SAMB for "01"

Generated on 2023-06-02 20:42 by MultiPie 1.1.2

- Group: No. 207 O^1 $P432$ [cubic]
- Associated point group: No. 30 O 432 [cubic]

- Generation condition
  - model type: `tight_binding`
  - time-reversal type: `electric`
  - irrep: `[A1]`
  - `spinful`

- Unit cell:
  $a = 1.0,\ b = 1.0,\ c = 1.0,\ \alpha = 90.0,\ \beta = 90.0,\ \gamma = 90.0$
- Lattice vectors:
  $\boldsymbol{a}_1 = \begin{pmatrix} 1.0 & 0 & 0 \end{pmatrix}$
  $\boldsymbol{a}_2 = \begin{pmatrix} 0 & 1.0 & 0 \end{pmatrix}$
  $\boldsymbol{a}_3 = \begin{pmatrix} 0 & 0 & 1.0 \end{pmatrix}$

Table 1: High-symmetry line: Γ-X.

	symbol	position	symbol	position
	Γ	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	X	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$

- Kets: dimension = 8

Table 2: Hilbert space for full matrix.

	No.	ket	No.	ket	No.	ket	No.	ket	No.	ket
	1	$(s,\uparrow)@\mathrm{A}_1$	2	$(s,\downarrow)@\mathrm{A}_1$	3	$(p_x,\uparrow)@\mathrm{A}_1$	4	$(p_x,\downarrow)@\mathrm{A}_1$	5	$(p_y,\uparrow)@\mathrm{A}_1$
	6	$(p_y,\downarrow)@\mathrm{A}_1$	7	$(p_z,\uparrow)@\mathrm{A}_1$	8	$(p_z,\downarrow)@\mathrm{A}_1$				

- Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position	mapping
S_1	A_1	$\begin{pmatrix}0 & 0 & 0\end{pmatrix}$	[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24]

- Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

	bond	tail	head	n	#	$\boldsymbol{b}@\boldsymbol{c}$	mapping
B_1	b_1	A_1	A_1	1	1	$\begin{pmatrix}0 & 0 & 1\end{pmatrix}@\begin{pmatrix}0 & 0 & \frac{1}{2}\end{pmatrix}$	[1,2,-3,-4,-5,-8,19,22]
	b_2	A_1	A_1	1	1	$\begin{pmatrix}1 & 0 & 0\end{pmatrix}@\begin{pmatrix}\frac{1}{2} & 0 & 0\end{pmatrix}$	[6,-9,11,-12,13,-14,21,-24]
	b_3	A_1	A_1	1	1	$\begin{pmatrix}0 & 1 & 0\end{pmatrix}@\begin{pmatrix}0 & \frac{1}{2} & 0\end{pmatrix}$	[7,-10,15,16,-17,-18,-20,23]
B_2	b_4	A_1	A_1	2	1	$\begin{pmatrix}0 & 1 & 1\end{pmatrix}@\begin{pmatrix}0 & \frac{1}{2} & \frac{1}{2}\end{pmatrix}$	[1,-3,7,-10]
	b_5	A_1	A_1	2	1	$\begin{pmatrix}0 & 1 & -1\end{pmatrix}@\begin{pmatrix}0 & \frac{1}{2} & \frac{1}{2}\end{pmatrix}$	[-2,4,-20,23]
	b_6	A_1	A_1	2	1	$\begin{pmatrix}1 & 0 & -1\end{pmatrix}@\begin{pmatrix}\frac{1}{2} & 0 & \frac{1}{2}\end{pmatrix}$	[5,-12,13,-19]
	b_7	A_1	A_1	2	1	$\begin{pmatrix}1 & -1 & 0\end{pmatrix}@\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & 0\end{pmatrix}$	[6,-16,18,-24]
	b_8	A_1	A_1	2	1	$\begin{pmatrix}1 & 0 & 1\end{pmatrix}@\begin{pmatrix}\frac{1}{2} & 0 & \frac{1}{2}\end{pmatrix}$	[-8,11,-14,22]
	b_9	A_1	A_1	2	1	$\begin{pmatrix}1 & 1 & 0\end{pmatrix}@\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & 0\end{pmatrix}$	[-9,15,-17,21]

- SAMB:

No. 1 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$$

$$\hat{\mathbb{Z}}_1(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 2 $\hat{\mathbb{G}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_2, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_2 = \mathbb{X}_5[\mathbb{G}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$$

$$\hat{\mathbb{Z}}_2(\boldsymbol{k}) = \mathbb{X}_5[\mathbb{G}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 3 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_3, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_3 = \mathbb{X}_{11}[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$$

$$\hat{\mathbb{Z}}_3(\boldsymbol{k}) = \mathbb{X}_{11}[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 4 $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_3, \mathrm{S}_1]$

$$\hat{\mathbb{Z}}_4 = \mathbb{X}_{12}[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$$

$$\hat{\mathbb{Z}}_4(\boldsymbol{k}) = \mathbb{X}_{12}[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 5 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_5 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_1)}]$$

$$\hat{\mathbb{Z}}_5(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_1)}]$$

No. 6 $\hat{\mathbb{Q}}_4^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_6 = -\frac{\sqrt{3}\mathbb{X}_2[\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_5[\mathbb{T}_{4,0}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_3[\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_6[\mathbb{T}_{4,1}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_7[\mathbb{T}_{4,2}^{(b,T_1)}]}{3}$$

$$\hat{\mathbb{Z}}_6(\boldsymbol{k}) = -\frac{\sqrt{3}\mathbb{X}_2[\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_4[\mathbb{T}_{4,0}^{(k,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_3[\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_5[\mathbb{T}_{4,1}^{(k,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_6[\mathbb{T}_{4,2}^{(k,T_1)}]}{3}$$

No. 7 $\hat{\mathbb{G}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_7 = \mathbb{X}_5[\mathbb{G}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_1)}]$$

$$\hat{\mathbb{Z}}_7(\boldsymbol{k}) = \mathbb{X}_5[\mathbb{G}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_1)}]$$

No. 8 $\hat{\mathbb{G}}_0^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_8 = \frac{\sqrt{2}\mathbb{X}_6[\mathbb{G}_{2,0}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_7[\mathbb{G}_{2,1}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_4[\mathbb{Q}_{2,1}^{(b,E)}]}{2}$$

$$\hat{\mathbb{Z}}_8(\boldsymbol{k}) = \frac{\sqrt{2}\mathbb{X}_6[\mathbb{G}_{2,0}^{(a,E)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_2[\mathbb{Q}_{2,0}^{(k,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_7[\mathbb{G}_{2,1}^{(a,E)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_3[\mathbb{Q}_{2,1}^{(k,E)}]}{2}$$

No. 9 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_9 = \mathbb{X}_{11}[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_1)}]$$

$$\hat{\mathbb{Z}}_9(\boldsymbol{k}) = \mathbb{X}_{11}[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_1)}]$$

No. 10 $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{10} = \mathbb{X}_{12}[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(b,A_1)}]$$

$$\hat{\mathbb{Z}}_{10}(\boldsymbol{k}) = \mathbb{X}_{12}[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_1[\mathbb{Q}_0^{(k,A_1)}]$$

No. 11 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{11} = \frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{Q}_{2,0}^{(a,E)}] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{Q}_{2,1}^{(a,E)}] \otimes \mathbb{Y}_4[\mathbb{Q}_{2,1}^{(b,E)}]}{2}$$

$$\hat{\mathbb{Z}}_{11}(\boldsymbol{k}) = \frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{Q}_{2,0}^{(a,E)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_2[\mathbb{Q}_{2,0}^{(k,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{Q}_{2,1}^{(a,E)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_3[\mathbb{Q}_{2,1}^{(k,E)}]}{2}$$

No. 12 $\hat{\mathbb{Q}}_0^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{12} = \frac{\sqrt{2}\mathbb{X}_{18}[\mathbb{Q}_{2,0}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_3[\mathbb{Q}_{2,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{19}[\mathbb{Q}_{2,1}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_4[\mathbb{Q}_{2,1}^{(b,E)}]}{2}$$

$$\hat{\mathbb{Z}}_{12}(\boldsymbol{k}) = \frac{\sqrt{2}\mathbb{X}_{18}[\mathbb{Q}_{2,0}^{(a,E)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_2[\mathbb{Q}_{2,0}^{(k,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{19}[\mathbb{Q}_{2,1}^{(a,E)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_3[\mathbb{Q}_{2,1}^{(k,E)}]}{2}$$

No. 13 $\hat{\mathbb{Q}}_4^{(A_1)}$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{13} = -\frac{\sqrt{3}\mathbb{X}_{23}[\mathbb{M}_{1,0}^{(a,T_1)}] \otimes \mathbb{Y}_5[\mathbb{T}_{4,0}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{24}[\mathbb{M}_{1,1}^{(a,T_1)}] \otimes \mathbb{Y}_6[\mathbb{T}_{4,1}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{25}[\mathbb{M}_{1,2}^{(a,T_1)}] \otimes \mathbb{Y}_7[\mathbb{T}_{4,2}^{(b,T_1)}]}{3}$$

$$\hat{\mathbb{Z}}_{13}(\boldsymbol{k}) = -\frac{\sqrt{3}\mathbb{X}_{23}[\mathbb{M}_{1,0}^{(a,T_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_4[\mathbb{T}_{4,0}^{(k,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{24}[\mathbb{M}_{1,1}^{(a,T_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_5[\mathbb{T}_{4,1}^{(k,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{25}[\mathbb{M}_{1,2}^{(a,T_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_6[\mathbb{T}_{4,2}^{(k,T_1)}]}{3}$$

No. 14 $\hat{\mathbb{Q}}_4^{(A_1)}(1,1)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{14} = -\frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{M}_{1,0}^{(a,T_1)}(1,1)] \otimes \mathbb{Y}_5[\mathbb{T}_{4,0}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{27}[\mathbb{M}_{1,1}^{(a,T_1)}(1,1)] \otimes \mathbb{Y}_6[\mathbb{T}_{4,1}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{28}[\mathbb{M}_{1,2}^{(a,T_1)}(1,1)] \otimes \mathbb{Y}_7[\mathbb{T}_{4,2}^{(b,T_1)}]}{3}$$

$$\hat{\mathbb{Z}}_{14}(\boldsymbol{k}) = -\frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{M}_{1,0}^{(a,T_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_4[\mathbb{T}_{4,0}^{(k,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{27}[\mathbb{M}_{1,1}^{(a,T_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_5[\mathbb{T}_{4,1}^{(k,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{28}[\mathbb{M}_{1,2}^{(a,T_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_6[\mathbb{T}_{4,2}^{(k,T_1)}]}{3}$$

No. 15 $\hat{\mathbb{Q}}_4^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{15} = -\frac{\sqrt{3}\mathbb{X}_{29}[\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_5[\mathbb{T}_{4,0}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{30}[\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_6[\mathbb{T}_{4,1}^{(b,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_7[\mathbb{T}_{4,2}^{(b,T_1)}]}{3}$$

$$\hat{\mathbb{Z}}_{15}(\boldsymbol{k}) = -\frac{\sqrt{3}\mathbb{X}_{29}[\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_4[\mathbb{T}_{4,0}^{(k,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{30}[\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_5[\mathbb{T}_{4,1}^{(k,T_1)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_6[\mathbb{T}_{4,2}^{(k,T_1)}]}{3}$$

No. 16 $\hat{\mathbb{Q}}_4^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{16} = \frac{\sqrt{3}\mathbb{X}_{32}[\mathbb{M}_{3,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_5[\mathbb{T}_{4,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{33}[\mathbb{M}_{3,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_6[\mathbb{T}_{4,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{34}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_7[\mathbb{T}_{4,2}^{(b,T_1)}]}{3}$$

$$\hat{\mathbb{Z}}_{16}(\boldsymbol{k}) = \frac{\sqrt{3}\mathbb{X}_{32}[\mathbb{M}_{3,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_4[\mathbb{T}_{4,0}^{(k,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{33}[\mathbb{M}_{3,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_5[\mathbb{T}_{4,1}^{(k,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{34}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_6[\mathbb{T}_{4,2}^{(k,T_1)}]}{3}$$

No. 17 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_1, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{17} = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_8[\mathbb{Q}_0^{(b,A_1)}]$$

$$\hat{\mathbb{Z}}_{17}(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_7[\mathbb{Q}_0^{(k,A_1)}]$$

No. 18 $\hat{\mathbb{G}}_0^{(A_1)}(1,-1)$ $[\mathrm{M}_1, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{18} = \frac{\sqrt{3}\mathbb{X}_2[\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{1,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_3[\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{15}[\mathbb{T}_{1,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(b,T_1)}]}{3}$$

$$\hat{\mathbb{Z}}_{18}(\boldsymbol{k}) = \frac{\sqrt{3}\mathbb{X}_2[\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{13}[\mathbb{T}_{1,0}^{(k,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_3[\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{14}[\mathbb{T}_{1,1}^{(k,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_4[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{15}[\mathbb{T}_{1,2}^{(k,T_1)}]}{3}$$

No. 19 $\hat{\mathbb{G}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_2, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{19} = \mathbb{X}_5[\mathbb{G}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_8[\mathbb{Q}_0^{(b,A_1)}]$$

$$\hat{\mathbb{Z}}_{19}(\boldsymbol{k}) = \mathbb{X}_5[\mathbb{G}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_7[\mathbb{Q}_0^{(k,A_1)}]$$

No. 20 $\hat{\mathbb{G}}_0^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{20} = \frac{\sqrt{2}\mathbb{X}_6[\mathbb{G}_{2,0}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_7[\mathbb{G}_{2,1}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,1}^{(b,E)}]}{2}$$

$$\hat{\mathbb{Z}}_{20}(\boldsymbol{k}) = \frac{\sqrt{2}\mathbb{X}_6[\mathbb{G}_{2,0}^{(a,E)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_8[\mathbb{Q}_{2,0}^{(k,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_7[\mathbb{G}_{2,1}^{(a,E)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_9[\mathbb{Q}_{2,1}^{(k,E)}]}{2}$$

No. 21 $\hat{\mathbb{Q}}_4^{(A_1)}(1,-1)$ $[\mathrm{M}_2, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{21} = \frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{G}_{2,2}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_8[\mathbb{G}_{2,0}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{3,0}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_9[\mathbb{G}_{2,1}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{3,1}^{(b,T_2)}]}{3}$$

$$\hat{\mathbb{Z}}_{21}(\boldsymbol{k}) = \frac{\sqrt{3}\mathbb{X}_{10}[\mathbb{G}_{2,2}^{(a,T_2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{12}[\mathbb{Q}_{3,2}^{(k,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_8[\mathbb{G}_{2,0}^{(a,T_2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{10}[\mathbb{Q}_{3,0}^{(k,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_9[\mathbb{G}_{2,1}^{(a,T_2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{11}[\mathbb{Q}_{3,1}^{(k,T_2)}]}{3}$$

No. 22 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{22} = \mathbb{X}_{11}[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_8[\mathbb{Q}_0^{(b,A_1)}]$$

$$\hat{\mathbb{Z}}_{22}(\boldsymbol{k}) = \mathbb{X}_{11}[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_7[\mathbb{Q}_0^{(k,A_1)}]$$

No. 23 $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{23} = \mathbb{X}_{12}[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_8[\mathbb{Q}_0^{(b,A_1)}]$$

$$\hat{\mathbb{Z}}_{23}(\boldsymbol{k}) = \mathbb{X}_{12}[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_7[\mathbb{Q}_0^{(k,A_1)}]$$

No. 24 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{24} = \frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{Q}_{2,0}^{(a,E)}] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{Q}_{2,1}^{(a,E)}] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,1}^{(b,E)}]}{2}$$

$$\hat{\mathbb{Z}}_{24}(\boldsymbol{k}) = \frac{\sqrt{2}\mathbb{X}_{13}[\mathbb{Q}_{2,0}^{(a,E)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_8[\mathbb{Q}_{2,0}^{(k,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{14}[\mathbb{Q}_{2,1}^{(a,E)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_9[\mathbb{Q}_{2,1}^{(k,E)}]}{2}$$

No. 25 $\hat{\mathbb{G}}_4^{(A_1)}$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{25} = \frac{\sqrt{3}\mathbb{X}_{15}[\mathbb{Q}_{2,0}^{(a,T_2)}] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{3,0}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{16}[\mathbb{Q}_{2,1}^{(a,T_2)}] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{3,1}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{17}[\mathbb{Q}_{2,2}^{(a,T_2)}] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3}$$

$$\hat{\mathbb{Z}}_{25}(\boldsymbol{k}) = \frac{\sqrt{3}\mathbb{X}_{15}[\mathbb{Q}_{2,0}^{(a,T_2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{10}[\mathbb{Q}_{3,0}^{(k,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{16}[\mathbb{Q}_{2,1}^{(a,T_2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{11}[\mathbb{Q}_{3,1}^{(k,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{17}[\mathbb{Q}_{2,2}^{(a,T_2)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{12}[\mathbb{Q}_{3,2}^{(k,T_2)}]}{3}$$

No. 26 $\hat{\mathbb{Q}}_0^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{26} = \frac{\sqrt{2}\mathbb{X}_{18}[\mathbb{Q}_{2,0}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_9[\mathbb{Q}_{2,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{19}[\mathbb{Q}_{2,1}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_{10}[\mathbb{Q}_{2,1}^{(b,E)}]}{2}$$

$$\hat{\mathbb{Z}}_{26}(\boldsymbol{k}) = \frac{\sqrt{2}\mathbb{X}_{18}[\mathbb{Q}_{2,0}^{(a,E)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_8[\mathbb{Q}_{2,0}^{(k,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{19}[\mathbb{Q}_{2,1}^{(a,E)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_9[\mathbb{Q}_{2,1}^{(k,E)}]}{2}$$

No. 27 $\hat{\mathbb{G}}_4^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{27} = \frac{\sqrt{3}\mathbb{X}_{20}[\mathbb{Q}_{2,0}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{11}[\mathbb{Q}_{3,0}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{21}[\mathbb{Q}_{2,1}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{12}[\mathbb{Q}_{3,1}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{22}[\mathbb{Q}_{2,2}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{13}[\mathbb{Q}_{3,2}^{(b,T_2)}]}{3}$$

$$\hat{\mathbb{Z}}_{27}(\boldsymbol{k}) = \frac{\sqrt{3}\mathbb{X}_{20}[\mathbb{Q}_{2,0}^{(a,T_2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{10}[\mathbb{Q}_{3,0}^{(k,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{21}[\mathbb{Q}_{2,1}^{(a,T_2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{11}[\mathbb{Q}_{3,1}^{(k,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{22}[\mathbb{Q}_{2,2}^{(a,T_2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{12}[\mathbb{Q}_{3,2}^{(k,T_2)}]}{3}$$

No. 28 $\hat{\mathbb{G}}_0^{(A_1)}$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{28} = \frac{\sqrt{3}\mathbb{X}_{23}[\mathbb{M}_{1,0}^{(a,T_1)}] \otimes \mathbb{Y}_{14}[\mathbb{T}_{1,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{24}[\mathbb{M}_{1,1}^{(a,T_1)}] \otimes \mathbb{Y}_{15}[\mathbb{T}_{1,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{25}[\mathbb{M}_{1,2}^{(a,T_1)}] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(b,T_1)}]}{3}$$

$$\hat{\mathbb{Z}}_{28}(\boldsymbol{k}) = \frac{\sqrt{3}\mathbb{X}_{23}[\mathbb{M}_{1,0}^{(a,T_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{13}[\mathbb{T}_{1,0}^{(k,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{24}[\mathbb{M}_{1,1}^{(a,T_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{14}[\mathbb{T}_{1,1}^{(k,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{25}[\mathbb{M}_{1,2}^{(a,T_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{15}[\mathbb{T}_{1,2}^{(k,T_1)}]}{3}$$

No. 29 $\hat{\mathbb{G}}_0^{(A_1)}(1,1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{29} = \frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{M}_{1,0}^{(a,T_1)}(1,1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{1,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{27}[\mathbb{M}_{1,1}^{(a,T_1)}(1,1)] \otimes \mathbb{Y}_{15}[\mathbb{T}_{1,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{28}[\mathbb{M}_{1,2}^{(a,T_1)}(1,1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(b,T_1)}]}{3}$$

$$\hat{\mathbb{Z}}_{29}(\boldsymbol{k}) = \frac{\sqrt{3}\mathbb{X}_{26}[\mathbb{M}_{1,0}^{(a,T_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{13}[\mathbb{T}_{1,0}^{(k,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{27}[\mathbb{M}_{1,1}^{(a,T_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{14}[\mathbb{T}_{1,1}^{(k,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{28}[\mathbb{M}_{1,2}^{(a,T_1)}(1,1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{15}[\mathbb{T}_{1,2}^{(k,T_1)}]}{3}$$

No. 30 $\hat{\mathbb{G}}_0^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{30} = \frac{\sqrt{3}\mathbb{X}_{29}[\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{1,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{30}[\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{15}[\mathbb{T}_{1,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(b,T_1)}]}{3}$$

$$\hat{\mathbb{Z}}_{30}(\boldsymbol{k}) = \frac{\sqrt{3}\mathbb{X}_{29}[\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{13}[\mathbb{T}_{1,0}^{(k,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{30}[\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{14}[\mathbb{T}_{1,1}^{(k,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{31}[\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{15}[\mathbb{T}_{1,2}^{(k,T_1)}]}{3}$$

No. 31 $\hat{\mathbb{G}}_4^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{31} = \frac{\sqrt{3}\mathbb{X}_{32}[\mathbb{M}_{3,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{14}[\mathbb{T}_{1,0}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{33}[\mathbb{M}_{3,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{15}[\mathbb{T}_{1,1}^{(b,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{34}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{Y}_{16}[\mathbb{T}_{1,2}^{(b,T_1)}]}{3}$$

$$\hat{\mathbb{Z}}_{31}(\boldsymbol{k}) = \frac{\sqrt{3}\mathbb{X}_{32}[\mathbb{M}_{3,0}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{13}[\mathbb{T}_{1,0}^{(k,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{33}[\mathbb{M}_{3,1}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{14}[\mathbb{T}_{1,1}^{(k,T_1)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{34}[\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{15}[\mathbb{T}_{1,2}^{(k,T_1)}]}{3}$$

No. 32 $\hat{\mathbb{Q}}_4^{(A_1)}(1,-1)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{32} = -\frac{\sqrt{3}\mathbb{X}_{35}[\mathbb{M}_{3,0}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{17}[\mathbb{T}_{2,0}^{(b,T_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{36}[\mathbb{M}_{3,1}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{18}[\mathbb{T}_{2,1}^{(b,T_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{37}[\mathbb{M}_{3,2}^{(a,T_2)}(1,-1)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{2,2}^{(b,T_2)}]}{3}$$

$$\hat{\mathbb{Z}}_{32}(\boldsymbol{k}) = -\frac{\sqrt{3}\mathbb{X}_{35}[\mathbb{M}_{3,0}^{(a,T_2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{16}[\mathbb{T}_{2,0}^{(k,T_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{36}[\mathbb{M}_{3,1}^{(a,T_2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{17}[\mathbb{T}_{2,1}^{(k,T_2)}]}{3} - \frac{\sqrt{3}\mathbb{X}_{37}[\mathbb{M}_{3,2}^{(a,T_2)}(1,-1)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{18}[\mathbb{T}_{2,2}^{(k,T_2)}]}{3}$$

No. 33 $\hat{\mathbb{Q}}_0^{(A_1)}(1,0)$ $[\mathrm{M}_3, \mathrm{B}_2]$

$$\hat{\mathbb{Z}}_{33} = \frac{\sqrt{3}\mathbb{X}_{38}[\mathbb{T}_{2,0}^{(a,T_2)}(1,0)] \otimes \mathbb{Y}_{17}[\mathbb{T}_{2,0}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{39}[\mathbb{T}_{2,1}^{(a,T_2)}(1,0)] \otimes \mathbb{Y}_{18}[\mathbb{T}_{2,1}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{40}[\mathbb{T}_{2,2}^{(a,T_2)}(1,0)] \otimes \mathbb{Y}_{19}[\mathbb{T}_{2,2}^{(b,T_2)}]}{3}$$

$$\hat{\mathbb{Z}}_{33}(\boldsymbol{k}) = \frac{\sqrt{3}\mathbb{X}_{38}[\mathbb{T}_{2,0}^{(a,T_2)}(1,0)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{16}[\mathbb{T}_{2,0}^{(k,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{39}[\mathbb{T}_{2,1}^{(a,T_2)}(1,0)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{17}[\mathbb{T}_{2,1}^{(k,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_{40}[\mathbb{T}_{2,2}^{(a,T_2)}(1,0)] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}] \otimes \mathbb{F}_{18}[\mathbb{T}_{2,2}^{(k,T_2)}]}{3}$$

Table 5: Atomic SAMB group.

group	bra	ket
M_1	$(s,\uparrow),(s,\downarrow)$	$(s,\uparrow),(s,\downarrow)$
M_2	$(s,\uparrow),(s,\downarrow)$	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$
M_3	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow),(p_z,\uparrow),(p_z,\downarrow)$

Table 6: Atomic SAMB.

symbol	type	group	form
$\mathbb{X}_1$	$\mathbb{Q}_0^{(a,A_1)}$	M_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{X}_2$	$\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$
$\mathbb{X}_3$	$\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & -\frac{\sqrt{2}i}{2} \\ \frac{\sqrt{2}i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_4$	$\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)$	M_1	$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{X}_5$	$\mathbb{G}_0^{(a,A_1)}(1,1)$	M_2	$\begin{pmatrix} 0 & \frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}i}{6} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}i}{6} \end{pmatrix}$
$\mathbb{X}_6$	$\mathbb{G}_{2,0}^{(a,E)}(1,-1)$	M_2	$\begin{pmatrix} 0 & -\frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}i}{3} & 0 \\ -\frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}i}{3} \end{pmatrix}$
$\mathbb{X}_7$	$\mathbb{G}_{2,1}^{(a,E)}(1,-1)$	M_2	$\begin{pmatrix} 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ \frac{i}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_8$	$\mathbb{G}_{2,0}^{(a,T_2)}(1,-1)$	M_2	$\begin{pmatrix} 0 & 0 & \frac{i}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & -\frac{i}{2} & -\frac{1}{2} & 0 \end{pmatrix}$
$\mathbb{X}_9$	$\mathbb{G}_{2,1}^{(a,T_2)}(1,-1)$	M_2	$\begin{pmatrix} \frac{i}{2} & 0 & 0 & 0 & 0 & \frac{i}{2} \\ 0 & -\frac{i}{2} & 0 & 0 & \frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{X}_{10}$	$\mathbb{G}_{2,2}^{(a,T_2)}(1,-1)$	M_2	$\begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{11}$	$\mathbb{Q}_0^{(a,A_1)}$	M_3	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_{12}$	$\mathbb{Q}_0^{(a,A_1)}(1,1)$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ \frac{\sqrt{3}i}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{13}$	$\mathbb{Q}_{2,0}^{(a,E)}$	M_3	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{X}_{14}$	$\mathbb{Q}_{2,1}^{(a,E)}$	M_3	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{15}$	$\mathbb{Q}_{2,0}^{(a,T_2)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{16}$	$\mathbb{Q}_{2,1}^{(a,T_2)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{17}$	$\mathbb{Q}_{2,2}^{(a,T_2)}$	M_3	$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{18}$	$\mathbb{Q}_{2,0}^{(a,E)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{12} \\ 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & \frac{\sqrt{6}i}{12} & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{19}$	$\mathbb{Q}_{2,1}^{(a,E)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{20}$	$\mathbb{Q}_{2,0}^{(a,T_2)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} \\ 0 & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{21}$	$\mathbb{Q}_{2,1}^{(a,T_2)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ \frac{\sqrt{2}i}{4} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & \frac{\sqrt{2}i}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}i}{4} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{22}$	$\mathbb{Q}_{2,2}^{(a,T_2)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{4} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & 0 \\ 0 & -\frac{\sqrt{2}i}{4} & 0 & \frac{\sqrt{2}}{4} & 0 & 0 \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{23}$	$\mathbb{M}_{1,0}^{(a,T_1)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{24}$	$\mathbb{M}_{1,1}^{(a,T_1)}$	M_3	$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{25}$	$\mathbb{M}_{1,2}^{(a,T_1)}$	M_3	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{26}$	$\mathbb{M}_{1,0}^{(a,T_1)}(1,1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{30}}{15} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{20} & 0 \\ \frac{\sqrt{30}}{15} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{20} \\ 0 & -\frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 \\ \frac{\sqrt{30}i}{20} & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & 0 \\ \frac{\sqrt{30}}{20} & 0 & 0 & 0 & 0 & -\frac{\sqrt{30}}{30} \\ 0 & -\frac{\sqrt{30}}{20} & 0 & 0 & -\frac{\sqrt{30}}{30} & 0 \end{pmatrix}$
$\mathbb{X}_{27}$	$\mathbb{M}_{1,1}^{(a,T_1)}(1,1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{30}i}{30} & 0 & \frac{\sqrt{30}}{20} & 0 & 0 \\ -\frac{\sqrt{30}i}{30} & 0 & \frac{\sqrt{30}}{20} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{15} & \frac{\sqrt{30}}{20} & 0 \\ \frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{15} & 0 & 0 & -\frac{\sqrt{30}}{20} \\ 0 & 0 & \frac{\sqrt{30}}{20} & 0 & 0 & \frac{\sqrt{30}i}{30} \\ 0 & 0 & 0 & -\frac{\sqrt{30}}{20} & -\frac{\sqrt{30}i}{30} & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{28}$	$\mathbb{M}_{1,2}^{(a,T_1)}(1,1)$	M_3	$\begin{pmatrix} -\frac{\sqrt{30}}{30} & 0 & 0 & 0 & 0 & \frac{\sqrt{30}}{20} \\ 0 & \frac{\sqrt{30}}{30} & 0 & 0 & \frac{\sqrt{30}}{20} & 0 \\ 0 & 0 & -\frac{\sqrt{30}}{30} & 0 & 0 & -\frac{\sqrt{30}i}{20} \\ 0 & 0 & 0 & \frac{\sqrt{30}}{30} & \frac{\sqrt{30}i}{20} & 0 \\ 0 & \frac{\sqrt{30}}{20} & 0 & -\frac{\sqrt{30}i}{20} & \frac{\sqrt{30}}{15} & 0 \\ \frac{\sqrt{30}}{20} & 0 & \frac{\sqrt{30}i}{20} & 0 & 0 & -\frac{\sqrt{30}}{15} \end{pmatrix}$
$\mathbb{X}_{29}$	$\mathbb{M}_{1,0}^{(a,T_1)}(1,-1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{30}$	$\mathbb{M}_{1,1}^{(a,T_1)}(1,-1)$	M_3	$\begin{pmatrix} 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{6}i}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}i}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{31}$	$\mathbb{M}_{1,2}^{(a,T_1)}(1,-1)$	M_3	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{X}_{32}$	$\mathbb{M}_{3,0}^{(a,T_1)}(1,-1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{5}}{5} & 0 & \frac{\sqrt{5}i}{10} & -\frac{\sqrt{5}}{10} & 0 \\ \frac{\sqrt{5}}{5} & 0 & -\frac{\sqrt{5}i}{10} & 0 & 0 & \frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 \\ -\frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & 0 \\ -\frac{\sqrt{5}}{10} & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}}{10} & 0 & 0 & -\frac{\sqrt{5}}{10} & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{33}$	$\mathbb{M}_{3,1}^{(a,T_1)}(1,-1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 \\ -\frac{\sqrt{5}i}{10} & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{5}}{10} & 0 & -\frac{\sqrt{5}i}{5} & -\frac{\sqrt{5}}{10} & 0 \\ -\frac{\sqrt{5}}{10} & 0 & \frac{\sqrt{5}i}{5} & 0 & 0 & \frac{\sqrt{5}}{10} \\ 0 & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & \frac{\sqrt{5}i}{10} \\ 0 & 0 & 0 & \frac{\sqrt{5}}{10} & -\frac{\sqrt{5}i}{10} & 0 \end{pmatrix}$
$\mathbb{X}_{34}$	$\mathbb{M}_{3,2}^{(a,T_1)}(1,-1)$	M_3	$\begin{pmatrix} -\frac{\sqrt{5}}{10} & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}}{10} \\ 0 & \frac{\sqrt{5}}{10} & 0 & 0 & -\frac{\sqrt{5}}{10} & 0 \\ 0 & 0 & -\frac{\sqrt{5}}{10} & 0 & 0 & \frac{\sqrt{5}i}{10} \\ 0 & 0 & 0 & \frac{\sqrt{5}}{10} & -\frac{\sqrt{5}i}{10} & 0 \\ 0 & -\frac{\sqrt{5}}{10} & 0 & \frac{\sqrt{5}i}{10} & \frac{\sqrt{5}}{5} & 0 \\ -\frac{\sqrt{5}}{10} & 0 & -\frac{\sqrt{5}i}{10} & 0 & 0 & -\frac{\sqrt{5}}{5} \end{pmatrix}$
$\mathbb{X}_{35}$	$\mathbb{M}_{3,0}^{(a,T_2)}(1,-1)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{3}i}{6} & -\frac{\sqrt{3}}{6} & 0 \\ 0 & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & -\frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 \\ \frac{\sqrt{3}i}{6} & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \\ 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{36}$	$\mathbb{M}_{3,1}^{(a,T_2)}(1,-1)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 \\ -\frac{\sqrt{3}i}{6} & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \\ -\frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{6} \\ 0 & 0 & \frac{\sqrt{3}}{6} & 0 & 0 & -\frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}i}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{37}$	$\mathbb{M}_{3,2}^{(a,T_2)}(1,-1)$	M_3	$\begin{pmatrix} \frac{\sqrt{3}}{6} & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{6} \\ 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}}{6} & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{6} & 0 & 0 & \frac{\sqrt{3}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{6} & -\frac{\sqrt{3}i}{6} & 0 \\ 0 & \frac{\sqrt{3}}{6} & 0 & \frac{\sqrt{3}i}{6} & 0 & 0 \\ \frac{\sqrt{3}}{6} & 0 & -\frac{\sqrt{3}i}{6} & 0 & 0 & 0 \end{pmatrix}$

continued ...

Table 6

symbol	type	group	form
$\mathbb{X}_{38}$	$\mathbb{T}^{(a,T_2)}_{2,0}(1,0)$	M_3	$\begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{6}i}{12} & \frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & \frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 \\ -\frac{\sqrt{6}i}{12} & 0 & \frac{\sqrt{6}}{6} & 0 & 0 & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{6} \\ 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{39}$	$\mathbb{T}^{(a,T_2)}_{2,1}(1,0)$	M_3	$\begin{pmatrix} 0 & \frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 \\ -\frac{\sqrt{6}i}{6} & 0 & \frac{\sqrt{6}}{12} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}}{12} & 0 \\ \frac{\sqrt{6}}{12} & 0 & 0 & 0 & 0 & \frac{\sqrt{6}}{12} \\ 0 & 0 & -\frac{\sqrt{6}}{12} & 0 & 0 & -\frac{\sqrt{6}i}{6} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{12} & \frac{\sqrt{6}i}{6} & 0 \end{pmatrix}$
$\mathbb{X}_{40}$	$\mathbb{T}^{(a,T_2)}_{2,2}(1,0)$	M_3	$\begin{pmatrix} \frac{\sqrt{6}}{6} & 0 & 0 & 0 & 0 & -\frac{\sqrt{6}}{12} \\ 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}}{12} & 0 \\ 0 & 0 & -\frac{\sqrt{6}}{6} & 0 & 0 & -\frac{\sqrt{6}i}{12} \\ 0 & 0 & 0 & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}i}{12} & 0 \\ 0 & -\frac{\sqrt{6}}{12} & 0 & -\frac{\sqrt{6}i}{12} & 0 & 0 \\ -\frac{\sqrt{6}}{12} & 0 & \frac{\sqrt{6}i}{12} & 0 & 0 & 0 \end{pmatrix}$

Table 7: Cluster SAMB.

symbol	type	cluster	form
$\mathbb{Y}_1$	$\mathbb{Q}^{(s,A_1)}_0$	S_1	$\begin{pmatrix} 1 \end{pmatrix}$
$\mathbb{Y}_2$	$\mathbb{Q}^{(b,A_1)}_0$	B_1	$\begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{Y}_3$	$\mathbb{Q}^{(b,E)}_{2,0}$	B_1	$\begin{pmatrix} -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_4$	$\mathbb{Q}^{(b,E)}_{2,1}$	B_1	$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{Y}_5$	$\mathbb{T}^{(b,T_1)}_{4,0}$	B_1	$\begin{pmatrix} 0 & i & 0 \end{pmatrix}$
$\mathbb{Y}_6$	$\mathbb{T}^{(b,T_1)}_{4,1}$	B_1	$\begin{pmatrix} 0 & 0 & i \end{pmatrix}$

continued ...

Table 7

symbol	type	cluster	form
$\mathbb{Y}_7$	$\mathbb{T}_{4,2}^{(b,T_1)}$	B_1	$\begin{pmatrix} i & 0 & 0 \end{pmatrix}$
$\mathbb{Y}_8$	$\mathbb{Q}_{0}^{(b,A_1)}$	B_2	$\begin{pmatrix} \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_9$	$\mathbb{Q}_{2,0}^{(b,E)}$	B_2	$\begin{pmatrix} -\frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{Y}_{10}$	$\mathbb{Q}_{2,1}^{(b,E)}$	B_2	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 \end{pmatrix}$
$\mathbb{Y}_{11}$	$\mathbb{Q}_{3,0}^{(b,T_2)}$	B_2	$\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{Y}_{12}$	$\mathbb{Q}_{3,1}^{(b,T_2)}$	B_2	$\begin{pmatrix} 0 & 0 & -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$
$\mathbb{Y}_{13}$	$\mathbb{Q}_{3,2}^{(b,T_2)}$	B_2	$\begin{pmatrix} 0 & 0 & 0 & -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{Y}_{14}$	$\mathbb{T}_{1,0}^{(b,T_1)}$	B_2	$\begin{pmatrix} 0 & 0 & \frac{i}{2} & \frac{i}{2} & \frac{i}{2} & \frac{i}{2} \end{pmatrix}$
$\mathbb{Y}_{15}$	$\mathbb{T}_{1,1}^{(b,T_1)}$	B_2	$\begin{pmatrix} \frac{i}{2} & \frac{i}{2} & 0 & -\frac{i}{2} & 0 & \frac{i}{2} \end{pmatrix}$
$\mathbb{Y}_{16}$	$\mathbb{T}_{1,2}^{(b,T_1)}$	B_2	$\begin{pmatrix} \frac{i}{2} & -\frac{i}{2} & -\frac{i}{2} & 0 & \frac{i}{2} & 0 \end{pmatrix}$
$\mathbb{Y}_{17}$	$\mathbb{T}_{2,0}^{(b,T_2)}$	B_2	$\begin{pmatrix} 0 & 0 & \frac{i}{2} & -\frac{i}{2} & \frac{i}{2} & -\frac{i}{2} \end{pmatrix}$
$\mathbb{Y}_{18}$	$\mathbb{T}_{2,1}^{(b,T_2)}$	B_2	$\begin{pmatrix} -\frac{i}{2} & -\frac{i}{2} & 0 & -\frac{i}{2} & 0 & \frac{i}{2} \end{pmatrix}$
$\mathbb{Y}_{19}$	$\mathbb{T}_{2,2}^{(b,T_2)}$	B_2	$\begin{pmatrix} \frac{i}{2} & -\frac{i}{2} & \frac{i}{2} & 0 & -\frac{i}{2} & 0 \end{pmatrix}$

Table 8: Uniform SAMB.

symbol	type	cluster	form
$\mathbb{U}_1$	$\mathbb{Q}_{0}^{(s,A_1)}$	S_1	$\begin{pmatrix} 1 \end{pmatrix}$

Table 9: Structure SAMB.

symbol	type	cluster	form
$\mathbb{F}_1$	$\mathbb{Q}_{0}^{(k,A_1)}$	B_1	$\frac{\sqrt{6}c_{001}}{3} + \frac{\sqrt{6}c_{002}}{3} + \frac{\sqrt{6}c_{003}}{3}$
$\mathbb{F}_2$	$\mathbb{Q}_{2,0}^{(k,E)}$	B_1	$-\frac{2\sqrt{3}c_{001}}{3} + \frac{\sqrt{3}c_{002}}{3} + \frac{\sqrt{3}c_{003}}{3}$

continued ...

Table 9

symbol	type	cluster	form
$\mathbb{F}_3$	$\mathbb{Q}_{2,1}^{(k,E)}$	B_1	$-c_{002}+c_{003}$
$\mathbb{F}_4$	$\mathbb{T}_{4,0}^{(k,T_1)}$	B_1	$\sqrt{2}s_{002}$
$\mathbb{F}_5$	$\mathbb{T}_{4,1}^{(k,T_1)}$	B_1	$\sqrt{2}s_{003}$
$\mathbb{F}_6$	$\mathbb{T}_{4,2}^{(k,T_1)}$	B_1	$\sqrt{2}s_{001}$
$\mathbb{F}_7$	$\mathbb{Q}_{0}^{(k,A_1)}$	B_2	$\frac{\sqrt{3}c_{004}}{3}+\frac{\sqrt{3}c_{005}}{3}+\frac{\sqrt{3}c_{006}}{3}+\frac{\sqrt{3}c_{007}}{3}+\frac{\sqrt{3}c_{008}}{3}+\frac{\sqrt{3}c_{009}}{3}$
$\mathbb{F}_8$	$\mathbb{Q}_{2,0}^{(k,E)}$	B_2	$-\frac{\sqrt{6}c_{004}}{6}-\frac{\sqrt{6}c_{005}}{6}-\frac{\sqrt{6}c_{006}}{6}+\frac{\sqrt{6}c_{007}}{3}-\frac{\sqrt{6}c_{008}}{6}+\frac{\sqrt{6}c_{009}}{3}$
$\mathbb{F}_9$	$\mathbb{Q}_{2,1}^{(k,E)}$	B_2	$\frac{\sqrt{2}c_{004}}{2}+\frac{\sqrt{2}c_{005}}{2}-\frac{\sqrt{2}c_{006}}{2}-\frac{\sqrt{2}c_{008}}{2}$
$\mathbb{F}_{10}$	$\mathbb{Q}_{3,0}^{(k,T_2)}$	B_2	$c_{004}-c_{005}$
$\mathbb{F}_{11}$	$\mathbb{Q}_{3,1}^{(k,T_2)}$	B_2	$-c_{006}+c_{008}$
$\mathbb{F}_{12}$	$\mathbb{Q}_{3,2}^{(k,T_2)}$	B_2	$-c_{007}+c_{009}$
$\mathbb{F}_{13}$	$\mathbb{T}_{1,0}^{(k,T_1)}$	B_2	$\frac{\sqrt{2}s_{006}}{2}+\frac{\sqrt{2}s_{007}}{2}+\frac{\sqrt{2}s_{008}}{2}+\frac{\sqrt{2}s_{009}}{2}$
$\mathbb{F}_{14}$	$\mathbb{T}_{1,1}^{(k,T_1)}$	B_2	$\frac{\sqrt{2}s_{004}}{2}+\frac{\sqrt{2}s_{005}}{2}-\frac{\sqrt{2}s_{007}}{2}+\frac{\sqrt{2}s_{009}}{2}$
$\mathbb{F}_{15}$	$\mathbb{T}_{1,2}^{(k,T_1)}$	B_2	$\frac{\sqrt{2}s_{004}}{2}-\frac{\sqrt{2}s_{005}}{2}-\frac{\sqrt{2}s_{006}}{2}+\frac{\sqrt{2}s_{008}}{2}$
$\mathbb{F}_{16}$	$\mathbb{T}_{2,0}^{(k,T_2)}$	B_2	$\frac{\sqrt{2}s_{006}}{2}-\frac{\sqrt{2}s_{007}}{2}+\frac{\sqrt{2}s_{008}}{2}-\frac{\sqrt{2}s_{009}}{2}$
$\mathbb{F}_{17}$	$\mathbb{T}_{2,1}^{(k,T_2)}$	B_2	$-\frac{\sqrt{2}s_{004}}{2}-\frac{\sqrt{2}s_{005}}{2}-\frac{\sqrt{2}s_{007}}{2}+\frac{\sqrt{2}s_{009}}{2}$
$\mathbb{F}_{18}$	$\mathbb{T}_{2,2}^{(k,T_2)}$	B_2	$\frac{\sqrt{2}s_{004}}{2}-\frac{\sqrt{2}s_{005}}{2}+\frac{\sqrt{2}s_{006}}{2}-\frac{\sqrt{2}s_{008}}{2}$

Table 10: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_{0}^{(A_1)}$	0	A_1	–	–	1
2	$\mathbb{Q}_{1,0}^{(T_1)}$	1	T_1	–	0	x
3	$\mathbb{Q}_{1,1}^{(T_1)}$	1	T_1	–	1	y
4	$\mathbb{Q}_{1,2}^{(T_1)}$	1	T_1	–	2	z
5	$\mathbb{Q}_{2,0}^{(E)}$	2	E	–	0	$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$
6	$\mathbb{Q}_{2,1}^{(E)}$	2	E	–	1	$\frac{\sqrt{3}(x-y)(x+y)}{2}$
7	$\mathbb{Q}_{2,0}^{(T_2)}$	2	T_2	–	0	$\sqrt{3}yz$

continued ...

Table 10

No.	symbol	rank	irrep.	mul.	comp.	form
8	$\mathbb{Q}_{2,1}^{(T_2)}$	2	T_2	–	1	$\sqrt{3}xz$
9	$\mathbb{Q}_{2,2}^{(T_2)}$	2	T_2	–	2	$\sqrt{3}xy$
10	$\mathbb{Q}_{3,0}^{(T_2)}$	3	T_2	–	0	$\frac{\sqrt{15}x(y-z)(y+z)}{2}$
11	$\mathbb{Q}_{3,1}^{(T_2)}$	3	T_2	–	1	$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$
12	$\mathbb{Q}_{3,2}^{(T_2)}$	3	T_2	–	2	$\frac{\sqrt{15}z(x-y)(x+y)}{2}$
13	$\mathbb{Q}_{4,0}^{(T_1)}$	4	T_1	–	0	$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$
14	$\mathbb{Q}_{4,1}^{(T_1)}$	4	T_1	–	1	$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$
15	$\mathbb{Q}_{4,2}^{(T_1)}$	4	T_1	–	2	$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$

Table 11: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_{0}^{(A_1)}$	0	A_1	–	–	1
2	$\mathbb{G}_{1,0}^{(T_1)}$	1	T_1	–	0	X
3	$\mathbb{G}_{1,1}^{(T_1)}$	1	T_1	–	1	Y
4	$\mathbb{G}_{1,2}^{(T_1)}$	1	T_1	–	2	Z
5	$\mathbb{G}_{2,0}^{(E)}$	2	E	–	0	$-\frac{X^2}{2}-\frac{Y^2}{2}+Z^2$
6	$\mathbb{G}_{2,1}^{(E)}$	2	E	–	1	$\frac{\sqrt{3}(X-Y)(X+Y)}{2}$
7	$\mathbb{G}_{2,0}^{(T_2)}$	2	T_2	–	0	$\sqrt{3}YZ$
8	$\mathbb{G}_{2,1}^{(T_2)}$	2	T_2	–	1	$\sqrt{3}XZ$
9	$\mathbb{G}_{2,2}^{(T_2)}$	2	T_2	–	2	$\sqrt{3}XY$
10	$\mathbb{G}_{3,0}^{(T_1)}$	3	T_1	–	0	$\frac{X\left(2X^2-3Y^2-3Z^2\right)}{2}$
11	$\mathbb{G}_{3,1}^{(T_1)}$	3	T_1	–	1	$-\frac{Y\left(3X^2-2Y^2+3Z^2\right)}{2}$
12	$\mathbb{G}_{3,2}^{(T_1)}$	3	T_1	–	2	$-\frac{Z\left(3X^2+3Y^2-2Z^2\right)}{2}$
13	$\mathbb{G}_{3,0}^{(T_2)}$	3	T_2	–	0	$\frac{\sqrt{15}X(Y-Z)(Y+Z)}{2}$
14	$\mathbb{G}_{3,1}^{(T_2)}$	3	T_2	–	1	$-\frac{\sqrt{15}Y(X-Z)(X+Z)}{2}$
15	$\mathbb{G}_{3,2}^{(T_2)}$	3	T_2	–	2	$\frac{\sqrt{15}Z(X-Y)(X+Y)}{2}$

- Group info.: Generator = $\{2_{001}|0\},\ \{2_{010}|0\},\ \{3^{+}_{111}|0\},\ \{2_{110}|0\}$

Table 12: Conjugacy class (point-group part).

rep. SO	symmetry operations
$\{1\|0\}$	$\{1\|0\}$
$\{2_{001}\|0\}$	$\{2_{001}\|0\},\ \{2_{100}\|0\},\ \{2_{010}\|0\}$
$\{2_{110}\|0\}$	$\{2_{110}\|0\},\ \{2_{101}\|0\},\ \{2_{011}\|0\},\ \{2_{1-10}\|0\},\ \{2_{-101}\|0\},\ \{2_{01-1}\|0\}$
$\{3^{+}_{111}\|0\}$	$\{3^{+}_{111}\|0\},\ \{3^{+}_{1-1-1}\|0\},\ \{3^{+}_{-11-1}\|0\},\ \{3^{+}_{-1-11}\|0\},\ \{3^{-}_{111}\|0\},\ \{3^{-}_{1-1-1}\|0\},\ \{3^{-}_{-11-1}\|0\},\ \{3^{-}_{-1-11}\|0\}$
$\{4^{+}_{001}\|0\}$	$\{4^{+}_{001}\|0\},\ \{4^{+}_{100}\|0\},\ \{4^{+}_{010}\|0\},\ \{4^{-}_{001}\|0\},\ \{4^{-}_{100}\|0\},\ \{4^{-}_{010}\|0\}$

Table 13: Symmetry operations.

	No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
	1	$\{1\|0\}$	2	$\{2_{001}\|0\}$	3	$\{2_{100}\|0\}$	4	$\{2_{010}\|0\}$	5	$\{2_{110}\|0\}$
	6	$\{2_{101}\|0\}$	7	$\{2_{011}\|0\}$	8	$\{2_{1-10}\|0\}$	9	$\{2_{-101}\|0\}$	10	$\{2_{01-1}\|0\}$
	11	$\{3^{+}_{111}\|0\}$	12	$\{3^{+}_{1-1-1}\|0\}$	13	$\{3^{+}_{-11-1}\|0\}$	14	$\{3^{+}_{-1-11}\|0\}$	15	$\{3^{-}_{111}\|0\}$
	16	$\{3^{-}_{1-1-1}\|0\}$	17	$\{3^{-}_{-11-1}\|0\}$	18	$\{3^{-}_{-1-11}\|0\}$	19	$\{4^{+}_{001}\|0\}$	20	$\{4^{+}_{100}\|0\}$
	21	$\{4^{+}_{010}\|0\}$	22	$\{4^{-}_{001}\|0\}$	23	$\{4^{-}_{100}\|0\}$	24	$\{4^{-}_{010}\|0\}$		

Table 14: Character table (point-group part).

	1	2_{001}	2_{110}	3^{+}_{111}	4^{+}_{001}
A_1	1	1	1	1	1
A_2	1	1	−1	1	−1
E	2	2	0	−1	0
T_1	3	−1	−1	0	1
T_2	3	−1	1	0	−1

Table 15: Parity conversion.

	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$
	A_1 (A_1)	A_2 (A_2)	E (E)	T_1 (T_1)	T_2 (T_2)

Table 16: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

	A_1	A_2	E	T_1	T_2
A_1	A_1	A_2	E	T_1	T_2
A_2		A_1	E	T_2	T_1
E			$A_1 + E$	$T_1 + T_2$	$T_1 + T_2$
T_1				$A_1 + E + T_2$	$A_2 + E + T_1 + T_2$
T_2					$A_1 + E + T_2$

Table 17: Anti-symmetric product, $[\Gamma \otimes \Gamma]_-$.

A_1	A_2	E	T_1	T_2
$-$	$-$	A_2	T_1	T_1

Table 18: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix} 2 & 1 & 0 \end{pmatrix}$	2	$\begin{pmatrix} -2 & -1 & 0 \end{pmatrix}$	3	$\begin{pmatrix} 2 & -1 & 0 \end{pmatrix}$	4	$\begin{pmatrix} -2 & 1 & 0 \end{pmatrix}$
5	$\begin{pmatrix} 1 & 2 & 0 \end{pmatrix}$	6	$\begin{pmatrix} 0 & -1 & 2 \end{pmatrix}$	7	$\begin{pmatrix} -2 & 0 & 1 \end{pmatrix}$	8	$\begin{pmatrix} -1 & -2 & 0 \end{pmatrix}$
9	$\begin{pmatrix} 0 & -1 & -2 \end{pmatrix}$	10	$\begin{pmatrix} -2 & 0 & -1 \end{pmatrix}$	11	$\begin{pmatrix} 0 & 2 & 1 \end{pmatrix}$	12	$\begin{pmatrix} 0 & -2 & 1 \end{pmatrix}$
13	$\begin{pmatrix} 0 & -2 & -1 \end{pmatrix}$	14	$\begin{pmatrix} 0 & 2 & -1 \end{pmatrix}$	15	$\begin{pmatrix} 1 & 0 & 2 \end{pmatrix}$	16	$\begin{pmatrix} -1 & 0 & -2 \end{pmatrix}$

continued ...

Table 18

No.	position	No.	position	No.	position	No.	position
17	$\begin{pmatrix}-1 & 0 & 2\end{pmatrix}$	18	$\begin{pmatrix}1 & 0 & -2\end{pmatrix}$	19	$\begin{pmatrix}-1 & 2 & 0\end{pmatrix}$	20	$\begin{pmatrix}2 & 0 & 1\end{pmatrix}$
21	$\begin{pmatrix}0 & 1 & -2\end{pmatrix}$	22	$\begin{pmatrix}1 & -2 & 0\end{pmatrix}$	23	$\begin{pmatrix}2 & 0 & -1\end{pmatrix}$	24	$\begin{pmatrix}0 & 1 & 2\end{pmatrix}$

Table 19: Virtual-cluster basis.

symbol	1	2	3	4	5	6	7	8	9	10
$\mathbb{Q}_0^{(A_1)}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$						
$\mathbb{Q}_{1,0}^{(T_1)}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$
	0	0	0	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$
	0	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0						
$\mathbb{Q}_{1,1}^{(T_1)}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0
	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	0	0	0	0	$\frac{\sqrt{10}}{10}$	0
	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$						
$\mathbb{Q}_{1,2}^{(T_1)}$	0	0	0	0	0	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$
	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$
	$-\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$						
$\mathbb{Q}_{2,0}^{(E)}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$	$-\frac{5\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$
	$-\frac{\sqrt{39}}{78}$	$-\frac{\sqrt{39}}{78}$	$-\frac{\sqrt{39}}{78}$	$-\frac{\sqrt{39}}{78}$	$\frac{7\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$
	$\frac{7\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$	$\frac{7\sqrt{39}}{156}$						
$\mathbb{Q}_{2,1}^{(E)}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$-\frac{3\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{13}$	$-\frac{3\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{13}$
	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$-\frac{3\sqrt{13}}{52}$	$\frac{\sqrt{13}}{13}$
	$-\frac{\sqrt{13}}{52}$	$-\frac{3\sqrt{13}}{52}$	$\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{52}$						
$\mathbb{Q}_{2,0}^{(T_2)}$	0	0	0	0	0	$-\frac{\sqrt{2}}{4}$	0	0	$\frac{\sqrt{2}}{4}$	0
	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	0	0	0	0	0

continued ...

Table 19

symbol	1	2	3	4	5	6	7	8	9	10
	$-\frac{\sqrt{2}}{4}$	0	0	$\frac{\sqrt{2}}{4}$						
$\mathbb{Q}_{2,1}^{(T_2)}$	0	0	0	0	0	0	$-\frac{\sqrt{2}}{4}$	0	0	$\frac{\sqrt{2}}{4}$
	0	0	0	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	$\frac{\sqrt{2}}{4}$
	0	0	$-\frac{\sqrt{2}}{4}$	0						
$\mathbb{Q}_{2,2}^{(T_2)}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	0	0	$\frac{\sqrt{2}}{4}$	0	0
	0	0	0	0	0	0	0	0	$-\frac{\sqrt{2}}{4}$	0
	0	$-\frac{\sqrt{2}}{4}$	0	0						
$\mathbb{Q}_{3,0}^{(T_1)}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$
	0	0	0	0	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$
	0	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0						
$\mathbb{Q}_{3,1}^{(T_1)}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0
	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	0	0	0	0	$\frac{\sqrt{10}}{20}$	0
	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$						
$\mathbb{Q}_{3,2}^{(T_1)}$	0	0	0	0	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$
	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$
	$-\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$						
$\mathbb{Q}_{3,0}^{(T_2)}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$
	0	0	0	0	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$
	0	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0						
$\mathbb{Q}_{3,1}^{(T_2)}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0
	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	0	0	0	0	$-\frac{\sqrt{10}}{20}$	0
	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$						
$\mathbb{Q}_{3,2}^{(T_2)}$	0	0	0	0	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$
	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$
	$\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$						
$\mathbb{Q}_{4,0}^{(E)}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{13}$	$\frac{3\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{13}$
	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{13}$
	$\frac{\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{13}$	$\frac{\sqrt{13}}{52}$						
$\mathbb{Q}_{4,1}^{(E)}$	$\frac{5\sqrt{39}}{156}$	$\frac{5\sqrt{39}}{156}$	$\frac{5\sqrt{39}}{156}$	$\frac{5\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$	$-\frac{5\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$

continued ...

Table 19

symbol	1	2	3	4	5	6	7	8	9	10
	$\frac{\sqrt{39}}{78}$	$\frac{\sqrt{39}}{78}$	$\frac{\sqrt{39}}{78}$	$\frac{\sqrt{39}}{78}$	$-\frac{7\sqrt{39}}{156}$	$-\frac{7\sqrt{39}}{156}$	$-\frac{7\sqrt{39}}{156}$	$-\frac{7\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$
	$\frac{7\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$	$\frac{7\sqrt{39}}{156}$						
$\mathbb{Q}_{4,0}^{(T_1)}$	0	0	0	0	0	$\frac{\sqrt{2}}{4}$	0	0	$-\frac{\sqrt{2}}{4}$	0
	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	0	0	0	0	0
	$\frac{\sqrt{2}}{4}$	0	0	$-\frac{\sqrt{2}}{4}$						
$\mathbb{Q}_{4,1}^{(T_1)}$	0	0	0	0	0	0	$\frac{\sqrt{2}}{4}$	0	0	$-\frac{\sqrt{2}}{4}$
	0	0	0	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	$-\frac{\sqrt{2}}{4}$
	0	0	$\frac{\sqrt{2}}{4}$	0						
$\mathbb{Q}_{4,2}^{(T_1)}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	0	$-\frac{\sqrt{2}}{4}$	0	0
	0	0	0	0	0	0	0	0	$\frac{\sqrt{2}}{4}$	0
	0	$\frac{\sqrt{2}}{4}$	0	0						
$\mathbb{Q}_{5,0}^{(T_2)}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$
	0	0	0	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$
	0	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0						
$\mathbb{Q}_{5,1}^{(T_2)}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0
	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	0	0	0	0	$-\frac{\sqrt{10}}{10}$	0
	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$						
$\mathbb{Q}_{5,2}^{(T_2)}$	0	0	0	0	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$
	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$
	$\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$						
$\mathbb{Q}_{6}^{(A_2)}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$
	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$						