SAMB for "C3v1"

- Group: No. 156 C_{3v}^1 $P3m1$ [trigonal]
- Associated point group: No. 19 C_{3v} $3m1$ (3m1 setting) [trigonal]

- Generation condition
  - model type: `tight_binding`
  - time-reversal type: `electric`
  - irrep: `[A1]`
  - `spinful`

- Unit cell:
  $a = 1.0,\ b = 1.0,\ c = 1.0,\ \alpha = 90.0,\ \beta = 90.0,\ \gamma = 120.0$
- Lattice vectors:
  $\boldsymbol{a}_1 = \begin{pmatrix} 1.0 & 0 & 0 \end{pmatrix}$
  $\boldsymbol{a}_2 = \begin{pmatrix} -0.5 & 0.86602540378444 & 0 \end{pmatrix}$
  $\boldsymbol{a}_3 = \begin{pmatrix} 0 & 0 & 1.0 \end{pmatrix}$

Table 1: High-symmetry line: Γ-X.

symbol	position	symbol	position
Γ	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	X	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$

- Kets: dimension = 8

Table 2: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket	No.	ket	No.	ket
1	$(p_x,\uparrow)@\mathrm{A}_1$	2	$(p_x,\downarrow)@\mathrm{A}_1$	3	$(p_y,\uparrow)@\mathrm{A}_1$	4	$(p_y,\downarrow)@\mathrm{A}_1$	5	$(p_x,\uparrow)@\mathrm{B}_1$
6	$(p_x,\downarrow)@\mathrm{B}_1$	7	$(p_y,\uparrow)@\mathrm{B}_1$	8	$(p_y,\downarrow)@\mathrm{B}_1$				

- Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position	mapping
S_1 [1b: 3m.]	A_1	$\left(\begin{array}{ccc}\frac{1}{3} & \frac{2}{3} & 0\end{array}\right)$	[1,2,3,4,5,6]
S_2 [1c: 3m.]	B_1	$\left(\begin{array}{ccc}\frac{2}{3} & \frac{1}{3} & 0\end{array}\right)$	[1,2,3,4,5,6]

- Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

	bond	tail	head	n	#	$\boldsymbol{b}@\boldsymbol{c}$	mapping
B_1 [3d: .m.]	b_1	B_1	A_1	1	1	$\left(\begin{array}{ccc}\frac{1}{3} & \frac{2}{3} & 0\end{array}\right)@\left(\begin{array}{ccc}\frac{1}{2} & 0 & 0\end{array}\right)$	[1,4]
	b_2	B_1	A_1	1	1	$\left(\begin{array}{ccc}-\frac{2}{3} & -\frac{1}{3} & 0\end{array}\right)@\left(\begin{array}{ccc}0 & \frac{1}{2} & 0\end{array}\right)$	[2,6]
	b_3	B_1	A_1	1	1	$\left(\begin{array}{ccc}\frac{1}{3} & -\frac{1}{3} & 0\end{array}\right)@\left(\begin{array}{ccc}\frac{1}{2} & \frac{1}{2} & 0\end{array}\right)$	[3,5]

- SAMB:

No. 1 $\hat{\mathbb{Q}}_0^{(A_1)}$ [M$_1$, S$_1$]

$\hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$

No. 2 $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ $[M_1, S_1]$

$\hat{\mathbb{Z}}_2 = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$

No. 3 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[M_1, S_2]$

$\hat{\mathbb{Z}}_3 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(s,A_1)}]$

No. 4 $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ $[M_1, S_2]$

$\hat{\mathbb{Z}}_4 = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(s,A_1)}]$

No. 5 $\hat{\mathbb{Q}}_0^{(A_1)}$ $[M_1, B_1]$

$\hat{\mathbb{Z}}_5 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_1)}]$

No. 6 $\hat{\mathbb{Q}}_0^{(A_1)}(1,1)$ $[M_1, B_1]$

$\hat{\mathbb{Z}}_6 = \mathbb{X}_2[\mathbb{Q}_0^{(a,A_1)}(1,1)] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_1)}]$

No. 7 $\hat{\mathbb{Q}}_3^{(A_1,2)}$ $[M_1, B_1]$

$\hat{\mathbb{Z}}_7 = -\dfrac{\sqrt{2}\mathbb{X}_3[\mathbb{Q}_{2,0}^{(a,E,2)}] \otimes \mathbb{Y}_4[\mathbb{Q}_{1,0}^{(b,E)}]}{2} - \dfrac{\sqrt{2}\mathbb{X}_4[\mathbb{Q}_{2,1}^{(a,E,2)}] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,1}^{(b,E)}]}{2}$

No. 8 $\hat{\mathbb{Q}}_1^{(A_1)}(1,-1)$ $[M_1, B_1]$

$\hat{\mathbb{Z}}_8 = \dfrac{\sqrt{2}\mathbb{X}_5[\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_4[\mathbb{Q}_{1,0}^{(b,E)}]}{2} + \dfrac{\sqrt{2}\mathbb{X}_6[\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,1}^{(b,E)}]}{2}$

No. 9 $\hat{\mathbb{Q}}_1^{(A_1)}(1,1)$ $[M_1, B_1]$

$\hat{\mathbb{Z}}_9 = \dfrac{\sqrt{2}\mathbb{X}_7[\mathbb{M}_{1,0}^{(a,E)}(1,1)] \otimes \mathbb{Y}_7[\mathbb{T}_{1,0}^{(b,E)}]}{2} + \dfrac{\sqrt{2}\mathbb{X}_8[\mathbb{M}_{1,1}^{(a,E)}(1,1)] \otimes \mathbb{Y}_8[\mathbb{T}_{1,1}^{(b,E)}]}{2}$

No. 10 $\hat{\mathbb{G}}_3^{(A_1)}(1,-1)$ $[M_1, B_1]$

$\hat{\mathbb{Z}}_{10} = \mathbb{X}_{13}[\mathbb{M}_3^{(a,A_1)}(1,-1)] \otimes \mathbb{Y}_6[\mathbb{T}_0^{(b,A_1)}]$

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No. 11 $\hat{\mathbb{Q}}_3^{(A_1,2)}(1,-1)\ [\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{11} = \frac{\sqrt{2}\mathbb{X}_{11}[\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_7[\mathbb{T}_{1,0}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_{12}[\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)] \otimes \mathbb{Y}_8[\mathbb{T}_{1,1}^{(b,E)}]}{2}$$

No. 12 $\hat{\mathbb{Q}}_1^{(A_1)}(1,-1)\ [\mathrm{M}_1,\mathrm{B}_1]$

$$\hat{\mathbb{Z}}_{12} = \frac{\sqrt{2}\mathbb{X}_{10}[\mathbb{M}_{1,1}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_8[\mathbb{T}_{1,1}^{(b,E)}]}{2} + \frac{\sqrt{2}\mathbb{X}_9[\mathbb{M}_{1,0}^{(a,E)}(1,-1)] \otimes \mathbb{Y}_7[\mathbb{T}_{1,0}^{(b,E)}]}{2}$$

Table 5: Atomic SAMB group.

group	bra	ket
M_1	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow)$	$(p_x,\uparrow),(p_x,\downarrow),(p_y,\uparrow),(p_y,\downarrow)$

Table 6: Atomic SAMB.

symbol	type	group	form
$\mathbb{X}_1$	$\mathbb{Q}_0^{(a,A_1)}$	M_1	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$
$\mathbb{X}_2$	$\mathbb{Q}_0^{(a,A_1)}(1,1)$	M_1	$\begin{pmatrix} 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & \frac{i}{2} \\ \frac{i}{2} & 0 & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_3$	$\mathbb{Q}_{2,0}^{(a,E,2)}$	M_1	$\begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \end{pmatrix}$

continued …

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Table 6

symbol	type	group	form
$\mathbb{X}_4$	$\mathbb{Q}_{2,1}^{(a,E,2)}$	M$_1$	$\begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$
$\mathbb{X}_5$	$\mathbb{Q}_{2,0}^{(a,E,1)}(1,-1)$	M$_1$	$\begin{pmatrix} 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & -\frac{i}{2} & 0 \\ 0 & \frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_6$	$\mathbb{Q}_{2,1}^{(a,E,1)}(1,-1)$	M$_1$	$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$
$\mathbb{X}_7$	$\mathbb{M}_{1,0}^{(a,E)}(1,1)$	M$_1$	$\begin{pmatrix} 0 & -\frac{\sqrt{19}i}{19} & 0 & -\frac{3\sqrt{19}}{38} \\ \frac{\sqrt{19}i}{19} & 0 & -\frac{3\sqrt{19}}{38} & 0 \\ 0 & -\frac{3\sqrt{19}}{38} & 0 & \frac{2\sqrt{19}i}{19} \\ -\frac{3\sqrt{19}}{38} & 0 & -\frac{2\sqrt{19}i}{19} & 0 \end{pmatrix}$
$\mathbb{X}_8$	$\mathbb{M}_{1,1}^{(a,E)}(1,1)$	M$_1$	$\begin{pmatrix} 0 & \frac{2\sqrt{19}}{19} & 0 & -\frac{3\sqrt{19}i}{38} \\ \frac{2\sqrt{19}}{19} & 0 & \frac{3\sqrt{19}i}{38} & 0 \\ 0 & -\frac{3\sqrt{19}i}{38} & 0 & -\frac{\sqrt{19}}{19} \\ \frac{3\sqrt{19}i}{38} & 0 & -\frac{\sqrt{19}}{19} & 0 \end{pmatrix}$
$\mathbb{X}_9$	$\mathbb{M}_{1,0}^{(a,E)}(1,-1)$	M$_1$	$\begin{pmatrix} 0 & \frac{7\sqrt{38}i}{76} & 0 & \frac{\sqrt{38}}{76} \\ -\frac{7\sqrt{38}i}{76} & 0 & \frac{\sqrt{38}}{76} & 0 \\ 0 & \frac{\sqrt{38}}{76} & 0 & \frac{5\sqrt{38}i}{76} \\ \frac{\sqrt{38}}{76} & 0 & -\frac{5\sqrt{38}i}{76} & 0 \end{pmatrix}$
$\mathbb{X}_{10}$	$\mathbb{M}_{1,1}^{(a,E)}(1,-1)$	M$_1$	$\begin{pmatrix} 0 & \frac{5\sqrt{38}}{76} & 0 & \frac{\sqrt{38}i}{76} \\ \frac{5\sqrt{38}}{76} & 0 & -\frac{\sqrt{38}i}{76} & 0 \\ 0 & \frac{\sqrt{38}i}{76} & 0 & \frac{7\sqrt{38}}{76} \\ -\frac{\sqrt{38}i}{76} & 0 & \frac{7\sqrt{38}}{76} & 0 \end{pmatrix}$
$\mathbb{X}_{11}$	$\mathbb{M}_{3,0}^{(a,E,2)}(1,-1)$	M$_1$	$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$

continued …

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Table 6

symbol	type	group	form
$\mathbb{X}_{12}$	$\mathbb{M}_{3,1}^{(a,E,2)}(1,-1)$	M_1	$\begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$
$\mathbb{X}_{13}$	$\mathbb{M}_{3}^{(a,A_1)}(1,-1)$	M_1	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}i}{4} \\ \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}i}{4} & 0 \\ 0 & \frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} \\ -\frac{\sqrt{2}i}{4} & 0 & -\frac{\sqrt{2}}{4} & 0 \end{pmatrix}$

Table 7: Cluster SAMB.

symbol	type	cluster	form
$\mathbb{Y}_1$	$\mathbb{Q}_0^{(s,A_1)}$	S_1	$\begin{pmatrix} 1 \end{pmatrix}$
$\mathbb{Y}_2$	$\mathbb{Q}_0^{(s,A_1)}$	S_2	$\begin{pmatrix} 1 \end{pmatrix}$
$\mathbb{Y}_3$	$\mathbb{Q}_0^{(b,A_1)}$	B_1	$\begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$
$\mathbb{Y}_4$	$\mathbb{Q}_{1,0}^{(b,E)}$	B_1	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$
$\mathbb{Y}_5$	$\mathbb{Q}_{1,1}^{(b,E)}$	B_1	$\begin{pmatrix} -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} \end{pmatrix}$
$\mathbb{Y}_6$	$\mathbb{T}_0^{(b,A_1)}$	B_1	$\begin{pmatrix} \frac{\sqrt{3}i}{3} & \frac{\sqrt{3}i}{3} & \frac{\sqrt{3}i}{3} \end{pmatrix}$
$\mathbb{Y}_7$	$\mathbb{T}_{1,0}^{(b,E)}$	B_1	$\begin{pmatrix} 0 & \frac{\sqrt{2}i}{2} & -\frac{\sqrt{2}i}{2} \end{pmatrix}$
$\mathbb{Y}_8$	$\mathbb{T}_{1,1}^{(b,E)}$	B_1	$\begin{pmatrix} -\frac{\sqrt{6}i}{3} & \frac{\sqrt{6}i}{6} & \frac{\sqrt{6}i}{6} \end{pmatrix}$

Table 8: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_0^{(A_1)}$	0	A_1	–	–	1
2	$\mathbb{Q}_{1,0}^{(E)}$	1	E	–	0	x
3	$\mathbb{Q}_{1,1}^{(E)}$	1	E	–	1	y
4	$\mathbb{Q}_{2,0}^{(E,1)}$	2	E	1	0	$\sqrt{3}xz$
5	$\mathbb{Q}_{2,1}^{(E,1)}$	2	E	1	1	$\sqrt{3}yz$
6	$\mathbb{Q}_{2,0}^{(E,2)}$	2	E	2	0	$-\sqrt{3}xy$
7	$\mathbb{Q}_{2,1}^{(E,2)}$	2	E	2	1	$-\frac{\sqrt{3}(x-y)(x+y)}{2}$

Table 9: Axial harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{G}_{1,0}^{(E)}$	1	E	–	0	$-Y$
2	$\mathbb{G}_{1,1}^{(E)}$	1	E	–	1	X
3	$\mathbb{G}_3^{(A_1)}$	3	A_1	–	–	$\frac{\sqrt{10}X(X^2-3Y^2)}{4}$
4	$\mathbb{G}_{3,0}^{(E,2)}$	3	E	2	0	$\frac{\sqrt{15}Z(X-Y)(X+Y)}{2}$
5	$\mathbb{G}_{3,1}^{(E,2)}$	3	E	2	1	$-\sqrt{15}XYZ$

- Group info.: Generator = $\{3_{001}^+|0\}$, $\{m_{110}|0\}$

Table 10: Conjugacy class (point-group part).

rep. SO	symmetry operations
$\{1|0\}$	$\{1|0\}$
$\{3_{001}^+|0\}$	$\{3_{001}^+|0\}$, $\{3_{001}^-|0\}$

continued …

Table 10

rep. SO	symmetry operations
$\{m_{100}|0\}$	$\{m_{100}|0\}$, $\{m_{010}|0\}$, $\{m_{110}|0\}$

Table 11: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	$\{1|0\}$	2	$\{3^+_{001}|0\}$	3	$\{3^-_{001}|0\}$	4	$\{m_{100}|0\}$	5	$\{m_{010}|0\}$
6	$\{m_{110}|0\}$								

Table 12: Character table (point-group part).

	1	3^+_{001}	m_{100}
A_1	1	1	1
A_2	1	1	-1
E	2	-1	0

Table 13: Parity conversion.

$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$
A_1 (A_2)	A_2 (A_1)	E (E)

Table 14: Symmetric product, $[\Gamma \otimes \Gamma']_+$.

	A_1	A_2	E
A_1	A_1	A_2	E
A_2		A_1	E
E			$A_1 + E$

Table 15: Anti-symmetric product, $[\Gamma \otimes \Gamma]_-$.

A_1	A_2	E
$-$	$-$	A_2

Table 16: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix} -1 & -1 & 0 \end{pmatrix}$	2	$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$	3	$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$	4	$\begin{pmatrix} 0 & -1 & 0 \end{pmatrix}$
5	$\begin{pmatrix} -1 & 0 & 0 \end{pmatrix}$	6	$\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}$				

Table 17: Virtual-cluster basis.

symbol	1	2	3	4	5	6
$\mathbb{Q}_0^{(A_1)}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$
$\mathbb{Q}_{1,0}^{(E)}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{6}$
$\mathbb{Q}_{1,1}^{(E)}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$
$\mathbb{Q}_{2,0}^{(E,2)}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$

continued ...

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Table 17

symbol	1	2	3	4	5	6
$\mathbb{Q}_{2,1}^{(E,2)}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{6}$
$\mathbb{Q}_3^{(A_2)}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$	$-\frac{\sqrt{6}}{6}$