

# No. 1 $C_1$ 1 [ triclinic ] (polar)

表 1 rank 0

| No. | irrep. | (tag) | mul. | comp. | harmonics     | (tag)          | definition |
|-----|--------|-------|------|-------|---------------|----------------|------------|
| 1   | $A$    | $A$   | —    | —     | $Q_0^{(h,A)}$ | $Qh(0, A, , )$ | $C_0$      |

表 2 rank 1

| No. | irrep. | (tag) | mul. | comp. | harmonics       | (tag)           | definition |
|-----|--------|-------|------|-------|-----------------|-----------------|------------|
| 2   | $A$    | $A$   | 1    | —     | $Q_1^{(h,A,1)}$ | $Qh(1, A, 1, )$ | $C_1$      |
| 3   | $A$    | $A$   | 2    | —     | $Q_1^{(h,A,2)}$ | $Qh(1, A, 2, )$ | $S_1$      |
| 4   | $A$    | $A$   | 3    | —     | $Q_1^{(h,A,3)}$ | $Qh(1, A, 3, )$ | $C_0$      |

表 3 rank 2

| No. | irrep. | (tag) | mul. | comp. | harmonics       | (tag)           | definition |
|-----|--------|-------|------|-------|-----------------|-----------------|------------|
| 5   | $A$    | $A$   | 1    | —     | $Q_2^{(h,A,1)}$ | $Qh(2, A, 1, )$ | $C_0$      |
| 6   | $A$    | $A$   | 2    | —     | $Q_2^{(h,A,2)}$ | $Qh(2, A, 2, )$ | $C_2$      |
| 7   | $A$    | $A$   | 3    | —     | $Q_2^{(h,A,3)}$ | $Qh(2, A, 3, )$ | $S_1$      |
| 8   | $A$    | $A$   | 4    | —     | $Q_2^{(h,A,4)}$ | $Qh(2, A, 4, )$ | $C_1$      |
| 9   | $A$    | $A$   | 5    | —     | $Q_2^{(h,A,5)}$ | $Qh(2, A, 5, )$ | $S_2$      |

表 4 rank 3

| No. | irrep. | (tag) | mul. | comp. | harmonics       | (tag)           | definition  |
|-----|--------|-------|------|-------|-----------------|-----------------|---|
| 10  | $A$    | $A$   | 1    | —     | $Q_3^{(h,A,1)}$ | $Qh(3, A, 1, )$ | $S_2$   |
| 11  | $A$    | $A$   | 2    | —     | $Q_3^{(h,A,2)}$ | $Qh(3, A, 2, )$ | $-\frac{\sqrt{6}C_1}{4} + \frac{\sqrt{10}C_3}{4}$ |
| 12  | $A$    | $A$   | 3    | —     | $Q_3^{(h,A,3)}$ | $Qh(3, A, 3, )$ | $-\frac{\sqrt{6}S_1}{4} - \frac{\sqrt{10}S_3}{4}$ |
| 13  | $A$    | $A$   | 4    | —     | $Q_3^{(h,A,4)}$ | $Qh(3, A, 4, )$ | $C_0$   |
| 14  | $A$    | $A$   | 5    | —     | $Q_3^{(h,A,5)}$ | $Qh(3, A, 5, )$ | $-\frac{\sqrt{10}C_1}{4} - \frac{\sqrt{6}C_3}{4}$ |
| 15  | $A$    | $A$   | 6    | —     | $Q_3^{(h,A,6)}$ | $Qh(3, A, 6, )$ | $\frac{\sqrt{10}S_1}{4} - \frac{\sqrt{6}S_3}{4}$  |
| 16  | $A$    | $A$   | 7    | —     | $Q_3^{(h,A,7)}$ | $Qh(3, A, 7, )$ | $C_2$   |

表 5 rank 4

| No. | irrep. | (tag) | mul. | comp. | harmonics       | (tag)           | definition  |
|-----|--------|-------|------|-------|-----------------|-----------------|---|
| 17  | $A$    | $A$   | 1    | —     | $Q_4^{(h,A,1)}$ | $Qh(4, A, 1, )$ | $\frac{\sqrt{21}C_0}{6} + \frac{\sqrt{15}C_4}{6}$ |
| 18  | $A$    | $A$   | 2    | —     | $Q_4^{(h,A,2)}$ | $Qh(4, A, 2, )$ | $\frac{\sqrt{15}C_0}{6} - \frac{\sqrt{21}C_4}{6}$ |
| 19  | $A$    | $A$   | 3    | —     | $Q_4^{(h,A,3)}$ | $Qh(4, A, 3, )$ | $-C_2$  |
| 20  | $A$    | $A$   | 4    | —     | $Q_4^{(h,A,4)}$ | $Qh(4, A, 4, )$ | $-\frac{\sqrt{14}S_1}{4} - \frac{\sqrt{2}S_3}{4}$ |
| 21  | $A$    | $A$   | 5    | —     | $Q_4^{(h,A,5)}$ | $Qh(4, A, 5, )$ | $\frac{\sqrt{14}C_1}{4} - \frac{\sqrt{2}C_3}{4}$  |
| 22  | $A$    | $A$   | 6    | —     | $Q_4^{(h,A,6)}$ | $Qh(4, A, 6, )$ | $S_4$   |
| 23  | $A$    | $A$   | 7    | —     | $Q_4^{(h,A,7)}$ | $Qh(4, A, 7, )$ | $-\frac{\sqrt{2}S_1}{4} + \frac{\sqrt{14}S_3}{4}$ |
| 24  | $A$    | $A$   | 8    | —     | $Q_4^{(h,A,8)}$ | $Qh(4, A, 8, )$ | $-\frac{\sqrt{2}C_1}{4} - \frac{\sqrt{14}C_3}{4}$ |
| 25  | $A$    | $A$   | 9    | —     | $Q_4^{(h,A,9)}$ | $Qh(4, A, 9, )$ | $S_2$   |

表 6 rank 5

| No. | irrep. | (tag) | mul. | comp. | harmonics        | (tag)          | definition  |
|-----|--------|-------|------|-------|------------------|----------------|---|
| 26  | A      | A     | 1    | —     | $Q_5^{(h,A,1)}$  | Qh(5, A, 1, )  | $S_4$   |
| 27  | A      | A     | 2    | —     | $Q_5^{(h,A,2)}$  | Qh(5, A, 2, )  | $-S_2$  |
| 28  | A      | A     | 3    | —     | $Q_5^{(h,A,3)}$  | Qh(5, A, 3, )  | $\frac{\sqrt{15}C_1}{8} - \frac{\sqrt{70}C_3}{16} + \frac{3\sqrt{14}C_5}{16}$ |
| 29  | A      | A     | 4    | —     | $Q_5^{(h,A,4)}$  | Qh(5, A, 4, )  | $\frac{\sqrt{15}S_1}{8} + \frac{\sqrt{70}S_3}{16} + \frac{3\sqrt{14}S_5}{16}$ |
| 30  | A      | A     | 5    | —     | $Q_5^{(h,A,5)}$  | Qh(5, A, 5, )  | $C_0$   |
| 31  | A      | A     | 6    | —     | $Q_5^{(h,A,6)}$  | Qh(5, A, 6, )  | $\frac{\sqrt{21}C_1}{8} + \frac{9\sqrt{2}C_3}{16} + \frac{\sqrt{10}C_5}{16}$  |
| 32  | A      | A     | 7    | —     | $Q_5^{(h,A,7)}$  | Qh(5, A, 7, )  | $\frac{\sqrt{21}S_1}{8} - \frac{9\sqrt{2}S_3}{16} + \frac{\sqrt{10}S_5}{16}$  |
| 33  | A      | A     | 8    | —     | $Q_5^{(h,A,8)}$  | Qh(5, A, 8, )  | $C_4$   |
| 34  | A      | A     | 9    | —     | $Q_5^{(h,A,9)}$  | Qh(5, A, 9, )  | $\frac{\sqrt{7}C_1}{4} - \frac{\sqrt{6}C_3}{8} - \frac{\sqrt{30}C_5}{8}$      |
| 35  | A      | A     | 10   | —     | $Q_5^{(h,A,10)}$ | Qh(5, A, 10, ) | $-\frac{\sqrt{7}S_1}{4} - \frac{\sqrt{6}S_3}{8} + \frac{\sqrt{30}S_5}{8}$     |
| 36  | A      | A     | 11   | —     | $Q_5^{(h,A,11)}$ | Qh(5, A, 11, ) | $C_2$   |

表 7 rank 6

| No. | irrep. | (tag) | mul. | comp. | harmonics        | (tag)          | definition  |
|-----|--------|-------|------|-------|------------------|----------------|---|
| 37  | A      | A     | 1    | —     | $Q_6^{(h,A,1)}$  | Qh(6, A, 1, )  | $\frac{\sqrt{2}C_0}{4} - \frac{\sqrt{14}C_4}{4}$                              |
| 38  | A      | A     | 2    | —     | $Q_6^{(h,A,2)}$  | Qh(6, A, 2, )  | $\frac{\sqrt{11}C_2}{4} - \frac{\sqrt{5}C_6}{4}$                              |
| 39  | A      | A     | 3    | —     | $Q_6^{(h,A,3)}$  | Qh(6, A, 3, )  | $\frac{\sqrt{14}C_0}{4} + \frac{\sqrt{2}C_4}{4}$                              |
| 40  | A      | A     | 4    | —     | $Q_6^{(h,A,4)}$  | Qh(6, A, 4, )  | $\frac{\sqrt{5}C_2}{4} + \frac{\sqrt{11}C_6}{4}$                              |
| 41  | A      | A     | 5    | —     | $Q_6^{(h,A,5)}$  | Qh(6, A, 5, )  | $\frac{\sqrt{3}S_1}{4} - \frac{\sqrt{30}S_3}{8} - \frac{\sqrt{22}S_5}{8}$     |
| 42  | A      | A     | 6    | —     | $Q_6^{(h,A,6)}$  | Qh(6, A, 6, )  | $-\frac{\sqrt{3}C_1}{4} - \frac{\sqrt{30}C_3}{8} + \frac{\sqrt{22}C_5}{8}$    |
| 43  | A      | A     | 7    | —     | $Q_6^{(h,A,7)}$  | Qh(6, A, 7, )  | $S_4$   |
| 44  | A      | A     | 8    | —     | $Q_6^{(h,A,8)}$  | Qh(6, A, 8, )  | $\frac{3\sqrt{22}S_1}{16} + \frac{\sqrt{55}S_3}{16} + \frac{\sqrt{3}S_5}{16}$ |
| 45  | A      | A     | 9    | —     | $Q_6^{(h,A,9)}$  | Qh(6, A, 9, )  | $\frac{3\sqrt{22}C_1}{16} - \frac{\sqrt{55}C_3}{16} + \frac{\sqrt{3}C_5}{16}$ |
| 46  | A      | A     | 10   | —     | $Q_6^{(h,A,10)}$ | Qh(6, A, 10, ) | $S_6$   |
| 47  | A      | A     | 11   | —     | $Q_6^{(h,A,11)}$ | Qh(6, A, 11, ) | $\frac{\sqrt{10}S_1}{16} - \frac{9S_3}{16} + \frac{\sqrt{165}S_5}{16}$        |
| 48  | A      | A     | 12   | —     | $Q_6^{(h,A,12)}$ | Qh(6, A, 12, ) | $\frac{\sqrt{10}C_1}{16} + \frac{9C_3}{16} + \frac{\sqrt{165}C_5}{16}$        |
| 49  | A      | A     | 13   | —     | $Q_6^{(h,A,13)}$ | Qh(6, A, 13, ) | $S_2$   |

表 8 rank 7

| No. | irrep. | (tag) | mul. | comp. | harmonics        | (tag)          | definition   |
|-----|--------|-------|------|-------|------------------|----------------|--|
| 50  | A      | A     | 1    | —     | $Q_7^{(h,A,1)}$  | Qh(7, A, 1, )  | $\frac{\sqrt{78}S_2}{12} + \frac{\sqrt{66}S_6}{12}$  |
| 51  | A      | A     | 2    | —     | $Q_7^{(h,A,2)}$  | Qh(7, A, 2, )  | $S_4$  |
| 52  | A      | A     | 3    | —     | $Q_7^{(h,A,3)}$  | Qh(7, A, 3, )  | $\frac{\sqrt{66}S_2}{12} - \frac{\sqrt{78}S_6}{12}$  |
| 53  | A      | A     | 4    | —     | $Q_7^{(h,A,4)}$  | Qh(7, A, 4, )  | $-\frac{5\sqrt{7}C_1}{32} + \frac{3\sqrt{21}C_3}{32} - \frac{\sqrt{231}C_5}{32} + \frac{\sqrt{429}C_7}{32}$  |
| 54  | A      | A     | 5    | —     | $Q_7^{(h,A,5)}$  | Qh(7, A, 5, )  | $-\frac{5\sqrt{7}S_1}{32} - \frac{3\sqrt{21}S_3}{32} - \frac{\sqrt{231}S_5}{32} - \frac{\sqrt{429}S_7}{32}$  |
| 55  | A      | A     | 6    | —     | $Q_7^{(h,A,6)}$  | Qh(7, A, 6, )  | $C_0$  |
| 56  | A      | A     | 7    | —     | $Q_7^{(h,A,7)}$  | Qh(7, A, 7, )  | $-\frac{3\sqrt{33}C_1}{32} - \frac{\sqrt{11}C_3}{32} + \frac{25C_5}{32} + \frac{\sqrt{91}C_7}{32}$           |
| 57  | A      | A     | 8    | —     | $Q_7^{(h,A,8)}$  | Qh(7, A, 8, )  | $-\frac{3\sqrt{33}S_1}{32} + \frac{\sqrt{11}S_3}{32} + \frac{25S_5}{32} - \frac{\sqrt{91}S_7}{32}$           |
| 58  | A      | A     | 9    | —     | $Q_7^{(h,A,9)}$  | Qh(7, A, 9, )  | $C_4$  |
| 59  | A      | A     | 10   | —     | $Q_7^{(h,A,10)}$ | Qh(7, A, 10, ) | $-\frac{\sqrt{858}C_1}{64} - \frac{3\sqrt{286}C_3}{64} - \frac{5\sqrt{26}C_5}{64} - \frac{\sqrt{14}C_7}{64}$ |
| 60  | A      | A     | 11   | —     | $Q_7^{(h,A,11)}$ | Qh(7, A, 11, ) | $\frac{\sqrt{858}S_1}{64} - \frac{3\sqrt{286}S_3}{64} + \frac{5\sqrt{26}S_5}{64} - \frac{\sqrt{14}S_7}{64}$  |
| 61  | A      | A     | 12   | —     | $Q_7^{(h,A,12)}$ | Qh(7, A, 12, ) | $C_6$  |
| 62  | A      | A     | 13   | —     | $Q_7^{(h,A,13)}$ | Qh(7, A, 13, ) | $-\frac{15\sqrt{6}C_1}{64} + \frac{19\sqrt{2}C_3}{64} - \frac{\sqrt{22}C_5}{64} - \frac{\sqrt{2002}C_7}{64}$ |
| 63  | A      | A     | 14   | —     | $Q_7^{(h,A,14)}$ | Qh(7, A, 14, ) | $\frac{15\sqrt{6}S_1}{64} + \frac{19\sqrt{2}S_3}{64} + \frac{\sqrt{22}S_5}{64} - \frac{\sqrt{2002}S_7}{64}$  |
| 64  | A      | A     | 15   | —     | $Q_7^{(h,A,15)}$ | Qh(7, A, 15, ) | $C_2$  |

表 9 rank 8

| No. | irrep. | (tag) | mul. | comp. | harmonics        | (tag)          | definition  |
|-----|--------|-------|------|-------|------------------|----------------|---|
| 65  | A      | A     | 1    | —     | $Q_8^{(h,A,1)}$  | Qh(8, A, 1, )  | $\frac{\sqrt{33}C_0}{8} + \frac{\sqrt{21}C_4}{12} + \frac{\sqrt{195}C_8}{24}$                                 |
| 66  | A      | A     | 2    | —     | $Q_8^{(h,A,2)}$  | Qh(8, A, 2, )  | $-\frac{\sqrt{286}C_0}{32} + \frac{\sqrt{182}C_4}{16} + \frac{\sqrt{10}C_8}{32}$                              |
| 67  | A      | A     | 3    | —     | $Q_8^{(h,A,3)}$  | Qh(8, A, 3, )  | $C_6$   |
| 68  | A      | A     | 4    | —     | $Q_8^{(h,A,4)}$  | Qh(8, A, 4, )  | $-\frac{\sqrt{210}C_0}{32} - \frac{\sqrt{330}C_4}{48} + \frac{\sqrt{6006}C_8}{96}$                            |
| 69  | A      | A     | 5    | —     | $Q_8^{(h,A,5)}$  | Qh(8, A, 5, )  | $C_2$   |
| 70  | A      | A     | 6    | —     | $Q_8^{(h,A,6)}$  | Qh(8, A, 6, )  | $-\frac{\sqrt{715}S_1}{32} - \frac{\sqrt{273}S_3}{32} - \frac{\sqrt{35}S_5}{32} - \frac{S_7}{32}$             |
| 71  | A      | A     | 7    | —     | $Q_8^{(h,A,7)}$  | Qh(8, A, 7, )  | $\frac{\sqrt{715}C_1}{32} - \frac{\sqrt{273}C_3}{32} + \frac{\sqrt{35}C_5}{32} - \frac{C_7}{32}$              |
| 72  | A      | A     | 8    | —     | $Q_8^{(h,A,8)}$  | Qh(8, A, 8, )  | $S_8$   |
| 73  | A      | A     | 9    | —     | $Q_8^{(h,A,9)}$  | Qh(8, A, 9, )  | $-\frac{\sqrt{77}S_1}{32} + \frac{5\sqrt{15}S_3}{32} - \frac{3\sqrt{13}S_5}{32} - \frac{\sqrt{455}S_7}{32}$   |
| 74  | A      | A     | 10   | —     | $Q_8^{(h,A,10)}$ | Qh(8, A, 10, ) | $\frac{\sqrt{77}C_1}{32} + \frac{5\sqrt{15}C_3}{32} + \frac{3\sqrt{13}C_5}{32} - \frac{\sqrt{455}C_7}{32}$    |
| 75  | A      | A     | 11   | —     | $Q_8^{(h,A,11)}$ | Qh(8, A, 11, ) | $S_4$   |
| 76  | A      | A     | 12   | —     | $Q_8^{(h,A,12)}$ | Qh(8, A, 12, ) | $-\frac{\sqrt{858}S_1}{64} + \frac{\sqrt{910}S_3}{64} + \frac{7\sqrt{42}S_5}{64} + \frac{3\sqrt{30}S_7}{64}$  |
| 77  | A      | A     | 13   | —     | $Q_8^{(h,A,13)}$ | Qh(8, A, 13, ) | $-\frac{\sqrt{858}C_1}{64} - \frac{\sqrt{910}C_3}{64} + \frac{7\sqrt{42}C_5}{64} - \frac{3\sqrt{30}C_7}{64}$  |
| 78  | A      | A     | 14   | —     | $Q_8^{(h,A,14)}$ | Qh(8, A, 14, ) | $S_6$   |
| 79  | A      | A     | 15   | —     | $Q_8^{(h,A,15)}$ | Qh(8, A, 15, ) | $-\frac{\sqrt{70}S_1}{64} + \frac{3\sqrt{66}S_3}{64} - \frac{\sqrt{1430}S_5}{64} + \frac{\sqrt{2002}S_7}{64}$ |
| 80  | A      | A     | 16   | —     | $Q_8^{(h,A,16)}$ | Qh(8, A, 16, ) | $-\frac{\sqrt{70}C_1}{64} - \frac{3\sqrt{66}C_3}{64} - \frac{\sqrt{1430}C_5}{64} - \frac{\sqrt{2002}C_7}{64}$ |
| 81  | A      | A     | 17   | —     | $Q_8^{(h,A,17)}$ | Qh(8, A, 17, ) | $S_2$   |

表 10 rank 9

| No. | irrep. | (tag) | mul. | comp. | harmonics        | (tag)            | definition  |
|-----|--------|-------|------|-------|------------------|------------------|---|
| 82  | $A$    | $A$   | 1    | —     | $Q_9^{(h,A,1)}$  | $Qh(9, A, 1, )$  | $\frac{\sqrt{102}S_4}{12} - \frac{\sqrt{42}S_8}{12}$  |
| 83  | $A$    | $A$   | 2    | —     | $Q_9^{(h,A,2)}$  | $Qh(9, A, 2, )$  | $\frac{\sqrt{3}S_2}{4} - \frac{\sqrt{13}S_6}{4}$  |
| 84  | $A$    | $A$   | 3    | —     | $Q_9^{(h,A,3)}$  | $Qh(9, A, 3, )$  | $\frac{\sqrt{42}S_4}{12} + \frac{\sqrt{102}S_8}{12}$  |
| 85  | $A$    | $A$   | 4    | —     | $Q_9^{(h,A,4)}$  | $Qh(9, A, 4, )$  | $-\frac{\sqrt{13}S_2}{4} - \frac{\sqrt{3}S_6}{4}$   |
| 86  | $A$    | $A$   | 5    | —     | $Q_9^{(h,A,5)}$  | $Qh(9, A, 5, )$  | $\frac{21\sqrt{5}C_1}{128} - \frac{\sqrt{2310}C_3}{128} + \frac{3\sqrt{286}C_5}{128} - \frac{3\sqrt{1430}C_7}{256} + \frac{\sqrt{24310}C_9}{256}$ |
| 87  | $A$    | $A$   | 6    | —     | $Q_9^{(h,A,6)}$  | $Qh(9, A, 6, )$  | $\frac{21\sqrt{5}S_1}{128} + \frac{\sqrt{2310}S_3}{128} + \frac{3\sqrt{286}S_5}{128} + \frac{3\sqrt{1430}S_7}{256} + \frac{\sqrt{24310}S_9}{256}$ |
| 88  | $A$    | $A$   | 7    | —     | $Q_9^{(h,A,7)}$  | $Qh(9, A, 7, )$  | $C_0$   |
| 89  | $A$    | $A$   | 8    | —     | $Q_9^{(h,A,8)}$  | $Qh(9, A, 8, )$  | $\frac{\sqrt{2431}C_1}{128} + \frac{\sqrt{9282}C_3}{128} + \frac{5\sqrt{170}C_5}{128} + \frac{7\sqrt{34}C_7}{256} + \frac{3\sqrt{2}C_9}{256}$     |
| 90  | $A$    | $A$   | 9    | —     | $Q_9^{(h,A,9)}$  | $Qh(9, A, 9, )$  | $\frac{\sqrt{2431}S_1}{128} - \frac{\sqrt{9282}S_3}{128} + \frac{5\sqrt{170}S_5}{128} - \frac{7\sqrt{34}S_7}{256} + \frac{3\sqrt{2}S_9}{256}$     |
| 91  | $A$    | $A$   | 10   | —     | $Q_9^{(h,A,10)}$ | $Qh(9, A, 10, )$ | $C_8$   |
| 92  | $A$    | $A$   | 11   | —     | $Q_9^{(h,A,11)}$ | $Qh(9, A, 11, )$ | $\frac{\sqrt{1001}C_1}{64} - \frac{\sqrt{78}C_3}{64} - \frac{3\sqrt{70}C_5}{64} + \frac{23\sqrt{14}C_7}{128} + \frac{3\sqrt{238}C_9}{128}$        |
| 93  | $A$    | $A$   | 12   | —     | $Q_9^{(h,A,12)}$ | $Qh(9, A, 12, )$ | $\frac{\sqrt{1001}S_1}{64} + \frac{\sqrt{78}S_3}{64} - \frac{3\sqrt{70}S_5}{64} - \frac{23\sqrt{14}S_7}{128} + \frac{3\sqrt{238}S_9}{128}$        |
| 94  | $A$    | $A$   | 13   | —     | $Q_9^{(h,A,13)}$ | $Qh(9, A, 13, )$ | $C_4$   |
| 95  | $A$    | $A$   | 14   | —     | $Q_9^{(h,A,14)}$ | $Qh(9, A, 14, )$ | $\frac{\sqrt{858}C_1}{64} + \frac{\sqrt{91}C_3}{32} - \frac{5\sqrt{15}C_5}{32} - \frac{21\sqrt{3}C_7}{64} - \frac{\sqrt{51}C_9}{64}$              |
| 96  | $A$    | $A$   | 15   | —     | $Q_9^{(h,A,15)}$ | $Qh(9, A, 15, )$ | $-\frac{\sqrt{858}S_1}{64} + \frac{\sqrt{91}S_3}{32} + \frac{5\sqrt{15}S_5}{32} - \frac{21\sqrt{3}S_7}{64} + \frac{\sqrt{51}S_9}{64}$             |
| 97  | $A$    | $A$   | 16   | —     | $Q_9^{(h,A,16)}$ | $Qh(9, A, 16, )$ | $C_6$   |
| 98  | $A$    | $A$   | 17   | —     | $Q_9^{(h,A,17)}$ | $Qh(9, A, 17, )$ | $\frac{7\sqrt{22}C_1}{64} - \frac{3\sqrt{21}C_3}{32} + \frac{\sqrt{65}C_5}{32} + \frac{\sqrt{13}C_7}{64} - \frac{3\sqrt{221}C_9}{64}$             |
| 99  | $A$    | $A$   | 18   | —     | $Q_9^{(h,A,18)}$ | $Qh(9, A, 18, )$ | $-\frac{7\sqrt{22}S_1}{64} - \frac{3\sqrt{21}S_3}{32} - \frac{\sqrt{65}S_5}{32} + \frac{\sqrt{13}S_7}{64} + \frac{3\sqrt{221}S_9}{64}$            |
| 100 | $A$    | $A$   | 19   | —     | $Q_9^{(h,A,19)}$ | $Qh(9, A, 19, )$ | $C_2$   |

表 11 rank 10

| No. | irrep. | (tag) | mul. | comp. | harmonics           | (tag)             | definition   |
|-----|--------|-------|------|-------|---------------------|-------------------|--|
| 101 | $A$    | $A$   | 1    | —     | $Q_{10}^{(h,A,1)}$  | $Qh(10, A, 1, )$  | $\frac{\sqrt{390}C_0}{48} - \frac{\sqrt{22}C_4}{8} - \frac{\sqrt{1122}C_8}{48}$  |
| 102 | $A$    | $A$   | 2    | —     | $Q_{10}^{(h,A,2)}$  | $Qh(10, A, 2, )$  | $-\frac{\sqrt{85}C_{10}}{16} + \frac{\sqrt{1482}C_2}{48} + \frac{\sqrt{57}C_6}{48}$  |
| 103 | $A$    | $A$   | 3    | —     | $Q_{10}^{(h,A,3)}$  | $Qh(10, A, 3, )$  | $\frac{11\sqrt{420189}C_0}{8988} + \frac{\sqrt{827645}C_4}{1498} - \frac{\sqrt{146055}C_8}{8988}$  |
| 104 | $A$    | $A$   | 4    | —     | $Q_{10}^{(h,A,4)}$  | $Qh(10, A, 4, )$  | $\frac{\sqrt{370006}C_{10}}{749} + \frac{\sqrt{190995}C_2}{749}$   |
| 105 | $A$    | $A$   | 5    | —     | $Q_{10}^{(h,A,5)}$  | $Qh(10, A, 5, )$  | $\frac{3\sqrt{3213210}C_0}{11984} - \frac{83\sqrt{1498}C_4}{5992} + \frac{31\sqrt{76398}C_8}{11984}$   |
| 106 | $A$    | $A$   | 6    | —     | $Q_{10}^{(h,A,6)}$  | $Qh(10, A, 6, )$  | $\frac{\sqrt{1209635}C_{10}}{11984} - \frac{19\sqrt{58422}C_2}{35952} + \frac{\sqrt{2247}C_6}{48}$   |
| 107 | $A$    | $A$   | 7    | —     | $Q_{10}^{(h,A,7)}$  | $Qh(10, A, 7, )$  | $\frac{\sqrt{221}S_1}{32} - \frac{\sqrt{102}S_3}{32} - \frac{\sqrt{510}S_5}{32} - \frac{11\sqrt{6}S_7}{64} - \frac{\sqrt{38}S_9}{64}$          |
| 108 | $A$    | $A$   | 8    | —     | $Q_{10}^{(h,A,8)}$  | $Qh(10, A, 8, )$  | $-\frac{\sqrt{221}C_1}{32} - \frac{\sqrt{102}C_3}{32} + \frac{\sqrt{510}C_5}{32} - \frac{11\sqrt{6}C_7}{64} + \frac{\sqrt{38}C_9}{64}$         |
| 109 | $A$    | $A$   | 9    | —     | $Q_{10}^{(h,A,9)}$  | $Qh(10, A, 9, )$  | $S_8$  |
| 110 | $A$    | $A$   | 10   | —     | $Q_{10}^{(h,A,10)}$ | $Qh(10, A, 10, )$ | $\frac{\sqrt{39}S_1}{32} - \frac{11\sqrt{2}S_3}{32} + \frac{5\sqrt{10}S_5}{32} - \frac{\sqrt{34}S_7}{64} - \frac{\sqrt{1938}S_9}{64}$          |
| 111 | $A$    | $A$   | 11   | —     | $Q_{10}^{(h,A,11)}$ | $Qh(10, A, 11, )$ | $-\frac{\sqrt{39}C_1}{32} - \frac{11\sqrt{2}C_3}{32} - \frac{5\sqrt{10}C_5}{32} - \frac{\sqrt{34}C_7}{64} + \frac{\sqrt{1938}C_9}{64}$         |
| 112 | $A$    | $A$   | 12   | —     | $Q_{10}^{(h,A,12)}$ | $Qh(10, A, 12, )$ | $S_4$  |
| 113 | $A$    | $A$   | 13   | —     | $Q_{10}^{(h,A,13)}$ | $Qh(10, A, 13, )$ | $\frac{\sqrt{41990}S_1}{256} + \frac{\sqrt{4845}S_3}{128} + \frac{\sqrt{969}S_5}{128} + \frac{\sqrt{285}S_7}{256} + \frac{\sqrt{5}S_9}{256}$   |
| 114 | $A$    | $A$   | 14   | —     | $Q_{10}^{(h,A,14)}$ | $Qh(10, A, 14, )$ | $\frac{\sqrt{41990}C_1}{256} - \frac{\sqrt{4845}C_3}{128} + \frac{\sqrt{969}C_5}{128} - \frac{\sqrt{285}C_7}{256} + \frac{\sqrt{5}C_9}{256}$   |
| 115 | $A$    | $A$   | 15   | —     | $Q_{10}^{(h,A,15)}$ | $Qh(10, A, 15, )$ | $S_{10}$   |
| 116 | $A$    | $A$   | 16   | —     | $Q_{10}^{(h,A,16)}$ | $Qh(10, A, 16, )$ | $\frac{9\sqrt{78}S_1}{256} - \frac{69S_3}{128} - \frac{\sqrt{5}S_5}{128} + \frac{43\sqrt{17}S_7}{256} + \frac{3\sqrt{969}S_9}{256}$            |
| 117 | $A$    | $A$   | 17   | —     | $Q_{10}^{(h,A,17)}$ | $Qh(10, A, 17, )$ | $\frac{9\sqrt{78}C_1}{256} + \frac{69C_3}{128} - \frac{\sqrt{5}C_5}{128} - \frac{43\sqrt{17}C_7}{256} + \frac{3\sqrt{969}C_9}{256}$            |
| 118 | $A$    | $A$   | 18   | —     | $Q_{10}^{(h,A,18)}$ | $Qh(10, A, 18, )$ | $S_6$  |
| 119 | $A$    | $A$   | 19   | —     | $Q_{10}^{(h,A,19)}$ | $Qh(10, A, 19, )$ | $\frac{7\sqrt{3}S_1}{128} - \frac{7\sqrt{26}S_3}{128} + \frac{5\sqrt{130}S_5}{128} - \frac{7\sqrt{442}S_7}{256} + \frac{\sqrt{25194}S_9}{256}$ |
| 120 | $A$    | $A$   | 20   | —     | $Q_{10}^{(h,A,20)}$ | $Qh(10, A, 20, )$ | $\frac{7\sqrt{3}C_1}{128} + \frac{7\sqrt{26}C_3}{128} + \frac{5\sqrt{130}C_5}{128} + \frac{7\sqrt{442}C_7}{256} + \frac{\sqrt{25194}C_9}{256}$ |
| 121 | $A$    | $A$   | 21   | —     | $Q_{10}^{(h,A,21)}$ | $Qh(10, A, 21, )$ | $S_2$  |

表 12 rank 11

| No. | irrep. | (tag) | mul. | comp. | harmonics                    | (tag)                    | definition  |
|-----|--------|-------|------|-------|------------------------------|--------------------------|---|
| 122 | $A$    | $A$   | 1    | —     | $\mathbb{Q}_{11}^{(h,A,1)}$  | $\text{Qh}(11, A, 1, )$  | $\frac{\sqrt{798}S_{10}}{48} + \frac{\sqrt{255}S_2}{24} + \frac{3\sqrt{6}S_6}{16}$  |
| 123 | $A$    | $A$   | 2    | —     | $\mathbb{Q}_{11}^{(h,A,2)}$  | $\text{Qh}(11, A, 2, )$  | $S_8$   |
| 124 | $A$    | $A$   | 3    | —     | $\mathbb{Q}_{11}^{(h,A,3)}$  | $\text{Qh}(11, A, 3, )$  | $-\frac{\sqrt{210}S_{10}}{96} + \frac{\sqrt{969}S_2}{48} - \frac{\sqrt{570}S_6}{32}$  |
| 125 | $A$    | $A$   | 4    | —     | $\mathbb{Q}_{11}^{(h,A,4)}$  | $\text{Qh}(11, A, 4, )$  | $S_4$   |
| 126 | $A$    | $A$   | 5    | —     | $\mathbb{Q}_{11}^{(h,A,5)}$  | $\text{Qh}(11, A, 5, )$  | $-\frac{\sqrt{646}S_{10}}{32} + \frac{\sqrt{35}S_2}{16} + \frac{\sqrt{238}S_6}{32}$   |
| 127 | $A$    | $A$   | 6    | —     | $\mathbb{Q}_{11}^{(h,A,6)}$  | $\text{Qh}(11, A, 6, )$  | $-\frac{21\sqrt{66}C_1}{512} + \frac{\sqrt{88179}C_{11}}{512} + \frac{\sqrt{30030}C_3}{512} - \frac{15\sqrt{143}C_5}{512} + \frac{\sqrt{36465}C_7}{512} - \frac{\sqrt{46189}C_9}{512}$    |
| 128 | $A$    | $A$   | 7    | —     | $\mathbb{Q}_{11}^{(h,A,7)}$  | $\text{Qh}(11, A, 7, )$  | $-\frac{21\sqrt{66}S_1}{512} - \frac{\sqrt{88179}S_{11}}{512} - \frac{\sqrt{30030}S_3}{512} - \frac{15\sqrt{143}S_5}{512} - \frac{\sqrt{36465}S_7}{512} - \frac{\sqrt{46189}S_9}{512}$    |
| 129 | $A$    | $A$   | 8    | —     | $\mathbb{Q}_{11}^{(h,A,8)}$  | $\text{Qh}(11, A, 8, )$  | $C_0$   |
| 130 | $A$    | $A$   | 9    | —     | $\mathbb{Q}_{11}^{(h,A,9)}$  | $\text{Qh}(11, A, 9, )$  | $-\frac{\sqrt{41990}C_1}{512} + \frac{\sqrt{385}C_{11}}{512} - \frac{3\sqrt{4522}C_3}{512} + \frac{3\sqrt{4845}C_5}{512} + \frac{77\sqrt{19}C_7}{512} + \frac{39\sqrt{15}C_9}{512}$       |
| 131 | $A$    | $A$   | 10   | —     | $\mathbb{Q}_{11}^{(h,A,10)}$ | $\text{Qh}(11, A, 10, )$ | $-\frac{\sqrt{41990}S_1}{512} - \frac{\sqrt{385}S_{11}}{512} + \frac{3\sqrt{4522}S_3}{512} + \frac{3\sqrt{4845}S_5}{512} - \frac{77\sqrt{19}S_7}{512} + \frac{39\sqrt{15}S_9}{512}$       |
| 132 | $A$    | $A$   | 11   | —     | $\mathbb{Q}_{11}^{(h,A,11)}$ | $\text{Qh}(11, A, 11, )$ | $C_8$   |
| 133 | $A$    | $A$   | 12   | —     | $\mathbb{Q}_{11}^{(h,A,12)}$ | $\text{Qh}(11, A, 12, )$ | $-\frac{5\sqrt{546}C_1}{256} + \frac{\sqrt{10659}C_{11}}{256} + \frac{11\sqrt{30}C_3}{256} + \frac{13\sqrt{7}C_5}{256} - \frac{3\sqrt{1785}C_7}{256} + \frac{3\sqrt{2261}C_9}{256}$       |
| 134 | $A$    | $A$   | 13   | —     | $\mathbb{Q}_{11}^{(h,A,13)}$ | $\text{Qh}(11, A, 13, )$ | $-\frac{5\sqrt{546}S_1}{256} - \frac{\sqrt{10659}S_{11}}{256} - \frac{11\sqrt{30}S_3}{256} + \frac{13\sqrt{7}S_5}{256} + \frac{3\sqrt{1785}S_7}{256} + \frac{3\sqrt{2261}S_9}{256}$       |
| 135 | $A$    | $A$   | 14   | —     | $\mathbb{Q}_{11}^{(h,A,14)}$ | $\text{Qh}(11, A, 14, )$ | $C_4$   |
| 136 | $A$    | $A$   | 15   | —     | $\mathbb{Q}_{11}^{(h,A,15)}$ | $\text{Qh}(11, A, 15, )$ | $-\frac{\sqrt{29393}C_1}{512} - \frac{\sqrt{22}C_{11}}{1024} - \frac{9\sqrt{1615}C_3}{512} - \frac{5\sqrt{13566}C_5}{1024} - \frac{7\sqrt{1330}C_7}{1024} - \frac{9\sqrt{42}C_9}{1024}$   |
| 137 | $A$    | $A$   | 16   | —     | $\mathbb{Q}_{11}^{(h,A,16)}$ | $\text{Qh}(11, A, 16, )$ | $\frac{\sqrt{29393}S_1}{512} - \frac{\sqrt{22}S_{11}}{1024} - \frac{9\sqrt{1615}S_3}{512} + \frac{5\sqrt{13566}S_5}{1024} - \frac{7\sqrt{1330}S_7}{1024} + \frac{9\sqrt{42}S_9}{1024}$    |
| 138 | $A$    | $A$   | 17   | —     | $\mathbb{Q}_{11}^{(h,A,17)}$ | $\text{Qh}(11, A, 17, )$ | $C_{10}$  |
| 139 | $A$    | $A$   | 18   | —     | $\mathbb{Q}_{11}^{(h,A,18)}$ | $\text{Qh}(11, A, 18, )$ | $-\frac{15\sqrt{221}C_1}{512} - \frac{3\sqrt{2926}C_{11}}{1024} - \frac{\sqrt{595}C_3}{512} + \frac{53\sqrt{102}C_5}{1024} - \frac{105\sqrt{10}C_7}{1024} - \frac{61\sqrt{114}C_9}{1024}$ |
| 140 | $A$    | $A$   | 19   | —     | $\mathbb{Q}_{11}^{(h,A,19)}$ | $\text{Qh}(11, A, 19, )$ | $\frac{15\sqrt{221}S_1}{512} - \frac{3\sqrt{2926}S_{11}}{1024} - \frac{\sqrt{595}S_3}{512} - \frac{53\sqrt{102}S_5}{1024} - \frac{105\sqrt{10}S_7}{1024} + \frac{61\sqrt{114}S_9}{1024}$  |
| 141 | $A$    | $A$   | 20   | —     | $\mathbb{Q}_{11}^{(h,A,20)}$ | $\text{Qh}(11, A, 20, )$ | $C_6$   |
| 142 | $A$    | $A$   | 21   | —     | $\mathbb{Q}_{11}^{(h,A,21)}$ | $\text{Qh}(11, A, 21, )$ | $-\frac{21\sqrt{130}C_1}{512} - \frac{\sqrt{124355}C_{11}}{512} + \frac{57\sqrt{14}C_3}{512} - \frac{41\sqrt{15}C_5}{512} + \frac{17\sqrt{17}C_7}{512} + \frac{\sqrt{4845}C_9}{512}$      |
| 143 | $A$    | $A$   | 22   | —     | $\mathbb{Q}_{11}^{(h,A,22)}$ | $\text{Qh}(11, A, 22, )$ | $\frac{21\sqrt{130}S_1}{512} - \frac{\sqrt{124355}S_{11}}{512} + \frac{57\sqrt{14}S_3}{512} + \frac{41\sqrt{15}S_5}{512} + \frac{17\sqrt{17}S_7}{512} - \frac{\sqrt{4845}S_9}{512}$       |
| 144 | $A$    | $A$   | 23   | —     | $\mathbb{Q}_{11}^{(h,A,23)}$ | $\text{Qh}(11, A, 23, )$ | $C_2$   |