

PG No. 8 D_{2h} mmm [orthorhombic] (axial, internal polar quadrupole)

* Harmonics for rank 0

* Harmonics for rank 1

$$\vec{G}_1^{(2,1)}[q](B_{1g})$$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{30}Q_vxy}{5} - \frac{\sqrt{30}Q_{xy}(x-y)(x+y)}{10} + \frac{\sqrt{30}Q_{xz}yz}{10} - \frac{\sqrt{30}Q_{yz}xz}{10}$$

$$\vec{G}_1^{(2,1)}[q](B_{2g})$$

** symmetry

$$y$$

** expression

$$\frac{3\sqrt{10}Q_{uxz}}{10} - \frac{\sqrt{30}Q_vxz}{10} - \frac{\sqrt{30}Q_{xy}yz}{10} + \frac{\sqrt{30}Q_{xz}(x-z)(x+z)}{10} + \frac{\sqrt{30}Q_{yz}xy}{10}$$

$$\vec{G}_1^{(2,1)}[q](B_{3g})$$

** symmetry

$$x$$

** expression

$$-\frac{3\sqrt{10}Q_{uxz}}{10} - \frac{\sqrt{30}Q_vyz}{10} + \frac{\sqrt{30}Q_{xy}xz}{10} - \frac{\sqrt{30}Q_{xz}xy}{10} - \frac{\sqrt{30}Q_{yz}(y-z)(y+z)}{10}$$

* Harmonics for rank 2

$$\vec{G}_2^{(2,-1)}[q](A_u, 1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{2}Q_{xz}y}{2} - \frac{\sqrt{2}Q_{yz}x}{2}$$

$$\vec{G}_2^{(2,-1)}[q](A_u, 2)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{6}Q_{xyz}}{3} - \frac{\sqrt{6}Q_{xz}y}{6} - \frac{\sqrt{6}Q_{yz}x}{6}$$

$$\vec{G}_2^{(2,1)}[q](A_u, 1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{5\sqrt{42}Q_vxyz}{14} - \frac{5\sqrt{42}Q_{xyz}(x-y)(x+y)}{28} - \frac{\sqrt{42}Q_{xz}y(x^2+y^2-4z^2)}{28} + \frac{\sqrt{42}Q_{yz}x(x^2+y^2-4z^2)}{28}$$

$$\vec{G}_2^{(2,1)}[q](A_u, 2)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$-\frac{5\sqrt{42}Q_{xyz}}{14} + \frac{\sqrt{14}Q_{xy}z(3x^2+3y^2-2z^2)}{28} - \frac{\sqrt{14}Q_{xz}y(9x^2-y^2-6z^2)}{28} + \frac{\sqrt{14}Q_{yz}x(x^2-9y^2+6z^2)}{28}$$

$$\vec{G}_2^{(2,-1)}[q](B_{1u})$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{6}Q_v z}{3} + \frac{\sqrt{6}Q_{xz}x}{6} - \frac{\sqrt{6}Q_{yz}y}{6}$$

$$\vec{\mathbb{G}}_2^{(2,1)}[q](B_{1u})$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{5\sqrt{42}Q_u z (x-y)(x+y)}{28} - \frac{\sqrt{14}Q_v z (3x^2 + 3y^2 - 2z^2)}{28} + \frac{\sqrt{14}Q_{xz}x (2x^2 - 3y^2 - 3z^2)}{14} + \frac{\sqrt{14}Q_{yz}y (3x^2 - 2y^2 + 3z^2)}{14}$$

$$\vec{\mathbb{G}}_2^{(2,-1)}[q](B_{2u})$$

** symmetry

$$\sqrt{3}xz$$

** expression

$$-\frac{\sqrt{2}Q_u y}{2} + \frac{\sqrt{6}Q_v y}{6} - \frac{\sqrt{6}Q_{xy}x}{6} + \frac{\sqrt{6}Q_{yz}z}{6}$$

$$\vec{\mathbb{G}}_2^{(2,1)}[q](B_{2u})$$

** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{42}Q_u y (x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{14}Q_v y (9x^2 - y^2 - 6z^2)}{28} - \frac{\sqrt{14}Q_{xy}x (2x^2 - 3y^2 - 3z^2)}{14} - \frac{\sqrt{14}Q_{yz}z (3x^2 + 3y^2 - 2z^2)}{14}$$

$$\vec{\mathbb{G}}_2^{(2,-1)}[q](B_{3u})$$

** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{2}Q_u x}{2} + \frac{\sqrt{6}Q_v x}{6} + \frac{\sqrt{6}Q_{xy}y}{6} - \frac{\sqrt{6}Q_{xz}z}{6}$$

$$\vec{\mathbb{G}}_2^{(2,1)}[q](B_{3u})$$

** symmetry

$$\sqrt{3}yz$$

** expression

$$-\frac{\sqrt{42}Q_u x (x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{14}Q_v x (x^2 - 9y^2 + 6z^2)}{28} - \frac{\sqrt{14}Q_{xy}y (3x^2 - 2y^2 + 3z^2)}{14} + \frac{\sqrt{14}Q_{xz}z (3x^2 + 3y^2 - 2z^2)}{14}$$

* Harmonics for rank 3

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](A_g)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{6}Q_u (x-y)(x+y)}{4} + \frac{\sqrt{2}Q_v (x^2 + y^2 - 2z^2)}{4}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](A_g)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$-\frac{\sqrt{3}Q_u (x-y)(x+y)(x^2 + y^2 - 6z^2)}{6} - \frac{Q_v (x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{6} \\ - \frac{7Q_{xy}xy(x-y)(x+y)}{6} + \frac{7Q_{xz}xz(x-z)(x+z)}{6} - \frac{7Q_{yz}yz(y-z)(y+z)}{6}$$

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](B_{1g}, 1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{\sqrt{30}Q_vxy}{10} + \frac{\sqrt{30}Q_{xy}(x-y)(x+y)}{20} + \frac{\sqrt{30}Q_{xz}yz}{5} - \frac{\sqrt{30}Q_{yz}xz}{5}$$

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](B_{1g}, 2)$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$-\frac{\sqrt{6}Q_vxy}{2} - \frac{\sqrt{2}Q_{xy}(x^2 + y^2 - 2z^2)}{4}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](B_{1g}, 1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_vxy(x^2 + y^2 - 6z^2)}{6} + \frac{\sqrt{15}Q_{xy}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{12} - \frac{\sqrt{15}Q_{xz}yz(3x^2 + 3y^2 - 4z^2)}{12} + \frac{\sqrt{15}Q_{yz}xz(3x^2 + 3y^2 - 4z^2)}{12}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](B_{1g}, 2)$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{3}Q_vxy(x^2 + y^2 - 6z^2)}{3} + \frac{7Q_vxy(x-y)(x+y)}{6} - \frac{Q_{xy}(5x^4 - 18x^2y^2 - 12x^2z^2 + 5y^4 - 12y^2z^2 + 4z^4)}{12} - \frac{7Q_{xz}yz(3x^2 + y^2 - 2z^2)}{12} - \frac{7Q_{yz}xz(x^2 + 3y^2 - 2z^2)}{12}$$

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](B_{2g}, 1)$$

** symmetry

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$-\frac{3\sqrt{10}Q_vxz}{20} + \frac{\sqrt{30}Q_vxz}{20} - \frac{\sqrt{30}Q_{xy}yz}{5} - \frac{\sqrt{30}Q_{xz}(x-z)(x+z)}{20} + \frac{\sqrt{30}Q_{yz}xy}{5}$$

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](B_{2g}, 2)$$

** symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

** expression

$$\frac{\sqrt{6}Q_vxz}{4} + \frac{3\sqrt{2}Q_vxz}{4} - \frac{\sqrt{2}Q_{xz}(x^2 - 2y^2 + z^2)}{4}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](B_{2g}, 1)$$

** symmetry

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$-\frac{\sqrt{5}Q_vxz(x^2 - 6y^2 + z^2)}{4} + \frac{\sqrt{15}Q_vxz(x^2 - 6y^2 + z^2)}{12} + \frac{\sqrt{15}Q_{xy}yz(3x^2 - 4y^2 + 3z^2)}{12} - \frac{\sqrt{15}Q_{xz}(x-z)(x+z)(x^2 - 6y^2 + z^2)}{12} - \frac{\sqrt{15}Q_{yz}xy(3x^2 - 4y^2 + 3z^2)}{12}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](B_{2g}, 2)$$

** symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_u x z (9x^2 - 12y^2 - 5z^2)}{12} + \frac{Q_v x z (x^2 + 36y^2 - 13z^2)}{12} - \frac{7Q_{xy} y z (3x^2 - 2y^2 + z^2)}{12} \\ - \frac{Q_{xz} (5x^4 - 12x^2 y^2 - 18x^2 z^2 + 4y^4 - 12y^2 z^2 + 5z^4)}{12} - \frac{7Q_{yz} x y (x^2 - 2y^2 + 3z^2)}{12}$$

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](B_{3g}, 1)$$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

** expression

$$\frac{3\sqrt{10}Q_u y z}{20} + \frac{\sqrt{30}Q_v y z}{20} + \frac{\sqrt{30}Q_{xy} x z}{5} - \frac{\sqrt{30}Q_{xz} x y}{5} + \frac{\sqrt{30}Q_{yz} (y-z)(y+z)}{20}$$

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](B_{3g}, 2)$$

** symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

** expression

$$\frac{\sqrt{6}Q_u y z}{4} - \frac{3\sqrt{2}Q_v y z}{4} + \frac{\sqrt{2}Q_{yz} (2x^2 - y^2 - z^2)}{4}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](B_{3g}, 1)$$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

** expression

$$-\frac{\sqrt{5}Q_u y z (6x^2 - y^2 - z^2)}{4} - \frac{\sqrt{15}Q_v y z (6x^2 - y^2 - z^2)}{12} + \frac{\sqrt{15}Q_{xy} x z (4x^2 - 3y^2 - 3z^2)}{12} \\ - \frac{\sqrt{15}Q_{xz} x y (4x^2 - 3y^2 - 3z^2)}{12} - \frac{\sqrt{15}Q_{yz} (y-z)(y+z)(6x^2 - y^2 - z^2)}{12}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](B_{3g}, 2)$$

** symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

** expression

$$\frac{\sqrt{3}Q_u y z (12x^2 - 9y^2 + 5z^2)}{12} - \frac{Q_v y z (36x^2 + y^2 - 13z^2)}{12} + \frac{7Q_{xy} x z (2x^2 - 3y^2 - z^2)}{12} \\ + \frac{7Q_{xz} x y (2x^2 - y^2 - 3z^2)}{12} - \frac{Q_{yz} (4x^4 - 12x^2 y^2 - 12x^2 z^2 + 5y^4 - 18y^2 z^2 + 5z^4)}{12}$$

* Harmonics for rank 4

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](A_u, 1)$$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2 y^2 - 3x^2 z^2 + y^4 - 3y^2 z^2 + z^4)}{6}$$

** expression

$$\frac{\sqrt{5}Q_{xy} z (x-y)(x+y)}{2} - \frac{\sqrt{5}Q_{xz} y (x-z)(x+z)}{2} + \frac{\sqrt{5}Q_{yz} x (y-z)(y+z)}{2}$$

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](A_u, 2)$$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2 y^2 + 6x^2 z^2 + y^4 + 6y^2 z^2 - 2z^4)}{12}$$

** expression

$$-\frac{6\sqrt{7}Q_v x y z}{7} - \frac{\sqrt{7}Q_{xy} z (x-y)(x+y)}{14} + \frac{\sqrt{7}Q_{xz} y (4x^2 - 3y^2 + 5z^2)}{14} + \frac{\sqrt{7}Q_{yz} x (3x^2 - 4y^2 - 5z^2)}{14}$$

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](A_u, 3)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\frac{6\sqrt{7}Q_uxyz}{7} + \frac{\sqrt{21}Q_{xy}z(3x^2+3y^2-2z^2)}{14} - \frac{\sqrt{21}Q_{xz}y(2x^2-y^2+z^2)}{14} + \frac{\sqrt{21}Q_{yz}x(x^2-2y^2-z^2)}{14}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](A_u, 1)$$

** symmetry

$$\frac{\sqrt{21}(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4)}{6}$$

** expression

$$\begin{aligned} & -\frac{3\sqrt{2310}Q_uxyz(x-y)(x+y)}{44} - \frac{3\sqrt{770}Q_vxyz(x^2+y^2-2z^2)}{44} + \frac{\sqrt{770}Q_{xy}z(x-y)(x+y)(x^2+y^2-2z^2)}{22} \\ & - \frac{\sqrt{770}Q_{xz}y(x-z)(x+z)(x^2-2y^2+z^2)}{22} - \frac{\sqrt{770}Q_{yz}x(y-z)(y+z)(2x^2-y^2-z^2)}{22} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](A_u, 2)$$

** symmetry

$$-\frac{\sqrt{15}(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4)}{12}$$

** expression

$$\begin{aligned} & \frac{21\sqrt{66}Q_uxyz(x-y)(x+y)}{44} - \frac{21\sqrt{22}Q_vxyz(x^2+y^2-2z^2)}{44} + \frac{7\sqrt{22}Q_{xy}z(x-y)(x+y)(x^2+y^2-2z^2)}{44} \\ & + \frac{\sqrt{22}Q_{xz}y(17x^4-22x^2y^2-36x^2z^2+3y^4-8y^2z^2+10z^4)}{44} - \frac{\sqrt{22}Q_{yz}x(3x^4-22x^2y^2-8x^2z^2+17y^4-36y^2z^2+10z^4)}{44} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](A_u, 3)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\begin{aligned} & -\frac{21\sqrt{22}Q_uxyz(x^2+y^2-2z^2)}{44} - \frac{21\sqrt{66}Q_vxyz(x-y)(x+y)}{44} + \frac{\sqrt{66}Q_{xy}z(9x^4-24x^2y^2-10x^2z^2+9y^4-10y^2z^2+2z^4)}{44} \\ & - \frac{\sqrt{66}Q_{xz}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44} - \frac{\sqrt{66}Q_{yz}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](B_{1u}, 1)$$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_vz(x-y)(x+y)}{2} + \sqrt{3}Q_{xy}xyz + \frac{\sqrt{3}Q_{xz}x(x^2-3y^2)}{4} - \frac{\sqrt{3}Q_{yz}y(3x^2-y^2)}{4}$$

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](B_{1u}, 2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$\frac{3\sqrt{7}Q_uz(x-y)(x+y)}{7} + \frac{\sqrt{21}Q_vz(3x^2+3y^2-2z^2)}{14} - \frac{\sqrt{21}Q_{xz}x(x^2-5y^2+2z^2)}{28} - \frac{\sqrt{21}Q_{yz}y(5x^2-y^2-2z^2)}{28}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](B_{1u}, 1)$$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{9\sqrt{154}Q_u z (x^2 - 2xy - y^2) (x^2 + 2xy - y^2)}{88} - \frac{\sqrt{462}Q_v z (x - y) (x + y) (x^2 + y^2 - 2z^2)}{88} + \frac{\sqrt{462}Q_{xy}xyz (x^2 + y^2 - 2z^2)}{44} \\ + \frac{\sqrt{462}Q_{xz}x (x^4 - 8x^2y^2 - 2x^2z^2 + 3y^4 + 6y^2z^2)}{44} + \frac{\sqrt{462}Q_{yz}y (3x^4 - 8x^2y^2 + 6x^2z^2 + y^4 - 2y^2z^2)}{44}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](B_{1u}, 2)$

** symmetry

$$-\frac{\sqrt{5}xy (x^2 + y^2 - 6z^2)}{2}$$

** expression

$$-\frac{21\sqrt{22}Q_u z (x - y) (x + y) (x^2 + y^2 - 2z^2)}{88} - \frac{\sqrt{66}Q_v z (3x^4 - 78x^2y^2 + 20x^2z^2 + 3y^4 + 20y^2z^2 - 4z^4)}{88} - \frac{21\sqrt{66}Q_{xy}xyz (x - y) (x + y)}{44} \\ - \frac{\sqrt{66}Q_{xz}x (x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44} + \frac{\sqrt{66}Q_{yz}y (x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44}$$

$\vec{\mathbb{G}}_4^{(2,-1)}[q](B_{2u}, 1)$

** symmetry

$$-\frac{\sqrt{35}xz (x - z) (x + z)}{2}$$

** expression

$$\frac{3Q_u y (x - z) (x + z)}{4} - \frac{\sqrt{3}Q_v y (x - z) (x + z)}{4} + \frac{\sqrt{3}Q_{xy}x (x^2 - 3z^2)}{4} + \sqrt{3}Q_{xz}xyz - \frac{\sqrt{3}Q_{yz}z (3x^2 - z^2)}{4}$$

$\vec{\mathbb{G}}_4^{(2,-1)}[q](B_{2u}, 2)$

** symmetry

$$-\frac{\sqrt{5}xz (x^2 - 6y^2 + z^2)}{2}$$

** expression

$$\frac{3\sqrt{7}Q_u y (5x^2 - 2y^2 + z^2)}{28} + \frac{\sqrt{21}Q_v y (3x^2 + 2y^2 - 9z^2)}{28} + \frac{\sqrt{21}Q_{xy}x (x^2 + 2y^2 - 5z^2)}{28} + \frac{\sqrt{21}Q_{yz}z (5x^2 - 2y^2 - z^2)}{28}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](B_{2u}, 1)$

** symmetry

$$-\frac{\sqrt{35}xz (x - z) (x + z)}{2}$$

** expression

$$-\frac{3\sqrt{154}Q_u y (x^4 + x^2y^2 - 9x^2z^2 - y^2z^2 + 2z^4)}{88} - \frac{\sqrt{462}Q_v y (5x^4 - x^2y^2 - 27x^2z^2 + y^2z^2 + 4z^4)}{88} \\ + \frac{\sqrt{462}Q_{xy}x (x^4 - 2x^2y^2 - 8x^2z^2 + 6y^2z^2 + 3z^4)}{44} + \frac{\sqrt{462}Q_{xz}xyz (x^2 - 2y^2 + z^2)}{44} + \frac{\sqrt{462}Q_{yz}z (3x^4 + 6x^2y^2 - 8x^2z^2 - 2y^2z^2 + z^4)}{44}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](B_{2u}, 2)$

** symmetry

$$-\frac{\sqrt{5}xz (x^2 - 6y^2 + z^2)}{2}$$

** expression

$$-\frac{3\sqrt{22}Q_u y (5x^4 + 3x^2y^2 - 39x^2z^2 - 2y^4 + 17y^2z^2 - 2z^4)}{88} - \frac{\sqrt{66}Q_v y (9x^4 - 31x^2y^2 + 39x^2z^2 + 2y^4 + 11y^2z^2 - 12z^4)}{88} \\ + \frac{\sqrt{66}Q_{xy}x (x^4 - 12x^2y^2 + 2x^2z^2 + 8y^4 - 12y^2z^2 + z^4)}{44} + \frac{21\sqrt{66}Q_{xz}xyz (x - z) (x + z)}{44} - \frac{\sqrt{66}Q_{yz}z (x^4 - 12x^2y^2 + 2x^2z^2 + 8y^4 - 12y^2z^2 + z^4)}{44}$$

$\vec{\mathbb{G}}_4^{(2,-1)}[q](B_{3u}, 1)$

** symmetry

$$\frac{\sqrt{35}yz (y - z) (y + z)}{2}$$

** expression

$$\frac{3Q_u x (y - z) (y + z)}{4} + \frac{\sqrt{3}Q_v x (y - z) (y + z)}{4} + \frac{\sqrt{3}Q_{xy}y (y^2 - 3z^2)}{4} - \frac{\sqrt{3}Q_{xz}z (3y^2 - z^2)}{4} + \sqrt{3}Q_{yz}xyz$$

$\vec{\mathbb{G}}_4^{(2,-1)}[q](B_{3u}, 2)$

** symmetry

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

** expression

$$\frac{3\sqrt{7}Q_{ux}(2x^2 - 5y^2 - z^2)}{28} + \frac{\sqrt{21}Q_{vx}(2x^2 + 3y^2 - 9z^2)}{28} - \frac{\sqrt{21}Q_{xyy}(2x^2 + y^2 - 5z^2)}{28} + \frac{\sqrt{21}Q_{xzz}(2x^2 - 5y^2 + z^2)}{28}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](B_{3u}, 1)$$

** symmetry

$$\frac{\sqrt{35}yz(y - z)(y + z)}{2}$$

** expression

$$\begin{aligned} & - \frac{3\sqrt{154}Q_{ux}(x^2y^2 - x^2z^2 + y^4 - 9y^2z^2 + 2z^4)}{88} - \frac{\sqrt{462}Q_{vx}(x^2y^2 - x^2z^2 - 5y^4 + 27y^2z^2 - 4z^4)}{88} \\ & - \frac{\sqrt{462}Q_{xyy}(2x^2y^2 - 6x^2z^2 - y^4 + 8y^2z^2 - 3z^4)}{44} + \frac{\sqrt{462}Q_{xzz}(6x^2y^2 - 2x^2z^2 + 3y^4 - 8y^2z^2 + z^4)}{44} - \frac{\sqrt{462}Q_{yzzxyz}(2x^2 - y^2 - z^2)}{44} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](B_{3u}, 2)$$

** symmetry

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

** expression

$$\begin{aligned} & - \frac{3\sqrt{22}Q_{ux}(2x^4 - 3x^2y^2 - 17x^2z^2 - 5y^4 + 39y^2z^2 + 2z^4)}{88} - \frac{\sqrt{66}Q_{vx}(2x^4 - 31x^2y^2 + 11x^2z^2 + 9y^4 + 39y^2z^2 - 12z^4)}{88} \\ & - \frac{\sqrt{66}Q_{xyy}(8x^4 - 12x^2y^2 - 12x^2z^2 + y^4 + 2y^2z^2 + z^4)}{44} + \frac{\sqrt{66}Q_{xzz}(8x^4 - 12x^2y^2 - 12x^2z^2 + y^4 + 2y^2z^2 + z^4)}{44} - \frac{21\sqrt{66}Q_{yzzxyz}(y - z)(y + z)}{44} \end{aligned}$$