

\* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

No.	tag	matrix (polar)	det	TR
1	$\{1 0\}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	1	1
2	$\{3_{001}^+ 0\}$	$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	1	1
3	$\{3_{001}^- 0\}$	$\begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	1	1
4	$\{2_{100} 00\frac{1}{2}\}$	$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$	1	1
5	$\{2_{110} 00\frac{1}{2}\}$	$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$	1	1
6	$\{2_{010} 00\frac{1}{2}\}$	$\begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$	1	1
7	$\{-1 0\}$	$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$	-1	1
8	$\{-3_{001}^+ 0\}$	$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$	-1	1
9	$\{-3_{001}^- 0\}$	$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$	-1	1
10	$\{m_{100} 00\frac{1}{2}\}$	$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$	-1	1
11	$\{m_{110} 00\frac{1}{2}\}$	$\begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$	-1	1
12	$\{m_{010} 00\frac{1}{2}\}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$	-1	1
13	$\{1 \frac{2}{3}\frac{1}{3}\frac{1}{3}\}$	$\begin{bmatrix} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix}$	1	1

*continued ...*

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No.	tag	matrix (polar)	det	TR
14	$\{3_{001}^+ \frac{2}{3}\frac{1}{3}\frac{1}{3}\}$	$\begin{bmatrix} 0 & -1 & 0 & \frac{2}{3} \\ 1 & -1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix}$	1	1
15	$\{3_{001}^- \frac{2}{3}\frac{1}{3}\frac{1}{3}\}$	$\begin{bmatrix} -1 & 1 & 0 & \frac{2}{3} \\ -1 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix}$	1	1
16	$\{2_{100} \frac{2}{3}\frac{1}{3}\frac{5}{6}\}$	$\begin{bmatrix} 1 & -1 & 0 & \frac{2}{3} \\ 0 & -1 & 0 & \frac{1}{3} \\ 0 & 0 & -1 & \frac{5}{6} \end{bmatrix}$	1	1
17	$\{2_{110} \frac{2}{3}\frac{1}{3}\frac{5}{6}\}$	$\begin{bmatrix} 0 & 1 & 0 & \frac{2}{3} \\ 1 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & -1 & \frac{5}{6} \end{bmatrix}$	1	1
18	$\{2_{010} \frac{2}{3}\frac{1}{3}\frac{5}{6}\}$	$\begin{bmatrix} -1 & 0 & 0 & \frac{2}{3} \\ -1 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & -1 & \frac{5}{6} \end{bmatrix}$	1	1
19	$\{-1 \frac{2}{3}\frac{1}{3}\frac{1}{3}\}$	$\begin{bmatrix} -1 & 0 & 0 & \frac{2}{3} \\ 0 & -1 & 0 & \frac{1}{3} \\ 0 & 0 & -1 & \frac{1}{3} \end{bmatrix}$	-1	1
20	$\{-3_{001}^+ \frac{2}{3}\frac{1}{3}\frac{1}{3}\}$	$\begin{bmatrix} 0 & 1 & 0 & \frac{2}{3} \\ -1 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & -1 & \frac{1}{3} \end{bmatrix}$	-1	1
21	$\{-3_{001}^- \frac{2}{3}\frac{1}{3}\frac{1}{3}\}$	$\begin{bmatrix} 1 & -1 & 0 & \frac{2}{3} \\ 1 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & -1 & \frac{1}{3} \end{bmatrix}$	-1	1
22	$\{m_{100} \frac{2}{3}\frac{1}{3}\frac{5}{6}\}$	$\begin{bmatrix} -1 & 1 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{5}{6} \end{bmatrix}$	-1	1
23	$\{m_{110} \frac{2}{3}\frac{1}{3}\frac{5}{6}\}$	$\begin{bmatrix} 0 & -1 & 0 & \frac{2}{3} \\ -1 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{5}{6} \end{bmatrix}$	-1	1
24	$\{m_{010} \frac{2}{3}\frac{1}{3}\frac{5}{6}\}$	$\begin{bmatrix} 1 & 0 & 0 & \frac{2}{3} \\ 1 & -1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{5}{6} \end{bmatrix}$	-1	1
25	$\{1 \frac{1}{3}\frac{2}{3}\frac{2}{3}\}$	$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{3} \end{bmatrix}$	1	1
26	$\{3_{001}^+ \frac{1}{3}\frac{2}{3}\frac{2}{3}\}$	$\begin{bmatrix} 0 & -1 & 0 & \frac{1}{3} \\ 1 & -1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{3} \end{bmatrix}$	1	1
27	$\{3_{001}^- \frac{1}{3}\frac{2}{3}\frac{2}{3}\}$	$\begin{bmatrix} -1 & 1 & 0 & \frac{1}{3} \\ -1 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{3} \end{bmatrix}$	1	1

continued ...

Table 1

No.	tag	matrix (polar)	det	TR
28	$\{2_{100} \frac{1}{3}\frac{2}{3}\frac{1}{6}\}$	$\begin{bmatrix} 1 & -1 & 0 & \frac{1}{3} \\ 0 & -1 & 0 & \frac{2}{3} \\ 0 & 0 & -1 & \frac{1}{6} \end{bmatrix}$	1	1
29	$\{2_{110} \frac{1}{3}\frac{2}{3}\frac{1}{6}\}$	$\begin{bmatrix} 0 & 1 & 0 & \frac{1}{3} \\ 1 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & -1 & \frac{1}{6} \end{bmatrix}$	1	1
30	$\{2_{010} \frac{1}{3}\frac{2}{3}\frac{1}{6}\}$	$\begin{bmatrix} -1 & 0 & 0 & \frac{1}{3} \\ -1 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & -1 & \frac{1}{6} \end{bmatrix}$	1	1
31	$\{-1 \frac{1}{3}\frac{2}{3}\frac{2}{3}\}$	$\begin{bmatrix} -1 & 0 & 0 & \frac{1}{3} \\ 0 & -1 & 0 & \frac{2}{3} \\ 0 & 0 & -1 & \frac{2}{3} \end{bmatrix}$	-1	1
32	$\{-3_{001}^+ \frac{1}{3}\frac{2}{3}\frac{2}{3}\}$	$\begin{bmatrix} 0 & 1 & 0 & \frac{1}{3} \\ -1 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & -1 & \frac{2}{3} \end{bmatrix}$	-1	1
33	$\{-3_{001}^- \frac{1}{3}\frac{2}{3}\frac{2}{3}\}$	$\begin{bmatrix} 1 & -1 & 0 & \frac{1}{3} \\ 1 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & -1 & \frac{2}{3} \end{bmatrix}$	-1	1
34	$\{m_{100} \frac{1}{3}\frac{2}{3}\frac{1}{6}\}$	$\begin{bmatrix} -1 & 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{6} \end{bmatrix}$	-1	1
35	$\{m_{110} \frac{1}{3}\frac{2}{3}\frac{1}{6}\}$	$\begin{bmatrix} 0 & -1 & 0 & \frac{1}{3} \\ -1 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{6} \end{bmatrix}$	-1	1
36	$\{m_{010} \frac{1}{3}\frac{2}{3}\frac{1}{6}\}$	$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 1 & -1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{6} \end{bmatrix}$	-1	1
37	$\{1' 0\}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	1	-1
38	$\{3_{001}^+' 0\}$	$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	1	-1
39	$\{3_{001}^-' 0\}$	$\begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	1	-1
40	$\{2_{100}' 00\frac{1}{2}\}$	$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$	1	-1
41	$\{2_{110}' 00\frac{1}{2}\}$	$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$	1	-1

continued ...

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No.	tag	matrix (polar)	det	TR
42	$\{2_{010}' 00\frac{1}{2}\}$	$\begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$	1	-1
43	$\{-1' 0\}$	$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$	-1	-1
44	$\{-3_{001}^+' 0\}$	$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$	-1	-1
45	$\{-3_{001}^-' 0\}$	$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$	-1	-1
46	$\{m_{100}' 00\frac{1}{2}\}$	$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$	-1	-1
47	$\{m_{110}' 00\frac{1}{2}\}$	$\begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$	-1	-1
48	$\{m_{010}' 00\frac{1}{2}\}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$	-1	-1
49	$\{1' \frac{2}{3}\frac{1}{3}\frac{1}{3}\}$	$\begin{bmatrix} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix}$	1	-1
50	$\{3_{001}^+' \frac{2}{3}\frac{1}{3}\frac{1}{3}\}$	$\begin{bmatrix} 0 & -1 & 0 & \frac{2}{3} \\ 1 & -1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix}$	1	-1
51	$\{3_{001}^-' \frac{2}{3}\frac{1}{3}\frac{1}{3}\}$	$\begin{bmatrix} -1 & 1 & 0 & \frac{2}{3} \\ -1 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix}$	1	-1
52	$\{2_{100}' \frac{2}{3}\frac{1}{3}\frac{5}{6}\}$	$\begin{bmatrix} 1 & -1 & 0 & \frac{2}{3} \\ 0 & -1 & 0 & \frac{1}{3} \\ 0 & 0 & -1 & \frac{5}{6} \end{bmatrix}$	1	-1
53	$\{2_{110}' \frac{2}{3}\frac{1}{3}\frac{5}{6}\}$	$\begin{bmatrix} 0 & 1 & 0 & \frac{2}{3} \\ 1 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & -1 & \frac{5}{6} \end{bmatrix}$	1	-1
54	$\{2_{010}' \frac{2}{3}\frac{1}{3}\frac{5}{6}\}$	$\begin{bmatrix} -1 & 0 & 0 & \frac{2}{3} \\ -1 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & -1 & \frac{5}{6} \end{bmatrix}$	1	-1
55	$\{-1' \frac{2}{3}\frac{1}{3}\frac{1}{3}\}$	$\begin{bmatrix} -1 & 0 & 0 & \frac{2}{3} \\ 0 & -1 & 0 & \frac{1}{3} \\ 0 & 0 & -1 & \frac{1}{3} \end{bmatrix}$	-1	-1

continued ...

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No.	tag	matrix (polar)	det	TR
56	$\{-3_{001}^+   \frac{2}{3} \frac{1}{3} \frac{1}{3}\}$	$\begin{bmatrix} 0 & 1 & 0 & \frac{2}{3} \\ -1 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & -1 & \frac{1}{3} \end{bmatrix}$	-1	-1
57	$\{-3_{001}^-   \frac{2}{3} \frac{1}{3} \frac{1}{3}\}$	$\begin{bmatrix} 1 & -1 & 0 & \frac{2}{3} \\ 1 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & -1 & \frac{1}{3} \end{bmatrix}$	-1	-1
58	$\{m_{100}'   \frac{2}{3} \frac{1}{3} \frac{5}{6}\}$	$\begin{bmatrix} -1 & 1 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{5}{6} \end{bmatrix}$	-1	-1
59	$\{m_{110}'   \frac{2}{3} \frac{1}{3} \frac{5}{6}\}$	$\begin{bmatrix} 0 & -1 & 0 & \frac{2}{3} \\ -1 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{5}{6} \end{bmatrix}$	-1	-1
60	$\{m_{010}'   \frac{2}{3} \frac{1}{3} \frac{5}{6}\}$	$\begin{bmatrix} 1 & 0 & 0 & \frac{2}{3} \\ 1 & -1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{5}{6} \end{bmatrix}$	-1	-1
61	$\{1'   \frac{1}{3} \frac{2}{3} \frac{2}{3}\}$	$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{3} \end{bmatrix}$	1	-1
62	$\{3_{001}^+   \frac{1}{3} \frac{2}{3} \frac{2}{3}\}$	$\begin{bmatrix} 0 & -1 & 0 & \frac{1}{3} \\ 1 & -1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{3} \end{bmatrix}$	1	-1
63	$\{3_{001}^-   \frac{1}{3} \frac{2}{3} \frac{2}{3}\}$	$\begin{bmatrix} -1 & 1 & 0 & \frac{1}{3} \\ -1 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{3} \end{bmatrix}$	1	-1
64	$\{2_{100}'   \frac{1}{3} \frac{2}{3} \frac{1}{6}\}$	$\begin{bmatrix} 1 & -1 & 0 & \frac{1}{3} \\ 0 & -1 & 0 & \frac{2}{3} \\ 0 & 0 & -1 & \frac{1}{6} \end{bmatrix}$	1	-1
65	$\{2_{110}'   \frac{1}{3} \frac{2}{3} \frac{1}{6}\}$	$\begin{bmatrix} 0 & 1 & 0 & \frac{1}{3} \\ 1 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & -1 & \frac{1}{6} \end{bmatrix}$	1	-1
66	$\{2_{010}'   \frac{1}{3} \frac{2}{3} \frac{1}{6}\}$	$\begin{bmatrix} -1 & 0 & 0 & \frac{1}{3} \\ -1 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & -1 & \frac{1}{6} \end{bmatrix}$	1	-1
67	$\{-1'   \frac{1}{3} \frac{2}{3} \frac{2}{3}\}$	$\begin{bmatrix} -1 & 0 & 0 & \frac{1}{3} \\ 0 & -1 & 0 & \frac{2}{3} \\ 0 & 0 & -1 & \frac{2}{3} \end{bmatrix}$	-1	-1
68	$\{-3_{001}^+   \frac{1}{3} \frac{2}{3} \frac{2}{3}\}$	$\begin{bmatrix} 0 & 1 & 0 & \frac{1}{3} \\ -1 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & -1 & \frac{2}{3} \end{bmatrix}$	-1	-1
69	$\{-3_{001}^-   \frac{1}{3} \frac{2}{3} \frac{2}{3}\}$	$\begin{bmatrix} 1 & -1 & 0 & \frac{1}{3} \\ 1 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & -1 & \frac{2}{3} \end{bmatrix}$	-1	-1

continued ...

Table 1

No.	tag	matrix (polar)	det	TR
70	$\{m_{100}' \frac{1}{3}\frac{2}{3}\frac{1}{6}\}$	$\begin{bmatrix} -1 & 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{6} \end{bmatrix}$	-1	-1
71	$\{m_{110}' \frac{1}{3}\frac{2}{3}\frac{1}{6}\}$	$\begin{bmatrix} 0 & -1 & 0 & \frac{1}{3} \\ -1 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{6} \end{bmatrix}$	-1	-1
72	$\{m_{010}' \frac{1}{3}\frac{2}{3}\frac{1}{6}\}$	$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 1 & -1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{6} \end{bmatrix}$	-1	-1