4 - 21x^2y^2 + 5x^2z^2 - 4y^4 + 15y^2z^2 - 2z^4)}{2288} \\
& + \frac{\sqrt{15015}G_3y(51x^4 - 80x^2y^2 - 66x^2z^2 + 16y^4 - 80y^2z^2 + 51z^4)}{4004} - \frac{\sqrt{6006}G_{3x}z(187x^4 + 213x^2y^2 - 445x^2z^2 - 268y^4 + 465y^2z^2 - 2z^4)}{16016} \\
& + \frac{3\sqrt{6006}G_{3y}xyz(9x^2 + 2y^2 - 11z^2)}{1144} + \frac{3\sqrt{1001}G_{az}x(17x^4 - 15x^2y^2 - 155x^2z^2 - 32y^4 + 237y^2z^2 + 38z^4)}{8008} \\
& + \frac{\sqrt{15015}G_{bz}x(11x^4 - 195x^2y^2 + 85x^2z^2 + 88y^4 + 57y^2z^2 - 52z^4)}{8008}
\end{aligned}$$

$\vec{\mathbb{G}}_{4,1}^{(3,3)}[g](E_u, 1), \vec{\mathbb{G}}_{4,2}^{(3,3)}[g](E_u, 1)$

\*\* symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

\*\* expression

$$\begin{aligned}
& \frac{\sqrt{1430}G_1z(2x^6 - 345x^4y^2 + 105x^4z^2 + 1185x^2y^4 - 1680x^2y^2z^2 + 105x^2z^4 - 184y^6 + 525y^4z^2 - 147y^2z^4 + 2z^6)}{5720} \\
& - \frac{3\sqrt{1430}G_2xyz(18x^4 - 305x^2y^2 + 245x^2z^2 + 249y^4 - 525y^2z^2 + 84z^4)}{5720} \\
& + \frac{3\sqrt{2145}G_3x(y-z)(y+z)(3x^4 - 5x^2y^2 - 5x^2z^2 - 8y^4 + 127y^2z^2 - 8z^4)}{1430} - \frac{3\sqrt{858}G_{3x}xyz(6x^4 + 45x^2y^2 - 65x^2z^2 + 39y^4 - 175y^2z^2 + 72z^4)}{1144} \\
& + \frac{\sqrt{858}G_{3y}z(2x^6 + 15x^4y^2 - 15x^4z^2 - 75x^2y^4 + 120x^2y^2z^2 - 3x^2z^4 - 88y^6 + 465y^4z^2 - 291y^2z^4 + 14z^6)}{1144} \\
& + \frac{\sqrt{143}G_{az}y(2x^6 + 15x^4y^2 - 75x^4z^2 + 24x^2y^4 - 330x^2y^2z^2 + 240x^2z^4 + 11y^6 - 255y^4z^2 + 480y^2z^4 - 112z^6)}{572} \\
& + \frac{\sqrt{2145}G_{bz}y(2x^6 - 45x^4y^2 + 105x^4z^2 - 30x^2y^4 + 570x^2y^2z^2 - 390x^2z^4 + 17y^6 - 327y^4z^2 + 450y^2z^4 - 64z^6)}{2860} \\
& - \frac{3\sqrt{1430}G_1xyz(249x^4 - 305x^2y^2 - 525x^2z^2 + 18y^4 + 245y^2z^2 + 84z^4)}{5720} \\
& - \frac{\sqrt{1430}G_2z(184x^6 - 1185x^4y^2 - 525x^4z^2 + 345x^2y^4 + 1680x^2y^2z^2 + 147x^2z^4 - 2y^6 - 105y^4z^2 - 105y^2z^4 - 2z^6)}{5720} \\
& + \frac{3\sqrt{2145}G_3y(x-z)(x+z)(8x^4 + 5x^2y^2 - 127x^2z^2 - 3y^4 + 5y^2z^2 + 8z^4)}{1430} \\
& + \frac{\sqrt{858}G_{3x}z(88x^6 + 75x^4y^2 - 465x^4z^2 - 15x^2y^4 - 120x^2y^2z^2 + 291x^2z^4 - 2y^6 + 15y^4z^2 + 3y^2z^4 - 14z^6)}{1144} \\
& + \frac{3\sqrt{858}G_{3y}xyz(39x^4 + 45x^2y^2 - 175x^2z^2 + 6y^4 - 65y^2z^2 + 72z^4)}{1144} \\
& - \frac{\sqrt{143}G_{az}x(11x^6 + 24x^4y^2 - 255x^4z^2 + 15x^2y^4 - 330x^2y^2z^2 + 480x^2z^4 + 2y^6 - 75y^4z^2 + 240y^2z^4 - 112z^6)}{572} \\
& + \frac{\sqrt{2145}G_{bz}x(17x^6 - 30x^4y^2 - 327x^4z^2 - 45x^2y^4 + 570x^2y^2z^2 + 450x^2z^4 + 2y^6 + 105y^4z^2 - 390y^2z^4 - 64z^6)}{2860}
\end{aligned}$$

$\vec{\mathbb{G}}_{4,1}^{(3,3)}[g](E_u, 2), \vec{\mathbb{G}}_{4,2}^{(3,3)}[g](E_u, 2)$

\*\* symmetry

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

$$\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

\*\* expression

$$\begin{aligned}
& - \frac{\sqrt{10010}G_1z(56x^6 - 885x^4y^2 + 15x^4z^2 + 735x^2y^4 + 300x^2y^2z^2 - 39x^2z^4 - 40y^6 - 45y^4z^2 - 3y^2z^4 + 2z^6)}{5720} \\
& + \frac{3\sqrt{10010}G_2xyz(120x^4 - 365x^2y^2 - 35x^2z^2 + 87y^4 + 75y^2z^2 - 12z^4)}{5720} \\
& + \frac{\sqrt{15015}G_3x(2x^6 - 21x^4y^2 - 21x^4z^2 - 15x^2y^4 + 300x^2y^2z^2 - 15x^2z^4 + 8y^6 - 75y^4z^2 - 75y^2z^4 + 8z^6)}{1430} \\
& - \frac{3\sqrt{6006}G_{3x}xyz(24x^4 + 15x^2y^2 - 95x^2z^2 - 9y^4 + 15y^2z^2 + 24z^4)}{1144} \\
& + \frac{\sqrt{6006}G_{3y}z(8x^6 - 75x^4y^2 - 15x^4z^2 - 75x^2y^4 + 300x^2y^2z^2 - 21x^2z^4 + 8y^6 - 15y^4z^2 - 21y^2z^4 + 2z^6)}{1144} \\
& + \frac{\sqrt{1001}G_{az}y(8x^6 + 15x^4y^2 - 165x^4z^2 + 6x^2y^4 - 150x^2y^2z^2 + 240x^2z^4 - y^6 + 15y^4z^2 - 16z^6)}{572} \\
& - \frac{3\sqrt{15015}G_{bz}y(8x^6 - 5x^4y^2 - 105x^4z^2 - 12x^2y^4 + 150x^2y^2z^2 + 30x^2z^4 + y^6 - 9y^4z^2 - 10y^2z^4)}{2860}
\end{aligned}$$

$$\begin{aligned}
& \frac{3\sqrt{10010}G_1xyz(87x^4 - 365x^2y^2 + 75x^2z^2 + 120y^4 - 35y^2z^2 - 12z^4)}{5720} \\
& + \frac{\sqrt{10010}G_2z(40x^6 - 735x^4y^2 + 45x^4z^2 + 885x^2y^4 - 300x^2y^2z^2 + 3x^2z^4 - 56y^6 - 15y^4z^2 + 39y^2z^4 - 2z^6)}{5720} \\
& - \frac{\sqrt{15015}G_3y(8x^6 - 15x^4y^2 - 75x^4z^2 - 21x^2y^4 + 300x^2y^2z^2 - 75x^2z^4 + 2y^6 - 21y^4z^2 - 15y^2z^4 + 8z^6)}{1430} \\
& - \frac{\sqrt{6006}G_{3x}z(8x^6 - 75x^4y^2 - 15x^4z^2 - 75x^2y^4 + 300x^2y^2z^2 - 21x^2z^4 + 8y^6 - 15y^4z^2 - 21y^2z^4 + 2z^6)}{1144} \\
& - \frac{3\sqrt{6006}G_{3y}xyz(9x^4 - 15x^2y^2 - 15x^2z^2 - 24y^4 + 95y^2z^2 - 24z^4)}{1144} \\
& + \frac{\sqrt{1001}G_{az}x(x^6 - 6x^4y^2 - 15x^4z^2 - 15x^2y^4 + 150x^2y^2z^2 - 8y^6 + 165y^4z^2 - 240y^2z^4 + 16z^6)}{572} \\
& - \frac{3\sqrt{15015}G_{bz}x(x^6 - 12x^4y^2 - 9x^4z^2 - 5x^2y^4 + 150x^2y^2z^2 - 10x^2z^4 + 8y^6 - 105y^4z^2 + 30y^2z^4)}{2860}
\end{aligned}$$