

PG No. 10 $S_4 \bar{4}$ [tetragonal] (polar, internal polar octupole)

* Harmonics for rank 0

$$\vec{\mathbb{Q}}_0^{(3,3)}[q](A)$$

** symmetry

1

** expression

$$\begin{aligned} & \frac{\sqrt{70}Q_1y(3x^2-y^2)}{28} + \frac{\sqrt{70}Q_2x(x^2-3y^2)}{28} + \frac{\sqrt{105}Q_3xyz}{7} - \frac{\sqrt{42}Q_{3x}x(x^2+y^2-4z^2)}{28} \\ & - \frac{\sqrt{42}Q_{3y}y(x^2+y^2-4z^2)}{28} - \frac{\sqrt{7}Q_{az}z(3x^2+3y^2-2z^2)}{14} + \frac{\sqrt{105}Q_{bz}z(x-y)(x+y)}{14} \end{aligned}$$

* Harmonics for rank 1

$$\vec{\mathbb{Q}}_1^{(3,1)}[q](B)$$

** symmetry

z

** expression

$$\frac{\sqrt{21}Q_3xy}{7} + \frac{2\sqrt{210}Q_{3x}xz}{35} + \frac{2\sqrt{210}Q_{3y}yz}{35} - \frac{3\sqrt{35}Q_{az}(x^2+y^2-2z^2)}{70} + \frac{\sqrt{21}Q_{bz}(x-y)(x+y)}{14}$$

$$\vec{\mathbb{Q}}_1^{(3,3)}[q](B)$$

** symmetry

z

** expression

$$\begin{aligned} & \frac{\sqrt{210}Q_1yz(3x^2-y^2)}{24} + \frac{\sqrt{210}Q_2xz(x^2-3y^2)}{24} - \frac{\sqrt{35}Q_3xy(x^2+y^2-6z^2)}{14} - \frac{5\sqrt{14}Q_{3x}xz(3x^2+3y^2-4z^2)}{56} \\ & - \frac{5\sqrt{14}Q_{3y}yz(3x^2+3y^2-4z^2)}{56} + \frac{\sqrt{21}Q_{az}(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4)}{84} - \frac{\sqrt{35}Q_{bz}(x-y)(x+y)(x^2+y^2-6z^2)}{28} \end{aligned}$$

$$\vec{\mathbb{Q}}_{1,1}^{(3,1)}[q](E), \vec{\mathbb{Q}}_{1,2}^{(3,1)}[q](E)$$

** symmetry

x

y

** expression

$$\begin{aligned} & \frac{3\sqrt{14}Q_1xy}{14} + \frac{3\sqrt{14}Q_2(x-y)(x+y)}{28} + \frac{\sqrt{21}Q_3yz}{7} - \frac{\sqrt{210}Q_{3x}(3x^2+y^2-4z^2)}{140} - \frac{\sqrt{210}Q_{3y}xy}{70} - \frac{3\sqrt{35}Q_{az}xz}{35} + \frac{\sqrt{21}Q_{bz}xz}{7} \\ & \frac{3\sqrt{14}Q_1(x-y)(x+y)}{28} - \frac{3\sqrt{14}Q_2xy}{14} + \frac{\sqrt{21}Q_3xz}{7} - \frac{\sqrt{210}Q_{3x}xy}{70} - \frac{\sqrt{210}Q_{3y}(x^2+3y^2-4z^2)}{140} - \frac{3\sqrt{35}Q_{az}yz}{35} - \frac{\sqrt{21}Q_{bz}yz}{7} \end{aligned}$$

$$\vec{\mathbb{Q}}_{1,1}^{(3,3)}[q](E), \vec{\mathbb{Q}}_{1,2}^{(3,3)}[q](E)$$

** symmetry

x

y

** expression

$$\begin{aligned} & \frac{\sqrt{210}Q_1xy(15x^2-13y^2-6z^2)}{168} + \frac{\sqrt{210}Q_2(4x^4-21x^2y^2-3x^2z^2+3y^4+3y^2z^2)}{168} \\ & + \frac{\sqrt{35}Q_3yz(6x^2-y^2-z^2)}{14} - \frac{\sqrt{14}Q_{3x}(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4)}{56} \\ & - \frac{5\sqrt{14}Q_{3y}xy(x^2+y^2-6z^2)}{56} - \frac{5\sqrt{21}Q_{az}xz(3x^2+3y^2-4z^2)}{84} + \frac{\sqrt{35}Q_{bz}xz(5x^2-9y^2-2z^2)}{28} \\ & - \frac{\sqrt{210}Q_1(3x^4-21x^2y^2+3x^2z^2+4y^4-3y^2z^2)}{168} + \frac{\sqrt{210}Q_2xy(13x^2-15y^2+6z^2)}{168} - \frac{\sqrt{35}Q_{3x}xz(x^2-6y^2+z^2)}{14} - \frac{5\sqrt{14}Q_{3x}xy(x^2+y^2-6z^2)}{56} \\ & + \frac{\sqrt{14}Q_{3y}(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4)}{56} - \frac{5\sqrt{21}Q_{az}yz(3x^2+3y^2-4z^2)}{84} + \frac{\sqrt{35}Q_{bz}yz(9x^2-5y^2+2z^2)}{28} \end{aligned}$$

* Harmonics for rank 2

$$\vec{\mathbb{Q}}_2^{(3,-1)}[q](A)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{14}Q_{3xx}}{7} + \frac{\sqrt{14}Q_{3yy}}{7} + \frac{\sqrt{21}Q_{azz}}{7}$$

$\tilde{\mathbb{Q}}_2^{(3,1)}[q](A)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{5\sqrt{210}Q_1y(3x^2 - y^2)}{168} - \frac{5\sqrt{210}Q_2x(x^2 - 3y^2)}{168} - \frac{3\sqrt{14}Q_{3xx}(x^2 + y^2 - 4z^2)}{56} - \frac{3\sqrt{14}Q_{3yy}(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{21}Q_{azz}(3x^2 + 3y^2 - 2z^2)}{21}$$

$\tilde{\mathbb{Q}}_2^{(3,3)}[q](A)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\begin{aligned} & -\frac{\sqrt{1155}Q_1y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{264} - \frac{\sqrt{1155}Q_2x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{264} - \frac{3\sqrt{770}Q_3xyz(x^2 + y^2 - 2z^2)}{44} \\ & + \frac{15\sqrt{77}Q_{3xx}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{616} + \frac{15\sqrt{77}Q_{3yy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{616} \\ & + \frac{5\sqrt{462}Q_{azz}(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{1848} - \frac{3\sqrt{770}Q_{bz}(x-y)(x+y)(x^2 + y^2 - 2z^2)}{88} \end{aligned}$$

$\tilde{\mathbb{Q}}_2^{(3,-1)}[q](B, 1)$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{70}Q_1y}{14} + \frac{\sqrt{70}Q_2x}{14} - \frac{\sqrt{42}Q_{3xx}}{42} + \frac{\sqrt{42}Q_{3yy}}{42} + \frac{\sqrt{105}Q_{bz}}{21}$$

$\tilde{\mathbb{Q}}_2^{(3,-1)}[q](B, 2)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{70}Q_1x}{14} - \frac{\sqrt{70}Q_2y}{14} + \frac{\sqrt{105}Q_3z}{21} - \frac{\sqrt{42}Q_{3xy}}{42} - \frac{\sqrt{42}Q_{3yz}}{42}$$

$\tilde{\mathbb{Q}}_2^{(3,1)}[q](B, 1)$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{70}Q_1y(x^2 + y^2 - 4z^2)}{56} + \frac{\sqrt{70}Q_2x(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{42}Q_{3xx}(11x^2 - 9y^2 - 24z^2)}{168} \\ & - \frac{\sqrt{42}Q_{3yy}(9x^2 - 11y^2 + 24z^2)}{168} - \frac{5\sqrt{7}Q_{azz}(x-y)(x+y)}{14} + \frac{\sqrt{105}Q_{bz}(3x^2 + 3y^2 - 2z^2)}{42} \end{aligned}$$

$\tilde{\mathbb{Q}}_2^{(3,1)}[q](B, 2)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\begin{aligned} & \frac{\sqrt{70}Q_1x(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{70}Q_2y(x^2 + y^2 - 4z^2)}{56} + \frac{\sqrt{105}Q_3z(3x^2 + 3y^2 - 2z^2)}{42} \\ & - \frac{\sqrt{42}Q_{3xy}(21x^2 + y^2 - 24z^2)}{168} - \frac{\sqrt{42}Q_{3yz}(x^2 + 21y^2 - 24z^2)}{168} - \frac{5\sqrt{7}Q_{az}xyz}{7} \end{aligned}$$

$\vec{\mathbb{Q}}_2^{(3,3)}[q](B, 1)$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{385}Q_{1y}(53x^4 - 104x^2y^2 - 6x^2z^2 + 11y^4 - 6y^2z^2 + 4z^4)}{616} + \frac{\sqrt{385}Q_{2x}(11x^4 - 104x^2y^2 - 6x^2z^2 + 53y^4 - 6y^2z^2 + 4z^4)}{616} \\ & + \frac{3\sqrt{2310}Q_3xyz(x-y)(x+y)}{44} - \frac{5\sqrt{231}Q_{3xx}(5x^4 - 4x^2y^2 - 46x^2z^2 - 9y^4 + 66y^2z^2 + 12z^4)}{1848} \\ & - \frac{5\sqrt{231}Q_{3yy}(9x^4 + 4x^2y^2 - 66x^2z^2 - 5y^4 + 46y^2z^2 - 12z^4)}{1848} - \frac{5\sqrt{154}Q_{azz}(x-y)(x+y)(x^2 + y^2 - 2z^2)}{88} \\ & + \frac{\sqrt{2310}Q_{bz}z(39x^4 - 174x^2y^2 - 20x^2z^2 + 39y^4 - 20y^2z^2 + 4z^4)}{1848} \end{aligned}$$

$\vec{\mathbb{Q}}_2^{(3,3)}[q](B, 2)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\begin{aligned} & -\frac{\sqrt{385}Q_{1x}(5x^4 - 53x^2y^2 + 3x^2z^2 + 26y^4 + 3y^2z^2 - 2z^4)}{308} + \frac{\sqrt{385}Q_{2y}(26x^4 - 53x^2y^2 + 3x^2z^2 + 5y^4 + 3y^2z^2 - 2z^4)}{308} \\ & - \frac{\sqrt{2310}Q_3z(6x^4 - 51x^2y^2 + 5x^2z^2 + 6y^4 + 5y^2z^2 - z^4)}{462} - \frac{5\sqrt{231}Q_{3xy}(6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4)}{924} \\ & + \frac{5\sqrt{231}Q_{3yx}(x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{924} - \frac{5\sqrt{154}Q_{azxyz}(x^2 + y^2 - 2z^2)}{44} + \frac{3\sqrt{2310}Q_{bzxyz}(x-y)(x+y)}{44} \end{aligned}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,-1)}[q](E), \vec{\mathbb{Q}}_{2,2}^{(3,-1)}[q](E)$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{105}Q_{3x}}{21} + \frac{2\sqrt{42}Q_{3y}z}{21} - \frac{\sqrt{7}Q_{az}y}{7} - \frac{\sqrt{105}Q_{bz}y}{21}$$

$$\frac{\sqrt{105}Q_{3y}}{21} + \frac{2\sqrt{42}Q_{3x}z}{21} - \frac{\sqrt{7}Q_{az}x}{7} + \frac{\sqrt{105}Q_{bz}x}{21}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,1)}[q](E), \vec{\mathbb{Q}}_{2,2}^{(3,1)}[q](E)$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

** expression

$$\begin{aligned} & \frac{5\sqrt{70}Q_{1z}(x-y)(x+y)}{56} - \frac{5\sqrt{70}Q_{2xyz}}{28} - \frac{\sqrt{105}Q_{3x}(2x^2 - 3y^2 - 3z^2)}{42} + \frac{5\sqrt{42}Q_{3xy}yz}{28} \\ & - \frac{\sqrt{42}Q_{3yz}(21x^2 - 9y^2 - 4z^2)}{168} - \frac{\sqrt{7}Q_{azy}(x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{105}Q_{bz}y(9x^2 - y^2 - 6z^2)}{84} \end{aligned}$$

$$\begin{aligned} & \frac{5\sqrt{70}Q_{1xyz}}{28} + \frac{5\sqrt{70}Q_{2z}(x-y)(x+y)}{56} + \frac{\sqrt{105}Q_{3y}(3x^2 - 2y^2 + 3z^2)}{42} + \frac{\sqrt{42}Q_{3xz}(9x^2 - 21y^2 + 4z^2)}{168} \\ & + \frac{5\sqrt{42}Q_{3yz}xyz}{28} - \frac{\sqrt{7}Q_{azx}(x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{105}Q_{bz}x(x^2 - 9y^2 + 6z^2)}{84} \end{aligned}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,3)}[q](E), \vec{\mathbb{Q}}_{2,2}^{(3,3)}[q](E)$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

** expression

$$\begin{aligned}
& - \frac{\sqrt{385}Q_1z(x^4 - 9x^2y^2 + x^2z^2 + 2y^4 - y^2z^2)}{44} + \frac{\sqrt{385}Q_2xyz(5x^2 - 7y^2 + 2z^2)}{44} + \frac{\sqrt{2310}Q_3x(x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{462} \\
& - \frac{5\sqrt{231}Q_{3x}xyz(x^2 + y^2 - 2z^2)}{44} + \frac{5\sqrt{231}Q_{3y}z(3x^4 - 15x^2y^2 - x^2z^2 - 18y^4 + 41y^2z^2 - 4z^4)}{44} \\
& + \frac{5\sqrt{154}Q_{az}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{308} - \frac{\sqrt{2310}Q_{bz}y(9x^4 + 4x^2y^2 - 66x^2z^2 - 5y^4 + 46y^2z^2 - 12z^4)}{924}
\end{aligned}$$

* Harmonics for rank 3

$$\vec{\mathbb{Q}}_3^{(3,-3)}[q](A, 1)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$Q_3$$

$$\vec{\mathbb{Q}}_3^{(3,-3)}[q](A, 2)$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$Q_{bz}$$

$$\vec{\mathbb{Q}}_3^{(3,-1)}[q](A, 1)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{10}Q_1xz}{4} - \frac{\sqrt{10}Q_2yz}{4} + \frac{\sqrt{6}Q_{3x}yz}{4} + \frac{\sqrt{6}Q_{3y}xz}{4} - Q_{az}xy$$

$$\vec{\mathbb{Q}}_3^{(3,-1)}[q](A, 2)$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{10}Q_1yz}{4} + \frac{\sqrt{10}Q_2xz}{4} + \frac{\sqrt{6}Q_{3x}xz}{4} - \frac{\sqrt{6}Q_{3y}yz}{4} - \frac{Q_{az}(x-y)(x+y)}{2}$$

$$\vec{\mathbb{Q}}_3^{(3,1)}[q](A, 1)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\begin{aligned}
& \frac{5\sqrt{33}Q_1xz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{5\sqrt{33}Q_2yz(3x^2 + 3y^2 - 4z^2)}{132} - \frac{7\sqrt{22}Q_3(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{66} \\
& + \frac{\sqrt{55}Q_{3x}yz(9x^2 - 19y^2 + 16z^2)}{132} - \frac{\sqrt{55}Q_{3y}xz(19x^2 - 9y^2 - 16z^2)}{132} + \frac{\sqrt{330}Q_{az}xy(x^2 + y^2 - 6z^2)}{132} + \frac{35\sqrt{22}Q_{bz}xy(x-y)(x+y)}{132}
\end{aligned}$$

$$\vec{\mathbb{Q}}_3^{(3,1)}[q](A, 2)$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned}
& \frac{5\sqrt{33}Q_1yz(3x^2 + 3y^2 - 4z^2)}{132} + \frac{5\sqrt{33}Q_2xz(3x^2 + 3y^2 - 4z^2)}{132} + \frac{35\sqrt{22}Q_3xy(x-y)(x+y)}{132} - \frac{\sqrt{55}Q_{3x}xz(5x^2 + 33y^2 - 16z^2)}{132} \\
& + \frac{\sqrt{55}Q_{3y}yz(33x^2 + 5y^2 - 16z^2)}{132} + \frac{\sqrt{330}Q_{az}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{264} + \frac{7\sqrt{22}Q_{bz}(x^4 - 18x^2y^2 + 12x^2z^2 + y^4 + 12y^2z^2 - 4z^4)}{264}
\end{aligned}$$

$$\tilde{\mathbb{Q}}_3^{(3,3)}[q](A, 1)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$-\frac{7\sqrt{286}Q_1xz(7x^4 - 85x^2y^2 + 5x^2z^2 + 40y^4 + 5y^2z^2 - 2z^4)}{1144} + \frac{7\sqrt{286}Q_2yz(40x^4 - 85x^2y^2 + 5x^2z^2 + 7y^4 + 5y^2z^2 - 2z^4)}{1144} \\ + \frac{\sqrt{429}Q_3(2x^6 - 15x^4y^2 - 15x^4z^2 - 15x^2y^4 + 180x^2y^2z^2 - 15x^2z^4 + 2y^6 - 15y^4z^2 - 15y^2z^4 + 2z^6)}{286} \\ - \frac{7\sqrt{429}Q_{3xy}yz(8x^4 + 7x^2y^2 - 23x^2z^2 - y^4 + y^2z^2 + 2z^4)}{1144} + \frac{7\sqrt{429}Q_{3y}xzx(4x^4 - 7x^2y^2 - x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4)}{1144} \\ + \frac{7\sqrt{715}Q_{az}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} - \frac{21\sqrt{429}Q_{bz}xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572}$$

$$\tilde{\mathbb{Q}}_3^{(3,3)}[q](A, 2)$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$-\frac{7\sqrt{286}Q_1yz(85x^4 - 160x^2y^2 - 10x^2z^2 + 19y^4 - 10y^2z^2 + 4z^4)}{2288} + \frac{7\sqrt{286}Q_2xz(19x^4 - 160x^2y^2 - 10x^2z^2 + 85y^4 - 10y^2z^2 + 4z^4)}{2288} \\ - \frac{21\sqrt{429}Q_3xy(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572} - \frac{7\sqrt{429}Q_{3x}xz(7x^4 - 4x^2y^2 - 22x^2z^2 - 11y^4 + 26y^2z^2 + 4z^4)}{2288} \\ - \frac{7\sqrt{429}Q_{3y}yz(11x^4 + 4x^2y^2 - 26x^2z^2 - 7y^4 + 22y^2z^2 - 4z^4)}{2288} + \frac{7\sqrt{715}Q_{az}(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144} \\ - \frac{\sqrt{429}Q_{bz}(13x^6 - 45x^4y^2 - 150x^4z^2 - 45x^2y^4 + 540x^2y^2z^2 + 60x^2z^4 + 13y^6 - 150y^4z^2 + 60y^2z^4 - 8z^6)}{1144}$$

$$\tilde{\mathbb{Q}}_3^{(3,-3)}[q](B)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$Q_{az}$$

$$\tilde{\mathbb{Q}}_3^{(3,-1)}[q](B)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-Q_3xy + \frac{\sqrt{10}Q_{3x}xz}{10} + \frac{\sqrt{10}Q_{3y}yz}{10} - \frac{\sqrt{15}Q_{az}(x^2 + y^2 - 2z^2)}{15} - \frac{Q_{bz}(x-y)(x+y)}{2}$$

$$\tilde{\mathbb{Q}}_3^{(3,1)}[q](B)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{7\sqrt{55}Q_1yz(3x^2 - y^2)}{44} - \frac{7\sqrt{55}Q_2xz(x^2 - 3y^2)}{44} + \frac{\sqrt{330}Q_3xy(x^2 + y^2 - 6z^2)}{132} - \frac{5\sqrt{33}Q_{3x}xz(3x^2 + 3y^2 - 4z^2)}{132} \\ - \frac{5\sqrt{33}Q_{3y}yz(3x^2 + 3y^2 - 4z^2)}{132} + \frac{\sqrt{22}Q_{az}(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{88} + \frac{\sqrt{330}Q_{bz}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{264}$$

$$\tilde{\mathbb{Q}}_3^{(3,3)}[q](B)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{7\sqrt{429}Q_1yz(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{2288} - \frac{7\sqrt{429}Q_2xz(x^2 - 3y^2)(3x^2 + 3y^2 - 8z^2)}{2288} \\ + \frac{7\sqrt{715}Q_3xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} + \frac{35\sqrt{286}Q_{3x}xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\ + \frac{35\sqrt{286}Q_{3y}yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\ - \frac{5\sqrt{429}Q_{az}(5x^6 + 15x^4y^2 - 90x^4z^2 + 15x^2y^4 - 180x^2y^2z^2 + 120x^2z^4 + 5y^6 - 90y^4z^2 + 120y^2z^4 - 16z^6)}{3432} \\ + \frac{7\sqrt{715}Q_{bz}(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144}$$

$$\vec{\mathbb{Q}}_{3,1}^{(3,-3)}[q](E,1), \vec{\mathbb{Q}}_{3,2}^{(3,-3)}[q](E,1)$$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$\frac{\sqrt{10}Q_2}{4} - \frac{\sqrt{6}Q_{3x}}{4}$$

$$-\frac{\sqrt{10}Q_1}{4} - \frac{\sqrt{6}Q_{3y}}{4}$$

$$\vec{\mathbb{Q}}_{3,1}^{(3,-3)}[q](E,2), \vec{\mathbb{Q}}_{3,2}^{(3,-3)}[q](E,2)$$

** symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

** expression

$$-\frac{\sqrt{6}Q_1}{4} + \frac{\sqrt{10}Q_{3y}}{4}$$

$$-\frac{\sqrt{6}Q_2}{4} - \frac{\sqrt{10}Q_{3x}}{4}$$

$$\vec{\mathbb{Q}}_{3,1}^{(3,-1)}[q](E,1), \vec{\mathbb{Q}}_{3,2}^{(3,-1)}[q](E,1)$$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$\frac{\sqrt{6}Q_1xy}{8} + \frac{\sqrt{6}Q_2(4x^2 + y^2 - 5z^2)}{24} - Q_3yz - \frac{\sqrt{10}Q_{3x}(4x^2 - 7y^2 + 3z^2)}{40} - \frac{\sqrt{10}Q_{3y}xy}{40} - \frac{\sqrt{15}Q_{az}xz}{20} + \frac{Q_{bz}xz}{4}$$

$$-\frac{\sqrt{6}Q_1(x^2 + 4y^2 - 5z^2)}{24} - \frac{\sqrt{6}Q_2xy}{8} - Q_3xz - \frac{\sqrt{10}Q_{3x}xy}{40} + \frac{\sqrt{10}Q_{3y}(7x^2 - 4y^2 - 3z^2)}{40} - \frac{\sqrt{15}Q_{az}yz}{20} - \frac{Q_{bz}yz}{4}$$

$$\vec{\mathbb{Q}}_{3,1}^{(3,-1)}[q](E,2), \vec{\mathbb{Q}}_{3,2}^{(3,-1)}[q](E,2)$$

** symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

** expression

$$-\frac{\sqrt{10}Q_1(x-z)(x+z)}{8} + \frac{\sqrt{10}Q_2xy}{8} + \frac{3\sqrt{6}Q_{3x}xy}{8} - \frac{\sqrt{6}Q_{3y}(x-z)(x+z)}{8} + \frac{Q_{az}yz}{4} - \frac{\sqrt{15}Q_{bz}yz}{4}$$

$$\frac{\sqrt{10}Q_1xy}{8} - \frac{\sqrt{10}Q_2(y-z)(y+z)}{8} + \frac{\sqrt{6}Q_{3x}(y-z)(y+z)}{8} - \frac{3\sqrt{6}Q_{3y}xy}{8} - \frac{Q_{az}xz}{4} - \frac{\sqrt{15}Q_{bz}xz}{4}$$

$$\vec{\mathbb{Q}}_{3,1}^{(3,1)}[q](E,1), \vec{\mathbb{Q}}_{3,2}^{(3,1)}[q](E,1)$$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{55}Q_1xy(5x^2 - 2y^2 - 9z^2)}{44} + \frac{\sqrt{55}Q_2(4x^4 - 9x^2y^2 - 15x^2z^2 + y^4 + 3y^2z^2 + 2z^4)}{88} \\ & - \frac{\sqrt{330}Q_3yz(6x^2 - y^2 - z^2)}{132} - \frac{\sqrt{33}Q_{3x}(12x^4 - 51x^2y^2 - 21x^2z^2 + 7y^4 + 9y^2z^2 + 2z^4)}{264} \\ & - \frac{5\sqrt{33}Q_{3y}xy(x^2 - 6y^2 + 15z^2)}{132} - \frac{5\sqrt{22}Q_{az}xz(x^2 - 6y^2 + z^2)}{44} + \frac{\sqrt{330}Q_{bz}xz(5x^2 + 12y^2 - 9z^2)}{132} \\ & - \frac{\sqrt{55}Q_1(x^4 - 9x^2y^2 + 3x^2z^2 + 4y^4 - 15y^2z^2 + 2z^4)}{88} + \frac{\sqrt{55}Q_2xy(2x^2 - 5y^2 + 9z^2)}{44} + \frac{\sqrt{330}Q_3xz(x^2 - 6y^2 + z^2)}{132} + \frac{5\sqrt{33}Q_{3x}xy(6x^2 - y^2 - 15z^2)}{132} \\ & - \frac{\sqrt{33}Q_{3y}(7x^4 - 51x^2y^2 + 9x^2z^2 + 12y^4 - 21y^2z^2 + 2z^4)}{264} + \frac{5\sqrt{22}Q_{az}yz(6x^2 - y^2 - z^2)}{44} - \frac{\sqrt{330}Q_{bz}yz(12x^2 + 5y^2 - 9z^2)}{132} \end{aligned}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,1)}[q](E, 2), \vec{\mathbb{Q}}_{3,2}^{(3,1)}[q](E, 2)$

** symmetry

$$-\frac{\sqrt{15}y(x - z)(x + z)}{2}$$

$$\frac{\sqrt{15}x(y - z)(y + z)}{2}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{33}Q_1(x^4 + 57x^2y^2 - 63x^2z^2 - 14y^4 + 27y^2z^2 + 6z^4)}{264} - \frac{5\sqrt{33}Q_2xy(2x^2 - 5y^2 + 9z^2)}{132} - \frac{35\sqrt{22}Q_3xz(x - z)(x + z)}{132} \\ & + \frac{\sqrt{55}Q_{3x}xy(10x^2 - 11y^2 + 3z^2)}{132} + \frac{\sqrt{55}Q_{3y}(5x^4 + 33x^2y^2 - 63x^2z^2 - 14y^4 + 51y^2z^2 + 2z^4)}{264} \\ & + \frac{\sqrt{330}Q_{az}yz(12x^2 - 9y^2 + 5z^2)}{132} + \frac{5\sqrt{22}Q_{bz}yz(6x^2 - y^2 - z^2)}{132} \\ & - \frac{5\sqrt{33}Q_1xy(5x^2 - 2y^2 - 9z^2)}{132} + \frac{\sqrt{33}Q_2(14x^4 - 57x^2y^2 - 27x^2z^2 - y^4 + 63y^2z^2 - 6z^4)}{264} \\ & + \frac{35\sqrt{22}Q_3yz(y - z)(y + z)}{132} + \frac{\sqrt{55}Q_{3x}(14x^4 - 33x^2y^2 - 51x^2z^2 - 5y^4 + 63y^2z^2 - 2z^4)}{264} \\ & + \frac{\sqrt{55}Q_{3y}xy(11x^2 - 10y^2 - 3z^2)}{132} + \frac{\sqrt{330}Q_{az}xz(9x^2 - 12y^2 - 5z^2)}{132} - \frac{5\sqrt{22}Q_{bz}xz(x^2 - 6y^2 + z^2)}{132} \end{aligned}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,3)}[q](E, 1), \vec{\mathbb{Q}}_{3,2}^{(3,3)}[q](E, 1)$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$-\frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$\begin{aligned} & 7\sqrt{4290}Q_1xy(10x^4 - 27x^2y^2 - 19x^2z^2 + 7y^4 + 11y^2z^2 + 4z^4) \\ & + \frac{7\sqrt{4290}Q_2(40x^6 - 468x^4y^2 - 132x^4z^2 + 393x^2y^4 + 450x^2y^2z^2 + 57x^2z^4 - 23y^6 - 48y^4z^2 - 27y^2z^4 - 2z^6)}{6864} \\ & + \frac{7\sqrt{715}Q_3yz(16x^4 - 16x^2y^2 - 16x^2z^2 + y^4 + 2y^2z^2 + z^4)}{572} \\ & - \frac{5\sqrt{286}Q_{3x}(8x^6 - 4x^4y^2 - 116x^4z^2 - 11x^2y^4 + 90x^2y^2z^2 + 101x^2z^4 + y^6 - 4y^4z^2 - 11y^2z^4 - 6z^6)}{2288} \\ & - \frac{35\sqrt{286}Q_{3y}xy(2x^4 + x^2y^2 - 23x^2z^2 - y^4 + 7y^2z^2 + 8z^4)}{2288} - \frac{35\sqrt{429}Q_{az}xz(2x^4 + x^2y^2 - 7x^2z^2 - y^4 + y^2z^2 + 2z^4)}{1144} \\ & + \frac{7\sqrt{715}Q_{bz}xz(10x^4 - 43x^2y^2 - 19x^2z^2 + 13y^4 + 17y^2z^2 + 4z^4)}{1144} \end{aligned}$$

$$\begin{aligned} & \sqrt{4290}Q_1(23x^6 - 393x^4y^2 + 48x^4z^2 + 468x^2y^4 - 450x^2y^2z^2 + 27x^2z^4 - 40y^6 + 132y^4z^2 - 57y^2z^4 + 2z^6) \\ & - \frac{7\sqrt{4290}Q_2xy(7x^4 - 27x^2y^2 + 11x^2z^2 + 10y^4 - 19y^2z^2 + 4z^4)}{2288} \\ & + \frac{7\sqrt{715}Q_3xz(x^4 - 16x^2y^2 + 2x^2z^2 + 16y^4 - 16y^2z^2 + z^4)}{572} + \frac{35\sqrt{286}Q_{3x}xy(x^4 - x^2y^2 - 7x^2z^2 - 2y^4 + 23y^2z^2 - 8z^4)}{2288} \\ & - \frac{5\sqrt{286}Q_{3y}(x^6 - 11x^4y^2 - 4x^4z^2 - 4x^2y^4 + 90x^2y^2z^2 - 11x^2z^4 + 8y^6 - 116y^4z^2 + 101y^2z^4 - 6z^6)}{2288} \\ & + \frac{35\sqrt{429}Q_{az}yz(x^4 - x^2y^2 - x^2z^2 - 2y^4 + 7y^2z^2 - 2z^4)}{1144} - \frac{7\sqrt{715}Q_{bz}yz(13x^4 - 43x^2y^2 + 17x^2z^2 + 10y^4 - 19y^2z^2 + 4z^4)}{1144} \end{aligned}$$

$$\vec{\mathbb{Q}}_{3,1}^{(3,3)}[q](E,2), \vec{\mathbb{Q}}_{3,2}^{(3,3)}[q](E,2)$$

** symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{286}Q_1(37x^6 - 505x^4y^2 - 50x^4z^2 + 370x^2y^4 + 810x^2y^2z^2 - 85x^2z^4 - 12y^6 - 190y^4z^2 + 55y^2z^4 + 2z^6)}{2288} \\ & - \frac{7\sqrt{286}Q_2xy(31x^4 - 85x^2y^2 - 55x^2z^2 + 16y^4 + 95y^2z^2 - 20z^4)}{2288} \\ & + \frac{21\sqrt{429}Q_3xz(x-z)(x+z)(x^2 - 10y^2 + z^2)}{572} + \frac{21\sqrt{4290}Q_{3x}xy(x^4 + x^2y^2 - 13x^2z^2 - 3y^2z^2 + 8z^4)}{2288} \\ & - \frac{\sqrt{4290}Q_{3y}(3x^6 - 19x^4y^2 - 26x^4z^2 - 26x^2y^4 + 270x^2y^2z^2 - 19x^2z^4 - 4y^6 + 86y^4z^2 - 131y^2z^4 + 10z^6)}{2288} \\ & + \frac{7\sqrt{715}Q_{az}yz(13x^4 + 17x^2y^2 - 43x^2z^2 + 4y^4 - 19y^2z^2 + 10z^4)}{1144} - \frac{7\sqrt{429}Q_{bz}yz(35x^4 - 35x^2y^2 - 35x^2z^2 - 4y^4 + 25y^2z^2 - 4z^4)}{1144} \\ & - \frac{7\sqrt{286}Q_1xy(16x^4 - 85x^2y^2 + 95x^2z^2 + 31y^4 - 55y^2z^2 - 20z^4)}{2288} \\ & - \frac{\sqrt{286}Q_2(12x^6 - 370x^4y^2 + 190x^4z^2 + 505x^2y^4 - 810x^2y^2z^2 - 55x^2z^4 - 37y^6 + 50y^4z^2 + 85y^2z^4 - 2z^6)}{2288} \\ & + \frac{21\sqrt{429}Q_3yz(y-z)(y+z)(10x^2 - y^2 - z^2)}{572} \\ & - \frac{\sqrt{4290}Q_{3x}(4x^6 + 26x^4y^2 - 86x^4z^2 + 19x^2y^4 - 270x^2y^2z^2 + 131x^2z^4 - 3y^6 + 26y^4z^2 + 19y^2z^4 - 10z^6)}{2288} \\ & - \frac{21\sqrt{4290}Q_{3y}xy(x^2y^2 - 3x^2z^2 + y^4 - 13y^2z^2 + 8z^4)}{2288} - \frac{7\sqrt{715}Q_{az}xz(4x^4 + 17x^2y^2 - 19x^2z^2 + 13y^4 - 43y^2z^2 + 10z^4)}{1144} \\ & + \frac{7\sqrt{429}Q_{bz}xz(4x^4 + 35x^2y^2 - 25x^2z^2 - 35y^4 + 35y^2z^2 + 4z^4)}{1144} \end{aligned}$$

* Harmonics for rank 4

$$\vec{\mathbb{Q}}_4^{(3,-3)}[q](A,1)$$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$-\frac{\sqrt{30}Q_1y}{12} + \frac{\sqrt{30}Q_2x}{12} - \frac{\sqrt{2}Q_{3x}x}{4} - \frac{\sqrt{2}Q_{3y}y}{4} + \frac{\sqrt{3}Q_{az}z}{3}$$

$$\vec{\mathbb{Q}}_4^{(3,-3)}[q](A,2)$$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\frac{\sqrt{42}Q_1y}{12} - \frac{\sqrt{42}Q_2x}{12} - \frac{\sqrt{70}Q_{3x}x}{28} - \frac{\sqrt{70}Q_{3y}y}{28} + \frac{\sqrt{105}Q_{az}z}{21}$$

$$\vec{\mathbb{Q}}_4^{(3,-3)}[q](A,3)$$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}Q_1x}{2} + \frac{\sqrt{2}Q_2y}{2}$$

$$\vec{\mathbb{Q}}_4^{(3,-1)}[q](A,1)$$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\begin{aligned} & \frac{\sqrt{165}Q_{1y}(x^2 - y^2 + 2z^2)}{44} + \frac{\sqrt{165}Q_{2x}(x^2 - y^2 - 2z^2)}{44} - \frac{3\sqrt{110}Q_3xyz}{11} - \frac{\sqrt{11}Q_{3xx}(3x^2 - 7y^2 - 2z^2)}{44} \\ & + \frac{\sqrt{11}Q_{3yy}(7x^2 - 3y^2 + 2z^2)}{44} - \frac{\sqrt{66}Q_{azz}(3x^2 + 3y^2 - 2z^2)}{44} - \frac{\sqrt{110}Q_{bz}(x-y)(x+y)}{44} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,-1)}[q](A, 2)$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\begin{aligned} & \frac{\sqrt{231}Q_{1y}(11x^2 + y^2 - 14z^2)}{308} - \frac{\sqrt{231}Q_{2x}(x^2 + 11y^2 - 14z^2)}{308} + \frac{\sqrt{385}Q_{3xx}(3x^2 - 11y^2 + 2z^2)}{308} \\ & - \frac{\sqrt{385}Q_{3yy}(11x^2 - 3y^2 - 2z^2)}{308} - \frac{\sqrt{2310}Q_{azz}(3x^2 + 3y^2 - 2z^2)}{308} - \frac{5\sqrt{154}Q_{bz}(x-y)(x+y)}{44} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,-1)}[q](A, 3)$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{3\sqrt{11}Q_{1x}(x^2 + y^2 - 4z^2)}{44} + \frac{3\sqrt{11}Q_{2y}(x^2 + y^2 - 4z^2)}{44} + \frac{5\sqrt{66}Q_{3z}(x-y)(x+y)}{44} - \frac{\sqrt{165}Q_{3xy}(3x^2 - y^2)}{44} - \frac{\sqrt{165}Q_{3yx}(x^2 - 3y^2)}{44} + \frac{5\sqrt{66}Q_{bz}xyz}{22}$$

$\tilde{\mathbb{Q}}_4^{(3,1)}[q](A, 1)$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\begin{aligned} & \frac{5\sqrt{858}Q_{1y}(5x^4 + 3x^2y^2 - 39x^2z^2 - 2y^4 + 17y^2z^2 - 2z^4)}{1144} + \frac{5\sqrt{858}Q_{2x}(2x^4 - 3x^2y^2 - 17x^2z^2 - 5y^4 + 39y^2z^2 + 2z^4)}{1144} \\ & - \frac{\sqrt{1430}Q_{3xx}(6x^4 - 65x^2y^2 + 5x^2z^2 + 13y^4 + 117y^2z^2 - 22z^4)}{1144} - \frac{\sqrt{1430}Q_{3yy}(13x^4 - 65x^2y^2 + 117x^2z^2 + 6y^4 + 5y^2z^2 - 22z^4)}{1144} \\ & - \frac{\sqrt{2145}Q_{azz}(3x^4 - 78x^2y^2 + 20x^2z^2 + 3y^4 + 20y^2z^2 - 4z^4)}{572} + \frac{35\sqrt{143}Q_{bz}(x-y)(x+y)(x^2 + y^2 - 2z^2)}{572} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,1)}[q](A, 2)$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\begin{aligned} & \frac{\sqrt{30030}Q_{1y}(4x^4 + 3x^2y^2 - 33x^2z^2 - y^4 + 7y^2z^2 + 2z^4)}{1144} + \frac{\sqrt{30030}Q_{2x}(x^4 - 3x^2y^2 - 7x^2z^2 - 4y^4 + 33y^2z^2 - 2z^4)}{1144} + \frac{3\sqrt{5005}Q_{3xy}(x^2 + y^2 - 2z^2)}{143} \\ & + \frac{\sqrt{2002}Q_{3xx}(75x^4 - 389x^2y^2 - 361x^2z^2 + 124y^4 + 423y^2z^2 + 110z^4)}{8008} + \frac{\sqrt{2002}Q_{3yy}(124x^4 - 389x^2y^2 + 423x^2z^2 + 75y^4 - 361y^2z^2 + 110z^4)}{8008} \\ & + \frac{\sqrt{3003}Q_{azz}(111x^4 - 366x^2y^2 - 100x^2z^2 + 111y^4 - 100y^2z^2 + 20z^4)}{4004} - \frac{\sqrt{5005}Q_{bz}(x-y)(x+y)(x^2 + y^2 - 2z^2)}{572} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,1)}[q](A, 3)$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{1430}Q_{1x}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} + \frac{3\sqrt{1430}Q_{2y}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} \\ & + \frac{7\sqrt{2145}Q_{3z}(x-y)(x+y)(x^2 + y^2 - 2z^2)}{572} - \frac{7\sqrt{858}Q_{3xy}(21x^4 - 26x^2y^2 - 48x^2z^2 + y^4 + 16y^2z^2)}{2288} \\ & + \frac{7\sqrt{858}Q_{3yy}(x^4 - 26x^2y^2 + 16x^2z^2 + 21y^4 - 48y^2z^2)}{2288} - \frac{63\sqrt{143}Q_{azxyz}(x-y)(x+y)}{143} + \frac{7\sqrt{2145}Q_{bzxyz}(x^2 + y^2 - 2z^2)}{286} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,3)}[q](A, 1)$

** symmetry

$$\frac{\sqrt{21} (x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\begin{aligned} & \frac{\sqrt{858}Q_1y (211x^6 - 900x^4y^2 - 465x^4z^2 + 570x^2y^4 - 300x^2y^2z^2 + 615x^2z^4 - 35y^6 + 165y^4z^2 - 225y^2z^4 + 4z^6)}{3432} \\ & + \frac{\sqrt{858}Q_2x (35x^6 - 570x^4y^2 - 165x^4z^2 + 900x^2y^4 + 300x^2y^2z^2 + 225x^2z^4 - 211y^6 + 465y^4z^2 - 615y^2z^4 - 4z^6)}{3432} \\ & + \frac{3\sqrt{143}Q_3xyz (3x^4 - 5x^2y^2 - 5x^2z^2 + 3y^4 - 5y^2z^2 + 3z^4)}{22} \\ & - \frac{\sqrt{1430}Q_{3xx}(7x^6 - 6x^4y^2 - 141x^4z^2 + 60x^2y^2z^2 + 225x^2z^4 + 13y^6 - 195y^4z^2 + 165y^2z^4 - 56z^6)}{1144} \\ & - \frac{\sqrt{1430}Q_{3yy}(13x^6 - 195x^4z^2 - 6x^2y^4 + 60x^2y^2z^2 + 165x^2z^4 + 7y^6 - 141y^4z^2 + 225y^2z^4 - 56z^6)}{1144} \\ & - \frac{\sqrt{2145}Q_{azz}(43x^6 + 30x^4y^2 - 225x^4z^2 + 30x^2y^4 - 120x^2y^2z^2 + 147x^2z^4 + 43y^6 - 225y^4z^2 + 147y^2z^4 - 14z^6)}{1716} \\ & + \frac{3\sqrt{143}Q_{bzz}(x-y)(x+y)(23x^4 - 97x^2y^2 - 75x^2z^2 + 23y^4 - 75y^2z^2 + 45z^4)}{572} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,3)}[q](A, 2)$

** symmetry

$$-\frac{\sqrt{15} (x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\begin{aligned} & - \frac{\sqrt{30030}Q_1y (341x^6 - 1935x^4y^2 + 690x^4z^2 + 1113x^2y^4 + 480x^2y^2z^2 - 930x^2z^4 - 43y^6 - 210y^4z^2 + 270y^2z^4 + 8z^6)}{34320} \\ & - \frac{\sqrt{30030}Q_2x (43x^6 - 1113x^4y^2 + 210x^4z^2 + 1935x^2y^4 - 480x^2y^2z^2 - 270x^2z^4 - 341y^6 - 690y^4z^2 + 930y^2z^4 - 8z^6)}{34320} \\ & - \frac{3\sqrt{5005}Q_3xyz (3x^4 - 20x^2y^2 + 10x^2z^2 + 3y^4 + 10y^2z^2 - 6z^4)}{260} \\ & - \frac{\sqrt{2002}Q_{3xx}(x^6 + 57x^4y^2 - 78x^4z^2 + 45x^2y^4 - 840x^2y^2z^2 + 270x^2z^4 - 11y^6 + 30y^4z^2 + 390y^2z^4 - 80z^6)}{2288} \\ & + \frac{\sqrt{2002}Q_{3yy}(11x^6 - 45x^4y^2 - 30x^4z^2 - 57x^2y^4 + 840x^2y^2z^2 - 390x^2z^4 - y^6 + 78y^4z^2 - 270y^2z^4 + 80z^6)}{2288} \\ & - \frac{\sqrt{3003}Q_{azz}(19x^6 + 255x^4y^2 - 180x^4z^2 + 255x^2y^4 - 1020x^2y^2z^2 + 210x^2z^4 + 19y^6 - 180y^4z^2 + 210y^2z^4 - 20z^6)}{3432} \\ & - \frac{3\sqrt{5005}Q_{bzz}(x-y)(x+y)(x^4 - 284x^2y^2 + 90x^2z^2 + y^4 + 90y^2z^2 - 54z^4)}{5720} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,3)}[q](A, 3)$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & - \frac{\sqrt{1430}Q_1x (53x^6 - 1128x^4y^2 + 15x^4z^2 + 1875x^2y^4 + 30x^2y^2z^2 - 30x^2z^4 - 376y^6 + 15y^4z^2 - 30y^2z^4 + 8z^6)}{5720} \\ & + \frac{\sqrt{1430}Q_2y (376x^6 - 1875x^4y^2 - 15x^4z^2 + 1128x^2y^4 - 30x^2y^2z^2 + 30x^2z^4 - 53y^6 - 15y^4z^2 + 30y^2z^4 - 8z^6)}{5720} \\ & - \frac{3\sqrt{2145}Q_3z(x-y)(x+y)(8x^4 - 127x^2y^2 + 5x^2z^2 + 8y^4 + 5y^2z^2 - 3z^4)}{1430} \\ & - \frac{3\sqrt{858}Q_{3xy}(8x^6 - 5x^4y^2 - 105x^4z^2 - 12x^2y^4 + 150x^2y^2z^2 + 30x^2z^4 + y^6 - 9y^4z^2 - 10y^2z^4)}{1144} \\ & + \frac{3\sqrt{858}Q_{3yx}(x^6 - 12x^4y^2 - 9x^4z^2 - 5x^2y^4 + 150x^2y^2z^2 - 10x^2z^4 + 8y^6 - 105y^4z^2 + 30y^2z^4)}{1144} \\ & - \frac{3\sqrt{143}Q_{azxyz}(x-y)(x+y)(3x^2 + 3y^2 - 10z^2)}{52} + \frac{3\sqrt{2145}Q_{bzyxyz}(111x^4 - 350x^2y^2 - 20x^2z^2 + 111y^4 - 20y^2z^2 + 12z^4)}{2860} \end{aligned}$$

$\tilde{\mathbb{Q}}_4^{(3,-3)}[q](B, 1)$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\frac{\sqrt{14}Q_1y}{28} + \frac{\sqrt{14}Q_2x}{28} - \frac{\sqrt{210}Q_{3xx}}{28} + \frac{\sqrt{210}Q_{3yy}}{28} - \frac{\sqrt{21}Q_{bzz}}{7}$$

$$\tilde{\mathbb{Q}}_4^{(3,-3)}[q](B, 2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$-\frac{\sqrt{14}Q_1x}{28} + \frac{\sqrt{14}Q_2y}{28} + \frac{\sqrt{21}Q_3z}{7} + \frac{\sqrt{210}Q_{3xy}}{28} + \frac{\sqrt{210}Q_{3y}x}{28}$$

$$\tilde{\mathbb{Q}}_4^{(3,-1)}[q](B, 1)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{9\sqrt{77}Q_1y(x^2 + y^2 - 4z^2)}{308} + \frac{9\sqrt{77}Q_2x(x^2 + y^2 - 4z^2)}{308} - \frac{\sqrt{1155}Q_{3x}x(x^2 - 11y^2 + 8z^2)}{308} \\ & - \frac{\sqrt{1155}Q_{3y}y(11x^2 - y^2 - 8z^2)}{308} + \frac{3\sqrt{770}Q_{az}z(x-y)(x+y)}{308} - \frac{\sqrt{462}Q_{bz}z(3x^2 + 3y^2 - 2z^2)}{308} \end{aligned}$$

$$\tilde{\mathbb{Q}}_4^{(3,-1)}[q](B, 2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\begin{aligned} & -\frac{9\sqrt{77}Q_1x(x^2 + y^2 - 4z^2)}{308} + \frac{9\sqrt{77}Q_2y(x^2 + y^2 - 4z^2)}{308} + \frac{\sqrt{462}Q_3z(3x^2 + 3y^2 - 2z^2)}{308} \\ & + \frac{\sqrt{1155}Q_{3xy}(7x^2 - 5y^2 + 8z^2)}{308} - \frac{\sqrt{1155}Q_{3y}x(5x^2 - 7y^2 - 8z^2)}{308} - \frac{3\sqrt{770}Q_{az}xyz}{154} \end{aligned}$$

$$\tilde{\mathbb{Q}}_4^{(3,1)}[q](B, 1)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{9\sqrt{10010}Q_1y(10x^4 - 15x^2y^2 - 15x^2z^2 + 3y^4 - 15y^2z^2 + 10z^4)}{8008} + \frac{9\sqrt{10010}Q_2x(3x^4 - 15x^2y^2 - 15x^2z^2 + 10y^4 - 15y^2z^2 + 10z^4)}{8008} \\ & - \frac{3\sqrt{15015}Q_3xyz(x-y)(x+y)}{143} + \frac{\sqrt{6006}Q_{3x}x(5x^4 - 53x^2y^2 + 3x^2z^2 - 58y^4 + 507y^2z^2 - 86z^4)}{8008} \\ & + \frac{\sqrt{6006}Q_{3y}y(58x^4 + 53x^2y^2 - 507x^2z^2 - 5y^4 - 3y^2z^2 + 86z^4)}{8008} + \frac{3\sqrt{1001}Q_{az}z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{572} \\ & + \frac{\sqrt{15015}Q_{bz}z(9x^4 + 186x^2y^2 - 80x^2z^2 + 9y^4 - 80y^2z^2 + 16z^4)}{4004} \end{aligned}$$

$$\tilde{\mathbb{Q}}_4^{(3,1)}[q](B, 2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{9\sqrt{10010}Q_1x(x^4 - 40x^2y^2 + 30x^2z^2 + 15y^4 + 30y^2z^2 - 20z^4)}{16016} - \frac{9\sqrt{10010}Q_2y(15x^4 - 40x^2y^2 + 30x^2z^2 + y^4 + 30y^2z^2 - 20z^4)}{16016} \\ & - \frac{\sqrt{15015}Q_3z(51x^4 - 66x^2y^2 - 80x^2z^2 + 51y^4 - 80y^2z^2 + 16z^4)}{4004} - \frac{\sqrt{6006}Q_{3x}y(73x^4 + 20x^2y^2 - 498x^2z^2 - 53y^4 + 510y^2z^2 - 172z^4)}{16016} \\ & + \frac{\sqrt{6006}Q_{3y}x(53x^4 - 20x^2y^2 - 510x^2z^2 - 73y^4 + 498y^2z^2 + 172z^4)}{16016} - \frac{3\sqrt{1001}Q_{az}xyz(x^2 + y^2 - 2z^2)}{286} + \frac{3\sqrt{15015}Q_{bz}xyz(x-y)(x+y)}{143} \end{aligned}$$

$$\tilde{\mathbb{Q}}_4^{(3,3)}[q](B, 1)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$\begin{aligned}
& \frac{\sqrt{10010}Q_{1y}(85x^6 - 75x^4y^2 - 1050x^4z^2 - 141x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 19y^6 - 258y^4z^2 + 120y^2z^4 - 32z^6)}{11440} \\
& + \frac{\sqrt{10010}Q_{2x}(19x^6 - 141x^4y^2 - 258x^4z^2 - 75x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 85y^6 - 1050y^4z^2 + 120y^2z^4 - 32z^6)}{11440} \\
& + \frac{3\sqrt{15015}Q_{3xyz}(x-y)(x+y)(3x^2+3y^2-10z^2)}{260} \\
& - \frac{\sqrt{6006}Q_{3xx}(7x^6+3x^4y^2-150x^4z^2-15x^2y^4+60x^2y^2z^2+240x^2z^4-11y^6+210y^4z^2-240y^2z^4-32z^6)}{2288} \\
& - \frac{\sqrt{6006}Q_{3yy}(11x^6+15x^4y^2-210x^4z^2-3x^2y^4-60x^2y^2z^2+240x^2z^4-7y^6+150y^4z^2-240y^2z^4+32z^6)}{2288} \\
& - \frac{3\sqrt{1001}Q_{azz}(x-y)(x+y)(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4)}{1144} \\
& + \frac{\sqrt{15015}Q_{bzz}(67x^6-195x^4y^2-270x^4z^2-195x^2y^4+780x^2y^2z^2+84x^2z^4+67y^6-270y^4z^2+84y^2z^4-8z^6)}{5720}
\end{aligned}$$

$\vec{\mathbb{Q}}_4^{(3,3)}[q](B, 2)$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$\begin{aligned}
& \frac{\sqrt{10010}Q_{1x}(7x^6-78x^4y^2-69x^4z^2-45x^2y^4+1050x^2y^2z^2-60x^2z^4+40y^6-465y^4z^2-60y^2z^4+16z^6)}{5720} \\
& - \frac{\sqrt{10010}Q_{2y}(40x^6-45x^4y^2-465x^4z^2-78x^2y^4+1050x^2y^2z^2-60x^2z^4+7y^6-69y^4z^2-60y^2z^4+16z^6)}{5720} \\
& + \frac{\sqrt{15015}Q_{3zz}(8x^6-75x^4y^2-15x^4z^2-75x^2y^4+300x^2y^2z^2-21x^2z^4+8y^6-15y^4z^2-21y^2z^4+2z^6)}{1430} \\
& + \frac{\sqrt{6006}Q_{3xy}(8x^6+15x^4y^2-165x^4z^2+6x^2y^4-150x^2y^2z^2+240x^2z^4-y^6+15y^4z^2-16z^6)}{1144} \\
& - \frac{\sqrt{6006}Q_{3yx}(x^6-6x^4y^2-15x^4z^2-15x^2y^4+150x^2y^2z^2-8y^6+165y^4z^2-240y^2z^4+16z^6)}{1144} \\
& + \frac{3\sqrt{1001}Q_{azxyz}(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4)}{572} - \frac{3\sqrt{15015}Q_{bzyxyz}(x-y)(x+y)(3x^2+3y^2-10z^2)}{260}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,-3)}[q](E, 1), \vec{\mathbb{Q}}_{4,2}^{(3,-3)}[q](E, 1)$

** symmetry

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$\begin{aligned}
& -\frac{\sqrt{2}Q_{2z}}{8} + \frac{\sqrt{30}Q_{3xz}}{8} + \frac{\sqrt{5}Q_{azx}}{4} - \frac{\sqrt{3}Q_{bzx}}{4} \\
& -\frac{\sqrt{2}Q_{1z}}{8} - \frac{\sqrt{30}Q_{3yz}}{8} - \frac{\sqrt{5}Q_{azy}}{4} - \frac{\sqrt{3}Q_{bzy}}{4}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,-3)}[q](E, 2), \vec{\mathbb{Q}}_{4,2}^{(3,-3)}[q](E, 2)$

** symmetry

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

** expression

$$\begin{aligned}
& \frac{\sqrt{14}Q_{1z}}{8} + \frac{\sqrt{21}Q_{3x}}{7} - \frac{\sqrt{210}Q_{3yz}}{56} - \frac{\sqrt{35}Q_{azy}}{28} + \frac{3\sqrt{21}Q_{bzy}}{28} \\
& - \frac{\sqrt{14}Q_{2z}}{8} + \frac{\sqrt{21}Q_{3y}}{7} - \frac{\sqrt{210}Q_{3xz}}{56} - \frac{\sqrt{35}Q_{azx}}{28} - \frac{3\sqrt{21}Q_{bzx}}{28}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,-1)}[q](E, 1), \vec{\mathbb{Q}}_{4,2}^{(3,-1)}[q](E, 1)$

** symmetry

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$-\frac{15\sqrt{11}Q_1xyz}{44} - \frac{3\sqrt{11}Q_2z(4x^2-y^2-z^2)}{44} - \frac{5\sqrt{66}Q_3y(x-z)(x+z)}{44} - \frac{\sqrt{165}Q_{3x}z(3y^2-z^2)}{44} \\ + \frac{5\sqrt{165}Q_{3y}xyz}{44} - \frac{3\sqrt{110}Q_{az}x(y-z)(y+z)}{44} - \frac{\sqrt{66}Q_{bz}x(2x^2-3y^2-3z^2)}{44}$$

$$\frac{3\sqrt{11}Q_1z(x^2-4y^2+z^2)}{44} - \frac{15\sqrt{11}Q_2xyz}{44} + \frac{5\sqrt{66}Q_3x(y-z)(y+z)}{44} - \frac{5\sqrt{165}Q_{3x}xyz}{44} \\ + \frac{\sqrt{165}Q_{3y}z(3x^2-z^2)}{44} + \frac{3\sqrt{110}Q_{az}y(x-z)(x+z)}{44} + \frac{\sqrt{66}Q_{bz}y(3x^2-2y^2+3z^2)}{44}$$

$$\vec{\mathbb{Q}}_{4,1}^{(3,-1)}[q](E,2), \vec{\mathbb{Q}}_{4,2}^{(3,-1)}[q](E,2)$$

** symmetry

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

** expression

$$\frac{3\sqrt{77}Q_1z(13x^2+8y^2-7z^2)}{308} - \frac{15\sqrt{77}Q_2xyz}{308} - \frac{\sqrt{462}Q_3x(2x^2-3y^2-3z^2)}{308} + \frac{3\sqrt{1155}Q_{3x}xyz}{308} \\ + \frac{\sqrt{1155}Q_{3y}z(7x^2-4y^2-z^2)}{308} - \frac{3\sqrt{770}Q_{az}y(5x^2-2y^2+z^2)}{308} + \frac{\sqrt{462}Q_{bz}y(x^2-4y^2+11z^2)}{308}$$

$$\frac{15\sqrt{77}Q_1xyz}{308} - \frac{3\sqrt{77}Q_2z(8x^2+13y^2-7z^2)}{308} + \frac{\sqrt{462}Q_3y(3x^2-2y^2+3z^2)}{308} - \frac{\sqrt{1155}Q_{3x}z(4x^2-7y^2+z^2)}{308} \\ + \frac{3\sqrt{1155}Q_{3y}xyz}{308} + \frac{3\sqrt{770}Q_{az}x(2x^2-5y^2-z^2)}{308} + \frac{\sqrt{462}Q_{bz}x(4x^2-y^2-11z^2)}{308}$$

$$\vec{\mathbb{Q}}_{4,1}^{(3,1)}[q](E,1), \vec{\mathbb{Q}}_{4,2}^{(3,1)}[q](E,1)$$

** symmetry

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$\frac{21\sqrt{1430}Q_1xyz(7x^2-2y^2-5z^2)}{1144} - \frac{3\sqrt{1430}Q_2z(37x^4-87x^2y^2-45x^2z^2+2y^4+25y^2z^2+2z^4)}{2288} - \frac{7\sqrt{2145}Q_3y(x-z)(x+z)(x^2-2y^2+z^2)}{572} \\ - \frac{\sqrt{858}Q_{3x}z(33x^4-39x^2y^2-53x^2z^2-30y^4+73y^2z^2-2z^4)}{2288} - \frac{7\sqrt{858}Q_{3y}xyz(13x^2+10y^2-23z^2)}{1144} \\ + \frac{3\sqrt{143}Q_{az}x(3x^4+13x^2y^2-43x^2z^2+10y^4-99y^2z^2+38z^4)}{1144} + \frac{\sqrt{2145}Q_{bz}x(x^4+37x^2y^2-47x^2z^2-6y^4-75y^2z^2+36z^4)}{1144}$$

$$-\frac{3\sqrt{1430}Q_1z(2x^4-87x^2y^2+25x^2z^2+37y^4-45y^2z^2+2z^4)}{2288} + \frac{21\sqrt{1430}Q_2xyz(2x^2-7y^2+5z^2)}{1144} - \frac{7\sqrt{2145}Q_3x(y-z)(y+z)(2x^2-y^2-z^2)}{572} \\ + \frac{7\sqrt{858}Q_{3x}xyz(10x^2+13y^2-23z^2)}{1144} - \frac{\sqrt{858}Q_{3y}z(30x^4+39x^2y^2-73x^2z^2-33y^4+53y^2z^2+2z^4)}{2288} \\ - \frac{3\sqrt{143}Q_{az}y(10x^4+13x^2y^2-99x^2z^2+3y^4-43y^2z^2+38z^4)}{1144} - \frac{\sqrt{2145}Q_{bz}y(6x^4-37x^2y^2+75x^2z^2-y^4+47y^2z^2-36z^4)}{1144}$$

$$\vec{\mathbb{Q}}_{4,1}^{(3,1)}[q](E,2), \vec{\mathbb{Q}}_{4,2}^{(3,1)}[q](E,2)$$

** symmetry

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{10010}Q_1z(4x^4 + 21x^2y^2 - 15x^2z^2 - y^4 - 5y^2z^2 + 2z^4)}{2288} + \frac{3\sqrt{10010}Q_2xyz(2x^2 - 7y^2 + 5z^2)}{1144} \\
& - \frac{\sqrt{15015}Q_3x(16x^4 - 80x^2y^2 - 80x^2z^2 + 51y^4 - 66y^2z^2 + 51z^4)}{4004} - \frac{3\sqrt{6006}Q_{3x}xyz(2x^2 + 9y^2 - 11z^2)}{1144} \\
& - \frac{\sqrt{6006}Q_{3y}z(268x^4 - 213x^2y^2 - 465x^2z^2 - 187y^4 + 445y^2z^2 + 2z^4)}{16016} \\
& + \frac{3\sqrt{1001}Q_{az}y(32x^4 + 15x^2y^2 - 237x^2z^2 - 17y^4 + 155y^2z^2 - 38z^4)}{8008} + \frac{\sqrt{15015}Q_{bz}y(88x^4 - 195x^2y^2 + 57x^2z^2 + 11y^4 + 85y^2z^2 - 52z^4)}{8008}
\end{aligned}$$

$$\begin{aligned}
& \frac{3\sqrt{10010}Q_1xyz(7x^2 - 2y^2 - 5z^2)}{1144} + \frac{3\sqrt{10010}Q_2z(x^4 - 21x^2y^2 + 5x^2z^2 - 4y^4 + 15y^2z^2 - 2z^4)}{2288} \\
& - \frac{\sqrt{15015}Q_3y(51x^4 - 80x^2y^2 - 66x^2z^2 + 16y^4 - 80y^2z^2 + 51z^4)}{4004} + \frac{\sqrt{6006}Q_{3x}z(187x^4 + 213x^2y^2 - 445x^2z^2 - 268y^4 + 465y^2z^2 - 2z^4)}{16016} \\
& - \frac{3\sqrt{6006}Q_{3y}xyz(9x^2 + 2y^2 - 11z^2)}{1144} - \frac{3\sqrt{1001}Q_{az}x(17x^4 - 15x^2y^2 - 155x^2z^2 - 32y^4 + 237y^2z^2 + 38z^4)}{8008} \\
& - \frac{\sqrt{15015}Q_{bz}x(11x^4 - 195x^2y^2 + 85x^2z^2 + 88y^4 + 57y^2z^2 - 52z^4)}{8008}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,3)}[q](E, 1), \vec{\mathbb{Q}}_{4,2}^{(3,3)}[q](E, 1)$

** symmetry

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{1430}Q_1xyz(249x^4 - 305x^2y^2 - 525x^2z^2 + 18y^4 + 245y^2z^2 + 84z^4)}{5720} \\
& - \frac{\sqrt{1430}Q_2z(184x^6 - 1185x^4y^2 - 525x^4z^2 + 345x^2y^4 + 1680x^2y^2z^2 + 147x^2z^4 - 2y^6 - 105y^4z^2 - 105y^2z^4 - 2z^6)}{5720} \\
& + \frac{3\sqrt{2145}Q_3y(x-z)(x+z)(8x^4 + 5x^2y^2 - 127x^2z^2 - 3y^4 + 5y^2z^2 + 8z^4)}{1430} \\
& + \frac{\sqrt{858}Q_{3x}z(88x^6 + 75x^4y^2 - 465x^4z^2 - 15x^2y^4 - 120x^2y^2z^2 + 291x^2z^4 - 2y^6 + 15y^4z^2 + 3y^2z^4 - 14z^6)}{1144} \\
& + \frac{3\sqrt{858}Q_{3y}xyz(39x^4 + 45x^2y^2 - 175x^2z^2 + 6y^4 - 65y^2z^2 + 72z^4)}{2860} \\
& - \frac{\sqrt{143}Q_{az}x(11x^6 + 24x^4y^2 - 255x^4z^2 + 15x^2y^4 - 330x^2y^2z^2 + 480x^2z^4 + 2y^6 - 75y^4z^2 + 240y^2z^4 - 112z^6)}{572} \\
& + \frac{\sqrt{2145}Q_{bz}x(17x^6 - 30x^4y^2 - 327x^4z^2 - 45x^2y^4 + 570x^2y^2z^2 + 450x^2z^4 + 2y^6 + 105y^4z^2 - 390y^2z^4 - 64z^6)}{2860}
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{1430}Q_1z(2x^6 - 345x^4y^2 + 105x^4z^2 + 1185x^2y^4 - 1680x^2y^2z^2 + 105x^2z^4 - 184y^6 + 525y^4z^2 - 147y^2z^4 + 2z^6)}{5720} \\
& - \frac{3\sqrt{1430}Q_2xyz(18x^4 - 305x^2y^2 + 245x^2z^2 + 249y^4 - 525y^2z^2 + 84z^4)}{5720}
\end{aligned}$$

$$\begin{aligned}
& + \frac{3\sqrt{2145}Q_3x(y-z)(y+z)(3x^4 - 5x^2y^2 - 5x^2z^2 - 8y^4 + 127y^2z^2 - 8z^4)}{1430} - \frac{3\sqrt{858}Q_{3x}xyz(6x^4 + 45x^2y^2 - 65x^2z^2 + 39y^4 - 175y^2z^2 + 72z^4)}{1144} \\
& + \frac{\sqrt{858}Q_{3y}z(2x^6 + 15x^4y^2 - 15x^4z^2 - 75x^2y^4 + 120x^2y^2z^2 - 3x^2z^4 - 88y^6 + 465y^4z^2 - 291y^2z^4 + 14z^6)}{1144} \\
& + \frac{\sqrt{143}Q_{az}y(2x^6 + 15x^4y^2 - 75x^4z^2 + 24x^2y^4 - 330x^2y^2z^2 + 240x^2z^4 + 11y^6 - 255y^4z^2 + 480y^2z^4 - 112z^6)}{572} \\
& + \frac{\sqrt{2145}Q_{bz}y(2x^6 - 45x^4y^2 + 105x^4z^2 - 30x^2y^4 + 570x^2y^2z^2 - 390x^2z^4 + 17y^6 - 327y^4z^2 + 450y^2z^4 - 64z^6)}{2860}
\end{aligned}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,3)}[q](E, 2), \vec{\mathbb{Q}}_{4,2}^{(3,3)}[q](E, 2)$

** symmetry

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

$$-\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

** expression

$$\begin{aligned}
& - \frac{\sqrt{10010}Q_1z(56x^6 - 885x^4y^2 + 15x^4z^2 + 735x^2y^4 + 300x^2y^2z^2 - 39x^2z^4 - 40y^6 - 45y^4z^2 - 3y^2z^4 + 2z^6)}{5720} \\
& + \frac{3\sqrt{10010}Q_2xyz(120x^4 - 365x^2y^2 - 35x^2z^2 + 87y^4 + 75y^2z^2 - 12z^4)}{5720} \\
& + \frac{\sqrt{15015}Q_3x(2x^6 - 21x^4y^2 - 21x^4z^2 - 15x^2y^4 + 300x^2y^2z^2 - 15x^2z^4 + 8y^6 - 75y^4z^2 - 75y^2z^4 + 8z^6)}{1430} \\
& - \frac{3\sqrt{6006}Q_{3x}xyz(24x^4 + 15x^2y^2 - 95x^2z^2 - 9y^4 + 15y^2z^2 + 24z^4)}{1144} \\
& + \frac{\sqrt{6006}Q_{3y}z(8x^6 - 75x^4y^2 - 15x^4z^2 - 75x^2y^4 + 300x^2y^2z^2 - 21x^2z^4 + 8y^6 - 15y^4z^2 - 21y^2z^4 + 2z^6)}{1144} \\
& + \frac{\sqrt{1001}Q_{az}y(8x^6 + 15x^4y^2 - 165x^4z^2 + 6x^2y^4 - 150x^2y^2z^2 + 240x^2z^4 - y^6 + 15y^4z^2 - 16z^6)}{572} \\
& - \frac{3\sqrt{15015}Q_{bz}y(8x^6 - 5x^4y^2 - 105x^4z^2 - 12x^2y^4 + 150x^2y^2z^2 + 30x^2z^4 + y^6 - 9y^4z^2 - 10y^2z^4)}{2860} \\
& - \frac{3\sqrt{10010}Q_1xyz(87x^4 - 365x^2y^2 + 75x^2z^2 + 120y^4 - 35y^2z^2 - 12z^4)}{5720} \\
& - \frac{\sqrt{10010}Q_{2z}(40x^6 - 735x^4y^2 + 45x^4z^2 + 885x^2y^4 - 300x^2y^2z^2 + 3x^2z^4 - 56y^6 - 15y^4z^2 + 39y^2z^4 - 2z^6)}{5720} \\
& + \frac{\sqrt{15015}Q_{3y}(8x^6 - 15x^4y^2 - 75x^4z^2 - 21x^2y^4 + 300x^2y^2z^2 - 75x^2z^4 + 2y^6 - 21y^4z^2 - 15y^2z^4 + 8z^6)}{1430} \\
& + \frac{\sqrt{6006}Q_{3x}z(8x^6 - 75x^4y^2 - 15x^4z^2 - 75x^2y^4 + 300x^2y^2z^2 - 21x^2z^4 + 8y^6 - 15y^4z^2 - 21y^2z^4 + 2z^6)}{1144} \\
& + \frac{3\sqrt{6006}Q_{3y}xyz(9x^4 - 15x^2y^2 - 15x^2z^2 - 24y^4 + 95y^2z^2 - 24z^4)}{1144} \\
& - \frac{\sqrt{1001}Q_{az}x(x^6 - 6x^4y^2 - 15x^4z^2 - 15x^2y^4 + 150x^2y^2z^2 - 8y^6 + 165y^4z^2 - 240y^2z^4 + 16z^6)}{572} \\
& + \frac{3\sqrt{15015}Q_{bz}x(x^6 - 12x^4y^2 - 9x^4z^2 - 5x^2y^4 + 150x^2y^2z^2 - 10x^2z^4 + 8y^6 - 105y^4z^2 + 30y^2z^4)}{2860}
\end{aligned}$$