# SAMB for "GaAs"

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- Generation condition
  - model type: tight\_binding
  - time-reversal type: electric
  - irrep: [A1]
  - spinless
- Unit cell:

$$a=1.0,\ b=1.0,\ c=1.0,\ \alpha=90.0,\ \beta=90.0,\ \gamma=90.0$$

- Lattice vectors:
  - $\boldsymbol{a}_1 = \begin{pmatrix} 1.0 & 0 & 0 \end{pmatrix}$

$$\mathbf{a}_2 = \begin{pmatrix} 0 & 1.0 & 0 \end{pmatrix}$$

$$\mathbf{a}_3 = \begin{pmatrix} 0 & 0 & 1.0 \end{pmatrix}$$

- Plus sets:
  - $+(0 \quad 0 \quad 0)$
  - $+\begin{pmatrix}0&\frac{1}{2}&\frac{1}{2}\end{pmatrix}$

  - $+\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \\ +\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$

Table 1: High-symmetry line:  $\Gamma$ -X.

symbol	position	symbol	position
Γ	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	X	$\begin{pmatrix} \frac{1}{2} & 0 & 0 \end{pmatrix}$

### • Kets: dimension = 6

Table 2: Hilbert space for full matrix.

No.	ket	No.	ket	No.	ket	No.	ket	No.	ket
1	$p_x@\mathrm{Ga}_1$	2	$p_y@{\mathrm{Ga}_1}$	3	$p_z@\mathrm{Ga}_1$	4	$p_x@\mathrm{As}_1$	5	$p_y@As_1$
6	$p_z@\mathrm{As}_1$								

### • Sites in (primitive) unit cell:

Table 3: Site-clusters.

	site	position	mapping
S <sub>1</sub> [4a: -43m]	$Ga_1$	$\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$	[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24]
S <sub>2</sub> [4c: -43m]	$As_1$	$     \begin{pmatrix}       \frac{1}{4} & \frac{1}{4} & \frac{1}{4}     \end{pmatrix} $	[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24]

## • Bonds in (primitive) unit cell:

Table 4: Bond-clusters.

	bond	tail	head	n	#	b@c	mapping
B <sub>1</sub> [16e: .3m]	$b_1$	$As_1$	$Ga_1$	1	1	$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} @ \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$	[1,5,9,16,17,18]
	$b_2$	$As_1$	$Ga_1$	1	1	$\begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix} @ \begin{pmatrix} \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{pmatrix}$	[2,6,11,13,21,23]
	$b_3$	$As_1$	$Ga_1$	1	1	$\left(\frac{1}{4} - \frac{1}{4} - \frac{1}{4}\right) @ \left(\frac{1}{8} - \frac{3}{8} - \frac{3}{8}\right)$	$[3,\!7,\!12,\!15,\!19,\!24]$
	$b_4$	$As_1$	$Ga_1$	1	1	$\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{pmatrix} @ \begin{pmatrix} \frac{3}{8} & \frac{1}{8} & \frac{3}{8} \end{pmatrix}$	[4,8,10,14,20,22]

#### • SAMB:

No. 1 
$$\hat{\mathbb{Q}}_0^{(A_1)}$$
 [M<sub>1</sub>, S<sub>1</sub>]

$$\hat{\mathbb{Z}}_1 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_1[\mathbb{Q}_0^{(s,A_1)}]$$

$$\hat{\mathbb{Z}}_1(\mathbf{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_1[\mathbb{Q}_0^{(s,A_1)}]$$

No. 2 
$$\hat{\mathbb{Q}}_0^{(A_1)}$$
 [M<sub>1</sub>, S<sub>2</sub>]

$$\hat{\mathbb{Z}}_2 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_2[\mathbb{Q}_0^{(s,A_1)}]$$

$$\hat{\mathbb{Z}}_2(\boldsymbol{k}) = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{U}_2[\mathbb{Q}_0^{(s,A_1)}]$$

No. 3 
$$\hat{\mathbb{Q}}_0^{(A_1)}$$
 [M<sub>1</sub>, B<sub>1</sub>]

$$\hat{\mathbb{Z}}_3 = \mathbb{X}_1[\mathbb{Q}_0^{(a,A_1)}] \otimes \mathbb{Y}_3[\mathbb{Q}_0^{(b,A_1)}]$$

$$\hat{\mathbb{Z}}_{3}(\mathbf{k}) = \frac{\sqrt{2}\mathbb{X}_{1}[\mathbb{Q}_{0}^{(a,A_{1})}] \otimes \mathbb{U}_{3}[\mathbb{Q}_{0}^{(u,A_{1})}] \otimes \mathbb{F}_{1}[\mathbb{Q}_{0}^{(k,A_{1})}]}{2} - \frac{\sqrt{2}\mathbb{X}_{1}[\mathbb{Q}_{0}^{(a,A_{1})}] \otimes \mathbb{U}_{4}[\mathbb{T}_{0}^{(u,A_{1})}] \otimes \mathbb{F}_{5}[\mathbb{T}_{0}^{(k,A_{1})}]}{2}$$

No. 4 
$$\hat{\mathbb{Q}}_{3}^{(A_1)}$$
 [M<sub>1</sub>, B<sub>1</sub>]

$$\hat{\mathbb{Z}}_4 = \frac{\sqrt{3}\mathbb{X}_2[\mathbb{Q}_{2,0}^{(a,T_2)}] \otimes \mathbb{Y}_4[\mathbb{Q}_{1,0}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_3[\mathbb{Q}_{2,1}^{(a,T_2)}] \otimes \mathbb{Y}_5[\mathbb{Q}_{1,1}^{(b,T_2)}]}{3} + \frac{\sqrt{3}\mathbb{X}_4[\mathbb{Q}_{2,2}^{(a,T_2)}] \otimes \mathbb{Y}_6[\mathbb{Q}_{1,2}^{(b,T_2)}]}{3}$$

$$\hat{\mathbb{Z}}_{4}(\boldsymbol{k}) = \frac{\sqrt{6}\mathbb{X}_{2}[\mathbb{Q}_{2,0}^{(a,T_{2})}] \otimes \mathbb{U}_{3}[\mathbb{Q}_{0}^{(u,A_{1})}] \otimes \mathbb{F}_{2}[\mathbb{Q}_{1,0}^{(k,T_{2})}]}{6} - \frac{\sqrt{6}\mathbb{X}_{2}[\mathbb{Q}_{2,0}^{(a,T_{2})}] \otimes \mathbb{U}_{4}[\mathbb{T}_{0}^{(u,A_{1})}] \otimes \mathbb{F}_{6}[\mathbb{T}_{1,0}^{(k,T_{2})}]}{6} + \frac{\sqrt{6}\mathbb{X}_{3}[\mathbb{Q}_{2,1}^{(a,T_{2})}] \otimes \mathbb{U}_{3}[\mathbb{Q}_{0}^{(u,A_{1})}] \otimes \mathbb{F}_{3}[\mathbb{Q}_{1,1}^{(k,T_{2})}]}{6} \\ - \frac{\sqrt{6}\mathbb{X}_{3}[\mathbb{Q}_{2,1}^{(a,T_{2})}] \otimes \mathbb{U}_{4}[\mathbb{T}_{0}^{(u,A_{1})}] \otimes \mathbb{F}_{7}[\mathbb{T}_{1,1}^{(k,T_{2})}]}{6} + \frac{\sqrt{6}\mathbb{X}_{4}[\mathbb{Q}_{2,2}^{(a,T_{2})}] \otimes \mathbb{U}_{3}[\mathbb{Q}_{0}^{(u,A_{1})}] \otimes \mathbb{F}_{4}[\mathbb{Q}_{1,2}^{(k,T_{2})}]}{6} - \frac{\sqrt{6}\mathbb{X}_{4}[\mathbb{Q}_{2,2}^{(a,T_{2})}] \otimes \mathbb{U}_{4}[\mathbb{T}_{0}^{(u,A_{1})}] \otimes \mathbb{F}_{8}[\mathbb{T}_{1,2}^{(k,T_{2})}]}{6} \\ - \frac{\sqrt{6}\mathbb{X}_{4}[\mathbb{Q}_{2,1}^{(a,T_{2})}] \otimes \mathbb{U}_{4}[\mathbb{T}_{0}^{(u,A_{1})}] \otimes \mathbb{F}_{7}[\mathbb{T}_{1,1}^{(k,T_{2})}]}{6} \\ - \frac{\sqrt{6}\mathbb{X}_{4}[\mathbb{Q}_{2,1}^{(a,T_{2})}] \otimes \mathbb{U}_{4}[\mathbb{T}_{0}^{(u,A_{1})}] \otimes \mathbb{F}_{8}[\mathbb{T}_{1,2}^{(k,T_{2})}]}{6} \\ - \frac{\sqrt{6}\mathbb{X}_{4}[\mathbb{Q}_{2,1}^{(a,T_{2})}] \otimes \mathbb{U}_{4}[\mathbb{T}_{0}^{(u,A_{1})}] \otimes \mathbb{F}_{7}[\mathbb{T}_{1,1}^{(k,T_{2})}]}{6} \\ - \frac{\sqrt{6}\mathbb{X}_{4}[\mathbb{Q}_{2,1}^{(u,A_{1})}] \otimes \mathbb{F}_{7}[\mathbb{T}_{1,1}^{(u,A_{1})}] \otimes \mathbb{F}_{7}[\mathbb{T}_{1,1}^{(u,A_{1})}]}{6} \\ - \frac{\sqrt{6}\mathbb{X}_{4}[\mathbb{Q}_{2,1}^{(u,A_{1})}] \otimes \mathbb{F}_{8}[\mathbb{T}_{1,1}^{(u,A_{1})}]}{6} \\ - \frac{\sqrt{6}\mathbb{X}_{4}[\mathbb{Q}_{2,1}^{(u,A_{1})}] \otimes \mathbb{F}_{8}[\mathbb{T}_{1,1}^{(u,A_{1})}]}{6} \\ - \frac{\sqrt{6}\mathbb{X}_{4}[\mathbb{Q}_{2,1}^{(u,A_{1})}] \otimes \mathbb{F}_{7}[\mathbb{Q}_{1,1}^{(u,A_{1})}]}{6} \\ - \frac{\sqrt{6}\mathbb{X}_{4}[\mathbb{Q}_{2,1}^{(u,A_{1})}] \otimes \mathbb{F}_{8}[\mathbb{Q}_{1,1}^{(u,A_{1})}]}{6} \\ - \frac{\sqrt{6}\mathbb{X}_{4}[\mathbb{Q}_{1,1}^{(u,A_{1})}] \otimes \mathbb{F}_{8}[\mathbb{Q}_{1,1$$

Table 5: Atomic SAMB group.

group	bra	ket
$M_1$	$p_x, p_y, p_z$	$p_x, p_y, p_z$

Table 6: Atomic SAMB.

symbol	type	group	form
$\mathbb{X}_1$	$\mathbb{Q}_0^{(a,A_1)}$	$M_1$	$ \begin{pmatrix} \frac{\sqrt{3}}{3} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{3} & 0 \\ 0 & 0 & \frac{\sqrt{3}}{3} \end{pmatrix} $
$\mathbb{X}_2$	$\mathbb{Q}_{2,0}^{(a,T_2)}$	$ m M_1$	$ \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & 0 \end{array} \right) $
$\mathbb{X}_3$	$\mathbb{Q}_{2,1}^{(a,T_2)}$	$ m M_1$	$ \left[ \begin{array}{cccc} 0 & 0 & \frac{\sqrt{2}}{2} \\ 0 & 0 & 0 \\ \frac{\sqrt{2}}{2} & 0 & 0 \end{array} \right] $
$\mathbb{X}_4$	$\mathbb{Q}_{2,2}^{(a,T_2)}$	$M_1$	$ \begin{pmatrix} 0 & \frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} $

Table 7: Cluster SAMB.

symbol	type	cluster	form
$\mathbb{Y}_1$	$\mathbb{Q}_0^{(s,A_1)}$	$S_1$	(1)
$\mathbb{Y}_2$	$\mathbb{Q}_0^{(s,A_1)}$	$S_2$	(1)
$\mathbb{Y}_3$	$\mathbb{Q}_0^{(b,A_1)}$	$\mathrm{B}_1$	$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$
$\mathbb{Y}_4$	$\mathbb{Q}_{1,0}^{(b,T_2)}$	$B_1$	$\left[\begin{array}{cccc} \left(\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{array}\right)\right]$

Table 7

symbol	type	cluster	form
$\mathbb{Y}_5$	$\mathbb{Q}_{1,1}^{(b,T_2)}$	$B_1$	$\left[\begin{array}{cccc} \left(\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \right) \end{array}\right]$
$\mathbb{Y}_6$	$\mathbb{Q}_{1,2}^{(b,T_2)}$	$\mathrm{B}_1$	$\left[\begin{array}{cccc} \left(\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \right) \end{array}\right]$

Table 8: Uniform SAMB.

symbol	type	cluster	form
$\mathbb{U}_1$	$\mathbb{Q}_0^{(s,A_1)}$	$S_1$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
$\mathbb{U}_2$	$\mathbb{Q}_0^{(s,A_1)}$	$S_2$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
$\mathbb{U}_3$	$\mathbb{Q}_0^{(u,A_1)}$	$\mathrm{B}_1$	$\begin{pmatrix} 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$
$\mathbb{U}_4$	$\mathbb{T}_0^{(u,A_1)}$	$B_1$	$\begin{pmatrix} 0 & -\frac{\sqrt{2}i}{2} \\ \frac{\sqrt{2}i}{2} & 0 \end{pmatrix}$

Table 9: Structure SAMB.

symbol	type	cluster	form
$\mathbb{F}_1$	$\mathbb{Q}_0^{(k,A_1)}$	$B_1$	$\frac{\sqrt{2}c_{001}}{2} + \frac{\sqrt{2}c_{002}}{2} + \frac{\sqrt{2}c_{003}}{2} + \frac{\sqrt{2}c_{004}}{2}$
$\mathbb{F}_2$	$\mathbb{Q}_{1,0}^{(k,T_2)}$	$B_1$	$\frac{\sqrt{2}c_{001}}{2} - \frac{\sqrt{2}c_{002}}{2} + \frac{\sqrt{2}c_{003}}{2} - \frac{\sqrt{2}c_{004}}{2}$
$\mathbb{F}_3$	$\mathbb{Q}_{1,1}^{(k,T_2)}$	$B_1$	$\frac{\sqrt{2}c_{001}}{2} - \frac{\sqrt{2}c_{002}}{2} - \frac{\sqrt{2}c_{003}}{2} + \frac{\sqrt{2}c_{004}}{2}$
$\mathbb{F}_4$	$\mathbb{Q}_{1,2}^{(k,T_2)}$	$B_1$	$\frac{\sqrt{2}c_{001}}{2} + \frac{\sqrt{2}c_{002}}{2} - \frac{\sqrt{2}c_{003}}{2} - \frac{\sqrt{2}c_{004}}{2}$
$\mathbb{F}_5$	$\mathbb{T}_0^{(k,A_1)}$	$B_1$	$\frac{\sqrt{2}s_{001}}{2} + \frac{\sqrt{2}s_{002}}{2} + \frac{\sqrt{2}s_{003}}{2} + \frac{\sqrt{2}s_{004}}{2}$
$\mathbb{F}_6$	$\mathbb{T}_{1,0}^{(k,T_2)}$	$B_1$	$\frac{\sqrt{2}s_{001}}{2} - \frac{\sqrt{2}s_{002}}{2} + \frac{\sqrt{2}s_{003}}{2} - \frac{\sqrt{2}s_{004}}{2}$
$\mathbb{F}_7$	$\mathbb{T}_{1,1}^{(k,T_2)}$	$B_1$	$\frac{\sqrt{2}s_{001}}{2} - \frac{\sqrt{2}s_{002}}{2} - \frac{\sqrt{2}s_{003}}{2} + \frac{\sqrt{2}s_{004}}{2}$

Table 9

symbol	type	cluster	form				
$\mathbb{F}_8$	$\mathbb{T}_{1,2}^{(k,T_2)}$	$B_1$	$\frac{\sqrt{2}s_{001}}{2} + \frac{\sqrt{2}s_{002}}{2} - \frac{\sqrt{2}s_{003}}{2} - \frac{\sqrt{2}s_{004}}{2}$				

Table 10: Polar harmonics.

No.	symbol	rank	irrep.	mul.	comp.	form
1	$\mathbb{Q}_0^{(A_1)}$	0	$A_1$	_	_	1
2	$\mathbb{Q}_{1,0}^{(T_2)}$	1	$T_2$	_	0	x
3	$\mathbb{Q}_{1,1}^{(T_2)}$	1	$T_2$	_	1	y
4	$\mathbb{Q}_{1,2}^{(T_2)}$	1	$T_2$	_	2	z
5	$\mathbb{Q}_{2,0}^{(T_2)}$	2	$T_2$	_	0	$\sqrt{3}yz$
6	$\mathbb{Q}_{2,1}^{(T_2)}$	2	$T_2$	_	1	$\sqrt{3}xz$
7	$\mathbb{Q}_{2,2}^{(T_2)}$	2	$T_2$	_	2	$\sqrt{3}xy$

 $\bullet \ \, \text{Group info.: Generator} = \{2_{001}|0\}, \ \{2_{010}|0\}, \ \{3_{\ 111}^{+}|0\}, \ \{m_{1-10}|0\}$ 

Table 11: Conjugacy class (point-group part).

rep. SO	symmetry operations
{1 0}	{1 0}
$\{2_{001} 0\}$	$\{2_{001} 0\}, \{2_{100} 0\}, \{2_{010} 0\}$
${\{3^{+}_{111} 0\}}$	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$
$\{m_{110} 0\}$	$\{m_{110} 0\}, \{m_{101} 0\}, \{m_{011} 0\}, \{m_{1-10} 0\}, \{m_{-101} 0\}, \{m_{01-1} 0\}$
$\{-4^{+}_{001} 0\}$	$\{-4^{+}_{001} 0\}, \ \{-4^{+}_{100} 0\}, \ \{-4^{+}_{010} 0\}, \ \{-4^{-}_{001} 0\}, \ \{-4^{-}_{100} 0\}, \ \{-4^{-}_{010} 0\}$

Table 12: Symmetry operations.

No.	SO	No.	SO	No.	SO	No.	SO	No.	SO
1	$\{1 0\}$	2	$\{2_{001} 0\}$	3	$\{2_{100} 0\}$	4	$\{2_{010} 0\}$	5	$\{3^{+}_{111} 0\}$
6	$\{3^+_{1-1-1} 0\}$	7	${3^{+}_{-11-1} 0}$	8	$\{3^{+}_{-1-11} 0\}$	9	$\{3^{-}_{111} 0\}$	10	$\{3^{1-1-1} 0\}$
11	$\{3^{-}_{-11-1} 0\}$	12	$\{3^{-}_{-1-11} 0\}$	13	$\{m_{110} 0\}$	14	$\{m_{101} 0\}$	15	$\{m_{011} 0\}$
16	$\{m_{1-10} 0\}$	17	$\{m_{-101} 0\}$	18	$\{m_{01-1} 0\}$	19	$\{-4^{+}_{001} 0\}$	20	$\{-4^{+}_{100} 0\}$
21	$\{-4^{+}_{010} 0\}$	22	$\{-4^{-}_{001} 0\}$	23	$\{-4^{-}_{100} 0\}$	24	$\{-4^{-}_{\ 010} 0\}$		

Table 13: Character table (point-group part).

	1	2001	3 <sup>+</sup>	m <sub>110</sub>	$-4^{+}_{001}$
$\overline{A_1}$	1	1	1	1	1
$A_2$	1	1	1	-1	-1
E	2	2	-1	0	0
$T_1$	3	-1	0	-1	1
$T_2$	3	-1	0	1	-1

Table 14: Parity conversion.

$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$	$\leftrightarrow$
$A_1 (A_2)$	$A_2(A_1)$	E(E)	$T_1$ $(T_2)$	$T_2$ $(T_1)$

Table 15: Symmetric product,  $[\Gamma \otimes \Gamma']_+$ .

	$A_1$	$A_2$	E	$T_1$	$T_2$
$\overline{A_1}$	$A_1$	$A_2$	E	$T_1$	$T_2$
$A_2$		$A_1$	E	$T_2$	$T_1$
E			$A_1 + E$	$T_1 + T_2$	$T_1 + T_2$

Table 15

	$A_1$	$A_2$	E	$T_1$	$T_2$
$T_1$				$A_1 + E + T_2$	$A_2 + E + T_1 + T_2$
$T_2$					$A_1 + E + T_2$

Table 16: Anti-symmetric product,  $[\Gamma \otimes \Gamma]_{-}$ .

$A_1$	$A_2$	E	$T_1$	$T_2$
_	_	$A_2$	$T_1$	$T_1$

Table 17: Virtual-cluster sites.

No.	position	No.	position	No.	position	No.	position
1	$\begin{pmatrix} 2 & 1 & 0 \end{pmatrix}$	2	$\begin{pmatrix} -2 & -1 & 0 \end{pmatrix}$	3	$\begin{pmatrix} 2 & -1 & 0 \end{pmatrix}$	4	$\begin{pmatrix} -2 & 1 & 0 \end{pmatrix}$
5	$\begin{pmatrix} 0 & 2 & 1 \end{pmatrix}$	6	$\begin{pmatrix} 0 & -2 & 1 \end{pmatrix}$	7	$\begin{pmatrix} 0 & -2 & -1 \end{pmatrix}$	8	$\begin{pmatrix} 0 & 2 & -1 \end{pmatrix}$
9	$\begin{pmatrix} 1 & 0 & 2 \end{pmatrix}$	10	$\begin{pmatrix} -1 & 0 & -2 \end{pmatrix}$	11	$\begin{pmatrix} -1 & 0 & 2 \end{pmatrix}$	12	$\begin{pmatrix} 1 & 0 & -2 \end{pmatrix}$
13	$\begin{pmatrix} -1 & -2 & 0 \end{pmatrix}$	14	$\begin{pmatrix} 0 & 1 & -2 \end{pmatrix}$	15	$\begin{pmatrix} 2 & 0 & -1 \end{pmatrix}$	16	$\begin{pmatrix} 1 & 2 & 0 \end{pmatrix}$
17	$\begin{pmatrix} 0 & 1 & 2 \end{pmatrix}$	18	$\begin{pmatrix} 2 & 0 & 1 \end{pmatrix}$	19	$\begin{pmatrix} 1 & -2 & 0 \end{pmatrix}$	20	$\begin{pmatrix} -2 & 0 & -1 \end{pmatrix}$
21	$\begin{pmatrix} 0 & -1 & 2 \end{pmatrix}$	22	$\begin{pmatrix} -1 & 2 & 0 \end{pmatrix}$	23	$\begin{pmatrix} -2 & 0 & 1 \end{pmatrix}$	24	$\begin{pmatrix} 0 & -1 & -2 \end{pmatrix}$

Table 18: Virtual-cluster basis.

symbol	1	2	3	4	5	6	7	8	9	10
$\mathbb{Q}_0^{(A_1)}$	$\frac{\sqrt{6}}{12}$									

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		10
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{1}}{20}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{1}}{10}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0	0
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$-\frac{\sqrt{10}}{10}$	0
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{1}}{1}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0	$-\frac{\sqrt{2}}{2}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\frac{7\sqrt{39}}{156}$	$\frac{7\sqrt{3}}{156}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$-\frac{5\sqrt{39}}{156}$	$-\frac{\sqrt{3}}{7}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$-\frac{3\sqrt{13}}{52}$	$\frac{\sqrt{13}}{13}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0	0
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0	0
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$
$\mathbb{Q}_{2,2}^{(T_2)}  \frac{\sqrt{2}}{4}  \frac{\sqrt{2}}{4}  -\frac{\sqrt{2}}{4}  -\frac{\sqrt{2}}{4}  0  0  0  0  0  0  0  0  0  $	0	$\frac{\sqrt{2}}{4}$
$\mathbb{Q}_{2,2}^{(T_2)}  \frac{\sqrt{2}}{4}  \frac{\sqrt{2}}{4}  -\frac{\sqrt{2}}{4}  -\frac{\sqrt{2}}{4}  0  0  0  0  0  0  0  0  0  $		
$0   0   \frac{\sqrt{2}}{4}   0   0   \frac{\sqrt{2}}{4}   0$	0	0
	$-\frac{\sqrt{2}}{4}$	0
$0 - \frac{1}{4}$ $0$ $0$		
$\mathbb{Q}_{3,0}^{(T_1)}  \frac{\sqrt{10}}{20}  -\frac{\sqrt{10}}{20}  \frac{\sqrt{10}}{20}  -\frac{\sqrt{10}}{20}  0  0  0$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$

symbol	1	2	3	4	5	6	7	8	9
$\mathbb{Q}_{3,1}^{(T_1)}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	0
,	0	0	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$
	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$					
$\mathbb{Q}_{3,2}^{(T_1)}$	0	0	0	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$
	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0
	$-\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$					
$\mathbb{Q}_{3,0}^{(T_2)}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	0	0	0	0	$-\frac{\sqrt{10}}{10}$
	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$
	0	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0					
$\mathbb{Q}_{3,1}^{(T_2)}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	0
	0	0	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$
	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$					
$\mathbb{Q}_{3,2}^{(T_2)}$	0	0	0	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$
	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0
	$\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$					
$\mathbb{Q}_{4,0}^{(E)}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$-\frac{\sqrt{13}}{13}$	$\frac{\sqrt{13}}{52}$
	$\frac{\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{13}$	$\frac{3\sqrt{13}}{52}$	$\frac{\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{13}$	$\frac{3\sqrt{13}}{52}$
	$\frac{\sqrt{13}}{52}$	$\frac{3\sqrt{13}}{52}$	$-\frac{\sqrt{13}}{13}$	$\frac{\sqrt{13}}{52}$					
$\mathbb{Q}_{4,1}^{(E)}$	$\frac{5\sqrt{39}}{156}$	$\frac{5\sqrt{39}}{156}$	$\frac{5\sqrt{39}}{156}$	$\frac{5\sqrt{39}}{156}$	$\frac{\sqrt{39}}{78}$	$\frac{\sqrt{39}}{78}$	$\frac{\sqrt{39}}{78}$	$\frac{\sqrt{39}}{78}$	$-\frac{7\sqrt{39}}{156}$
	$-\frac{7\sqrt{39}}{156}$	$-\frac{7\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$	$-\frac{5\sqrt{39}}{156}$	$\frac{7\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$	$-\frac{5\sqrt{39}}{156}$
	$\frac{7\sqrt{39}}{156}$	$-\frac{5\sqrt{39}}{156}$	$-\frac{\sqrt{39}}{78}$	$\frac{7\sqrt{39}}{156}$					
$\mathbb{Q}_{4,0}^{(T_1)}$	0	0	0	0	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0
	0	0	0	$\frac{\sqrt{2}}{4}$	0	0	$-\frac{\sqrt{2}}{4}$	0	0
	$\frac{\sqrt{2}}{4}$	0	0	$-\frac{\sqrt{2}}{4}$					
$\mathbb{Q}_{4,1}^{(T_1)}$	0	0	0	0	0	0	0	0	$\frac{\sqrt{2}}{4}$
	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	0	$\frac{\sqrt{2}}{4}$	0	0	$-\frac{\sqrt{2}}{4}$	0
	0	0	$\frac{\sqrt{2}}{4}$	0					
$\mathbb{Q}_{4,2}^{(T_1)}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	0	0	0	0	0
	0	0	$-\frac{\sqrt{2}}{4}$	0	0	$-\frac{\sqrt{2}}{4}$	0	0	$\frac{\sqrt{2}}{4}$

Table 18

symbol	1	2	3	4	5	6	7	8	9	10
	0	$\frac{\sqrt{2}}{4}$	0	0						
$\mathbb{Q}_{5,0}^{(T_1)}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	0	0	0	0	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$
	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$
	0	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	0						
$\mathbb{Q}_{5,1}^{(T_1)}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	0	0
	0	0	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{10}$	0
	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{20}$						
$\mathbb{Q}_{5,2}^{(T_1)}$	0	0	0	0	$\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$
	$\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{10}$	0	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{10}}{20}$	0	$-\frac{\sqrt{10}}{10}$	$-\frac{\sqrt{10}}{20}$	0	$\frac{\sqrt{10}}{20}$
	$-\frac{\sqrt{10}}{10}$	0	$-\frac{\sqrt{10}}{20}$	$\frac{\sqrt{10}}{10}$						
$\mathbb{Q}_6^{(A_2)}$	$\frac{\sqrt{6}}{12}$									
	$\frac{\sqrt{6}}{12}$	$\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$							
	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$	$-\frac{\sqrt{6}}{12}$						