

* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

| No. | tag | matrix (polar) | det | TR |
|-----|---|---|-----|----|
| 1 | $\{1 0\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | 1 | 1 |
| 2 | $\{2_{100} 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | 1 | 1 |
| 3 | $\{2_{010} \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | 1 | 1 |
| 4 | $\{2_{001} \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | 1 | 1 |
| 5 | $\{3_{111}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | 1 | 1 |
| 6 | $\{3_{111}^- 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | 1 | 1 |
| 7 | $\{3_{1-1-1}^- \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | 1 | 1 |
| 8 | $\{3_{1-1-1}^+ \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | 1 | 1 |
| 9 | $\{3_{-11-1}^- \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | 1 | 1 |
| 10 | $\{3_{-11-1}^+ 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \end{bmatrix}$ | 1 | 1 |
| 11 | $\{3_{-1-11}^- 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | 1 | 1 |
| 12 | $\{3_{-1-11}^+ \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{2} \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \end{bmatrix}$ | 1 | 1 |
| 13 | $\{-1 0\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | -1 | 1 |

continued ...

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|-----|--|--|-----|----|
| 14 | $\{m_{100} 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | -1 | 1 |
| 15 | $\{m_{010} \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | -1 | 1 |
| 16 | $\{m_{001} \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | -1 | 1 |
| 17 | $\{-3_{111}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | -1 | 1 |
| 18 | $\{-3_{111}^- 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | -1 | 1 |
| 19 | $\{-3_{1-1-1}^- \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | -1 | 1 |
| 20 | $\{-3_{1-1-1}^+ \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | -1 | 1 |
| 21 | $\{-3_{-11-1}^- \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | -1 | 1 |
| 22 | $\{-3_{-11-1}^+ 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \end{bmatrix}$ | -1 | 1 |
| 23 | $\{-3_{-1-11}^- 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | -1 | 1 |
| 24 | $\{-3_{-1-11}^+ \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \end{bmatrix}$ | -1 | 1 |
| 25 | $\{4_{100}^+ \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \end{bmatrix}$ | 1 | -1 |
| 26 | $\{4_{100}^- \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | 1 | -1 |
| 27 | $\{4_{010}^+ \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | 1 | -1 |

continued ...

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| No. | tag | matrix (polar) | det | TR |
|-----|--|---|-----|----|
| 28 | $\{4_{010}^{-\prime} 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | 1 | -1 |
| 29 | $\{4_{001}^{+\prime} 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | 1 | -1 |
| 30 | $\{4_{001}^{-\prime} \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$ | 1 | -1 |
| 31 | $\{2_{110}^{\prime} \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | 1 | -1 |
| 32 | $\{2_{1-10}^{\prime} 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | 1 | -1 |
| 33 | $\{2_{011}^{\prime} 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \end{bmatrix}$ | 1 | -1 |
| 34 | $\{2_{01-1}^{\prime} 0\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | 1 | -1 |
| 35 | $\{2_{101}^{\prime} \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | 1 | -1 |
| 36 | $\{2_{-101}^{\prime} 0\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | 1 | -1 |
| 37 | $\{-4_{100}^{+\prime} \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \end{bmatrix}$ | -1 | -1 |
| 38 | $\{-4_{100}^{-\prime} \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | -1 | -1 |
| 39 | $\{-4_{010}^{+\prime} \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | -1 | -1 |
| 40 | $\{-4_{010}^{-\prime} 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | -1 | -1 |
| 41 | $\{-4_{001}^{+\prime} 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1 | -1 |

continued ...

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| No. | tag | matrix (polar) | det | TR |
|-----|--|---|-----|----|
| 42 | $\{-4_{001}^- \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1 | -1 |
| 43 | $\{m_{110}' \frac{1}{2}\frac{1}{2}0\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | -1 | -1 |
| 44 | $\{m_{1-10}' 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | -1 | -1 |
| 45 | $\{m_{011}' 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \end{bmatrix}$ | -1 | -1 |
| 46 | $\{m_{01-1}' 0\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | -1 | -1 |
| 47 | $\{m_{101}' \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | -1 | -1 |
| 48 | $\{m_{-101}' 0\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | -1 | -1 |