Response Tensors up to 4th rank in D_2

— polar tensors —

$$C^{(0,Q)} = \begin{pmatrix} C^{(0,Q)} \end{pmatrix}$$

$$C^{(0,Q)} = Q_0$$

$$S^{(2,Q)} = \begin{pmatrix} S^{(2,Q)}_{xx} & 0 & 0 \\ 0 & S^{(2,Q)}_{yy} & 0 \\ 0 & 0 & S^{(2,Q)}_{zz} \end{pmatrix}$$

$$\begin{aligned} S^{(2,Q)}_{xx} &= Q_0 - Q_u + Q_v \\ S^{(2,Q)}_{yy} &= Q_0 - Q_u - Q_v \\ S^{(2,Q)}_{zz} &= Q_0 + 2Q_u \end{aligned}$$

$$S^{(3,Q)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ S^{(3,Q)}_{4x} & 0 & 0 \\ 0 & S^{(3,Q)}_{5y} & 0 \\ 0 & 0 & S^{(3,Q)}_{6z} \end{pmatrix}$$

$$\begin{aligned} S^{(3,Q)}_{4x} &= -3G_u[1] - G_v[1] + Q_{xyz} \\ S^{(3,Q)}_{5y} &= 3G_u[1] - G_v[1] + Q_{xyz} \\ S^{(3,Q)}_{6z} &= 2G_v[1] + Q_{xyz} \end{aligned}$$

$$A^{(3,Q)} = \begin{pmatrix} A^{(3,Q)}_{4x} & 0 & 0 \\ 0 & A^{(3,Q)}_{5y} & 0 \\ 0 & 0 & A^{(3,Q)}_{6z} \end{pmatrix}$$

$$\begin{aligned} A^{(3,Q)}_{4x} &= G_0 - G_u[2] + G_v[2] \\ A^{(3,Q)}_{5y} &= G_0 - G_u[2] - G_v[2] \\ A^{(3,Q)}_{6z} &= G_0 + 2G_u[2] \end{aligned}$$

$$S^{(4,Q)} = \begin{pmatrix} S^{(4,Q)}_{11} & S^{(4,Q)}_{12} & S^{(4,Q)}_{13} & 0 & 0 & 0 \\ S^{(4,Q)}_{12} & S^{(4,Q)}_{22} & S^{(4,Q)}_{23} & 0 & 0 & 0 \\ S^{(4,Q)}_{13} & S^{(4,Q)}_{23} & S^{(4,Q)}_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S^{(4,Q)}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S^{(4,Q)}_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S^{(4,Q)}_{66} \end{pmatrix}$$

$$
\begin{aligned}
S_{11}^{(4,Q)} &= Q_0[1] + 2Q_0[2] - Q_{4u} + Q_{4v} + 2Q_4 - 2Q_u[1] - 4Q_u[2] + 2Q_v[1] + 4Q_v[2] \\
S_{12}^{(4,Q)} &= Q_0[1] + 2Q_{4u} - Q_4 - 2Q_u[1] \\
S_{13}^{(4,Q)} &= Q_0[1] - Q_{4u} - Q_{4v} - Q_4 + Q_u[1] + Q_v[1] \\
S_{22}^{(4,Q)} &= Q_0[1] + 2Q_0[2] - Q_{4u} - Q_{4v} + 2Q_4 - 2Q_u[1] - 4Q_u[2] - 2Q_v[1] - 4Q_v[2] \\
S_{23}^{(4,Q)} &= Q_0[1] - Q_{4u} + Q_{4v} - Q_4 + Q_u[1] - Q_v[1] \\
S_{33}^{(4,Q)} &= Q_0[1] + 2Q_0[2] + 2Q_{4u} + 2Q_4 + 4Q_u[1] + 8Q_u[2] \\
S_{44}^{(4,Q)} &= Q_0[2] - Q_{4u} + Q_{4v} - Q_4 + Q_u[2] - Q_v[2] \\
S_{55}^{(4,Q)} &= Q_0[2] - Q_{4u} - Q_{4v} - Q_4 + Q_u[2] + Q_v[2] \\
S_{66}^{(4,Q)} &= Q_0[2] + 2Q_{4u} - Q_4 - 2Q_u[2]
\end{aligned}
$$

$$
\bar{S}^{(4,Q)} = \begin{pmatrix}
0 & \bar{S}_{12}^{(4,Q)} & \bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 \\
-\bar{S}_{12}^{(4,Q)} & 0 & \bar{S}_{23}^{(4,Q)} & 0 & 0 & 0 \\
-\bar{S}_{13}^{(4,Q)} & -\bar{S}_{23}^{(4,Q)} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
\begin{aligned}
\bar{S}_{12}^{(4,Q)} &= 4G_{xyz}[1] - 2Q_v[3] \\
\bar{S}_{13}^{(4,Q)} &= -4G_{xyz}[1] + 3Q_u[3] - Q_v[3] \\
\bar{S}_{23}^{(4,Q)} &= 4G_{xyz}[1] + 3Q_u[3] + Q_v[3]
\end{aligned}
$$

$$
A^{(4,Q)} = \begin{pmatrix}
A_{xx}^{(4,Q)} & 0 & 0 \\
0 & A_{yy}^{(4,Q)} & 0 \\
0 & 0 & A_{zz}^{(4,Q)}
\end{pmatrix}
$$

$$
\begin{aligned}
A_{xx}^{(4,Q)} &= Q_0[3] - 2Q_u[6] + 2Q_v[6] \\
A_{yy}^{(4,Q)} &= Q_0[3] - 2Q_u[6] - 2Q_v[6] \\
A_{zz}^{(4,Q)} &= Q_0[3] + 4Q_u[6]
\end{aligned}
$$

$$
M^{(4,Q)} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
M_{4x}^{(4,Q)} & 0 & 0 \\
0 & M_{5y}^{(4,Q)} & 0 \\
0 & 0 & M_{6z}^{(4,Q)}
\end{pmatrix}
$$

$$
\begin{aligned}
M_{4x}^{(4,Q)} &= G_{xyz}[2] - 3Q_u[4] - Q_v[4] \\
M_{5y}^{(4,Q)} &= G_{xyz}[2] + 3Q_u[4] - Q_v[4] \\
M_{6z}^{(4,Q)} &= G_{xyz}[2] + 2Q_v[4]
\end{aligned}
$$

$$
\bar{M}^{(4,Q)} = \begin{pmatrix}
0 & 0 & 0 & \bar{M}_{x4}^{(4,Q)} & 0 & 0 \\
0 & 0 & 0 & 0 & \bar{M}_{y5}^{(4,Q)} & 0 \\
0 & 0 & 0 & 0 & 0 & \bar{M}_{z6}^{(4,Q)}
\end{pmatrix}
$$

$$\bar{M}_{x4}^{(4,Q)} = G_{xyz}[3] - 3Q_u[5] - Q_v[5]$$
$$\bar{M}_{y5}^{(4,Q)} = G_{xyz}[3] + 3Q_u[5] - Q_v[5]$$
$$\bar{M}_{z6}^{(4,Q)} = G_{xyz}[3] + 2Q_v[5]$$

— axial tensors —

$$C^{(0,G)} = \begin{pmatrix} C^{(0,G)} \end{pmatrix}$$

$$C^{(0,G)} = G_0$$

$$S^{(2,G)} = \begin{pmatrix} S^{(2,G)}_{xx} & 0 & 0 \\ 0 & S^{(2,G)}_{yy} & 0 \\ 0 & 0 & S^{(2,G)}_{zz} \end{pmatrix}$$

$$S^{(2,G)}_{xx} = G_0 - G_u + G_v$$
$$S^{(2,G)}_{yy} = G_0 - G_u - G_v$$
$$S^{(2,G)}_{zz} = G_0 + 2G_u$$

$$S^{(3,G)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ S^{(3,G)}_{4x} & 0 & 0 \\ 0 & S^{(3,G)}_{5y} & 0 \\ 0 & 0 & S^{(3,G)}_{6z} \end{pmatrix}$$

$$S^{(3,G)}_{4x} = G_{xyz} - 3Q_u[1] - Q_v[1]$$
$$S^{(3,G)}_{5y} = G_{xyz} + 3Q_u[1] - Q_v[1]$$
$$S^{(3,G)}_{6z} = G_{xyz} + 2Q_v[1]$$

$$A^{(3,G)} = \begin{pmatrix} A^{(3,G)}_{4x} & 0 & 0 \\ 0 & A^{(3,G)}_{5y} & 0 \\ 0 & 0 & A^{(3,G)}_{6z} \end{pmatrix}$$

$$A^{(3,G)}_{4x} = Q_0 - Q_u[2] + Q_v[2]$$
$$A^{(3,G)}_{5y} = Q_0 - Q_u[2] - Q_v[2]$$
$$A^{(3,G)}_{6z} = Q_0 + 2Q_u[2]$$

$$S^{(4,G)} = \begin{pmatrix} S^{(4,G)}_{11} & S^{(4,G)}_{12} & S^{(4,G)}_{13} & 0 & 0 & 0 \\ S^{(4,G)}_{12} & S^{(4,G)}_{22} & S^{(4,G)}_{23} & 0 & 0 & 0 \\ S^{(4,G)}_{13} & S^{(4,G)}_{23} & S^{(4,G)}_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S^{(4,G)}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S^{(4,G)}_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S^{(4,G)}_{66} \end{pmatrix}$$

$$
\begin{aligned}
S^{(4,G)}_{11} &= G_0[1] + 2G_0[2] - G_{4u} + G_{4v} + 2G_4 - 2G_u[1] - 4G_u[2] + 2G_v[1] + 4G_v[2] \\
S^{(4,G)}_{12} &= G_0[1] + 2G_{4u} - G_4 - 2G_u[1] \\
S^{(4,G)}_{13} &= G_0[1] - G_{4u} - G_{4v} - G_4 + G_u[1] + G_v[1] \\
S^{(4,G)}_{22} &= G_0[1] + 2G_0[2] - G_{4u} - G_{4v} + 2G_4 - 2G_u[1] - 4G_u[2] - 2G_v[1] - 4G_v[2] \\
S^{(4,G)}_{23} &= G_0[1] - G_{4u} + G_{4v} - G_4 + G_u[1] - G_v[1] \\
S^{(4,G)}_{33} &= G_0[1] + 2G_0[2] + 2G_{4u} + 2G_4 + 4G_u[1] + 8G_u[2] \\
S^{(4,G)}_{44} &= G_0[2] - G_{4u} + G_{4v} - G_4 + G_u[2] - G_v[2] \\
S^{(4,G)}_{55} &= G_0[2] - G_{4u} - G_{4v} - G_4 + G_u[2] + G_v[2] \\
S^{(4,G)}_{66} &= G_0[2] + 2G_{4u} - G_4 - 2G_u[2]
\end{aligned}
$$

$$
\bar{S}^{(4,G)} = \begin{pmatrix}
0 & \bar{S}^{(4,G)}_{12} & \bar{S}^{(4,G)}_{13} & 0 & 0 & 0 \\
-\bar{S}^{(4,G)}_{12} & 0 & \bar{S}^{(4,G)}_{23} & 0 & 0 & 0 \\
-\bar{S}^{(4,G)}_{13} & -\bar{S}^{(4,G)}_{23} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
\begin{aligned}
\bar{S}^{(4,G)}_{12} &= -2G_v[3] + 4Q_{xyz}[1] \\
\bar{S}^{(4,G)}_{13} &= 3G_u[3] - G_v[3] - 4Q_{xyz}[1] \\
\bar{S}^{(4,G)}_{23} &= 3G_u[3] + G_v[3] + 4Q_{xyz}[1]
\end{aligned}
$$

$$
A^{(4,G)} = \begin{pmatrix}
A^{(4,G)}_{xx} & 0 & 0 \\
0 & A^{(4,G)}_{yy} & 0 \\
0 & 0 & A^{(4,G)}_{zz}
\end{pmatrix}
$$

$$
\begin{aligned}
A^{(4,G)}_{xx} &= G_0[3] - 2G_u[6] + 2G_v[6] \\
A^{(4,G)}_{yy} &= G_0[3] - 2G_u[6] - 2G_v[6] \\
A^{(4,G)}_{zz} &= G_0[3] + 4G_u[6]
\end{aligned}
$$

$$
M^{(4,G)} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
M^{(4,G)}_{4x} & 0 & 0 \\
0 & M^{(4,G)}_{5y} & 0 \\
0 & 0 & M^{(4,G)}_{6z}
\end{pmatrix}
$$

$$
\begin{aligned}
M^{(4,G)}_{4x} &= -3G_u[4] - G_v[4] + Q_{xyz}[2] \\
M^{(4,G)}_{5y} &= 3G_u[4] - G_v[4] + Q_{xyz}[2] \\
M^{(4,G)}_{6z} &= 2G_v[4] + Q_{xyz}[2]
\end{aligned}
$$

$$
\bar{M}^{(4,G)} = \begin{pmatrix}
0 & 0 & 0 & \bar{M}^{(4,G)}_{x4} & 0 & 0 \\
0 & 0 & 0 & 0 & \bar{M}^{(4,G)}_{y5} & 0 \\
0 & 0 & 0 & 0 & 0 & \bar{M}^{(4,G)}_{z6}
\end{pmatrix}
$$

$$\bar{M}_{x4}^{(4,G)} = -3G_u[5] - G_v[5] + Q_{xyz}[3]$$
$$\bar{M}_{y5}^{(4,G)} = 3G_u[5] - G_v[5] + Q_{xyz}[3]$$
$$\bar{M}_{z6}^{(4,G)} = 2G_v[5] + Q_{xyz}[3]$$