

\* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

| No. | tag   | matrix (polar)  | det | TR |
|-----|---|---|-----|----|
| 1   | $\{1 0\}$   | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                 | 1   | 1  |
| 2   | $\{2_{001} 0\}$                                     | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                               | 1   | 1  |
| 3   | $\{2_{110} 00\frac{1}{2}\}$                         | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$                      | 1   | 1  |
| 4   | $\{2_{1-10} 00\frac{1}{2}\}$                        | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$                    | 1   | 1  |
| 5   | $\{-4_{001}^+ 0\}$                                  | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                               | -1  | 1  |
| 6   | $\{-4_{001}^- 0\}$                                  | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                               | -1  | 1  |
| 7   | $\{m_{100} 00\frac{1}{2}\}$                         | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$                      | -1  | 1  |
| 8   | $\{m_{010} 00\frac{1}{2}\}$                         | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$                      | -1  | 1  |
| 9   | $\{1' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$          | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$   | 1   | -1 |
| 10  | $\{2_{001}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$    | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 11  | $\{2_{110}' \frac{1}{2}\frac{1}{2}0\}$              | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$            | 1   | -1 |
| 12  | $\{2_{1-10}' \frac{1}{2}\frac{1}{2}0\}$             | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$          | 1   | -1 |
| 13  | $\{-4_{001}^+' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |

*continued ...*

Table 1

| No. | tag  | matrix (polar)  | det | TR |
|-----|--|---|-----|----|
| 14  | $\{-4_{001}^-   \frac{1}{2} \frac{1}{2} \frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 15  | $\{m_{100}'   \frac{1}{2} \frac{1}{2} 0\}$             | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$            | -1  | -1 |
| 16  | $\{m_{010}'   \frac{1}{2} \frac{1}{2} 0\}$             | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$            | -1  | -1 |