

PG No. 21 C_6 6 [hexagonal] (axial, internal axial dipole)

* Harmonics for rank 0

$$\vec{\mathbb{G}}_0^{(1,1)}[g](A)$$

** symmetry

1

** expression

$$\frac{\sqrt{3}G_x x}{3} + \frac{\sqrt{3}G_y y}{3} + \frac{\sqrt{3}G_z z}{3}$$

* Harmonics for rank 1

$$\vec{\mathbb{G}}_1^{(1,-1)}[g](A)$$

** symmetry

z

** expression

G_z

$$\vec{\mathbb{G}}_1^{(1,1)}[g](A)$$

** symmetry

z

** expression

$$\frac{3\sqrt{10}G_x x z}{10} + \frac{3\sqrt{10}G_y y z}{10} - \frac{\sqrt{10}G_z (x^2 + y^2 - 2z^2)}{10}$$

$$\vec{\mathbb{G}}_{1,1}^{(1,-1)}[g](E_1), \vec{\mathbb{G}}_{1,2}^{(1,-1)}[g](E_1)$$

** symmetry

x

y

** expression

G_x

G_y

$$\vec{\mathbb{G}}_{1,1}^{(1,1)}[g](E_1), \vec{\mathbb{G}}_{1,2}^{(1,1)}[g](E_1)$$

** symmetry

x

y

** expression

$$\frac{\sqrt{10}G_x (2x^2 - y^2 - z^2)}{10} + \frac{3\sqrt{10}G_y x y}{10} + \frac{3\sqrt{10}G_z x z}{10}$$

$$\frac{3\sqrt{10}G_x x y}{10} - \frac{\sqrt{10}G_y (x^2 - 2y^2 + z^2)}{10} + \frac{3\sqrt{10}G_z y z}{10}$$

* Harmonics for rank 2

$$\vec{\mathbb{G}}_2^{(1,-1)}[g](A)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{6}G_x x}{6} - \frac{\sqrt{6}G_y y}{6} + \frac{\sqrt{6}G_z z}{3}$$

$$\vec{\mathbb{G}}_2^{(1,1)}[g](A)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{21}G_xx(x^2+y^2-4z^2)}{14}-\frac{\sqrt{21}G_yy(x^2+y^2-4z^2)}{14}-\frac{\sqrt{21}G_zz(3x^2+3y^2-2z^2)}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(1,-1)}[g](E_1), \vec{\mathbb{G}}_{2,2}^{(1,-1)}[g](E_1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}G_yz}{2}+\frac{\sqrt{2}G_zy}{2}$$

$$-\frac{\sqrt{2}G_xz}{2}-\frac{\sqrt{2}G_zx}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,1)}[g](E_1), \vec{\mathbb{G}}_{2,2}^{(1,1)}[g](E_1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{5\sqrt{7}G_xxyz}{7}-\frac{\sqrt{7}G_yz(x^2-4y^2+z^2)}{7}-\frac{\sqrt{7}G_zy(x^2+y^2-4z^2)}{7}$$

$$-\frac{\sqrt{7}G_xz(4x^2-y^2-z^2)}{7}-\frac{5\sqrt{7}G_yxyz}{7}+\frac{\sqrt{7}G_zx(x^2+y^2-4z^2)}{7}$$

$\vec{\mathbb{G}}_{2,1}^{(1,-1)}[g](E_2), \vec{\mathbb{G}}_{2,2}^{(1,-1)}[g](E_2)$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}G_xx}{2}-\frac{\sqrt{2}G_yy}{2}$$

$$-\frac{\sqrt{2}G_xy}{2}-\frac{\sqrt{2}G_yx}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,1)}[g](E_2), \vec{\mathbb{G}}_{2,2}^{(1,1)}[g](E_2)$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{7}G_xx(3x^2-7y^2-2z^2)}{14}+\frac{\sqrt{7}G_yy(7x^2-3y^2+2z^2)}{14}+\frac{5\sqrt{7}G_zz(x-y)(x+y)}{14}$$

$$-\frac{\sqrt{7}G_xy(4x^2-y^2-z^2)}{7}+\frac{\sqrt{7}G_yx(x^2-4y^2+z^2)}{7}-\frac{5\sqrt{7}G_zxyz}{7}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A)$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$-\frac{\sqrt{15}G_xxz}{5} - \frac{\sqrt{15}G_yyz}{5} - \frac{\sqrt{15}G_z(x^2 + y^2 - 2z^2)}{10}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A)$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{5G_xxz(3x^2 + 3y^2 - 4z^2)}{12} - \frac{5G_yyz(3x^2 + 3y^2 - 4z^2)}{12} + \frac{G_z(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{12}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B, 1)$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$\frac{\sqrt{6}G_xxy}{2} + \frac{\sqrt{6}G_y(x-y)(x+y)}{4}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B, 2)$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{6}G_x(x-y)(x+y)}{4} - \frac{\sqrt{6}G_yxy}{2}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B, 1)$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$\frac{\sqrt{10}G_xxy(15x^2 - 13y^2 - 6z^2)}{24} - \frac{\sqrt{10}G_y(3x^4 - 21x^2y^2 + 3x^2z^2 + 4y^4 - 3y^2z^2)}{24} + \frac{7\sqrt{10}G_yyz(3x^2 - y^2)}{24}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B, 2)$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{10}G_x(4x^4 - 21x^2y^2 - 3x^2z^2 + 3y^4 + 3y^2z^2)}{24} + \frac{\sqrt{10}G_yxy(13x^2 - 15y^2 + 6z^2)}{24} + \frac{7\sqrt{10}G_zxz(x^2 - 3y^2)}{24}$$

$\vec{\mathbb{G}}_{3,1}^{(1,-1)}[g](E_1), \vec{\mathbb{G}}_{3,2}^{(1,-1)}[g](E_1)$

** symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$-\frac{\sqrt{10}G_x(3x^2 + y^2 - 4z^2)}{20} - \frac{\sqrt{10}G_yxy}{10} + \frac{2\sqrt{10}G_zxz}{5}$$

$$-\frac{\sqrt{10}G_xxy}{10} - \frac{\sqrt{10}G_y(x^2 + 3y^2 - 4z^2)}{20} + \frac{2\sqrt{10}G_yyz}{5}$$

$\vec{\mathbb{G}}_{3,1}^{(1,1)}[g](E_1), \vec{\mathbb{G}}_{3,2}^{(1,1)}[g](E_1)$

** symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$-\frac{\sqrt{6}G_x(4x^4 + 3x^2y^2 - 27x^2z^2 - y^4 + 3y^2z^2 + 4z^4)}{24} - \frac{5\sqrt{6}G_xy(x^2 + y^2 - 6z^2)}{24} - \frac{5\sqrt{6}G_zxz(3x^2 + 3y^2 - 4z^2)}{24}$$

$$-\frac{5\sqrt{6}G_xy(x^2 + y^2 - 6z^2)}{24} + \frac{\sqrt{6}G_y(x^4 - 3x^2y^2 - 3x^2z^2 - 4y^4 + 27y^2z^2 - 4z^4)}{24} - \frac{5\sqrt{6}G_zyz(3x^2 + 3y^2 - 4z^2)}{24}$$

$$\vec{\mathbb{G}}_{3,1}^{(1,-1)}[g](E_2), \vec{\mathbb{G}}_{3,2}^{(1,-1)}[g](E_2)$$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

** expression

$$G_xyz + G_yxz + G_zyy$$

$$G_xxz - G_yyz + \frac{G_z(x - y)(x + y)}{2}$$

$$\vec{\mathbb{G}}_{3,1}^{(1,1)}[g](E_2), \vec{\mathbb{G}}_{3,2}^{(1,1)}[g](E_2)$$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x - y)(x + y)}{2}$$

** expression

$$\frac{\sqrt{15}G_xy(x^2 - y^2 - z^2)}{6} - \frac{\sqrt{15}G_yxz(x^2 - 6y^2 + z^2)}{6} - \frac{\sqrt{15}G_zxy(x^2 + y^2 - 6z^2)}{6}$$

$$\frac{\sqrt{15}G_xxz(5x^2 - 9y^2 - 2z^2)}{12} + \frac{\sqrt{15}G_yyz(9x^2 - 5y^2 + 2z^2)}{12} - \frac{\sqrt{15}G_z(x - y)(x + y)(x^2 + y^2 - 6z^2)}{12}$$

* Harmonics for rank 4

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](A)$$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\frac{3\sqrt{7}G_xx(x^2 + y^2 - 4z^2)}{28} + \frac{3\sqrt{7}G_yy(x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{7}G_zz(3x^2 + 3y^2 - 2z^2)}{7}$$

$$\vec{\mathbb{G}}_4^{(1,1)}[g](A)$$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\begin{aligned} & \frac{3\sqrt{55}G_xx(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} + \frac{3\sqrt{55}G_yy(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} \\ & + \frac{\sqrt{55}G_zz(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{88} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(1,-1)}[g](B, 1)$$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\frac{3\sqrt{10}G_xz(x-y)(x+y)}{8} - \frac{3\sqrt{10}G_yxyz}{4} + \frac{\sqrt{10}G_zx(x^2-3y^2)}{8}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](B, 2)$

** symmetry

$$\frac{\sqrt{70}yz(3x^2-y^2)}{4}$$

** expression

$$\frac{3\sqrt{10}G_xxyz}{4} + \frac{3\sqrt{10}G_yz(x-y)(x+y)}{8} + \frac{\sqrt{10}G_zy(3x^2-y^2)}{8}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](B, 1)$

** symmetry

$$\frac{\sqrt{70}xz(x^2-3y^2)}{4}$$

** expression

$$\frac{3\sqrt{154}G_xz(2x^4-9x^2y^2-x^2z^2+y^4+y^2z^2)}{44} + \frac{3\sqrt{154}G_yxyz(5x^2-7y^2+2z^2)}{44} - \frac{\sqrt{154}G_zx(x^2-3y^2)(x^2+y^2-8z^2)}{44}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](B, 2)$

** symmetry

$$\frac{\sqrt{70}yz(3x^2-y^2)}{4}$$

** expression

$$\frac{3\sqrt{154}G_xxyz(7x^2-5y^2-2z^2)}{44} - \frac{3\sqrt{154}G_yz(x^4-9x^2y^2+x^2z^2+2y^4-y^2z^2)}{44} - \frac{\sqrt{154}G_zy(3x^2-y^2)(x^2+y^2-8z^2)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E_1), \vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E_1)$

** symmetry

$$-\frac{\sqrt{10}yz(3x^2+3y^2-4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2+3y^2-4z^2)}{4}$$

** expression

$$-\frac{3\sqrt{70}G_xxyz}{28} - \frac{\sqrt{70}G_yz(3x^2+9y^2-4z^2)}{56} - \frac{3\sqrt{70}G_zy(x^2+y^2-4z^2)}{56}$$

$$\frac{\sqrt{70}G_xz(9x^2+3y^2-4z^2)}{56} + \frac{3\sqrt{70}G_yxyz}{28} + \frac{3\sqrt{70}G_zx(x^2+y^2-4z^2)}{56}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E_1), \vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E_1)$

** symmetry

$$-\frac{\sqrt{10}yz(3x^2+3y^2-4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2+3y^2-4z^2)}{4}$$

** expression

$$-\frac{21\sqrt{22}G_xxyz(x^2+y^2-2z^2)}{44} + \frac{\sqrt{22}G_yz(3x^4-15x^2y^2-x^2z^2-18y^4+41y^2z^2-4z^4)}{44} + \frac{3\sqrt{22}G_zy(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44}$$

$$\frac{\sqrt{22}G_xz(18x^4+15x^2y^2-41x^2z^2-3y^4+y^2z^2+4z^4)}{44} + \frac{21\sqrt{22}G_yxyz(x^2+y^2-2z^2)}{44} - \frac{3\sqrt{22}G_zx(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E_2, 1), \vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E_2, 1)$

** symmetry

$$\frac{\sqrt{35}(x^2-2xy-y^2)(x^2+2xy-y^2)}{8}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{5}G_x x (x^2 - 3y^2)}{4} - \frac{\sqrt{5}G_y y (3x^2 - y^2)}{4}$$

$$\frac{\sqrt{5}G_x y (3x^2 - y^2)}{4} + \frac{\sqrt{5}G_y x (x^2 - 3y^2)}{4}$$

$$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E_2, 2), \vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E_2, 2)$$

** symmetry

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$-\frac{\sqrt{35}G_x x (x^2 - 3z^2)}{14} + \frac{\sqrt{35}G_y y (y^2 - 3z^2)}{14} + \frac{3\sqrt{35}G_z z (x-y)(x+y)}{14}$$

$$\frac{\sqrt{35}G_x y (3x^2 + y^2 - 6z^2)}{28} + \frac{\sqrt{35}G_y x (x^2 + 3y^2 - 6z^2)}{28} - \frac{3\sqrt{35}G_z xyz}{7}$$

$$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E_2, 1), \vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E_2, 1)$$

** symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{77}G_x x (5x^4 - 46x^2y^2 - 4x^2z^2 + 21y^4 + 12y^2z^2)}{88} + \frac{\sqrt{77}G_y y (21x^4 - 46x^2y^2 + 12x^2z^2 + 5y^4 - 4y^2z^2)}{88} + \frac{9\sqrt{77}G_z z (x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{88}$$

$$\frac{\sqrt{77}G_x y (6x^4 - 11x^2y^2 - 3x^2z^2 + y^4 + y^2z^2)}{22} - \frac{\sqrt{77}G_y x (x^4 - 11x^2y^2 + x^2z^2 + 6y^4 - 3y^2z^2)}{22} + \frac{9\sqrt{77}G_z xyz (x-y)(x+y)}{22}$$

$$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E_2, 2), \vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E_2, 2)$$

** symmetry

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$-\frac{\sqrt{11}G_x x (5x^4 - 4x^2y^2 - 46x^2z^2 - 9y^4 + 66y^2z^2 + 12z^4)}{44}$$

$$-\frac{\sqrt{11}G_y y (9x^4 + 4x^2y^2 - 66x^2z^2 - 5y^4 + 46y^2z^2 - 12z^4)}{44} - \frac{21\sqrt{11}G_z z (x-y)(x+y)(x^2 + y^2 - 2z^2)}{44}$$

$$\frac{\sqrt{11}G_x y (6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4)}{22} - \frac{\sqrt{11}G_y x (x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{22} + \frac{21\sqrt{11}G_z xyz (x^2 + y^2 - 2z^2)}{22}$$