

Response Tensors up to 4th rank in $D_{2d} - 1$

— polar tensors —

$$C^{(0,Q)} = (C^{(0,Q)})$$

$$C^{(0,Q)} = Q_0$$

$$S^{(2,Q)} = \begin{pmatrix} S_{xx}^{(2,Q)} & 0 & 0 \\ 0 & S_{xx}^{(2,Q)} & 0 \\ 0 & 0 & S_{zz}^{(2,Q)} \end{pmatrix}$$

$$S_{xx}^{(2,Q)} = Q_0 - Q_u$$

$$S_{zz}^{(2,Q)} = Q_0 + 2Q_u$$

$$S^{(3,Q)} = \begin{pmatrix} 0 & 0 & S_{1z}^{(3,Q)} \\ 0 & 0 & -S_{1z}^{(3,Q)} \\ 0 & 0 & 0 \\ 0 & S_{4y}^{(3,Q)} & 0 \\ -S_{4y}^{(3,Q)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_{1z}^{(3,Q)} = -2G_{xy}[1] + Q_z^\beta$$

$$S_{4y}^{(3,Q)} = -G_{xy}[1] - Q_z^\beta$$

$$A^{(3,Q)} = \begin{pmatrix} 0 & A_{4y}^{(3,Q)} & 0 \\ A_{4y}^{(3,Q)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{4y}^{(3,Q)} = G_{xy}[2]$$

$$S^{(4,Q)} = \begin{pmatrix} S_{11}^{(4,Q)} & S_{12}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & 0 & 0 \\ S_{12}^{(4,Q)} & S_{11}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & 0 & 0 \\ S_{13}^{(4,Q)} & S_{13}^{(4,Q)} & S_{33}^{(4,Q)} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66}^{(4,Q)} \end{pmatrix}$$

$$S_{11}^{(4,Q)} = Q_0[1] + 2Q_0[2] - Q_{4u} + 2Q_4 - 2Q_u[1] - 4Q_u[2]$$

$$S_{12}^{(4,Q)} = Q_0[1] + 2Q_{4u} - Q_4 - 2Q_u[1]$$

$$S_{13}^{(4,Q)} = Q_0[1] - Q_{4u} - Q_4 + Q_u[1]$$

$$S_{33}^{(4,Q)} = Q_0[1] + 2Q_0[2] + 2Q_{4u} + 2Q_4 + 4Q_u[1] + 8Q_u[2]$$

$$S_{44}^{(4,Q)} = Q_0[2] - Q_{4u} - Q_4 + Q_u[2]$$

$$S_{66}^{(4,Q)} = Q_0[2] + 2Q_{4u} - Q_4 - 2Q_u[2]$$

$$\bar{S}^{(4,Q)} = \begin{pmatrix} 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 \\ 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 \\ -\bar{S}_{13}^{(4,Q)} & -\bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{S}_{13}^{(4,Q)} = 3Q_u[3]$$

$$A^{(4,Q)} = \begin{pmatrix} A_{xx}^{(4,Q)} & 0 & 0 \\ 0 & A_{xx}^{(4,Q)} & 0 \\ 0 & 0 & A_{zz}^{(4,Q)} \end{pmatrix}$$

$$A_{xx}^{(4,Q)} = Q_0[3] - 2Q_u[6]$$

$$A_{zz}^{(4,Q)} = Q_0[3] + 4Q_u[6]$$

$$M^{(4,Q)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ M_{4x}^{(4,Q)} & 0 & 0 \\ 0 & -M_{4x}^{(4,Q)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$M_{4x}^{(4,Q)} = -3Q_u[4]$$

$$\bar{M}^{(4,Q)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}_{x4}^{(4,Q)} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\bar{M}_{x4}^{(4,Q)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{M}_{x4}^{(4,Q)} = -3Q_u[5]$$

— axial tensors —

$$S^{(2,G)} = \begin{pmatrix} 0 & S_{xy}^{(2,G)} & 0 \\ S_{xy}^{(2,G)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_{xy}^{(2,G)} = G_{xy}$$

$$S^{(3,G)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ S_{4x}^{(3,G)} & 0 & 0 \\ 0 & -S_{4x}^{(3,G)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_{4x}^{(3,G)} = -3Q_u[1]$$

$$A^{(3,G)} = \begin{pmatrix} A_{4x}^{(3,G)} & 0 & 0 \\ 0 & A_{4x}^{(3,G)} & 0 \\ 0 & 0 & A_{6z}^{(3,G)} \end{pmatrix}$$

$$A_{4x}^{(3,G)} = Q_0 - Q_u[2]$$

$$A_{6z}^{(3,G)} = Q_0 + 2Q_u[2]$$

$$S^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & S_{16}^{(4,G)} \\ 0 & 0 & 0 & 0 & 0 & S_{16}^{(4,G)} \\ 0 & 0 & 0 & 0 & 0 & S_{36}^{(4,G)} \\ 0 & 0 & 0 & 0 & S_{45}^{(4,G)} & 0 \\ 0 & 0 & 0 & S_{45}^{(4,G)} & 0 & 0 \\ S_{16}^{(4,G)} & S_{16}^{(4,G)} & S_{36}^{(4,G)} & 0 & 0 & 0 \end{pmatrix}$$

$$S_{16}^{(4,G)} = -G_{4z}^\beta + G_{xy}[1] + 2G_{xy}[2]$$

$$S_{36}^{(4,G)} = 2G_{4z}^\beta + G_{xy}[1]$$

$$S_{45}^{(4,G)} = 2G_{4z}^\beta + G_{xy}[2]$$

$$\bar{S}^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \bar{S}_{16}^{(4,G)} \\ 0 & 0 & 0 & 0 & 0 & \bar{S}_{16}^{(4,G)} \\ 0 & 0 & 0 & 0 & 0 & \bar{S}_{36}^{(4,G)} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\bar{S}_{16}^{(4,G)} & -\bar{S}_{16}^{(4,G)} & -\bar{S}_{36}^{(4,G)} & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{S}_{16}^{(4,G)} = G_{xy}[3] + 2Q_z^\beta[1]$$

$$\bar{S}_{36}^{(4,G)} = G_{xy}[3] - 4Q_z^\beta[1]$$

$$A^{(4,G)} = \begin{pmatrix} 0 & A_{xy}^{(4,G)} & 0 \\ A_{xy}^{(4,G)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{xy}^{(4,G)} = 2G_{xy}[6]$$

$$M^{(4,G)} = \begin{pmatrix} 0 & 0 & M_{1z}^{(4,G)} \\ 0 & 0 & -M_{1z}^{(4,G)} \\ 0 & 0 & 0 \\ 0 & M_{4y}^{(4,G)} & 0 \\ -M_{4y}^{(4,G)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$M_{1z}^{(4,G)} = -2G_{xy}[4] + Q_z^\beta[2]$$

$$M_{4y}^{(4,G)} = -G_{xy}[4] - Q_z^\beta[2]$$

$$\bar{M}^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & 0 & \bar{M}_{x5}^{(4,G)} & 0 \\ 0 & 0 & 0 & -\bar{M}_{x5}^{(4,G)} & 0 & 0 \\ \bar{M}_{z1}^{(4,G)} & -\bar{M}_{z1}^{(4,G)} & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{M}_{x5}^{(4,G)} = G_{xy}[5] + Q_z^\beta[3]$$

$$\bar{M}_{z1}^{(4,G)} = -2G_{xy}[5] + Q_z^\beta[3]$$