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Probabilistic
Seismic Hazard & Fragility Analysis
Hands-on: Matlab Tutorials

Ph.D. Student Chiara Nardin – M.Sc., Eng. in Civil Engineering

https://github.com/kia13nn/ISPS.git

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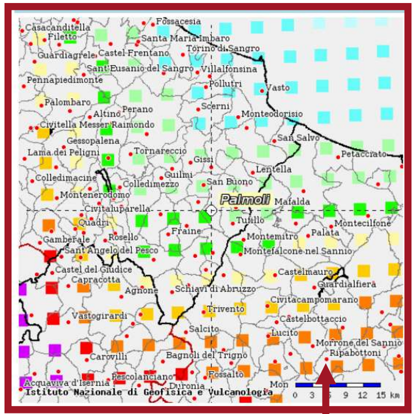
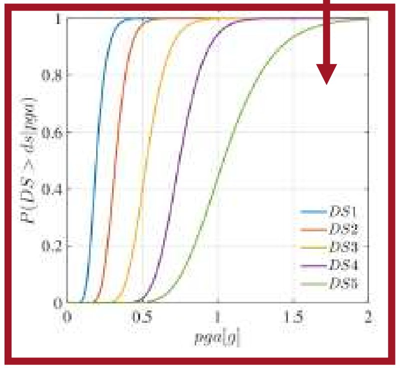
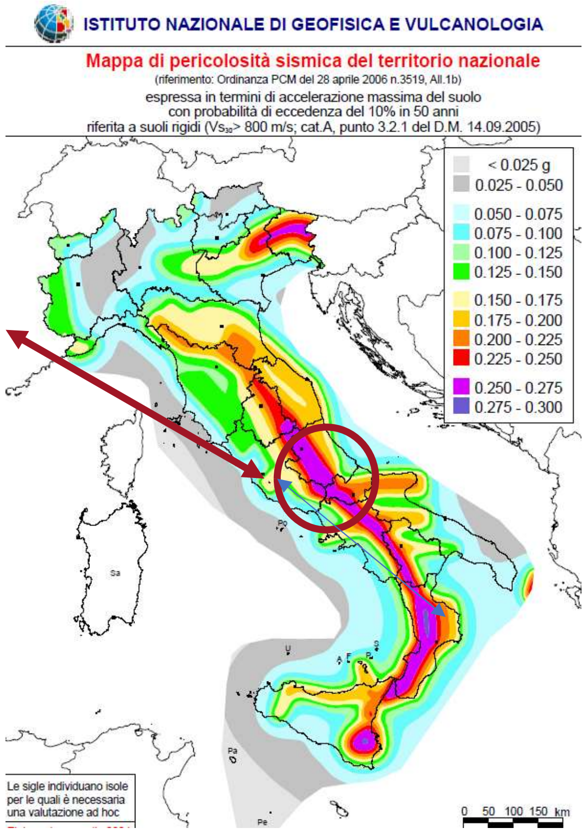
Course outline:

- Introduction
  - i. PBEE overview
  - ii. the PEER-PBEE framework
- Probabilistic Seismic Hazard Model
  - i. Concept and Formulation
  - ii. Hazard Computation (MatLab)
- Vulnerability and Fragility Analysis
  - i. Concept and Formulation
  - ii. Fragility Computation (MatLab)
- References







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Introduction

Performance Based Earthquake Engineering

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https://github.com/kia13nn/ISPS.git

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PBEE: **P**erformance **B**ased **E**arthquake **E**ngineering

- Probabilistic framework for *(i)* assessing design, *(ii)* evaluation and *(iii)* planning of civil system.

- load-and-resistence-factor design (LRFD)

  ↕

  performance based design (PBD):

  3Ds – i.e. downtime, dollars, death

- 2 axes define the domain of acceptable system's response:

- System performance objectives
- Seismic hazard level, w.r.t. return periods

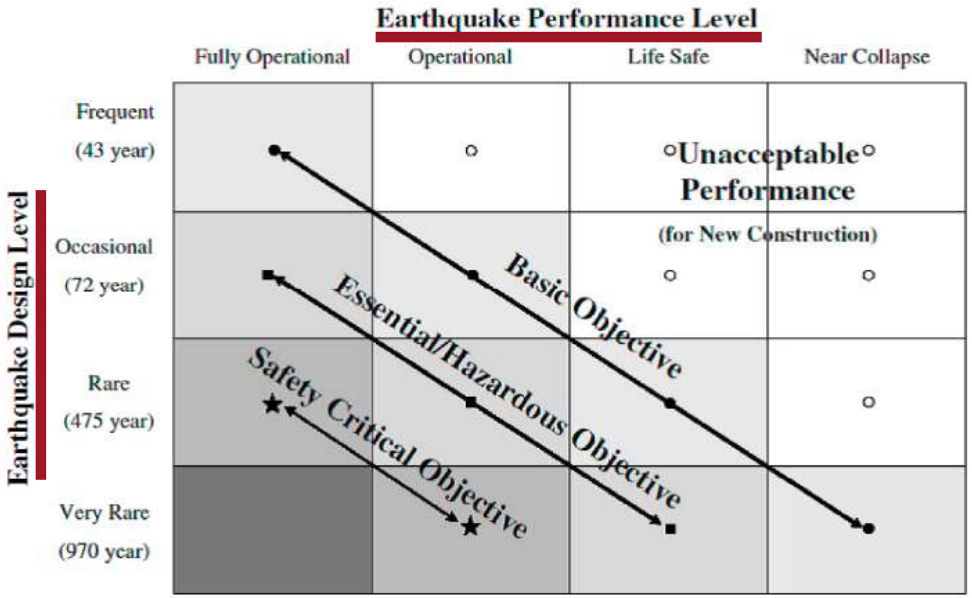

Fig.1 – PBEE concept: seismic performance objectives *vs* seismic hazard level. ©*Poland et al.,(1995)-Vision 2000: Performance Based Earthquake Engineering of buildings. Structural Engineers Association of California, Sacramento, CA.*

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The **PEER-PBEE** Framework

- PEER ~ Pacific Earthquake Engineering Research center – analytical approaches **based on total probability theorem** for the *yearly mean number of events of a selected decision variable*

$$IM \longrightarrow EDP \longrightarrow DM \longrightarrow DV$$

$$\lambda(dv) = \int_d \int_{edp} \int_{im} G(dv|d)|dG(d|edp)||dG(edp|im)||d\lambda(im)| \qquad (1)$$

- with the underlying assumptions of:
  - Markovian structure → $DV \perp\!\!\!\perp EDP, IM | DM = dm$; $DM \perp\!\!\!\perp IM | EDP = edp$;
  - No-aging effects on structures;
  - Poisson's processes → memoryless seismic events.

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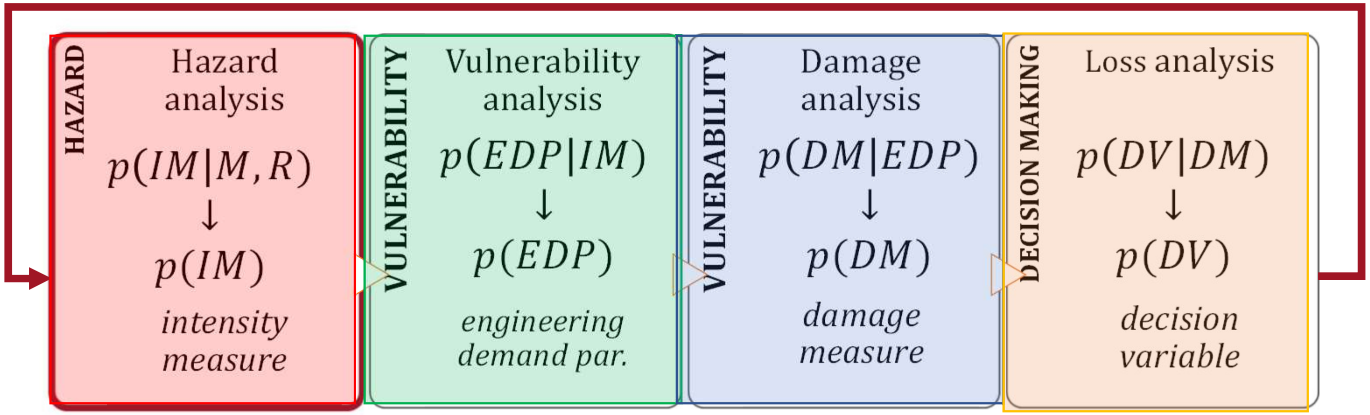
The **PEER-PBEE** Framework

$IM \longrightarrow EDP \longrightarrow DM \longrightarrow DV$



(i) hazard analysis

(ii) fragility analysis

(iii) damage analysis

(iv) loss analysis

$$\lambda(dv) = \int_d \int_{edp} \int_{im} G(dv|d)|dG(d|edp)||dG(edp|im)||d\lambda(im)|$$

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The **PEER-PBEE** Framework

$$IM \longrightarrow EDP \longrightarrow DM \longrightarrow DV$$

$$\lambda(dv) = \int_d \int_{edp} \int_{im} G(dv|d)|dG(d|edp)||dG(edp|im)||d\lambda(im)|$$

where

– im is an intensity measure (e.g., peak ground acceleration, spectral acceleration, etc.);

– edp is an engineering demand parameter (e.g., interstorey drift);

– d is a damage measure (e.g., minor, medium, extensive, collapse);

– dv is a decision variable (e.g., monetary losses, fatalities, etc.);

– $\lambda(x)$ is the mean annual rate of events exceeding a given threshold for a given variable x;

– $G(y|x) = P(Y \geq y|X = x)$ is the conditional complementary cumulative distribution function (CCDF)

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Probabilistic Hazard Analysis

Formulation and MatLab Computation

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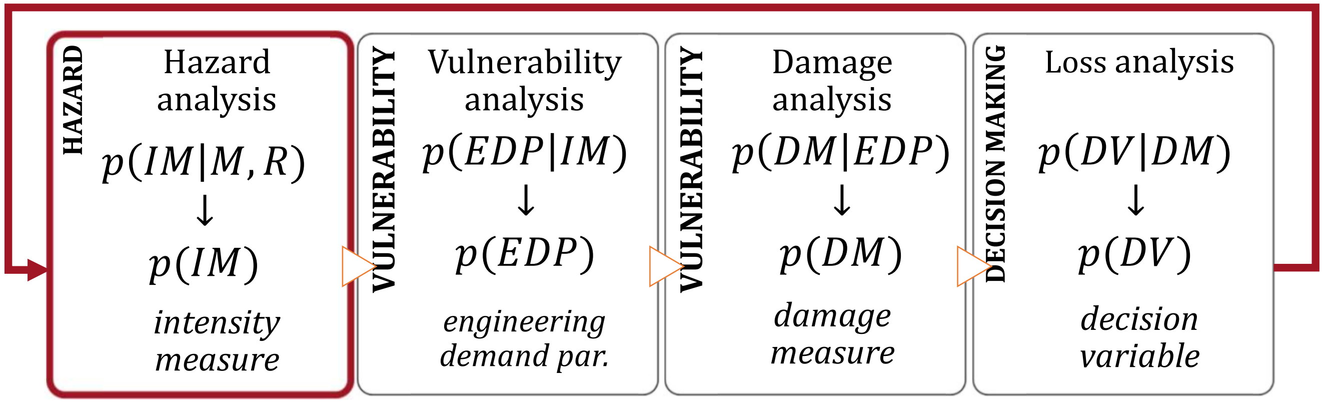
Seismic Hazard Analysis

SHA is the basis for many earthquake **design codes** and for **seismic risk analysis** of civil systems.



Useful definitions:

seismic hazard := the exceedance (or occurrence) probability for a given ground motion intensity measure threshold, site and time interval

seismic risk:= the exceedance (or occurrence) probability for a given loss threshold, site and time interval.

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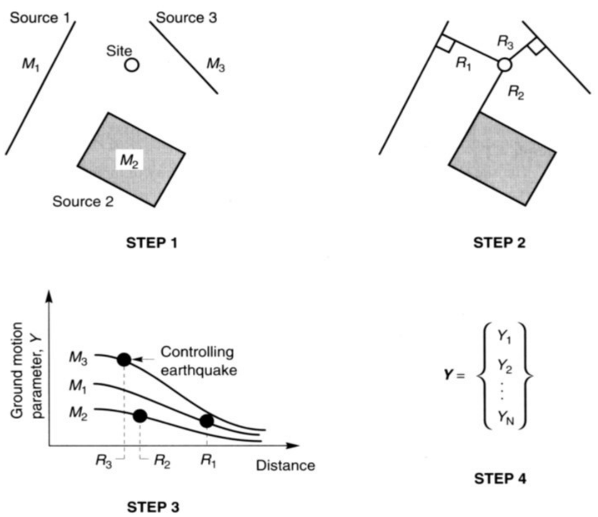
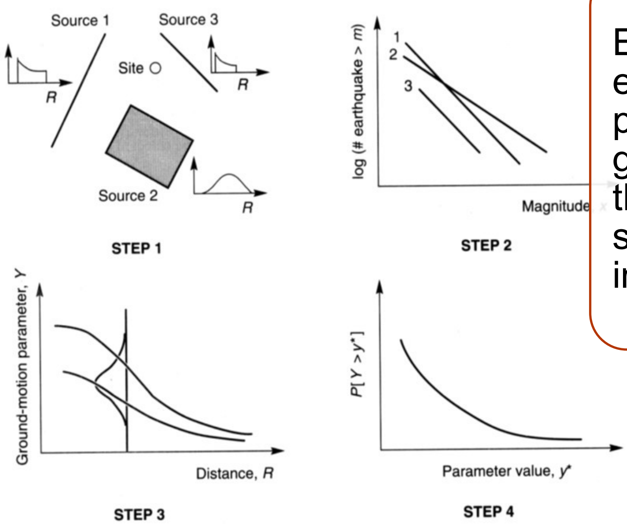
Seismic Hazard Analysis – DSHA vs PSHA

DSHA



Evaluate the worst-case scenario

PSHA



Evaluate the exceedance probability of a given g.m. IM threshold at given site and time interval

Four steps of a DSHA and PSHA analyses - Kramer, 1996

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PSHA

Probabilistic Seismic Hazard Analysis (PSHA) evaluates the exceedance (or occurrence) probability of a given ground motion intensity measure threshold at given site and time interval.

PSHA provides a framework in which uncertainties, typically include magnitude size, earthquake location, soil condition, and rate of occurrence of earthquakes, are quantified.

The calculation of seismic hazard is based on the Total Probability Theorem*

$$P(IM > im) = \int_S P(IM > im \mid M = m, R = r) \, f_{R|M}^{(n)} \, f_M^{(n)} dr \, dm \tag{1}$$

<< the probability that a fixed value of ground motion im is exceeded at a given site, given the occurrence of random earthquake from the seismic source n >>

**see Notes at the end of Presentation*

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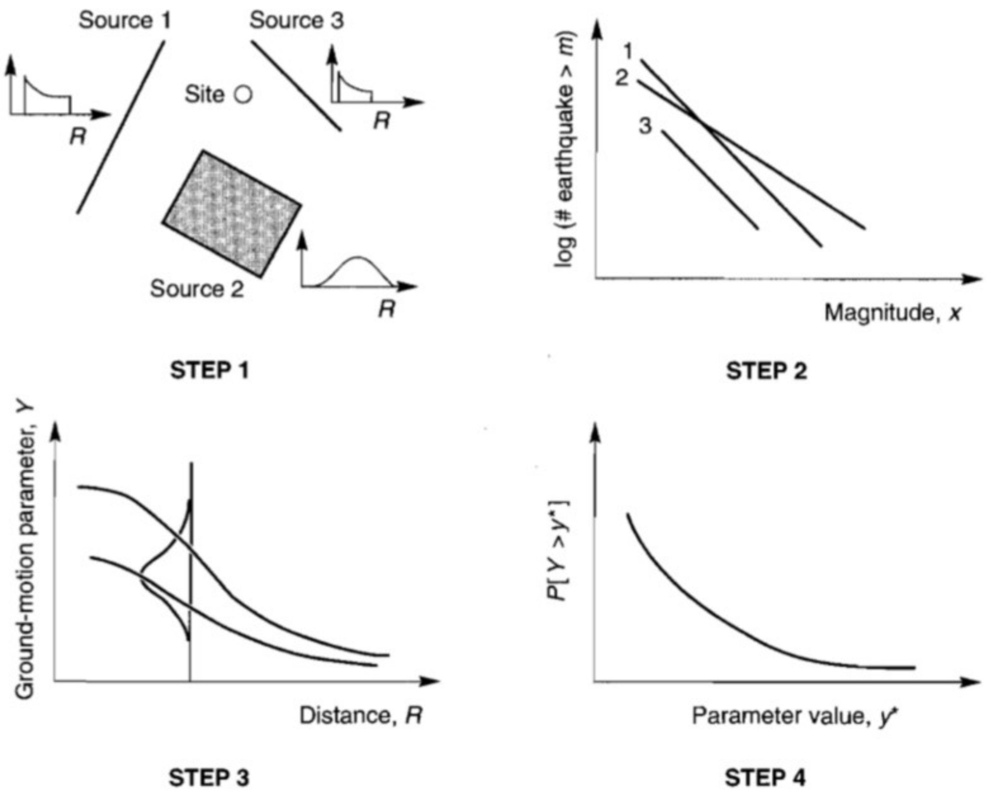
PSHA - Steps

i. **Source Characterization**: Identification and classification of the Ns source → Definition of $f_{R|M}^{(n)}$

ii. **Earthquake Size**: for each source based on magnitude recurrence relationship $f_M^{(n)}$

iii. **Ground Motion Estimation**: empirical regression models named ground motion prediction equations (GMPE) → Definition of $P(IM > im \,|M = m, R = r)$

iv. **Hazard Computation**: solution of the integral *(1)* $\forall\, N_s$



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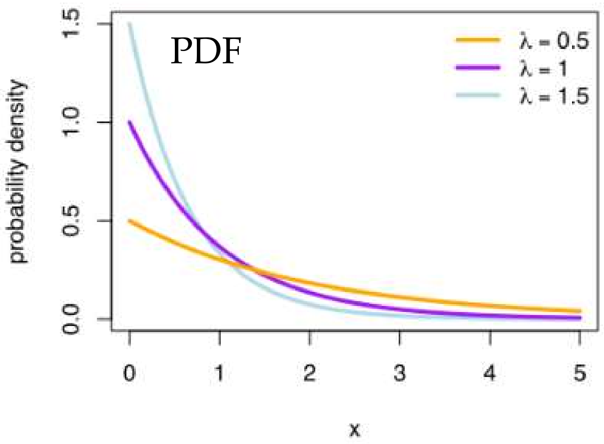
PSHA – Assumptions:

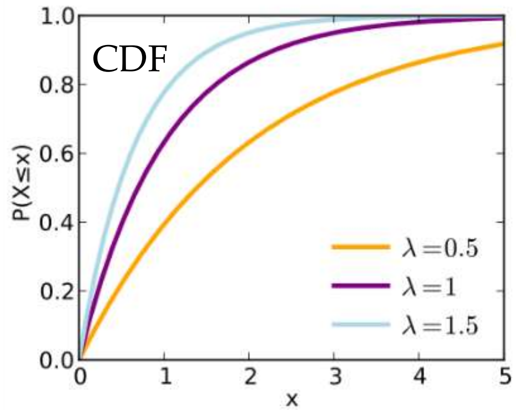
Eq. *(1)* is based on the following assumptions:

- *Earthquakes form a stochastic process;*
- *Earthquakes can be considered instantaneous events and memory-less*

the occurrence of earthquakes can be defined as Homogeneus Poisson Process (**HPP**)

exponential distribution of earthquake recurrence





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PSHA – Step 1: Earthquake source characterization

SOURCES

Goal: **to identify**

- *Fault sources:* individual or multiple identified faults
- *Area sources:* defined by polygons in which seismicity is assumed uniform

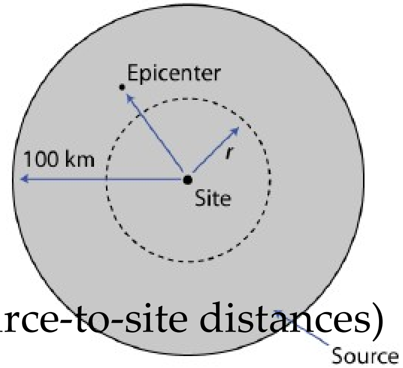
(Identification based upon the interpretation of geological, geophysical and seismological – historical data)

DISTANCES

and **to characterize seismic sources**

- *Point source*
- *Linear source*
- *Area source*

(the geometry of the source is used to identify the probability distribution of source-to-site distances)



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PSHA – Step 2: Earthquake size (Recurrence law)

Goal: **to define distribution of** $f_M^{(n)}$

the chance of an earthquake of a given size occurring anywhere inside the source during a specified period of time

The Gutenberg-Richter law (G-R law) expresses the relationship between the magnitude and rate of cumulative number of earthquakes in any given region:

$$\log \lambda(m) = a - b \cdot m$$

$\log \lambda(m)$ logarithm base 10 of the mean annual rate of exceedance of magnitude m, a overall rate of earthquakes of the source and b relative ratio of small vs large magnitudes

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PSHA – Step 2: Earthquake size (Recurrence law)

Observations:

- $m_{min} - m_{max}$ linked to the minimum magnitude capable of causing damages and to physically capability of seismic zone to generate magnitude with these values
- G-R law *bounded*
- given a seismic source, $a - b$ parameters are estimated through statistical analysis of historical data with constraints from geological evidence
- paramount aspect regarding completeness and undistortion of the reference catalogue in terms of intensity/magnitude range and time intervals

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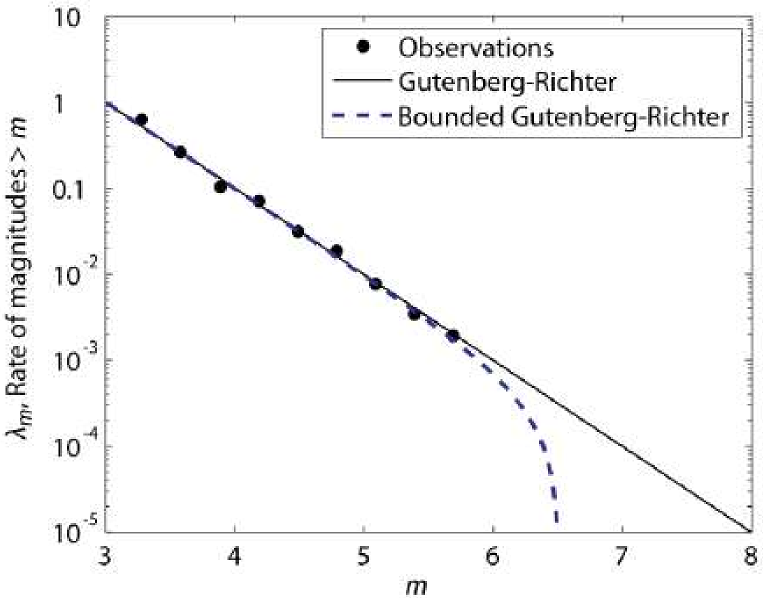
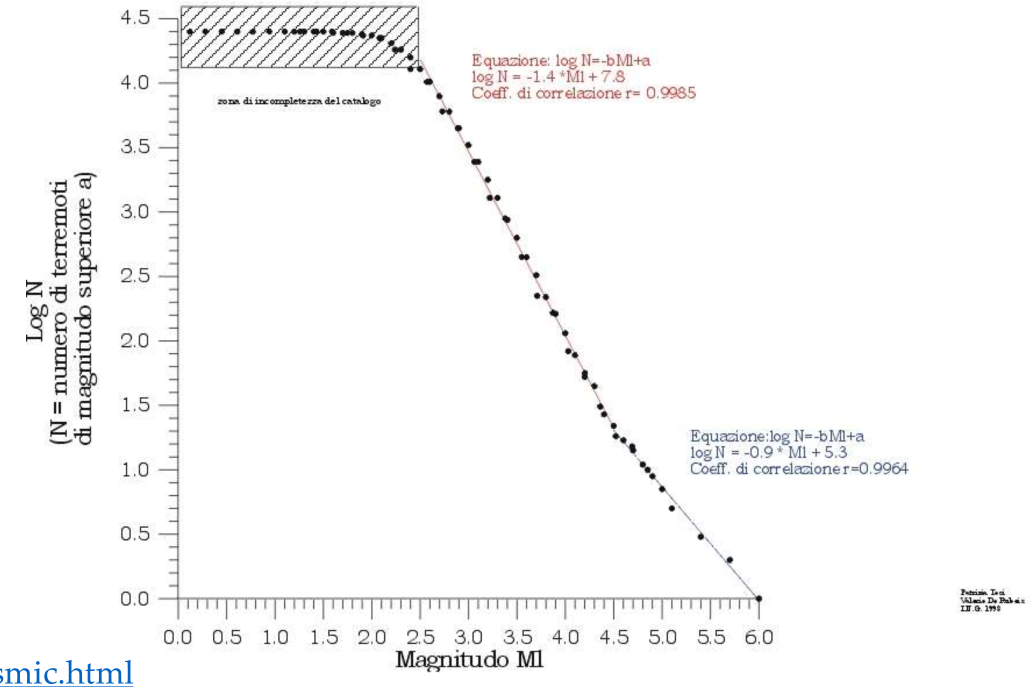
PSHA – Step 2: Earthquake size (Recurrence law)



Distribuzione cumulata degli eventi sismici in funzione della magnitudo dal 1 gen. 1983 al 5 ott. 1997 (Regione italiana)



ttp://legacy.ingv.it/roma/attivita/pererischio/macrosismica/Seismic/seismic.html

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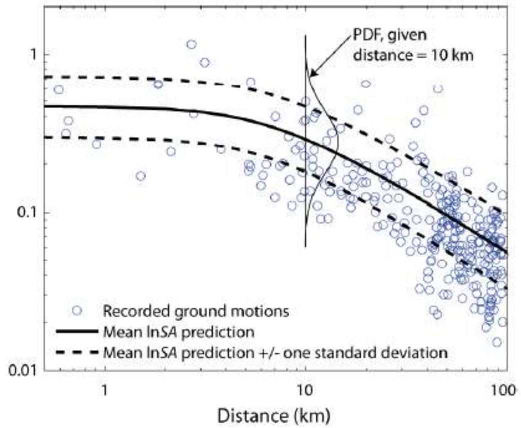
PSHA – Step 3: Ground motion predictive equations (GMPE)

Goal: **estimate ground motion at the site**

- Identify the IM of interest for the situation and purposes;
- Estimation of the PDF of the selected IM by referring to predictor variables such as the earthquake source properties $(M, R\ \ldots)$

GMPEs are usually adopted to evaluate the probability that a particular IM exceeds a certain value, im, for a given earthquake $M = m$, occurring at a given distance, $R = r$



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PSHA – Step 3: Ground motion predictive equations (GMPE)

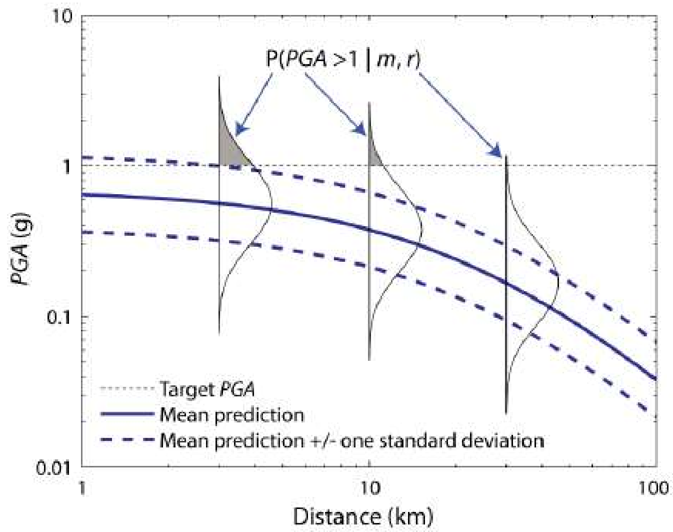
In probabilistic terms

$$P[IM > im|R = r, M = m] = 1 - F_{IM|RM}(im|r, m)$$

Usually, the conditional distribution of the ground motion intensity measure, i.e. $F(im|r, m)$ is assumed log normal.

Observations:

- GMPEs are developed independently for each region;
- Faulting mechanism, effects of local site conditions are of paramount importance



Schematic illustration of conditional probability of exceeding a particular value of a ground motion parameter for a given magnitude and distance.

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PSHA – Step 4: Hazard Computation

The seismic hazard curve is a function representing the annual frequency of exceeding various levels of ground shaking (i.e. the IM) at a specific site. The curve is obtained by integration of the previously three steps over all possible magnitudes and earthquakes locations.

Seismic hazard curves are obtained for individual sources and, then, combined to express the aggregate hazard at a particular site.

Then

$$\lambda(im) = \sum_{n=1}^{N_s} \lambda_{min}^{(n)} \left[\int_{r_{min}}^{r_{max}} \int_{m_{min}}^{m_{max}} P(IM > im \mid M = m , R = r) \; f_R^{(n)}(r) f_M^{(n)}(m) dr dm \right] \tag{2}$$

Numerically

$$\lambda(im) \approx \sum_{n=1}^{N_s} \sum_{m=1}^{N_r} \sum_{l=1}^{N_m} \lambda_{min}^{(n)} P\left(IM > im \middle| M^{(n)} = m_l, R^{(n)} = r_m\right) f_R^{(n)}(r_m|m_l) f_M^{(n)}(m_l) \Delta r \, \Delta m \tag{3}$$

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Probabilistic Hazard Analysis

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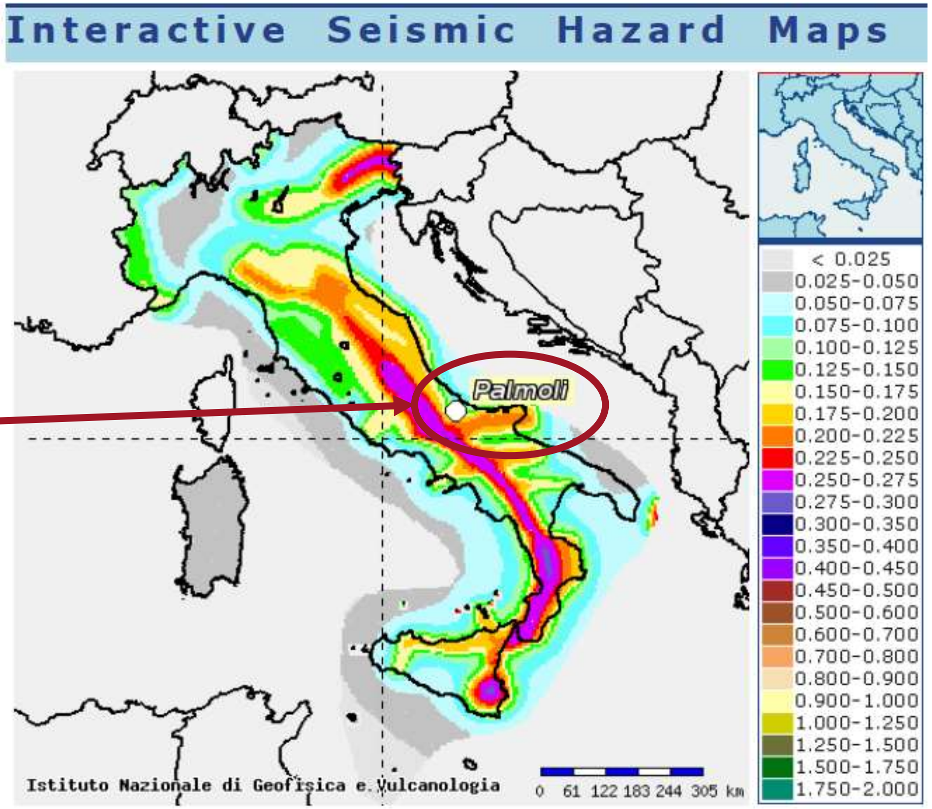
Goal: to perform a PSHA analysis

Through the scheme depicted in *(1)*, compute:

- the annual hazard curve for each fault;
- the 50 years hazard curve for each fault;
- the 475 years hazard curve for each fault

for the highlighted seismic site.



http://esse1-gis.mi.ingv.it/

Seismic hazard map: shaking parameter PGA, site Palmoli (CH) Latitude 41,527 – Longitude 14,483.

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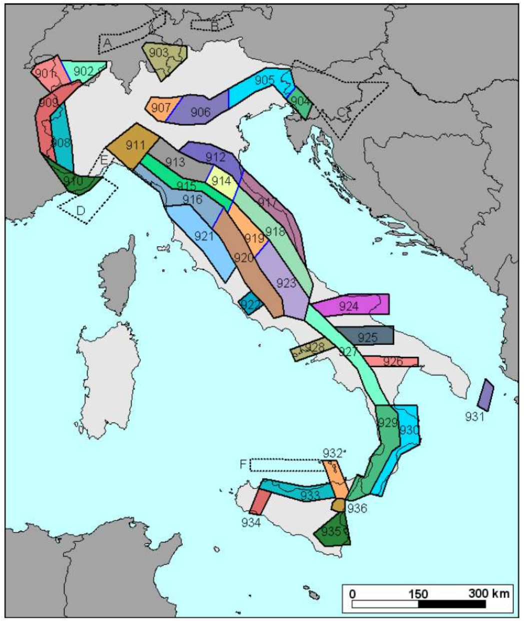
Step 1: Source characterization

I. Localize seismogenetic zones: Italian ZS9* model for application of the attenuation law

↓

Italian database and catalogue of the overall seismicity
http://zonesismiche.mi.ingv.it/



Different characteristics of zonation by:

- Seismogenetic faults and mechanisms (direct, inverse, strike-slip …)
- Hypocenter depth (shallow, intermediate, deep)
- ect.

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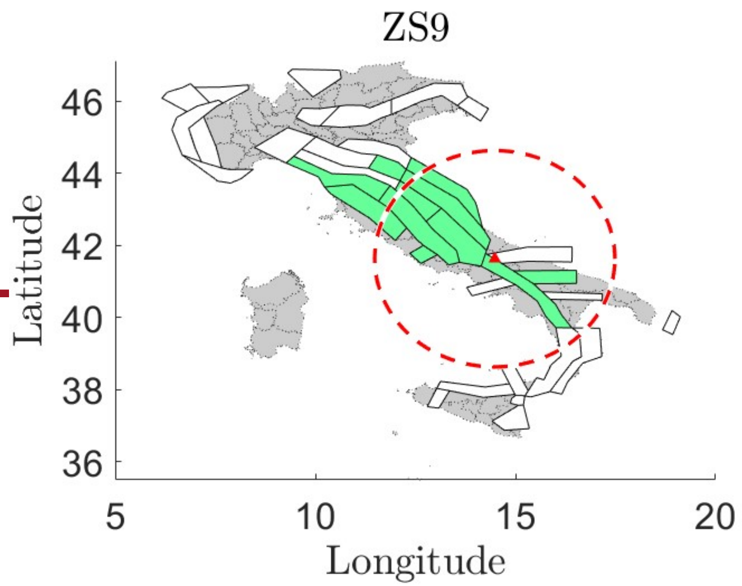
Step 1: Source characterization

ZS Name	ZS9	MwMax AR	Tassi Mwmax Co-04.2 AR	Tassi Mwmax Co-04.4 AR	b Co-04.2	b Co-04.4	MwMax GR	Tassi Mwmax (Co.04.2) GR
Savoia	901	5,91	0,21	0,21	-1,18	-1,26	6,14	0,11
Vallese	902	6,14			-1,26	-1,05	6,14	0,14
Grigioni - Valtellina	903	5,91	0,21	0,21	-1,26	-1,05	6,14	0,14
Trieste - Monte Nevoso	904	5,68			-1,12	-1,32	6,14	0,14
Friuli - Veneto Orientale	905	6,60			-1,06	-1,12	6,60	0,37
Garda - Veronese	906	6,60		0,14	-1,14	-1,70	6,60	0,11
Bergamasco	907	5,91	0,14	0,14	-1,71	-1,48	6,14	0,04
Piemonte	908	5,68			-1,91	-1,67	6,14	0,04
Alpi Occidentali	909	5,68	0,21	0,33	-1,27	-1,38	6,14	0,10
Nizza - Sanremo	910	6,37			-1,12	-1,06	6,37	0,14
Tortona - Bobbio	911	5,68			-1,47	-1,33	6,14	0,05
Dorsale Ferrarese	912	6,14	0,12	0,12	-1,35	-1,32	6,14	0,12
Appennino Emiliano-Romagnolo	913	5,91		0,21	-1,80	-1,53	6,14	0,07
Forlivese	914	5,91			-1,33	-1,23	6,14	0,14
Garfagnana - Mugello	915	6,60			-1,34	-1,36	6,60	0,11
Versilia-Chianti	916	5,68	0,21	0,33	-1,96	-1,58	6,14	0,04
Rimini - Ancona	917	6,14	0,12	0,12	[illegible]	[illegible]	6,14	0,12
Medio-Marchigiana/Abruzzese	918	6,37	0,14	0,21	-1,10	-1,11	6,37	0,14
Appennino Umbro	919	6,37			-1,22	[illegible]	6,37	[illegible]
Val di Chiana - Ciociaria	920	5,68	0,28	0,33	-1,96	-1,58	6,14	0,06
Etruria	921	5,91		0,08	-2,00	-2,01	6,14	0,05
Colli Albani	922	5,45			-2,00	-2,01	5,45	0,37
Appennino Abruzzese	923	7,06			-1,05	-1,09	7,06	0,14
Molise-Gargano	924	6,83			-1,04	-1,06	6,83	0,13
Ofanto	925	6,83			-0,67	-0,75	6,83	0,17
Basento	926	5,91			-1,28	-1,38	6,14	0,10
Sannio - Irpinia - Basilicata	927	7,06			-0,74	-0,72	7,06	0,43
Ischia - Vesuvio	928	5,91	0,21	0,21	-1,04	-0,66	5,91	0,21
Calabria tirrenica	929	7,29			-0,82	-0,79	7,29	0,17
Calabria ionica	930	6,60			-0,98	-0,89	6,60	0,17
Canale d'Otranto	931	6,83			-0,63	-0,63	6,83	0,21
Eolie - Patti	932	6,14			-1,21	-1,08	6,14	0,21
Sicilia settentrionale	933	6,14	0,21	0,33	-1,39	-1,24	6,14	0,20
Belice	934	6,14			-0,96	-0,93	6,14	0,20
Iblei	935	7,29			-0,72	-0,69	7,29	0,12
Etna	936	5,45	0,33	0,33	-1,63	-1,22	5,45	0,33

I. Localize seismogenetic zones: Italian ZS9* model for application of the attenuation law



http://zonesismiche.mi.ingv.it/

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Step 1: Source characterization

Extrapolate information regarding the source and its bound limits: convert coordinates from *deg2utm* for each sources

	Coordinates				
	WGS 84		UTM		UTM−Z
Location	**long (x)**	**lat (y)**			
Palmoli	14,48	41,63	465.293	4643124	33 T

Source	b	Mw Max	Mw Min	Rate
S1	−1,11	6,37	4,76	0,21
S2	−1,09	7,06	4,76	0,14
…	…	…	…	…

ZS9−918	S1					
WGS 84		UTM		UTM−Z	UTM−INPORT	
12,11651	43,79397	268015,9	4853032	33 T	−197.277	209.907
12,27012	44,10645	281523,5	4887320	33 T	−183.769	244.196
13,54116	43,2943	381660,3	4794530	33 T	−83.633	151.405
14,181	42,47483	432678,9	4702824	33 T	−32.614	59.699
14,37249	42,14993	448153,3	4666614	33 T	−17.140	23.489
14,27923	41,85719	440174,4	4634171	33 T	−25.118	−8.953
13,1518	43,02822	349423	4765606	33 T	−115.870	122.481
12,11651	43,79397	268015,9	4853032	33 T	−197.277	209.907

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Step 1: Source characterization - Location

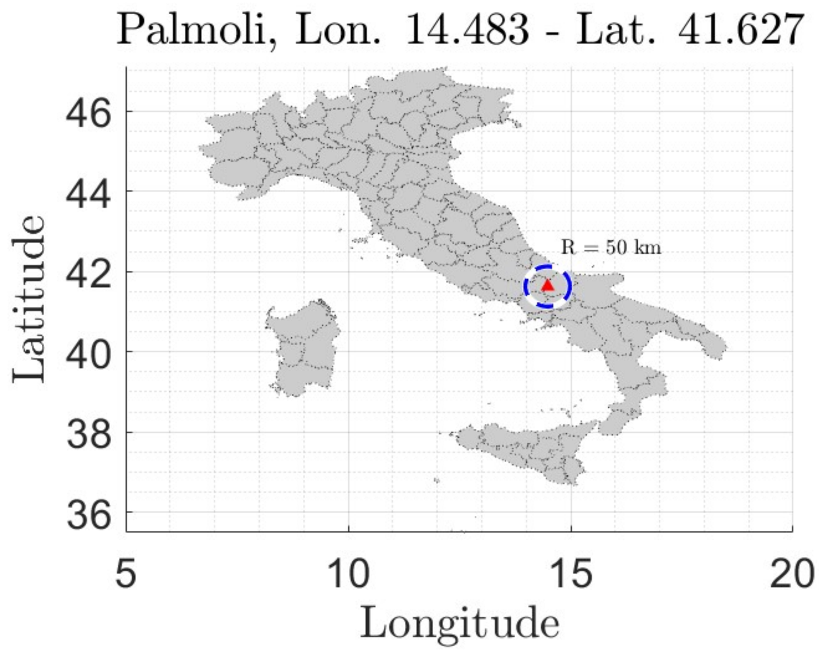
Define the distribution of $f_R(r)$

To simplify, consider 1 source with a regular zonation of $R_{fix} = 50\ km$.

So, the distribution of an epicenter being located at a distance of less than r is:

CDF: $$F(r) = P(R \leq r) = \frac{area\ of\ circle\ with\ radius\ r}{area\ of\ circle\ with\ radius\ R_{fix}}$$
$$= \frac{r^2}{R_{fix}^2}, 0 \leq r \leq R_{fix} \quad \wedge \quad 1, r \geq R_{fix}$$

PDF: $$f(r) = \frac{r}{2\ R_{fix}^2}, 0 \leq r \leq R_{fix} \quad \wedge \quad 0, r \geq R_{fix}$$

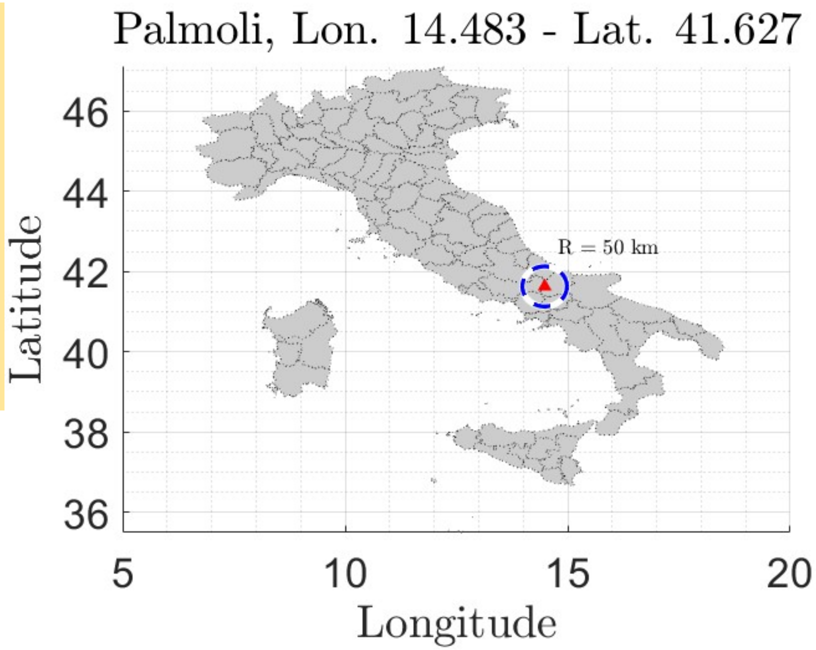


Step 1: Source characterization

Codes:

i. Read *source.xls* file

ii. Run section *Earthquake source characterization* in the attached Matlab code or load variable *seismic_source.mat* already prepared with the previously seen data



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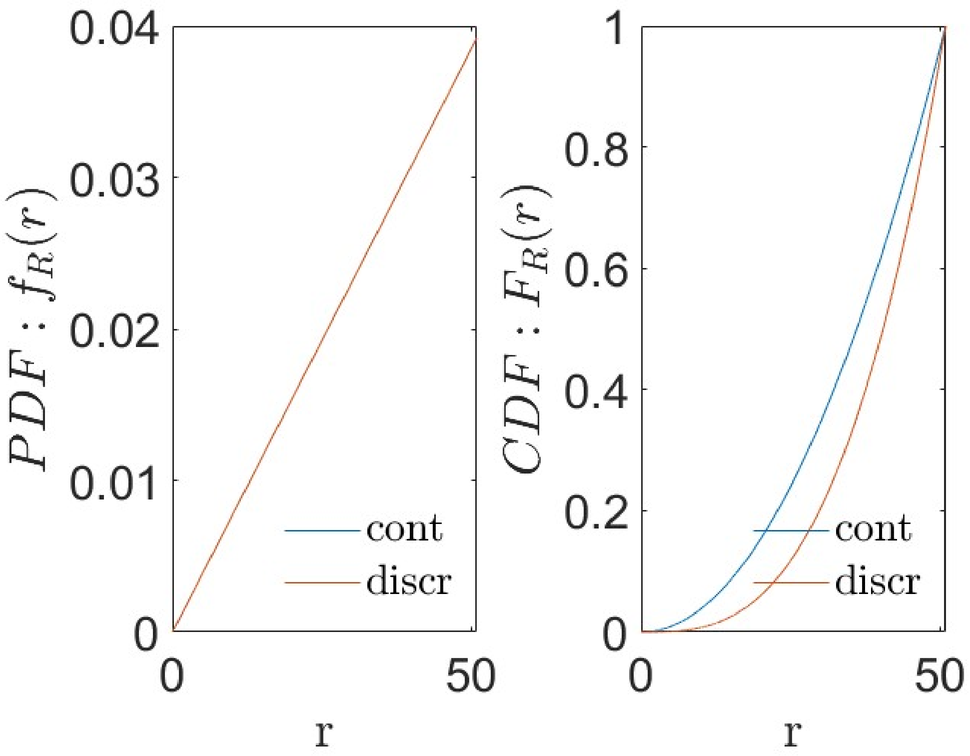
Step 1: Source characterization - Location

Codes:

```
% PDF of R
  rmax = Rmax*100;    % 50km
  rmin = Rmin.*100;   % site

  r_step = R_step;
  r = rmin:r_step:(rmax-r_step);
  Fr = r.^2./rmax.^2;

  fr = 2*r./rmax.^2;
  DFr = r_step;
  frdiscr = zeros(1,numel(Fr));
  Frdiscr = zeros(1,numel(Fr));
  for idr = 2:numel(Fr)
  frdiscr(idr) = -(Fr(idr-1)-Fr(idr))./(DFr);
  Frdiscr(idr) = Frdiscr(idr-1)+Fr(idr);
  end
```



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Step 2: Earthquake size

Define the distribution of $f_M(m)$

$$\lambda(m) = \exp(a - b \cdot m)$$

Gutenberg-Richter bounded defines the relationship between the magnitude and rate of cumulative number of earthquakes

CDF:

$$F_M(m) = P(M \leq m | m_{max} \geq M \geq m_{min})$$

$$= \frac{\lambda_{min} - \lambda(m)}{\lambda_{min} - \lambda_{max}} = \dots = \frac{1 - 10^{-b(m - m_{min})}}{1 - 10^{-b(m_{max} - m_{min})}} = \frac{1 - \exp[-\beta(m - m_{min})]}{1 - \exp[-\beta(m_{max} - m_{min})]}$$

PDF:

$$f(m) = \frac{d}{dm} F(m) = \frac{\beta \exp[-\beta(m - m_{min})]}{1 - \exp[-\beta(m_{max} - m_{min})]}$$

$$\beta = \ln(10)\, b$$

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Step 2: Earthquake size

Define the distribution of $f_M(m)$: Discretize

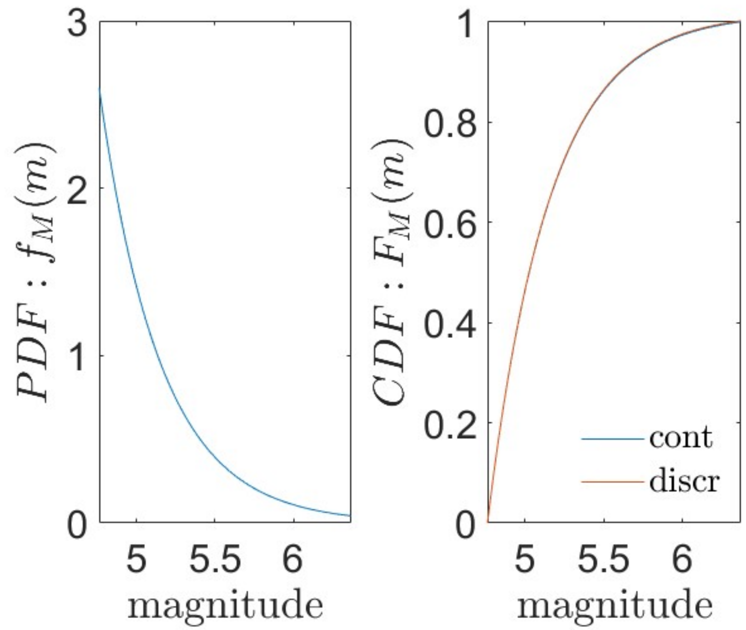
Converting the continuous distribution of magnitudes into a discrete set of magnitudes holds

$$P(M = m_j) = F_M(m_{j+1}) - F_M(m_j)$$

where m_j are the discrete set of magnitudes ordered so that $m_{j+1} < m_j$

<u>Codes:</u>

```
% PDF of M
Nm = 1000; % No. of discretized points between m0 and mu for numerical integration
M = linspace(m0,mu,Nm);   %mo = m_min
fM = beta*exp(-beta*(M-m0))/(1-exp(-beta*(mu-m0)));
dM=M(2)-M(1);
Mdiscr=fM*dM;
```



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Step 3: Ground motion estimation

Define effects $P(IM > x|m, r)$: Attenuation models or prediction models

These models predict the probability distribution of ground motion intensity, as a function of many predictor variables such as the earthquake's magnitude, distance, faulting mechanism, the near-surface site conditions, etc.

To describe this probability distribution, prediction models take the following general form:

$$\ln(IM) = \overline{\ln IM(M, R, \theta)} + \sigma(M, R, \theta) \cdot \varepsilon$$

where

- $\ln(IM)$ is the natural log of the ground motion *intensity measure* of interest;
- $\ln(IM)$ is modeled as a random variable and is represented by a normal distribution;
- $\overline{\ln IM(M, R, \theta)}$ and $\sigma(M, R, \theta)$ are the predicted mean and standard deviation of $\ln(IM)$;
- $\overline{\ln IM(M, R, \theta)}$ and $\sigma(M, R, \theta)$ are functions of the earthquake's magnitude (M), distance (R) and other parameters (θ)

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Step 3: Ground motion estimation

Define effects $P(IM > x|m, r)$: Attenuation models or prediction models

Here, for clarity, it is assumed Cornell (1979) for the mean of log peak ground acceleration (in units of g):

$$\overline{\ln PGA} = -0{,}152 + 0{,}859M \; - 1{,}803 \ln(R + 25)$$

with

- $\sigma = 0{,}57$ of $\ln(PGA)$;
- $\ln(PGA)$ normally distributed

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Step 3: Ground motion estimation

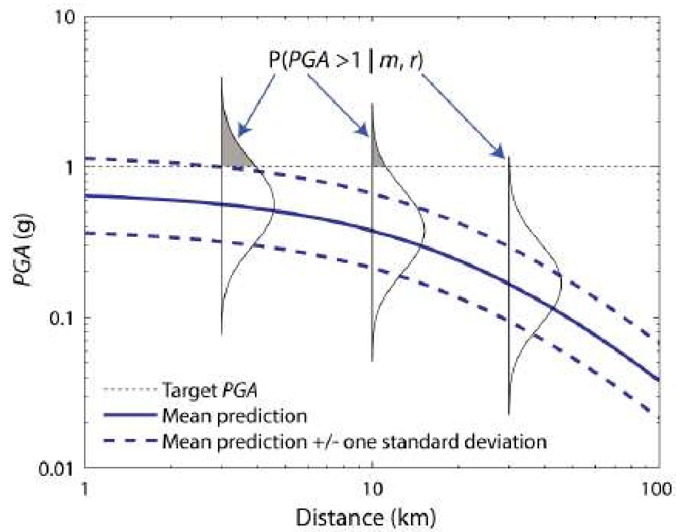
Define effects $P(IM > x|m, r)$: Attenuation models or prediction models

We can thereby compute the probability of exceeding any *PGA* level using knowledge of this mean and standard deviation:

$$P(PGA > x|m, r) = 1 - \Phi\left(\frac{\ln x \; - \ln \overline{PGA}}{\sigma_{\ln(PGA)}}\right)$$

where $\Phi(\,)$ is the standard normal cumulative distribution function.

N.B.: these probabilities correspond to the fraction of the corresponding PDFs that are shaded.



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Step 3: Ground motion estimation

Define effects $P(IM > x|m, r)$: Attenuation models or prediction models

Codes:

```
function [mean_im, sigma_im] = GMPE(magnitude,R_distance)
%Cornell et al. (1979): Ground motion predictive equation for PGA
mean_lnPGA = -0.152+0.859*magnitude -1.803*log(R_distance+25);
mean_PGA = exp(mean_lnPGA);

mean_im = mean_PGA;
sigma_im = 0.57;
end
```

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Step 4: Hazard Computation

Combine all information: compute $\int\int\int$

I. Compute the probability of exceeding an *IM* intensity level *x*, *given* occurrence of a future earthquake from a *single source*:

$$P(IM > x) = \int_{r_{min}}^{r_{max}} \int_{m_{min}}^{m_{max}} P(IM > im \,|M = m\,, R = r)\; f_R^{(n)}(r) f_M^{(n)}(m) dr dm$$

$from\ GMPE$ $PDF\ of\ R$ $PDF\ of\ M$

II. Compute the rate of *IM* > *x* for each single source:

$$\lambda\,(IM > x) = \lambda(M > m_{min}) \int_{r_{min}}^{r_{max}} \int_{m_{min}}^{m_{max}} P(IM > im \,|M = m\,, R = r)\; f_R^{(n)}(r) f_M^{(n)}(m) dr dm$$

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Step 4: Hazard Computation

Combine all information: compute $\int \int \int$

III. Considering all sources by the sum of the rates of $IM > x$ from each individual source, we can write:

$$\lambda(IM > x) = \sum_{i=1}^{n_{sources}} \lambda(M_i > m_{min}) \int_{r_{min}}^{r_{max}} \int_{m_{min}}^{m_{max}} P(IM > im \,|M = m\,, R = r)\; f_{R_i}^{(n)}(r) f_{M_i}^{(n)}(m) dr dm$$

where $n_{sources}$ is the number of sources considered, and $M_i \sim R_i$ denote the magnitude ~ distance distributions for source i

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Step 4: Hazard Computation

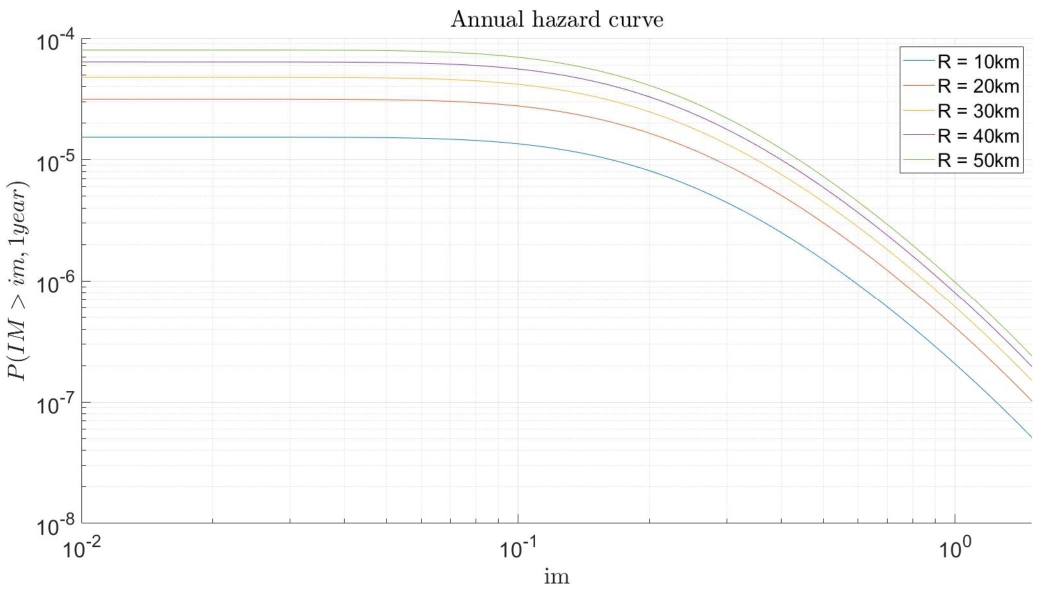
Combine all information: compute $\int\int\int$ - Discretize

IV. By discretizing:

$$\lambda(im) \approx \sum_{n=1}^{N_s} \sum_{m=1}^{N_r} \sum_{l=1}^{N_m} \lambda_{min}^{(n)} P\left(IM > im \middle| M^{(n)} = m_l, R^{(n)} = r_m\right) P\left(R^{(n)} = r_m \middle| m_l\right) P\left(M^{(n)} = m_l\right)$$

where the range of possible M_i and R_i have been discretized into n_M and n_R intervals, respectively

Step 4: Hazard Computation



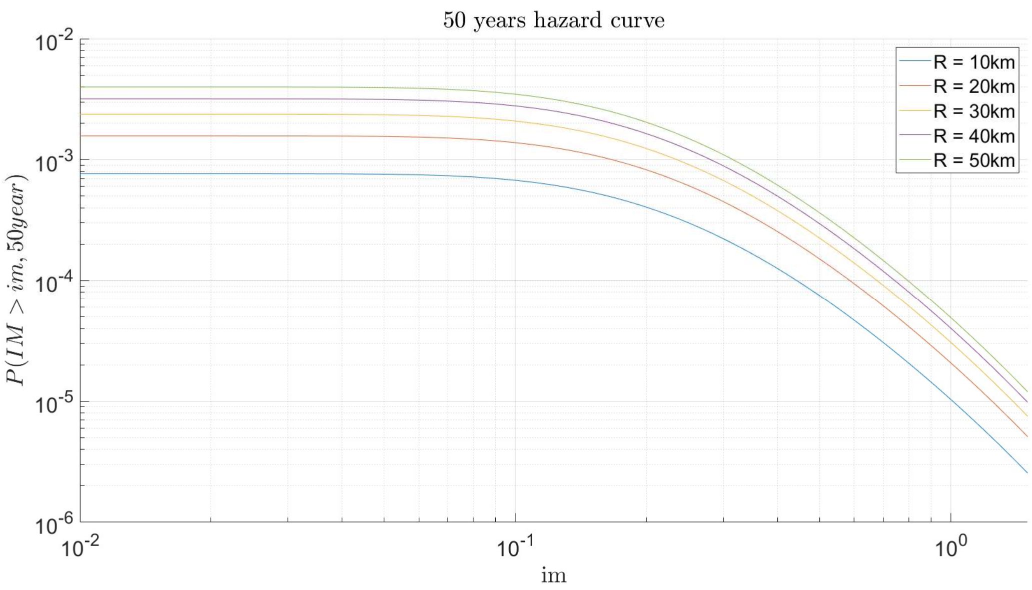
Goal and results

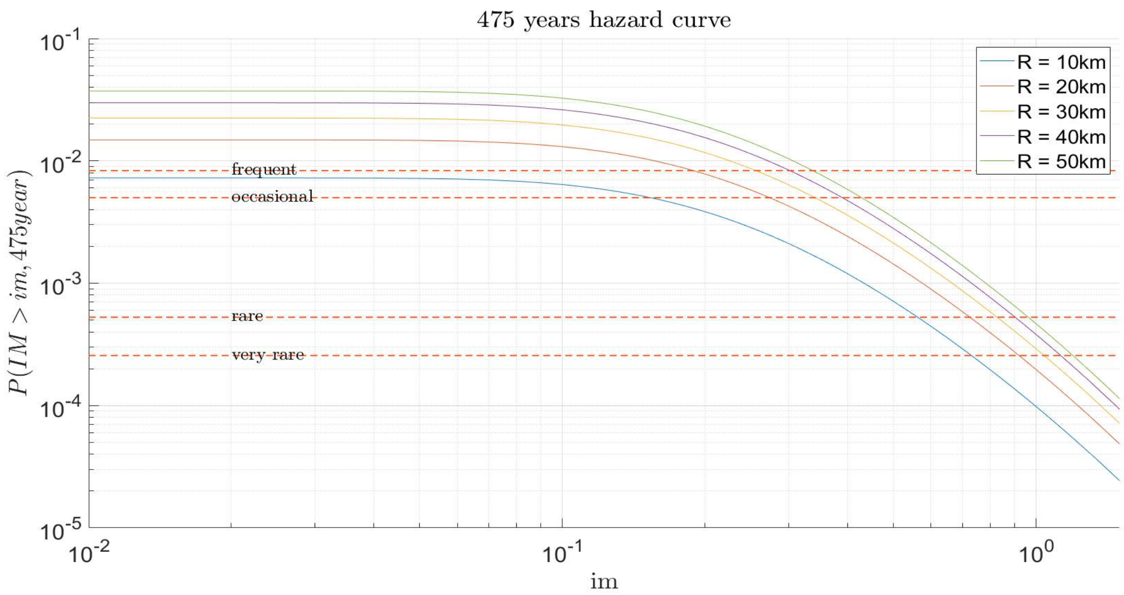
Through the scheme depicted in *(1)*, compute:

- the annual hazard curve for each fault;
- the 50 years hazard curve for each fault;
- the 475 years hazard curve for each fault

for the highlighted seismic site.

Step 4: Hazard Computation





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Summary

Location
- $f_R(r)$ source site distance

Size
- $f_M(m)$ magnitude distribution

GMPE
- $P(IM > im\,|M = m\,, R = r)$ attenuation law

Time
- $P = 1 - e^{-\lambda t}$ Poisson model

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Fragility Analysis

Formulation and MatLab Computation

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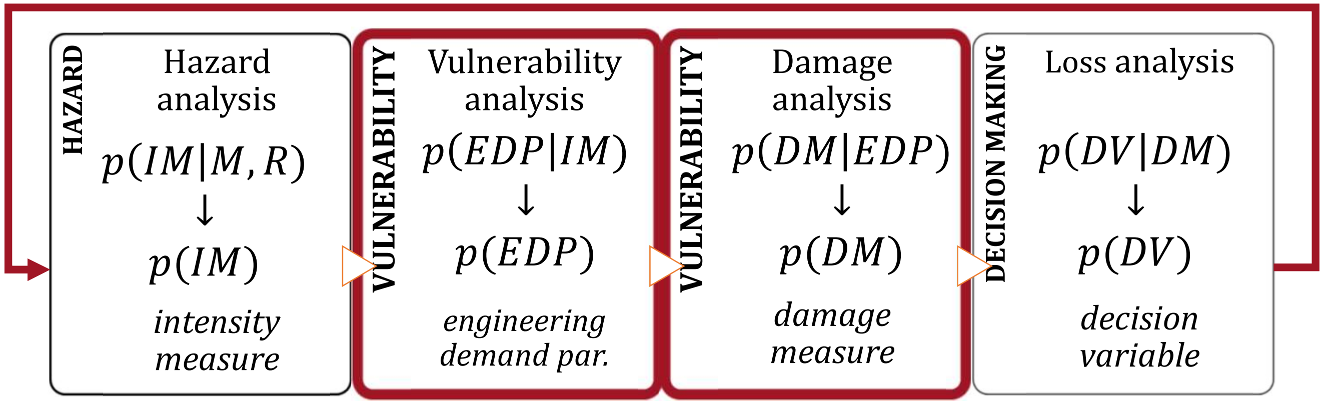
Fragility Analysis

Fragility (or Vulnerability) Analysis is the second step of the PBEE-PEER framework



Useful definitions:

seismic fragility function := the conditional probability of an event (e.g. a defined limit/damage state) given the observation of an intensity measure which describe the seismic event.

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Fragility Analysis

VULNERABILITY

Vulnerability analysis

$p(EDP|IM)$

$\downarrow$

$p(EDP)$

engineering demand par.

VULNERABILITY

Damage analysis

$p(DM|EDP)$

$\downarrow$

$p(DM)$

damage measure

The fragility curve is defined as the conditional probability of failure of a structure, or its critical components, at given values of seismic intensity measures (*IMs*).

$$\lambda(dv) = \sum_d \int_{edp} \int_{im} G(dv|d) P(d|im) |d\lambda(im)|$$

In practice, a fragility curve is calculated as the conditional probability that the damage measure (*D*) exceeds a critical threshold, for a given seismic *IM*.

$$P\ (D > d_{threshold}\ |\ IM = im)$$

Fragility Analysis

VULNERABILITY

Vulnerability analysis

$p(EDP|IM)$
$\downarrow$
$p(EDP)$

engineering demand par.

VULNERABILITY

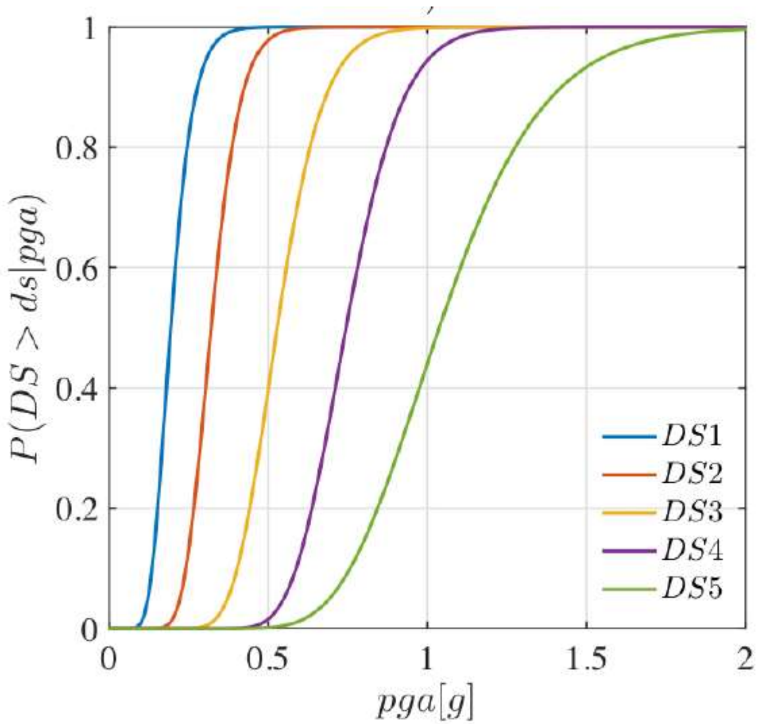
Damage analysis

$p(DM|EDP)$
$\downarrow$
$p(DM)$

damage measure

$$P\ (D > d_{threshold} \mid IM = im)$$

Fragility curves for different damage limit states or thresholds.



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Fragility Analysis: key aspects

VULNERABILITY

Vulnerability analysis

$p(EDP|IM)$

$\downarrow$

$p(EDP)$

engineering demand par.

VULNERABILITY

Damage analysis

$p(DM|EDP)$

$\downarrow$

$p(DM)$

damage measure

INTENSITY MEASURE (*IM*)	DAMAGE STATE (*D*)
in terms of: • ***Efficiency***, i.e. variability of an *EDP* for a given *IM*; • ***Robustness***, i.e. efficiency between *IM-EDP* at different period ranges; • ***Practicality***, i.e. correlation to known and easy identifiable engineering quantities; • ***Sufficiency***, i.e. validity of *EDP*|*IM* as statistically independent from gm site characteristics; • ***Effectiveness***, i.e. ability to evaluate an analytical relation.	Should suit the specific structural problem → associate each damage state to a specific *EDP* • Categorical variables, i.e. D_0 *no damages* – D_1 *minor* – D_2 *moderate* – ... – $D_f = C$ *collapse* ; • Probabilistic or deterministic relationship between *EDP* and *D*

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Class of fragilities

EMPIRICAL	ANALYTICAL	EXPERT OPINION	HYBRID
• by fitting a function to observational data from past earthquakes or lab tests ↕ collections pairs of level of excitation and categorical variables of damage or collapse	• by defining analytical structural model and analyzing its performance under different levels of the seismic hazard ↕ ***static***, i.e. hazard as response spectrum and push-over analysis *vs* ***dynamic***, i.e. collection of gms and simulations on FEM via NLA	• by polling one or more experts of the given structural asset ↕ to guess or estimate the failure probability for a given hazard level	• based on combination of the different methods

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Dynamic based fragility functions: steps

i) Definition of a numerical model: $y(t) = \mathcal{M}\left[\ddot{x}_g(t|IM = im); \boldsymbol{\theta}_{\mathcal{M}}(t)\right]$

ii) Selection of a suitable IM given the structure

iii) Selection of a suitable set of N gms for the location

iv) Selection of an EDP of interest

v) Definition of damage limit states D via EDP thresholds

vi) Scale each gm based on the given IM eventually until collapse

- *a) Full IDA*
- *b) Truncated IDA*

vii) Save each $[EDP \rightarrow D]$ threshold-im_n pair for each gm

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Computation of fragility function

Hp: - assume a *lognormal probability distribution* for the random variable IM associated with given D

FULL IDA

$\forall$ damage state $\rightarrow N$ results, since $\forall\, IM_n = im_n$, $y(t)$ reached the given damage state

$$P(D > d | IM = im) = \Phi\left(\frac{\ln(im_n) - \hat{\mu}}{\hat{\sigma}}\right)$$

where $\Phi(\cdot)$ is the CDF of the normal distribution and

$$\hat{\mu} = \frac{1}{N}\sum_{n=1}^{N} \ln(im_n)$$

$$\hat{\sigma} = \sqrt{\frac{1}{N}\sum_{n=1}^{N} (\ln(im_n) - \hat{\mu})^2}.$$

TRUNCATED IDA

$\overline{IM}$ upper limit;

a) data that causes collapse $n \in [1, \bar{N}]$,

b) data that do not cause collapse $n \in [\bar{N}+1, N]$

a) $\rightarrow$ $$\underline{\mathcal{L}}(\mu,\sigma) \alpha \prod_{n=1}^{\bar{N}} \varphi\left(\frac{\ln(im_n) - \mu}{\sigma}\right)$$

b) $\rightarrow$ $$\overline{\mathcal{L}}(\mu,\sigma) \alpha \prod_{n=\bar{N}+1}^{N} \left[1 - \Phi\left(\frac{\ln(\overline{IM}) - \mu}{\sigma}\right)\right] = \left[1 - \Phi\left(\frac{\ln(\overline{IM}) - \mu}{\sigma}\right)\right]^{N-\bar{N}}$$

a)+b) $\rightarrow$

$$\mathcal{L}(\mu,\sigma) = \underline{\mathcal{L}}(\mu,\sigma)\overline{\mathcal{L}}(\mu,\sigma) =$$

$$= \prod_{n=1}^{\bar{N}} \varphi\left(\frac{\ln(im_n) - \mu}{\sigma}\right)\left[1 - \Phi\left(\frac{\ln(\overline{IM}) - \mu}{\sigma}\right)\right]^{N-N}$$

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Computation of fragility function

TRUNCATED IDA

The likelihood for the entire set of data

$$\mathcal{L}(\mu,\sigma) = \underline{\mathcal{L}}(\mu,\sigma)\overline{\mathcal{L}}(\mu,\sigma) = \prod_{n=1}^{\bar{N}} \varphi\left(\frac{\ln(im_n)-\mu}{\sigma}\right)\left[1-\Phi\left(\frac{\ln(\overline{IM})-\mu}{\sigma}\right)\right]^{N-\overline{N}} \tag{1}$$

And the log likelihood

$$\ln\mathcal{L}(\mu,\sigma) = \sum_{n=1}^{\bar{N}} \varphi\left(\frac{\ln(im_n)-\mu}{\sigma}\right) + (N-\overline{N})\left[1-\Phi\left(\frac{\ln(\overline{IM})-\mu}{\sigma}\right)\right] \tag{2}$$

Estimation of parameters by optimization

$$[\hat{\mu},\hat{\sigma}] = \underset{\mu,\sigma}{\operatorname{argmin}}[-\ln\mathcal{L}(\mu,\sigma)].$$

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Computation of fragility function

For a generic $f(x;\boldsymbol{\theta})$

$$\mathcal{L}(\boldsymbol{\theta}) = \prod_{n=1}^{\bar{N}} f\left(im_n;\boldsymbol{\theta}\right)\left[1 - F(\overline{IM};\boldsymbol{\theta})\right]^{N-\overline{N}}$$

And the log likelihood

$$\ln \mathcal{L}(\boldsymbol{\theta}) = \sum_{n=1}^{\bar{N}} f\left(im_n;\boldsymbol{\theta}\right) + (N - \overline{N})\left[1 - F(\overline{IM};\boldsymbol{\theta})\right]$$

Estimation of parameters by optimization

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}[-\ln \mathcal{L}(\boldsymbol{\theta})]$$

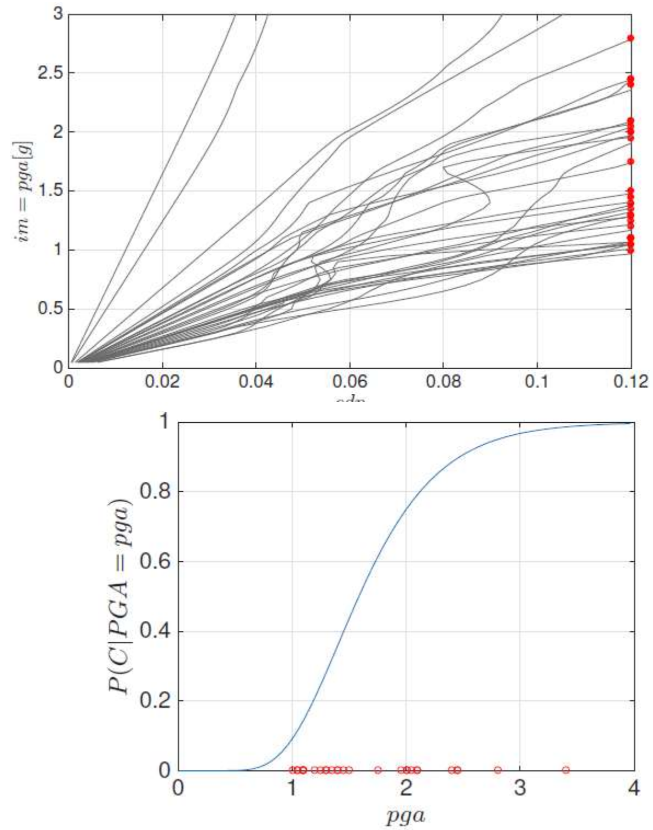

Fig.1 – Fragility function computed via IDA, see Broccardo, M. (2018) *Probabilistic seismic risk analysis for civil systems*, Lecture Notes.

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Fragility Analysis

Formulation and MatLab Computation

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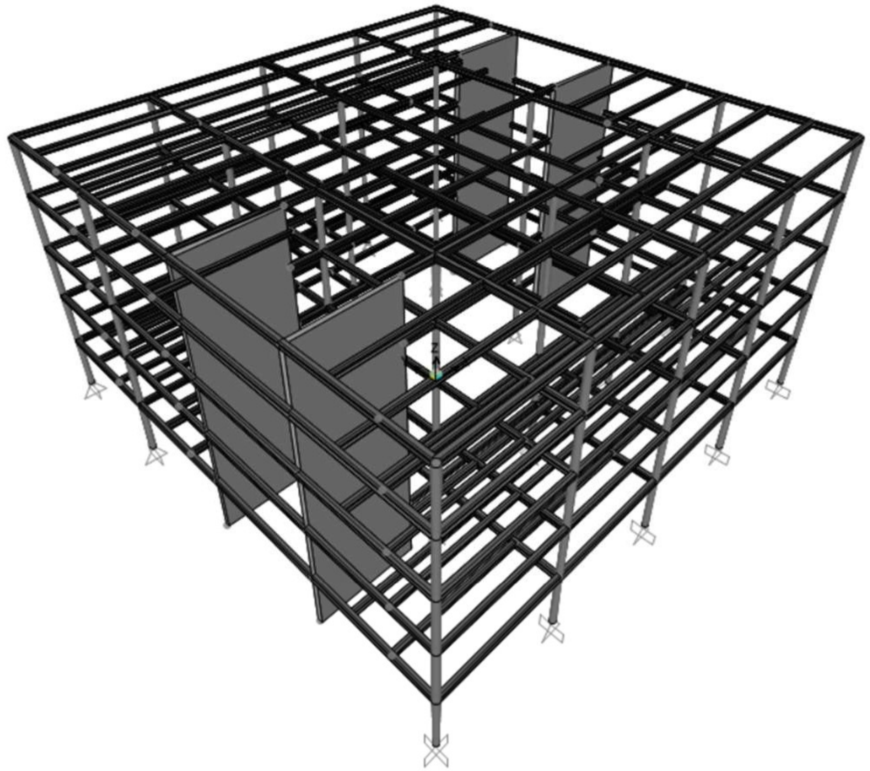
Goal: to perform fragility analysis

Given the provided set of ground motions, perform a classical and truncated incremental dynamic analysis (*IDA*) and determine fragility curves for:

- ATTEL – moment resistant frame (*MRF*);
- ATTEL – braced frame (*BF*)

by considering both

- Linear elastic behaviour
- Bouc – Wen model for hysteresis



3D model of the case study ATTEL – SERA project.

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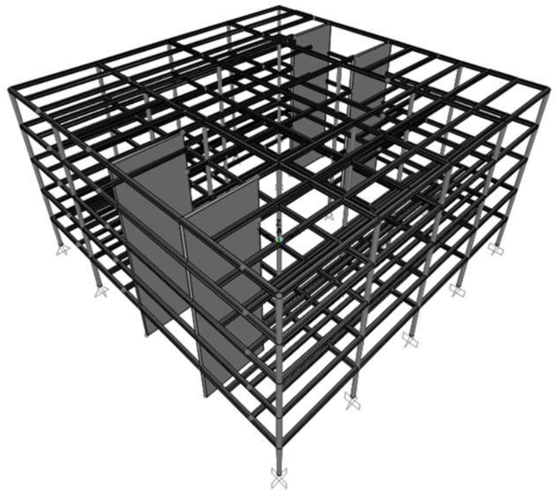
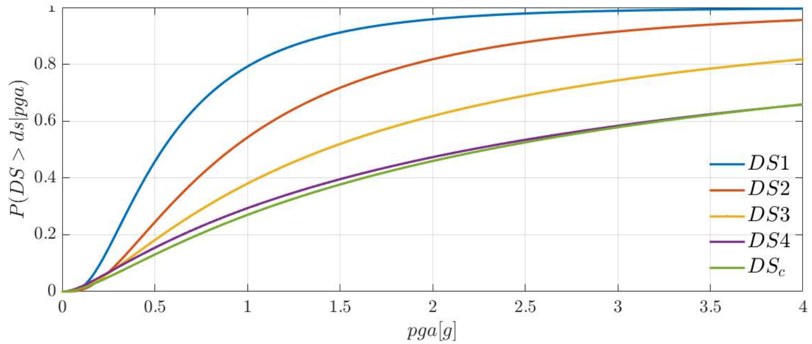
Main steps:

1) Definition of the numerical model
2) Input and *IM* selection
3) Definition of *damage limit states* and reference *EDP*
4) Performing non-linear time histories analysis (IDA, truncated IDA, cloud, MSA …)
5) Collecting results pairs and computing fragility





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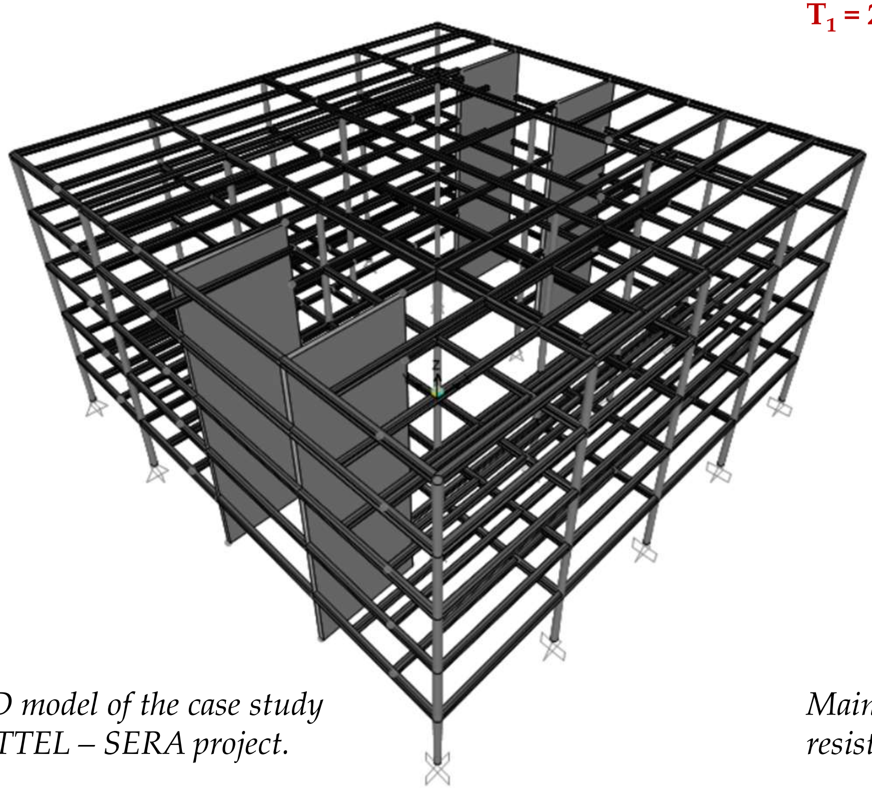
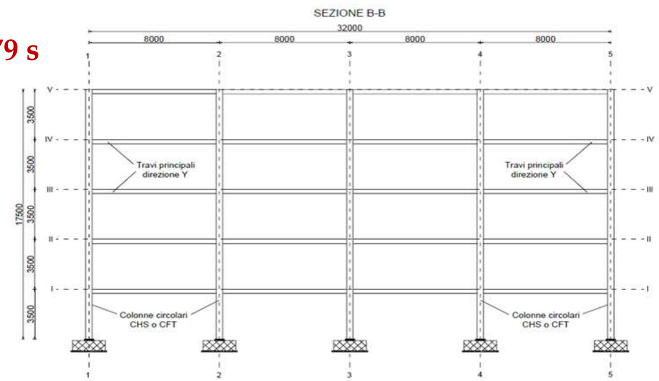
Step 1: the case study ATTEL[(1)]

[1]BURSI, PUCINOTTI, TONDINI, ZANON, Tests and model calibration of high strength steel tubular beam-to-column and column-base composite joints for moment-resisting structures, Earthquake Engineering and Structural Dynamics, (2015).

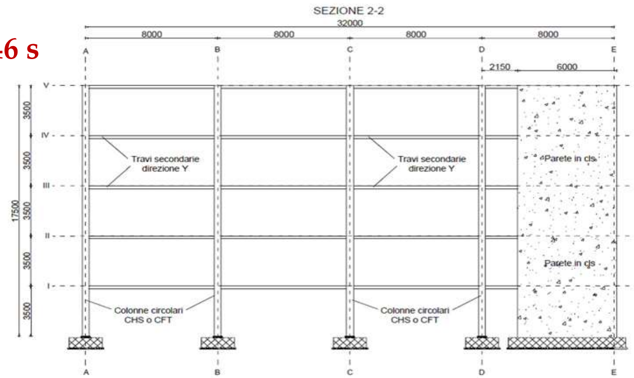


3D model of the case study ATTEL – SERA project.

T_1 = 2,79 s



T_1 = 0,46 s



Main sections of the moment resistent frame and the braced one.

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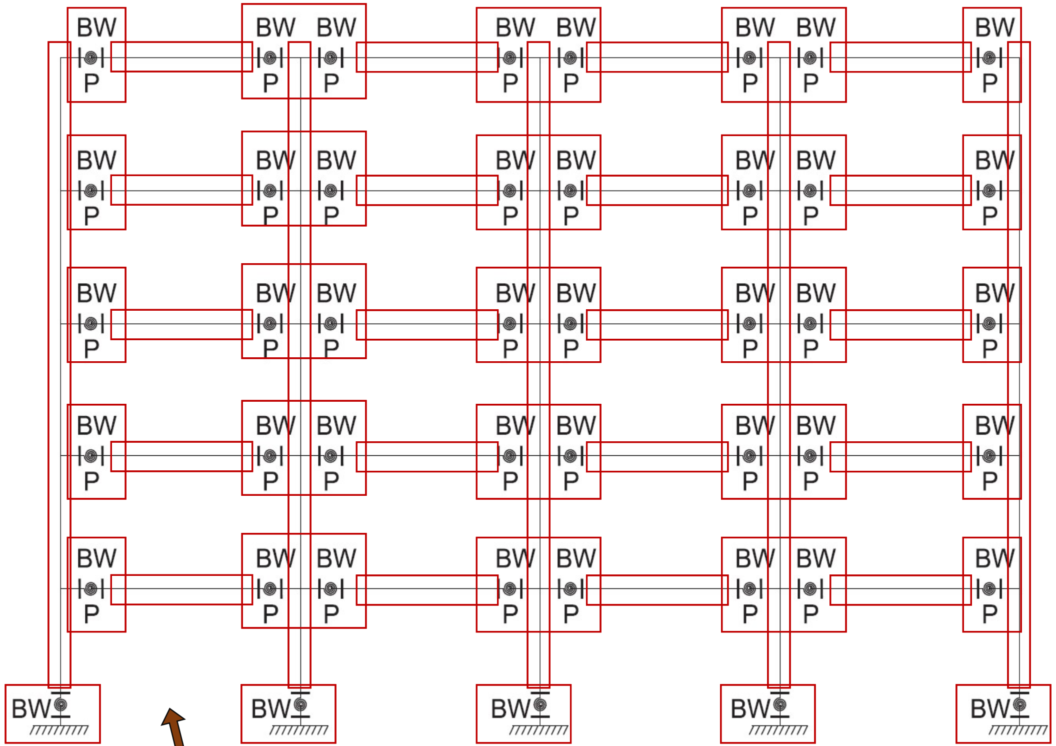


Model of the structure in OpenSees.

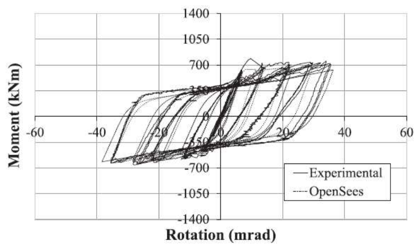
Designed according to EC8 and modelled in OS:

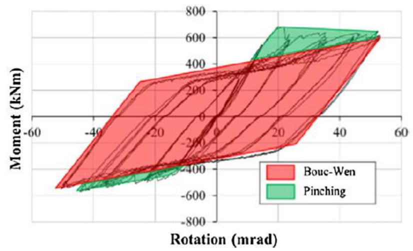
- beam and column elements with linear elastic behavior

```
elasticBeamColumn
```

- mechanical nonlinearities

```
uniaxialMaterial BoucWen
uniaxialMaterial Pinching4
uniaxialMaterial Parallel
```






```
e uniaxialMaterial Parallel $matBoucWen $matPinching  $Iz $transfTag
```

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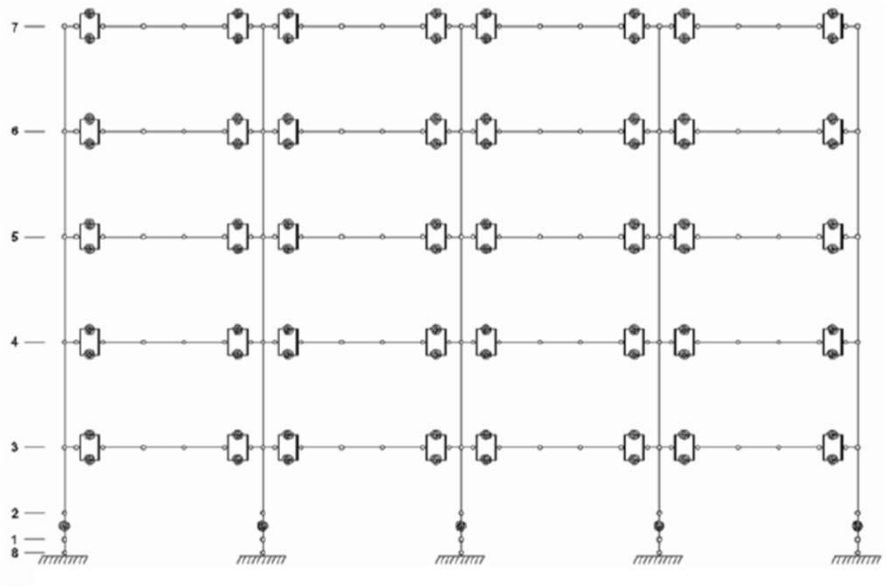
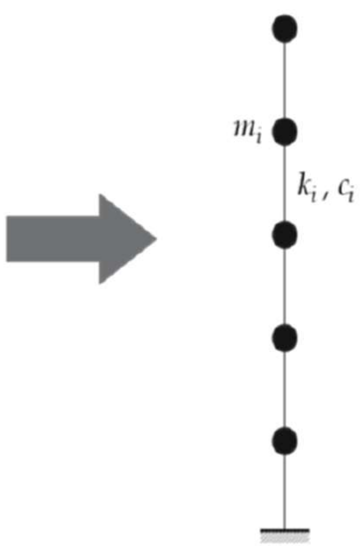
High number of analysis for seismic simulations →

- to reduce computational burden
- to reduce required simulation times





Calibration oriented to correspondence of:

- main periods
- modes of vibrating
- dissipative behavior

high fidelity model in *OpenSees - OS* → simplified model MDOF in *MATLAB® - ML*

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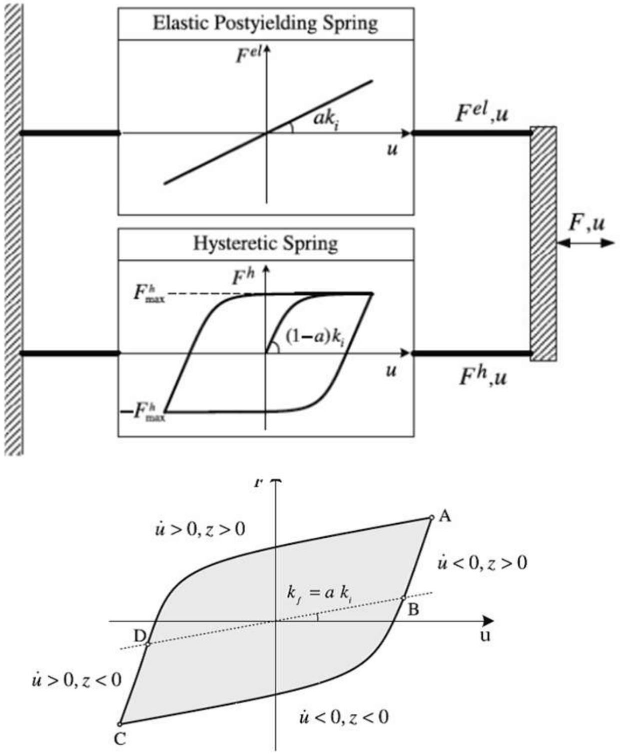


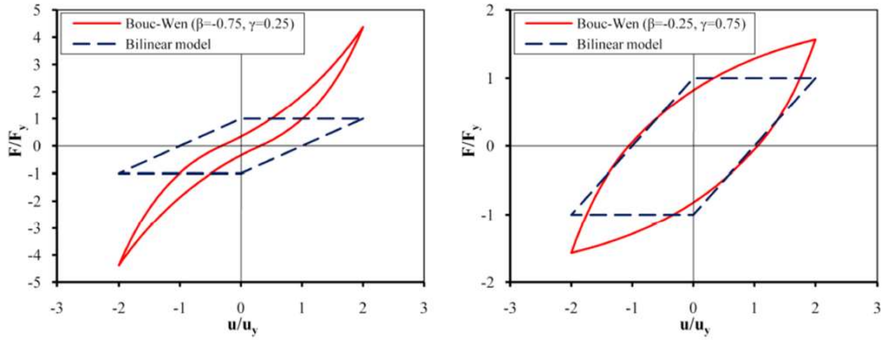
Hysteretic model of Bouc Wen.

Hysteretic model of Bouc Wen

$$m\ddot u(t) + c\dot u(t) + F_s(t) = F(t)$$

$$F_s(u(t), \dot u(t), z(t)) = F_{el}(t) + F_h(t) = \alpha\, k_i\, u(t) + (1-\alpha)\, k_i\, z(t)$$

$$\dot z = \frac{A\dot u - \left\{\beta\,|\dot x|\,z\,|z|^{n-1} + \gamma \dot u\,|z|^{n}\right\} v}{\eta}$$



Formulation of the problem and examples of hysteretic cycles.

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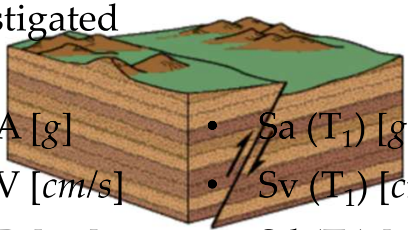
Step 2: input and *IM* selection

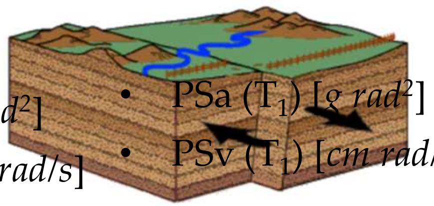
Dataset NGA-WEST 2 → *206 ground motions*

main features:
- crustal earthquakes
- $M_w > 6$
- $R_{rup} > 10$ km
- $V_{s30} > 600$ m/s

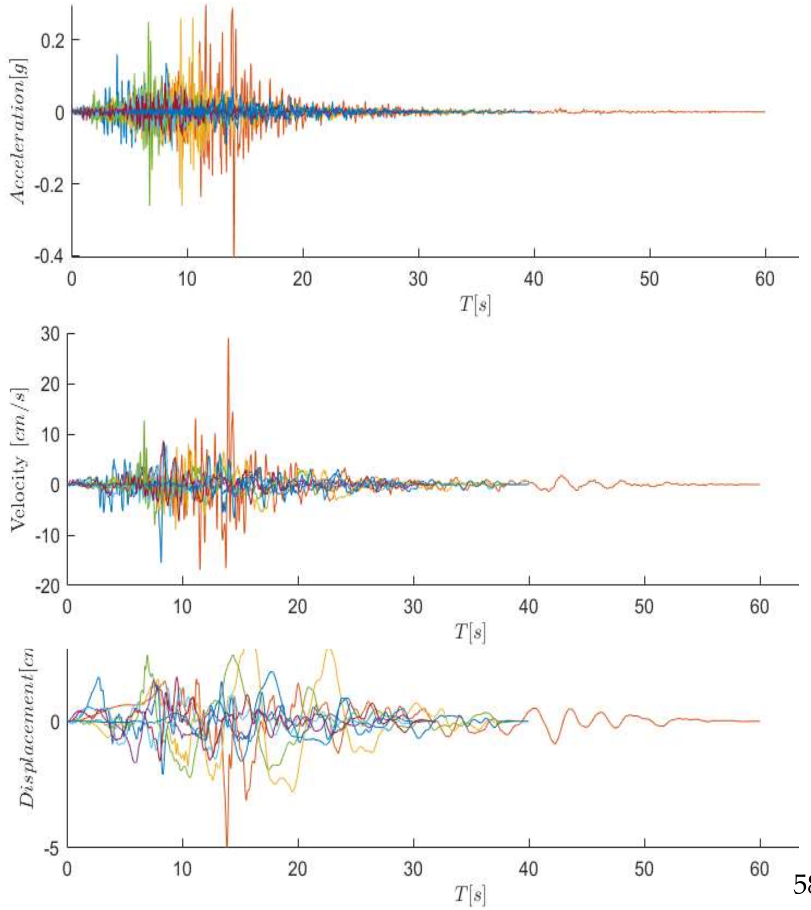
fault mechanism:
- *reverse REV*
- *strike slip SS*

IMs investigated





- PGA [*g*]
- PGV [*cm/s*]
- PGD [*cm*]
- Sa (T_1) [*g rad*^{2}]
- Sv (T_1) [*cm rad/s*]
- Sd (T_1) [*cm*]
- PSa (T_1) [*g rad*^{2}]
- PSv (T_1) [*cm rad/s*]



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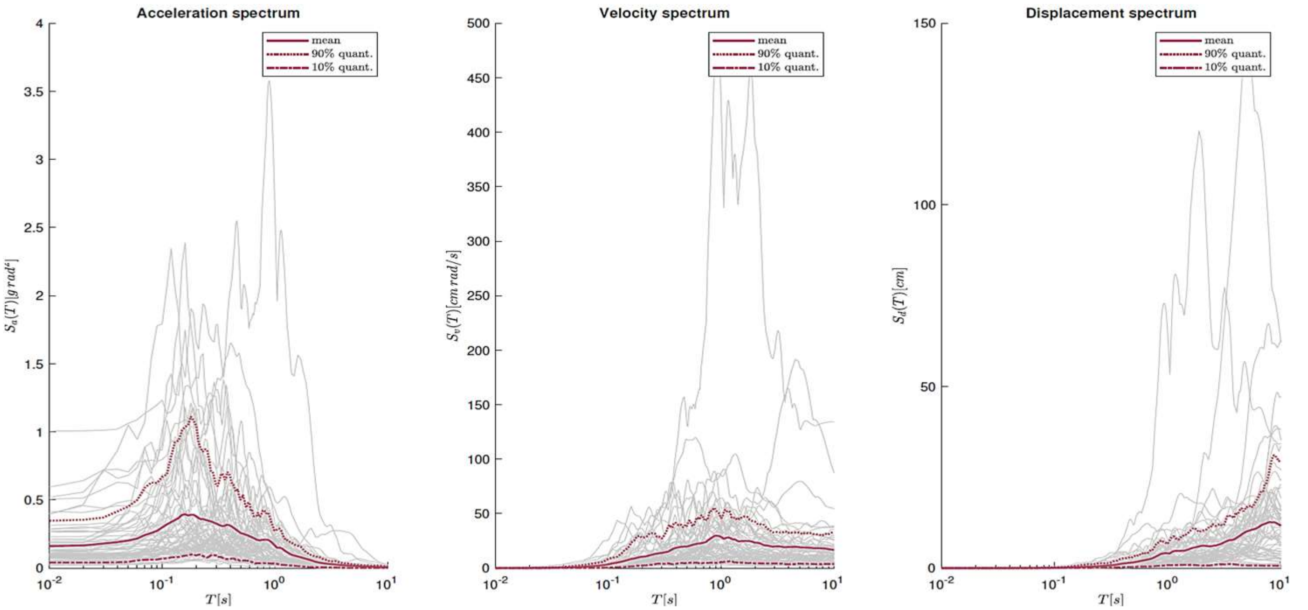
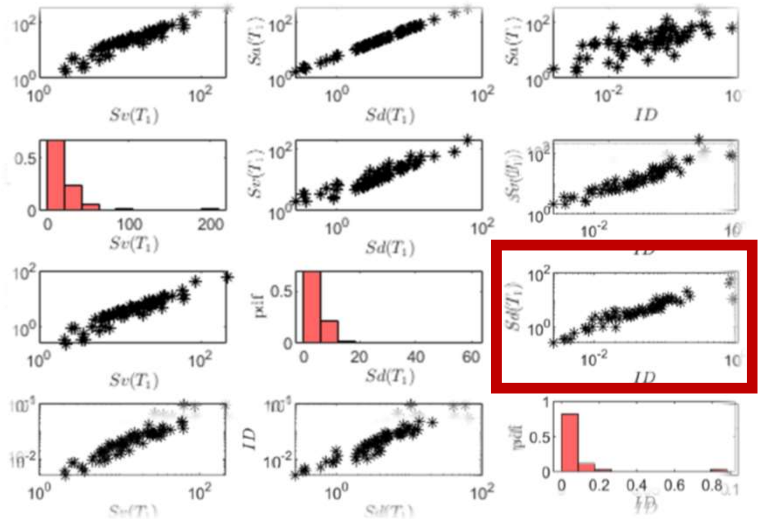
Step 2: input and *IM* selection

1) Data exploration of recorded gms
2) Scatter plot and statistic tools to evaluate proper *IM*



Acceleration, velocity and displacement response spectra with mean value, 10-th and 90-th quantile.



Scatter plot for correlation.

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Step 2: input and *IM* selection

Codes:

```
%%  Ground motions
Ground_motions = load('accelrot_cellarray.mat');
%
NN = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 19, 21, 23, 24,...
    25, 26, 28, 29, 30, 31, 32, 33, 34, 36, 37]; % number ID of the SS ground motions
DT = [0.01 , 0.01 , 0.005, 0.005, 0.005, 0.005, 0.02 , 0.02, 0.02, 0.01,...
    0.01 , 0.01 , 0.02 , 0.02 , 0.02 , 0.02 , 0.02 , 0.02, 0.01, 0.01,...
    0.005, 0.005, 0.005, 0.005, 0.02 , 0.005, 0.005, 0.01, 0.01, 0.01,...
    0.01 , 0.005, 0.005, 0.02 , 0.005, 0.01 , 0.01]; % integration time step
```

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Step 3: Definition of *damage limit states* and reference *EDP*

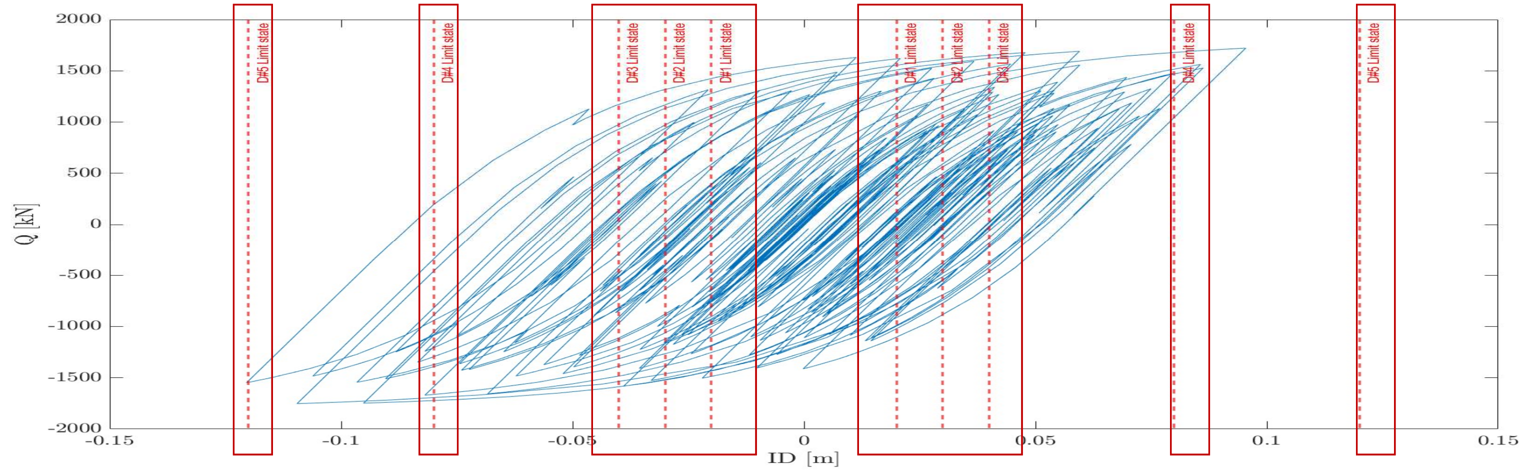
MRF

		Structural Performance Levels			
		Collapse Prevention	**Life Safety**	**Service**	**Immediate Occupancy**
Drift	[%]	5%	2,50%	1%	0,70%
	[m]	0,175	0,088	0,035	0,025

BF

		Structural Performance Levels			
		Collapse Prevention	**Life Safety**	**Service**	**Immediate Occupancy**
Drift	[%]	2,00%	1,00%	0,50%	0,30%
	[m]	0,07	0,035	0,0175	0,0105

Document *FEMA 356 - Prestandard and Commentary for the Seismic Rehabilitation of Buildings; Table C1-3 - Structural Performance Levels and Damage.*



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Step 3: Definition of *damage limit states* and reference *EDP*

Codes:

```
%% MDOF Properties
% Choose between the structural system
MDOF_properties_BW_MRF
MDOF_properties_BW_BF
% and  between linear or hysteretic behaviour
%% Structural behaviour
System_type = 'le'; % 'bw'
% bw = bouc-wen
% le = linear elastic
```

```
…
%% Limit States
LS = [0.50 0.75 1 2 3]*4/100;
for ls_i = 1:numel(LS)
ls_val = LS(ls_i)
Main_IDA_o_t
ls_i = ls_i + 1;
end
```

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Step 4: Performing non-linear time histories analysis

Codes:

```
%% Initial condition
Mat.dFe=zeros(Mat.NDOF,numel(a_g_norm));    % Preallocation for the load for the time series
a_g = a_g_norm*scale;                      % Scaled ground motion
Mat.Fe=Mat.M*Mat.r'*a_g'*g;
%% Computation response
[HistVarBw]=ResponceMDF_Bw(Mat);
edp = max(abs(HistVarBw.eps(1,:)));
EDP(i) = edp; %store the EDP for each time history analysis
SCALE(i) = scale; %store the scale factor for each time history analysis…
```

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Step 5: Computing fragilities

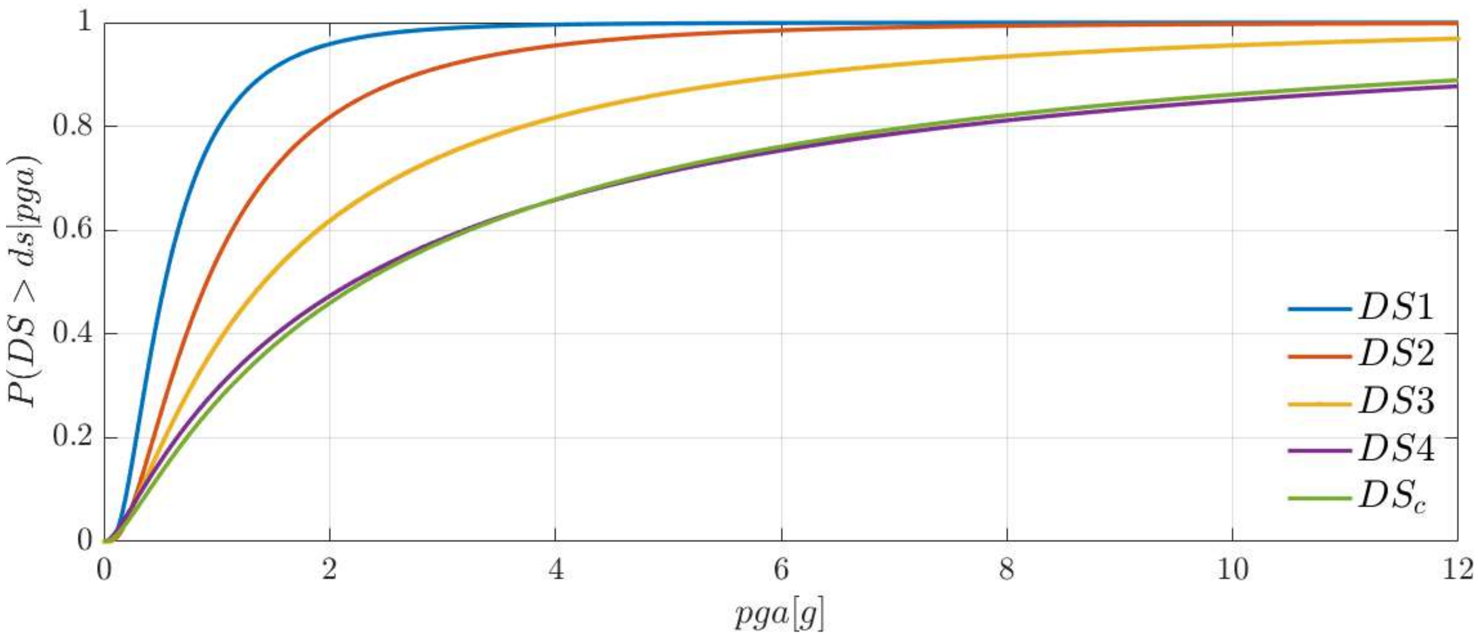
$$\mathcal{L}(\mu,\sigma) = \underline{\mathcal{L}}(\mu,\sigma)\overline{\mathcal{L}}(\mu,\sigma) = \prod_{n=1}^{\bar{N}} \varphi\left(\frac{\ln(im_n)-\mu}{\sigma}\right)\left[1-\Phi\left(\frac{\ln(\overline{IM})-\mu}{\sigma}\right)\right]^{N-\overline{N}} \quad (1)$$

$$\ln\mathcal{L}(\mu,\sigma) = \sum_{n=1}^{\bar{N}} \varphi\left(\frac{\ln(im_n)-\mu}{\sigma}\right) + (N-\overline{N})\left[1-\Phi\left(\frac{\ln(\overline{IM})-\mu}{\sigma}\right)\right] \quad (2)$$

<u>Codes:</u>

```
%% Untruncated IDA
[parmhat,parmci] = lognfit(IM_t_c ,0.01);
mu_IDA = parmhat(1);
sigma_IDA = parmhat(2);
%% Truncated IDA
IM_max = 2.2;
IM_trunc = IM_t_c(IM_t_c < IM_max); % take only the results with IM < IM_max
eq_over = sum(IM_t_c >= IM_max);    % number of analyses reached IM_max without collapsing
% Maximum likelihood fit, using equation (1) and (2) of previously slides
[mu_IDA_t, sigma_IDA_t ] = truncated_ida(IM_trunc, IM_max, eq_over);
```

Step 5: Computing fragilities - Results



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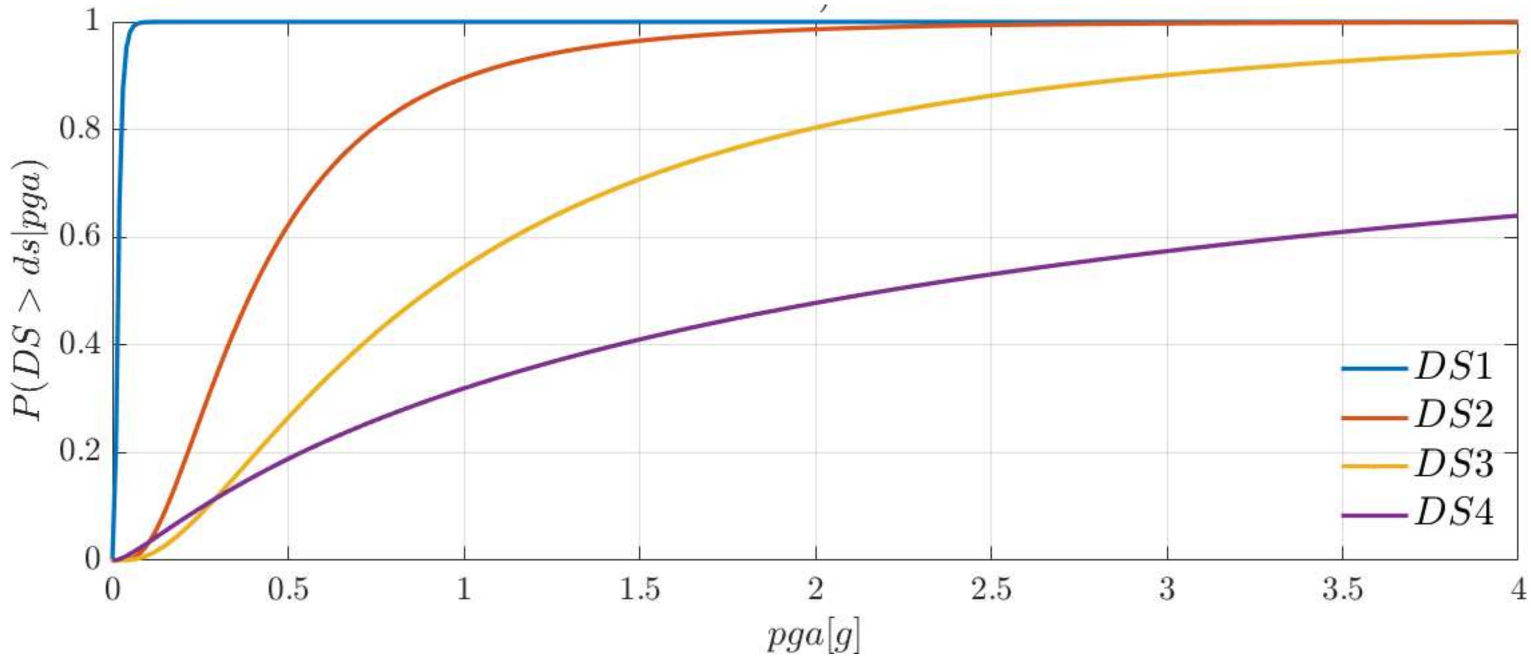
Step 5: Computing fragilities - Results



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References

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References

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