

No. 19 $C_{3v} - 1$ $31m$ (31m setting) [trigonal] (polar)

表 1 rank 0

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
1	A_1	A1	—	—	$\mathbb{Q}_0^{(h,A_1)}$	Qh(0, A1, ,)	C_0

表 2 rank 1

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
2	A_1	A1	—	—	$\mathbb{Q}_1^{(h,A_1)}$	Qh(1, A1, ,)	C_0
3	E	E	—	0	$\mathbb{Q}_{1,0}^{(h,E)}$	Qh(1, E, , 0)	C_1
4	E	E	—	1	$\mathbb{Q}_{1,1}^{(h,E)}$	Qh(1, E, , 1)	S_1

表 3 rank 2

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
5	A_1	A1	—	—	$\mathbb{Q}_2^{(h,A_1)}$	Qh(2, A1, ,)	C_0
6	E	E	1	0	$\mathbb{Q}_{2,0}^{(h,E,1)}$	Qh(2, E, 1, 0)	C_1
7	E	E	1	1	$\mathbb{Q}_{2,1}^{(h,E,1)}$	Qh(2, E, 1, 1)	S_1
8	E	E	2	0	$\mathbb{Q}_{2,0}^{(h,E,2)}$	Qh(2, E, 2, 0)	C_2
9	E	E	2	1	$\mathbb{Q}_{2,1}^{(h,E,2)}$	Qh(2, E, 2, 1)	$-S_2$

表 4 rank 3

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
10	A_1	A1	1	—	$\mathbb{Q}_3^{(h,A_1,1)}$	Qh(3, A1, 1,)	C_0
11	A_1	A1	2	—	$\mathbb{Q}_3^{(h,A_1,2)}$	Qh(3, A1, 2,)	C_3
12	A_2	A2	—	—	$\mathbb{Q}_3^{(h,A_2)}$	Qh(3, A2, ,)	S_3
13	E	E	1	0	$\mathbb{Q}_{3,0}^{(h,E,1)}$	Qh(3, E, 1, 0)	C_1
14	E	E	1	1	$\mathbb{Q}_{3,1}^{(h,E,1)}$	Qh(3, E, 1, 1)	S_1
15	E	E	2	0	$\mathbb{Q}_{3,0}^{(h,E,2)}$	Qh(3, E, 2, 0)	C_2
16	E	E	2	1	$\mathbb{Q}_{3,1}^{(h,E,2)}$	Qh(3, E, 2, 1)	$-S_2$

表 5 rank 4

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
17	A_1	A1	1	—	$\mathbb{Q}_4^{(h,A_1,1)}$	Qh(4, A1, 1,)	C_0
18	A_1	A1	2	—	$\mathbb{Q}_4^{(h,A_1,2)}$	Qh(4, A1, 2,)	C_3
19	A_2	A2	—	—	$\mathbb{Q}_4^{(h,A_2)}$	Qh(4, A2, ,)	S_3
20	E	E	1	0	$\mathbb{Q}_{4,0}^{(h,E,1)}$	Qh(4, E, 1, 0)	C_1
21	E	E	1	1	$\mathbb{Q}_{4,1}^{(h,E,1)}$	Qh(4, E, 1, 1)	S_1
22	E	E	2	0	$\mathbb{Q}_{4,0}^{(h,E,2)}$	Qh(4, E, 2, 0)	C_4
23	E	E	2	1	$\mathbb{Q}_{4,1}^{(h,E,2)}$	Qh(4, E, 2, 1)	S_4
24	E	E	3	0	$\mathbb{Q}_{4,0}^{(h,E,3)}$	Qh(4, E, 3, 0)	C_2
25	E	E	3	1	$\mathbb{Q}_{4,1}^{(h,E,3)}$	Qh(4, E, 3, 1)	$-S_2$

表 6 rank 5

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
26	A_1	A1	1	—	$\mathbb{Q}_5^{(h,A_1,1)}$	Qh(5, A1, 1,)	C_0
27	A_1	A1	2	—	$\mathbb{Q}_5^{(h,A_1,2)}$	Qh(5, A1, 2,)	C_3
28	A_2	A2	—	—	$\mathbb{Q}_5^{(h,A_2)}$	Qh(5, A2, ,)	S_3
29	E	E	1	0	$\mathbb{Q}_{5,0}^{(h,E,1)}$	Qh(5, E, 1, 0)	C_5
30	E	E	1	1	$\mathbb{Q}_{5,1}^{(h,E,1)}$	Qh(5, E, 1, 1)	$-S_5$
31	E	E	2	0	$\mathbb{Q}_{5,0}^{(h,E,2)}$	Qh(5, E, 2, 0)	C_1
32	E	E	2	1	$\mathbb{Q}_{5,1}^{(h,E,2)}$	Qh(5, E, 2, 1)	S_1
33	E	E	3	0	$\mathbb{Q}_{5,0}^{(h,E,3)}$	Qh(5, E, 3, 0)	C_4
34	E	E	3	1	$\mathbb{Q}_{5,1}^{(h,E,3)}$	Qh(5, E, 3, 1)	S_4
35	E	E	4	0	$\mathbb{Q}_{5,0}^{(h,E,4)}$	Qh(5, E, 4, 0)	C_2
36	E	E	4	1	$\mathbb{Q}_{5,1}^{(h,E,4)}$	Qh(5, E, 4, 1)	$-S_2$

表 7 rank 6

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
37	A_1	A1	1	—	$\mathbb{Q}_6^{(h,A_1,1)}$	Qh(6, A1, 1,)	C_0
38	A_1	A1	2	—	$\mathbb{Q}_6^{(h,A_1,2)}$	Qh(6, A1, 2,)	C_6
39	A_1	A1	3	—	$\mathbb{Q}_6^{(h,A_1,3)}$	Qh(6, A1, 3,)	C_3
40	A_2	A2	1	—	$\mathbb{Q}_6^{(h,A_2,1)}$	Qh(6, A2, 1,)	S_6
41	A_2	A2	2	—	$\mathbb{Q}_6^{(h,A_2,2)}$	Qh(6, A2, 2,)	S_3
42	E	E	1	0	$\mathbb{Q}_{6,0}^{(h,E,1)}$	Qh(6, E, 1, 0)	C_5
43	E	E	1	1	$\mathbb{Q}_{6,1}^{(h,E,1)}$	Qh(6, E, 1, 1)	$-S_5$
44	E	E	2	0	$\mathbb{Q}_{6,0}^{(h,E,2)}$	Qh(6, E, 2, 0)	C_1
45	E	E	2	1	$\mathbb{Q}_{6,1}^{(h,E,2)}$	Qh(6, E, 2, 1)	S_1
46	E	E	3	0	$\mathbb{Q}_{6,0}^{(h,E,3)}$	Qh(6, E, 3, 0)	C_4
47	E	E	3	1	$\mathbb{Q}_{6,1}^{(h,E,3)}$	Qh(6, E, 3, 1)	S_4
48	E	E	4	0	$\mathbb{Q}_{6,0}^{(h,E,4)}$	Qh(6, E, 4, 0)	C_2
49	E	E	4	1	$\mathbb{Q}_{6,1}^{(h,E,4)}$	Qh(6, E, 4, 1)	$-S_2$

表 8 rank 7

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
50	A_1	A1	1	—	$\mathbb{Q}_7^{(h,A_1,1)}$	$\text{Qh}(7, \text{A1}, 1,)$	C_0
51	A_1	A1	2	—	$\mathbb{Q}_7^{(h,A_1,2)}$	$\text{Qh}(7, \text{A1}, 2,)$	C_6
52	A_1	A1	3	—	$\mathbb{Q}_7^{(h,A_1,3)}$	$\text{Qh}(7, \text{A1}, 3,)$	C_3
53	A_2	A2	1	—	$\mathbb{Q}_7^{(h,A_2,1)}$	$\text{Qh}(7, \text{A2}, 1,)$	S_6
54	A_2	A2	2	—	$\mathbb{Q}_7^{(h,A_2,2)}$	$\text{Qh}(7, \text{A2}, 2,)$	S_3
55	E	E	1	0	$\mathbb{Q}_{7,0}^{(h,E,1)}$	$\text{Qh}(7, \text{E}, 1, 0)$	C_7
56	E	E	1	1	$\mathbb{Q}_{7,1}^{(h,E,1)}$	$\text{Qh}(7, \text{E}, 1, 1)$	S_7
57	E	E	2	0	$\mathbb{Q}_{7,0}^{(h,E,2)}$	$\text{Qh}(7, \text{E}, 2, 0)$	C_5
58	E	E	2	1	$\mathbb{Q}_{7,1}^{(h,E,2)}$	$\text{Qh}(7, \text{E}, 2, 1)$	$-S_5$
59	E	E	3	0	$\mathbb{Q}_{7,0}^{(h,E,3)}$	$\text{Qh}(7, \text{E}, 3, 0)$	C_1
60	E	E	3	1	$\mathbb{Q}_{7,1}^{(h,E,3)}$	$\text{Qh}(7, \text{E}, 3, 1)$	S_1
61	E	E	4	0	$\mathbb{Q}_{7,0}^{(h,E,4)}$	$\text{Qh}(7, \text{E}, 4, 0)$	C_4
62	E	E	4	1	$\mathbb{Q}_{7,1}^{(h,E,4)}$	$\text{Qh}(7, \text{E}, 4, 1)$	S_4
63	E	E	5	0	$\mathbb{Q}_{7,0}^{(h,E,5)}$	$\text{Qh}(7, \text{E}, 5, 0)$	C_2
64	E	E	5	1	$\mathbb{Q}_{7,1}^{(h,E,5)}$	$\text{Qh}(7, \text{E}, 5, 1)$	$-S_2$

表 9 rank 8

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
65	A_1	A1	1	—	$\mathbb{Q}_8^{(h,A_1,1)}$	$\text{Qh}(8, \text{A1}, 1,)$	C_0
66	A_1	A1	2	—	$\mathbb{Q}_8^{(h,A_1,2)}$	$\text{Qh}(8, \text{A1}, 2,)$	C_6
67	A_1	A1	3	—	$\mathbb{Q}_8^{(h,A_1,3)}$	$\text{Qh}(8, \text{A1}, 3,)$	C_3
68	A_2	A2	1	—	$\mathbb{Q}_8^{(h,A_2,1)}$	$\text{Qh}(8, \text{A2}, 1,)$	S_6
69	A_2	A2	2	—	$\mathbb{Q}_8^{(h,A_2,2)}$	$\text{Qh}(8, \text{A2}, 2,)$	S_3
70	E	E	1	0	$\mathbb{Q}_{8,0}^{(h,E,1)}$	$\text{Qh}(8, \text{E}, 1, 0)$	C_7
71	E	E	1	1	$\mathbb{Q}_{8,1}^{(h,E,1)}$	$\text{Qh}(8, \text{E}, 1, 1)$	S_7
72	E	E	2	0	$\mathbb{Q}_{8,0}^{(h,E,2)}$	$\text{Qh}(8, \text{E}, 2, 0)$	C_5
73	E	E	2	1	$\mathbb{Q}_{8,1}^{(h,E,2)}$	$\text{Qh}(8, \text{E}, 2, 1)$	$-S_5$
74	E	E	3	0	$\mathbb{Q}_{8,0}^{(h,E,3)}$	$\text{Qh}(8, \text{E}, 3, 0)$	C_1
75	E	E	3	1	$\mathbb{Q}_{8,1}^{(h,E,3)}$	$\text{Qh}(8, \text{E}, 3, 1)$	S_1
76	E	E	4	0	$\mathbb{Q}_{8,0}^{(h,E,4)}$	$\text{Qh}(8, \text{E}, 4, 0)$	C_8
77	E	E	4	1	$\mathbb{Q}_{8,1}^{(h,E,4)}$	$\text{Qh}(8, \text{E}, 4, 1)$	$-S_8$
78	E	E	5	0	$\mathbb{Q}_{8,0}^{(h,E,5)}$	$\text{Qh}(8, \text{E}, 5, 0)$	C_4
79	E	E	5	1	$\mathbb{Q}_{8,1}^{(h,E,5)}$	$\text{Qh}(8, \text{E}, 5, 1)$	S_4
80	E	E	6	0	$\mathbb{Q}_{8,0}^{(h,E,6)}$	$\text{Qh}(8, \text{E}, 6, 0)$	C_2
81	E	E	6	1	$\mathbb{Q}_{8,1}^{(h,E,6)}$	$\text{Qh}(8, \text{E}, 6, 1)$	$-S_2$

表 10 rank 9

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
82	A_1	A1	1	—	$\mathbb{Q}_9^{(h,A_1,1)}$	Qh(9, A1, 1,)	C_0
83	A_1	A1	2	—	$\mathbb{Q}_9^{(h,A_1,2)}$	Qh(9, A1, 2,)	C_6
84	A_1	A1	3	—	$\mathbb{Q}_9^{(h,A_1,3)}$	Qh(9, A1, 3,)	C_9
85	A_1	A1	4	—	$\mathbb{Q}_9^{(h,A_1,4)}$	Qh(9, A1, 4,)	C_3
86	A_2	A2	1	—	$\mathbb{Q}_9^{(h,A_2,1)}$	Qh(9, A2, 1,)	S_6
87	A_2	A2	2	—	$\mathbb{Q}_9^{(h,A_2,2)}$	Qh(9, A2, 2,)	S_9
88	A_2	A2	3	—	$\mathbb{Q}_9^{(h,A_2,3)}$	Qh(9, A2, 3,)	S_3
89	E	E	1	0	$\mathbb{Q}_{9,0}^{(h,E,1)}$	Qh(9, E, 1, 0)	C_7
90	E	E	1	1	$\mathbb{Q}_{9,1}^{(h,E,1)}$	Qh(9, E, 1, 1)	S_7
91	E	E	2	0	$\mathbb{Q}_{9,0}^{(h,E,2)}$	Qh(9, E, 2, 0)	C_5
92	E	E	2	1	$\mathbb{Q}_{9,1}^{(h,E,2)}$	Qh(9, E, 2, 1)	$-S_5$
93	E	E	3	0	$\mathbb{Q}_{9,0}^{(h,E,3)}$	Qh(9, E, 3, 0)	C_1
94	E	E	3	1	$\mathbb{Q}_{9,1}^{(h,E,3)}$	Qh(9, E, 3, 1)	S_1
95	E	E	4	0	$\mathbb{Q}_{9,0}^{(h,E,4)}$	Qh(9, E, 4, 0)	C_8
96	E	E	4	1	$\mathbb{Q}_{9,1}^{(h,E,4)}$	Qh(9, E, 4, 1)	$-S_8$
97	E	E	5	0	$\mathbb{Q}_{9,0}^{(h,E,5)}$	Qh(9, E, 5, 0)	C_4
98	E	E	5	1	$\mathbb{Q}_{9,1}^{(h,E,5)}$	Qh(9, E, 5, 1)	S_4
99	E	E	6	0	$\mathbb{Q}_{9,0}^{(h,E,6)}$	Qh(9, E, 6, 0)	C_2
100	E	E	6	1	$\mathbb{Q}_{9,1}^{(h,E,6)}$	Qh(9, E, 6, 1)	$-S_2$

表 11 rank 10

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
101	A_1	A1	1	—	$\mathbb{Q}_{10}^{(h,A_1,1)}$	Qh(10, A1, 1,)	C_0
102	A_1	A1	2	—	$\mathbb{Q}_{10}^{(h,A_1,2)}$	Qh(10, A1, 2,)	C_6
103	A_1	A1	3	—	$\mathbb{Q}_{10}^{(h,A_1,3)}$	Qh(10, A1, 3,)	C_9
104	A_1	A1	4	—	$\mathbb{Q}_{10}^{(h,A_1,4)}$	Qh(10, A1, 4,)	C_3
105	A_2	A2	1	—	$\mathbb{Q}_{10}^{(h,A_2,1)}$	Qh(10, A2, 1,)	S_6
106	A_2	A2	2	—	$\mathbb{Q}_{10}^{(h,A_2,2)}$	Qh(10, A2, 2,)	S_9
107	A_2	A2	3	—	$\mathbb{Q}_{10}^{(h,A_2,3)}$	Qh(10, A2, 3,)	S_3
108	E	E	1	0	$\mathbb{Q}_{10,0}^{(h,E,1)}$	Qh(10, E, 1, 0)	C_7
109	E	E	1	1	$\mathbb{Q}_{10,1}^{(h,E,1)}$	Qh(10, E, 1, 1)	S_7
110	E	E	2	0	$\mathbb{Q}_{10,0}^{(h,E,2)}$	Qh(10, E, 2, 0)	C_5
111	E	E	2	1	$\mathbb{Q}_{10,1}^{(h,E,2)}$	Qh(10, E, 2, 1)	$-S_5$
112	E	E	3	0	$\mathbb{Q}_{10,0}^{(h,E,3)}$	Qh(10, E, 3, 0)	C_1
113	E	E	3	1	$\mathbb{Q}_{10,1}^{(h,E,3)}$	Qh(10, E, 3, 1)	S_1
114	E	E	4	0	$\mathbb{Q}_{10,0}^{(h,E,4)}$	Qh(10, E, 4, 0)	C_{10}
115	E	E	4	1	$\mathbb{Q}_{10,1}^{(h,E,4)}$	Qh(10, E, 4, 1)	S_{10}
116	E	E	5	0	$\mathbb{Q}_{10,0}^{(h,E,5)}$	Qh(10, E, 5, 0)	C_8
117	E	E	5	1	$\mathbb{Q}_{10,1}^{(h,E,5)}$	Qh(10, E, 5, 1)	$-S_8$
118	E	E	6	0	$\mathbb{Q}_{10,0}^{(h,E,6)}$	Qh(10, E, 6, 0)	C_4
119	E	E	6	1	$\mathbb{Q}_{10,1}^{(h,E,6)}$	Qh(10, E, 6, 1)	S_4
120	E	E	7	0	$\mathbb{Q}_{10,0}^{(h,E,7)}$	Qh(10, E, 7, 0)	C_2
121	E	E	7	1	$\mathbb{Q}_{10,1}^{(h,E,7)}$	Qh(10, E, 7, 1)	$-S_2$

表 12 rank 11

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
122	A_1	A1	1	—	$\mathbb{Q}_{11}^{(h,A_1,1)}$	$\text{Qh}(11, \text{A1}, 1,)$	C_0
123	A_1	A1	2	—	$\mathbb{Q}_{11}^{(h,A_1,2)}$	$\text{Qh}(11, \text{A1}, 2,)$	C_6
124	A_1	A1	3	—	$\mathbb{Q}_{11}^{(h,A_1,3)}$	$\text{Qh}(11, \text{A1}, 3,)$	C_9
125	A_1	A1	4	—	$\mathbb{Q}_{11}^{(h,A_1,4)}$	$\text{Qh}(11, \text{A1}, 4,)$	C_3
126	A_2	A2	1	—	$\mathbb{Q}_{11}^{(h,A_2,1)}$	$\text{Qh}(11, \text{A2}, 1,)$	S_6
127	A_2	A2	2	—	$\mathbb{Q}_{11}^{(h,A_2,2)}$	$\text{Qh}(11, \text{A2}, 2,)$	S_9
128	A_2	A2	3	—	$\mathbb{Q}_{11}^{(h,A_2,3)}$	$\text{Qh}(11, \text{A2}, 3,)$	S_3
129	E	E	1	0	$\mathbb{Q}_{11,0}^{(h,E,1)}$	$\text{Qh}(11, \text{E}, 1, 0)$	C_{11}
130	E	E	1	1	$\mathbb{Q}_{11,1}^{(h,E,1)}$	$\text{Qh}(11, \text{E}, 1, 1)$	$-S_{11}$
131	E	E	2	0	$\mathbb{Q}_{11,0}^{(h,E,2)}$	$\text{Qh}(11, \text{E}, 2, 0)$	C_7
132	E	E	2	1	$\mathbb{Q}_{11,1}^{(h,E,2)}$	$\text{Qh}(11, \text{E}, 2, 1)$	S_7
133	E	E	3	0	$\mathbb{Q}_{11,0}^{(h,E,3)}$	$\text{Qh}(11, \text{E}, 3, 0)$	C_5
134	E	E	3	1	$\mathbb{Q}_{11,1}^{(h,E,3)}$	$\text{Qh}(11, \text{E}, 3, 1)$	$-S_5$
135	E	E	4	0	$\mathbb{Q}_{11,0}^{(h,E,4)}$	$\text{Qh}(11, \text{E}, 4, 0)$	C_1
136	E	E	4	1	$\mathbb{Q}_{11,1}^{(h,E,4)}$	$\text{Qh}(11, \text{E}, 4, 1)$	S_1
137	E	E	5	0	$\mathbb{Q}_{11,0}^{(h,E,5)}$	$\text{Qh}(11, \text{E}, 5, 0)$	C_{10}
138	E	E	5	1	$\mathbb{Q}_{11,1}^{(h,E,5)}$	$\text{Qh}(11, \text{E}, 5, 1)$	S_{10}
139	E	E	6	0	$\mathbb{Q}_{11,0}^{(h,E,6)}$	$\text{Qh}(11, \text{E}, 6, 0)$	C_8
140	E	E	6	1	$\mathbb{Q}_{11,1}^{(h,E,6)}$	$\text{Qh}(11, \text{E}, 6, 1)$	$-S_8$
141	E	E	7	0	$\mathbb{Q}_{11,0}^{(h,E,7)}$	$\text{Qh}(11, \text{E}, 7, 0)$	C_4
142	E	E	7	1	$\mathbb{Q}_{11,1}^{(h,E,7)}$	$\text{Qh}(11, \text{E}, 7, 1)$	S_4
143	E	E	8	0	$\mathbb{Q}_{11,0}^{(h,E,8)}$	$\text{Qh}(11, \text{E}, 8, 0)$	C_2
144	E	E	8	1	$\mathbb{Q}_{11,1}^{(h,E,8)}$	$\text{Qh}(11, \text{E}, 8, 1)$	$-S_2$