

PG No. 13 C_{4v} $4mm$ [tetragonal] (axial, internal axial quadrupole)

* Harmonics for rank 0

$$\vec{\mathbb{G}}_0^{(2,2)}[g](A_2)$$

** symmetry

1

** expression

$$-\frac{\sqrt{5}G_u(x^2 + y^2 - 2z^2)}{10} + \frac{\sqrt{15}G_v(x - y)(x + y)}{10} + \frac{\sqrt{15}G_{xy}xy}{5} + \frac{\sqrt{15}G_{xz}xz}{5} + \frac{\sqrt{15}G_{yz}yz}{5}$$

* Harmonics for rank 1

$$\vec{\mathbb{G}}_1^{(2,0)}[g](A_2)$$

** symmetry

z

** expression

$$\frac{\sqrt{10}G_uz}{5} + \frac{\sqrt{30}G_{xz}x}{10} + \frac{\sqrt{30}G_{yz}y}{10}$$

$$\vec{\mathbb{G}}_1^{(2,2)}[g](A_2)$$

** symmetry

z

** expression

$$-\frac{3\sqrt{35}G_uz(3x^2 + 3y^2 - 2z^2)}{70} + \frac{\sqrt{105}G_vz(x - y)(x + y)}{14} + \frac{\sqrt{105}G_{xy}xyz}{7} - \frac{\sqrt{105}G_{xz}x(x^2 + y^2 - 4z^2)}{35} - \frac{\sqrt{105}G_{yz}y(x^2 + y^2 - 4z^2)}{35}$$

$$\vec{\mathbb{G}}_{1,1}^{(2,0)}[g](E), \vec{\mathbb{G}}_{1,2}^{(2,0)}[g](E)$$

** symmetry

$-y$

x

** expression

$$\frac{\sqrt{10}G_uy}{10} + \frac{\sqrt{30}G_vy}{10} - \frac{\sqrt{30}G_{xy}x}{10} - \frac{\sqrt{30}G_{yz}z}{10}$$

$$-\frac{\sqrt{10}G_ux}{10} + \frac{\sqrt{30}G_vx}{10} + \frac{\sqrt{30}G_{xy}y}{10} + \frac{\sqrt{30}G_{xz}z}{10}$$

$$\vec{\mathbb{G}}_{1,1}^{(2,2)}[g](E), \vec{\mathbb{G}}_{1,2}^{(2,2)}[g](E)$$

** symmetry

$-y$

x

** expression

$$\frac{3\sqrt{35}G_uy(x^2 + y^2 - 4z^2)}{70} - \frac{\sqrt{105}G_vy(7x^2 - 3y^2 + 2z^2)}{70} + \frac{\sqrt{105}G_{xy}x(x^2 - 4y^2 + z^2)}{35} - \frac{\sqrt{105}G_{xz}xyz}{7} + \frac{\sqrt{105}G_{yz}z(x^2 - 4y^2 + z^2)}{35}$$

$$-\frac{3\sqrt{35}G_ux(x^2 + y^2 - 4z^2)}{70} + \frac{\sqrt{105}G_vx(3x^2 - 7y^2 - 2z^2)}{70} + \frac{\sqrt{105}G_{xy}y(4x^2 - y^2 - z^2)}{35} + \frac{\sqrt{105}G_{xz}z(4x^2 - y^2 - z^2)}{35} + \frac{\sqrt{105}G_{yz}xyz}{7}$$

* Harmonics for rank 2

$$\vec{\mathbb{G}}_2^{(2,-2)}[g](A_2)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

G_u

$$\vec{\mathbb{G}}_2^{(2,0)}[g](A_2)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{14}G_u(x^2 + y^2 - 2z^2)}{14} - \frac{\sqrt{42}G_v(x - y)(x + y)}{14} - \frac{\sqrt{42}G_{xy}xy}{7} + \frac{\sqrt{42}G_{xz}xz}{14} + \frac{\sqrt{42}G_{yz}yz}{14}$$

$\tilde{\mathbb{G}}_2^{(2,2)}[g](A_2)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\begin{aligned} & \frac{\sqrt{14}G_u(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{56} - \frac{5\sqrt{42}G_v(x - y)(x + y)(x^2 + y^2 - 6z^2)}{168} \\ & - \frac{5\sqrt{42}G_{xy}xy(x^2 + y^2 - 6z^2)}{84} - \frac{5\sqrt{42}G_{xz}xz(3x^2 + 3y^2 - 4z^2)}{84} - \frac{5\sqrt{42}G_{yz}yz(3x^2 + 3y^2 - 4z^2)}{84} \end{aligned}$$

$\tilde{\mathbb{G}}_2^{(2,-2)}[g](B_1)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$G_{xy}$$

$\tilde{\mathbb{G}}_2^{(2,0)}[g](B_1)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{42}G_uxy}{7} + \frac{\sqrt{14}G_{xy}(x^2 + y^2 - 2z^2)}{14} + \frac{3\sqrt{14}G_{xz}yz}{14} + \frac{3\sqrt{14}G_{yz}xz}{14}$$

$\tilde{\mathbb{G}}_2^{(2,2)}[g](B_1)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\begin{aligned} & -\frac{5\sqrt{42}G_uxy(x^2 + y^2 - 6z^2)}{84} + \frac{5\sqrt{14}G_vxy(x - y)(x + y)}{12} - \frac{\sqrt{14}G_{xy}(4x^4 - 27x^2y^2 + 3x^2z^2 + 4y^4 + 3y^2z^2 - z^4)}{42} \\ & + \frac{5\sqrt{14}G_{xz}yz(6x^2 - y^2 - z^2)}{42} - \frac{5\sqrt{14}G_{yz}xz(x^2 - 6y^2 + z^2)}{42} \end{aligned}$$

$\tilde{\mathbb{G}}_2^{(2,-2)}[g](B_2)$

** symmetry

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

** expression

$$G_v$$

$\tilde{\mathbb{G}}_2^{(2,0)}[g](B_2)$

** symmetry

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

** expression

$$-\frac{\sqrt{42}G_u(x - y)(x + y)}{14} + \frac{\sqrt{14}G_v(x^2 + y^2 - 2z^2)}{14} + \frac{3\sqrt{14}G_{xz}xz}{14} - \frac{3\sqrt{14}G_{yz}yz}{14}$$

$\tilde{\mathbb{G}}_2^{(2,2)}[g](B_2)$

** symmetry

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

** expression

$$\begin{aligned} & -\frac{5\sqrt{42}G_u(x - y)(x + y)(x^2 + y^2 - 6z^2)}{168} + \frac{\sqrt{14}G_v(19x^4 - 102x^2y^2 - 12x^2z^2 + 19y^4 - 12y^2z^2 + 4z^4)}{168} \\ & + \frac{5\sqrt{14}G_{xy}xy(x - y)(x + y)}{12} + \frac{5\sqrt{14}G_{xz}xz(5x^2 - 9y^2 - 2z^2)}{84} + \frac{5\sqrt{14}G_{yz}yz(9x^2 - 5y^2 + 2z^2)}{84} \end{aligned}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,-2)}[g](E), \vec{\mathbb{G}}_{2,2}^{(2,-2)}[g](E)$$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$G_{yz}$$

$$-G_{xz}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,0)}[g](E), \vec{\mathbb{G}}_{2,2}^{(2,0)}[g](E)$$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{42}G_uyz}{14} - \frac{3\sqrt{14}G_vyz}{14} + \frac{3\sqrt{14}G_{xy}xz}{14} + \frac{3\sqrt{14}G_{xz}xy}{14} - \frac{\sqrt{14}G_{yz}(2x^2 - y^2 - z^2)}{14}$$

$$- \frac{\sqrt{42}G_uxz}{14} - \frac{3\sqrt{14}G_vxz}{14} - \frac{3\sqrt{14}G_{xy}yz}{14} - \frac{\sqrt{14}G_{xz}(x^2 - 2y^2 + z^2)}{14} - \frac{3\sqrt{14}G_{yz}xy}{14}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,2)}[g](E), \vec{\mathbb{G}}_{2,2}^{(2,2)}[g](E)$$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$- \frac{5\sqrt{42}G_uyz(3x^2 + 3y^2 - 4z^2)}{84} + \frac{5\sqrt{14}G_vyz(9x^2 - 5y^2 + 2z^2)}{84} - \frac{5\sqrt{14}G_{xy}xz(x^2 - 6y^2 + z^2)}{42} \\ - \frac{5\sqrt{14}G_{xz}xy(x^2 + y^2 - 6z^2)}{42} + \frac{\sqrt{14}G_{yz}(x^4 - 3x^2y^2 - 3x^2z^2 - 4y^4 + 27y^2z^2 - 4z^4)}{42}$$

$$\frac{5\sqrt{42}G_uxz(3x^2 + 3y^2 - 4z^2)}{84} - \frac{5\sqrt{14}G_vxz(5x^2 - 9y^2 - 2z^2)}{84} - \frac{5\sqrt{14}G_{xy}yz(6x^2 - y^2 - z^2)}{42} \\ + \frac{\sqrt{14}G_{xz}(4x^4 + 3x^2y^2 - 27x^2z^2 - y^4 + 3y^2z^2 + 4z^4)}{42} + \frac{5\sqrt{14}G_{yz}xy(x^2 + y^2 - 6z^2)}{42}$$

* Harmonics for rank 3

$$\vec{\mathbb{G}}_3^{(2,-2)}[g](A_2)$$

** symmetry

$$- \frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$\frac{\sqrt{15}G_uz}{5} - \frac{\sqrt{5}G_{xz}x}{5} - \frac{\sqrt{5}G_{yz}y}{5}$$

$$\vec{\mathbb{G}}_3^{(2,0)}[g](A_2)$$

** symmetry

$$- \frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$- \frac{\sqrt{15}G_uz(3x^2 + 3y^2 - 2z^2)}{15} - \frac{\sqrt{5}G_vz(x - y)(x + y)}{2} - \sqrt{5}G_{xy}xyz - \frac{\sqrt{5}G_{xz}x(x^2 + y^2 - 4z^2)}{20} - \frac{\sqrt{5}G_{yz}y(x^2 + y^2 - 4z^2)}{20}$$

$$\vec{\mathbb{G}}_3^{(2,2)}[g](A_2)$$

** symmetry

$$- \frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{330}G_uz(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{264} - \frac{7\sqrt{110}G_vz(x-y)(x+y)(x^2 + y^2 - 2z^2)}{88} - \frac{7\sqrt{110}G_{xy}xyz(x^2 + y^2 - 2z^2)}{44} \\ & + \frac{\sqrt{110}G_{xz}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44} + \frac{\sqrt{110}G_{yz}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44} \end{aligned}$$

$\tilde{\mathbb{G}}_3^{(2,-2)}[g](B_1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{3}G_{xy}z}{3} + \frac{\sqrt{3}G_{xz}y}{3} + \frac{\sqrt{3}G_{yz}x}{3}$$

$\tilde{\mathbb{G}}_3^{(2,0)}[g](B_1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{3}G_{xy}z(3x^2 + 3y^2 - 2z^2)}{6} + \frac{\sqrt{3}G_{xz}y(3x^2 - 2y^2 + 3z^2)}{6} - \frac{\sqrt{3}G_{yz}x(2x^2 - 3y^2 - 3z^2)}{6}$$

$\tilde{\mathbb{G}}_3^{(2,2)}[g](B_1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\begin{aligned} & -\frac{21\sqrt{22}G_uxyz(x^2 + y^2 - 2z^2)}{44} + \frac{21\sqrt{66}G_vxyz(x-y)(x+y)}{44} - \frac{\sqrt{66}G_{xy}z(6x^4 - 51x^2y^2 + 5x^2z^2 + 6y^4 + 5y^2z^2 - z^4)}{66} \\ & - \frac{\sqrt{66}G_{xz}y(6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4)}{66} + \frac{\sqrt{66}G_{yz}x(x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{66} \end{aligned}$$

$\tilde{\mathbb{G}}_3^{(2,-2)}[g](B_2)$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{3}G_vz}{3} + \frac{\sqrt{3}G_{xz}x}{3} - \frac{\sqrt{3}G_{yz}y}{3}$$

$\tilde{\mathbb{G}}_3^{(2,0)}[g](B_2)$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{3}G_vz(3x^2 + 3y^2 - 2z^2)}{6} + \frac{\sqrt{3}G_{xz}x(x^2 - 9y^2 + 6z^2)}{12} + \frac{\sqrt{3}G_{yz}y(9x^2 - y^2 - 6z^2)}{12}$$

$\tilde{\mathbb{G}}_3^{(2,2)}[g](B_2)$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & -\frac{21\sqrt{22}G_uz(x-y)(x+y)(x^2 + y^2 - 2z^2)}{88} + \frac{\sqrt{66}G_vz(39x^4 - 174x^2y^2 - 20x^2z^2 + 39y^4 - 20y^2z^2 + 4z^4)}{264} + \frac{21\sqrt{66}G_{xy}xyz(x-y)(x+y)}{44} \\ & - \frac{\sqrt{66}G_{xz}x(5x^4 - 4x^2y^2 - 46x^2z^2 - 9y^4 + 66y^2z^2 + 12z^4)}{132} - \frac{\sqrt{66}G_{yz}y(9x^4 + 4x^2y^2 - 66x^2z^2 - 5y^4 + 46y^2z^2 - 12z^4)}{132} \end{aligned}$$

$\tilde{\mathbb{G}}_{3,1}^{(2,-2)}[g](E,1), \tilde{\mathbb{G}}_{3,2}^{(2,-2)}[g](E,1)$

** symmetry

$$\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

** expression

$$\frac{\sqrt{15}G_u y}{10} + \frac{3\sqrt{5}G_v y}{10} + \frac{\sqrt{5}G_{xy}x}{5} + \frac{\sqrt{5}G_{yz}z}{5}$$

$$-\frac{\sqrt{15}G_u x}{10} + \frac{3\sqrt{5}G_v x}{10} - \frac{\sqrt{5}G_{xy}y}{5} - \frac{\sqrt{5}G_{xz}z}{5}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-2)}[g](E,2), \vec{\mathbb{G}}_{3,2}^{(2,-2)}[g](E,2)$$

** symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

** expression

$$\frac{G_u y}{2} - \frac{\sqrt{3}G_v y}{6} - \frac{\sqrt{3}G_{xy}x}{3} + \frac{\sqrt{3}G_{yz}z}{3}$$

$$-\frac{G_u x}{2} - \frac{\sqrt{3}G_v x}{6} + \frac{\sqrt{3}G_{xy}y}{3} - \frac{\sqrt{3}G_{xz}z}{3}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,0)}[g](E,1), \vec{\mathbb{G}}_{3,2}^{(2,0)}[g](E,1)$$

** symmetry

$$\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

** expression

$$-\frac{\sqrt{15}G_u y(21x^2 - 4y^2 - 9z^2)}{60} - \frac{\sqrt{5}G_v y(x^2 - 4y^2 + 11z^2)}{20} + \frac{\sqrt{5}G_{xy}x(x^2 - 4y^2 + z^2)}{20} + \sqrt{5}G_{xz}xyz + \frac{\sqrt{5}G_{yz}z(x^2 - 4y^2 + z^2)}{20}$$

$$-\frac{\sqrt{15}G_u x(4x^2 - 21y^2 + 9z^2)}{60} + \frac{\sqrt{5}G_v x(4x^2 - y^2 - 11z^2)}{20} + \frac{\sqrt{5}G_{xy}y(4x^2 - y^2 - z^2)}{20} + \frac{\sqrt{5}G_{xz}z(4x^2 - y^2 - z^2)}{20} - \sqrt{5}G_{yz}xyz$$

$$\vec{\mathbb{G}}_{3,1}^{(2,0)}[g](E,2), \vec{\mathbb{G}}_{3,2}^{(2,0)}[g](E,2)$$

** symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

** expression

$$\frac{G_u y(3x^2 - 2y^2 + 3z^2)}{4} - \frac{\sqrt{3}G_v y(3x^2 - 2y^2 + 3z^2)}{12} - \frac{\sqrt{3}G_{xy}x(x^2 + 6y^2 - 9z^2)}{12} - \frac{\sqrt{3}G_{yz}z(9x^2 - 6y^2 - z^2)}{12}$$

$$\frac{G_u x(2x^2 - 3y^2 - 3z^2)}{4} + \frac{\sqrt{3}G_v x(2x^2 - 3y^2 - 3z^2)}{12} + \frac{\sqrt{3}G_{xy}y(6x^2 + y^2 - 9z^2)}{12} - \frac{\sqrt{3}G_{xz}z(6x^2 - 9y^2 + z^2)}{12}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,2)}[g](E,1), \vec{\mathbb{G}}_{3,2}^{(2,2)}[g](E,1)$$

** symmetry

$$\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

** expression

$$-\frac{\sqrt{330}G_u y(3x^4 - x^2y^2 - 15x^2z^2 - 4y^4 + 41y^2z^2 - 18z^4)}{264} + \frac{\sqrt{110}G_v y(11x^4 - 27x^2y^2 + 15x^2z^2 + 4y^4 - 13y^2z^2 + 4z^4)}{88}$$

$$-\frac{\sqrt{110}G_{xy}x(x^4 - 12x^2y^2 + 2x^2z^2 + 8y^4 - 12y^2z^2 + z^4)}{44} + \frac{7\sqrt{110}G_{xz}xyz(x^2 - 2y^2 + z^2)}{44} - \frac{\sqrt{110}G_{yz}z(x^4 - 12x^2y^2 + 2x^2z^2 + 8y^4 - 12y^2z^2 + z^4)}{44}$$

$$-\frac{\sqrt{330}G_u x \left(4x^4 + x^2y^2 - 41x^2z^2 - 3y^4 + 15y^2z^2 + 18z^4\right)}{264} + \frac{\sqrt{110}G_v x \left(4x^4 - 27x^2y^2 - 13x^2z^2 + 11y^4 + 15y^2z^2 + 4z^4\right)}{88} \\ + \frac{\sqrt{110}G_{xy} y \left(8x^4 - 12x^2y^2 - 12x^2z^2 + y^4 + 2y^2z^2 + z^4\right)}{44} + \frac{\sqrt{110}G_{xz} z \left(8x^4 - 12x^2y^2 - 12x^2z^2 + y^4 + 2y^2z^2 + z^4\right)}{44} + \frac{7\sqrt{110}G_{yz} xyz \left(2x^2 - y^2 - z^2\right)}{44}$$

$\vec{\mathbb{G}}_{3,1}^{(2,2)}[g](E, 2), \vec{\mathbb{G}}_{3,2}^{(2,2)}[g](E, 2)$

** symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

** expression

$$-\frac{\sqrt{22}G_u y \left(9x^4 + 11x^2y^2 - 87x^2z^2 + 2y^4 - 31y^2z^2 + 30z^4\right)}{88} - \frac{\sqrt{66}G_v y \left(51x^4 - 73x^2y^2 - 87x^2z^2 + 2y^4 + 53y^2z^2 - 12z^4\right)}{264} \\ + \frac{\sqrt{66}G_{xy} x \left(5x^4 - 46x^2y^2 - 4x^2z^2 + 12y^4 + 66y^2z^2 - 9z^4\right)}{132} - \frac{21\sqrt{66}G_{xz}xyz(x-z)(x+z)}{44} \\ + \frac{\sqrt{66}G_{yz} z \left(9x^4 - 66x^2y^2 + 4x^2z^2 - 12y^4 + 46y^2z^2 - 5z^4\right)}{132}$$

$$-\frac{\sqrt{22}G_u x \left(2x^4 + 11x^2y^2 - 31x^2z^2 + 9y^4 - 87y^2z^2 + 30z^4\right)}{88} - \frac{\sqrt{66}G_v x \left(2x^4 - 73x^2y^2 + 53x^2z^2 + 51y^4 - 87y^2z^2 - 12z^4\right)}{264} \\ - \frac{\sqrt{66}G_{xy} y \left(12x^4 - 46x^2y^2 + 66x^2z^2 + 5y^4 - 4y^2z^2 - 9z^4\right)}{132} + \frac{\sqrt{66}G_{xz} z \left(12x^4 + 66x^2y^2 - 46x^2z^2 - 9y^4 - 4y^2z^2 + 5z^4\right)}{132} + \frac{21\sqrt{66}G_{yz}xyz(y-z)(y+z)}{44}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(2,-2)}[g](A_1)$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{6}G_v xy}{2} + \frac{\sqrt{6}G_{xy} (x-y)(x+y)}{4}$$

$\vec{\mathbb{G}}_4^{(2,0)}[g](A_1)$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$-\frac{7\sqrt{11}G_u xy(x-y)(x+y)}{11} + \frac{\sqrt{33}G_v xy(x^2 + y^2 - 6z^2)}{22} \\ + \frac{\sqrt{33}G_{xy} (x-y)(x+y)(x^2 + y^2 - 6z^2)}{44} + \frac{7\sqrt{33}G_{xz}yz(3x^2 - y^2)}{44} + \frac{7\sqrt{33}G_{yz}xz(x^2 - 3y^2)}{44}$$

$\vec{\mathbb{G}}_4^{(2,2)}[g](A_1)$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$-\frac{9\sqrt{1001}G_u xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572} + \frac{\sqrt{3003}G_v xy(25x^4 - 82x^2y^2 - 4x^2z^2 + 25y^4 - 4y^2z^2 + 4z^4)}{572} \\ - \frac{\sqrt{3003}G_{xy} (x-y)(x+y)(2x^4 - 29x^2y^2 + x^2z^2 + 2y^4 + y^2z^2 - z^4)}{286} \\ + \frac{3\sqrt{3003}G_{xz}yz(8x^4 - 13x^2y^2 - 3x^2z^2 + y^4 + y^2z^2)}{286} - \frac{3\sqrt{3003}G_{yz}xz(x^4 - 13x^2y^2 + x^2z^2 + 8y^4 - 3y^2z^2)}{286}$$

$\vec{\mathbb{G}}_4^{(2,-2)}[g](A_2, 1)$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$-\frac{\sqrt{30}G_u(x^2 + y^2 - 2z^2)}{20} + \frac{3\sqrt{10}G_v(x - y)(x + y)}{20} - \frac{\sqrt{10}G_{xy}xy}{5} - \frac{\sqrt{10}G_{xz}xz}{5} - \frac{\sqrt{10}G_{yz}yz}{5}$$

$\tilde{\mathbb{G}}_4^{(2,-2)}[g](A_2, 2)$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$-\frac{\sqrt{42}G_u(x^2 + y^2 - 2z^2)}{28} - \frac{3\sqrt{14}G_v(x - y)(x + y)}{28} + \frac{2\sqrt{14}G_{xy}xy}{7} - \frac{\sqrt{14}G_{xz}xz}{7} - \frac{\sqrt{14}G_{yz}yz}{7}$$

$\tilde{\mathbb{G}}_4^{(2,0)}[g](A_2, 1)$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$-\frac{\sqrt{165}G_u(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{66} + \frac{\sqrt{55}G_v(x - y)(x + y)(x^2 + y^2 - 6z^2)}{22} \\ + \frac{\sqrt{55}G_{xy}xy(x^2 + y^2 - 6z^2)}{22} + \frac{\sqrt{55}G_{xz}xz(x^2 - 6y^2 + z^2)}{22} - \frac{\sqrt{55}G_{yz}yz(6x^2 - y^2 - z^2)}{22}$$

$\tilde{\mathbb{G}}_4^{(2,0)}[g](A_2, 2)$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\frac{\sqrt{231}G_u(8x^4 - 33x^2y^2 - 15x^2z^2 + 8y^4 - 15y^2z^2 + 5z^4)}{231} + \frac{\sqrt{77}G_v(x - y)(x + y)(x^2 + y^2 - 6z^2)}{77} \\ + \frac{\sqrt{77}G_{xy}xy(x^2 + y^2 - 6z^2)}{14} - \frac{\sqrt{77}G_{xz}xz(16x^2 - 33y^2 - 5z^2)}{154} + \frac{\sqrt{77}G_{yz}yz(33x^2 - 16y^2 + 5z^2)}{154}$$

$\tilde{\mathbb{G}}_4^{(2,2)}[g](A_2, 1)$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$-\frac{\sqrt{15015}G_u(x^6 - 15x^4z^2 + 15x^2z^4 + y^6 - 15y^4z^2 + 15y^2z^4 - 2z^6)}{572} \\ + \frac{3\sqrt{5005}G_v(x - y)(x + y)(x^4 - 9x^2y^2 - 5x^2z^2 + y^4 - 5y^2z^2 + 5z^4)}{572} + \frac{\sqrt{5005}G_{xy}xy(7x^4 - 19x^2y^2 - 13x^2z^2 + 7y^4 - 13y^2z^2 + 13z^4)}{286} \\ + \frac{\sqrt{5005}G_{xz}xz(7x^4 - 13x^2y^2 - 19x^2z^2 + 13y^4 - 13y^2z^2 + 7z^4)}{286} + \frac{\sqrt{5005}G_{yz}yz(13x^4 - 13x^2y^2 - 13x^2z^2 + 7y^4 - 19y^2z^2 + 7z^4)}{286}$$

$\tilde{\mathbb{G}}_4^{(2,2)}[g](A_2, 2)$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$-\frac{\sqrt{429}G_u(x^6 + 45x^4y^2 - 60x^4z^2 + 45x^2y^4 - 540x^2y^2z^2 + 150x^2z^4 + y^6 - 60y^4z^2 + 150y^2z^4 - 20z^6)}{1144} \\ - \frac{21\sqrt{143}G_v(x - y)(x + y)(x^4 - 20x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 6z^4)}{1144} - \frac{7\sqrt{143}G_{xy}xy(x^4 - 4x^2y^2 + 2x^2z^2 + y^4 + 2y^2z^2 - 2z^4)}{52} \\ + \frac{7\sqrt{143}G_{xz}xz(x^4 + 56x^2y^2 - 22x^2z^2 - 11y^4 - 34y^2z^2 + 10z^4)}{572} - \frac{7\sqrt{143}G_{yz}yz(11x^4 - 56x^2y^2 + 34x^2z^2 - y^4 + 22y^2z^2 - 10z^4)}{572}$$

$\tilde{\mathbb{G}}_4^{(2,-2)}[g](B_1)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\frac{3\sqrt{14}G_{u}xy}{14} - \frac{\sqrt{42}G_{xy}(x^2 + y^2 - 2z^2)}{28} + \frac{\sqrt{42}G_{xz}yz}{7} + \frac{\sqrt{42}G_{yz}xz}{7}$$

$\tilde{\mathbb{G}}_4^{(2,0)}[g](B_1)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$-\frac{2\sqrt{77}G_{u}xy(x^2 + y^2 - 6z^2)}{77} - \frac{\sqrt{231}G_{v}xy(x-y)(x+y)}{22} - \frac{\sqrt{231}G_{xy}(x^4 + 30x^2y^2 - 36x^2z^2 + y^4 - 36y^2z^2 + 12z^4)}{308} \\ + \frac{\sqrt{231}G_{xz}yz(39x^2 - 31y^2 + 18z^2)}{308} - \frac{\sqrt{231}G_{yz}xz(31x^2 - 39y^2 - 18z^2)}{308}$$

$\tilde{\mathbb{G}}_4^{(2,2)}[g](B_1)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\frac{21\sqrt{143}G_{u}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} - \frac{21\sqrt{429}G_{v}xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572} \\ + \frac{\sqrt{429}G_{xy}(2x^6 - 15x^4y^2 - 15x^4z^2 - 15x^2y^4 + 180x^2y^2z^2 - 15x^2z^4 + 2y^6 - 15y^4z^2 - 15y^2z^4 + 2z^6)}{286} \\ - \frac{7\sqrt{429}G_{xz}yz(8x^4 + 7x^2y^2 - 23x^2z^2 - y^4 + y^2z^2 + 2z^4)}{286} + \frac{7\sqrt{429}G_{yz}xz(x^4 - 7x^2y^2 - x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4)}{286}$$

$\tilde{\mathbb{G}}_4^{(2,-2)}[g](B_2)$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$-\frac{3\sqrt{14}G_u(x-y)(x+y)}{28} + \frac{\sqrt{42}G_v(x^2 + y^2 - 2z^2)}{28} - \frac{\sqrt{42}G_{xz}xz}{7} + \frac{\sqrt{42}G_{yz}yz}{7}$$

$\tilde{\mathbb{G}}_4^{(2,0)}[g](B_2)$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\frac{\sqrt{77}G_u(x-y)(x+y)(x^2 + y^2 - 6z^2)}{77} + \frac{\sqrt{231}G_v(2x^4 - 3x^2y^2 - 9x^2z^2 + 2y^4 - 9y^2z^2 + 3z^4)}{77} \\ + \frac{\sqrt{231}G_{xy}xy(x-y)(x+y)}{22} - \frac{\sqrt{231}G_{xz}xz(2x^2 - 33y^2 + 9z^2)}{154} - \frac{\sqrt{231}G_{yz}yz(33x^2 - 2y^2 - 9z^2)}{154}$$

$\tilde{\mathbb{G}}_4^{(2,2)}[g](B_2)$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$-\frac{21\sqrt{143}G_u(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144} \\ + \frac{\sqrt{429}G_v(13x^6 - 45x^4y^2 - 150x^4z^2 - 45x^2y^4 + 540x^2y^2z^2 + 60x^2z^4 + 13y^6 - 150y^4z^2 + 60y^2z^4 - 8z^6)}{1144} \\ + \frac{21\sqrt{429}G_{xy}xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572} + \frac{7\sqrt{429}G_{xz}xz(7x^4 - 4x^2y^2 - 22x^2z^2 - 11y^4 + 26y^2z^2 + 4z^4)}{572} \\ + \frac{7\sqrt{429}G_{yz}yz(11x^4 + 4x^2y^2 - 26x^2z^2 - 7y^4 + 22y^2z^2 - 4z^4)}{572}$$

$\tilde{\mathbb{G}}_{4,1}^{(2,-2)}[g](E,1), \tilde{\mathbb{G}}_{4,2}^{(2,-2)}[g](E,1)$

** symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

** expression

$$-\frac{3\sqrt{2}G_uyz}{4} - \frac{\sqrt{6}G_vyz}{4} + \frac{\sqrt{6}G_{yz}(y-z)(y+z)}{4}$$

$$\frac{3\sqrt{2}G_uxz}{4} - \frac{\sqrt{6}G_vxz}{4} - \frac{\sqrt{6}G_{xz}(x-z)(x+z)}{4}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E,2), \vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E,2)$$

** symmetry

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

$$\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

** expression

$$-\frac{3\sqrt{14}G_uyz}{28} + \frac{3\sqrt{42}G_vyz}{28} + \frac{\sqrt{42}G_{xy}xz}{7} + \frac{\sqrt{42}G_{xz}xy}{7} + \frac{\sqrt{42}G_{yz}(2x^2-y^2-z^2)}{28}$$

$$\frac{3\sqrt{14}G_uxz}{28} + \frac{3\sqrt{42}G_vxz}{28} - \frac{\sqrt{42}G_{xy}yz}{7} + \frac{\sqrt{42}G_{xz}(x^2-2y^2+z^2)}{28} - \frac{\sqrt{42}G_{yz}xy}{7}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E,1), \vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E,1)$$

** symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{11}G_uyz(18x^2+11y^2-17z^2)}{44} + \frac{\sqrt{33}G_vyz(6x^2-15y^2+13z^2)}{44} + \frac{7\sqrt{33}G_{xy}xz(3y^2-z^2)}{44} \\ & + \frac{7\sqrt{33}G_{xz}xy(y^2-3z^2)}{44} - \frac{\sqrt{33}G_{yz}(y-z)(y+z)(6x^2-y^2-z^2)}{44} \end{aligned}$$

$$\begin{aligned} & -\frac{\sqrt{11}G_uxz(11x^2+18y^2-17z^2)}{44} - \frac{\sqrt{33}G_vxz(15x^2-6y^2-13z^2)}{44} - \frac{7\sqrt{33}G_{xy}yz(3x^2-z^2)}{44} \\ & - \frac{\sqrt{33}G_{xz}(x-z)(x+z)(x^2-6y^2+z^2)}{44} - \frac{7\sqrt{33}G_{yz}xy(x^2-3z^2)}{44} \end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E,2), \vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E,2)$$

** symmetry

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

$$\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{77}G_uyz(24x^2-25y^2+17z^2)}{308} + \frac{\sqrt{231}G_vyz(24x^2+3y^2-11z^2)}{308} + \frac{\sqrt{231}G_{xy}xz(18x^2+39y^2-31z^2)}{308} \\ & + \frac{\sqrt{231}G_{xz}xy(18x^2-31y^2+39z^2)}{308} - \frac{\sqrt{231}G_{yz}(12x^4-36x^2y^2-36x^2z^2+y^4+30y^2z^2+z^4)}{308} \end{aligned}$$

$$\begin{aligned} & -\frac{\sqrt{77}G_uxz(25x^2-24y^2-17z^2)}{308} + \frac{\sqrt{231}G_vxz(3x^2+24y^2-11z^2)}{308} - \frac{\sqrt{231}G_{xy}yz(39x^2+18y^2-31z^2)}{308} \\ & + \frac{\sqrt{231}G_{xz}(x^4-36x^2y^2+30x^2z^2+12y^4-36y^2z^2+z^4)}{308} + \frac{\sqrt{231}G_{yz}xy(31x^2-18y^2-39z^2)}{308} \end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E,1), \vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E,1)$$

** symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

** expression

$$\begin{aligned} & -\frac{3\sqrt{1001}G_uyz(2x^4 + 13x^2y^2 - 17x^2z^2 + 11y^4 - 41y^2z^2 + 14z^4)}{572} - \frac{\sqrt{3003}G_vyz(2x^4 - 47x^2y^2 + 43x^2z^2 + 17y^4 - 41y^2z^2 + 8z^4)}{572} \\ & - \frac{3\sqrt{3003}G_{xy}xz(3x^2y^2 - x^2z^2 - 8y^4 + 13y^2z^2 - z^4)}{286} - \frac{3\sqrt{3003}G_{xz}xy(x^2y^2 - 3x^2z^2 + y^4 - 13y^2z^2 + 8z^4)}{286} \\ & + \frac{\sqrt{3003}G_{yz}(y-z)(y+z)(x^4 - x^2y^2 - x^2z^2 - 2y^4 + 29y^2z^2 - 2z^4)}{286} \end{aligned}$$

$$\begin{aligned} & \frac{3\sqrt{1001}G_uxz(11x^4 + 13x^2y^2 - 41x^2z^2 + 2y^4 - 17y^2z^2 + 14z^4)}{572} \\ & - \frac{\sqrt{3003}G_vxz(17x^4 - 47x^2y^2 - 41x^2z^2 + 2y^4 + 43y^2z^2 + 8z^4)}{572} - \frac{3\sqrt{3003}G_{xy}yz(8x^4 - 3x^2y^2 - 13x^2z^2 + y^2z^2 + z^4)}{286} \\ & + \frac{\sqrt{3003}G_{xz}(x-z)(x+z)(2x^4 + x^2y^2 - 29x^2z^2 - y^4 + y^2z^2 + 2z^4)}{286} + \frac{3\sqrt{3003}G_{yz}xy(x^4 + x^2y^2 - 13x^2z^2 - 3y^2z^2 + 8z^4)}{286} \end{aligned}$$

$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E, 2), \vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E, 2)$

** symmetry

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

$$\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

** expression

$$\begin{aligned} & -\frac{21\sqrt{143}G_uyz(8x^4 + 7x^2y^2 - 23x^2z^2 - y^4 + y^2z^2 + 2z^4)}{572} + \frac{21\sqrt{429}G_vyz(8x^4 - 13x^2y^2 - 3x^2z^2 + y^4 + y^2z^2)}{572} \\ & - \frac{7\sqrt{429}G_{xy}xz(2x^4 - 23x^2y^2 + x^2z^2 + 8y^4 + 7y^2z^2 - z^4)}{286} - \frac{7\sqrt{429}G_{xz}xy(2x^4 + x^2y^2 - 23x^2z^2 - y^4 + 7y^2z^2 + 8z^4)}{286} \\ & + \frac{\sqrt{429}G_{yz}(2x^6 - 15x^4y^2 - 15x^4z^2 - 15x^2y^4 + 180x^2y^2z^2 - 15x^2z^4 + 2y^6 - 15y^4z^2 - 15y^2z^4 + 2z^6)}{286} \\ & - \frac{21\sqrt{143}G_uxz(x^4 - 7x^2y^2 - x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4)}{572} + \frac{21\sqrt{429}G_vxz(x^4 - 13x^2y^2 + x^2z^2 + 8y^4 - 3y^2z^2)}{572} \\ & + \frac{7\sqrt{429}G_{xy}yz(8x^4 - 23x^2y^2 + 7x^2z^2 + 2y^4 + y^2z^2 - z^4)}{286} \\ & - \frac{\sqrt{429}G_{xz}(2x^6 - 15x^4y^2 - 15x^4z^2 - 15x^2y^4 + 180x^2y^2z^2 - 15x^2z^4 + 2y^6 - 15y^4z^2 - 15y^2z^4 + 2z^6)}{286} \\ & - \frac{7\sqrt{429}G_{yz}xy(x^4 - x^2y^2 - 7x^2z^2 - 2y^4 + 23y^2z^2 - 8z^4)}{286} \end{aligned}$$