

MPG No. 9.1.29 4 [ Type I, tetragonal ] [T tensor]

\* Rank 0 tensor.

$$[T]$$

$$T = T_s^{(1)}$$

\* Rank 1 tensor.

$$[0 \ 0 \ T_z]$$

$$T_z = T_{pz}^{(1)}$$

\* Rank 2 tensor (s).

$$\begin{bmatrix} T_{xx} & 0 & 0 \\ 0 & T_{xx} & 0 \\ 0 & 0 & T_{zz} \end{bmatrix}$$

$$T_{xx} = -T_{du}^{(1)} + T_s^{(1)}$$

$$T_{zz} = 2T_{du}^{(1)} + T_s^{(1)}$$

\* Rank 2 tensor (a).

$$\begin{bmatrix} 0 & T_{xy} & 0 \\ -T_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T_{xy} = M_{pz}^{(1)}$$

\* Rank 3 tensor (s).

$$\begin{bmatrix} 0 & 0 & T_{xxz} \\ 0 & 0 & T_{xxz} \\ 0 & 0 & T_{zzz} \\ T_{yzz} & T_{yzy} & 0 \\ T_{yzy} & -T_{yzz} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T_{xxz} = -T_{faz}^{(1)} + T_{pz}^{(1)}$$

$$T_{zzz} = 2T_{faz}^{(1)} + T_{pz}^{(1)} + 2T_{pz}^{(2)}$$

$$T_{yzz} = -3M_{du}^{(1)}$$

$$T_{yzy} = -T_{faz}^{(1)} + T_{pz}^{(2)}$$

\* Rank 3 tensor (a).

$$\begin{bmatrix} T_{yzz} & T_{yzy} & 0 \\ -T_{yzy} & T_{yzz} & 0 \\ 0 & 0 & T_{xyz} \end{bmatrix}$$

$$T_{yzz} = -M_{du}^{(2)} + M_s^{(1)}$$

$$T_{yzy} = T_{pz}^{(3)}$$

$$T_{xyz} = 2M_{du}^{(2)} + M_s^{(1)}$$

\* Rank 4 tensor (sss).

$$\begin{bmatrix} T_{xxxx} & T_{xxyy} & T_{xxzz} & 0 & 0 & T_{xxxz} \\ T_{xxyy} & T_{xxxx} & T_{xxzz} & 0 & 0 & -T_{xxxz} \\ T_{xxzz} & T_{xxzz} & T_{zzzz} & 0 & 0 & 0 \\ 0 & 0 & 0 & T_{yzyz} & 0 & 0 \\ 0 & 0 & 0 & 0 & T_{yzyz} & 0 \\ T_{xxxz} & -T_{xxxz} & 0 & 0 & 0 & T_{xyxy} \end{bmatrix}$$

$$T_{xxxx} = -2T_{du}^{(1)} - 4T_{du}^{(2)} - T_{gu}^{(1)} + 2T_g^{(1)} + T_s^{(1)} + 2T_s^{(2)}$$

$$T_{xxyy} = -2T_{du}^{(1)} + 2T_{gu}^{(1)} - T_g^{(1)} + T_s^{(1)}$$

$$T_{xxzz} = T_{du}^{(1)} - T_{gu}^{(1)} - T_g^{(1)} + T_s^{(1)}$$

$$T_{xxxz} = T_{gaz}^{(1)}$$

$$T_{zzzz} = 4T_{du}^{(1)} + 8T_{du}^{(2)} + 2T_{gu}^{(1)} + 2T_g^{(1)} + T_s^{(1)} + 2T_s^{(2)}$$

$$T_{yzyz} = T_{du}^{(2)} - T_{gu}^{(1)} - T_g^{(1)} + T_s^{(2)}$$

$$T_{xyxy} = -2T_{du}^{(2)} + 2T_{gu}^{(1)} - T_g^{(1)} + T_s^{(2)}$$

\* Rank 4 tensor (ssa).

$$\begin{bmatrix} 0 & 0 & T_{xxzz} & 0 & 0 & T_{xxxz} \\ 0 & 0 & T_{xxzz} & 0 & 0 & -T_{xxxz} \\ -T_{xxzz} & -T_{xxzz} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & T_{yzzx} & 0 \\ 0 & 0 & 0 & -T_{yzzx} & 0 & 0 \\ -T_{xxxz} & T_{xxxz} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T_{xxzz} = 3T_{du}^{(3)}$$

$$T_{xxxz} = -2M_{faz}^{(1)} + 2M_{pz}^{(1)}$$

$$T_{yzzx} = -4M_{faz}^{(1)} - M_{pz}^{(1)}$$

\* Rank 4 tensor (aas).

$$\begin{bmatrix} T_{yzyz} & 0 & 0 \\ 0 & T_{yzyz} & 0 \\ 0 & 0 & T_{xyxy} \end{bmatrix}$$

$$T_{yzyz} = -2T_{du}^{(4)} + T_s^{(3)}$$

$$T_{xyxy} = 4T_{du}^{(4)} + T_s^{(3)}$$

\* Rank 4 tensor (aaa).

$$\begin{bmatrix} 0 & T_{yzzx} & 0 \\ -T_{yzzx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T_{yzzx} = -M_{pz}^{(2)}$$

\* Rank 4 tensor (sa).

$$\begin{bmatrix} 0 & 0 & T_{xxxz} \\ 0 & 0 & T_{xxxz} \\ 0 & 0 & T_{zzxy} \\ T_{yzyz} & T_{yzzx} & 0 \\ T_{yzzx} & -T_{yzyz} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}
T_{xxxy} &= -M_{faz}^{(2)} + M_{pz}^{(3)} \\
T_{zzxy} &= 2M_{faz}^{(2)} + M_{pz}^{(3)} + 2M_{pz}^{(4)} \\
T_{yzyz} &= -3T_{du}^{(5)} \\
T_{yzxz} &= -M_{faz}^{(2)} + M_{pz}^{(4)}
\end{aligned}$$

\* Rank 4 tensor (as).

$$\begin{bmatrix}
0 & 0 & 0 & T_{yzyz} & T_{yzxz} & 0 \\
0 & 0 & 0 & T_{yzxz} & -T_{yzyz} & 0 \\
T_{xyxx} & T_{xyxx} & T_{xyzz} & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{aligned}
T_{yzyz} &= -3T_{du}^{(6)} \\
T_{yzxz} &= -M_{faz}^{(3)} + M_{pz}^{(6)} \\
T_{xyxx} &= -M_{faz}^{(3)} + M_{pz}^{(5)} \\
T_{xyzz} &= 2M_{faz}^{(3)} + M_{pz}^{(5)} + 2M_{pz}^{(6)}
\end{aligned}$$

\* Rank 4 tensor (s).

$$\begin{bmatrix}
T_{xxxx} & T_{xxyy} & T_{xxzz} & 0 & 0 & T_{xxxy} & 0 & 0 & T_{xxyx} \\
T_{xxyy} & T_{xxxx} & T_{xxzz} & 0 & 0 & -T_{xxyx} & 0 & 0 & -T_{xxxy} \\
T_{zzxx} & T_{zzxx} & T_{zzzz} & 0 & 0 & T_{zzxy} & 0 & 0 & -T_{zzxy} \\
0 & 0 & 0 & T_{yzyz} & T_{yzxz} & 0 & T_{yzyy} & T_{yzxz} & 0 \\
0 & 0 & 0 & -T_{yzxz} & T_{yzyy} & 0 & -T_{yzxz} & T_{yzyz} & 0 \\
T_{xyxx} & -T_{xyxx} & 0 & 0 & 0 & T_{xyxy} & 0 & 0 & T_{xyxy}
\end{bmatrix}$$

$$\begin{aligned}
T_{xxxx} &= -2T_{du}^{(1)} - 4T_{du}^{(2)} - T_{gu}^{(1)} + 2T_g^{(1)} + T_s^{(1)} + 2T_s^{(2)} \\
T_{xxyy} &= -2T_{du}^{(1)} + 2T_{gu}^{(1)} - T_g^{(1)} + T_s^{(1)} \\
T_{xxzz} &= T_{du}^{(1)} + 3T_{du}^{(3)} - T_{gu}^{(1)} - T_g^{(1)} + T_s^{(1)} \\
T_{xxxy} &= -2M_{faz}^{(1)} - M_{faz}^{(2)} + 2M_{pz}^{(1)} + M_{pz}^{(3)} + T_{gaz}^{(1)} \\
T_{xxyx} &= -2M_{faz}^{(1)} + M_{faz}^{(2)} + 2M_{pz}^{(1)} - M_{pz}^{(3)} + T_{gaz}^{(1)} \\
T_{zzxx} &= T_{du}^{(1)} - 3T_{du}^{(3)} - T_{gu}^{(1)} - T_g^{(1)} + T_s^{(1)} \\
T_{zzzz} &= 4T_{du}^{(1)} + 8T_{du}^{(2)} + 2T_{gu}^{(1)} + 2T_g^{(1)} + T_s^{(1)} + 2T_s^{(2)} \\
T_{zzxy} &= 2M_{faz}^{(2)} + M_{pz}^{(3)} + 2M_{pz}^{(4)} \\
T_{yzyz} &= T_{du}^{(2)} - 3T_{du}^{(5)} - T_{gu}^{(1)} - T_g^{(1)} + T_s^{(2)} \\
T_{yzzx} &= -4M_{faz}^{(1)} - M_{faz}^{(2)} - M_{pz}^{(1)} + M_{pz}^{(4)} \\
T_{yzyy} &= T_{du}^{(2)} + 3T_{du}^{(5)} - T_{gu}^{(1)} - T_g^{(1)} + T_s^{(2)} \\
T_{yzxz} &= -4M_{faz}^{(1)} + M_{faz}^{(2)} - M_{pz}^{(1)} - M_{pz}^{(4)} \\
T_{xyxx} &= 2M_{faz}^{(1)} - 2M_{pz}^{(1)} + T_{gaz}^{(1)} \\
T_{xyxy} &= -2T_{du}^{(2)} + 2T_{gu}^{(1)} - T_g^{(1)} + T_s^{(2)}
\end{aligned}$$

\* Rank 4 tensor (a).

$$\begin{bmatrix}
0 & 0 & 0 & T_{yzyz} & T_{yzxz} & 0 & T_{yzyy} & T_{yzxz} & 0 \\
0 & 0 & 0 & T_{yzxz} & -T_{yzyy} & 0 & T_{yzzx} & -T_{yzyz} & 0 \\
T_{xyxx} & T_{xyxx} & T_{xyzz} & 0 & 0 & T_{xyxy} & 0 & 0 & -T_{xyxy}
\end{bmatrix}$$

$$\begin{aligned}
T_{yzyz} &= -2T_{du}^{(4)} - 3T_{du}^{(6)} + T_s^{(3)} \\
T_{yzxx} &= -M_{faz}^{(3)} - M_{pz}^{(2)} + M_{pz}^{(6)} \\
T_{yzyy} &= 2T_{du}^{(4)} - 3T_{du}^{(6)} - T_s^{(3)} \\
T_{yzxz} &= -M_{faz}^{(3)} + M_{pz}^{(2)} + M_{pz}^{(6)} \\
T_{xyxx} &= -M_{faz}^{(3)} + M_{pz}^{(5)} \\
T_{xyzz} &= 2M_{faz}^{(3)} + M_{pz}^{(5)} + 2M_{pz}^{(6)} \\
T_{xyxy} &= 4T_{du}^{(4)} + T_s^{(3)}
\end{aligned}$$

\* Rank 4 tensor (t).

$$\left[ \begin{array}{ccc}
T_{xxxx} & T_{xxxz} & 0 \\
-T_{xxxy} & T_{xxxx} & 0 \\
0 & 0 & T_{zzzz} \\
0 & 0 & T_{yyzz} \\
T_{zzxx} & 0 & 0 \\
T_{xxxy} & T_{xxyy} & 0 \\
0 & T_{yzyy} & 0 \\
0 & 0 & T_{zxxz} \\
T_{xyyy} + T_{yyzz} - T_{yzyy} - T_{zzxx} + T_{zxxz} & -T_{xxxy} & 0 \\
0 & 0 & 0
\end{array} \right]$$

$$\begin{aligned}
T_{xxxx} &= -2T_{du}^{(1)} - 4T_{du}^{(2)} - T_{gu}^{(1)} + 2T_g^{(1)} + 3T_s^{(1)} \\
T_{xxxy} &= T_{gaz}^{(1)} \\
T_{zzzz} &= 4T_{du}^{(1)} + 8T_{du}^{(2)} + 2T_{gu}^{(1)} + 2T_g^{(1)} + 3T_s^{(1)} \\
T_{yyzz} &= T_{du}^{(1)} + 3T_{du}^{(3)} - T_{gu}^{(1)} - T_g^{(1)} + T_s^{(1)} \\
T_{zzxx} &= T_{du}^{(1)} - 3T_{du}^{(3)} - T_{gu}^{(1)} - T_g^{(1)} + T_s^{(1)} \\
T_{xxyy} &= -2T_{du}^{(1)} + 2T_{gu}^{(1)} - T_g^{(1)} + T_s^{(1)} \\
T_{yzyy} &= T_{du}^{(2)} + 3T_{du}^{(5)} - T_{gu}^{(1)} - T_g^{(1)} + T_s^{(1)} \\
T_{zxxz} &= T_{du}^{(2)} - 3T_{du}^{(5)} - T_{gu}^{(1)} - T_g^{(1)} + T_s^{(1)}
\end{aligned}$$