

\* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

| No. | tag                 | matrix (polar)  | det | TR |
|-----|---------------------|---|-----|----|
| 1   | $\{1 0\}$           | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$   | 1   | 1  |
| 2   | $\{2_{100} 0\}$     | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | 1   | 1  |
| 3   | $\{2_{010} 0\}$     | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | 1   | 1  |
| 4   | $\{2_{001} 0\}$     | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | 1   | 1  |
| 5   | $\{3_{111}^+ 0\}$   | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$   | 1   | 1  |
| 6   | $\{3_{111}^- 0\}$   | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$   | 1   | 1  |
| 7   | $\{3_{1-1-1}^- 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | 1   | 1  |
| 8   | $\{3_{1-1-1}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | 1   | 1  |
| 9   | $\{3_{-11-1}^- 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | 1   | 1  |
| 10  | $\{3_{-11-1}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | 1   | 1  |
| 11  | $\{3_{-1-11}^- 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | 1   | 1  |
| 12  | $\{3_{-1-11}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | 1   | 1  |
| 13  | $\{-4_{100}^+ 0\}$  | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | -1  | 1  |

*continued ...*

Table 1

| No. | tag  | matrix (polar)  | det | TR |
|-----|--|---|-----|----|
| 14  | $\{-4_{100}^- 0\}$                               | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$                               | -1  | 1  |
| 15  | $\{-4_{010}^+ 0\}$                               | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$                               | -1  | 1  |
| 16  | $\{-4_{010}^- 0\}$                               | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$                               | -1  | 1  |
| 17  | $\{-4_{001}^+ 0\}$                               | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                               | -1  | 1  |
| 18  | $\{-4_{001}^- 0\}$                               | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                               | -1  | 1  |
| 19  | $\{m_{110} 0\}$                                  | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                               | -1  | 1  |
| 20  | $\{m_{1-10} 0\}$                                 | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                 | -1  | 1  |
| 21  | $\{m_{011} 0\}$                                  | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$                               | -1  | 1  |
| 22  | $\{m_{01-1} 0\}$                                 | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$                                 | -1  | 1  |
| 23  | $\{m_{101} 0\}$                                  | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$                               | -1  | 1  |
| 24  | $\{m_{-101} 0\}$                                 | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$                                 | -1  | 1  |
| 25  | $\{1' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$       | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$   | 1   | -1 |
| 26  | $\{2_{100}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 27  | $\{2_{010}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |

continued ...

Table 1

| No. | tag   | matrix (polar)  | det | TR |
|-----|---|---|-----|----|
| 28  | $\{2_{001}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$    | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 29  | $\{3_{111}^+ \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$   | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \end{bmatrix}$   | 1   | -1 |
| 30  | $\{3_{111}^- \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$   | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$   | 1   | -1 |
| 31  | $\{3_{1-1-1}^- \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 32  | $\{3_{1-1-1}^+ \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 33  | $\{3_{-11-1}^- \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 34  | $\{3_{-11-1}^+ \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 35  | $\{3_{-1-11}^- \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 36  | $\{3_{-1-11}^+ \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 37  | $\{-4_{100}^+ \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 38  | $\{-4_{100}^- \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 39  | $\{-4_{010}^+ \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 40  | $\{-4_{010}^- \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 41  | $\{-4_{001}^+ \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |

continued ...

Table 1

| No. | tag   | matrix (polar)  | det | TR |
|-----|---|---|-----|----|
| 42  | $\{-4_{001}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 43  | $\{m_{110}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 44  | $\{m_{1-10}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$   | -1  | -1 |
| 45  | $\{m_{011}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 46  | $\{m_{01-1}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \end{bmatrix}$   | -1  | -1 |
| 47  | $\{m_{101}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 48  | $\{m_{-101}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$   | -1  | -1 |