Inferential analysis

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Summary: Analytical Approaches

Typically Less Effort → *Typically More Effort*

Descriptive Analysis

- 1st thing you do on new data
- Summarize the data
- univariate plots of variables

Exploratory Analysis

- Exploring relationships
- Asking/definning questions
- univariate/bivariate/multivariate analysis and plotting
- formulate hypothesis

Inferential Analysis

- Estimating uncertainty
- test theories (infer) about the population (data gen. process)
- Building inference models

Tool: statistics

Predictive Analysis

- Building predictive models
- Use historical knowledge to predict future events
- Finding patterns

Mechanistic Analysis

- Understand precise changes one variable has on another
- typically modeled using deterministic equations
- break down complex systems into constituent parts

Causal Analysis

- Determine the average change in one variable when you alter another
- typically requires experiments (e.g. randomized studies)
- manipulate one variable observe effect on other

Population

We want to learn something about this...

Sampling

Inference

....but we can only *actually* collect data from this

Sample

1million comments from 2020

NIH Public Access
Author Manuscript
Epidemiology. Author manuscript; available in PMC 2014 January 01.

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Epidemiology. 2013 January ; 24(1): 23–31. doi:10.1097/EDE.0b013e3182770237.

The Effect of Air Pollution Control on Life Expectancy in the United States: An Analysis of 545 US counties for the period 2000 to 2007

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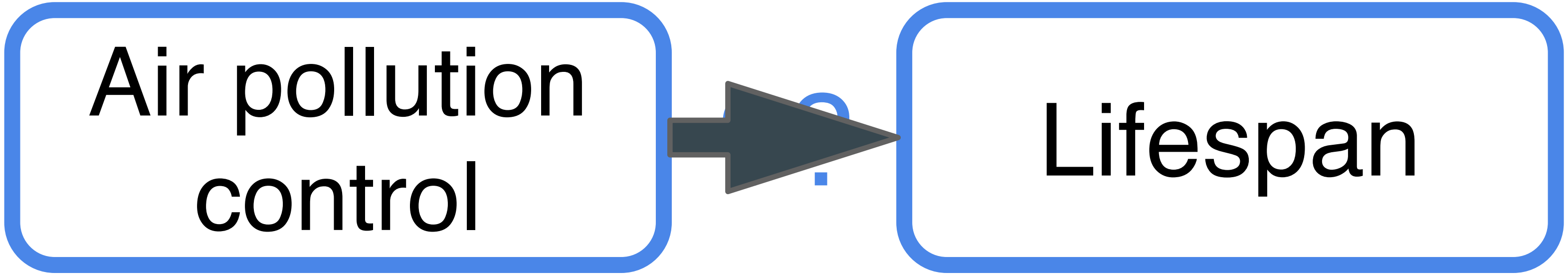
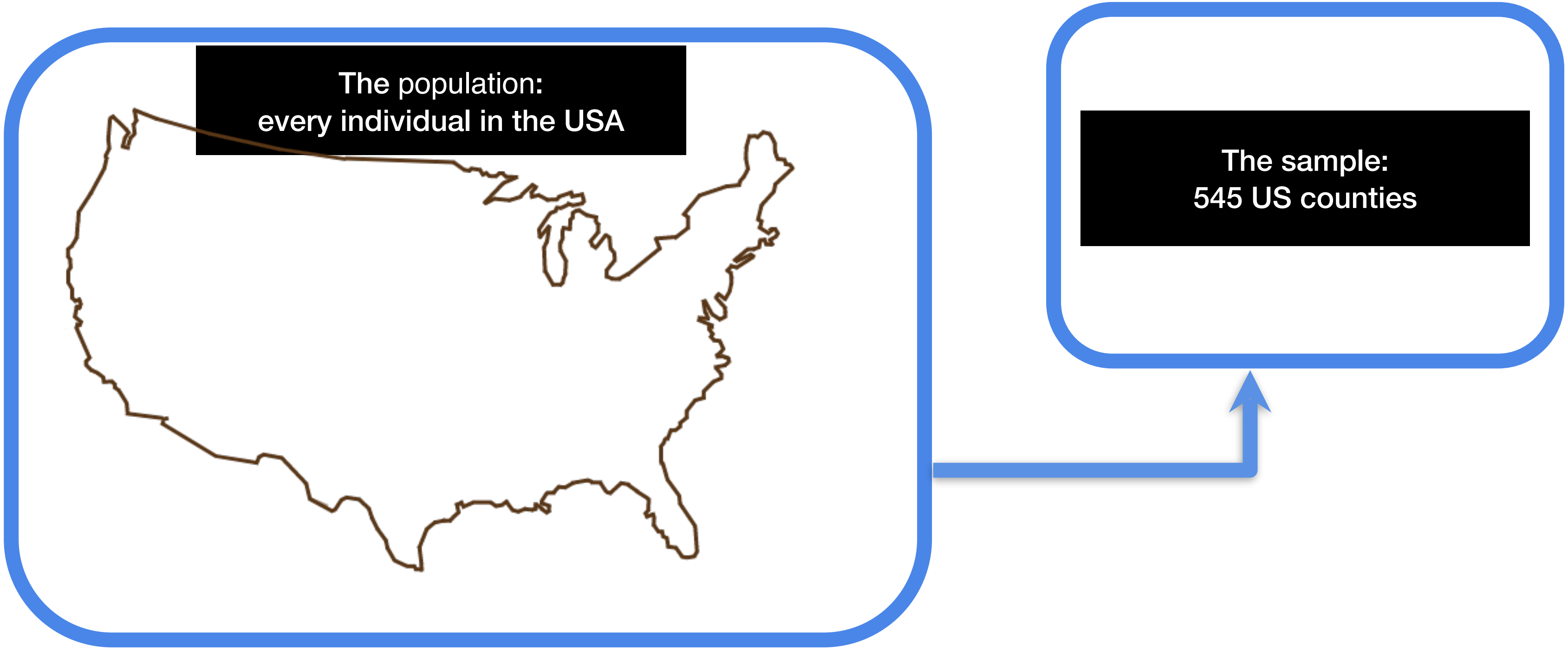


Is there a relationship between air pollution control and lifespan?

A decrease of 10 μg/m3 in the concentration of PM2.5 was associated with an increase in mean life expectancy of 0.35 years SD= 0.16 years, p = 0.033). This association was stronger in more urban and densely populated counties.

What if we want to know the effect of air pollution on everyone in the United States?


The population:
every individual in the USA
The sample:
545 US counties

#	State	Total Number of Counties	Total Area
1	Texas	254	268,596 mi^2
2	Georgia	159	59,425 mi^2
3	Virginia	133	42,775 mi^2
4	Kentucky	120	40,408 mi^2
5	Missouri	115	69,707 mi^2
6	Kansas	105	82,278 mi^2
7	Illinois	102	57,914 mi^2
8	North Carolina	100	53,819 mi^2
9	Iowa	99	56,273 mi^2
10	Tennessee	95	42,144 mi^2

3143 counties in the US

CA is #27

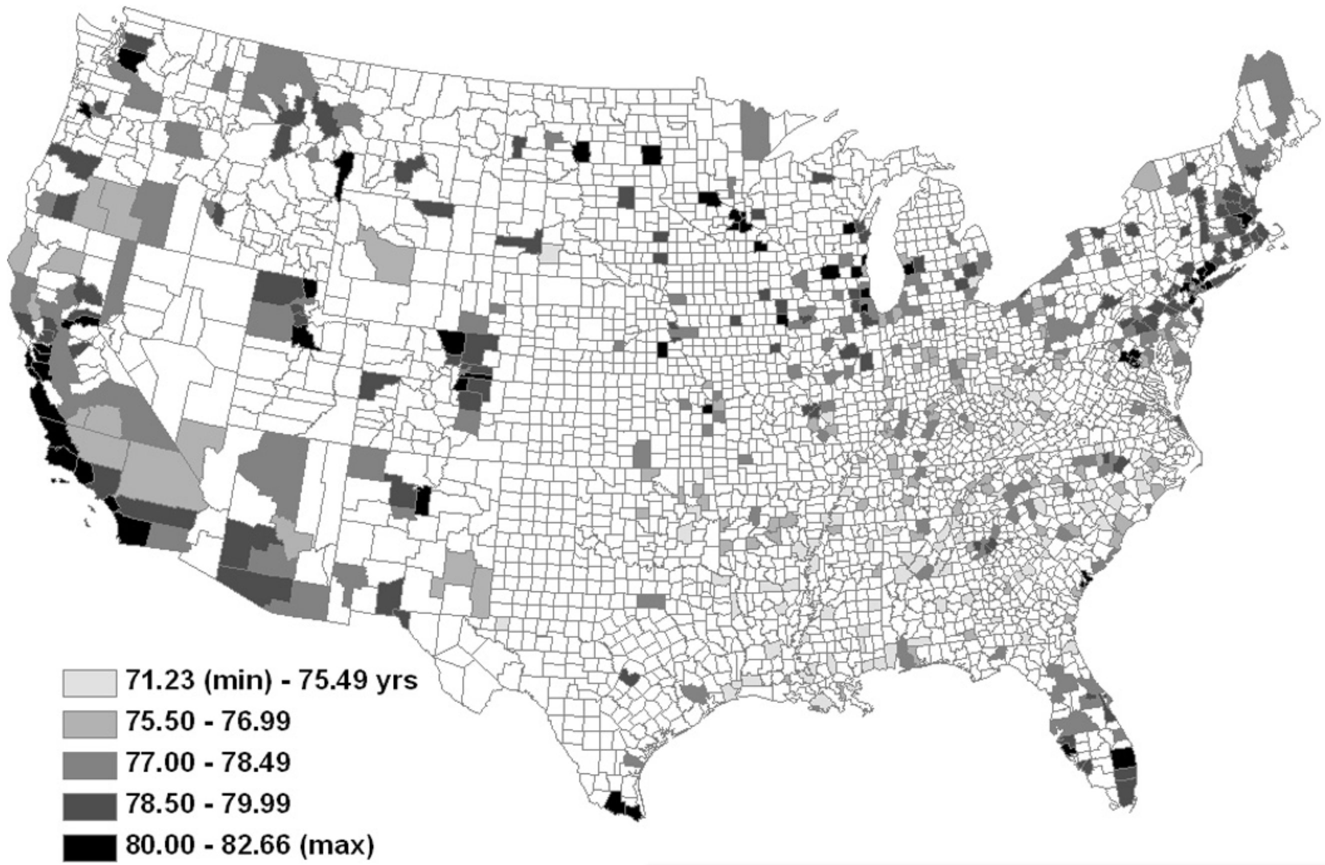
58 counties on 163k mi^2

Random Sampler

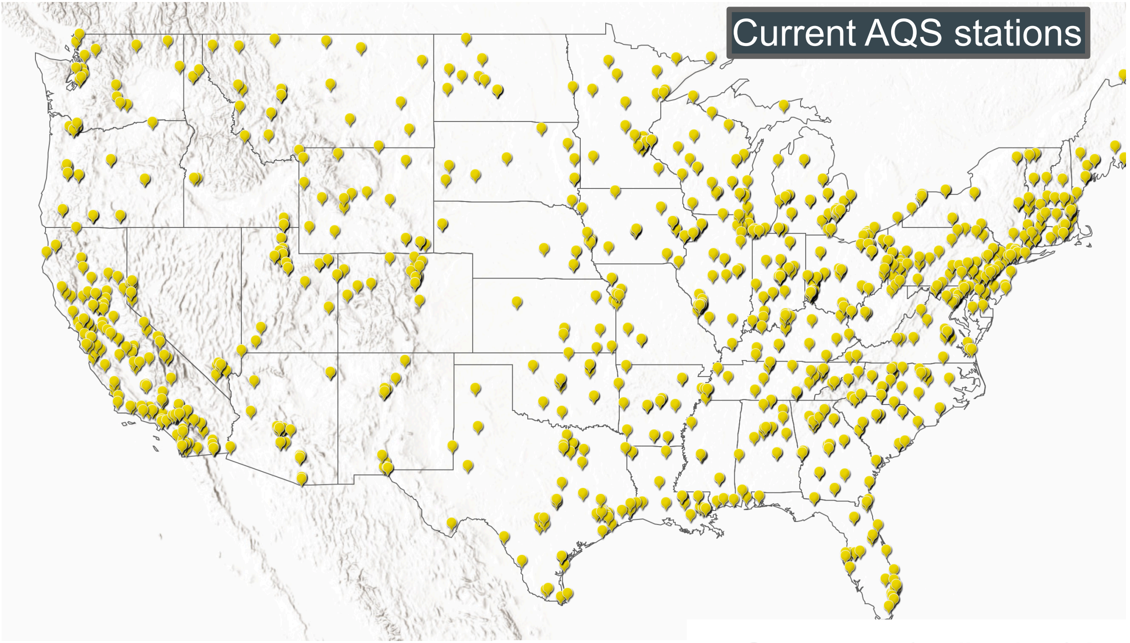
How would you want to select those 545 counties?

What criteria are ideal?

What do you think they did?



All counties with with available matching PM2.5 data for 2000 and 2007 from the EPA's Air Quality System. Includes both metropolitan and non-metro counties



States typically decide where monitors are placed based on areas of relatively high population and/or areas believed to have relatively higher pollutant concentrations. Each state is responsible for developing its own monitoring plan, which is then reviewed and revised every five years. Aug 28, 2023

United States Environmental Protection Agency (.gov)
https://www.epa.gov › outdoor-air-quality-data › who-de...

Who decides where monitors get placed? | US EPA

Approaches to Inference

CORRELATION

COMPARISON OF MEANS

REGRESSION

NON-PARAMETRIC TESTS

CORRELATION

ASSOCIATION BETWEEN VARIABLES

i.e. Pearson Correlation, Spearman Correlation, chi-square test

COMPARISON OF MEANS

DIFFERENCE IN MEANS BETWEEN VARIABLES

i.e. t-test, ANOVA

REGRESSION

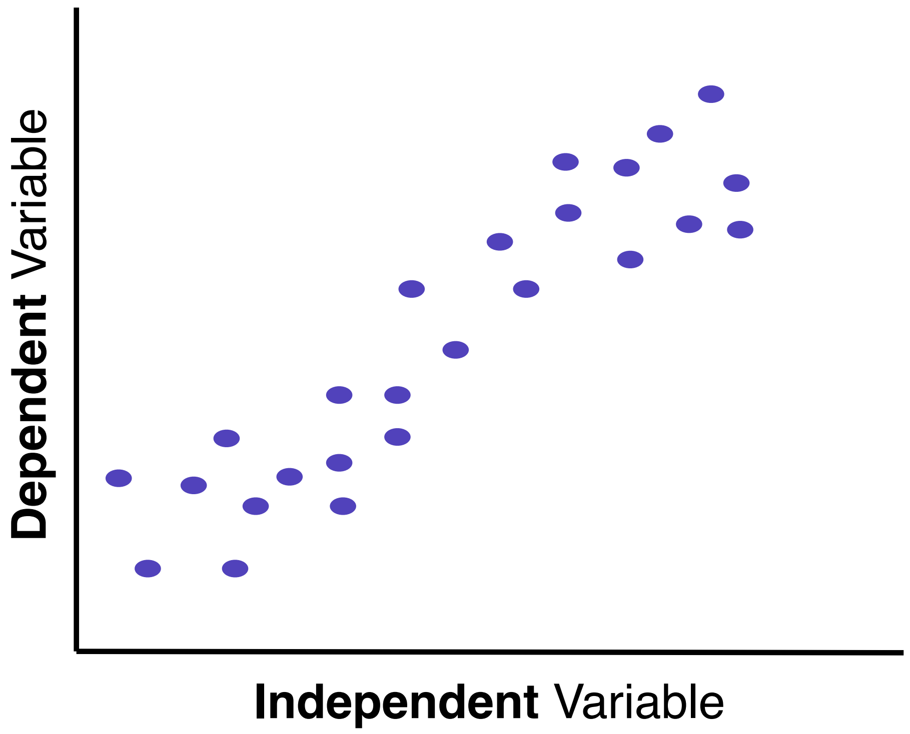
DOES CHANGE IN ONE VARIABLE MEAN CHANGE IN ANOTHER?

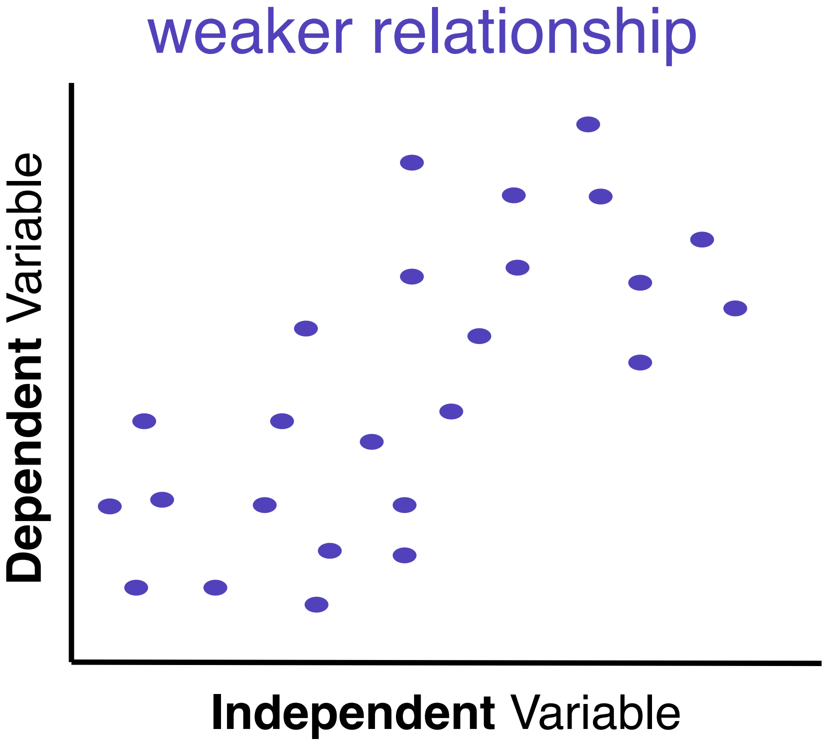
I.e. simple regression, multiple regression

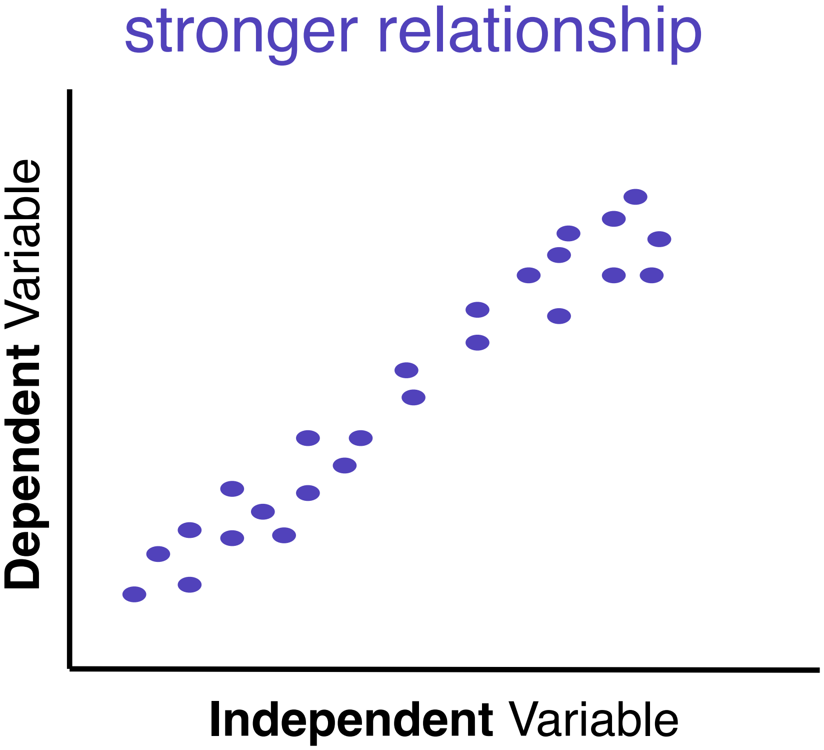
NON-PARAMETRIC TESTS

FOR WHEN ASSUMPTIONS IN THESE OTHER 3 CATEGORIES ARE NOT MET

i.e. Wilcoxon rank-sum test, Wilcoxon sign-rank test, sign test

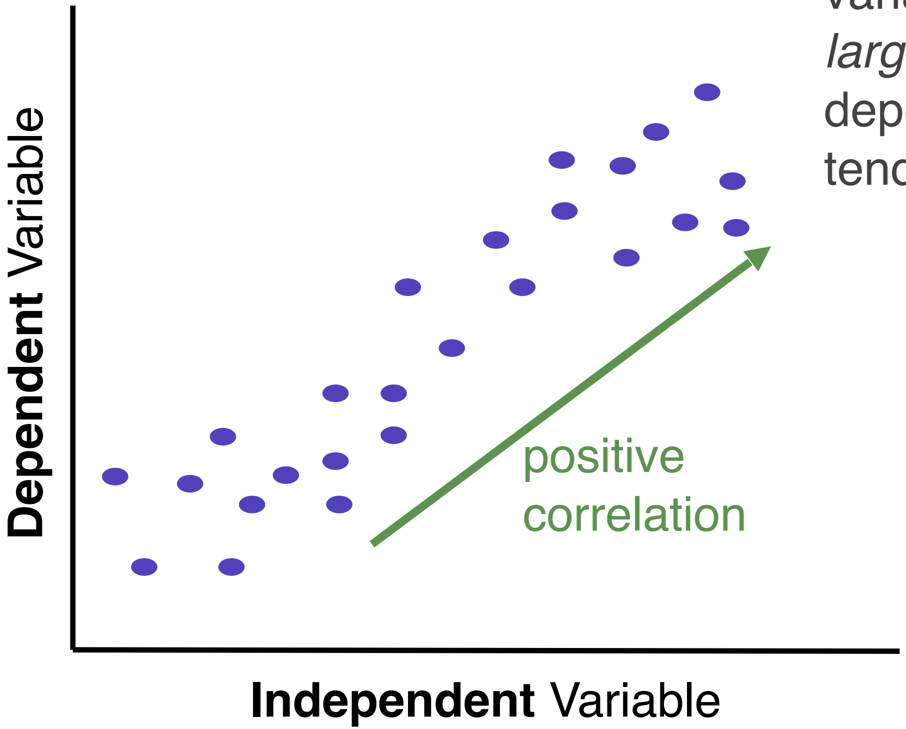

Dependent Variable
Independent Variable





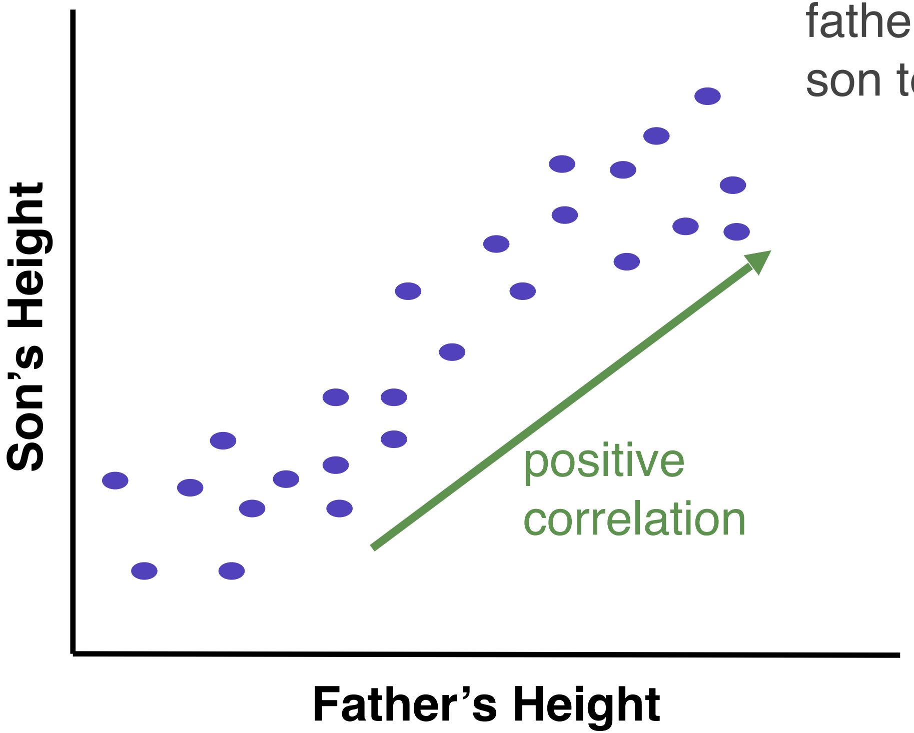
stronger relationship = higher correlation

The *larger* the independent variable value, the *larger* the dependent variable tends to be



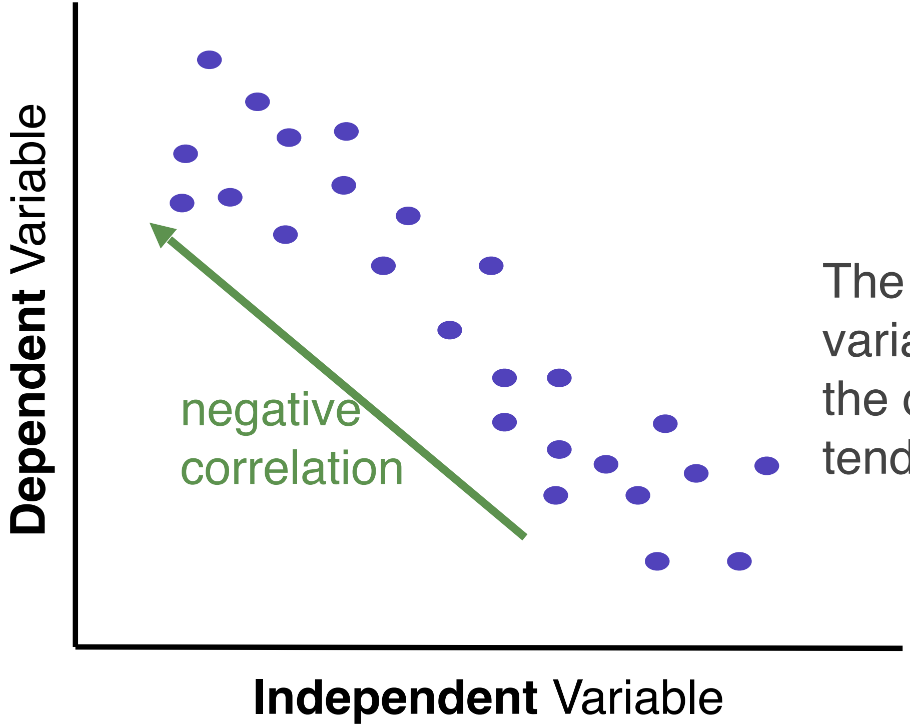
The *smaller* the independent variable value, the *smaller* the dependent variable tends to be

The *taller* the father, the taller his son tends to be



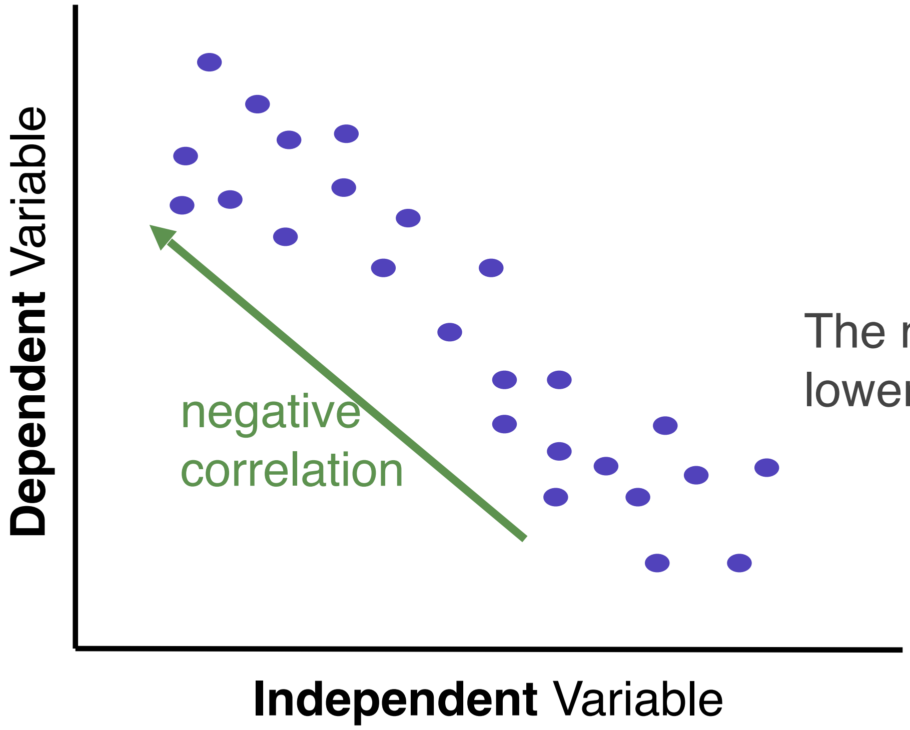
The *shorter* the father, the shorter his son tends to be

The *smaller* the independent variable value, the *larger* the dependent variable tends to be



The *larger* the independent variable value, the *smaller* the dependent variable tends to be

The *lower* the number of absences, the *higher* the students' grades tend to be



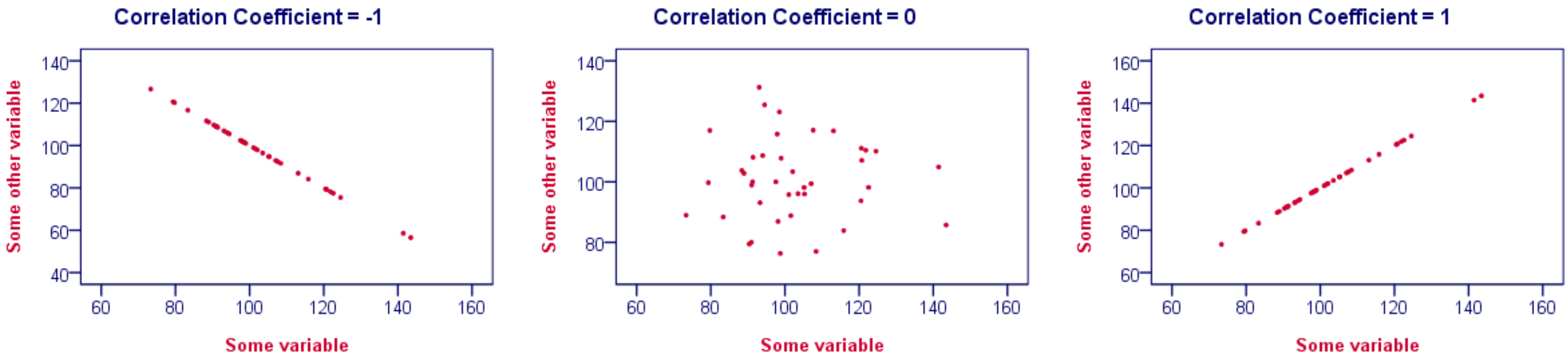
The more absences, the lower the students' grades

Pearson's *r* :

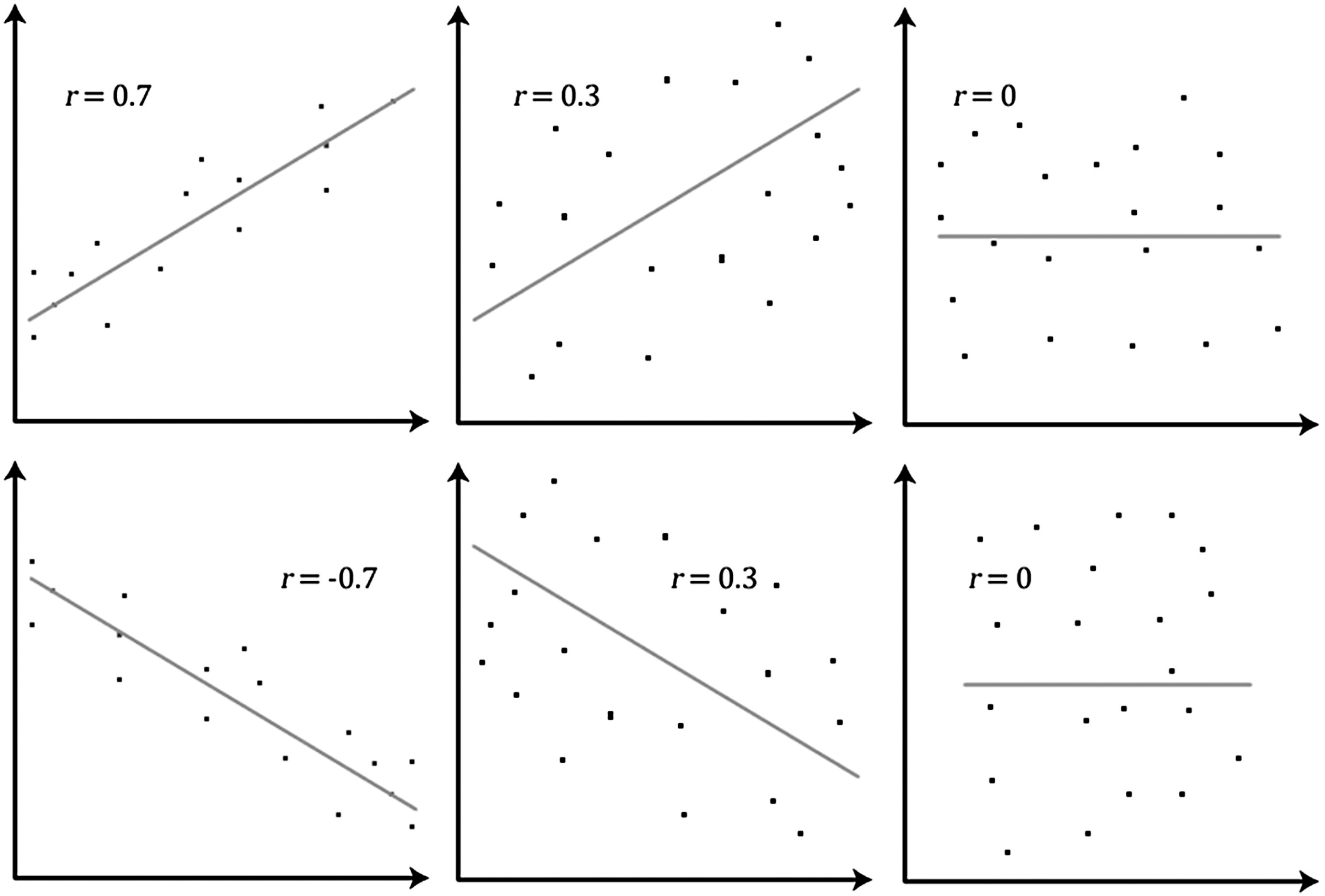
linear correlation between two variables

takes values [-1,1]

Correlation is how close the data are to being in a line…
BUT IT HAS NOTHING TO DO WITH THE SLOPE

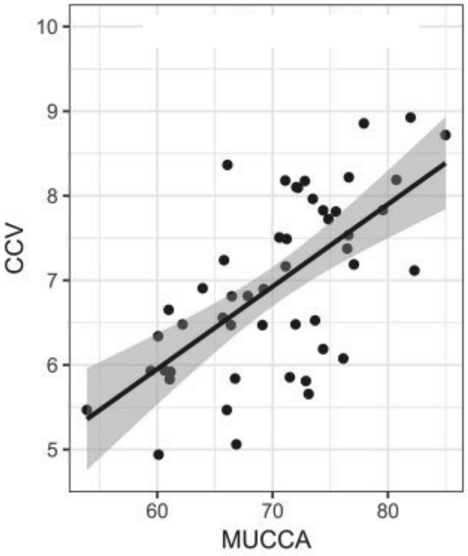


https://www.spss-tutorials.com/pearson-correlation-coefficient/


r = 0.7
r = 0.3
r = 0
r = -0.7
r = 0.3
r = 0

https://www.guessthecorrelation.com/

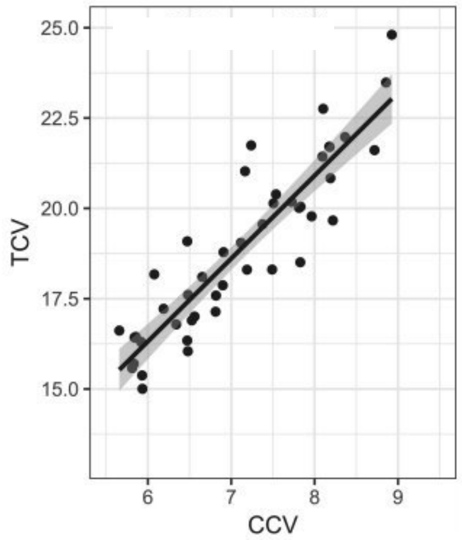
Correlation Champ



Which of the following is the Pearson correlation coefficient (*r*) for this relationship?

A	B	C	D	E
- 0.92	- 0.64	0.03	0.64	0.92

Correlation Champ



Which of the following is the Pearson correlation coefficient (r) for this relationship?

A
- 0.92

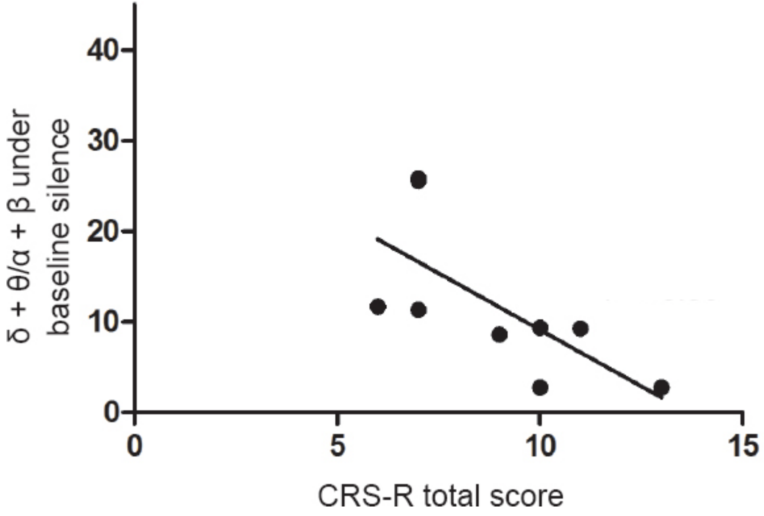
B
- 0.64

C
0.03

D
0.64

E
0.92

Correlation Champ



Which of the following is the Pearson correlation coefficient (r) for this relationship?

A	B	C	D	E
- 0.91	- 0.68	0.03	0.68	0.90

Correlation != Causation

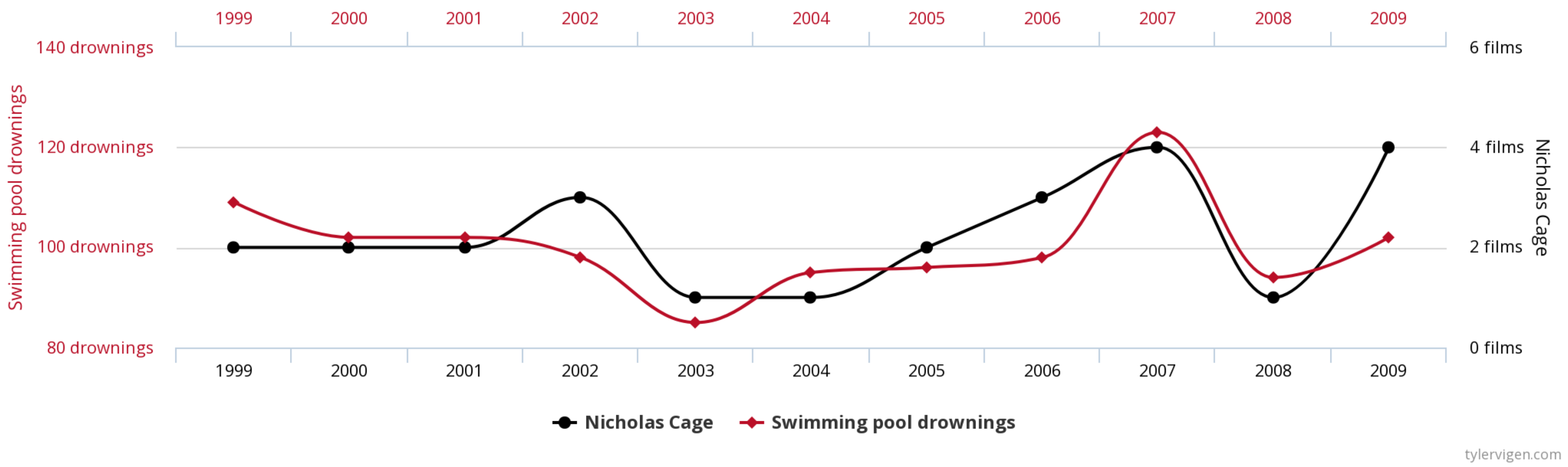
Correlation establishes a relationship.

It does <u>NOT</u> establish causation.

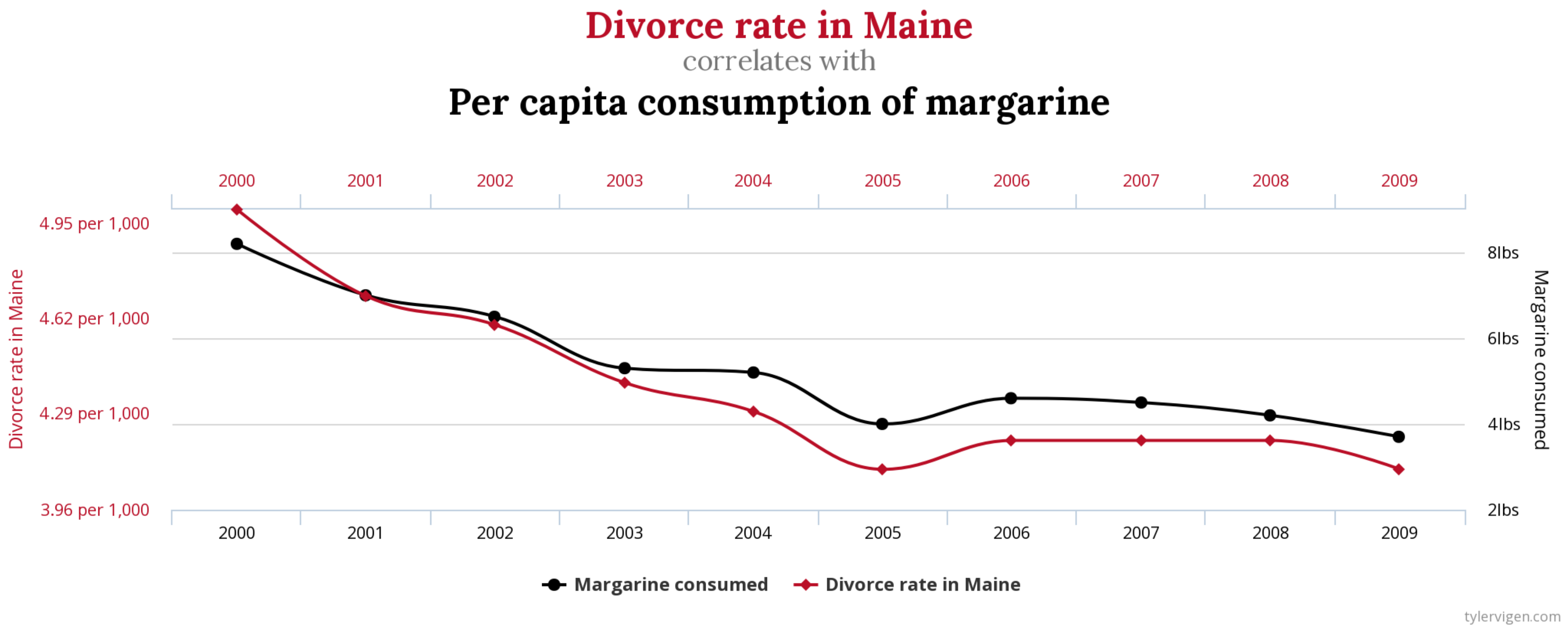
Number of people who drowned by falling into a pool

correlates with

Films Nicolas Cage appeared in



http://www.tylervigen.com/spurious-correlations



http://www.tylervigen.com/spurious-correlations

What about correlation between categorical variables?

- Make a contingency table …

Handed-ness / Sex	Right-handed	Left-handed	Total
Male	43	9	52
Female	44	4	48
Total	87	13	100

- Then calculate phi (based on chi-squared statistic) …

Phi coefficient [edit]

Main article: Phi coefficient

A simple measure, applicable only to the case of 2 × 2 contingency tables, is the phi coefficient (ϕ) defined by

$$\phi = \pm\sqrt{\frac{\chi^2}{N}},$$

where χ^2 is computed as in Pearson's chi-squared test, and N is the grand total of observations. ϕ varies from 0 (corresponding to no association between the variables) to 1 or −1 (complete association or complete inverse association), provided it is based on frequency data represented in 2 × 2 tables. Then its sign equals the sign of the product of the main diagonal elements of the table minus the product of the off–diagonal elements. ϕ takes on the minimum value −1.0 or the maximum value of +1.0 if and only if every marginal proportion is equal to 0.5 (and two diagonal cells are empty).[2]

- Yes there are ways to do this in pandas/numpy and other libraries!

CORRELATION

ASSOCIATION BETWEEN VARIABLES

i.e. Pearson Correlation, Spearman Correlation, chi-square test

COMPARISON OF MEANS

DIFFERENCE IN MEANS BETWEEN CONDITIONS

i.e. t-test, ANOVA

REGRESSION

DOES CHANGE IN ONE VARIABLE MEAN CHANGE IN ANOTHER?

I.e. simple regression, multiple regression

NON-PARAMETRIC TESTS

FOR WHEN ASSUMPTIONS IN THESE OTHER 3 CATEGORIES ARE NOT MET

i.e. Wilcoxon rank-sum test, Wilcoxon sign-rank test, sign test

t-test:

tests for difference in means between groups

William Sealy Gosset (13 June 1876 – 16 October 1937) was an English statistician, chemist and brewer who served as Head Brewer of Guinness and Head Experimental Brewer of Guinness and was a pioneer of modern statistics. He pioneered small sample experimental design and analysis with an economic approach to the logic of uncertainty. Gosset published under the pen name **Student** and developed most famously Student's t-distribution – originally called Student's "z" – and "Student's test of statistical significance".[1]

Contents [hide]

Life and career [edit]

Born in Canterbury, England the eldest son of Agnes Sealy Vidal and Colonel Frederic Gosset, R.E. Royal Engineers, Gosset attended Winchester College before matriculating as Winchester Scholar in natural sciences and mathematics at New College, Oxford. Upon graduating in 1899, he joined the brewery of Arthur Guinness & Son in Dublin, Ireland; he spent the rest of his 38-year career at Guinness.[1][2]

Gosset had three children with Marjory Gosset (née Phillpotts). Harry Gosset (1907–1965) was a consultant paediatrician; Bertha Marian Gosset (1909–2004) was a geographer and nurse; the youngest, Ruth Gosset (1911–1953) married the Oxford mathematician Douglas Roaf and had five children.

In his job as Head Experimental Brewer at Guinness, the self-trained Gosset developed new statistical methods – both in the brewery and on the farm – now central to the design of experiments, to proper use of significance testing on repeated trials, and to analysis of economic significance (an early instance of decision theory interpretation of statistics) and more, such as his small-sample, stratified, and repeated balanced experiments on barley for proving the best yielding varieties.[3] Gosset acquired that knowledge by study, by trial and error, by cooperating with others, and by spending two terms in 1906–1907 in the Biometrics laboratory of Karl Pearson.[4] Gosset and Pearson had a good relationship.[4] Pearson helped Gosset with the mathematics of his papers, including the 1908 papers, but had little appreciation of their importance. The papers addressed the brewer's concern with small samples; biometricians like Pearson, on the other hand, typically had hundreds of observations and saw no urgency in developing small-sample methods.[2]

Gosset's first publication came in 1907, "On the Error of Counting with a Haemacytometer," in which – unbeknownst to Gosset aka "Student" – he rediscovered the Poisson distribution.[3] Another researcher at Guinness had previously published a paper containing trade secrets of the Guinness

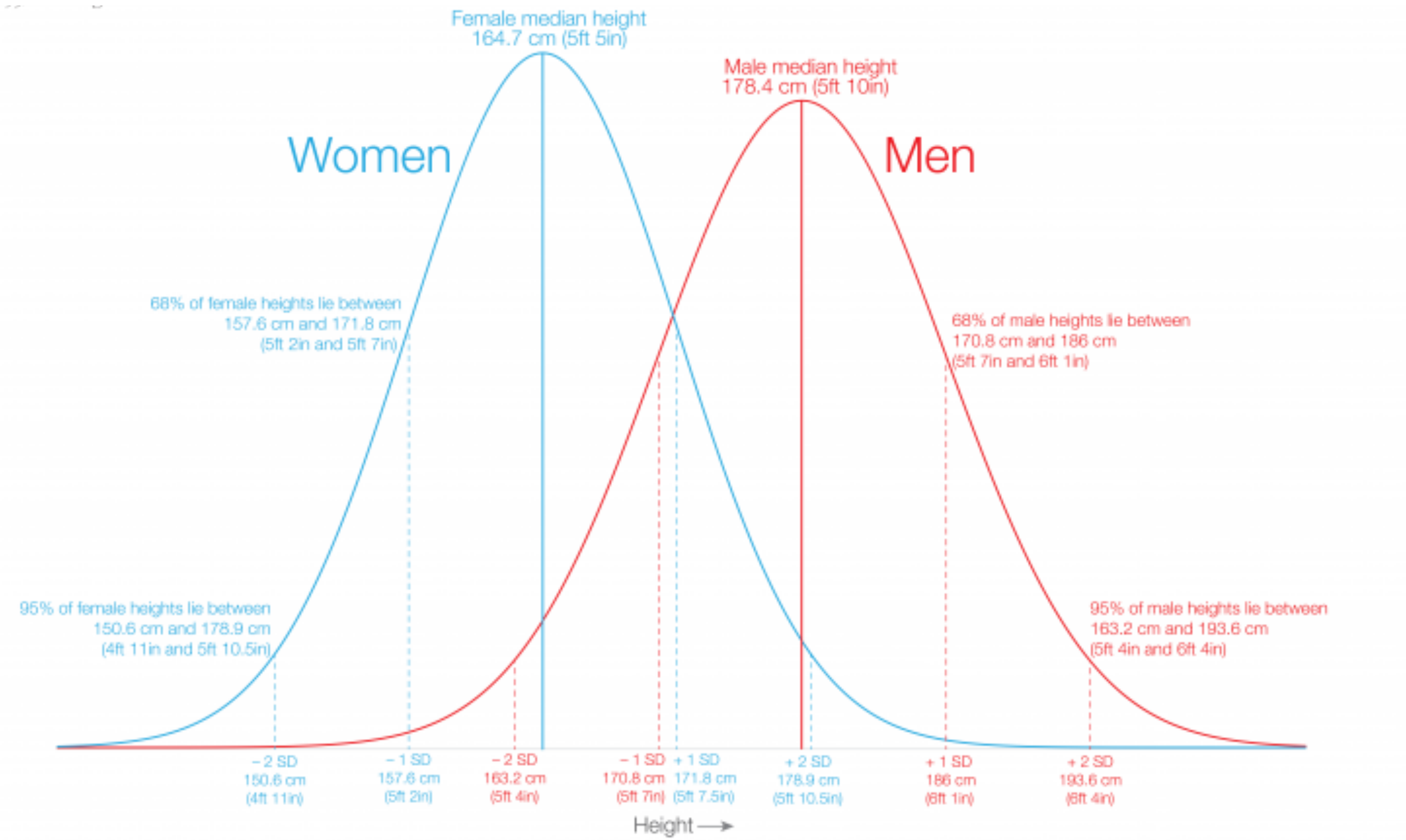
William Sealy Gosset

William Sealy Gosset (aka *Student*) in 1908 (age 32)

Born	13 June 1876 Canterbury, Kent, England
Died	16 October 1937 (aged 61) Beaconsfield, Buckinghamshire, England
Other names	Student
Alma mater	New College, Oxford, Winchester College
Known for	Student's t-distribution, statistical significance, design of experiments, Monte Carlo method, quality control, Modern synthesis, agricultural economics, econometrics
Children	5, including Isaac Henry Gosset

Scientific career

Do the heights between males and females differ?

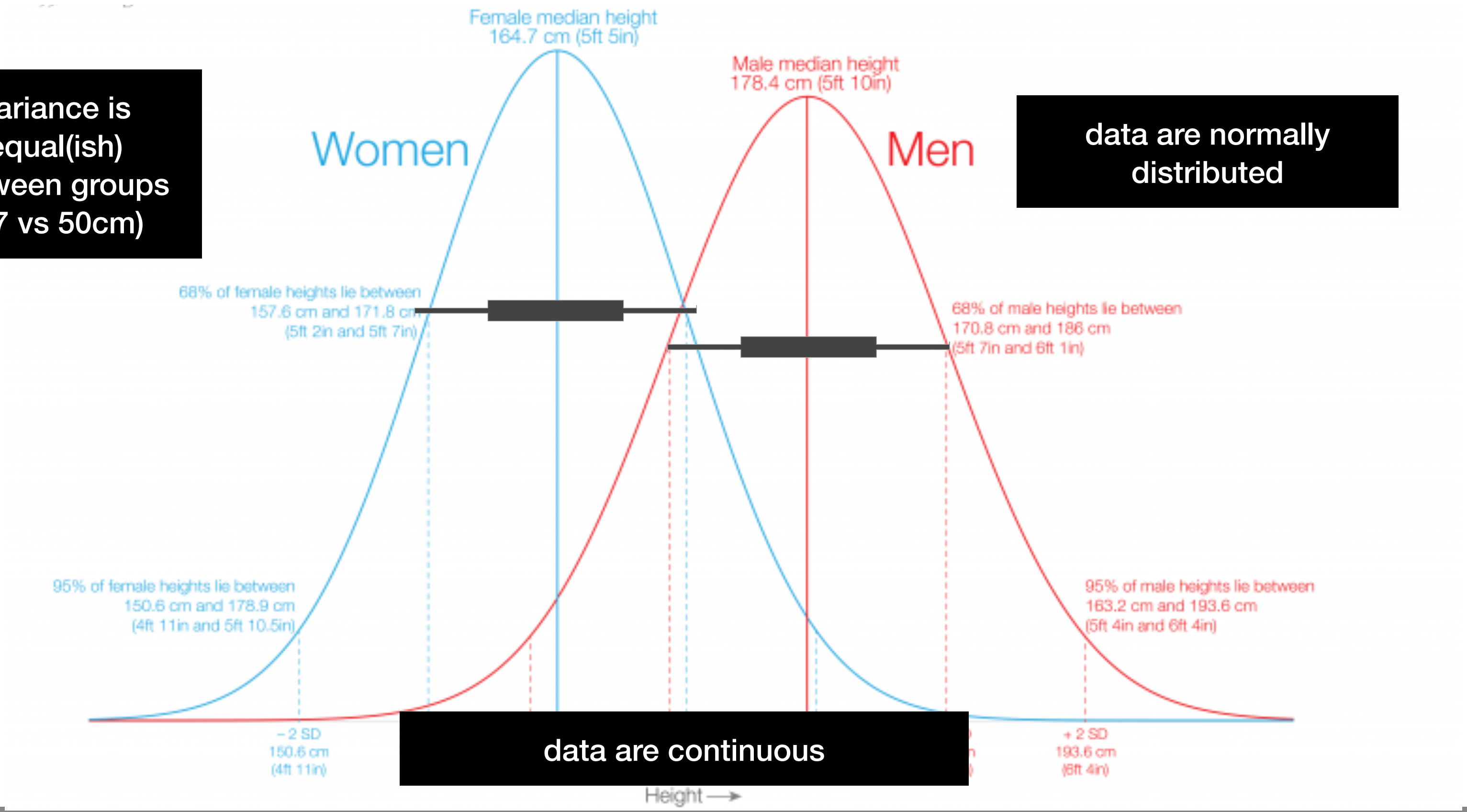


https://ourworldindata.org/human-height

t-test Assumptions

1. Data are continuous
2. Normally distributed
3. Equal variance b/w groups (but can use Welch's test!)
4. Not paired (will talk more about this later)

Do the heights between males and females differ?



variance is equal(ish) between groups (57 vs 50cm)

data are normally distributed

sample size affects statistic

N=10,000

data are continuous

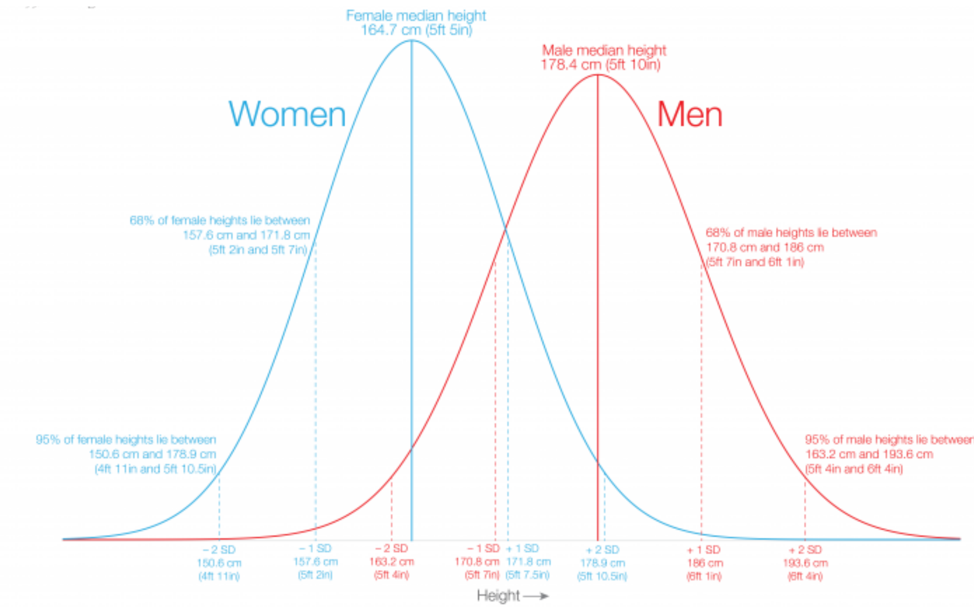
Do the heights between males and females differ?

t-statistic: -95.6

p-value << 0.001

95% CI for true difference in means
[-5.43, -5.21]

Yes.

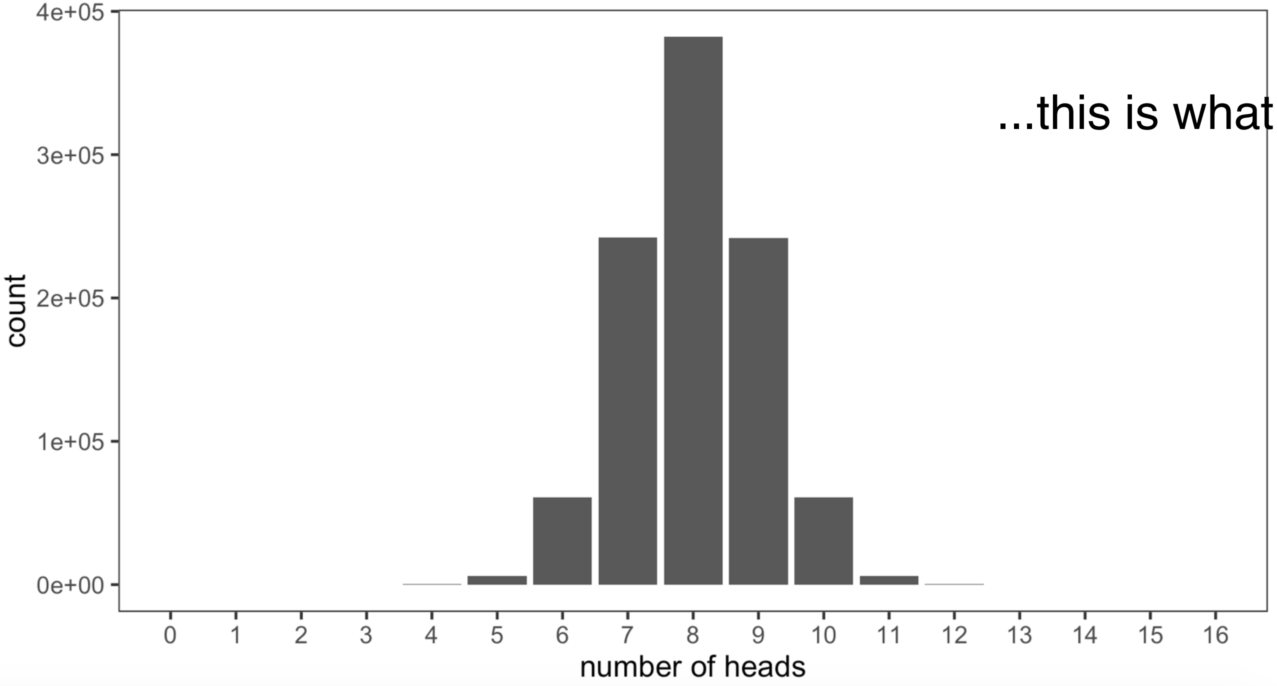


p-value : the probability of getting the observed results (or results more extreme) by chance alone

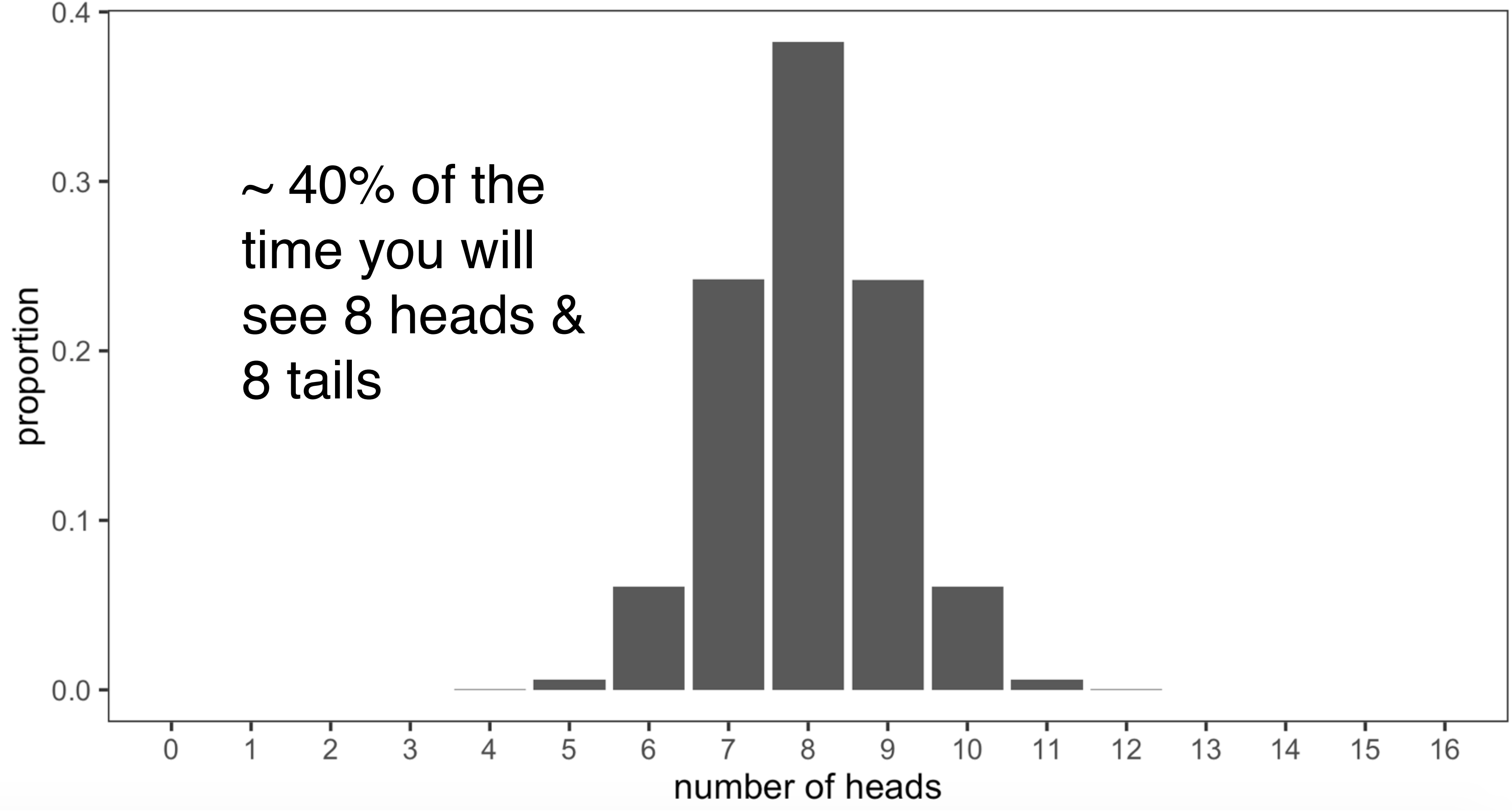
https://forms.gle/6MCyp7qFsaHgGKi5A

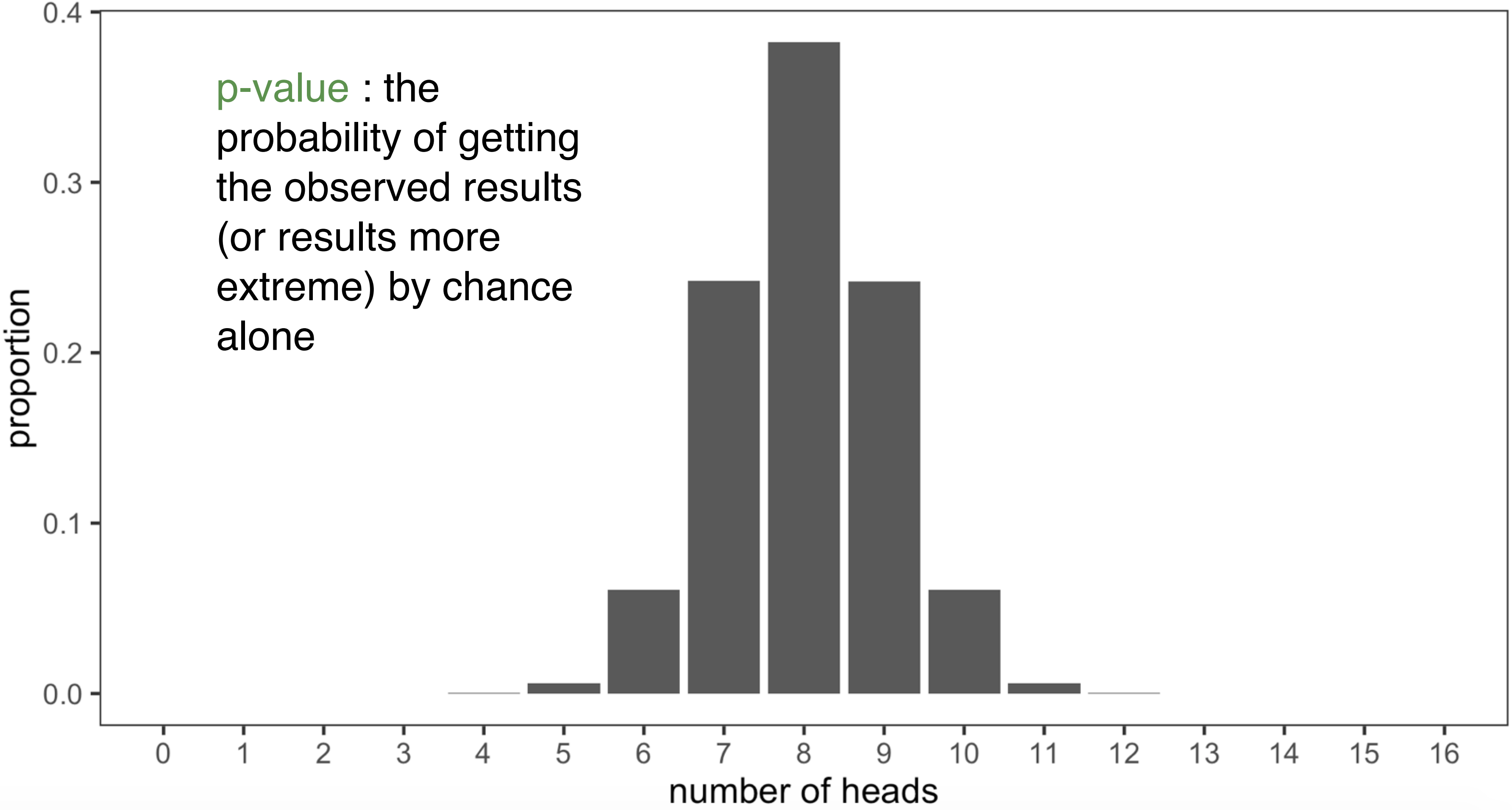
If we flip a coin 16 times and record the number of heads....

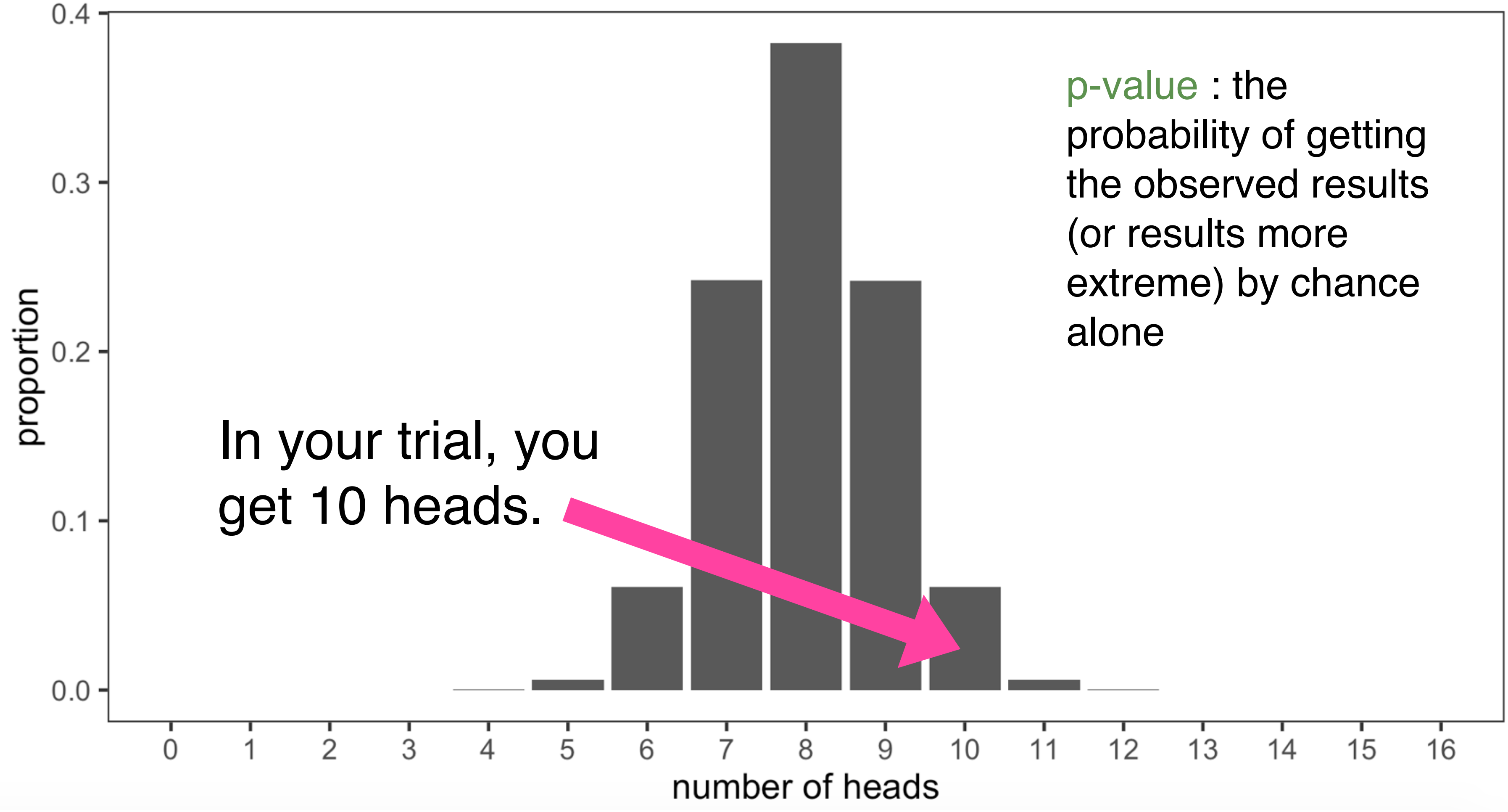
….and then do that 1M times

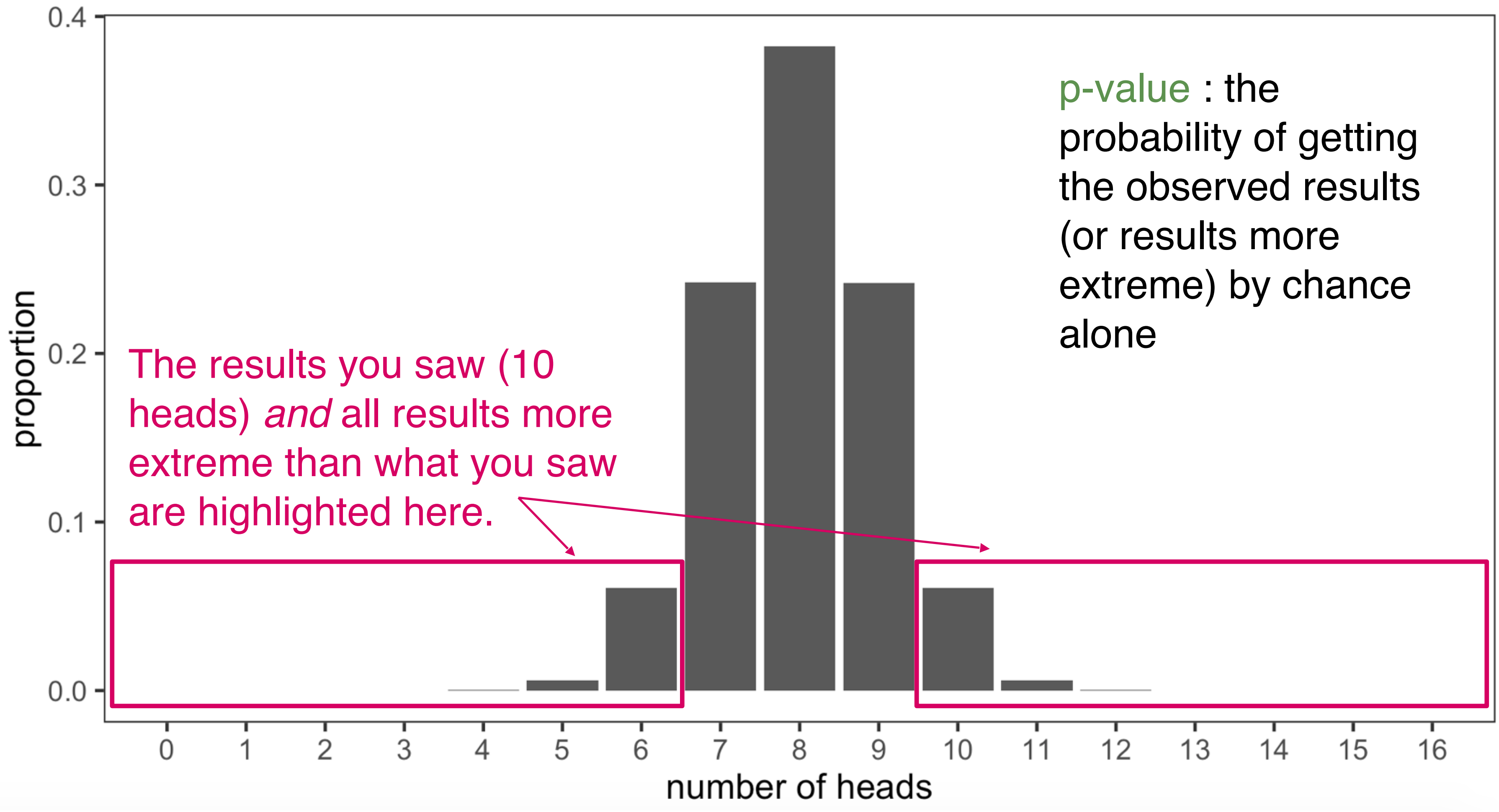


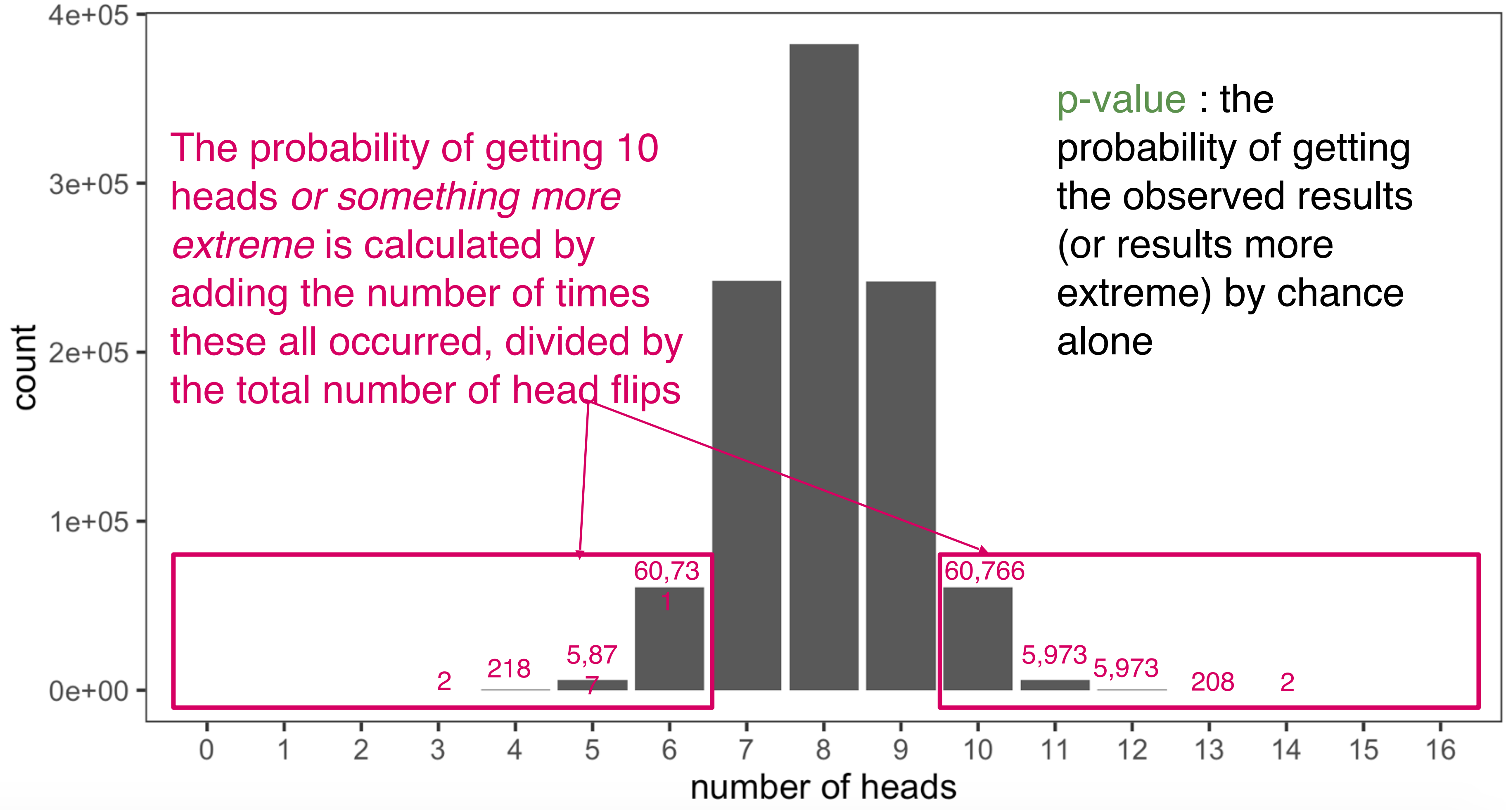
...this is what happens by chance alone.

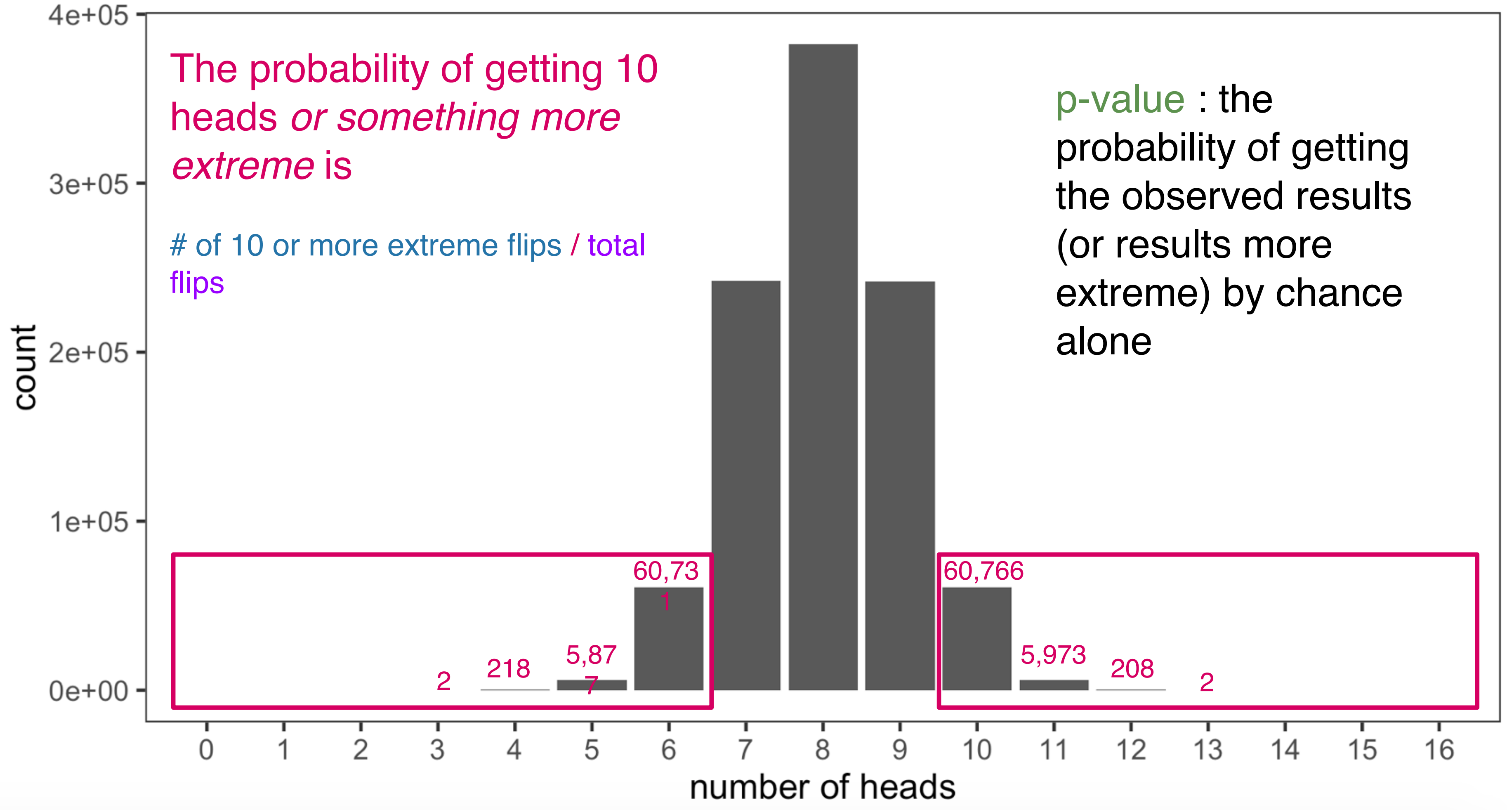

~ 40% of the time you will see 8 heads & 8 tails
proportion
0.0
0.1
0.2
0.3
0.4
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
number of heads


p-value : the probability of getting the observed results (or results more extreme) by chance alone
0.4
0.3
0.2
0.1
0.0
proportion
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
number of heads


0.4
0.3
0.2
0.1
0.0
proportion
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
number of heads
In your trial, you get 10 heads.
p-value : the probability of getting the observed results (or results more extreme) by chance alone


p-value : the probability of getting the observed results (or results more extreme) by chance alone
The results you saw (10 heads) *and* all results more extreme than what you saw are highlighted here.
proportion
0.4
0.3
0.2
0.1
0.0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
number of heads


The probability of getting 10 heads *or something more extreme* is calculated by adding the number of times these all occurred, divided by the total number of head flips
p-value : the probability of getting the observed results (or results more extreme) by chance alone
count
4e+05
3e+05
2e+05
1e+05
0e+00
2
218
5,87
7
60,73
1
60,766
5,973
5,973
208
2
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
number of heads



The probability of getting 10 heads *or something more extreme* is
# of 10 or more extreme flips / total flips
p-value : the probability of getting the observed results (or results more extreme) by chance alone
count
4e+05
3e+05
2e+05
1e+05
0e+00
60,731
60,766
5,877
5,973
218
208
2
2
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
number of heads

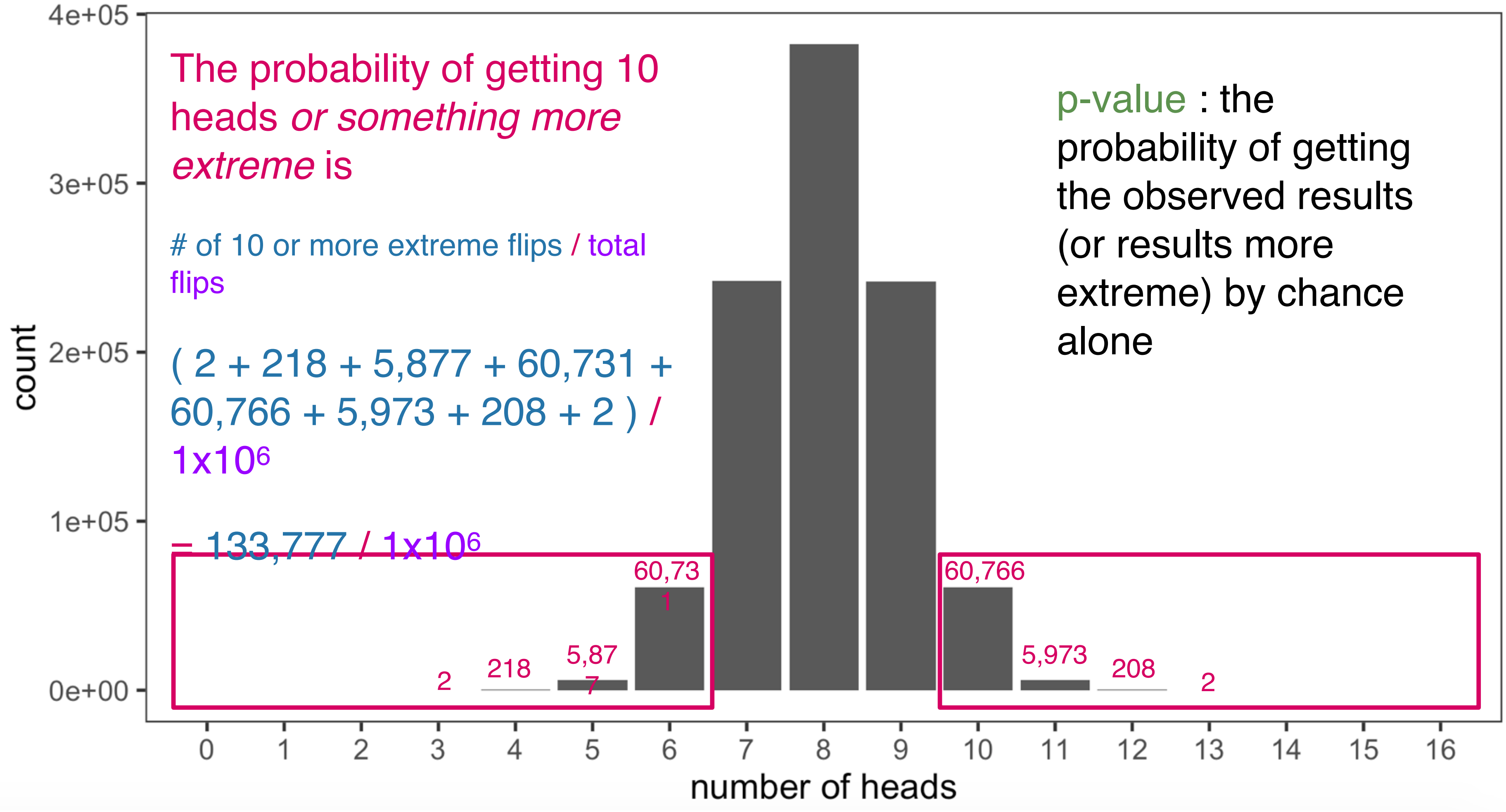
The probability of getting 10 heads *or something more extreme* is

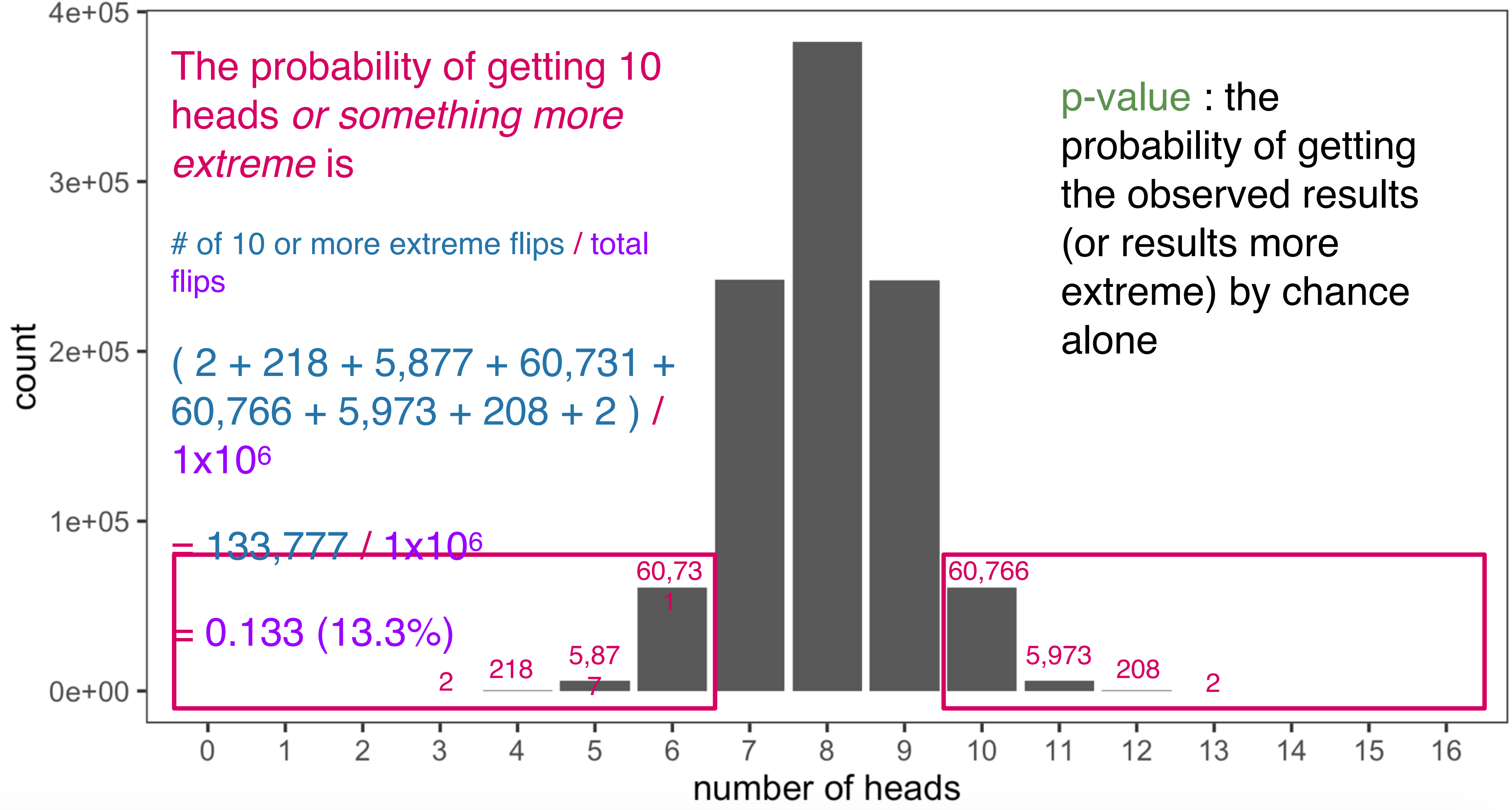
# of 10 or more extreme flips / total flips

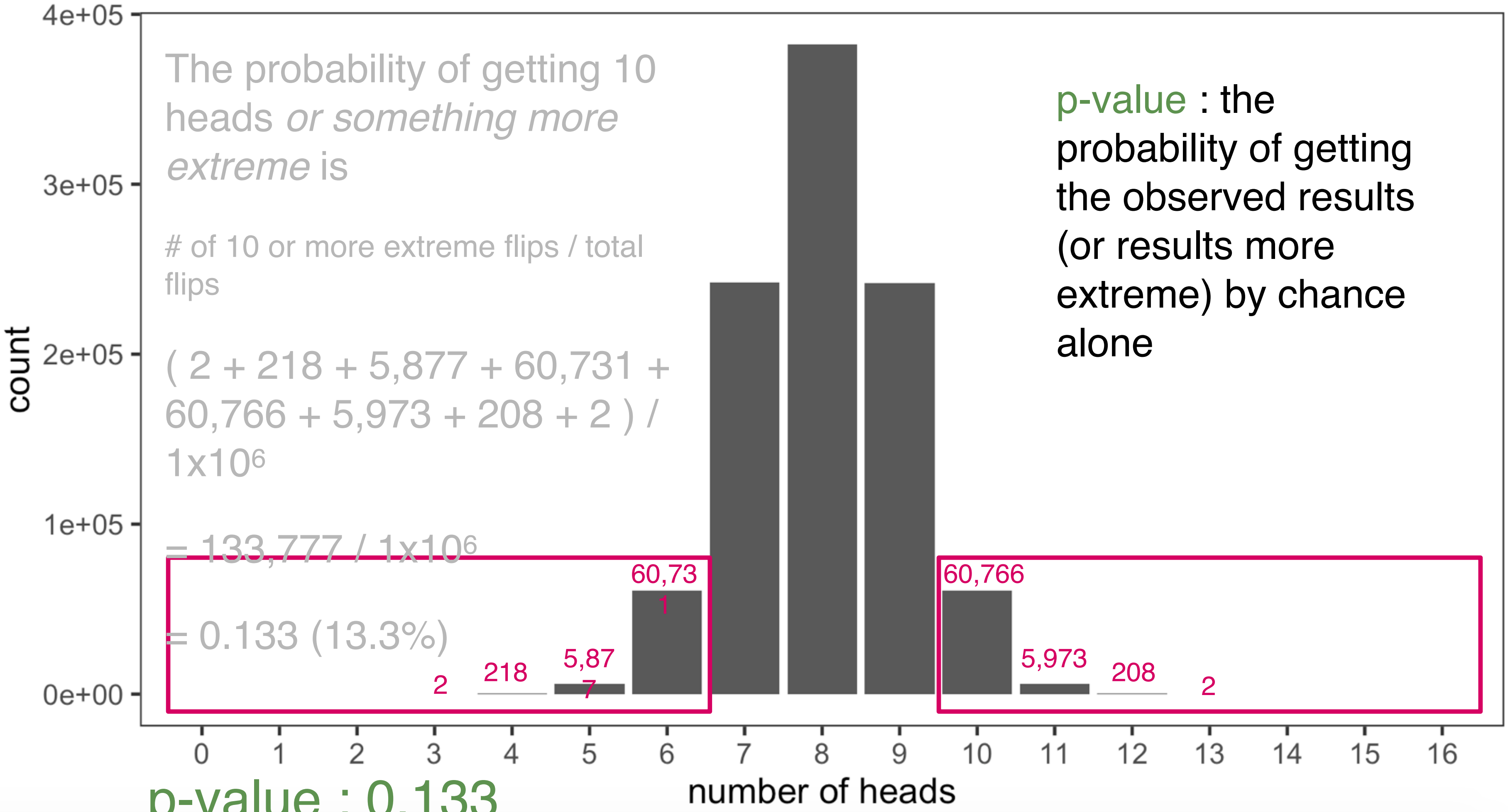
(2 + 218 + 5,877 + 60,731 + 60,766 + 5,973 + 208 + 2) / 1×10^6

= 133,777 / 1×10^6

p-value : the probability of getting the observed results (or results more extreme) by chance alone

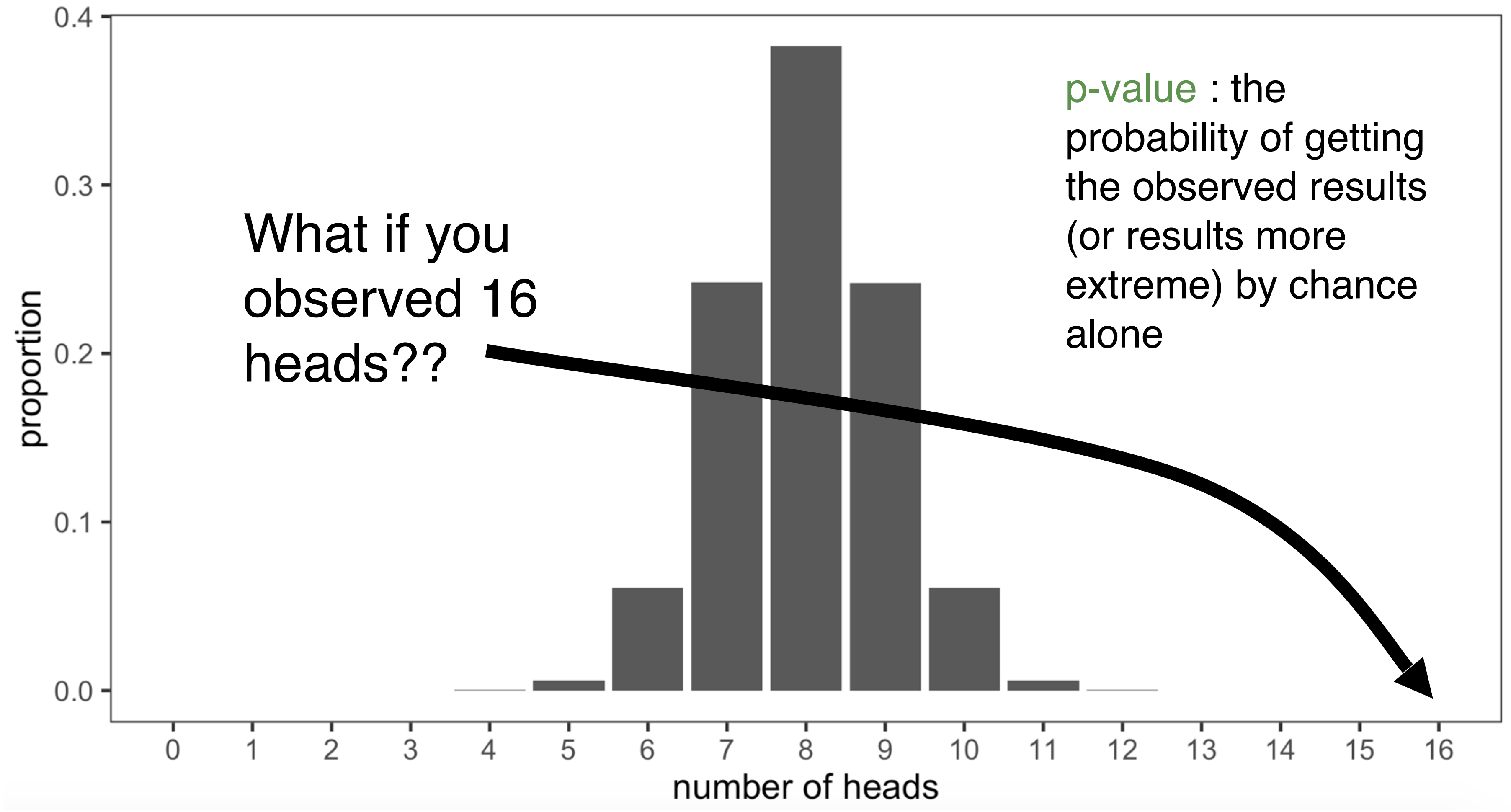


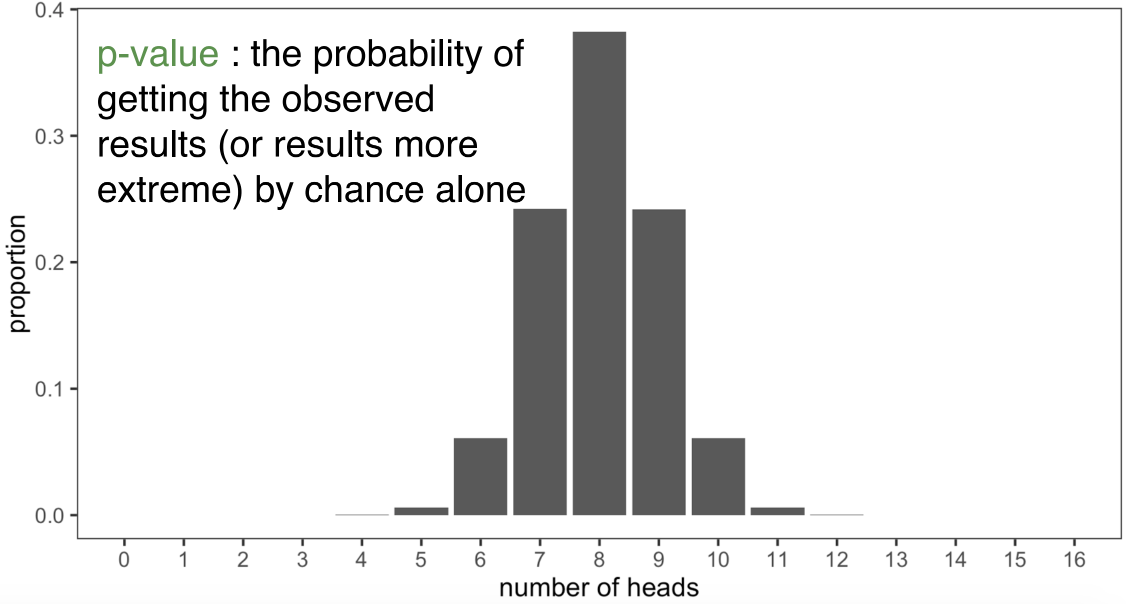

The probability of getting 10 heads *or something more extreme* is
# of 10 or more extreme flips / total flips
(2 + 218 + 5,877 + 60,731 + 60,766 + 5,973 + 208 + 2) / 1x10^6
= 133,777 / 1x10^6
= 0.133 (13.3%)
p-value : the probability of getting the observed results (or results more extreme) by chance alone
2
218
5,877
60,731
60,766
5,973
208
2
count
4e+05
3e+05
2e+05
1e+05
0e+00
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
number of heads



The probability of getting 10 heads *or something more extreme* is
# of 10 or more extreme flips / total flips
(2 + 218 + 5,877 + 60,731 + 60,766 + 5,973 + 208 + 2) / 1x10^6
= 133,777 / 1x10^6
= 0.133 (13.3%)
p-value : the probability of getting the observed results (or results more extreme) by chance alone
count
4e+05
3e+05
2e+05
1e+05
0e+00
2
218
5,877
60,731
60,766
5,973
208
2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
number of heads

p-value : 0.133


0.4
0.3
0.2
0.1
0.0
proportion
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
number of heads
What if you observed 16 heads??
p-value : the probability of getting the observed results (or results more extreme) by chance alone

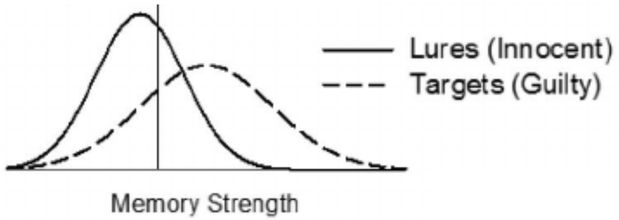


What would be the p-value of you flipping 16 heads?

A
< 0.13

B
> 0.13

Difference in Means



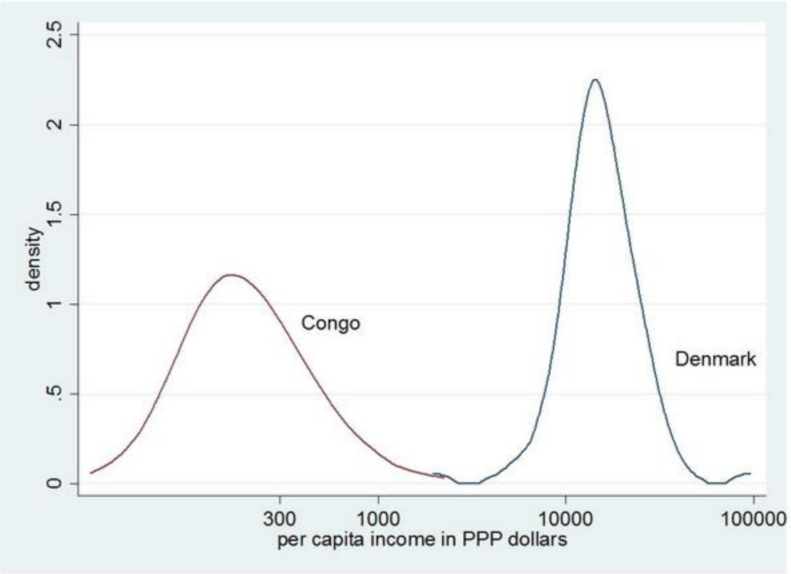
Why would a t-test *not* be appropriate for these data?

A
Not normally distributed

B
Unequal variances

C
Small sample size

D
Data are not continuous



Would a t-test find a significant difference in means?

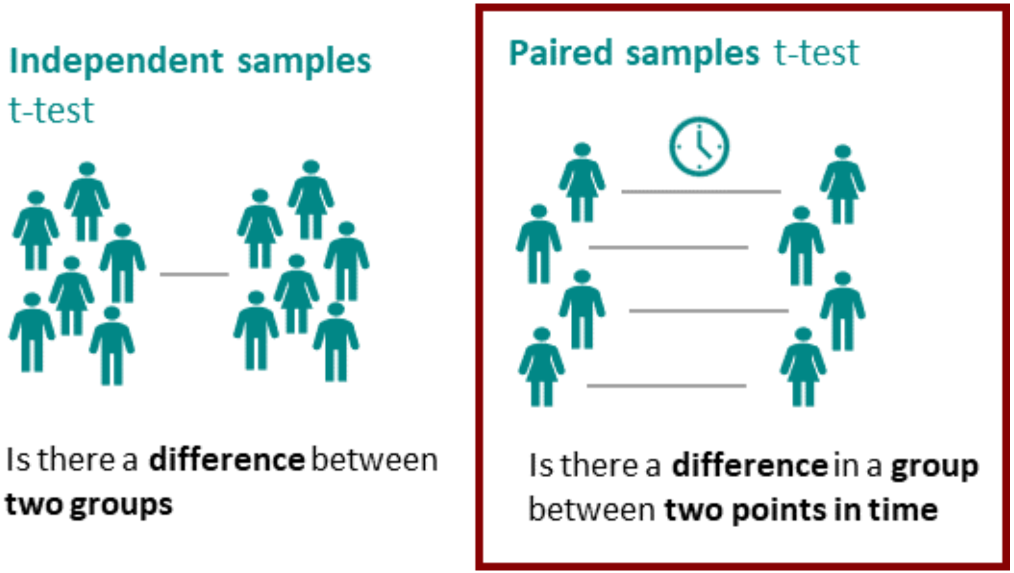
A
t-test not appropriate

B
Yes

C
No

D
Need more information

Paired data



https://datatab.net/tutorial/paired-t-test

CORRELATION

ASSOCIATION BETWEEN VARIABLES

i.e. Pearson Correlation, Spearman Correlation, chi-square test

COMPARISON OF MEANS

DIFFERENCE IN MEANS BETWEEN VARIABLES

i.e. t-test, ANOVA

REGRESSION

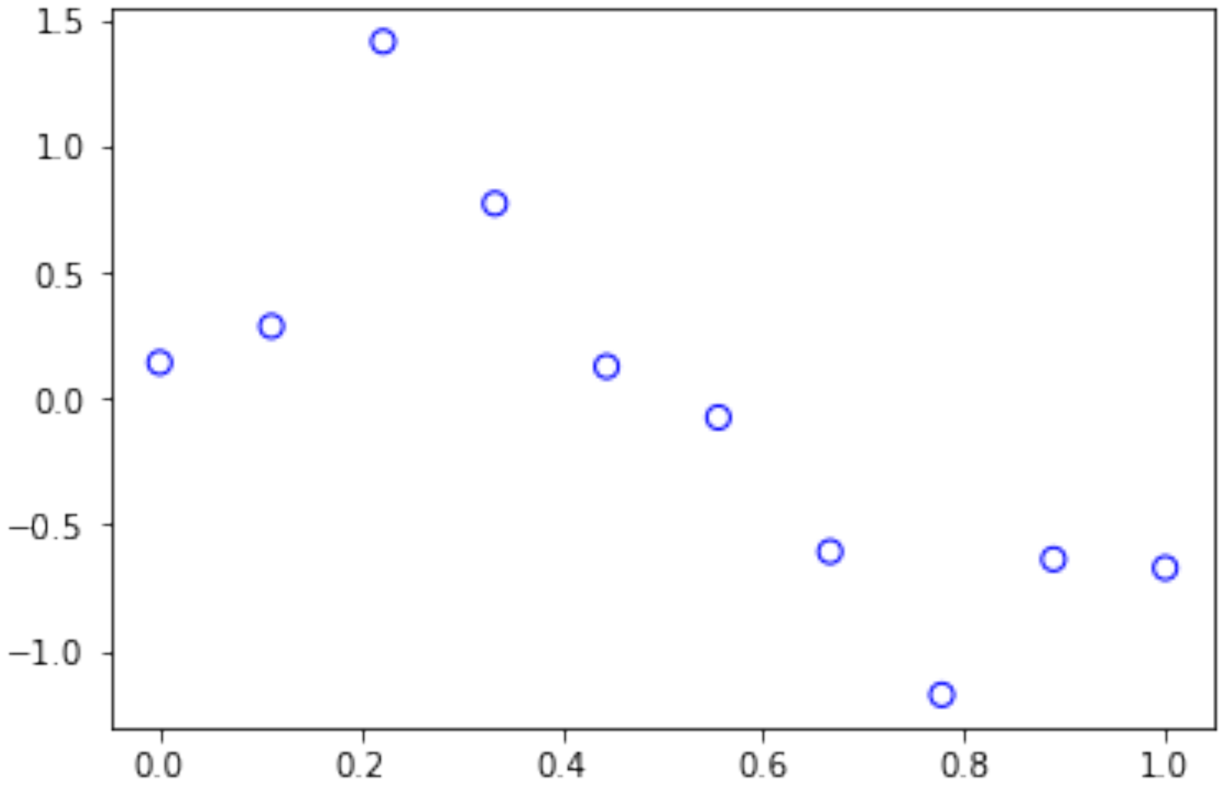
DOES CHANGE IN ONE VARIABLE MEAN CHANGE IN ANOTHER?

I.e. simple regression, multiple regression

NON-PARAMETRIC TESTS

FOR WHEN ASSUMPTIONS IN THESE OTHER 3 CATEGORIES ARE NOT MET

i.e. Wilcoxon rank-sum test, Wilcoxon sign-rank test, sign test


1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0

CORRELATION

ASSOCIATION BETWEEN VARIABLES

i.e. Pearson Correlation, Spearman Correlation, chi-square test

COMPARISON OF MEANS

DIFFERENCE IN MEANS BETWEEN VARIABLES

i.e. t-test, ANOVA

REGRESSION

DOES CHANGE IN ONE VARIABLE MEAN CHANGE IN ANOTHER?

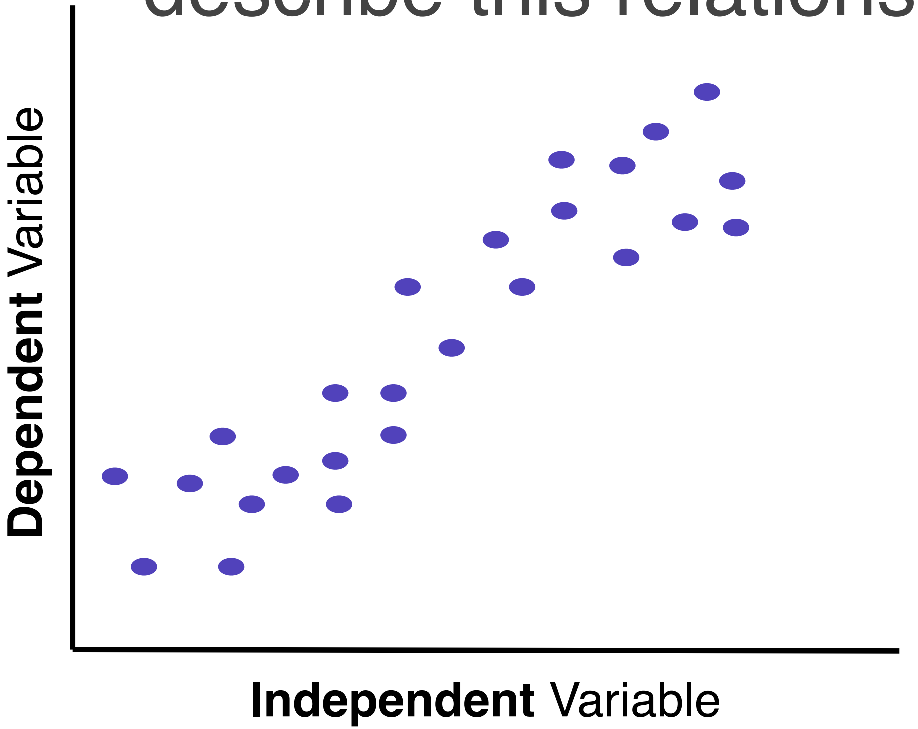
I.e. simple regression, multiple regression

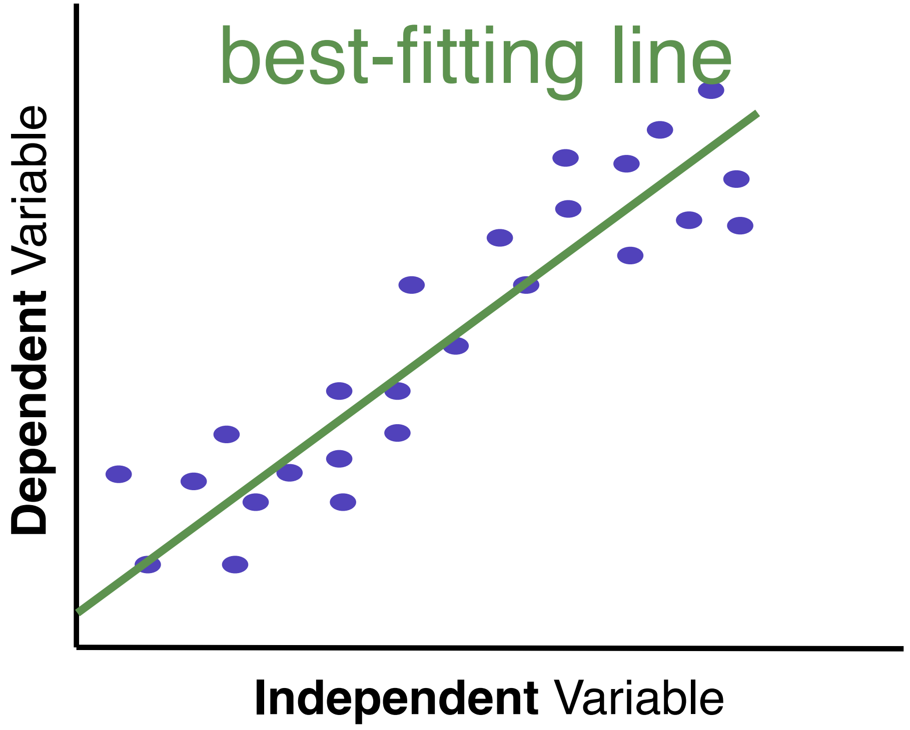
NON-PARAMETRIC TESTS

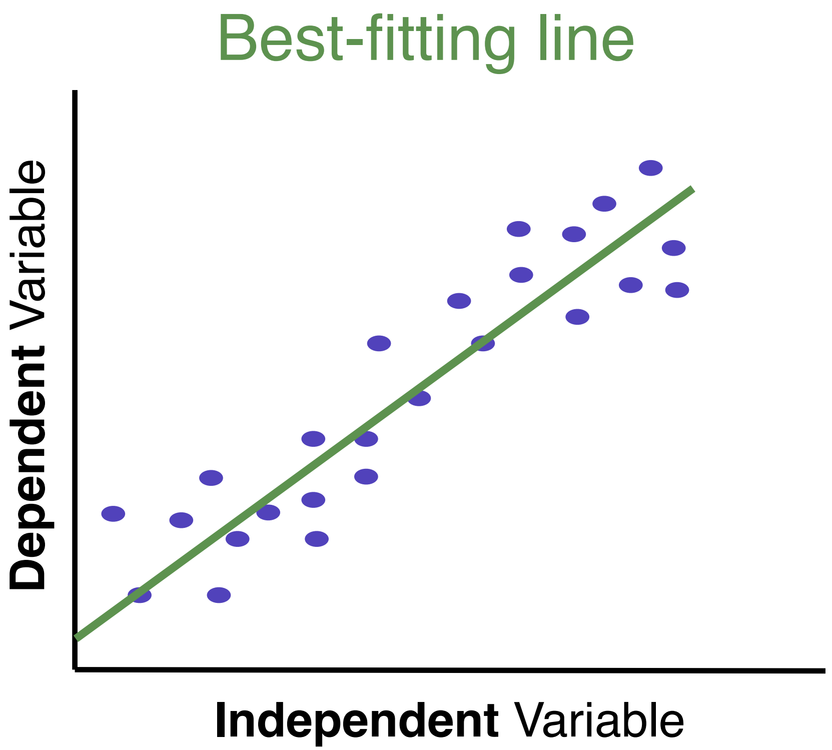
FOR WHEN ASSUMPTIONS IN THESE OTHER 3 CATEGORIES ARE NOT MET

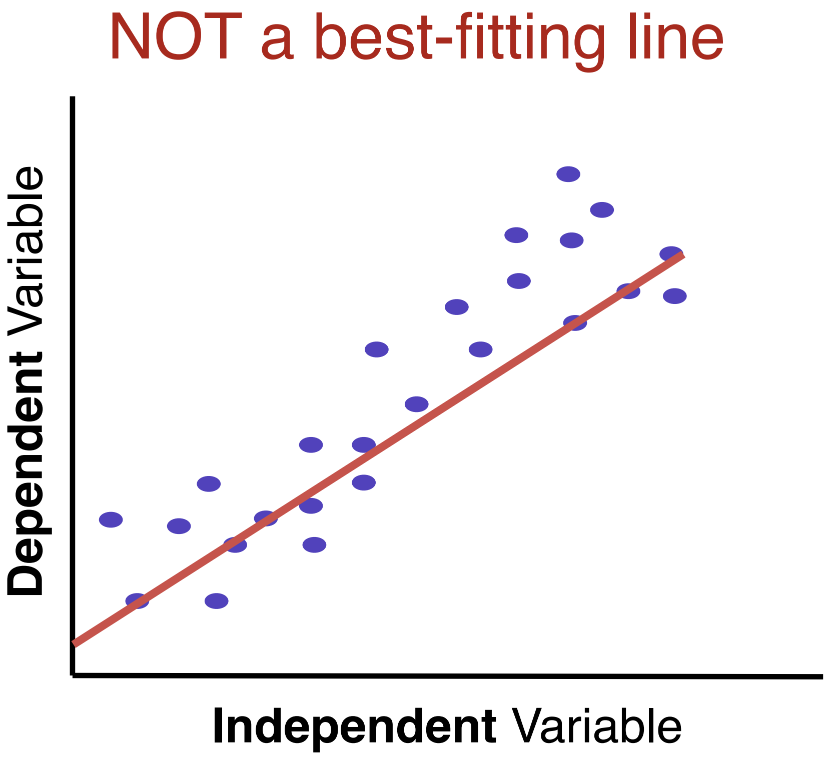
i.e. Wilcoxon rank-sum test, Wilcoxon sign-rank test, sign test

Linear regression can be used to describe this relationship

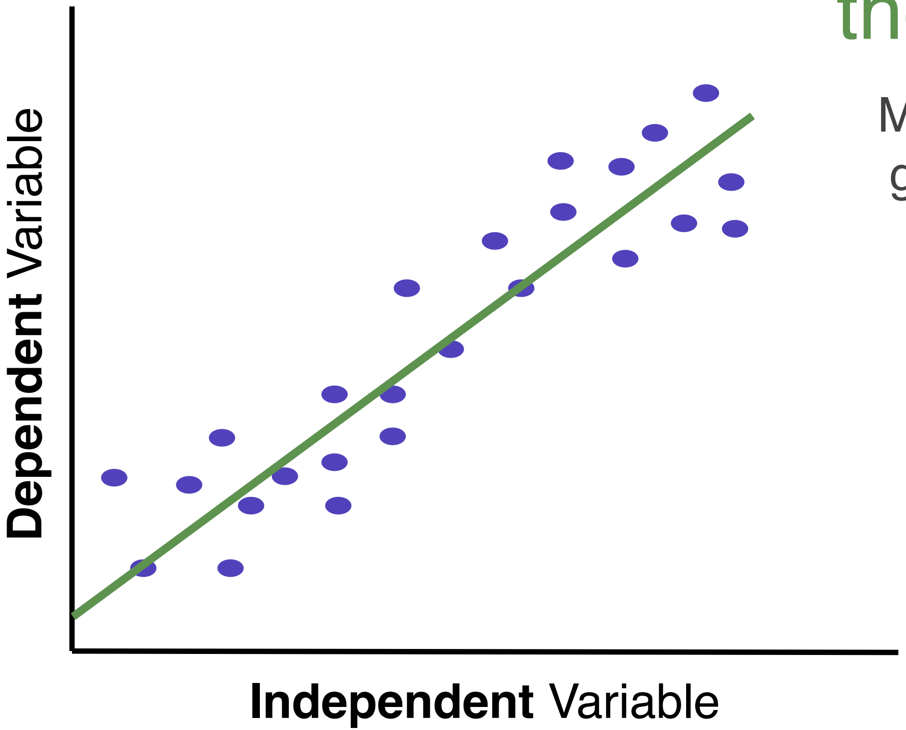



best-fitting line
Dependent Variable
Independent Variable

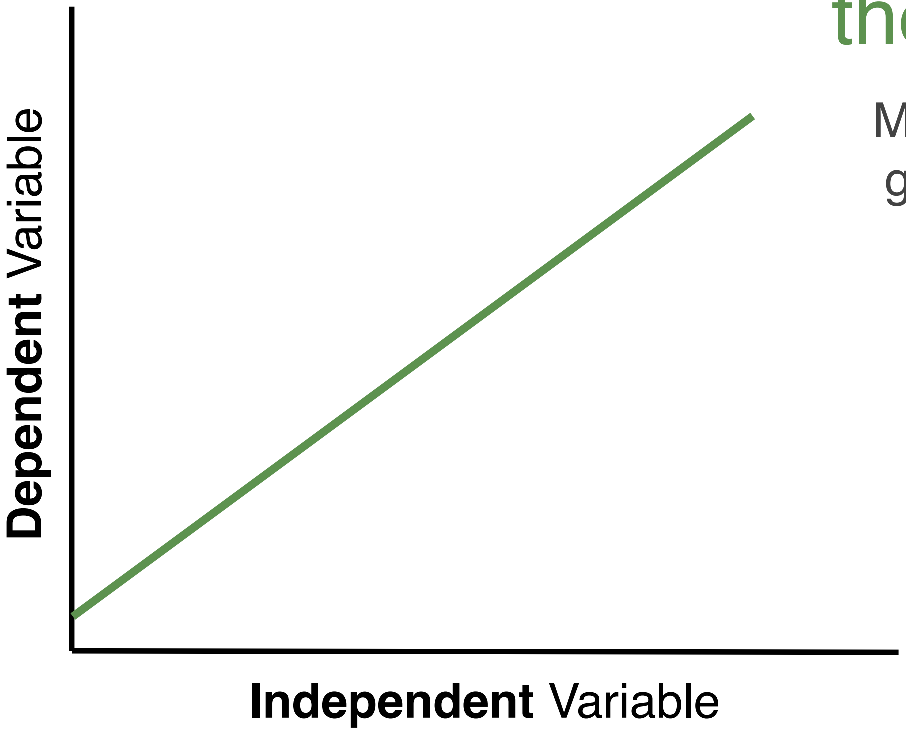

Best-fitting line
Dependent Variable
Independent Variable


NOT a best-fitting line
Dependent Variable
Independent Variable

This line is a model of the data



Models are mathematical equations generated to *represent* the real life situation



This line is a model of the data

Models are mathematical equations generated to *represent* the real life situation

2.3 Parsimony

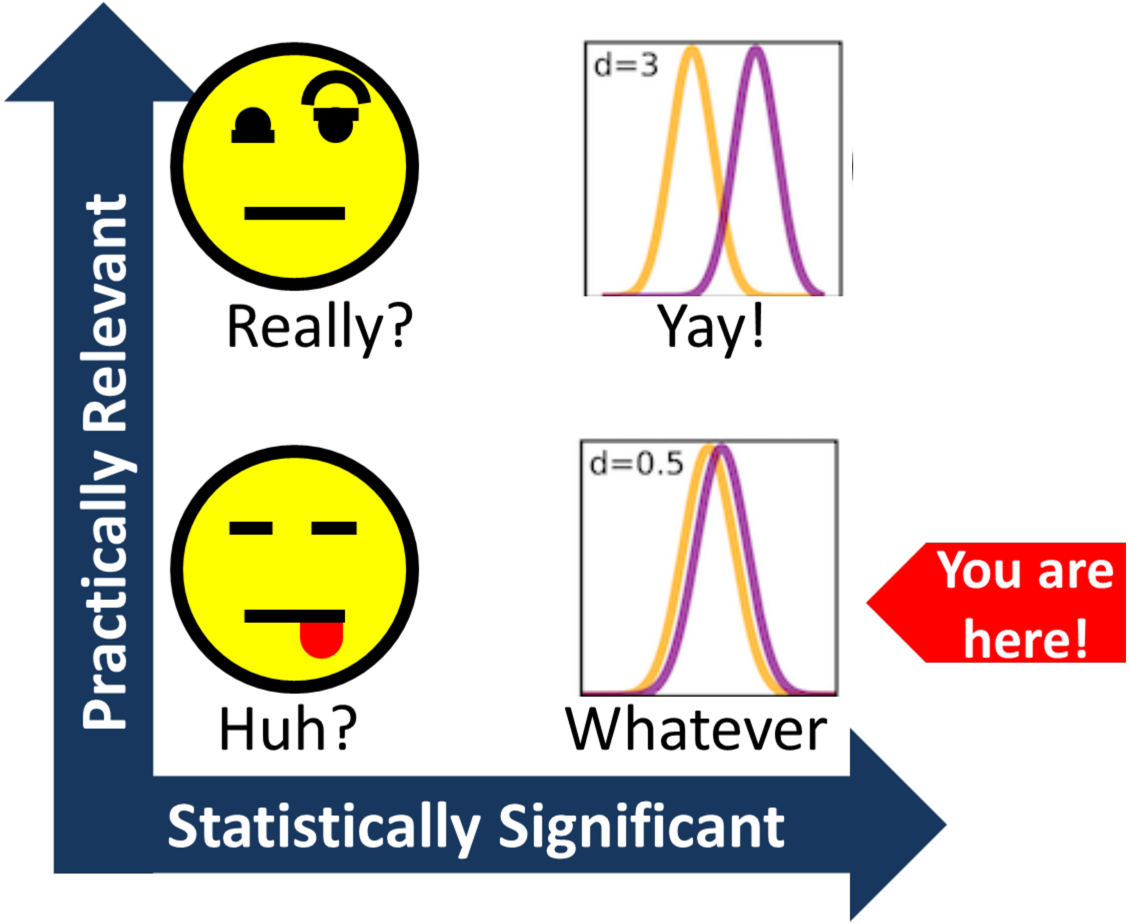
Since all models are wrong the scientist cannot obtain a "correct" one by excessive elaboration. On the contrary following William of Occam he should seek an economical description of natural phenomena. Just as the ability to devise simple but evocative models is the signature of the great scientist so overelaboration and overparameterization is often the mark of mediocrity.

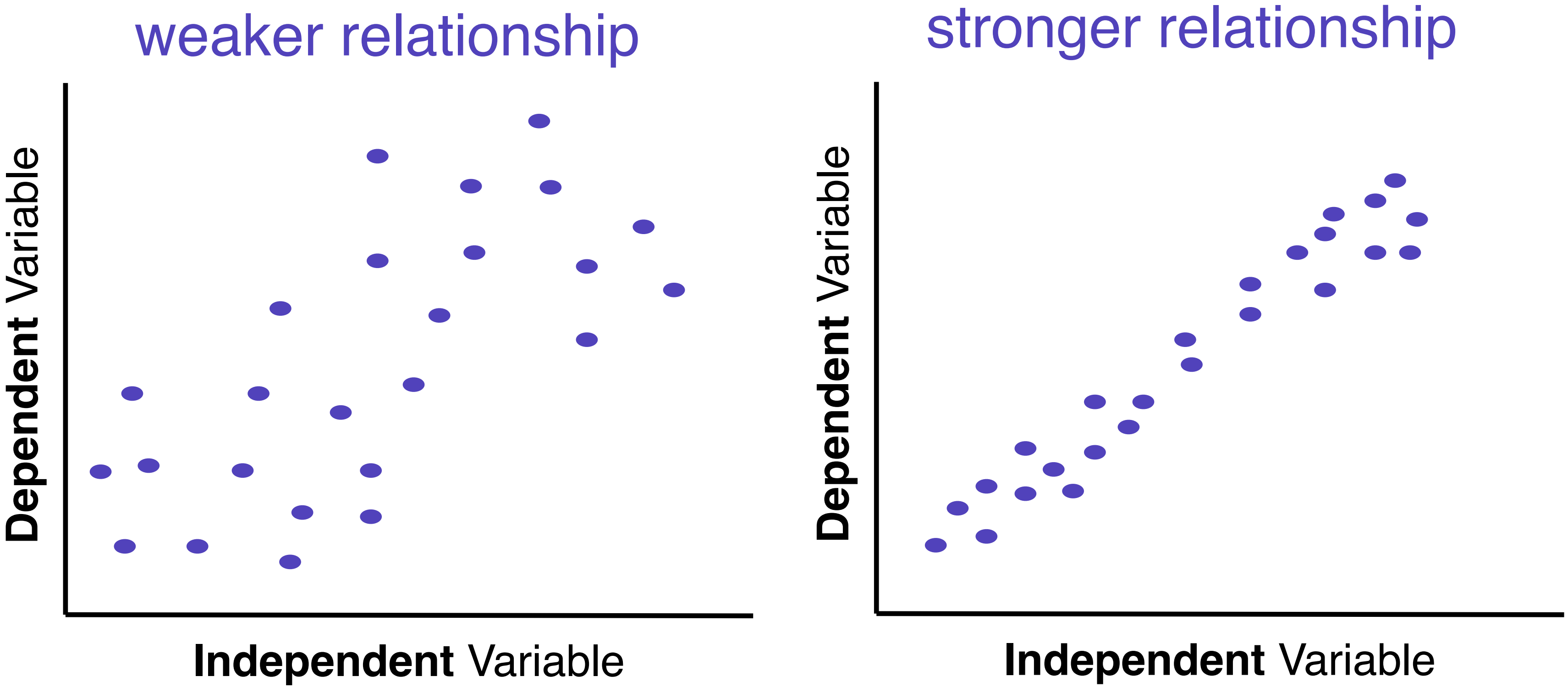
2.4 Worrying Selectively

Since all models are wrong the scientist must be alert to what is importantly wrong. It is inappropriate to be concerned about mice when there are tigers abroad.

George Box

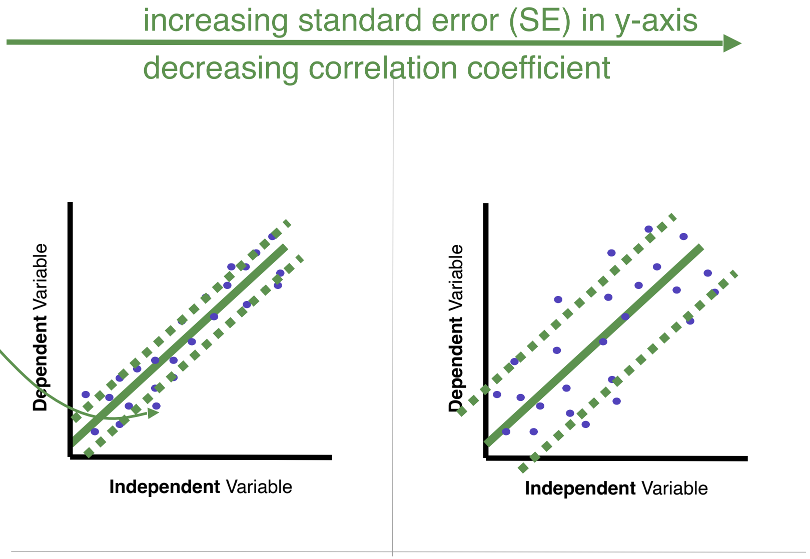
Effect sizes!





stronger relationship = higher correlation

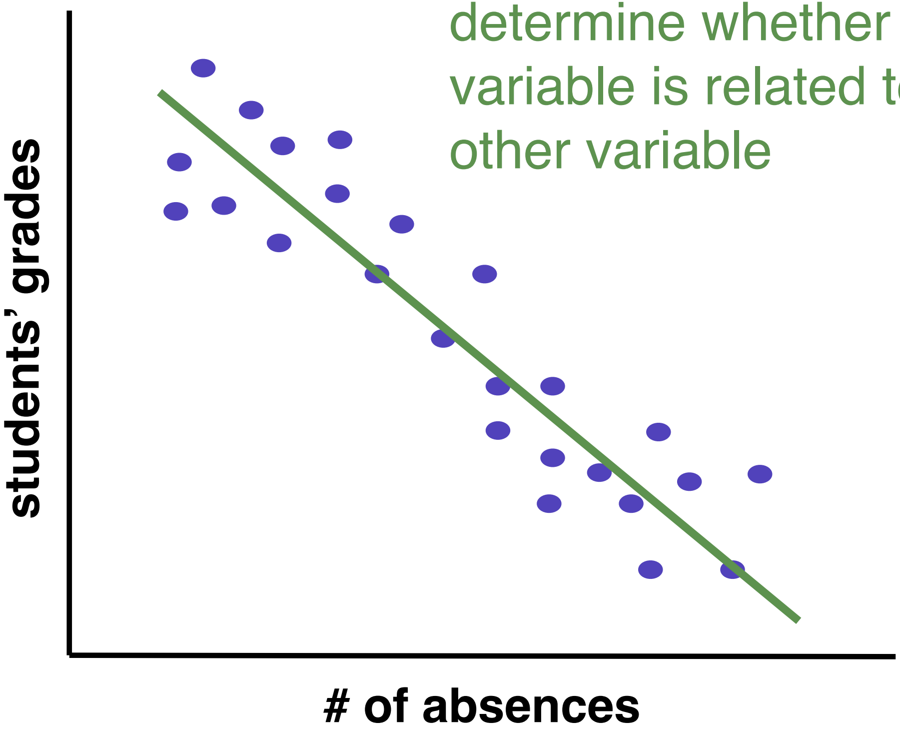
This is a kind of effect size



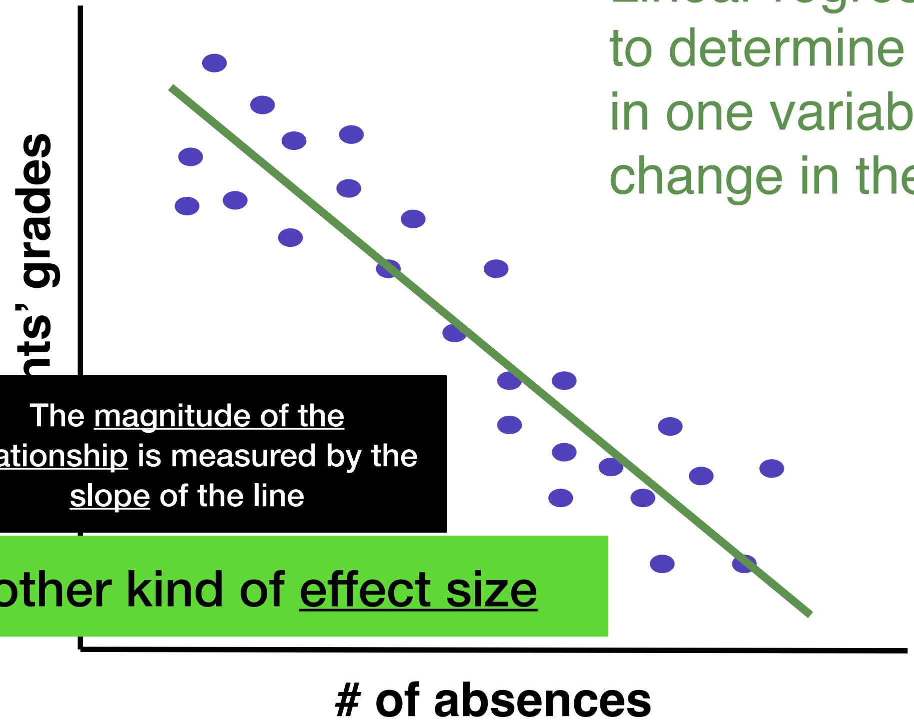
The *closer* the points are to the regression line, the *less uncertain* we are in our estimate

Standard error is standard deviation / sqrt(n)

Linear regression can be used to determine whether a change in one variable is related to the change in the other variable



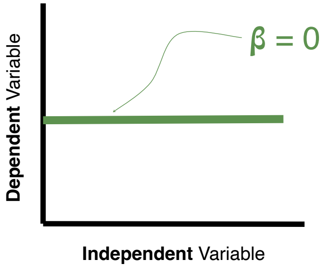
Linear regression can be used to determine whether a change in one variable is related to the change in the other variable



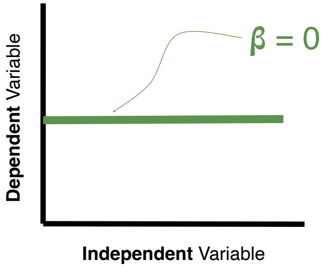
The <u>magnitude of the relationship</u> is measured by the <u>slope</u> of the line

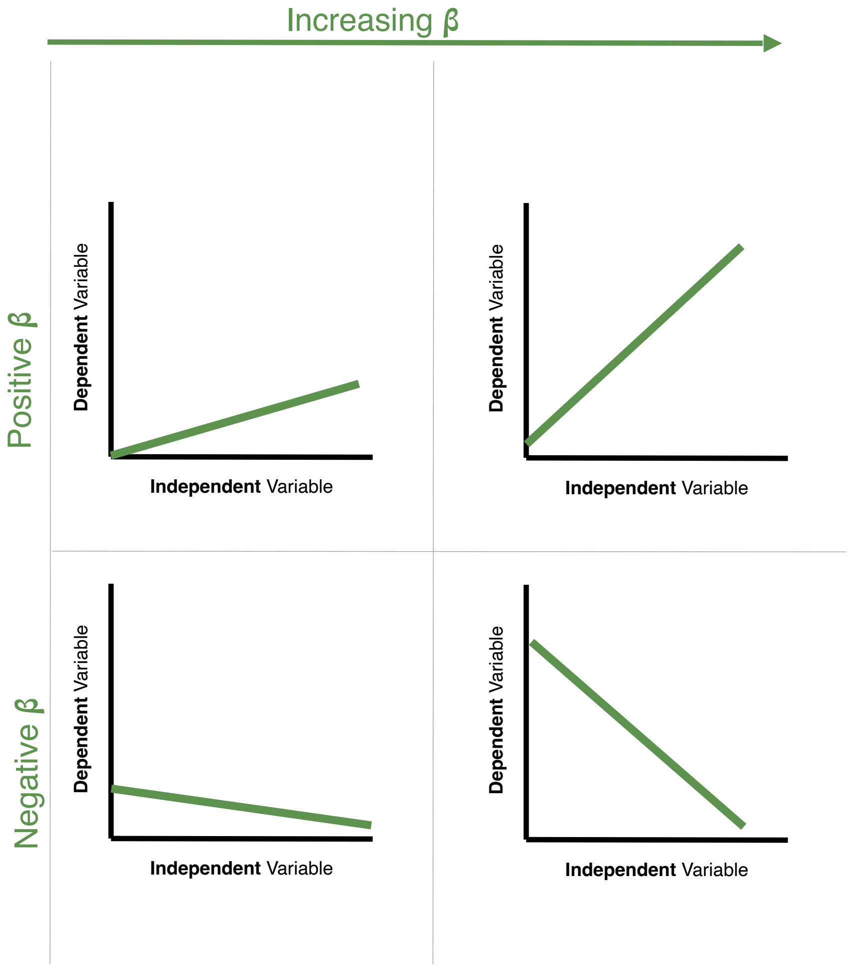
This is another kind of <u>effect size</u>

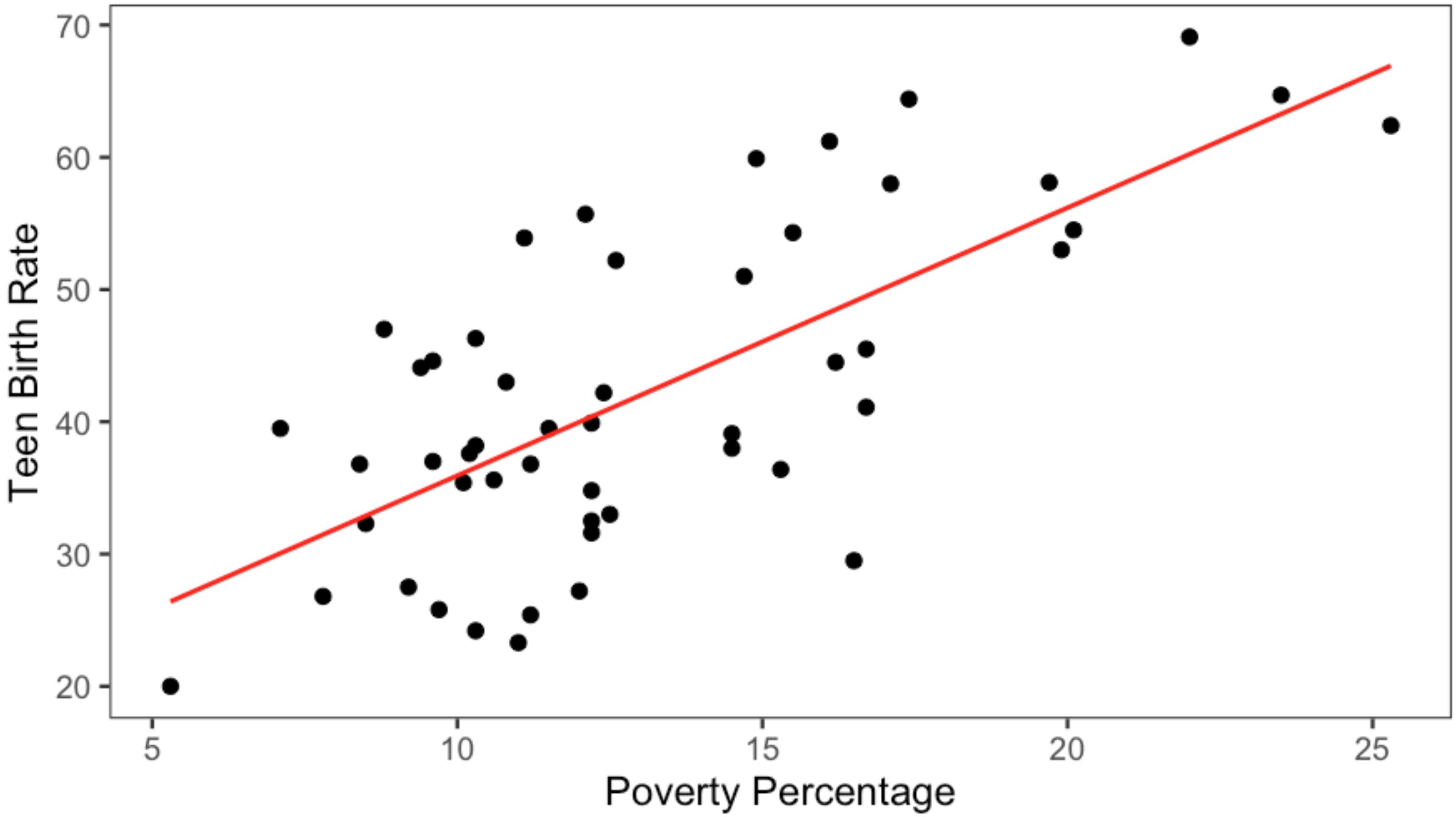
Effect size (β) can be estimated using the slope of the line



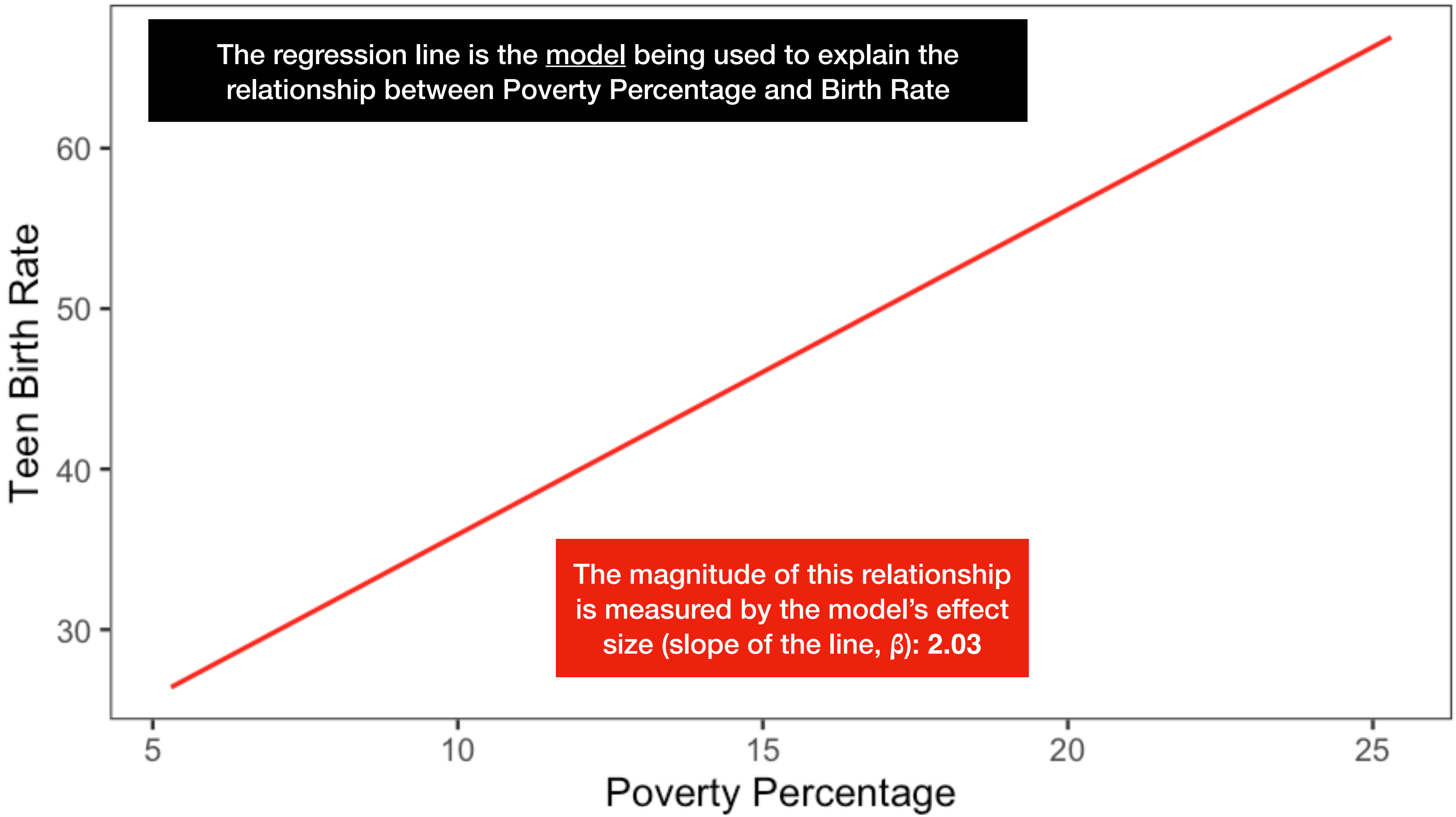
Effect size (β) can be estimated using the slope of the line

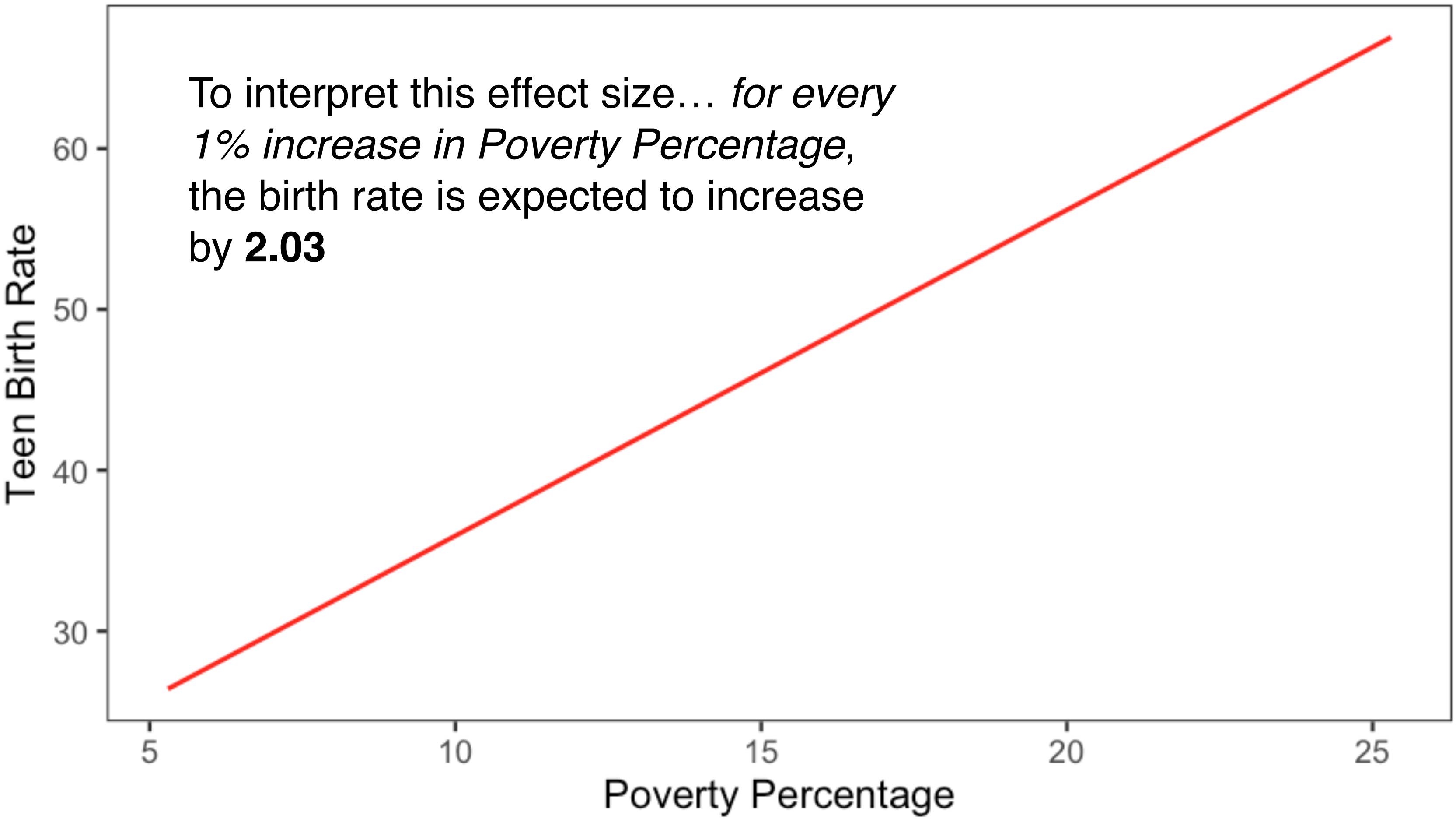


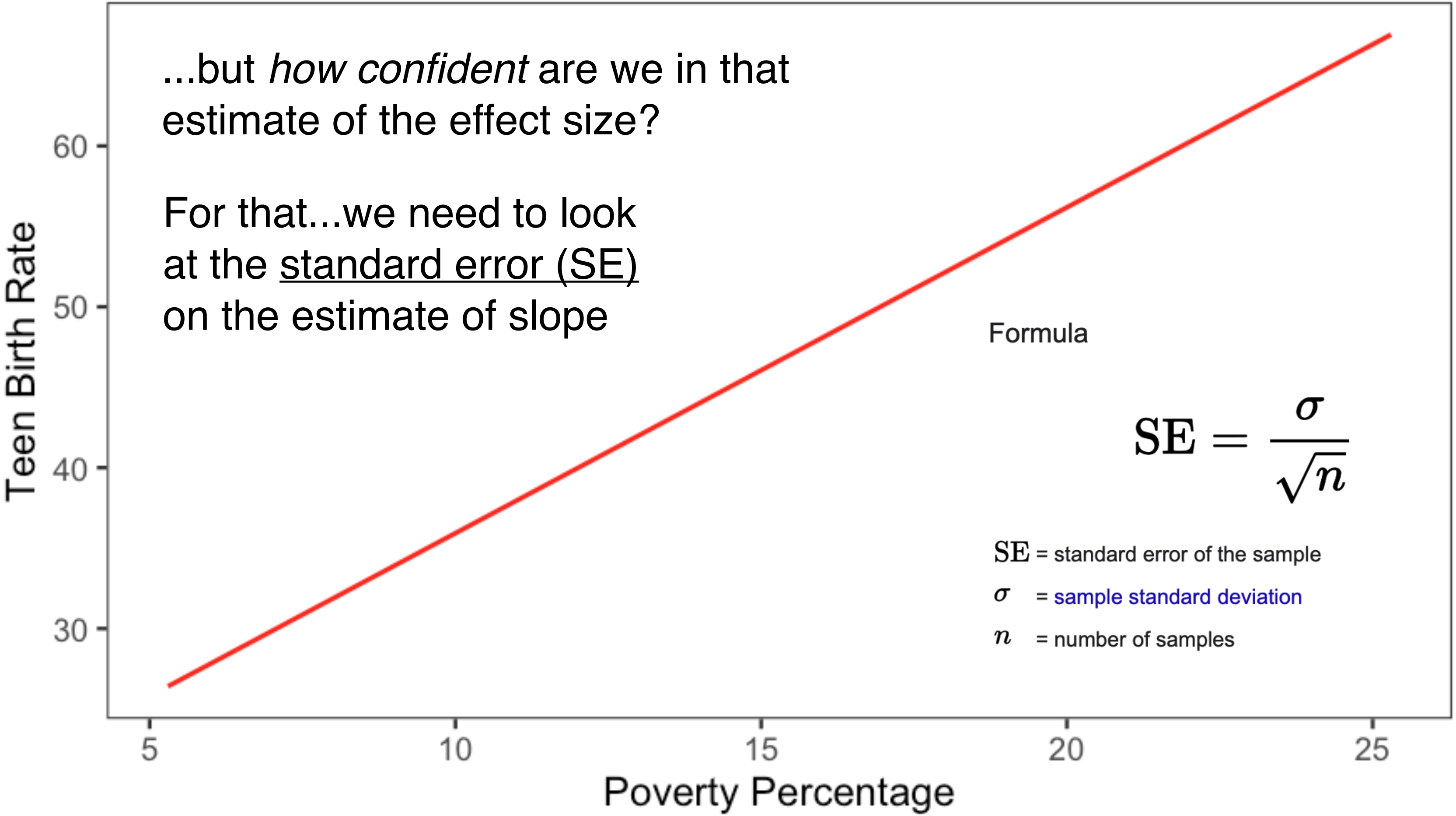




Teen Birth Rate
70
60
50
40
30
20
5
10
15
20
25
Poverty Percentage


The regression line is the model being used to explain the relationship between Poverty Percentage and Birth Rate
The magnitude of this relationship is measured by the model's effect size (slope of the line, β): 2.03
Teen Birth Rate
60
50
40
30
5
10
15
20
25
Poverty Percentage


To interpret this effect size… *for every 1% increase in Poverty Percentage*, the birth rate is expected to increase by **2.03**
Teen Birth Rate
60
50
40
30
5
10
15
20
25
Poverty Percentage



...but *how confident* are we in that estimate of the effect size?

For that...we need to look at the standard error (SE) on the estimate of slope

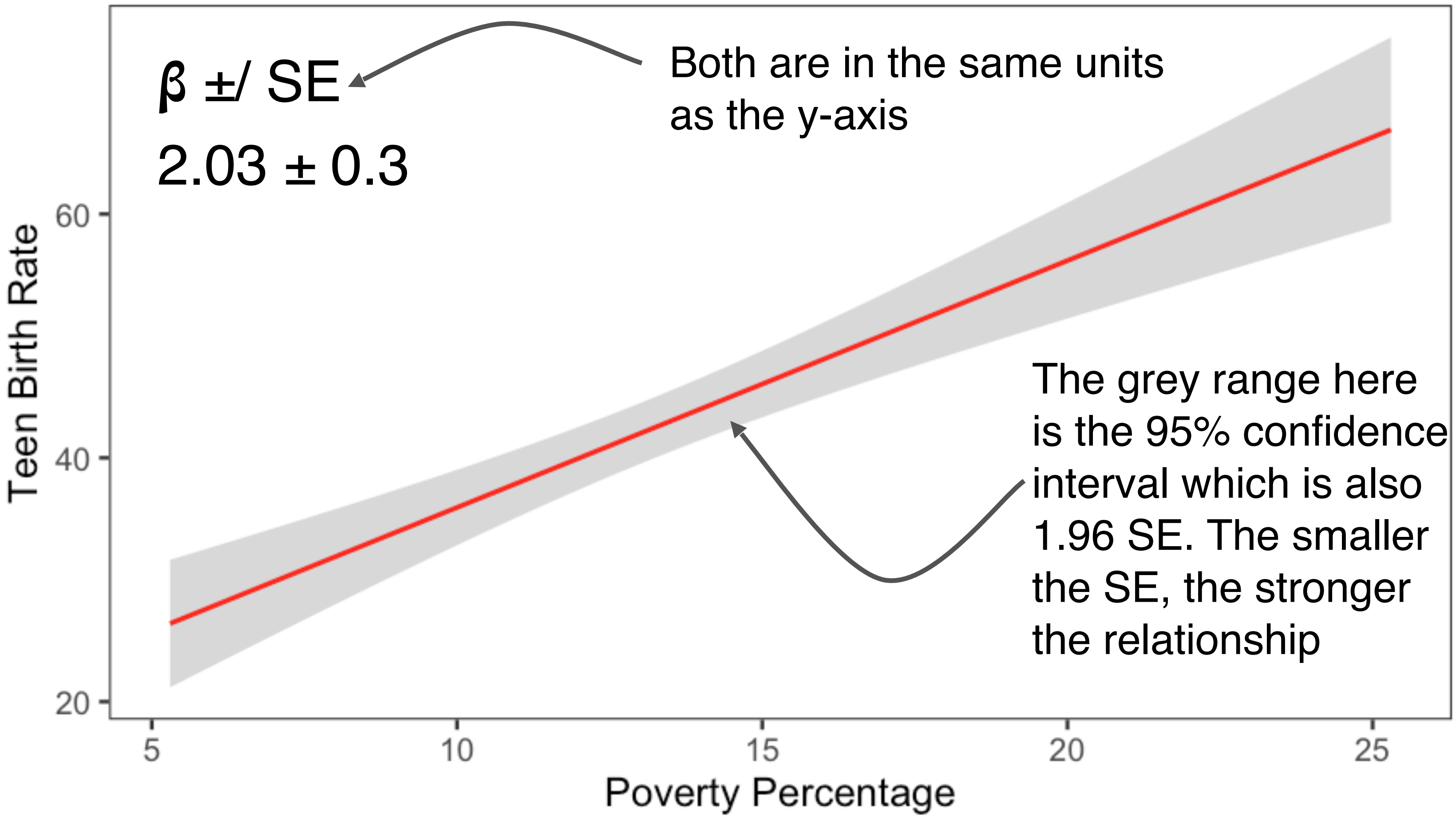
Formula

$$\mathbf{SE} = \frac{\sigma}{\sqrt{n}}$$

SE = standard error of the sample

σ = sample standard deviation

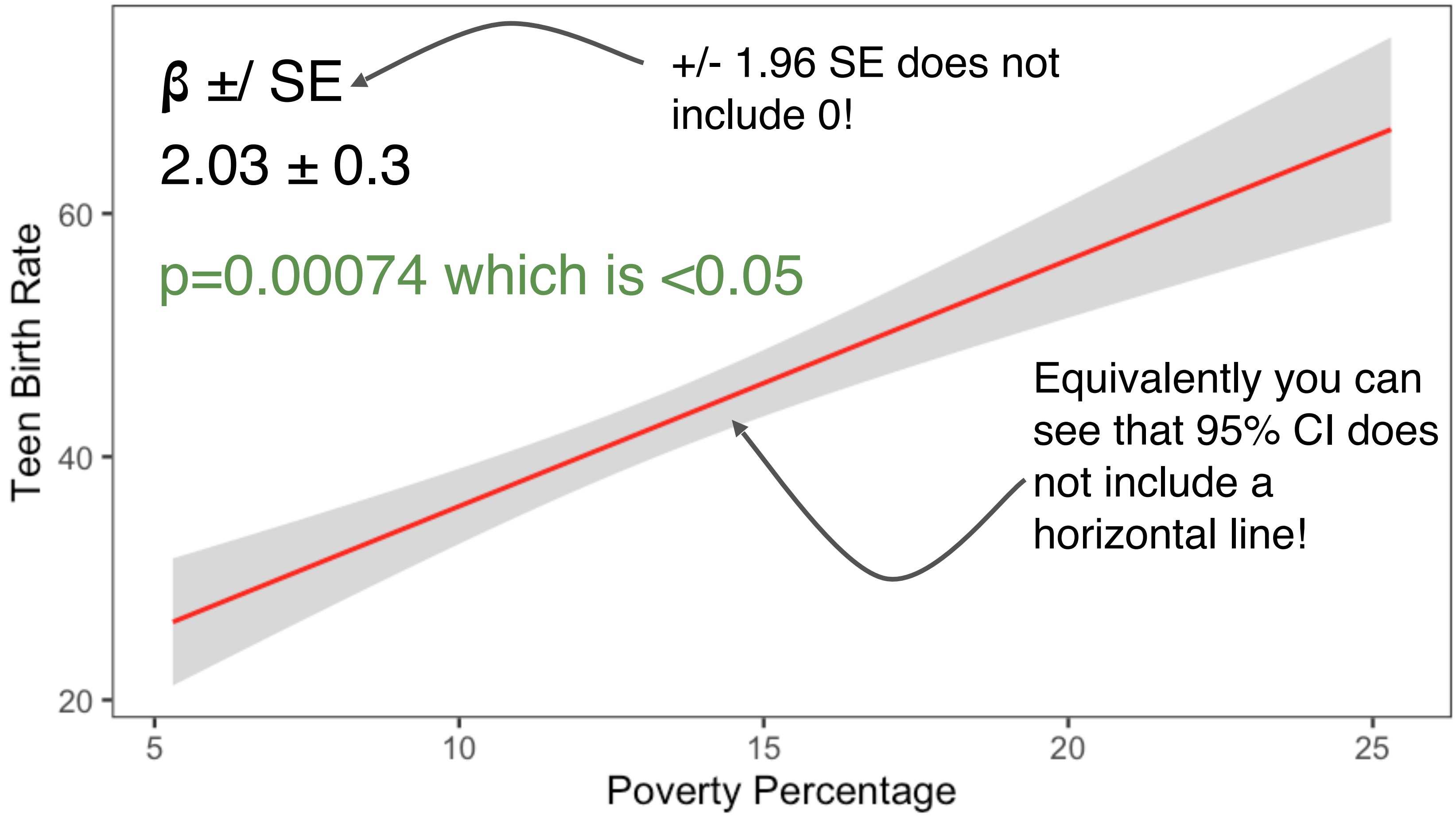
n = number of samples



β ±/ SE
2.03 ± 0.3
Both are in the same units as the y-axis
The grey range here is the 95% confidence interval which is also 1.96 SE. The smaller the SE, the stronger the relationship
Teen Birth Rate
20
40
60
5
10
15
20
25
Poverty Percentage

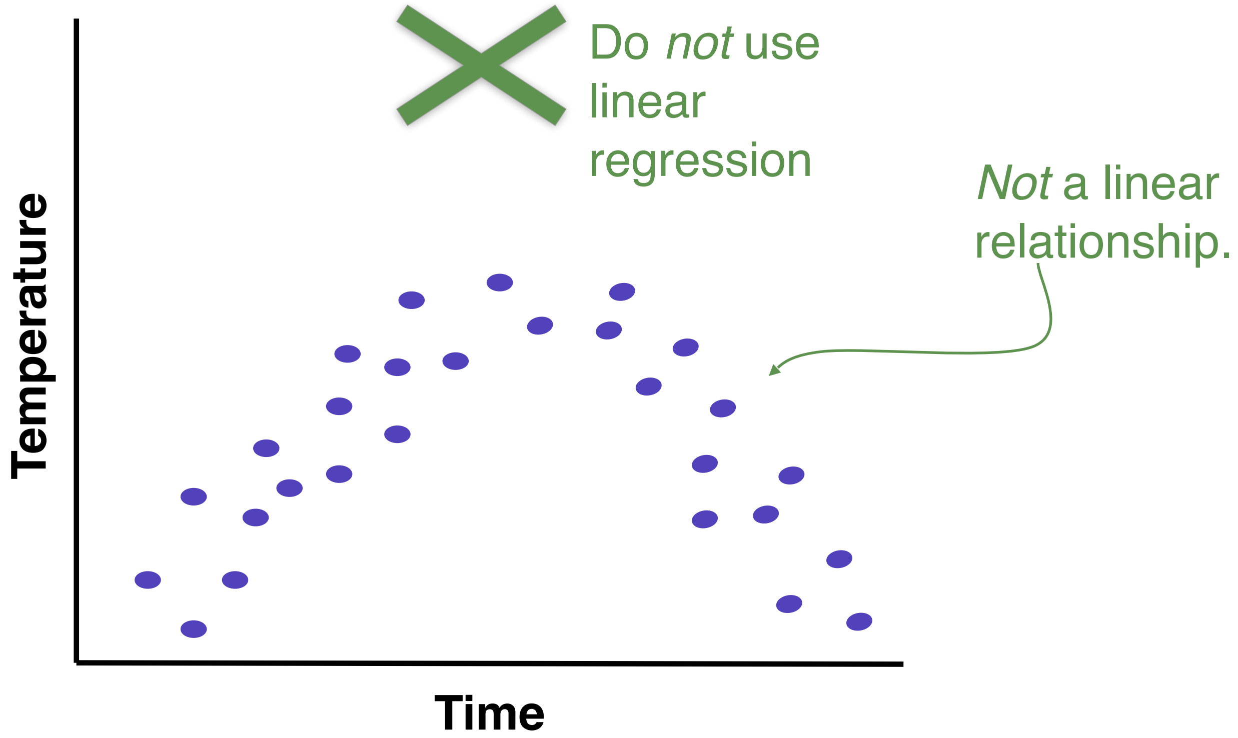
p-value : the probability of getting the observed results (or results more extreme) by chance alone

Takes into account the effect size (β) and the SE

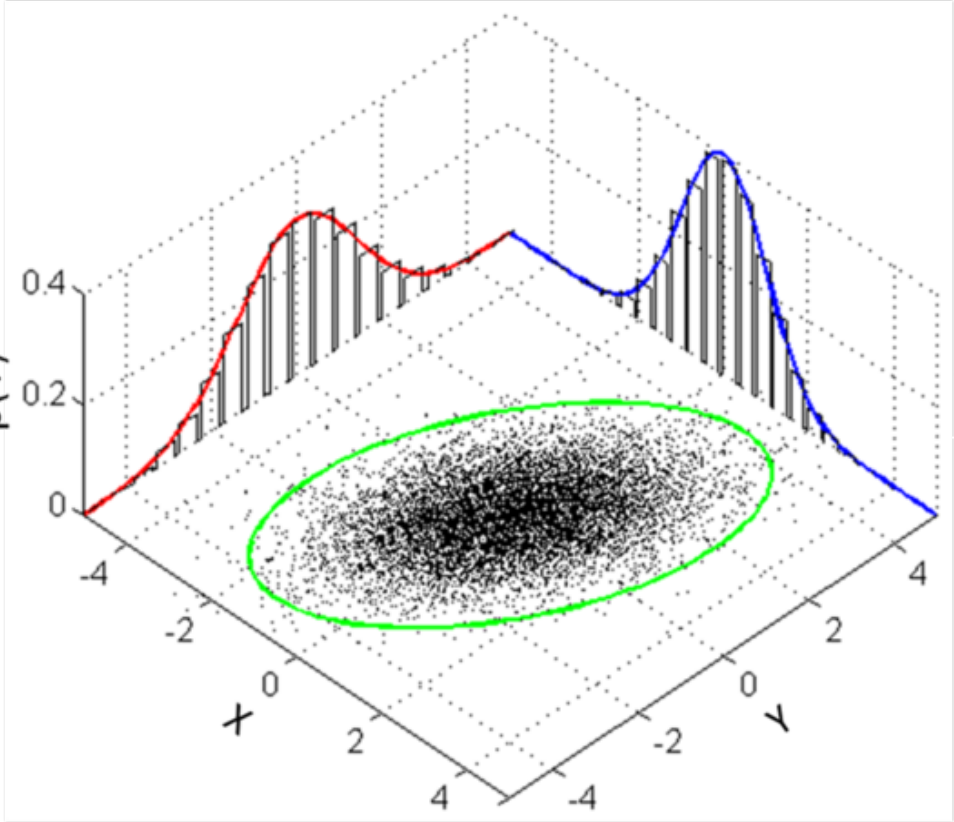

β ±/ SE
2.03 ± 0.3
+/- 1.96 SE does not include 0!
p=0.00074 which is <0.05
Equivalently you can see that 95% CI does not include a horizontal line!
Teen Birth Rate
60
40
20
5
10
15
20
25
Poverty Percentage

Assumptions of linear regression

1. Linear relationship
2. Multivariate normality
3. No multicollinearity
4. No autocorrelation
5. Homoscedasticity

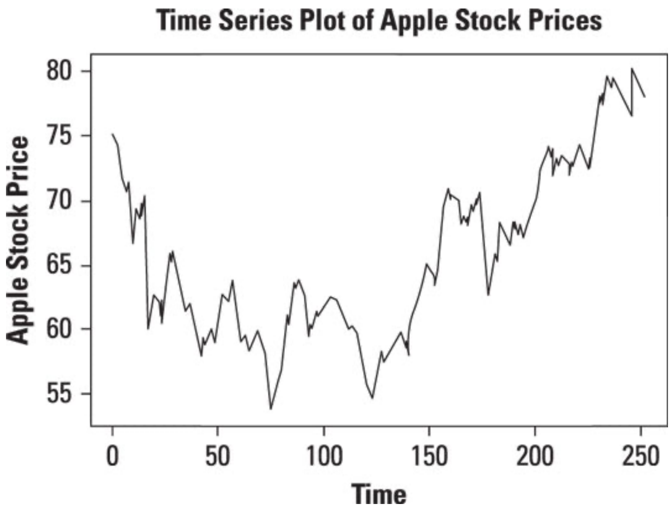

Do *not* use linear regression
Not a linear relationship.
Temperature
Time

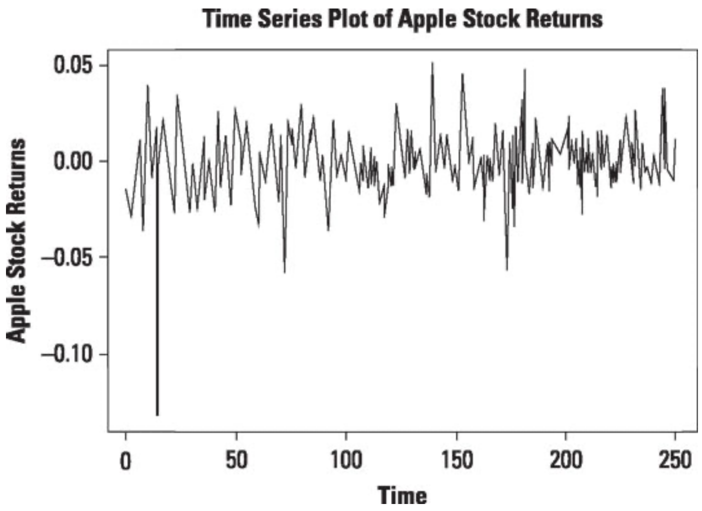
A multivariate normal probability distribution (joint normal)



Linear regression assumes no multicollinearity. Multicollinearity occurs when the independent variables (in multiple linear regression) are too highly correlated with each other.

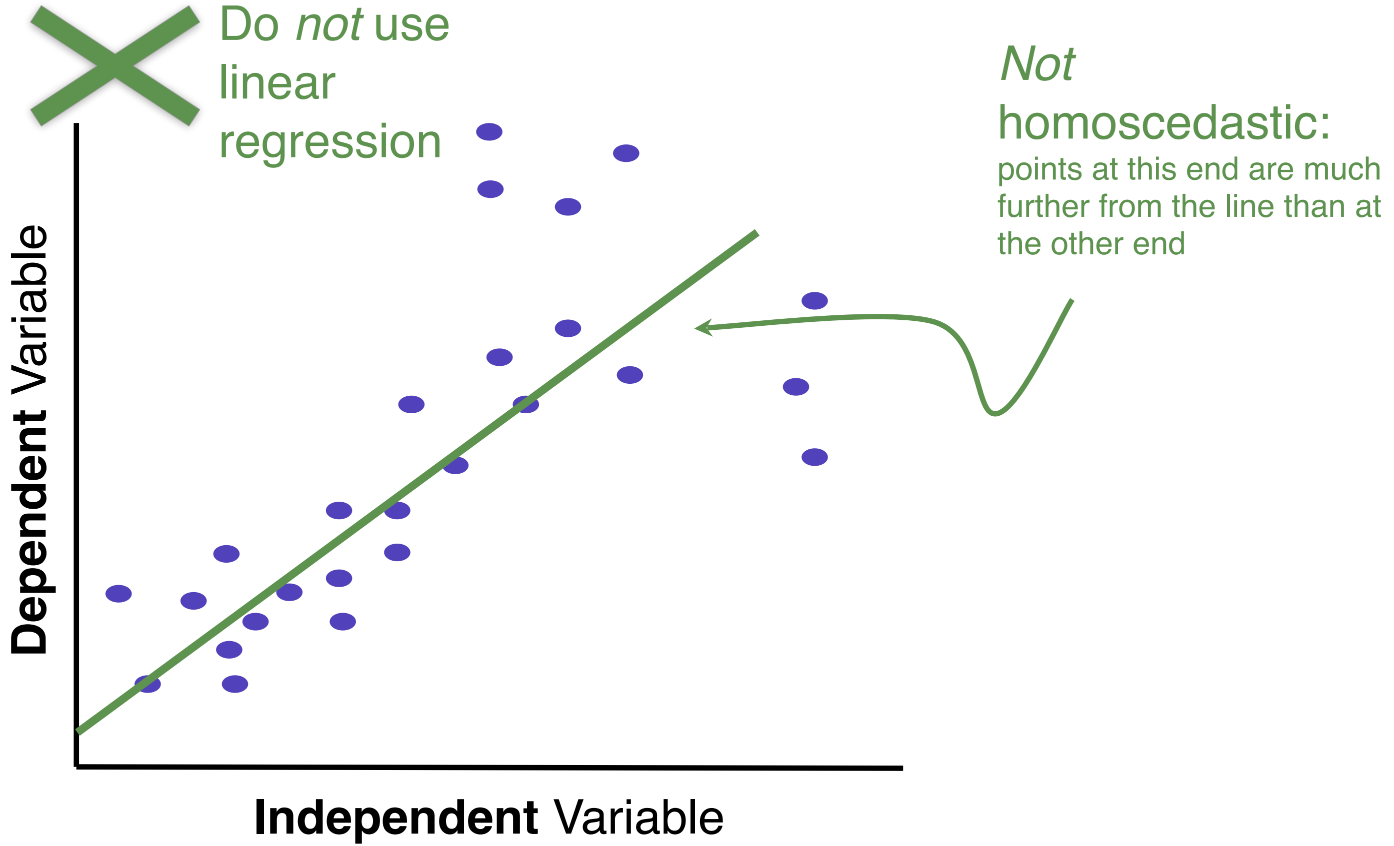
Daily returns are
$\ln(\text{price}_t / \text{price}_{t-1})$



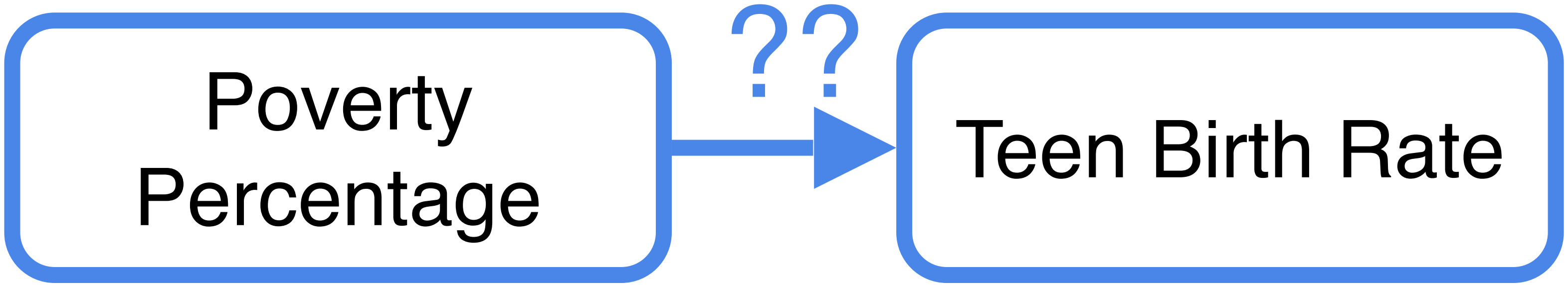


Autocorrelation occurs when the observations are *not* independent of one another (i.e. stock prices)

https://www.dummies.com/programming/big-data/data-science/autocorrelation-plots-graphical-technique-for-statistical-data//


Do *not* use linear regression
Not homoscedastic:
points at this end are much further from the line than at the other end
Dependent Variable
Independent Variable

Does Poverty Percentage affect Teen Birth Rate?


Poverty Percentage
??
Teen Birth Rate

What is the relationship between Poverty Percentage & Teen Birth Rate?

What's your hypothesis?

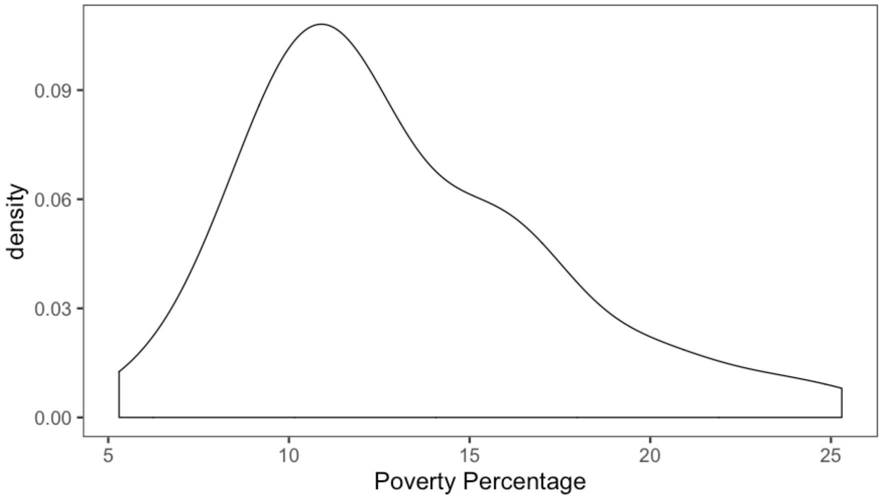
A
No
relationship

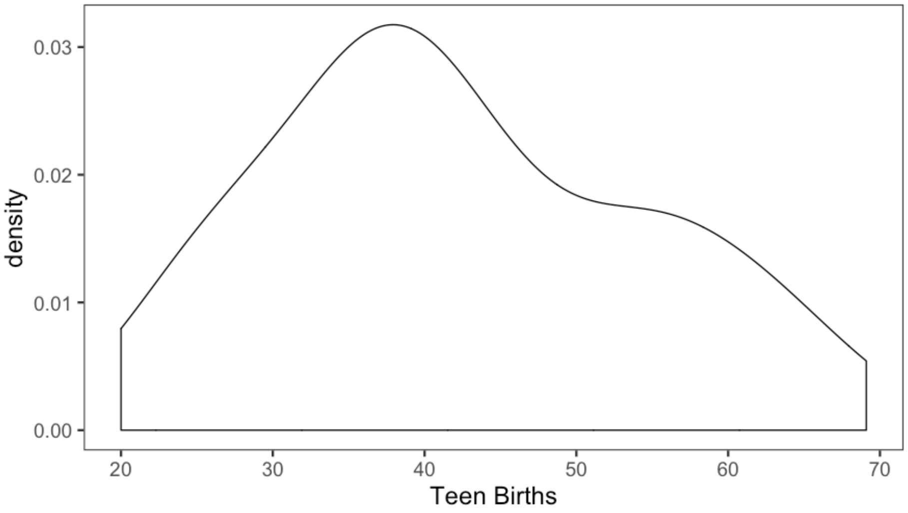
B
Positive
relationship

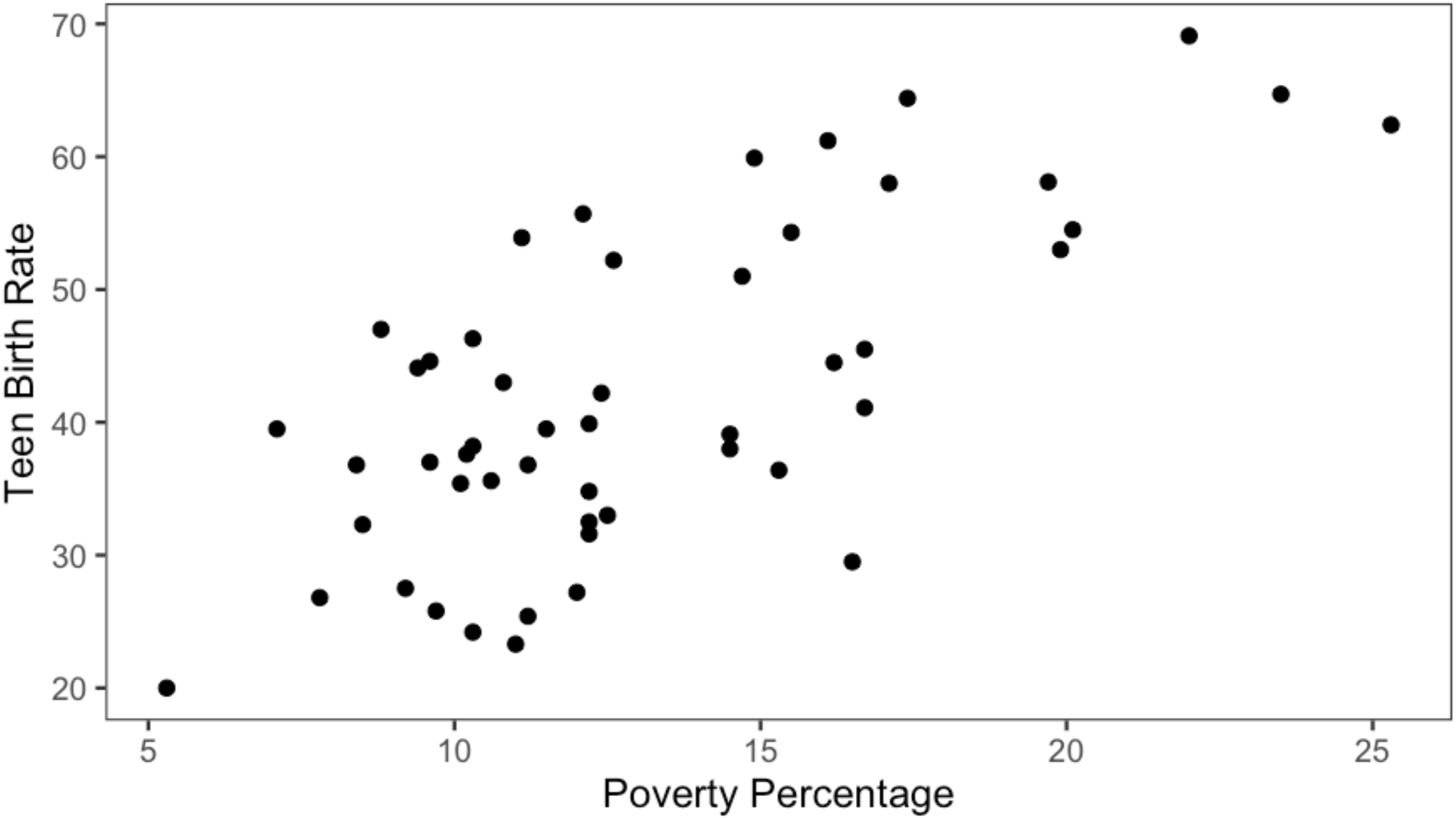
C
Negative
relationship

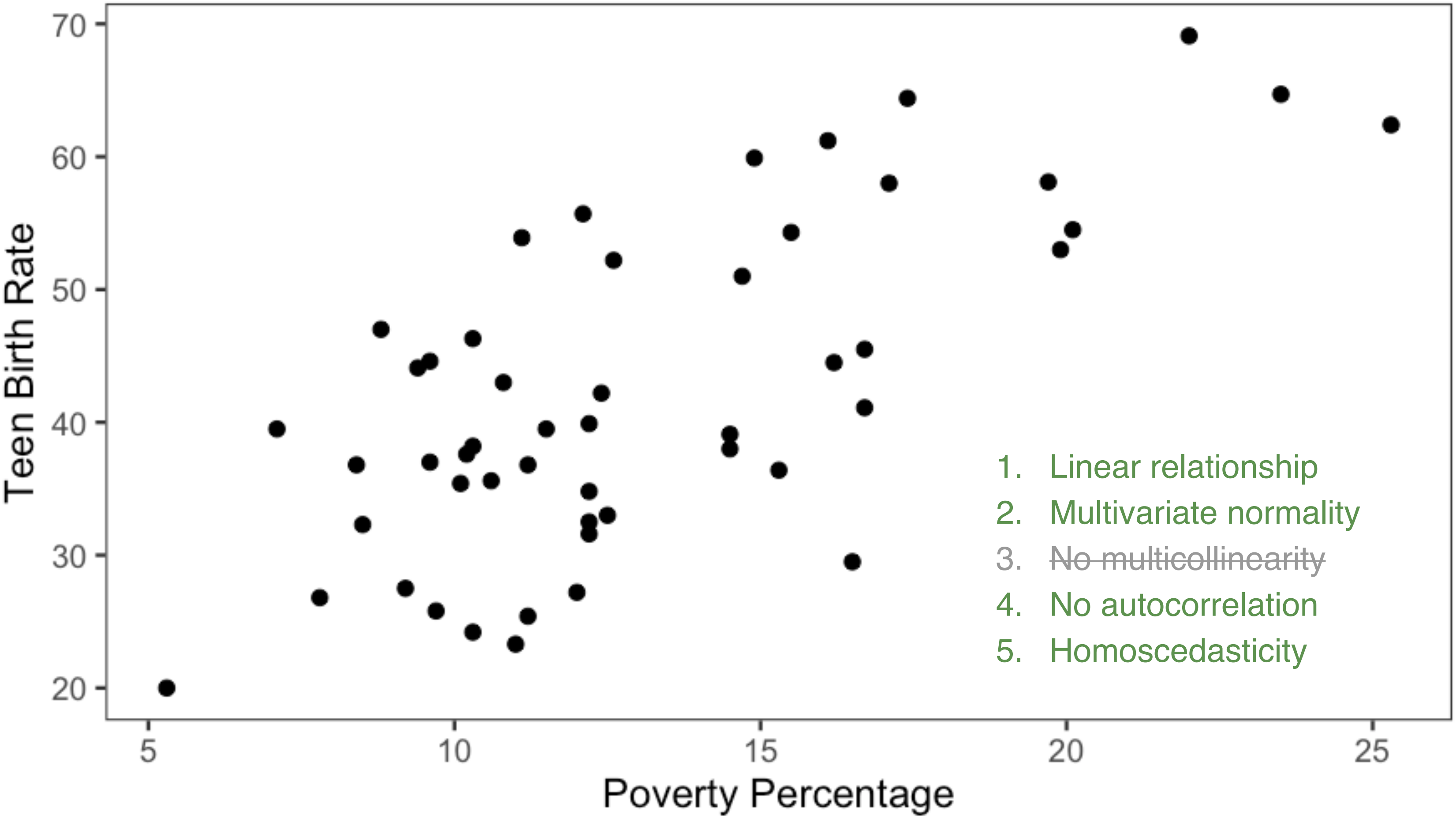
	Location	PovPct	Brth15to17	Brth18to19	ViolCrime	TeenBrth
1	Alabama	20.1	31.5	88.7	11.2	54.5
2	Alaska	7.1	18.9	73.7	9.1	39.5
3	Arizona	16.1	35.0	102.5	10.4	61.2
4	Arkansas	14.9	31.6	101.7	10.4	59.9
5	California	16.7	22.6	69.1	11.2	41.1
6	Colorado	8.8	26.2	79.1	5.8	47.0
7	Connecticut	9.7	14.1	45.1	4.6	25.8
8	Delaware	10.3	24.7	77.8	3.5	46.3
9	District_of_Columbia	22.0	44.8	101.5	65.0	69.1
10	Florida	16.2	23.2	78.4	7.3	44.5
11	Georgia	12.1	31.4	92.8	9.5	55.7
12	Hawaii	10.3	17.7	66.4	4.7	38.2
13	Idaho	14.5	18.4	69.1	4.1	39.1
14	Illinois	12.4	23.4	70.5	10.3	42.2
15	Indiana	9.6	22.6	78.5	8.0	44.6
16	Iowa	12.2	16.4	55.4	1.8	32.5
17	Kansas	10.8	21.4	74.2	6.2	43.0

Normal(ish) distributions

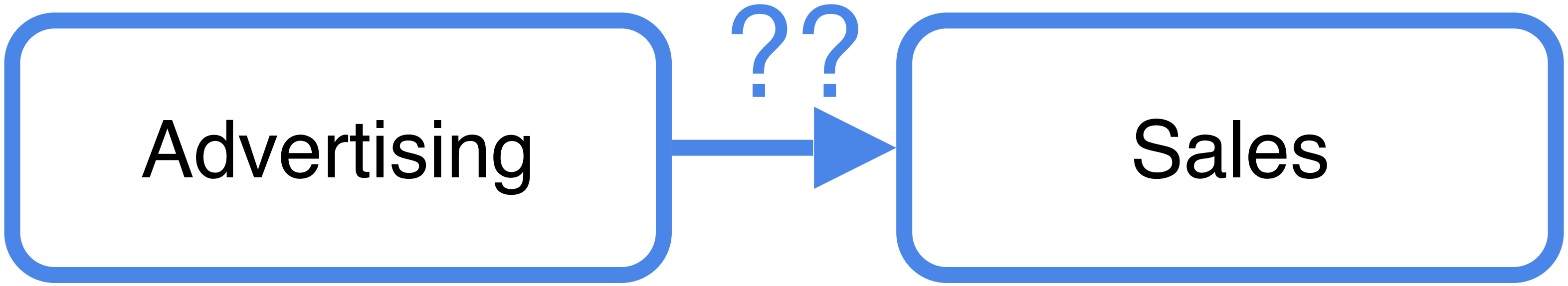





Teen Birth Rate
70
60
50
40
30
20
5
10
15
20
25
Poverty Percentage

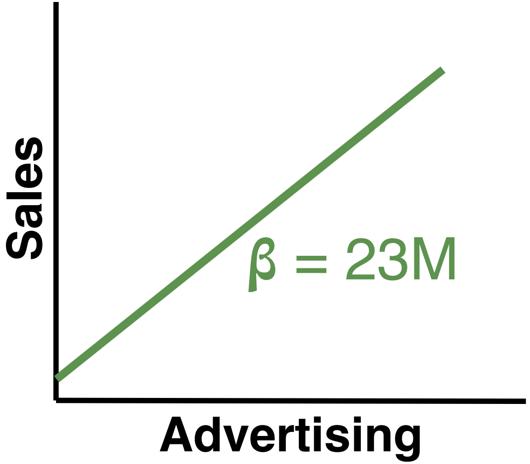


1. Linear relationship
2. Multivariate normality
3. ~~No multicollinearity~~
4. No autocorrelation
5. Homoscedasticity


Advertising
??
Sales

Effect size interpretation

Sales (Million Euro)	Advertising (Million Euro)
651	23
762	26
856	30
1,063	34
1,190	43
1,298	48
1,421	52
1,440	57
1,518	58



The effect size (β) between the advertising and sales is 23M. What does this mean?

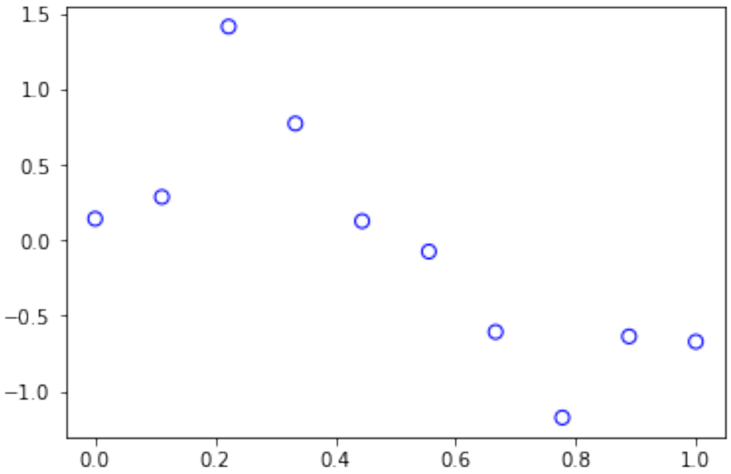
A
For every 1M Euro spent on advertising, the company sees 23M more in sales

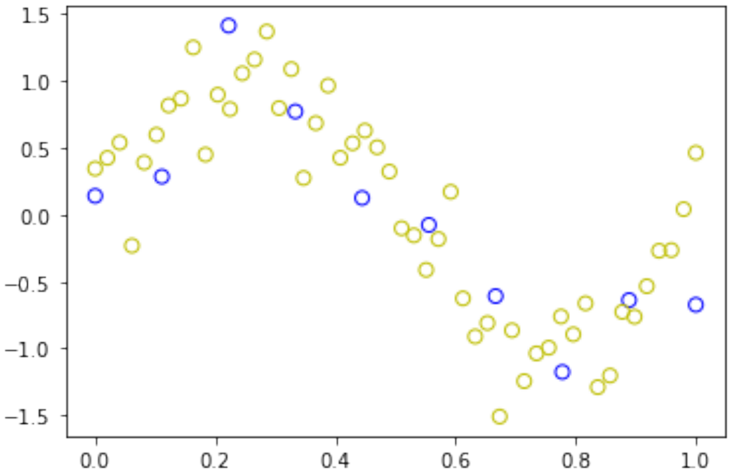
B
For every 1M Euro spent in sales, the company spends 23M more in advertising

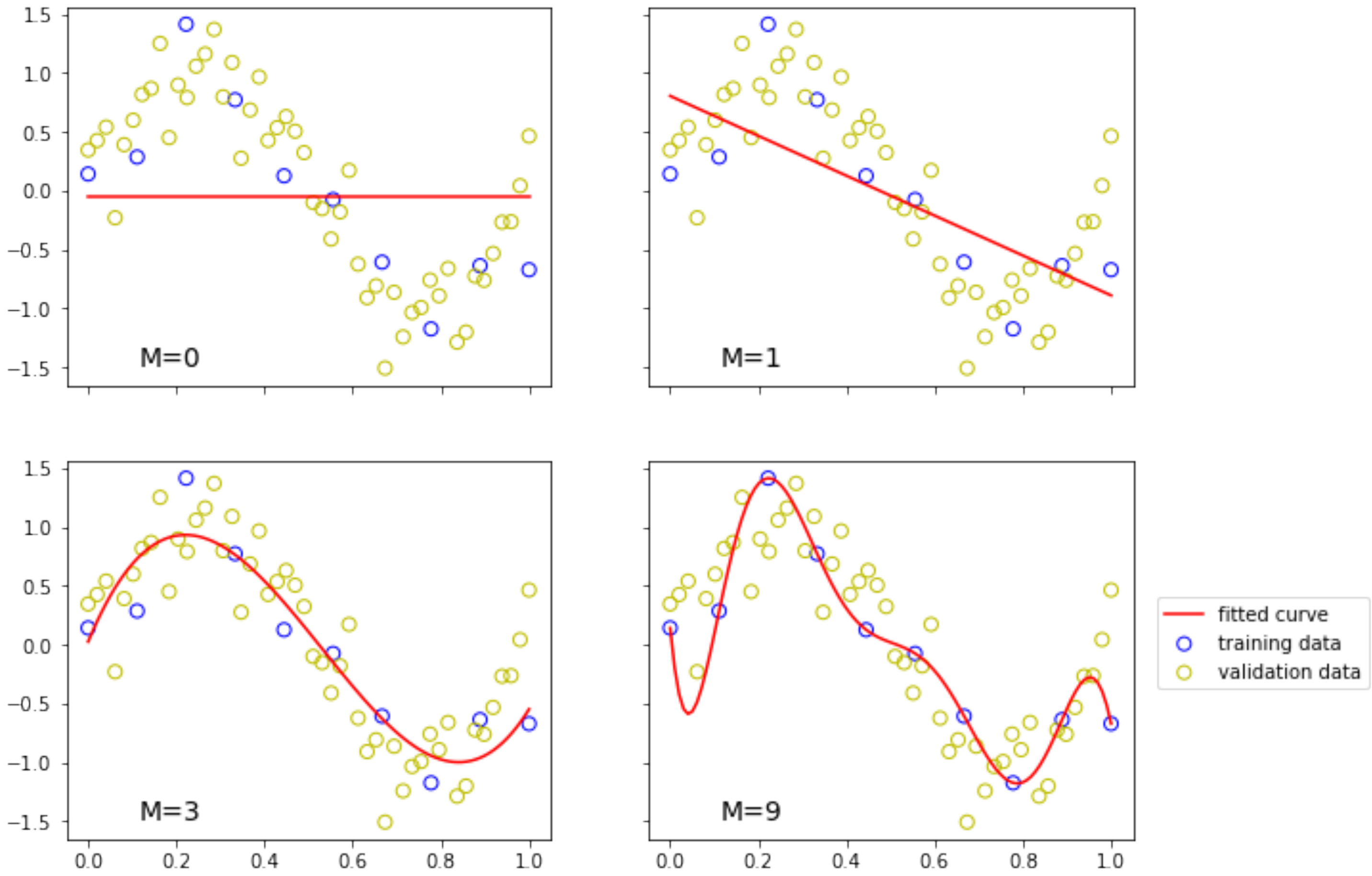
C
For every 1M Euro spent on advertising, the company sees 24M less in sales

D
For every 1M Euro spent in sales, the company spends 23M less in advertising

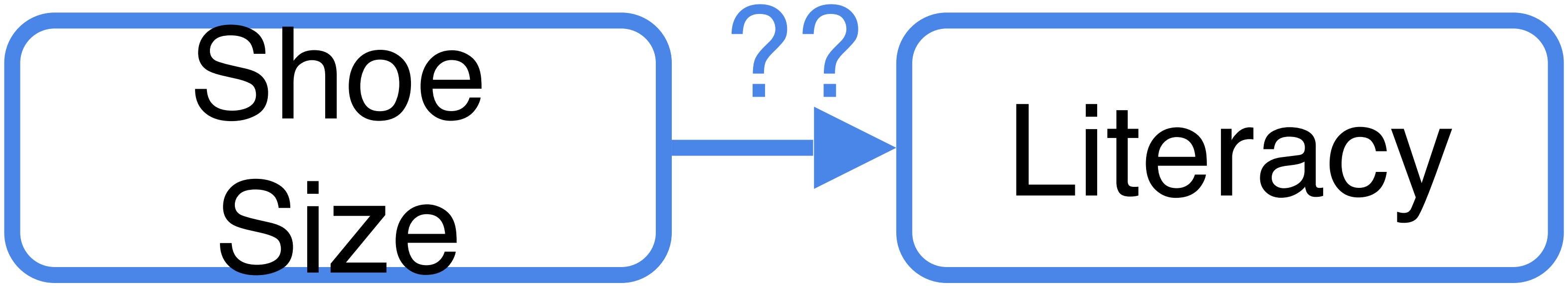
Regression problems


1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.0
0.2
0.4
0.6
0.8
1.0

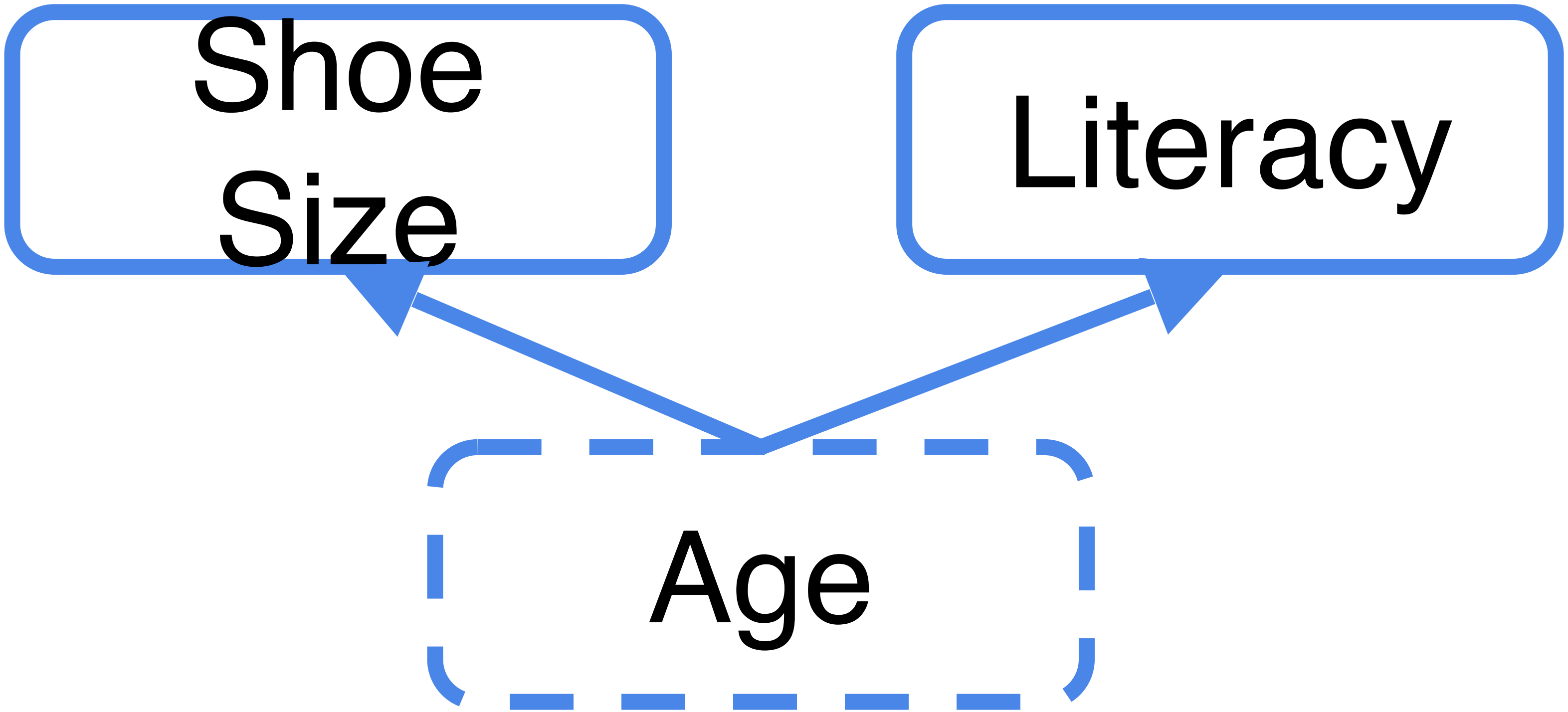

M=0
M=1
M=3
M=9
fitted curve
training data
validation data

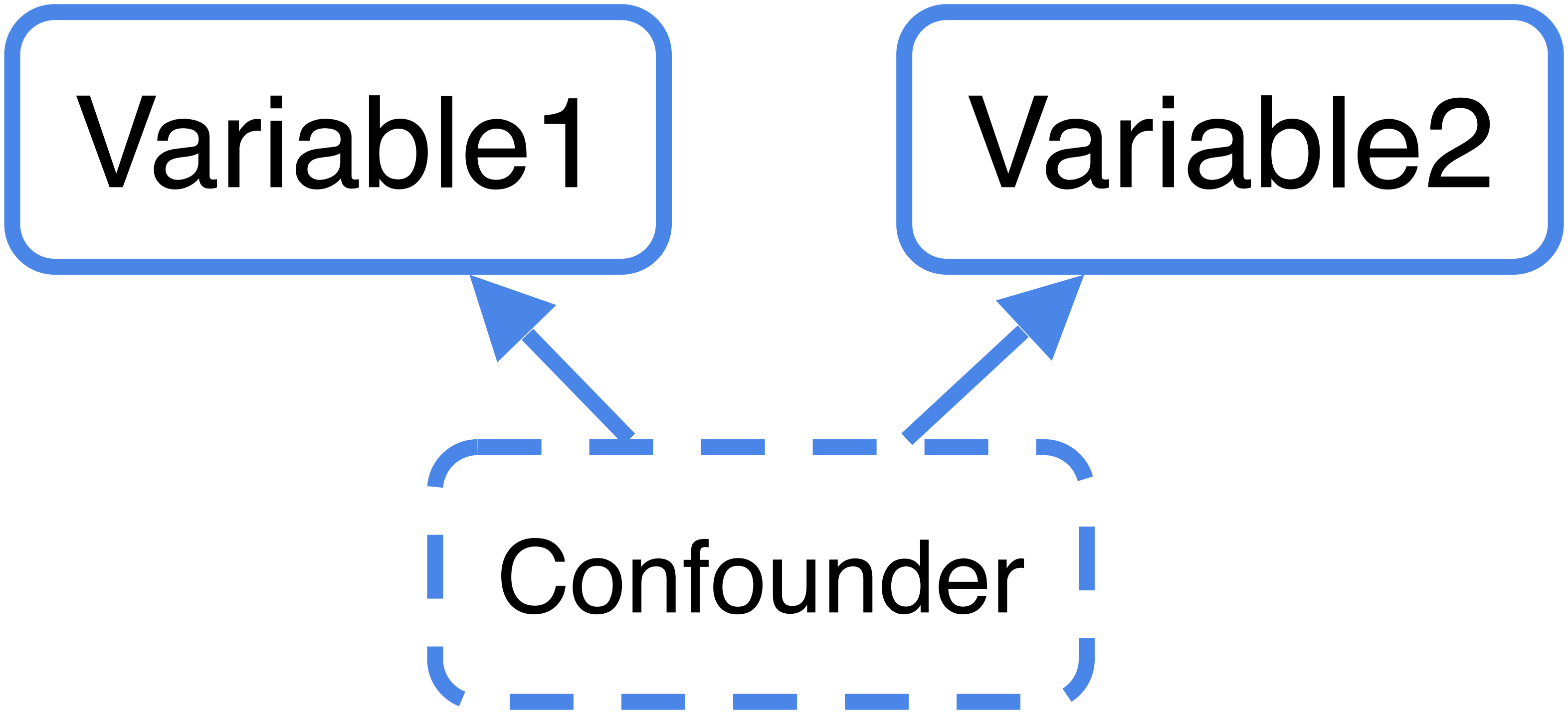
Confounding


Shoe Size
??
Literacy

Small shoes
Not literate
Child

Big shoes
Literate
Adult


Shoe Size
Literacy
Age


Variable1
Variable2
Confounder

Confounding

popsicles → crime rate

Your analysis sees an increase in crime rate whenever popsicle sales increase. What could confound this analysis?

A
popsicle preference

B
new gun laws

C
temperature

D
changes in popsicle prices

E
new law enforcement officers

You can plan ahead to avoid confounding and/or include confounders in your models to account for their role on the outcome variable.

Ignoring confounders will lead you to draw incorrect conclusions

Stratification changes results

Sample: 400 patients with index vertebral fractures

…looks like vertebroplasty was *way* worse for patients!

Vertebroplasty	Conservative care	Relative risk (95% confidence interval)
30/200 (15%)	15/200 (7.5%)	2.0 (1.1–3.6)

subsequent fractures

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3503514/

But wait...at time of initial fracture...

	Vertebroplasty N = 200	Conservative care N = 200
Age, y, mean ± SD	78.2 ± 4.1	79.0 ± 5.2
Weight, kg, mean ± SD	54.4 ± 2.3	53.9 ± 2.1
Smoking status, No. (%)	110 (55)	16 (8)

Age and weight are similar between groups. **Smoking Status** differs vastly.

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3503514/

So...let's stratify those results real quick

Smoke			No smoke		
Vertebroplasty	Conservative	RR (95% confidence interval)	Vertebroplasty	Conservative	RR (95% confidence interval)
23/110 (21%)	3/16 (19%)	1.1 (0.4, 3.3)	7/90 (8%)	12/184(7%)	1.2 (0.5, 2.9)

Risk of re-fracture is now similar within group

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3503514/