

* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

| No. | tag | matrix (polar) | det | TR |
|-----|------------------------------|--|-----|----|
| 1 | $\{1 0\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | 1 | 1 |
| 2 | $\{4_{001}^+ 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | 1 | 1 |
| 3 | $\{4_{001}^- 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | 1 | 1 |
| 4 | $\{2_{100} 00\frac{1}{2}\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | 1 | 1 |
| 5 | $\{2_{010} 00\frac{1}{2}\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | 1 | 1 |
| 6 | $\{2_{001} 0\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | 1 | 1 |
| 7 | $\{2_{110} 00\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | 1 | 1 |
| 8 | $\{2_{1-10} 00\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | 1 | 1 |
| 9 | $\{-1 0\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | -1 | 1 |
| 10 | $\{-4_{001}^+ 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | -1 | 1 |
| 11 | $\{-4_{001}^- 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | -1 | 1 |
| 12 | $\{m_{100} 00\frac{1}{2}\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | -1 | 1 |
| 13 | $\{m_{010} 00\frac{1}{2}\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | -1 | 1 |

continued ...

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| 14 | $\{m_{001} 0\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | -1 | 1 |
| 15 | $\{m_{110} 00\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | -1 | 1 |
| 16 | $\{m_{1-10} 00\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | -1 | 1 |
| 17 | $\{1' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | 1 | -1 |
| 18 | $\{4_{001}^+ \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | 1 | -1 |
| 19 | $\{4_{001}^- \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | 1 | -1 |
| 20 | $\{2_{100}' \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | 1 | -1 |
| 21 | $\{2_{010}' \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | 1 | -1 |
| 22 | $\{2_{001}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | 1 | -1 |
| 23 | $\{2_{110}' \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | 1 | -1 |
| 24 | $\{2_{1-10}' \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | 1 | -1 |
| 25 | $\{-1' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1 | -1 |
| 26 | $\{-4_{001}^+ \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1 | -1 |
| 27 | $\{-4_{001}^- \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1 | -1 |

continued ...

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| No. | tag | matrix (polar) | det | TR |
|-----|--|--|-----|----|
| 28 | $\{m_{100}' \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | -1 | -1 |
| 29 | $\{m_{010}' \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | -1 | -1 |
| 30 | $\{m_{001}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1 | -1 |
| 31 | $\{m_{110}' \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | -1 | -1 |
| 32 | $\{m_{1-10}' \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | -1 | -1 |