Response Tensors up to 4th rank in S_4

— polar tensors —

$$C^{(0,Q)} = \begin{pmatrix} C^{(0,Q)} \end{pmatrix}$$

$$C^{(0,Q)} = Q_0$$

$$S^{(2,Q)} = \begin{pmatrix} S_{xx}^{(2,Q)} & 0 & 0 \\ 0 & S_{xx}^{(2,Q)} & 0 \\ 0 & 0 & S_{zz}^{(2,Q)} \end{pmatrix}$$

$$S_{xx}^{(2,Q)} = Q_0 - Q_u$$
$$S_{zz}^{(2,Q)} = Q_0 + 2Q_u$$

$$A^{(2,Q)} = \begin{pmatrix} 0 & A_{xy}^{(2,Q)} & 0 \\ -A_{xy}^{(2,Q)} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{xy}^{(2,Q)} = G_z$$

$$S^{(3,Q)} = \begin{pmatrix} 0 & 0 & S_{1z}^{(3,Q)} \\ 0 & 0 & -S_{1z}^{(3,Q)} \\ 0 & 0 & 0 \\ S_{4x}^{(3,Q)} & S_{4y}^{(3,Q)} & 0 \\ -S_{4y}^{(3,Q)} & S_{4x}^{(3,Q)} & 0 \\ 0 & 0 & S_{6z}^{(3,Q)} \end{pmatrix}$$

$$S_{1z}^{(3,Q)} = -2G_{xy}[1] + Q_z^{\beta}$$
$$S_{4x}^{(3,Q)} = -G_v[1] + Q_{xyz}$$
$$S_{4y}^{(3,Q)} = -G_{xy}[1] - Q_z^{\beta}$$
$$S_{6z}^{(3,Q)} = 2G_v[1] + Q_{xyz}$$

$$A^{(3,Q)} = \begin{pmatrix} A_{4x}^{(3,Q)} & A_{4y}^{(3,Q)} & 0 \\ A_{4y}^{(3,Q)} & -A_{4x}^{(3,Q)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{4x}^{(3,Q)} = G_v[2]$$
$$A_{4y}^{(3,Q)} = G_{xy}[2]$$

$$S^{(4,Q)} = \begin{pmatrix} S_{11}^{(4,Q)} & S_{12}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & 0 & S_{16}^{(4,Q)} \\ S_{12}^{(4,Q)} & S_{11}^{(4,Q)} & S_{13}^{(4,Q)} & 0 & 0 & -S_{16}^{(4,Q)} \\ S_{13}^{(4,Q)} & S_{13}^{(4,Q)} & S_{33}^{(4,Q)} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44}^{(4,Q)} & 0 \\ S_{16}^{(4,Q)} & -S_{16}^{(4,Q)} & 0 & 0 & 0 & S_{66}^{(4,Q)} \end{pmatrix}$$

$$
\begin{aligned}
S_{11}^{(4,Q)} &= Q_0[1] + 2Q_0[2] - Q_{4u} + 2Q_4 - 2Q_u[1] - 4Q_u[2] \\
S_{12}^{(4,Q)} &= Q_0[1] + 2Q_{4u} - Q_4 - 2Q_u[1] \\
S_{13}^{(4,Q)} &= Q_0[1] - Q_{4u} - Q_4 + Q_u[1] \\
S_{16}^{(4,Q)} &= Q_{4z}^{\alpha} \\
S_{33}^{(4,Q)} &= Q_0[1] + 2Q_0[2] + 2Q_{4u} + 2Q_4 + 4Q_u[1] + 8Q_u[2] \\
S_{44}^{(4,Q)} &= Q_0[2] - Q_{4u} - Q_4 + Q_u[2] \\
S_{66}^{(4,Q)} &= Q_0[2] + 2Q_{4u} - Q_4 - 2Q_u[2]
\end{aligned}
$$

$$
\bar{S}^{(4,Q)} = \begin{pmatrix}
0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & 0 & \bar{S}_{16}^{(4,Q)} \\
0 & 0 & \bar{S}_{13}^{(4,Q)} & 0 & 0 & -\bar{S}_{16}^{(4,Q)} \\
-\bar{S}_{13}^{(4,Q)} & -\bar{S}_{13}^{(4,Q)} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \bar{S}_{45}^{(4,Q)} & 0 \\
0 & 0 & 0 & -\bar{S}_{45}^{(4,Q)} & 0 & 0 \\
-\bar{S}_{16}^{(4,Q)} & \bar{S}_{16}^{(4,Q)} & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
\begin{aligned}
\bar{S}_{13}^{(4,Q)} &= 3Q_u[3] \\
\bar{S}_{16}^{(4,Q)} &= 2G_z[1] - 2G_z^{\alpha}[1] \\
\bar{S}_{45}^{(4,Q)} &= -G_z[1] - 4G_z^{\alpha}[1]
\end{aligned}
$$

$$
A^{(4,Q)} = \begin{pmatrix}
A_{xx}^{(4,Q)} & 0 & 0 \\
0 & A_{xx}^{(4,Q)} & 0 \\
0 & 0 & A_{zz}^{(4,Q)}
\end{pmatrix}
$$

$$
\begin{aligned}
A_{xx}^{(4,Q)} &= Q_0[3] - 2Q_u[6] \\
A_{zz}^{(4,Q)} &= Q_0[3] + 4Q_u[6]
\end{aligned}
$$

$$
\bar{A}^{(4,Q)} = \begin{pmatrix}
0 & \bar{A}_{xy}^{(4,Q)} & 0 \\
-\bar{A}_{xy}^{(4,Q)} & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
$$

$$
\bar{A}_{xy}^{(4,Q)} = G_z[6]
$$

$$
M^{(4,Q)} = \begin{pmatrix}
0 & 0 & M_{1z}^{(4,Q)} \\
0 & 0 & M_{1z}^{(4,Q)} \\
0 & 0 & M_{3z}^{(4,Q)} \\
M_{4x}^{(4,Q)} & M_{4y}^{(4,Q)} & 0 \\
M_{4y}^{(4,Q)} & -M_{4x}^{(4,Q)} & 0 \\
0 & 0 & 0
\end{pmatrix}
$$

$$
\begin{aligned}
M_{1z}^{(4,Q)} &= G_z[3] - G_z^{\alpha}[2] \\
M_{3z}^{(4,Q)} &= 2G_z[2] + G_z[3] + 2G_z^{\alpha}[2] \\
M_{4x}^{(4,Q)} &= -3Q_u[4] \\
M_{4y}^{(4,Q)} &= G_z[2] - G_z^{\alpha}[2]
\end{aligned}
$$

$$\bar{M}^{(4,Q)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}_{x4}^{(4,Q)} & \bar{M}_{x5}^{(4,Q)} & 0 \\ 0 & 0 & 0 & \bar{M}_{x5}^{(4,Q)} & -\bar{M}_{x4}^{(4,Q)} & 0 \\ \bar{M}_{z1}^{(4,Q)} & \bar{M}_{z1}^{(4,Q)} & \bar{M}_{z3}^{(4,Q)} & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \bar{M}_{x4}^{(4,Q)} &= -3Q_u[5] \\ \bar{M}_{x5}^{(4,Q)} &= G_z[4] - G_z^{\alpha}[3] \\ \bar{M}_{z1}^{(4,Q)} &= G_z[5] - G_z^{\alpha}[3] \\ \bar{M}_{z3}^{(4,Q)} &= 2G_z[4] + G_z[5] + 2G_z^{\alpha}[3] \end{aligned}$$

— axial tensors —

$$C^{(1,G)} = \begin{pmatrix} 0 & 0 & C_z^{(1,G)} \end{pmatrix}$$

$$C_z^{(1,G)} = G_z$$

$$S^{(2,G)} = \begin{pmatrix} S_{xx}^{(2,G)} & S_{xy}^{(2,G)} & 0 \\ S_{xy}^{(2,G)} & -S_{xx}^{(2,G)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_{xx}^{(2,G)} = G_v$$
$$S_{xy}^{(2,G)} = G_{xy}$$

$$S^{(3,G)} = \begin{pmatrix} 0 & 0 & S_{1z}^{(3,G)} \\ 0 & 0 & S_{1z}^{(3,G)} \\ 0 & 0 & S_{3z}^{(3,G)} \\ S_{4x}^{(3,G)} & S_{4y}^{(3,G)} & 0 \\ S_{4y}^{(3,G)} & -S_{4x}^{(3,G)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_{1z}^{(3,G)} = G_z[2] - G_z^{\alpha}$$
$$S_{3z}^{(3,G)} = 2G_z[1] + G_z[2] + 2G_z^{\alpha}$$
$$S_{4x}^{(3,G)} = -3Q_u[1]$$
$$S_{4y}^{(3,G)} = G_z[1] - G_z^{\alpha}$$

$$A^{(3,G)} = \begin{pmatrix} A_{4x}^{(3,G)} & A_{4y}^{(3,G)} & 0 \\ -A_{4y}^{(3,G)} & A_{4x}^{(3,G)} & 0 \\ 0 & 0 & A_{6z}^{(3,G)} \end{pmatrix}$$

$$A_{4x}^{(3,G)} = Q_0 - Q_u[2]$$
$$A_{4y}^{(3,G)} = G_z[3]$$
$$A_{6z}^{(3,G)} = Q_0 + 2Q_u[2]$$

$$S^{(4,G)} = \begin{pmatrix} S_{11}^{(4,G)} & 0 & S_{13}^{(4,G)} & 0 & 0 & S_{16}^{(4,G)} \\ 0 & -S_{11}^{(4,G)} & -S_{13}^{(4,G)} & 0 & 0 & S_{16}^{(4,G)} \\ S_{13}^{(4,G)} & -S_{13}^{(4,G)} & 0 & 0 & 0 & S_{36}^{(4,G)} \\ 0 & 0 & 0 & S_{44}^{(4,G)} & S_{45}^{(4,G)} & 0 \\ 0 & 0 & 0 & S_{45}^{(4,G)} & -S_{44}^{(4,G)} & 0 \\ S_{16}^{(4,G)} & S_{16}^{(4,G)} & S_{36}^{(4,G)} & 0 & 0 & 0 \end{pmatrix}$$

$$S_{11}^{(4,G)} = G_{4v} + 2G_v[1] + 4G_v[2]$$
$$S_{13}^{(4,G)} = -G_{4v} + G_v[1]$$
$$S_{16}^{(4,G)} = -G_{4z}^{\beta} + G_{xy}[1] + 2G_{xy}[2]$$
$$S_{36}^{(4,G)} = 2G_{4z}^{\beta} + G_{xy}[1]$$
$$S_{44}^{(4,G)} = G_{4v} - G_v[2]$$
$$S_{45}^{(4,G)} = 2G_{4z}^{\beta} + G_{xy}[2]$$

$$\bar{S}^{(4,G)} = \begin{pmatrix} 0 & \bar{S}_{12}^{(4,G)} & \bar{S}_{13}^{(4,G)} & 0 & 0 & \bar{S}_{16}^{(4,G)} \\ -\bar{S}_{12}^{(4,G)} & 0 & -\bar{S}_{13}^{(4,G)} & 0 & 0 & \bar{S}_{16}^{(4,G)} \\ -\bar{S}_{13}^{(4,G)} & \bar{S}_{13}^{(4,G)} & 0 & 0 & 0 & \bar{S}_{36}^{(4,G)} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\bar{S}_{16}^{(4,G)} & -\bar{S}_{16}^{(4,G)} & -\bar{S}_{36}^{(4,G)} & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{S}_{12}^{(4,G)} = -2G_v[3] + 4Q_{xyz}[1]$$
$$\bar{S}_{13}^{(4,G)} = -G_v[3] - 4Q_{xyz}[1]$$
$$\bar{S}_{16}^{(4,G)} = G_{xy}[3] + 2Q_z^{\beta}[1]$$
$$\bar{S}_{36}^{(4,G)} = G_{xy}[3] - 4Q_z^{\beta}[1]$$

$$A^{(4,G)} = \begin{pmatrix} A_{xx}^{(4,G)} & A_{xy}^{(4,G)} & 0 \\ A_{xy}^{(4,G)} & -A_{xx}^{(4,G)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{xx}^{(4,G)} = 2G_v[6]$$
$$A_{xy}^{(4,G)} = 2G_{xy}[6]$$

$$M^{(4,G)} = \begin{pmatrix} 0 & 0 & M_{1z}^{(4,G)} \\ 0 & 0 & -M_{1z}^{(4,G)} \\ 0 & 0 & 0 \\ M_{4x}^{(4,G)} & M_{4y}^{(4,G)} & 0 \\ -M_{4y}^{(4,G)} & M_{4x}^{(4,G)} & 0 \\ 0 & 0 & M_{6z}^{(4,G)} \end{pmatrix}$$

$$M_{1z}^{(4,G)} = -2G_{xy}[4] + Q_z^{\beta}[2]$$
$$M_{4x}^{(4,G)} = -G_v[4] + Q_{xyz}[2]$$
$$M_{4y}^{(4,G)} = -G_{xy}[4] - Q_z^{\beta}[2]$$
$$M_{6z}^{(4,G)} = 2G_v[4] + Q_{xyz}[2]$$

$$\bar{M}^{(4,G)} = \begin{pmatrix} 0 & 0 & 0 & \bar{M}_{x4}^{(4,G)} & \bar{M}_{x5}^{(4,G)} & 0 \\ 0 & 0 & 0 & -\bar{M}_{x5}^{(4,G)} & \bar{M}_{x4}^{(4,G)} & 0 \\ \bar{M}_{z1}^{(4,G)} & -\bar{M}_{z1}^{(4,G)} & 0 & 0 & 0 & \bar{M}_{z6}^{(4,G)} \end{pmatrix}$$

$$\bar{M}_{x4}^{(4,G)} = -G_v[5] + Q_{xyz}[3]$$
$$\bar{M}_{x5}^{(4,G)} = G_{xy}[5] + Q_z^{\beta}[3]$$
$$\bar{M}_{z1}^{(4,G)} = -2G_{xy}[5] + Q_z^{\beta}[3]$$
$$\bar{M}_{z6}^{(4,G)} = 2G_v[5] + Q_{xyz}[3]$$