

MPG No. 17.3.64 3' [ Type III, trigonal ] [M tensor]

\* Rank 0 tensor.

$$[M]$$

$$M = M_s^{(1)}$$

\* Rank 1 tensor. \* Rank 2 tensor (s).

$$\begin{bmatrix} M_{xx} & 0 & 0 \\ 0 & M_{xx} & 0 \\ 0 & 0 & M_{zz} \end{bmatrix}$$

$$M_{xx} = -M_{du}^{(1)} + M_s^{(1)}$$

$$M_{zz} = 2M_{du}^{(1)} + M_s^{(1)}$$

\* Rank 2 tensor (a).

$$\begin{bmatrix} 0 & M_{xy} & 0 \\ -M_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{xy} = T_{pz}^{(1)}$$

\* Rank 3 tensor (s). \* Rank 3 tensor (a). \* Rank 4 tensor (sss).

$$\begin{bmatrix} M_{xxxx} & M_{xxyy} & M_{xxzz} & M_{xxyz} & M_{xxzx} & 0 \\ M_{xxyy} & M_{xxxx} & M_{xxzz} & -M_{xxyz} & -M_{xxzx} & 0 \\ M_{xxzz} & M_{xxzz} & M_{zzzz} & 0 & 0 & 0 \\ M_{xxyz} & -M_{xxyz} & 0 & M_{yzyz} & 0 & -M_{xxzx} \\ M_{xxzx} & -M_{xxzx} & 0 & 0 & M_{yzyz} & M_{xxyz} \\ 0 & 0 & 0 & -M_{xxzx} & M_{xxyz} & \frac{M_{xxxx}}{2} - \frac{M_{xxyy}}{2} \end{bmatrix}$$

$$M_{xxxx} = -2M_{du}^{(1)} - 4M_{du}^{(2)} + 3M_{g0}^{(1)} + M_s^{(1)} + 2M_s^{(2)}$$

$$M_{xxyy} = -2M_{du}^{(1)} + M_{g0}^{(1)} + M_s^{(1)}$$

$$M_{xxzz} = M_{du}^{(1)} - 4M_{g0}^{(1)} + M_s^{(1)}$$

$$M_{xxyz} = M_{gb}^{(1)}$$

$$M_{xxzx} = M_{ga}^{(1)}$$

$$M_{zzzz} = 4M_{du}^{(1)} + 8M_{du}^{(2)} + 8M_{g0}^{(1)} + M_s^{(1)} + 2M_s^{(2)}$$

$$M_{yzyz} = M_{du}^{(2)} - 4M_{g0}^{(1)} + M_s^{(2)}$$

\* Rank 4 tensor (ssa).

$$\begin{bmatrix} 0 & 0 & M_{xxzz} & M_{xxyz} & M_{xxzx} & M_{xxyy} \\ 0 & 0 & M_{xxzz} & -M_{xxyz} & -M_{xxzx} & -M_{xxyy} \\ -M_{xxzz} & -M_{xxzz} & 0 & 0 & 0 & 0 \\ -M_{xxyz} & M_{xxyz} & 0 & 0 & M_{yzzx} & M_{xxzx} \\ -M_{xxzx} & M_{xxzx} & 0 & -M_{yzzx} & 0 & -M_{xxyz} \\ -M_{xxyy} & M_{xxyy} & 0 & -M_{xxzx} & M_{xxyz} & 0 \end{bmatrix}$$

$$M_{xxzz} = 3M_{du}^{(3)}$$

$$M_{xxyz} = 2T_{f2}^{(1)}$$

$$M_{xxzx} = -2T_{f1}^{(1)}$$

$$M_{xxyy} = -2T_{faz}^{(1)} + 2T_{pz}^{(1)}$$

$$M_{yzzx} = -4T_{faz}^{(1)} - T_{pz}^{(1)}$$

\* Rank 4 tensor (aas).

$$\begin{bmatrix} M_{yzyz} & 0 & 0 \\ 0 & M_{yzyz} & 0 \\ 0 & 0 & M_{xyxy} \end{bmatrix}$$

$$M_{yzyz} = -2M_{du}^{(4)} + M_s^{(3)}$$

$$M_{xyxy} = 4M_{du}^{(4)} + M_s^{(3)}$$

\* Rank 4 tensor (aaa).

$$\begin{bmatrix} 0 & M_{yzxx} & 0 \\ -M_{yzxx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{yzxx} = -T_{pz}^{(2)}$$

\* Rank 4 tensor (sa).

$$\begin{bmatrix} M_{xxyz} & M_{xxzx} & M_{xxxz} \\ -M_{xxyz} & -M_{xxzx} & M_{zzxy} \\ 0 & 0 & M_{zzxy} \\ M_{yzyz} & M_{yzxx} & 0 \\ M_{yzxx} & -M_{yzyz} & 0 \\ M_{xxzx} & -M_{xxyz} & 0 \end{bmatrix}$$

$$M_{xxyz} = T_{f2}^{(2)}$$

$$M_{xxzx} = T_{f1}^{(2)}$$

$$M_{xxxz} = -T_{faz}^{(2)} + T_{pz}^{(3)}$$

$$M_{zzxy} = 2T_{faz}^{(2)} + T_{pz}^{(3)} + 2T_{pz}^{(4)}$$

$$M_{yzyz} = -3M_{du}^{(5)}$$

$$M_{yzxx} = -T_{faz}^{(2)} + T_{pz}^{(4)}$$

\* Rank 4 tensor (as).

$$\begin{bmatrix} M_{yzxx} & -M_{yzxx} & 0 & M_{yzyz} & M_{yzxx} & M_{yzxy} \\ M_{yzxy} & -M_{yzxy} & 0 & M_{yzxx} & -M_{yzyz} & -M_{yzxx} \\ M_{xyxx} & M_{xyxx} & M_{xyzz} & 0 & 0 & 0 \end{bmatrix}$$

$$M_{yzxx} = T_{f2}^{(3)}$$

$$M_{yzyz} = -3M_{du}^{(6)}$$

$$M_{yzxx} = -T_{faz}^{(3)} + T_{pz}^{(6)}$$

$$M_{yzxy} = T_{f1}^{(3)}$$

$$M_{xyxx} = -T_{faz}^{(3)} + T_{pz}^{(5)}$$

$$M_{xyzz} = 2T_{faz}^{(3)} + T_{pz}^{(5)} + 2T_{pz}^{(6)}$$

\* Rank 4 tensor (s).

$$\begin{bmatrix} M_{xxxx} & M_{xxyy} & M_{xxzz} & M_{xxyz} & M_{xxzx} & M_{xxxy} & M_{xxzy} & M_{xxxz} & M_{xxyx} \\ M_{xxyy} & M_{xxxx} & M_{xxzz} & -M_{xxyz} & -M_{xxzx} & -M_{xxxy} & -M_{xxzy} & -M_{xxxz} & -M_{xxyx} \\ M_{zzxx} & M_{zzxx} & M_{zzzz} & 0 & 0 & M_{zzxy} & 0 & 0 & -M_{zzxy} \\ M_{yzzx} & -M_{yzxx} & 0 & M_{yzyz} & M_{yzxx} & M_{yzyy} & M_{yzzz} & M_{yzxx} & M_{yzxy} \\ -M_{yzxy} & M_{yzxy} & 0 & -M_{yzxx} & M_{yzyy} & M_{yzxx} & -M_{yzzz} & M_{yzyz} & M_{yzxx} \\ -\frac{M_{xxyy}}{2} - \frac{M_{xxxy}}{2} & \frac{M_{xxyy}}{2} + \frac{M_{xxxy}}{2} & 0 & -M_{xxzx} & M_{xxzy} & \frac{M_{xxxx} - M_{xxyy}}{2} & -M_{xxzx} & M_{xxyz} & \frac{M_{xxxx} - M_{xxyy}}{2} \end{bmatrix}$$

$$\begin{aligned}
M_{xxxx} &= -2M_{du}^{(1)} - 4M_{du}^{(2)} + 3M_{g0}^{(1)} + M_s^{(1)} + 2M_s^{(2)} \\
M_{xxyy} &= -2M_{du}^{(1)} + M_{g0}^{(1)} + M_s^{(1)} \\
M_{xxzz} &= M_{du}^{(1)} + 3M_{du}^{(3)} - 4M_{g0}^{(1)} + M_s^{(1)} \\
M_{xxyz} &= M_{gb}^{(1)} + 2T_{f2}^{(1)} + T_{f2}^{(2)} \\
M_{xxzx} &= M_{ga}^{(1)} - 2T_{f1}^{(1)} + T_{f1}^{(2)} \\
M_{xxxz} &= -2T_{faz}^{(1)} - T_{faz}^{(2)} + 2T_{pz}^{(1)} + T_{pz}^{(3)} \\
M_{xxzy} &= M_{gb}^{(1)} + 2T_{f2}^{(1)} - T_{f2}^{(2)} \\
M_{xxxz} &= M_{ga}^{(1)} - 2T_{f1}^{(1)} - T_{f1}^{(2)} \\
M_{xxyx} &= -2T_{faz}^{(1)} + T_{faz}^{(2)} + 2T_{pz}^{(1)} - T_{pz}^{(3)} \\
M_{zzxx} &= M_{du}^{(1)} - 3M_{du}^{(3)} - 4M_{g0}^{(1)} + M_s^{(1)} \\
M_{zzzz} &= 4M_{du}^{(1)} + 8M_{du}^{(2)} + 8M_{g0}^{(1)} + M_s^{(1)} + 2M_s^{(2)} \\
M_{zzxy} &= 2T_{faz}^{(2)} + T_{pz}^{(3)} + 2T_{pz}^{(4)} \\
M_{yzxx} &= M_{gb}^{(1)} - 2T_{f2}^{(1)} \\
M_{yzyz} &= M_{du}^{(2)} - 3M_{du}^{(5)} - 4M_{g0}^{(1)} + M_s^{(2)} \\
M_{yzzx} &= -4T_{faz}^{(1)} - T_{faz}^{(2)} - T_{pz}^{(1)} + T_{pz}^{(4)} \\
M_{yzxy} &= -M_{ga}^{(1)} - 2T_{f1}^{(1)} \\
M_{yzyy} &= M_{du}^{(2)} + 3M_{du}^{(5)} - 4M_{g0}^{(1)} + M_s^{(2)} \\
M_{yzxz} &= -4T_{faz}^{(1)} + T_{faz}^{(2)} - T_{pz}^{(1)} - T_{pz}^{(4)}
\end{aligned}$$

\* Rank 4 tensor (a).

$$\begin{bmatrix}
M_{yzxx} & -M_{yzxx} & 0 & M_{yzyz} & M_{yzzx} & M_{yzxy} & M_{yzyz} & M_{yzzx} & M_{yzyy} \\
M_{yzxy} & -M_{yzxy} & 0 & M_{yzzx} & -M_{yzyz} & -M_{yzxx} & M_{yzzx} & -M_{yzyz} & -M_{yzxx} \\
M_{xyxx} & M_{xyxx} & M_{xyzz} & 0 & 0 & M_{xyxy} & 0 & 0 & -M_{xyxy}
\end{bmatrix}$$

$$\begin{aligned}
M_{yzxx} &= T_{f2}^{(3)} \\
M_{yzyz} &= -2M_{du}^{(4)} - 3M_{du}^{(6)} + M_s^{(3)} \\
M_{yzzx} &= -T_{faz}^{(3)} - T_{pz}^{(2)} + T_{pz}^{(6)} \\
M_{yzxy} &= T_{f1}^{(3)} \\
M_{yzyy} &= 2M_{du}^{(4)} - 3M_{du}^{(6)} - M_s^{(3)} \\
M_{yzxz} &= -T_{faz}^{(3)} + T_{pz}^{(2)} + T_{pz}^{(6)} \\
M_{xyxx} &= -T_{faz}^{(3)} + T_{pz}^{(5)} \\
M_{xyzz} &= 2T_{faz}^{(3)} + T_{pz}^{(5)} + 2T_{pz}^{(6)} \\
M_{xyxy} &= 4M_{du}^{(4)} + M_s^{(3)}
\end{aligned}$$

\* Rank 4 tensor (t).

$$\left[ \begin{array}{ccc}
M_{xxxx} & 0 & M_{xxxx} \\
0 & M_{xxxx} & M_{yyyz} \\
0 & 0 & M_{zzzz} \\
-M_{xxxx} & M_{yyyy} & M_{yyzz} \\
M_{zzxx} & 0 & 0 \\
0 & M_{xxyy} & -M_{yyyz} \\
0 & M_{yzyy} & 0 \\
M_{xxxz} & -M_{yyyz} & -\frac{M_{xxxx}}{2} + \frac{3M_{xxyy}}{2} + M_{yyzz} - M_{yzyy} + M_{zzxx} \\
\frac{M_{xxxx}}{2} - \frac{M_{xxyy}}{2} & 0 & -M_{xxxx} \\
-M_{yyyz} & -M_{xxxx} & 0
\end{array} \right]$$

$$M_{xxxx} = -2M_{du}^{(1)} - 4M_{du}^{(2)} + 3M_{g0}^{(1)} + 3M_s^{(1)}$$

$$M_{xxxz} = M_{ga}^{(1)}$$

$$M_{yyyz} = -M_{gb}^{(1)}$$

$$M_{zzzz} = 4M_{du}^{(1)} + 8M_{du}^{(2)} + 8M_{g0}^{(1)} + 3M_s^{(1)}$$

$$M_{yyzz} = M_{du}^{(1)} + 3M_{du}^{(3)} - 4M_{g0}^{(1)} + M_s^{(1)}$$

$$M_{zzxx} = M_{du}^{(1)} - 3M_{du}^{(3)} - 4M_{g0}^{(1)} + M_s^{(1)}$$

$$M_{xxyy} = -2M_{du}^{(1)} + M_{g0}^{(1)} + M_s^{(1)}$$

$$M_{yzyy} = M_{du}^{(2)} + 3M_{du}^{(5)} - 4M_{g0}^{(1)} + M_s^{(1)}$$