

PG No. 30 O 432 [cubic]

Table 1 Harmonics for rank 0.

No.	multipole	expression
1	$\mathbb{Q}_0(A_1)$	1

Table 2 Harmonics for rank 1.

No.	multipole	expression
2	$\mathbb{Q}_{1,1}(T_1)$	x
3	$\mathbb{Q}_{1,2}(T_1)$	y
4	$\mathbb{Q}_{1,3}(T_1)$	z

Table 3 Harmonics for rank 2.

No.	multipole	expression
5	$\mathbb{Q}_{2,1}(E)$	$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$
6	$\mathbb{Q}_{2,2}(E)$	$\frac{\sqrt{3}(x-y)(x+y)}{2}$
7	$\mathbb{Q}_{2,1}(T_2)$	$\sqrt{3}yz$
8	$\mathbb{Q}_{2,2}(T_2)$	$\sqrt{3}xz$
9	$\mathbb{Q}_{2,3}(T_2)$	$\sqrt{3}xy$

Table 4 Harmonics for rank 3.

No.	multipole	expression
10	$\mathbb{Q}_3(A_2)$	$\sqrt{15}xyz$
11	$\mathbb{Q}_{3,1}(T_1)$	$\frac{x(2x^2-3y^2-3z^2)}{2}$
12	$\mathbb{Q}_{3,2}(T_1)$	$-\frac{y(3x^2-2y^2+3z^2)}{2}$
13	$\mathbb{Q}_{3,3}(T_1)$	$-\frac{z(3x^2+3y^2-2z^2)}{2}$
14	$\mathbb{Q}_{3,1}(T_2)$	$\frac{\sqrt{15}x(y-z)(y+z)}{2}$
15	$\mathbb{Q}_{3,2}(T_2)$	$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$
16	$\mathbb{Q}_{3,3}(T_2)$	$\frac{\sqrt{15}z(x-y)(x+y)}{2}$

Table 5 Harmonics for rank 4.

No.	multipole	expression
17	$\mathbb{Q}_4(A_1)$	$\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$
18	$\mathbb{Q}_{4,1}(E)$	$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$
19	$\mathbb{Q}_{4,2}(E)$	$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$
20	$\mathbb{Q}_{4,1}(T_1)$	$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$
21	$\mathbb{Q}_{4,2}(T_1)$	$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$
22	$\mathbb{Q}_{4,3}(T_1)$	$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$
23	$\mathbb{Q}_{4,1}(T_2)$	$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$
24	$\mathbb{Q}_{4,2}(T_2)$	$-\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$
25	$\mathbb{Q}_{4,3}(T_2)$	$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$

Table 6 Harmonics for rank 5.

No.	multipole	expression
26	$\mathbb{Q}_{5,1}(E)$	$\frac{3\sqrt{35}xyz(x-y)(x+y)}{2}$
27	$\mathbb{Q}_{5,2}(E)$	$\frac{\sqrt{105}xyz(x^2 + y^2 - 2z^2)}{2}$
28	$\mathbb{Q}_{5,1}(T_1, 1)$	$\frac{x(8x^4 - 40x^2y^2 - 40x^2z^2 + 15y^4 + 30y^2z^2 + 15z^4)}{8}$
29	$\mathbb{Q}_{5,2}(T_1, 1)$	$\frac{y(15x^4 - 40x^2y^2 + 30x^2z^2 + 8y^4 - 40y^2z^2 + 15z^4)}{8}$
30	$\mathbb{Q}_{5,3}(T_1, 1)$	$\frac{z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{8}$
31	$\mathbb{Q}_{5,1}(T_1, 2)$	$\frac{3\sqrt{35}x(y^2 - 2yz - z^2)(y^2 + 2yz - z^2)}{8}$
32	$\mathbb{Q}_{5,2}(T_1, 2)$	$\frac{3\sqrt{35}y(x^2 - 2xz - z^2)(x^2 + 2xz - z^2)}{8}$
33	$\mathbb{Q}_{5,3}(T_1, 2)$	$\frac{3\sqrt{35}z(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$
34	$\mathbb{Q}_{5,1}(T_2)$	$\frac{\sqrt{105}x(y-z)(y+z)(2x^2 - y^2 - z^2)}{4}$
35	$\mathbb{Q}_{5,2}(T_2)$	$\frac{\sqrt{105}y(x-z)(x+z)(x^2 - 2y^2 + z^2)}{4}$
36	$\mathbb{Q}_{5,3}(T_2)$	$-\frac{\sqrt{105}z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{4}$

Table 7 Harmonics for rank 6.

No.	multipole	expression
37	$\mathbb{Q}_6(A_1)$	$\frac{\sqrt{2}(2x^6 - 15x^4y^2 - 15x^4z^2 - 15x^2y^4 + 180x^2y^2z^2 - 15x^2z^4 + 2y^6 - 15y^4z^2 - 15y^2z^4 + 2z^6)}{8}$
38	$\mathbb{Q}_6(A_2)$	$-\frac{\sqrt{2310}(x-y)(x+y)(x-z)(x+z)(y-z)(y+z)}{8}$
39	$\mathbb{Q}_{6,1}(E)$	$-\frac{\sqrt{14}(x^6 - 15x^4z^2 + 15x^2z^4 + y^6 - 15y^4z^2 + 15y^2z^4 - 2z^6)}{8}$
40	$\mathbb{Q}_{6,2}(E)$	$\frac{\sqrt{42}(x-y)(x+y)(x^4 - 9x^2y^2 - 5x^2z^2 + y^4 - 5y^2z^2 + 5z^4)}{8}$
41	$\mathbb{Q}_{6,1}(T_1)$	$\frac{3\sqrt{7}yz(y-z)(y+z)(10x^2 - y^2 - z^2)}{4}$
42	$\mathbb{Q}_{6,2}(T_1)$	$\frac{3\sqrt{7}xz(x-z)(x+z)(x^2 - 10y^2 + z^2)}{4}$
43	$\mathbb{Q}_{6,3}(T_1)$	$-\frac{3\sqrt{7}xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{4}$
44	$\mathbb{Q}_{6,1}(T_2, 1)$	$\frac{\sqrt{462}yz(y^2 - 3z^2)(3y^2 - z^2)}{16}$
45	$\mathbb{Q}_{6,2}(T_2, 1)$	$\frac{\sqrt{462}xz(x^2 - 3z^2)(3x^2 - z^2)}{16}$
46	$\mathbb{Q}_{6,3}(T_2, 1)$	$\frac{\sqrt{462}xy(x^2 - 3y^2)(3x^2 - y^2)}{16}$
47	$\mathbb{Q}_{6,1}(T_2, 2)$	$\frac{\sqrt{210}yz(16x^4 - 16x^2y^2 - 16x^2z^2 + y^4 + 2y^2z^2 + z^4)}{16}$
48	$\mathbb{Q}_{6,2}(T_2, 2)$	$\frac{\sqrt{210}xz(x^4 - 16x^2y^2 + 2x^2z^2 + 16y^4 - 16y^2z^2 + z^4)}{16}$
49	$\mathbb{Q}_{6,3}(T_2, 2)$	$\frac{\sqrt{210}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{16}$

Table 8 Harmonics for rank 7.

No.	multipole	expression
50	$\mathbb{Q}_7(A_2)$	$\frac{\sqrt{91}xyz(3x^4 - 5x^2y^2 - 5x^2z^2 + 3y^4 - 5y^2z^2 + 3z^4)}{2}$
51	$\mathbb{Q}_{7,1}(E)$	$-\frac{\sqrt{231}xyz(x-y)(x+y)(3x^2 + 3y^2 - 10z^2)}{4}$
52	$\mathbb{Q}_{7,2}(E)$	$-\frac{\sqrt{77}xyz(3x^4 - 20x^2y^2 + 10x^2z^2 + 3y^4 + 10y^2z^2 - 6z^4)}{4}$
53	$\mathbb{Q}_{7,1}(T_1, 1)$	$\frac{x(16x^6 - 168x^4y^2 - 168x^4z^2 + 210x^2y^4 + 420x^2y^2z^2 + 210x^2z^4 - 35y^6 - 105y^4z^2 - 105y^2z^4 - 35z^6)}{16}$
54	$\mathbb{Q}_{7,2}(T_1, 1)$	$-\frac{y(35x^6 - 210x^4y^2 + 105x^4z^2 + 168x^2y^4 - 420x^2y^2z^2 + 105x^2z^4 - 16y^6 + 168y^4z^2 - 210y^2z^4 + 35z^6)}{16}$
55	$\mathbb{Q}_{7,3}(T_1, 1)$	$-\frac{z(35x^6 + 105x^4y^2 - 210x^4z^2 + 105x^2y^4 - 420x^2y^2z^2 + 168x^2z^4 + 35y^6 - 210y^4z^2 + 168y^2z^4 - 16z^6)}{16}$
56	$\mathbb{Q}_{7,1}(T_1, 2)$	$\frac{\sqrt{231}x(10x^2 - 3y^2 - 3z^2)(y^2 - 2yz - z^2)(y^2 + 2yz - z^2)}{16}$
57	$\mathbb{Q}_{7,2}(T_1, 2)$	$-\frac{\sqrt{231}y(x^2 - 2xz - z^2)(x^2 + 2xz - z^2)(3x^2 - 10y^2 + 3z^2)}{16}$
58	$\mathbb{Q}_{7,3}(T_1, 2)$	$-\frac{\sqrt{231}z(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)(3x^2 + 3y^2 - 10z^2)}{16}$
59	$\mathbb{Q}_{7,1}(T_2, 1)$	$\frac{\sqrt{6006}x(y-z)(y+z)(y^2 - 4yz + z^2)(y^2 + 4yz + z^2)}{32}$
60	$\mathbb{Q}_{7,2}(T_2, 1)$	$-\frac{\sqrt{6006}y(x-z)(x+z)(x^2 - 4xz + z^2)(x^2 + 4xz + z^2)}{32}$
61	$\mathbb{Q}_{7,3}(T_2, 1)$	$\frac{\sqrt{6006}z(x-y)(x+y)(x^2 - 4xy + y^2)(x^2 + 4xy + y^2)}{32}$
62	$\mathbb{Q}_{7,1}(T_2, 2)$	$\frac{\sqrt{42}x(y-z)(y+z)(48x^4 - 80x^2y^2 - 80x^2z^2 + 15y^4 + 30y^2z^2 + 15z^4)}{32}$
63	$\mathbb{Q}_{7,2}(T_2, 2)$	$-\frac{\sqrt{42}y(x-z)(x+z)(15x^4 - 80x^2y^2 + 30x^2z^2 + 48y^4 - 80y^2z^2 + 15z^4)}{32}$
64	$\mathbb{Q}_{7,3}(T_2, 2)$	$\frac{\sqrt{42}z(x-y)(x+y)(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{32}$

Table 9 Harmonics for rank 8.

No.	multipole	expression
65	$\mathbb{Q}_8(A_1)$	$\frac{\sqrt{33}(x^8 - 14x^6y^2 - 14x^6z^2 + 35x^4y^4 + 35x^4z^4 - 14x^2y^6 - 14x^2z^6 + y^8 - 14y^6z^2 + 35y^4z^4 - 14y^2z^6 + z^8)}{8}$
66	$\mathbb{Q}_{8,1}(E, 1)$	$\frac{\sqrt{286}(x^8 - 14x^6y^2 - 14x^6z^2 + 210x^4y^2z^2 - 14x^2y^6 + 210x^2y^4z^2 - 420x^2y^2z^4 + 28x^2z^6 + y^8 - 14y^6z^2 + 28y^2z^6 - 2z^8)}{64}$
67	$\mathbb{Q}_{8,2}(E, 1)$	$-\frac{\sqrt{858}(x-y)(x+y)(x^2+y^2-14z^2)(x^2-4xy+y^2)(x^2+4xy+y^2)}{64}$
68	$\mathbb{Q}_{8,1}(E, 2)$	$\frac{\sqrt{210}(x^8 - 62x^6y^2 + 34x^6z^2 + 160x^4y^4 - 30x^4y^2z^2 - 80x^4z^4 - 62x^2y^6 - 30x^2y^4z^2 + 60x^2y^2z^4 + 28x^2z^6 + y^8 + 34y^6z^2 - 80y^4z^4 + 28y^2z^6 - 2z^8)}{64}$
69	$\mathbb{Q}_{8,2}(E, 2)$	$-\frac{3\sqrt{70}(x-y)(x+y)(x^6+3x^4y^2-30x^4z^2+3x^2y^4-60x^2y^2z^2+80x^2z^4+y^6-30y^4z^2+80y^2z^4-32z^6)}{64}$
70	$\mathbb{Q}_{8,1}(T_1, 1)$	$\frac{3\sqrt{715}yz(y-z)(y+z)(y^2-2yz-z^2)(y^2+2yz-z^2)}{16}$
71	$\mathbb{Q}_{8,2}(T_1, 1)$	$-\frac{3\sqrt{715}xz(x-z)(x+z)(x^2-2xz-z^2)(x^2+2xz-z^2)}{16}$
72	$\mathbb{Q}_{8,3}(T_1, 1)$	$\frac{3\sqrt{715}xy(x-y)(x+y)(x^2-2xy-y^2)(x^2+2xy-y^2)}{16}$
73	$\mathbb{Q}_{8,1}(T_1, 2)$	$\frac{3\sqrt{77}yz(y-z)(y+z)(40x^4-24x^2y^2-24x^2z^2+y^4+2y^2z^2+z^4)}{16}$
74	$\mathbb{Q}_{8,2}(T_1, 2)$	$-\frac{3\sqrt{77}xz(x-z)(x+z)(x^4-24x^2y^2+2x^2z^2+40y^4-24y^2z^2+z^4)}{16}$
75	$\mathbb{Q}_{8,3}(T_1, 2)$	$\frac{3\sqrt{77}xy(x-y)(x+y)(x^4+2x^2y^2-24x^2z^2+y^4-24y^2z^2+40z^4)}{16}$
76	$\mathbb{Q}_{8,1}(T_2, 1)$	$\frac{\sqrt{858}yz(y^2-3z^2)(3y^2-z^2)(14x^2-y^2-z^2)}{32}$
77	$\mathbb{Q}_{8,2}(T_2, 1)$	$-\frac{\sqrt{858}xz(x^2-3z^2)(3x^2-z^2)(x^2-14y^2+z^2)}{32}$
78	$\mathbb{Q}_{8,3}(T_2, 1)$	$-\frac{\sqrt{858}xy(x^2-3y^2)(3x^2-y^2)(x^2+y^2-14z^2)}{32}$
79	$\mathbb{Q}_{8,1}(T_2, 2)$	$\frac{3\sqrt{70}yz(32x^6-80x^4y^2-80x^4z^2+30x^2y^4+60x^2y^2z^2+30x^2z^4-y^6-3y^4z^2-3y^2z^4-z^6)}{32}$
80	$\mathbb{Q}_{8,2}(T_2, 2)$	$-\frac{3\sqrt{70}xz(x^6-30x^4y^2+3x^4z^2+80x^2y^4-60x^2y^2z^2+3x^2z^4-32y^6+80y^4z^2-30y^2z^4+z^6)}{32}$
81	$\mathbb{Q}_{8,3}(T_2, 2)$	$-\frac{3\sqrt{70}xy(x^6+3x^4y^2-30x^4z^2+3x^2y^4-60x^2y^2z^2+80x^2z^4+y^6-30y^4z^2+80y^2z^4-32z^6)}{32}$

Table 10 Harmonics for rank 9.

No.	multipole	expression
82	$\mathbb{Q}_9(A_1)$	$\frac{-\sqrt{5}10510xyz(x-y)(x+y)(x-z)(x+z)(y-z)(y+z)}{8}$
83	$\mathbb{Q}_9(A_2)$	$\frac{\sqrt{330}xyz(6x^6-21x^4y^2-21x^4z^2-21x^2y^4+140x^2y^2z^2-21x^2z^4+6y^6-21y^4z^2-21y^2z^4+6z^6)}{8}$
84	$\mathbb{Q}_{9,1}(E)$	$\frac{\sqrt{4290}xyz(x-y)(x+y)(3x^4-11x^2y^2-7x^2z^2+3y^4-7y^2z^2+7z^4)}{8}$
85	$\mathbb{Q}_{9,2}(E)$	$\frac{3\sqrt{1430}xyz(x^6-7x^4z^2+7x^2z^4+y^6-7y^4z^2+7y^2z^4-2z^6)}{8}$
86	$\mathbb{Q}_{9,1}(T_1, 1)$	$\frac{x(128x^8-2304x^6y^2-2304x^6z^2+6048x^4y^4+12096x^4y^2z^2+6048x^4z^4-3360x^2y^6-10080x^2y^4z^2-10080x^2y^2z^4-3360x^2z^6+315y^8+1260y^6z^2+1890y^4z^4+1260y^2z^6+315z^8)}{128}$
87	$\mathbb{Q}_{9,2}(T_1, 1)$	$\frac{y(315x^8-3360x^6y^2+1260x^6z^2+6048x^4y^4-10080x^4y^2z^2+1890x^4z^4-2304x^2y^6+12096x^2y^4z^2-10080x^2y^2z^4+1260x^2z^6+128y^8-2304y^6z^2+6048y^4z^4-3360y^2z^6+315z^8)}{128}$
88	$\mathbb{Q}_{9,3}(T_1, 1)$	$\frac{z(315x^8+1260x^6y^2-3360x^6z^2+1890x^4y^4-10080x^4y^2z^2+6048x^4z^4+1260x^2y^6-10080x^2y^4z^2+12096x^2y^2z^4-2304x^2z^6+315y^8-3360y^6z^2+6048y^4z^4-2304y^2z^6+128z^8)}{128}$
89	$\mathbb{Q}_{9,1}(T_1, 2)$	$\frac{3\sqrt{12155}x(y^4-4y^3z-6y^2z^2+4yz^3+z^4)(y^4+4y^3z-6y^2z^2-4yz^3+z^4)}{128}$
90	$\mathbb{Q}_{9,2}(T_1, 2)$	$\frac{3\sqrt{12155}y(x^4-4x^3z-6x^2z^2+4xz^3+z^4)(x^4+4x^3z-6x^2z^2-4xz^3+z^4)}{128}$
91	$\mathbb{Q}_{9,3}(T_1, 2)$	$\frac{3\sqrt{12155}z(x^4-4x^3y-6x^2y^2+4xy^3+y^4)(x^4+4x^3y-6x^2y^2-4xy^3+y^4)}{128}$
92	$\mathbb{Q}_{9,1}(T_1, 3)$	$\frac{3\sqrt{5005}x(y^2-2yz-z^2)(y^2+2yz-z^2)(8x^4-8x^2y^2-8x^2z^2+y^4+2y^2z^2+z^4)}{64}$
93	$\mathbb{Q}_{9,2}(T_1, 3)$	$\frac{3\sqrt{5005}y(x^2-2xz-z^2)(x^2+2xz-z^2)(x^4-8x^2y^2+2x^2z^2+8y^4-8y^2z^2+z^4)}{64}$
94	$\mathbb{Q}_{9,3}(T_1, 3)$	$\frac{3\sqrt{5005}z(x^2-2xy-y^2)(x^2+2xy-y^2)(x^4+2x^2y^2-8x^2z^2+y^4-8y^2z^2+8z^4)}{64}$
95	$\mathbb{Q}_{9,1}(T_2, 1)$	$\frac{\sqrt{4290}x(y-z)(y+z)(14x^2-3y^2-3z^2)(y^2-4yz+z^2)(y^2+4yz+z^2)}{64}$
96	$\mathbb{Q}_{9,2}(T_2, 1)$	$\frac{\sqrt{4290}y(x-z)(x+z)(x^2-4xz+z^2)(x^2+4xz+z^2)(3x^2-14y^2+3z^2)}{64}$
97	$\mathbb{Q}_{9,3}(T_2, 1)$	$-\frac{\sqrt{4290}z(x-y)(x+y)(x^2-4xy+y^2)(x^2+4xy+y^2)(3x^2+3y^2-14z^2)}{64}$
98	$\mathbb{Q}_{9,1}(T_2, 2)$	$\frac{3\sqrt{110}x(y-z)(y+z)(32x^6-112x^4y^2-112x^4z^2+70x^2y^4+140x^2y^2z^2+70x^2z^4-7y^6-21y^4z^2-21y^2z^4-7z^6)}{64}$
99	$\mathbb{Q}_{9,2}(T_2, 2)$	$\frac{3\sqrt{110}y(x-z)(x+z)(7x^6-70x^4y^2+21x^4z^2+112x^2y^4-140x^2y^2z^2+21x^2z^4-32y^6+112y^4z^2-70y^2z^4+7z^6)}{64}$
100	$\mathbb{Q}_{9,3}(T_2, 2)$	$-\frac{3\sqrt{110}z(x-y)(x+y)(7x^6+21x^4y^2-70x^4z^2+21x^2y^4-140x^2y^2z^2+112x^2z^4+7y^6-70y^4z^2+112y^2z^4-32z^6)}{64}$

Table 11 Harmonics for rank 10.

No.	multipole	expression
101	$\mathbb{Q}_{10}(A_1)$	$\frac{\sqrt{390}(2x^{10}-45x^8y^2-45x^8z^2+42x^6y^4+1008x^6y^2z^2+42x^6z^4+42x^4y^6-1260x^4y^4z^2-1260x^4y^2z^4+42x^4z^6-45x^2y^8+1008x^2y^6z^2-1260x^2y^4z^4+1008x^2y^2z^6-45x^2z^8+2y^{10}-45y^8z^2+42y^6z^4+42y^4z^6-45y^2z^8+2z^{10})}{96}$
102	$\mathbb{Q}_{10}(A_2)$	$-\frac{\sqrt{27170}(x-y)(x+y)(x-z)(x+z)(y-z)(y+z)(3x^4-11x^2y^2-11x^2z^2+3y^4-11y^2z^2+3z^4)}{32}$
103	$\mathbb{Q}_{10,1}(E, 1)$	$-\frac{\sqrt{420189}(22x^{10}+45x^8y^2-1035x^8z^2-105x^6y^4-630x^6y^2z^2+4935x^6z^4-105x^4y^6+3150x^4y^4z^2-1575x^4y^2z^4-4830x^4z^6+45x^2y^8-630x^2y^6z^2-1575x^2y^4z^4+1260x^2y^2z^6+990x^2z^8+22y^{10}-1035y^8z^2+4935y^6z^4-4830y^4z^6+990y^2z^8-44z^{10})}{35952}$
104	$\mathbb{Q}_{10,2}(E, 1)$	$\frac{\sqrt{140063}(x-y)(x+y)(22x^8-653x^6y^2-315x^6z^2+2602x^4y^4-945x^4y^2z^2+1575x^4z^4-653x^2y^6-945x^2y^4z^2+3150x^2y^2z^4-1680x^2z^6+22y^8-315y^6z^2+1575y^4z^4-1680y^2z^6+360z^8)}{11984}$
105	$\mathbb{Q}_{10,1}(E, 2)$	$-\frac{\sqrt{3213210}(3x^{10}-96x^8y^2-39x^8z^2+224x^6y^4+1344x^6y^2z^2-42x^6z^4+224x^4y^6-6720x^4y^4z^2+3360x^4y^2z^4-182x^4z^6-96x^2y^8+1344x^2y^6z^2+3360x^2y^4z^4-2688x^2y^2z^6+135x^2z^8+3y^{10}-39y^8z^2-42y^6z^4-182y^4z^6+135y^2z^8-6z^{10})}{23968}$
106	$\mathbb{Q}_{10,2}(E, 2)$	$\frac{\sqrt{1071070}(x-y)(x+y)(9x^8-165x^6y^2-231x^6z^2-25x^4y^4+3801x^4y^2z^2+406x^4z^4-165x^2y^6+3801x^2y^4z^2-9674x^2y^2z^4+266x^2z^6+9y^8-231y^6z^2+406y^4z^4+266y^2z^6-57z^8)}{23968}$
107	$\mathbb{Q}_{10,1}(T_1, 1)$	$\frac{\sqrt{12155}yz(y-z)(y+z)(18x^2-y^2-z^2)(y^2-2yz-z^2)(y^2+2yz-z^2)}{32}$
108	$\mathbb{Q}_{10,2}(T_1, 1)$	$\frac{\sqrt{12155}xz(x-z)(x+z)(x^2-18y^2+z^2)(x^2-2xz-z^2)(x^2+2xz-z^2)}{32}$
109	$\mathbb{Q}_{10,3}(T_1, 1)$	$-\frac{\sqrt{12155}xy(x-y)(x+y)(x^2-18z^2)(x^2-2xy-y^2)(x^2+2xy-y^2)}{32}$
110	$\mathbb{Q}_{10,1}(T_1, 2)$	$\frac{\sqrt{2145}yz(y-z)(y+z)(112x^6-168x^4y^2-168x^4z^2+42x^2y^4+84x^2y^2z^2+42x^2z^4-y^6-3y^4z^2-3y^2z^4-z^6)}{32}$
111	$\mathbb{Q}_{10,2}(T_1, 2)$	$\frac{\sqrt{2145}xz(x-z)(x+z)(x^6-42x^4y^2+3x^4z^2+168x^2y^4-84x^2y^2z^2+3x^2z^4-112y^6+168y^4z^2-42y^2z^4+z^6)}{32}$
112	$\mathbb{Q}_{10,3}(T_1, 2)$	$-\frac{\sqrt{2145}xy(x-y)(x+y)(x^6+3x^4y^2-42x^4z^2+3x^2y^4-84x^2y^2z^2+168x^2z^4+y^6-42y^4z^2+168y^2z^4-112z^6)}{32}$
113	$\mathbb{Q}_{10,1}(T_2, 1)$	$\frac{\sqrt{92378}yz(y^4-10y^2z^2+5z^4)(5y^4-10y^2z^2+z^4)}{256}$
114	$\mathbb{Q}_{10,2}(T_2, 1)$	$\frac{\sqrt{92378}xz(x^4-10x^2z^2+5z^4)(5x^4-10x^2z^2+z^4)}{256}$
115	$\mathbb{Q}_{10,3}(T_2, 1)$	$\frac{\sqrt{92378}xy(x^4-10x^2y^2+5y^4)(5x^4-10x^2y^2+y^4)}{256}$
116	$\mathbb{Q}_{10,1}(T_2, 2)$	$\frac{\sqrt{4290}yz(y^2-3z^2)(3y^2-z^2)(224x^4-96x^2y^2-96x^2z^2+3y^4+6y^2z^2+3z^4)}{256}$
117	$\mathbb{Q}_{10,2}(T_2, 2)$	$\frac{\sqrt{4290}xz(x^2-3z^2)(3x^2-z^2)(3x^4-96x^2y^2+6x^2z^2+224y^4-96y^2z^2+3z^4)}{256}$
118	$\mathbb{Q}_{10,3}(T_2, 2)$	$\frac{\sqrt{4290}xy(x^2-3y^2)(3x^2-y^2)(3x^4+6x^2y^2-96x^2z^2+3y^4-96y^2z^2+224z^4)}{256}$
119	$\mathbb{Q}_{10,1}(T_2, 3)$	$\frac{\sqrt{165}yz(384x^8-1792x^6y^2-1792x^6z^2+1680x^4y^4+3360x^4y^2z^2+1680x^4z^4-336x^2y^6-1008x^2y^4z^2-1008x^2y^2z^4-336x^2z^6+7y^8+28y^6z^2+42y^4z^4+28y^2z^6+7z^8)}{128}$
120	$\mathbb{Q}_{10,2}(T_2, 3)$	$\frac{\sqrt{165}xz(7x^8-336x^6y^2+28x^6z^2+1680x^4y^4-1008x^4y^2z^2+42x^4z^4-1792x^2y^6+3360x^2y^4z^2-1008x^2y^2z^4+28x^2z^6+384y^8-1792y^6z^2+1680y^4z^4-336y^2z^6+7z^8)}{128}$
121	$\mathbb{Q}_{10,3}(T_2, 3)$	$\frac{\sqrt{165}xy(7x^8+28x^6y^2-336x^6z^2+42x^4y^4-1008x^4y^2z^2+1680x^4z^4+28x^2y^6-1008x^2y^2z^4+3360x^2y^2z^4-1792x^2z^6+7y^8-336y^6z^2+1680y^4z^4-1792y^2z^6+384z^8)}{128}$

Table 12 Harmonics for rank 11.

No.	multipole	expression
122	$\mathbb{Q}_{11}(A_2)$	$\frac{\sqrt{243} \bar{1} x y z (5 x^8 - 30 x^6 y^2 - 30 x^6 z^2 + 63 x^4 y^4 + 63 x^4 z^4 - 30 x^2 y^6 - 30 x^2 z^6 + 5 y^8 - 30 y^6 z^2 + 63 y^4 z^4 - 30 y^2 z^6 + 5 z^8)}{8}$
123	$\mathbb{Q}_{11,1}(E, 1)$	$-\frac{\sqrt{692835} x y z (x - y) (x + y) (x^2 + y^2 - 6 z^2) (x^2 - 2 x y - y^2) (x^2 + 2 x y - y^2)}{32}$
124	$\mathbb{Q}_{11,2}(E, 1)$	$-\frac{\sqrt{230945} x y z (x^8 - 6 x^6 y^2 - 6 x^6 z^2 + 42 x^4 y^2 z^2 - 6 x^2 y^6 + 42 x^2 y^4 z^2 - 84 x^2 y^2 z^4 + 12 x^2 z^6 + y^8 - 6 y^6 z^2 + 12 y^2 z^6 - 2 z^8)}{32}$
125	$\mathbb{Q}_{11,1}(E, 2)$	$-\frac{3 \sqrt{1001} x y z (x - y) (x + y) (5 x^6 + 15 x^4 y^2 - 70 x^4 z^2 + 15 x^2 y^4 - 140 x^2 y^2 z^2 + 168 x^2 z^4 + 5 y^6 - 70 y^4 z^2 + 168 y^2 z^4 - 80 z^6)}{32}$
126	$\mathbb{Q}_{11,2}(E, 2)$	$-\frac{\sqrt{3003} x y z (5 x^8 - 150 x^6 y^2 + 90 x^6 z^2 + 336 x^4 y^4 - 70 x^4 y^2 z^2 - 168 x^4 z^4 - 150 x^2 y^6 - 70 x^2 y^4 z^2 + 140 x^2 y^2 z^4 + 60 x^2 z^6 + 5 y^8 + 90 y^6 z^2 - 168 y^4 z^4 + 60 y^2 z^6 - 10 z^8)}{32}$
127	$\mathbb{Q}_{11,1}(T_1, 1)$	$\frac{x (256 x^{10} - 7040 x^8 y^2 - 7040 x^8 z^2 + 31680 x^6 y^4 + 63360 x^6 y^2 z^2 + 31680 x^6 z^4 - 36960 x^4 y^6 - 110880 x^4 y^4 z^2 - 110880 x^4 y^2 z^4 - 36960 x^4 z^6 + 11550 x^2 y^8 + 46200 x^2 y^6 z^2 + 69300 x^2 y^4 z^4 + 46200 x^2 y^2 z^6 + 11550 x^2 z^8 - 693 y^{10} - 3465 y^8 z^2 - 6930 y^6 z^4 - 6930 y^4 z^6 - 3465 y^2 z^8 - 693 z^{10})}{256}$
128	$\mathbb{Q}_{11,2}(T_1, 1)$	$-\frac{y (693 x^{10} - 11550 x^8 y^2 + 3465 x^8 z^2 + 36960 x^6 y^4 - 46200 x^6 y^2 z^2 + 6930 x^6 z^4 - 31680 x^4 y^6 + 110880 x^4 y^4 z^2 - 69300 x^4 y^2 z^4 + 6930 x^4 z^6 + 7040 x^2 y^8 - 63360 x^2 y^6 z^2 + 110880 x^2 y^4 z^4 - 46200 x^2 y^2 z^6 + 3465 x^2 z^8 - 256 y^{10} + 7040 y^8 z^2 - 31680 y^6 z^4 + 36960 y^4 z^6 - 11550 y^2 z^8 + 693 z^{10})}{256}$
129	$\mathbb{Q}_{11,3}(T_1, 1)$	$-\frac{z (693 x^{10} + 3465 x^8 y^2 - 11550 x^8 z^2 + 6930 x^6 y^4 - 46200 x^6 y^2 z^2 + 36960 x^6 z^4 + 6930 x^4 y^6 - 69300 x^4 y^4 z^2 + 110880 x^4 y^2 z^4 - 31680 x^4 z^6 + 3465 x^2 y^8 - 46200 x^2 y^6 z^2 + 110880 x^2 y^4 z^4 - 63360 x^2 y^2 z^6 + 7040 x^2 z^8 + 693 y^{10} - 11550 y^8 z^2 + 36960 y^6 z^4 - 31680 y^4 z^6 + 7040 y^2 z^8 - 256 z^{10})}{256}$
130	$\mathbb{Q}_{11,1}(T_1, 2)$	$\frac{\sqrt{692835} x (6 x^2 - y^2 - z^2) (y^4 - 4 y^3 z - 6 y^2 z^2 + 4 y z^3 + z^4) (y^4 + 4 y^3 z - 6 y^2 z^2 - 4 y z^3 + z^4)}{256}$
131	$\mathbb{Q}_{11,2}(T_1, 2)$	$-\frac{\sqrt{692835} y (x^2 - 6 y^2 + z^2) (x^4 - 4 x^3 z - 6 x^2 z^2 + 4 x z^3 + z^4) (x^4 + 4 x^3 z - 6 x^2 z^2 - 4 x z^3 + z^4)}{256}$
132	$\mathbb{Q}_{11,3}(T_1, 2)$	$-\frac{\sqrt{692835} z (x^2 + y^2 - 6 z^2) (x^4 - 4 x^3 y - 6 x^2 y^2 + 4 x y^3 + y^4) (x^4 + 4 x^3 y - 6 x^2 y^2 - 4 x y^3 + y^4)}{256}$
133	$\mathbb{Q}_{11,1}(T_1, 3)$	$\frac{3 \sqrt{1001} x (y^2 - 2 y z - z^2) (y^2 + 2 y z - z^2) (80 x^6 - 168 x^4 y^2 - 168 x^4 z^2 + 70 x^2 y^4 + 140 x^2 y^2 z^2 + 70 x^2 z^4 - 5 y^6 - 15 y^4 z^2 - 15 y^2 z^4 - 5 z^6)}{128}$
134	$\mathbb{Q}_{11,2}(T_1, 3)$	$-\frac{3 \sqrt{1001} y (x^2 - 2 x z - z^2) (x^2 + 2 x z - z^2) (5 x^6 - 70 x^4 y^2 + 15 x^4 z^2 + 168 x^2 y^4 - 140 x^2 y^2 z^2 + 15 x^2 z^4 - 80 y^6 + 168 y^4 z^2 - 70 y^2 z^4 + 5 z^6)}{128}$
135	$\mathbb{Q}_{11,3}(T_1, 3)$	$-\frac{3 \sqrt{1001} z (x^2 - 2 x y - y^2) (x^2 + 2 x y - y^2) (5 x^6 + 15 x^4 y^2 - 70 x^4 z^2 + 15 x^2 y^4 - 140 x^2 y^2 z^2 + 168 x^2 z^4 + 5 y^6 - 70 y^4 z^2 + 168 y^2 z^4 - 80 z^6)}{128}$
136	$\mathbb{Q}_{11,1}(T_2, 1)$	$\frac{\sqrt{1939938} x (y - z) (y + z) (y^4 - 4 y^3 z - 14 y^2 z^2 - 4 y z^3 + z^4) (y^4 + 4 y^3 z - 14 y^2 z^2 + 4 y z^3 + z^4)}{512}$
137	$\mathbb{Q}_{11,2}(T_2, 1)$	$-\frac{\sqrt{1939938} y (x - z) (x + z) (x^4 - 4 x^3 z - 14 x^2 z^2 - 4 x z^3 + z^4) (x^4 + 4 x^3 z - 14 x^2 z^2 + 4 x z^3 + z^4)}{512}$
138	$\mathbb{Q}_{11,3}(T_2, 1)$	$\frac{\sqrt{1939938} z (x - y) (x + y) (x^4 - 4 x^3 y - 14 x^2 y^2 - 4 x y^3 + y^4) (x^4 + 4 x^3 y - 14 x^2 y^2 + 4 x y^3 + y^4)}{512}$
139	$\mathbb{Q}_{11,1}(T_2, 2)$	$\frac{\sqrt{14586} x (y - z) (y + z) (y^2 - 4 y z + z^2) (y^2 + 4 y z + z^2) (224 x^4 - 160 x^2 y^2 - 160 x^2 z^2 + 15 y^4 + 30 y^2 z^2 + 15 z^4)}{512}$
140	$\mathbb{Q}_{11,2}(T_2, 2)$	$-\frac{\sqrt{14586} y (x - z) (x + z) (x^2 - 4 x z + z^2) (x^2 + 4 x z + z^2) (15 x^4 - 160 x^2 y^2 + 30 x^2 z^2 + 224 y^4 - 160 y^2 z^2 + 15 z^4)}{512}$
141	$\mathbb{Q}_{11,3}(T_2, 2)$	$\frac{\sqrt{14586} z (x - y) (x + y) (x^2 - 4 x y + y^2) (x^2 + 4 x y + y^2) (15 x^4 + 30 x^2 y^2 - 160 x^2 z^2 + 15 y^4 - 160 y^2 z^2 + 224 z^4)}{512}$
142	$\mathbb{Q}_{11,1}(T_2, 3)$	$\frac{\sqrt{2145} x (y - z) (y + z) (128 x^8 - 768 x^6 y^2 - 768 x^6 z^2 + 1008 x^4 y^4 + 2016 x^4 y^2 z^2 + 1008 x^4 z^4 - 336 x^2 y^6 - 1008 x^2 y^4 z^2 - 1008 x^2 y^2 z^4 - 336 x^2 z^6 + 21 y^8 + 84 y^6 z^2 + 126 y^4 z^4 + 84 y^2 z^6 + 21 z^8)}{256}$
143	$\mathbb{Q}_{11,2}(T_2, 3)$	$-\frac{\sqrt{2145} y (x - z) (x + z) (21 x^8 - 336 x^6 y^2 + 84 x^6 z^2 + 1008 x^4 y^4 - 1008 x^4 y^2 z^2 + 126 x^4 z^4 - 768 x^2 y^6 + 2016 x^2 y^4 z^2 - 1008 x^2 y^2 z^4 + 84 x^2 z^6 + 128 y^8 - 768 y^6 z^2 + 1008 y^4 z^4 - 336 y^2 z^6 + 21 z^8)}{256}$
144	$\mathbb{Q}_{11,3}(T_2, 3)$	$\frac{\sqrt{2145} z (x - y) (x + y) (21 x^8 + 84 x^6 y^2 - 336 x^6 z^2 + 126 x^4 y^4 - 1008 x^4 y^2 z^2 + 1008 x^4 z^4 + 84 x^2 y^6 - 1008 x^2 y^4 z^2 + 2016 x^2 y^2 z^4 - 768 x^2 z^6 + 21 y^8 - 336 y^6 z^2 + 1008 y^4 z^4 - 768 y^2 z^6 + 128 z^8)}{256}$