1 Eigenvectors and eigenvalues

2 Similar matrices

Two $n \times n$ matrices A and C are **similar** if

Proposition: If A and C are similar matrices, they have the same eigenvalues.

3 Diagonalizable matrices

An $n \times n$ matrix A is **diagonalizable** if it is

Proposition: An $n \times n$ matrix A has n independent eigenvectors $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$, with eigenvalues $\{\lambda_1, \ldots, \lambda_n\}$, if and only if

4 Diagonalization as change of basis

If $A = X\Lambda X^{-1}$, A and Λ are connected by the **change of basis** matrix X.

If $A = X\Lambda X^{-1}$, A and Λ are the same transformation with respect to different bases.

Example:

$$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 1/2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} -2/3 & 1/3 \\ 2/3 & 2/3 \end{pmatrix}$$

- $n \times n$ matrix A
- n independent eigenvectors $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$
- eigenvalues $\{\lambda_1, \ldots, \lambda_n\}$

- $A(c_1\mathbf{x}_1 + \cdots + c_n\mathbf{x}_n) =$
- $\Lambda(c_1\mathbf{e}_1 + \cdots + c_n\mathbf{e}_n) =$

5 Powers of a matrix

Suppose $A = X\Lambda X^{-1}$.

Then $A^n =$