

PG No. 1  $C_1$  1 [ triclinic ] (axial, internal axial dipole)

\* Harmonics for rank 0

$$\vec{G}_0^{(1,1)}[g](A)$$

\*\* symmetry

$$1$$

\*\* expression

$$\frac{\sqrt{3}G_x x}{3} + \frac{\sqrt{3}G_y y}{3} + \frac{\sqrt{3}G_z z}{3}$$

\* Harmonics for rank 1

$$\vec{G}_1^{(1,-1)}[g](A, 1)$$

\*\* symmetry

$$x$$

\*\* expression

$$G_x$$

$$\vec{G}_1^{(1,-1)}[g](A, 2)$$

\*\* symmetry

$$y$$

\*\* expression

$$G_y$$

$$\vec{G}_1^{(1,-1)}[g](A, 3)$$

\*\* symmetry

$$z$$

\*\* expression

$$G_z$$

$$\vec{G}_1^{(1,1)}[g](A, 1)$$

\*\* symmetry

$$x$$

\*\* expression

$$\frac{\sqrt{10}G_x (2x^2 - y^2 - z^2)}{10} + \frac{3\sqrt{10}G_y xy}{10} + \frac{3\sqrt{10}G_z xz}{10}$$

$$\vec{G}_1^{(1,1)}[g](A, 2)$$

\*\* symmetry

$$y$$

\*\* expression

$$\frac{3\sqrt{10}G_x xy}{10} - \frac{\sqrt{10}G_y (x^2 - 2y^2 + z^2)}{10} + \frac{3\sqrt{10}G_z yz}{10}$$

$$\vec{G}_1^{(1,1)}[g](A, 3)$$

\*\* symmetry

$$z$$

\*\* expression

$$\frac{3\sqrt{10}G_x xz}{10} + \frac{3\sqrt{10}G_y yz}{10} - \frac{\sqrt{10}G_z (x^2 + y^2 - 2z^2)}{10}$$

\* Harmonics for rank 2

$$\vec{G}_2^{(1,-1)}[g](A, 1)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$-\frac{\sqrt{6}G_x x}{6} - \frac{\sqrt{6}G_y y}{6} + \frac{\sqrt{6}G_z z}{3}$$

$$\vec{\mathbb{G}}_2^{(1,-1)}[g](A, 2)$$

\*\* symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

\*\* expression

$$\frac{\sqrt{2}G_x x}{2} - \frac{\sqrt{2}G_y y}{2}$$

$$\vec{\mathbb{G}}_2^{(1,-1)}[g](A, 3)$$

\*\* symmetry

$$\sqrt{3}yz$$

\*\* expression

$$\frac{\sqrt{2}G_y z}{2} + \frac{\sqrt{2}G_z y}{2}$$

$$\vec{\mathbb{G}}_2^{(1,-1)}[g](A, 4)$$

\*\* symmetry

$$\sqrt{3}xz$$

\*\* expression

$$\frac{\sqrt{2}G_x z}{2} + \frac{\sqrt{2}G_z x}{2}$$

$$\vec{\mathbb{G}}_2^{(1,-1)}[g](A, 5)$$

\*\* symmetry

$$\sqrt{3}xy$$

\*\* expression

$$\frac{\sqrt{2}G_x y}{2} + \frac{\sqrt{2}G_y x}{2}$$

$$\vec{\mathbb{G}}_2^{(1,1)}[g](A, 1)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$-\frac{\sqrt{21}G_x x (x^2 + y^2 - 4z^2)}{14} - \frac{\sqrt{21}G_y y (x^2 + y^2 - 4z^2)}{14} - \frac{\sqrt{21}G_z z (3x^2 + 3y^2 - 2z^2)}{14}$$

$$\vec{\mathbb{G}}_2^{(1,1)}[g](A, 2)$$

\*\* symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

\*\* expression

$$\frac{\sqrt{7}G_x x (3x^2 - 7y^2 - 2z^2)}{14} + \frac{\sqrt{7}G_y y (7x^2 - 3y^2 + 2z^2)}{14} + \frac{5\sqrt{7}G_z z (x-y)(x+y)}{14}$$

$$\vec{\mathbb{G}}_2^{(1,1)}[g](A, 3)$$

\*\* symmetry

$$\sqrt{3}yz$$

\*\* expression

$$\frac{5\sqrt{7}G_x x y z}{7} - \frac{\sqrt{7}G_y z (x^2 - 4y^2 + z^2)}{7} - \frac{\sqrt{7}G_z y (x^2 + y^2 - 4z^2)}{7}$$

$$\vec{\mathbb{G}}_2^{(1,1)}[g](A, 4)$$

\*\* symmetry

$$\sqrt{3}xz$$

\*\* expression

$$\frac{\sqrt{7}G_x z (4x^2 - y^2 - z^2)}{7} + \frac{5\sqrt{7}G_y x y z}{7} - \frac{\sqrt{7}G_z x (x^2 + y^2 - 4z^2)}{7}$$

$$\vec{\mathbb{G}}_2^{(1,1)}[g](A, 5)$$

\*\* symmetry

$$\sqrt{3}xy$$

\*\* expression

$$\frac{\sqrt{7}G_xy(4x^2 - y^2 - z^2)}{7} - \frac{\sqrt{7}G_yx(x^2 - 4y^2 + z^2)}{7} + \frac{5\sqrt{7}G_zxyz}{7}$$

\* Harmonics for rank 3

$$\vec{\mathbb{G}}_3^{(1,-1)}[g](A, 1)$$

\*\* symmetry

$$\sqrt{15}xyz$$

\*\* expression

$$G_xyz + G_yxz + G_zxy$$

$$\vec{\mathbb{G}}_3^{(1,-1)}[g](A, 2)$$

\*\* symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

\*\* expression

$$\frac{\sqrt{15}G_x(2x^2 - y^2 - z^2)}{10} - \frac{\sqrt{15}G_yxy}{5} - \frac{\sqrt{15}G_zxz}{5}$$

$$\vec{\mathbb{G}}_3^{(1,-1)}[g](A, 3)$$

\*\* symmetry

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{15}G_xxy}{5} - \frac{\sqrt{15}G_y(x^2 - 2y^2 + z^2)}{10} - \frac{\sqrt{15}G_zyz}{5}$$

$$\vec{\mathbb{G}}_3^{(1,-1)}[g](A, 4)$$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{15}G_xxz}{5} - \frac{\sqrt{15}G_yyz}{5} - \frac{\sqrt{15}G_z(x^2 + y^2 - 2z^2)}{10}$$

$$\vec{\mathbb{G}}_3^{(1,-1)}[g](A, 5)$$

\*\* symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

\*\* expression

$$\frac{G_x(y-z)(y+z)}{2} + G_yxy - G_zxz$$

$$\vec{\mathbb{G}}_3^{(1,-1)}[g](A, 6)$$

\*\* symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

\*\* expression

$$-G_xxy - \frac{G_y(x-z)(x+z)}{2} + G_zyz$$

$$\vec{\mathbb{G}}_3^{(1,-1)}[g](A, 7)$$

\*\* symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

\*\* expression

$$G_x x z - G_y y z + \frac{G_z (x - y) (x + y)}{2}$$

$$\vec{\mathbb{G}}_3^{(1,1)}[g](A, 1)$$

\*\* symmetry

$$\sqrt{15} x y z$$

\*\* expression

$$\frac{\sqrt{15} G_x y z (6x^2 - y^2 - z^2)}{6} - \frac{\sqrt{15} G_y x z (x^2 - 6y^2 + z^2)}{6} - \frac{\sqrt{15} G_z x y (x^2 + y^2 - 6z^2)}{6}$$

$$\vec{\mathbb{G}}_3^{(1,1)}[g](A, 2)$$

\*\* symmetry

$$\frac{x (2x^2 - 3y^2 - 3z^2)}{2}$$

\*\* expression

$$\frac{G_x (8x^4 - 24x^2 y^2 - 24x^2 z^2 + 3y^4 + 6y^2 z^2 + 3z^4)}{12} + \frac{5G_y x y (4x^2 - 3y^2 - 3z^2)}{12} + \frac{5G_z x z (4x^2 - 3y^2 - 3z^2)}{12}$$

$$\vec{\mathbb{G}}_3^{(1,1)}[g](A, 3)$$

\*\* symmetry

$$-\frac{y (3x^2 - 2y^2 + 3z^2)}{2}$$

\*\* expression

$$-\frac{5G_x x y (3x^2 - 4y^2 + 3z^2)}{12} + \frac{G_y (3x^4 - 24x^2 y^2 + 6x^2 z^2 + 8y^4 - 24y^2 z^2 + 3z^4)}{12} - \frac{5G_z y z (3x^2 - 4y^2 + 3z^2)}{12}$$

$$\vec{\mathbb{G}}_3^{(1,1)}[g](A, 4)$$

\*\* symmetry

$$-\frac{z (3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$-\frac{5G_x x z (3x^2 + 3y^2 - 4z^2)}{12} - \frac{5G_y y z (3x^2 + 3y^2 - 4z^2)}{12} + \frac{G_z (3x^4 + 6x^2 y^2 - 24x^2 z^2 + 3y^4 - 24y^2 z^2 + 8z^4)}{12}$$

$$\vec{\mathbb{G}}_3^{(1,1)}[g](A, 5)$$

\*\* symmetry

$$\frac{\sqrt{15} x (y - z) (y + z)}{2}$$

\*\* expression

$$\frac{\sqrt{15} G_x (y - z) (y + z) (6x^2 - y^2 - z^2)}{12} - \frac{\sqrt{15} G_y x y (2x^2 - 5y^2 + 9z^2)}{12} + \frac{\sqrt{15} G_z x z (2x^2 + 9y^2 - 5z^2)}{12}$$

$$\vec{\mathbb{G}}_3^{(1,1)}[g](A, 6)$$

\*\* symmetry

$$-\frac{\sqrt{15} y (x - z) (x + z)}{2}$$

\*\* expression

$$-\frac{\sqrt{15} G_x x y (5x^2 - 2y^2 - 9z^2)}{12} + \frac{\sqrt{15} G_y (x - z) (x + z) (x^2 - 6y^2 + z^2)}{12} - \frac{\sqrt{15} G_z y z (9x^2 + 2y^2 - 5z^2)}{12}$$

$$\vec{\mathbb{G}}_3^{(1,1)}[g](A, 7)$$

\*\* symmetry

$$\frac{\sqrt{15} z (x - y) (x + y)}{2}$$

\*\* expression

$$\frac{\sqrt{15} G_x x z (5x^2 - 9y^2 - 2z^2)}{12} + \frac{\sqrt{15} G_y y z (9x^2 - 5y^2 + 2z^2)}{12} - \frac{\sqrt{15} G_z (x - y) (x + y) (x^2 + y^2 - 6z^2)}{12}$$

\* Harmonics for rank 4

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](A, 1)$$

\*\* symmetry

$$\frac{\sqrt{21} \left( x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4 \right)}{6}$$

\*\* expression

$$\frac{\sqrt{3}G_{xx} \left( 2x^2 - 3y^2 - 3z^2 \right)}{6} - \frac{\sqrt{3}G_{yy} \left( 3x^2 - 2y^2 + 3z^2 \right)}{6} - \frac{\sqrt{3}G_{zz} \left( 3x^2 + 3y^2 - 2z^2 \right)}{6}$$

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](A, 2)$$

\*\* symmetry

$$- \frac{\sqrt{15} \left( x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4 \right)}{12}$$

\*\* expression

$$- \frac{\sqrt{105}G_{xx} \left( x^2 - 6y^2 + 3z^2 \right)}{42} + \frac{\sqrt{105}G_{yy} \left( 6x^2 - y^2 - 3z^2 \right)}{42} - \frac{\sqrt{105}G_{zz} \left( 3x^2 + 3y^2 - 2z^2 \right)}{42}$$

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](A, 3)$$

\*\* symmetry

$$\frac{\sqrt{5} \left( x - y \right) \left( x + y \right) \left( x^2 + y^2 - 6z^2 \right)}{4}$$

\*\* expression

$$\frac{\sqrt{35}G_{xx} \left( x^2 - 3z^2 \right)}{14} - \frac{\sqrt{35}G_{yy} \left( y^2 - 3z^2 \right)}{14} - \frac{3\sqrt{35}G_{zz} \left( x - y \right) \left( x + y \right)}{14}$$

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](A, 4)$$

\*\* symmetry

$$\frac{\sqrt{35}yz \left( y - z \right) \left( y + z \right)}{2}$$

\*\* expression

$$\frac{\sqrt{5}G_{yz} \left( 3y^2 - z^2 \right)}{4} + \frac{\sqrt{5}G_{zy} \left( y^2 - 3z^2 \right)}{4}$$

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](A, 5)$$

\*\* symmetry

$$- \frac{\sqrt{35}xz \left( x - z \right) \left( x + z \right)}{2}$$

\*\* expression

$$- \frac{\sqrt{5}G_{xz} \left( 3x^2 - z^2 \right)}{4} - \frac{\sqrt{5}G_{zx} \left( x^2 - 3z^2 \right)}{4}$$

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](A, 6)$$

\*\* symmetry

$$\frac{\sqrt{35}xy \left( x - y \right) \left( x + y \right)}{2}$$

\*\* expression

$$\frac{\sqrt{5}G_{xy} \left( 3x^2 - y^2 \right)}{4} + \frac{\sqrt{5}G_{yx} \left( x^2 - 3y^2 \right)}{4}$$

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](A, 7)$$

\*\* symmetry

$$\frac{\sqrt{5}yz \left( 6x^2 - y^2 - z^2 \right)}{2}$$

\*\* expression

$$\frac{3\sqrt{35}G_{xyz}}{7} + \frac{\sqrt{35}G_{yz} \left( 6x^2 - 3y^2 - z^2 \right)}{28} + \frac{\sqrt{35}G_{zy} \left( 6x^2 - y^2 - 3z^2 \right)}{28}$$

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](A, 8)$$

\*\* symmetry

$$- \frac{\sqrt{5}xz \left( x^2 - 6y^2 + z^2 \right)}{2}$$

\*\* expression

$$-\frac{\sqrt{35}G_x z (3x^2 - 6y^2 + z^2)}{28} + \frac{3\sqrt{35}G_y x y z}{7} - \frac{\sqrt{35}G_z x (x^2 - 6y^2 + 3z^2)}{28}$$

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](A, 9)$$

\*\* symmetry

$$-\frac{\sqrt{5}xy (x^2 + y^2 - 6z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{35}G_x y (3x^2 + y^2 - 6z^2)}{28} - \frac{\sqrt{35}G_y x (x^2 + 3y^2 - 6z^2)}{28} + \frac{3\sqrt{35}G_z x y z}{7}$$

$$\vec{\mathbb{G}}_4^{(1,1)}[g](A, 1)$$

\*\* symmetry

$$\frac{\sqrt{21} (x^4 - 3x^2 y^2 - 3x^2 z^2 + y^4 - 3y^2 z^2 + z^4)}{6}$$

\*\* expression

$$\frac{\sqrt{1155}G_x x (x^4 - 5x^2 y^2 - 5x^2 z^2 + 3y^4 - 3y^2 z^2 + 3z^4)}{66} + \frac{\sqrt{1155}G_y y (3x^4 - 5x^2 y^2 - 3x^2 z^2 + y^4 - 5y^2 z^2 + 3z^4)}{66} + \frac{\sqrt{1155}G_z z (3x^4 - 3x^2 y^2 - 5x^2 z^2 + 3y^4 - 5y^2 z^2 + z^4)}{66}$$

$$\vec{\mathbb{G}}_4^{(1,1)}[g](A, 2)$$

\*\* symmetry

$$-\frac{\sqrt{15} (x^4 - 12x^2 y^2 + 6x^2 z^2 + y^4 + 6y^2 z^2 - 2z^4)}{12}$$

\*\* expression

$$-\frac{\sqrt{33}G_x x (5x^4 - 88x^2 y^2 + 38x^2 z^2 + 33y^4 + 66y^2 z^2 - 30z^4)}{132} - \frac{\sqrt{33}G_y y (33x^4 - 88x^2 y^2 + 66x^2 z^2 + 5y^4 + 38y^2 z^2 - 30z^4)}{132} + \frac{\sqrt{33}G_z z (3x^4 + 132x^2 y^2 - 50x^2 z^2 + 3y^4 - 50y^2 z^2 + 10z^4)}{132}$$

$$\vec{\mathbb{G}}_4^{(1,1)}[g](A, 3)$$

\*\* symmetry

$$\frac{\sqrt{5} (x - y) (x + y) (x^2 + y^2 - 6z^2)}{4}$$

\*\* expression

$$\frac{\sqrt{11}G_x x (5x^4 - 4x^2 y^2 - 46x^2 z^2 - 9y^4 + 66y^2 z^2 + 12z^4)}{44} + \frac{\sqrt{11}G_y y (9x^4 + 4x^2 y^2 - 66x^2 z^2 - 5y^4 + 46y^2 z^2 - 12z^4)}{44} + \frac{21\sqrt{11}G_z z (x - y) (x + y) (x^2 + y^2 - 2z^2)}{44}$$

$$\vec{\mathbb{G}}_4^{(1,1)}[g](A, 4)$$

\*\* symmetry

$$\frac{\sqrt{35}yz (y - z) (y + z)}{2}$$

\*\* expression

$$\frac{9\sqrt{77}G_x x y z (y - z) (y + z)}{22} - \frac{\sqrt{77}G_y z (3x^2 y^2 - x^2 z^2 - 6y^4 + 11y^2 z^2 - z^4)}{22} - \frac{\sqrt{77}G_z y (x^2 y^2 - 3x^2 z^2 + y^4 - 11y^2 z^2 + 6z^4)}{22}$$

$$\vec{\mathbb{G}}_4^{(1,1)}[g](A, 5)$$

\*\* symmetry

$$-\frac{\sqrt{35}xz (x - z) (x + z)}{2}$$

\*\* expression

$$-\frac{\sqrt{77}G_x z (6x^4 - 3x^2 y^2 - 11x^2 z^2 + y^2 z^2 + z^4)}{22} - \frac{9\sqrt{77}G_y x y z (x - z) (x + z)}{22} + \frac{\sqrt{77}G_z x (x^4 + x^2 y^2 - 11x^2 z^2 - 3y^2 z^2 + 6z^4)}{22}$$

$$\vec{\mathbb{G}}_4^{(1,1)}[g](A, 6)$$

\*\* symmetry

$$\frac{\sqrt{35}xy (x - y) (x + y)}{2}$$

\*\* expression

$$\frac{\sqrt{77}G_{xy}(6x^4 - 11x^2y^2 - 3x^2z^2 + y^4 + y^2z^2)}{22} - \frac{\sqrt{77}G_yx(x^4 - 11x^2y^2 + x^2z^2 + 6y^4 - 3y^2z^2)}{22} + \frac{9\sqrt{77}G_zxyz(x-y)(x+y)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A, 7)$

\*\* symmetry

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

\*\* expression

$$\frac{21\sqrt{11}G_{xyz}(2x^2 - y^2 - z^2)}{22} - \frac{\sqrt{11}G_yz(6x^4 - 51x^2y^2 + 5x^2z^2 + 6y^4 + 5y^2z^2 - z^4)}{22} - \frac{\sqrt{11}G_zy(6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A, 8)$

\*\* symmetry

$$-\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{11}G_{xz}(6x^4 - 51x^2y^2 + 5x^2z^2 + 6y^4 + 5y^2z^2 - z^4)}{22} - \frac{21\sqrt{11}G_yxyz(x^2 - 2y^2 + z^2)}{22} + \frac{\sqrt{11}G_zx(x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A, 9)$

\*\* symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{11}G_{xy}(6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4)}{22} + \frac{\sqrt{11}G_yx(x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{22} - \frac{21\sqrt{11}G_zxyz(x^2 + y^2 - 2z^2)}{22}$$