Response Tensors up to 4th rank in C_4

— polar tensors —

$$C^{(0,Q)} = \begin{pmatrix} C^{(0,Q)} \end{pmatrix}$$

$$C^{(0,Q)} = Q_0$$

$$C^{(1,Q)} = \begin{pmatrix} 0 & 0 & C_z^{(1,Q)} \end{pmatrix}$$

$$C_z^{(1,Q)} = Q_z$$

$$S^{(2,Q)} = \begin{pmatrix} S_{xx}^{(2,Q)} & 0 & 0 \\ 0 & S_{xx}^{(2,Q)} & 0 \\ 0 & 0 & S_{zz}^{(2,Q)} \end{pmatrix}$$

$$S_{xx}^{(2,Q)} = Q_0 - Q_u$$
$$S_{zz}^{(2,Q)} = Q_0 + 2Q_u$$

$$A^{(2,Q)} = \begin{pmatrix} 0 & A_{xy}^{(2,Q)} & 0 \\ -A_{xy}^{(2,Q)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{xy}^{(2,Q)} = G_z$$

$$S^{(3,Q)} = \begin{pmatrix} 0 & 0 & S_{1z}^{(3,Q)} \\ 0 & 0 & S_{1z}^{(3,Q)} \\ 0 & 0 & S_{3z}^{(3,Q)} \\ S_{4x}^{(3,Q)} & S_{4y}^{(3,Q)} & 0 \\ S_{4y}^{(3,Q)} & -S_{4x}^{(3,Q)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_{1z}^{(3,Q)} = Q_z[2] - Q_z^{\alpha}$$
$$S_{3z}^{(3,Q)} = 2Q_z[1] + Q_z[2] + 2Q_z^{\alpha}$$
$$S_{4x}^{(3,Q)} = -3G_u[1]$$
$$S_{4y}^{(3,Q)} = Q_z[1] - Q_z^{\alpha}$$

$$A^{(3,Q)} = \begin{pmatrix} A_{4x}^{(3,Q)} & A_{4y}^{(3,Q)} & 0 \\ -A_{4y}^{(3,Q)} & A_{4x}^{(3,Q)} & 0 \\ 0 & 0 & A_{6z}^{(3,Q)} \end{pmatrix}$$

$$A_{4x}^{(3,Q)} = G_0 - G_u[2]$$
$$A_{4y}^{(3,Q)} = Q_z[3]$$
$$A_{6z}^{(3,Q)} = G_0 + 2G_u[2]$$

$$S^{(4,Q)} = \begin{pmatrix} S_{11}^{(4,Q)} & S_{12}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & 0 & S_{16}^{(4,Q)} \\ S_{12}^{(4,Q)} & S_{11}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & 0 & -S_{16}^{(4,Q)} \\ S_{13}^{(4,Q)} & S_{13}^{(4,Q)} & S_{33}^{(4,Q)} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 \\ S_{16}^{(4,Q)} & -S_{16}^{(4,Q)} & 0 & 0 & 0 & S_{66}^{(4,Q)} \end{pmatrix}$$

$$S_{11}^{(4,Q)} = Q_0[1] + 2Q_0[2] - Q_{4u} + 2Q_4 - 2Q_u[1] - 4Q_u[2]$$
$$S_{12}^{(4,Q)} = Q_0[1] + 2Q_{4u} - Q_4 - 2Q_u[1]$$
$$S_{13}^{(4,Q)} = Q_0[1] - Q_{4u} - Q_4 + Q_u[1]$$
$$S_{16}^{(4,Q)} = Q_{4z}^{\alpha}$$
$$S_{33}^{(4,Q)} = Q_0[1] + 2Q_0[2] + 2Q_{4u} + 2Q_4 + 4Q_u[1] + 8Q_u[2]$$
$$S_{44}^{(4,Q)} = Q_0[2] - Q_{4u} - Q_4 + Q_u[2]$$
$$S_{66}^{(4,Q)} = Q_0[2] + 2Q_{4u} - Q_4 - 2Q_u[2]$$

$$\bar{S}^{(4,Q)} = \begin{pmatrix} 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & 0 & \bar{S}_{16}^{(4,Q)} \\ 0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & 0 & -\bar{S}_{16}^{(4,Q)} \\ -\bar{S}_{13}^{(4,Q)} & -\bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{S}_{45}^{(4,Q)} & 0 \\ 0 & 0 & 0 & -\bar{S}_{45}^{(4,Q)} & 0 & 0 \\ -\bar{S}_{16}^{(4,Q)} & \bar{S}_{16}^{(4,Q)} & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{S}_{13}^{(4,Q)} = 3Q_u[3]$$
$$\bar{S}_{16}^{(4,Q)} = 2G_z[1] - 2G_z^{\alpha}[1]$$
$$\bar{S}_{45}^{(4,Q)} = -G_z[1] - 4G_z^{\alpha}[1]$$

$$A^{(4,Q)} = \begin{pmatrix} A_{xx}^{(4,Q)} & 0 & 0 \\ 0 & A_{xx}^{(4,Q)} & 0 \\ 0 & 0 & A_{zz}^{(4,Q)} \end{pmatrix}$$

$$A_{xx}^{(4,Q)} = Q_0[3] - 2Q_u[6]$$
$$A_{zz}^{(4,Q)} = Q_0[3] + 4Q_u[6]$$

$$\bar{A}^{(4,Q)} = \begin{pmatrix} 0 & \bar{A}_{xy}^{(4,Q)} & 0 \\ -\bar{A}_{xy}^{(4,Q)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\bar{A}_{xy}^{(4,Q)} = G_z[6]$$

$$M^{(4,Q)} = \begin{pmatrix} 0 & 0 & M_{1z}^{(4,Q)} \\ 0 & 0 & M_{1z}^{(4,Q)} \\ 0 & 0 & M_{3z}^{(4,Q)} \\ M_{4x}^{(4,Q)} & M_{4y}^{(4,Q)} & 0 \\ M_{4y}^{(4,Q)} & -M_{4x}^{(4,Q)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} M_{1z}^{(4,Q)} &= G_z[3] - G_z^{\alpha}[2] \\ M_{3z}^{(4,Q)} &= 2G_z[2] + G_z[3] + 2G_z^{\alpha}[2] \\ M_{4x}^{(4,Q)} &= -3Q_u[4] \\ M_{4y}^{(4,Q)} &= G_z[2] - G_z^{\alpha}[2] \end{aligned}$$

$$\bar{M}^{(4,Q)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}_{x4}^{(4,Q)} & \bar{M}_{x5}^{(4,Q)} & 0 \\ 0 & 0 & 0 & \bar{M}_{x5}^{(4,Q)} & -\bar{M}_{x4}^{(4,Q)} & 0 \\ \bar{M}_{z1}^{(4,Q)} & \bar{M}_{z1}^{(4,Q)} & \bar{M}_{z3}^{(4,Q)} & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \bar{M}_{x4}^{(4,Q)} &= -3Q_u[5] \\ \bar{M}_{x5}^{(4,Q)} &= G_z[4] - G_z^{\alpha}[3] \\ \bar{M}_{z1}^{(4,Q)} &= G_z[5] - G_z^{\alpha}[3] \\ \bar{M}_{z3}^{(4,Q)} &= 2G_z[4] + G_z[5] + 2G_z^{\alpha}[3] \end{aligned}$$

— axial tensors —

$$C^{(0,G)} = \begin{pmatrix} C^{(0,G)} \end{pmatrix}$$

$$C^{(0,G)} = G_0$$

$$C^{(1,G)} = \begin{pmatrix} 0 & 0 & C_z^{(1,G)} \end{pmatrix}$$

$$C_z^{(1,G)} = G_z$$

$$S^{(2,G)} = \begin{pmatrix} S_{xx}^{(2,G)} & 0 & 0 \\ 0 & S_{xx}^{(2,G)} & 0 \\ 0 & 0 & S_{zz}^{(2,G)} \end{pmatrix}$$

$$S_{xx}^{(2,G)} = G_0 - G_u$$
$$S_{zz}^{(2,G)} = G_0 + 2G_u$$

$$A^{(2,G)} = \begin{pmatrix} 0 & A_{xy}^{(2,G)} & 0 \\ -A_{xy}^{(2,G)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{xy}^{(2,G)} = Q_z$$

$$S^{(3,G)} = \begin{pmatrix} 0 & 0 & S_{1z}^{(3,G)} \\ 0 & 0 & S_{1z}^{(3,G)} \\ 0 & 0 & S_{3z}^{(3,G)} \\ S_{4x}^{(3,G)} & S_{4y}^{(3,G)} & 0 \\ S_{4y}^{(3,G)} & -S_{4x}^{(3,G)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_{1z}^{(3,G)} = G_z[2] - G_z^{\alpha}$$
$$S_{3z}^{(3,G)} = 2G_z[1] + G_z[2] + 2G_z^{\alpha}$$
$$S_{4x}^{(3,G)} = -3Q_u[1]$$
$$S_{4y}^{(3,G)} = G_z[1] - G_z^{\alpha}$$

$$A^{(3,G)} = \begin{pmatrix} A_{4x}^{(3,G)} & A_{4y}^{(3,G)} & 0 \\ -A_{4y}^{(3,G)} & A_{4x}^{(3,G)} & 0 \\ 0 & 0 & A_{6z}^{(3,G)} \end{pmatrix}$$

$$A_{4x}^{(3,G)} = Q_0 - Q_u[2]$$
$$A_{4y}^{(3,G)} = G_z[3]$$
$$A_{6z}^{(3,G)} = Q_0 + 2Q_u[2]$$

$$S^{(4,G)} = \begin{pmatrix} S_{11}^{(4,G)} & S_{12}^{(4,G)} & S_{13}^{(4,G)} & 0 & 0 & S_{16}^{(4,G)} \\ S_{12}^{(4,G)} & S_{11}^{(4,G)} & S_{13}^{(4,G)} & 0 & 0 & -S_{16}^{(4,G)} \\ S_{13}^{(4,G)} & S_{13}^{(4,G)} & S_{33}^{(4,G)} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^{(4,G)} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44}^{(4,G)} & 0 \\ S_{16}^{(4,G)} & -S_{16}^{(4,G)} & 0 & 0 & 0 & S_{66}^{(4,G)} \end{pmatrix}$$

$$\begin{aligned} S_{11}^{(4,G)} &= G_0[1] + 2G_0[2] - G_{4u} + 2G_4 - 2G_u[1] - 4G_u[2] \\ S_{12}^{(4,G)} &= G_0[1] + 2G_{4u} - G_4 - 2G_u[1] \\ S_{13}^{(4,G)} &= G_0[1] - G_{4u} - G_4 + G_u[1] \\ S_{16}^{(4,G)} &= G_{4z}^{\alpha} \\ S_{33}^{(4,G)} &= G_0[1] + 2G_0[2] + 2G_{4u} + 2G_4 + 4G_u[1] + 8G_u[2] \\ S_{44}^{(4,G)} &= G_0[2] - G_{4u} - G_4 + G_u[2] \\ S_{66}^{(4,G)} &= G_0[2] + 2G_{4u} - G_4 - 2G_u[2] \end{aligned}$$

$$\bar{S}^{(4,G)} = \begin{pmatrix} 0 & 0 & \bar{S}_{13}^{(4,G)} & 0 & 0 & \bar{S}_{16}^{(4,G)} \\ 0 & 0 & \bar{S}_{13}^{(4,G)} & 0 & 0 & -\bar{S}_{16}^{(4,G)} \\ -\bar{S}_{13}^{(4,G)} & -\bar{S}_{13}^{(4,G)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{S}_{45}^{(4,G)} & 0 \\ 0 & 0 & 0 & -\bar{S}_{45}^{(4,G)} & 0 & 0 \\ -\bar{S}_{16}^{(4,G)} & \bar{S}_{16}^{(4,G)} & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \bar{S}_{13}^{(4,G)} &= 3G_u[3] \\ \bar{S}_{16}^{(4,G)} &= 2Q_z[1] - 2Q_z^{\alpha}[1] \\ \bar{S}_{45}^{(4,G)} &= -Q_z[1] - 4Q_z^{\alpha}[1] \end{aligned}$$

$$A^{(4,G)} = \begin{pmatrix} A_{xx}^{(4,G)} & 0 & 0 \\ 0 & A_{xx}^{(4,G)} & 0 \\ 0 & 0 & A_{zz}^{(4,G)} \end{pmatrix}$$

$$\begin{aligned} A_{xx}^{(4,G)} &= G_0[3] - 2G_u[6] \\ A_{zz}^{(4,G)} &= G_0[3] + 4G_u[6] \end{aligned}$$

$$\bar{A}^{(4,G)} = \begin{pmatrix} 0 & \bar{A}_{xy}^{(4,G)} & 0 \\ -\bar{A}_{xy}^{(4,G)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\bar{A}_{xy}^{(4,G)} = Q_z[6]$$

$$M^{(4,G)} = \begin{pmatrix} 0 & 0 & M_{1z}^{(4,G)} \\ 0 & 0 & M_{1z}^{(4,G)} \\ 0 & 0 & M_{3z}^{(4,G)} \\ M_{4x}^{(4,G)} & M_{4y}^{(4,G)} & 0 \\ M_{4y}^{(4,G)} & -M_{4x}^{(4,G)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$
\begin{aligned}
M_{1z}^{(4,G)} &= Q_z[3] - Q_z^{\alpha}[2] \\
M_{3z}^{(4,G)} &= 2Q_z[2] + Q_z[3] + 2Q_z^{\alpha}[2] \\
M_{4x}^{(4,G)} &= -3G_u[4] \\
M_{4y}^{(4,G)} &= Q_z[2] - Q_z^{\alpha}[2]
\end{aligned}
$$

$$
\bar{M}^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}_{x4}^{(4,G)} & \bar{M}_{x5}^{(4,G)} & 0 \\ 0 & 0 & 0 & \bar{M}_{x5}^{(4,G)} & -\bar{M}_{x4}^{(4,G)} & 0 \\ \bar{M}_{z1}^{(4,G)} & \bar{M}_{z1}^{(4,G)} & \bar{M}_{z3}^{(4,G)} & 0 & 0 & 0 \end{pmatrix}
$$

$$
\begin{aligned}
\bar{M}_{x4}^{(4,G)} &= -3G_u[5] \\
\bar{M}_{x5}^{(4,G)} &= Q_z[4] - Q_z^{\alpha}[3] \\
\bar{M}_{z1}^{(4,G)} &= Q_z[5] - Q_z^{\alpha}[3] \\
\bar{M}_{z3}^{(4,G)} &= 2Q_z[4] + Q_z[5] + 2Q_z^{\alpha}[3]
\end{aligned}
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