

PG No. 19  $C_{3v}$   $3m$  (3m1 setting) [ trigonal ] (axial, internal axial octupole)

\* Harmonics for rank 0

$$\vec{G}_0^{(3,3)}[g](A_2)$$

\*\* symmetry

$$1$$

\*\* expression

$$\frac{\sqrt{70}G_1y(3x^2-y^2)}{28} + \frac{\sqrt{70}G_2x(x^2-3y^2)}{28} + \frac{\sqrt{105}G_3xyz}{7} - \frac{\sqrt{42}G_{3x}x(x^2+y^2-4z^2)}{28} \\ - \frac{\sqrt{42}G_{3y}y(x^2+y^2-4z^2)}{28} - \frac{\sqrt{7}G_{az}z(3x^2+3y^2-2z^2)}{14} + \frac{\sqrt{105}G_{bz}z(x-y)(x+y)}{14}$$

\* Harmonics for rank 1

$$\vec{G}_1^{(3,1)}[g](A_2)$$

\*\* symmetry

$$z$$

\*\* expression

$$\frac{\sqrt{21}G_3xy}{7} + \frac{2\sqrt{210}G_{3x}xz}{35} + \frac{2\sqrt{210}G_{3y}yz}{35} - \frac{3\sqrt{35}G_{az}(x^2+y^2-2z^2)}{70} + \frac{\sqrt{21}G_{bz}(x-y)(x+y)}{14}$$

$$\vec{G}_1^{(3,3)}[g](A_2)$$

\*\* symmetry

$$z$$

\*\* expression

$$\frac{\sqrt{210}G_1yz(3x^2-y^2)}{24} + \frac{\sqrt{210}G_2xz(x^2-3y^2)}{24} - \frac{\sqrt{35}G_3xy(x^2+y^2-6z^2)}{14} - \frac{5\sqrt{14}G_{3x}xz(3x^2+3y^2-4z^2)}{56} \\ - \frac{5\sqrt{14}G_{3y}yz(3x^2+3y^2-4z^2)}{56} + \frac{\sqrt{21}G_{az}(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4)}{84} - \frac{\sqrt{35}G_{bz}(x-y)(x+y)(x^2+y^2-6z^2)}{28}$$

$$\vec{G}_{1,1}^{(3,1)}[g](E), \vec{G}_{1,2}^{(3,1)}[g](E)$$

\*\* symmetry

$$-y$$

$$x$$

\*\* expression

$$-\frac{3\sqrt{14}G_1(x-y)(x+y)}{28} + \frac{3\sqrt{14}G_2xy}{14} - \frac{\sqrt{21}G_{3x}z}{7} + \frac{\sqrt{210}G_{3x}xy}{70} + \frac{\sqrt{210}G_{3y}(x^2+3y^2-4z^2)}{140} + \frac{3\sqrt{35}G_{az}yz}{35} + \frac{\sqrt{21}G_{bz}yz}{7} \\ \frac{3\sqrt{14}G_1xy}{14} + \frac{3\sqrt{14}G_2(x-y)(x+y)}{28} + \frac{\sqrt{21}G_{3y}z}{7} - \frac{\sqrt{210}G_{3x}(3x^2+y^2-4z^2)}{140} - \frac{\sqrt{210}G_{3y}xy}{70} - \frac{3\sqrt{35}G_{az}xz}{35} + \frac{\sqrt{21}G_{bz}xz}{7}$$

$$\vec{G}_{1,1}^{(3,3)}[g](E), \vec{G}_{1,2}^{(3,3)}[g](E)$$

\*\* symmetry

$$-y$$

$$x$$

\*\* expression

$$\frac{\sqrt{210}G_1(3x^4-21x^2y^2+3x^2z^2+4y^4-3y^2z^2)}{168} - \frac{\sqrt{210}G_2xy(13x^2-15y^2+6z^2)}{168} + \frac{\sqrt{35}G_{3x}z(x^2-6y^2+z^2)}{14} + \frac{5\sqrt{14}G_{3x}xy(x^2+y^2-6z^2)}{56} \\ - \frac{\sqrt{14}G_{3y}(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4)}{56} + \frac{5\sqrt{21}G_{az}yz(3x^2+3y^2-4z^2)}{84} - \frac{\sqrt{35}G_{bz}yz(9x^2-5y^2+2z^2)}{28} \\ \frac{\sqrt{210}G_1xy(15x^2-13y^2-6z^2)}{168} + \frac{\sqrt{210}G_2(4x^4-21x^2y^2-3x^2z^2+3y^4+3y^2z^2)}{168} \\ + \frac{\sqrt{35}G_{3y}z(6x^2-y^2-z^2)}{14} - \frac{\sqrt{14}G_{3x}(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4)}{56} \\ - \frac{5\sqrt{14}G_{3y}xy(x^2+y^2-6z^2)}{56} - \frac{5\sqrt{21}G_{az}xz(3x^2+3y^2-4z^2)}{84} + \frac{\sqrt{35}G_{bz}xz(5x^2-9y^2-2z^2)}{28}$$

\* Harmonics for rank 2

$$\vec{G}_2^{(3,-1)}[g](A_2)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$\frac{\sqrt{14}G_{3x}x}{7} + \frac{\sqrt{14}G_{3y}y}{7} + \frac{\sqrt{21}G_{az}z}{7}$$

$$\vec{\mathbb{G}}_2^{(3,1)}[g](A_2)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$-\frac{5\sqrt{210}G_{1y}(3x^2 - y^2)}{168} - \frac{5\sqrt{210}G_{2x}(x^2 - 3y^2)}{168} - \frac{3\sqrt{14}G_{3xx}(x^2 + y^2 - 4z^2)}{56} - \frac{3\sqrt{14}G_{3yy}(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{21}G_{az}z(3x^2 + 3y^2 - 2z^2)}{21}$$

$$\vec{\mathbb{G}}_2^{(3,3)}[g](A_2)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$\begin{aligned} & -\frac{\sqrt{1155}G_{1y}(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{264} - \frac{\sqrt{1155}G_{2x}(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{264} - \frac{3\sqrt{770}G_{3xyz}(x^2 + y^2 - 2z^2)}{44} \\ & + \frac{15\sqrt{77}G_{3xx}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{616} + \frac{15\sqrt{77}G_{3yy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{616} \\ & + \frac{5\sqrt{462}G_{az}z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{1848} - \frac{3\sqrt{770}G_{bzz}(x - y)(x + y)(x^2 + y^2 - 2z^2)}{88} \end{aligned}$$

$$\vec{\mathbb{G}}_{2,1}^{(3,-1)}[g](E, 1), \vec{\mathbb{G}}_{2,2}^{(3,-1)}[g](E, 1)$$

\*\* symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{105}G_{3x}x}{21} + \frac{2\sqrt{42}G_{3yz}}{21} - \frac{\sqrt{7}G_{az}y}{7} - \frac{\sqrt{105}G_{bzy}}{21} \\ & - \frac{\sqrt{105}G_{3y}}{21} - \frac{2\sqrt{42}G_{3xz}}{21} + \frac{\sqrt{7}G_{az}x}{7} - \frac{\sqrt{105}G_{bzx}}{21} \end{aligned}$$

$$\vec{\mathbb{G}}_{2,1}^{(3,-1)}[g](E, 2), \vec{\mathbb{G}}_{2,2}^{(3,-1)}[g](E, 2)$$

\*\* symmetry

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

$$-\sqrt{3}xy$$

\*\* expression

$$\begin{aligned} & \frac{\sqrt{70}G_{1y}}{14} + \frac{\sqrt{70}G_{2x}}{14} - \frac{\sqrt{42}G_{3xx}}{42} + \frac{\sqrt{42}G_{3yy}}{42} + \frac{\sqrt{105}G_{bzz}}{21} \\ & - \frac{\sqrt{70}G_{1x}}{14} + \frac{\sqrt{70}G_{2y}}{14} - \frac{\sqrt{105}G_{3z}}{21} + \frac{\sqrt{42}G_{3xy}}{42} + \frac{\sqrt{42}G_{3yx}}{42} \end{aligned}$$

$$\vec{\mathbb{G}}_{2,1}^{(3,1)}[g](E, 1), \vec{\mathbb{G}}_{2,2}^{(3,1)}[g](E, 1)$$

\*\* symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

\*\* expression

$$\begin{aligned} & \frac{5\sqrt{70}G_{1z}(x - y)(x + y)}{56} - \frac{5\sqrt{70}G_{2xyz}}{28} - \frac{\sqrt{105}G_{3x}(2x^2 - 3y^2 - 3z^2)}{42} + \frac{5\sqrt{42}G_{3xyz}}{28} \\ & - \frac{\sqrt{42}G_{3yz}(21x^2 - 9y^2 - 4z^2)}{168} - \frac{\sqrt{7}G_{az}y(x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{105}G_{bzy}(9x^2 - y^2 - 6z^2)}{84} \end{aligned}$$

$$\begin{aligned}
& -\frac{5\sqrt{70}G_1xyz}{28} - \frac{5\sqrt{70}G_2z(x-y)(x+y)}{56} - \frac{\sqrt{105}G_3y(3x^2-2y^2+3z^2)}{42} - \frac{\sqrt{42}G_{3x}z(9x^2-21y^2+4z^2)}{168} \\
& - \frac{5\sqrt{42}G_{3y}xyz}{28} + \frac{\sqrt{7}G_{az}x(x^2+y^2-4z^2)}{28} - \frac{\sqrt{105}G_{bz}x(x^2-9y^2+6z^2)}{84}
\end{aligned}$$

$$\vec{\mathbb{G}}_{2,1}^{(3,1)}[g](E, 2), \vec{\mathbb{G}}_{2,2}^{(3,1)}[g](E, 2)$$

\*\* symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

\*\* expression

$$\begin{aligned}
& \frac{\sqrt{70}G_1y(x^2+y^2-4z^2)}{56} + \frac{\sqrt{70}G_2x(x^2+y^2-4z^2)}{56} - \frac{\sqrt{42}G_{3x}x(11x^2-9y^2-24z^2)}{168} \\
& - \frac{\sqrt{42}G_{3y}y(9x^2-11y^2+24z^2)}{168} - \frac{5\sqrt{7}G_{az}z(x-y)(x+y)}{14} + \frac{\sqrt{105}G_{bz}z(3x^2+3y^2-2z^2)}{42} \\
& - \frac{\sqrt{70}G_1x(x^2+y^2-4z^2)}{56} + \frac{\sqrt{70}G_2y(x^2+y^2-4z^2)}{56} - \frac{\sqrt{105}G_3z(3x^2+3y^2-2z^2)}{42} \\
& + \frac{\sqrt{42}G_{3x}y(21x^2+y^2-24z^2)}{168} + \frac{\sqrt{42}G_{3y}x(x^2+21y^2-24z^2)}{168} + \frac{5\sqrt{7}G_{az}xyz}{7}
\end{aligned}$$

$$\vec{\mathbb{G}}_{2,1}^{(3,3)}[g](E, 1), \vec{\mathbb{G}}_{2,2}^{(3,3)}[g](E, 1)$$

\*\* symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

\*\* expression

$$\begin{aligned}
& -\frac{\sqrt{385}G_1z(x^4-9x^2y^2+x^2z^2+2y^4-y^2z^2)}{44} + \frac{\sqrt{385}G_2xyz(5x^2-7y^2+2z^2)}{44} + \frac{\sqrt{2310}G_3x(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{462} \\
& - \frac{5\sqrt{231}G_{3x}xyz(x^2+y^2-2z^2)}{44} + \frac{5\sqrt{231}G_{3y}z(3x^4-15x^2y^2-x^2z^2-18y^4+41y^2z^2-4z^4)}{924} \\
& + \frac{5\sqrt{154}G_{az}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{308} - \frac{\sqrt{2310}G_{bz}y(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4)}{924} \\
& - \frac{\sqrt{385}G_1xyz(7x^2-5y^2-2z^2)}{44} - \frac{\sqrt{385}G_2z(2x^4-9x^2y^2-x^2z^2+y^4+y^2z^2)}{44} + \frac{\sqrt{2310}G_3y(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{462} \\
& + \frac{5\sqrt{231}G_{3x}z(18x^4+15x^2y^2-41x^2z^2-3y^4+y^2z^2+4z^4)}{924} + \frac{5\sqrt{231}G_{3y}xyz(x^2+y^2-2z^2)}{44} \\
& - \frac{5\sqrt{154}G_{az}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{308} + \frac{\sqrt{2310}G_{bz}x(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4)}{924}
\end{aligned}$$

$$\vec{\mathbb{G}}_{2,1}^{(3,3)}[g](E, 2), \vec{\mathbb{G}}_{2,2}^{(3,3)}[g](E, 2)$$

\*\* symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

\*\* expression

$$\begin{aligned}
& \frac{\sqrt{385}G_1y(53x^4-104x^2y^2-6x^2z^2+11y^4-6y^2z^2+4z^4)}{616} + \frac{\sqrt{385}G_2x(11x^4-104x^2y^2-6x^2z^2+53y^4-6y^2z^2+4z^4)}{616} \\
& + \frac{3\sqrt{2310}G_{3xy}z(x-y)(x+y)}{44} - \frac{5\sqrt{231}G_{3x}x(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4)}{1848} \\
& - \frac{5\sqrt{231}G_{3y}y(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4)}{1848} - \frac{5\sqrt{154}G_{az}z(x-y)(x+y)(x^2+y^2-2z^2)}{88} \\
& + \frac{\sqrt{2310}G_{bz}z(39x^4-174x^2y^2-20x^2z^2+39y^4-20y^2z^2+4z^4)}{1848} \\
& \frac{\sqrt{385}G_1x(5x^4-53x^2y^2+3x^2z^2+26y^4+3y^2z^2-2z^4)}{308} - \frac{\sqrt{385}G_2y(26x^4-53x^2y^2+3x^2z^2+5y^4+3y^2z^2-2z^4)}{308} \\
& + \frac{\sqrt{2310}G_3z(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4)}{462} + \frac{5\sqrt{231}G_{3xy}(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{924} \\
& - \frac{5\sqrt{231}G_{3y}x(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{924} + \frac{5\sqrt{154}G_{az}xyz(x^2+y^2-2z^2)}{44} - \frac{3\sqrt{2310}G_{bzx}yz(x-y)(x+y)}{44}
\end{aligned}$$

\* Harmonics for rank 3

$$\vec{\mathbb{G}}_3^{(3,-3)}[g](A_1)$$

\*\* symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

\*\* expression

$$G_2$$

$$\vec{\mathbb{G}}_3^{(3,-1)}[g](A_1)$$

\*\* symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

\*\* expression

$$\frac{\sqrt{15}G_2(x^2 + y^2 - 2z^2)}{12} - \frac{\sqrt{10}G_3yz}{4} - \frac{G_{3x}(x-y)(x+y)}{4} + \frac{G_{3y}xy}{2} + \frac{\sqrt{10}G_{bz}xz}{4}$$

$$\vec{\mathbb{G}}_3^{(3,1)}[g](A_1)$$

\*\* symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

\*\* expression

$$\frac{\sqrt{22}G_2(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{176} - \frac{5\sqrt{33}G_3yz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{\sqrt{330}G_{3x}(13x^4 - 42x^2y^2 - 36x^2z^2 + y^4 + 36y^2z^2)}{528}$$

$$- \frac{\sqrt{330}G_{3y}xy(2x^2 - 5y^2 + 9z^2)}{66} - \frac{7\sqrt{55}G_{az}xz(x^2 - 3y^2)}{44} + \frac{5\sqrt{33}G_{bz}xz(3x^2 + 3y^2 - 4z^2)}{132}$$

$$\vec{\mathbb{G}}_3^{(3,3)}[g](A_1)$$

\*\* symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

\*\* expression

$$\frac{7\sqrt{429}G_1xy(x^2 - 3y^2)(3x^2 - y^2)}{208} + \frac{\sqrt{429}G_2(118x^6 - 1725x^4y^2 - 45x^4z^2 + 1740x^2y^4 - 90x^2y^2z^2 + 60x^2z^4 - 113y^6 - 45y^4z^2 + 60y^2z^4 - 8z^6)}{6864}$$

$$+ \frac{7\sqrt{286}G_3yz(40x^4 - 85x^2y^2 + 5x^2z^2 + 7y^4 + 5y^2z^2 - 2z^4)}{1144}$$

$$- \frac{7\sqrt{715}G_{3x}(2x^6 - 7x^4y^2 - 23x^4z^2 - 8x^2y^4 + 90x^2y^2z^2 + 8x^2z^4 + y^6 - 7y^4z^2 - 8y^2z^4)}{2288}$$

$$- \frac{7\sqrt{715}G_{3y}xy(5x^4 - 2x^2y^2 - 44x^2z^2 - 7y^4 + 76y^2z^2 - 16z^4)}{2288} - \frac{7\sqrt{4290}G_{az}xz(x^2 - 3y^2)(3x^2 + 3y^2 - 8z^2)}{2288}$$

$$+ \frac{7\sqrt{286}G_{bz}xz(19x^4 - 160x^2y^2 - 10x^2z^2 + 85y^4 - 10y^2z^2 + 4z^4)}{2288}$$

$$\vec{\mathbb{G}}_3^{(3,-3)}[g](A_2, 1)$$

\*\* symmetry

$$- \frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$G_{az}$$

$$\vec{\mathbb{G}}_3^{(3,-3)}[g](A_2, 2)$$

\*\* symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

\*\* expression

$$G_1$$

$$\vec{\mathbb{G}}_3^{(3,-1)}[g](A_2, 1)$$

\*\* symmetry

$$- \frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$-G_3xy + \frac{\sqrt{10}G_3xxz}{10} + \frac{\sqrt{10}G_3yyz}{10} - \frac{\sqrt{15}G_{az}(x^2 + y^2 - 2z^2)}{15} - \frac{G_{bz}(x - y)(x + y)}{2}$$

$$\vec{\mathbb{G}}_3^{(3,-1)}[g](A_2, 2)$$

\*\* symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

\*\* expression

$$\frac{\sqrt{15}G_1(x^2 + y^2 - 2z^2)}{12} + \frac{\sqrt{10}G_3xz}{4} - \frac{G_3xy}{2} - \frac{G_3y(x - y)(x + y)}{4} + \frac{\sqrt{10}G_{bz}yz}{4}$$

$$\vec{\mathbb{G}}_3^{(3,1)}[g](A_2, 1)$$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$-\frac{7\sqrt{55}G_1yz(3x^2 - y^2)}{44} - \frac{7\sqrt{55}G_2xz(x^2 - 3y^2)}{44} + \frac{\sqrt{330}G_3xy(x^2 + y^2 - 6z^2)}{132} - \frac{5\sqrt{33}G_3xxz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{5\sqrt{33}G_3yyz(3x^2 + 3y^2 - 4z^2)}{132} + \frac{\sqrt{22}G_{az}(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{88} + \frac{\sqrt{330}G_{bz}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{264}$$

$$\vec{\mathbb{G}}_3^{(3,1)}[g](A_2, 2)$$

\*\* symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

\*\* expression

$$\frac{\sqrt{22}G_1(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{176} + \frac{5\sqrt{33}G_3xz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{\sqrt{330}G_3xy(5x^2 - 2y^2 - 9z^2)}{66} + \frac{\sqrt{330}G_3y(x^4 - 42x^2y^2 + 36x^2z^2 + 13y^4 - 36y^2z^2)}{528} - \frac{7\sqrt{55}G_{az}yz(3x^2 - y^2)}{44} + \frac{5\sqrt{33}G_{bz}yz(3x^2 + 3y^2 - 4z^2)}{132}$$

$$\vec{\mathbb{G}}_3^{(3,3)}[g](A_2, 1)$$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$-\frac{7\sqrt{4290}G_1yz(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{2288} - \frac{7\sqrt{4290}G_2xz(x^2 - 3y^2)(3x^2 + 3y^2 - 8z^2)}{2288} + \frac{7\sqrt{715}G_3xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} + \frac{35\sqrt{286}G_3xxz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} + \frac{35\sqrt{286}G_3yyz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} - \frac{5\sqrt{429}G_{az}(5x^6 + 15x^4y^2 - 90x^4z^2 + 15x^2y^4 - 180x^2y^2z^2 + 120x^2z^4 + 5y^6 - 90y^4z^2 + 120y^2z^4 - 16z^6)}{3432} + \frac{7\sqrt{715}G_{bz}(x - y)(x + y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144}$$

$$\vec{\mathbb{G}}_3^{(3,3)}[g](A_2, 2)$$

\*\* symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

\*\* expression

$$-\frac{\sqrt{429}G_1(113x^6 - 1740x^4y^2 + 45x^4z^2 + 1725x^2y^4 + 90x^2y^2z^2 - 60x^2z^4 - 118y^6 + 45y^4z^2 - 60y^2z^4 + 8z^6)}{6864} + \frac{7\sqrt{429}G_2xy(x^2 - 3y^2)(3x^2 - y^2)}{208} - \frac{7\sqrt{286}G_3xz(7x^4 - 85x^2y^2 + 5x^2z^2 + 40y^4 + 5y^2z^2 - 2z^4)}{1144} - \frac{7\sqrt{715}G_3xy(7x^4 + 2x^2y^2 - 76x^2z^2 - 5y^4 + 44y^2z^2 + 16z^4)}{2288} + \frac{7\sqrt{715}G_3y(x^6 - 8x^4y^2 - 7x^4z^2 - 7x^2y^4 + 90x^2y^2z^2 - 8x^2z^4 + 2y^6 - 23y^4z^2 + 8y^2z^4)}{2288} - \frac{7\sqrt{4290}G_{az}yz(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{2288} + \frac{7\sqrt{286}G_{bz}yz(85x^4 - 160x^2y^2 - 10x^2z^2 + 19y^4 - 10y^2z^2 + 4z^4)}{2288}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,-3)}[g](E, 1), \vec{\mathbb{G}}_{3,2}^{(3,-3)}[g](E, 1)$$

\*\* symmetry

$$\frac{\sqrt{6}y (x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}x (x^2 + y^2 - 4z^2)}{4}$$

\*\* expression

$$-G_{3y}$$

$$G_{3x}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,-3)}[g](E, 2), \vec{\mathbb{G}}_{3,2}^{(3,-3)}[g](E, 2)$$

\*\* symmetry

$$-\frac{\sqrt{15}z (x - y) (x + y)}{2}$$

$$\sqrt{15}xyz$$

\*\* expression

$$-G_{bz}$$

$$G_3$$

$$\vec{\mathbb{G}}_{3,1}^{(3,-1)}[g](E, 1), \vec{\mathbb{G}}_{3,2}^{(3,-1)}[g](E, 1)$$

\*\* symmetry

$$\frac{\sqrt{6}y (x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}x (x^2 + y^2 - 4z^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{G_1 (x - y) (x + y)}{4} - \frac{G_2 xy}{2} - \frac{\sqrt{6}G_3 xz}{4} - \frac{\sqrt{15}G_3 xy}{5} + \frac{\sqrt{15}G_3 y (3x^2 - y^2 - 2z^2)}{20} - \frac{\sqrt{10}G_{az} yz}{10} + \frac{\sqrt{6}G_{bz} yz}{4} \\ & - \frac{G_1 xy}{2} - \frac{G_2 (x - y) (x + y)}{4} + \frac{\sqrt{6}G_3 yz}{4} + \frac{\sqrt{15}G_3 x (x^2 - 3y^2 + 2z^2)}{20} + \frac{\sqrt{15}G_3 xy}{5} + \frac{\sqrt{10}G_{az} xz}{10} + \frac{\sqrt{6}G_{bz} xz}{4} \end{aligned}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,-1)}[g](E, 2), \vec{\mathbb{G}}_{3,2}^{(3,-1)}[g](E, 2)$$

\*\* symmetry

$$-\frac{\sqrt{15}z (x - y) (x + y)}{2}$$

$$\sqrt{15}xyz$$

\*\* expression

$$\begin{aligned} & -\frac{\sqrt{10}G_1 yz}{4} - \frac{\sqrt{10}G_2 xz}{4} - \frac{\sqrt{6}G_3 xz}{4} + \frac{\sqrt{6}G_3 yz}{4} + \frac{G_{az} (x - y) (x + y)}{2} \\ & \frac{\sqrt{10}G_1 xz}{4} - \frac{\sqrt{10}G_2 yz}{4} + \frac{\sqrt{6}G_3 xz}{4} + \frac{\sqrt{6}G_3 yx}{4} - G_{az} xy \end{aligned}$$

$$\vec{\mathbb{G}}_{3,1}^{(3,1)}[g](E, 1), \vec{\mathbb{G}}_{3,2}^{(3,1)}[g](E, 1)$$

\*\* symmetry

$$\frac{\sqrt{6}y (x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}x (x^2 + y^2 - 4z^2)}{4}$$

\*\* expression

$$\begin{aligned} & -\frac{\sqrt{330}G_1 (x^4 - 42x^2 y^2 + 36x^2 z^2 + 13y^4 - 36y^2 z^2)}{528} + \frac{\sqrt{330}G_2 xy (2x^2 - 5y^2 + 9z^2)}{66} + \frac{\sqrt{55}G_3 xz (19x^2 - 9y^2 - 16z^2)}{132} + \frac{5\sqrt{22}G_3 xy (x^2 + y^2 - 6z^2)}{66} \\ & - \frac{\sqrt{22}G_3 y (23x^4 + 6x^2 y^2 - 144x^2 z^2 - 17y^4 + 96y^2 z^2 + 8z^4)}{528} + \frac{5\sqrt{33}G_{az} yz (3x^2 + 3y^2 - 4z^2)}{132} - \frac{\sqrt{55}G_{bz} yz (33x^2 + 5y^2 - 16z^2)}{132} \end{aligned}$$

$$\begin{aligned}
& - \frac{\sqrt{330}G_1xy(5x^2 - 2y^2 - 9z^2)}{66} - \frac{\sqrt{330}G_2(13x^4 - 42x^2y^2 - 36x^2z^2 + y^4 + 36y^2z^2)}{528} \\
& + \frac{\sqrt{55}G_3yz(9x^2 - 19y^2 + 16z^2)}{132} - \frac{\sqrt{22}G_{3x}(17x^4 - 6x^2y^2 - 96x^2z^2 - 23y^4 + 144y^2z^2 - 8z^4)}{528} \\
& - \frac{5\sqrt{22}G_{3y}xy(x^2 + y^2 - 6z^2)}{66} - \frac{5\sqrt{33}G_{az}xz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{\sqrt{55}G_{bz}xz(5x^2 + 33y^2 - 16z^2)}{132} \\
& \vec{\mathbb{G}}_{3,1}^{(3,1)}[g](E, 2), \vec{\mathbb{G}}_{3,2}^{(3,1)}[g](E, 2) \\
& ** \text{ symmetry} \\
& - \frac{\sqrt{15}z(x-y)(x+y)}{2} \\
& \sqrt{15}xyz \\
& ** \text{ expression} \\
& - \frac{5\sqrt{33}G_1yz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{5\sqrt{33}G_2xz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{35\sqrt{22}G_3xy(x-y)(x+y)}{132} + \frac{\sqrt{55}G_{3x}xz(5x^2 + 33y^2 - 16z^2)}{132} \\
& - \frac{\sqrt{55}G_{3y}yz(33x^2 + 5y^2 - 16z^2)}{132} - \frac{\sqrt{330}G_{az}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{264} - \frac{7\sqrt{22}G_{bz}(x^4 - 18x^2y^2 + 12x^2z^2 + y^4 + 12y^2z^2 - 4z^4)}{264} \\
& \frac{5\sqrt{33}G_1xz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{5\sqrt{33}G_2yz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{7\sqrt{22}G_3(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{66} \\
& + \frac{\sqrt{55}G_{3x}yz(9x^2 - 19y^2 + 16z^2)}{132} - \frac{\sqrt{55}G_{3y}xz(19x^2 - 9y^2 - 16z^2)}{132} + \frac{\sqrt{330}G_{az}xy(x^2 + y^2 - 6z^2)}{132} + \frac{35\sqrt{22}G_{bz}xy(x-y)(x+y)}{132} \\
& \vec{\mathbb{G}}_{3,1}^{(3,3)}[g](E, 1), \vec{\mathbb{G}}_{3,2}^{(3,3)}[g](E, 1) \\
& ** \text{ symmetry} \\
& \frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4} \\
& - \frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4} \\
& ** \text{ expression} \\
& - \frac{7\sqrt{715}G_1(x^6 - 8x^4y^2 - 7x^4z^2 - 7x^2y^4 + 90x^2y^2z^2 - 8x^2z^4 + 2y^6 - 23y^4z^2 + 8y^2z^4)}{2288} \\
& + \frac{7\sqrt{715}G_2xy(5x^4 - 2x^2y^2 - 44x^2z^2 - 7y^4 + 76y^2z^2 - 16z^4)}{2288} \\
& - \frac{7\sqrt{4290}G_3xz(x^4 - 7x^2y^2 - x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4)}{1144} - \frac{35\sqrt{429}G_{3x}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{2288} \\
& + \frac{5\sqrt{429}G_{3y}(x^6 - 4x^4y^2 - 11x^4z^2 - 11x^2y^4 + 90x^2y^2z^2 - 4x^2z^4 - 6y^6 + 101y^4z^2 - 116y^2z^4 + 8z^6)}{2288} \\
& - \frac{35\sqrt{286}G_{az}yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} + \frac{7\sqrt{4290}G_{bz}yz(11x^4 + 4x^2y^2 - 26x^2z^2 - 7y^4 + 22y^2z^2 - 4z^4)}{2288} \\
& - \frac{7\sqrt{715}G_1xy(7x^4 + 2x^2y^2 - 76x^2z^2 - 5y^4 + 44y^2z^2 + 16z^4)}{2288} \\
& 7\sqrt{715}G_2(2x^6 - 7x^4y^2 - 23x^4z^2 - 8x^2y^4 + 90x^2y^2z^2 + 8x^2z^4 + y^6 - 7y^4z^2 - 8y^2z^4) - \frac{7\sqrt{4290}G_3yz(8x^4 + 7x^2y^2 - 23x^2z^2 - y^4 + y^2z^2 + 2z^4)}{1144} \\
& + \frac{5\sqrt{429}G_{3x}(6x^6 + 11x^4y^2 - 101x^4z^2 + 4x^2y^4 - 90x^2y^2z^2 + 116x^2z^4 - y^6 + 11y^4z^2 + 4y^2z^4 - 8z^6)}{2288} \\
& + \frac{35\sqrt{429}G_{3y}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{2288} + \frac{35\sqrt{286}G_{az}xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\
& - \frac{7\sqrt{4290}G_{bz}xz(7x^4 - 4x^2y^2 - 22x^2z^2 - 11y^4 + 26y^2z^2 + 4z^4)}{2288} \\
& \vec{\mathbb{G}}_{3,1}^{(3,3)}[g](E, 2), \vec{\mathbb{G}}_{3,2}^{(3,3)}[g](E, 2) \\
& ** \text{ symmetry} \\
& - \frac{\sqrt{15}z(x-y)(x+y)}{2} \\
& \sqrt{15}xyz
\end{aligned}$$

\*\* expression

$$\begin{aligned}
& - \frac{7\sqrt{286}G_{1yz}(85x^4 - 160x^2y^2 - 10x^2z^2 + 19y^4 - 10y^2z^2 + 4z^4)}{2288} - \frac{7\sqrt{286}G_{2xz}(19x^4 - 160x^2y^2 - 10x^2z^2 + 85y^4 - 10y^2z^2 + 4z^4)}{2288} \\
& + \frac{21\sqrt{429}G_{3xy}(x-y)(x+y)(x^2+y^2-10z^2)}{572} + \frac{7\sqrt{4290}G_{3xxz}(7x^4 - 4x^2y^2 - 22x^2z^2 - 11y^4 + 26y^2z^2 + 4z^4)}{2288} \\
& + \frac{7\sqrt{4290}G_{3yyz}(11x^4 + 4x^2y^2 - 26x^2z^2 - 7y^4 + 22y^2z^2 - 4z^4)}{2288} - \frac{7\sqrt{715}G_{az}(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144} \\
& + \frac{\sqrt{429}G_{bz}(13x^6 - 45x^4y^2 - 150x^4z^2 - 45x^2y^4 + 540x^2y^2z^2 + 60x^2z^4 + 13y^6 - 150y^4z^2 + 60y^2z^4 - 8z^6)}{1144} \\
& - \frac{7\sqrt{286}G_{1xz}(7x^4 - 85x^2y^2 + 5x^2z^2 + 40y^4 + 5y^2z^2 - 2z^4)}{1144} + \frac{7\sqrt{286}G_{2yz}(40x^4 - 85x^2y^2 + 5x^2z^2 + 7y^4 + 5y^2z^2 - 2z^4)}{1144} \\
& + \frac{\sqrt{429}G_3(2x^6 - 15x^4y^2 - 15x^4z^2 - 15x^2y^4 + 180x^2y^2z^2 - 15x^2z^4 + 2y^6 - 15y^4z^2 - 15y^2z^4 + 2z^6)}{286} \\
& - \frac{7\sqrt{4290}G_{3xyz}(8x^4 + 7x^2y^2 - 23x^2z^2 - y^4 + y^2z^2 + 2z^4)}{286} + \frac{7\sqrt{4290}G_{3yxz}(x^4 - 7x^2y^2 - x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4)}{1144} \\
& + \frac{7\sqrt{715}G_{azxy}(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} - \frac{21\sqrt{429}G_{bzxxy}(x-y)(x+y)(x^2+y^2-10z^2)}{572}
\end{aligned}$$

\* Harmonics for rank 4

$$\vec{G}_4^{(3,-3)}[g](A_1)$$

\*\* symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

\*\* expression

$$\frac{G_2z}{2} - \frac{\sqrt{6}G_3y}{4} + \frac{\sqrt{6}G_{bz}x}{4}$$

$$\vec{G}_4^{(3,-1)}[g](A_1)$$

\*\* symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

\*\* expression

$$\begin{aligned}
& \frac{3\sqrt{22}G_2z(3x^2 + 3y^2 - 2z^2)}{44} + \frac{\sqrt{33}G_3y(x^2 + y^2 - 4z^2)}{44} + \frac{\sqrt{330}G_{3xz}(x-y)(x+y)}{44} \\
& - \frac{\sqrt{330}G_{3yxyz}}{22} - \frac{3\sqrt{55}G_{azx}(x^2 - 3y^2)}{44} - \frac{\sqrt{33}G_{bzx}(x^2 + y^2 - 4z^2)}{44}
\end{aligned}$$

$$\vec{G}_4^{(3,1)}[g](A_1)$$

\*\* symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

\*\* expression

$$\begin{aligned}
& \frac{3\sqrt{715}G_2z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{2288} + \frac{\sqrt{4290}G_3y(29x^4 - 47x^2y^2 - 33x^2z^2 + 8y^4 - 33y^2z^2 + 22z^4)}{1144} \\
& - \frac{7\sqrt{429}G_{3xz}(11x^4 + 18x^2y^2 - 28x^2z^2 - 17y^4 + 28y^2z^2)}{2288} + \frac{7\sqrt{429}G_{3yxyz}(5x^2 + 2y^2 - 7z^2)}{286} \\
& + \frac{21\sqrt{286}G_{azx}(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{2288} + \frac{\sqrt{4290}G_{bzx}(5x^4 - 116x^2y^2 + 66x^2z^2 + 47y^4 + 66y^2z^2 - 44z^4)}{2288}
\end{aligned}$$

$$\vec{G}_4^{(3,3)}[g](A_1)$$

\*\* symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

\*\* expression

$$\begin{aligned}
& \frac{3\sqrt{715}G_1xyz(x^2 - 3y^2)(3x^2 - y^2)}{40} \\
& + \frac{\sqrt{715}G_2z(232x^6 - 3165x^4y^2 - 105x^4z^2 + 3270x^2y^4 - 210x^2y^2z^2 + 84x^2z^4 - 197y^6 - 105y^4z^2 + 84y^2z^4 - 8z^6)}{5720} \\
& - \frac{\sqrt{4290}G_3y(40x^6 - 45x^4y^2 - 465x^4z^2 - 78x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 7y^6 - 69y^4z^2 - 60y^2z^4 + 16z^6)}{2860} \\
& - \frac{3\sqrt{429}G_{3xz}(24x^6 - 75x^4y^2 - 95x^4z^2 - 90x^2y^4 + 330x^2y^2z^2 + 24x^2z^4 + 9y^6 - 15y^4z^2 - 24y^2z^4)}{1144} \\
& - \frac{3\sqrt{429}G_{3yxyz}(51x^4 - 30x^2y^2 - 140x^2z^2 - 81y^4 + 300y^2z^2 - 48z^4)}{1144} + \frac{3\sqrt{286}G_{azx}(x^2 - 3y^2)(3x^4 + 6x^2y^2 - 60x^2z^2 + 3y^4 - 60y^2z^2 + 80z^4)}{1144} \\
& - \frac{\sqrt{4290}G_{bzx}(19x^6 - 141x^4y^2 - 258x^4z^2 - 75x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 85y^6 - 1050y^4z^2 + 120y^2z^4 - 32z^6)}{5720}
\end{aligned}$$



$$\vec{\mathbb{G}}_4^{(3,-3)}[g](A_2, 1)$$

\*\* symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

\*\* expression

$$-\frac{\sqrt{42}G_{3xx}}{14} - \frac{\sqrt{42}G_{3yy}}{14} + \frac{2\sqrt{7}G_{azz}}{7}$$

$$\vec{\mathbb{G}}_4^{(3,-3)}[g](A_2, 2)$$

\*\* symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

\*\* expression

$$\frac{G_{1z}}{2} + \frac{\sqrt{6}G_{3x}}{4} + \frac{\sqrt{6}G_{bz}y}{4}$$

$$\vec{\mathbb{G}}_4^{(3,-1)}[g](A_2, 1)$$

\*\* symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{385}G_{1y}(3x^2 - y^2)}{308} + \frac{3\sqrt{385}G_{2x}(x^2 - 3y^2)}{308} - \frac{\sqrt{2310}G_{3xyz}}{22} - \frac{\sqrt{231}G_{3xx}(x^2 + y^2 - 4z^2)}{308} \\ & - \frac{\sqrt{231}G_{3yy}(x^2 + y^2 - 4z^2)}{308} - \frac{3\sqrt{154}G_{azz}(3x^2 + 3y^2 - 2z^2)}{154} - \frac{\sqrt{2310}G_{bzz}(x - y)(x + y)}{44} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(3,-1)}[g](A_2, 2)$$

\*\* symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{22}G_{1z}(3x^2 + 3y^2 - 2z^2)}{44} - \frac{\sqrt{33}G_{3x}(x^2 + y^2 - 4z^2)}{44} + \frac{\sqrt{330}G_{3xyz}}{22} \\ & + \frac{\sqrt{330}G_{3yz}(x - y)(x + y)}{44} - \frac{3\sqrt{55}G_{azz}(3x^2 - y^2)}{44} - \frac{\sqrt{33}G_{bzy}(x^2 + y^2 - 4z^2)}{44} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(3,1)}[g](A_2, 1)$$

\*\* symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

\*\* expression

$$\begin{aligned} & \frac{15\sqrt{2002}G_{1y}(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{2288} + \frac{15\sqrt{2002}G_{2x}(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{2288} + \frac{5\sqrt{3003}G_{3xyz}(x^2 + y^2 - 2z^2)}{286} \\ & + \frac{\sqrt{30030}G_{3xx}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{1456} + \frac{\sqrt{30030}G_{3yy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{1456} \\ & + \frac{3\sqrt{5005}G_{azz}(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{4004} + \frac{5\sqrt{3003}G_{bzz}(x - y)(x + y)(x^2 + y^2 - 2z^2)}{572} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(3,1)}[g](A_2, 2)$$

\*\* symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{715}G_{1z}(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{2288} - \frac{\sqrt{4290}G_{3x}(8x^4 - 47x^2y^2 - 33x^2z^2 + 29y^4 - 33y^2z^2 + 22z^4)}{1144} \\ & - \frac{7\sqrt{429}G_{3xyz}(2x^2 + 5y^2 - 7z^2)}{286} - \frac{7\sqrt{429}G_{3yz}(17x^4 - 18x^2y^2 - 28x^2z^2 - 11y^4 + 28y^2z^2)}{2288} \\ & + \frac{21\sqrt{286}G_{azz}(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{2288} + \frac{\sqrt{4290}G_{bzy}(47x^4 - 116x^2y^2 + 66x^2z^2 + 5y^4 + 66y^2z^2 - 44z^4)}{2288} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(3,3)}[g](A_2, 1)$$

\*\* symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

\*\* expression

$$\begin{aligned} & \frac{3\sqrt{2002}G_1y(3x^2 - y^2)(3x^4 + 6x^2y^2 - 60x^2z^2 + 3y^4 - 60y^2z^2 + 80z^4)}{4576} \\ & + \frac{3\sqrt{2002}G_2x(x^2 - 3y^2)(3x^4 + 6x^2y^2 - 60x^2z^2 + 3y^4 - 60y^2z^2 + 80z^4)}{4576} + \frac{3\sqrt{3003}G_3xyz(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{1144} \\ & - \frac{\sqrt{30030}G_{3x}x(5x^6 + 15x^4y^2 - 120x^4z^2 + 15x^2y^4 - 240x^2y^2z^2 + 240x^2z^4 + 5y^6 - 120y^4z^2 + 240y^2z^4 - 64z^6)}{4576} \\ & - \frac{\sqrt{30030}G_{3y}y(5x^6 + 15x^4y^2 - 120x^4z^2 + 15x^2y^4 - 240x^2y^2z^2 + 240x^2z^4 + 5y^6 - 120y^4z^2 + 240y^2z^4 - 64z^6)}{4576} \\ & - \frac{\sqrt{5005}G_{az}z(35x^6 + 105x^4y^2 - 210x^4z^2 + 105x^2y^4 - 420x^2y^2z^2 + 168x^2z^4 + 35y^6 - 210y^4z^2 + 168y^2z^4 - 16z^6)}{2288} \\ & + \frac{3\sqrt{3003}G_{bz}z(x - y)(x + y)(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{2288} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(3,3)}[g](A_2, 2)$$

\*\* symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

\*\* expression

$$\begin{aligned} & - \frac{\sqrt{715}G_1z(197x^6 - 3270x^4y^2 + 105x^4z^2 + 3165x^2y^4 + 210x^2y^2z^2 - 84x^2z^4 - 232y^6 + 105y^4z^2 - 84y^2z^4 + 8z^6)}{5720} \\ & + \frac{3\sqrt{715}G_2xyz(x^2 - 3y^2)(3x^2 - y^2)}{40} \\ & + \frac{\sqrt{4290}G_3x(7x^6 - 78x^4y^2 - 69x^4z^2 - 45x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 40y^6 - 465y^4z^2 - 60y^2z^4 + 16z^6)}{2860} \\ & - \frac{3\sqrt{429}G_{3x}xyz(81x^4 + 30x^2y^2 - 300x^2z^2 - 51y^4 + 140y^2z^2 + 48z^4)}{1144} \\ & + \frac{3\sqrt{429}G_{3y}z(9x^6 - 90x^4y^2 - 15x^4z^2 - 75x^2y^4 + 330x^2y^2z^2 - 24x^2z^4 + 24y^6 - 95y^4z^2 + 24y^2z^4)}{1144} \\ & + \frac{3\sqrt{286}G_{az}y(3x^2 - y^2)(3x^4 + 6x^2y^2 - 60x^2z^2 + 3y^4 - 60y^2z^2 + 80z^4)}{1144} \\ & - \frac{\sqrt{4290}G_{bz}y(85x^6 - 75x^4y^2 - 1050x^4z^2 - 141x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 19y^6 - 258y^4z^2 + 120y^2z^4 - 32z^6)}{5720} \end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,-3)}[g](E, 1), \vec{\mathbb{G}}_{4,2}^{(3,-3)}[g](E, 1)$$

\*\* symmetry

$$- \frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

\*\* expression

$$\begin{aligned} & - \frac{\sqrt{42}G_3x}{28} + \frac{\sqrt{105}G_{3y}z}{14} + \frac{\sqrt{70}G_{az}y}{14} + \frac{\sqrt{42}G_{bz}y}{28} \\ & \frac{\sqrt{42}G_3y}{28} - \frac{\sqrt{105}G_{3x}z}{14} - \frac{\sqrt{70}G_{az}x}{14} + \frac{\sqrt{42}G_{bz}x}{28} \end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,-3)}[g](E, 2), \vec{\mathbb{G}}_{4,2}^{(3,-3)}[g](E, 2)$$

\*\* symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x - y)(x + y)}{2}$$

\*\* expression

$$- \frac{\sqrt{2}G_1y}{2} + \frac{\sqrt{2}G_2x}{2}$$

$$\frac{\sqrt{2}G_1x}{2} + \frac{\sqrt{2}G_2y}{2}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,-3)}[g](E, 3), \vec{\mathbb{G}}_{4,2}^{(3,-3)}[g](E, 3)$$

\*\* symmetry

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{14}G_1y}{28}-\frac{\sqrt{14}G_2x}{28}+\frac{\sqrt{210}G_{3x}x}{28}-\frac{\sqrt{210}G_{3y}y}{28}+\frac{\sqrt{21}G_{bz}z}{7}$$

$$\frac{\sqrt{14}G_1x}{28}-\frac{\sqrt{14}G_2y}{28}-\frac{\sqrt{21}G_3z}{7}-\frac{\sqrt{210}G_{3x}y}{28}-\frac{\sqrt{210}G_{3y}x}{28}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,-1)}[g](E, 1), \vec{\mathbb{G}}_{4,2}^{(3,-1)}[g](E, 1)$$

\*\* symmetry

$$-\frac{\sqrt{10}yz(3x^2+3y^2-4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2+3y^2-4z^2)}{4}$$

\*\* expression

$$-\frac{15\sqrt{154}G_1z(x-y)(x+y)}{308}+\frac{15\sqrt{154}G_2xyz}{154}+\frac{\sqrt{231}G_3x(x^2-19y^2+16z^2)}{308}+\frac{2\sqrt{2310}G_{3x}xyz}{77}$$

$$-\frac{\sqrt{2310}G_{3y}z(7x^2-y^2-2z^2)}{308}-\frac{3\sqrt{385}G_{az}y(x^2+y^2-4z^2)}{308}-\frac{\sqrt{231}G_{bz}y(11x^2-9y^2+16z^2)}{308}$$

$$\frac{15\sqrt{154}G_1xyz}{154}+\frac{15\sqrt{154}G_2z(x-y)(x+y)}{308}+\frac{\sqrt{231}G_3y(19x^2-y^2-16z^2)}{308}-\frac{\sqrt{2310}G_{3x}z(x^2-7y^2+2z^2)}{308}$$

$$-\frac{2\sqrt{2310}G_{3y}xyz}{77}+\frac{3\sqrt{385}G_{az}x(x^2+y^2-4z^2)}{308}+\frac{\sqrt{231}G_{bz}x(9x^2-11y^2-16z^2)}{308}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,-1)}[g](E, 2), \vec{\mathbb{G}}_{4,2}^{(3,-1)}[g](E, 2)$$

\*\* symmetry

$$\frac{\sqrt{35}(x^2-2xy-y^2)(x^2+2xy-y^2)}{8}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

\*\* expression

$$-\frac{3\sqrt{11}G_1y(x^2+y^2-4z^2)}{44}+\frac{3\sqrt{11}G_2x(x^2+y^2-4z^2)}{44}-\frac{5\sqrt{66}G_3xyz}{22}-\frac{\sqrt{165}G_{3x}x(x^2-3y^2)}{44}+\frac{\sqrt{165}G_{3y}y(3x^2-y^2)}{44}+\frac{5\sqrt{66}G_{bz}z(x-y)(x+y)}{44}$$

$$\frac{3\sqrt{11}G_1x(x^2+y^2-4z^2)}{44}+\frac{3\sqrt{11}G_2y(x^2+y^2-4z^2)}{44}+\frac{5\sqrt{66}G_3z(x-y)(x+y)}{44}-\frac{\sqrt{165}G_{3x}y(3x^2-y^2)}{44}-\frac{\sqrt{165}G_{3y}x(x^2-3y^2)}{44}+\frac{5\sqrt{66}G_{bz}xyz}{22}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,-1)}[g](E, 3), \vec{\mathbb{G}}_{4,2}^{(3,-1)}[g](E, 3)$$

\*\* symmetry

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

\*\* expression

$$-\frac{9\sqrt{77}G_1y(x^2+y^2-4z^2)}{308}-\frac{9\sqrt{77}G_2x(x^2+y^2-4z^2)}{308}+\frac{\sqrt{1155}G_{3x}x(x^2-11y^2+8z^2)}{308}$$

$$+\frac{\sqrt{1155}G_{3y}y(11x^2-y^2-8z^2)}{308}-\frac{3\sqrt{770}G_{az}z(x-y)(x+y)}{308}+\frac{\sqrt{462}G_{bz}z(3x^2+3y^2-2z^2)}{308}$$

$$\frac{9\sqrt{77}G_1x(x^2+y^2-4z^2)}{308}-\frac{9\sqrt{77}G_2y(x^2+y^2-4z^2)}{308}-\frac{\sqrt{462}G_3z(3x^2+3y^2-2z^2)}{308}$$

$$-\frac{\sqrt{1155}G_{3x}y(7x^2-5y^2+8z^2)}{308}+\frac{\sqrt{1155}G_{3y}x(5x^2-7y^2-8z^2)}{308}+\frac{3\sqrt{770}G_{az}xyz}{154}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,1)}[g](E, 1), \vec{\mathbb{G}}_{4,2}^{(3,1)}[g](E, 1)$$

\*\* symmetry

$$-\frac{\sqrt{10}yz(3x^2+3y^2-4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2+3y^2-4z^2)}{4}$$

\*\* expression

$$\begin{aligned} & -\frac{3\sqrt{5005}G_1z(x^4+54x^2y^2-20x^2z^2-19y^4+20y^2z^2)}{2288} -\frac{3\sqrt{5005}G_2xyz(2x^2-7y^2+5z^2)}{286} \\ & +\frac{\sqrt{30030}G_3x(8x^4+9x^2y^2-89x^2z^2+y^4-33y^2z^2+50z^4)}{8008} -\frac{4\sqrt{3003}G_{3x}xyz(x^2+y^2-2z^2)}{143} \\ & +\frac{\sqrt{3003}G_{3y}z(239x^4+30x^2y^2-488x^2z^2-209y^4+408y^2z^2+8z^4)}{16016} \\ & +\frac{57\sqrt{2002}G_{az}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{16016} -\frac{\sqrt{30030}G_{bz}y(23x^4+32x^2y^2-234x^2z^2+9y^4-122y^2z^2+100z^4)}{16016} \\ & \frac{3\sqrt{5005}G_1xyz(7x^2-2y^2-5z^2)}{286} +\frac{3\sqrt{5005}G_2z(19x^4-54x^2y^2-20x^2z^2-y^4+20y^2z^2)}{2288} \\ & -\frac{\sqrt{30030}G_3y(x^4+9x^2y^2-33x^2z^2+8y^4-89y^2z^2+50z^4)}{8008} +\frac{\sqrt{3003}G_{3x}z(209x^4-30x^2y^2-408x^2z^2-239y^4+488y^2z^2-8z^4)}{16016} \\ & +\frac{4\sqrt{3003}G_{3y}xyz(x^2+y^2-2z^2)}{143} -\frac{57\sqrt{2002}G_{az}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{16016} \\ & -\frac{\sqrt{30030}G_{bz}x(9x^4+32x^2y^2-122x^2z^2+23y^4-234y^2z^2+100z^4)}{16016} \end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,1)}[g](E, 2), \vec{\mathbb{G}}_{4,2}^{(3,1)}[g](E, 2)$$

\*\* symmetry

$$\frac{\sqrt{35}(x^2-2xy-y^2)(x^2+2xy-y^2)}{8}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

\*\* expression

$$\begin{aligned} & -\frac{3\sqrt{1430}G_1y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{2288} +\frac{3\sqrt{1430}G_2x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{2288} \\ & -\frac{7\sqrt{2145}G_3xyz(x^2+y^2-2z^2)}{286} -\frac{7\sqrt{858}G_{3x}x(5x^4-34x^2y^2-16x^2z^2+9y^4+48y^2z^2)}{2288} -\frac{7\sqrt{858}G_{3y}y(9x^4-34x^2y^2+48x^2z^2+5y^4-16y^2z^2)}{2288} \\ & -\frac{63\sqrt{143}G_{az}z(x^2-2xy-y^2)(x^2+2xy-y^2)}{572} +\frac{7\sqrt{2145}G_{bz}z(x-y)(x+y)(x^2+y^2-2z^2)}{572} \\ & \frac{3\sqrt{1430}G_1x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{2288} +\frac{3\sqrt{1430}G_2y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{2288} \\ & +\frac{7\sqrt{2145}G_3z(x-y)(x+y)(x^2+y^2-2z^2)}{572} -\frac{7\sqrt{858}G_{3x}y(21x^4-26x^2y^2-48x^2z^2+y^4+16y^2z^2)}{2288} \\ & +\frac{7\sqrt{858}G_{3y}x(x^4-26x^2y^2+16x^2z^2+21y^4-48y^2z^2)}{2288} -\frac{63\sqrt{143}G_{az}xyz(x-y)(x+y)}{143} +\frac{7\sqrt{2145}G_{bz}xyz(x^2+y^2-2z^2)}{286} \end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,1)}[g](E, 3), \vec{\mathbb{G}}_{4,2}^{(3,1)}[g](E, 3)$$

\*\* symmetry

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

\*\* expression

$$\begin{aligned} & -\frac{9\sqrt{10010}G_1y(10x^4-15x^2y^2-15x^2z^2+3y^4-15y^2z^2+10z^4)}{8008} -\frac{9\sqrt{10010}G_2x(3x^4-15x^2y^2-15x^2z^2+10y^4-15y^2z^2+10z^4)}{8008} \\ & +\frac{3\sqrt{15015}G_3xyz(x-y)(x+y)}{143} -\frac{\sqrt{6006}G_{3x}x(5x^4-53x^2y^2+3x^2z^2-58y^4+507y^2z^2-86z^4)}{8008} \\ & -\frac{\sqrt{6006}G_{3y}y(58x^4+53x^2y^2-507x^2z^2-5y^4-3y^2z^2+86z^4)}{8008} -\frac{3\sqrt{1001}G_{az}z(x-y)(x+y)(x^2+y^2-2z^2)}{572} \\ & -\frac{\sqrt{15015}G_{bz}z(9x^4+186x^2y^2-80x^2z^2+9y^4-80y^2z^2+16z^4)}{4004} \end{aligned}$$

$$\begin{aligned}
& - \frac{9\sqrt{10010}G_1x(x^4 - 40x^2y^2 + 30x^2z^2 + 15y^4 + 30y^2z^2 - 20z^4)}{16016} + \frac{9\sqrt{10010}G_2y(15x^4 - 40x^2y^2 + 30x^2z^2 + y^4 + 30y^2z^2 - 20z^4)}{16016} \\
& + \frac{\sqrt{15015}G_3z(51x^4 - 66x^2y^2 - 80x^2z^2 + 51y^4 - 80y^2z^2 + 16z^4)}{4004} + \frac{\sqrt{6006}G_{3x}y(73x^4 + 20x^2y^2 - 498x^2z^2 - 53y^4 + 510y^2z^2 - 172z^4)}{16016} \\
& - \frac{\sqrt{6006}G_{3y}x(53x^4 - 20x^2y^2 - 510x^2z^2 - 73y^4 + 498y^2z^2 + 172z^4)}{16016} + \frac{3\sqrt{1001}G_{az}xyz(x^2 + y^2 - 2z^2)}{286} - \frac{3\sqrt{15015}G_{bz}xyz(x-y)(x+y)}{143}
\end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,3)}[g](E, 1), \vec{\mathbb{G}}_{4,2}^{(3,3)}[g](E, 1)$$

\*\* symmetry

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

\*\* expression

$$\begin{aligned}
& \frac{3\sqrt{5005}G_1z(9x^6 - 90x^4y^2 - 15x^4z^2 - 75x^2y^4 + 330x^2y^2z^2 - 24x^2z^4 + 24y^6 - 95y^4z^2 + 24y^2z^4)}{5720} \\
& - \frac{3\sqrt{5005}G_2xyz(51x^4 - 30x^2y^2 - 140x^2z^2 - 81y^4 + 300y^2z^2 - 48z^4)}{5720} \\
& - \frac{\sqrt{30030}G_3x(x^6 - 6x^4y^2 - 15x^4z^2 - 15x^2y^4 + 150x^2y^2z^2 - 8y^6 + 165y^4z^2 - 240y^2z^4 + 16z^6)}{2860} \\
& + \frac{3\sqrt{3003}G_{3x}xyz(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{1144} \\
& - \frac{\sqrt{3003}G_{3y}z(5x^6 - 30x^4y^2 - 15x^4z^2 - 75x^2y^4 + 210x^2y^2z^2 - 12x^2z^4 - 40y^6 + 225y^4z^2 - 156y^2z^4 + 8z^6)}{1144} \\
& - \frac{\sqrt{2002}G_{az}y(5x^6 + 15x^4y^2 - 120x^4z^2 + 15x^2y^4 - 240x^2y^2z^2 + 240x^2z^4 + 5y^6 - 120y^4z^2 + 240y^2z^4 - 64z^6)}{1144} \\
& + \frac{\sqrt{30030}G_{bz}y(11x^6 + 15x^4y^2 - 210x^4z^2 - 3x^2y^4 - 60x^2y^2z^2 + 240x^2z^4 - 7y^6 + 150y^4z^2 - 240y^2z^4 + 32z^6)}{5720}
\end{aligned}$$

$$\begin{aligned}
& \frac{3\sqrt{5005}G_1xyz(81x^4 + 30x^2y^2 - 300x^2z^2 - 51y^4 + 140y^2z^2 + 48z^4)}{5720} \\
& + \frac{3\sqrt{5005}G_2z(24x^6 - 75x^4y^2 - 95x^4z^2 - 90x^2y^4 + 330x^2y^2z^2 + 24x^2z^4 + 9y^6 - 15y^4z^2 - 24y^2z^4)}{5720} \\
& - \frac{\sqrt{30030}G_3y(8x^6 + 15x^4y^2 - 165x^4z^2 + 6x^2y^4 - 150x^2y^2z^2 + 240x^2z^4 - y^6 + 15y^4z^2 - 16z^6)}{2860} \\
& - \frac{\sqrt{3003}G_{3x}z(40x^6 + 75x^4y^2 - 225x^4z^2 + 30x^2y^4 - 210x^2y^2z^2 + 156x^2z^4 - 5y^6 + 15y^4z^2 + 12y^2z^4 - 8z^6)}{1144} \\
& - \frac{3\sqrt{3003}G_{3y}xyz(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{1144} \\
& + \frac{\sqrt{2002}G_{az}x(5x^6 + 15x^4y^2 - 120x^4z^2 + 15x^2y^4 - 240x^2y^2z^2 + 240x^2z^4 + 5y^6 - 120y^4z^2 + 240y^2z^4 - 64z^6)}{1144} \\
& - \frac{\sqrt{30030}G_{bz}x(7x^6 + 3x^4y^2 - 150x^4z^2 - 15x^2y^4 + 60x^2y^2z^2 + 240x^2z^4 - 11y^6 + 210y^4z^2 - 240y^2z^4 - 32z^6)}{5720}
\end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,3)}[g](E, 2), \vec{\mathbb{G}}_{4,2}^{(3,3)}[g](E, 2)$$

\*\* symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

\*\* expression

$$\begin{aligned}
& \frac{\sqrt{1430}G_1y(1499x^6 - 7515x^4y^2 + 60x^4z^2 + 4497x^2y^4 + 120x^2y^2z^2 - 120x^2z^4 - 217y^6 + 60y^4z^2 - 120y^2z^4 + 32z^6)}{22880} \\
& + \frac{\sqrt{1430}G_2x(217x^6 - 4497x^4y^2 - 60x^4z^2 + 7515x^2y^4 - 120x^2y^2z^2 + 120x^2z^4 - 1499y^6 - 60y^4z^2 + 120y^2z^4 - 32z^6)}{22880} \\
& + \frac{3\sqrt{2145}G_3xyz(207x^4 - 730x^2y^2 + 40x^2z^2 + 207y^4 + 40y^2z^2 - 24z^4)}{5720} \\
& - \frac{3\sqrt{858}G_{3x}x(7x^6 - 51x^4y^2 - 96x^4z^2 - 35x^2y^4 + 720x^2y^2z^2 + 40x^2z^4 + 23y^6 - 240y^4z^2 - 120y^2z^4)}{4576} \\
& - \frac{3\sqrt{858}G_{3y}y(23x^6 - 35x^4y^2 - 240x^4z^2 - 51x^2y^4 + 720x^2y^2z^2 - 120x^2z^4 + 7y^6 - 96y^4z^2 + 40y^2z^4)}{4576} \\
& - \frac{3\sqrt{143}G_{az}z(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)(3x^2 + 3y^2 - 10z^2)}{208} \\
& + \frac{3\sqrt{2145}G_{bz}z(x-y)(x+y)(79x^4 - 986x^2y^2 - 40x^2z^2 + 79y^4 - 40y^2z^2 + 24z^4)}{11440}
\end{aligned}$$

$$\begin{aligned}
& - \frac{\sqrt{1430}G_1x(53x^6 - 1128x^4y^2 + 15x^4z^2 + 1875x^2y^4 + 30x^2y^2z^2 - 30x^2z^4 - 376y^6 + 15y^4z^2 - 30y^2z^4 + 8z^6)}{5720} \\
& + \frac{\sqrt{1430}G_2y(376x^6 - 1875x^4y^2 - 15x^4z^2 + 1128x^2y^4 - 30x^2y^2z^2 + 30x^2z^4 - 53y^6 - 15y^4z^2 + 30y^2z^4 - 8z^6)}{5720} \\
& - \frac{3\sqrt{2145}G_3z(x-y)(x+y)(8x^4 - 127x^2y^2 + 5x^2z^2 + 8y^4 + 5y^2z^2 - 3z^4)}{1430} \\
& - \frac{3\sqrt{858}G_{3xy}(8x^6 - 5x^4y^2 - 105x^4z^2 - 12x^2y^4 + 150x^2y^2z^2 + 30x^2z^4 + y^6 - 9y^4z^2 - 10y^2z^4)}{1144} \\
& + \frac{3\sqrt{858}G_{3yx}(x^6 - 12x^4y^2 - 9x^4z^2 - 5x^2y^4 + 150x^2y^2z^2 - 10x^2z^4 + 8y^6 - 105y^4z^2 + 30y^2z^4)}{1144} \\
& - \frac{3\sqrt{143}G_{az}xyz(x-y)(x+y)(3x^2 + 3y^2 - 10z^2)}{52} + \frac{3\sqrt{2145}G_{bz}xyz(111x^4 - 350x^2y^2 - 20x^2z^2 + 111y^4 - 20y^2z^2 + 12z^4)}{2860}
\end{aligned}$$

$$\vec{\mathbb{G}}_{4,1}^{(3,3)}[g](E, 3), \vec{\mathbb{G}}_{4,2}^{(3,3)}[g](E, 3)$$

\*\* symmetry

$$- \frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

\*\* expression

$$\begin{aligned}
& - \frac{\sqrt{10010}G_1y(85x^6 - 75x^4y^2 - 1050x^4z^2 - 141x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 19y^6 - 258y^4z^2 + 120y^2z^4 - 32z^6)}{11440} \\
& - \frac{\sqrt{10010}G_2x(19x^6 - 141x^4y^2 - 258x^4z^2 - 75x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 85y^6 - 1050y^4z^2 + 120y^2z^4 - 32z^6)}{11440} \\
& - \frac{3\sqrt{15015}G_3xyz(x-y)(x+y)(3x^2 + 3y^2 - 10z^2)}{260} \\
& + \frac{\sqrt{6006}G_{3xz}(7x^6 + 3x^4y^2 - 150x^4z^2 - 15x^2y^4 + 60x^2y^2z^2 + 240x^2z^4 - 11y^6 + 210y^4z^2 - 240y^2z^4 - 32z^6)}{2288} \\
& + \frac{\sqrt{6006}G_{3xy}(11x^6 + 15x^4y^2 - 210x^4z^2 - 3x^2y^4 - 60x^2y^2z^2 + 240x^2z^4 - 7y^6 + 150y^4z^2 - 240y^2z^4 + 32z^6)}{2288} \\
& + \frac{3\sqrt{1001}G_{az}z(x-y)(x+y)(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{1144} \\
& - \frac{\sqrt{15015}G_{bz}z(67x^6 - 195x^4y^2 - 270x^4z^2 - 195x^2y^4 + 780x^2y^2z^2 + 84x^2z^4 + 67y^6 - 270y^4z^2 + 84y^2z^4 - 8z^6)}{5720} \\
& - \frac{\sqrt{10010}G_1x(7x^6 - 78x^4y^2 - 69x^4z^2 - 45x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 40y^6 - 465y^4z^2 - 60y^2z^4 + 16z^6)}{5720} \\
& + \frac{\sqrt{10010}G_2y(40x^6 - 45x^4y^2 - 465x^4z^2 - 78x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 7y^6 - 69y^4z^2 - 60y^2z^4 + 16z^6)}{5720} \\
& - \frac{\sqrt{15015}G_3z(8x^6 - 75x^4y^2 - 15x^4z^2 - 75x^2y^4 + 300x^2y^2z^2 - 21x^2z^4 + 8y^6 - 15y^4z^2 - 21y^2z^4 + 2z^6)}{1430} \\
& - \frac{\sqrt{6006}G_{3xy}(8x^6 + 15x^4y^2 - 165x^4z^2 + 6x^2y^4 - 150x^2y^2z^2 + 240x^2z^4 - y^6 + 15y^4z^2 - 16z^6)}{1144} \\
& + \frac{\sqrt{6006}G_{3yx}(x^6 - 6x^4y^2 - 15x^4z^2 - 15x^2y^4 + 150x^2y^2z^2 - 8y^6 + 165y^4z^2 - 240y^2z^4 + 16z^6)}{1144} \\
& - \frac{3\sqrt{1001}G_{az}xyz(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4)}{572} + \frac{3\sqrt{15015}G_{bz}xyz(x-y)(x+y)(3x^2 + 3y^2 - 10z^2)}{260}
\end{aligned}$$