

PG No. 13 C_{4v} $4mm$ [tetragonal] (polar, internal polar octupole)

* Harmonics for rank 0

$$\bar{Q}_0^{(3,3)}[q](A_1)$$

** symmetry

$$1$$

** expression

$$\frac{\sqrt{70}Q_{1y}(3x^2 - y^2)}{28} + \frac{\sqrt{70}Q_{2x}(x^2 - 3y^2)}{28} + \frac{\sqrt{105}Q_{3xyz}}{7} - \frac{\sqrt{42}Q_{3xx}(x^2 + y^2 - 4z^2)}{28} \\ - \frac{\sqrt{42}Q_{3yy}(x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{7}Q_{azz}(3x^2 + 3y^2 - 2z^2)}{14} + \frac{\sqrt{105}Q_{bzz}(x - y)(x + y)}{14}$$

* Harmonics for rank 1

$$\bar{Q}_1^{(3,1)}[q](A_1)$$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{21}Q_{3xy}}{7} + \frac{2\sqrt{210}Q_{3xxz}}{35} + \frac{2\sqrt{210}Q_{3yyz}}{35} - \frac{3\sqrt{35}Q_{az}(x^2 + y^2 - 2z^2)}{70} + \frac{\sqrt{21}Q_{bz}(x - y)(x + y)}{14}$$

$$\bar{Q}_1^{(3,3)}[q](A_1)$$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{210}Q_{1yz}(3x^2 - y^2)}{24} + \frac{\sqrt{210}Q_{2xz}(x^2 - 3y^2)}{24} - \frac{\sqrt{35}Q_{3xy}(x^2 + y^2 - 6z^2)}{14} - \frac{5\sqrt{14}Q_{3xxz}(3x^2 + 3y^2 - 4z^2)}{56} \\ - \frac{5\sqrt{14}Q_{3yyz}(3x^2 + 3y^2 - 4z^2)}{56} + \frac{\sqrt{21}Q_{az}(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{84} - \frac{\sqrt{35}Q_{bz}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{28}$$

$$\bar{Q}_{1,1}^{(3,1)}[q](E), \bar{Q}_{1,2}^{(3,1)}[q](E)$$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{3\sqrt{14}Q_{1xy}}{14} + \frac{3\sqrt{14}Q_{2x}(x - y)(x + y)}{28} + \frac{\sqrt{21}Q_{3yz}}{7} - \frac{\sqrt{210}Q_{3xx}(3x^2 + y^2 - 4z^2)}{140} - \frac{\sqrt{210}Q_{3yy}}{70} - \frac{3\sqrt{35}Q_{azz}}{35} + \frac{\sqrt{21}Q_{bzx}}{7} \\ \frac{3\sqrt{14}Q_{1x}(x - y)(x + y)}{28} - \frac{3\sqrt{14}Q_{2xy}}{14} + \frac{\sqrt{21}Q_{3xz}}{7} - \frac{\sqrt{210}Q_{3xy}}{70} - \frac{\sqrt{210}Q_{3y}(x^2 + 3y^2 - 4z^2)}{140} - \frac{3\sqrt{35}Q_{ayz}}{35} - \frac{\sqrt{21}Q_{bzy}}{7}$$

$$\bar{Q}_{1,1}^{(3,3)}[q](E), \bar{Q}_{1,2}^{(3,3)}[q](E)$$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{\sqrt{210}Q_{1xy}(15x^2 - 13y^2 - 6z^2)}{168} + \frac{\sqrt{210}Q_{2x}(4x^4 - 21x^2y^2 - 3x^2z^2 + 3y^4 + 3y^2z^2)}{168} \\ + \frac{\sqrt{35}Q_{3yz}(6x^2 - y^2 - z^2)}{14} - \frac{\sqrt{14}Q_{3xx}(4x^4 + 3x^2y^2 - 27x^2z^2 - y^4 + 3y^2z^2 + 4z^4)}{56} \\ - \frac{5\sqrt{14}Q_{3xy}(x^2 + y^2 - 6z^2)}{56} - \frac{5\sqrt{21}Q_{azz}(3x^2 + 3y^2 - 4z^2)}{84} + \frac{\sqrt{35}Q_{bzx}(5x^2 - 9y^2 - 2z^2)}{28} \\ - \frac{\sqrt{210}Q_{1x}(3x^4 - 21x^2y^2 + 3x^2z^2 + 4y^4 - 3y^2z^2)}{168} + \frac{\sqrt{210}Q_{2xy}(13x^2 - 15y^2 + 6z^2)}{168} - \frac{\sqrt{35}Q_{3xz}(x^2 - 6y^2 + z^2)}{14} - \frac{5\sqrt{14}Q_{3xy}(x^2 + y^2 - 6z^2)}{56} \\ + \frac{\sqrt{14}Q_{3y}(x^4 - 3x^2y^2 - 3x^2z^2 - 4y^4 + 27y^2z^2 - 4z^4)}{56} - \frac{5\sqrt{21}Q_{ayz}(3x^2 + 3y^2 - 4z^2)}{84} + \frac{\sqrt{35}Q_{bzy}(9x^2 - 5y^2 + 2z^2)}{28}$$

* Harmonics for rank 2

$$\bar{Q}_2^{(3,-1)}[q](A_1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{14}Q_{3x}x}{7} + \frac{\sqrt{14}Q_{3y}y}{7} + \frac{\sqrt{21}Q_{az}z}{7}$$

$$\bar{Q}_2^{(3,1)}[q](A_1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{5\sqrt{210}Q_{1y}(3x^2 - y^2)}{168} - \frac{5\sqrt{210}Q_{2x}(x^2 - 3y^2)}{168} - \frac{3\sqrt{14}Q_{3x}x(x^2 + y^2 - 4z^2)}{56} - \frac{3\sqrt{14}Q_{3y}y(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{21}Q_{az}z(3x^2 + 3y^2 - 2z^2)}{21}$$

$$\bar{Q}_2^{(3,3)}[q](A_1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\begin{aligned} & -\frac{\sqrt{1155}Q_{1y}(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{264} - \frac{\sqrt{1155}Q_{2x}(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{264} - \frac{3\sqrt{770}Q_{3xyz}(x^2 + y^2 - 2z^2)}{44} \\ & + \frac{15\sqrt{77}Q_{3xx}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{616} + \frac{15\sqrt{77}Q_{3yy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{616} \\ & + \frac{5\sqrt{462}Q_{az}z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{1848} - \frac{3\sqrt{770}Q_{bzz}(x - y)(x + y)(x^2 + y^2 - 2z^2)}{88} \end{aligned}$$

$$\bar{Q}_2^{(3,-1)}[q](B_1)$$

** symmetry

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

** expression

$$\frac{\sqrt{70}Q_{1y}}{14} + \frac{\sqrt{70}Q_{2x}}{14} - \frac{\sqrt{42}Q_{3xx}}{42} + \frac{\sqrt{42}Q_{3yy}}{42} + \frac{\sqrt{105}Q_{bzz}}{21}$$

$$\bar{Q}_2^{(3,1)}[q](B_1)$$

** symmetry

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{70}Q_{1y}(x^2 + y^2 - 4z^2)}{56} + \frac{\sqrt{70}Q_{2x}(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{42}Q_{3xx}(11x^2 - 9y^2 - 24z^2)}{168} \\ & - \frac{\sqrt{42}Q_{3yy}(9x^2 - 11y^2 + 24z^2)}{168} - \frac{5\sqrt{7}Q_{az}z(x - y)(x + y)}{14} + \frac{\sqrt{105}Q_{bzz}(3x^2 + 3y^2 - 2z^2)}{42} \end{aligned}$$

$$\bar{Q}_2^{(3,3)}[q](B_1)$$

** symmetry

$$\frac{\sqrt{3}(x - y)(x + y)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{385}Q_{1y}(53x^4 - 104x^2y^2 - 6x^2z^2 + 11y^4 - 6y^2z^2 + 4z^4)}{616} + \frac{\sqrt{385}Q_{2x}(11x^4 - 104x^2y^2 - 6x^2z^2 + 53y^4 - 6y^2z^2 + 4z^4)}{616} \\ & + \frac{3\sqrt{2310}Q_{3xyz}(x - y)(x + y)}{44} - \frac{5\sqrt{231}Q_{3xx}(5x^4 - 4x^2y^2 - 46x^2z^2 - 9y^4 + 66y^2z^2 + 12z^4)}{1848} \\ & - \frac{5\sqrt{231}Q_{3yy}(9x^4 + 4x^2y^2 - 66x^2z^2 - 5y^4 + 46y^2z^2 - 12z^4)}{1848} - \frac{5\sqrt{154}Q_{az}z(x - y)(x + y)(x^2 + y^2 - 2z^2)}{88} \\ & + \frac{\sqrt{2310}Q_{bzz}(39x^4 - 174x^2y^2 - 20x^2z^2 + 39y^4 - 20y^2z^2 + 4z^4)}{1848} \end{aligned}$$

$$\bar{Q}_2^{(3,-1)}[q](B_2)$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{70}Q_1x}{14} - \frac{\sqrt{70}Q_2y}{14} + \frac{\sqrt{105}Q_3z}{21} - \frac{\sqrt{42}Q_{3x}y}{42} - \frac{\sqrt{42}Q_{3y}x}{42}$$

$$\tilde{Q}_2^{(3,1)}[q](B_2)$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{70}Q_1x(x^2+y^2-4z^2)}{56} - \frac{\sqrt{70}Q_2y(x^2+y^2-4z^2)}{56} + \frac{\sqrt{105}Q_3z(3x^2+3y^2-2z^2)}{42} \\ - \frac{\sqrt{42}Q_{3x}y(21x^2+y^2-24z^2)}{168} - \frac{\sqrt{42}Q_{3y}x(x^2+21y^2-24z^2)}{168} - \frac{5\sqrt{7}Q_{az}xyz}{7}$$

$$\tilde{Q}_2^{(3,3)}[q](B_2)$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$- \frac{\sqrt{385}Q_1x(5x^4-53x^2y^2+3x^2z^2+26y^4+3y^2z^2-2z^4)}{308} + \frac{\sqrt{385}Q_2y(26x^4-53x^2y^2+3x^2z^2+5y^4+3y^2z^2-2z^4)}{308} \\ - \frac{\sqrt{2310}Q_3z(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4)}{462} - \frac{5\sqrt{231}Q_{3x}y(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{924} \\ + \frac{5\sqrt{231}Q_{3y}x(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{924} - \frac{5\sqrt{154}Q_{az}xyz(x^2+y^2-2z^2)}{44} + \frac{3\sqrt{2310}Q_{bz}xyz(x-y)(x+y)}{44}$$

$$\tilde{Q}_{2,1}^{(3,-1)}[q](E), \tilde{Q}_{2,2}^{(3,-1)}[q](E)$$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{105}Q_3y}{21} + \frac{2\sqrt{42}Q_{3x}z}{21} - \frac{\sqrt{7}Q_{az}x}{7} + \frac{\sqrt{105}Q_{bz}x}{21} \\ \frac{\sqrt{105}Q_3x}{21} + \frac{2\sqrt{42}Q_{3y}z}{21} - \frac{\sqrt{7}Q_{az}y}{7} - \frac{\sqrt{105}Q_{bz}y}{21}$$

$$\tilde{Q}_{2,1}^{(3,1)}[q](E), \tilde{Q}_{2,2}^{(3,1)}[q](E)$$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$\frac{5\sqrt{70}Q_1xyz}{28} + \frac{5\sqrt{70}Q_2z(x-y)(x+y)}{56} + \frac{\sqrt{105}Q_3y(3x^2-2y^2+3z^2)}{42} + \frac{\sqrt{42}Q_{3x}z(9x^2-21y^2+4z^2)}{168} \\ + \frac{5\sqrt{42}Q_{3y}xyz}{28} - \frac{\sqrt{7}Q_{az}x(x^2+y^2-4z^2)}{28} + \frac{\sqrt{105}Q_{bz}x(x^2-9y^2+6z^2)}{84} \\ \frac{5\sqrt{70}Q_1z(x-y)(x+y)}{56} - \frac{5\sqrt{70}Q_2xyz}{28} - \frac{\sqrt{105}Q_3x(2x^2-3y^2-3z^2)}{42} + \frac{5\sqrt{42}Q_{3x}xyz}{28} \\ - \frac{\sqrt{42}Q_{3y}z(21x^2-9y^2-4z^2)}{168} - \frac{\sqrt{7}Q_{az}y(x^2+y^2-4z^2)}{28} + \frac{\sqrt{105}Q_{bz}y(9x^2-y^2-6z^2)}{84}$$

$$\tilde{Q}_{2,1}^{(3,3)}[q](E), \tilde{Q}_{2,2}^{(3,3)}[q](E)$$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$\begin{aligned} & \frac{\sqrt{385}Q_1xyz(7x^2-5y^2-2z^2)}{44} + \frac{\sqrt{385}Q_2z(2x^4-9x^2y^2-x^2z^2+y^4+y^2z^2)}{44} - \frac{\sqrt{2310}Q_3y(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{462} \\ & - \frac{5\sqrt{231}Q_{3x}z(18x^4+15x^2y^2-41x^2z^2-3y^4+y^2z^2+4z^4)}{924} - \frac{5\sqrt{231}Q_{3y}xyz(x^2+y^2-2z^2)}{44} \\ & + \frac{5\sqrt{154}Q_{az}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{308} - \frac{\sqrt{2310}Q_{bz}x(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4)}{924} \\ & - \frac{\sqrt{385}Q_1z(x^4-9x^2y^2+x^2z^2+2y^4-y^2z^2)}{44} + \frac{\sqrt{385}Q_2xyz(5x^2-7y^2+2z^2)}{44} + \frac{\sqrt{2310}Q_3x(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{462} \\ & - \frac{5\sqrt{231}Q_{3x}xyz(x^2+y^2-2z^2)}{44} + \frac{5\sqrt{231}Q_{3y}z(3x^4-15x^2y^2-x^2z^2-18y^4+41y^2z^2-4z^4)}{924} \\ & + \frac{5\sqrt{154}Q_{az}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{308} - \frac{\sqrt{2310}Q_{bz}y(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4)}{924} \end{aligned}$$

* Harmonics for rank 3

$$\tilde{Q}_3^{(3,-3)}[q](A_1)$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$Q_{az}$$

$$\tilde{Q}_3^{(3,-1)}[q](A_1)$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$-Q_3xy + \frac{\sqrt{10}Q_{3x}xz}{10} + \frac{\sqrt{10}Q_{3y}yz}{10} - \frac{\sqrt{15}Q_{az}(x^2+y^2-2z^2)}{15} - \frac{Q_{bz}(x-y)(x+y)}{2}$$

$$\tilde{Q}_3^{(3,1)}[q](A_1)$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$\begin{aligned} & -\frac{7\sqrt{55}Q_1yz(3x^2-y^2)}{44} - \frac{7\sqrt{55}Q_2xz(x^2-3y^2)}{44} + \frac{\sqrt{330}Q_3xy(x^2+y^2-6z^2)}{132} - \frac{5\sqrt{33}Q_{3x}xz(3x^2+3y^2-4z^2)}{132} \\ & - \frac{5\sqrt{33}Q_{3y}yz(3x^2+3y^2-4z^2)}{132} + \frac{\sqrt{22}Q_{az}(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4)}{88} + \frac{\sqrt{330}Q_{bz}(x-y)(x+y)(x^2+y^2-6z^2)}{264} \end{aligned}$$

$$\tilde{Q}_3^{(3,3)}[q](A_1)$$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$\begin{aligned} & -\frac{7\sqrt{4290}Q_1yz(3x^2-y^2)(3x^2+3y^2-8z^2)}{2288} - \frac{7\sqrt{4290}Q_2xz(x^2-3y^2)(3x^2+3y^2-8z^2)}{2288} \\ & + \frac{7\sqrt{715}Q_3xy(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{572} + \frac{35\sqrt{286}Q_{3x}xz(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4)}{2288} \\ & + \frac{35\sqrt{286}Q_{3y}yz(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4)}{2288} \\ & - \frac{5\sqrt{429}Q_{az}(5x^6+15x^4y^2-90x^4z^2+15x^2y^4-180x^2y^2z^2+120x^2z^4+5y^6-90y^4z^2+120y^2z^4-16z^6)}{3432} \\ & + \frac{7\sqrt{715}Q_{bz}(x-y)(x+y)(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{1144} \end{aligned}$$

$$\tilde{Q}_3^{(3,-3)}[q](B_1)$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$Q_{bz}$$

$$\bar{Q}_3^{(3,-1)}[q](B_1)$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{10}Q_1yz}{4} + \frac{\sqrt{10}Q_2xz}{4} + \frac{\sqrt{6}Q_3xxz}{4} - \frac{\sqrt{6}Q_3yyz}{4} - \frac{Q_{az}(x-y)(x+y)}{2}$$

$$\bar{Q}_3^{(3,1)}[q](B_1)$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & \frac{5\sqrt{33}Q_1yz(3x^2+3y^2-4z^2)}{132} + \frac{5\sqrt{33}Q_2xz(3x^2+3y^2-4z^2)}{132} + \frac{35\sqrt{22}Q_3xy(x-y)(x+y)}{132} - \frac{\sqrt{55}Q_{3x}xz(5x^2+33y^2-16z^2)}{132} \\ & + \frac{\sqrt{55}Q_{3y}yz(33x^2+5y^2-16z^2)}{132} + \frac{\sqrt{330}Q_{az}(x-y)(x+y)(x^2+y^2-6z^2)}{264} + \frac{7\sqrt{22}Q_{bz}(x^4-18x^2y^2+12x^2z^2+y^4+12y^2z^2-4z^4)}{264} \end{aligned}$$

$$\bar{Q}_3^{(3,3)}[q](B_1)$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & \frac{7\sqrt{286}Q_1yz(85x^4-160x^2y^2-10x^2z^2+19y^4-10y^2z^2+4z^4)}{2288} + \frac{7\sqrt{286}Q_2xz(19x^4-160x^2y^2-10x^2z^2+85y^4-10y^2z^2+4z^4)}{2288} \\ & - \frac{21\sqrt{429}Q_3xy(x-y)(x+y)(x^2+y^2-10z^2)}{572} - \frac{7\sqrt{4290}Q_{3x}xz(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4)}{2288} \\ & - \frac{7\sqrt{4290}Q_{3y}yz(11x^4+4x^2y^2-26x^2z^2-7y^4+22y^2z^2-4z^4)}{2288} + \frac{7\sqrt{715}Q_{az}(x-y)(x+y)(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{1144} \\ & - \frac{\sqrt{429}Q_{bz}(13x^6-45x^4y^2-150x^4z^2-45x^2y^4+540x^2y^2z^2+60x^2z^4+13y^6-150y^4z^2+60y^2z^4-8z^6)}{1144} \end{aligned}$$

$$\bar{Q}_3^{(3,-3)}[q](B_2)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$Q_3$$

$$\bar{Q}_3^{(3,-1)}[q](B_2)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{10}Q_1xz}{4} - \frac{\sqrt{10}Q_2yz}{4} + \frac{\sqrt{6}Q_3xyz}{4} + \frac{\sqrt{6}Q_3yxz}{4} - Q_{az}xy$$

$$\bar{Q}_3^{(3,1)}[q](B_2)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\begin{aligned} & \frac{5\sqrt{33}Q_1xz(3x^2+3y^2-4z^2)}{132} - \frac{5\sqrt{33}Q_2yz(3x^2+3y^2-4z^2)}{132} - \frac{7\sqrt{22}Q_3(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4)}{66} \\ & + \frac{\sqrt{55}Q_{3x}yz(9x^2-19y^2+16z^2)}{132} - \frac{\sqrt{55}Q_{3y}xz(19x^2-9y^2-16z^2)}{132} + \frac{\sqrt{330}Q_{az}xy(x^2+y^2-6z^2)}{132} + \frac{35\sqrt{22}Q_{bz}xy(x-y)(x+y)}{132} \end{aligned}$$

$$\bar{Q}_3^{(3,3)}[q](B_2)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\begin{aligned}
& -\frac{7\sqrt{286}Q_1xz(7x^4-85x^2y^2+5x^2z^2+40y^4+5y^2z^2-2z^4)}{1144} + \frac{7\sqrt{286}Q_2yz(40x^4-85x^2y^2+5x^2z^2+7y^4+5y^2z^2-2z^4)}{1144} \\
& + \frac{\sqrt{429}Q_3(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6)}{286} \\
& - \frac{7\sqrt{4290}Q_{3x}yz(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4)}{1144} + \frac{7\sqrt{4290}Q_{3y}xz(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4)}{1144} \\
& + \frac{7\sqrt{715}Q_{az}xy(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{572} - \frac{21\sqrt{429}Q_{bz}xy(x-y)(x+y)(x^2+y^2-10z^2)}{572}
\end{aligned}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(3,-3)}[q](E,1), \tilde{\mathbb{Q}}_{3,2}^{(3,-3)}[q](E,1)$$

** symmetry

$$\begin{aligned}
& \frac{x(2x^2-3y^2-3z^2)}{2} \\
& - \frac{y(3x^2-2y^2+3z^2)}{2}
\end{aligned}$$

** expression

$$\begin{aligned}
& \frac{\sqrt{10}Q_2}{4} - \frac{\sqrt{6}Q_{3x}}{4} \\
& - \frac{\sqrt{10}Q_1}{4} - \frac{\sqrt{6}Q_{3y}}{4}
\end{aligned}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(3,-3)}[q](E,2), \tilde{\mathbb{Q}}_{3,2}^{(3,-3)}[q](E,2)$$

** symmetry

$$\begin{aligned}
& \frac{\sqrt{15}x(y-z)(y+z)}{2} \\
& \frac{\sqrt{15}y(x-z)(x+z)}{2}
\end{aligned}$$

** expression

$$\begin{aligned}
& -\frac{\sqrt{6}Q_2}{4} - \frac{\sqrt{10}Q_{3x}}{4} \\
& \frac{\sqrt{6}Q_1}{4} - \frac{\sqrt{10}Q_{3y}}{4}
\end{aligned}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(3,-1)}[q](E,1), \tilde{\mathbb{Q}}_{3,2}^{(3,-1)}[q](E,1)$$

** symmetry

$$\begin{aligned}
& \frac{x(2x^2-3y^2-3z^2)}{2} \\
& - \frac{y(3x^2-2y^2+3z^2)}{2}
\end{aligned}$$

** expression

$$\begin{aligned}
& \frac{\sqrt{6}Q_1xy}{8} + \frac{\sqrt{6}Q_2(4x^2+y^2-5z^2)}{24} - Q_{3yz} - \frac{\sqrt{10}Q_{3x}(4x^2-7y^2+3z^2)}{40} - \frac{\sqrt{10}Q_{3y}xy}{40} - \frac{\sqrt{15}Q_{az}xz}{20} + \frac{Q_{bz}xz}{4} \\
& - \frac{\sqrt{6}Q_1(4x^2+y^2-5z^2)}{24} - \frac{\sqrt{6}Q_2xy}{8} - Q_{3xz} - \frac{\sqrt{10}Q_{3x}xy}{40} + \frac{\sqrt{10}Q_{3y}(7x^2-4y^2-3z^2)}{40} - \frac{\sqrt{15}Q_{az}yz}{20} - \frac{Q_{bz}yz}{4}
\end{aligned}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(3,-1)}[q](E,2), \tilde{\mathbb{Q}}_{3,2}^{(3,-1)}[q](E,2)$$

** symmetry

$$\begin{aligned}
& \frac{\sqrt{15}x(y-z)(y+z)}{2} \\
& \frac{\sqrt{15}y(x-z)(x+z)}{2}
\end{aligned}$$

** expression

$$\begin{aligned}
& \frac{\sqrt{10}Q_1xy}{8} - \frac{\sqrt{10}Q_2(y-z)(y+z)}{8} + \frac{\sqrt{6}Q_{3x}(y-z)(y+z)}{8} - \frac{3\sqrt{6}Q_{3y}xy}{8} - \frac{Q_{az}xz}{4} - \frac{\sqrt{15}Q_{bz}xz}{4}
\end{aligned}$$

$$\frac{\sqrt{10}Q_1(x-z)(x+z)}{8} - \frac{\sqrt{10}Q_2xy}{8} - \frac{3\sqrt{6}Q_{3x}xy}{8} + \frac{\sqrt{6}Q_{3y}(x-z)(x+z)}{8} - \frac{Q_{az}yz}{4} + \frac{\sqrt{15}Q_{bz}yz}{4}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(3,1)}[q](E, 1), \tilde{\mathbb{Q}}_{3,2}^{(3,1)}[q](E, 1)$$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$- \frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{55}Q_1xy(5x^2 - 2y^2 - 9z^2)}{44} + \frac{\sqrt{55}Q_2(4x^4 - 9x^2y^2 - 15x^2z^2 + y^4 + 3y^2z^2 + 2z^4)}{88} \\ & - \frac{\sqrt{330}Q_{3y}z(6x^2 - y^2 - z^2)}{132} - \frac{\sqrt{33}Q_{3x}(12x^4 - 51x^2y^2 - 21x^2z^2 + 7y^4 + 9y^2z^2 + 2z^4)}{88} \\ & - \frac{5\sqrt{33}Q_{3y}xy(x^2 - 6y^2 + 15z^2)}{132} - \frac{5\sqrt{22}Q_{az}xz(x^2 - 6y^2 + z^2)}{44} + \frac{\sqrt{330}Q_{bz}xz(5x^2 + 12y^2 - 9z^2)}{132} \\ & - \frac{\sqrt{55}Q_1(x^4 - 9x^2y^2 + 3x^2z^2 + 4y^4 - 15y^2z^2 + 2z^4)}{88} + \frac{\sqrt{55}Q_2xy(2x^2 - 5y^2 + 9z^2)}{44} + \frac{\sqrt{330}Q_{3xz}(x^2 - 6y^2 + z^2)}{132} + \frac{5\sqrt{33}Q_{3x}xy(6x^2 - y^2 - 15z^2)}{132} \\ & - \frac{\sqrt{33}Q_{3y}(7x^4 - 51x^2y^2 + 9x^2z^2 + 12y^4 - 21y^2z^2 + 2z^4)}{264} + \frac{5\sqrt{22}Q_{az}yz(6x^2 - y^2 - z^2)}{44} - \frac{\sqrt{330}Q_{bz}yz(12x^2 + 5y^2 - 9z^2)}{132} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(3,1)}[q](E, 2), \tilde{\mathbb{Q}}_{3,2}^{(3,1)}[q](E, 2)$$

** symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

** expression

$$\begin{aligned} & \frac{5\sqrt{33}Q_1xy(5x^2 - 2y^2 - 9z^2)}{132} + \frac{\sqrt{33}Q_2(14x^4 - 57x^2y^2 - 27x^2z^2 - y^4 + 63y^2z^2 - 6z^4)}{264} \\ & + \frac{35\sqrt{22}Q_{3y}z(y-z)(y+z)}{132} + \frac{\sqrt{55}Q_{3x}(14x^4 - 33x^2y^2 - 51x^2z^2 - 5y^4 + 63y^2z^2 - 2z^4)}{264} \\ & + \frac{\sqrt{55}Q_{3y}xy(11x^2 - 10y^2 - 3z^2)}{132} + \frac{\sqrt{330}Q_{az}xz(9x^2 - 12y^2 - 5z^2)}{132} - \frac{5\sqrt{22}Q_{bz}xz(x^2 - 6y^2 + z^2)}{132} \\ & \frac{\sqrt{33}Q_1(x^4 + 57x^2y^2 - 63x^2z^2 - 14y^4 + 27y^2z^2 + 6z^4)}{264} + \frac{5\sqrt{33}Q_2xy(2x^2 - 5y^2 + 9z^2)}{132} + \frac{35\sqrt{22}Q_{3xz}(x-z)(x+z)}{132} \\ & - \frac{\sqrt{55}Q_{3x}xy(10x^2 - 11y^2 + 3z^2)}{132} - \frac{\sqrt{55}Q_{3y}(5x^4 + 33x^2y^2 - 63x^2z^2 - 14y^4 + 51y^2z^2 + 2z^4)}{264} \\ & - \frac{\sqrt{330}Q_{az}yz(12x^2 - 9y^2 + 5z^2)}{132} - \frac{5\sqrt{22}Q_{bz}yz(6x^2 - y^2 - z^2)}{132} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{3,1}^{(3,3)}[q](E, 1), \tilde{\mathbb{Q}}_{3,2}^{(3,3)}[q](E, 1)$$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$- \frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{7\sqrt{4290}Q_1xy(10x^4 - 27x^2y^2 - 19x^2z^2 + 7y^4 + 11y^2z^2 + 4z^4)}{2288} \\ & + \frac{\sqrt{4290}Q_2(40x^6 - 468x^4y^2 - 132x^4z^2 + 393x^2y^4 + 450x^2y^2z^2 + 57x^2z^4 - 23y^6 - 48y^4z^2 - 27y^2z^4 - 2z^6)}{6864} \\ & + \frac{7\sqrt{715}Q_{3y}z(16x^4 - 16x^2y^2 - 16x^2z^2 + y^4 + 2y^2z^2 + z^4)}{572} \\ & - \frac{5\sqrt{286}Q_{3x}(8x^6 - 4x^4y^2 - 116x^4z^2 - 11x^2y^4 + 90x^2y^2z^2 + 101x^2z^4 + y^6 - 4y^4z^2 - 11y^2z^4 - 6z^6)}{2288} \\ & - \frac{35\sqrt{286}Q_{3y}xy(2x^4 + x^2y^2 - 23x^2z^2 - y^4 + 7y^2z^2 + 8z^4)}{2288} - \frac{35\sqrt{429}Q_{az}xz(2x^4 + x^2y^2 - 7x^2z^2 - y^4 + y^2z^2 + 2z^4)}{1144} \\ & + \frac{7\sqrt{715}Q_{bz}xz(10x^4 - 43x^2y^2 - 19x^2z^2 + 13y^4 + 17y^2z^2 + 4z^4)}{1144} \end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{4290}Q_1 (23x^6 - 393x^4y^2 + 48x^4z^2 + 468x^2y^4 - 450x^2y^2z^2 + 27x^2z^4 - 40y^6 + 132y^4z^2 - 57y^2z^4 + 2z^6)}{6864} \\
& - \frac{7\sqrt{4290}Q_2xy (7x^4 - 27x^2y^2 + 11x^2z^2 + 10y^4 - 19y^2z^2 + 4z^4)}{2288} \\
& + \frac{7\sqrt{715}Q_3xz (x^4 - 16x^2y^2 + 2x^2z^2 + 16y^4 - 16y^2z^2 + z^4)}{572} + \frac{35\sqrt{286}Q_{3x}xy (x^4 - x^2y^2 - 7x^2z^2 - 2y^4 + 23y^2z^2 - 8z^4)}{2288} \\
& - \frac{5\sqrt{286}Q_{3y} (x^6 - 11x^4y^2 - 4x^4z^2 - 4x^2y^4 + 90x^2y^2z^2 - 11x^2z^4 + 8y^6 - 116y^4z^2 + 101y^2z^4 - 6z^6)}{2288} \\
& + \frac{35\sqrt{429}Q_{az}yz (x^4 - x^2y^2 - x^2z^2 - 2y^4 + 7y^2z^2 - 2z^4)}{1144} - \frac{7\sqrt{715}Q_{bz}yz (13x^4 - 43x^2y^2 + 17x^2z^2 + 10y^4 - 19y^2z^2 + 4z^4)}{1144}
\end{aligned}$$

$$\bar{Q}_{3,1}^{(3,3)}[q](E, 2), \bar{Q}_{3,2}^{(3,3)}[q](E, 2)$$

** symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

** expression

$$\begin{aligned}
& - \frac{7\sqrt{286}Q_1xy (16x^4 - 85x^2y^2 + 95x^2z^2 + 31y^4 - 55y^2z^2 - 20z^4)}{2288} \\
& - \frac{\sqrt{286}Q_2 (12x^6 - 370x^4y^2 + 190x^4z^2 + 505x^2y^4 - 810x^2y^2z^2 - 55x^2z^4 - 37y^6 + 50y^4z^2 + 85y^2z^4 - 2z^6)}{2288} \\
& + \frac{21\sqrt{429}Q_3yz (y-z)(y+z)(10x^2 - y^2 - z^2)}{572} \\
& - \frac{\sqrt{4290}Q_{3x} (4x^6 + 26x^4y^2 - 86x^4z^2 + 19x^2y^4 - 270x^2y^2z^2 + 131x^2z^4 - 3y^6 + 26y^4z^2 + 19y^2z^4 - 10z^6)}{2288} \\
& - \frac{21\sqrt{4290}Q_{3y}xy (x^2y^2 - 3x^2z^2 + y^4 - 13y^2z^2 + 8z^4)}{2288} - \frac{7\sqrt{715}Q_{az}xz (4x^4 + 17x^2y^2 - 19x^2z^2 + 13y^4 - 43y^2z^2 + 10z^4)}{1144} \\
& + \frac{7\sqrt{429}Q_{bz}xz (4x^4 + 35x^2y^2 - 25x^2z^2 - 35y^4 + 35y^2z^2 + 4z^4)}{1144} \\
& - \frac{\sqrt{286}Q_1 (37x^6 - 505x^4y^2 - 50x^4z^2 + 370x^2y^4 + 810x^2y^2z^2 - 85x^2z^4 - 12y^6 - 190y^4z^2 + 55y^2z^4 + 2z^6)}{2288} \\
& + \frac{7\sqrt{286}Q_2xy (31x^4 - 85x^2y^2 - 55x^2z^2 + 16y^4 + 95y^2z^2 - 20z^4)}{2288} \\
& - \frac{21\sqrt{429}Q_3xz (x-z)(x+z)(x^2 - 10y^2 + z^2)}{572} - \frac{21\sqrt{4290}Q_{3x}xy (x^4 + x^2y^2 - 13x^2z^2 - 3y^2z^2 + 8z^4)}{2288} \\
& + \frac{\sqrt{4290}Q_{3y} (3x^6 - 19x^4y^2 - 26x^4z^2 - 26x^2y^4 + 270x^2y^2z^2 - 19x^2z^4 - 4y^6 + 86y^4z^2 - 131y^2z^4 + 10z^6)}{2288} \\
& - \frac{7\sqrt{715}Q_{az}yz (13x^4 + 17x^2y^2 - 43x^2z^2 + 4y^4 - 19y^2z^2 + 10z^4)}{1144} + \frac{7\sqrt{429}Q_{bz}yz (35x^4 - 35x^2y^2 - 35x^2z^2 - 4y^4 + 25y^2z^2 - 4z^4)}{1144}
\end{aligned}$$

* Harmonics for rank 4

$$\bar{Q}_4^{(3,-3)}[q](A_1, 1)$$

** symmetry

$$\frac{\sqrt{21} (x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$-\frac{\sqrt{30}Q_1y}{12} + \frac{\sqrt{30}Q_2x}{12} - \frac{\sqrt{2}Q_{3x}x}{4} - \frac{\sqrt{2}Q_{3y}y}{4} + \frac{\sqrt{3}Q_{az}z}{3}$$

$$\bar{Q}_4^{(3,-3)}[q](A_1, 2)$$

** symmetry

$$-\frac{\sqrt{15} (x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\frac{\sqrt{42}Q_1y}{12} - \frac{\sqrt{42}Q_2x}{12} - \frac{\sqrt{70}Q_{3x}x}{28} - \frac{\sqrt{70}Q_{3y}y}{28} + \frac{\sqrt{105}Q_{az}z}{21}$$

$$\bar{Q}_4^{(3,-1)}[q](A_1, 1)$$

** symmetry

$$\frac{\sqrt{21} (x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\frac{\sqrt{165}Q_1y(x^2 - y^2 + 2z^2)}{44} + \frac{\sqrt{165}Q_2x(x^2 - y^2 - 2z^2)}{44} - \frac{3\sqrt{110}Q_3xyz}{11} - \frac{\sqrt{11}Q_{3xx}(3x^2 - 7y^2 - 2z^2)}{44} \\ + \frac{\sqrt{11}Q_{3yy}(7x^2 - 3y^2 + 2z^2)}{44} - \frac{\sqrt{66}Q_{azz}(3x^2 + 3y^2 - 2z^2)}{44} - \frac{\sqrt{110}Q_{bzz}(x - y)(x + y)}{44}$$

$$\bar{Q}_4^{(3,-1)}[q](A_1, 2)$$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\frac{\sqrt{231}Q_1y(11x^2 + y^2 - 14z^2)}{308} - \frac{\sqrt{231}Q_2x(x^2 + 11y^2 - 14z^2)}{308} + \frac{\sqrt{385}Q_{3xx}(3x^2 - 11y^2 + 2z^2)}{308} \\ - \frac{\sqrt{385}Q_{3yy}(11x^2 - 3y^2 - 2z^2)}{308} - \frac{\sqrt{2310}Q_{azz}(3x^2 + 3y^2 - 2z^2)}{308} - \frac{5\sqrt{154}Q_{bzz}(x - y)(x + y)}{44}$$

$$\bar{Q}_4^{(3,1)}[q](A_1, 1)$$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\frac{5\sqrt{858}Q_1y(5x^4 + 3x^2y^2 - 39x^2z^2 - 2y^4 + 17y^2z^2 - 2z^4)}{1144} + \frac{5\sqrt{858}Q_2x(2x^4 - 3x^2y^2 - 17x^2z^2 - 5y^4 + 39y^2z^2 + 2z^4)}{1144} \\ - \frac{\sqrt{1430}Q_{3xx}(6x^4 - 65x^2y^2 + 5x^2z^2 + 13y^4 + 117y^2z^2 - 22z^4)}{1144} - \frac{\sqrt{1430}Q_{3yy}(13x^4 - 65x^2y^2 + 117x^2z^2 + 6y^4 + 5y^2z^2 - 22z^4)}{1144} \\ - \frac{\sqrt{2145}Q_{azz}(3x^4 - 78x^2y^2 + 20x^2z^2 + 3y^4 + 20y^2z^2 - 4z^4)}{572} + \frac{35\sqrt{143}Q_{bzz}(x - y)(x + y)(x^2 + y^2 - 2z^2)}{572}$$

$$\bar{Q}_4^{(3,1)}[q](A_1, 2)$$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\frac{\sqrt{30030}Q_1y(4x^4 + 3x^2y^2 - 33x^2z^2 - y^4 + 7y^2z^2 + 2z^4)}{1144} + \frac{\sqrt{30030}Q_2x(x^4 - 3x^2y^2 - 7x^2z^2 - 4y^4 + 33y^2z^2 - 2z^4)}{1144} + \frac{3\sqrt{5005}Q_3xyz(x^2 + y^2 - 2z^2)}{143} \\ + \frac{\sqrt{2002}Q_{3xx}(75x^4 - 389x^2y^2 - 361x^2z^2 + 124y^4 + 423y^2z^2 + 110z^4)}{8008} + \frac{\sqrt{2002}Q_{3yy}(124x^4 - 389x^2y^2 + 423x^2z^2 + 75y^4 - 361y^2z^2 + 110z^4)}{8008} \\ + \frac{\sqrt{3003}Q_{azz}(111x^4 - 366x^2y^2 - 100x^2z^2 + 111y^4 - 100y^2z^2 + 20z^4)}{4004} - \frac{\sqrt{5005}Q_{bzz}(x - y)(x + y)(x^2 + y^2 - 2z^2)}{572}$$

$$\bar{Q}_4^{(3,3)}[q](A_1, 1)$$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$\frac{\sqrt{858}Q_1y(211x^6 - 900x^4y^2 - 465x^4z^2 + 570x^2y^4 - 300x^2y^2z^2 + 615x^2z^4 - 35y^6 + 165y^4z^2 - 225y^2z^4 + 4z^6)}{3432} \\ + \frac{\sqrt{858}Q_2x(35x^6 - 570x^4y^2 - 165x^4z^2 + 900x^2y^4 + 300x^2y^2z^2 + 225x^2z^4 - 211y^6 + 465y^4z^2 - 615y^2z^4 - 4z^6)}{3432} \\ + \frac{3\sqrt{143}Q_3xyz(3x^4 - 5x^2y^2 - 5x^2z^2 + 3y^4 - 5y^2z^2 + 3z^4)}{22} \\ - \frac{\sqrt{1430}Q_{3xx}(7x^6 - 6x^4y^2 - 141x^4z^2 + 60x^2y^2z^2 + 225x^2z^4 + 13y^6 - 195y^4z^2 + 165y^2z^4 - 56z^6)}{1144} \\ - \frac{\sqrt{1430}Q_{3yy}(13x^6 - 195x^4z^2 - 6x^2y^4 + 60x^2y^2z^2 + 165x^2z^4 + 7y^6 - 141y^4z^2 + 225y^2z^4 - 56z^6)}{1144} \\ - \frac{\sqrt{2145}Q_{azz}(43x^6 + 30x^4y^2 - 225x^4z^2 + 30x^2y^4 - 120x^2y^2z^2 + 147x^2z^4 + 43y^6 - 225y^4z^2 + 147y^2z^4 - 14z^6)}{1716} \\ + \frac{3\sqrt{143}Q_{bzz}(x - y)(x + y)(23x^4 - 97x^2y^2 - 75x^2z^2 + 23y^4 - 75y^2z^2 + 45z^4)}{572}$$

$$\bar{Q}_4^{(3,3)}[q](A_1, 2)$$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

** expression

$$\begin{aligned}
& - \frac{\sqrt{30030}Q_1y (341x^6 - 1935x^4y^2 + 690x^4z^2 + 1113x^2y^4 + 480x^2y^2z^2 - 930x^2z^4 - 43y^6 - 210y^4z^2 + 270y^2z^4 + 8z^6)}{34320} \\
& - \frac{\sqrt{30030}Q_2x (43x^6 - 1113x^4y^2 + 210x^4z^2 + 1935x^2y^4 - 480x^2y^2z^2 - 270x^2z^4 - 341y^6 - 690y^4z^2 + 930y^2z^4 - 8z^6)}{34320} \\
& - \frac{3\sqrt{5005}Q_3xyz (3x^4 - 20x^2y^2 + 10x^2z^2 + 3y^4 + 10y^2z^2 - 6z^4)}{260} \\
& - \frac{\sqrt{2002}Q_{3x}x (x^6 + 57x^4y^2 - 78x^4z^2 + 45x^2y^4 - 840x^2y^2z^2 + 270x^2z^4 - 11y^6 + 30y^4z^2 + 390y^2z^4 - 80z^6)}{2288} \\
& + \frac{\sqrt{2002}Q_{3y}y (11x^6 - 45x^4y^2 - 30x^4z^2 - 57x^2y^4 + 840x^2y^2z^2 - 390x^2z^4 - y^6 + 78y^4z^2 - 270y^2z^4 + 80z^6)}{2288} \\
& - \frac{\sqrt{3003}Q_{az}z (19x^6 + 255x^4y^2 - 180x^4z^2 + 255x^2y^4 - 1020x^2y^2z^2 + 210x^2z^4 + 19y^6 - 180y^4z^2 + 210y^2z^4 - 20z^6)}{3432} \\
& - \frac{3\sqrt{5005}Q_{bz}z (x - y) (x + y) (x^4 - 284x^2y^2 + 90x^2z^2 + y^4 + 90y^2z^2 - 54z^4)}{5720}
\end{aligned}$$

$\bar{Q}_4^{(3,-3)}[q](A_2)$

** symmetry

$$\frac{\sqrt{35}xy (x - y) (x + y)}{2}$$

** expression

$$\frac{\sqrt{2}Q_1x}{2} + \frac{\sqrt{2}Q_2y}{2}$$

$\bar{Q}_4^{(3,-1)}[q](A_2)$

** symmetry

$$\frac{\sqrt{35}xy (x - y) (x + y)}{2}$$

** expression

$$\frac{3\sqrt{11}Q_1x (x^2 + y^2 - 4z^2)}{44} + \frac{3\sqrt{11}Q_2y (x^2 + y^2 - 4z^2)}{44} + \frac{5\sqrt{66}Q_3z (x - y) (x + y)}{44} - \frac{\sqrt{165}Q_{3x}y (3x^2 - y^2)}{44} - \frac{\sqrt{165}Q_{3y}x (x^2 - 3y^2)}{44} + \frac{5\sqrt{66}Q_{bz}xyz}{22}$$

$\bar{Q}_4^{(3,1)}[q](A_2)$

** symmetry

$$\frac{\sqrt{35}xy (x - y) (x + y)}{2}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{1430}Q_1x (x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} + \frac{3\sqrt{1430}Q_2y (x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{2288} \\
& + \frac{7\sqrt{2145}Q_3z (x - y) (x + y) (x^2 + y^2 - 2z^2)}{572} - \frac{7\sqrt{858}Q_{3x}y (21x^4 - 26x^2y^2 - 48x^2z^2 + y^4 + 16y^2z^2)}{2288} \\
& + \frac{7\sqrt{858}Q_{3y}x (x^4 - 26x^2y^2 + 16x^2z^2 + 21y^4 - 48y^2z^2)}{2288} - \frac{63\sqrt{143}Q_{az}xyz (x - y) (x + y)}{143} + \frac{7\sqrt{2145}Q_{bz}xyz (x^2 + y^2 - 2z^2)}{286}
\end{aligned}$$

$\bar{Q}_4^{(3,3)}[q](A_2)$

** symmetry

$$\frac{\sqrt{35}xy (x - y) (x + y)}{2}$$

** expression

$$\begin{aligned}
& - \frac{\sqrt{1430}Q_1x (53x^6 - 1128x^4y^2 + 15x^4z^2 + 1875x^2y^4 + 30x^2y^2z^2 - 30x^2z^4 - 376y^6 + 15y^4z^2 - 30y^2z^4 + 8z^6)}{5720} \\
& + \frac{\sqrt{1430}Q_2y (376x^6 - 1875x^4y^2 - 15x^4z^2 + 1128x^2y^4 - 30x^2y^2z^2 + 30x^2z^4 - 53y^6 - 15y^4z^2 + 30y^2z^4 - 8z^6)}{5720} \\
& - \frac{3\sqrt{2145}Q_3z (x - y) (x + y) (8x^4 - 127x^2y^2 + 5x^2z^2 + 8y^4 + 5y^2z^2 - 3z^4)}{1430} \\
& - \frac{3\sqrt{858}Q_{3x}y (8x^6 - 5x^4y^2 - 105x^4z^2 - 12x^2y^4 + 150x^2y^2z^2 + 30x^2z^4 + y^6 - 9y^4z^2 - 10y^2z^4)}{1144} \\
& + \frac{3\sqrt{858}Q_{3y}x (x^6 - 12x^4y^2 - 9x^4z^2 - 5x^2y^4 + 150x^2y^2z^2 - 10x^2z^4 + 8y^6 - 105y^4z^2 + 30y^2z^4)}{1144} \\
& - \frac{3\sqrt{143}Q_{az}xyz (x - y) (x + y) (3x^2 + 3y^2 - 10z^2)}{52} + \frac{3\sqrt{2145}Q_{bz}xyz (111x^4 - 350x^2y^2 - 20x^2z^2 + 111y^4 - 20y^2z^2 + 12z^4)}{2860}
\end{aligned}$$

$\bar{Q}_4^{(3,-3)}[q](B_1)$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\frac{\sqrt{14}Q_1y}{28} + \frac{\sqrt{14}Q_2x}{28} - \frac{\sqrt{210}Q_{3xx}}{28} + \frac{\sqrt{210}Q_{3yy}}{28} - \frac{\sqrt{21}Q_{bzz}}{7}$$

$$\bar{Q}_4^{(3,-1)}[q](B_1)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\frac{9\sqrt{77}Q_1y(x^2+y^2-4z^2)}{308} + \frac{9\sqrt{77}Q_2x(x^2+y^2-4z^2)}{308} - \frac{\sqrt{1155}Q_{3xx}(x^2-11y^2+8z^2)}{308} \\ - \frac{\sqrt{1155}Q_{3yy}(11x^2-y^2-8z^2)}{308} + \frac{3\sqrt{770}Q_{azz}(x-y)(x+y)}{308} - \frac{\sqrt{462}Q_{bzz}(3x^2+3y^2-2z^2)}{308}$$

$$\bar{Q}_4^{(3,1)}[q](B_1)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\frac{9\sqrt{10010}Q_1y(10x^4-15x^2y^2-15x^2z^2+3y^4-15y^2z^2+10z^4)}{8008} + \frac{9\sqrt{10010}Q_2x(3x^4-15x^2y^2-15x^2z^2+10y^4-15y^2z^2+10z^4)}{8008} \\ - \frac{3\sqrt{15015}Q_{3xyz}(x-y)(x+y)}{143} + \frac{\sqrt{6006}Q_{3xx}(5x^4-53x^2y^2+3x^2z^2-58y^4+507y^2z^2-86z^4)}{8008} \\ + \frac{\sqrt{6006}Q_{3yy}(58x^4+53x^2y^2-507x^2z^2-5y^4-3y^2z^2+86z^4)}{8008} + \frac{3\sqrt{1001}Q_{azz}(x-y)(x+y)(x^2+y^2-2z^2)}{572} \\ + \frac{\sqrt{15015}Q_{bzz}(9x^4+186x^2y^2-80x^2z^2+9y^4-80y^2z^2+16z^4)}{4004}$$

$$\bar{Q}_4^{(3,3)}[q](B_1)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\frac{\sqrt{10010}Q_1y(85x^6-75x^4y^2-1050x^4z^2-141x^2y^4+1860x^2y^2z^2+120x^2z^4+19y^6-258y^4z^2+120y^2z^4-32z^6)}{11440} \\ + \frac{\sqrt{10010}Q_2x(19x^6-141x^4y^2-258x^4z^2-75x^2y^4+1860x^2y^2z^2+120x^2z^4+85y^6-1050y^4z^2+120y^2z^4-32z^6)}{11440} \\ + \frac{3\sqrt{15015}Q_{3xyz}(x-y)(x+y)(3x^2+3y^2-10z^2)}{260} \\ - \frac{\sqrt{6006}Q_{3xx}(7x^6+3x^4y^2-150x^4z^2-15x^2y^4+60x^2y^2z^2+240x^2z^4-11y^6+210y^4z^2-240y^2z^4-32z^6)}{2288} \\ - \frac{\sqrt{6006}Q_{3yy}(11x^6+15x^4y^2-210x^4z^2-3x^2y^4-60x^2y^2z^2+240x^2z^4-7y^6+150y^4z^2-240y^2z^4+32z^6)}{2288} \\ - \frac{3\sqrt{1001}Q_{azz}(x-y)(x+y)(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4)}{1144} \\ + \frac{\sqrt{15015}Q_{bzz}(67x^6-195x^4y^2-270x^4z^2-195x^2y^4+780x^2y^2z^2+84x^2z^4+67y^6-270y^4z^2+84y^2z^4-8z^6)}{5720}$$

$$\bar{Q}_4^{(3,-3)}[q](B_2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$-\frac{\sqrt{14}Q_1x}{28} + \frac{\sqrt{14}Q_2y}{28} + \frac{\sqrt{21}Q_3z}{7} + \frac{\sqrt{210}Q_{3xy}}{28} + \frac{\sqrt{210}Q_{3yx}}{28}$$

$$\bar{Q}_4^{(3,-1)}[q](B_2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$-\frac{9\sqrt{77}Q_1x(x^2+y^2-4z^2)}{308} + \frac{9\sqrt{77}Q_2y(x^2+y^2-4z^2)}{308} + \frac{\sqrt{462}Q_3z(3x^2+3y^2-2z^2)}{308} \\ + \frac{\sqrt{1155}Q_{3x}y(7x^2-5y^2+8z^2)}{308} - \frac{\sqrt{1155}Q_{3y}x(5x^2-7y^2-8z^2)}{308} - \frac{3\sqrt{770}Q_{az}xyz}{154}$$

$$\bar{Q}_4^{(3,1)}[q](B_2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$\frac{9\sqrt{10010}Q_1x(x^4-40x^2y^2+30x^2z^2+15y^4+30y^2z^2-20z^4)}{16016} - \frac{9\sqrt{10010}Q_2y(15x^4-40x^2y^2+30x^2z^2+y^4+30y^2z^2-20z^4)}{16016} \\ - \frac{\sqrt{15015}Q_3z(51x^4-66x^2y^2-80x^2z^2+51y^4-80y^2z^2+16z^4)}{4004} - \frac{\sqrt{6006}Q_{3x}y(73x^4+20x^2y^2-498x^2z^2-53y^4+510y^2z^2-172z^4)}{16016} \\ + \frac{\sqrt{6006}Q_{3y}x(53x^4-20x^2y^2-510x^2z^2-73y^4+498y^2z^2+172z^4)}{16016} - \frac{3\sqrt{1001}Q_{az}xyz(x^2+y^2-2z^2)}{286} + \frac{3\sqrt{15015}Q_{bz}xyz(x-y)(x+y)}{143}$$

$$\bar{Q}_4^{(3,3)}[q](B_2)$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$\frac{\sqrt{10010}Q_1x(7x^6-78x^4y^2-69x^4z^2-45x^2y^4+1050x^2y^2z^2-60x^2z^4+40y^6-465y^4z^2-60y^2z^4+16z^6)}{5720} \\ - \frac{\sqrt{10010}Q_2y(40x^6-45x^4y^2-465x^4z^2-78x^2y^4+1050x^2y^2z^2-60x^2z^4+7y^6-69y^4z^2-60y^2z^4+16z^6)}{5720} \\ + \frac{\sqrt{15015}Q_3z(8x^6-75x^4y^2-15x^4z^2-75x^2y^4+300x^2y^2z^2-21x^2z^4+8y^6-15y^4z^2-21y^2z^4+2z^6)}{1430} \\ + \frac{\sqrt{6006}Q_{3x}y(8x^6+15x^4y^2-165x^4z^2+6x^2y^4-150x^2y^2z^2+240x^2z^4-y^6+15y^4z^2-16z^6)}{1144} \\ - \frac{\sqrt{6006}Q_{3y}x(x^6-6x^4y^2-15x^4z^2-15x^2y^4+150x^2y^2z^2-8y^6+165y^4z^2-240y^2z^4+16z^6)}{1144} \\ + \frac{3\sqrt{1001}Q_{az}xyz(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4)}{572} - \frac{3\sqrt{15015}Q_{bz}xyz(x-y)(x+y)(3x^2+3y^2-10z^2)}{260}$$

$$\bar{Q}_{4,1}^{(3,-3)}[q](E,1), \bar{Q}_{4,2}^{(3,-3)}[q](E,1)$$

** symmetry

$$\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$\frac{\sqrt{2}Q_2z}{8} - \frac{\sqrt{30}Q_{3x}z}{8} - \frac{\sqrt{5}Q_{az}x}{4} + \frac{\sqrt{3}Q_{bz}x}{4} \\ - \frac{\sqrt{2}Q_1z}{8} - \frac{\sqrt{30}Q_{3y}z}{8} - \frac{\sqrt{5}Q_{az}y}{4} - \frac{\sqrt{3}Q_{bz}y}{4}$$

$$\bar{Q}_{4,1}^{(3,-3)}[q](E,2), \bar{Q}_{4,2}^{(3,-3)}[q](E,2)$$

** symmetry

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

** expression

$$-\frac{\sqrt{14}Q_2z}{8} + \frac{\sqrt{21}Q_3y}{7} - \frac{\sqrt{210}Q_{3x}z}{56} - \frac{\sqrt{35}Q_{az}x}{28} - \frac{3\sqrt{21}Q_{bz}x}{28} \\ \frac{\sqrt{14}Q_1z}{8} + \frac{\sqrt{21}Q_3x}{7} - \frac{\sqrt{210}Q_{3y}z}{56} - \frac{\sqrt{35}Q_{az}y}{28} + \frac{3\sqrt{21}Q_{bz}y}{28}$$

$$\tilde{Q}_{4,1}^{(3,-1)}[q](E, 1), \tilde{Q}_{4,2}^{(3,-1)}[q](E, 1)$$

** symmetry

$$\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$\begin{aligned} & \frac{15\sqrt{11}Q_1xyz}{44} + \frac{3\sqrt{11}Q_2z(4x^2 - y^2 - z^2)}{44} + \frac{5\sqrt{66}Q_3y(x-z)(x+z)}{44} + \frac{\sqrt{165}Q_{3x}z(3y^2 - z^2)}{44} \\ & - \frac{5\sqrt{165}Q_{3y}xyz}{44} + \frac{3\sqrt{110}Q_{az}x(y-z)(y+z)}{44} + \frac{\sqrt{66}Q_{bz}x(2x^2 - 3y^2 - 3z^2)}{44} \\ & \frac{3\sqrt{11}Q_1z(x^2 - 4y^2 + z^2)}{44} - \frac{15\sqrt{11}Q_2xyz}{44} + \frac{5\sqrt{66}Q_3x(y-z)(y+z)}{44} - \frac{5\sqrt{165}Q_{3x}xyz}{44} \\ & + \frac{\sqrt{165}Q_{3y}z(3x^2 - z^2)}{44} + \frac{3\sqrt{110}Q_{az}y(x-z)(x+z)}{44} + \frac{\sqrt{66}Q_{bz}y(3x^2 - 2y^2 + 3z^2)}{44} \end{aligned}$$

$$\tilde{Q}_{4,1}^{(3,-1)}[q](E, 2), \tilde{Q}_{4,2}^{(3,-1)}[q](E, 2)$$

** symmetry

$$-\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{15\sqrt{77}Q_1xyz}{308} - \frac{3\sqrt{77}Q_2z(8x^2 + 13y^2 - 7z^2)}{308} + \frac{\sqrt{462}Q_3y(3x^2 - 2y^2 + 3z^2)}{308} - \frac{\sqrt{1155}Q_{3x}z(4x^2 - 7y^2 + z^2)}{308} \\ & + \frac{3\sqrt{1155}Q_{3y}xyz}{308} + \frac{3\sqrt{770}Q_{az}x(2x^2 - 5y^2 - z^2)}{308} + \frac{\sqrt{462}Q_{bz}x(4x^2 - y^2 - 11z^2)}{308} \\ & \frac{3\sqrt{77}Q_1z(13x^2 + 8y^2 - 7z^2)}{308} - \frac{15\sqrt{77}Q_2xyz}{308} - \frac{\sqrt{462}Q_3x(2x^2 - 3y^2 - 3z^2)}{308} + \frac{3\sqrt{1155}Q_{3x}xyz}{308} \\ & + \frac{\sqrt{1155}Q_{3y}z(7x^2 - 4y^2 - z^2)}{308} - \frac{3\sqrt{770}Q_{az}y(5x^2 - 2y^2 + z^2)}{308} + \frac{\sqrt{462}Q_{bz}y(x^2 - 4y^2 + 11z^2)}{308} \end{aligned}$$

$$\tilde{Q}_{4,1}^{(3,1)}[q](E, 1), \tilde{Q}_{4,2}^{(3,1)}[q](E, 1)$$

** symmetry

$$\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$\begin{aligned} & \frac{21\sqrt{1430}Q_1xyz(7x^2 - 2y^2 - 5z^2)}{1144} + \frac{3\sqrt{1430}Q_2z(37x^4 - 87x^2y^2 - 45x^2z^2 + 2y^4 + 25y^2z^2 + 2z^4)}{2288} + \frac{7\sqrt{2145}Q_3y(x-z)(x+z)(x^2 - 2y^2 + z^2)}{572} \\ & + \frac{\sqrt{858}Q_{3x}z(33x^4 - 39x^2y^2 - 53x^2z^2 - 30y^4 + 73y^2z^2 - 2z^4)}{2288} + \frac{7\sqrt{858}Q_{3y}xyz(13x^2 + 10y^2 - 23z^2)}{1144} \\ & - \frac{3\sqrt{143}Q_{az}x(3x^4 + 13x^2y^2 - 43x^2z^2 + 10y^4 - 99y^2z^2 + 38z^4)}{1144} - \frac{\sqrt{2145}Q_{bz}x(x^4 + 37x^2y^2 - 47x^2z^2 - 6y^4 - 75y^2z^2 + 36z^4)}{1144} \\ & - \frac{3\sqrt{1430}Q_1z(2x^4 - 87x^2y^2 + 25x^2z^2 + 37y^4 - 45y^2z^2 + 2z^4)}{2288} + \frac{21\sqrt{1430}Q_2xyz(2x^2 - 7y^2 + 5z^2)}{1144} - \frac{7\sqrt{2145}Q_3x(y-z)(y+z)(2x^2 - y^2 - z^2)}{572} \\ & + \frac{7\sqrt{858}Q_{3x}xyz(10x^2 + 13y^2 - 23z^2)}{1144} - \frac{\sqrt{858}Q_{3y}z(30x^4 + 39x^2y^2 - 73x^2z^2 - 33y^4 + 53y^2z^2 + 2z^4)}{2288} \\ & - \frac{3\sqrt{143}Q_{az}y(10x^4 + 13x^2y^2 - 99x^2z^2 + 3y^4 - 43y^2z^2 + 38z^4)}{1144} - \frac{\sqrt{2145}Q_{bz}y(6x^4 - 37x^2y^2 + 75x^2z^2 - y^4 + 47y^2z^2 - 36z^4)}{1144} \end{aligned}$$

$$\tilde{Q}_{4,1}^{(3,1)}[q](E, 2), \tilde{Q}_{4,2}^{(3,1)}[q](E, 2)$$

** symmetry

$$-\frac{\sqrt{5}xz(x^2 - 6y^2 + z^2)}{2}$$

$$\frac{\sqrt{5}yz(6x^2 - y^2 - z^2)}{2}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{10010}Q_1xyz(7x^2-2y^2-5z^2)}{1144} + \frac{3\sqrt{10010}Q_2z(x^4-21x^2y^2+5x^2z^2-4y^4+15y^2z^2-2z^4)}{2288} \\
& - \frac{\sqrt{15015}Q_{3y}(51x^4-80x^2y^2-66x^2z^2+16y^4-80y^2z^2+51z^4)}{4004} + \frac{\sqrt{6006}Q_{3xz}(187x^4+213x^2y^2-445x^2z^2-268y^4+465y^2z^2-2z^4)}{16016} \\
& - \frac{3\sqrt{6006}Q_{3yxyz}(9x^2+2y^2-11z^2)}{1144} - \frac{3\sqrt{1001}Q_{azx}(17x^4-15x^2y^2-155x^2z^2-32y^4+237y^2z^2+38z^4)}{8008} \\
& - \frac{\sqrt{15015}Q_{bzx}(11x^4-195x^2y^2+85x^2z^2+88y^4+57y^2z^2-52z^4)}{8008} \\
& \frac{3\sqrt{10010}Q_1z(4x^4+21x^2y^2-15x^2z^2-y^4-5y^2z^2+2z^4)}{2288} + \frac{3\sqrt{10010}Q_2xyz(2x^2-7y^2+5z^2)}{1144} \\
& - \frac{\sqrt{15015}Q_{3x}(16x^4-80x^2y^2-80x^2z^2+51y^4-66y^2z^2+51z^4)}{4004} - \frac{3\sqrt{6006}Q_{3xyz}(2x^2+9y^2-11z^2)}{1144} \\
& - \frac{\sqrt{6006}Q_{3yz}(268x^4-213x^2y^2-465x^2z^2-187y^4+445y^2z^2+2z^4)}{16016} \\
& + \frac{3\sqrt{1001}Q_{azx}(32x^4+15x^2y^2-237x^2z^2-17y^4+155y^2z^2-38z^4)}{8008} + \frac{\sqrt{15015}Q_{bzy}(88x^4-195x^2y^2+57x^2z^2+11y^4+85y^2z^2-52z^4)}{8008}
\end{aligned}$$

$$\tilde{\mathbb{Q}}_{4,1}^{(3,3)}[q](E, 1), \tilde{\mathbb{Q}}_{4,2}^{(3,3)}[q](E, 1)$$

** symmetry

$$\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{1430}Q_1xyz(249x^4-305x^2y^2-525x^2z^2+18y^4+245y^2z^2+84z^4)}{5720} \\
& + \frac{\sqrt{1430}Q_2z(184x^6-1185x^4y^2-525x^4z^2+345x^2y^4+1680x^2y^2z^2+147x^2z^4-2y^6-105y^4z^2-105y^2z^4-2z^6)}{5720} \\
& - \frac{3\sqrt{2145}Q_{3y}(x-z)(x+z)(8x^4+5x^2y^2-127x^2z^2-3y^4+5y^2z^2+8z^4)}{1430} \\
& - \frac{\sqrt{858}Q_{3xz}(88x^6+75x^4y^2-465x^4z^2-15x^2y^4-120x^2y^2z^2+291x^2z^4-2y^6+15y^4z^2+3y^2z^4-14z^6)}{1144} \\
& - \frac{3\sqrt{858}Q_{3yxyz}(39x^4+45x^2y^2-175x^2z^2+6y^4-65y^2z^2+72z^4)}{1144} \\
& + \frac{\sqrt{143}Q_{azx}(11x^6+24x^4y^2-255x^4z^2+15x^2y^4-330x^2y^2z^2+480x^2z^4+2y^6-75y^4z^2+240y^2z^4-112z^6)}{572} \\
& - \frac{\sqrt{2145}Q_{bzx}(17x^6-30x^4y^2-327x^4z^2-45x^2y^4+570x^2y^2z^2+450x^2z^4+2y^6+105y^4z^2-390y^2z^4-64z^6)}{2860} \\
& \frac{\sqrt{1430}Q_1z(2x^6-345x^4y^2+105x^4z^2+1185x^2y^4-1680x^2y^2z^2+105x^2z^4-184y^6+525y^4z^2-147y^2z^4+2z^6)}{5720} \\
& - \frac{3\sqrt{1430}Q_2xyz(18x^4-305x^2y^2+245x^2z^2+249y^4-525y^2z^2+84z^4)}{5720} \\
& + \frac{3\sqrt{2145}Q_{3x}(y-z)(y+z)(3x^4-5x^2y^2-5x^2z^2-8y^4+127y^2z^2-8z^4)}{1430} - \frac{3\sqrt{858}Q_{3xyz}(6x^4+45x^2y^2-65x^2z^2+39y^4-175y^2z^2+72z^4)}{1144} \\
& + \frac{\sqrt{858}Q_{3yz}(2x^6+15x^4y^2-15x^4z^2-75x^2y^4+120x^2y^2z^2-3x^2z^4-88y^6+465y^4z^2-291y^2z^4+14z^6)}{1144} \\
& + \frac{\sqrt{143}Q_{azy}(2x^6+15x^4y^2-75x^4z^2+24x^2y^4-330x^2y^2z^2+240x^2z^4+11y^6-255y^4z^2+480y^2z^4-112z^6)}{572} \\
& + \frac{\sqrt{2145}Q_{bzy}(2x^6-45x^4y^2+105x^4z^2-30x^2y^4+570x^2y^2z^2-390x^2z^4+17y^6-327y^4z^2+450y^2z^4-64z^6)}{2860}
\end{aligned}$$

$$\tilde{\mathbb{Q}}_{4,1}^{(3,3)}[q](E, 2), \tilde{\mathbb{Q}}_{4,2}^{(3,3)}[q](E, 2)$$

** symmetry

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

** expression

$$\begin{aligned}
& - \frac{3\sqrt{10010}Q_1xyz(87x^4 - 365x^2y^2 + 75x^2z^2 + 120y^4 - 35y^2z^2 - 12z^4)}{5720} \\
& - \frac{\sqrt{10010}Q_2z(40x^6 - 735x^4y^2 + 45x^4z^2 + 885x^2y^4 - 300x^2y^2z^2 + 3x^2z^4 - 56y^6 - 15y^4z^2 + 39y^2z^4 - 2z^6)}{5720} \\
& + \frac{\sqrt{15015}Q_3y(8x^6 - 15x^4y^2 - 75x^4z^2 - 21x^2y^4 + 300x^2y^2z^2 - 75x^2z^4 + 2y^6 - 21y^4z^2 - 15y^2z^4 + 8z^6)}{1430} \\
& + \frac{\sqrt{6006}Q_3xz(8x^6 - 75x^4y^2 - 15x^4z^2 - 75x^2y^4 + 300x^2y^2z^2 - 21x^2z^4 + 8y^6 - 15y^4z^2 - 21y^2z^4 + 2z^6)}{1144} \\
& + \frac{3\sqrt{6006}Q_3yxyz(9x^4 - 15x^2y^2 - 15x^2z^2 - 24y^4 + 95y^2z^2 - 24z^4)}{1144} \\
& - \frac{\sqrt{1001}Q_{az}x(x^6 - 6x^4y^2 - 15x^4z^2 - 15x^2y^4 + 150x^2y^2z^2 - 8y^6 + 165y^4z^2 - 240y^2z^4 + 16z^6)}{572} \\
& + \frac{3\sqrt{15015}Q_{bz}x(x^6 - 12x^4y^2 - 9x^4z^2 - 5x^2y^4 + 150x^2y^2z^2 - 10x^2z^4 + 8y^6 - 105y^4z^2 + 30y^2z^4)}{2860} \\
& - \frac{\sqrt{10010}Q_1z(56x^6 - 885x^4y^2 + 15x^4z^2 + 735x^2y^4 + 300x^2y^2z^2 - 39x^2z^4 - 40y^6 - 45y^4z^2 - 3y^2z^4 + 2z^6)}{5720} \\
& + \frac{3\sqrt{10010}Q_2xyz(120x^4 - 365x^2y^2 - 35x^2z^2 + 87y^4 + 75y^2z^2 - 12z^4)}{5720} \\
& + \frac{\sqrt{15015}Q_3x(2x^6 - 21x^4y^2 - 21x^4z^2 - 15x^2y^4 + 300x^2y^2z^2 - 15x^2z^4 + 8y^6 - 75y^4z^2 - 75y^2z^4 + 8z^6)}{1430} \\
& - \frac{3\sqrt{6006}Q_3xxyz(24x^4 + 15x^2y^2 - 95x^2z^2 - 9y^4 + 15y^2z^2 + 24z^4)}{1144} \\
& + \frac{\sqrt{6006}Q_3yz(8x^6 - 75x^4y^2 - 15x^4z^2 - 75x^2y^4 + 300x^2y^2z^2 - 21x^2z^4 + 8y^6 - 15y^4z^2 - 21y^2z^4 + 2z^6)}{1144} \\
& + \frac{\sqrt{1001}Q_{az}y(8x^6 + 15x^4y^2 - 165x^4z^2 + 6x^2y^4 - 150x^2y^2z^2 + 240x^2z^4 - y^6 + 15y^4z^2 - 16z^6)}{572} \\
& - \frac{3\sqrt{15015}Q_{bz}y(8x^6 - 5x^4y^2 - 105x^4z^2 - 12x^2y^4 + 150x^2y^2z^2 + 30x^2z^4 + y^6 - 9y^4z^2 - 10y^2z^4)}{2860}
\end{aligned}$$