

PG No. 10 S_4 $\bar{4}$ [tetragonal] (polar, internal axial dipole)

* Harmonics for rank 0

* Harmonics for rank 1

$$\bar{Q}_1^{(1,0)}[g](B)$$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}G_x y}{2} - \frac{\sqrt{2}G_y x}{2}$$

$$\bar{Q}_{1,1}^{(1,0)}[g](E), \bar{Q}_{1,2}^{(1,0)}[g](E)$$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{\sqrt{2}G_y z}{2} - \frac{\sqrt{2}G_z y}{2}$$

$$-\frac{\sqrt{2}G_x z}{2} + \frac{\sqrt{2}G_z x}{2}$$

* Harmonics for rank 2

$$\bar{Q}_2^{(1,0)}[g](A)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{6}G_x y z}{2} - \frac{\sqrt{6}G_y x z}{2}$$

$$\bar{Q}_2^{(1,0)}[g](B, 1)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}G_x y z}{2} + \frac{\sqrt{2}G_y x z}{2} - \sqrt{2}G_z x y$$

$$\bar{Q}_2^{(1,0)}[g](B, 2)$$

** symmetry

$$\sqrt{3}x y$$

** expression

$$-\frac{\sqrt{2}G_x x z}{2} + \frac{\sqrt{2}G_y y z}{2} + \frac{\sqrt{2}G_z (x-y)(x+y)}{2}$$

$$\bar{Q}_{2,1}^{(1,0)}[g](E), \bar{Q}_{2,2}^{(1,0)}[g](E)$$

** symmetry

$$\sqrt{3}y z$$

$$\sqrt{3}x z$$

** expression

$$\frac{\sqrt{2}G_x (y-z)(y+z)}{2} - \frac{\sqrt{2}G_y x y}{2} + \frac{\sqrt{2}G_z x z}{2}$$

$$\frac{\sqrt{2}G_x x y}{2} - \frac{\sqrt{2}G_y (x-z)(x+z)}{2} - \frac{\sqrt{2}G_z y z}{2}$$

* Harmonics for rank 3

$$\bar{Q}_3^{(1,0)}[g](A, 1)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{5}G_x x (y-z)(y+z)}{2} - \frac{\sqrt{5}G_y y (x-z)(x+z)}{2} + \frac{\sqrt{5}G_z z (x-y)(x+y)}{2}$$

$$\tilde{\mathbb{Q}}_3^{(1,0)}[g](A, 2)$$

** symmetry

$$\frac{\sqrt{15}z (x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{5}G_x y (x^2 - y^2 + 2z^2)}{4} - \frac{\sqrt{5}G_y x (x^2 - y^2 - 2z^2)}{4} - \sqrt{5}G_z xyz$$

$$\tilde{\mathbb{Q}}_3^{(1,0)}[g](B)$$

** symmetry

$$-\frac{z (3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{\sqrt{3}G_x y (x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{3}G_y x (x^2 + y^2 - 4z^2)}{4}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(1,0)}[g](E, 1), \tilde{\mathbb{Q}}_{3,2}^{(1,0)}[g](E, 1)$$

** symmetry

$$\frac{x (2x^2 - 3y^2 - 3z^2)}{2}$$

$$-\frac{y (3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$\frac{\sqrt{3}G_y z (4x^2 - y^2 - z^2)}{4} - \frac{\sqrt{3}G_z y (4x^2 - y^2 - z^2)}{4}$$

$$\frac{\sqrt{3}G_x z (x^2 - 4y^2 + z^2)}{4} - \frac{\sqrt{3}G_z x (x^2 - 4y^2 + z^2)}{4}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(1,0)}[g](E, 2), \tilde{\mathbb{Q}}_{3,2}^{(1,0)}[g](E, 2)$$

** symmetry

$$-\frac{\sqrt{15}y (x-z)(x+z)}{2}$$

$$\frac{\sqrt{15}x (y-z)(y+z)}{2}$$

** expression

$$\frac{\sqrt{5}G_x z (x^2 + 2y^2 - z^2)}{4} - \sqrt{5}G_y xyz - \frac{\sqrt{5}G_z x (x^2 - 2y^2 - z^2)}{4}$$

$$-\sqrt{5}G_x xyz + \frac{\sqrt{5}G_y z (2x^2 + y^2 - z^2)}{4} + \frac{\sqrt{5}G_z y (2x^2 - y^2 + z^2)}{4}$$

* Harmonics for rank 4

$$\tilde{\mathbb{Q}}_4^{(1,0)}[g](A, 1)$$

** symmetry

$$\frac{\sqrt{21} (x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$-\frac{\sqrt{105}G_x y z (y-z)(y+z)}{6} + \frac{\sqrt{105}G_y x z (x-z)(x+z)}{6} - \frac{\sqrt{105}G_z x y (x-y)(x+y)}{6}$$

$$\tilde{\mathbb{Q}}_4^{(1,0)}[g](A, 2)$$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$-\frac{\sqrt{3}G_xyz(9x^2 + 2y^2 - 5z^2)}{6} + \frac{\sqrt{3}G_yxz(2x^2 + 9y^2 - 5z^2)}{6} + \frac{7\sqrt{3}G_zxy(x-y)(x+y)}{6}$$

$$\tilde{\mathbb{Q}}_4^{(1,0)}[g](A, 3)$$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$-\frac{\sqrt{7}G_xxz(x^2 - 3y^2)}{4} + \frac{\sqrt{7}G_yyz(3x^2 - y^2)}{4} + \frac{\sqrt{7}G_z(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{4}$$

$$\tilde{\mathbb{Q}}_4^{(1,0)}[g](B, 1)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$-\frac{G_xyz(3x^2 - 4y^2 + 3z^2)}{2} + \frac{G_yxz(4x^2 - 3y^2 - 3z^2)}{2} - \frac{G_zxy(x^2 + y^2 - 6z^2)}{2}$$

$$\tilde{\mathbb{Q}}_4^{(1,0)}[g](B, 2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\frac{G_xxz(x^2 + 15y^2 - 6z^2)}{4} - \frac{G_yyz(15x^2 + y^2 - 6z^2)}{4} - \frac{G_z(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

$$\tilde{\mathbb{Q}}_{4,1}^{(1,0)}[g](E, 1), \tilde{\mathbb{Q}}_{4,2}^{(1,0)}[g](E, 1)$$

** symmetry

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$-\frac{\sqrt{7}G_xxy(x^2 - 3z^2)}{4} + \frac{\sqrt{7}G_y(x^2 - 2xz - z^2)(x^2 + 2xz - z^2)}{4} + \frac{\sqrt{7}G_zyz(3x^2 - z^2)}{4}$$

$$\frac{\sqrt{7}G_x(y^2 - 2yz - z^2)(y^2 + 2yz - z^2)}{4} - \frac{\sqrt{7}G_yxy(y^2 - 3z^2)}{4} + \frac{\sqrt{7}G_zxz(3y^2 - z^2)}{4}$$

$$\tilde{\mathbb{Q}}_{4,1}^{(1,0)}[g](E, 2), \tilde{\mathbb{Q}}_{4,2}^{(1,0)}[g](E, 2)$$

** symmetry

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

$$-\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

** expression

$$\frac{G_x(y-z)(y+z)(6x^2 - y^2 - z^2)}{4} - \frac{G_yxy(6x^2 - y^2 - 15z^2)}{4} + \frac{G_zxz(6x^2 - 15y^2 - z^2)}{4}$$

$$-\frac{G_xxy(x^2 - 6y^2 + 15z^2)}{4} + \frac{G_y(x-z)(x+z)(x^2 - 6y^2 + z^2)}{4} + \frac{G_zyz(15x^2 - 6y^2 + z^2)}{4}$$