SAMB for "D2h1"

Generated on 2023-06-02 20:28 by MultiPie 1.1.2

- Associated point group: No. 8 D_{2h} mmm [orthorhombic]
- Generation condition

 - time-reversal type: electric
 - irrep: [Ag]
 - spinful
- Unit cell:

$$a=1.0,\ b=1.2,\ c=1.5,\ \alpha=90.0,\ \beta=90.0,\ \gamma=90.0$$

- Lattice vectors:
 - $\boldsymbol{a}_1 = \begin{pmatrix} 1.0 & 0 & 0 \end{pmatrix}$
 - $\boldsymbol{a}_2 = \begin{pmatrix} 0 & 1.2 & 0 \end{pmatrix}$
 - $\boldsymbol{a}_3 = \begin{pmatrix} 0 & 0 & 1.5 \end{pmatrix}$

Table 1: High-symmetry line: Γ -X.

symbol	position	n	symbol	pc	sitio	on
Γ	$\begin{pmatrix} 0 & 0 \end{pmatrix}$	0)	X	$\left(\frac{1}{2}\right)$	0	0)

• Kets: dimension = 8

Table 2: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket	No.	ket	No.	ket
 1	(s,\uparrow) @A ₁	2	(s,\downarrow) @A ₁	3	(p_x,\uparrow) @A ₁	4	(p_x,\downarrow) @A ₁	5	(p_y,\uparrow) @A ₁
6	(p_y,\downarrow) @A ₁	7	(p_z,\uparrow) @A ₁	8	(p_z,\downarrow) @A ₁				

• Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position	mapping
S_1	A_1	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	[1,2,3,4,5,6,7,8]

• Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

	bond	tail	head	n	#					b@c	;				mapping
B_1	b_1	A_1	A_1	1	1	(1	0	0) @ ($\left(\frac{1}{2}\right)$	0	0)	[1,-2,-3,4,-5,6,7,-8]
B_2	b_2	A_1	A_1	2	1	(0	1	0) @ (0	$\frac{1}{2}$	0)	[1,-2,3,-4,-5,6,-7,8]
B_3	b_3	A_1	A_1	3	1	(0	0	1	(@	0	0	$\left(\frac{1}{2}\right)$)	[1,2,-3,-4,-5,-6,7,8]
B_4	b_4	A_1	A_1	4	1	(1	1	0) @ ($\frac{1}{2}$	$\frac{1}{2}$	0)	[1,-2,-5,6]
	b_5	A_1	A_1	4	1	(1	Ì	-1	0	0	$\frac{1}{2}$	$\frac{1}{2}$	Ć)	[-3,4,7,-8]

• SAMB:

$$\begin{split} & \boxed{ \text{No. 1} } \quad \hat{\mathbb{Q}}_0^{(A_g)} \ [\text{M}_1, \text{S}_1] \\ & \hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a, A_g)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s, A_g)}] \end{split}$$

$$\hat{\mathbb{Z}}_1(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}]$$

No. 2
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₃, S₁]

$$\hat{\mathbb{Z}}_2 = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_g)}]$$

$$\hat{\mathbb{Z}}_2(\mathbf{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}]$$

No. 3
$$\hat{\mathbb{Q}}_{2}^{(A_{g},1)}$$
 [M₃, S₁]

$$\hat{\mathbb{Z}}_3 = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_g,1)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_g)}]$$

$$\hat{\mathbb{Z}}_3(\boldsymbol{k}) = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_g,1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}]$$

No. 4
$$\hat{\mathbb{Q}}_{2}^{(A_{g},2)}$$
 [M₃,S₁]

$$\hat{\mathbb{Z}}_4 = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_g)}]$$

$$\hat{\mathbb{Z}}_4(\boldsymbol{k}) = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}]$$

No. 5
$$\hat{\mathbb{Q}}_0^{(A_g)}(1,1)$$
 [M₃, S₁]

$$\hat{\mathbb{Z}}_5 = \mathbb{X}_5[\mathbb{Q}_0^{(a,A_g)}(1,1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_g)}]$$

$$\hat{\mathbb{Z}}_5(\mathbf{k}) = \mathbb{X}_5[\mathbb{Q}_0^{(a,A_g)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}]$$

No. 6
$$\hat{\mathbb{Q}}_2^{(A_g,1)}(1,-1)$$
 [M₃,S₁]

$$\hat{\mathbb{Z}}_6 = \mathbb{X}_6[\mathbb{Q}_2^{(a,A_g,1)}(1,-1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_g)}]$$

$$\hat{\mathbb{Z}}_6(\mathbf{k}) = \mathbb{X}_6[\mathbb{Q}_2^{(a,A_g,1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}]$$

No. 7
$$\hat{\mathbb{Q}}_2^{(A_g,2)}(1,-1)$$
 [M₃,S₁]

$$\hat{\mathbb{Z}}_7 = \mathbb{X}_7[\mathbb{Q}_2^{(a,A_g,2)}(1,-1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_g)}]$$

$$\hat{\mathbb{Z}}_7(\boldsymbol{k}) = \mathbb{X}_7[\mathbb{Q}_2^{(a,A_g,2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}]$$

No. 8
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₁, B₁]

$$\hat{\mathbb{Z}}_8 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_8(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_g)}]$$

No. 9
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₃, B₁]

$$\hat{\mathbb{Z}}_9 = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_9(\boldsymbol{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_g)}]$$

No. 10
$$\hat{\mathbb{Q}}_{2}^{(A_g,1)}$$
 [M₃, B₁]

$$\hat{\mathbb{Z}}_{10} = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_g,1)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{10}(\boldsymbol{k}) = \mathbb{X}_{3}[\mathbb{Q}_{2}^{(a,A_{g},1)}] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{1}[\mathbb{Q}_{0}^{(k,A_{g})}]$$

No. 11
$$\hat{\mathbb{Q}}_{2}^{(A_g,2)}$$
 [M₃, B₁]

$$\hat{\mathbb{Z}}_{11} = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{11}(\boldsymbol{k}) = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_g)}]$$

No. 12
$$\hat{\mathbb{Q}}_0^{(A_g)}(1,1)$$
 [M₃, B₁]

$$\hat{\mathbb{Z}}_{12} = \mathbb{X}_{5}[\mathbb{Q}_{0}^{(a,A_{g})}(1,1)] \otimes \mathbb{Y}_{2}[\mathbb{Q}_{0}^{(b,A_{g})}]$$

$$\hat{\mathbb{Z}}_{12}(\boldsymbol{k}) = \mathbb{X}_{5}[\mathbb{Q}_{0}^{(a,A_g)}(1,1)] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_g)}] \otimes \mathbb{F}_{1}[\mathbb{Q}_{0}^{(k,A_g)}]$$

No. 13
$$\hat{\mathbb{Q}}_2^{(A_g,1)}(1,-1)$$
 [M₃,B₁]

$$\hat{\mathbb{Z}}_{13} = \mathbb{X}_{6}[\mathbb{Q}_{2}^{(a,A_{g},1)}(1,-1)] \otimes \mathbb{Y}_{2}[\mathbb{Q}_{0}^{(b,A_{g})}]$$

$$\hat{\mathbb{Z}}_{13}(\boldsymbol{k}) = \mathbb{X}_{6}[\mathbb{Q}_{2}^{(a,A_{g},1)}(1,-1)] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{1}[\mathbb{Q}_{0}^{(k,A_{g})}]$$

No. 14
$$\hat{\mathbb{Q}}_2^{(A_g,2)}(1,-1)$$
 [M₃,B₁]

$$\hat{\mathbb{Z}}_{14} = \mathbb{X}_7[\mathbb{Q}_2^{(a,A_g,2)}(1,-1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{14}(\boldsymbol{k}) = \mathbb{X}_{7}[\mathbb{Q}_{2}^{(a,A_{g},2)}(1,-1)] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{1}[\mathbb{Q}_{0}^{(k,A_{g})}]$$

No. 15
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₁, B₂]

$$\hat{\mathbb{Z}}_{15} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{15}(\textbf{\textit{k}}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_2[\mathbb{Q}_0^{(k,A_g)}]$$

No. 16
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{16} = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{16}(\boldsymbol{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_2[\mathbb{Q}_0^{(k,A_g)}]$$

No. 17
$$\hat{\mathbb{Q}}_{2}^{(A_g,1)}$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{17} = \mathbb{X}_{3}[\mathbb{Q}_{2}^{(a,A_{g},1)}] \otimes \mathbb{Y}_{3}[\mathbb{Q}_{0}^{(b,A_{g})}]$$

$$\hat{\mathbb{Z}}_{17}(\boldsymbol{k}) = \mathbb{X}_{3}[\mathbb{Q}_{2}^{(a,A_{g},1)}] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{2}[\mathbb{Q}_{0}^{(k,A_{g})}]$$

No. 18
$$\hat{\mathbb{Q}}_{2}^{(A_g,2)}$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{18} = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{18}(\mathbf{k}) = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_2[\mathbb{Q}_0^{(k,A_g)}]$$

No. 19
$$\hat{\mathbb{Q}}_0^{(A_g)}(1,1)$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{19} = \mathbb{X}_{5}[\mathbb{Q}_{0}^{(a,A_{g})}(1,1)] \otimes \mathbb{Y}_{3}[\mathbb{Q}_{0}^{(b,A_{g})}]$$

$$\hat{\mathbb{Z}}_{19}(\boldsymbol{k}) = \mathbb{X}_{5}[\mathbb{Q}_{0}^{(a,A_g)}(1,1)] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_g)}] \otimes \mathbb{F}_{2}[\mathbb{Q}_{0}^{(k,A_g)}]$$

No. 20
$$\hat{\mathbb{Q}}_2^{(A_g,1)}(1,-1)$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{20} = \mathbb{X}_{6}[\mathbb{Q}_{2}^{(a,A_{g},1)}(1,-1)] \otimes \mathbb{Y}_{3}[\mathbb{Q}_{0}^{(b,A_{g})}]$$

$$\hat{\mathbb{Z}}_{20}(\textbf{\textit{k}}) = \mathbb{X}_{6}[\mathbb{Q}_{2}^{(a,A_{g},1)}(1,-1)] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{2}[\mathbb{Q}_{0}^{(k,A_{g})}]$$

No. 21
$$\hat{\mathbb{Q}}_2^{(A_g,2)}(1,-1)$$
 [M₃, B₂]

$$\hat{\mathbb{Z}}_{21} = \mathbb{X}_7[\mathbb{Q}_2^{(a,A_g,2)}(1,-1)] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{21}(\pmb{k}) = \mathbb{X}_{7}[\mathbb{Q}_{2}^{(a,A_{g},2)}(1,-1)] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{2}[\mathbb{Q}_{0}^{(k,A_{g})}]$$

No. 22
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₁, B₃]

$$\hat{\mathbb{Z}}_{22} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{22}(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k,A_g)}]$$

No. 23
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₃, B₃]

$$\hat{\mathbb{Z}}_{23} = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{23}(\boldsymbol{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k,A_g)}]$$

No. 24
$$\hat{\mathbb{Q}}_2^{(A_g,1)}$$
 [M₃, B₃]

$$\hat{\mathbb{Z}}_{24} = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_g,1)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{24}(\boldsymbol{k}) = \mathbb{X}_{3}[\mathbb{Q}_{2}^{(a,A_{g},1)}] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{3}[\mathbb{Q}_{0}^{(k,A_{g})}]$$

No. 25
$$\hat{\mathbb{Q}}_2^{(A_g,2)}$$
 [M₃, B₃]

$$\hat{\mathbb{Z}}_{25} = \mathbb{X}_4[\mathbb{Q}_2^{(a, A_g, 2)}] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{\mathbb{Z}}_{25}(\boldsymbol{k}) = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_3[\mathbb{Q}_0^{(k,A_g)}]$$

No. 26
$$\hat{\mathbb{Q}}_0^{(A_g)}(1,1)$$
 [M₃, B₃]

$$\hat{\mathbb{Z}}_{26} = \mathbb{X}_{5}[\mathbb{Q}_{0}^{(a,A_g)}(1,1)] \otimes \mathbb{Y}_{4}[\mathbb{Q}_{0}^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{26}(\boldsymbol{k}) = \mathbb{X}_{5}[\mathbb{Q}_{0}^{(a,A_g)}(1,1)] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_g)}] \otimes \mathbb{F}_{3}[\mathbb{Q}_{0}^{(k,A_g)}]$$

No. 27
$$\hat{\mathbb{Q}}_2^{(A_g,1)}(1,-1)$$
 [M₃, B₃]

$$\hat{\mathbb{Z}}_{27} = \mathbb{X}_{6}[\mathbb{Q}_{2}^{(a,A_{g},1)}(1,-1)] \otimes \mathbb{Y}_{4}[\mathbb{Q}_{0}^{(b,A_{g})}]$$

$$\hat{\mathbb{Z}}_{27}(\pmb{k}) = \mathbb{X}_{6}[\mathbb{Q}_{2}^{(a,A_g,1)}(1,-1)] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_g)}] \otimes \mathbb{F}_{3}[\mathbb{Q}_{0}^{(k,A_g)}]$$

No. 28
$$\hat{\mathbb{Q}}_2^{(A_g,2)}(1,-1)$$
 [M₃, B₃]

$$\hat{\mathbb{Z}}_{28} = \mathbb{X}_7[\mathbb{Q}_2^{(a,A_g,2)}(1,-1)] \otimes \mathbb{Y}_4[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{28}(\textbf{\textit{k}}) = \mathbb{X}_{7}[\mathbb{Q}_{2}^{(a,A_{g},2)}(1,-1)] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{3}[\mathbb{Q}_{0}^{(k,A_{g})}]$$

No. 29
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₁, B₄]

$$\hat{\mathbb{Z}}_{29} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{29}(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_4[\mathbb{Q}_0^{(k,A_g)}]$$

No. 30
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₃, B₄]

$$\hat{\mathbb{Z}}_{30} = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{30}(\boldsymbol{k}) = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_g)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_4[\mathbb{Q}_0^{(k,A_g)}]$$

No. 31
$$\hat{\mathbb{Q}}_{2}^{(A_g,1)}$$
 [M₃, B₄]

$$\hat{\mathbb{Z}}_{31} = \mathbb{X}_3[\mathbb{Q}_2^{(a,A_g,1)}] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{31}(\boldsymbol{k}) = \mathbb{X}_{3}[\mathbb{Q}_{2}^{(a,A_{g},1)}] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{4}[\mathbb{Q}_{0}^{(k,A_{g})}]$$

No. 32
$$\hat{\mathbb{Q}}_2^{(A_g,2)}$$
 [M₃, B₄]

$$\hat{\mathbb{Z}}_{32} = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{32}(\boldsymbol{k}) = \mathbb{X}_4[\mathbb{Q}_2^{(a,A_g,2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_4[\mathbb{Q}_0^{(k,A_g)}]$$

No. 33
$$\hat{\mathbb{Q}}_0^{(A_g)}$$
 [M₃, B₄]

$$\hat{\mathbb{Z}}_{33} = \mathbb{X}_8[\mathbb{Q}_2^{(a,B_{1g})}] \otimes \mathbb{Y}_6[\mathbb{Q}_2^{(b,B_{1g})}]$$

$$\hat{\mathbb{Z}}_{33}(\boldsymbol{k}) = \mathbb{X}_{8}[\mathbb{Q}_{2}^{(a,B_{1g})}] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{5}[\mathbb{Q}_{2}^{(k,B_{1g})}]$$

No. 34
$$\hat{\mathbb{Q}}_0^{(A_g)}(1,1)$$
 [M₃, B₄]

$$\hat{\mathbb{Z}}_{34} = \mathbb{X}_5[\mathbb{Q}_0^{(a,A_g)}(1,1)] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{34}(\boldsymbol{k}) = \mathbb{X}_{5}[\mathbb{Q}_{0}^{(a,A_g)}(1,1)] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_g)}] \otimes \mathbb{F}_{4}[\mathbb{Q}_{0}^{(k,A_g)}]$$

No. 35
$$\hat{\mathbb{Q}}_2^{(A_g,1)}(1,-1)$$
 [M₃, B₄]

$$\hat{\mathbb{Z}}_{35} = \mathbb{X}_6[\mathbb{Q}_2^{(a, A_g, 1)}(1, -1)] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b, A_g)}]$$

$$\hat{\mathbb{Z}}_{35}(\mathbf{k}) = \mathbb{X}_{6}[\mathbb{Q}_{2}^{(a,A_{g},1)}(1,-1)] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{4}[\mathbb{Q}_{0}^{(k,A_{g})}]$$

No. 36
$$\hat{\mathbb{Q}}_2^{(A_g,2)}(1,-1)$$
 [M₃, B₄]

$$\hat{\mathbb{Z}}_{36} = \mathbb{X}_7[\mathbb{Q}_2^{(a,A_g,2)}(1,-1)] \otimes \mathbb{Y}_5[\mathbb{Q}_0^{(b,A_g)}]$$

$$\hat{\mathbb{Z}}_{36}(\pmb{k}) = \mathbb{X}_{7}[\mathbb{Q}_{2}^{(a,A_{g},2)}(1,-1)] \otimes \mathbb{U}_{1}[\mathbb{Q}_{0}^{(s,A_{g})}] \otimes \mathbb{F}_{4}[\mathbb{Q}_{0}^{(k,A_{g})}]$$

No. 37
$$\hat{\mathbb{Q}}_0^{(A_g)}(1,-1)$$
 [M₃, B₄]

$$\hat{\mathbb{Z}}_{37} = \mathbb{X}_9[\mathbb{Q}_2^{(a,B_{1g})}(1,-1)] \otimes \mathbb{Y}_6[\mathbb{Q}_2^{(b,B_{1g})}]$$

$$\hat{\mathbb{Z}}_{37}(\mathbf{k}) = \mathbb{X}_9[\mathbb{Q}_2^{(a,B_{1g})}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_5[\mathbb{Q}_2^{(k,B_{1g})}]$$

No. 38
$$\hat{\mathbb{Q}}_2^{(A_g,2)}(1,0)$$
 [M₃, B₄]

$$\hat{\mathbb{Z}}_{38} = -\mathbb{X}_{10}[\mathbb{G}_{1}^{(a,B_{1g})}(1,0)] \otimes \mathbb{Y}_{6}[\mathbb{Q}_{2}^{(b,B_{1g})}]$$

$$\hat{\mathbb{Z}}_{38}(\pmb{k}) = -\mathbb{X}_{10}[\mathbb{G}_1^{(a,B_{1g})}(1,0)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_g)}] \otimes \mathbb{F}_5[\mathbb{Q}_2^{(k,B_{1g})}]$$

Table 5: Atomic SAMB group.

group	bra	ket
M_1	$(s,\uparrow),(s,\downarrow)$	$(s,\uparrow),(s,\downarrow)$
M_2	$(s,\uparrow),(s,\downarrow)$	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$
M_3	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$

Table 6: Atomic SAMB.

symbol	type	group	form
\mathbb{X}_1	$\mathbb{Q}_0^{(a,A_g)}$	M_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0\\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{X}_2	$\mathbb{Q}_0^{(a,A_g)}$	M ₃	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \end{pmatrix}$

 $continued\ \dots$

Table 6

symbol	type	group	form
X 3	$\mathbb{Q}_2^{(a,A_g,1)}$	М3	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} \end{pmatrix}$
\mathbb{X}_4	$\mathbb{Q}_2^{(a,A_g,2)}$	$ m M_3$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 &$
\mathbb{X}_5	$\mathbb{Q}_0^{(a,A_g)}(1,1)$	$ m M_3$	$ \begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ \frac{\sqrt{3}i}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix} $
\mathbb{X}_6	$\mathbb{Q}_{2}^{(a,A_{g},1)}(1,-1)$	$ m M_3$	$\begin{bmatrix} 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{12} & 0\\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12}\\ 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} & 0\\ 0 & \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0\\ -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{bmatrix}$
\mathbb{X}_7	$\mathbb{Q}_{2}^{(a,A_{g},2)}(1,-1)$	$ m M_3$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \end{pmatrix}$

 $continued\ \dots$

Table 6

symbol	type	group	form
\mathbb{X}_8	$\mathbb{Q}_2^{(a,B_{1g})}$	$ m M_3$	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0$
\mathbb{X}_9	$\mathbb{Q}_2^{(a,B_{1g})}(1,-1)$	$ m M_3$	$ \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix} $
\mathbb{X}_{10}	$\mathbb{G}_{1}^{(a,B_{1g})}(1,0)$	$ m M_3$	$ \begin{pmatrix} -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix} $

Table 7: Cluster SAMB.

symbol	type	cluster	form
\mathbb{Y}_1	$\mathbb{Q}_0^{(s,A_g)}$	S_1	(1)
\mathbb{Y}_2	$\mathbb{Q}_0^{(b,A_g)}$	B_1	(1)
\mathbb{Y}_3	$\mathbb{Q}_0^{(b,A_g)}$	B_2	(1)
\mathbb{Y}_4	$\mathbb{Q}_0^{(b,A_g)}$	B_3	(1)
\mathbb{Y}_5	$\mathbb{Q}_0^{(b,A_g)}$	B_4	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{Y}_6	$\mathbb{Q}_0^{(b,A_g)}$ $\mathbb{Q}_2^{(b,B_{1g})}$	$_{ m B_4}$	$\begin{pmatrix} \sqrt{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}$

Table 8: Uniform SAMB.

symbol	type	cluster	form
\mathbb{U}_1	$\mathbb{Q}_0^{(s,A_g)}$	S_1	(1)

Table 9: Structure SAMB.

symbol	type	cluster	form
\mathbb{F}_1	$\mathbb{Q}_0^{(k,A_g)}$	B_1	$\sqrt{2}c_{001}$
\mathbb{F}_2	$\mathbb{Q}_0^{(k,A_g)}$	B_2	$\sqrt{2}c_{002}$
\mathbb{F}_3	$\mathbb{Q}_0^{(k,A_g)}$	B_3	$\sqrt{2}c_{003}$
\mathbb{F}_4	$\mathbb{Q}_0^{(k,A_g)}$	B_4	$c_{004} + c_{005}$
\mathbb{F}_5	$\mathbb{Q}_2^{(k,B_{1g})}$	B_4	$c_{004} - c_{005}$

Table 10: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_0^{(A_g)}$	0	A_g	_	_	1
2	$\mathbb{Q}_2^{(A_g,1)}$	2	A_g	1	_	$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$
3	$\mathbb{Q}_2^{(A_g,2)}$	2	A_g	2	_	$\frac{2}{\sqrt{3}(x-y)(x+y)}$
4	$\mathbb{Q}_2^{(B_{1g})}$	2	B_{1g}	_	_	$\sqrt{3}xy$

Table 11: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_1^{(B_{1g})}$	1	B_{1g}	_	_	Z

• Group info.: Generator = $\{2_{001}|0\}$, $\{2_{010}|0\}$, $\{-1|0\}$

Table 12: Conjugacy class (point-group part).

rep. SO	symmetry operations
{1 0}	{1 0}
$\{2_{001} 0\}$	$\{2_{001} 0\}$
$\{2_{010} 0\}$	$\{2_{010} 0\}$
$\{2_{100} 0\}$	$\{2_{100} 0\}$
$\{-1 0\}$	{-1 0}
${\{m_{001} 0\}}$	$\{m_{001} 0\}$
${\{m_{010} 0\}}$	$\{m_{010} 0\}$
$\{m_{100} 0\}$	$\{m_{100} 0\}$

Table 13: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	{1 0}	2	$\{2_{001} 0\}$	3	$\{2_{010} 0\}$	4	$\{2_{100} 0\}$	5	$\{-1 0\}$
6	$\{m_{001} 0\}$	7	$\{m_{010} 0\}$	8	$\{m_{100} 0\}$				

Table 14: Character table (point-group part).

	1	2001	2010	2100	-1	m ₀₀₁	m ₀₁₀	m ₁₀₀
$\overline{A_g}$	1	1	1	1	1	1	1	1
B_{1g}	1	1	-1	-1	1	1	-1	-1
B_{2g}	1	-1	1	-1	1	-1	1	-1
B_{3g}	1	-1	-1	1	1	-1	-1	1
A_u	1	1	1	1	-1	-1	-1	-1
B_{1u}	1	1	-1	-1	-1	-1	1	1
B_{2u}	1	-1	1	-1	-1	1	-1	1
B_{3u}	1	-1	-1	1	-1	1	1	-1

Table 15: Parity conversion.

\leftrightarrow	\leftrightarrow	\leftrightarrow	\leftrightarrow	\leftrightarrow
$A_g (A_u)$	$B_{3g} (B_{3u})$	$B_{2g} (B_{2u})$	B_{1g} (B_{1u})	$A_u (A_g)$
$B_{3u} (B_{3g})$	B_{2u} (B_{2g})	$B_{1u} (B_{1g})$		

Table 16: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

	A_g	B_{1g}	B_{2g}	B_{3g}	A_u	B_{1u}	B_{2u}	B_{3u}
A_g	A_g	B_{1g}	B_{2g}	B_{3g}	A_u	B_{1u}	B_{2u}	B_{3u}
B_{1g}		A_g	B_{3g}	B_{2g}	B_{1u}	A_u	B_{3u}	B_{2u}
B_{2g}			A_g	B_{1g}	B_{2u}	B_{3u}	A_u	B_{1u}
B_{3g}				A_g	B_{3u}	B_{2u}	B_{1u}	A_u
A_u					A_g	B_{1g}	B_{2g}	B_{3g}
B_{1u}						A_g	B_{3g}	B_{2g}
B_{2u}							A_g	B_{1g}
B_{3u}								A_g

Table 17: Anti-symmetric product, $[\Gamma \otimes \Gamma]_{-}$.

A_g	B_{1g}	B_{2g}	B_{3g}	A_u	B_{1u}	B_{2u}	B_{3u}
_	_	_	_	_	_	_	_

Table 18: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$	2	$\begin{pmatrix} -1 & -1 & 1 \end{pmatrix}$	3	$\begin{pmatrix} -1 & 1 & -1 \end{pmatrix}$	4	$\begin{pmatrix} 1 & -1 & -1 \end{pmatrix}$
5	$\begin{pmatrix} -1 & -1 & -1 \end{pmatrix}$	6	$\begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$	7	$\begin{pmatrix} 1 & -1 & 1 \end{pmatrix}$	8	$\begin{pmatrix} -1 & 1 & 1 \end{pmatrix}$

Table 19: Virtual-cluster basis.

symbol	1	2	3	4	5	6	7	8
$\mathbb{Q}_0^{(A_g)}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$
$\mathbb{Q}_1^{(B_{1u})}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$
$\mathbb{Q}_1^{(B_{2u})}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$
$\mathbb{Q}_1^{(B_{3u})}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$
$\mathbb{Q}_2^{(B_{1g})}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$
$\mathbb{Q}_2^{(B_{2g})}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$
$\mathbb{Q}_2^{(B_{3g})}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$
$\mathbb{Q}_3^{(A_u)}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$