

Table 1 Harmonics for rank 0.

No.	multipole	definition
1	$\mathbb{Q}_0(A_{1g})$	C_0

Table 2 Harmonics for rank 1.

No.	multipole	definition
2	$\mathbb{Q}_1(A_{2u})$	C_0
3	$\mathbb{Q}_{1,1}(E_{1u})$	C_1
4	$\mathbb{Q}_{1,2}(E_{1u})$	S_1

Table 3 Harmonics for rank 2.

No.	multipole	definition
5	$\mathbb{Q}_2(A_{1g})$	C_0
6	$\mathbb{Q}_{2,1}(E_{1g})$	S_1
7	$\mathbb{Q}_{2,2}(E_{1g})$	$-C_1$
8	$\mathbb{Q}_{2,1}(E_{2g})$	C_2
9	$\mathbb{Q}_{2,2}(E_{2g})$	$-S_2$

Table 4 Harmonics for rank 3.

No.	multipole	definition
10	$\mathbb{Q}_3(A_{2u})$	C_0
11	$\mathbb{Q}_3(B_{1u})$	S_3
12	$\mathbb{Q}_3(B_{2u})$	C_3
13	$\mathbb{Q}_{3,1}(E_{1u})$	C_1
14	$\mathbb{Q}_{3,2}(E_{1u})$	S_1
15	$\mathbb{Q}_{3,1}(E_{2u})$	S_2
16	$\mathbb{Q}_{3,2}(E_{2u})$	C_2

Table 5 Harmonics for rank 4.

No.	multipole	definition
17	$\mathbb{Q}_4(A_{1g})$	C_0
18	$\mathbb{Q}_4(B_{1g})$	C_3
19	$\mathbb{Q}_4(B_{2g})$	S_3
20	$\mathbb{Q}_{4,1}(E_{1g})$	S_1
21	$\mathbb{Q}_{4,2}(E_{1g})$	$-C_1$
22	$\mathbb{Q}_{4,1}(E_{2g}, 1)$	C_4
23	$\mathbb{Q}_{4,2}(E_{2g}, 1)$	S_4
24	$\mathbb{Q}_{4,1}(E_{2g}, 2)$	C_2
25	$\mathbb{Q}_{4,2}(E_{2g}, 2)$	$-S_2$

Table 6 Harmonics for rank 5.

No.	multipole	definition
26	$\mathbb{Q}_5(A_{2u})$	C_0
27	$\mathbb{Q}_5(B_{1u})$	S_3
28	$\mathbb{Q}_5(B_{2u})$	C_3
29	$\mathbb{Q}_{5,1}(E_{1u}, 1)$	C_5
30	$\mathbb{Q}_{5,2}(E_{1u}, 1)$	$-S_5$
31	$\mathbb{Q}_{5,1}(E_{1u}, 2)$	C_1
32	$\mathbb{Q}_{5,2}(E_{1u}, 2)$	S_1
33	$\mathbb{Q}_{5,1}(E_{2u}, 1)$	$-S_4$
34	$\mathbb{Q}_{5,2}(E_{2u}, 1)$	C_4
35	$\mathbb{Q}_{5,1}(E_{2u}, 2)$	S_2
36	$\mathbb{Q}_{5,2}(E_{2u}, 2)$	C_2

Table 7 Harmonics for rank 6.

No.	multipole	definition
37	$\mathbb{Q}_6(A_{1g}, 1)$	C_0
38	$\mathbb{Q}_6(A_{1g}, 2)$	C_6
39	$\mathbb{Q}_6(A_{2g})$	S_6
40	$\mathbb{Q}_6(B_{1g})$	C_3
41	$\mathbb{Q}_6(B_{2g})$	S_3
42	$\mathbb{Q}_{6,1}(E_{1g}, 1)$	S_5
43	$\mathbb{Q}_{6,2}(E_{1g}, 1)$	C_5
44	$\mathbb{Q}_{6,1}(E_{1g}, 2)$	S_1
45	$\mathbb{Q}_{6,2}(E_{1g}, 2)$	$-C_1$
46	$\mathbb{Q}_{6,1}(E_{2g}, 1)$	C_4
47	$\mathbb{Q}_{6,2}(E_{2g}, 1)$	S_4
48	$\mathbb{Q}_{6,1}(E_{2g}, 2)$	C_2
49	$\mathbb{Q}_{6,2}(E_{2g}, 2)$	$-S_2$

Table 8 Harmonics for rank 7.

No.	multipole	definition
50	$\mathbb{Q}_7(A_{1u})$	S_6
51	$\mathbb{Q}_7(A_{2u}, 1)$	C_0
52	$\mathbb{Q}_7(A_{2u}, 2)$	C_6
53	$\mathbb{Q}_7(B_{1u})$	S_3
54	$\mathbb{Q}_7(B_{2u})$	C_3
55	$\mathbb{Q}_{7,1}(E_{1u}, 1)$	C_7
56	$\mathbb{Q}_{7,2}(E_{1u}, 1)$	S_7
57	$\mathbb{Q}_{7,1}(E_{1u}, 2)$	C_5
58	$\mathbb{Q}_{7,2}(E_{1u}, 2)$	$-S_5$
59	$\mathbb{Q}_{7,1}(E_{1u}, 3)$	C_1
60	$\mathbb{Q}_{7,2}(E_{1u}, 3)$	S_1
61	$\mathbb{Q}_{7,1}(E_{2u}, 1)$	$-S_4$
62	$\mathbb{Q}_{7,2}(E_{2u}, 1)$	C_4
63	$\mathbb{Q}_{7,1}(E_{2u}, 2)$	S_2
64	$\mathbb{Q}_{7,2}(E_{2u}, 2)$	C_2

Table 9 Harmonics for rank 8.

No.	multipole	definition
65	$\mathbb{Q}_8(A_{1g}, 1)$	C_0
66	$\mathbb{Q}_8(A_{1g}, 2)$	C_6
67	$\mathbb{Q}_8(A_{2g})$	S_6
68	$\mathbb{Q}_8(B_{1g})$	C_3
69	$\mathbb{Q}_8(B_{2g})$	S_3
70	$\mathbb{Q}_{8,1}(E_{1g}, 1)$	S_7
71	$\mathbb{Q}_{8,2}(E_{1g}, 1)$	$-C_7$
72	$\mathbb{Q}_{8,1}(E_{1g}, 2)$	S_5
73	$\mathbb{Q}_{8,2}(E_{1g}, 2)$	C_5
74	$\mathbb{Q}_{8,1}(E_{1g}, 3)$	S_1
75	$\mathbb{Q}_{8,2}(E_{1g}, 3)$	$-C_1$
76	$\mathbb{Q}_{8,1}(E_{2g}, 1)$	C_8
77	$\mathbb{Q}_{8,2}(E_{2g}, 1)$	$-S_8$
78	$\mathbb{Q}_{8,1}(E_{2g}, 2)$	C_4
79	$\mathbb{Q}_{8,2}(E_{2g}, 2)$	S_4
80	$\mathbb{Q}_{8,1}(E_{2g}, 3)$	C_2
81	$\mathbb{Q}_{8,2}(E_{2g}, 3)$	$-S_2$

Table 10 Harmonics for rank 9.

No.	multipole	definition
82	$\mathbb{Q}_9(A_{1u})$	S_6
83	$\mathbb{Q}_9(A_{2u}, 1)$	C_0
84	$\mathbb{Q}_9(A_{2u}, 2)$	C_6
85	$\mathbb{Q}_9(B_{1u}, 1)$	S_9
86	$\mathbb{Q}_9(B_{1u}, 2)$	S_3
87	$\mathbb{Q}_9(B_{2u}, 1)$	C_9
88	$\mathbb{Q}_9(B_{2u}, 2)$	C_3
89	$\mathbb{Q}_{9,1}(E_{1u}, 1)$	C_7
90	$\mathbb{Q}_{9,2}(E_{1u}, 1)$	S_7
91	$\mathbb{Q}_{9,1}(E_{1u}, 2)$	C_5
92	$\mathbb{Q}_{9,2}(E_{1u}, 2)$	$-S_5$
93	$\mathbb{Q}_{9,1}(E_{1u}, 3)$	C_1
94	$\mathbb{Q}_{9,2}(E_{1u}, 3)$	S_1
95	$\mathbb{Q}_{9,1}(E_{2u}, 1)$	S_8
96	$\mathbb{Q}_{9,2}(E_{2u}, 1)$	C_8
97	$\mathbb{Q}_{9,1}(E_{2u}, 2)$	$-S_4$
98	$\mathbb{Q}_{9,2}(E_{2u}, 2)$	C_4
99	$\mathbb{Q}_{9,1}(E_{2u}, 3)$	S_2
100	$\mathbb{Q}_{9,2}(E_{2u}, 3)$	C_2

Table 11 Harmonics for rank 10.

No.	multipole	definition
101	$\mathbb{Q}_{10}(A_{1g}, 1)$	C_0
102	$\mathbb{Q}_{10}(A_{1g}, 2)$	C_6
103	$\mathbb{Q}_{10}(A_{2g})$	S_6
104	$\mathbb{Q}_{10}(B_{1g}, 1)$	C_9
105	$\mathbb{Q}_{10}(B_{1g}, 2)$	C_3
106	$\mathbb{Q}_{10}(B_{2g}, 1)$	S_9
107	$\mathbb{Q}_{10}(B_{2g}, 2)$	S_3
108	$\mathbb{Q}_{10,1}(E_{1g}, 1)$	S_7
109	$\mathbb{Q}_{10,2}(E_{1g}, 1)$	$-C_7$
110	$\mathbb{Q}_{10,1}(E_{1g}, 2)$	S_5
111	$\mathbb{Q}_{10,2}(E_{1g}, 2)$	C_5
112	$\mathbb{Q}_{10,1}(E_{1g}, 3)$	S_1
113	$\mathbb{Q}_{10,2}(E_{1g}, 3)$	$-C_1$
114	$\mathbb{Q}_{10,1}(E_{2g}, 1)$	C_{10}
115	$\mathbb{Q}_{10,2}(E_{2g}, 1)$	S_{10}
116	$\mathbb{Q}_{10,1}(E_{2g}, 2)$	C_8
117	$\mathbb{Q}_{10,2}(E_{2g}, 2)$	$-S_8$
118	$\mathbb{Q}_{10,1}(E_{2g}, 3)$	C_4
119	$\mathbb{Q}_{10,2}(E_{2g}, 3)$	S_4
120	$\mathbb{Q}_{10,1}(E_{2g}, 4)$	C_2
121	$\mathbb{Q}_{10,2}(E_{2g}, 4)$	$-S_2$

Table 12 Harmonics for rank 11.

No.	multipole	definition
122	$\mathbb{Q}_{11}(A_{1u})$	S_6
123	$\mathbb{Q}_{11}(A_{2u}, 1)$	C_0
124	$\mathbb{Q}_{11}(A_{2u}, 2)$	C_6
125	$\mathbb{Q}_{11}(B_{1u}, 1)$	S_9
126	$\mathbb{Q}_{11}(B_{1u}, 2)$	S_3
127	$\mathbb{Q}_{11}(B_{2u}, 1)$	C_9
128	$\mathbb{Q}_{11}(B_{2u}, 2)$	C_3
129	$\mathbb{Q}_{11,1}(E_{1u}, 1)$	C_{11}
130	$\mathbb{Q}_{11,2}(E_{1u}, 1)$	$-S_{11}$
131	$\mathbb{Q}_{11,1}(E_{1u}, 2)$	C_7
132	$\mathbb{Q}_{11,2}(E_{1u}, 2)$	S_7
133	$\mathbb{Q}_{11,1}(E_{1u}, 3)$	C_5
134	$\mathbb{Q}_{11,2}(E_{1u}, 3)$	$-S_5$
135	$\mathbb{Q}_{11,1}(E_{1u}, 4)$	C_1
136	$\mathbb{Q}_{11,2}(E_{1u}, 4)$	S_1
137	$\mathbb{Q}_{11,1}(E_{2u}, 1)$	$-S_{10}$
138	$\mathbb{Q}_{11,2}(E_{2u}, 1)$	C_{10}
139	$\mathbb{Q}_{11,1}(E_{2u}, 2)$	S_8
140	$\mathbb{Q}_{11,2}(E_{2u}, 2)$	C_8
141	$\mathbb{Q}_{11,1}(E_{2u}, 3)$	$-S_4$
142	$\mathbb{Q}_{11,2}(E_{2u}, 3)$	C_4
143	$\mathbb{Q}_{11,1}(E_{2u}, 4)$	S_2
144	$\mathbb{Q}_{11,2}(E_{2u}, 4)$	C_2