No. 20 D_{3d} $-31m$ (-31m setting) [trigonal] (axial)

表 1 rank 0

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
1	A_{1u}	A1u	–	–	$\mathbb{G}_0^{(h,A_{1u})}$	Gh(0, A1u, ,)	C_0

表 2 rank 1

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
2	A_{2g}	A2g	–	–	$\mathbb{G}_1^{(h,A_{2g})}$	Gh(1, A2g, ,)	C_0
3	E_g	Eg	–	0	$\mathbb{G}_{1,0}^{(h,E_g)}$	Gh(1, Eg, , 0)	$-S_1$
4	E_g	Eg	–	1	$\mathbb{G}_{1,1}^{(h,E_g)}$	Gh(1, Eg, , 1)	C_1

表 3 rank 2

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
5	A_{1u}	A1u	–	–	$\mathbb{G}_2^{(h,A_{1u})}$	Gh(2, A1u, ,)	C_0
6	E_u	Eu	1	0	$\mathbb{G}_{2,0}^{(h,E_u,1)}$	Gh(2, Eu, 1, 0)	$-S_1$
7	E_u	Eu	1	1	$\mathbb{G}_{2,1}^{(h,E_u,1)}$	Gh(2, Eu, 1, 1)	C_1
8	E_u	Eu	2	0	$\mathbb{G}_{2,0}^{(h,E_u,2)}$	Gh(2, Eu, 2, 0)	S_2
9	E_u	Eu	2	1	$\mathbb{G}_{2,1}^{(h,E_u,2)}$	Gh(2, Eu, 2, 1)	C_2

表 4 rank 3

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
10	A_{1g}	A1g	–	–	$\mathbb{G}_3^{(h,A_{1g})}$	Gh(3, A1g, ,)	S_3
11	A_{2g}	A2g	1	–	$\mathbb{G}_3^{(h,A_{2g},1)}$	Gh(3, A2g, 1,)	C_0
12	A_{2g}	A2g	2	–	$\mathbb{G}_3^{(h,A_{2g},2)}$	Gh(3, A2g, 2,)	C_3
13	E_g	Eg	1	0	$\mathbb{G}_{3,0}^{(h,E_g,1)}$	Gh(3, Eg, 1, 0)	$-S_1$
14	E_g	Eg	1	1	$\mathbb{G}_{3,1}^{(h,E_g,1)}$	Gh(3, Eg, 1, 1)	C_1
15	E_g	Eg	2	0	$\mathbb{G}_{3,0}^{(h,E_g,2)}$	Gh(3, Eg, 2, 0)	S_2
16	E_g	Eg	2	1	$\mathbb{G}_{3,1}^{(h,E_g,2)}$	Gh(3, Eg, 2, 1)	C_2

表 5 rank 4

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
17	A_{1u}	A1u	1	–	$\mathbb{G}_4^{(h,A_{1u},1)}$	Gh(4, A1u, 1,)	C_0
18	A_{1u}	A1u	2	–	$\mathbb{G}_4^{(h,A_{1u},2)}$	Gh(4, A1u, 2,)	C_3
19	A_{2u}	A2u	–	–	$\mathbb{G}_4^{(h,A_{2u})}$	Gh(4, A2u, ,)	S_3
20	E_u	Eu	1	0	$\mathbb{G}_{4,0}^{(h,E_u,1)}$	Gh(4, Eu, 1, 0)	$-S_1$
21	E_u	Eu	1	1	$\mathbb{G}_{4,1}^{(h,E_u,1)}$	Gh(4, Eu, 1, 1)	C_1
22	E_u	Eu	2	0	$\mathbb{G}_{4,0}^{(h,E_u,2)}$	Gh(4, Eu, 2, 0)	$-S_4$
23	E_u	Eu	2	1	$\mathbb{G}_{4,1}^{(h,E_u,2)}$	Gh(4, Eu, 2, 1)	C_4
24	E_u	Eu	3	0	$\mathbb{G}_{4,0}^{(h,E_u,3)}$	Gh(4, Eu, 3, 0)	S_2
25	E_u	Eu	3	1	$\mathbb{G}_{4,1}^{(h,E_u,3)}$	Gh(4, Eu, 3, 1)	C_2

表 6　rank 5

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
26	A_{1g}	A1g	–	–	$\mathbb{G}_5^{(h,A_{1g})}$	Gh(5, A1g, ,)	S_3
27	A_{2g}	A2g	1	–	$\mathbb{G}_5^{(h,A_{2g},1)}$	Gh(5, A2g, 1,)	C_0
28	A_{2g}	A2g	2	–	$\mathbb{G}_5^{(h,A_{2g},2)}$	Gh(5, A2g, 2,)	C_3
29	E_g	Eg	1	0	$\mathbb{G}_{5,0}^{(h,E_g,1)}$	Gh(5, Eg, 1, 0)	S_5
30	E_g	Eg	1	1	$\mathbb{G}_{5,1}^{(h,E_g,1)}$	Gh(5, Eg, 1, 1)	C_5
31	E_g	Eg	2	0	$\mathbb{G}_{5,0}^{(h,E_g,2)}$	Gh(5, Eg, 2, 0)	$-S_1$
32	E_g	Eg	2	1	$\mathbb{G}_{5,1}^{(h,E_g,2)}$	Gh(5, Eg, 2, 1)	C_1
33	E_g	Eg	3	0	$\mathbb{G}_{5,0}^{(h,E_g,3)}$	Gh(5, Eg, 3, 0)	$-S_4$
34	E_g	Eg	3	1	$\mathbb{G}_{5,1}^{(h,E_g,3)}$	Gh(5, Eg, 3, 1)	C_4
35	E_g	Eg	4	0	$\mathbb{G}_{5,0}^{(h,E_g,4)}$	Gh(5, Eg, 4, 0)	S_2
36	E_g	Eg	4	1	$\mathbb{G}_{5,1}^{(h,E_g,4)}$	Gh(5, Eg, 4, 1)	C_2

表 7　rank 6

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
37	A_{1u}	A1u	1	–	$\mathbb{G}_6^{(h,A_{1u},1)}$	Gh(6, A1u, 1,)	C_0
38	A_{1u}	A1u	2	–	$\mathbb{G}_6^{(h,A_{1u},2)}$	Gh(6, A1u, 2,)	C_6
39	A_{1u}	A1u	3	–	$\mathbb{G}_6^{(h,A_{1u},3)}$	Gh(6, A1u, 3,)	C_3
40	A_{2u}	A2u	1	–	$\mathbb{G}_6^{(h,A_{2u},1)}$	Gh(6, A2u, 1,)	S_6
41	A_{2u}	A2u	2	–	$\mathbb{G}_6^{(h,A_{2u},2)}$	Gh(6, A2u, 2,)	S_3
42	E_u	Eu	1	0	$\mathbb{G}_{6,0}^{(h,E_u,1)}$	Gh(6, Eu, 1, 0)	S_5
43	E_u	Eu	1	1	$\mathbb{G}_{6,1}^{(h,E_u,1)}$	Gh(6, Eu, 1, 1)	C_5
44	E_u	Eu	2	0	$\mathbb{G}_{6,0}^{(h,E_u,2)}$	Gh(6, Eu, 2, 0)	$-S_1$
45	E_u	Eu	2	1	$\mathbb{G}_{6,1}^{(h,E_u,2)}$	Gh(6, Eu, 2, 1)	C_1
46	E_u	Eu	3	0	$\mathbb{G}_{6,0}^{(h,E_u,3)}$	Gh(6, Eu, 3, 0)	$-S_4$
47	E_u	Eu	3	1	$\mathbb{G}_{6,1}^{(h,E_u,3)}$	Gh(6, Eu, 3, 1)	C_4
48	E_u	Eu	4	0	$\mathbb{G}_{6,0}^{(h,E_u,4)}$	Gh(6, Eu, 4, 0)	S_2
49	E_u	Eu	4	1	$\mathbb{G}_{6,1}^{(h,E_u,4)}$	Gh(6, Eu, 4, 1)	C_2

表 8　rank 7

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
50	A_{1g}	A1g	1	–	$\mathbb{G}_7^{(h,A_{1g},1)}$	Gh(7, A1g, 1,)	S_6
51	A_{1g}	A1g	2	–	$\mathbb{G}_7^{(h,A_{1g},2)}$	Gh(7, A1g, 2,)	S_3
52	A_{2g}	A2g	1	–	$\mathbb{G}_7^{(h,A_{2g},1)}$	Gh(7, A2g, 1,)	C_0
53	A_{2g}	A2g	2	–	$\mathbb{G}_7^{(h,A_{2g},2)}$	Gh(7, A2g, 2,)	C_6
54	A_{2g}	A2g	3	–	$\mathbb{G}_7^{(h,A_{2g},3)}$	Gh(7, A2g, 3,)	C_3
55	E_g	Eg	1	0	$\mathbb{G}_{7,0}^{(h,E_g,1)}$	Gh(7, Eg, 1, 0)	$-S_7$
56	E_g	Eg	1	1	$\mathbb{G}_{7,1}^{(h,E_g,1)}$	Gh(7, Eg, 1, 1)	C_7
57	E_g	Eg	2	0	$\mathbb{G}_{7,0}^{(h,E_g,2)}$	Gh(7, Eg, 2, 0)	S_5
58	E_g	Eg	2	1	$\mathbb{G}_{7,1}^{(h,E_g,2)}$	Gh(7, Eg, 2, 1)	C_5
59	E_g	Eg	3	0	$\mathbb{G}_{7,0}^{(h,E_g,3)}$	Gh(7, Eg, 3, 0)	$-S_1$
60	E_g	Eg	3	1	$\mathbb{G}_{7,1}^{(h,E_g,3)}$	Gh(7, Eg, 3, 1)	C_1
61	E_g	Eg	4	0	$\mathbb{G}_{7,0}^{(h,E_g,4)}$	Gh(7, Eg, 4, 0)	$-S_4$
62	E_g	Eg	4	1	$\mathbb{G}_{7,1}^{(h,E_g,4)}$	Gh(7, Eg, 4, 1)	C_4
63	E_g	Eg	5	0	$\mathbb{G}_{7,0}^{(h,E_g,5)}$	Gh(7, Eg, 5, 0)	S_2
64	E_g	Eg	5	1	$\mathbb{G}_{7,1}^{(h,E_g,5)}$	Gh(7, Eg, 5, 1)	C_2

表 9　rank 8

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
65	A_{1u}	A1u	1	–	$\mathbb{G}_8^{(h,A_{1u},1)}$	Gh(8, A1u, 1,)	C_0
66	A_{1u}	A1u	2	–	$\mathbb{G}_8^{(h,A_{1u},2)}$	Gh(8, A1u, 2,)	C_6
67	A_{1u}	A1u	3	–	$\mathbb{G}_8^{(h,A_{1u},3)}$	Gh(8, A1u, 3,)	C_3
68	A_{2u}	A2u	1	–	$\mathbb{G}_8^{(h,A_{2u},1)}$	Gh(8, A2u, 1,)	S_6
69	A_{2u}	A2u	2	–	$\mathbb{G}_8^{(h,A_{2u},2)}$	Gh(8, A2u, 2,)	S_3
70	E_u	Eu	1	0	$\mathbb{G}_{8,0}^{(h,E_u,1)}$	Gh(8, Eu, 1, 0)	$-S_7$
71	E_u	Eu	1	1	$\mathbb{G}_{8,1}^{(h,E_u,1)}$	Gh(8, Eu, 1, 1)	C_7
72	E_u	Eu	2	0	$\mathbb{G}_{8,0}^{(h,E_u,2)}$	Gh(8, Eu, 2, 0)	S_5
73	E_u	Eu	2	1	$\mathbb{G}_{8,1}^{(h,E_u,2)}$	Gh(8, Eu, 2, 1)	C_5
74	E_u	Eu	3	0	$\mathbb{G}_{8,0}^{(h,E_u,3)}$	Gh(8, Eu, 3, 0)	$-S_1$
75	E_u	Eu	3	1	$\mathbb{G}_{8,1}^{(h,E_u,3)}$	Gh(8, Eu, 3, 1)	C_1
76	E_u	Eu	4	0	$\mathbb{G}_{8,0}^{(h,E_u,4)}$	Gh(8, Eu, 4, 0)	S_8
77	E_u	Eu	4	1	$\mathbb{G}_{8,1}^{(h,E_u,4)}$	Gh(8, Eu, 4, 1)	C_8
78	E_u	Eu	5	0	$\mathbb{G}_{8,0}^{(h,E_u,5)}$	Gh(8, Eu, 5, 0)	$-S_4$
79	E_u	Eu	5	1	$\mathbb{G}_{8,1}^{(h,E_u,5)}$	Gh(8, Eu, 5, 1)	C_4
80	E_u	Eu	6	0	$\mathbb{G}_{8,0}^{(h,E_u,6)}$	Gh(8, Eu, 6, 0)	S_2
81	E_u	Eu	6	1	$\mathbb{G}_{8,1}^{(h,E_u,6)}$	Gh(8, Eu, 6, 1)	C_2

表 10　rank 9

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
82	A_{1g}	A1g	1	–	$\mathbb{G}_9^{(h,A_{1g},1)}$	Gh(9, A1g, 1,)	S_6
83	A_{1g}	A1g	2	–	$\mathbb{G}_9^{(h,A_{1g},2)}$	Gh(9, A1g, 2,)	S_9
84	A_{1g}	A1g	3	–	$\mathbb{G}_9^{(h,A_{1g},3)}$	Gh(9, A1g, 3,)	S_3
85	A_{2g}	A2g	1	–	$\mathbb{G}_9^{(h,A_{2g},1)}$	Gh(9, A2g, 1,)	C_0
86	A_{2g}	A2g	2	–	$\mathbb{G}_9^{(h,A_{2g},2)}$	Gh(9, A2g, 2,)	C_6
87	A_{2g}	A2g	3	–	$\mathbb{G}_9^{(h,A_{2g},3)}$	Gh(9, A2g, 3,)	C_9
88	A_{2g}	A2g	4	–	$\mathbb{G}_9^{(h,A_{2g},4)}$	Gh(9, A2g, 4,)	C_3
89	E_g	Eg	1	0	$\mathbb{G}_{9,0}^{(h,E_g,1)}$	Gh(9, Eg, 1, 0)	$-S_7$
90	E_g	Eg	1	1	$\mathbb{G}_{9,1}^{(h,E_g,1)}$	Gh(9, Eg, 1, 1)	C_7
91	E_g	Eg	2	0	$\mathbb{G}_{9,0}^{(h,E_g,2)}$	Gh(9, Eg, 2, 0)	S_5
92	E_g	Eg	2	1	$\mathbb{G}_{9,1}^{(h,E_g,2)}$	Gh(9, Eg, 2, 1)	C_5
93	E_g	Eg	3	0	$\mathbb{G}_{9,0}^{(h,E_g,3)}$	Gh(9, Eg, 3, 0)	$-S_1$
94	E_g	Eg	3	1	$\mathbb{G}_{9,1}^{(h,E_g,3)}$	Gh(9, Eg, 3, 1)	C_1
95	E_g	Eg	4	0	$\mathbb{G}_{9,0}^{(h,E_g,4)}$	Gh(9, Eg, 4, 0)	S_8
96	E_g	Eg	4	1	$\mathbb{G}_{9,1}^{(h,E_g,4)}$	Gh(9, Eg, 4, 1)	C_8
97	E_g	Eg	5	0	$\mathbb{G}_{9,0}^{(h,E_g,5)}$	Gh(9, Eg, 5, 0)	$-S_4$
98	E_g	Eg	5	1	$\mathbb{G}_{9,1}^{(h,E_g,5)}$	Gh(9, Eg, 5, 1)	C_4
99	E_g	Eg	6	0	$\mathbb{G}_{9,0}^{(h,E_g,6)}$	Gh(9, Eg, 6, 0)	S_2
100	E_g	Eg	6	1	$\mathbb{G}_{9,1}^{(h,E_g,6)}$	Gh(9, Eg, 6, 1)	C_2

表 11　rank 10

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
101	A_{1u}	A1u	1	–	$\mathbb{G}_{10}^{(h,A_{1u},1)}$	Gh(10, A1u, 1,)	C_0
102	A_{1u}	A1u	2	–	$\mathbb{G}_{10}^{(h,A_{1u},2)}$	Gh(10, A1u, 2,)	C_6
103	A_{1u}	A1u	3	–	$\mathbb{G}_{10}^{(h,A_{1u},3)}$	Gh(10, A1u, 3,)	C_9
104	A_{1u}	A1u	4	–	$\mathbb{G}_{10}^{(h,A_{1u},4)}$	Gh(10, A1u, 4,)	C_3
105	A_{2u}	A2u	1	–	$\mathbb{G}_{10}^{(h,A_{2u},1)}$	Gh(10, A2u, 1,)	S_6
106	A_{2u}	A2u	2	–	$\mathbb{G}_{10}^{(h,A_{2u},2)}$	Gh(10, A2u, 2,)	S_9
107	A_{2u}	A2u	3	–	$\mathbb{G}_{10}^{(h,A_{2u},3)}$	Gh(10, A2u, 3,)	S_3
108	E_u	Eu	1	0	$\mathbb{G}_{10,0}^{(h,E_u,1)}$	Gh(10, Eu, 1, 0)	$-S_7$
109	E_u	Eu	1	1	$\mathbb{G}_{10,1}^{(h,E_u,1)}$	Gh(10, Eu, 1, 1)	C_7
110	E_u	Eu	2	0	$\mathbb{G}_{10,0}^{(h,E_u,2)}$	Gh(10, Eu, 2, 0)	S_5
111	E_u	Eu	2	1	$\mathbb{G}_{10,1}^{(h,E_u,2)}$	Gh(10, Eu, 2, 1)	C_5
112	E_u	Eu	3	0	$\mathbb{G}_{10,0}^{(h,E_u,3)}$	Gh(10, Eu, 3, 0)	$-S_1$
113	E_u	Eu	3	1	$\mathbb{G}_{10,1}^{(h,E_u,3)}$	Gh(10, Eu, 3, 1)	C_1
114	E_u	Eu	4	0	$\mathbb{G}_{10,0}^{(h,E_u,4)}$	Gh(10, Eu, 4, 0)	$-S_{10}$
115	E_u	Eu	4	1	$\mathbb{G}_{10,1}^{(h,E_u,4)}$	Gh(10, Eu, 4, 1)	C_{10}
116	E_u	Eu	5	0	$\mathbb{G}_{10,0}^{(h,E_u,5)}$	Gh(10, Eu, 5, 0)	S_8
117	E_u	Eu	5	1	$\mathbb{G}_{10,1}^{(h,E_u,5)}$	Gh(10, Eu, 5, 1)	C_8
118	E_u	Eu	6	0	$\mathbb{G}_{10,0}^{(h,E_u,6)}$	Gh(10, Eu, 6, 0)	$-S_4$
119	E_u	Eu	6	1	$\mathbb{G}_{10,1}^{(h,E_u,6)}$	Gh(10, Eu, 6, 1)	C_4
120	E_u	Eu	7	0	$\mathbb{G}_{10,0}^{(h,E_u,7)}$	Gh(10, Eu, 7, 0)	S_2
121	E_u	Eu	7	1	$\mathbb{G}_{10,1}^{(h,E_u,7)}$	Gh(10, Eu, 7, 1)	C_2

表 12　rank 11

No.	irrep.	(tag)	mul.	comp.	harmonics	(tag)	definition
122	A_{1g}	A1g	1	–	$\mathbb{G}_{11}^{(h,A_{1g},1)}$	Gh(11, A1g, 1,)	S_6
123	A_{1g}	A1g	2	–	$\mathbb{G}_{11}^{(h,A_{1g},2)}$	Gh(11, A1g, 2,)	S_9
124	A_{1g}	A1g	3	–	$\mathbb{G}_{11}^{(h,A_{1g},3)}$	Gh(11, A1g, 3,)	S_3
125	A_{2g}	A2g	1	–	$\mathbb{G}_{11}^{(h,A_{2g},1)}$	Gh(11, A2g, 1,)	C_0
126	A_{2g}	A2g	2	–	$\mathbb{G}_{11}^{(h,A_{2g},2)}$	Gh(11, A2g, 2,)	C_6
127	A_{2g}	A2g	3	–	$\mathbb{G}_{11}^{(h,A_{2g},3)}$	Gh(11, A2g, 3,)	C_9
128	A_{2g}	A2g	4	–	$\mathbb{G}_{11}^{(h,A_{2g},4)}$	Gh(11, A2g, 4,)	C_3
129	E_g	Eg	1	0	$\mathbb{G}_{11,0}^{(h,E_g,1)}$	Gh(11, Eg, 1, 0)	S_{11}
130	E_g	Eg	1	1	$\mathbb{G}_{11,1}^{(h,E_g,1)}$	Gh(11, Eg, 1, 1)	C_{11}
131	E_g	Eg	2	0	$\mathbb{G}_{11,0}^{(h,E_g,2)}$	Gh(11, Eg, 2, 0)	$-S_7$
132	E_g	Eg	2	1	$\mathbb{G}_{11,1}^{(h,E_g,2)}$	Gh(11, Eg, 2, 1)	C_7
133	E_g	Eg	3	0	$\mathbb{G}_{11,0}^{(h,E_g,3)}$	Gh(11, Eg, 3, 0)	S_5
134	E_g	Eg	3	1	$\mathbb{G}_{11,1}^{(h,E_g,3)}$	Gh(11, Eg, 3, 1)	C_5
135	E_g	Eg	4	0	$\mathbb{G}_{11,0}^{(h,E_g,4)}$	Gh(11, Eg, 4, 0)	$-S_1$
136	E_g	Eg	4	1	$\mathbb{G}_{11,1}^{(h,E_g,4)}$	Gh(11, Eg, 4, 1)	C_1
137	E_g	Eg	5	0	$\mathbb{G}_{11,0}^{(h,E_g,5)}$	Gh(11, Eg, 5, 0)	$-S_{10}$
138	E_g	Eg	5	1	$\mathbb{G}_{11,1}^{(h,E_g,5)}$	Gh(11, Eg, 5, 1)	C_{10}
139	E_g	Eg	6	0	$\mathbb{G}_{11,0}^{(h,E_g,6)}$	Gh(11, Eg, 6, 0)	S_8
140	E_g	Eg	6	1	$\mathbb{G}_{11,1}^{(h,E_g,6)}$	Gh(11, Eg, 6, 1)	C_8
141	E_g	Eg	7	0	$\mathbb{G}_{11,0}^{(h,E_g,7)}$	Gh(11, Eg, 7, 0)	$-S_4$
142	E_g	Eg	7	1	$\mathbb{G}_{11,1}^{(h,E_g,7)}$	Gh(11, Eg, 7, 1)	C_4
143	E_g	Eg	8	0	$\mathbb{G}_{11,0}^{(h,E_g,8)}$	Gh(11, Eg, 8, 0)	S_2
144	E_g	Eg	8	1	$\mathbb{G}_{11,1}^{(h,E_g,8)}$	Gh(11, Eg, 8, 1)	C_2