Response Tensors up to 4th rank in D_{2d}

— polar tensors —

$$C^{(0,Q)} = \left(C^{(0,Q)}\right)$$

$$C^{(0,Q)} = Q_0$$

$$S^{(2,Q)} = \begin{pmatrix} S_{xx}^{(2,Q)} & 0 & 0 \\ 0 & S_{xx}^{(2,Q)} & 0 \\ 0 & 0 & S_{zz}^{(2,Q)} \end{pmatrix}$$

$$S_{xx}^{(2,Q)} = Q_0 - Q_u$$
$$S_{zz}^{(2,Q)} = Q_0 + 2Q_u$$

$$S^{(3,Q)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ S_{4x}^{(3,Q)} & 0 & 0 \\ 0 & S_{4x}^{(3,Q)} & 0 \\ 0 & 0 & S_{6z}^{(3,Q)} \end{pmatrix}$$

$$S_{4x}^{(3,Q)} = -G_v[1] + Q_{xyz}$$
$$S_{6z}^{(3,Q)} = 2G_v[1] + Q_{xyz}$$

$$A^{(3,Q)} = \begin{pmatrix} A_{4x}^{(3,Q)} & 0 & 0 \\ 0 & -A_{4x}^{(3,Q)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{4x}^{(3,Q)} = G_v[2]$$

$$S^{(4,Q)} = \begin{pmatrix} S_{11}^{(4,Q)} & S_{12}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & 0 & 0 \\ S_{12}^{(4,Q)} & S_{11}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & 0 & 0 \\ S_{13}^{(4,Q)} & S_{13}^{(4,Q)} & S_{33}^{(4,Q)} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66}^{(4,Q)} \end{pmatrix}$$

$$S_{11}^{(4,Q)} = Q_0[1] + 2Q_0[2] - Q_{4u} + 2Q_4 - 2Q_u[1] - 4Q_u[2]$$
$$S_{12}^{(4,Q)} = Q_0[1] + 2Q_{4u} - Q_4 - 2Q_u[1]$$
$$S_{13}^{(4,Q)} = Q_0[1] - Q_{4u} - Q_4 + Q_u[1]$$
$$S_{33}^{(4,Q)} = Q_0[1] + 2Q_0[2] + 2Q_{4u} + 2Q_4 + 4Q_u[1] + 8Q_u[2]$$
$$S_{44}^{(4,Q)} = Q_0[2] - Q_{4u} - Q_4 + Q_u[2]$$
$$S_{66}^{(4,Q)} = Q_0[2] + 2Q_{4u} - Q_4 - 2Q_u[2]$$

$$\bar{S}^{(4,Q)} = \begin{pmatrix} 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 \\ 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 \\ -\bar{S}_{13}^{(4,Q)} & -\bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{S}_{13}^{(4,Q)} = 3Q_u[3]$$

$$A^{(4,Q)} = \begin{pmatrix} A_{xx}^{(4,Q)} & 0 & 0 \\ 0 & A_{xx}^{(4,Q)} & 0 \\ 0 & 0 & A_{zz}^{(4,Q)} \end{pmatrix}$$

$$A_{xx}^{(4,Q)} = Q_0[3] - 2Q_u[6]$$
$$A_{zz}^{(4,Q)} = Q_0[3] + 4Q_u[6]$$

$$M^{(4,Q)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ M_{4x}^{(4,Q)} & 0 & 0 \\ 0 & -M_{4x}^{(4,Q)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$M_{4x}^{(4,Q)} = -3Q_u[4]$$

$$\bar{M}^{(4,Q)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}_{x4}^{(4,Q)} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\bar{M}_{x4}^{(4,Q)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{M}_{x4}^{(4,Q)} = -3Q_u[5]$$

<center>— axial tensors —</center>

$$S^{(2,G)} = \begin{pmatrix} S_{xx}^{(2,G)} & 0 & 0 \\ 0 & -S_{xx}^{(2,G)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_{xx}^{(2,G)} = G_v$$

$$S^{(3,G)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ S_{4x}^{(3,G)} & 0 & 0 \\ 0 & -S_{4x}^{(3,G)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_{4x}^{(3,G)} = -3Q_u[1]$$

$$A^{(3,G)} = \begin{pmatrix} A_{4x}^{(3,G)} & 0 & 0 \\ 0 & A_{4x}^{(3,G)} & 0 \\ 0 & 0 & A_{6z}^{(3,G)} \end{pmatrix}$$

$$A_{4x}^{(3,G)} = Q_0 - Q_u[2]$$
$$A_{6z}^{(3,G)} = Q_0 + 2Q_u[2]$$

$$S^{(4,G)} = \begin{pmatrix} S_{11}^{(4,G)} & 0 & S_{13}^{(4,G)} & 0 & 0 & 0 \\ 0 & -S_{11}^{(4,G)} & -S_{13}^{(4,G)} & 0 & 0 & 0 \\ S_{13}^{(4,G)} & -S_{13}^{(4,G)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^{(4,G)} & 0 & 0 \\ 0 & 0 & 0 & 0 & -S_{44}^{(4,G)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$S_{11}^{(4,G)} = G_{4v} + 2G_v[1] + 4G_v[2]$$
$$S_{13}^{(4,G)} = -G_{4v} + G_v[1]$$
$$S_{44}^{(4,G)} = G_{4v} - G_v[2]$$

$$\bar{S}^{(4,G)} = \begin{pmatrix} 0 & \bar{S}_{12}^{(4,G)} & \bar{S}_{13}^{(4,G)} & 0 & 0 & 0 \\ -\bar{S}_{12}^{(4,G)} & 0 & -\bar{S}_{13}^{(4,G)} & 0 & 0 & 0 \\ -\bar{S}_{13}^{(4,G)} & \bar{S}_{13}^{(4,G)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{S}_{12}^{(4,G)} = -2G_v[3] + 4Q_{xyz}[1]$$
$$\bar{S}_{13}^{(4,G)} = -G_v[3] - 4Q_{xyz}[1]$$

$$A^{(4,G)} = \begin{pmatrix} A_{xx}^{(4,G)} & 0 & 0 \\ 0 & -A_{xx}^{(4,G)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{xx}^{(4,G)} = 2G_v[6]$$

$$M^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ M_{4x}^{(4,G)} & 0 & 0 \\ 0 & M_{4x}^{(4,G)} & 0 \\ 0 & 0 & M_{6z}^{(4,G)} \end{pmatrix}$$

$$M_{4x}^{(4,G)} = -G_v[4] + Q_{xyz}[2]$$
$$M_{6z}^{(4,G)} = 2G_v[4] + Q_{xyz}[2]$$

$$\bar{M}^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}_{x4}^{(4,G)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{M}_{x4}^{(4,G)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{M}_{z6}^{(4,G)} \end{pmatrix}$$

$$\bar{M}_{x4}^{(4,G)} = -G_v[5] + Q_{xyz}[3]$$
$$\bar{M}_{z6}^{(4,G)} = 2G_v[5] + Q_{xyz}[3]$$