

PG No. 25 C_{6v} $6mm$ [hexagonal] (polar, internal axial dipole)

* Harmonics for rank 0

* Harmonics for rank 1

$$\vec{Q}_1^{(1,0)}[g](A_1)$$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}G_x y}{2} - \frac{\sqrt{2}G_y x}{2}$$

$$\vec{Q}_{1,1}^{(1,0)}[g](E_1), \vec{Q}_{1,2}^{(1,0)}[g](E_1)$$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{\sqrt{2}G_y z}{2} - \frac{\sqrt{2}G_z y}{2}$$

$$-\frac{\sqrt{2}G_x z}{2} + \frac{\sqrt{2}G_z x}{2}$$

* Harmonics for rank 2

$$\vec{Q}_2^{(1,0)}[g](A_1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{6}G_x y z}{2} - \frac{\sqrt{6}G_y x z}{2}$$

$$\vec{Q}_{2,1}^{(1,0)}[g](E_1), \vec{Q}_{2,2}^{(1,0)}[g](E_1)$$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{2}G_x x y}{2} - \frac{\sqrt{2}G_y (x - z)(x + z)}{2} - \frac{\sqrt{2}G_z y z}{2}$$

$$\frac{\sqrt{2}G_x (y - z)(y + z)}{2} - \frac{\sqrt{2}G_y x y}{2} + \frac{\sqrt{2}G_z x z}{2}$$

$$\vec{Q}_{2,1}^{(1,0)}[g](E_2), \vec{Q}_{2,2}^{(1,0)}[g](E_2)$$

** symmetry

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}G_x y z}{2} + \frac{\sqrt{2}G_y x z}{2} - \sqrt{2}G_z x y$$

$$\frac{\sqrt{2}G_x x z}{2} - \frac{\sqrt{2}G_y y z}{2} - \frac{\sqrt{2}G_z (x - y)(x + y)}{2}$$

* Harmonics for rank 3

$$\vec{Q}_3^{(1,0)}[g](A_1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{\sqrt{3}G_x y (x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{3}G_y x (x^2 + y^2 - 4z^2)}{4}$$

$$\tilde{\mathbb{Q}}_3^{(1,0)}[g](B_1)$$

** symmetry

$$\frac{\sqrt{10}y (3x^2 - y^2)}{4}$$

** expression

$$-\frac{\sqrt{30}G_x z (x - y) (x + y)}{8} + \frac{\sqrt{30}G_y x y z}{4} + \frac{\sqrt{30}G_z x (x^2 - 3y^2)}{8}$$

$$\tilde{\mathbb{Q}}_3^{(1,0)}[g](B_2)$$

** symmetry

$$\frac{\sqrt{10}x (x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{30}G_x x y z}{4} + \frac{\sqrt{30}G_y z (x - y) (x + y)}{8} - \frac{\sqrt{30}G_z y (3x^2 - y^2)}{8}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(1,0)}[g](E_1), \tilde{\mathbb{Q}}_{3,2}^{(1,0)}[g](E_1)$$

** symmetry

$$-\frac{\sqrt{6}x (x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y (x^2 + y^2 - 4z^2)}{4}$$

** expression

$$\frac{5\sqrt{2}G_x x y z}{4} - \frac{\sqrt{2}G_y z (11x^2 + y^2 - 4z^2)}{8} + \frac{\sqrt{2}G_z y (x^2 + y^2 - 4z^2)}{8}$$

$$\frac{\sqrt{2}G_x z (x^2 + 11y^2 - 4z^2)}{8} - \frac{5\sqrt{2}G_y x y z}{4} - \frac{\sqrt{2}G_z x (x^2 + y^2 - 4z^2)}{8}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(1,0)}[g](E_2), \tilde{\mathbb{Q}}_{3,2}^{(1,0)}[g](E_2)$$

** symmetry

$$-\frac{\sqrt{15}z (x - y) (x + y)}{2}$$

$$\sqrt{15}x y z$$

** expression

$$-\frac{\sqrt{5}G_x y (x^2 - y^2 + 2z^2)}{4} + \frac{\sqrt{5}G_y x (x^2 - y^2 - 2z^2)}{4} + \sqrt{5}G_z x y z$$

$$\frac{\sqrt{5}G_x x (y - z) (y + z)}{2} - \frac{\sqrt{5}G_y y (x - z) (x + z)}{2} + \frac{\sqrt{5}G_z z (x - y) (x + y)}{2}$$

* Harmonics for rank 4

$$\tilde{\mathbb{Q}}_4^{(1,0)}[g](A_1)$$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2 y^2}{4} - 3x^2 z^2 + \frac{3y^4}{8} - 3y^2 z^2 + z^4$$

** expression

$$-\frac{\sqrt{5}G_x y z (3x^2 + 3y^2 - 4z^2)}{4} + \frac{\sqrt{5}G_y x z (3x^2 + 3y^2 - 4z^2)}{4}$$

$$\tilde{\mathbb{Q}}_4^{(1,0)}[g](B_1)$$

** symmetry

$$\frac{\sqrt{70}y z (3x^2 - y^2)}{4}$$

** expression

$$\frac{\sqrt{14}G_x(3x^2y^2 - 3x^2z^2 - y^4 + 3y^2z^2)}{8} - \frac{\sqrt{14}G_yxy(3x^2 - y^2 - 6z^2)}{8} + \frac{3\sqrt{14}G_zxz(x^2 - 3y^2)}{8}$$

$$\tilde{Q}_4^{(1,0)}[g](B_2)$$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{14}G_xxy(x^2 - 3y^2 + 6z^2)}{8} - \frac{\sqrt{14}G_y(x^4 - 3x^2y^2 - 3x^2z^2 + 3y^2z^2)}{8} - \frac{3\sqrt{14}G_zyz(3x^2 - y^2)}{8}$$

$$\tilde{Q}_{4,1}^{(1,0)}[g](E_1), \tilde{Q}_{4,2}^{(1,0)}[g](E_1)$$

** symmetry

$$-\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$-\frac{3\sqrt{2}G_xxy(x^2 + y^2 - 6z^2)}{8} + \frac{\sqrt{2}G_y(3x^4 + 3x^2y^2 - 21x^2z^2 - 3y^2z^2 + 4z^4)}{8} + \frac{\sqrt{2}G_zyz(3x^2 + 3y^2 - 4z^2)}{8}$$

$$-\frac{\sqrt{2}G_x(3x^2y^2 - 3x^2z^2 + 3y^4 - 21y^2z^2 + 4z^4)}{8} + \frac{3\sqrt{2}G_yxy(x^2 + y^2 - 6z^2)}{8} - \frac{\sqrt{2}G_zxz(3x^2 + 3y^2 - 4z^2)}{8}$$

$$\tilde{Q}_{4,1}^{(1,0)}[g](E_2, 1), \tilde{Q}_{4,2}^{(1,0)}[g](E_2, 1)$$

** symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x - y)(x + y)}{2}$$

** expression

$$\frac{\sqrt{7}G_xyz(3x^2 - y^2)}{4} + \frac{\sqrt{7}G_yxz(x^2 - 3y^2)}{4} - \sqrt{7}G_zxy(x - y)(x + y)$$

$$-\frac{\sqrt{7}G_xxz(x^2 - 3y^2)}{4} + \frac{\sqrt{7}G_yyz(3x^2 - y^2)}{4} + \frac{\sqrt{7}G_z(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{4}$$

$$\tilde{Q}_{4,1}^{(1,0)}[g](E_2, 2), \tilde{Q}_{4,2}^{(1,0)}[g](E_2, 2)$$

** symmetry

$$-\frac{\sqrt{5}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\frac{G_xyz(3x^2 - 4y^2 + 3z^2)}{2} - \frac{G_yxz(4x^2 - 3y^2 - 3z^2)}{2} + \frac{G_zxy(x^2 + y^2 - 6z^2)}{2}$$

$$-\frac{G_xxz(x^2 + 15y^2 - 6z^2)}{4} + \frac{G_yyz(15x^2 + y^2 - 6z^2)}{4} + \frac{G_z(x - y)(x + y)(x^2 + y^2 - 6z^2)}{4}$$