

No. 20 D_{3d} $-31m$ ($-31m$ setting) [trigonal] (polar)

表 1 rank 0

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
1	A_{1g}	$A1g$	—	—	$Q_0^{(h,A_{1g})}$	$Qh(0, A1g, ,)$	C_0

表 2 rank 1

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
2	A_{2u}	$A2u$	—	—	$Q_1^{(h,A_{2u})}$	$Qh(1, A2u, ,)$	C_0
3	E_u	Eu	—	0	$Q_{1,0}^{(h,E_u)}$	$Qh(1, Eu, , 0)$	C_1
4	E_u	Eu	—	1	$Q_{1,1}^{(h,E_u)}$	$Qh(1, Eu, , 1)$	S_1

表 3 rank 2

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
5	A_{1g}	$A1g$	—	—	$Q_2^{(h,A_{1g})}$	$Qh(2, A1g, ,)$	C_0
6	E_g	Eg	1	0	$Q_{2,0}^{(h,E_g,1)}$	$Qh(2, Eg, 1, 0)$	C_1
7	E_g	Eg	1	1	$Q_{2,1}^{(h,E_g,1)}$	$Qh(2, Eg, 1, 1)$	S_1
8	E_g	Eg	2	0	$Q_{2,0}^{(h,E_g,2)}$	$Qh(2, Eg, 2, 0)$	C_2
9	E_g	Eg	2	1	$Q_{2,1}^{(h,E_g,2)}$	$Qh(2, Eg, 2, 1)$	$-S_2$

表 4 rank 3

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
10	A_{1u}	$A1u$	—	—	$Q_3^{(h,A_{1u})}$	$Qh(3, A1u, ,)$	S_3
11	A_{2u}	$A2u$	1	—	$Q_3^{(h,A_{2u},1)}$	$Qh(3, A2u, 1,)$	C_0
12	A_{2u}	$A2u$	2	—	$Q_3^{(h,A_{2u},2)}$	$Qh(3, A2u, 2,)$	C_3
13	E_u	Eu	1	0	$Q_{3,0}^{(h,E_u,1)}$	$Qh(3, Eu, 1, 0)$	C_1
14	E_u	Eu	1	1	$Q_{3,1}^{(h,E_u,1)}$	$Qh(3, Eu, 1, 1)$	S_1
15	E_u	Eu	2	0	$Q_{3,0}^{(h,E_u,2)}$	$Qh(3, Eu, 2, 0)$	C_2
16	E_u	Eu	2	1	$Q_{3,1}^{(h,E_u,2)}$	$Qh(3, Eu, 2, 1)$	$-S_2$

表 5 rank 4

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
17	A_{1g}	$A1g$	1	—	$Q_4^{(h,A_{1g},1)}$	$Qh(4, A1g, 1,)$	C_0
18	A_{1g}	$A1g$	2	—	$Q_4^{(h,A_{1g},2)}$	$Qh(4, A1g, 2,)$	C_3
19	A_{2g}	$A2g$	—	—	$Q_4^{(h,A_{2g})}$	$Qh(4, A2g, ,)$	S_3
20	E_g	Eg	1	0	$Q_{4,0}^{(h,E_g,1)}$	$Qh(4, Eg, 1, 0)$	C_1
21	E_g	Eg	1	1	$Q_{4,1}^{(h,E_g,1)}$	$Qh(4, Eg, 1, 1)$	S_1
22	E_g	Eg	2	0	$Q_{4,0}^{(h,E_g,2)}$	$Qh(4, Eg, 2, 0)$	C_4
23	E_g	Eg	2	1	$Q_{4,1}^{(h,E_g,2)}$	$Qh(4, Eg, 2, 1)$	S_4
24	E_g	Eg	3	0	$Q_{4,0}^{(h,E_g,3)}$	$Qh(4, Eg, 3, 0)$	C_2
25	E_g	Eg	3	1	$Q_{4,1}^{(h,E_g,3)}$	$Qh(4, Eg, 3, 1)$	$-S_2$

表 6 rank 5

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
26	A_{1u}	A1u	—	—	$\mathbb{Q}_5^{(h,A_{1u})}$	$\text{Qh}(5, \text{A1u},)$	S_3
27	A_{2u}	A2u	1	—	$\mathbb{Q}_5^{(h,A_{2u},1)}$	$\text{Qh}(5, \text{A2u}, 1,)$	C_0
28	A_{2u}	A2u	2	—	$\mathbb{Q}_5^{(h,A_{2u},2)}$	$\text{Qh}(5, \text{A2u}, 2,)$	C_3
29	E_u	Eu	1	0	$\mathbb{Q}_{5,0}^{(h,E_u,1)}$	$\text{Qh}(5, \text{Eu}, 1, 0)$	C_5
30	E_u	Eu	1	1	$\mathbb{Q}_{5,1}^{(h,E_u,1)}$	$\text{Qh}(5, \text{Eu}, 1, 1)$	$-S_5$
31	E_u	Eu	2	0	$\mathbb{Q}_{5,0}^{(h,E_u,2)}$	$\text{Qh}(5, \text{Eu}, 2, 0)$	C_1
32	E_u	Eu	2	1	$\mathbb{Q}_{5,1}^{(h,E_u,2)}$	$\text{Qh}(5, \text{Eu}, 2, 1)$	S_1
33	E_u	Eu	3	0	$\mathbb{Q}_{5,0}^{(h,E_u,3)}$	$\text{Qh}(5, \text{Eu}, 3, 0)$	C_4
34	E_u	Eu	3	1	$\mathbb{Q}_{5,1}^{(h,E_u,3)}$	$\text{Qh}(5, \text{Eu}, 3, 1)$	S_4
35	E_u	Eu	4	0	$\mathbb{Q}_{5,0}^{(h,E_u,4)}$	$\text{Qh}(5, \text{Eu}, 4, 0)$	C_2
36	E_u	Eu	4	1	$\mathbb{Q}_{5,1}^{(h,E_u,4)}$	$\text{Qh}(5, \text{Eu}, 4, 1)$	$-S_2$

表 7 rank 6

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
37	A_{1g}	A1g	1	—	$\mathbb{Q}_6^{(h,A_{1g},1)}$	$\text{Qh}(6, \text{A1g}, 1,)$	C_0
38	A_{1g}	A1g	2	—	$\mathbb{Q}_6^{(h,A_{1g},2)}$	$\text{Qh}(6, \text{A1g}, 2,)$	C_6
39	A_{1g}	A1g	3	—	$\mathbb{Q}_6^{(h,A_{1g},3)}$	$\text{Qh}(6, \text{A1g}, 3,)$	C_3
40	A_{2g}	A2g	1	—	$\mathbb{Q}_6^{(h,A_{2g},1)}$	$\text{Qh}(6, \text{A2g}, 1,)$	S_6
41	A_{2g}	A2g	2	—	$\mathbb{Q}_6^{(h,A_{2g},2)}$	$\text{Qh}(6, \text{A2g}, 2,)$	S_3
42	E_g	Eg	1	0	$\mathbb{Q}_{6,0}^{(h,E_g,1)}$	$\text{Qh}(6, \text{Eg}, 1, 0)$	C_5
43	E_g	Eg	1	1	$\mathbb{Q}_{6,1}^{(h,E_g,1)}$	$\text{Qh}(6, \text{Eg}, 1, 1)$	$-S_5$
44	E_g	Eg	2	0	$\mathbb{Q}_{6,0}^{(h,E_g,2)}$	$\text{Qh}(6, \text{Eg}, 2, 0)$	C_1
45	E_g	Eg	2	1	$\mathbb{Q}_{6,1}^{(h,E_g,2)}$	$\text{Qh}(6, \text{Eg}, 2, 1)$	S_1
46	E_g	Eg	3	0	$\mathbb{Q}_{6,0}^{(h,E_g,3)}$	$\text{Qh}(6, \text{Eg}, 3, 0)$	C_4
47	E_g	Eg	3	1	$\mathbb{Q}_{6,1}^{(h,E_g,3)}$	$\text{Qh}(6, \text{Eg}, 3, 1)$	S_4
48	E_g	Eg	4	0	$\mathbb{Q}_{6,0}^{(h,E_g,4)}$	$\text{Qh}(6, \text{Eg}, 4, 0)$	C_2
49	E_g	Eg	4	1	$\mathbb{Q}_{6,1}^{(h,E_g,4)}$	$\text{Qh}(6, \text{Eg}, 4, 1)$	$-S_2$

表 8 rank 7

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
50	A_{1u}	A1u	1	—	$Q_7^{(h,A_{1u},1)}$	$Qh(7, A1u, 1,)$	S_6
51	A_{1u}	A1u	2	—	$Q_7^{(h,A_{1u},2)}$	$Qh(7, A1u, 2,)$	S_3
52	A_{2u}	A2u	1	—	$Q_7^{(h,A_{2u},1)}$	$Qh(7, A2u, 1,)$	C_0
53	A_{2u}	A2u	2	—	$Q_7^{(h,A_{2u},2)}$	$Qh(7, A2u, 2,)$	C_6
54	A_{2u}	A2u	3	—	$Q_7^{(h,A_{2u},3)}$	$Qh(7, A2u, 3,)$	C_3
55	E_u	Eu	1	0	$Q_{7,0}^{(h,E_u,1)}$	$Qh(7, Eu, 1, 0)$	C_7
56	E_u	Eu	1	1	$Q_{7,1}^{(h,E_u,1)}$	$Qh(7, Eu, 1, 1)$	S_7
57	E_u	Eu	2	0	$Q_{7,0}^{(h,E_u,2)}$	$Qh(7, Eu, 2, 0)$	C_5
58	E_u	Eu	2	1	$Q_{7,1}^{(h,E_u,2)}$	$Qh(7, Eu, 2, 1)$	$-S_5$
59	E_u	Eu	3	0	$Q_{7,0}^{(h,E_u,3)}$	$Qh(7, Eu, 3, 0)$	C_1
60	E_u	Eu	3	1	$Q_{7,1}^{(h,E_u,3)}$	$Qh(7, Eu, 3, 1)$	S_1
61	E_u	Eu	4	0	$Q_{7,0}^{(h,E_u,4)}$	$Qh(7, Eu, 4, 0)$	C_4
62	E_u	Eu	4	1	$Q_{7,1}^{(h,E_u,4)}$	$Qh(7, Eu, 4, 1)$	S_4
63	E_u	Eu	5	0	$Q_{7,0}^{(h,E_u,5)}$	$Qh(7, Eu, 5, 0)$	C_2
64	E_u	Eu	5	1	$Q_{7,1}^{(h,E_u,5)}$	$Qh(7, Eu, 5, 1)$	$-S_2$

表 9 rank 8

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
65	A_{1g}	A1g	1	—	$Q_8^{(h,A_{1g},1)}$	$Qh(8, A1g, 1,)$	C_0
66	A_{1g}	A1g	2	—	$Q_8^{(h,A_{1g},2)}$	$Qh(8, A1g, 2,)$	C_6
67	A_{1g}	A1g	3	—	$Q_8^{(h,A_{1g},3)}$	$Qh(8, A1g, 3,)$	C_3
68	A_{2g}	A2g	1	—	$Q_8^{(h,A_{2g},1)}$	$Qh(8, A2g, 1,)$	S_6
69	A_{2g}	A2g	2	—	$Q_8^{(h,A_{2g},2)}$	$Qh(8, A2g, 2,)$	S_3
70	E_g	Eg	1	0	$Q_{8,0}^{(h,E_g,1)}$	$Qh(8, Eg, 1, 0)$	C_7
71	E_g	Eg	1	1	$Q_{8,1}^{(h,E_g,1)}$	$Qh(8, Eg, 1, 1)$	S_7
72	E_g	Eg	2	0	$Q_{8,0}^{(h,E_g,2)}$	$Qh(8, Eg, 2, 0)$	C_5
73	E_g	Eg	2	1	$Q_{8,1}^{(h,E_g,2)}$	$Qh(8, Eg, 2, 1)$	$-S_5$
74	E_g	Eg	3	0	$Q_{8,0}^{(h,E_g,3)}$	$Qh(8, Eg, 3, 0)$	C_1
75	E_g	Eg	3	1	$Q_{8,1}^{(h,E_g,3)}$	$Qh(8, Eg, 3, 1)$	S_1
76	E_g	Eg	4	0	$Q_{8,0}^{(h,E_g,4)}$	$Qh(8, Eg, 4, 0)$	C_8
77	E_g	Eg	4	1	$Q_{8,1}^{(h,E_g,4)}$	$Qh(8, Eg, 4, 1)$	$-S_8$
78	E_g	Eg	5	0	$Q_{8,0}^{(h,E_g,5)}$	$Qh(8, Eg, 5, 0)$	C_4
79	E_g	Eg	5	1	$Q_{8,1}^{(h,E_g,5)}$	$Qh(8, Eg, 5, 1)$	S_4
80	E_g	Eg	6	0	$Q_{8,0}^{(h,E_g,6)}$	$Qh(8, Eg, 6, 0)$	C_2
81	E_g	Eg	6	1	$Q_{8,1}^{(h,E_g,6)}$	$Qh(8, Eg, 6, 1)$	$-S_2$

表 10 rank 9

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
82	A_{1u}	A1u	1	—	$\mathbb{Q}_9^{(h,A_{1u},1)}$	Qh(9, A1u, 1,)	S_6
83	A_{1u}	A1u	2	—	$\mathbb{Q}_9^{(h,A_{1u},2)}$	Qh(9, A1u, 2,)	S_9
84	A_{1u}	A1u	3	—	$\mathbb{Q}_9^{(h,A_{1u},3)}$	Qh(9, A1u, 3,)	S_3
85	A_{2u}	A2u	1	—	$\mathbb{Q}_9^{(h,A_{2u},1)}$	Qh(9, A2u, 1,)	C_0
86	A_{2u}	A2u	2	—	$\mathbb{Q}_9^{(h,A_{2u},2)}$	Qh(9, A2u, 2,)	C_6
87	A_{2u}	A2u	3	—	$\mathbb{Q}_9^{(h,A_{2u},3)}$	Qh(9, A2u, 3,)	C_9
88	A_{2u}	A2u	4	—	$\mathbb{Q}_9^{(h,A_{2u},4)}$	Qh(9, A2u, 4,)	C_3
89	E_u	Eu	1	0	$\mathbb{Q}_{9,0}^{(h,E_u,1)}$	Qh(9, Eu, 1, 0)	C_7
90	E_u	Eu	1	1	$\mathbb{Q}_{9,1}^{(h,E_u,1)}$	Qh(9, Eu, 1, 1)	S_7
91	E_u	Eu	2	0	$\mathbb{Q}_{9,0}^{(h,E_u,2)}$	Qh(9, Eu, 2, 0)	C_5
92	E_u	Eu	2	1	$\mathbb{Q}_{9,1}^{(h,E_u,2)}$	Qh(9, Eu, 2, 1)	$-S_5$
93	E_u	Eu	3	0	$\mathbb{Q}_{9,0}^{(h,E_u,3)}$	Qh(9, Eu, 3, 0)	C_1
94	E_u	Eu	3	1	$\mathbb{Q}_{9,1}^{(h,E_u,3)}$	Qh(9, Eu, 3, 1)	S_1
95	E_u	Eu	4	0	$\mathbb{Q}_{9,0}^{(h,E_u,4)}$	Qh(9, Eu, 4, 0)	C_8
96	E_u	Eu	4	1	$\mathbb{Q}_{9,1}^{(h,E_u,4)}$	Qh(9, Eu, 4, 1)	$-S_8$
97	E_u	Eu	5	0	$\mathbb{Q}_{9,0}^{(h,E_u,5)}$	Qh(9, Eu, 5, 0)	C_4
98	E_u	Eu	5	1	$\mathbb{Q}_{9,1}^{(h,E_u,5)}$	Qh(9, Eu, 5, 1)	S_4
99	E_u	Eu	6	0	$\mathbb{Q}_{9,0}^{(h,E_u,6)}$	Qh(9, Eu, 6, 0)	C_2
100	E_u	Eu	6	1	$\mathbb{Q}_{9,1}^{(h,E_u,6)}$	Qh(9, Eu, 6, 1)	$-S_2$

表 11 rank 10

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
101	A_{1g}	A1g	1	—	$\mathbb{Q}_{10}^{(h,A_{1g},1)}$	Qh(10, A1g, 1,)	C_0
102	A_{1g}	A1g	2	—	$\mathbb{Q}_{10}^{(h,A_{1g},2)}$	Qh(10, A1g, 2,)	C_6
103	A_{1g}	A1g	3	—	$\mathbb{Q}_{10}^{(h,A_{1g},3)}$	Qh(10, A1g, 3,)	C_9
104	A_{1g}	A1g	4	—	$\mathbb{Q}_{10}^{(h,A_{1g},4)}$	Qh(10, A1g, 4,)	C_3
105	A_{2g}	A2g	1	—	$\mathbb{Q}_{10}^{(h,A_{2g},1)}$	Qh(10, A2g, 1,)	S_6
106	A_{2g}	A2g	2	—	$\mathbb{Q}_{10}^{(h,A_{2g},2)}$	Qh(10, A2g, 2,)	S_9
107	A_{2g}	A2g	3	—	$\mathbb{Q}_{10}^{(h,A_{2g},3)}$	Qh(10, A2g, 3,)	S_3
108	E_g	Eg	1	0	$\mathbb{Q}_{10,0}^{(h,E_g,1)}$	Qh(10, Eg, 1, 0)	C_7
109	E_g	Eg	1	1	$\mathbb{Q}_{10,1}^{(h,E_g,1)}$	Qh(10, Eg, 1, 1)	S_7
110	E_g	Eg	2	0	$\mathbb{Q}_{10,0}^{(h,E_g,2)}$	Qh(10, Eg, 2, 0)	C_5
111	E_g	Eg	2	1	$\mathbb{Q}_{10,1}^{(h,E_g,2)}$	Qh(10, Eg, 2, 1)	$-S_5$
112	E_g	Eg	3	0	$\mathbb{Q}_{10,0}^{(h,E_g,3)}$	Qh(10, Eg, 3, 0)	C_1
113	E_g	Eg	3	1	$\mathbb{Q}_{10,1}^{(h,E_g,3)}$	Qh(10, Eg, 3, 1)	S_1
114	E_g	Eg	4	0	$\mathbb{Q}_{10,0}^{(h,E_g,4)}$	Qh(10, Eg, 4, 0)	C_{10}
115	E_g	Eg	4	1	$\mathbb{Q}_{10,1}^{(h,E_g,4)}$	Qh(10, Eg, 4, 1)	S_{10}
116	E_g	Eg	5	0	$\mathbb{Q}_{10,0}^{(h,E_g,5)}$	Qh(10, Eg, 5, 0)	C_8
117	E_g	Eg	5	1	$\mathbb{Q}_{10,1}^{(h,E_g,5)}$	Qh(10, Eg, 5, 1)	$-S_8$
118	E_g	Eg	6	0	$\mathbb{Q}_{10,0}^{(h,E_g,6)}$	Qh(10, Eg, 6, 0)	C_4
119	E_g	Eg	6	1	$\mathbb{Q}_{10,1}^{(h,E_g,6)}$	Qh(10, Eg, 6, 1)	S_4
120	E_g	Eg	7	0	$\mathbb{Q}_{10,0}^{(h,E_g,7)}$	Qh(10, Eg, 7, 0)	C_2
121	E_g	Eg	7	1	$\mathbb{Q}_{10,1}^{(h,E_g,7)}$	Qh(10, Eg, 7, 1)	$-S_2$

表 12 rank 11

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
122	A_{1u}	A1u	1	—	$Q_{11}^{(h,A_{1u},1)}$	$Qh(11, A1u, 1,)$	S_6
123	A_{1u}	A1u	2	—	$Q_{11}^{(h,A_{1u},2)}$	$Qh(11, A1u, 2,)$	S_9
124	A_{1u}	A1u	3	—	$Q_{11}^{(h,A_{1u},3)}$	$Qh(11, A1u, 3,)$	S_3
125	A_{2u}	A2u	1	—	$Q_{11}^{(h,A_{2u},1)}$	$Qh(11, A2u, 1,)$	C_0
126	A_{2u}	A2u	2	—	$Q_{11}^{(h,A_{2u},2)}$	$Qh(11, A2u, 2,)$	C_6
127	A_{2u}	A2u	3	—	$Q_{11}^{(h,A_{2u},3)}$	$Qh(11, A2u, 3,)$	C_9
128	A_{2u}	A2u	4	—	$Q_{11}^{(h,A_{2u},4)}$	$Qh(11, A2u, 4,)$	C_3
129	E_u	Eu	1	0	$Q_{11,0}^{(h,E_u,1)}$	$Qh(11, Eu, 1, 0)$	C_{11}
130	E_u	Eu	1	1	$Q_{11,1}^{(h,E_u,1)}$	$Qh(11, Eu, 1, 1)$	$-S_{11}$
131	E_u	Eu	2	0	$Q_{11,0}^{(h,E_u,2)}$	$Qh(11, Eu, 2, 0)$	C_7
132	E_u	Eu	2	1	$Q_{11,1}^{(h,E_u,2)}$	$Qh(11, Eu, 2, 1)$	S_7
133	E_u	Eu	3	0	$Q_{11,0}^{(h,E_u,3)}$	$Qh(11, Eu, 3, 0)$	C_5
134	E_u	Eu	3	1	$Q_{11,1}^{(h,E_u,3)}$	$Qh(11, Eu, 3, 1)$	$-S_5$
135	E_u	Eu	4	0	$Q_{11,0}^{(h,E_u,4)}$	$Qh(11, Eu, 4, 0)$	C_1
136	E_u	Eu	4	1	$Q_{11,1}^{(h,E_u,4)}$	$Qh(11, Eu, 4, 1)$	S_1
137	E_u	Eu	5	0	$Q_{11,0}^{(h,E_u,5)}$	$Qh(11, Eu, 5, 0)$	C_{10}
138	E_u	Eu	5	1	$Q_{11,1}^{(h,E_u,5)}$	$Qh(11, Eu, 5, 1)$	S_{10}
139	E_u	Eu	6	0	$Q_{11,0}^{(h,E_u,6)}$	$Qh(11, Eu, 6, 0)$	C_8
140	E_u	Eu	6	1	$Q_{11,1}^{(h,E_u,6)}$	$Qh(11, Eu, 6, 1)$	$-S_8$
141	E_u	Eu	7	0	$Q_{11,0}^{(h,E_u,7)}$	$Qh(11, Eu, 7, 0)$	C_4
142	E_u	Eu	7	1	$Q_{11,1}^{(h,E_u,7)}$	$Qh(11, Eu, 7, 1)$	S_4
143	E_u	Eu	8	0	$Q_{11,0}^{(h,E_u,8)}$	$Qh(11, Eu, 8, 0)$	C_2
144	E_u	Eu	8	1	$Q_{11,1}^{(h,E_u,8)}$	$Qh(11, Eu, 8, 1)$	$-S_2$