

PG No. 22 C_{3h} $\bar{6}$ [hexagonal] (axial, internal axial quadrupole)

* Harmonics for rank 0

$$\vec{\mathbb{G}}_0^{(2,2)}[g](A'')$$

** symmetry

1

** expression

$$-\frac{\sqrt{5}G_u(x^2 + y^2 - 2z^2)}{10} + \frac{\sqrt{15}G_v(x - y)(x + y)}{10} + \frac{\sqrt{15}G_{xy}xy}{5} + \frac{\sqrt{15}G_{xz}xz}{5} + \frac{\sqrt{15}G_{yz}yz}{5}$$

* Harmonics for rank 1

$$\vec{\mathbb{G}}_1^{(2,0)}[g](A')$$

** symmetry

z

** expression

$$\frac{\sqrt{10}G_uz}{5} + \frac{\sqrt{30}G_{xz}x}{10} + \frac{\sqrt{30}G_{yz}y}{10}$$

$$\vec{\mathbb{G}}_1^{(2,2)}[g](A')$$

** symmetry

z

** expression

$$-\frac{3\sqrt{35}G_uz(3x^2 + 3y^2 - 2z^2)}{70} + \frac{\sqrt{105}G_vz(x - y)(x + y)}{14} + \frac{\sqrt{105}G_{xy}xyz}{7} - \frac{\sqrt{105}G_{xz}x(x^2 + y^2 - 4z^2)}{35} - \frac{\sqrt{105}G_{yz}y(x^2 + y^2 - 4z^2)}{35}$$

$$\vec{\mathbb{G}}_{1,1}^{(2,0)}[g](E''), \vec{\mathbb{G}}_{1,2}^{(2,0)}[g](E'')$$

** symmetry

x

y

** expression

$$-\frac{\sqrt{10}G_ux}{10} + \frac{\sqrt{30}G_vx}{10} + \frac{\sqrt{30}G_{xy}y}{10} + \frac{\sqrt{30}G_{xz}z}{10}$$

$$-\frac{\sqrt{10}G_uy}{10} - \frac{\sqrt{30}G_vy}{10} + \frac{\sqrt{30}G_{xy}x}{10} + \frac{\sqrt{30}G_{yz}z}{10}$$

$$\vec{\mathbb{G}}_{1,1}^{(2,2)}[g](E''), \vec{\mathbb{G}}_{1,2}^{(2,2)}[g](E'')$$

** symmetry

x

y

** expression

$$-\frac{3\sqrt{35}G_ux(x^2 + y^2 - 4z^2)}{70} + \frac{\sqrt{105}G_vx(3x^2 - 7y^2 - 2z^2)}{70} + \frac{\sqrt{105}G_{xy}y(4x^2 - y^2 - z^2)}{35} + \frac{\sqrt{105}G_{xz}z(4x^2 - y^2 - z^2)}{35} + \frac{\sqrt{105}G_{yz}xyz}{7}$$

$$-\frac{3\sqrt{35}G_uy(x^2 + y^2 - 4z^2)}{70} + \frac{\sqrt{105}G_vy(7x^2 - 3y^2 + 2z^2)}{70} - \frac{\sqrt{105}G_{xy}x(x^2 - 4y^2 + z^2)}{35} + \frac{\sqrt{105}G_{xz}xyz}{7} - \frac{\sqrt{105}G_{yz}z(x^2 - 4y^2 + z^2)}{35}$$

* Harmonics for rank 2

$$\vec{\mathbb{G}}_2^{(2,-2)}[g](A'')$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

G_u

$$\vec{\mathbb{G}}_2^{(2,0)}[g](A'')$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{14}G_u(x^2+y^2-2z^2)}{14}-\frac{\sqrt{42}G_v(x-y)(x+y)}{14}-\frac{\sqrt{42}G_{xy}xy}{7}+\frac{\sqrt{42}G_{xz}xz}{14}+\frac{\sqrt{42}G_{yz}yz}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(2,2)}[g](A'')$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$\begin{aligned} & \frac{\sqrt{14}G_u(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4)}{56}-\frac{5\sqrt{42}G_v(x-y)(x+y)(x^2+y^2-6z^2)}{168} \\ & -\frac{5\sqrt{42}G_{xy}xy(x^2+y^2-6z^2)}{84}-\frac{5\sqrt{42}G_{xz}xz(3x^2+3y^2-4z^2)}{84}-\frac{5\sqrt{42}G_{yz}yz(3x^2+3y^2-4z^2)}{84} \end{aligned}$$

$\vec{\mathbb{G}}_{2,1}^{(2,-2)}[g](E''), \vec{\mathbb{G}}_{2,2}^{(2,-2)}[g](E')$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$G_{yz}$$

$$-G_{xz}$$

$\vec{\mathbb{G}}_{2,1}^{(2,0)}[g](E'), \vec{\mathbb{G}}_{2,2}^{(2,0)}[g](E')$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\begin{aligned} & \frac{\sqrt{42}G_uyz}{14}-\frac{3\sqrt{14}G_vyz}{14}+\frac{3\sqrt{14}G_{xy}xz}{14}+\frac{3\sqrt{14}G_{xz}xy}{14}-\frac{\sqrt{14}G_{yz}(2x^2-y^2-z^2)}{14} \\ & -\frac{\sqrt{42}G_uxz}{14}-\frac{3\sqrt{14}G_vxz}{14}-\frac{3\sqrt{14}G_{xy}yz}{14}-\frac{\sqrt{14}G_{xz}(x^2-2y^2+z^2)}{14}-\frac{3\sqrt{14}G_{yz}xy}{14} \end{aligned}$$

$\vec{\mathbb{G}}_{2,1}^{(2,2)}[g](E'), \vec{\mathbb{G}}_{2,2}^{(2,2)}[g](E')$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\begin{aligned} & -\frac{5\sqrt{42}G_uyz(3x^2+3y^2-4z^2)}{84}+\frac{5\sqrt{14}G_vyz(9x^2-5y^2+2z^2)}{84}-\frac{5\sqrt{14}G_{xy}xz(x^2-6y^2+z^2)}{42} \\ & -\frac{5\sqrt{14}G_{xz}xy(x^2+y^2-6z^2)}{42}+\frac{\sqrt{14}G_{yz}(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4)}{42} \\ & \frac{5\sqrt{42}G_uxz(3x^2+3y^2-4z^2)}{84}-\frac{5\sqrt{14}G_vxz(5x^2-9y^2-2z^2)}{84}-\frac{5\sqrt{14}G_{xy}yz(6x^2-y^2-z^2)}{42} \\ & +\frac{\sqrt{14}G_{xz}(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4)}{42}+\frac{5\sqrt{14}G_{yz}xy(x^2+y^2-6z^2)}{42} \end{aligned}$$

$\vec{\mathbb{G}}_{2,1}^{(2,-2)}[g](E''), \vec{\mathbb{G}}_{2,2}^{(2,-2)}[g](E'')$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$G_v$$

$$-G_{xy}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,0)}[g](E''), \vec{\mathbb{G}}_{2,2}^{(2,0)}[g](E'')$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{42}G_u(x-y)(x+y)}{14} + \frac{\sqrt{14}G_v(x^2+y^2-2z^2)}{14} + \frac{3\sqrt{14}G_{xz}xz}{14} - \frac{3\sqrt{14}G_{yz}yz}{14}$$

$$\frac{\sqrt{42}G_uxy}{7} - \frac{\sqrt{14}G_{xy}(x^2+y^2-2z^2)}{14} - \frac{3\sqrt{14}G_{xz}yz}{14} - \frac{3\sqrt{14}G_{yz}xz}{14}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,2)}[g](E''), \vec{\mathbb{G}}_{2,2}^{(2,2)}[g](E'')$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$-\frac{5\sqrt{42}G_u(x-y)(x+y)(x^2+y^2-6z^2)}{168} + \frac{\sqrt{14}G_v(19x^4-102x^2y^2-12x^2z^2+19y^4-12y^2z^2+4z^4)}{168} \\ + \frac{5\sqrt{14}G_{xy}xy(x-y)(x+y)}{12} + \frac{5\sqrt{14}G_{xz}xz(5x^2-9y^2-2z^2)}{84} + \frac{5\sqrt{14}G_{yz}yz(9x^2-5y^2+2z^2)}{84}$$

$$\frac{5\sqrt{42}G_uxy(x^2+y^2-6z^2)}{84} - \frac{5\sqrt{14}G_vxy(x-y)(x+y)}{12} + \frac{\sqrt{14}G_{xy}(4x^4-27x^2y^2+3x^2z^2+4y^4+3y^2z^2-z^4)}{42} \\ - \frac{5\sqrt{14}G_{xz}yz(6x^2-y^2-z^2)}{42} + \frac{5\sqrt{14}G_{yz}xz(x^2-6y^2+z^2)}{42}$$

* Harmonics for rank 3

$$\vec{\mathbb{G}}_3^{(2,-2)}[g](A')$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$\frac{\sqrt{15}G_uz}{5} - \frac{\sqrt{5}G_{xz}x}{5} - \frac{\sqrt{5}G_{yz}y}{5}$$

$$\vec{\mathbb{G}}_3^{(2,0)}[g](A')$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$-\frac{\sqrt{15}G_uz(3x^2+3y^2-2z^2)}{15} - \frac{\sqrt{5}G_vz(x-y)(x+y)}{2} - \sqrt{5}G_{xy}xyz - \frac{\sqrt{5}G_{xz}x(x^2+y^2-4z^2)}{20} - \frac{\sqrt{5}G_{yz}y(x^2+y^2-4z^2)}{20}$$

$$\vec{\mathbb{G}}_3^{(2,2)}[g](A')$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$\frac{\sqrt{330}G_uz(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4)}{264} - \frac{7\sqrt{110}G_vz(x-y)(x+y)(x^2+y^2-2z^2)}{88} - \frac{7\sqrt{110}G_{xy}xyz(x^2+y^2-2z^2)}{44} \\ + \frac{\sqrt{110}G_{xz}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44} + \frac{\sqrt{110}G_{yz}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44}$$

$$\vec{\mathbb{G}}_3^{(2,-2)}[g](A'', 1)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$\frac{\sqrt{2}G_vy}{2} + \frac{\sqrt{2}G_{xy}x}{2}$$

$$\vec{\mathbb{G}}_3^{(2,-2)}[g](A'', 2)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{2}G_vx}{2} - \frac{\sqrt{2}G_{xy}y}{2}$$

$$\vec{\mathbb{G}}_3^{(2,0)}[g](A'', 1)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$-\frac{5\sqrt{6}G_{uy}(3x^2 - y^2)}{24} + \frac{\sqrt{2}G_vy(x^2 + y^2 - 4z^2)}{8} + \frac{\sqrt{2}G_{xy}x(x^2 + y^2 - 4z^2)}{8} + \frac{5\sqrt{2}G_{xz}xyz}{4} + \frac{5\sqrt{2}G_{yz}z(x - y)(x + y)}{8}$$

$$\vec{\mathbb{G}}_3^{(2,0)}[g](A'', 2)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$-\frac{5\sqrt{6}G_u x(x^2 - 3y^2)}{24} + \frac{\sqrt{2}G_v x(x^2 + y^2 - 4z^2)}{8} - \frac{\sqrt{2}G_{xy}y(x^2 + y^2 - 4z^2)}{8} + \frac{5\sqrt{2}G_{xz}z(x - y)(x + y)}{8} - \frac{5\sqrt{2}G_{yz}xyz}{4}$$

$$\vec{\mathbb{G}}_3^{(2,2)}[g](A'', 1)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$-\frac{7\sqrt{33}G_{uy}(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{264} + \frac{\sqrt{11}G_{vy}(53x^4 - 104x^2y^2 - 6x^2z^2 + 11y^4 - 6y^2z^2 + 4z^4)}{88} \\ - \frac{\sqrt{11}G_{xy}x(5x^4 - 53x^2y^2 + 3x^2z^2 + 26y^4 + 3y^2z^2 - 2z^4)}{44} + \frac{7\sqrt{11}G_{xz}xyz(7x^2 - 5y^2 - 2z^2)}{44} - \frac{7\sqrt{11}G_{yz}z(x^4 - 9x^2y^2 + x^2z^2 + 2y^4 - y^2z^2)}{44}$$

$$\vec{\mathbb{G}}_3^{(2,2)}[g](A'', 2)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$-\frac{7\sqrt{33}G_u x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{264} + \frac{\sqrt{11}G_v x(11x^4 - 104x^2y^2 - 6x^2z^2 + 53y^4 - 6y^2z^2 + 4z^4)}{88} \\ + \frac{\sqrt{11}G_{xy}y(26x^4 - 53x^2y^2 + 3x^2z^2 + 5y^4 + 3y^2z^2 - 2z^4)}{44} + \frac{7\sqrt{11}G_{xz}z(2x^4 - 9x^2y^2 - x^2z^2 + y^4 + y^2z^2)}{44} + \frac{7\sqrt{11}G_{yz}xyz(5x^2 - 7y^2 + 2z^2)}{44}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-2)}[g](E'), \vec{\mathbb{G}}_{3,2}^{(2,-2)}[g](E')$$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

** expression

$$\frac{\sqrt{3}G_{xy}z}{3} + \frac{\sqrt{3}G_{xz}y}{3} + \frac{\sqrt{3}G_{yz}x}{3}$$

$$\frac{\sqrt{3}G_vz}{3} + \frac{\sqrt{3}G_{xz}x}{3} - \frac{\sqrt{3}G_{yz}y}{3}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,0)}[g](E'), \vec{\mathbb{G}}_{3,2}^{(2,0)}[g](E')$$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{3}G_{xy}z(3x^2+3y^2-2z^2)}{6} + \frac{\sqrt{3}G_{xz}y(3x^2-2y^2+3z^2)}{6} - \frac{\sqrt{3}G_{yz}x(2x^2-3y^2-3z^2)}{6}$$

$$\frac{\sqrt{3}G_vz(3x^2+3y^2-2z^2)}{6} + \frac{\sqrt{3}G_{xz}x(x^2-9y^2+6z^2)}{12} + \frac{\sqrt{3}G_{yz}y(9x^2-y^2-6z^2)}{12}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,2)}[g](E'), \vec{\mathbb{G}}_{3,2}^{(2,2)}[g](E')$$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$-\frac{21\sqrt{22}G_uxyz(x^2+y^2-2z^2)}{44} + \frac{21\sqrt{66}G_vxyz(x-y)(x+y)}{44} - \frac{\sqrt{66}G_{xy}z(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4)}{66}$$

$$-\frac{\sqrt{66}G_{xz}y(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{66} + \frac{\sqrt{66}G_{yz}x(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{66}$$

$$-\frac{21\sqrt{22}G_uz(x-y)(x+y)(x^2+y^2-2z^2)}{88} + \frac{\sqrt{66}G_vz(39x^4-174x^2y^2-20x^2z^2+39y^4-20y^2z^2+4z^4)}{264} + \frac{21\sqrt{66}G_{xy}xyz(x-y)(x+y)}{44}$$

$$-\frac{\sqrt{66}G_{xz}x(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4)}{132} - \frac{\sqrt{66}G_{yz}y(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4)}{132}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-2)}[g](E''), \vec{\mathbb{G}}_{3,2}^{(2,-2)}[g](E'')$$

** symmetry

$$-\frac{\sqrt{6}x(x^2+y^2-4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2+y^2-4z^2)}{4}$$

** expression

$$\frac{\sqrt{10}G_ux}{5} - \frac{\sqrt{30}G_vx}{30} - \frac{\sqrt{30}G_{xy}y}{30} + \frac{2\sqrt{30}G_{xz}z}{15}$$

$$\frac{\sqrt{10}G_uy}{5} + \frac{\sqrt{30}G_vy}{30} - \frac{\sqrt{30}G_{xy}x}{30} + \frac{2\sqrt{30}G_{yz}z}{15}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,0)}[g](E''), \vec{\mathbb{G}}_{3,2}^{(2,0)}[g](E'')$$

** symmetry

$$-\frac{\sqrt{6}x(x^2+y^2-4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2+y^2-4z^2)}{4}$$

** expression

$$-\frac{3\sqrt{10}G_ux(x^2+y^2-4z^2)}{40} - \frac{\sqrt{30}G_vx(11x^2-9y^2-24z^2)}{120} - \frac{\sqrt{30}G_{xy}y(21x^2+y^2-24z^2)}{120} + \frac{\sqrt{30}G_{xz}z(9x^2-21y^2+4z^2)}{120} + \frac{\sqrt{30}G_{yz}xyz}{4}$$

$$-\frac{3\sqrt{10}G_uy(x^2+y^2-4z^2)}{40} - \frac{\sqrt{30}G_vy(9x^2-11y^2+24z^2)}{120} - \frac{\sqrt{30}G_{xy}x(x^2+21y^2-24z^2)}{120} + \frac{\sqrt{30}G_{xz}xyz}{4} - \frac{\sqrt{30}G_{yz}z(21x^2-9y^2-4z^2)}{120}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,2)}[g](E''), \vec{\mathbb{G}}_{3,2}^{(2,2)}[g](E'')$$

** symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{55}G_{ux}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} - \frac{\sqrt{165}G_vx(5x^4 - 4x^2y^2 - 46x^2z^2 - 9y^4 + 66y^2z^2 + 12z^4)}{264} \\ & - \frac{\sqrt{165}G_{xy}y(6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4)}{132} \\ & - \frac{\sqrt{165}G_{xz}z(18x^4 + 15x^2y^2 - 41x^2z^2 - 3y^4 + y^2z^2 + 4z^4)}{132} - \frac{7\sqrt{165}G_{yz}xyz(x^2 + y^2 - 2z^2)}{44} \end{aligned}$$

$$\begin{aligned} & \frac{3\sqrt{55}G_{uy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} - \frac{\sqrt{165}G_vy(9x^4 + 4x^2y^2 - 66x^2z^2 - 5y^4 + 46y^2z^2 - 12z^4)}{264} \\ & + \frac{\sqrt{165}G_{xy}x(x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{132} - \frac{7\sqrt{165}G_{xz}xyz(x^2 + y^2 - 2z^2)}{44} \\ & + \frac{\sqrt{165}G_{yz}z(3x^4 - 15x^2y^2 - x^2z^2 - 18y^4 + 41y^2z^2 - 4z^4)}{132} \end{aligned}$$

* Harmonics for rank 4

$$\vec{\mathbb{G}}_4^{(2,-2)}[g](A', 1)$$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{3}G_vxz}{2} - \frac{\sqrt{3}G_{xy}yz}{2} + \frac{\sqrt{3}G_{xz}(x - y)(x + y)}{4} - \frac{\sqrt{3}G_{yz}xy}{2}$$

$$\vec{\mathbb{G}}_4^{(2,-2)}[g](A', 2)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\frac{\sqrt{3}G_vyz}{2} + \frac{\sqrt{3}G_{xy}xz}{2} + \frac{\sqrt{3}G_{xz}xy}{2} + \frac{\sqrt{3}G_{yz}(x - y)(x + y)}{4}$$

$$\vec{\mathbb{G}}_4^{(2,0)}[g](A', 1)$$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\begin{aligned} & -\frac{7\sqrt{22}G_{ux}z(x^2 - 3y^2)}{88} + \frac{3\sqrt{66}G_vxz(3x^2 + 3y^2 - 4z^2)}{88} - \frac{3\sqrt{66}G_{xy}yz(3x^2 + 3y^2 - 4z^2)}{88} \\ & + \frac{\sqrt{66}G_{xz}(x^4 - 21x^2y^2 + 15x^2z^2 + 6y^4 - 15y^2z^2)}{88} + \frac{\sqrt{66}G_{yz}xy(19x^2 - 9y^2 - 30z^2)}{88} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(2,0)}[g](A', 2)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\begin{aligned} & -\frac{7\sqrt{22}G_{uy}z(3x^2 - y^2)}{88} + \frac{3\sqrt{66}G_vyz(3x^2 + 3y^2 - 4z^2)}{88} + \frac{3\sqrt{66}G_{xy}xz(3x^2 + 3y^2 - 4z^2)}{88} \\ & + \frac{\sqrt{66}G_{xz}xy(9x^2 - 19y^2 + 30z^2)}{88} - \frac{\sqrt{66}G_{yz}(6x^4 - 21x^2y^2 - 15x^2z^2 + y^4 + 15y^2z^2)}{88} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(2,2)}[g](A', 1)$$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\begin{aligned} & -\frac{9\sqrt{2002}G_uxz(x^2-3y^2)(3x^2+3y^2-8z^2)}{1144} + \frac{\sqrt{6006}G_vxz(19x^4-160x^2y^2-10x^2z^2+85y^4-10y^2z^2+4z^4)}{1144} \\ & + \frac{\sqrt{6006}G_{xy}yz(40x^4-85x^2y^2+5x^2z^2+7y^4+5y^2z^2-2z^4)}{572} \\ & - \frac{\sqrt{6006}G_{xz}(2x^6-7x^4y^2-23x^4z^2-8x^2y^4+90x^2y^2z^2+8x^2z^4+y^6-7y^4z^2-8y^2z^4)}{572} \\ & - \frac{\sqrt{6006}G_{yz}xy(5x^4-2x^2y^2-44x^2z^2-7y^4+76y^2z^2-16z^4)}{572} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(2,2)}[g](A', 2)$

** symmetry

$$\frac{\sqrt{70}yz(3x^2-y^2)}{4}$$

** expression

$$\begin{aligned} & -\frac{9\sqrt{2002}G_uyz(3x^2-y^2)(3x^2+3y^2-8z^2)}{1144} + \frac{\sqrt{6006}G_vyz(85x^4-160x^2y^2-10x^2z^2+19y^4-10y^2z^2+4z^4)}{1144} \\ & - \frac{\sqrt{6006}G_{xy}xz(7x^4-85x^2y^2+5x^2z^2+40y^4+5y^2z^2-2z^4)}{572} - \frac{\sqrt{6006}G_{xz}xy(7x^4+2x^2y^2-76x^2z^2-5y^4+44y^2z^2+16z^4)}{572} \\ & + \frac{\sqrt{6006}G_{yz}(x^6-8x^4y^2-7x^4z^2-7x^2y^4+90x^2y^2z^2-8x^2z^4+2y^6-23y^4z^2+8y^2z^4)}{572} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(2,-2)}[g](A'')$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$-\frac{3\sqrt{70}G_u(x^2+y^2-2z^2)}{70} + \frac{\sqrt{210}G_v(x-y)(x+y)}{140} + \frac{\sqrt{210}G_{xy}xy}{70} - \frac{2\sqrt{210}G_{xz}xz}{35} - \frac{2\sqrt{210}G_{yz}yz}{35}$$

$\vec{\mathbb{G}}_4^{(2,0)}[g](A'')$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\begin{aligned} & \frac{\sqrt{385}G_u(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4)}{308} + \frac{3\sqrt{1155}G_v(x-y)(x+y)(x^2+y^2-6z^2)}{308} \\ & + \frac{3\sqrt{1155}G_{xy}xy(x^2+y^2-6z^2)}{154} - \frac{\sqrt{1155}G_{xz}xz(3x^2+3y^2-4z^2)}{308} - \frac{\sqrt{1155}G_{yz}yz(3x^2+3y^2-4z^2)}{308} \end{aligned}$$

$\vec{\mathbb{G}}_4^{(2,2)}[g](A'')$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\begin{aligned} & -\frac{3\sqrt{715}G_u(5x^6+15x^4y^2-90x^4z^2+15x^2y^4-180x^2y^2z^2+120x^2z^4+5y^6-90y^4z^2+120y^2z^4-16z^6)}{2288} \\ & + \frac{7\sqrt{2145}G_v(x-y)(x+y)(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{2288} + \frac{7\sqrt{2145}G_{xy}xy(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{1144} \\ & + \frac{7\sqrt{2145}G_{xz}xz(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4)}{1144} + \frac{7\sqrt{2145}G_{yz}yz(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4)}{1144} \end{aligned}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E''), \vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E')$

** symmetry

$$-\frac{\sqrt{10}yz(3x^2+3y^2-4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2+3y^2-4z^2)}{4}$$

** expression

$$\frac{3\sqrt{7}G_uyz}{7} + \frac{\sqrt{21}G_vyz}{14} - \frac{\sqrt{21}G_{xy}xz}{14} - \frac{\sqrt{21}G_{xz}xy}{14} - \frac{\sqrt{21}G_{yz}(x^2+3y^2-4z^2)}{28}$$

$$-\frac{3\sqrt{7}G_{uxz}}{7} + \frac{\sqrt{21}G_{vzx}}{14} + \frac{\sqrt{21}G_{xyyz}}{14} + \frac{\sqrt{21}G_{xz}(3x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{21}G_{yzxy}}{14}$$

$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E')$, $\vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E')$

** symmetry

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$-\frac{17\sqrt{154}G_{uyz}(3x^2 + 3y^2 - 4z^2)}{616} - \frac{\sqrt{462}G_{vyz}(33x^2 - 51y^2 + 40z^2)}{616} - \frac{\sqrt{462}G_{xyxz}(9x^2 + 93y^2 - 40z^2)}{616}$$

$$-\frac{9\sqrt{462}G_{xz}xy(x^2 + y^2 - 6z^2)}{616} + \frac{\sqrt{462}G_{yz}(6x^4 + 3x^2y^2 - 39x^2z^2 - 3y^4 + 15y^2z^2 + 4z^4)}{616}$$

$$\frac{17\sqrt{154}G_{uxz}(3x^2 + 3y^2 - 4z^2)}{616} + \frac{\sqrt{462}G_{vxz}(51x^2 - 33y^2 - 40z^2)}{616} + \frac{\sqrt{462}G_{xyyz}(93x^2 + 9y^2 - 40z^2)}{616}$$

$$+ \frac{\sqrt{462}G_{xz}(3x^4 - 3x^2y^2 - 15x^2z^2 - 6y^4 + 39y^2z^2 - 4z^4)}{616} + \frac{9\sqrt{462}G_{yzxy}(x^2 + y^2 - 6z^2)}{616}$$

$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E')$, $\vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E')$

** symmetry

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$-\frac{21\sqrt{286}G_{uyz}(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{1144} - \frac{7\sqrt{858}G_{vyz}(11x^4 + 4x^2y^2 - 26x^2z^2 - 7y^4 + 22y^2z^2 - 4z^4)}{1144}$$

$$+ \frac{7\sqrt{858}G_{xyxz}(x^4 - 7x^2y^2 - x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4)}{572} + \frac{7\sqrt{858}G_{xz}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572}$$

$$-\frac{\sqrt{858}G_{yz}(x^6 - 4x^4y^2 - 11x^4z^2 - 11x^2y^4 + 90x^2y^2z^2 - 4x^2z^4 - 6y^6 + 101y^4z^2 - 116y^2z^4 + 8z^6)}{572}$$

$$-\frac{21\sqrt{286}G_{uxz}(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{1144}$$

$$+ \frac{7\sqrt{858}G_{vxz}(7x^4 - 4x^2y^2 - 22x^2z^2 - 11y^4 + 26y^2z^2 + 4z^4)}{1144} + \frac{7\sqrt{858}G_{xyyz}(8x^4 + 7x^2y^2 - 23x^2z^2 - y^4 + y^2z^2 + 2z^4)}{572}$$

$$-\frac{\sqrt{858}G_{xz}(6x^6 + 11x^4y^2 - 101x^4z^2 + 4x^2y^4 - 90x^2y^2z^2 + 116x^2z^4 - y^6 + 11y^4z^2 + 4y^2z^4 - 8z^6)}{572}$$

$$-\frac{7\sqrt{858}G_{yzxy}(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E'', 1)$, $\vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E'', 1)$

** symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x - y)(x + y)}{2}$$

** expression

$$\frac{\sqrt{6}G_v(x - y)(x + y)}{4} - \frac{\sqrt{6}G_{xyxy}}{2}$$

$$\frac{\sqrt{6}G_vxy}{2} + \frac{\sqrt{6}G_{xy}(x - y)(x + y)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E'', 2)$, $\vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E'', 2)$

** symmetry

$$-\frac{\sqrt{5}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\frac{3\sqrt{14}G_u(x-y)(x+y)}{28} - \frac{\sqrt{42}G_v(x^2 + y^2 - 2z^2)}{28} + \frac{\sqrt{42}G_{xz}xz}{7} - \frac{\sqrt{42}G_{yz}yz}{7}$$

$$- \frac{3\sqrt{14}G_uxy}{14} + \frac{\sqrt{42}G_{xy}(x^2 + y^2 - 2z^2)}{28} - \frac{\sqrt{42}G_{xz}yz}{7} - \frac{\sqrt{42}G_{yz}xz}{7}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E'', 1), \vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E'', 1)$$

** symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$- \frac{7\sqrt{11}G_u(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{44} + \frac{\sqrt{33}G_v(x-y)(x+y)(x^2 + y^2 - 6z^2)}{44}$$

$$- \frac{\sqrt{33}G_{xy}xy(x^2 + y^2 - 6z^2)}{22} + \frac{7\sqrt{33}G_{xz}xz(x^2 - 3y^2)}{44} - \frac{7\sqrt{33}G_{yz}yz(3x^2 - y^2)}{44}$$

$$- \frac{7\sqrt{11}G_uxy(x-y)(x+y)}{11} + \frac{\sqrt{33}G_vxy(x^2 + y^2 - 6z^2)}{22}$$

$$+ \frac{\sqrt{33}G_{xy}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{44} + \frac{7\sqrt{33}G_{xz}yz(3x^2 - y^2)}{44} + \frac{7\sqrt{33}G_{yz}xz(x^2 - 3y^2)}{44}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E'', 2), \vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E'', 2)$$

** symmetry

$$- \frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$- \frac{\sqrt{77}G_u(x-y)(x+y)(x^2 + y^2 - 6z^2)}{77} - \frac{\sqrt{231}G_v(2x^4 - 3x^2y^2 - 9x^2z^2 + 2y^4 - 9y^2z^2 + 3z^4)}{77}$$

$$- \frac{\sqrt{231}G_{xy}xy(x-y)(x+y)}{22} + \frac{\sqrt{231}G_{xz}xz(2x^2 - 33y^2 + 9z^2)}{154} + \frac{\sqrt{231}G_{yz}yz(33x^2 - 2y^2 - 9z^2)}{154}$$

$$- \frac{2\sqrt{77}G_uxy(x^2 + y^2 - 6z^2)}{77} + \frac{\sqrt{231}G_vxy(x-y)(x+y)}{22} + \frac{\sqrt{231}G_{xy}(x^4 + 30x^2y^2 - 36x^2z^2 + y^4 - 36y^2z^2 + 12z^4)}{308}$$

$$- \frac{\sqrt{231}G_{xz}yz(39x^2 - 31y^2 + 18z^2)}{308} + \frac{\sqrt{231}G_{yz}xz(31x^2 - 39y^2 - 18z^2)}{308}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E'', 1), \vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E'', 1)$$

** symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$- \frac{9\sqrt{1001}G_u(x^2 + y^2 - 10z^2)(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{2288} + \frac{\sqrt{3003}G_v(x-y)(x+y)(17x^4 - 230x^2y^2 - 8x^2z^2 + 17y^4 - 8y^2z^2 + 8z^4)}{2288}$$

$$+ \frac{\sqrt{3003}G_{xy}xy(49x^4 - 166x^2y^2 + 8x^2z^2 + 49y^4 + 8y^2z^2 - 8z^4)}{1144}$$

$$+ \frac{3\sqrt{3003}G_{xz}xz(7x^4 - 58x^2y^2 - 4x^2z^2 + 23y^4 + 12y^2z^2)}{1144} + \frac{3\sqrt{3003}G_{yz}yz(23x^4 - 58x^2y^2 + 12x^2z^2 + 7y^4 - 4y^2z^2)}{1144}$$

$$- \frac{9\sqrt{1001}G_uxy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572} + \frac{\sqrt{3003}G_vxy(25x^4 - 82x^2y^2 - 4x^2z^2 + 25y^4 - 4y^2z^2 + 4z^4)}{572}$$

$$- \frac{\sqrt{3003}G_{xy}(x-y)(x+y)(2x^4 - 29x^2y^2 + x^2z^2 + 2y^4 + y^2z^2 - z^4)}{286}$$

$$+ \frac{3\sqrt{3003}G_{xz}yz(8x^4 - 13x^2y^2 - 3x^2z^2 + y^4 + y^2z^2)}{286} - \frac{3\sqrt{3003}G_{yz}xz(x^4 - 13x^2y^2 + x^2z^2 + 8y^4 - 3y^2z^2)}{286}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E'', 2), \vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E'', 2)$$

** symmetry

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{21\sqrt{143}G_u(x-y)(x+y)(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{1144} \\ & - \frac{\sqrt{429}G_v(13x^6-45x^4y^2-150x^4z^2-45x^2y^4+540x^2y^2z^2+60x^2z^4+13y^6-150y^4z^2+60y^2z^4-8z^6)}{572} \\ & - \frac{21\sqrt{429}G_{xy}xy(x-y)(x+y)(x^2+y^2-10z^2)}{572} - \frac{1144}{7\sqrt{429}G_{xz}xz(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4)} \\ & - \frac{7\sqrt{429}G_{yz}yz(11x^4+4x^2y^2-26x^2z^2-7y^4+22y^2z^2-4z^4)}{572} \\ & - \frac{21\sqrt{143}G_uxy(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{572} + \frac{21\sqrt{429}G_vxy(x-y)(x+y)(x^2+y^2-10z^2)}{572} \\ & - \frac{\sqrt{429}G_{xy}(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6)}{286} \\ & + \frac{7\sqrt{429}G_{xz}yz(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4)}{286} - \frac{7\sqrt{429}G_{yz}xz(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4)}{286} \end{aligned}$$