

\* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

| No. | tag   | matrix (polar)   | det | TR |
|-----|---|--|-----|----|
| 1   | {1 0}   | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                  | 1   | 1  |
| 2   | {4 <sup>+</sup> <sub>001</sub>   $\frac{1}{4}\frac{3}{4}\frac{1}{4}$ }  | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 3   | {4 <sup>-</sup> <sub>001</sub>   $\frac{3}{4}\frac{3}{4}\frac{1}{4}$ }  | $\begin{bmatrix} 0 & 1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 4   | {2 <sub>100</sub>   $\frac{1}{2}\frac{1}{2}0$ }                         | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$            | 1   | 1  |
| 5   | {2 <sub>010</sub>   $\frac{1}{2}00$ }                                   | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                      | 1   | 1  |
| 6   | {2 <sub>001</sub>   $0\frac{1}{2}0$ }                                   | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$                      | 1   | 1  |
| 7   | {2 <sub>110</sub>   $\frac{3}{4}\frac{1}{4}\frac{1}{4}$ }               | $\begin{bmatrix} 0 & 1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 8   | {2 <sub>1-10</sub>   $\frac{1}{4}\frac{1}{4}\frac{1}{4}$ }              | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$ | 1   | 1  |
| 9   | {-1 0}  | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                               | -1  | 1  |
| 10  | {-4 <sup>+</sup> <sub>001</sub>   $\frac{3}{4}\frac{1}{4}\frac{3}{4}$ } | $\begin{bmatrix} 0 & 1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$  | -1  | 1  |
| 11  | {-4 <sup>-</sup> <sub>001</sub>   $\frac{1}{4}\frac{1}{4}\frac{3}{4}$ } | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$  | -1  | 1  |
| 12  | {m <sub>100</sub>   $00\frac{1}{2}$ }                                   | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$                       | -1  | 1  |
| 13  | {m <sub>010</sub>   $0\frac{1}{2}\frac{1}{2}$ }                         | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$             | -1  | 1  |

continued ...

Table 1

| No. | tag  | matrix (polar)   | det | TR |
|-----|--|--|-----|----|
| 14  | $\{m_{001} 0\frac{1}{2}0\}$                        | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$                       | -1  | 1  |
| 15  | $\{m_{110} \frac{3}{4}\frac{1}{4}\frac{1}{4}\}$    | $\begin{bmatrix} 0 & -1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$  | -1  | 1  |
| 16  | $\{m_{1-10} \frac{1}{4}\frac{1}{4}\frac{1}{4}\}$   | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$    | -1  | 1  |
| 17  | $\{1 \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$          | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$    | 1   | 1  |
| 18  | $\{4_{001}^+ \frac{3}{4}\frac{1}{4}\frac{3}{4}\}$  | $\begin{bmatrix} 0 & -1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 19  | $\{4_{001}^- \frac{1}{4}\frac{1}{4}\frac{3}{4}\}$  | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 20  | $\{2_{100} 00\frac{1}{2}\}$                        | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$                      | 1   | 1  |
| 21  | $\{2_{010} 0\frac{1}{2}\frac{1}{2}\}$              | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$            | 1   | 1  |
| 22  | $\{2_{001} \frac{1}{2}0\frac{1}{2}\}$              | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$            | 1   | 1  |
| 23  | $\{2_{110} \frac{1}{4}\frac{3}{4}\frac{3}{4}\}$    | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 24  | $\{2_{1-10} \frac{3}{4}\frac{3}{4}\frac{3}{4}\}$   | $\begin{bmatrix} 0 & -1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$ | 1   | 1  |
| 25  | $\{-1 \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$         | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1  | 1  |
| 26  | $\{-4_{001}^+ \frac{1}{4}\frac{3}{4}\frac{1}{4}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$  | -1  | 1  |
| 27  | $\{-4_{001}^- \frac{3}{4}\frac{3}{4}\frac{1}{4}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$  | -1  | 1  |

continued ...

Table 1

| No. | tag  | matrix (polar)   | det | TR |
|-----|--|--|-----|----|
| 28  | $\{m_{100} \frac{1}{2}\frac{1}{2}0\}$              | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$             | -1  | 1  |
| 29  | $\{m_{010} \frac{1}{2}00\}$                        | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                       | -1  | 1  |
| 30  | $\{m_{001} \frac{1}{2}0\frac{1}{2}\}$              | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$             | -1  | 1  |
| 31  | $\{m_{110} \frac{1}{4}\frac{3}{4}\frac{3}{4}\}$    | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$  | -1  | 1  |
| 32  | $\{m_{1-10} \frac{3}{4}\frac{3}{4}\frac{3}{4}\}$   | $\begin{bmatrix} 0 & 1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$    | -1  | 1  |
| 33  | $\{1' 0\}$   | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                  | 1   | -1 |
| 34  | $\{4^+_{001}' \frac{1}{4}\frac{3}{4}\frac{1}{4}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$   | 1   | -1 |
| 35  | $\{4^-_{001}' \frac{3}{4}\frac{3}{4}\frac{1}{4}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$   | 1   | -1 |
| 36  | $\{2_{100}' \frac{1}{2}\frac{1}{2}0\}$             | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$            | 1   | -1 |
| 37  | $\{2_{010}' \frac{1}{2}00\}$                       | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                      | 1   | -1 |
| 38  | $\{2_{001}' 0\frac{1}{2}0\}$                       | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$                      | 1   | -1 |
| 39  | $\{2_{110}' \frac{3}{4}\frac{1}{4}\frac{1}{4}\}$   | $\begin{bmatrix} 0 & 1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$   | 1   | -1 |
| 40  | $\{2_{1-10}' \frac{1}{4}\frac{1}{4}\frac{1}{4}\}$  | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$ | 1   | -1 |
| 41  | $\{-1' 0\}$  | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                               | -1  | -1 |

continued ...

Table 1

| No. | tag  | matrix (polar)  | det | TR |
|-----|--|---|-----|----|
| 42  | $\{-4_{001}' \frac{3}{4}\frac{1}{4}\frac{3}{4}\}$        | $\begin{bmatrix} 0 & 1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$ | -1  | -1 |
| 43  | $\{-4_{001}' \frac{1}{4}\frac{1}{4}\frac{3}{4}\}$        | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$ | -1  | -1 |
| 44  | $\{\text{m}_{100}' 00\frac{1}{2}\}$                      | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$                      | -1  | -1 |
| 45  | $\{\text{m}_{010}' 0\frac{1}{2}\frac{1}{2}\}$            | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$            | -1  | -1 |
| 46  | $\{\text{m}_{001}' 0\frac{1}{2}0\}$                      | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$                      | -1  | -1 |
| 47  | $\{\text{m}_{110}' \frac{3}{4}\frac{1}{4}\frac{1}{4}\}$  | $\begin{bmatrix} 0 & -1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$ | -1  | -1 |
| 48  | $\{\text{m}_{1-10}' \frac{1}{4}\frac{1}{4}\frac{1}{4}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$   | -1  | -1 |
| 49  | $\{1' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$               | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$   | 1   | -1 |
| 50  | $\{4_{001}' \frac{3}{4}\frac{1}{4}\frac{3}{4}\}$         | $\begin{bmatrix} 0 & -1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$  | 1   | -1 |
| 51  | $\{4_{001}' \frac{1}{4}\frac{1}{4}\frac{3}{4}\}$         | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$  | 1   | -1 |
| 52  | $\{\text{2}_{100}' 00\frac{1}{2}\}$                      | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$                     | 1   | -1 |
| 53  | $\{\text{2}_{010}' 0\frac{1}{2}\frac{1}{2}\}$            | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$           | 1   | -1 |
| 54  | $\{\text{2}_{001}' \frac{1}{2}0\frac{1}{2}\}$            | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$           | 1   | -1 |
| 55  | $\{\text{2}_{110}' \frac{1}{4}\frac{3}{4}\frac{3}{4}\}$  | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$  | 1   | -1 |

continued ...

Table 1

| No. | tag  | matrix (polar)   | det | TR |
|-----|--|--|-----|----|
| 56  | $\{2_{1-10}' \frac{3}{4}\frac{3}{4}\frac{3}{4}\}$        | $\begin{bmatrix} 0 & -1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$ | 1   | -1 |
| 57  | $\{-1' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$              | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 58  | $\{-4_{001}' \frac{1}{4}\frac{3}{4}\frac{1}{4}\}$        | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$  | -1  | -1 |
| 59  | $\{-4_{001}' \frac{3}{4}\frac{3}{4}\frac{1}{4}\}$        | $\begin{bmatrix} 0 & -1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$  | -1  | -1 |
| 60  | $\{\text{m}_{100}' \frac{1}{2}\frac{1}{2}0\}$            | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$             | -1  | -1 |
| 61  | $\{\text{m}_{010}' \frac{1}{2}00\}$                      | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                       | -1  | -1 |
| 62  | $\{\text{m}_{001}' \frac{1}{2}0\frac{1}{2}\}$            | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$             | -1  | -1 |
| 63  | $\{\text{m}_{110}' \frac{1}{4}\frac{3}{4}\frac{3}{4}\}$  | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$  | -1  | -1 |
| 64  | $\{\text{m}_{1-10}' \frac{3}{4}\frac{3}{4}\frac{3}{4}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$    | -1  | -1 |