

PG No. 37 $D_{3h}(1) \quad \bar{6}m2 \quad (-62m \text{ setting}) \quad [\text{hexagonal}] \quad (\text{axial, internal polar dipole})$

* Harmonics for rank 0

* Harmonics for rank 1

$$\vec{G}_1^{(1,0)}[q](A'_2)$$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}Q_x y}{2} - \frac{\sqrt{2}Q_y x}{2}$$

$$\vec{G}_{1,1}^{(1,0)}[q](E''), \vec{G}_{1,2}^{(1,0)}[q](E'')$$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{\sqrt{2}Q_y z}{2} - \frac{\sqrt{2}Q_z y}{2}$$

$$-\frac{\sqrt{2}Q_x z}{2} + \frac{\sqrt{2}Q_z x}{2}$$

* Harmonics for rank 2

$$\vec{G}_2^{(1,0)}[q](A'_1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{6}Q_x y z}{2} - \frac{\sqrt{6}Q_y x z}{2}$$

$$\vec{G}_{2,1}^{(1,0)}[q](E'), \vec{G}_{2,2}^{(1,0)}[q](E')$$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_x (y-z)(y+z)}{2} - \frac{\sqrt{2}Q_y xy}{2} + \frac{\sqrt{2}Q_z xz}{2}$$

$$-\frac{\sqrt{2}Q_x xy}{2} + \frac{\sqrt{2}Q_y (x-z)(x+z)}{2} + \frac{\sqrt{2}Q_z yz}{2}$$

$$\vec{G}_{2,1}^{(1,0)}[q](E''), \vec{G}_{2,2}^{(1,0)}[q](E'')$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}Q_x y z}{2} + \frac{\sqrt{2}Q_y x z}{2} - \sqrt{2}Q_z xy$$

$$\frac{\sqrt{2}Q_x x z}{2} - \frac{\sqrt{2}Q_y y z}{2} - \frac{\sqrt{2}Q_z (x-y)(x+y)}{2}$$

* Harmonics for rank 3

$$\vec{G}_3^{(1,0)}[q](A'_1)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{30}Q_xxyz}{4} + \frac{\sqrt{30}Q_yz(x-y)(x+y)}{8} - \frac{\sqrt{30}Q_z y(3x^2 - y^2)}{8}$$

$$\vec{\mathbb{G}}_3^{(1,0)}[q](A'_2)$$

** symmetry

$$- \frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$- \frac{\sqrt{3}Q_{xy}(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{3}Q_yx(x^2 + y^2 - 4z^2)}{4}$$

$$\vec{\mathbb{G}}_3^{(1,0)}[q](A''_2)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$- \frac{\sqrt{30}Q_xz(x-y)(x+y)}{8} + \frac{\sqrt{30}Q_yxyz}{4} + \frac{\sqrt{30}Q_zx(x^2 - 3y^2)}{8}$$

$$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E'), \vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E')$$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{5}Q_{xx}(y-z)(y+z)}{2} - \frac{\sqrt{5}Q_yy(x-z)(x+z)}{2} + \frac{\sqrt{5}Q_zz(x-y)(x+y)}{2}$$

$$\frac{\sqrt{5}Q_{xy}(x^2 - y^2 + 2z^2)}{4} - \frac{\sqrt{5}Q_yx(x^2 - y^2 - 2z^2)}{4} - \sqrt{5}Q_zxyz$$

$$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E''), \vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E'')$$

** symmetry

$$- \frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$- \frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$\frac{5\sqrt{2}Q_xxyz}{4} - \frac{\sqrt{2}Q_yz(11x^2 + y^2 - 4z^2)}{8} + \frac{\sqrt{2}Q_z y(x^2 + y^2 - 4z^2)}{8}$$

$$\frac{\sqrt{2}Q_xz(x^2 + 11y^2 - 4z^2)}{8} - \frac{5\sqrt{2}Q_yxyz}{4} - \frac{\sqrt{2}Q_zx(x^2 + y^2 - 4z^2)}{8}$$

* Harmonics for rank 4

$$\vec{\mathbb{G}}_4^{(1,0)}[q](A'_1)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\frac{\sqrt{14}Q_x(3x^2y^2 - 3x^2z^2 - y^4 + 3y^2z^2)}{8} - \frac{\sqrt{14}Q_yxy(3x^2 - y^2 - 6z^2)}{8} + \frac{3\sqrt{14}Q_zxz(x^2 - 3y^2)}{8}$$

$$\vec{\mathbb{G}}_4^{(1,0)}[q](A''_1)$$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$-\frac{\sqrt{5}Q_x y z (3x^2 + 3y^2 - 4z^2)}{4} + \frac{\sqrt{5}Q_y x z (3x^2 + 3y^2 - 4z^2)}{4}$$

$$\vec{\mathbb{G}}_4^{(1,0)}[q](A'_2)$$

** symmetry

$$\frac{\sqrt{70}x z (x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{14}Q_x x y (x^2 - 3y^2 + 6z^2)}{8} - \frac{\sqrt{14}Q_y (x^4 - 3x^2 y^2 - 3x^2 z^2 + 3y^2 z^2)}{8} - \frac{3\sqrt{14}Q_z y z (3x^2 - y^2)}{8}$$

$$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E'), \vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E')$$

** symmetry

$$-\frac{\sqrt{10}y z (3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}x z (3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$-\frac{\sqrt{2}Q_x (3x^2 y^2 - 3x^2 z^2 + 3y^4 - 21y^2 z^2 + 4z^4)}{8} + \frac{3\sqrt{2}Q_y x y (x^2 + y^2 - 6z^2)}{8} - \frac{\sqrt{2}Q_z x z (3x^2 + 3y^2 - 4z^2)}{8}$$

$$\frac{3\sqrt{2}Q_x x y (x^2 + y^2 - 6z^2)}{8} - \frac{\sqrt{2}Q_y (3x^4 + 3x^2 y^2 - 21x^2 z^2 - 3y^2 z^2 + 4z^4)}{8} - \frac{\sqrt{2}Q_z y z (3x^2 + 3y^2 - 4z^2)}{8}$$

$$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E'', 1), \vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E'', 1)$$

** symmetry

$$\frac{\sqrt{35} (x^2 - 2xy - y^2) (x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy (x - y) (x + y)}{2}$$

** expression

$$\frac{\sqrt{7}Q_x y z (3x^2 - y^2)}{4} + \frac{\sqrt{7}Q_y x z (x^2 - 3y^2)}{4} - \sqrt{7}Q_z x y (x - y) (x + y)$$

$$-\frac{\sqrt{7}Q_x x z (x^2 - 3y^2)}{4} + \frac{\sqrt{7}Q_y y z (3x^2 - y^2)}{4} + \frac{\sqrt{7}Q_z (x^2 - 2xy - y^2) (x^2 + 2xy - y^2)}{4}$$

$$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E'', 2), \vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E'', 2)$$

** symmetry

$$-\frac{\sqrt{5} (x - y) (x + y) (x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy (x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\frac{Q_x y z (3x^2 - 4y^2 + 3z^2)}{2} - \frac{Q_y x z (4x^2 - 3y^2 - 3z^2)}{2} + \frac{Q_z x y (x^2 + y^2 - 6z^2)}{2}$$

$$-\frac{Q_x x z (x^2 + 15y^2 - 6z^2)}{4} + \frac{Q_y y z (15x^2 + y^2 - 6z^2)}{4} + \frac{Q_z (x - y) (x + y) (x^2 + y^2 - 6z^2)}{4}$$