

Response Tensors up to 4th rank in C_1

— polar tensors —

$$C^{(0,Q)} = (C^{(0,Q)})$$

$$C^{(0,Q)} = Q_0$$

$$C^{(1,Q)} = \begin{pmatrix} C_x^{(1,Q)} & C_y^{(1,Q)} & C_z^{(1,Q)} \end{pmatrix}$$

$$C_x^{(1,Q)} = Q_x$$

$$C_y^{(1,Q)} = Q_y$$

$$C_z^{(1,Q)} = Q_z$$

$$S^{(2,Q)} = \begin{pmatrix} S_{xx}^{(2,Q)} & S_{xy}^{(2,Q)} & S_{xz}^{(2,Q)} \\ S_{xy}^{(2,Q)} & S_{yy}^{(2,Q)} & S_{yz}^{(2,Q)} \\ S_{xz}^{(2,Q)} & S_{yz}^{(2,Q)} & S_{zz}^{(2,Q)} \end{pmatrix}$$

$$S_{xx}^{(2,Q)} = Q_0 - Q_u + Q_v$$

$$S_{xy}^{(2,Q)} = Q_{xy}$$

$$S_{xz}^{(2,Q)} = Q_{zx}$$

$$S_{yy}^{(2,Q)} = Q_0 - Q_u - Q_v$$

$$S_{yz}^{(2,Q)} = Q_{yz}$$

$$S_{zz}^{(2,Q)} = Q_0 + 2Q_u$$

$$A^{(2,Q)} = \begin{pmatrix} 0 & A_{xy}^{(2,Q)} & A_{xz}^{(2,Q)} \\ -A_{xy}^{(2,Q)} & 0 & A_{yz}^{(2,Q)} \\ -A_{xz}^{(2,Q)} & -A_{yz}^{(2,Q)} & 0 \end{pmatrix}$$

$$A_{xy}^{(2,Q)} = G_z$$

$$A_{xz}^{(2,Q)} = -G_y$$

$$A_{yz}^{(2,Q)} = G_x$$

$$S^{(3,Q)} = \begin{pmatrix} S_{1x}^{(3,Q)} & S_{1y}^{(3,Q)} & S_{1z}^{(3,Q)} \\ S_{2x}^{(3,Q)} & S_{2y}^{(3,Q)} & S_{2z}^{(3,Q)} \\ S_{3x}^{(3,Q)} & S_{3y}^{(3,Q)} & S_{3z}^{(3,Q)} \\ S_{4x}^{(3,Q)} & S_{4y}^{(3,Q)} & S_{4z}^{(3,Q)} \\ S_{5x}^{(3,Q)} & S_{5y}^{(3,Q)} & S_{5z}^{(3,Q)} \\ S_{6x}^{(3,Q)} & S_{6y}^{(3,Q)} & S_{6z}^{(3,Q)} \end{pmatrix}$$

$$\begin{aligned}
S_{1x}^{(3,Q)} &= 2Q_x[1] + Q_x[2] + 2Q_x^\alpha \\
S_{1y}^{(3,Q)} &= 2G_{zx}[1] + Q_y[2] - Q_y^\alpha - Q_y^\beta \\
S_{1z}^{(3,Q)} &= -2G_{xy}[1] + Q_z[2] - Q_z^\alpha + Q_z^\beta \\
S_{2x}^{(3,Q)} &= -2G_{yz}[1] + Q_x[2] - Q_x^\alpha + Q_x^\beta \\
S_{2y}^{(3,Q)} &= 2Q_y[1] + Q_y[2] + 2Q_y^\alpha \\
S_{2z}^{(3,Q)} &= 2G_{xy}[1] + Q_z[2] - Q_z^\alpha - Q_z^\beta \\
S_{3x}^{(3,Q)} &= 2G_{yz}[1] + Q_x[2] - Q_x^\alpha - Q_x^\beta \\
S_{3y}^{(3,Q)} &= -2G_{zx}[1] + Q_y[2] - Q_y^\alpha + Q_y^\beta \\
S_{3z}^{(3,Q)} &= 2Q_z[1] + Q_z[2] + 2Q_z^\alpha \\
S_{4x}^{(3,Q)} &= -3G_u[1] - G_v[1] + Q_{xyz} \\
S_{4y}^{(3,Q)} &= -G_{xy}[1] + Q_z[1] - Q_z^\alpha - Q_z^\beta \\
S_{4z}^{(3,Q)} &= G_{zx}[1] + Q_y[1] - Q_y^\alpha + Q_y^\beta \\
S_{5x}^{(3,Q)} &= G_{xy}[1] + Q_z[1] - Q_z^\alpha + Q_z^\beta \\
S_{5y}^{(3,Q)} &= 3G_u[1] - G_v[1] + Q_{xyz} \\
S_{5z}^{(3,Q)} &= -G_{yz}[1] + Q_x[1] - Q_x^\alpha - Q_x^\beta \\
S_{6x}^{(3,Q)} &= -G_{zx}[1] + Q_y[1] - Q_y^\alpha - Q_y^\beta \\
S_{6y}^{(3,Q)} &= G_{yz}[1] + Q_x[1] - Q_x^\alpha + Q_x^\beta \\
S_{6z}^{(3,Q)} &= 2G_v[1] + Q_{xyz}
\end{aligned}$$

$$A^{(3,Q)} = \begin{pmatrix} A_{4x}^{(3,Q)} & A_{4y}^{(3,Q)} & A_{4z}^{(3,Q)} \\ A_{5x}^{(3,Q)} & A_{5y}^{(3,Q)} & A_{5z}^{(3,Q)} \\ A_{6x}^{(3,Q)} & A_{6y}^{(3,Q)} & A_{6z}^{(3,Q)} \end{pmatrix}$$

$$\begin{aligned}
A_{4x}^{(3,Q)} &= G_0 - G_u[2] + G_v[2] \\
A_{4y}^{(3,Q)} &= G_{xy}[2] + Q_z[3] \\
A_{4z}^{(3,Q)} &= G_{zx}[2] - Q_y[3] \\
A_{5x}^{(3,Q)} &= G_{xy}[2] - Q_z[3] \\
A_{5y}^{(3,Q)} &= G_0 - G_u[2] - G_v[2] \\
A_{5z}^{(3,Q)} &= G_{yz}[2] + Q_x[3] \\
A_{6x}^{(3,Q)} &= G_{zx}[2] + Q_y[3] \\
A_{6y}^{(3,Q)} &= G_{yz}[2] - Q_x[3] \\
A_{6z}^{(3,Q)} &= G_0 + 2G_u[2]
\end{aligned}$$

$$S^{(4,Q)} = \begin{pmatrix} S_{11}^{(4,Q)} & S_{12}^{(4,Q)} & S_{13}^{(4,Q)} & S_{14}^{(4,Q)} & S_{15}^{(4,Q)} & S_{16}^{(4,Q)} \\ S_{12}^{(4,Q)} & S_{22}^{(4,Q)} & S_{23}^{(4,Q)} & S_{24}^{(4,Q)} & S_{25}^{(4,Q)} & S_{26}^{(4,Q)} \\ S_{13}^{(4,Q)} & S_{23}^{(4,Q)} & S_{33}^{(4,Q)} & S_{34}^{(4,Q)} & S_{35}^{(4,Q)} & S_{36}^{(4,Q)} \\ S_{14}^{(4,Q)} & S_{24}^{(4,Q)} & S_{34}^{(4,Q)} & S_{44}^{(4,Q)} & S_{45}^{(4,Q)} & S_{46}^{(4,Q)} \\ S_{15}^{(4,Q)} & S_{25}^{(4,Q)} & S_{35}^{(4,Q)} & S_{45}^{(4,Q)} & S_{55}^{(4,Q)} & S_{56}^{(4,Q)} \\ S_{16}^{(4,Q)} & S_{26}^{(4,Q)} & S_{36}^{(4,Q)} & S_{46}^{(4,Q)} & S_{56}^{(4,Q)} & S_{66}^{(4,Q)} \end{pmatrix}$$

$$\begin{aligned}
S_{11}^{(4,Q)} &= Q_0[1] + 2Q_0[2] - Q_{4u} + Q_{4v} + 2Q_4 - 2Q_u[1] - 4Q_u[2] + 2Q_v[1] + 4Q_v[2] \\
S_{12}^{(4,Q)} &= Q_0[1] + 2Q_{4u} - Q_4 - 2Q_u[1] \\
S_{13}^{(4,Q)} &= Q_0[1] - Q_{4u} - Q_{4v} - Q_4 + Q_u[1] + Q_v[1] \\
S_{14}^{(4,Q)} &= 2Q_{4x}^\beta + Q_{yz}[1] \\
S_{15}^{(4,Q)} &= -Q_{4y}^\alpha - Q_{4y}^\beta + Q_{zx}[1] + 2Q_{zx}[2] \\
S_{16}^{(4,Q)} &= Q_{4z}^\alpha - Q_{4z}^\beta + Q_{xy}[1] + 2Q_{xy}[2] \\
S_{22}^{(4,Q)} &= Q_0[1] + 2Q_0[2] - Q_{4u} - Q_{4v} + 2Q_4 - 2Q_u[1] - 4Q_u[2] - 2Q_v[1] - 4Q_v[2] \\
S_{23}^{(4,Q)} &= Q_0[1] - Q_{4u} + Q_{4v} - Q_4 + Q_u[1] - Q_v[1] \\
S_{24}^{(4,Q)} &= Q_{4x}^\alpha - Q_{4x}^\beta + Q_{yz}[1] + 2Q_{yz}[2] \\
S_{25}^{(4,Q)} &= 2Q_{4y}^\beta + Q_{zx}[1] \\
S_{26}^{(4,Q)} &= -Q_{4z}^\alpha - Q_{4z}^\beta + Q_{xy}[1] + 2Q_{xy}[2] \\
S_{33}^{(4,Q)} &= Q_0[1] + 2Q_0[2] + 2Q_{4u} + 2Q_4 + 4Q_u[1] + 8Q_u[2] \\
S_{34}^{(4,Q)} &= -Q_{4x}^\alpha - Q_{4x}^\beta + Q_{yz}[1] + 2Q_{yz}[2] \\
S_{35}^{(4,Q)} &= Q_{4y}^\alpha - Q_{4y}^\beta + Q_{zx}[1] + 2Q_{zx}[2] \\
S_{36}^{(4,Q)} &= 2Q_{4z}^\beta + Q_{xy}[1] \\
S_{44}^{(4,Q)} &= Q_0[2] - Q_{4u} + Q_{4v} - Q_4 + Q_u[2] - Q_v[2] \\
S_{45}^{(4,Q)} &= 2Q_{4z}^\beta + Q_{xy}[2] \\
S_{46}^{(4,Q)} &= 2Q_{4y}^\beta + Q_{zx}[2] \\
S_{55}^{(4,Q)} &= Q_0[2] - Q_{4u} - Q_{4v} - Q_4 + Q_u[2] + Q_v[2] \\
S_{56}^{(4,Q)} &= 2Q_{4x}^\beta + Q_{yz}[2] \\
S_{66}^{(4,Q)} &= Q_0[2] + 2Q_{4u} - Q_4 - 2Q_u[2]
\end{aligned}$$

$$\bar{S}^{(4,Q)} = \begin{pmatrix} 0 & \bar{S}_{12}^{(4,Q)} & \bar{S}_{13}^{(4,Q)} & \bar{S}_{14}^{(4,Q)} & \bar{S}_{15}^{(4,Q)} & \bar{S}_{16}^{(4,Q)} \\ -\bar{S}_{12}^{(4,Q)} & 0 & \bar{S}_{23}^{(4,Q)} & \bar{S}_{24}^{(4,Q)} & \bar{S}_{25}^{(4,Q)} & \bar{S}_{26}^{(4,Q)} \\ -\bar{S}_{13}^{(4,Q)} & -\bar{S}_{23}^{(4,Q)} & 0 & \bar{S}_{34}^{(4,Q)} & \bar{S}_{35}^{(4,Q)} & \bar{S}_{36}^{(4,Q)} \\ -\bar{S}_{14}^{(4,Q)} & -\bar{S}_{24}^{(4,Q)} & -\bar{S}_{34}^{(4,Q)} & 0 & \bar{S}_{45}^{(4,Q)} & \bar{S}_{46}^{(4,Q)} \\ -\bar{S}_{15}^{(4,Q)} & -\bar{S}_{25}^{(4,Q)} & -\bar{S}_{35}^{(4,Q)} & -\bar{S}_{45}^{(4,Q)} & 0 & \bar{S}_{56}^{(4,Q)} \\ -\bar{S}_{16}^{(4,Q)} & -\bar{S}_{26}^{(4,Q)} & -\bar{S}_{36}^{(4,Q)} & -\bar{S}_{46}^{(4,Q)} & -\bar{S}_{56}^{(4,Q)} & 0 \end{pmatrix}$$

$$\begin{aligned}
\bar{S}_{12}^{(4,Q)} &= 4G_{xyz}[1] - 2Q_v[3] \\
\bar{S}_{13}^{(4,Q)} &= -4G_{xyz}[1] + 3Q_u[3] - Q_v[3] \\
\bar{S}_{14}^{(4,Q)} &= -4G_x^\beta[1] + Q_{yz}[3] \\
\bar{S}_{15}^{(4,Q)} &= -2G_y[1] + 2G_y^\alpha[1] + 2G_y^\beta[1] + Q_{zx}[3] \\
\bar{S}_{16}^{(4,Q)} &= 2G_z[1] - 2G_z^\alpha[1] + 2G_z^\beta[1] + Q_{xy}[3] \\
\bar{S}_{23}^{(4,Q)} &= 4G_{xyz}[1] + 3Q_u[3] + Q_v[3] \\
\bar{S}_{24}^{(4,Q)} &= 2G_x[1] - 2G_x^\alpha[1] + 2G_x^\beta[1] + Q_{yz}[3] \\
\bar{S}_{25}^{(4,Q)} &= -4G_y^\beta[1] + Q_{zx}[3] \\
\bar{S}_{26}^{(4,Q)} &= -2G_z[1] + 2G_z^\alpha[1] + 2G_z^\beta[1] + Q_{xy}[3] \\
\bar{S}_{34}^{(4,Q)} &= -2G_x[1] + 2G_x^\alpha[1] + 2G_x^\beta[1] + Q_{yz}[3] \\
\bar{S}_{35}^{(4,Q)} &= 2G_y[1] - 2G_y^\alpha[1] + 2G_y^\beta[1] + Q_{zx}[3] \\
\bar{S}_{36}^{(4,Q)} &= -4G_z^\beta[1] + Q_{xy}[3] \\
\bar{S}_{45}^{(4,Q)} &= -G_z[1] - 4G_z^\alpha[1] \\
\bar{S}_{46}^{(4,Q)} &= G_y[1] + 4G_y^\alpha[1] \\
\bar{S}_{56}^{(4,Q)} &= -G_x[1] - 4G_x^\alpha[1]
\end{aligned}$$

$$A^{(4,Q)} = \begin{pmatrix} A_{xx}^{(4,Q)} & A_{xy}^{(4,Q)} & A_{xz}^{(4,Q)} \\ A_{xy}^{(4,Q)} & A_{yy}^{(4,Q)} & A_{yz}^{(4,Q)} \\ A_{xz}^{(4,Q)} & A_{yz}^{(4,Q)} & A_{zz}^{(4,Q)} \end{pmatrix}$$

$$\begin{aligned}
A_{xx}^{(4,Q)} &= Q_0[3] - 2Q_u[6] + 2Q_v[6] \\
A_{xy}^{(4,Q)} &= 2Q_{xy}[6] \\
A_{xz}^{(4,Q)} &= 2Q_{zx}[6] \\
A_{yy}^{(4,Q)} &= Q_0[3] - 2Q_u[6] - 2Q_v[6] \\
A_{yz}^{(4,Q)} &= 2Q_{yz}[6] \\
A_{zz}^{(4,Q)} &= Q_0[3] + 4Q_u[6]
\end{aligned}$$

$$\bar{A}^{(4,Q)} = \begin{pmatrix} 0 & \bar{A}_{xy}^{(4,Q)} & \bar{A}_{xz}^{(4,Q)} \\ -\bar{A}_{xy}^{(4,Q)} & 0 & \bar{A}_{yz}^{(4,Q)} \\ -\bar{A}_{xz}^{(4,Q)} & -\bar{A}_{yz}^{(4,Q)} & 0 \end{pmatrix}$$

$$\begin{aligned}
\bar{A}_{xy}^{(4,Q)} &= G_z[6] \\
\bar{A}_{xz}^{(4,Q)} &= -G_y[6] \\
\bar{A}_{yz}^{(4,Q)} &= G_x[6]
\end{aligned}$$

$$M^{(4,Q)} = \begin{pmatrix} M_{1x}^{(4,Q)} & M_{1y}^{(4,Q)} & M_{1z}^{(4,Q)} \\ M_{2x}^{(4,Q)} & M_{2y}^{(4,Q)} & M_{2z}^{(4,Q)} \\ M_{3x}^{(4,Q)} & M_{3y}^{(4,Q)} & M_{3z}^{(4,Q)} \\ M_{4x}^{(4,Q)} & M_{4y}^{(4,Q)} & M_{4z}^{(4,Q)} \\ M_{5x}^{(4,Q)} & M_{5y}^{(4,Q)} & M_{5z}^{(4,Q)} \\ M_{6x}^{(4,Q)} & M_{6y}^{(4,Q)} & M_{6z}^{(4,Q)} \end{pmatrix}$$

$$\begin{aligned}
M_{1x}^{(4,Q)} &= 2G_x[2] + G_x[3] + 2G_x^\alpha[2] \\
M_{1y}^{(4,Q)} &= G_y[3] - G_y^\alpha[2] - G_y^\beta[2] + 2Q_{zx}[4] \\
M_{1z}^{(4,Q)} &= G_z[3] - G_z^\alpha[2] + G_z^\beta[2] - 2Q_{xy}[4] \\
M_{2x}^{(4,Q)} &= G_x[3] - G_x^\alpha[2] + G_x^\beta[2] - 2Q_{yz}[4] \\
M_{2y}^{(4,Q)} &= 2G_y[2] + G_y[3] + 2G_y^\alpha[2] \\
M_{2z}^{(4,Q)} &= G_z[3] - G_z^\alpha[2] - G_z^\beta[2] + 2Q_{xy}[4] \\
M_{3x}^{(4,Q)} &= G_x[3] - G_x^\alpha[2] - G_x^\beta[2] + 2Q_{yz}[4] \\
M_{3y}^{(4,Q)} &= G_y[3] - G_y^\alpha[2] + G_y^\beta[2] - 2Q_{zx}[4] \\
M_{3z}^{(4,Q)} &= 2G_z[2] + G_z[3] + 2G_z^\alpha[2] \\
M_{4x}^{(4,Q)} &= G_{xyz}[2] - 3Q_u[4] - Q_v[4] \\
M_{4y}^{(4,Q)} &= G_z[2] - G_z^\alpha[2] - G_z^\beta[2] - Q_{xy}[4] \\
M_{4z}^{(4,Q)} &= G_y[2] - G_y^\alpha[2] + G_y^\beta[2] + Q_{zx}[4] \\
M_{5x}^{(4,Q)} &= G_z[2] - G_z^\alpha[2] + G_z^\beta[2] + Q_{xy}[4] \\
M_{5y}^{(4,Q)} &= G_{xyz}[2] + 3Q_u[4] - Q_v[4] \\
M_{5z}^{(4,Q)} &= G_x[2] - G_x^\alpha[2] - G_x^\beta[2] - Q_{yz}[4] \\
M_{6x}^{(4,Q)} &= G_y[2] - G_y^\alpha[2] - G_y^\beta[2] - Q_{zx}[4] \\
M_{6y}^{(4,Q)} &= G_x[2] - G_x^\alpha[2] + G_x^\beta[2] + Q_{yz}[4] \\
M_{6z}^{(4,Q)} &= G_{xyz}[2] + 2Q_v[4]
\end{aligned}$$

$$\bar{M}^{(4,Q)} = \begin{pmatrix} \bar{M}_{x1}^{(4,Q)} & \bar{M}_{x2}^{(4,Q)} & \bar{M}_{x3}^{(4,Q)} & \bar{M}_{x4}^{(4,Q)} & \bar{M}_{x5}^{(4,Q)} & \bar{M}_{x6}^{(4,Q)} \\ \bar{M}_{y1}^{(4,Q)} & \bar{M}_{y2}^{(4,Q)} & \bar{M}_{y3}^{(4,Q)} & \bar{M}_{y4}^{(4,Q)} & \bar{M}_{y5}^{(4,Q)} & \bar{M}_{y6}^{(4,Q)} \\ \bar{M}_{z1}^{(4,Q)} & \bar{M}_{z2}^{(4,Q)} & \bar{M}_{z3}^{(4,Q)} & \bar{M}_{z4}^{(4,Q)} & \bar{M}_{z5}^{(4,Q)} & \bar{M}_{z6}^{(4,Q)} \end{pmatrix}$$

$$\begin{aligned}
\bar{M}_{x1}^{(4,Q)} &= 2G_x[4] + G_x[5] + 2G_x^\alpha[3] \\
\bar{M}_{x2}^{(4,Q)} &= G_x[5] - G_x^\alpha[3] + G_x^\beta[3] - 2Q_{yz}[5] \\
\bar{M}_{x3}^{(4,Q)} &= G_x[5] - G_x^\alpha[3] - G_x^\beta[3] + 2Q_{yz}[5] \\
\bar{M}_{x4}^{(4,Q)} &= G_{xyz}[3] - 3Q_u[5] - Q_v[5] \\
\bar{M}_{x5}^{(4,Q)} &= G_z[4] - G_z^\alpha[3] + G_z^\beta[3] + Q_{xy}[5] \\
\bar{M}_{x6}^{(4,Q)} &= G_y[4] - G_y^\alpha[3] - G_y^\beta[3] - Q_{zx}[5] \\
\bar{M}_{y1}^{(4,Q)} &= G_y[5] - G_y^\alpha[3] - G_y^\beta[3] + 2Q_{zx}[5] \\
\bar{M}_{y2}^{(4,Q)} &= 2G_y[4] + G_y[5] + 2G_y^\alpha[3] \\
\bar{M}_{y3}^{(4,Q)} &= G_y[5] - G_y^\alpha[3] + G_y^\beta[3] - 2Q_{zx}[5] \\
\bar{M}_{y4}^{(4,Q)} &= G_z[4] - G_z^\alpha[3] - G_z^\beta[3] - Q_{xy}[5] \\
\bar{M}_{y5}^{(4,Q)} &= G_{xyz}[3] + 3Q_u[5] - Q_v[5] \\
\bar{M}_{y6}^{(4,Q)} &= G_x[4] - G_x^\alpha[3] + G_x^\beta[3] + Q_{yz}[5] \\
\bar{M}_{z1}^{(4,Q)} &= G_z[5] - G_z^\alpha[3] + G_z^\beta[3] - 2Q_{xy}[5] \\
\bar{M}_{z2}^{(4,Q)} &= G_z[5] - G_z^\alpha[3] - G_z^\beta[3] + 2Q_{xy}[5] \\
\bar{M}_{z3}^{(4,Q)} &= 2G_z[4] + G_z[5] + 2G_z^\alpha[3] \\
\bar{M}_{z4}^{(4,Q)} &= G_y[4] - G_y^\alpha[3] + G_y^\beta[3] + Q_{zx}[5] \\
\bar{M}_{z5}^{(4,Q)} &= G_x[4] - G_x^\alpha[3] - G_x^\beta[3] - Q_{yz}[5] \\
\bar{M}_{z6}^{(4,Q)} &= G_{xyz}[3] + 2Q_v[5]
\end{aligned}$$

— axial tensors —

$$C^{(0,G)} = (C^{(0,G)})$$

$$C^{(0,G)} = G_0$$

$$C^{(1,G)} = \begin{pmatrix} C_x^{(1,G)} & C_y^{(1,G)} & C_z^{(1,G)} \end{pmatrix}$$

$$C_x^{(1,G)} = G_x$$

$$C_y^{(1,G)} = G_y$$

$$C_z^{(1,G)} = G_z$$

$$S^{(2,G)} = \begin{pmatrix} S_{xx}^{(2,G)} & S_{xy}^{(2,G)} & S_{xz}^{(2,G)} \\ S_{xy}^{(2,G)} & S_{yy}^{(2,G)} & S_{yz}^{(2,G)} \\ S_{xz}^{(2,G)} & S_{yz}^{(2,G)} & S_{zz}^{(2,G)} \end{pmatrix}$$

$$S_{xx}^{(2,G)} = G_0 - G_u + G_v$$

$$S_{xy}^{(2,G)} = G_{xy}$$

$$S_{xz}^{(2,G)} = G_{zx}$$

$$S_{yy}^{(2,G)} = G_0 - G_u - G_v$$

$$S_{yz}^{(2,G)} = G_{yz}$$

$$S_{zz}^{(2,G)} = G_0 + 2G_u$$

$$A^{(2,G)} = \begin{pmatrix} 0 & A_{xy}^{(2,G)} & A_{xz}^{(2,G)} \\ -A_{xy}^{(2,G)} & 0 & A_{yz}^{(2,G)} \\ -A_{xz}^{(2,G)} & -A_{yz}^{(2,G)} & 0 \end{pmatrix}$$

$$A_{xy}^{(2,G)} = Q_z$$

$$A_{xz}^{(2,G)} = -Q_y$$

$$A_{yz}^{(2,G)} = Q_x$$

$$S^{(3,G)} = \begin{pmatrix} S_{1x}^{(3,G)} & S_{1y}^{(3,G)} & S_{1z}^{(3,G)} \\ S_{2x}^{(3,G)} & S_{2y}^{(3,G)} & S_{2z}^{(3,G)} \\ S_{3x}^{(3,G)} & S_{3y}^{(3,G)} & S_{3z}^{(3,G)} \\ S_{4x}^{(3,G)} & S_{4y}^{(3,G)} & S_{4z}^{(3,G)} \\ S_{5x}^{(3,G)} & S_{5y}^{(3,G)} & S_{5z}^{(3,G)} \\ S_{6x}^{(3,G)} & S_{6y}^{(3,G)} & S_{6z}^{(3,G)} \end{pmatrix}$$

$$\begin{aligned}
S_{1x}^{(3,G)} &= 2G_x[1] + G_x[2] + 2G_x^\alpha \\
S_{1y}^{(3,G)} &= G_y[2] - G_y^\alpha - G_y^\beta + 2Q_{zx}[1] \\
S_{1z}^{(3,G)} &= G_z[2] - G_z^\alpha + G_z^\beta - 2Q_{xy}[1] \\
S_{2x}^{(3,G)} &= G_x[2] - G_x^\alpha + G_x^\beta - 2Q_{yz}[1] \\
S_{2y}^{(3,G)} &= 2G_y[1] + G_y[2] + 2G_y^\alpha \\
S_{2z}^{(3,G)} &= G_z[2] - G_z^\alpha - G_z^\beta + 2Q_{xy}[1] \\
S_{3x}^{(3,G)} &= G_x[2] - G_x^\alpha - G_x^\beta + 2Q_{yz}[1] \\
S_{3y}^{(3,G)} &= G_y[2] - G_y^\alpha + G_y^\beta - 2Q_{zx}[1] \\
S_{3z}^{(3,G)} &= 2G_z[1] + G_z[2] + 2G_z^\alpha \\
S_{4x}^{(3,G)} &= G_{xyz} - 3Q_u[1] - Q_v[1] \\
S_{4y}^{(3,G)} &= G_z[1] - G_z^\alpha - G_z^\beta - Q_{xy}[1] \\
S_{4z}^{(3,G)} &= G_y[1] - G_y^\alpha + G_y^\beta + Q_{zx}[1] \\
S_{5x}^{(3,G)} &= G_z[1] - G_z^\alpha + G_z^\beta + Q_{xy}[1] \\
S_{5y}^{(3,G)} &= G_{xyz} + 3Q_u[1] - Q_v[1] \\
S_{5z}^{(3,G)} &= G_x[1] - G_x^\alpha - G_x^\beta - Q_{yz}[1] \\
S_{6x}^{(3,G)} &= G_y[1] - G_y^\alpha - G_y^\beta - Q_{zx}[1] \\
S_{6y}^{(3,G)} &= G_x[1] - G_x^\alpha + G_x^\beta + Q_{yz}[1] \\
S_{6z}^{(3,G)} &= G_{xyz} + 2Q_v[1]
\end{aligned}$$

$$A^{(3,G)} = \begin{pmatrix} A_{4x}^{(3,G)} & A_{4y}^{(3,G)} & A_{4z}^{(3,G)} \\ A_{5x}^{(3,G)} & A_{5y}^{(3,G)} & A_{5z}^{(3,G)} \\ A_{6x}^{(3,G)} & A_{6y}^{(3,G)} & A_{6z}^{(3,G)} \end{pmatrix}$$

$$\begin{aligned}
A_{4x}^{(3,G)} &= Q_0 - Q_u[2] + Q_v[2] \\
A_{4y}^{(3,G)} &= G_z[3] + Q_{xy}[2] \\
A_{4z}^{(3,G)} &= -G_y[3] + Q_{zx}[2] \\
A_{5x}^{(3,G)} &= -G_z[3] + Q_{xy}[2] \\
A_{5y}^{(3,G)} &= Q_0 - Q_u[2] - Q_v[2] \\
A_{5z}^{(3,G)} &= G_x[3] + Q_{yz}[2] \\
A_{6x}^{(3,G)} &= G_y[3] + Q_{zx}[2] \\
A_{6y}^{(3,G)} &= -G_x[3] + Q_{yz}[2] \\
A_{6z}^{(3,G)} &= Q_0 + 2Q_u[2]
\end{aligned}$$

$$S^{(4,G)} = \begin{pmatrix} S_{11}^{(4,G)} & S_{12}^{(4,G)} & S_{13}^{(4,G)} & S_{14}^{(4,G)} & S_{15}^{(4,G)} & S_{16}^{(4,G)} \\ S_{12}^{(4,G)} & S_{22}^{(4,G)} & S_{23}^{(4,G)} & S_{24}^{(4,G)} & S_{25}^{(4,G)} & S_{26}^{(4,G)} \\ S_{13}^{(4,G)} & S_{23}^{(4,G)} & S_{33}^{(4,G)} & S_{34}^{(4,G)} & S_{35}^{(4,G)} & S_{36}^{(4,G)} \\ S_{14}^{(4,G)} & S_{24}^{(4,G)} & S_{34}^{(4,G)} & S_{44}^{(4,G)} & S_{45}^{(4,G)} & S_{46}^{(4,G)} \\ S_{15}^{(4,G)} & S_{25}^{(4,G)} & S_{35}^{(4,G)} & S_{45}^{(4,G)} & S_{55}^{(4,G)} & S_{56}^{(4,G)} \\ S_{16}^{(4,G)} & S_{26}^{(4,G)} & S_{36}^{(4,G)} & S_{46}^{(4,G)} & S_{56}^{(4,G)} & S_{66}^{(4,G)} \end{pmatrix}$$

$$\begin{aligned}
S_{11}^{(4,G)} &= G_0[1] + 2G_0[2] - G_{4u} + G_{4v} + 2G_4 - 2G_u[1] - 4G_u[2] + 2G_v[1] + 4G_v[2] \\
S_{12}^{(4,G)} &= G_0[1] + 2G_{4u} - G_4 - 2G_u[1] \\
S_{13}^{(4,G)} &= G_0[1] - G_{4u} - G_{4v} - G_4 + G_u[1] + G_v[1] \\
S_{14}^{(4,G)} &= 2G_{4x}^\beta + G_{yz}[1] \\
S_{15}^{(4,G)} &= -G_{4y}^\alpha - G_{4y}^\beta + G_{zx}[1] + 2G_{zx}[2] \\
S_{16}^{(4,G)} &= G_{4z}^\alpha - G_{4z}^\beta + G_{xy}[1] + 2G_{xy}[2] \\
S_{22}^{(4,G)} &= G_0[1] + 2G_0[2] - G_{4u} - G_{4v} + 2G_4 - 2G_u[1] - 4G_u[2] - 2G_v[1] - 4G_v[2] \\
S_{23}^{(4,G)} &= G_0[1] - G_{4u} + G_{4v} - G_4 + G_u[1] - G_v[1] \\
S_{24}^{(4,G)} &= G_{4x}^\alpha - G_{4x}^\beta + G_{yz}[1] + 2G_{yz}[2] \\
S_{25}^{(4,G)} &= 2G_{4y}^\beta + G_{zx}[1] \\
S_{26}^{(4,G)} &= -G_{4z}^\alpha - G_{4z}^\beta + G_{xy}[1] + 2G_{xy}[2] \\
S_{33}^{(4,G)} &= G_0[1] + 2G_0[2] + 2G_{4u} + 2G_4 + 4G_u[1] + 8G_u[2] \\
S_{34}^{(4,G)} &= -G_{4x}^\alpha - G_{4x}^\beta + G_{yz}[1] + 2G_{yz}[2] \\
S_{35}^{(4,G)} &= G_{4y}^\alpha - G_{4y}^\beta + G_{zx}[1] + 2G_{zx}[2] \\
S_{36}^{(4,G)} &= 2G_{4z}^\beta + G_{xy}[1] \\
S_{44}^{(4,G)} &= G_0[2] - G_{4u} + G_{4v} - G_4 + G_u[2] - G_v[2] \\
S_{45}^{(4,G)} &= 2G_{4z}^\beta + G_{xy}[2] \\
S_{46}^{(4,G)} &= 2G_{4y}^\beta + G_{zx}[2] \\
S_{55}^{(4,G)} &= G_0[2] - G_{4u} - G_{4v} - G_4 + G_u[2] + G_v[2] \\
S_{56}^{(4,G)} &= 2G_{4x}^\beta + G_{yz}[2] \\
S_{66}^{(4,G)} &= G_0[2] + 2G_{4u} - G_4 - 2G_u[2]
\end{aligned}$$

$$\bar{S}^{(4,G)} = \begin{pmatrix} 0 & \bar{S}_{12}^{(4,G)} & \bar{S}_{13}^{(4,G)} & \bar{S}_{14}^{(4,G)} & \bar{S}_{15}^{(4,G)} & \bar{S}_{16}^{(4,G)} \\ -\bar{S}_{12}^{(4,G)} & 0 & \bar{S}_{23}^{(4,G)} & \bar{S}_{24}^{(4,G)} & \bar{S}_{25}^{(4,G)} & \bar{S}_{26}^{(4,G)} \\ -\bar{S}_{13}^{(4,G)} & -\bar{S}_{23}^{(4,G)} & 0 & \bar{S}_{34}^{(4,G)} & \bar{S}_{35}^{(4,G)} & \bar{S}_{36}^{(4,G)} \\ -\bar{S}_{14}^{(4,G)} & -\bar{S}_{24}^{(4,G)} & -\bar{S}_{34}^{(4,G)} & 0 & \bar{S}_{45}^{(4,G)} & \bar{S}_{46}^{(4,G)} \\ -\bar{S}_{15}^{(4,G)} & -\bar{S}_{25}^{(4,G)} & -\bar{S}_{35}^{(4,G)} & -\bar{S}_{45}^{(4,G)} & 0 & \bar{S}_{56}^{(4,G)} \\ -\bar{S}_{16}^{(4,G)} & -\bar{S}_{26}^{(4,G)} & -\bar{S}_{36}^{(4,G)} & -\bar{S}_{46}^{(4,G)} & -\bar{S}_{56}^{(4,G)} & 0 \end{pmatrix}$$

$$\begin{aligned}
\bar{S}_{12}^{(4,G)} &= -2G_v[3] + 4Q_{xyz}[1] \\
\bar{S}_{13}^{(4,G)} &= 3G_u[3] - G_v[3] - 4Q_{xyz}[1] \\
\bar{S}_{14}^{(4,G)} &= G_{yz}[3] - 4Q_x^\beta[1] \\
\bar{S}_{15}^{(4,G)} &= G_{zx}[3] - 2Q_y[1] + 2Q_y^\alpha[1] + 2Q_y^\beta[1] \\
\bar{S}_{16}^{(4,G)} &= G_{xy}[3] + 2Q_z[1] - 2Q_z^\alpha[1] + 2Q_z^\beta[1] \\
\bar{S}_{23}^{(4,G)} &= 3G_u[3] + G_v[3] + 4Q_{xyz}[1] \\
\bar{S}_{24}^{(4,G)} &= G_{yz}[3] + 2Q_x[1] - 2Q_x^\alpha[1] + 2Q_x^\beta[1] \\
\bar{S}_{25}^{(4,G)} &= G_{zx}[3] - 4Q_y^\beta[1] \\
\bar{S}_{26}^{(4,G)} &= G_{xy}[3] - 2Q_z[1] + 2Q_z^\alpha[1] + 2Q_z^\beta[1] \\
\bar{S}_{34}^{(4,G)} &= G_{yz}[3] - 2Q_x[1] + 2Q_x^\alpha[1] + 2Q_x^\beta[1] \\
\bar{S}_{35}^{(4,G)} &= G_{zx}[3] + 2Q_y[1] - 2Q_y^\alpha[1] + 2Q_y^\beta[1] \\
\bar{S}_{36}^{(4,G)} &= G_{xy}[3] - 4Q_z^\beta[1] \\
\bar{S}_{45}^{(4,G)} &= -Q_z[1] - 4Q_z^\alpha[1] \\
\bar{S}_{46}^{(4,G)} &= Q_y[1] + 4Q_y^\alpha[1] \\
\bar{S}_{56}^{(4,G)} &= -Q_x[1] - 4Q_x^\alpha[1]
\end{aligned}$$

$$A^{(4,G)} = \begin{pmatrix} A_{xx}^{(4,G)} & A_{xy}^{(4,G)} & A_{xz}^{(4,G)} \\ A_{xy}^{(4,G)} & A_{yy}^{(4,G)} & A_{yz}^{(4,G)} \\ A_{xz}^{(4,G)} & A_{yz}^{(4,G)} & A_{zz}^{(4,G)} \end{pmatrix}$$

$$\begin{aligned}
A_{xx}^{(4,G)} &= G_0[3] - 2G_u[6] + 2G_v[6] \\
A_{xy}^{(4,G)} &= 2G_{xy}[6] \\
A_{xz}^{(4,G)} &= 2G_{zx}[6] \\
A_{yy}^{(4,G)} &= G_0[3] - 2G_u[6] - 2G_v[6] \\
A_{yz}^{(4,G)} &= 2G_{yz}[6] \\
A_{zz}^{(4,G)} &= G_0[3] + 4G_u[6]
\end{aligned}$$

$$\bar{A}^{(4,G)} = \begin{pmatrix} 0 & \bar{A}_{xy}^{(4,G)} & \bar{A}_{xz}^{(4,G)} \\ -\bar{A}_{xy}^{(4,G)} & 0 & \bar{A}_{yz}^{(4,G)} \\ -\bar{A}_{xz}^{(4,G)} & -\bar{A}_{yz}^{(4,G)} & 0 \end{pmatrix}$$

$$\begin{aligned}
\bar{A}_{xy}^{(4,G)} &= Q_z[6] \\
\bar{A}_{xz}^{(4,G)} &= -Q_y[6] \\
\bar{A}_{yz}^{(4,G)} &= Q_x[6]
\end{aligned}$$

$$M^{(4,G)} = \begin{pmatrix} M_{1x}^{(4,G)} & M_{1y}^{(4,G)} & M_{1z}^{(4,G)} \\ M_{2x}^{(4,G)} & M_{2y}^{(4,G)} & M_{2z}^{(4,G)} \\ M_{3x}^{(4,G)} & M_{3y}^{(4,G)} & M_{3z}^{(4,G)} \\ M_{4x}^{(4,G)} & M_{4y}^{(4,G)} & M_{4z}^{(4,G)} \\ M_{5x}^{(4,G)} & M_{5y}^{(4,G)} & M_{5z}^{(4,G)} \\ M_{6x}^{(4,G)} & M_{6y}^{(4,G)} & M_{6z}^{(4,G)} \end{pmatrix}$$

$$\begin{aligned}
M_{1x}^{(4,G)} &= 2Q_x[2] + Q_x[3] + 2Q_x^\alpha[2] \\
M_{1y}^{(4,G)} &= 2G_{zx}[4] + Q_y[3] - Q_y^\alpha[2] - Q_y^\beta[2] \\
M_{1z}^{(4,G)} &= -2G_{xy}[4] + Q_z[3] - Q_z^\alpha[2] + Q_z^\beta[2] \\
M_{2x}^{(4,G)} &= -2G_{yz}[4] + Q_x[3] - Q_x^\alpha[2] + Q_x^\beta[2] \\
M_{2y}^{(4,G)} &= 2Q_y[2] + Q_y[3] + 2Q_y^\alpha[2] \\
M_{2z}^{(4,G)} &= 2G_{xy}[4] + Q_z[3] - Q_z^\alpha[2] - Q_z^\beta[2] \\
M_{3x}^{(4,G)} &= 2G_{yz}[4] + Q_x[3] - Q_x^\alpha[2] - Q_x^\beta[2] \\
M_{3y}^{(4,G)} &= -2G_{zx}[4] + Q_y[3] - Q_y^\alpha[2] + Q_y^\beta[2] \\
M_{3z}^{(4,G)} &= 2Q_z[2] + Q_z[3] + 2Q_z^\alpha[2] \\
M_{4x}^{(4,G)} &= -3G_u[4] - G_v[4] + Q_{xyz}[2] \\
M_{4y}^{(4,G)} &= -G_{xy}[4] + Q_z[2] - Q_z^\alpha[2] - Q_z^\beta[2] \\
M_{4z}^{(4,G)} &= G_{zx}[4] + Q_y[2] - Q_y^\alpha[2] + Q_y^\beta[2] \\
M_{5x}^{(4,G)} &= G_{xy}[4] + Q_z[2] - Q_z^\alpha[2] + Q_z^\beta[2] \\
M_{5y}^{(4,G)} &= 3G_u[4] - G_v[4] + Q_{xyz}[2] \\
M_{5z}^{(4,G)} &= -G_{yz}[4] + Q_x[2] - Q_x^\alpha[2] - Q_x^\beta[2] \\
M_{6x}^{(4,G)} &= -G_{zx}[4] + Q_y[2] - Q_y^\alpha[2] - Q_y^\beta[2] \\
M_{6y}^{(4,G)} &= G_{yz}[4] + Q_x[2] - Q_x^\alpha[2] + Q_x^\beta[2] \\
M_{6z}^{(4,G)} &= 2G_v[4] + Q_{xyz}[2]
\end{aligned}$$

$$\bar{M}^{(4,G)} = \begin{pmatrix} \bar{M}_{x1}^{(4,G)} & \bar{M}_{x2}^{(4,G)} & \bar{M}_{x3}^{(4,G)} & \bar{M}_{x4}^{(4,G)} & \bar{M}_{x5}^{(4,G)} & \bar{M}_{x6}^{(4,G)} \\ \bar{M}_{y1}^{(4,G)} & \bar{M}_{y2}^{(4,G)} & \bar{M}_{y3}^{(4,G)} & \bar{M}_{y4}^{(4,G)} & \bar{M}_{y5}^{(4,G)} & \bar{M}_{y6}^{(4,G)} \\ \bar{M}_{z1}^{(4,G)} & \bar{M}_{z2}^{(4,G)} & \bar{M}_{z3}^{(4,G)} & \bar{M}_{z4}^{(4,G)} & \bar{M}_{z5}^{(4,G)} & \bar{M}_{z6}^{(4,G)} \end{pmatrix}$$

$$\begin{aligned}
\bar{M}_{x1}^{(4,G)} &= 2Q_x[4] + Q_x[5] + 2Q_x^\alpha[3] \\
\bar{M}_{x2}^{(4,G)} &= -2G_{yz}[5] + Q_x[5] - Q_x^\alpha[3] + Q_x^\beta[3] \\
\bar{M}_{x3}^{(4,G)} &= 2G_{yz}[5] + Q_x[5] - Q_x^\alpha[3] - Q_x^\beta[3] \\
\bar{M}_{x4}^{(4,G)} &= -3G_u[5] - G_v[5] + Q_{xyz}[3] \\
\bar{M}_{x5}^{(4,G)} &= G_{xy}[5] + Q_z[4] - Q_z^\alpha[3] + Q_z^\beta[3] \\
\bar{M}_{x6}^{(4,G)} &= -G_{zx}[5] + Q_y[4] - Q_y^\alpha[3] - Q_y^\beta[3] \\
\bar{M}_{y1}^{(4,G)} &= 2G_{zx}[5] + Q_y[5] - Q_y^\alpha[3] - Q_y^\beta[3] \\
\bar{M}_{y2}^{(4,G)} &= 2Q_y[4] + Q_y[5] + 2Q_y^\alpha[3] \\
\bar{M}_{y3}^{(4,G)} &= -2G_{zx}[5] + Q_y[5] - Q_y^\alpha[3] + Q_y^\beta[3] \\
\bar{M}_{y4}^{(4,G)} &= -G_{xy}[5] + Q_z[4] - Q_z^\alpha[3] - Q_z^\beta[3] \\
\bar{M}_{y5}^{(4,G)} &= 3G_u[5] - G_v[5] + Q_{xyz}[3] \\
\bar{M}_{y6}^{(4,G)} &= G_{yz}[5] + Q_x[4] - Q_x^\alpha[3] + Q_x^\beta[3] \\
\bar{M}_{z1}^{(4,G)} &= -2G_{xy}[5] + Q_z[5] - Q_z^\alpha[3] + Q_z^\beta[3] \\
\bar{M}_{z2}^{(4,G)} &= 2G_{xy}[5] + Q_z[5] - Q_z^\alpha[3] - Q_z^\beta[3] \\
\bar{M}_{z3}^{(4,G)} &= 2Q_z[4] + Q_z[5] + 2Q_z^\alpha[3] \\
\bar{M}_{z4}^{(4,G)} &= G_{zx}[5] + Q_y[4] - Q_y^\alpha[3] + Q_y^\beta[3] \\
\bar{M}_{z5}^{(4,G)} &= -G_{yz}[5] + Q_x[4] - Q_x^\alpha[3] - Q_x^\beta[3] \\
\bar{M}_{z6}^{(4,G)} &= 2G_v[5] + Q_{xyz}[3]
\end{aligned}$$