SAMB for "01"

Generated on 2023-09-27 01:31 by MultiPie $1.1.14\,$

• Group: No. 207 O^1 P432 [cubic]

• Associated point group: No. 30 O 432 [cubic]

• Generation condition

model type: tight_bindingtime-reversal type: electric

- irrep: [A1]
- spinful

• Unit cell:

$$a=1.0,\ b=1.0,\ c=1.0,\ \alpha=90.0,\ \beta=90.0,\ \gamma=90.0$$

• Lattice vectors:

$$\boldsymbol{a}_1 = \begin{pmatrix} 1.0 & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{a}_2 = \begin{pmatrix} 0 & 1.0 & 0 \end{pmatrix}$$

$$\mathbf{a}_3 = \begin{pmatrix} 0 & 0 & 1.0 \end{pmatrix}$$

Table 1: High-symmetry line: Γ -X.

symbol	position	symbol	position
Γ	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	X	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$

• Kets: dimension = 8

Table 2: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket	No.	ket	No.	ket
 1	(s,\uparrow) @A ₁	2	(s,\downarrow) @A ₁	3	(p_x,\uparrow) @A ₁	4	(p_x,\downarrow) @A ₁	5	(p_y,\uparrow) @A ₁
6	(p_y,\downarrow) @A ₁	7	(p_z,\uparrow) @A ₁	8	(p_z,\downarrow) @A ₁				

• Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position	mapping
S_1 [1a: 432]	A_1	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24]

• Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

	bond	tail	head	n	#	b@c	mapping
B ₁ [3d: 42.2]	b_1	A_1	A_1	1	1	$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} @ \begin{pmatrix} 0 & 0 & \frac{1}{2} \end{pmatrix}$	[1,2,-3,-4,-5,-8,19,22]
	b_2	A_1	A_1	1	1	$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} $ $ \begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix} $	[6, -9, 11, -12, 13, -14, 21, -24]
	b_3	A_1	A_1	1	1	$ \left(\begin{array}{cccc} 0 & 1 & 0 \end{array} \right) @ \left(\begin{array}{cccc} 0 & \frac{1}{2} & 0 \end{array} \right) $	[7,-10,15,16,-17,-18,-20,23]
B ₂ [3c: 42.2]	b_4	A_1	A_1	2	1	$ \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} $	[1,-3,7,-10]
	b_5	A_1	A_1	2	1	$ \begin{pmatrix} 0 & 1 & -1 \end{pmatrix} @ \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} $	[-2,4,-20,23]
	b_6	A_1	A_1	2	1	$ \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} $	[5,-12,13,-19]
	b_7	A_1	A_1	2	1	$\begin{pmatrix} 1 & -1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	[6,-16,18,-24]
	b_8	A_1	A_1	2	1	$\begin{pmatrix} 1 & 0 & 1 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$	[-8,11,-14,22]
	b ₉	A_1	A_1	2	1	$\begin{pmatrix} 1 & 1 & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	[-9,15,-17,21]

• SAMB:

No. 1
$$\hat{\mathbb{Q}}_0^{(A_1)}$$
 [M₁, S₁]

$$\hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 2
$$\hat{\mathbb{G}}_0^{(A_1)}(1,1)$$
 [M₂, S₁]

$$\hat{\mathbb{Z}}_2 = \mathbb{X}_{14}[\mathbb{G}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 3
$$\hat{\mathbb{Q}}_0^{(A_1)}$$
 [M₃, S₁]

$$\hat{\mathbb{Z}}_3 = \mathbb{X}_{20}[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 4
$$\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$$
 [M₃, S₁]

$$\hat{\mathbb{Z}}_4 = \mathbb{X}_{21}[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 5
$$\hat{\mathbb{Q}}_0^{(A_1)}$$
 [M₁, B₁]

$$\hat{\mathbb{Z}}_5 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_1)}]$$

No. 6
$$\hat{\mathbb{Q}}_{4}^{(A_1)}(1,-1)$$
 [M₁, B₁]

$$\hat{\mathbb{Z}}_6 = -\frac{\sqrt{3}\mathbb{X}_2[\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_5[\mathbb{T}_{4,0}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_3[\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_6[\mathbb{T}_{4,1}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{T}_{4,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{T}_{4,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{T}_{4,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_7[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}$$

No. 7
$$\hat{\mathbb{G}}_4^{(A_1)}$$
 [M₂, B₁]

$$\hat{\mathbb{Z}}_7 = -\frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{T}_{1,2}^{(a,T_1)}] \otimes \mathbb{Y}_7[\mathbb{T}_{4,2}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_8[\mathbb{T}_{1,0}^{(a,T_1)}] \otimes \mathbb{Y}_5[\mathbb{T}_{4,0}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_9[\mathbb{T}_{1,1}^{(a,T_1)}] \otimes \mathbb{Y}_6[\mathbb{T}_{4,1}^{(b,T_1)}]}{3}$$

No. 8
$$\hat{\mathbb{G}}_{4}^{(A_1)}(1,0)$$
 [M₂, B₁]

$$\hat{\mathbb{Z}}_{8} = -\frac{\sqrt{3}\mathbb{X}_{11}[\mathbb{T}_{1,0}^{(a,T_{1})}(1,0)]\otimes\mathbb{Y}_{5}[\mathbb{T}_{4,0}^{(b,T_{1})}]}{3} - \frac{\sqrt{3}\mathbb{X}_{12}[\mathbb{T}_{1,1}^{(a,T_{1})}(1,0)]\otimes\mathbb{Y}_{6}[\mathbb{T}_{4,1}^{(b,T_{1})}]}{3} - \frac{\sqrt{3}\mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T_{1})}(1,0)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_{1})}]}{3} - \frac{\mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T_{1})}(1,0)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_{1})}]}{3} - \frac{\mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T_{1})}(1,0)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_{1})}]}{3} - \frac{\mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T_{1})}(1,0)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_{1})}]}{3} - \frac{\mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T_{1})}(1,0)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{1,2}^{(b,T_{1})}]}{3} - \frac{\mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T_{1})}(1,0)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{1,2}^{(b,T_{1})}]}{3} - \frac{\mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T_{1})}(1,0)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{1,2}^{(b,T_{1})}]}{3} - \frac{\mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T_{1})}(1,0)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{1,2}^{(b,T_{1})}]}{3} - \frac{\mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T_{1})}(1,0)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{1,2}^{(b,T_{1})}]}{3} - \mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T_{1})}(1,0)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{1,2}^{(b,T_{1})}]}$$

No. 9
$$\hat{\mathbb{G}}_0^{(A_1)}(1,1)$$
 [M₂, B₁]

$$\hat{\mathbb{Z}}_9 = \mathbb{X}_{14}[\mathbb{G}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_1)}]$$

No. 10
$$\hat{\mathbb{G}}_0^{(A_1)}(1,-1)$$
 [M₂, B₁]

$$\hat{\mathbb{Z}}_{10} = \frac{\sqrt{2}\mathbb{X}_{15}[\mathbb{G}_{2,0}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{3}[\mathbb{Q}_{2,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{16}[\mathbb{G}_{2,1}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{4}[\mathbb{Q}_{2,1}^{(b,E)}]}{2}$$

No. 11
$$\hat{\mathbb{Q}}_0^{(A_1)}$$
 [M₃, B₁]

$$\hat{\mathbb{Z}}_{11} = \mathbb{X}_{20}[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_1)}]$$

No. 12
$$\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$$
 [M₃, B₁]

$$\hat{\mathbb{Z}}_{12} = \mathbb{X}_{21}[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_1)}]$$

No. 13
$$\hat{\mathbb{Q}}_0^{(A_1)}$$
 [M₃, B₁]

$$\hat{\mathbb{Z}}_{13} = \frac{\sqrt{2}\mathbb{X}_{22}[\mathbb{Q}_{2,0}^{(a,E)}] \otimes \mathbb{Y}_{3}[\mathbb{Q}_{2,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{23}[\mathbb{Q}_{2,1}^{(a,E)}] \otimes \mathbb{Y}_{4}[\mathbb{Q}_{2,1}^{(b,E)}]}{2}$$

No. 14
$$\hat{\mathbb{Q}}_0^{(A_1)}(1,-1)$$
 [M₃, B₁]

$$\hat{\mathbb{Z}}_{14} = \frac{\sqrt{2}\mathbb{X}_{27}[\mathbb{Q}_{2,0}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{3}[\mathbb{Q}_{2,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{28}[\mathbb{Q}_{2,1}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{4}[\mathbb{Q}_{2,1}^{(b,E)}]}{2}$$

No. 15
$$\hat{\mathbb{Q}}_{4}^{(A_1)}$$
 [M₃, B₁]

$$\hat{\mathbb{Z}}_{15} = -\frac{\sqrt{3}\mathbb{X}_{32}[\mathbb{M}_{1,0}^{(a,T_1)}] \otimes \mathbb{Y}_{5}[\mathbb{T}_{4,0}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{33}[\mathbb{M}_{1,1}^{(a,T_1)}] \otimes \mathbb{Y}_{6}[\mathbb{T}_{4,1}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{34}[\mathbb{M}_{1,2}^{(a,T_1)}] \otimes \mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_1)}]}{3}$$

No. 16
$$\hat{\mathbb{Q}}_{4}^{(A_1)}(1,1)$$
 [M₃, B₁]

$$\hat{\mathbb{Z}}_{16} = -\frac{\sqrt{3}\mathbb{X}_{35}[\mathbb{M}_{1,0}^{(a,T_1)}(1,1)]\otimes\mathbb{Y}_{5}[\mathbb{T}_{4,0}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{36}[\mathbb{M}_{1,1}^{(a,T_1)}(1,1)]\otimes\mathbb{Y}_{6}[\mathbb{T}_{4,1}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{37}[\mathbb{M}_{1,2}^{(a,T_1)}(1,1)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{47}[\mathbb{M}_{1,2}^{(a,T_1)}(1,1)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{47}[\mathbb{M}_{1,2}^{(a,T_1)}(1,1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{1,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_{47}[\mathbb{M}_{1,2}^{(b,T_1)}(1,1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{1,2}^{(b,T_1)}(1,1)]}{3} - \frac{\mathbb{X}_{47}[\mathbb{M}_{1,2}^{(b,T_1)}(1,1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{1,2}^{(b,T_1)}(1,1)}}{3} - \frac{\mathbb{X}_{47}[\mathbb{M}_$$

No. 17
$$\hat{\mathbb{Q}}_4^{(A_1)}(1,-1)$$
 [M₃, B₁]

$$\hat{\mathbb{Z}}_{17} = -\frac{\sqrt{3}\mathbb{X}_{38}[\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{5}[\mathbb{T}_{4,0}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{39}[\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{6}[\mathbb{T}_{4,1}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{1,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{1,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{1,2}^{(b,T_1)}]}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{X}_{1,2}}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{X}_{1,2}}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{X}_{1,2}}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{X}_{1,2}}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{X}_{1,2}}{3} - \frac{\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{X}_{1,2}}{3} - \frac{\mathbb{X}_$$

No. 18 $\hat{\mathbb{Q}}_{4}^{(A_1)}(1,-1)$ [M₃, B₁]

$$\hat{\mathbb{Z}}_{18} = \frac{\sqrt{3}\mathbb{X}_{41}[\mathbb{M}_{3,0}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{5}[\mathbb{T}_{4,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{42}[\mathbb{M}_{3,1}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{6}[\mathbb{T}_{4,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{T}_{4,2}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{3,2}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{3,2}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{3,2}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{Y}_{7}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(b,T_1)}(1,-1)]\otimes\mathbb{Y}_$$

No. 19 $\hat{\mathbb{Q}}_0^{(A_1)}$ [M₁, B₂]

$$\hat{\mathbb{Z}}_{19} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_8[\mathbb{Q}_0^{(b,A_1)}]$$

No. 20
$$\hat{\mathbb{G}}_0^{(A_1)}(1,-1)$$
 [M₁, B₂]

$$\hat{\mathbb{Z}}_{20} = \frac{\sqrt{3}\mathbb{X}_2[\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{1,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_3[\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{15}[\mathbb{T}_{1,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes$$

No. 21
$$\hat{\mathbb{G}}_0^{(A_1)}(1,-1)$$
 [M₂, B₂]

$$\hat{\mathbb{Z}}_{21} = \frac{\sqrt{3}\mathbb{X}_{5}[\mathbb{M}_{2,0}^{(a,T_{2})}(1,-1)] \otimes \mathbb{Y}_{17}[\mathbb{T}_{2,0}^{(b,T_{2})}]}{3} + \frac{\sqrt{3}\mathbb{X}_{6}[\mathbb{M}_{2,1}^{(a,T_{2})}(1,-1)] \otimes \mathbb{Y}_{18}[\mathbb{T}_{2,1}^{(b,T_{2})}]}{3} + \frac{\sqrt{3}\mathbb{X}_{7}[\mathbb{M}_{2,2}^{(a,T_{2})}(1,-1)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{2,2}^{(b,T_{2})}]}{3} + \frac{\sqrt{3}\mathbb{X}_{7}[\mathbb{M}_{2,2}^{(a,T_{2})}(1,-1)] \otimes \mathbb{Y}_{19}[\mathbb{M}_{2,2}^{(a,T_{2})}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{7}[\mathbb{M}_{2,2}^{(a,T_{2})}(1,-1)] \otimes \mathbb{Y}_{19}[\mathbb{M}_{2,2}^{(a,T_{2})}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{7}[\mathbb{M}_{2,2}^{(a,T_{2})}(1,-1)] \otimes \mathbb{Y}_{19}[\mathbb{M}_{2,2}^{(a,T_{2})}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{7}[\mathbb{M}_{2,2}^{(a,T_{2})}(1,-1)] \otimes \mathbb{Y}_{19}[\mathbb{M}_{2,2}^{(a,T_{2})}(1,-1)]}{3} + \frac{\sqrt{$$

No. 22
$$\hat{\mathbb{Q}}_0^{(A_1)}$$
 [M₂, B₂]

$$\hat{\mathbb{Z}}_{22} = \frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{T}_{1,2}^{(a,T_1)}] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{8}[\mathbb{T}_{1,0}^{(a,T_1)}] \otimes \mathbb{Y}_{14}[\mathbb{T}_{1,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{9}[\mathbb{T}_{1,1}^{(a,T_1)}] \otimes \mathbb{Y}_{15}[\mathbb{T}_{1,1}^{(b,T_1)}]}{3}$$

No. 23
$$\hat{\mathbb{Q}}_0^{(A_1)}(1,0)$$
 [M₂, B₂]

$$\hat{\mathbb{Z}}_{23} = \frac{\sqrt{3}\mathbb{X}_{11}[\mathbb{T}_{1,0}^{(a,T_1)}(1,0)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{1,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{12}[\mathbb{T}_{1,1}^{(a,T_1)}(1,0)] \otimes \mathbb{Y}_{15}[\mathbb{T}_{1,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T_1)}(1,0)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T_1)}(1,0)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(a,T_1)}(1,0)]}{3} + \frac{\sqrt{3}\mathbb{X}_{13}[\mathbb{T}_{1,2}^{(a,T$$

No. 24
$$\hat{\mathbb{G}}_0^{(A_1)}(1,1)$$
 [M₂, B₂]

$$\hat{\mathbb{Z}}_{24} = \mathbb{X}_{14}[\mathbb{G}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_8[\mathbb{Q}_0^{(b,A_1)}]$$

No. 25
$$\hat{\mathbb{G}}_0^{(A_1)}(1,-1)$$
 [M₂, B₂]

$$\hat{\mathbb{Z}}_{25} = \frac{\sqrt{2}\mathbb{X}_{15}[\mathbb{G}_{2,0}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{9}[\mathbb{Q}_{2,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{16}[\mathbb{G}_{2,1}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,1}^{(b,E)}]}{2}$$

No. 26
$$\hat{\mathbb{Q}}_{4}^{(A_1)}(1,-1)$$
 [M₂, B₂]

$$\hat{\mathbb{Z}}_{26} = \frac{\sqrt{3}\mathbb{X}_{17}[\mathbb{G}_{2,0}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{3,0}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{18}[\mathbb{G}_{2,1}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{3,1}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{19}[\mathbb{G}_{2,2}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{19}[\mathbb{Q}_{3,2}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{19}[\mathbb{Q}_{3,2}^{(a,T_2)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{19}[\mathbb{Q}_{3,2}^{(a,T_2)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{19}[\mathbb{Q}_{3,2}^$$

No. 27
$$\hat{\mathbb{Q}}_0^{(A_1)}$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{27} = \mathbb{X}_{20}[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_8[\mathbb{Q}_0^{(b,A_1)}]$$

No. 28
$$\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{28} = \mathbb{X}_{21}[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_8[\mathbb{Q}_0^{(b,A_1)}]$$

No. 29
$$\hat{\mathbb{Q}}_0^{(A_1)}$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{29} = \frac{\sqrt{2}\mathbb{X}_{22}[\mathbb{Q}_{2,0}^{(a,E)}] \otimes \mathbb{Y}_{9}[\mathbb{Q}_{2,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{23}[\mathbb{Q}_{2,1}^{(a,E)}] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,1}^{(b,E)}]}{2}$$

No. 30
$$\hat{\mathbb{G}}_{4}^{(A_1)}$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{30} = \frac{\sqrt{3}\mathbb{X}_{24}[\mathbb{Q}_{2,0}^{(a,T_2)}] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{3,0}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{25}[\mathbb{Q}_{2,1}^{(a,T_2)}] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{3,1}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{Q}_{2,2}^{(a,T_2)}] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{Q}_{2,2}^{(b,T_2)}] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{Q}_{2,2}^{(b,T_2)}] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{Q}_{2,2}^{(b,T_2)}] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{Q}_{2,2}^{(b,T_2)}] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{Q}_{2,2}^{(b,T_2)}]}{3} +$$

No. 31
$$\hat{\mathbb{Q}}_0^{(A_1)}(1,-1)$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{31} = \frac{\sqrt{2}\mathbb{X}_{27}[\mathbb{Q}_{2,0}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{9}[\mathbb{Q}_{2,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{28}[\mathbb{Q}_{2,1}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,1}^{(b,E)}]}{2}$$

No. 32
$$\hat{\mathbb{G}}_{4}^{(A_1)}(1,-1)$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{32} = \frac{\sqrt{3}\mathbb{X}_{29}[\mathbb{Q}_{2,0}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{3,0}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{30}[\mathbb{Q}_{2,1}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{3,1}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{Q}_{2,2}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{Q}_{3,2}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{Q}_{3,2}^{(b,T_2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{Q}_{3,2}^{(b,T_2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{Q}_{3,2}^{(b,T_2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{Q}_{3,2}^{(b,T_2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{Q}_{3,2}^{(b,T_2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{Q}_{3,2}^{(b,T_2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{Q}_{3,2}^{(b,T_2)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{Q}_{3$$

No. 33
$$\hat{\mathbb{G}}_0^{(A_1)}$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{33} = \frac{\sqrt{3}\mathbb{X}_{32}[\mathbb{M}_{1,0}^{(a,T_1)}] \otimes \mathbb{Y}_{14}[\mathbb{T}_{1,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{33}[\mathbb{M}_{1,1}^{(a,T_1)}] \otimes \mathbb{Y}_{15}[\mathbb{T}_{1,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{34}[\mathbb{M}_{1,2}^{(a,T_1)}] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(b,T_1)}]}{3}$$

No. 34
$$\hat{\mathbb{G}}_0^{(A_1)}(1,1)$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{34} = \frac{\sqrt{3}\mathbb{X}_{35}[\mathbb{M}_{1,0}^{(a,T_1)}(1,1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{1,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{36}[\mathbb{M}_{1,1}^{(a,T_1)}(1,1)] \otimes \mathbb{Y}_{15}[\mathbb{T}_{1,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{37}[\mathbb{M}_{1,2}^{(a,T_1)}(1,1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{37}[\mathbb{M}_{1,2}^{(a,T_1)}(1,1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(a,T_1)}(1,1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{37}[\mathbb{M}_{1,2}^{(a,T_1)}(1,1)] \otimes \mathbb{Y}_{16}[\mathbb{M}_{1,2}^{(a,T_1)}(1,1)]}{3} + \frac{\sqrt{3}\mathbb$$

No. 35
$$\hat{\mathbb{G}}_0^{(A_1)}(1,-1)$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{35} = \frac{\sqrt{3}\mathbb{X}_{38}[\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{1,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{39}[\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{15}[\mathbb{T}_{1,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(a,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(a,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(a,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{40}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)]}{$$

No. 36
$$\hat{\mathbb{G}}_4^{(A_1)}(1,-1)$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{36} = \frac{\sqrt{3}\mathbb{X}_{41}[\mathbb{M}_{3,0}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{14}[\mathbb{T}_{1,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{42}[\mathbb{M}_{3,1}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{15}[\mathbb{T}_{1,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{16}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{16}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{16}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)]\otimes\mathbb{Y}_{16}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)]}{3} + \frac{\sqrt{3}\mathbb{X}_{43}[\mathbb{M}_{3,2}^{(a,T_1)}($$

No. 37
$$\hat{\mathbb{Q}}_4^{(A_1)}(1,-1)$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{37} = -\frac{\sqrt{3}\mathbb{X}_{44}[\mathbb{M}_{3,0}^{(a,T_2)}(1,-1)]\otimes\mathbb{Y}_{17}[\mathbb{T}_{2,0}^{(b,T_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{45}[\mathbb{M}_{3,1}^{(a,T_2)}(1,-1)]\otimes\mathbb{Y}_{18}[\mathbb{T}_{2,1}^{(b,T_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{46}[\mathbb{M}_{3,2}^{(a,T_2)}(1,-1)]\otimes\mathbb{Y}_{19}[\mathbb{T}_{2,2}^{(b,T_2)}]}{3} - \frac{\mathbb{X}_{46}[\mathbb{M}_{3,2}^{(a,T_2)}(1,-1)]\otimes\mathbb{Y}_{19}[\mathbb{T}_{2,2}^{(b,T_2)}]}{3} - \frac{\mathbb{X}_{46}[\mathbb{M}_{3,2}^{(a,T_2)}(1,-1)]\otimes\mathbb{Y}_{19}[\mathbb{M}_{3,2}^{(a,T_2)}(1,-1)]}{3} - \frac{\mathbb{X}_{46}[\mathbb{M}_{3,2}^{(a,T_2)}(1,-1)]\otimes\mathbb{Y}_{19}[\mathbb{M}_{3,2}^{(a,T_2)}(1,-$$

No. 38
$$\hat{\mathbb{Q}}_0^{(A_1)}(1,0)$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{38} = \frac{\sqrt{3}\mathbb{X}_{47}[\mathbb{T}_{2,0}^{(a,T_2)}(1,0)] \otimes \mathbb{Y}_{17}[\mathbb{T}_{2,0}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{48}[\mathbb{T}_{2,1}^{(a,T_2)}(1,0)] \otimes \mathbb{Y}_{18}[\mathbb{T}_{2,1}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{49}[\mathbb{T}_{2,2}^{(a,T_2)}(1,0)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{2,2}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{49}[\mathbb{T}_{2,2}^{(a,T_2)}(1,0)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{2,2}^{(a,T_2)}(1,0)]}{3} + \frac{\sqrt{3}\mathbb$$

Table 5: Atomic SAMB group.

group	bra	ket
M_1	$(s,\uparrow),(s,\downarrow)$	$(s,\uparrow),(s,\downarrow)$
M_2	$(s,\uparrow),(s,\downarrow)$	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$

Table 5

group	bra	ket
M_3	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$

Table 6: Atomic SAMB.

	ı		
symbol	type	group	form
\mathbb{X}_1	$\mathbb{Q}_0^{(a,A_1)}$	M_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0\\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$ $\begin{pmatrix} 0 & \frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$
\mathbb{X}_2	$\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$
\mathbb{X}_3	$\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)$	M_1	$\begin{pmatrix} 2 & 3\sqrt{2}i \\ 0 & -\frac{\sqrt{2}i}{2} \\ \frac{\sqrt{2}i}{2} & 0 \end{pmatrix}$ $\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{X}_4	$\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)$	M_1	
\mathbb{X}_5	$\mathbb{M}_{2,0}^{(a,T_2)}(1,-1)$	M_2	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{i}{2} & 0 \end{pmatrix}$
\mathbb{X}_6	$\mathbb{M}_{2,1}^{(a,T_2)}(1,-1)$	M_2	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$
\mathbb{X}_7	$\mathbb{M}_{2,2}^{(a,T_2)}(1,-1)$	M_2	$\begin{pmatrix} 0 & -\frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_8	$\mathbb{T}_{1,0}^{(a,T_1)}$	M_2	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{i}{2} & 0 \end{pmatrix}$ $\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & -\frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} \frac{\sqrt{2}i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & \frac{\sqrt{2}i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}i}{2} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_9	$\mathbb{T}_{1,1}^{(a,T_1)}$	M_2	$\begin{pmatrix} 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}i}{2} & 0 & 0 \end{pmatrix}$
\mathbb{X}_{10}	$\mathbb{T}_{1,2}^{(a,T_1)}$	M_2	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{2} \end{pmatrix}$
\mathbb{X}_{11}	$\mathbb{T}_{1,0}^{(a,T_1)}(1,0)$	M_2	$ \begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & 0 \end{pmatrix} $
\mathbb{X}_{12}	$\mathbb{T}_{1,1}^{(a,T_1)}(1,0)$	M_2	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 0 \end{pmatrix}$

Table 6

symbol	type	group	form
\mathbb{X}_{13}	$\mathbb{T}_{1,2}^{(a,T_1)}(1,0)$	M_2	$\begin{pmatrix} 0 & \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 \\ -\frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{14}	$\mathbb{G}_0^{(a,A_1)}(1,1)$	M_2	$ \begin{pmatrix} 0 & \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 \\ -\frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} $ $ \begin{pmatrix} 0 & \frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}i}{6} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}i}{6} \end{pmatrix} $ $ \begin{pmatrix} 0 & -\frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}i}{3} & 0 \\ -\frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}i}{3} \end{pmatrix} $ $ \begin{pmatrix} 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} $
\mathbb{X}_{15}	$\mathbb{G}_{2,0}^{(a,E)}(1,-1)$	M_2	$\begin{pmatrix} 6 & 0 & 6 & 0 & 6 \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}i}{3} & 0 \\ -\frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}i}{3} \end{pmatrix}$
\mathbb{X}_{16}	$\mathbb{G}_{2,1}^{(a,E)}(1,-1)$	M_2	$\begin{pmatrix} 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{17}	$\mathbb{G}_{2,0}^{(a,T_2)}(1,-1)$	M_2	$\left(egin{array}{ccccccc} 0 & 0 & rac{7}{2} & 0 & 0 & rac{7}{2} \ 0 & 0 & 0 & -rac{i}{2} & -rac{1}{2} & 0 \ \end{array} ight)$
\mathbb{X}_{18}	$\mathbb{G}_{2,1}^{(a,T_2)}(1,-1)$	M_2	$\left(egin{array}{cccccccccccccccccccccccccccccccccccc$
\mathbb{X}_{19}	$\mathbb{G}_{2,2}^{(a,T_2)}(1,-1)$	M_2	$\begin{pmatrix} -\frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{20}	$\mathbb{Q}_0^{(a,A_1)}$	$ m M_3$	$\begin{pmatrix} \frac{6}{0} & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \end{pmatrix}$
\mathbb{X}_{21}	$\mathbb{Q}_0^{(a,A_1)}(1,1)$	$ m M_3$	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ \frac{\sqrt{3}i}{6} & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{22}	$\mathbb{Q}_{2,0}^{(a,E)}$	$ m M_3$	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & 0\\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0\\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0\\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} \end{pmatrix}$

symbol	type	group	form
\mathbb{X}_{23}	$\mathbb{Q}_{2,1}^{(a,E)}$	$ m M_3$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 &$
\mathbb{X}_{24}	$\mathbb{Q}_{2,0}^{(a,T_2)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 &$
\mathbb{X}_{25}	$\mathbb{Q}_{2,1}^{(a,T_2)}$	$ m M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0$
\mathbb{X}_{26}	$\mathbb{Q}_{2,2}^{(a,T_2)}$	$ m M_3$	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0$
\mathbb{X}_{27}	$\mathbb{Q}_{2,0}^{(a,E)}(1,-1)$	$ m M_3$	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & \frac{\sqrt{6}i}{12} & 0 & 0 \end{pmatrix}$

Table 6

	ı		
symbol	type	group	form
\mathbb{X}_{28}	$\mathbb{Q}_{2,1}^{(a,E)}(1,-1)$	$ m M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{29}	$\mathbb{Q}_{2,0}^{(a,T_2)}(1,-1)$	$ m M_3$	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}i}{4} & 0\\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4}\\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0\\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0\\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{30}	$\mathbb{Q}_{2,1}^{(a,T_2)}(1,-1)$	$ m M_3$	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$
\mathbb{X}_{31}	$\mathbb{Q}_{2,2}^{(a,T_2)}(1,-1)$	$ m M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{32}	$\mathbb{M}_{1,0}^{(a,T_1)}$	$ m M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 &$

Table 6

symbol	type	group	form
\mathbb{X}_{33}	$\mathbb{M}_{1,1}^{(a,T_1)}$	$ m M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0$
\mathbb{X}_{34}	$\mathbb{M}_{1,2}^{(a,T_1)}$	$ m M_3$	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0$
\mathbb{X}_{35}	$\mathbb{M}_{1,0}^{(a,T_1)}(1,1)$	$ m M_3$	$ \begin{pmatrix} 0 & \frac{\sqrt{30}}{15} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{20} & 0\\ \frac{\sqrt{30}}{15} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{20}\\ 0 & -\frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0\\ \frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & 0\\ \frac{\sqrt{30}i}{20} & 0 & 0 & 0 & 0 & -\frac{\sqrt{30}}{30}\\ 0 & -\frac{\sqrt{30}}{20} & 0 & 0 & 0 & -\frac{\sqrt{30}}{30} & 0 \end{pmatrix} $
\mathbb{X}_{36}	$\mathbb{M}_{1,1}^{(a,T_1)}(1,1)$	$ m M_3$	$ \begin{pmatrix} 0 & \frac{\sqrt{30}i}{30} & 0 & \frac{\sqrt{30}}{20} & 0 & 0 \\ -\frac{\sqrt{30}i}{30} & 0 & \frac{\sqrt{30}}{20} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{15} & \frac{\sqrt{30}}{20} & 0 \\ \frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{15} & 0 & 0 & -\frac{\sqrt{30}}{20} \\ 0 & 0 & \frac{\sqrt{30}}{20} & 0 & 0 & \frac{\sqrt{30}i}{30} \\ 0 & 0 & 0 & -\frac{\sqrt{30}}{20} & -\frac{\sqrt{30}i}{30} & 0 \end{pmatrix} $
X ₃₇	$\mathbb{M}_{1,2}^{(a,T_1)}(1,1)$	$ m M_3$	$\begin{pmatrix} -\frac{\sqrt{30}}{30} & 0 & 0 & 0 & 0 & \frac{\sqrt{30}}{20} \\ 0 & \frac{\sqrt{30}}{30} & 0 & 0 & \frac{\sqrt{30}}{20} & 0 \\ 0 & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & -\frac{\sqrt{30}i}{20} \\ 0 & 0 & 0 & \frac{\sqrt{30}}{30} & \frac{\sqrt{30}i}{20} & 0 \\ 0 & \frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{15} & 0 \\ \frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{15} \end{pmatrix}$

symbol	type	group	form
X38	$\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)$	М3	$\begin{pmatrix} 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \end{pmatrix}$
\mathbb{X}_{39}	$\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)$	$ m M_3$	$\begin{pmatrix} 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0\\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0\\ 0 & 0 & \frac{\sqrt{6}i}{6} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}i}{6}\\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
\mathbb{X}_{40}	$\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)$	M_3	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \end{pmatrix}$
\mathbb{X}_{41}	$\mathbb{M}_{3,0}^{(a,T_1)}(1,-1)$	$ m M_3$	$\begin{pmatrix} 0 & \frac{\sqrt{5}}{5} & 0 & \frac{\sqrt{5}i}{10} & -\frac{\sqrt{5}}{10} & 0\\ \frac{\sqrt{5}}{5} & 0 & -\frac{\sqrt{5}i}{10} & 0 & 0 & \frac{\sqrt{5}}{10}\\ 0 & \frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0\\ -\frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & 0\\ -\frac{\sqrt{5}}{10} & 0 & 0 & 0 & -\frac{\sqrt{5}}{10}\\ 0 & \frac{\sqrt{5}}{5} & 0 & 0 & -\frac{\sqrt{5}}{5} & 0 \end{pmatrix}$
\mathbb{X}_{42}	$\mathbb{M}_{3,1}^{(a,T_1)}(1,-1)$	$ m M_3$	$\begin{pmatrix} 0 & \frac{10}{\sqrt{5}i} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0\\ -\frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & 0\\ 0 & -\frac{\sqrt{5}}{10} & 0 & -\frac{\sqrt{5}i}{5} & -\frac{\sqrt{5}}{10} & 0\\ -\frac{\sqrt{5}}{10} & 0 & \frac{\sqrt{5}i}{5} & 0 & 0 & \frac{\sqrt{5}i}{10}\\ 0 & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & \frac{\sqrt{5}i}{10} \end{pmatrix}$

symbol	type	group	form
\mathbb{X}_{43}	$\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)$	M_3	$\begin{pmatrix} -\frac{\sqrt{5}}{10} & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}}{10} & 0 & 0 & -\frac{\sqrt{5}}{10} & 0 \\ 0 & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & \frac{\sqrt{5}i}{10} \\ 0 & 0 & 0 & \frac{\sqrt{5}}{10} & -\frac{\sqrt{5}i}{10} & 0 \\ 0 & -\frac{\sqrt{5}}{10} & 0 & \frac{\sqrt{5}i}{10} & \frac{\sqrt{5}}{5} & 0 \\ -\frac{\sqrt{5}}{10} & 0 & -\frac{\sqrt{5}i}{10} & 0 & 0 & -\frac{\sqrt{5}}{5} \end{pmatrix}$
\mathbb{X}_{44}	$\mathbb{M}_{3,0}^{(a,T_2)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0\\ 0 & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6}\\ 0 & -\frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0\\ \frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & 0\\ -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6}\\ 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & \frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0\\ -\frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \end{pmatrix}$
\mathbb{X}_{45}	$\mathbb{M}_{3,1}^{(a,T_2)}(1,-1)$	M_3	$ \begin{pmatrix} 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \\ -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \\ 0 & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}i}{6} & 0 \end{pmatrix} $
\mathbb{X}_{46}	$\mathbb{M}_{3,2}^{(a,T_2)}(1,-1)$	M_3	$\begin{pmatrix} \frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 \end{pmatrix}$
\mathbb{X}_{47}	$\mathbb{T}_{2,0}^{(a,T_2)}(1,0)$	$ m M_3$	$\begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}}{12} & 0\\ 0 & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & -\frac{\sqrt{6}}{12}\\ 0 & \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0\\ -\frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0\\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6}\\ 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 \end{pmatrix}$

Table 6

symbol	type	group	form
X48	$\mathbb{T}_{2,1}^{(a,T_2)}(1,0)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 \\ -\frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{12} \\ 0 & 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{12} & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
\mathbb{X}_{49}	$\mathbb{T}_{2,2}^{(a,T_2)}(1,0)$	$ m M_3$	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}i}{12} & 0 \\ 0 & -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & \frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$

Table 7: Cluster SAMB.

symbol	type	cluster	form
\mathbb{Y}_1	$\mathbb{Q}_0^{(s,A_1)}$	S_1	(1)
\mathbb{Y}_2	$\mathbb{Q}_0^{(b,A_1)}$	B_1	$ \begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} $
\mathbb{Y}_3	$\mathbb{Q}_{2,0}^{(b,E)}$	B_1	$\begin{pmatrix} 3 & 3 & 3 \\ -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix}$
\mathbb{Y}_4	$\mathbb{Q}_{2,1}^{(b,E)}$ $\mathbb{T}_{4,0}^{(b,T_1)}$ $\mathbb{T}_{4,1}^{(b,T_1)}$	B_1	$\left(0 - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)'$
\mathbb{Y}_5	$\mathbb{T}_{4,0}^{(b,T_1)}$	B_1	$\begin{pmatrix} 0 & i & 0 \end{pmatrix}$
\mathbb{Y}_6	$\mathbb{T}_{4,1}^{(b,T_1)}$	B_1	$\begin{pmatrix} 0 & 0 & i \end{pmatrix}$
\mathbb{Y}_7	$\mathbb{T}_{4,0}^{(b,T_1)}$	B_1	$\begin{pmatrix} i & 0 & 0 \end{pmatrix}$
\mathbb{Y}_8	$\mathbb{Q}_0^{(b,A_1)}$	B_2	$ \begin{pmatrix} \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix} $
\mathbb{Y}_9	$\bigcap^{(b,E)}$	B_2	$ \left(-\frac{\sqrt{3}}{6} -\frac{\sqrt{3}}{6} -\frac{\sqrt{3}}{6} \frac{\sqrt{3}}{3} -\frac{\sqrt{3}}{6} \frac{\sqrt{3}}{3} \right) $
\mathbb{Y}_{10}	$\mathbb{Q}_{2,0}^{(b,E)}$ $\mathbb{Q}_{2,1}^{(b,T_2)}$	B_2	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 \end{pmatrix}$
\mathbb{Y}_{11}	$\mathbb{Q}_{3,0}^{(b,T_2)}$ $\mathbb{Q}_{3,1}^{(b,T_2)}$	B_2	$\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
\mathbb{Y}_{12}	$\mathbb{Q}_{3,1}^{(b,T_2)}$	B_2	$\begin{pmatrix} 2 & 2 & 2 & 0 & \sqrt{2} & 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & $

Table 7

symbol	type	cluster	form
\mathbb{Y}_{13}	$\mathbb{Q}_{3,2}^{(b,T_2)}$	B_2	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{Y}_{14}	$\mathbb{T}_{1,0}^{(b,T_1)}$	$_{ m B_2}$	$\begin{pmatrix} 0 & 0 & \frac{i}{2} & \frac{i}{2} & \frac{i}{2} & \frac{i}{2} \end{pmatrix}$
\mathbb{Y}_{15}	$\mathbb{T}_{1,1}^{(b,T_1)}$	$_{\mathrm{B}_{2}}$	$\left(\begin{array}{cccccccccccccccccccccccccccccccccccc$
\mathbb{Y}_{16}	$\mathbb{T}_{1,2}^{(b,T_1)}$	$_{\mathrm{B}_{2}}$	$\left(egin{array}{ccccc} rac{i}{2} & -rac{i}{2} & -rac{i}{2} & 0 & rac{i}{2} & 0 \end{array} ight)$
\mathbb{Y}_{17}	$\mathbb{T}_{2,0}^{(b,T_2)}$	$_{\mathrm{B}_{2}}$	$\left(egin{matrix} 0 & 0 & rac{i}{2} & -rac{i}{2} & rac{i}{2} & -rac{i}{2} \end{matrix} ight)$
\mathbb{Y}_{18}	$\mathbb{T}_{2,1}^{(b,T_2)}$	$_{\mathrm{B}_{2}}$	$\left(egin{array}{ccccc} -rac{i}{2} & -rac{i}{2} & 0 & -rac{i}{2} & 0 & rac{i}{2} \end{array} ight)$
\mathbb{Y}_{19}	$\mathbb{T}_{2,2}^{(b,T_2)}$	B_2	$\left(rac{i}{2} -rac{i}{2} rac{i}{2} 0 -rac{i}{2} 0 ight)$

Table 8: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_0^{(A_1)}$	0	A_1	_	_	1
2	$\mathbb{Q}_{1,0}^{(T_1)}$	1	T_1	_	0	x
3	$\mathbb{Q}_{1,1}^{(T_1)}$	1	T_1	_	1	y
4	$\mathbb{Q}_{1,2}^{(T_1)}$	1	T_1	_	2	z
5	$\mathbb{Q}_{2,0}^{(E)}$	2	E	-	0	$-\frac{x^{2}}{\frac{2}{2}} - \frac{y^{2}}{2} + z^{2}$ $\frac{\sqrt{3}(x-y)(x+y)}{\sqrt{3}}$
6	$\mathbb{Q}_{2,1}^{(E)}$	2	E	_	1	$\frac{\sqrt{3}(x-y)(x+y)}{2}$
7	$\mathbb{Q}_{2,0}^{(\hat{T}_2)}$	2	T_2	_	0	$\sqrt{3}yz$
8	$\mathbb{Q}_{2,1}^{(T_2)}$	2	T_2	_	1	$\sqrt{3}xz$
9	$\mathbb{Q}_{2,2}^{(T_2)}$	2	T_2	_	2	$\sqrt{3}xy$
10	$\mathbb{Q}_{3,0}^{(T_2)}$	3	T_2	_	0	$\frac{\sqrt{15}x(y-z)(y+z)}{2}$
11	$\mathbb{Q}_{3,1}^{(T_2)}$	3	T_2	_	1	$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$
12	$\mathbb{Q}_{3,2}^{(T_2)}$	3	T_2	_	2	$\frac{\sqrt{15}z(x-y)(x+y)}{2}$
13	$\mathbb{Q}_{4,0}^{(T_1)}$	4	T_1	_	0	$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$
14	$\mathbb{Q}_{4,1}^{(T_1)}$	4	T_1	_	1	$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$
15	$\mathbb{Q}_{4,2}^{(T_1)}$	4	T_1	_	2	$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$

Table 9: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_0^{(A_1)}$	0	A_1	-	_	1
2	$\mathbb{G}_{1,0}^{(T_1)}$	1	T_1	-	0	X
3	$\mathbb{G}_{1,1}^{(T_1)}$	1	T_1	_	1	Y
4	$\mathbb{G}_{1,2}^{(T_1)}$	1	T_1	_	2	Z
5	$\mathbb{G}_{2,0}^{(E)}$	2	E	-	0	$-\frac{X^2}{2} - \frac{Y^2}{2} + Z^2$
6	$\mathbb{G}_{2,1}^{(E)}$	2	E	_	1	$\frac{\sqrt{3}(X-Y)(X+Y)}{2}$
7	$\mathbb{G}_{2,0}^{(T_2)}$	2	T_2	_	0	$\sqrt{3}YZ$
8	$\mathbb{G}_{2,1}^{(T_2)}$	2	T_2	_	1	$\sqrt{3}XZ$
9	$\mathbb{G}_{2,2}^{(T_2)}$	2	T_2	_	2	$\sqrt{3}XY$
10	$\mathbb{G}_{3,0}^{(T_1)}$	3	T_1	_	0	$\frac{X(2X^2-3Y^2-3Z^2)}{2}$
11	$\mathbb{G}_{3,1}^{(T_1)}$	3	T_1	_	1	$-\frac{\frac{2}{Y(3X^2-2Y^2+3Z^2)}}{\frac{Z(3X^2+3Y^2-2Z^2)}{2}}$
12	$\mathbb{G}_{3,2}^{(T_1)}$	3	T_1	_	2	$-\frac{Z(3X^2+3Y^2-2Z^2)}{2}$
13	$\mathbb{G}_{3,0}^{3,2}$	3	T_2	_	0	$\frac{\sqrt{15}X(Y-Z)(Y+Z)}{2}$
14	$\mathbb{G}_{3,1}^{(T_2)}$	3	T_2	_	1	$-\frac{\sqrt{15}Y(X-Z)(X+Z)}{2}$
15	$\mathbb{G}_{3,2}^{(T_2)}$	3	T_2	_	2	$\sqrt{15Z(X-Y)(X+Y)}$
15	$\mathbb{G}_{3,2}^{(I_2)}$	3	T_2	_	2	$\frac{\sqrt{15Z(X-Y)(X+Y)}}{2}$

 \bullet Group info.: Generator = $\{2_{001}|0\},~\{2_{010}|0\},~\{3^{+}_{~111}|0\},~\{2_{110}|0\}$

Table 10: Conjugacy class (point-group part).

rep. SO	symmetry operations
{1 0}	$\{1 0\}$
$\{2_{001} 0\}$	$\{2_{001} 0\}, \{2_{100} 0\}, \{2_{010} 0\}$
$\{2_{110} 0\}$	$\{2_{110} 0\}, \{2_{101} 0\}, \{2_{011} 0\}, \{2_{1-10} 0\}, \{2_{-101} 0\}, \{2_{01-1} 0\}$

Table 10

rep. SO	symmetry operations
$\{3^{+}_{111} 0\}$	$\left \ \{3^{+}_{111} 0\}, \ \{3^{+}_{1-1-1} 0\}, \ \{3^{+}_{-11-1} 0\}, \ \{3^{+}_{-1-11} 0\}, \ \{3^{-}_{111} 0\}, \ \{3^{-}_{-11-1} 0\}, \ $
$\{4^{+}_{001} 0\}$	

Table 11: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
 1	$\{1 0\}$	2	$\{2_{001} 0\}$	3	$\{2_{100} 0\}$	4	$\{2_{010} 0\}$	5	$\{2_{110} 0\}$
6	$\{2_{101} 0\}$	7	$\{2_{011} 0\}$	8	$\{2_{1-10} 0\}$	9	$\{2_{-101} 0\}$	10	$\{2_{01-1} 0\}$
11	$\{3^{+}_{111} 0\}$	12	$\{3^+_{1-1-1} 0\}$	13	$\{3^{+}_{-11-1} 0\}$	14	$\{3^{+}_{-1-11} 0\}$	15	$\{3^{-}_{111} 0\}$
16	$\{3^{1-1-1} 0\}$	17	$\{3^{-}_{-11-1} 0\}$	18	$\{3^{-}_{-1-11} 0\}$	19	$\{4^{+}_{001} 0\}$	20	$\{4^{+}_{100} 0\}$
21	$\{4^{+}_{010} 0\}$	22	$\{4^{-}_{001} 0\}$	23	$\{4_{100}^{-} 0\}$	24	$\{4^{-}_{010} 0\}$		

Table 12: Character table (point-group part).

	1	2001	2110	3 ⁺ ₁₁₁	4 ⁺ ₀₀₁
A_1	1	1	1	1	1
A_2	1	1	-1	1	-1
E	2	2	0	-1	0
T_1	3	-1	-1	0	1
T_2	3	-1	1	0	-1

Table 13: Parity conversion.

\leftrightarrow	\leftrightarrow	\leftrightarrow	\leftrightarrow	\leftrightarrow
$A_1 (A_1)$	$A_2 (A_2)$	E(E)	T_1 (T_1)	T_2 (T_2)

Table 14: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

	A_1	A_2	E	T_1	T_2
A_1	A_1	A_2	E	T_1	T_2
A_2		A_1	E	T_2	T_1
E			$A_1 + E$	$T_1 + T_2$	$T_1 + T_2$
T_1				$A_1 + E + T_2$	$A_2 + E + T_1 + T_2$
T_2					$A_1 + E + T_2$

Table 15: Anti-symmetric product, $[\Gamma \otimes \Gamma]_-.$

A_1	A_2	E	T_1	T_2
_	_	A_2	T_1	T_1

Table 16: Virtual-cluster sites.

No.	position No.		position	No.	position	No.	position
1	$\begin{pmatrix} 2 & 1 & 0 \end{pmatrix}$	2	$\begin{pmatrix} -2 & -1 & 0 \end{pmatrix}$	3	$\begin{pmatrix} 2 & -1 & 0 \end{pmatrix}$	4	$\begin{pmatrix} -2 & 1 & 0 \end{pmatrix}$
5	$\begin{pmatrix} 1 & 2 & 0 \end{pmatrix}$	6	$\begin{pmatrix} 0 & -1 & 2 \end{pmatrix}$	7	$\begin{pmatrix} -2 & 0 & 1 \end{pmatrix}$	8	$\begin{pmatrix} -1 & -2 & 0 \end{pmatrix}$
9	$\begin{pmatrix} 0 & -1 & -2 \end{pmatrix}$	10	$\begin{pmatrix} -2 & 0 & -1 \end{pmatrix}$	11	$\begin{pmatrix} 0 & 2 & 1 \end{pmatrix}$	12	$\begin{pmatrix} 0 & -2 & 1 \end{pmatrix}$
13	$\begin{pmatrix} 0 & -2 & -1 \end{pmatrix}$	14	$\begin{pmatrix} 0 & 2 & -1 \end{pmatrix}$	15	$\begin{pmatrix} 1 & 0 & 2 \end{pmatrix}$	16	$\begin{pmatrix} -1 & 0 & -2 \end{pmatrix}$
17	$\begin{pmatrix} -1 & 0 & 2 \end{pmatrix}$	18	$\begin{pmatrix} 1 & 0 & -2 \end{pmatrix}$	19	$\begin{pmatrix} -1 & 2 & 0 \end{pmatrix}$	20	$\begin{pmatrix} 2 & 0 & 1 \end{pmatrix}$
21	$\begin{pmatrix} 0 & 1 & -2 \end{pmatrix}$	22	$\begin{pmatrix} 1 & -2 & 0 \end{pmatrix}$	23	$\begin{pmatrix} 2 & 0 & -1 \end{pmatrix}$	24	$\begin{pmatrix} 0 & 1 & 2 \end{pmatrix}$

Table 17: Virtual-cluster basis.

symbol	1	2	3	4	5	6	7	8	9	10
$\mathbb{Q}_0^{(A_1)}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$						
$\mathbb{Q}_{1,0}^{(T_1)}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$
	0	0	0	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$
	0	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0						
$\mathbb{Q}_{1,1}^{(T_1)}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0
	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	0	0	0	0	$\frac{\sqrt{10}}{10}$	0
	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$						
$\mathbb{Q}_{1,2}^{(T_1)}$	0	0	0	0	0	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$
	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$
	$-\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$						
$\mathbb{Q}_{2,0}^{(E)}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$	$-\frac{5\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$
	$-\frac{\sqrt{39}}{78}$	$-\frac{\sqrt{39}}{78}$	$-\frac{\sqrt{39}}{78}$	$-\frac{\sqrt{39}}{78}$	$\frac{7\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$
-	$\frac{7\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$	$\frac{7\sqrt{39}}{156}$						
$\mathbb{Q}_{2,1}^{(E)}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$-\frac{3\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{13}$	$-\frac{3\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{13}$
	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$-\frac{3\sqrt{13}}{52}$	$\frac{\sqrt{13}}{13}$
	$-\frac{\sqrt{13}}{52}$	$-\frac{3\sqrt{13}}{52}$	$\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{52}$						
$\mathbb{Q}_{2,0}^{(T_2)}$	0	0	0	0	0	$-\frac{\sqrt{2}}{4}$	0	0	$\frac{\sqrt{2}}{4}$	0
	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	0	0	0	0	0
	$-\frac{\sqrt{2}}{4}$	0	0	$\frac{\sqrt{2}}{4}$						
$\mathbb{Q}_{2,1}^{(T_2)}$	0	0	0	0	0	0	$-\frac{\sqrt{2}}{4}$	0	0	$\frac{\sqrt{2}}{4}$
	0	0	0	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	$\frac{\sqrt{2}}{4}$
	0	0	$-\frac{\sqrt{2}}{4}$	0						
$\mathbb{Q}_{2,2}^{(T_2)}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	0	0	$\frac{\sqrt{2}}{4}$	0	0
	0	0	0	0	0	0	0	0	$-\frac{\sqrt{2}}{4}$	0
	0	$-\frac{\sqrt{2}}{4}$	0	0						
$\mathbb{Q}_{3,0}^{(T_1)}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$
	0	0	0	0	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$

Table 17

symbol	1	2	3	4	5	6	7	8	9	10
	0	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0						
$\mathbb{Q}_{3,1}^{(T_1)}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0
	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	0	0	0	0	$\frac{\sqrt{10}}{20}$	0
	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$						
$\mathbb{Q}_{3,2}^{(T_1)}$	0	0	0	0	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$
	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$
	$-\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$						
$\mathbb{Q}_{3,0}^{(T_2)}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$
	0	0	0	0	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$
	0	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0						
$\mathbb{Q}_{3,1}^{(T_2)}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0
	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	0	0	0	0	$-\frac{\sqrt{10}}{20}$	0
	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$						
$\mathbb{Q}_{3,2}^{(T_2)}$	0	0	0	0	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$
	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$
	$\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$						
$\mathbb{Q}_{4,0}^{(E)}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{13}$	$\frac{3\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{13}$
	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{13}$
	$\frac{\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{13}$	$\frac{\sqrt{13}}{52}$						
$\mathbb{Q}_{4,1}^{(E)}$	$\frac{5\sqrt{39}}{156}$	$\frac{5\sqrt{39}}{156}$	$\frac{5\sqrt{39}}{156}$	$\frac{5\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$	$-\frac{5\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$
	$\frac{\sqrt{39}}{78}$	$\frac{\sqrt{39}}{78}$	$\frac{\sqrt{39}}{78}$	$\frac{\sqrt{39}}{78}$	$-\frac{7\sqrt{39}}{156}$	$-\frac{7\sqrt{39}}{156}$	$-\frac{7\sqrt{39}}{156}$	$-\frac{7\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$
	$\frac{7\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$	$\frac{7\sqrt{39}}{156}$						
$\mathbb{Q}_{4,0}^{(T_1)}$	0	0	0	0	0	$\frac{\sqrt{2}}{4}$	0	0	$-\frac{\sqrt{2}}{4}$	0
	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	0	0	0	0	0
	$\frac{\sqrt{2}}{4}$	0	0	$-\frac{\sqrt{2}}{4}$						
$\mathbb{Q}_{4,1}^{(T_1)}$	0	0	0	0	0	0	$\frac{\sqrt{2}}{4}$	0_	0	$-\frac{\sqrt{2}}{4}$
	0	0	0	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	$-\frac{\sqrt{2}}{4}$
	0	0	$\frac{\sqrt{2}}{4}$	0						
$\mathbb{Q}_{4,2}^{(T_1)}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	0	$-\frac{\sqrt{2}}{4}$	0	0

Table 17

symbol	1	2	3	4	5	6	7	8	9	10
	0	0	0	0	0	0	0	0	$\frac{\sqrt{2}}{4}$	0
	0	$\frac{\sqrt{2}}{4}$	0	0						
$\mathbb{Q}_{5,0}^{(T_2)}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$
	0	0	0	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$
	0	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0						
$\mathbb{Q}_{5,1}^{(T_2)}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0
	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	0	0	0	0	$-\frac{\sqrt{10}}{10}$	0
	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$						
$\mathbb{Q}_{5,2}^{(T_2)}$	0	0	0	0	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$
	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$
	$\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$						
$\mathbb{Q}_6^{(A_2)}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$							
	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$						