

* Harmonics for rank 0

$$\bar{Q}_0^{(1,1)}[q](A_g)$$

** symmetry

$$1$$

** expression

$$\frac{\sqrt{3}Q_x x}{3} + \frac{\sqrt{3}Q_y y}{3} + \frac{\sqrt{3}Q_z z}{3}$$

* Harmonics for rank 1

$$\bar{Q}_1^{(1,-1)}[q](A_u)$$

** symmetry

$$z$$

** expression

$$Q_z$$

$$\bar{Q}_1^{(1,1)}[q](A_u)$$

** symmetry

$$z$$

** expression

$$\frac{3\sqrt{10}Q_x xz}{10} + \frac{3\sqrt{10}Q_y yz}{10} - \frac{\sqrt{10}Q_z (x^2 + y^2 - 2z^2)}{10}$$

$$\bar{Q}_{1,1}^{(1,-1)}[q](E_u), \bar{Q}_{1,2}^{(1,-1)}[q](E_u)$$

** symmetry

$$x$$

$$y$$

** expression

$$Q_x$$

$$Q_y$$

$$\bar{Q}_{1,1}^{(1,1)}[q](E_u), \bar{Q}_{1,2}^{(1,1)}[q](E_u)$$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{\sqrt{10}Q_x (2x^2 - y^2 - z^2)}{10} + \frac{3\sqrt{10}Q_y xy}{10} + \frac{3\sqrt{10}Q_z xz}{10}$$

$$\frac{3\sqrt{10}Q_x xy}{10} - \frac{\sqrt{10}Q_y (x^2 - 2y^2 + z^2)}{10} + \frac{3\sqrt{10}Q_z yz}{10}$$

* Harmonics for rank 2

$$\bar{Q}_2^{(1,-1)}[q](A_g)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{6}Q_x x}{6} - \frac{\sqrt{6}Q_y y}{6} + \frac{\sqrt{6}Q_z z}{3}$$

$$\bar{Q}_2^{(1,1)}[q](A_g)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{21}Q_{xx}(x^2+y^2-4z^2)}{14}-\frac{\sqrt{21}Q_{yy}(x^2+y^2-4z^2)}{14}-\frac{\sqrt{21}Q_{zz}(3x^2+3y^2-2z^2)}{14}$$

$$\tilde{\mathbb{Q}}_2^{(1,-1)}[q](B_g,1)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}Q_{xx}}{2}-\frac{\sqrt{2}Q_{yy}}{2}$$

$$\tilde{\mathbb{Q}}_2^{(1,-1)}[q](B_g,2)$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}Q_{xy}}{2}+\frac{\sqrt{2}Q_{yx}}{2}$$

$$\tilde{\mathbb{Q}}_2^{(1,1)}[q](B_g,1)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{7}Q_{xx}(3x^2-7y^2-2z^2)}{14}+\frac{\sqrt{7}Q_{yy}(7x^2-3y^2+2z^2)}{14}+\frac{5\sqrt{7}Q_{zz}(x-y)(x+y)}{14}$$

$$\tilde{\mathbb{Q}}_2^{(1,1)}[q](B_g,2)$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{7}Q_{xy}(4x^2-y^2-z^2)}{7}-\frac{\sqrt{7}Q_{yx}(x^2-4y^2+z^2)}{7}+\frac{5\sqrt{7}Q_{zxyz}}{7}$$

$$\tilde{\mathbb{Q}}_{2,1}^{(1,-1)}[q](E_g), \tilde{\mathbb{Q}}_{2,2}^{(1,-1)}[q](E_g)$$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_{yz}}{2}+\frac{\sqrt{2}Q_{zy}}{2}$$

$$\frac{\sqrt{2}Q_{xz}}{2}+\frac{\sqrt{2}Q_{zx}}{2}$$

$$\tilde{\mathbb{Q}}_{2,1}^{(1,1)}[q](E_g), \tilde{\mathbb{Q}}_{2,2}^{(1,1)}[q](E_g)$$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

** expression

$$\frac{5\sqrt{7}Q_{xxyz}}{7}-\frac{\sqrt{7}Q_{yz}(x^2-4y^2+z^2)}{7}-\frac{\sqrt{7}Q_{zy}(x^2+y^2-4z^2)}{7}$$

$$\frac{\sqrt{7}Q_{xz}(4x^2-y^2-z^2)}{7}+\frac{5\sqrt{7}Q_{yxyz}}{7}-\frac{\sqrt{7}Q_{zx}(x^2+y^2-4z^2)}{7}$$

* Harmonics for rank 3

$$\bar{Q}_3^{(1,-1)}[q](A_u)$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_x xz}{5} - \frac{\sqrt{15}Q_y yz}{5} - \frac{\sqrt{15}Q_z(x^2+y^2-2z^2)}{10}$$

$$\bar{Q}_3^{(1,1)}[q](A_u)$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$-\frac{5Q_x xz(3x^2+3y^2-4z^2)}{12} - \frac{5Q_y yz(3x^2+3y^2-4z^2)}{12} + \frac{Q_z(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4)}{12}$$

$$\bar{Q}_3^{(1,-1)}[q](B_u, 1)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$Q_x yz + Q_y xz + Q_z xy$$

$$\bar{Q}_3^{(1,-1)}[q](B_u, 2)$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$Q_x xz - Q_y yz + \frac{Q_z(x-y)(x+y)}{2}$$

$$\bar{Q}_3^{(1,1)}[q](B_u, 1)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{15}Q_x yz(6x^2-y^2-z^2)}{6} - \frac{\sqrt{15}Q_y xz(x^2-6y^2+z^2)}{6} - \frac{\sqrt{15}Q_z xy(x^2+y^2-6z^2)}{6}$$

$$\bar{Q}_3^{(1,1)}[q](B_u, 2)$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{15}Q_x xz(5x^2-9y^2-2z^2)}{12} + \frac{\sqrt{15}Q_y yz(9x^2-5y^2+2z^2)}{12} - \frac{\sqrt{15}Q_z(x-y)(x+y)(x^2+y^2-6z^2)}{12}$$

$$\bar{Q}_{3,1}^{(1,-1)}[q](E_u, 1), \bar{Q}_{3,2}^{(1,-1)}[q](E_u, 1)$$

** symmetry

$$\frac{x(2x^2-3y^2-3z^2)}{2}$$

$$-\frac{y(3x^2-2y^2+3z^2)}{2}$$

** expression

$$\frac{\sqrt{15}Q_x(2x^2-y^2-z^2)}{10} - \frac{\sqrt{15}Q_y xy}{5} - \frac{\sqrt{15}Q_z xz}{5}$$

$$-\frac{\sqrt{15}Q_x xy}{5} - \frac{\sqrt{15}Q_y(x^2-2y^2+z^2)}{10} - \frac{\sqrt{15}Q_z yz}{5}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(1,-1)}[q](E_u, 2), \tilde{\mathbb{Q}}_{3,2}^{(1,-1)}[q](E_u, 2)$$

** symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

** expression

$$-Q_{xx}xy - \frac{Q_y(x-z)(x+z)}{2} + Q_z yz$$

$$\frac{Q_x(y-z)(y+z)}{2} + Q_y xy - Q_z xz$$

$$\tilde{\mathbb{Q}}_{3,1}^{(1,1)}[q](E_u, 1), \tilde{\mathbb{Q}}_{3,2}^{(1,1)}[q](E_u, 1)$$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{Q_x(8x^4 - 24x^2y^2 - 24x^2z^2 + 3y^4 + 6y^2z^2 + 3z^4)}{12} + \frac{5Q_yxy(4x^2 - 3y^2 - 3z^2)}{12} + \frac{5Q_zxz(4x^2 - 3y^2 - 3z^2)}{12} \\ & - \frac{5Q_{xx}y(3x^2 - 4y^2 + 3z^2)}{12} + \frac{Q_y(3x^4 - 24x^2y^2 + 6x^2z^2 + 8y^4 - 24y^2z^2 + 3z^4)}{12} - \frac{5Q_zyz(3x^2 - 4y^2 + 3z^2)}{12} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(1,1)}[q](E_u, 2), \tilde{\mathbb{Q}}_{3,2}^{(1,1)}[q](E_u, 2)$$

** symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{15}Q_{xx}y(5x^2 - 2y^2 - 9z^2)}{12} + \frac{\sqrt{15}Q_y(x-z)(x+z)(x^2 - 6y^2 + z^2)}{12} - \frac{\sqrt{15}Q_zyz(9x^2 + 2y^2 - 5z^2)}{12} \\ & \frac{\sqrt{15}Q_x(y-z)(y+z)(6x^2 - y^2 - z^2)}{12} - \frac{\sqrt{15}Q_yxy(2x^2 - 5y^2 + 9z^2)}{12} + \frac{\sqrt{15}Q_zxz(2x^2 + 9y^2 - 5z^2)}{12} \end{aligned}$$

* Harmonics for rank 4

$$\tilde{\mathbb{Q}}_4^{(1,-1)}[q](A_g, 1)$$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\frac{\sqrt{3}Q_{xx}(2x^2 - 3y^2 - 3z^2)}{6} - \frac{\sqrt{3}Q_yy(3x^2 - 2y^2 + 3z^2)}{6} - \frac{\sqrt{3}Q_zz(3x^2 + 3y^2 - 2z^2)}{6}$$

$$\tilde{\mathbb{Q}}_4^{(1,-1)}[q](A_g, 2)$$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$-\frac{\sqrt{105}Q_{xx}(x^2 - 6y^2 + 3z^2)}{42} + \frac{\sqrt{105}Q_yy(6x^2 - y^2 - 3z^2)}{42} - \frac{\sqrt{105}Q_zz(3x^2 + 3y^2 - 2z^2)}{42}$$

$$\tilde{\mathbb{Q}}_4^{(1,-1)}[q](A_g, 3)$$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{5}Q_{xy}(3x^2-y^2)}{4} + \frac{\sqrt{5}Q_{yx}(x^2-3y^2)}{4}$$

$$\tilde{Q}_4^{(1,1)}[q](A_g, 1)$$

** symmetry

$$\frac{\sqrt{21}(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4)}{6}$$

** expression

$$\frac{\sqrt{1155}Q_{xx}(x^4-5x^2y^2-5x^2z^2+3y^4-3y^2z^2+3z^4)}{66} + \frac{\sqrt{1155}Q_{yy}(3x^4-5x^2y^2-3x^2z^2+y^4-5y^2z^2+3z^4)}{66} \\ + \frac{\sqrt{1155}Q_{zz}(3x^4-3x^2y^2-5x^2z^2+3y^4-5y^2z^2+z^4)}{66}$$

$$\tilde{Q}_4^{(1,1)}[q](A_g, 2)$$

** symmetry

$$-\frac{\sqrt{15}(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4)}{12}$$

** expression

$$-\frac{\sqrt{33}Q_{xx}(5x^4-88x^2y^2+38x^2z^2+33y^4+66y^2z^2-30z^4)}{132} - \frac{\sqrt{33}Q_{yy}(33x^4-88x^2y^2+66x^2z^2+5y^4+38y^2z^2-30z^4)}{132} \\ + \frac{\sqrt{33}Q_{zz}(3x^4+132x^2y^2-50x^2z^2+3y^4-50y^2z^2+10z^4)}{132}$$

$$\tilde{Q}_4^{(1,1)}[q](A_g, 3)$$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{77}Q_{xy}(6x^4-11x^2y^2-3x^2z^2+y^4+y^2z^2)}{22} - \frac{\sqrt{77}Q_{yx}(x^4-11x^2y^2+x^2z^2+6y^4-3y^2z^2)}{22} + \frac{9\sqrt{77}Q_{zxyz}(x-y)(x+y)}{22}$$

$$\tilde{Q}_4^{(1,-1)}[q](B_g, 1)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\frac{\sqrt{35}Q_{xx}(x^2-3z^2)}{14} - \frac{\sqrt{35}Q_{yy}(y^2-3z^2)}{14} - \frac{3\sqrt{35}Q_{zz}(x-y)(x+y)}{14}$$

$$\tilde{Q}_4^{(1,-1)}[q](B_g, 2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$-\frac{\sqrt{35}Q_{xy}(3x^2+y^2-6z^2)}{28} - \frac{\sqrt{35}Q_{yx}(x^2+3y^2-6z^2)}{28} + \frac{3\sqrt{35}Q_{zxyz}}{7}$$

$$\tilde{Q}_4^{(1,1)}[q](B_g, 1)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\frac{\sqrt{11}Q_{xx}(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4)}{44} \\ + \frac{\sqrt{11}Q_{yy}(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4)}{44} + \frac{21\sqrt{11}Q_{zz}(x-y)(x+y)(x^2+y^2-2z^2)}{44}$$

$$\tilde{Q}_4^{(1,1)}[q](B_g, 2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$-\frac{\sqrt{11}Q_x y(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{22} + \frac{\sqrt{11}Q_y x(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{22} - \frac{21\sqrt{11}Q_z xyz(x^2+y^2-2z^2)}{22}$$

$$\tilde{Q}_{4,1}^{(1,-1)}[q](E_g, 1), \tilde{Q}_{4,2}^{(1,-1)}[q](E_g, 1)$$

** symmetry

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$-\frac{\sqrt{5}Q_x z(3x^2-z^2)}{4} - \frac{\sqrt{5}Q_z x(x^2-3z^2)}{4}$$

$$\frac{\sqrt{5}Q_y z(3y^2-z^2)}{4} + \frac{\sqrt{5}Q_z y(y^2-3z^2)}{4}$$

$$\tilde{Q}_{4,1}^{(1,-1)}[q](E_g, 2), \tilde{Q}_{4,2}^{(1,-1)}[q](E_g, 2)$$

** symmetry

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

** expression

$$\frac{3\sqrt{35}Q_x xyz}{7} + \frac{\sqrt{35}Q_y z(6x^2-3y^2-z^2)}{28} + \frac{\sqrt{35}Q_z y(6x^2-y^2-3z^2)}{28}$$

$$-\frac{\sqrt{35}Q_x z(3x^2-6y^2+z^2)}{28} + \frac{3\sqrt{35}Q_y xyz}{7} - \frac{\sqrt{35}Q_z x(x^2-6y^2+3z^2)}{28}$$

$$\tilde{Q}_{4,1}^{(1,1)}[q](E_g, 1), \tilde{Q}_{4,2}^{(1,1)}[q](E_g, 1)$$

** symmetry

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$-\frac{\sqrt{77}Q_x z(6x^4-3x^2y^2-11x^2z^2+y^2z^2+z^4)}{22} - \frac{9\sqrt{77}Q_y xyz(x-z)(x+z)}{22} + \frac{\sqrt{77}Q_z x(x^4+x^2y^2-11x^2z^2-3y^2z^2+6z^4)}{22}$$

$$\frac{9\sqrt{77}Q_x xyz(y-z)(y+z)}{22} - \frac{\sqrt{77}Q_y z(3x^2y^2-x^2z^2-6y^4+11y^2z^2-z^4)}{22} - \frac{\sqrt{77}Q_z y(x^2y^2-3x^2z^2+y^4-11y^2z^2+6z^4)}{22}$$

$$\tilde{Q}_{4,1}^{(1,1)}[q](E_g, 2), \tilde{Q}_{4,2}^{(1,1)}[q](E_g, 2)$$

** symmetry

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

** expression

$$\frac{21\sqrt{11}Q_x xyz(2x^2-y^2-z^2)}{22} - \frac{\sqrt{11}Q_y z(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4)}{22} - \frac{\sqrt{11}Q_z y(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{22}$$

$$-\frac{\sqrt{11}Q_x z(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4)}{22} - \frac{21\sqrt{11}Q_y xyz(x^2-2y^2+z^2)}{22} + \frac{\sqrt{11}Q_z x(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{22}$$