SAMB for "graphene"

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- Generation condition

 - time-reversal type: electric
 - irrep: [A1g]
 - spinful
- Unit cell:

$$a=1.0,\ b=1.0,\ c=1.0,\ \alpha=90.0,\ \beta=90.0,\ \gamma=120.0$$

• Lattice vectors:

$$\boldsymbol{a}_1 = \begin{pmatrix} 1.0 & 0 & 0 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -0.5 & 0.86602540378444 & 0 \end{pmatrix}$$

$$\boldsymbol{a}_3 = \begin{pmatrix} 0 & 0 & 1.0 \end{pmatrix}$$

Table 1: High-symmetry line: Γ -M-K- Γ -K'.

symbol	position	symbol	position	symbol	position
Г К'	$ \begin{pmatrix} 0 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{3} & 0 \end{pmatrix} $	М	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$	K	$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$

• Kets: dimension = 4

Table 2: Hilbert space for full matrix.

No.	ket	No. ket		No.	ket	No.	ket
1	(p_z,\uparrow) @A ₁	2	(p_z,\downarrow) @A ₁	3	(p_z,\uparrow) @A ₂	4	(p_z,\downarrow) @A ₂

• Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position	mapping
$S_1 [2c: -6m2]$	A_1	$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix}$	[1,6,7,8,9,10,14,15,16,17,23,24]
	A_2	$\begin{pmatrix} \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}$	[2,3,4,5,11,12,13,18,19,20,21,22]

• Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

	bond	tail	head	n	#	b@c	mapping
B ₁ [3f: mmm]	b_1	A_2	A_1	1	1	$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$	[1, -2, -3, 6, -13, 14, 17, -18]
	b_2	A_2	A_1	1	1	$\begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & 0 \end{pmatrix} @ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$	[-4,7,10,-11,15,-19,-22,23]
	b_3	A_2	A_1	1	1	$\begin{pmatrix} -\frac{2}{3} & -\frac{1}{3} & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & \frac{1}{2} & 0 \end{pmatrix}$	$[-5,\!8,\!9,\!-12,\!16,\!-20,\!-21,\!24]$

• SAMB:

Table 5: Atomic SAMB group.

group	bra	ket			
M_1	$(p_z,\uparrow),(p_z,\downarrow)$	$(p_z,\uparrow),(p_z,\downarrow)$			

Table 6: Atomic SAMB.

symbol	type	group	form
\mathbb{X}_1	$\mathbb{Q}_0^{(a,A_{1g})}$	M_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0\\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$

Table 7: Cluster SAMB.

symbol	type	cluster	form
\mathbb{Y}_1	$\mathbb{Q}_0^{(s,A_{1g})}$	S_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$
\mathbb{Y}_2	$\mathbb{Q}_0^{(b,A_{1g})}$	B_1	$\begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$

Table 8: Polar harmonics.

No.	No. symbol rank		irrep.	mul.	mul. comp.		
1	$\mathbb{Q}_0^{(A_{1g})}$	0	A_{1g}	_	_	1	

 \bullet Group info.: Generator = $\{3^{+}_{\ 001}|0\},\ \{2_{001}|0\},\ \{2_{110}|0\},\ \{-1|0\}$

Table 9: Conjugacy class (point-group part).

rep. SO	symmetry operations
{1 0}	{1 0}
$\{2_{001} 0\}$	$\{2_{001} 0\}$
$\{2_{100} 0\}$	$\{2_{100} 0\}, \{2_{010} 0\}, \{2_{110} 0\}$
$\{2_{120} 0\}$	${2_{120} 0}, {2_{210} 0}, {2_{1-10} 0}$
$\{3^{+}_{001} 0\}$	$\{3^{+}_{001} 0\}, \{3^{-}_{001} 0\}$
$\{6^{+}_{001} 0\}$	$\{6^{+}_{001} 0\}, \{6^{-}_{001} 0\}$
$\{-1 0\}$	$\{-1 0\}$
$\{m_{100} 0\}$	$\{m_{100} 0\}, \{m_{010} 0\}, \{m_{110} 0\}$
$\{m_{001} 0\}$	$\{m_{001} 0\}$
$\{m_{120} 0\}$	$\{m_{120} 0\}, \{m_{210} 0\}, \{m_{1-10} 0\}$
$\{-3^{+}_{001} 0\}$	$\{-3^{+}_{001} 0\}, \{-3^{-}_{001} 0\}$
$\{-6^{+}_{001} 0\}$	$\{-6^{+}_{001} 0\}, \{-6^{-}_{001} 0\}$

Table 10: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	$\{1 0\}$	2	$\{2_{001} 0\}$	3	$\{2_{100} 0\}$	4	$\{2_{010} 0\}$	5	$\{2_{110} 0\}$

 $continued \dots$

Table 10

-	No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
	6	$\{2_{120} 0\}$	7	$\{2_{210} 0\}$	8	$\{2_{1-10} 0\}$	9	$\{3^{+}_{001} 0\}$	10	$\{3^{-}_{001} 0\}$
	11	$\{6^{+}_{001} 0\}$	12	$\{6^{-}_{001} 0\}$	13	$\{-1 0\}$	14	$\{m_{100} 0\}$	15	$\{m_{010} 0\}$
	16	$\{m_{110} 0\}$	17	$\{m_{001} 0\}$	18	$\{m_{120} 0\}$	19	$\{m_{210} 0\}$	20	$\{m_{1-10} 0\}$
	21	$\{-3^{+}_{001} 0\}$	22	$\{-3^{-}_{001} 0\}$	23	$\{-6^{+}_{001} 0\}$	24	$\{-6^{-}_{001} 0\}$		

Table 11: Character table (point-group part).

	1	2001	2100	2120	3 ⁺ ₀₀₁	6 ⁺ ₀₀₁	-1	m ₁₀₀	m ₀₀₁	m ₁₂₀	-3^{+}_{001}	-6^{+}_{001}
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1
A_{2g}	1	1	-1	-1	1	1	1	-1	1	-1	1	1
B_{1g}	1	-1	-1	1	1	-1	1	-1	-1	1	1	-1
B_{2g}	1	-1	1	-1	1	-1	1	1	-1	-1	1	-1
E_{1g}	2	-2	0	0	-1	1	2	0	-2	0	-1	1
E_{2g}	2	2	0	0	-1	-1	2	0	2	0	-1	-1
A_{1u}	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
A_{2u}	1	1	-1	-1	1	1	-1	1	-1	1	-1	-1
B_{1u}	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
B_{2u}	1	-1	1	-1	1	-1	-1	-1	1	1	-1	1
E_{1u}	2	-2	0	0	-1	1	-2	0	2	0	1	-1
E_{2u}	2	2	0	0	-1	-1	-2	0	-2	0	1	1

Table 12: Parity conversion.

\leftrightarrow	\leftrightarrow	\leftrightarrow	\leftrightarrow	\leftrightarrow
$A_{1g} (A_{1u})$	$A_{2g} (A_{2u})$	B_{1g} (B_{1u})	$B_{2g} (B_{2u})$	E_{1g} (E_{1u})
$E_{2g} (E_{2u})$	$A_{1u} (A_{1g})$	$A_{2u} (A_{2g})$	$B_{1u} (B_{1g})$	$B_{2u} (B_{2g})$
E_{1u} (E_{1g})	$E_{2u} (E_{2g})$			

Table 13: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_{1u}	E_{2u}
A_{1g}	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_{1u}	E_{2u}
A_{2g}		A_{1g}	B_{2g}	B_{1g}	E_{1g}	E_{2g}	A_{2u}	A_{1u}	B_{2u}	B_{1u}	E_{1u}	E_{2u}
B_{1g}			A_{1g}	A_{2g}	E_{2g}	E_{1g}	B_{1u}	B_{2u}	A_{1u}	A_{2u}	E_{2u}	E_{1u}
B_{2g}				A_{1g}	E_{2g}	E_{1g}	B_{2u}	B_{1u}	A_{2u}	A_{1u}	E_{2u}	E_{1u}
E_{1g}					$A_{1g} + E_{2g}$	$B_{1g} + B_{2g} + E_{1g}$	E_{1u}	E_{1u}	E_{2u}	E_{2u}	$A_{1u} + A_{2u} + E_{2u}$	$B_{1u} + B_{2u} + E_{1u}$
E_{2g}						$A_{1g} + E_{2g}$	E_{2u}	E_{2u}	E_{1u}	E_{1u}	$B_{1u} + B_{2u} + E_{1u}$	$A_{1u} + A_{2u} + E_{2u}$
A_{1u}							A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}
A_{2u}								A_{1g}	B_{2g}	B_{1g}	E_{1g}	E_{2g}
B_{1u}									A_{1g}	A_{2g}	E_{2g}	E_{1g}
B_{2u}										A_{1g}	E_{2g}	E_{1g}
E_{1u}											$A_{1g} + E_{2g}$	$B_{1g} + B_{2g} + E_{1g}$
E_{2u}												$A_{1g} + E_{2g}$

Table 14: Anti-symmetric product, $[\Gamma \otimes \Gamma]_-$.

A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_{1u}	E_{2u}
	_	_	_	A_{2g}	A_{2g}	_	_	_	_	A_{2g}	A_{2g}

Table 15: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix} 1+\sqrt{3} & -1+\sqrt{3} & 1 \end{pmatrix}$	2	$\left(-\sqrt{3}-1 1-\sqrt{3} 1\right)$	3	$\begin{pmatrix} 2 & 1 - \sqrt{3} & -1 \end{pmatrix}$	4	$\left(-\sqrt{3}-1 -2 -1\right)$
5	$\begin{pmatrix} -1 + \sqrt{3} & 1 + \sqrt{3} & -1 \end{pmatrix}$	6	$\begin{pmatrix} -2 & -1 + \sqrt{3} & -1 \end{pmatrix}$	7	$\begin{pmatrix} 1+\sqrt{3} & 2 & -1 \end{pmatrix}$	8	$\begin{pmatrix} 1 - \sqrt{3} & -\sqrt{3} - 1 & -1 \end{pmatrix}$
9	$\begin{pmatrix} 1 - \sqrt{3} & 2 & 1 \end{pmatrix}$	10	$\begin{pmatrix} -2 & -\sqrt{3} - 1 & 1 \end{pmatrix}$	11	$\begin{pmatrix} 2 & 1 + \sqrt{3} & 1 \end{pmatrix}$	12	$\begin{pmatrix} -1 + \sqrt{3} & -2 & 1 \end{pmatrix}$
13	$\left(-\sqrt{3}-1 1-\sqrt{3} -1\right)$	14	$\begin{pmatrix} -2 & -1 + \sqrt{3} & 1 \end{pmatrix}$	15	$\begin{pmatrix} 1+\sqrt{3} & 2 & 1 \end{pmatrix}$	16	$\begin{pmatrix} 1 - \sqrt{3} & -\sqrt{3} - 1 & 1 \end{pmatrix}$
17	$\begin{pmatrix} 1+\sqrt{3} & -1+\sqrt{3} & -1 \end{pmatrix}$	18	$\begin{pmatrix} 2 & 1 - \sqrt{3} & 1 \end{pmatrix}$	19	$\left(-\sqrt{3}-1 -2 1\right)$	20	$\begin{pmatrix} -1 + \sqrt{3} & 1 + \sqrt{3} & 1 \end{pmatrix}$
21	$\begin{pmatrix} -1 + \sqrt{3} & -2 & -1 \end{pmatrix}$	22	$\begin{pmatrix} 2 & 1 + \sqrt{3} & -1 \end{pmatrix}$	23	$\begin{pmatrix} -2 & -\sqrt{3} - 1 & -1 \end{pmatrix}$	24	$\begin{pmatrix} 1 - \sqrt{3} & 2 & -1 \end{pmatrix}$

Table 16: Virtual-cluster basis.

symbol	1	2	3	4	5	6	7	8	9	10
$\mathbb{Q}_0^{(A_{1g})}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$						
$\mathbb{Q}_1^{(A_{2u})}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$						
$\mathbb{Q}_{1,0}^{(E_{1u})}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$
	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$
(F)	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{12}$						
$\mathbb{Q}_{1,1}^{(E_{1u})}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$
	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$
(E _{1,0})	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$	<i>(</i> 2		/6	/o /o	<u> </u>	
$\mathbb{Q}_{2,0}^{(E_{1g})}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$
	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$
${(E_{1a})}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\frac{\sqrt{6}}{12}}{\frac{\sqrt{6}}{12}}$,/2 ,/6	√2 · √6	<u>√6</u>	√6 √2	<u>√6</u>	
$\mathbb{Q}_{2,1}^{(E_{1g})}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$ $\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24} \\ -\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$		$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24} \\ -\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{12} \\ -\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$\frac{\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}}{-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$ $\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$
	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{5}}{12}$ $-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$ $\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8} \\ -\frac{\sqrt{6}}{12}$	$\frac{\sqrt{5}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{3}}{24}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{5}}{24}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{3}}{24} + \frac{\sqrt{2}}{8}$
$\mathbb{Q}_{2,0}^{(E_{2g})}$	$\frac{1}{12}$	$\frac{-\frac{1}{8} - \frac{1}{24}}{\frac{1}{4}}$		$-\frac{1}{12}$	$-\frac{1}{4}$	1	0	1	0	1
$\mathbb{Q}_{2,0}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$ $\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$ $-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$ $\frac{1}{4}$	0	$-\frac{1}{4}$ $-\frac{1}{4}$
	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{4}{4}$	$\overline{4}$	U	$-\frac{1}{4}$	$\overline{4}$	$\overline{4}$	U	$-\frac{1}{4}$
$\mathbb{Q}_{2,1}^{(E_{2g})}$	$-\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{12}$	$\frac{\overline{4}}{\frac{\sqrt{3}}{12}}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{12}$
$\mathcal{Q}_{2,1}$	$-\frac{\sqrt{3}}{12}$	$\frac{12}{\frac{\sqrt{3}}{6}}$	$-\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{6}$	$\frac{12}{\sqrt{3}}$	$-\frac{6}{6}$ $-\frac{\sqrt{3}}{12}$	$\frac{12}{\sqrt{3}}$	$-\frac{\sqrt{3}}{6}$	$\frac{-\frac{1}{12}}{\frac{\sqrt{3}}{12}}$
	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{12}$	$\frac{12}{\sqrt{3}}$	6	12	12	12	6	12
$\mathbb{Q}_{3}^{(B_{1u})}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
*-3	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$
	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	12	12	12	12	12	12
$\mathbb{Q}_{3}^{(B_{2u})}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
-	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$

 $continued\ \dots$

Table 16

symbol	1	2	3	4	5	6	7	8	9	10
	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$						
$\mathbb{Q}_{3,0}^{(E_{2u})}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	$-\frac{1}{4}$
	$-\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	$-\frac{1}{4}$
	0	$\frac{1}{4}$	$\frac{1}{4}$	0						
$\mathbb{Q}_{3,1}^{(E_{2u})}$	$-\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{12}$
	$-\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{12}$
(P.)	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{6}$						
$\mathbb{Q}_4^{(B_{1g})}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
$\overline{\mathbb{Q}_{4}^{(B_{2g})}}$	$\frac{\frac{\sqrt{6}}{12}}{\frac{\sqrt{6}}{12}}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$ $\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$ $\frac{\sqrt{6}}{12}$	<u>√6</u>	√ <u>6</u>	<u>√6</u>	$\sqrt{6}$	√ <u>6</u>	$\frac{\sqrt{6}}{12}$
\mathbb{Q}_4^{-g}	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$ $-\frac{\sqrt{6}}{12}$	$\frac{\frac{\sqrt{6}}{12}}{\frac{\sqrt{6}}{12}}$	$\frac{\sqrt{6}}{12}$ $\frac{\sqrt{6}}{12}$	$\frac{\frac{\sqrt{6}}{12}}{\frac{\sqrt{6}}{12}}$	$-\frac{\sqrt{6}}{12}$ $\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$ $-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$ $-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$ $-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$
	$-\frac{1}{12}$ $\frac{\sqrt{6}}{12}$	$-\frac{1}{12}$ $\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	12	12	$-\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{1}{12}$
$\mathbb{Q}_{4,0}^{(E_{2g},1)}$	$\frac{12}{\frac{\sqrt{3}}{12}}$	$\frac{12}{\frac{\sqrt{3}}{12}}$	$\frac{\sqrt{3}}{12}$	$-\frac{12}{6}$	$\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{12}$
₹4,0	$\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{6}$	$\begin{array}{r} 12 \\ \frac{\sqrt{3}}{12} \end{array}$	$\frac{6}{\sqrt{3}}$	$-\frac{\sqrt{3}}{6}$	$\frac{12}{\sqrt{3}}$	$\begin{array}{c} 6\\ \frac{\sqrt{3}}{12} \end{array}$	$\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{12}$
	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{6}$	6	12	12	12	6	12
$\mathbb{Q}_{4,1}^{(E_{2g},1)}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	$-\frac{1}{4}$
	$-\frac{1}{4}$	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$
	0	$-\frac{1}{4}$	$-\frac{1}{4}$	0						
$\mathbb{Q}_{5,0}^{(E_{1u},1)}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$
	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$
	$-\frac{\sqrt{6}}{12}$	$\frac{\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}}{\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$						
$\mathbb{Q}_{5,1}^{(E_{1u},1)}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$
	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$
	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{12}$./2	./2	./2	./2	./2	./2
$\mathbb{Q}_{5,0}^{(E_{2u},1)}$	$\frac{\sqrt{3}}{12}$ $\sqrt{3}$	$\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{12}$ $\sqrt{3}$	$\frac{\sqrt{3}}{6}$ $\sqrt{3}$	$-\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{12}$ $\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{12}$ $\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{12}$
	$\frac{\frac{\sqrt{3}}{12}}{\frac{\sqrt{3}}{6}}$	$-\frac{\sqrt{3}}{6}$ $-\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{3}}{12}$ $-\frac{\sqrt{3}}{12}$	$\frac{\frac{\sqrt{3}}{12}}{\frac{\sqrt{3}}{6}}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt[4]{5}}{12}$	$-\frac{\sqrt{3}}{12}$	$\frac{\sqrt[4]{5}}{12}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{12}$
$\mathbb{Q}_{5,1}^{(E_{2u},1)}$		$-\frac{\sqrt{3}}{12}$	$-\frac{\sqrt{5}}{12}$ $\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	$-\frac{1}{4}$	0	$-\frac{1}{4}$
₹5,1	$\overline{4}$	4	4		4	4		4		$\overline{4}$

 $continued\ \dots$

Table 16

symbol	1	2	3	4	5	6	7	8	9	10
	$-\frac{1}{4}$	0	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$
	0	$\frac{1}{4}$	$\frac{1}{4}$	0						
$\mathbb{Q}_6^{(A_{2g})}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$						
$\mathbb{Q}_{6,0}^{(E_{1g},1)}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$
	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$
	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$						
$\mathbb{Q}_{6,1}^{(E_{1g},1)}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$
	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{2}}{8} - \frac{\sqrt{6}}{24}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$
	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{24} + \frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8} + \frac{\sqrt{6}}{24}$	$-\frac{\sqrt{6}}{12}$						
$\mathbb{Q}_7^{(A_{1u})}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$							
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$
	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$						