

Introduction

Performance Based Earthquake Engineering

Ph.D. Student Chiara Nardin – M.Sc., Eng. in Civil Engineering

<https://github.com/kia13nn/ISPS.git>

Introduction


Probabilistic Hazard Analysis

Fragility Analysis

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PBEE: Performance Based Earthquake Engineering

- Probabilistic framework for (i) assessing design, (ii) evaluation and (iii) planning of civil system.
- load-and-resistance-factor design (LRFD)



 performance based design (PBD):
 3Ds – i.e. downtime, dollars, death
- 2 axes define the domain of acceptable system's response:
 - System performance objectives
 - Seismic hazard level, w.r.t. return periods

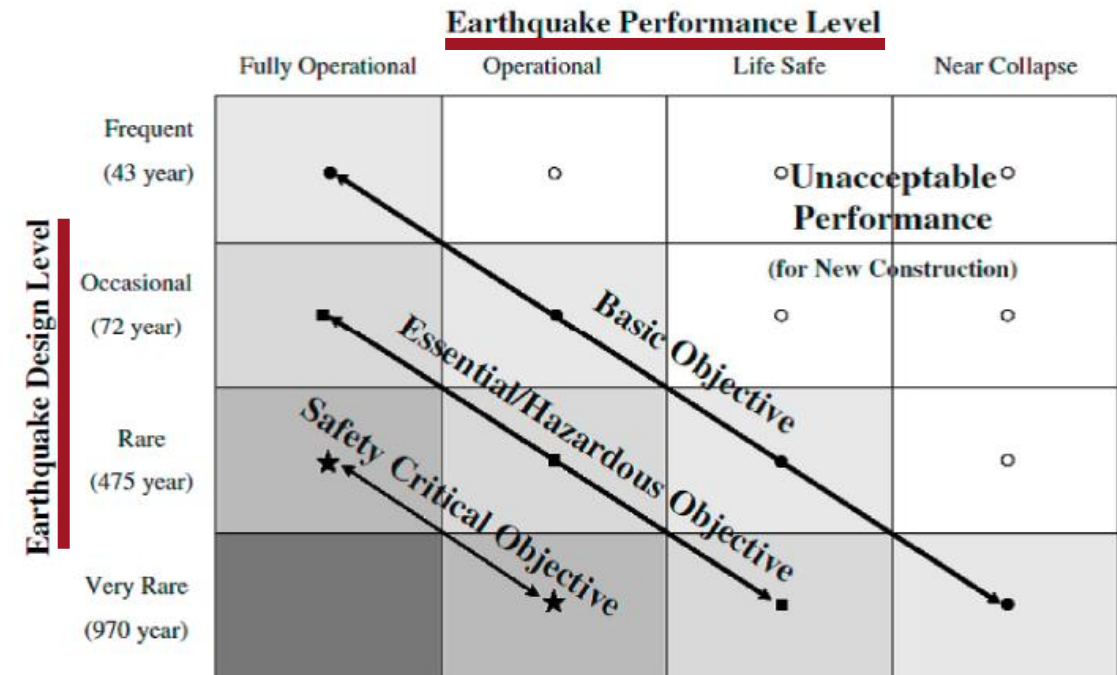


Fig.1 – PBEE concept: seismic performance objectives *vs* seismic hazard level.
©Poland et al.,(1995)-Vision 2000: Performance Based Earthquake Engineering of buildings. Structural Engineers Association of California, Sacramento, CA.

The PEER-PBEE Framework

- PEER ~ Pacific Earthquake Engineering Research center – analytical approaches **based on total probability theorem** for the *yearly mean number of events of a selected decision variable*

$$IM \longrightarrow EDP \longrightarrow DM \longrightarrow DV$$

$$\lambda(dv) = \int_d \int_{edp} \int_{im} G(dv|d) |dG(d|edp)| |dG(edp|im)| |d\lambda(im)| \quad (1)$$

- with the underlying assumptions of:
 - Markovian structure $\rightarrow DV \perp\!\!\!\perp EDP, IM | DM = dm$; $DM \perp\!\!\!\perp IM | EDP = edp$;
 - No-aging effects on structures;
 - Poisson's processes \rightarrow memoryless seismic events.

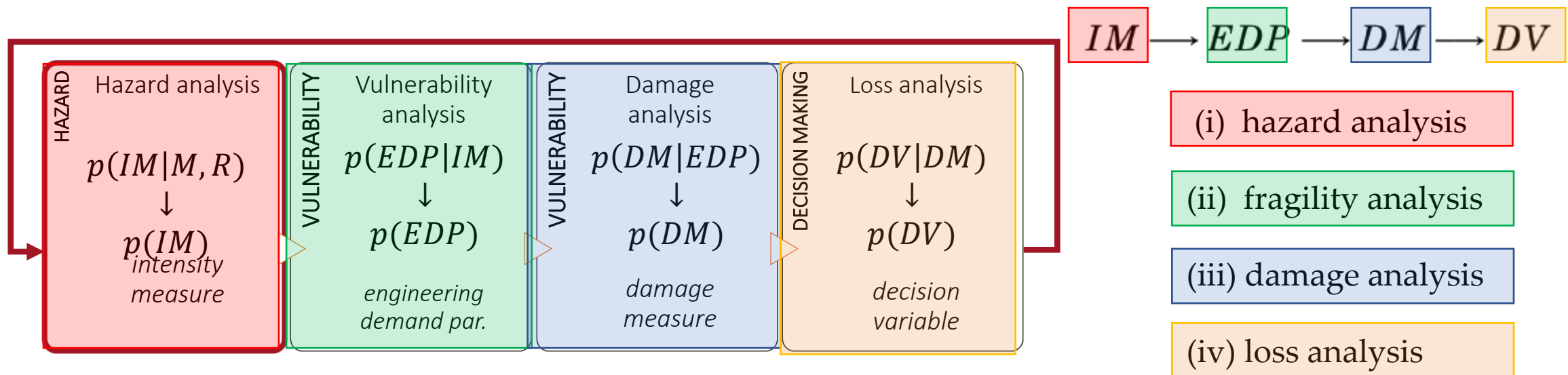
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The PEER-PBEE Framework



$$\lambda(dv) = \int_d \int_{edp} \int_{im} G(dv|d) \|dG(d|edp)\| \|dG(edp|im)\| d\lambda(im)$$

The PEER-PBEE Framework

$$IM \longrightarrow EDP \longrightarrow DM \longrightarrow DV$$

$$\lambda(dv) = \int_d \int_{edp} \int_{im} G(dv|d) |dG(d|edp)| |dG(edp|im)| |d\lambda(im)|$$

where

- im is an intensity measure (e.g., peak ground acceleration, spectral acceleration, etc.);
- edp is an engineering demand parameter (e.g., interstorey drift);
- d is a damage measure (e.g., minor, medium, extensive, collapse);
- dv is a decision variable (e.g., monetary losses, fatalities, etc.);
- $\lambda(x)$ is the mean annual rate of events exceeding a given threshold for a given variable x ;
- $G(y|x) = P(Y \geq y|X = x)$ is the conditional complementary cumulative distribution function (CCDF)

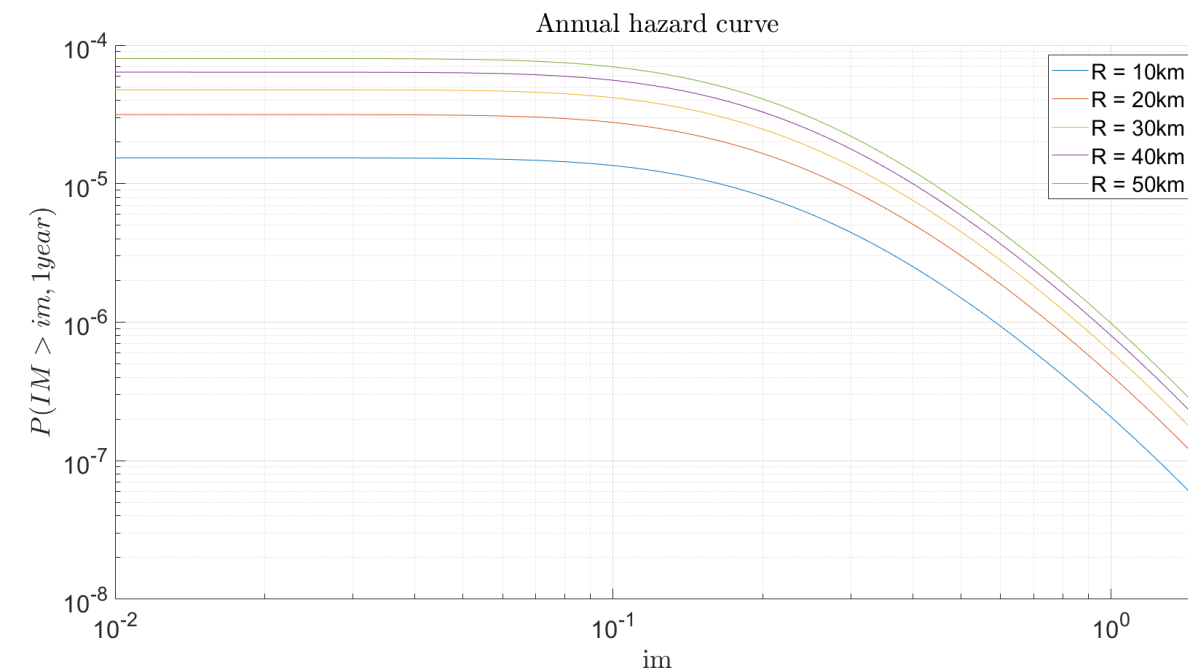
Probabilistic Hazard Analysis

Formulation and MatLab Computation

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Step 4: Hazard Computation

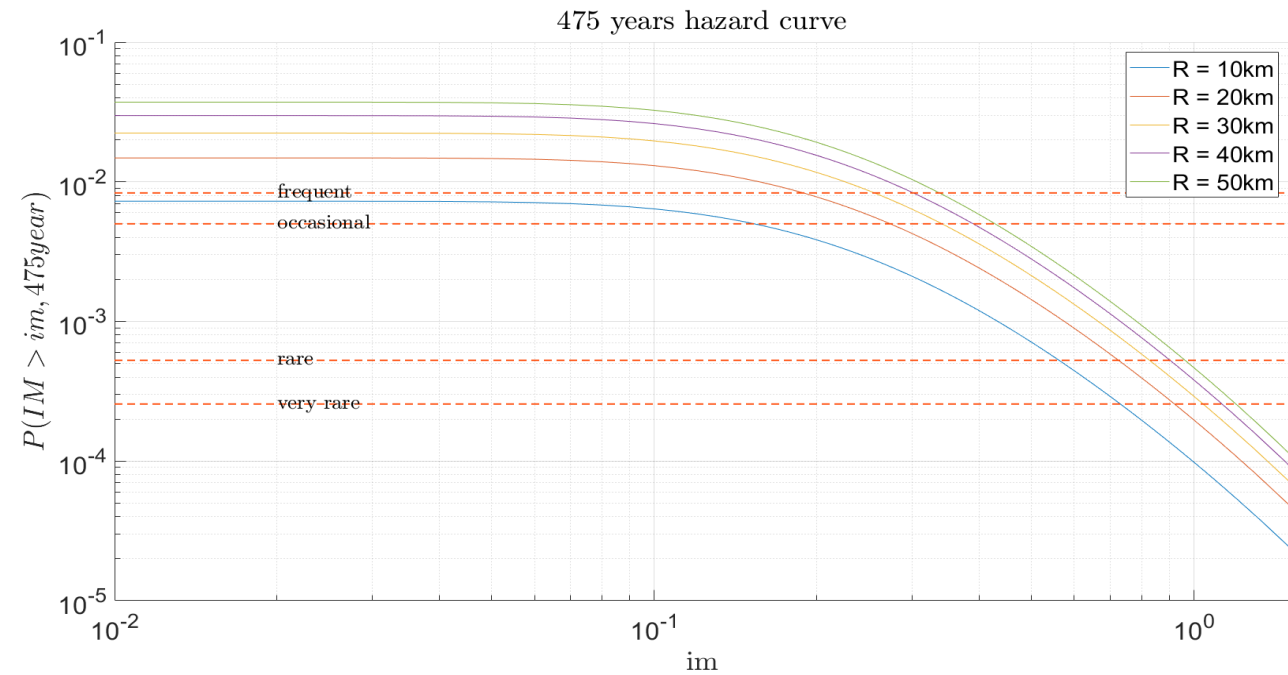
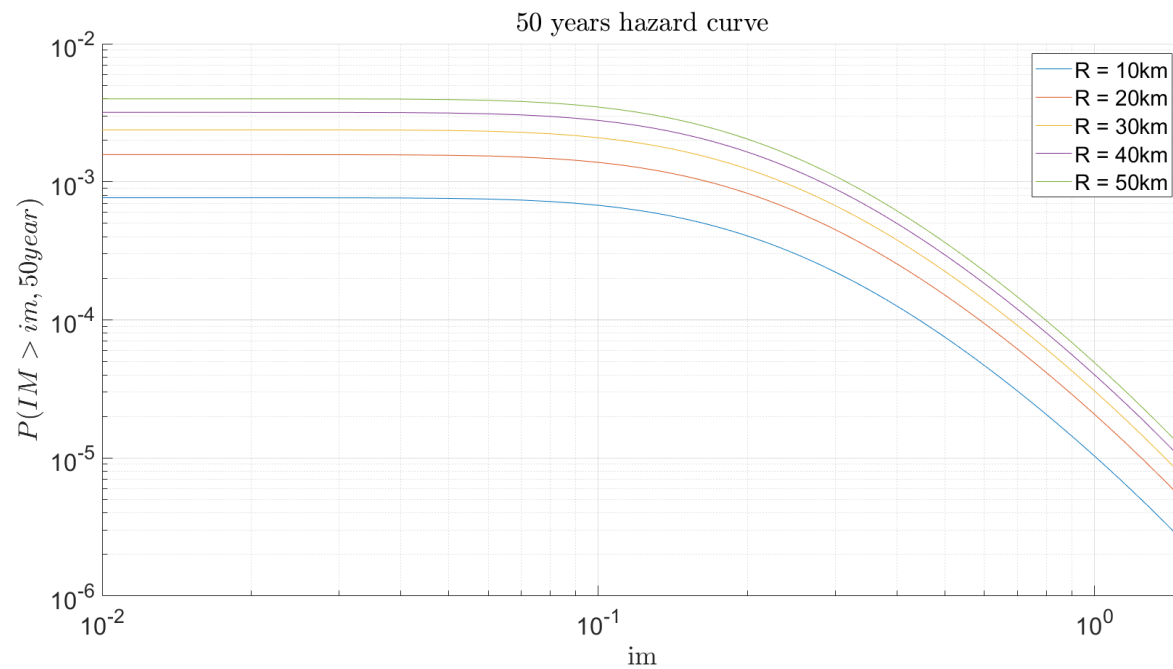


Goal and results

Through the scheme depicted in (1), compute:

- the annual hazard curve for each fault;
- the 50 years hazard curve for each fault;
- the 475 years hazard curve for each fault for the highlighted seismic site.

Step 4: Hazard Computation



Fragility Analysis

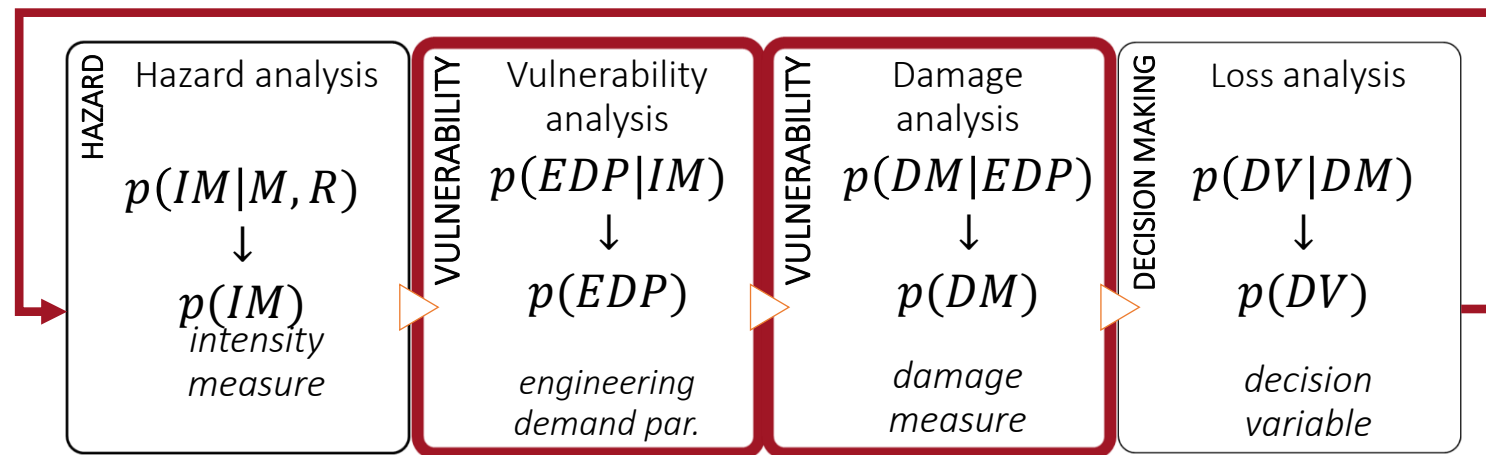
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Fragility Analysis

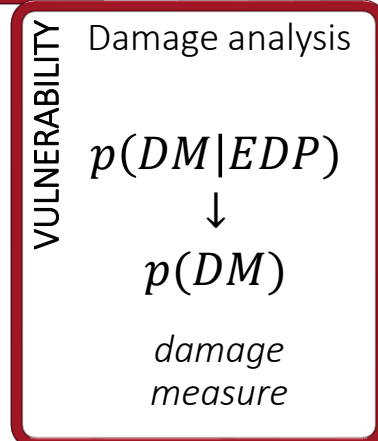
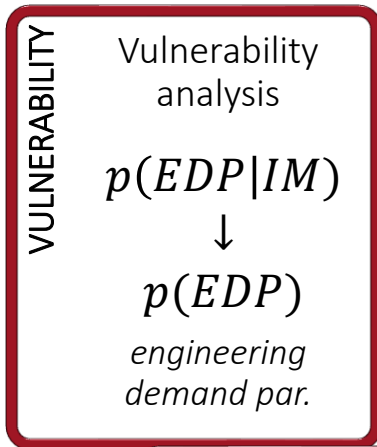
Fragility (or Vulnerability) Analysis is the second step of the PBEE-PEER framework



Useful definitions:

seismic fragility function := the conditional probability of an event (e.g. a defined limit/damage state) given the observation of an intensity measure which describe the seismic event.

Fragility Analysis



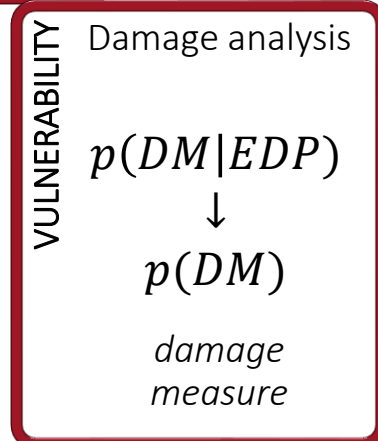
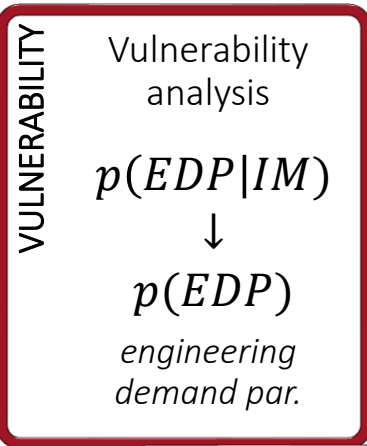
The fragility curve is defined as the conditional probability of failure of a structure, or its critical components, at given values of seismic intensity measures (IMs).

$$\lambda(dv) = \sum_d \int_{edp} \int_{im} G(dv|d) P(d|im) d\lambda(im)$$

In practice, a fragility curve is calculated as the conditional probability that the damage measure (D) exceeds a critical threshold, for a given seismic IM .

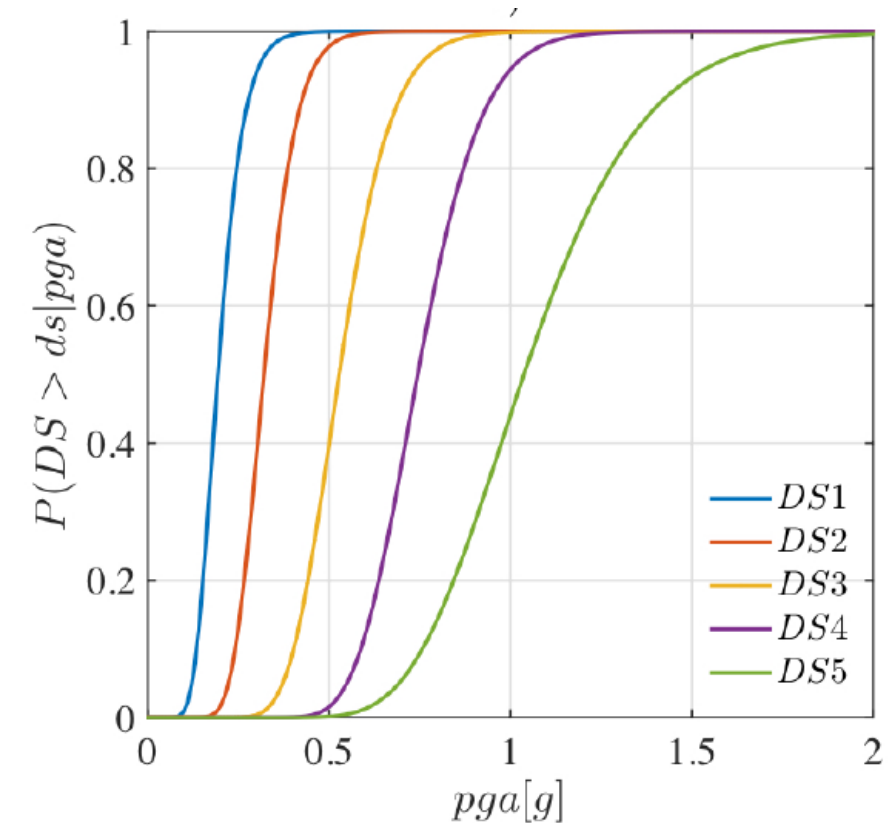
$$P(D > d_{threshold} | IM = im)$$

Fragility Analysis

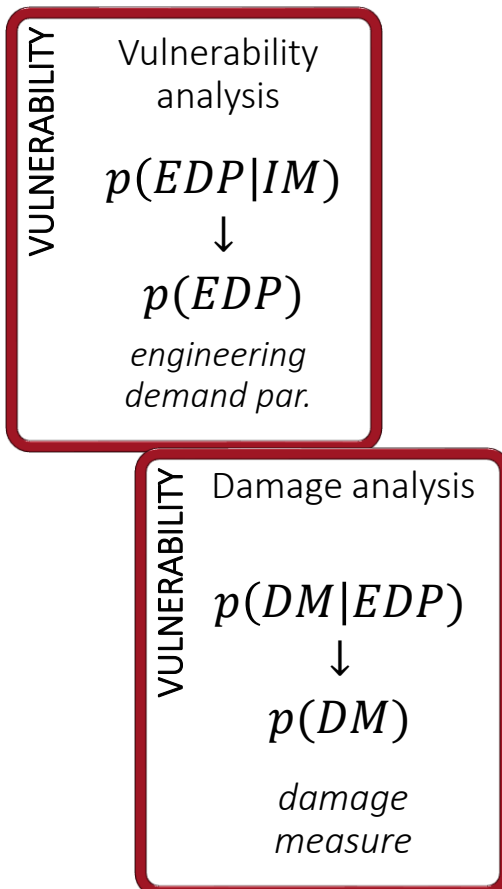


$$P(D > d_{threshold} | IM = im)$$

Fragility curves for different damage limit states or thresholds.



Fragility Analysis: key aspects



INTENSITY MEASURE (*IM*)

in terms of:

- **Efficiency**, i.e. variability of an *EDP* for a given *IM*;
- **Robustness**, i.e. efficiency between *IM-EDP* at different period ranges;
- **Practicality**, i.e. correlation to known and easy identifiable engineering quantities;
- **Sufficiency**, i.e. validity of *EDP|IM* as statistically independent from gm site characteristics;
- **Effectiveness**, i.e. ability to evaluate an analytical relation.

DAMAGE STATE (*D*)

Should suit the specific structural problem → associate each damage state to a specific *EDP*

- Categorical variables, i.e. D_0 no damages – D_1 minor – D_2 moderate – ... – $D_f = C$ collapse ;
- Probabilistic or deterministic relationship between *EDP* and *D*

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Probabilistic Hazard Analysis

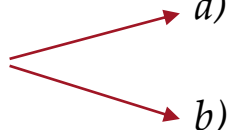
Fragility Analysis

References

Class of fragilities

EMPIRICAL	ANALYTICAL	EXPERT OPINION	HYBRID
<ul style="list-style-type: none"> by fitting a function to observational data from past earthquakes or lab tests <p>↕</p> <p>collections pairs of level of excitation and categorical variables of damage or collapse</p>	<ul style="list-style-type: none"> by defining analytical structural model and analyzing its performance under different levels of the seismic hazard <p>↕</p> <p><u>static</u>, i.e. hazard as response spectrum and push-over analysis vs <u>dynamic</u>, i.e. collection of gms and simulations on FEM via NLA</p>	<ul style="list-style-type: none"> by polling one or more experts of the given structural asset <p>↕</p> <p>to guess or estimate the failure probability for a given hazard level</p>	<ul style="list-style-type: none"> based on combination of the different methods

Dynamic based fragility functions: steps

- i) Definition of a numerical model: $y(t) = \mathcal{M}[\ddot{x}_g(t|IM = im); \theta_{\mathcal{M}}(t)]$
 - ii) Selection of a suitable IM given the structure
 - iii) Selection of a suitable set of N gms for the location
 - iv) Selection of an EDP of interest
 - v) Definition of damage limit states D via EDP thresholds
 - vi) Scale each gm based on the given IM eventually until collapse
 - vii) Save each $[EDP \rightarrow D]$ threshold- im_n pair for each gm
- 
 - a) Full IDA
 - b) Truncated IDA

Computation of fragility function

Hp: - assume a *lognormal probability distribution* for the random variable IM associated with given D

FULL IDA

\forall damage state $\rightarrow N$ results, since $\forall IM_n = im_n$,
 $y(t)$ reached the given damage state

$$P(D > d | IM = im) = \Phi \left(\frac{\ln(im_n) - \hat{\mu}}{\hat{\sigma}} \right)$$

where $\Phi(\cdot)$ is the CDF of the normal distribution and

$$\hat{\mu} = \frac{1}{N} \sum_{n=1}^N \ln(im_n)$$

$$\hat{\sigma} = \sqrt{\frac{1}{N} \sum_{n=1}^N (\ln(im_n) - \hat{\mu})^2}$$

TRUNCATED IDA

\overline{IM} upper limit;

- a) data that causes collapse $n \in [1, \bar{N}]$,
- b) data that do not cause collapse $n \in [\bar{N} + 1, N]$

$$a) \rightarrow \underline{\mathcal{L}}(\mu, \sigma) \alpha \prod_{n=1}^{\bar{N}} \varphi \left(\frac{\ln(im_n) - \mu}{\sigma} \right)$$

$$b) \rightarrow \overline{\mathcal{L}}(\mu, \sigma) \alpha \prod_{n=\bar{N}+1}^N \left[1 - \Phi \left(\frac{\ln(\overline{IM}) - \mu}{\sigma} \right) \right] = \left[1 - \Phi \left(\frac{\ln(\overline{IM}) - \mu}{\sigma} \right) \right]^{N-\bar{N}}$$

a)+b) \rightarrow

$$\begin{aligned} \mathcal{L}(\mu, \sigma) &= \underline{\mathcal{L}}(\mu, \sigma) \overline{\mathcal{L}}(\mu, \sigma) = \\ &= \prod_{n=1}^{\bar{N}} \varphi \left(\frac{\ln(im_n) - \mu}{\sigma} \right) \left[1 - \Phi \left(\frac{\ln(\overline{IM}) - \mu}{\sigma} \right) \right]^{N-\bar{N}} \end{aligned}$$

Computation of fragility function

TRUNCATED IDA

The likelihood for the entire set of data

$$\mathcal{L}(\mu, \sigma) = \underline{\mathcal{L}}(\mu, \sigma) \overline{\mathcal{L}}(\mu, \sigma) = \prod_{n=1}^{\bar{N}} \varphi \left(\frac{\ln(im_n) - \mu}{\sigma} \right) \left[1 - \Phi \left(\frac{\ln(\overline{IM}) - \mu}{\sigma} \right) \right]^{N - \bar{N}} \quad (1)$$

And the log likelihood

$$\ln \mathcal{L}(\mu, \sigma) = \sum_{n=1}^{\bar{N}} \varphi \left(\frac{\ln(im_n) - \mu}{\sigma} \right) + (N - \bar{N}) \left[1 - \Phi \left(\frac{\ln(\overline{IM}) - \mu}{\sigma} \right) \right] \quad (2)$$

Estimation of parameters by optimization

$$[\hat{\mu}, \hat{\sigma}] = \operatorname{argmin}_{\mu, \sigma} [-\ln \mathcal{L}(\mu, \sigma)].$$

Computation of fragility function

For a generic $f(x; \theta)$

$$\mathcal{L}(\theta) = \prod_{n=1}^N f(im_n; \theta) [1 - F(\overline{IM}; \theta)]^{N-\bar{N}}$$

And the log likelihood

$$\ln \mathcal{L}(\theta) = \sum_{n=1}^{\bar{N}} f(im_n; \theta) + (N - \bar{N}) [1 - F(\overline{IM}; \theta)]$$

Estimation of parameters by optimization

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} [-\ln \mathcal{L}(\theta)]$$

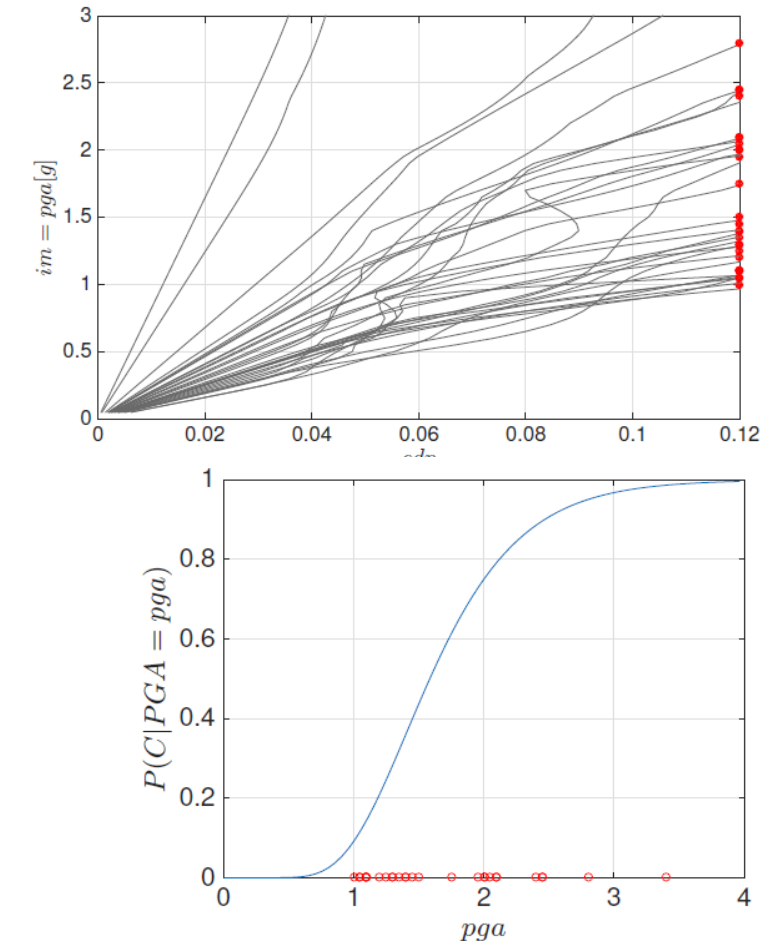


Fig.1 – Fragility function computed via IDA, see Broccardo, M. (2018)
Probabilistic seismic risk analysis for civil systems, Lecture Notes.

Fragility Analysis

Formulation and MatLab Computation

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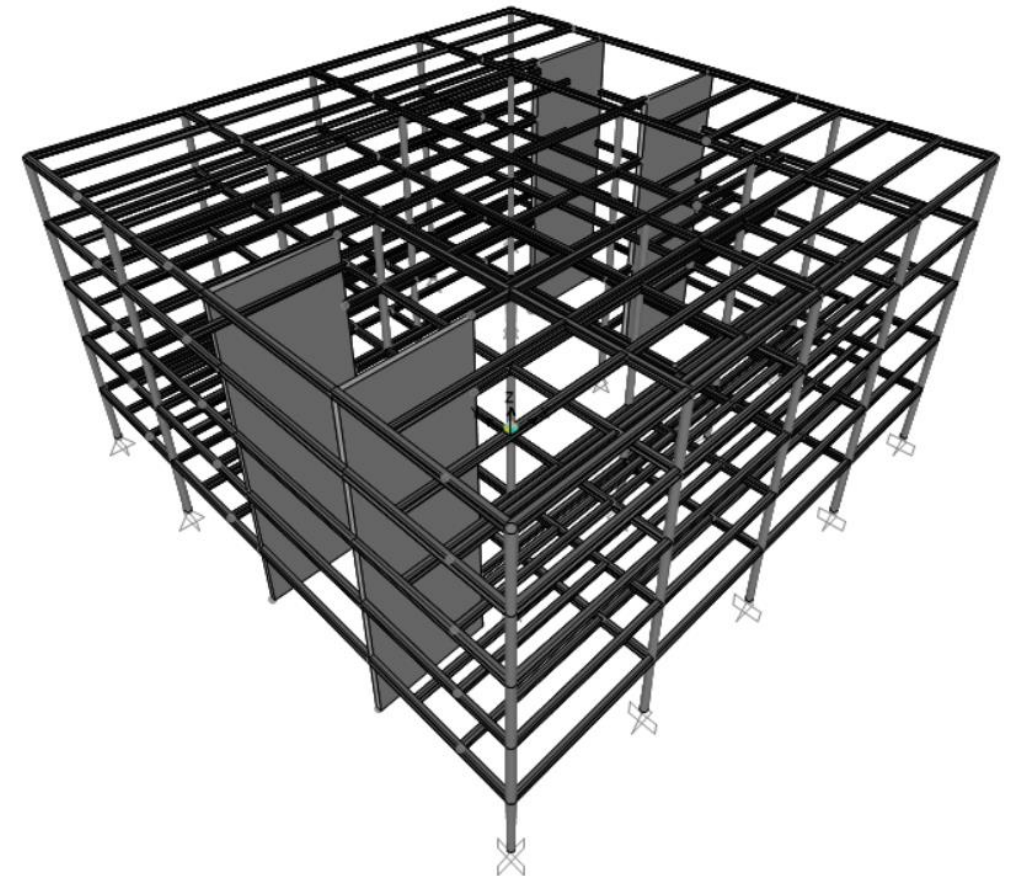
Goal: to perform fragility analysis

Given the provided set of ground motions, perform a classical and truncated incremental dynamic analysis (*IDA*) and determine fragility curves for:

- ATTEL – moment resistant frame (*MRF*);
- ATTEL – braced frame (*BF*)

by considering both

- Linear elastic behaviour
- Bouc – Wen model for hysteresis



3D model of the case study ATTEL – SERA project.

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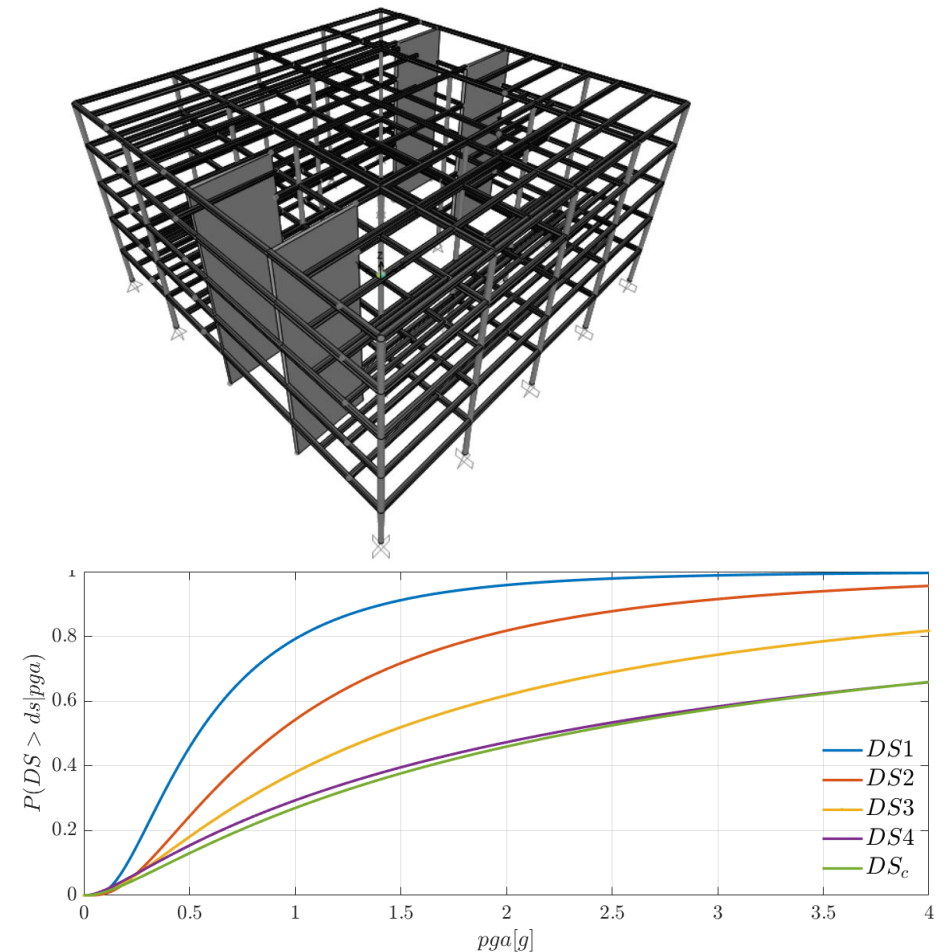
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Main steps:

- 1) Definition of the numerical model
- 2) Input and *IM* selection
- 3) Definition of *damage limit states* and reference *EDP*
- 4) Performing non-linear time histories analysis (IDA, truncated IDA, cloud, MSA ...)
- 5) Collecting results pairs and computing fragility



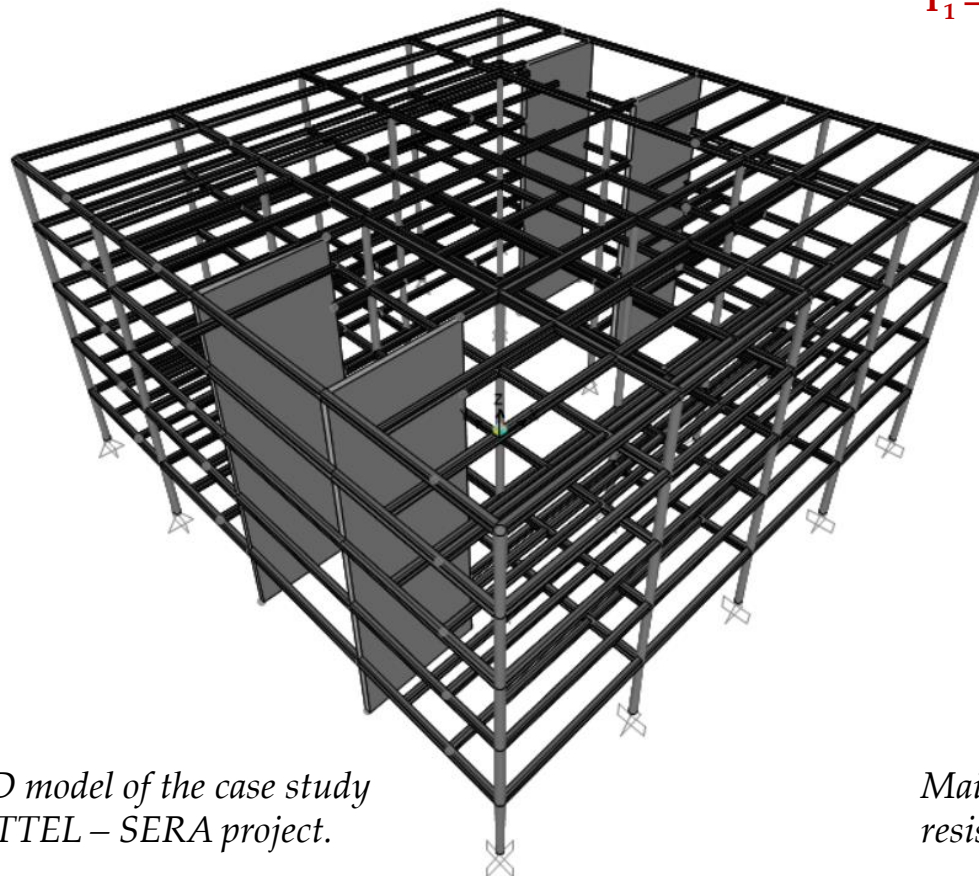
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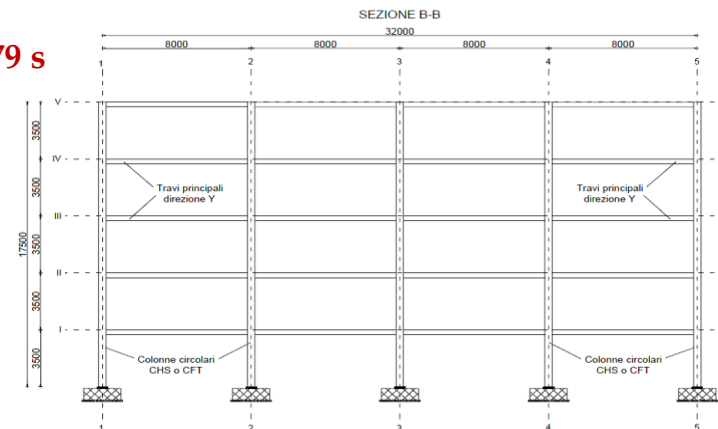
References

Step 1: the case study ATTEL⁽¹⁾



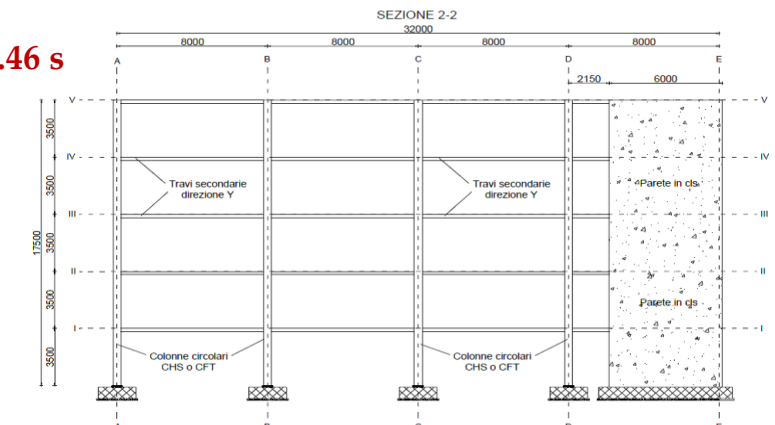
3D model of the case study
ATTEL – SERA project.

$$T_1 = 2.79 \text{ s}$$



Main sections of the moment
resistent frame and the braced one.

$$T_1 = 0.46 \text{ s}$$



^[1]BURSI, PUCINOTTI, TONDINI, ZANON, Tests and model calibration of high strength steel tubular beam-to-column and column-base composite joints for moment-resisting structures, Earthquake Engineering and Structural Dynamics, (2015).

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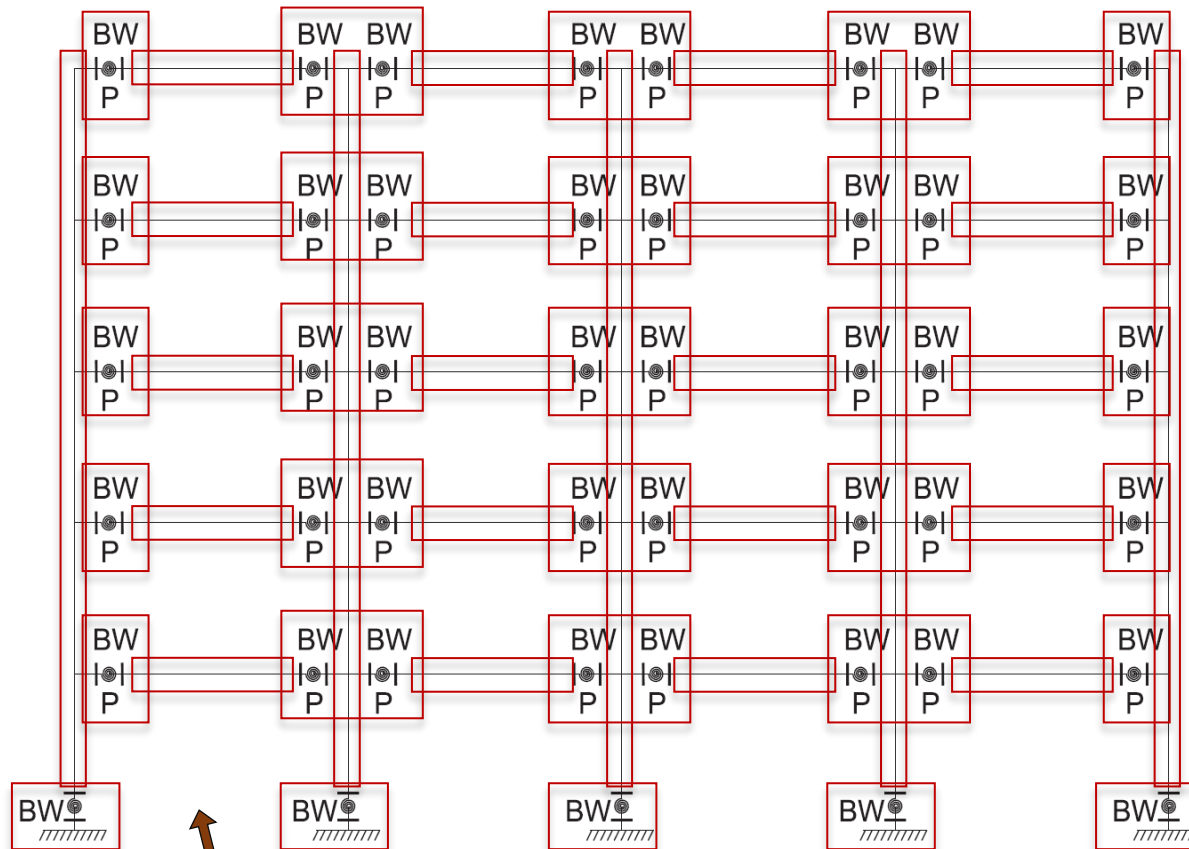
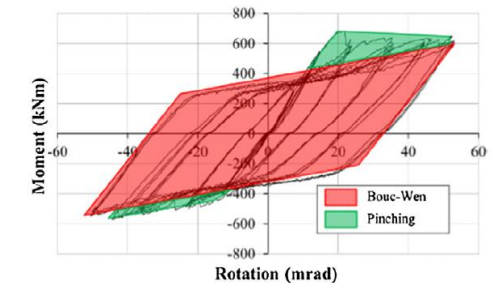
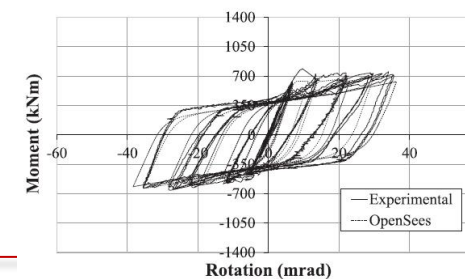
Designed according to EC8 and modelled in OS:

- beam and column elements with linear elastic behavior

`elasticBeamColumn`

- mechanical nonlinearities

`uniaxialMaterial BoucWen`
`uniaxialMaterial Pinching4`
`uniaxialMaterial Parallel`



Model of the structure in OpenSees.

```
e uniaxialMaterial Parallel $matBoucWen $matPinching $Iz $transfTag
```

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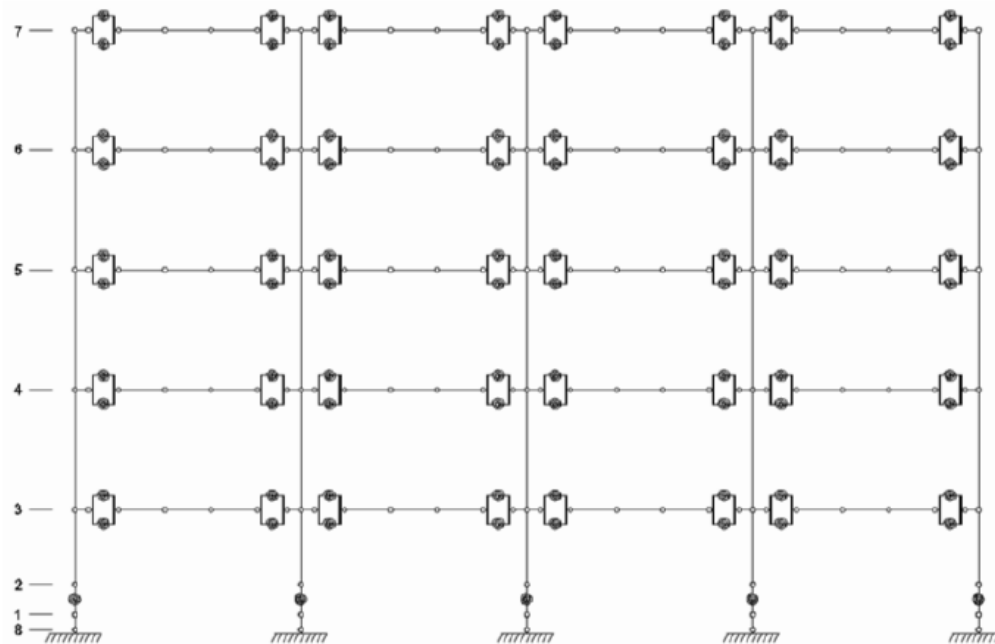
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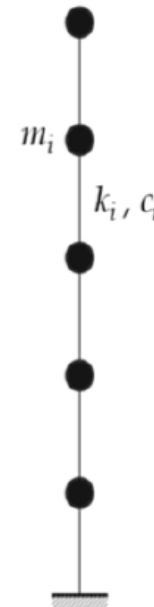
High number of analysis
for seismic simulations



- to reduce computational burden
- to reduce required simulation times



high fidelity model in *OpenSees* - OS



simplified model MDOF in *MATLAB*® - ML

Calibration oriented to
correspondence of:

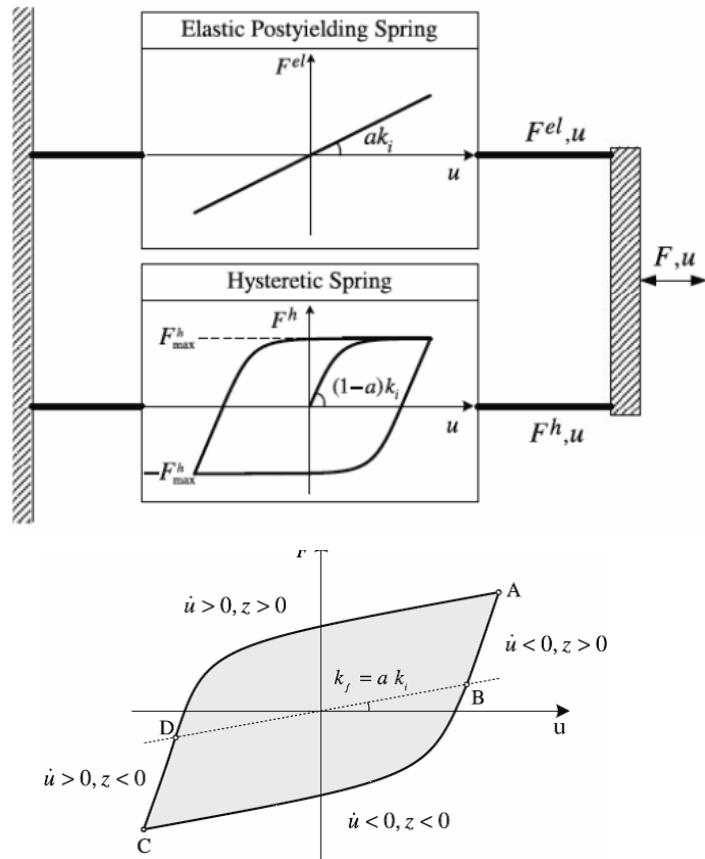
- main periods
- modes of vibrating
- dissipative behavior

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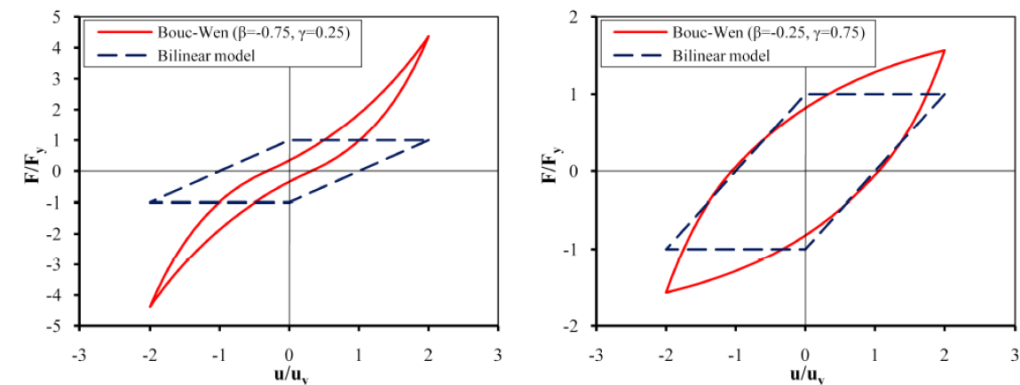
Hysteretic model of Bouc Wen.

Hysteretic model of Bouc Wen

$$m\ddot{u}(t) + c\dot{u}(t) + F_s(t) = F(t)$$

$$F_s(u(t), \dot{u}(t), z(t)) = F_{el}(t) + F_h(t) = \alpha k_i u(t) + (1 - \alpha) k_i z(t)$$

$$\dot{z} = \frac{A\dot{u} - \{\beta |\dot{x}| z |z|^{n-1} + \gamma \dot{u} |z|^n\} v}{\eta}$$



Formulation of the problem and examples of hysteretic cycles.

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Step 2: input and IM selection

Dataset NGA-WEST 2 → 206 ground motions

main features {

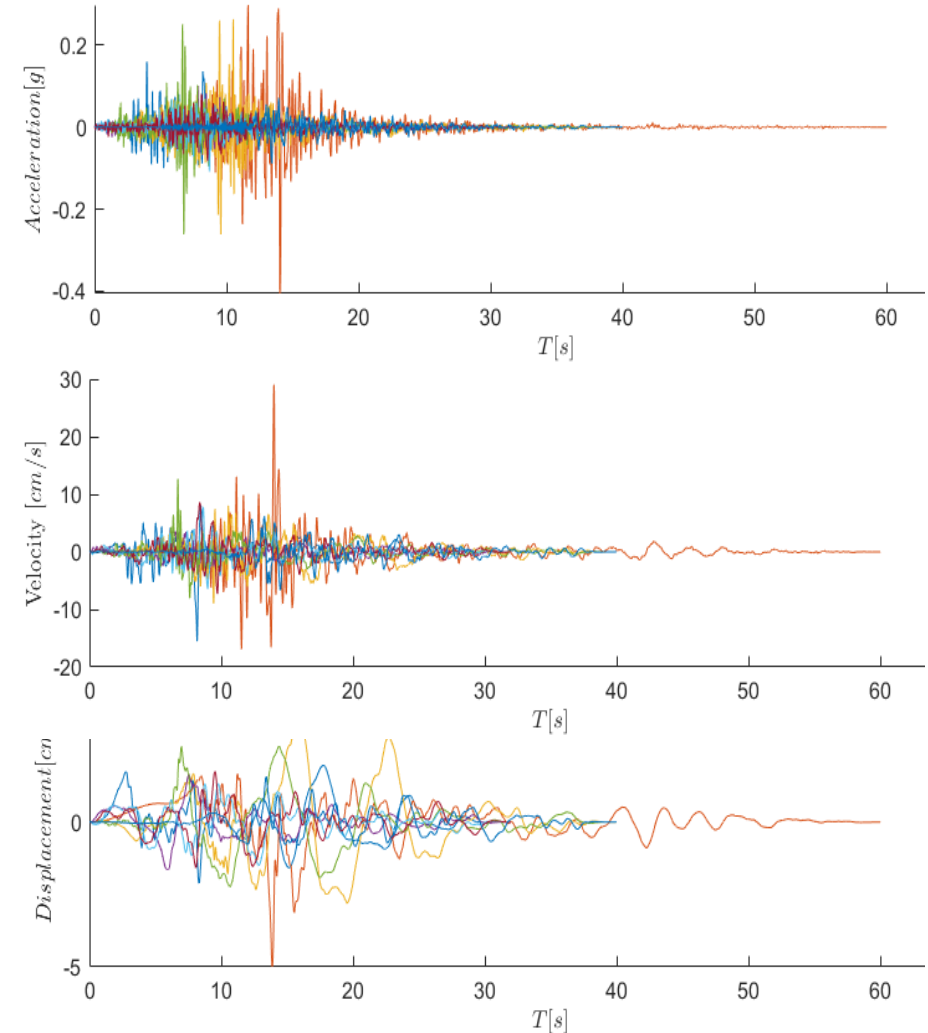
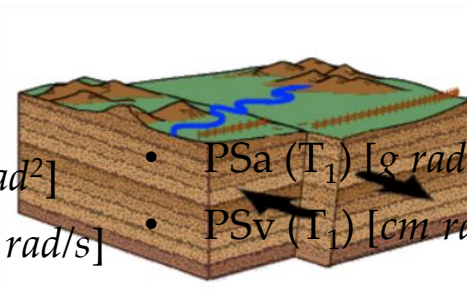
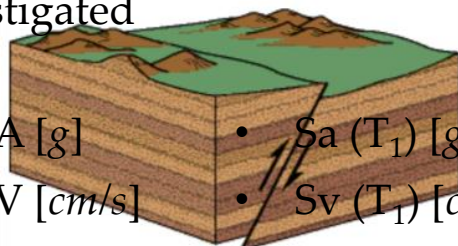
- crustal earthquakes
- $M_w > 6$
- $R_{rup} > 10$ km
- $V_{s30} > 600$ m/s

 fault mechanism {

- reverse REV
- strike slip SS

IMs investigated

- PGA [g]
- PGV [cm/s]
- PGD [cm]
- $S_a(T_1)$ [g rad²]
- $S_v(T_1)$ [cm rad/s]
- $S_d(T_1)$ [cm]
- $PS_a(T_1)$ [g rad²]
- $PS_v(T_1)$ [cm rad/s]



Introduction

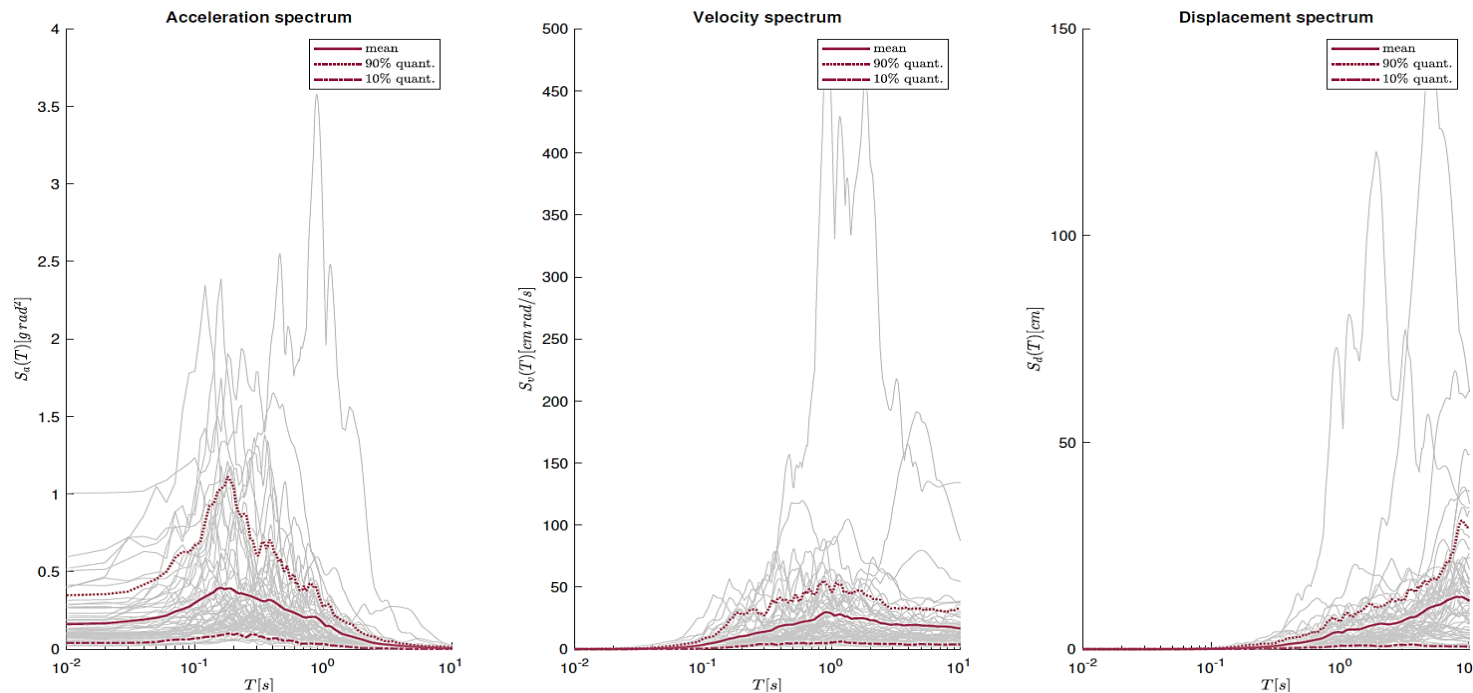
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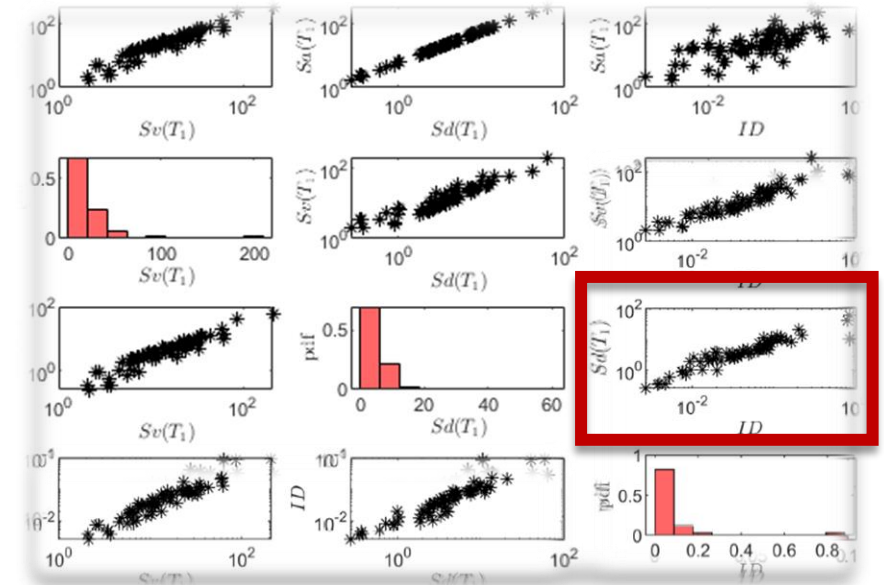
References

Step 2: input and *IM* selection

- 1) Data exploration of recorded gms
- 2) Scatter plot and statistic tools to evaluate proper *IM*



Acceleration, velocity and displacement response spectra with mean value, 10th and 90th quantile.



Scatter plot for correlation.

Step 2: input and *IM* selection

Codes:

```
%% Ground motions
Ground_motions = load('accelrot_cellarray.mat');
%
NN = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 19, 21, 23, 24,...
      25, 26, 28, 29, 30, 31, 32, 33, 34, 36, 37]; % number ID of the SS ground motions
DT = [0.01 , 0.01 , 0.005, 0.005, 0.005, 0.005, 0.02 , 0.02, 0.02, 0.01,...
      0.01 , 0.01 , 0.02 , 0.02 , 0.02 , 0.02 , 0.02 , 0.02, 0.01, 0.01,...
      0.005, 0.005, 0.005, 0.005, 0.02 , 0.005, 0.005, 0.01, 0.01, 0.01,...
      0.01 , 0.005, 0.005, 0.02 , 0.005, 0.01 , 0.01]; % integration time step
```

Step 3: Definition of *damage limit states* and reference *EDP*

MRF

Structural Performance Levels

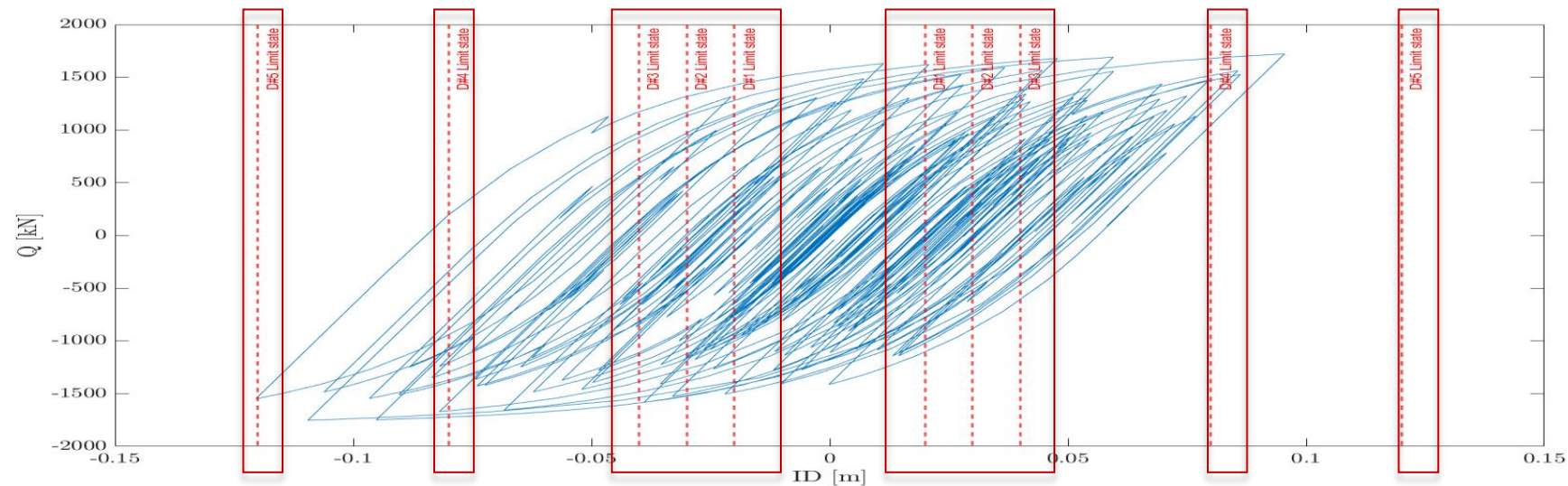
		Collapse Prevention	Life Safety	Service	Immediate Occupancy
Drift	[%]	5%	2,50%	1%	0,70%
	[m]	0,175	0,088	0,035	0,025

BF

Structural Performance Levels

		Collapse Prevention	Life Safety	Service	Immediate Occupancy
Drift	[%]	2,00%	1,00%	0,50%	0,30%
	[m]	0,07	0,035	0,0175	0,0105

Document *FEMA 356 - Prestandard and Commentary for the Seismic Rehabilitation of Buildings*; Table C1-3 - Structural Performance Levels and Damage.



Step 3: Definition of *damage limit states* and reference *EDP*

Codes:

```
%% MDOF Properties
% Choose between the structural system
MDOF_properties_BW_MRF
MDOF_properties_BW_BF
% and between linear or hysteretic
behaviour
%% Structural behaviour
System_type = 'le'; % 'bw'
% bw = bouc-wen
% le = linear elastic
```

```
...
%% Limit States
LS = [0.50 0.75 1 2 3]*4/100;
for ls_i = 1:numel(LS)
    ls_val = LS(ls_i)
    Main_IDA_o_t
    ls_i = ls_i + 1;
end
```

Step 4: Performing non-linear time histories analysis

Codes:

```
%% Initial condition
Mat.dFe=zeros(Mat.NDOF,numel(a_g_norm)); % Preallocation for the load for the time series
a_g = a_g_norm*scale; % Scaled ground motion
Mat.Fe=Mat.M*Mat.r'*a_g'*g;
%% Computation response
[HistVarBw]=ResponceMDF_Bw(Mat);
edp = max(abs(HistVarBw.eps(1,:)));
EDP(i) = edp; %store the EDP for each time history analysis
SCALE(i) = scale; %store the scale factor for each time history analysis...
```

Step 5: Computing fragilities

Codes:

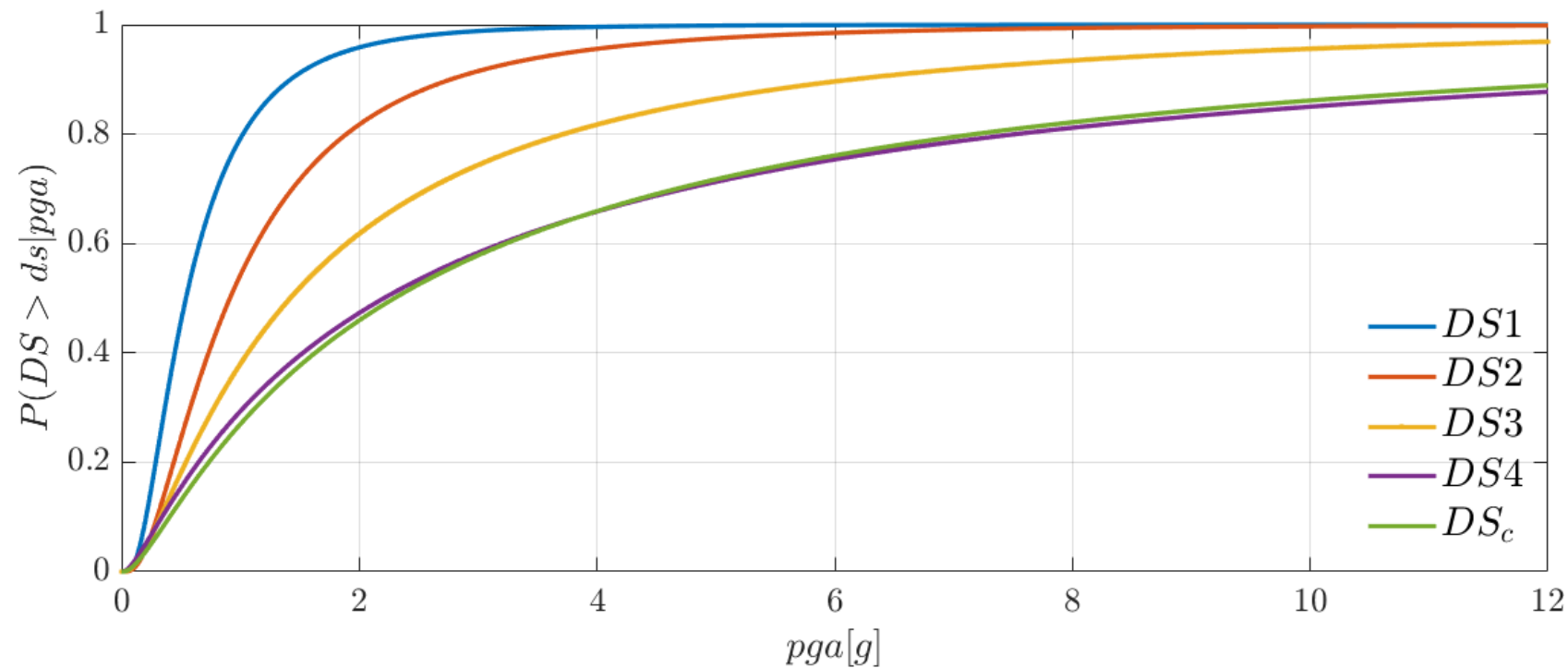
```
%% Untruncated IDA
[parmhat,parmci] = lognfit(IM_t_c ,0.01);
mu_IDA = parmhat(1);
sigma_IDA = parmhat(2);
%% Truncated IDA
IM_max = 2.2;
IM_trunc = IM_t_c(IM_t_c < IM_max); % take only the results with IM < IM_max
eq_over = sum(IM_t_c >= IM_max); % number of analyses reached IM_max without collapsing
% Maximum likelihood fit, using equation (1) and (2) of previously slides
[mu_IDA_t, sigma_IDA_t ] = truncated_ida(IM_trunc, IM_max, eq_over);
```

$$\mathcal{L}(\mu, \sigma) = \underline{\mathcal{L}}(\mu, \sigma) \overline{\mathcal{L}}(\mu, \sigma) = \prod_{n=1}^N \varphi \left(\frac{\ln(im_n) - \mu}{\sigma} \right) \left[1 - \Phi \left(\frac{\ln(\overline{IM}) - \mu}{\sigma} \right) \right]^{N - \overline{N}} \quad (1)$$

$$\ln \mathcal{L}(\mu, \sigma) = \sum_{n=1}^N \varphi \left(\frac{\ln(im_n) - \mu}{\sigma} \right) + (N - \overline{N}) \left[1 - \Phi \left(\frac{\ln(\overline{IM}) - \mu}{\sigma} \right) \right] \quad (2)$$

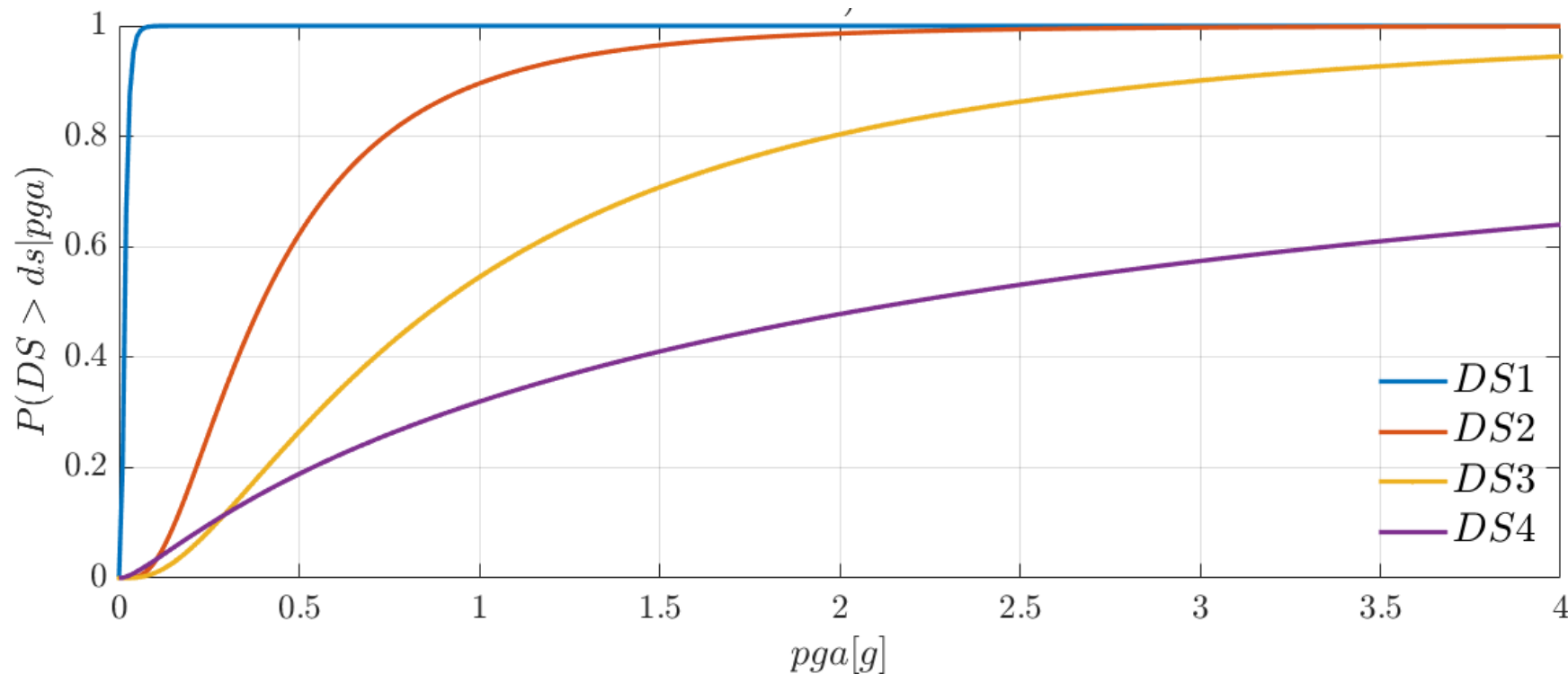
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Step 5: Computing fragilities - Results



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Step 5: Computing fragilities - Results



References

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<https://github.com/kia13nn/ISPS.git>

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Thanks for the attention!

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