

PG No. 13 C_{4v} 4mm [tetragonal] (polar, internal axial octupole)

* Harmonics for rank 0

* Harmonics for rank 1

$$\vec{Q}_1^{(3,2)}[g](A_1)$$

** symmetry

z

** expression

$$-\frac{3\sqrt{70}G_1x(x^2 - 3y^2)}{56} + \frac{3\sqrt{70}G_2y(3x^2 - y^2)}{56} - \frac{\sqrt{105}G_3z(x-y)(x+y)}{14} - \frac{\sqrt{42}G_{3x}y(x^2 + y^2 - 4z^2)}{56} + \frac{\sqrt{42}G_{3y}x(x^2 + y^2 - 4z^2)}{56} + \frac{\sqrt{105}G_{bz}xyz}{7}$$

$$\vec{Q}_{1,1}^{(3,2)}[g](E), \vec{Q}_{1,2}^{(3,2)}[g](E)$$

** symmetry

x

y

** expression

$$\begin{aligned} & \frac{3\sqrt{70}G_1z(x-y)(x+y)}{56} - \frac{3\sqrt{70}G_2xyz}{28} - \frac{\sqrt{105}G_3x(y-z)(y+z)}{14} - \frac{5\sqrt{42}G_{3x}xyz}{28} \\ & - \frac{\sqrt{42}G_{3y}z(x^2 + 11y^2 - 4z^2)}{56} + \frac{3\sqrt{7}G_{az}y(x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{105}G_{bz}y(x^2 - y^2 + 2z^2)}{28} \end{aligned}$$

$$\begin{aligned} & -\frac{3\sqrt{70}G_1xyz}{28} - \frac{3\sqrt{70}G_2z(x-y)(x+y)}{56} + \frac{\sqrt{105}G_3y(x-z)(x+z)}{14} + \frac{\sqrt{42}G_{3x}z(11x^2 + y^2 - 4z^2)}{56} \\ & + \frac{5\sqrt{42}G_{3y}xyz}{28} - \frac{3\sqrt{7}G_{az}x(x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{105}G_{bz}x(x^2 - y^2 - 2z^2)}{28} \end{aligned}$$

* Harmonics for rank 2

$$\vec{Q}_2^{(3,0)}[g](A_1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{210}G_3(x-y)(x+y)}{28} + \frac{\sqrt{21}G_{3x}yz}{7} - \frac{\sqrt{21}G_{3y}xz}{7} + \frac{\sqrt{210}G_{bz}xy}{14}$$

$$\vec{Q}_2^{(3,2)}[g](A_1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\begin{aligned} & -\frac{\sqrt{70}G_1xz(x^2 - 3y^2)}{8} + \frac{\sqrt{70}G_2yz(3x^2 - y^2)}{8} + \frac{\sqrt{105}G_3(x-y)(x+y)(x^2 + y^2 - 6z^2)}{42} \\ & - \frac{5\sqrt{42}G_{3x}yz(3x^2 + 3y^2 - 4z^2)}{168} + \frac{5\sqrt{42}G_{3y}xz(3x^2 + 3y^2 - 4z^2)}{168} - \frac{\sqrt{105}G_{bz}xy(x^2 + y^2 - 6z^2)}{21} \end{aligned}$$

$$\vec{Q}_2^{(3,0)}[g](B_1)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{105}G_1xz}{14} - \frac{\sqrt{105}G_2yz}{14} - \frac{\sqrt{70}G_3(x^2 + y^2 - 2z^2)}{28} - \frac{3\sqrt{7}G_{3x}yz}{14} - \frac{3\sqrt{7}G_{3y}xz}{14} + \frac{\sqrt{42}G_{az}xy}{14}$$

$$\vec{Q}_2^{(3,2)}[g](B_1)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{210}G_1xz(3x^2 + 3y^2 - 4z^2)}{168} - \frac{\sqrt{210}G_2yz(3x^2 + 3y^2 - 4z^2)}{168} + \frac{\sqrt{35}G_3(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{42} \\ & - \frac{5\sqrt{14}G_{3x}yz(27x^2 - y^2 - 8z^2)}{168} + \frac{5\sqrt{14}G_{3y}xz(x^2 - 27y^2 + 8z^2)}{168} + \frac{5\sqrt{21}G_{az}xy(x^2 + y^2 - 6z^2)}{42} - \frac{\sqrt{35}G_{bz}xy(x-y)(x+y)}{6} \end{aligned}$$

$$\vec{\mathbb{Q}}_2^{(3,0)}[g](B_2)$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{105}G_1yz}{14} - \frac{\sqrt{105}G_2xz}{14} + \frac{3\sqrt{7}G_{3x}xz}{14} - \frac{3\sqrt{7}G_{3y}yz}{14} - \frac{\sqrt{42}G_{az}(x-y)(x+y)}{28} + \frac{\sqrt{70}G_{bz}(x^2+y^2-2z^2)}{28}$$

$$\vec{\mathbb{Q}}_2^{(3,2)}[g](B_2)$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{210}G_1yz(3x^2+3y^2-4z^2)}{168} - \frac{\sqrt{210}G_2xz(3x^2+3y^2-4z^2)}{168} + \frac{\sqrt{35}G_3xy(x-y)(x+y)}{6} + \frac{5\sqrt{14}G_{3x}xz(13x^2-15y^2-8z^2)}{168} \\ + \frac{5\sqrt{14}G_{3y}yz(15x^2-13y^2+8z^2)}{168} - \frac{5\sqrt{21}G_{az}(x-y)(x+y)(x^2+y^2-6z^2)}{84} + \frac{\sqrt{35}G_{bz}(5x^4-18x^2y^2-12x^2z^2+5y^4-12y^2z^2+4z^4)}{84}$$

$$\vec{\mathbb{Q}}_{2,1}^{(3,0)}[g](E), \vec{\mathbb{Q}}_{2,2}^{(3,0)}[g](E)$$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$-\frac{\sqrt{105}G_1(x-y)(x+y)}{28} + \frac{\sqrt{105}G_2xy}{14} - \frac{3\sqrt{7}G_{3x}xy}{14} + \frac{\sqrt{7}G_{3y}(x^2-5y^2+4z^2)}{28} - \frac{\sqrt{42}G_{az}yz}{7} \\ \frac{\sqrt{105}G_1xy}{14} + \frac{\sqrt{105}G_2(x-y)(x+y)}{28} + \frac{\sqrt{7}G_{3x}(5x^2-y^2-4z^2)}{28} + \frac{3\sqrt{7}G_{3y}xy}{14} + \frac{\sqrt{42}G_{az}xz}{7}$$

$$\vec{\mathbb{Q}}_{2,1}^{(3,2)}[g](E), \vec{\mathbb{Q}}_{2,2}^{(3,2)}[g](E)$$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$-\frac{\sqrt{210}G_1(5x^4-21x^2y^2-9x^2z^2+2y^4+9y^2z^2)}{168} + \frac{\sqrt{210}G_2xy(17x^2-11y^2-18z^2)}{168} - \frac{\sqrt{35}G_3xz(x-z)(x+z)}{6} + \frac{5\sqrt{14}G_{3x}xy(x^2+y^2-6z^2)}{168} \\ + \frac{5\sqrt{14}G_{3y}(x^4+3x^2y^2-9x^2z^2+2y^4-15y^2z^2+4z^4)}{168} + \frac{5\sqrt{21}G_{az}yz(3x^2+3y^2-4z^2)}{84} + \frac{\sqrt{35}G_{bz}yz(3x^2+y^2-2z^2)}{12} \\ - \frac{\sqrt{210}G_1xy(11x^2-17y^2+18z^2)}{168} - \frac{\sqrt{210}G_2(2x^4-21x^2y^2+9x^2z^2+5y^4-9y^2z^2)}{168} \\ + \frac{\sqrt{35}G_3yz(y-z)(y+z)}{6} - \frac{5\sqrt{14}G_{3x}(2x^4+3x^2y^2-15x^2z^2+y^4-9y^2z^2+4z^4)}{168} \\ - \frac{5\sqrt{14}G_{3y}xy(x^2+y^2-6z^2)}{168} - \frac{5\sqrt{21}G_{az}xz(3x^2+3y^2-4z^2)}{84} + \frac{\sqrt{35}G_{bz}xz(x^2+3y^2-2z^2)}{12}$$

* Harmonics for rank 3

$$\vec{\mathbb{Q}}_3^{(3,-2)}[g](A_1)$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$\frac{\sqrt{2}G_{3x}y}{2} - \frac{\sqrt{2}G_{3y}x}{2}$$

$$\vec{\mathbb{Q}}_3^{(3,0)}[g](A_1)$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$\frac{\sqrt{15}G_1x(x^2 - 3y^2)}{12} - \frac{\sqrt{15}G_2y(3x^2 - y^2)}{12} - \frac{\sqrt{10}G_3z(x - y)(x + y)}{4} - \frac{G_{3xy}(x^2 + y^2 - 4z^2)}{4} + \frac{G_{3yx}(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{10}G_{bz}xyz}{2}$$

$\tilde{\mathbb{Q}}_3^{(3,2)}[g](A_1)$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{7\sqrt{330}G_1x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{528} - \frac{7\sqrt{330}G_2y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{528} + \frac{7\sqrt{55}G_3z(x - y)(x + y)(x^2 + y^2 - 2z^2)}{44} \\ & + \frac{5\sqrt{22}G_{3xy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{176} - \frac{5\sqrt{22}G_{3yx}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{176} - \frac{7\sqrt{55}G_{bz}xyz(x^2 + y^2 - 2z^2)}{22} \end{aligned}$$

$\tilde{\mathbb{Q}}_3^{(3,-2)}[g](B_1)$

** symmetry

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

** expression

$$-\frac{\sqrt{2}G_1x}{4} + \frac{\sqrt{2}G_2y}{4} + \frac{\sqrt{3}G_3z}{3} - \frac{\sqrt{30}G_{3xy}}{12} - \frac{\sqrt{30}G_{3yx}}{12}$$

$\tilde{\mathbb{Q}}_3^{(3,0)}[g](B_1)$

** symmetry

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

** expression

$$-\frac{G_1x(x^2 + y^2 - 4z^2)}{4} + \frac{G_2y(x^2 + y^2 - 4z^2)}{4} - \frac{\sqrt{6}G_3z(3x^2 + 3y^2 - 2z^2)}{12} - \frac{\sqrt{15}G_{3xy}(3x^2 - y^2)}{12} + \frac{\sqrt{15}G_{3yx}(x^2 - 3y^2)}{12} - \frac{\sqrt{10}G_{az}xyz}{2}$$

$\tilde{\mathbb{Q}}_3^{(3,2)}[g](B_1)$

** symmetry

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{22}G_1x(13x^4 - 100x^2y^2 - 30x^2z^2 + 55y^4 - 30y^2z^2 + 20z^4)}{176} + \frac{\sqrt{22}G_2y(55x^4 - 100x^2y^2 - 30x^2z^2 + 13y^4 - 30y^2z^2 + 20z^4)}{176} \\ & - \frac{\sqrt{33}G_3z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{132} + \frac{\sqrt{330}G_{3xy}(15x^4 + 16x^2y^2 - 138x^2z^2 + y^4 - 26y^2z^2 + 36z^4)}{528} \\ & + \frac{\sqrt{330}G_{3yx}(x^4 + 16x^2y^2 - 26x^2z^2 + 15y^4 - 138y^2z^2 + 36z^4)}{528} + \frac{7\sqrt{55}G_{az}xyz(x^2 + y^2 - 2z^2)}{22} \end{aligned}$$

$\tilde{\mathbb{Q}}_3^{(3,-2)}[g](B_2)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{2}G_1y}{4} + \frac{\sqrt{2}G_2x}{4} + \frac{\sqrt{30}G_{3xx}}{12} - \frac{\sqrt{30}G_{3yy}}{12} - \frac{\sqrt{3}G_{bz}z}{3}$$

$\tilde{\mathbb{Q}}_3^{(3,0)}[g](B_2)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{G_1y(x^2 + y^2 - 4z^2)}{4} + \frac{G_2x(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{15}G_{3xx}(x^2 - 3y^2)}{12} + \frac{\sqrt{15}G_{3yy}(3x^2 - y^2)}{12} + \frac{\sqrt{10}G_{az}z(x - y)(x + y)}{4} + \frac{\sqrt{6}G_{bz}z(3x^2 + 3y^2 - 2z^2)}{12}$$

$\tilde{\mathbb{Q}}_3^{(3,2)}[g](B_2)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$-\frac{\sqrt{22}G_1y(25x^4 - 55x^2y^2 + 15x^2z^2 + 4y^4 + 15y^2z^2 - 10z^4)}{88} - \frac{\sqrt{22}G_2x(4x^4 - 55x^2y^2 + 15x^2z^2 + 25y^4 + 15y^2z^2 - 10z^4)}{88}$$

$$-\frac{\sqrt{330}G_{3xx}(4x^4 + x^2y^2 - 41x^2z^2 - 3y^4 + 15y^2z^2 + 18z^4)}{264} - \frac{\sqrt{330}G_{3yy}(3x^4 - x^2y^2 - 15x^2z^2 - 4y^4 + 41y^2z^2 - 18z^4)}{264}$$

$$-\frac{7\sqrt{55}G_{az}z(x-y)(x+y)(x^2+y^2-2z^2)}{44} + \frac{\sqrt{33}G_{bz}z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{132}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,-2)}[g](E,1), \vec{\mathbb{Q}}_{3,2}^{(3,-2)}[g](E,1)$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$\frac{\sqrt{30}G_1z}{8} - \frac{\sqrt{2}G_{3y}z}{8} + \frac{\sqrt{3}G_{az}y}{4} - \frac{\sqrt{5}G_{bz}y}{4}$$

$$\frac{\sqrt{30}G_2z}{8} + \frac{\sqrt{2}G_{3xz}}{8} - \frac{\sqrt{3}G_{az}x}{4} - \frac{\sqrt{5}G_{bz}x}{4}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,-2)}[g](E,2), \vec{\mathbb{Q}}_{3,2}^{(3,-2)}[g](E,2)$

** symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

** expression

$$-\frac{3\sqrt{2}G_1z}{8} + \frac{\sqrt{3}G_3x}{3} - \frac{\sqrt{30}G_{3y}z}{24} + \frac{\sqrt{5}G_{az}y}{4} - \frac{\sqrt{3}G_{bz}y}{12}$$

$$-\frac{3\sqrt{2}G_2z}{8} - \frac{\sqrt{3}G_3y}{3} + \frac{\sqrt{30}G_{3xz}}{24} - \frac{\sqrt{5}G_{az}x}{4} - \frac{\sqrt{3}G_{bz}x}{12}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,0)}[g](E,1), \vec{\mathbb{Q}}_{3,2}^{(3,0)}[g](E,1)$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$\frac{\sqrt{15}G_1z(3x^2 - z^2)}{12} - \frac{\sqrt{15}G_2xyz}{4} - \frac{\sqrt{10}G_3x(y-z)(y+z)}{4} - \frac{5G_{3xy}yz}{4}$$

$$-\frac{G_{3yz}(x^2 - 4y^2 + z^2)}{4} + \frac{\sqrt{6}G_{az}y(3x^2 - 2y^2 + 3z^2)}{12} - \frac{\sqrt{10}G_{bz}y(x-z)(x+z)}{4}$$

$$-\frac{\sqrt{15}G_1xyz}{4} + \frac{\sqrt{15}G_2z(3y^2 - z^2)}{12} + \frac{\sqrt{10}G_3y(x-z)(x+z)}{4} - \frac{G_{3xz}(4x^2 - y^2 - z^2)}{4}$$

$$+ \frac{5G_{3y}xyz}{4} + \frac{\sqrt{6}G_{az}x(2x^2 - 3y^2 - 3z^2)}{12} - \frac{\sqrt{10}G_{bz}x(y-z)(y+z)}{4}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,0)}[g](E,2), \vec{\mathbb{Q}}_{3,2}^{(3,0)}[g](E,2)$

** symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

** expression

$$\begin{aligned} & \frac{G_1 z (x^2 - 4y^2 + z^2)}{4} - \frac{5G_2 xyz}{4} + \frac{\sqrt{6}G_3 x (2x^2 - 3y^2 - 3z^2)}{12} + \frac{\sqrt{15}G_{3x}xyz}{4} \\ & + \frac{\sqrt{15}G_{3y}z (3x^2 - z^2)}{12} - \frac{\sqrt{10}G_{az}y (x - z)(x + z)}{4} - \frac{\sqrt{6}G_{bz}y (3x^2 - 2y^2 + 3z^2)}{12} \\ & - \frac{5G_1 xyz}{4} - \frac{G_2 z (4x^2 - y^2 - z^2)}{4} + \frac{\sqrt{6}G_3 y (3x^2 - 2y^2 + 3z^2)}{12} - \frac{\sqrt{15}G_{3x}z (3y^2 - z^2)}{12} \\ & - \frac{\sqrt{15}G_{3y}xyz}{4} + \frac{\sqrt{10}G_{az}x (y - z)(y + z)}{4} + \frac{\sqrt{6}G_{bz}x (2x^2 - 3y^2 - 3z^2)}{12} \end{aligned}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,2)}[g](E, 1), \vec{\mathbb{Q}}_{3,2}^{(3,2)}[g](E, 1)$

** symmetry

$$\frac{x (2x^2 - 3y^2 - 3z^2)}{2}$$

$$-\frac{y (3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{330}G_1 z (30x^4 - 87x^2y^2 - 31x^2z^2 + 9y^4 + 11y^2z^2 + 2z^4)}{528} - \frac{7\sqrt{330}G_2 xyz (2x^2 - y^2 - z^2)}{88} - \frac{7\sqrt{55}G_3 x (y - z)(y + z)(2x^2 - y^2 - z^2)}{44} \\ & - \frac{35\sqrt{22}G_{3x}xyz (2x^2 - y^2 - z^2)}{88} - \frac{5\sqrt{22}G_{3y}z (2x^4 + 39x^2y^2 - 17x^2z^2 - 5y^4 - 3y^2z^2 + 2z^4)}{176} \\ & + \frac{5\sqrt{33}G_{az}y (6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4)}{264} - \frac{\sqrt{55}G_{bz}y (10x^4 - 29x^2y^2 + 27x^2z^2 + 3y^4 - y^2z^2 - 4z^4)}{88} \\ & \frac{7\sqrt{330}G_1 xyz (x^2 - 2y^2 + z^2)}{88} + \frac{\sqrt{330}G_2 z (9x^4 - 87x^2y^2 + 11x^2z^2 + 30y^4 - 31y^2z^2 + 2z^4)}{528} - \frac{7\sqrt{55}G_3 y (x - z)(x + z)(x^2 - 2y^2 + z^2)}{44} \\ & - \frac{5\sqrt{22}G_{3x}z (5x^4 - 39x^2y^2 + 3x^2z^2 - 2y^4 + 17y^2z^2 - 2z^4)}{176} - \frac{35\sqrt{22}G_{3y}xyz (x^2 - 2y^2 + z^2)}{88} \\ & + \frac{5\sqrt{33}G_{az}x (x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{264} - \frac{\sqrt{55}G_{bz}x (3x^4 - 29x^2y^2 - x^2z^2 + 10y^4 + 27y^2z^2 - 4z^4)}{88} \end{aligned}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,2)}[g](E, 2), \vec{\mathbb{Q}}_{3,2}^{(3,2)}[g](E, 2)$

** symmetry

$$\frac{\sqrt{15}x (y - z)(y + z)}{2}$$

$$\frac{\sqrt{15}y (x - z)(x + z)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{22}G_1 z (40x^4 - 165x^2y^2 - 25x^2z^2 + 5y^4 + 45y^2z^2 - 2z^4)}{176} - \frac{35\sqrt{22}G_2 xyz (2x^2 - y^2 - z^2)}{88} \\ & - \frac{\sqrt{33}G_3 x (8x^4 - 40x^2y^2 - 40x^2z^2 + 15y^4 + 30y^2z^2 + 15z^4)}{132} + \frac{7\sqrt{330}G_{3x}xyz (2x^2 - y^2 - z^2)}{88} \\ & - \frac{\sqrt{330}G_{3y}z (24x^4 - 57x^2y^2 - 29x^2z^2 + 45y^4 - 71y^2z^2 + 10z^4)}{528} \\ & - \frac{\sqrt{55}G_{az}y (4x^4 + x^2y^2 - 27x^2z^2 - 3y^4 + 29y^2z^2 - 10z^4)}{88} + \frac{\sqrt{33}G_{bz}y (60x^4 - 55x^2y^2 - 195x^2z^2 + 11y^4 - 55y^2z^2 + 60z^4)}{264} \end{aligned}$$

$$\begin{aligned} & \frac{35\sqrt{22}G_1 xyz (x^2 - 2y^2 + z^2)}{88} + \frac{\sqrt{22}G_2 z (5x^4 - 165x^2y^2 + 45x^2z^2 + 40y^4 - 25y^2z^2 - 2z^4)}{176} \\ & + \frac{\sqrt{33}G_3 y (15x^4 - 40x^2y^2 + 30x^2z^2 + 8y^4 - 40y^2z^2 + 15z^4)}{132} + \frac{\sqrt{330}G_{3x}z (45x^4 - 57x^2y^2 - 71x^2z^2 + 24y^4 - 29y^2z^2 + 10z^4)}{528} \\ & + \frac{7\sqrt{330}G_{3y}xyz (x^2 - 2y^2 + z^2)}{88} - \frac{\sqrt{55}G_{az}x (3x^4 - x^2y^2 - 29x^2z^2 - 4y^4 + 27y^2z^2 + 10z^4)}{88} \\ & + \frac{\sqrt{33}G_{bz}x (11x^4 - 55x^2y^2 - 55x^2z^2 + 60y^4 - 195y^2z^2 + 60z^4)}{264} \end{aligned}$$

* Harmonics for rank 4

$\vec{\mathbb{Q}}_4^{(3,-2)}[g](A_1, 1)$

** symmetry

$$\frac{\sqrt{21} (x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\frac{\sqrt{6}G_1xz}{4} + \frac{\sqrt{6}G_2yz}{4} + \frac{\sqrt{10}G_{3x}yz}{4} - \frac{\sqrt{10}G_{3y}xz}{4} - G_{bz}xy$$

$\tilde{\mathbb{Q}}_4^{(3,-2)}[g](A_1, 2)$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$-\frac{\sqrt{210}G_1xz}{20} - \frac{\sqrt{210}G_2yz}{20} + \frac{3\sqrt{35}G_3(x-y)(x+y)}{35} + \frac{5\sqrt{14}G_{3x}yz}{28} - \frac{5\sqrt{14}G_{3y}xz}{28} + \frac{\sqrt{35}G_{bz}xy}{35}$$

$\tilde{\mathbb{Q}}_4^{(3,0)}[g](A_1, 1)$

** symmetry

$$-\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\begin{aligned} & \frac{5\sqrt{33}G_1xz(5x^2 - 9y^2 - 2z^2)}{132} - \frac{5\sqrt{33}G_2yz(9x^2 - 5y^2 + 2z^2)}{132} - \frac{\sqrt{55}G_{3x}yz(15x^2 + y^2 - 6z^2)}{44} \\ & + \frac{\sqrt{55}G_{3y}xz(x^2 + 15y^2 - 6z^2)}{44} + \frac{7\sqrt{330}G_{az}xy(x-y)(x+y)}{132} - \frac{5\sqrt{22}G_{bz}xy(x^2 + y^2 - 6z^2)}{44} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,0)}[g](A_1, 2)$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\begin{aligned} & \frac{\sqrt{1155}G_1xz(x^2 - 9y^2 + 2z^2)}{132} - \frac{\sqrt{1155}G_2yz(9x^2 - y^2 - 2z^2)}{132} + \frac{3\sqrt{770}G_3(x-y)(x+y)(x^2 + y^2 - 6z^2)}{308} \\ & + \frac{\sqrt{77}G_{3x}yz(51x^2 - 47y^2 + 30z^2)}{308} + \frac{\sqrt{77}G_{3y}xz(47x^2 - 51y^2 - 30z^2)}{308} - \frac{7\sqrt{462}G_{az}xy(x-y)(x+y)}{132} + \frac{\sqrt{770}G_{bz}xy(x^2 + y^2 - 6z^2)}{308} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,2)}[g](A_1, 1)$

** symmetry

$$-\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\begin{aligned} & \frac{\sqrt{4290}G_1xz(17x^4 - 29x^2y^2 - 47x^2z^2 - 46y^4 + 121y^2z^2 + 2z^4)}{1144} - \frac{\sqrt{4290}G_2yz(46x^4 + 29x^2y^2 - 121x^2z^2 - 17y^4 + 47y^2z^2 - 2z^4)}{1144} \\ & - \frac{3\sqrt{715}G_3(x-y)(x+y)(x-z)(x+z)(y-z)(y+z)}{26} - \frac{5\sqrt{286}G_{3x}yz(46x^4 - 91x^2y^2 - x^2z^2 - 5y^4 + 47y^2z^2 - 14z^4)}{1144} \\ & - \frac{5\sqrt{286}G_{3y}xz(5x^4 + 91x^2y^2 - 47x^2z^2 - 46y^4 + y^2z^2 + 14z^4)}{1144} + \frac{15\sqrt{429}G_{az}xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572} \\ & - \frac{\sqrt{715}G_{bz}xy(19x^4 - 94x^2y^2 + 92x^2z^2 + 19y^4 + 92y^2z^2 - 92z^4)}{572} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,2)}[g](A_1, 2)$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\begin{aligned} & \frac{\sqrt{6006}G_1xz(10x^4 - 25x^2y^2 - 25x^2z^2 - 35y^4 + 95y^2z^2 - 2z^4)}{1144} - \frac{\sqrt{6006}G_2yz(35x^4 + 25x^2y^2 - 95x^2z^2 - 10y^4 + 25y^2z^2 + 2z^4)}{1144} \\ & - \frac{3\sqrt{1001}G_3(x-y)(x+y)(x^4 - 9x^2y^2 - 5x^2z^2 + y^4 - 5y^2z^2 + 5z^4)}{286} \\ & + \frac{\sqrt{10010}G_{3x}yz(61x^4 - 61x^2y^2 - 61x^2z^2 + 10y^4 - 13y^2z^2 + 10z^4)}{1144} - \frac{\sqrt{10010}G_{3y}xz(10x^4 - 61x^2y^2 - 13x^2z^2 + 61y^4 - 61y^2z^2 + 10z^4)}{1144} \\ & - \frac{3\sqrt{15015}G_{az}xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572} + \frac{\sqrt{1001}G_{bz}xy(31x^4 - 70x^2y^2 - 100x^2z^2 + 31y^4 - 100y^2z^2 + 100z^4)}{572} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,-2)}[g](A_2)$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{3\sqrt{10}G_1yz}{10} - \frac{3\sqrt{10}G_2xz}{10} - \frac{\sqrt{15}G_3xy}{5} + \frac{\sqrt{15}G_{bz}(x-y)(x+y)}{10}$$

$\vec{\mathbb{Q}}_4^{(3,0)}[g](A_2)$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{55}G_1yz(3x^2+3y^2-4z^2)}{44} - \frac{\sqrt{55}G_2xz(3x^2+3y^2-4z^2)}{44} - \frac{\sqrt{330}G_3xy(x^2+y^2-6z^2)}{44} + \frac{7\sqrt{33}G_{3x}xz(x^2-3y^2)}{44} \\ & - \frac{7\sqrt{33}G_{3y}yz(3x^2-y^2)}{44} - \frac{7\sqrt{22}G_{az}(x^2-2xy-y^2)(x^2+2xy-y^2)}{88} + \frac{\sqrt{330}G_{bz}(x-y)(x+y)(x^2+y^2-6z^2)}{88} \end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,2)}[g](A_2)$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{286}G_1yz(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4)}{2288} - \frac{3\sqrt{286}G_2xz(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4)}{2288} \\ & + \frac{\sqrt{429}G_3xy(47x^4-170x^2y^2+40x^2z^2+47y^4+40y^2z^2-40z^4)}{572} + \frac{3\sqrt{4290}G_{3x}xz(17x^4-122x^2y^2-16x^2z^2+37y^4+48y^2z^2)}{2288} \\ & + \frac{3\sqrt{4290}G_{3y}yz(37x^4-122x^2y^2+48x^2z^2+17y^4-16y^2z^2)}{2288} - \frac{9\sqrt{715}G_{az}(x^2+y^2-10z^2)(x^2-2xy-y^2)(x^2+2xy-y^2)}{1144} \\ & + \frac{\sqrt{429}G_{bz}(x-y)(x+y)(19x^4-226x^2y^2-40x^2z^2+19y^4-40y^2z^2+40z^4)}{1144} \end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,-2)}[g](B_1)$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\frac{9\sqrt{70}G_1xz}{140} - \frac{9\sqrt{70}G_2yz}{140} + \frac{\sqrt{105}G_3(x^2+y^2-2z^2)}{35} + \frac{\sqrt{42}G_{3x}yz}{28} + \frac{\sqrt{42}G_{3y}xz}{28} + \frac{3\sqrt{7}G_{az}xy}{7}$$

$\vec{\mathbb{Q}}_4^{(3,0)}[g](B_1)$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{5\sqrt{385}G_1xz(3x^2+3y^2-4z^2)}{308} - \frac{5\sqrt{385}G_2yz(3x^2+3y^2-4z^2)}{308} + \frac{\sqrt{2310}G_3(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4)}{308} \\ & + \frac{\sqrt{231}G_{3x}yz(27x^2-y^2-8z^2)}{308} - \frac{\sqrt{231}G_{3y}xz(x^2-27y^2+8z^2)}{308} - \frac{13\sqrt{154}G_{az}xy(x^2+y^2-6z^2)}{308} - \frac{\sqrt{2310}G_{bz}xy(x-y)(x+y)}{44} \end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,2)}[g](B_1)$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{9\sqrt{2002}G_1xz(4x^4-25x^2y^2-5x^2z^2+15y^4-5y^2z^2+2z^4)}{1144} - \frac{9\sqrt{2002}G_2yz(15x^4-25x^2y^2-5x^2z^2+4y^4-5y^2z^2+2z^4)}{1144} \\ & - \frac{\sqrt{3003}G_3(x^6-15x^4z^2+15x^2z^4+y^6-15y^4z^2+15y^2z^4-2z^6)}{286} \\ & - \frac{\sqrt{30030}G_{3x}yz(13x^4+17x^2y^2-43x^2z^2+4y^4-19y^2z^2+10z^4)}{1144} - \frac{\sqrt{30030}G_{3y}xz(4x^4+17x^2y^2-19x^2z^2+13y^4-43y^2z^2+10z^4)}{1144} \\ & + \frac{3\sqrt{5005}G_{az}xy(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{572} + \frac{3\sqrt{3003}G_{bz}xy(x-y)(x+y)(x^2+y^2-10z^2)}{572} \end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,-2)}[g](B_2)$
** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\frac{9\sqrt{70}G_1yz}{140} + \frac{9\sqrt{70}G_2xz}{140} + \frac{\sqrt{42}G_{3x}xz}{28} - \frac{\sqrt{42}G_{3y}yz}{28} + \frac{3\sqrt{7}G_{az}(x-y)(x+y)}{14} + \frac{\sqrt{105}G_{bz}(x^2 + y^2 - 2z^2)}{35}$$

$\vec{\mathbb{Q}}_4^{(3,0)}[g](B_2)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{5\sqrt{385}G_1yz(3x^2 + 3y^2 - 4z^2)}{308} + \frac{5\sqrt{385}G_2xz(3x^2 + 3y^2 - 4z^2)}{308} - \frac{\sqrt{2310}G_3xy(x-y)(x+y)}{44} + \frac{\sqrt{231}G_{3x}xz(13x^2 - 15y^2 - 8z^2)}{308} \\ & + \frac{\sqrt{231}G_{3y}yz(15x^2 - 13y^2 + 8z^2)}{308} - \frac{13\sqrt{154}G_{az}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{616} - \frac{\sqrt{2310}G_{bz}(5x^4 - 18x^2y^2 - 12x^2z^2 + 5y^4 - 12y^2z^2 + 4z^4)}{616} \end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,2)}[g](B_2)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\begin{aligned} & -\frac{9\sqrt{2002}G_1yz(25x^4 - 60x^2y^2 + 10x^2z^2 + 3y^4 + 10y^2z^2 - 4z^4)}{2288} - \frac{9\sqrt{2002}G_2xz(3x^4 - 60x^2y^2 + 10x^2z^2 + 25y^4 + 10y^2z^2 - 4z^4)}{2288} \\ & + \frac{3\sqrt{3003}G_3xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572} - \frac{\sqrt{30030}G_{3x}xz(17x^4 + 16x^2y^2 - 62x^2z^2 - y^4 - 14y^2z^2 + 20z^4)}{2288} \\ & - \frac{\sqrt{30030}G_{3y}yz(x^4 - 16x^2y^2 + 14x^2z^2 - 17y^4 + 62y^2z^2 - 20z^4)}{2288} + \frac{3\sqrt{5005}G_{az}(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144} \\ & - \frac{\sqrt{3003}G_{bz}(x^6 + 15x^4y^2 - 30x^4z^2 + 15x^2y^4 - 180x^2y^2z^2 + 60x^2z^4 + y^6 - 30y^4z^2 + 60y^2z^4 - 8z^6)}{1144} \end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,-2)}[g](E, 1), \vec{\mathbb{Q}}_{4,2}^{(3,-2)}[g](E, 1)$

** symmetry

$$\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$\begin{aligned} & -\frac{3\sqrt{10}G_1(x-z)(x+z)}{40} + \frac{3\sqrt{10}G_2xy}{40} + \frac{\sqrt{15}G_3xz}{5} - \frac{3\sqrt{6}G_{3x}xy}{8} + \frac{\sqrt{6}G_{3y}(x-z)(x+z)}{8} + \frac{3G_{az}yz}{4} - \frac{3\sqrt{15}G_{bz}yz}{20} \\ & - \frac{3\sqrt{10}G_1xy}{40} - \frac{3\sqrt{10}G_2(y-z)(y+z)}{40} - \frac{\sqrt{15}G_3yz}{5} - \frac{\sqrt{6}G_{3x}(y-z)(y+z)}{8} + \frac{3\sqrt{6}G_{3y}xy}{8} - \frac{3G_{az}xz}{4} - \frac{3\sqrt{15}G_{bz}xz}{20} \end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,-2)}[g](E, 2), \vec{\mathbb{Q}}_{4,2}^{(3,-2)}[g](E, 2)$

** symmetry

$$-\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{70}G_1(3x^2 + 4y^2 - 7z^2)}{280} + \frac{3\sqrt{70}G_2xy}{280} + \frac{5\sqrt{42}G_{3x}xy}{56} + \frac{\sqrt{42}G_{3y}(5x^2 - 4y^2 - z^2)}{56} + \frac{3\sqrt{7}G_{az}yz}{28} - \frac{\sqrt{105}G_{bz}yz}{20} \\ & - \frac{3\sqrt{70}G_1xy}{280} + \frac{3\sqrt{70}G_2(4x^2 + 3y^2 - 7z^2)}{280} + \frac{\sqrt{42}G_{3x}(4x^2 - 5y^2 + z^2)}{56} - \frac{5\sqrt{42}G_{3y}xy}{56} - \frac{3\sqrt{7}G_{az}xz}{28} - \frac{\sqrt{105}G_{bz}xz}{20} \end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,0)}[g](E, 1), \vec{\mathbb{Q}}_{4,2}^{(3,0)}[g](E, 1)$

** symmetry

$$\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{55}G_1(5x^4 - 9x^2y^2 - 21x^2z^2 + 9y^2z^2 + 2z^4)}{88} + \frac{\sqrt{55}G_2xy(6x^2 - y^2 - 15z^2)}{44} + \frac{\sqrt{330}G_3xz(x^2 - 6y^2 + z^2)}{44} - \frac{\sqrt{33}G_{3x}xy(2x^2 - 5y^2 + 9z^2)}{44} \\ & -\frac{\sqrt{33}G_{3y}(x^4 + 15x^2y^2 - 21x^2z^2 - 15y^2z^2 + 6z^4)}{88} - \frac{\sqrt{22}G_{az}yz(12x^2 + 5y^2 - 9z^2)}{44} - \frac{\sqrt{330}G_{bz}yz(6x^2 - y^2 - z^2)}{44} \\ & -\frac{\sqrt{55}G_1xy(x^2 - 6y^2 + 15z^2)}{44} + \frac{\sqrt{55}G_2(9x^2y^2 - 9x^2z^2 - 5y^4 + 21y^2z^2 - 2z^4)}{88} + \frac{\sqrt{330}G_3yz(6x^2 - y^2 - z^2)}{44} \\ & + \frac{\sqrt{33}G_{3x}(15x^2y^2 - 15x^2z^2 + y^4 - 21y^2z^2 + 6z^4)}{88} - \frac{\sqrt{33}G_{3y}xy(5x^2 - 2y^2 - 9z^2)}{44} + \frac{\sqrt{22}G_{az}xz(5x^2 + 12y^2 - 9z^2)}{44} + \frac{\sqrt{330}G_{bz}xz(x^2 - 6y^2 + z^2)}{44} \end{aligned}$$

$$\vec{\mathbb{Q}}_{4,1}^{(3,0)}[g](E, 2), \vec{\mathbb{Q}}_{4,2}^{(3,0)}[g](E, 2)$$

** symmetry

$$-\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{385}G_1(x^4 + 21x^2y^2 - 27x^2z^2 + 6y^4 - 57y^2z^2 + 14z^4)}{616} + \frac{\sqrt{385}G_2xy(6x^2 - y^2 - 15z^2)}{308} + \frac{\sqrt{2310}G_3xz(x-z)(x+z)}{44} \\ & + \frac{\sqrt{231}G_{3x}xy(22x^2 - 27y^2 + 15z^2)}{308} - \frac{\sqrt{231}G_{3y}(19x^4 - 69x^2y^2 - 45x^2z^2 + 10y^4 + 9y^2z^2 + 6z^4)}{616} \\ & + \frac{\sqrt{154}G_{az}yz(30x^2 - 19y^2 + 9z^2)}{308} + \frac{\sqrt{2310}G_{bz}yz(y-z)(y+z)}{44} \\ & -\frac{\sqrt{385}G_1xy(x^2 - 6y^2 + 15z^2)}{308} + \frac{\sqrt{385}G_2(6x^4 + 21x^2y^2 - 57x^2z^2 + y^4 - 27y^2z^2 + 14z^4)}{616} \\ & -\frac{\sqrt{2310}G_{3y}yz(y-z)(y+z)}{44} + \frac{\sqrt{231}G_{3x}(10x^4 - 69x^2y^2 + 9x^2z^2 + 19y^4 - 45y^2z^2 + 6z^4)}{616} \\ & + \frac{\sqrt{231}G_{3y}xy(27x^2 - 22y^2 - 15z^2)}{308} + \frac{\sqrt{154}G_{az}xz(19x^2 - 30y^2 - 9z^2)}{308} + \frac{\sqrt{2310}G_{bz}xz(x-z)(x+z)}{44} \end{aligned}$$

$$\vec{\mathbb{Q}}_{4,1}^{(3,2)}[g](E, 1), \vec{\mathbb{Q}}_{4,2}^{(3,2)}[g](E, 1)$$

** symmetry

$$\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{286}G_1(17x^6 - 95x^4y^2 - 160x^4z^2 + 20x^2y^4 + 450x^2y^2z^2 + 85x^2z^4 - 20y^4z^2 - 55y^2z^4 - 2z^6)}{2288} \\ & + \frac{3\sqrt{286}G_2xy(65x^4 - 65x^2y^2 - 455x^2z^2 + 2y^4 + 175y^2z^2 + 140z^4)}{2288} \\ & - \frac{\sqrt{429}G_3xz(47x^4 + 40x^2y^2 - 170x^2z^2 - 40y^4 + 40y^2z^2 + 47z^4)}{572} + \frac{3\sqrt{4290}G_{3x}xy(3x^4 + x^2y^2 - 33x^2z^2 - 2y^4 + 17y^2z^2 + 8z^4)}{2288} \\ & + \frac{\sqrt{4290}G_{3y}(5x^6 + 25x^4y^2 - 100x^4z^2 + 20x^2y^4 - 270x^2y^2z^2 + 145x^2z^4 - 20y^4z^2 + 65y^2z^4 - 14z^6)}{2288} \\ & + \frac{3\sqrt{715}G_{az}yz(29x^4 + 31x^2y^2 - 89x^2z^2 + 2y^4 - 17y^2z^2 + 14z^4)}{1144} + \frac{3\sqrt{429}G_{bz}yz(45x^4 + 65x^2y^2 - 155x^2z^2 - 2y^4 - 15y^2z^2 + 20z^4)}{1144} \\ & \frac{3\sqrt{286}G_1xy(2x^4 - 65x^2y^2 + 175x^2z^2 + 65y^4 - 455y^2z^2 + 140z^4)}{2288} \\ & - \frac{3\sqrt{286}G_2(20x^4y^2 - 20x^4z^2 - 95x^2y^4 + 450x^2y^2z^2 - 55x^2z^4 + 17y^6 - 160y^4z^2 + 85y^2z^4 - 2z^6)}{2288} \\ & - \frac{\sqrt{429}G_3yz(40x^4 - 40x^2y^2 - 40x^2z^2 - 47y^4 + 170y^2z^2 - 47z^4)}{572} \\ & - \frac{\sqrt{4290}G_{3x}(20x^4y^2 - 20x^4z^2 + 25x^2y^4 - 270x^2y^2z^2 + 65x^2z^4 + 5y^6 - 100y^4z^2 + 145y^2z^4 - 14z^6)}{2288} \\ & + \frac{3\sqrt{4290}G_{3y}xy(2x^4 - x^2y^2 - 17x^2z^2 - 3y^4 + 33y^2z^2 - 8z^4)}{2288} - \frac{3\sqrt{715}G_{az}xz(2x^4 + 31x^2y^2 - 17x^2z^2 + 29y^4 - 89y^2z^2 + 14z^4)}{1144} \\ & - \frac{3\sqrt{429}G_{bz}xz(2x^4 - 65x^2y^2 + 15x^2z^2 - 45y^4 + 155y^2z^2 - 20z^4)}{1144} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{4,1}^{(3,2)}[g](E,2), \tilde{\mathbb{Q}}_{4,2}^{(3,2)}[g](E,2)$$

** symmetry

$$-\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{2002}G_1(3x^6 - 55x^4y^2 + 10x^4z^2 + 70x^2y^4 - 90x^2y^2z^2 + 5x^2z^4 - 4y^6 - 10y^4z^2 + 25y^2z^4 - 2z^6)}{2288} \\ & - \frac{3\sqrt{2002}G_2xy(19x^4 - 85x^2y^2 + 65x^2z^2 + 28y^4 - 25y^2z^2 - 20z^4)}{2288} \\ & - \frac{3\sqrt{3003}G_3xz(x-z)(x+z)(x^2 - 10y^2 + z^2)}{572} - \frac{\sqrt{30030}G_{3x}xy(11x^4 + 7x^2y^2 - 131x^2z^2 - 4y^4 + 19y^2z^2 + 56z^4)}{2288} \\ & + \frac{\sqrt{30030}G_{3y}(x^6 - 9x^4y^2 - 6x^4z^2 - 6x^2y^4 + 90x^2y^2z^2 - 9x^2z^4 + 4y^6 - 54y^4z^2 + 39y^2z^4 - 2z^6)}{2288} \\ & - \frac{3\sqrt{5005}G_{az}yz(19x^4 + 11x^2y^2 - 49x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4)}{1144} + \frac{\sqrt{3003}G_{bz}yz(55x^4 + 5x^2y^2 - 115x^2z^2 + 16y^4 - 55y^2z^2 + 28z^4)}{1144} \\ & - \frac{3\sqrt{2002}G_1xy(28x^4 - 85x^2y^2 - 25x^2z^2 + 19y^4 + 65y^2z^2 - 20z^4)}{2288} \\ & - \frac{3\sqrt{2002}G_2(4x^6 - 70x^4y^2 + 10x^4z^2 + 55x^2y^4 + 90x^2y^2z^2 - 25x^2z^4 - 3y^6 - 10y^4z^2 - 5y^2z^4 + 2z^6)}{2288} \\ & - \frac{3\sqrt{3003}G_3yz(y-z)(y+z)(10x^2 - y^2 - z^2)}{572} - \frac{\sqrt{30030}G_{3x}(4x^6 - 6x^4y^2 - 54x^4z^2 - 9x^2y^4 + 90x^2y^2z^2 + 39x^2z^4 + y^6 - 6y^4z^2 - 9y^2z^4 - 2z^6)}{2288} \\ & - \frac{\sqrt{30030}G_{3y}xy(4x^4 - 7x^2y^2 - 19x^2z^2 - 11y^4 + 131y^2z^2 - 56z^4)}{2288} - \frac{3\sqrt{5005}G_{az}xz(8x^4 - 11x^2y^2 - 23x^2z^2 - 19y^4 + 49y^2z^2 + 2z^4)}{1144} \\ & + \frac{\sqrt{3003}G_{bz}xz(16x^4 + 5x^2y^2 - 55x^2z^2 + 55y^4 - 115y^2z^2 + 28z^4)}{1144} \end{aligned}$$