No. 32 O_h m-3m [cubic] (polar)

表 1 rank 0

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
1	A_{1g}	A1g	_	_	$\mathbb{Q}_0^{(h,A_{1g})}$	$\mathtt{Qh}(\mathtt{0},\mathtt{Alg},,)$	C_0

表 2 rank 1

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
2	T_{1u}	T1u	_	0	$\mathbb{Q}_{1,0}^{(h,T_{1u})}$	$\mathtt{Qh}(\mathtt{1},\mathtt{T1u},\mathtt{,0})$	C_1
3	T_{1u}	T1u	_	1	$\mathbb{Q}_{1,1}^{(h,T_{1u})}$	$\mathtt{Qh}(\mathtt{1},\mathtt{T1u},\mathtt{,1})$	S_1
4	T_{1u}	T1u	_	2	$\mathbb{Q}_{1,2}^{(h,T_{1u})}$	$\mathtt{Qh}(\mathtt{1},\mathtt{T1u},\mathtt{,2})$	C_0

表 3 rank 2

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
5	E_g	Eg	-	0	$\mathbb{Q}_{2,0}^{(h,E_g)}$	$\mathtt{Qh}(\mathtt{2},\mathtt{Eg},,\mathtt{0})$	C_0
6	E_g	Eg	_	1	$\mathbb{Q}_{2,1}^{(h,E_g)}$	$\mathtt{Qh}(\mathtt{2},\mathtt{Eg},,\mathtt{1})$	C_2
7	T_{2g}	T2g	_	0	$\mathbb{Q}_{2,0}^{(h,T_{2g})}$	$\mathtt{Qh}(\mathtt{2},\mathtt{T2g},,\mathtt{0})$	S_1
8	T_{2g}	T2g	_	1	$\mathbb{Q}_{2,1}^{(h,T_{2g})}$	$\mathtt{Qh}(\mathtt{2},\mathtt{T2g},,\mathtt{1})$	C_1
9	T_{2g}	T2g	_	2	$\mathbb{Q}_{2,2}^{(h,T_{2g})}$	$\mathtt{Qh}(\mathtt{2},\mathtt{T2g},,\mathtt{2})$	S_2

表 4 rank 3

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
10	A_{2u}	A2u	_	-	$\mathbb{Q}_3^{(h,A_{2u})}$	$\mathtt{Qh}(\mathtt{3},\mathtt{A2u},,)$	S_2
11	T_{1u}	T1u	_	0	$\mathbb{Q}_{3,0}^{(h,T_{1u})}$	$\mathtt{Qh}(\mathtt{3},\mathtt{T1u},,\mathtt{0})$	$-\frac{\sqrt{6}C_1}{4} + \frac{\sqrt{10}C_3}{4}$
12	T_{1u}	T1u	_	1	$\mathbb{Q}_{3,1}^{(h,T_{1u})}$	$\mathtt{Qh}(\mathtt{3},\mathtt{T1u},\mathtt{,1})$	$-\frac{\sqrt{6}S_1}{4} - \frac{\sqrt{10}S_3}{4}$
13	T_{1u}	T1u	_	2	$\mathbb{Q}_{3,2}^{(h,T_{1u})}$	$\mathtt{Qh}(\mathtt{3},\mathtt{T1u},\mathtt{,2})$	C_0
14	T_{2u}	T2u	_	0	$\mathbb{Q}_{3,0}^{(h,T_{2u})}$	$\mathtt{Qh}(3,\mathtt{T2u},,\mathtt{0})$	$-\frac{\sqrt{10}C_1}{4} - \frac{\sqrt{6}C_3}{4}$
15	T_{2u}	T2u	_	1	$\mathbb{Q}_{3,1}^{(h,T_{2u})}$	$\mathtt{Qh}(\mathtt{3},\mathtt{T2u},,\mathtt{1})$	$\frac{\sqrt{10}S_1}{4} - \frac{\sqrt{6}S_3}{4}$
16	T_{2u}	T2u	_	2	$\mathbb{Q}_{3,2}^{(h,T_{2u})}$	$\mathtt{Qh}(\mathtt{3},\mathtt{T2u},\mathtt{,2})$	C_2

表 5 rank 4

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
17	A_{1g}	A1g	_	_	$\mathbb{Q}_4^{(h,A_{1g})}$	$\mathtt{Qh}(\mathtt{4},\mathtt{A1g},,)$	$\frac{\sqrt{21}C_0}{6} + \frac{\sqrt{15}C_4}{6}$
18	E_g	Eg	_	0	$\mathbb{Q}_{4,0}^{(h,E_g)}$	$\mathtt{Qh}(\mathtt{4},\mathtt{Eg},,\mathtt{0})$	$\frac{\sqrt{15}C_0}{6} - \frac{\sqrt{21}C_4}{6}$
19	E_g	Eg	_	1	$\mathbb{Q}_{4,1}^{(h,E_g)}$	$\mathtt{Qh}(\mathtt{4},\mathtt{Eg},,\mathtt{1})$	$-C_2$
20	T_{1g}	T1g	_	0	$\mathbb{Q}_{4,0}^{(h,T_{1g})}$	$\mathtt{Qh}(\mathtt{4},\mathtt{T1g},,\mathtt{0})$	$-\frac{\sqrt{14}S_1}{4} - \frac{\sqrt{2}S_3}{4}$
21	T_{1g}	T1g	_	1	$\mathbb{Q}_{4,1}^{(h,T_{1g})}$	$\mathtt{Qh}(\mathtt{4},\mathtt{T1g},,\mathtt{1})$	$\frac{\sqrt{14}C_1}{4} - \frac{\sqrt{2}C_3}{4}$
22	T_{1g}	T1g	_	2	$\mathbb{Q}_{4,2}^{(h,T_{1g})}$	$\mathtt{Qh}(\mathtt{4},\mathtt{T1g},\mathtt{,2})$	S_4
23	T_{2g}	T2g	_	0	$\mathbb{Q}_{4,0}^{(h,T_{2g})}$	$\mathtt{Qh}(\mathtt{4},\mathtt{T2g},,\mathtt{0})$	$-\frac{\sqrt{2}S_1}{4} + \frac{\sqrt{14}S_3}{4}$
24	T_{2g}	T2g	_	1	$\mathbb{Q}_{4,1}^{(h,T_{2g})}$	$\mathtt{Qh}(\mathtt{4},\mathtt{T2g},,\mathtt{1})$	$-\frac{\sqrt{2}C_1}{4} - \frac{\sqrt{14}C_3}{4}$
25	T_{2g}	T2g	_	2	$\mathbb{Q}_{4,2}^{(h,T_{2g})}$	$\mathtt{Qh}(\mathtt{4},\mathtt{T2g},,\mathtt{2})$	S_2

表 6 rank 5

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
26	E_u	Eu	-	0	$\mathbb{Q}_{5,0}^{(h,E_u)}$	Qh(5, Eu,, 0)	S_4
27	E_u	Eu	-	1	$\mathbb{Q}_{5,1}^{(h,E_u)}$	$\mathtt{Qh}(\mathtt{5},\mathtt{Eu},,\mathtt{1})$	$-S_2$
28	T_{1u}	T1u	1	0	$\mathbb{Q}_{5,0}^{(h,T_{1u},1)}$	$\mathtt{Qh}(\mathtt{5},\mathtt{T1u},\mathtt{1},\mathtt{0})$	$\frac{\sqrt{15}C_1}{8} - \frac{\sqrt{70}C_3}{16} + \frac{3\sqrt{14}C_5}{16}$
29	T_{1u}	T1u	1	1	$\mathbb{Q}_{5,1}^{(h,T_{1u},1)}$	$\mathtt{Qh}(5,\mathtt{T1u},1,1)$	$\frac{\sqrt{15}S_1}{8} + \frac{\sqrt{70}S_3}{16} + \frac{3\sqrt{14}S_5}{16}$
30	T_{1u}	T1u	1	2	$\mathbb{Q}_{5,2}^{(h,T_{1u},1)}$	$\mathtt{Qh}(5,\mathtt{T1u},1,2)$	C_0
31	T_{1u}	T1u	2	0	$\mathbb{Q}_{5,0}^{(h,T_{1u},2)}$	$\mathtt{Qh}(5,\mathtt{T1u},2,0)$	$\frac{\sqrt{21}C_1}{8} + \frac{9\sqrt{2}C_3}{16} + \frac{\sqrt{10}C_5}{16}$
32	T_{1u}	T1u	2	1	$\mathbb{Q}_{5,1}^{(h,T_{1u},2)}$	$\mathtt{Qh}(5,\mathtt{T1u},2,1)$	$\frac{\sqrt{21}S_1}{8} - \frac{9\sqrt{2}S_3}{16} + \frac{\sqrt{10}S_5}{16}$
33	T_{1u}	T1u	2	2	$\mathbb{Q}_{5,2}^{(h,T_{1u},2)}$	$\mathtt{Qh}(5,\mathtt{T1u},2,2)$	C_4
34	T_{2u}	T2u	_	0	$\mathbb{Q}_{5,0}^{(h,T_{2u})}$	$\mathtt{Qh}(5,\mathtt{T2u},,0)$	$\frac{\sqrt{7}C_1}{4} - \frac{\sqrt{6}C_3}{8} - \frac{\sqrt{30}C_5}{8}$
35	T_{2u}	T2u	-	1	$\mathbb{Q}_{5,1}^{(h,T_{2u})}$	$\mathtt{Qh}(\mathtt{5},\mathtt{T2u},,\mathtt{1})$	$-\frac{\sqrt{7}S_1}{4} - \frac{\sqrt{6}S_3}{8} + \frac{\sqrt{30}S_5}{8}$
36	T_{2u}	T2u	_	2	$\mathbb{Q}_{5,2}^{(h,T_{2u})}$	$\mathtt{Qh}(\mathtt{5},\mathtt{T2u},\mathtt{,2})$	C_2

表 7 rank 6

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
37	A_{1g}	A1g	_	_	$\mathbb{Q}_6^{(h,A_{1g})}$	Qh(6, A1g,,)	$\frac{\sqrt{2}C_0}{4} - \frac{\sqrt{14}C_4}{4}$
38	A_{2g}	A2g	_	_	$\mathbb{Q}_{6}^{(h,A_{2g})}$	$\mathtt{Qh}(6,\mathtt{A2g},,)$	$\frac{\sqrt{11}C_2}{4} - \frac{\sqrt{5}C_6}{4}$
39	E_g	Eg	_	0	$\mathbb{Q}_{6,0}^{(h,E_g)}$	$\mathtt{Qh}(6,\mathtt{Eg},,\mathtt{0})$	$\frac{\sqrt{14}C_0}{4} + \frac{\sqrt{2}C_4}{4}$
40	E_g	Eg	_	1	$\mathbb{Q}_{6,1}^{(h,E_g)}$	$\mathtt{Qh}(6,\mathtt{Eg},,1)$	$\frac{\sqrt{5}C_2}{4} + \frac{\sqrt{11}C_6}{4}$
41	T_{1g}	T1g	_	0	$\mathbb{Q}_{6,0}^{(h,T_{1g})}$	$\mathtt{Qh}(6,\mathtt{T1g},,\mathtt{0})$	$\frac{\sqrt{3}S_1}{4} - \frac{\sqrt{30}S_3}{8} - \frac{\sqrt{22}S_5}{8}$
42	T_{1g}	T1g	_	1	$\mathbb{Q}_{6,1}^{(h,T_{1g})}$	$\mathtt{Qh}(6,\mathtt{T1g},,1)$	$-\frac{\sqrt{3}C_1}{4} - \frac{\sqrt{30}C_3}{8} + \frac{\sqrt{22}C_5}{8}$
43	T_{1g}	T1g	_	2	$\mathbb{Q}_{6,2}^{(h,T_{1g})}$	$\mathtt{Qh}(6,\mathtt{T1g},,2)$	S_4
44	T_{2g}	T2g	1	0	$\mathbb{Q}_{6,0}^{(h,T_{2g},1)}$	$\mathtt{Qh}(6,\mathtt{T2g},1,0)$	$\frac{3\sqrt{22}S_1}{16} + \frac{\sqrt{55}S_3}{16} + \frac{\sqrt{3}S_5}{16}$
45	T_{2g}	T2g	1	1	$\mathbb{Q}_{6,1}^{(h,T_{2g},1)}$	$\mathtt{Qh}(6,\mathtt{T2g},1,1)$	$\frac{3\sqrt{22}C_1}{16} - \frac{\sqrt{55}C_3}{16} + \frac{\sqrt{3}C_5}{16}$
46	T_{2g}	T2g	1	2	$\mathbb{Q}_{6,2}^{(h,T_{2g},1)}$	$\mathtt{Qh}(6,\mathtt{T2g},1,2)$	S_6
47	T_{2g}	T2g	2	0	$\mathbb{Q}_{6,0}^{(h,T_{2g},2)}$	$\mathtt{Qh}(6,\mathtt{T2g},2,0)$	$\frac{\sqrt{10}S_1}{16} - \frac{9S_3}{16} + \frac{\sqrt{165}S_5}{16}$
48	T_{2g}	T2g	2	1	$\mathbb{Q}_{6,1}^{(h,T_{2g},2)}$	$\mathtt{Qh}(6,\mathtt{T2g},2,1)$	$\frac{\sqrt{10}C_1}{16} + \frac{9C_3}{16} + \frac{\sqrt{165}C_5}{16}$
49	T_{2g}	T2g	2	2	$\mathbb{Q}_{6,2}^{(h,T_{2g},2)}$	$\mathtt{Qh}(6,\mathtt{T2g},2,2)$	S_2

表 8 $\operatorname{rank} 7$

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
50	A_{2u}	A2u	_	_	$\mathbb{Q}_7^{(h,A_{2u})}$	$\mathtt{Qh}(7,\mathtt{A2u},,)$	$\frac{\sqrt{78}S_2}{12} + \frac{\sqrt{66}S_6}{12}$
51	E_u	Eu	_	0	$\mathbb{Q}_{7,0}^{(h,E_u)}$	$\mathtt{Qh}(7,\mathtt{Eu},,\mathtt{0})$	S_4
52	E_u	Eu	_	1	$\mathbb{Q}_{7,1}^{(h,E_u)}$	$\mathtt{Qh}(7,\mathtt{Eu},,1)$	$\frac{\sqrt{66}S_2}{12} - \frac{\sqrt{78}S_6}{12}$
53	T_{1u}	T1u	1	0	$\mathbb{Q}_{7,0}^{(h,T_{1u},1)}$	$\mathtt{Qh}(7,\mathtt{T1u},1,\mathtt{0})$	$-\frac{5\sqrt{7}C_1}{32} + \frac{3\sqrt{21}C_3}{32} - \frac{\sqrt{231}C_5}{32} + \frac{\sqrt{429}C_7}{32}$
54	T_{1u}	T1u	1	1	$\mathbb{Q}_{7,1}^{(h,T_{1u},1)}$	$\mathtt{Qh}(7,\mathtt{T1u},1,1)$	$-\frac{5\sqrt{7}S_1}{32} - \frac{3\sqrt{21}S_3}{32} - \frac{\sqrt{231}S_5}{32} - \frac{\sqrt{429}S_7}{32}$
55	T_{1u}	T1u	1	2	$\mathbb{Q}_{7,2}^{(h,T_{1u},1)}$	$\mathtt{Qh}(7,\mathtt{T1u},1,2)$	C_0
56	T_{1u}	T1u	2	0	$\mathbb{Q}_{7,0}^{(h,T_{1u},2)}$	$\mathtt{Qh}(7,\mathtt{T1u},2,0)$	$-\frac{3\sqrt{33}C_1}{32} - \frac{\sqrt{11}C_3}{32} + \frac{25C_5}{32} + \frac{\sqrt{91}C_7}{32}$
57	T_{1u}	T1u	2	1	$\mathbb{Q}_{7,1}^{(h,T_{1u},2)}$	$\mathtt{Qh}(7,\mathtt{T1u},2,1)$	$-\frac{3\sqrt{33}S_1}{32} + \frac{\sqrt{11}S_3}{32} + \frac{25S_5}{32} - \frac{\sqrt{91}S_7}{32}$
58	T_{1u}	T1u	2	2	$\mathbb{Q}_{7,2}^{(h,T_{1u},2)}$	$\mathtt{Qh}(7,\mathtt{T1u},2,2)$	C_4
59	T_{2u}	T2u	1	0	$\mathbb{Q}_{7,0}^{(h,T_{2u},1)}$	$\mathtt{Qh}(7,\mathtt{T2u},1,\mathtt{0})$	$-\frac{\sqrt{858}C_1}{64} - \frac{3\sqrt{286}C_3}{64} - \frac{5\sqrt{26}C_5}{64} - \frac{\sqrt{14}C_7}{64}$
60	T_{2u}	T2u	1	1	$\mathbb{Q}_{7,1}^{(h,T_{2u},1)}$	$\mathtt{Qh}(7,\mathtt{T2u},1,1)$	$\frac{\sqrt{858}S_1}{64} - \frac{3\sqrt{286}S_3}{64} + \frac{5\sqrt{26}S_5}{64} - \frac{\sqrt{14}S_7}{64}$
61	T_{2u}	T2u	1	2	$\mathbb{Q}_{7,2}^{(h,T_{2u},1)}$	$\mathtt{Qh}(7,\mathtt{T2u},1,2)$	C_6
62	T_{2u}	T2u	2	0	$\mathbb{Q}_{7,0}^{(h,T_{2u},2)}$	$\mathtt{Qh}(7,\mathtt{T2u},2,0)$	$-\frac{15\sqrt{6}C_1}{64} + \frac{19\sqrt{2}C_3}{64} - \frac{\sqrt{22}C_5}{64} - \frac{\sqrt{2002}C_7}{64}$
63	T_{2u}	T2u	2	1	$\mathbb{Q}_{7,1}^{(h,T_{2u},2)}$	$\mathtt{Qh}(7,\mathtt{T2u},2,1)$	$\frac{15\sqrt{6}S_1}{64} + \frac{19\sqrt{2}S_3}{64} + \frac{\sqrt{22}S_5}{64} - \frac{\sqrt{2002}S_7}{64}$
64	T_{2u}	T2u	2	2	$\mathbb{Q}_{7,2}^{(h,T_{2u},2)}$	$\mathtt{Qh}(7,\mathtt{T2u},2,2)$	C_2

表 9 rank 8

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
65	A_{1g}	A1g	_	=	$\mathbb{Q}_8^{(h,A_{1g})}$	$\mathtt{Qh}(\mathtt{8},\mathtt{A1g},,)$	$\frac{\sqrt{33}C_0}{8} + \frac{\sqrt{21}C_4}{12} + \frac{\sqrt{195}C_8}{24}$
66	E_g	Eg	1	0	$\mathbb{Q}_{8,0}^{(h,E_g,1)}$	$\mathtt{Qh}(8,\mathtt{Eg},1,0)$	$-rac{\sqrt{286}C_0}{32}+rac{\sqrt{182}C_4}{16}+rac{\sqrt{10}C_8}{32}$
67	E_g	Eg	1	1	$\mathbb{Q}_{8,1}^{(h,E_g,1)}$	$\mathtt{Qh}(8,\mathtt{Eg},1,1)$	C_6
68	E_g	Eg	2	0	$\mathbb{Q}_{8,0}^{(h,E_g,2)}$	$\mathtt{Qh}(8,\mathtt{Eg},2,0)$	$-\frac{\sqrt{210}C_0}{32} - \frac{\sqrt{330}C_4}{48} + \frac{\sqrt{6006}C_8}{96}$
69	E_g	Eg	2	1	$\mathbb{Q}_{8,1}^{(h,E_g,2)}$	$\mathtt{Qh}(8,\mathtt{Eg},2,1)$	C_2
70	T_{1g}	T1g	1	0	$\mathbb{Q}_{8,0}^{(h,T_{1g},1)}$	$\mathtt{Qh}(8,\mathtt{T1g},1,\mathtt{0})$	$-\frac{\sqrt{715}S_1}{32} - \frac{\sqrt{273}S_3}{32} - \frac{\sqrt{35}S_5}{32} - \frac{S_7}{32}$
71	T_{1g}	T1g	1	1	$\mathbb{Q}_{8,1}^{(h,T_{1g},1)}$	$\mathtt{Qh}(8,\mathtt{T1g},1,1)$	$\frac{\sqrt{715}C_1}{32} - \frac{\sqrt{273}C_3}{32} + \frac{\sqrt{35}C_5}{32} - \frac{C_7}{32}$
72	T_{1g}	T1g	1	2	$\mathbb{Q}_{8,2}^{(h,T_{1g},1)}$	$\mathtt{Qh}(8,\mathtt{T1g},1,2)$	S_8
73	T_{1g}	T1g	2	0	$\mathbb{Q}_{8,0}^{(h,T_{1g},2)}$	$\mathtt{Qh}(8,\mathtt{T1g},2,0)$	$-\frac{\sqrt{77}S_1}{32} + \frac{5\sqrt{15}S_3}{32} - \frac{3\sqrt{13}S_5}{32} - \frac{\sqrt{455}S_7}{32}$
74	T_{1g}	T1g	2	1	$\mathbb{Q}_{8,1}^{(h,T_{1g},2)}$	$\mathtt{Qh}(8,\mathtt{T1g},2,1)$	$\frac{\sqrt{77}C_1}{32} + \frac{5\sqrt{15}C_3}{32} + \frac{3\sqrt{13}C_5}{32} - \frac{\sqrt{455}C_7}{32}$
75	T_{1g}	T1g	2	2	$\mathbb{Q}_{8,2}^{(h,T_{1g},2)}$	$\mathtt{Qh}(8,\mathtt{T1g},2,2)$	S_4
76	T_{2g}	T2g	1	0	$\mathbb{Q}_{8,0}^{(h,T_{2g},1)}$	$\mathtt{Qh}(8,\mathtt{T2g},1,0)$	$-\frac{\sqrt{858}S_1}{64} + \frac{\sqrt{910}S_3}{64} + \frac{7\sqrt{42}S_5}{64} + \frac{3\sqrt{30}S_7}{64}$
77	T_{2g}	T2g	1	1	$\mathbb{Q}_{8,1}^{(h,T_{2g},1)}$	$\mathtt{Qh}(8,\mathtt{T2g},1,1)$	$-\frac{\sqrt{858}C_1}{64} - \frac{\sqrt{910}C_3}{64} + \frac{7\sqrt{42}C_5}{64} - \frac{3\sqrt{30}C_7}{64}$
78	T_{2g}	T2g	1	2	$\mathbb{Q}_{8,2}^{(h,T_{2g},1)}$	$\mathtt{Qh}(8,\mathtt{T2g},1,2)$	S_6
79	T_{2g}	T2g	2	0	$\mathbb{Q}_{8,0}^{(h,T_{2g},2)}$	$\mathtt{Qh}(8,\mathtt{T2g},2,0)$	$-\frac{\sqrt{70}S_1}{64} + \frac{3\sqrt{66}S_3}{64} - \frac{\sqrt{1430}S_5}{64} + \frac{\sqrt{2002}S_7}{64}$
80	T_{2g}	T2g	2	1	$\mathbb{Q}_{8,1}^{(h,T_{2g},2)}$	$\mathtt{Qh}(8,\mathtt{T2g},2,1)$	$-\frac{\sqrt{70}C_1}{64} - \frac{3\sqrt{66}C_3}{64} - \frac{\sqrt{1430}C_5}{64} - \frac{\sqrt{2002}C_7}{64}$
81	T_{2g}	T2g	2	2	$\mathbb{Q}_{8,2}^{(h,T_{2g},2)}$	$\mathtt{Qh}(8,\mathtt{T2g},2,2)$	S_2

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
82	A_{1u}	A1u	-	-	$\mathbb{Q}_9^{(h,A_{1u})}$	$\mathtt{Qh}(9,\mathtt{A1u},,)$	$rac{\sqrt{102}S_4}{12} - rac{\sqrt{42}S_8}{12}$
83	A_{2u}	A2u	_	_	$\mathbb{Q}_9^{(h,A_{2u})}$	$\mathtt{Qh}(9,\mathtt{A2u},,)$	$rac{\sqrt{3}S_2}{4} - rac{\sqrt{13}S_6}{4}$
84	E_u	Eu	_	0	$\mathbb{Q}_{9,0}^{(h,E_u)}$	$\mathtt{Qh}(9,\mathtt{Eu},,0)$	$rac{\sqrt{42}S_4}{12} + rac{\sqrt{102}S_8}{12}$
85	E_u	Eu	_	1	$\mathbb{Q}_{9,1}^{(h,E_u)}$	$\mathtt{Qh}(9,\mathtt{Eu},,1)$	$-rac{\sqrt{13}S_2}{4} - rac{\sqrt{3}S_6}{4}$
86	T_{1u}	T1u	1	0	$\mathbb{Q}_{9,0}^{(h,T_{1u},1)}$	$\mathtt{Qh}(9,\mathtt{T1u},1,0)$	$\frac{21\sqrt{5}C_1}{128} - \frac{\sqrt{2310}C_3}{128} + \frac{3\sqrt{286}C_5}{128} - \frac{3\sqrt{1430}C_7}{256} + \frac{\sqrt{24310}C_9}{256}$
87	T_{1u}	T1u	1	1	$\mathbb{Q}_{9,1}^{(h,T_{1u},1)}$	$\mathtt{Qh}(9,\mathtt{T1u},1,1)$	$\frac{21\sqrt{5}S_1}{128} + \frac{\sqrt{2310}S_3}{128} + \frac{3\sqrt{286}S_5}{128} + \frac{3\sqrt{1430}S_7}{256} + \frac{\sqrt{24310}S_9}{256}$
88	T_{1u}	T1u	1	2	$\mathbb{Q}_{9,2}^{(h,T_{1u},1)}$	$\mathtt{Qh}(9,\mathtt{T1u},1,2)$	C_0
89	T_{1u}	T1u	2	0	$\mathbb{Q}_{9,0}^{(h,T_{1u},2)}$	$\mathtt{Qh}(9,\mathtt{T1u},2,0)$	$\frac{\sqrt{2431}C_1}{128} + \frac{\sqrt{9282}C_3}{128} + \frac{5\sqrt{170}C_5}{128} + \frac{7\sqrt{34}C_7}{256} + \frac{3\sqrt{2}C_9}{256}$
90	T_{1u}	T1u	2	1	$\mathbb{Q}_{9,1}^{(h,T_{1u},2)}$	$\mathtt{Qh}(9,\mathtt{T1u},2,1)$	$\frac{\sqrt{2431}S_1}{128} - \frac{\sqrt{9282}S_3}{128} + \frac{5\sqrt{170}S_5}{128} - \frac{7\sqrt{34}S_7}{256} + \frac{3\sqrt{2}S_9}{256}$
91	T_{1u}	T1u	2	2	$\mathbb{Q}_{9,2}^{(h,T_{1u},2)}$	$\mathtt{Qh}(9,\mathtt{T1u},2,2)$	C_8
92	T_{1u}	T1u	3	0	$\mathbb{Q}_{9,0}^{(h,T_{1u},3)}$	$\mathtt{Qh}(9,\mathtt{T1u},3,0)$	$\frac{\sqrt{1001}C_1}{64} - \frac{\sqrt{78}C_3}{64} - \frac{3\sqrt{70}C_5}{64} + \frac{23\sqrt{14}C_7}{128} + \frac{3\sqrt{238}C_9}{128}$
93	T_{1u}	T1u	3	1	$\mathbb{Q}_{9,1}^{(h,T_{1u},3)}$	$\mathtt{Qh}(9,\mathtt{T1u},3,1)$	$\frac{\sqrt{1001}S_1}{64} + \frac{\sqrt{78}S_3}{64} - \frac{3\sqrt{70}S_5}{64} - \frac{23\sqrt{14}S_7}{128} + \frac{3\sqrt{238}S_9}{128}$
94	T_{1u}	T1u	3	2	$\mathbb{Q}_{9,2}^{(h,T_{1u},3)}$	$\mathtt{Qh}(9,\mathtt{T1u},3,2)$	C_4
95	T_{2u}	T2u	1	0	$\mathbb{Q}_{9,0}^{(h,T_{2u},1)}$	$\mathtt{Qh}(9,\mathtt{T2u},1,0)$	$\frac{\sqrt{858}C_1}{64} + \frac{\sqrt{91}C_3}{32} - \frac{5\sqrt{15}C_5}{32} - \frac{21\sqrt{3}C_7}{64} - \frac{\sqrt{51}C_9}{64}$
96	T_{2u}	T2u	1	1	$\mathbb{Q}_{9,1}^{(h,T_{2u},1)}$	$\mathtt{Qh}(9,\mathtt{T2u},1,1)$	$-\frac{\sqrt{858}S_1}{64} + \frac{\sqrt{91}S_3}{32} + \frac{5\sqrt{15}S_5}{32} - \frac{21\sqrt{3}S_7}{64} + \frac{\sqrt{51}S_9}{64}$
97	T_{2u}	T2u	1	2	$\mathbb{Q}_{9,2}^{(h,T_{2u},1)}$	$\mathtt{Qh}(9,\mathtt{T2u},1,2)$	C_6
98	T_{2u}	T2u	2	0	$\mathbb{Q}_{9,0}^{(h,T_{2u},2)}$	$\mathtt{Qh}(9,\mathtt{T2u},2,0)$	$\frac{7\sqrt{22}C_1}{64} - \frac{3\sqrt{21}C_3}{32} + \frac{\sqrt{65}C_5}{32} + \frac{\sqrt{13}C_7}{64} - \frac{3\sqrt{221}C_9}{64}$
99	T_{2u}	T2u	2	1	$\mathbb{Q}_{9,1}^{(h,T_{2u},2)}$	$\mathtt{Qh}(9,\mathtt{T2u},2,1)$	$-\frac{7\sqrt{22}S_1}{64} - \frac{3\sqrt{21}S_3}{32} - \frac{\sqrt{65}S_5}{32} + \frac{\sqrt{13}S_7}{64} + \frac{3\sqrt{221}S_9}{64}$
100	T_{2u}	T2u	2	2	$\mathbb{Q}_{9,2}^{(h,T_{2u},2)}$	$\mathtt{Qh}(9,\mathtt{T2u},2,2)$	C_2

表 11 rank 10

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
101	A_{1g}	A1g	_	_	$\mathbb{Q}_{10}^{(h,A_{1g})}$	$\mathtt{Qh}(\mathtt{10},\mathtt{A1g},,)$	$\frac{\sqrt{390}C_0}{48} - \frac{\sqrt{22}C_4}{8} - \frac{\sqrt{1122}C_8}{48}$
102	A_{2g}	A2g	_	_	$\mathbb{Q}_{10}^{(h,A_{2g})}$	$\mathtt{Qh}(\mathtt{10},\mathtt{A2g},,)$	$-\frac{\sqrt{85}C_{10}}{16} + \frac{\sqrt{1482}C_2}{48} + \frac{\sqrt{57}C_6}{48}$
103	E_g	Eg	1	0	$\mathbb{Q}_{10,0}^{(h,E_g,1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{Eg},\mathtt{1},\mathtt{0})$	$\frac{11\sqrt{420189}C_0}{8988} + \frac{\sqrt{827645}C_4}{1498} - \frac{\sqrt{146055}C_8}{8988}$
104	E_g	Eg	1	1	$\mathbb{Q}_{10,1}^{(h,E_g,1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{Eg},\mathtt{1},\mathtt{1})$	$\frac{\sqrt{370006}C_{10}}{749} + \frac{\sqrt{190995}C_2}{749}$
105	E_g	Eg	2	0	$\mathbb{Q}_{10,0}^{(h,E_g,2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{Eg},\mathtt{2},\mathtt{0})$	$\frac{3\sqrt{3213210}C_0}{11984} - \frac{83\sqrt{1498}C_4}{5992} + \frac{31\sqrt{76398}C_8}{11984}$
106	E_g	Eg	2	1	$\mathbb{Q}_{10,1}^{(h,E_g,2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{Eg},\mathtt{2},\mathtt{1})$	$\frac{\sqrt{1209635}C_{10}}{11984} - \frac{19\sqrt{58422}C_2}{35952} + \frac{\sqrt{2247}C_6}{48}$
107	T_{1g}	T1g	1	0	$\mathbb{Q}_{10,0}^{(h,T_{1g},1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T1g},\mathtt{1},\mathtt{0})$	$\frac{\sqrt{221}S_1}{32} - \frac{\sqrt{102}S_3}{32} - \frac{\sqrt{510}S_5}{32} - \frac{11\sqrt{6}S_7}{64} - \frac{\sqrt{38}S_9}{64}$
108	T_{1g}	T1g	1	1	$\mathbb{Q}_{10,1}^{(h,T_{1g},1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T1g},\mathtt{1},\mathtt{1})$	$-\frac{\sqrt{221}C_1}{32} - \frac{\sqrt{102}C_3}{32} + \frac{\sqrt{510}C_5}{32} - \frac{11\sqrt{6}C_7}{64} + \frac{\sqrt{38}C_9}{64}$
109	T_{1g}	T1g	1	2	$\mathbb{Q}_{10,2}^{(h,T_{1g},1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T1g},\mathtt{1},\mathtt{2})$	S_8
110	T_{1g}	T1g	2	0	$\mathbb{Q}_{10,0}^{(h,T_{1g},2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T1g},\mathtt{2},\mathtt{0})$	$\frac{\sqrt{39}S_1}{32} - \frac{11\sqrt{2}S_3}{32} + \frac{5\sqrt{10}S_5}{32} - \frac{\sqrt{34}S_7}{64} - \frac{\sqrt{1938}S_9}{64}$
111	T_{1g}	T1g	2	1	$\mathbb{Q}_{10,1}^{(h,T_{1g},2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T1g},\mathtt{2},\mathtt{1})$	$-\frac{\sqrt{39}C_1}{32} - \frac{11\sqrt{2}C_3}{32} - \frac{5\sqrt{10}C_5}{32} - \frac{\sqrt{34}C_7}{64} + \frac{\sqrt{1938}C_9}{64}$
112	T_{1g}	T1g	2	2	$\mathbb{Q}_{10,2}^{(h,T_{1g},2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T1g},\mathtt{2},\mathtt{2})$	S_4
113	T_{2g}	T2g	1	0	$\mathbb{Q}_{10,0}^{(h,T_{2g},1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T2g},\mathtt{1},\mathtt{0})$	$\frac{\sqrt{41990}S_1}{256} + \frac{\sqrt{4845}S_3}{128} + \frac{\sqrt{969}S_5}{128} + \frac{\sqrt{285}S_7}{256} + \frac{\sqrt{5}S_9}{256}$
114	T_{2g}	T2g	1	1	$\mathbb{Q}_{10,1}^{(h,T_{2g},1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T2g},\mathtt{1},\mathtt{1})$	$\frac{\sqrt{41990}C_1}{256} - \frac{\sqrt{4845}C_3}{128} + \frac{\sqrt{969}C_5}{128} - \frac{\sqrt{285}C_7}{256} + \frac{\sqrt{5}C_9}{256}$
115	T_{2g}	T2g	1	2	$\mathbb{Q}_{10,2}^{(h,T_{2g},1)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T2g},\mathtt{1},\mathtt{2})$	S_{10}
116	T_{2g}	T2g	2	0	$\mathbb{Q}_{10,0}^{(h,T_{2g},2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T2g},\mathtt{2},\mathtt{0})$	$\frac{9\sqrt{78}S_1}{256} - \frac{69S_3}{128} - \frac{\sqrt{5}S_5}{128} + \frac{43\sqrt{17}S_7}{256} + \frac{3\sqrt{969}S_9}{256}$
117	T_{2g}	T2g	2	1	$\mathbb{Q}_{10,1}^{(h,T_{2g},2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T2g},\mathtt{2},\mathtt{1})$	$\frac{9\sqrt{78}C_1}{256} + \frac{69C_3}{128} - \frac{\sqrt{5}C_5}{128} - \frac{43\sqrt{17}C_7}{256} + \frac{3\sqrt{969}C_9}{256}$
118	T_{2g}	T2g	2	2	$\mathbb{Q}_{10,2}^{(h,T_{2g},2)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T2g},\mathtt{2},\mathtt{2})$	S_6
119	T_{2g}	T2g	3	0	$\mathbb{Q}_{10,0}^{(h,T_{2g},3)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T2g},\mathtt{3},\mathtt{0})$	$\frac{7\sqrt{3}S_1}{128} - \frac{7\sqrt{26}S_3}{128} + \frac{5\sqrt{130}S_5}{128} - \frac{7\sqrt{442}S_7}{256} + \frac{\sqrt{25194}S_9}{256}$
120	T_{2g}	T2g	3	1	$\mathbb{Q}_{10,1}^{(h,T_{2g},3)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T2g},\mathtt{3},\mathtt{1})$	$\frac{7\sqrt{3}C_1}{128} + \frac{7\sqrt{26}C_3}{128} + \frac{5\sqrt{130}C_5}{128} + \frac{7\sqrt{442}C_7}{256} + \frac{\sqrt{25194}C_9}{256}$
121	T_{2g}	T2g	3	2	$\mathbb{Q}_{10,2}^{(h,T_{2g},3)}$	$\mathtt{Qh}(\mathtt{10},\mathtt{T2g},\mathtt{3},\mathtt{2})$	S_2

表 12 rank 11

						12 12	TAHK 11
No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
122	A_{2u}	A2u	-	-	$\mathbb{Q}_{11}^{(h,A_{2u})}$	$\mathtt{Qh}(\mathtt{11},\mathtt{A2u},,)$	$\frac{\sqrt{798}S_{10}}{48} + \frac{\sqrt{255}S_2}{24} + \frac{3\sqrt{6}S_6}{16}$
123	E_u	Eu	1	0	$\mathbb{Q}_{11,0}^{(h,E_u,1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{Eu},\mathtt{1},\mathtt{0})$	S_8
124	E_u	Eu	1	1	$\mathbb{Q}_{11,1}^{(h,E_u,1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{Eu},\mathtt{1},\mathtt{1})$	$-\frac{\sqrt{210}S_{10}}{96} + \frac{\sqrt{969}S_2}{48} - \frac{\sqrt{570}S_6}{32}$
125	E_u	Eu	2	0	$\mathbb{Q}_{11,0}^{(h,E_u,2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{Eu},\mathtt{2},\mathtt{0})$	S_4
126	E_u	Eu	2	1	$\mathbb{Q}_{11,1}^{(h,E_u,2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{Eu},\mathtt{2},\mathtt{1})$	$-\frac{\sqrt{646}S_{10}}{32} + \frac{\sqrt{35}S_2}{16} + \frac{\sqrt{238}S_6}{32}$
127	T_{1u}	T1u	1	0	$\mathbb{Q}_{11,0}^{(h,T_{1u},1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T1u},\mathtt{1},\mathtt{0})$	$-\frac{21\sqrt{66}C_1}{512} + \frac{\sqrt{88179}C_{11}}{512} + \frac{\sqrt{30030}C_3}{512} - \frac{15\sqrt{143}C_5}{512} + \frac{\sqrt{36465}C_7}{512} - \frac{\sqrt{46189}C_9}{512}$
128	T_{1u}	T1u	1	1	$\mathbb{Q}_{11,1}^{(h,T_{1u},1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T1u},\mathtt{1},\mathtt{1})$	$-\frac{21\sqrt{66}S_1}{512}-\frac{\sqrt{88179}S_{11}}{512}-\frac{\sqrt{30030}S_3}{512}-\frac{15\sqrt{143}S_5}{512}-\frac{\sqrt{36465}S_7}{512}-\frac{\sqrt{46189}S_9}{512}$
129	T_{1u}	T1u	1	2	$\mathbb{Q}_{11,2}^{(h,T_{1u},1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T1u},\mathtt{1},\mathtt{2})$	C_0
130	T_{1u}	T1u	2	0	$\mathbb{Q}_{11,0}^{(h,T_{1u},2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T1u},\mathtt{2},\mathtt{0})$	$-\frac{\sqrt{41990}C_1}{512} + \frac{\sqrt{385}C_{11}}{512} - \frac{3\sqrt{4522}C_3}{512} + \frac{3\sqrt{4845}C_5}{512} + \frac{77\sqrt{19}C_7}{512} + \frac{39\sqrt{15}C_9}{512}$
131	T_{1u}	T1u	2	1	$\mathbb{Q}_{11,1}^{(h,T_{1u},2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T1u},\mathtt{2},\mathtt{1})$	$-\frac{\sqrt{41990}S_1}{512} - \frac{\sqrt{385}S_{11}}{512} + \frac{3\sqrt{4522}S_3}{512} + \frac{3\sqrt{4845}S_5}{512} - \frac{77\sqrt{19}S_7}{512} + \frac{39\sqrt{15}S_9}{512}$
132	T_{1u}	T1u	2	2	$\mathbb{Q}_{11,2}^{(h,T_{1u},2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T1u},\mathtt{2},\mathtt{2})$	C_8
133	T_{1u}	T1u	3	0	$\mathbb{Q}_{11,0}^{(h,T_{1u},3)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T1u},\mathtt{3},\mathtt{0})$	$-\frac{5\sqrt{546}C_1}{256} + \frac{\sqrt{10659}C_{11}}{256} + \frac{11\sqrt{30}C_3}{256} + \frac{13\sqrt{7}C_5}{256} - \frac{3\sqrt{1785}C_7}{256} + \frac{3\sqrt{2261}C_9}{256}$
134	T_{1u}	T1u	3	1	$\mathbb{Q}_{11,1}^{(h,T_{1u},3)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T1u},\mathtt{3},\mathtt{1})$	$-\frac{5\sqrt{546}S_1}{256} - \frac{\sqrt{10659}S_{11}}{256} - \frac{11\sqrt{30}S_3}{256} + \frac{13\sqrt{7}S_5}{256} + \frac{3\sqrt{1785}S_7}{256} + \frac{3\sqrt{2261}S_9}{256}$
135	T_{1u}	T1u	3	2	$\mathbb{Q}_{11,2}^{(h,T_{1u},3)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T1u},\mathtt{3},\mathtt{2})$	C_4
136	T_{2u}	T2u	1	0	$\mathbb{Q}_{11,0}^{(h,T_{2u},1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T2u},\mathtt{1},\mathtt{0})$	$-\frac{\sqrt{29393}C_1}{512}-\frac{\sqrt{22}C_{11}}{1024}-\frac{9\sqrt{1615}C_3}{512}-\frac{5\sqrt{13566}C_5}{1024}-\frac{7\sqrt{1330}C_7}{1024}-\frac{9\sqrt{42}C_9}{1024}$
137	T_{2u}	T2u	1	1	$\mathbb{Q}_{11,1}^{(h,T_{2u},1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T2u},\mathtt{1},\mathtt{1})$	$\frac{\sqrt{29393}S_1}{512} - \frac{\sqrt{22}S_{11}}{1024} - \frac{9\sqrt{1615}S_3}{512} + \frac{5\sqrt{13566}S_5}{1024} - \frac{7\sqrt{1330}S_7}{1024} + \frac{9\sqrt{42}S_9}{1024}$
138	T_{2u}	T2u	1	2	$\mathbb{Q}_{11,2}^{(h,T_{2u},1)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T2u},\mathtt{1},\mathtt{2})$	C_{10}
139	T_{2u}	T2u	2	0	$\mathbb{Q}_{11,0}^{(h,T_{2u},2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T2u},\mathtt{2},\mathtt{0})$	$-\frac{15\sqrt{221}C_1}{512}-\frac{3\sqrt{2926}C_{11}}{1024}-\frac{\sqrt{595}C_3}{512}+\frac{53\sqrt{102}C_5}{1024}-\frac{105\sqrt{10}C_7}{1024}-\frac{61\sqrt{114}C_9}{1024}$
140	T_{2u}	T2u	2	1	$\mathbb{Q}_{11,1}^{(h,T_{2u},2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T2u},\mathtt{2},\mathtt{1})$	$\frac{15\sqrt{221}S_1}{512} - \frac{3\sqrt{2926}S_{11}}{1024} - \frac{\sqrt{595}S_3}{512} - \frac{53\sqrt{102}S_5}{1024} - \frac{105\sqrt{10}S_7}{1024} + \frac{61\sqrt{114}S_9}{1024}$
141	T_{2u}	T2u	2	2	$\mathbb{Q}_{11,2}^{(h,T_{2u},2)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T2u},\mathtt{2},\mathtt{2})$	C_6
142	T_{2u}	T2u	3	0	$\mathbb{Q}_{11,0}^{(h,T_{2u},3)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T2u},\mathtt{3},\mathtt{0})$	$-\frac{21\sqrt{130}C_1}{512} - \frac{\sqrt{124355}C_{11}}{512} + \frac{57\sqrt{14}C_3}{512} - \frac{41\sqrt{15}C_5}{512} + \frac{17\sqrt{17}C_7}{512} + \frac{\sqrt{4845}C_9}{512}$
143	T_{2u}	T2u	3	1	$\mathbb{Q}_{11,1}^{(h,T_{2u},3)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T2u},\mathtt{3},\mathtt{1})$	$\tfrac{21\sqrt{130}S_1}{512} - \tfrac{\sqrt{124355}S_{11}}{512} + \tfrac{57\sqrt{14}S_3}{512} + \tfrac{41\sqrt{15}S_5}{512} + \tfrac{17\sqrt{17}S_7}{512} - \tfrac{\sqrt{4845}S_9}{512}$
144	T_{2u}	T2u	3	2	$\mathbb{Q}_{11,2}^{(h,T_{2u},3)}$	$\mathtt{Qh}(\mathtt{11},\mathtt{T2u},\mathtt{3},\mathtt{2})$	C_2