

MPG No. 9.3.31 4' [Type III, tetragonal] [Q tensor]

* Rank 0 tensor.

$$[Q]$$

$$Q = Q_s^{(1)}$$

* Rank 1 tensor.

$$[0 \ 0 \ Q_z]$$

$$Q_z = Q_{pz}^{(1)}$$

* Rank 2 tensor (s).

$$\begin{bmatrix} Q_{xx} & 0 & 0 \\ 0 & Q_{xx} & 0 \\ 0 & 0 & Q_{zz} \end{bmatrix}$$

$$Q_{xx} = -Q_{du}^{(1)} + Q_s^{(1)}$$

$$Q_{zz} = 2Q_{du}^{(1)} + Q_s^{(1)}$$

* Rank 2 tensor (a).

$$\begin{bmatrix} 0 & Q_{xy} & 0 \\ -Q_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Q_{xy} = G_{pz}^{(1)}$$

* Rank 3 tensor (s).

$$\begin{bmatrix} 0 & 0 & Q_{xxz} \\ 0 & 0 & Q_{xxz} \\ 0 & 0 & Q_{zzz} \\ Q_{yzx} & Q_{yzy} & 0 \\ Q_{yzy} & -Q_{yzx} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Q_{xxz} = -Q_{faz}^{(1)} + Q_{pz}^{(1)}$$

$$Q_{zzz} = 2Q_{faz}^{(1)} + Q_{pz}^{(1)} + 2Q_{pz}^{(2)}$$

$$Q_{yzx} = -3G_{du}^{(1)}$$

$$Q_{yzy} = -Q_{faz}^{(1)} + Q_{pz}^{(2)}$$

* Rank 3 tensor (a).

$$\begin{bmatrix} Q_{yzx} & Q_{yzy} & 0 \\ -Q_{yzy} & Q_{yzx} & 0 \\ 0 & 0 & Q_{xyz} \end{bmatrix}$$

$$Q_{yzx} = -G_{du}^{(2)} + G_s^{(1)}$$

$$Q_{yzy} = Q_{pz}^{(3)}$$

$$Q_{xyz} = 2G_{du}^{(2)} + G_s^{(1)}$$

* Rank 4 tensor (sss).

$$\begin{bmatrix} Q_{xxxx} & Q_{xxyy} & Q_{xxzz} & 0 & 0 & Q_{xxxz} \\ Q_{xxyy} & Q_{xxxx} & Q_{xxzz} & 0 & 0 & -Q_{xxxz} \\ Q_{xxzz} & Q_{xxzz} & Q_{zzzz} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{yzyz} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{yzyz} & 0 \\ Q_{xxxz} & -Q_{xxxz} & 0 & 0 & 0 & Q_{xyxy} \end{bmatrix}$$

$$Q_{xxxx} = -2Q_{du}^{(1)} - 4Q_{du}^{(2)} - Q_{gu}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)}$$

$$Q_{xxyy} = -2Q_{du}^{(1)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{xxzz} = Q_{du}^{(1)} - Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{xxxz} = Q_{gaz}^{(1)}$$

$$Q_{zzzz} = 4Q_{du}^{(1)} + 8Q_{du}^{(2)} + 2Q_{gu}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)}$$

$$Q_{yzyz} = Q_{du}^{(2)} - Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(2)}$$

$$Q_{xyxy} = -2Q_{du}^{(2)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(2)}$$

* Rank 4 tensor (ssa).

$$\begin{bmatrix} 0 & 0 & Q_{xxzz} & 0 & 0 & Q_{xxxz} \\ 0 & 0 & Q_{xxzz} & 0 & 0 & -Q_{xxxz} \\ -Q_{xxzz} & -Q_{xxzz} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{yzzx} & 0 \\ 0 & 0 & 0 & -Q_{yzzx} & 0 & 0 \\ -Q_{xxxz} & Q_{xxxz} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Q_{xxzz} = 3Q_{du}^{(3)}$$

$$Q_{xxxz} = -2G_{faz}^{(1)} + 2G_{pz}^{(1)}$$

$$Q_{yzzx} = -4G_{faz}^{(1)} - G_{pz}^{(1)}$$

* Rank 4 tensor (aas).

$$\begin{bmatrix} Q_{yzyz} & 0 & 0 \\ 0 & Q_{yzyz} & 0 \\ 0 & 0 & Q_{xyxy} \end{bmatrix}$$

$$Q_{yzyz} = -2Q_{du}^{(4)} + Q_s^{(3)}$$

$$Q_{xyxy} = 4Q_{du}^{(4)} + Q_s^{(3)}$$

* Rank 4 tensor (aaa).

$$\begin{bmatrix} 0 & Q_{yzzx} & 0 \\ -Q_{yzzx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Q_{yzzx} = -G_{pz}^{(2)}$$

* Rank 4 tensor (sa).

$$\begin{bmatrix} 0 & 0 & Q_{xxxz} \\ 0 & 0 & Q_{xxxz} \\ 0 & 0 & Q_{zzxy} \\ Q_{yzyz} & Q_{yzzx} & 0 \\ Q_{yzzx} & -Q_{yzyz} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}
Q_{xxxy} &= -G_{faz}^{(2)} + G_{pz}^{(3)} \\
Q_{zzyx} &= 2G_{faz}^{(2)} + G_{pz}^{(3)} + 2G_{pz}^{(4)} \\
Q_{yzyz} &= -3Q_{du}^{(5)} \\
Q_{yzzx} &= -G_{faz}^{(2)} + G_{pz}^{(4)}
\end{aligned}$$

* Rank 4 tensor (as).

$$\begin{bmatrix} 0 & 0 & 0 & Q_{yzyz} & Q_{yzzx} & 0 \\ 0 & 0 & 0 & Q_{yzzx} & -Q_{yzyz} & 0 \\ Q_{xyxx} & Q_{xyxx} & Q_{xyzz} & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}
Q_{yzyz} &= -3Q_{du}^{(6)} \\
Q_{yzzx} &= -G_{faz}^{(3)} + G_{pz}^{(6)} \\
Q_{xyxx} &= -G_{faz}^{(3)} + G_{pz}^{(5)} \\
Q_{xyzz} &= 2G_{faz}^{(3)} + G_{pz}^{(5)} + 2G_{pz}^{(6)}
\end{aligned}$$

* Rank 4 tensor (s).

$$\begin{bmatrix} Q_{xxxx} & Q_{xxyy} & Q_{xxzz} & 0 & 0 & Q_{xxxy} & 0 & 0 & Q_{xxyx} \\ Q_{xxyy} & Q_{xxxx} & Q_{xxzz} & 0 & 0 & -Q_{xxyx} & 0 & 0 & -Q_{xxxxy} \\ Q_{zzxx} & Q_{zzxx} & Q_{zzzz} & 0 & 0 & Q_{zzxy} & 0 & 0 & -Q_{zzxy} \\ 0 & 0 & 0 & Q_{yzyz} & Q_{yzzx} & 0 & Q_{yzzy} & Q_{yzxz} & 0 \\ 0 & 0 & 0 & -Q_{yzxz} & Q_{yzzy} & 0 & -Q_{yzxz} & Q_{yzyz} & 0 \\ Q_{xyxx} & -Q_{xyxx} & 0 & 0 & 0 & Q_{xyxy} & 0 & 0 & Q_{xyxy} \end{bmatrix}$$

$$\begin{aligned}
Q_{xxxx} &= -2Q_{du}^{(1)} - 4Q_{du}^{(2)} - Q_{gu}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)} \\
Q_{xxyy} &= -2Q_{du}^{(1)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)} \\
Q_{xxzz} &= Q_{du}^{(1)} + 3Q_{du}^{(3)} - Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)} \\
Q_{xxxy} &= -2G_{faz}^{(1)} - G_{faz}^{(2)} + 2G_{pz}^{(1)} + G_{pz}^{(3)} + Q_{gaz}^{(1)} \\
Q_{xxyx} &= -2G_{faz}^{(1)} + G_{faz}^{(2)} + 2G_{pz}^{(1)} - G_{pz}^{(3)} + Q_{gaz}^{(1)} \\
Q_{zzxx} &= Q_{du}^{(1)} - 3Q_{du}^{(3)} - Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)} \\
Q_{zzzz} &= 4Q_{du}^{(1)} + 8Q_{du}^{(2)} + 2Q_{gu}^{(1)} + 2Q_g^{(1)} + Q_s^{(1)} + 2Q_s^{(2)} \\
Q_{zzxy} &= 2G_{faz}^{(2)} + G_{pz}^{(3)} + 2G_{pz}^{(4)} \\
Q_{yzyz} &= Q_{du}^{(2)} - 3Q_{du}^{(5)} - Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(2)} \\
Q_{yzzx} &= -4G_{faz}^{(1)} - G_{faz}^{(2)} - G_{pz}^{(1)} + G_{pz}^{(4)} \\
Q_{yzyy} &= Q_{du}^{(2)} + 3Q_{du}^{(5)} - Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(2)} \\
Q_{yzxz} &= -4G_{faz}^{(1)} + G_{faz}^{(2)} - G_{pz}^{(1)} - G_{pz}^{(4)} \\
Q_{xyxx} &= 2G_{faz}^{(1)} - 2G_{pz}^{(1)} + Q_{gaz}^{(1)} \\
Q_{xyxy} &= -2Q_{du}^{(2)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(2)}
\end{aligned}$$

* Rank 4 tensor (a).

$$\begin{bmatrix} 0 & 0 & 0 & Q_{yzyz} & Q_{yzzx} & 0 & Q_{yzzy} & Q_{yzxz} & 0 \\ 0 & 0 & 0 & Q_{yzzx} & -Q_{yzzy} & 0 & Q_{yzzy} & -Q_{yzyz} & 0 \\ Q_{xyxx} & Q_{xyxx} & Q_{xyzz} & 0 & 0 & Q_{xyxy} & 0 & 0 & -Q_{xyxy} \end{bmatrix}$$

$$Q_{yzyz} = -2Q_{du}^{(4)} - 3Q_{du}^{(6)} + Q_s^{(3)}$$

$$Q_{yzzx} = -G_{faz}^{(3)} - G_{pz}^{(2)} + G_{pz}^{(6)}$$

$$Q_{yzyy} = 2Q_{du}^{(4)} - 3Q_{du}^{(6)} - Q_s^{(3)}$$

$$Q_{yzxz} = -G_{faz}^{(3)} + G_{pz}^{(2)} + G_{pz}^{(6)}$$

$$Q_{xyxx} = -G_{faz}^{(3)} + G_{pz}^{(5)}$$

$$Q_{xyzz} = 2G_{faz}^{(3)} + G_{pz}^{(5)} + 2G_{pz}^{(6)}$$

$$Q_{xyxy} = 4Q_{du}^{(4)} + Q_s^{(3)}$$

* Rank 4 tensor (t).

$$\begin{bmatrix} Q_{xxxx} & Q_{xxxz} & 0 \\ -Q_{xxxy} & Q_{xxxx} & 0 \\ 0 & 0 & Q_{zzzz} \\ 0 & 0 & Q_{yyzz} \\ Q_{zzxx} & 0 & 0 \\ Q_{xxyy} & Q_{xxyy} & 0 \\ 0 & Q_{yzyy} & 0 \\ 0 & 0 & Q_{zxxz} \\ Q_{xxyy} + Q_{yyzz} - Q_{yzyy} - Q_{zzxx} + Q_{zzxx} & -Q_{xxxy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Q_{xxxx} = -2Q_{du}^{(1)} - 4Q_{du}^{(2)} - Q_{gu}^{(1)} + 2Q_g^{(1)} + 3Q_s^{(1)}$$

$$Q_{xxxy} = Q_{gaz}^{(1)}$$

$$Q_{zzzz} = 4Q_{du}^{(1)} + 8Q_{du}^{(2)} + 2Q_{gu}^{(1)} + 2Q_g^{(1)} + 3Q_s^{(1)}$$

$$Q_{yyzz} = Q_{du}^{(1)} + 3Q_{du}^{(3)} - Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{zzxx} = Q_{du}^{(1)} - 3Q_{du}^{(3)} - Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{xxyy} = -2Q_{du}^{(1)} + 2Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{yzyy} = Q_{du}^{(2)} + 3Q_{du}^{(5)} - Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$

$$Q_{zxxz} = Q_{du}^{(2)} - 3Q_{du}^{(5)} - Q_{gu}^{(1)} - Q_g^{(1)} + Q_s^{(1)}$$