

# Response Tensors up to 4th rank in $D_3$

— polar tensors —

$$C^{(0,Q)} = (C^{(0,Q)})$$

$$C^{(0,Q)} = Q_0$$

$$S^{(2,Q)} = \begin{pmatrix} S_{xx}^{(2,Q)} & 0 & 0 \\ 0 & S_{xx}^{(2,Q)} & 0 \\ 0 & 0 & S_{zz}^{(2,Q)} \end{pmatrix}$$

$$S_{xx}^{(2,Q)} = Q_0 - Q_u$$

$$S_{zz}^{(2,Q)} = Q_0 + 2Q_u$$

$$S^{(3,Q)} = \begin{pmatrix} 0 & S_{1y}^{(3,Q)} & 0 \\ 0 & -S_{1y}^{(3,Q)} & 0 \\ 0 & 0 & 0 \\ S_{4x}^{(3,Q)} & 0 & 0 \\ 0 & -S_{4x}^{(3,Q)} & 0 \\ S_{1y}^{(3,Q)} & 0 & 0 \end{pmatrix}$$

$$S_{1y}^{(3,Q)} = Q_3^\beta$$

$$S_{4x}^{(3,Q)} = -3G_u[1]$$

$$A^{(3,Q)} = \begin{pmatrix} A_{4x}^{(3,Q)} & 0 & 0 \\ 0 & A_{4x}^{(3,Q)} & 0 \\ 0 & 0 & A_{6z}^{(3,Q)} \end{pmatrix}$$

$$A_{4x}^{(3,Q)} = G_0 - G_u[2]$$

$$A_{6z}^{(3,Q)} = G_0 + 2G_u[2]$$

$$S^{(4,Q)} = \begin{pmatrix} S_{11}^{(4,Q)} & S_{12}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & S_{15}^{(4,Q)} & 0 \\ S_{12}^{(4,Q)} & S_{11}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & -S_{15}^{(4,Q)} & 0 \\ S_{13}^{(4,Q)} & S_{13}^{(4,Q)} & S_{33}^{(4,Q)} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 & -S_{15}^{(4,Q)} \\ S_{15}^{(4,Q)} & -S_{15}^{(4,Q)} & 0 & 0 & S_{44}^{(4,Q)} & 0 \\ 0 & 0 & 0 & -S_{15}^{(4,Q)} & 0 & \frac{S_{11}^{(4,Q)}}{2} - \frac{S_{12}^{(4,Q)}}{2} \end{pmatrix}$$

$$S_{11}^{(4,Q)} = Q_0[1] + 2Q_0[2] + 3Q_4 - 2Q_u[1] - 4Q_u[2]$$

$$S_{12}^{(4,Q)} = Q_0[1] + Q_4 - 2Q_u[1]$$

$$S_{13}^{(4,Q)} = Q_0[1] - 4Q_4 + Q_u[1]$$

$$S_{15}^{(4,Q)} = Q_4^\alpha$$

$$S_{33}^{(4,Q)} = Q_0[1] + 2Q_0[2] + 8Q_4 + 4Q_u[1] + 8Q_u[2]$$

$$S_{44}^{(4,Q)} = Q_0[2] - 4Q_4 + Q_u[2]$$

$$\bar{S}^{(4,Q)} = \begin{pmatrix} 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & \bar{S}_{15}^{(4,Q)} & 0 \\ 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & -\bar{S}_{15}^{(4,Q)} & 0 \\ -\bar{S}_{13}^{(4,Q)} & -\bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{S}_{15}^{(4,Q)} \\ -\bar{S}_{15}^{(4,Q)} & \bar{S}_{15}^{(4,Q)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\bar{S}_{15}^{(4,Q)} & 0 & 0 \end{pmatrix}$$

$$\bar{S}_{13}^{(4,Q)} = 3Q_u[3]$$

$$\bar{S}_{15}^{(4,Q)} = -2G_3^\beta[1]$$

$$A^{(4,Q)} = \begin{pmatrix} A_{xx}^{(4,Q)} & 0 & 0 \\ 0 & A_{xx}^{(4,Q)} & 0 \\ 0 & 0 & A_{zz}^{(4,Q)} \end{pmatrix}$$

$$A_{xx}^{(4,Q)} = Q_0[3] - 2Q_u[6]$$

$$A_{zz}^{(4,Q)} = Q_0[3] + 4Q_u[6]$$

$$M^{(4,Q)} = \begin{pmatrix} 0 & M_{1y}^{(4,Q)} & 0 \\ 0 & -M_{1y}^{(4,Q)} & 0 \\ 0 & 0 & 0 \\ M_{4x}^{(4,Q)} & 0 & 0 \\ 0 & -M_{4x}^{(4,Q)} & 0 \\ M_{1y}^{(4,Q)} & 0 & 0 \end{pmatrix}$$

$$M_{1y}^{(4,Q)} = G_3^\beta[2]$$

$$M_{4x}^{(4,Q)} = -3Q_u[4]$$

$$\bar{M}^{(4,Q)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}_{x4}^{(4,Q)} & 0 & \bar{M}_{x6}^{(4,Q)} \\ \bar{M}_{x6}^{(4,Q)} & -\bar{M}_{x6}^{(4,Q)} & 0 & 0 & -\bar{M}_{x4}^{(4,Q)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{M}_{x4}^{(4,Q)} = -3Q_u[5]$$

$$\bar{M}_{x6}^{(4,Q)} = G_3^\beta[3]$$

— axial tensors —

$$C^{(0,G)} = (C^{(0,G)})$$

$$C^{(0,G)} = G_0$$

$$S^{(2,G)} = \begin{pmatrix} S_{xx}^{(2,G)} & 0 & 0 \\ 0 & S_{xx}^{(2,G)} & 0 \\ 0 & 0 & S_{zz}^{(2,G)} \end{pmatrix}$$

$$S_{xx}^{(2,G)} = G_0 - G_u$$

$$S_{zz}^{(2,G)} = G_0 + 2G_u$$

$$S^{(3,G)} = \begin{pmatrix} 0 & S_{1y}^{(3,G)} & 0 \\ 0 & -S_{1y}^{(3,G)} & 0 \\ 0 & 0 & 0 \\ S_{4x}^{(3,G)} & 0 & 0 \\ 0 & -S_{4x}^{(3,G)} & 0 \\ S_{1y}^{(3,G)} & 0 & 0 \end{pmatrix}$$

$$S_{1y}^{(3,G)} = G_3^\beta$$

$$S_{4x}^{(3,G)} = -3Q_u[1]$$

$$A^{(3,G)} = \begin{pmatrix} A_{4x}^{(3,G)} & 0 & 0 \\ 0 & A_{4x}^{(3,G)} & 0 \\ 0 & 0 & A_{6z}^{(3,G)} \end{pmatrix}$$

$$A_{4x}^{(3,G)} = Q_0 - Q_u[2]$$

$$A_{6z}^{(3,G)} = Q_0 + 2Q_u[2]$$

$$S^{(4,G)} = \begin{pmatrix} S_{11}^{(4,G)} & S_{12}^{(4,G)} & S_{13}^{(4,G)} & 0 & S_{15}^{(4,G)} & 0 \\ S_{12}^{(4,G)} & S_{11}^{(4,G)} & S_{13}^{(4,G)} & 0 & -S_{15}^{(4,G)} & 0 \\ S_{13}^{(4,G)} & S_{13}^{(4,G)} & S_{33}^{(4,G)} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^{(4,G)} & 0 & -S_{15}^{(4,G)} \\ S_{15}^{(4,G)} & -S_{15}^{(4,G)} & 0 & 0 & S_{44}^{(4,G)} & 0 \\ 0 & 0 & 0 & -S_{15}^{(4,G)} & 0 & \frac{S_{11}^{(4,G)}}{2} - \frac{S_{12}^{(4,G)}}{2} \end{pmatrix}$$

$$S_{11}^{(4,G)} = G_0[1] + 2G_0[2] + 3G_4 - 2G_u[1] - 4G_u[2]$$

$$S_{12}^{(4,G)} = G_0[1] + G_4 - 2G_u[1]$$

$$S_{13}^{(4,G)} = G_0[1] - 4G_4 + G_u[1]$$

$$S_{15}^{(4,G)} = G_4^\alpha$$

$$S_{33}^{(4,G)} = G_0[1] + 2G_0[2] + 8G_4 + 4G_u[1] + 8G_u[2]$$

$$S_{44}^{(4,G)} = G_0[2] - 4G_4 + G_u[2]$$

$$\bar{S}^{(4,G)} = \begin{pmatrix} 0 & 0 & \bar{S}_{13}^{(4,G)} & 0 & \bar{S}_{15}^{(4,G)} & 0 \\ 0 & 0 & \bar{S}_{13}^{(4,G)} & 0 & -\bar{S}_{15}^{(4,G)} & 0 \\ -\bar{S}_{13}^{(4,G)} & -\bar{S}_{13}^{(4,G)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{S}_{15}^{(4,G)} \\ -\bar{S}_{15}^{(4,G)} & \bar{S}_{15}^{(4,G)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\bar{S}_{15}^{(4,G)} & 0 & 0 \end{pmatrix}$$

$$\bar{S}_{13}^{(4,G)} = 3G_u[3]$$

$$\bar{S}_{15}^{(4,G)} = -2Q_3^\beta[1]$$

$$A^{(4,G)} = \begin{pmatrix} A_{xx}^{(4,G)} & 0 & 0 \\ 0 & A_{xx}^{(4,G)} & 0 \\ 0 & 0 & A_{zz}^{(4,G)} \end{pmatrix}$$

$$A_{xx}^{(4,G)} = G_0[3] - 2G_u[6]$$

$$A_{zz}^{(4,G)} = G_0[3] + 4G_u[6]$$

$$M^{(4,G)} = \begin{pmatrix} 0 & M_{1y}^{(4,G)} & 0 \\ 0 & -M_{1y}^{(4,G)} & 0 \\ 0 & 0 & 0 \\ M_{4x}^{(4,G)} & 0 & 0 \\ 0 & -M_{4x}^{(4,G)} & 0 \\ M_{1y}^{(4,G)} & 0 & 0 \end{pmatrix}$$

$$M_{1y}^{(4,G)} = Q_3^\beta[2]$$

$$M_{4x}^{(4,G)} = -3G_u[4]$$

$$\bar{M}^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}_{x4}^{(4,G)} & 0 & \bar{M}_{x6}^{(4,G)} \\ \bar{M}_{x6}^{(4,G)} & -\bar{M}_{x6}^{(4,G)} & 0 & 0 & -\bar{M}_{x4}^{(4,G)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{M}_{x4}^{(4,G)} = -3G_u[5]$$

$$\bar{M}_{x6}^{(4,G)} = Q_3^\beta[3]$$