Response Tensors up to 4th rank in $D_{3h}-1$

— polar tensors —

$$C^{(0,Q)} = \begin{pmatrix} C^{(0,Q)} \end{pmatrix}$$

$$C^{(0,Q)} = Q_0$$

$$S^{(2,Q)} = \begin{pmatrix} S_{xx}^{(2,Q)} & 0 & 0 \\ 0 & S_{xx}^{(2,Q)} & 0 \\ 0 & 0 & S_{zz}^{(2,Q)} \end{pmatrix}$$

$$S_{xx}^{(2,Q)} = Q_0 - Q_u$$
$$S_{zz}^{(2,Q)} = Q_0 + 2Q_u$$

$$S^{(3,Q)} = \begin{pmatrix} S_{1x}^{(3,Q)} & 0 & 0 \\ -S_{1x}^{(3,Q)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -S_{1x}^{(3,Q)} & 0 \end{pmatrix}$$

$$S_{1x}^{(3,Q)} = Q_3^{\gamma}$$

$$S^{(4,Q)} = \begin{pmatrix} S_{11}^{(4,Q)} & S_{12}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & 0 & 0 \\ S_{12}^{(4,Q)} & S_{11}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & 0 & 0 \\ S_{13}^{(4,Q)} & S_{13}^{(4,Q)} & S_{33}^{(4,Q)} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{S_{11}^{(4,Q)}}{2} - \frac{S_{12}^{(4,Q)}}{2} \end{pmatrix}$$

$$S_{11}^{(4,Q)} = Q_0[1] + 2Q_0[2] + 3Q_4 - 2Q_u[1] - 4Q_u[2]$$
$$S_{12}^{(4,Q)} = Q_0[1] + Q_4 - 2Q_u[1]$$
$$S_{13}^{(4,Q)} = Q_0[1] - 4Q_4 + Q_u[1]$$
$$S_{33}^{(4,Q)} = Q_0[1] + 2Q_0[2] + 8Q_4 + 4Q_u[1] + 8Q_u[2]$$
$$S_{44}^{(4,Q)} = Q_0[2] - 4Q_4 + Q_u[2]$$

$$\bar{S}^{(4,Q)} = \begin{pmatrix} 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 \\ 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 \\ -\bar{S}_{13}^{(4,Q)} & -\bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{S}^{(4,Q)}_{13} = 3Q_u[3]$$

$$A^{(4,Q)} = \begin{pmatrix} A^{(4,Q)}_{xx} & 0 & 0 \\ 0 & A^{(4,Q)}_{xx} & 0 \\ 0 & 0 & A^{(4,Q)}_{zz} \end{pmatrix}$$

$$A^{(4,Q)}_{xx} = Q_0[3] - 2Q_u[6]$$
$$A^{(4,Q)}_{zz} = Q_0[3] + 4Q_u[6]$$

$$M^{(4,Q)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ M^{(4,Q)}_{4x} & 0 & 0 \\ 0 & -M^{(4,Q)}_{4x} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$M^{(4,Q)}_{4x} = -3Q_u[4]$$

$$\bar{M}^{(4,Q)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}^{(4,Q)}_{x4} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\bar{M}^{(4,Q)}_{x4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{M}^{(4,Q)}_{x4} = -3Q_u[5]$$

— axial tensors —

$$S^{(3,G)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ S^{(3,G)}_{4x} & 0 & 0 \\ 0 & -S^{(3,G)}_{4x} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S^{(3,G)}_{4x} = -3Q_u[1]$$

$$A^{(3,G)} = \begin{pmatrix} A^{(3,G)}_{4x} & 0 & 0 \\ 0 & A^{(3,G)}_{4x} & 0 \\ 0 & 0 & A^{(3,G)}_{6z} \end{pmatrix}$$

$$A^{(3,G)}_{4x} = Q_0 - Q_u[2]$$
$$A^{(3,G)}_{6z} = Q_0 + 2Q_u[2]$$

$$S^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & S^{(4,G)}_{14} & 0 & 0 \\ 0 & 0 & 0 & -S^{(4,G)}_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ S^{(4,G)}_{14} & -S^{(4,G)}_{14} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & S^{(4,G)}_{14} \\ 0 & 0 & 0 & 0 & S^{(4,G)}_{14} & 0 \end{pmatrix}$$

$$S^{(4,G)}_{14} = G^{\beta}_{4}$$

$$\bar{S}^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & \bar{S}^{(4,G)}_{14} & 0 & 0 \\ 0 & 0 & 0 & -\bar{S}^{(4,G)}_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\bar{S}^{(4,G)}_{14} & \bar{S}^{(4,G)}_{14} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\bar{S}^{(4,G)}_{14} \\ 0 & 0 & 0 & 0 & \bar{S}^{(4,G)}_{14} & 0 \end{pmatrix}$$

$$\bar{S}^{(4,G)}_{14} = 2Q^{\gamma}_{3}[1]$$

$$M^{(4,G)} = \begin{pmatrix} M^{(4,G)}_{1x} & 0 & 0 \\ -M^{(4,G)}_{1x} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -M^{(4,G)}_{1x} & 0 \end{pmatrix}$$

$$M^{(4,G)}_{1x} = Q^{\gamma}_{3}[2]$$

$$\bar{M}^{(4,G)} = \begin{pmatrix} \bar{M}^{(4,G)}_{x1} & -\bar{M}^{(4,G)}_{x1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\bar{M}^{(4,G)}_{x1} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{M}^{(4,G)}_{x1} = Q^{\gamma}_{3}[3]$$