

MPG No. 7.4.23 $m'm'2$ [Type III, orthorhombic] [G tensor]

* Rank 0 tensor. * Rank 1 tensor. * Rank 2 tensor (s).

$$\begin{bmatrix} 0 & G_{xy} & 0 \\ G_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{xy} = G_{dxy}^{(1)}$$

* Rank 2 tensor (a).

$$\begin{bmatrix} 0 & G_{xy} & 0 \\ -G_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{xy} = Q_{pz}^{(1)}$$

* Rank 3 tensor (s).

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ G_{yzx} & 0 & 0 \\ 0 & G_{zxy} & 0 \\ 0 & 0 & G_{xyz} \end{bmatrix}$$

$$G_{yzx} = G_{f3}^{(1)} - 3Q_{du}^{(1)} - Q_{dv}^{(1)}$$

$$G_{zxy} = G_{f3}^{(1)} + 3Q_{du}^{(1)} - Q_{dv}^{(1)}$$

$$G_{xyz} = G_{f3}^{(1)} + 2Q_{dv}^{(1)}$$

* Rank 3 tensor (a).

$$\begin{bmatrix} G_{yzx} & 0 & 0 \\ 0 & G_{zxy} & 0 \\ 0 & 0 & G_{xyz} \end{bmatrix}$$

$$G_{yzx} = -Q_{du}^{(2)} + Q_{dv}^{(2)} + Q_s^{(1)}$$

$$G_{zxy} = -Q_{du}^{(2)} - Q_{dv}^{(2)} + Q_s^{(1)}$$

$$G_{xyz} = 2Q_{du}^{(2)} + Q_s^{(1)}$$

* Rank 4 tensor (sss).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & G_{xxxx} \\ 0 & 0 & 0 & 0 & 0 & G_{yyxy} \\ 0 & 0 & 0 & 0 & 0 & G_{zzxy} \\ 0 & 0 & 0 & 0 & G_{yzzx} & 0 \\ 0 & 0 & 0 & G_{yzzx} & 0 & 0 \\ G_{xxxx} & G_{yyxy} & G_{zzxy} & 0 & 0 & 0 \end{bmatrix}$$

$$G_{xxxx} = G_{dxy}^{(1)} + 2G_{dxy}^{(2)} + G_{gaz}^{(1)} - G_{gbz}^{(1)}$$

$$G_{yyxy} = G_{dxy}^{(1)} + 2G_{dxy}^{(2)} - G_{gaz}^{(1)} - G_{gbz}^{(1)}$$

$$G_{zzxy} = G_{dxy}^{(1)} + 2G_{gbz}^{(1)}$$

$$G_{yzzx} = G_{dxy}^{(2)} + 2G_{gbz}^{(1)}$$

* Rank 4 tensor (ssa).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & G_{xxyy} \\ 0 & 0 & 0 & 0 & 0 & G_{yyxy} \\ 0 & 0 & 0 & 0 & 0 & G_{zzxy} \\ 0 & 0 & 0 & 0 & 0 & G_{yzzx} \\ 0 & 0 & 0 & -G_{yzzx} & 0 & 0 \\ -G_{xxyy} & -G_{yyxy} & -G_{zzxy} & 0 & 0 & 0 \end{bmatrix}$$

$$G_{xxyy} = G_{dxy}^{(3)} - 2Q_{faz}^{(1)} + 2Q_{fbz}^{(1)} + 2Q_{pz}^{(1)}$$

$$G_{yyxy} = G_{dxy}^{(3)} + 2Q_{faz}^{(1)} + 2Q_{fbz}^{(1)} - 2Q_{pz}^{(1)}$$

$$G_{zzxy} = G_{dxy}^{(3)} - 4Q_{fbz}^{(1)}$$

$$G_{yzzx} = -4Q_{faz}^{(1)} - Q_{pz}^{(1)}$$

* Rank 4 tensor (aas).

$$\begin{bmatrix} 0 & G_{yzzx} & 0 \\ G_{yzzx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{yzzx} = 2G_{dxy}^{(4)}$$

* Rank 4 tensor (aaa).

$$\begin{bmatrix} 0 & G_{yzzx} & 0 \\ -G_{yzzx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{yzzx} = -Q_{pz}^{(2)}$$

* Rank 4 tensor (sa).

$$\begin{bmatrix} 0 & 0 & G_{xxyy} \\ 0 & 0 & G_{yyxy} \\ 0 & 0 & G_{zzxy} \\ 0 & G_{yzzx} & 0 \\ G_{zxyz} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{xxyy} = -2G_{dxy}^{(5)} - Q_{faz}^{(2)} + Q_{fbz}^{(2)} + Q_{pz}^{(3)}$$

$$G_{yyxy} = 2G_{dxy}^{(5)} - Q_{faz}^{(2)} - Q_{fbz}^{(2)} + Q_{pz}^{(3)}$$

$$G_{zzxy} = 2Q_{faz}^{(2)} + Q_{pz}^{(3)} + 2Q_{pz}^{(4)}$$

$$G_{yzzx} = -G_{dxy}^{(5)} - Q_{faz}^{(2)} - Q_{fbz}^{(2)} + Q_{pz}^{(4)}$$

$$G_{zxyz} = G_{dxy}^{(5)} - Q_{faz}^{(2)} + Q_{fbz}^{(2)} + Q_{pz}^{(4)}$$

* Rank 4 tensor (as).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & G_{yzzx} & 0 \\ 0 & 0 & 0 & G_{zxyz} & 0 & 0 \\ G_{xyxx} & G_{xyyy} & G_{xyzz} & 0 & 0 & 0 \end{bmatrix}$$

$$G_{yzzx} = G_{dxy}^{(6)} - Q_{faz}^{(3)} + Q_{fbz}^{(3)} + Q_{pz}^{(6)}$$

$$G_{zxyz} = -G_{dxy}^{(6)} - Q_{faz}^{(3)} - Q_{fbz}^{(3)} + Q_{pz}^{(6)}$$

$$G_{xyxx} = -2G_{dxy}^{(6)} - Q_{faz}^{(3)} + Q_{fbz}^{(3)} + Q_{pz}^{(5)}$$

$$G_{xyyy} = 2G_{dxy}^{(6)} - Q_{faz}^{(3)} - Q_{fbz}^{(3)} + Q_{pz}^{(5)}$$

$$G_{xyzz} = 2Q_{faz}^{(3)} + Q_{pz}^{(5)} + 2Q_{pz}^{(6)}$$

* Rank 4 tensor (s).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & G_{xxyy} & 0 & 0 & G_{xxyx} \\ 0 & 0 & 0 & 0 & 0 & G_{yyxy} & 0 & 0 & G_{yyyx} \\ 0 & 0 & 0 & 0 & 0 & G_{zzxy} & 0 & 0 & G_{zzyx} \\ 0 & 0 & 0 & 0 & G_{yzxz} & 0 & 0 & G_{yzzx} & 0 \\ 0 & 0 & 0 & G_{zxyz} & 0 & 0 & G_{zxzy} & 0 & 0 \\ G_{xyxx} & G_{xyyy} & G_{xyzz} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} G_{xxyy} &= G_{dxy}^{(1)} + 2G_{dxy}^{(2)} + G_{dxy}^{(3)} - 2G_{dxy}^{(5)} + G_{gaz}^{(1)} - G_{gbz}^{(1)} - 2Q_{faz}^{(1)} - Q_{fbz}^{(2)} + 2Q_{fbz}^{(1)} + Q_{pz}^{(2)} + 2Q_{pz}^{(1)} + Q_{pz}^{(3)} \\ G_{xxyx} &= G_{dxy}^{(1)} + 2G_{dxy}^{(2)} + G_{dxy}^{(3)} + 2G_{dxy}^{(5)} + G_{gaz}^{(1)} - G_{gbz}^{(1)} - 2Q_{faz}^{(1)} + Q_{fbz}^{(2)} + 2Q_{fbz}^{(1)} - Q_{fbz}^{(2)} + 2Q_{pz}^{(1)} - Q_{pz}^{(3)} \\ G_{yyxy} &= G_{dxy}^{(1)} + 2G_{dxy}^{(2)} + G_{dxy}^{(3)} + 2G_{dxy}^{(5)} - G_{gaz}^{(1)} - G_{gbz}^{(1)} + 2Q_{faz}^{(1)} - Q_{fbz}^{(2)} + 2Q_{fbz}^{(1)} - Q_{fbz}^{(2)} - 2Q_{pz}^{(1)} + Q_{pz}^{(3)} \\ G_{yyyx} &= G_{dxy}^{(1)} + 2G_{dxy}^{(2)} + G_{dxy}^{(3)} - 2G_{dxy}^{(5)} - G_{gaz}^{(1)} - G_{gbz}^{(1)} + 2Q_{faz}^{(1)} + Q_{fbz}^{(2)} + 2Q_{fbz}^{(1)} - Q_{fbz}^{(2)} - 2Q_{pz}^{(1)} - Q_{pz}^{(3)} \\ G_{zzxy} &= G_{dxy}^{(1)} + G_{dxy}^{(3)} + 2G_{dxy}^{(1)} + 2Q_{fbz}^{(2)} - 4Q_{faz}^{(1)} + Q_{pz}^{(3)} + 2Q_{pz}^{(4)} \\ G_{zzyx} &= G_{dxy}^{(1)} + G_{dxy}^{(3)} + 2G_{gbz}^{(2)} - 2Q_{faz}^{(1)} - 4Q_{fbz}^{(1)} - Q_{pz}^{(3)} - 2Q_{pz}^{(4)} \\ G_{yzzx} &= G_{dxy}^{(2)} - G_{dxy}^{(5)} + 2G_{gbz}^{(1)} - 4Q_{faz}^{(1)} - Q_{fbz}^{(2)} - Q_{fbz}^{(1)} - Q_{pz}^{(1)} + Q_{pz}^{(4)} \\ G_{yzzz} &= G_{dxy}^{(2)} + G_{dxy}^{(5)} + 2G_{gbz}^{(1)} - 4Q_{faz}^{(1)} + Q_{fbz}^{(2)} - Q_{fbz}^{(1)} - Q_{pz}^{(1)} - Q_{pz}^{(4)} \\ G_{zxyz} &= G_{dxy}^{(2)} + G_{dxy}^{(5)} + 2G_{gbz}^{(1)} + 4Q_{faz}^{(1)} - Q_{fbz}^{(2)} + Q_{fbz}^{(1)} + Q_{pz}^{(1)} + Q_{pz}^{(4)} \\ G_{zxzy} &= G_{dxy}^{(2)} - G_{dxy}^{(5)} + 2G_{gbz}^{(1)} + 4Q_{faz}^{(1)} + Q_{fbz}^{(2)} - Q_{fbz}^{(1)} + Q_{pz}^{(1)} - Q_{pz}^{(4)} \\ G_{xyxx} &= G_{dxy}^{(1)} + 2G_{dxy}^{(2)} - G_{dxy}^{(3)} + G_{gaz}^{(1)} - G_{gbz}^{(1)} + 2Q_{faz}^{(1)} - 2Q_{fbz}^{(1)} - 2Q_{pz}^{(1)} \\ G_{xyyy} &= G_{dxy}^{(1)} + 2G_{dxy}^{(2)} - G_{dxy}^{(3)} - G_{gaz}^{(1)} - G_{gbz}^{(1)} - 2Q_{faz}^{(1)} - 2Q_{fbz}^{(1)} + 2Q_{pz}^{(1)} \\ G_{xyzz} &= G_{dxy}^{(1)} - G_{dxy}^{(3)} + 2G_{gbz}^{(1)} + 4Q_{fbz}^{(1)} \end{aligned}$$

* Rank 4 tensor (a).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & G_{yzzx} & 0 & 0 & G_{yzzx} & 0 \\ 0 & 0 & 0 & G_{zxyz} & 0 & 0 & G_{zxzy} & 0 & 0 \\ G_{xyxx} & G_{xyyy} & G_{xyzz} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} G_{yzzx} &= 2G_{dxy}^{(4)} + G_{dxy}^{(6)} - Q_{faz}^{(3)} + Q_{fbz}^{(2)} - Q_{pz}^{(2)} + Q_{pz}^{(6)} \\ G_{yzzz} &= -2G_{dxy}^{(4)} + G_{dxy}^{(6)} - Q_{faz}^{(3)} + Q_{fbz}^{(2)} + Q_{pz}^{(2)} + Q_{pz}^{(6)} \\ G_{zxyz} &= 2G_{dxy}^{(4)} - G_{dxy}^{(6)} - Q_{faz}^{(3)} - Q_{fbz}^{(3)} + Q_{pz}^{(2)} + Q_{pz}^{(6)} \\ G_{zxzy} &= -2G_{dxy}^{(4)} - G_{dxy}^{(6)} - Q_{faz}^{(3)} - Q_{fbz}^{(2)} - Q_{pz}^{(2)} + Q_{pz}^{(6)} \\ G_{xyxx} &= -2G_{dxy}^{(6)} - Q_{faz}^{(3)} + Q_{fbz}^{(2)} + Q_{pz}^{(5)} \\ G_{xyyy} &= 2G_{dxy}^{(6)} - Q_{faz}^{(3)} - Q_{fbz}^{(2)} + Q_{pz}^{(5)} \\ G_{xyzz} &= 2Q_{faz}^{(3)} + Q_{pz}^{(5)} + 2Q_{pz}^{(6)} \end{aligned}$$

* Rank 4 tensor (t).

$$\begin{bmatrix} 0 & G_{xxyy} & 0 \\ G_{yyyx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & G_{zzxy} & 0 \\ G_{xxyx} & 0 & 0 \\ G_{yzzx} & 0 & 0 \\ 0 & G_{xyyy} & 0 \\ 0 & 0 & \frac{G_{xxyy}}{2} - \frac{G_{xxyx}}{2} + G_{xyyy} - G_{yyyx} + G_{zxyz} \end{bmatrix}$$

$$G_{xxyy} = G_{dxy}^{(1)} + 2G_{dxy}^{(2)} + G_{dxy}^{(3)} - 2G_{dxy}^{(5)} + G_{gaz}^{(1)} - G_{gbz}^{(1)}$$

$$G_{yyxx} = G_{dxy}^{(1)} + 2G_{dxy}^{(2)} + G_{dxy}^{(3)} - 2G_{dxy}^{(5)} - G_{gaz}^{(1)} - G_{gbz}^{(1)}$$

$$G_{zzxy} = G_{dxy}^{(1)} + G_{dxy}^{(3)} + 2G_{gbz}^{(1)}$$

$$G_{xxyx} = G_{dxy}^{(1)} + 2G_{dxy}^{(2)} + G_{dxy}^{(3)} + 2G_{dxy}^{(5)} + G_{gaz}^{(1)} - G_{gbz}^{(1)}$$

$$G_{yzzx} = G_{dxy}^{(2)} - G_{dxy}^{(5)} + 2G_{gbz}^{(1)}$$

$$G_{xyyy} = G_{dxy}^{(1)} + 2G_{dxy}^{(2)} - G_{dxy}^{(3)} - G_{gaz}^{(1)} - G_{gbz}^{(1)}$$