

PG No. 15 D_{4h} $4/mmm$ [tetragonal] (axial, internal polar quadrupole)

* Harmonics for rank 0

* Harmonics for rank 1

$$\vec{G}_1^{(2,1)}[q](A_{2g})$$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{30}Q_vxy}{5} - \frac{\sqrt{30}Q_{xy}(x-y)(x+y)}{10} + \frac{\sqrt{30}Q_{xz}yz}{10} - \frac{\sqrt{30}Q_{yz}xz}{10}$$

$$\vec{G}_{1,1}^{(2,1)}[q](E_g), \vec{G}_{1,2}^{(2,1)}[q](E_g)$$

** symmetry

$$x$$

$$-y$$

** expression

$$-\frac{3\sqrt{10}Q_{u}yz}{10} - \frac{\sqrt{30}Q_vyz}{10} + \frac{\sqrt{30}Q_{xy}xz}{10} - \frac{\sqrt{30}Q_{xz}xy}{10} - \frac{\sqrt{30}Q_{yz}(y-z)(y+z)}{10}$$

$$-\frac{3\sqrt{10}Q_{u}xz}{10} + \frac{\sqrt{30}Q_vxz}{10} + \frac{\sqrt{30}Q_{xy}yz}{10} - \frac{\sqrt{30}Q_{xz}(x-z)(x+z)}{10} - \frac{\sqrt{30}Q_{yz}xy}{10}$$

* Harmonics for rank 2

$$\vec{G}_2^{(2,-1)}[q](A_{1u})$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{2}Q_{xz}y}{2} - \frac{\sqrt{2}Q_{yz}x}{2}$$

$$\vec{G}_2^{(2,1)}[q](A_{1u})$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{5\sqrt{42}Q_vxyz}{14} - \frac{5\sqrt{42}Q_{xy}z(x-y)(x+y)}{28} - \frac{\sqrt{42}Q_{xz}y(x^2+y^2-4z^2)}{28} + \frac{\sqrt{42}Q_{yz}x(x^2+y^2-4z^2)}{28}$$

$$\vec{G}_2^{(2,-1)}[q](B_{1u})$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{6}Q_{xy}z}{3} - \frac{\sqrt{6}Q_{xz}y}{6} - \frac{\sqrt{6}Q_{yz}x}{6}$$

$$\vec{G}_2^{(2,1)}[q](B_{1u})$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$-\frac{5\sqrt{42}Q_{u}xyz}{14} + \frac{\sqrt{14}Q_{xy}z(3x^2+3y^2-2z^2)}{28} - \frac{\sqrt{14}Q_{xz}y(9x^2-y^2-6z^2)}{28} + \frac{\sqrt{14}Q_{yz}x(x^2-9y^2+6z^2)}{28}$$

$$\vec{G}_2^{(2,-1)}[q](B_{2u})$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{6}Q_v z}{3} + \frac{\sqrt{6}Q_{xz}x}{6} - \frac{\sqrt{6}Q_{yz}y}{6}$$

$$\vec{\mathbb{G}}_2^{(2,1)}[q](B_{2u})$$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{5\sqrt{42}Q_u z (x-y)(x+y)}{28} - \frac{\sqrt{14}Q_v z (3x^2 + 3y^2 - 2z^2)}{28} + \frac{\sqrt{14}Q_{xz}x (2x^2 - 3y^2 - 3z^2)}{14} + \frac{\sqrt{14}Q_{yz}y (3x^2 - 2y^2 + 3z^2)}{14}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,-1)}[q](E_u), \vec{\mathbb{G}}_{2,2}^{(2,-1)}[q](E_u)$$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_u x}{2} + \frac{\sqrt{6}Q_v x}{6} + \frac{\sqrt{6}Q_{xy}y}{6} - \frac{\sqrt{6}Q_{xz}z}{6}$$

$$\frac{\sqrt{2}Q_u y}{2} - \frac{\sqrt{6}Q_v y}{6} + \frac{\sqrt{6}Q_{xy}x}{6} - \frac{\sqrt{6}Q_{yz}z}{6}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,1)}[q](E_u), \vec{\mathbb{G}}_{2,2}^{(2,1)}[q](E_u)$$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$-\frac{\sqrt{42}Q_u x (x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{14}Q_v x (x^2 - 9y^2 + 6z^2)}{28} - \frac{\sqrt{14}Q_{xy}y (3x^2 - 2y^2 + 3z^2)}{14} + \frac{\sqrt{14}Q_{xz}z (3x^2 + 3y^2 - 2z^2)}{14}$$

$$-\frac{\sqrt{42}Q_u y (x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{14}Q_v y (9x^2 - y^2 - 6z^2)}{28} + \frac{\sqrt{14}Q_{xy}x (2x^2 - 3y^2 - 3z^2)}{14} + \frac{\sqrt{14}Q_{yz}z (3x^2 + 3y^2 - 2z^2)}{14}$$

* Harmonics for rank 3

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](A_{2g})$$

** symmetry

$$-\frac{z (3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{\sqrt{30}Q_v xy}{10} + \frac{\sqrt{30}Q_{xy} (x-y)(x+y)}{20} + \frac{\sqrt{30}Q_{xz}yz}{5} - \frac{\sqrt{30}Q_{yz}xz}{5}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](A_{2g})$$

** symmetry

$$-\frac{z (3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_v xy (x^2 + y^2 - 6z^2)}{6} + \frac{\sqrt{15}Q_{xy} (x-y)(x+y) (x^2 + y^2 - 6z^2)}{12} - \frac{\sqrt{15}Q_{xz}yz (3x^2 + 3y^2 - 4z^2)}{12} + \frac{\sqrt{15}Q_{yz}xz (3x^2 + 3y^2 - 4z^2)}{12}$$

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](B_{1g})$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{6}Q_u (x-y)(x+y)}{4} + \frac{\sqrt{2}Q_v (x^2 + y^2 - 2z^2)}{4}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](B_{1g})$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$-\frac{\sqrt{3}Q_u(x-y)(x+y)(x^2+y^2-6z^2)}{6}-\frac{Q_v(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4)}{6}-\frac{7Q_{xy}xy(x-y)(x+y)}{6}+\frac{7Q_{xz}xz(x-z)(x+z)}{6}-\frac{7Q_{yz}yz(y-z)(y+z)}{6}$$

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](B_{2g})$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$-\frac{\sqrt{6}Q_u xy}{2}-\frac{\sqrt{2}Q_{xy}(x^2+y^2-2z^2)}{4}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](B_{2g})$$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{3}Q_u xy(x^2+y^2-6z^2)}{3}+\frac{7Q_v xy(x-y)(x+y)}{6}-\frac{Q_{xy}(5x^4-18x^2y^2-12x^2z^2+5y^4-12y^2z^2+4z^4)}{12}-\frac{7Q_{xz}yz(3x^2+y^2-2z^2)}{12}-\frac{7Q_{yz}xz(x^2+3y^2-2z^2)}{12}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](E_g,1), \vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](E_g,1)$$

** symmetry

$$\frac{x(2x^2-3y^2-3z^2)}{2}$$

$$\frac{y(3x^2-2y^2+3z^2)}{2}$$

** expression

$$\frac{3\sqrt{10}Q_u yz}{20}+\frac{\sqrt{30}Q_v yz}{20}+\frac{\sqrt{30}Q_{xy}xz}{5}-\frac{\sqrt{30}Q_{xz}xy}{5}+\frac{\sqrt{30}Q_{yz}(y-z)(y+z)}{20}$$

$$\frac{3\sqrt{10}Q_u xz}{20}-\frac{\sqrt{30}Q_v xz}{20}+\frac{\sqrt{30}Q_{xy}yz}{5}+\frac{\sqrt{30}Q_{xz}(x-z)(x+z)}{20}-\frac{\sqrt{30}Q_{yz}xy}{5}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](E_g,2), \vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](E_g,2)$$

** symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

** expression

$$\frac{\sqrt{6}Q_u yz}{4}-\frac{3\sqrt{2}Q_v yz}{4}+\frac{\sqrt{2}Q_{yz}(2x^2-y^2-z^2)}{4}$$

$$\frac{\sqrt{6}Q_u xz}{4}+\frac{3\sqrt{2}Q_v xz}{4}-\frac{\sqrt{2}Q_{xz}(x^2-2y^2+z^2)}{4}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](E_g,1), \vec{\mathbb{G}}_{3,2}^{(2,1)}[q](E_g,1)$$

** symmetry

$$\frac{x(2x^2-3y^2-3z^2)}{2}$$

$$\frac{y(3x^2-2y^2+3z^2)}{2}$$

** expression

$$\begin{aligned} & -\frac{\sqrt{5}Q_u y z (6x^2 - y^2 - z^2)}{4} - \frac{\sqrt{15}Q_v y z (6x^2 - y^2 - z^2)}{12} + \frac{\sqrt{15}Q_{xy} x z (4x^2 - 3y^2 - 3z^2)}{12} \\ & - \frac{\sqrt{15}Q_{xz} x y (4x^2 - 3y^2 - 3z^2)}{12} - \frac{\sqrt{15}Q_{yz} (y - z) (y + z) (6x^2 - y^2 - z^2)}{12} \\ & \frac{\sqrt{5}Q_u x z (x^2 - 6y^2 + z^2)}{4} - \frac{\sqrt{15}Q_v x z (x^2 - 6y^2 + z^2)}{12} - \frac{\sqrt{15}Q_{xy} y z (3x^2 - 4y^2 + 3z^2)}{12} \\ & + \frac{\sqrt{15}Q_{xz} (x - z) (x + z) (x^2 - 6y^2 + z^2)}{12} + \frac{\sqrt{15}Q_{yz} x y (3x^2 - 4y^2 + 3z^2)}{12} \end{aligned}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](E_g, 2), \vec{\mathbb{G}}_{3,2}^{(2,1)}[q](E_g, 2)$$

** symmetry

$$\begin{aligned} & \frac{\sqrt{15}x(y-z)(y+z)}{2} \\ & - \frac{\sqrt{15}y(x-z)(x+z)}{2} \end{aligned}$$

** expression

$$\begin{aligned} & \frac{\sqrt{3}Q_u y z (12x^2 - 9y^2 + 5z^2)}{12} - \frac{Q_v y z (36x^2 + y^2 - 13z^2)}{12} + \frac{7Q_{xy} x z (2x^2 - 3y^2 - z^2)}{12} \\ & + \frac{7Q_{xz} x y (2x^2 - y^2 - 3z^2)}{12} - \frac{Q_{yz} (4x^4 - 12x^2 y^2 - 12x^2 z^2 + 5y^4 - 18y^2 z^2 + 5z^4)}{12} \\ & - \frac{\sqrt{3}Q_u x z (9x^2 - 12y^2 - 5z^2)}{12} + \frac{Q_v x z (x^2 + 36y^2 - 13z^2)}{12} - \frac{7Q_{xy} y z (3x^2 - 2y^2 + z^2)}{12} \\ & - \frac{Q_{xz} (5x^4 - 12x^2 y^2 - 18x^2 z^2 + 4y^4 - 12y^2 z^2 + 5z^4)}{12} - \frac{7Q_{yz} x y (x^2 - 2y^2 + 3z^2)}{12} \end{aligned}$$

* Harmonics for rank 4

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](A_{1u}, 1)$$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2 y^2 - 3x^2 z^2 + y^4 - 3y^2 z^2 + z^4)}{6}$$

** expression

$$\frac{\sqrt{5}Q_{xy} z (x - y) (x + y)}{2} - \frac{\sqrt{5}Q_{xz} y (x - z) (x + z)}{2} + \frac{\sqrt{5}Q_{yz} x (y - z) (y + z)}{2}$$

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](A_{1u}, 2)$$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2 y^2 + 6x^2 z^2 + y^4 + 6y^2 z^2 - 2z^4)}{12}$$

** expression

$$-\frac{6\sqrt{7}Q_v x y z}{7} - \frac{\sqrt{7}Q_{xy} z (x - y) (x + y)}{14} + \frac{\sqrt{7}Q_{xz} y (4x^2 - 3y^2 + 5z^2)}{14} + \frac{\sqrt{7}Q_{yz} x (3x^2 - 4y^2 - 5z^2)}{14}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](A_{1u}, 1)$$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2 y^2 - 3x^2 z^2 + y^4 - 3y^2 z^2 + z^4)}{6}$$

** expression

$$\begin{aligned} & -\frac{3\sqrt{2310}Q_u x y z (x - y) (x + y)}{44} - \frac{3\sqrt{770}Q_v x y z (x^2 + y^2 - 2z^2)}{44} + \frac{\sqrt{770}Q_{xy} z (x - y) (x + y) (x^2 + y^2 - 2z^2)}{22} \\ & - \frac{\sqrt{770}Q_{xz} y (x - z) (x + z) (x^2 - 2y^2 + z^2)}{22} - \frac{\sqrt{770}Q_{yz} x (y - z) (y + z) (2x^2 - y^2 - z^2)}{22} \end{aligned}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](A_{1u}, 2)$$

** symmetry

$$-\frac{\sqrt{15}(x^4 - 12x^2 y^2 + 6x^2 z^2 + y^4 + 6y^2 z^2 - 2z^4)}{12}$$

** expression

$$\frac{21\sqrt{66}Q_uxyz(x-y)(x+y)}{44} - \frac{21\sqrt{22}Q_vxyz(x^2+y^2-2z^2)}{44} + \frac{7\sqrt{22}Q_{xy}z(x-y)(x+y)(x^2+y^2-2z^2)}{44} + \frac{\sqrt{22}Q_{xz}y(17x^4-22x^2y^2-36x^2z^2+3y^4-8y^2z^2+10z^4)}{44} - \frac{\sqrt{22}Q_{yz}x(3x^4-22x^2y^2-8x^2z^2+17y^4-36y^2z^2+10z^4)}{44}$$

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](A_{2u})$$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_vz(x-y)(x+y)}{2} + \sqrt{3}Q_{xy}xyz + \frac{\sqrt{3}Q_{xz}x(x^2-3y^2)}{4} - \frac{\sqrt{3}Q_{yz}y(3x^2-y^2)}{4}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](A_{2u})$$

** symmetry

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{9\sqrt{154}Q_uz(x^2-2xy-y^2)(x^2+2xy-y^2)}{88} - \frac{\sqrt{462}Q_vz(x-y)(x+y)(x^2+y^2-2z^2)}{88} + \frac{\sqrt{462}Q_{xy}xyz(x^2+y^2-2z^2)}{44} + \frac{\sqrt{462}Q_{xz}x(x^4-8x^2y^2-2x^2z^2+3y^4+6y^2z^2)}{44} + \frac{\sqrt{462}Q_{yz}y(3x^4-8x^2y^2+6x^2z^2+y^4-2y^2z^2)}{44}$$

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](B_{1u})$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$\frac{6\sqrt{7}Q_uxyz}{7} + \frac{\sqrt{21}Q_{xy}z(3x^2+3y^2-2z^2)}{14} - \frac{\sqrt{21}Q_{xz}y(2x^2-y^2+z^2)}{14} + \frac{\sqrt{21}Q_{yz}x(x^2-2y^2-z^2)}{14}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](B_{1u})$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

** expression

$$-\frac{21\sqrt{22}Q_uxyz(x^2+y^2-2z^2)}{44} - \frac{21\sqrt{66}Q_vxyz(x-y)(x+y)}{44} + \frac{\sqrt{66}Q_{xy}z(9x^4-24x^2y^2-10x^2z^2+9y^4-10y^2z^2+2z^4)}{44} - \frac{\sqrt{66}Q_{xz}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44} - \frac{\sqrt{66}Q_{yz}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44}$$

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](B_{2u})$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$\frac{3\sqrt{7}Q_uz(x-y)(x+y)}{7} + \frac{\sqrt{21}Q_vz(3x^2+3y^2-2z^2)}{14} - \frac{\sqrt{21}Q_{xz}x(x^2-5y^2+2z^2)}{28} - \frac{\sqrt{21}Q_{yz}y(5x^2-y^2-2z^2)}{28}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](B_{2u})$$

** symmetry

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$-\frac{21\sqrt{22}Q_uz(x-y)(x+y)(x^2+y^2-2z^2)}{88} - \frac{\sqrt{66}Q_vz(3x^4-78x^2y^2+20x^2z^2+3y^4+20y^2z^2-4z^4)}{88} - \frac{21\sqrt{66}Q_{xy}xyz(x-y)(x+y)}{44} - \frac{\sqrt{66}Q_{xz}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44} + \frac{\sqrt{66}Q_{yz}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E_u, 1), \vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E_u, 1)$$

** symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

** expression

$$\frac{3Q_u x(y-z)(y+z)}{4} + \frac{\sqrt{3}Q_v x(y-z)(y+z)}{4} + \frac{\sqrt{3}Q_{xy} y(y^2-3z^2)}{4} - \frac{\sqrt{3}Q_{xz} z(3y^2-z^2)}{4} + \sqrt{3}Q_{yz} xyz$$

$$\frac{3Q_u y(x-z)(x+z)}{4} - \frac{\sqrt{3}Q_v y(x-z)(x+z)}{4} + \frac{\sqrt{3}Q_{xy} x(x^2-3z^2)}{4} + \sqrt{3}Q_{xz} xyz - \frac{\sqrt{3}Q_{yz} z(3x^2-z^2)}{4}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E_u, 2), \vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E_u, 2)$$

** symmetry

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

$$\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

** expression

$$\frac{3\sqrt{7}Q_u x(2x^2-5y^2-z^2)}{28} + \frac{\sqrt{21}Q_v x(2x^2+3y^2-9z^2)}{28} - \frac{\sqrt{21}Q_{xy} y(2x^2+y^2-5z^2)}{28} + \frac{\sqrt{21}Q_{xz} z(2x^2-5y^2+z^2)}{28}$$

$$-\frac{3\sqrt{7}Q_u y(5x^2-2y^2+z^2)}{28} - \frac{\sqrt{21}Q_v y(3x^2+2y^2-9z^2)}{28} - \frac{\sqrt{21}Q_{xy} x(x^2+2y^2-5z^2)}{28} - \frac{\sqrt{21}Q_{yz} z(5x^2-2y^2-z^2)}{28}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E_u, 1), \vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E_u, 1)$$

** symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

** expression

$$-\frac{3\sqrt{154}Q_u x(x^2y^2-x^2z^2+y^4-9y^2z^2+2z^4)}{88} - \frac{\sqrt{462}Q_v x(x^2y^2-x^2z^2-5y^4+27y^2z^2-4z^4)}{88}$$

$$-\frac{\sqrt{462}Q_{xy} y(2x^2y^2-6x^2z^2-y^4+8y^2z^2-3z^4)}{44} + \frac{\sqrt{462}Q_{xz} z(6x^2y^2-2x^2z^2+3y^4-8y^2z^2+z^4)}{44} - \frac{\sqrt{462}Q_{yz} xyz(2x^2-y^2-z^2)}{44}$$

$$-\frac{3\sqrt{154}Q_u y(x^4+x^2y^2-9x^2z^2-y^2z^2+2z^4)}{88} - \frac{\sqrt{462}Q_v y(5x^4-x^2y^2-27x^2z^2+y^2z^2+4z^4)}{88}$$

$$+\frac{\sqrt{462}Q_{xy} x(x^4-2x^2y^2-8x^2z^2+6y^2z^2+3z^4)}{44} + \frac{\sqrt{462}Q_{xz} xyz(x^2-2y^2+z^2)}{44} + \frac{\sqrt{462}Q_{yz} z(3x^4+6x^2y^2-8x^2z^2-2y^2z^2+z^4)}{44}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E_u, 2), \vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E_u, 2)$$

** symmetry

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

$$\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

** expression

$$-\frac{3\sqrt{22}Q_u x(2x^4-3x^2y^2-17x^2z^2-5y^4+39y^2z^2+2z^4)}{88} - \frac{\sqrt{66}Q_v x(2x^4-31x^2y^2+11x^2z^2+9y^4+39y^2z^2-12z^4)}{88}$$

$$-\frac{\sqrt{66}Q_{xy} y(8x^4-12x^2y^2-12x^2z^2+y^4+2y^2z^2+z^4)}{44} + \frac{\sqrt{66}Q_{xz} z(8x^4-12x^2y^2-12x^2z^2+y^4+2y^2z^2+z^4)}{44} - \frac{21\sqrt{66}Q_{yz} xyz(y-z)(y+z)}{44}$$

$$\frac{3\sqrt{22}Q_u y(5x^4+3x^2y^2-39x^2z^2-2y^4+17y^2z^2-2z^4)}{88} + \frac{\sqrt{66}Q_v y(9x^4-31x^2y^2+39x^2z^2+2y^4+11y^2z^2-12z^4)}{88}$$

$$-\frac{\sqrt{66}Q_{xy} x(x^4-12x^2y^2+2x^2z^2+8y^4-12y^2z^2+z^4)}{44} - \frac{21\sqrt{66}Q_{xz} xyz(x-z)(x+z)}{44} + \frac{\sqrt{66}Q_{yz} z(x^4-12x^2y^2+2x^2z^2+8y^4-12y^2z^2+z^4)}{44}$$