

\* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

| No. | tag  | matrix (polar)   | det | TR |
|-----|--|--|-----|----|
| 1   | $\{1 0\}$  | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                  | 1   | 1  |
| 2   | $\{2_{100} \frac{1}{2}00\}$                      | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                      | 1   | 1  |
| 3   | $\{2_{010} 0\}$                                  | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                                | 1   | 1  |
| 4   | $\{2_{001} \frac{1}{2}00\}$                      | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                      | 1   | 1  |
| 5   | $\{-1 0\}$                                       | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                               | -1  | 1  |
| 6   | $\{m_{100} \frac{1}{2}00\}$                      | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                       | -1  | 1  |
| 7   | $\{m_{010} 0\}$                                  | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                 | -1  | 1  |
| 8   | $\{m_{001} \frac{1}{2}00\}$                      | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$                       | -1  | 1  |
| 9   | $\{1' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$       | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$    | 1   | -1 |
| 10  | $\{2_{100}' 0\frac{1}{2}\frac{1}{2}\}$           | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$            | 1   | -1 |
| 11  | $\{2_{010}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$  | 1   | -1 |
| 12  | $\{2_{001}' 0\frac{1}{2}\frac{1}{2}\}$           | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$            | 1   | -1 |
| 13  | $\{-1' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$      | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |

*continued ...*

Table 1

| No. | tag  | matrix (polar)   | det | TR |
|-----|--|--|-----|----|
| 14  | $\{m_{100}' 0\frac{1}{2}\frac{1}{2}\}$           | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$           | -1  | -1 |
| 15  | $\{m_{010}' \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 16  | $\{m_{001}' 0\frac{1}{2}\frac{1}{2}\}$           | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$           | -1  | -1 |