

MPG No. 9.3.31 4' [Type III, tetragonal] [M tensor]

* Rank 0 tensor. * Rank 1 tensor. * Rank 2 tensor (s).

$$\begin{bmatrix} M_{xx} & M_{xy} & 0 \\ M_{xy} & -M_{xx} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{xx} = M_{dv}^{(1)}$$

$$M_{xy} = M_{dx}^{(1)}$$

* Rank 2 tensor (a). * Rank 3 tensor (s).

$$\begin{bmatrix} 0 & 0 & M_{xxz} \\ 0 & 0 & -M_{xxz} \\ 0 & 0 & 0 \\ M_{yzx} & M_{yzy} & 0 \\ -M_{yzy} & M_{yzx} & 0 \\ 0 & 0 & M_{xyz} \end{bmatrix}$$

$$M_{xxz} = M_{fbz}^{(1)} - 2T_{dxy}^{(1)}$$

$$M_{yzx} = M_{f3}^{(1)} - T_{dv}^{(1)}$$

$$M_{yzy} = -M_{fbz}^{(1)} - T_{dxy}^{(1)}$$

$$M_{xyz} = M_{f3}^{(1)} + 2T_{dv}^{(1)}$$

* Rank 3 tensor (a).

$$\begin{bmatrix} M_{yzx} & M_{yzy} & 0 \\ M_{yzy} & -M_{yzx} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{yzx} = T_{dv}^{(2)}$$

$$M_{yzy} = T_{dxy}^{(2)}$$

* Rank 4 tensor (sss).

$$\begin{bmatrix} M_{xxxx} & 0 & M_{xxzz} & 0 & 0 & M_{xxxz} \\ 0 & -M_{xxxx} & -M_{xxzz} & 0 & 0 & M_{xxxx} \\ M_{xxzz} & -M_{xxzz} & 0 & 0 & 0 & M_{zzxy} \\ 0 & 0 & 0 & M_{yzyz} & M_{yzzx} & 0 \\ 0 & 0 & 0 & M_{yzzx} & -M_{yzyz} & 0 \\ M_{xxxz} & M_{xxxz} & M_{zzxy} & 0 & 0 & 0 \end{bmatrix}$$

$$M_{xxxx} = 2M_{dv}^{(1)} + 4M_{dv}^{(2)} + M_{gv}^{(1)}$$

$$M_{xxzz} = M_{dv}^{(1)} - M_{gv}^{(1)}$$

$$M_{xxxz} = M_{dxy}^{(1)} + 2M_{dxy}^{(2)} - M_{gbz}^{(1)}$$

$$M_{zzxy} = M_{dxy}^{(1)} + 2M_{gbz}^{(1)}$$

$$M_{yzyz} = -M_{dv}^{(2)} + M_{gv}^{(1)}$$

$$M_{yzzx} = M_{dxy}^{(2)} + 2M_{gbz}^{(1)}$$

* Rank 4 tensor (ssa).

$$\begin{bmatrix} 0 & M_{xxyy} & M_{xxzz} & 0 & 0 & M_{xxxz} \\ -M_{xxyy} & 0 & -M_{xxzz} & 0 & 0 & M_{xxxz} \\ -M_{xxzz} & M_{xxzz} & 0 & 0 & 0 & M_{zzxy} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -M_{xxxz} & -M_{xxxz} & -M_{zzxy} & 0 & 0 & 0 \end{bmatrix}$$

$$M_{xxyy} = -2M_{dv}^{(3)} + 4T_{f3}^{(1)}$$

$$M_{xxzz} = -M_{dv}^{(3)} - 4T_{f3}^{(1)}$$

$$M_{xxxz} = M_{dxy}^{(3)} + 2T_{fbz}^{(1)}$$

$$M_{zzxy} = M_{dxy}^{(3)} - 4T_{fbz}^{(1)}$$

* Rank 4 tensor (aas).

$$\begin{bmatrix} M_{yzyz} & M_{yzzx} & 0 \\ M_{yzzx} & -M_{yzyz} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{yzyz} = 2M_{dv}^{(4)}$$

$$M_{yzzx} = 2M_{dxy}^{(4)}$$

* Rank 4 tensor (aaa). * Rank 4 tensor (sa).

$$\begin{bmatrix} 0 & 0 & M_{xxxz} \\ 0 & 0 & -M_{xxxy} \\ 0 & 0 & 0 \\ M_{yzyz} & M_{yzzx} & 0 \\ -M_{yzzx} & M_{yzyz} & 0 \\ 0 & 0 & M_{xyxy} \end{bmatrix}$$

$$M_{xxxy} = -2M_{dxy}^{(5)} + T_{fbz}^{(2)}$$

$$M_{yzyz} = -M_{dv}^{(5)} + T_{f3}^{(2)}$$

$$M_{yzzx} = -M_{dxy}^{(5)} - T_{fbz}^{(2)}$$

$$M_{xyxy} = 2M_{dv}^{(5)} + T_{f3}^{(2)}$$

* Rank 4 tensor (as).

$$\begin{bmatrix} 0 & 0 & 0 & M_{yzyz} & M_{yzzx} & 0 \\ 0 & 0 & 0 & -M_{yzzx} & M_{yzyz} & 0 \\ M_{xyxx} & -M_{xyxx} & 0 & 0 & 0 & M_{xyxy} \end{bmatrix}$$

$$M_{yzyz} = -M_{dv}^{(6)} + T_{f3}^{(3)}$$

$$M_{yzzx} = M_{dxy}^{(6)} + T_{fbz}^{(3)}$$

$$M_{xyxx} = -2M_{dxy}^{(6)} + T_{fbz}^{(3)}$$

$$M_{xyxy} = 2M_{dv}^{(6)} + T_{f3}^{(3)}$$

* Rank 4 tensor (s).

$$\begin{bmatrix} M_{xxxx} & M_{xxyy} & M_{xxzz} & 0 & 0 & M_{xxxz} & 0 & 0 & M_{xyxy} \\ -M_{xxyy} & -M_{xxxx} & -M_{xxzz} & 0 & 0 & M_{xxxy} & 0 & 0 & M_{xxxy} \\ M_{zzxx} & -M_{zzxx} & 0 & 0 & 0 & M_{zzxy} & 0 & 0 & M_{zzxy} \\ 0 & 0 & 0 & M_{yzyz} & M_{yzzx} & 0 & M_{yzyy} & M_{yzzx} & 0 \\ 0 & 0 & 0 & M_{yzzx} & -M_{yzyy} & 0 & M_{yzzx} & -M_{yzyz} & 0 \\ M_{xyxx} & M_{xyxx} & M_{xyzz} & 0 & 0 & M_{xyxy} & 0 & 0 & -M_{xyxy} \end{bmatrix}$$

$$\begin{aligned}
M_{xxxx} &= 2M_{dv}^{(1)} + 4M_{dv}^{(2)} + M_{gv}^{(1)} \\
M_{xxyy} &= -2M_{dv}^{(3)} + 4T_{f3}^{(1)} \\
M_{xxzz} &= M_{dv}^{(1)} - M_{dv}^{(3)} - M_{gv}^{(1)} - 4T_{f3}^{(1)} \\
M_{xxxz} &= M_{dx}^{(1)} + 2M_{dxy}^{(2)} + M_{dzy}^{(3)} - 2M_{dxy}^{(5)} - M_{gbz}^{(1)} + 2T_{fbz}^{(1)} + T_{fbz}^{(2)} \\
M_{xxyx} &= M_{dxy}^{(1)} + 2M_{dxy}^{(2)} + M_{dxy}^{(3)} + 2M_{dxy}^{(5)} - M_{gbz}^{(1)} + 2T_{fbz}^{(1)} - T_{fbz}^{(2)} \\
M_{zzxx} &= M_{dv}^{(1)} + M_{dv}^{(3)} - M_{gv}^{(1)} + 4T_{f3}^{(1)} \\
M_{zzxy} &= M_{dxy}^{(1)} + M_{dxy}^{(3)} + 2M_{gbz}^{(1)} - 4T_{fbz}^{(1)} \\
M_{yzyz} &= -M_{dv}^{(2)} - M_{dv}^{(5)} + M_{gv}^{(1)} + T_{f3}^{(2)} \\
M_{yzzx} &= M_{dxy}^{(2)} - M_{dxy}^{(5)} + 2M_{gbz}^{(1)} - T_{fbz}^{(2)} \\
M_{yzyz} &= -M_{dv}^{(2)} + M_{dv}^{(5)} + M_{gv}^{(1)} - T_{f3}^{(2)} \\
M_{yzxz} &= M_{dxy}^{(2)} + M_{dxy}^{(5)} + 2M_{gbz}^{(1)} + T_{fbz}^{(2)} \\
M_{xyxx} &= M_{dxy}^{(1)} + 2M_{dxy}^{(2)} - M_{dxy}^{(3)} - M_{gbz}^{(1)} - 2T_{fbz}^{(1)} \\
M_{xyzz} &= M_{dxy}^{(1)} - M_{dxy}^{(3)} + 2M_{gbz}^{(1)} + 4T_{fbz}^{(1)} \\
M_{xyxy} &= 2M_{dv}^{(5)} + T_{f3}^{(2)}
\end{aligned}$$

* Rank 4 tensor (a).

$$\begin{bmatrix} 0 & 0 & 0 & M_{yzyz} & M_{yzzx} & 0 & M_{yzzy} & M_{yzxz} & 0 \\ 0 & 0 & 0 & -M_{yzzx} & M_{yzyz} & 0 & -M_{yzzx} & M_{yzyz} & 0 \\ M_{xyxx} & -M_{xyxx} & 0 & 0 & 0 & M_{xyxy} & 0 & 0 & M_{xyxy} \end{bmatrix}$$

$$\begin{aligned}
M_{yzyz} &= 2M_{dv}^{(4)} - M_{dv}^{(6)} + T_{f3}^{(3)} \\
M_{yzzx} &= 2M_{dxy}^{(4)} + M_{dxy}^{(6)} + T_{fbz}^{(3)} \\
M_{yzzy} &= -2M_{dv}^{(4)} - M_{dv}^{(6)} + T_{f3}^{(3)} \\
M_{yzzx} &= -2M_{dxy}^{(4)} + M_{dxy}^{(6)} + T_{fbz}^{(3)} \\
M_{xyxx} &= -2M_{dxy}^{(6)} + T_{fbz}^{(3)} \\
M_{xyxy} &= 2M_{dv}^{(6)} + T_{f3}^{(3)}
\end{aligned}$$

* Rank 4 tensor (t).

$$\begin{bmatrix} M_{xxxx} & M_{xxyy} & 0 \\ M_{xxyy} & -M_{xxxx} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_{yyzz} \\ M_{zzxx} & M_{zzxy} & 0 \\ M_{xxyx} & -M_{yyzz} - M_{zzxx} & 0 \\ M_{yzzx} & M_{yzyz} & 0 \\ 0 & 0 & M_{zxxz} \\ -M_{yzyz} - M_{zzxx} & M_{xyyy} & 0 \\ 0 & 0 & -\frac{M_{xxyy}}{2} - \frac{M_{xxyx}}{2} + M_{xyyy} + M_{zzxy} \end{bmatrix}$$

$$M_{xxxx} = 2M_{dv}^{(1)} + 4M_{dv}^{(2)} + M_{gv}^{(1)}$$

$$M_{xxyy} = M_{dxy}^{(1)} + 2M_{dxy}^{(2)} + M_{dxy}^{(3)} - 2M_{dxy}^{(5)} - M_{gbz}^{(1)}$$

$$M_{yyzz} = -M_{dv}^{(1)} + M_{dv}^{(3)} + M_{gv}^{(1)}$$

$$M_{zzxx} = M_{dv}^{(1)} + M_{dv}^{(3)} - M_{gv}^{(1)}$$

$$M_{zzxy} = M_{dxy}^{(1)} + M_{dxy}^{(3)} + 2M_{gbz}^{(1)}$$

$$M_{xxyx} = M_{dxy}^{(1)} + 2M_{dxy}^{(2)} + M_{dxy}^{(3)} + 2M_{dxy}^{(5)} - M_{gbz}^{(1)}$$

$$M_{yzzx} = M_{dxy}^{(2)} - M_{dxy}^{(5)} + 2M_{gbz}^{(1)}$$

$$M_{yzzy} = -M_{dv}^{(2)} + M_{dv}^{(5)} + M_{gv}^{(1)}$$

$$M_{zxxz} = M_{dv}^{(2)} + M_{dv}^{(5)} - M_{gv}^{(1)}$$

$$M_{xyyy} = M_{dxy}^{(1)} + 2M_{dxy}^{(2)} - M_{dxy}^{(3)} - M_{gbz}^{(1)}$$