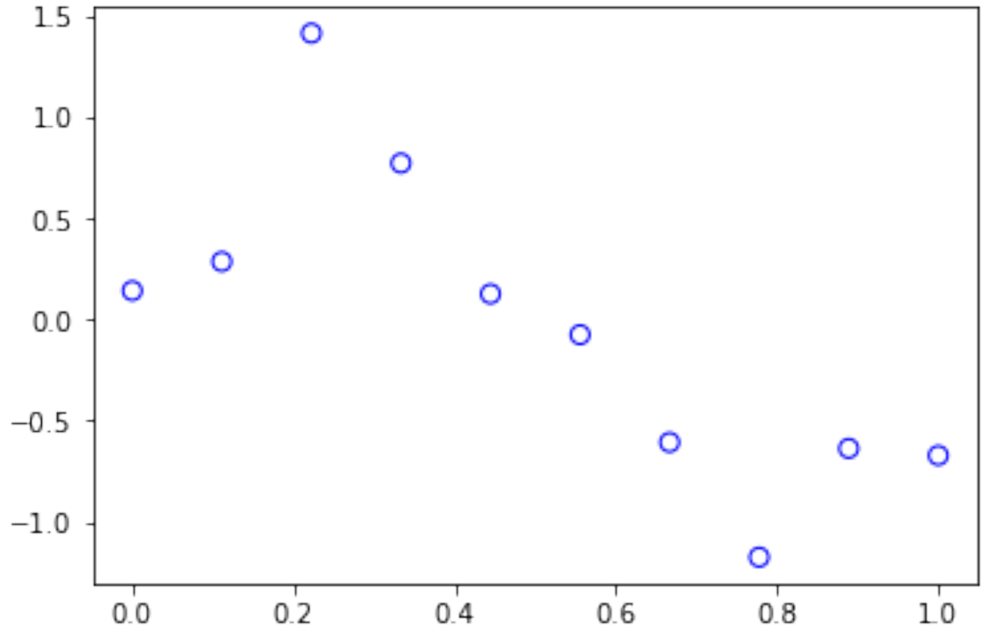
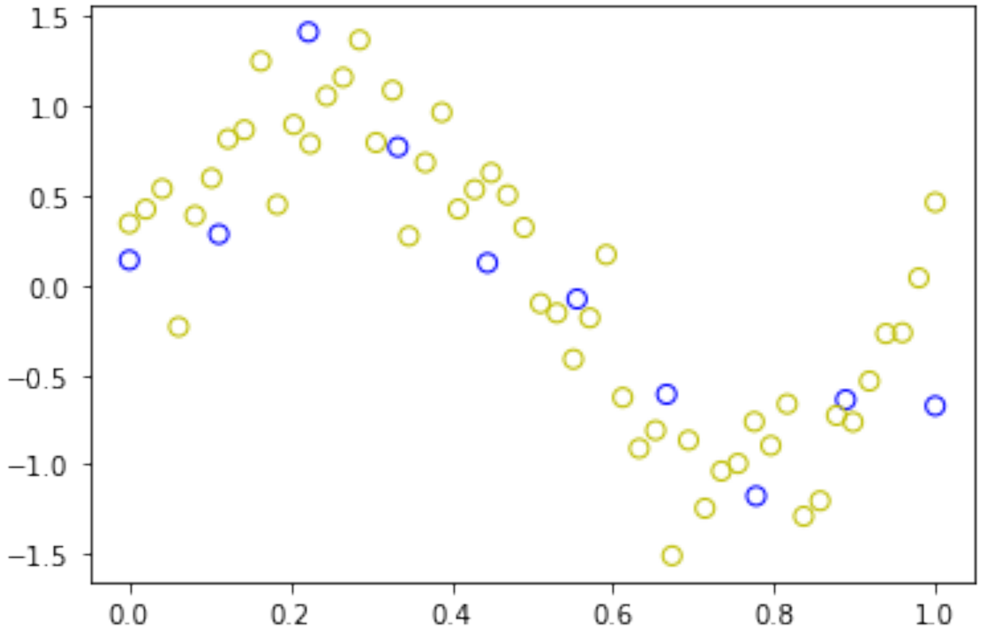
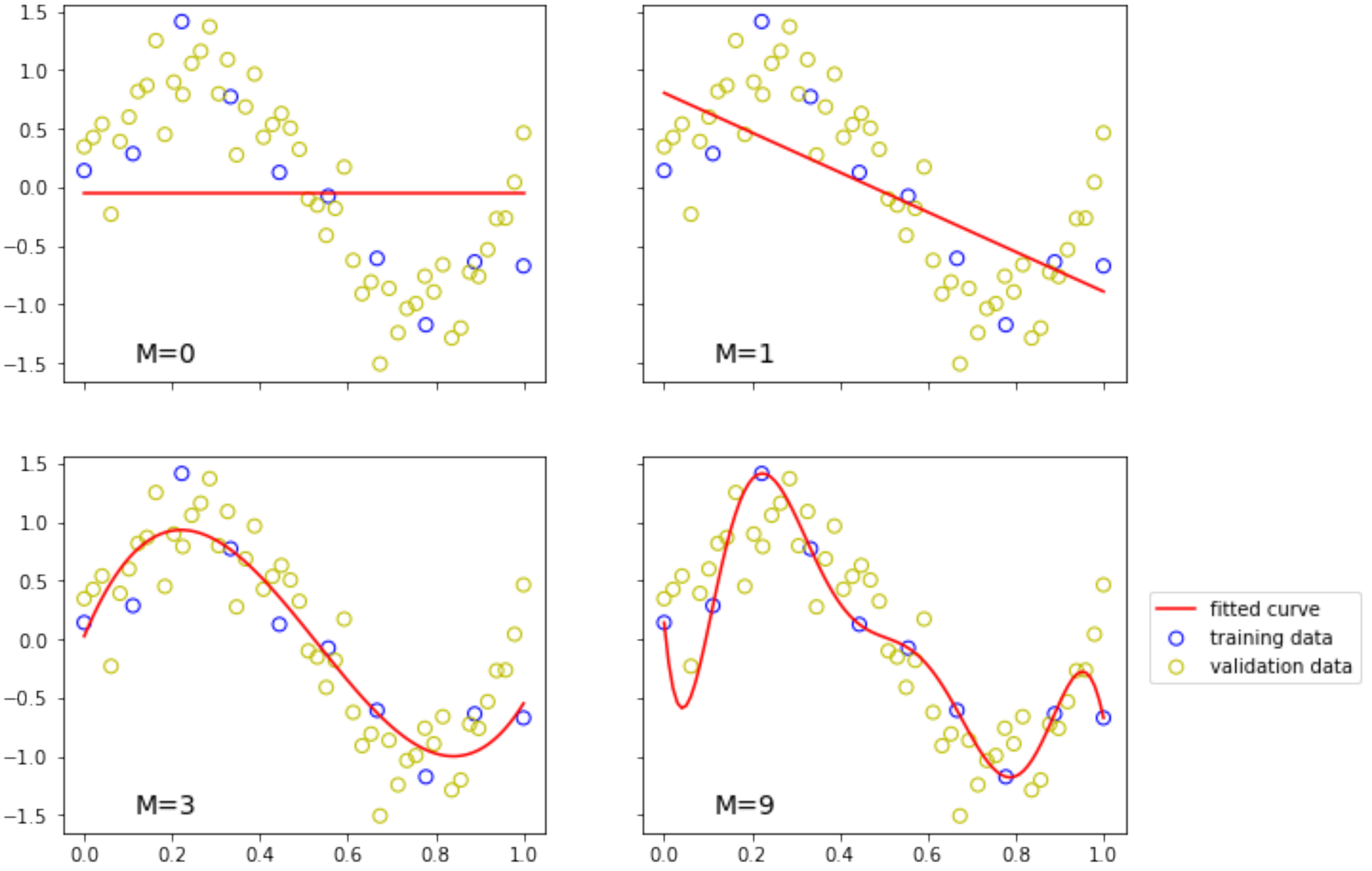
Model selection problems


1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.0
0.2
0.4
0.6
0.8
1.0


M=0
M=1
M=3
M=9
fitted curve
training data
validation data

How can I handle model selection?

- Out of sample methods, e.g.
  - Divide your one dataset into two or three separate pieces: Train / Validation (model selection) / Test sets
  - Select lowest SSE model on validation set
  - Think of the blue dots vs yellow dots in the previous slides
- In sample methods, e.g.
  - Adjusted R-squared
    - R-squared will ALWAYS get better with more parameters
    - Adjusted metric penalizes more parameters for the same level of variance explained

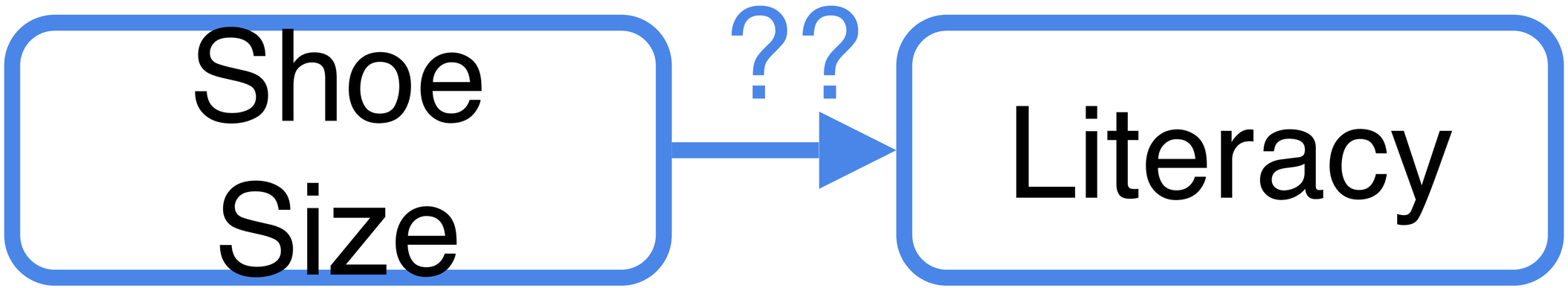
OLS Regression Results

Dep. Variable:	TeenBrth	R-squared:	0.670
Model:	OLS	Adj. R-squared:	0.649
Method:	Least Squares	F-statistic:	31.75
Date:	Mon, 06 May 2024	Prob (F-statistic):	2.29e-11
Time:	09:51:48	Log-Likelihood:	-171.69
No. Observations:	51	AIC:	351.4
Df Residuals:	47	BIC:	359.1
Df Model:	3		
Covariance Type:	nonrobust		

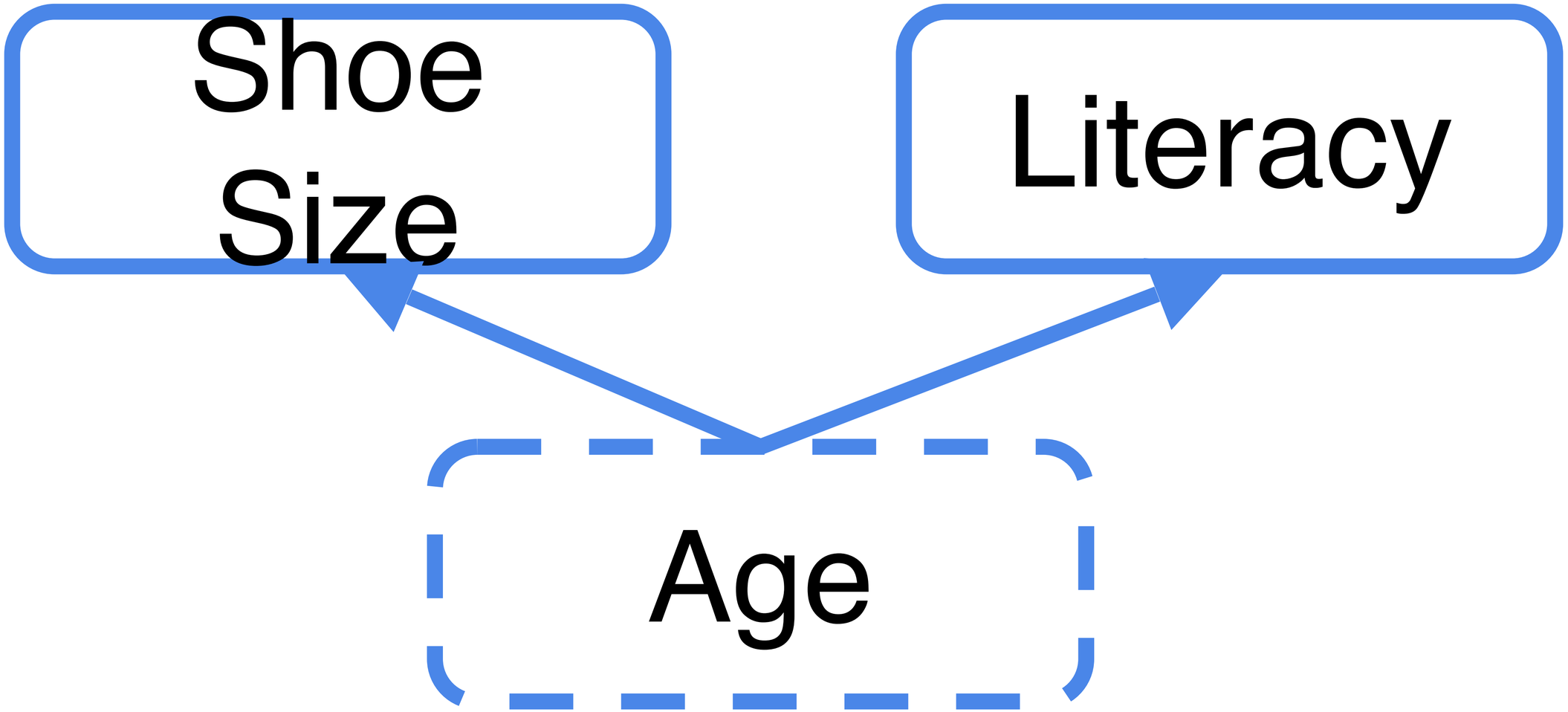
	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	0.8212	5.472	0.150	0.881	-10.186	11.829
PovPct	2.4365	0.342	7.126	0.000	1.749	3.124
ViolCrime	3.4119	0.777	4.391	0.000	1.849	4.975
PovPct:ViolCrime	-0.1438	0.037	-3.928	0.000	-0.217	-0.070

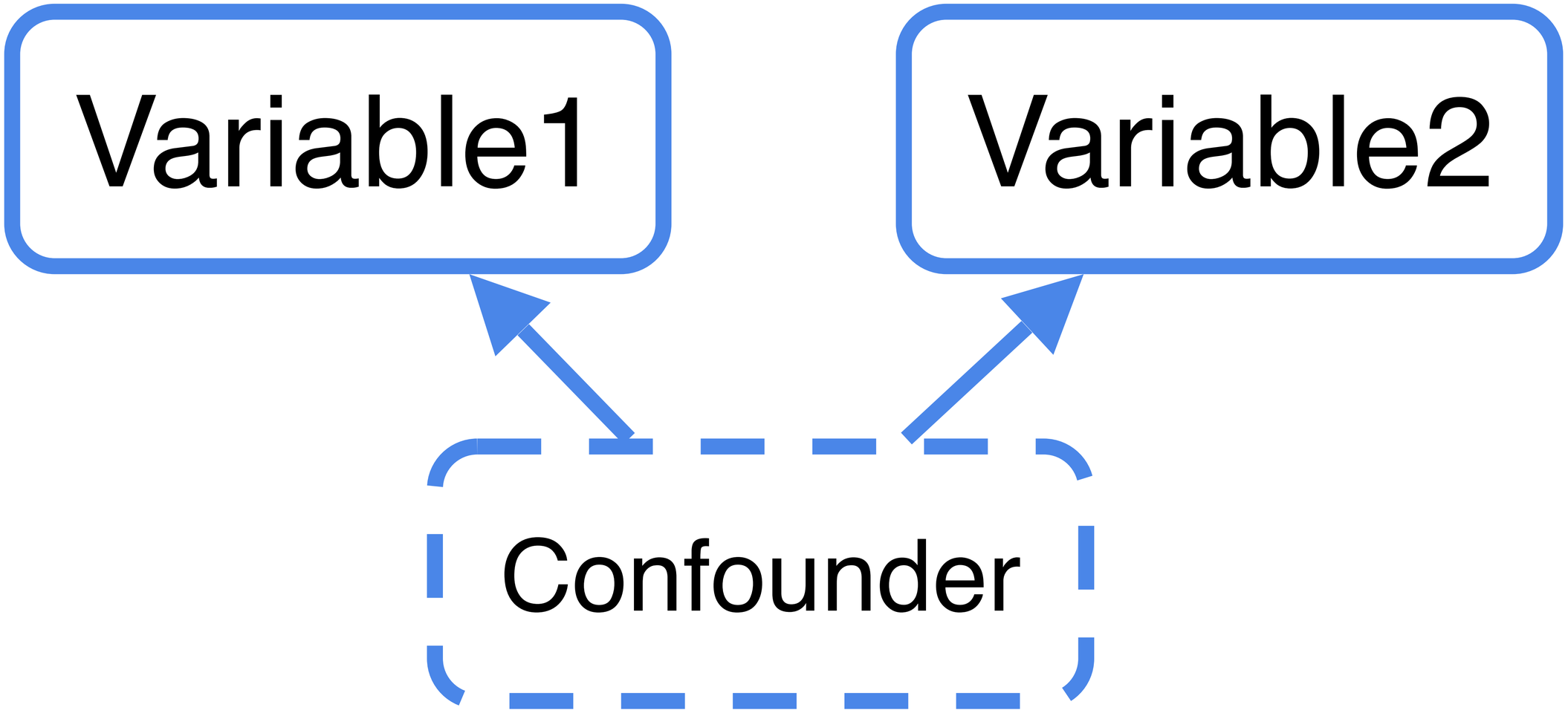
Omnibus:	1.616	Durbin-Watson:	2.081
Prob(Omnibus):	0.446	Jarque-Bera (JB):	1.053
Skew:	-0.345	Prob(JB):	0.591
Kurtosis:	3.141	Cond. No.	1.26e+03

Confounding


Shoe Size
??
Literacy


Small shoes
Not literate
Child
Big shoes
Literate
Adult

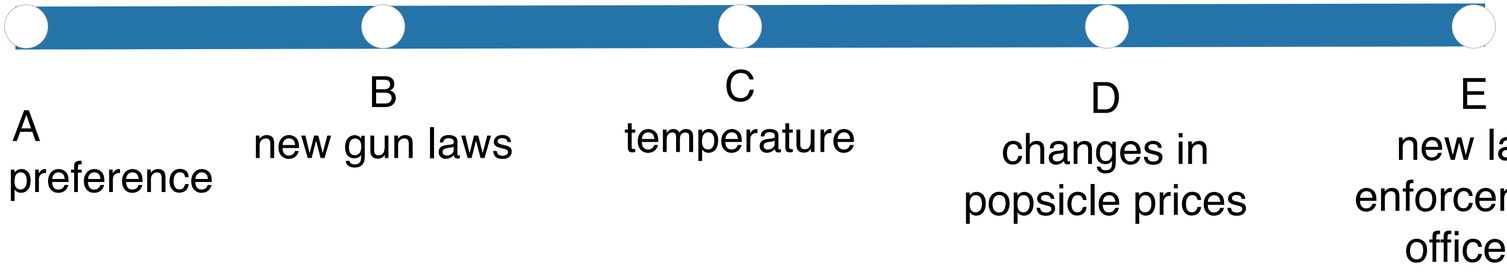

Shoe Size
Literacy
Age


Variable1
Variable2
Confounder

Confounding

popsicles → crime rate

Your analysis sees an increase in crime rate whenever popsicle sales increase. What could confound this analysis?



You can plan ahead to avoid confounding and/or include confounders in your models to account for their role on the outcome variable.

Ignoring confounders will lead you to draw incorrect conclusions

Stratification changes results

Sample: 400 patients with index vertebral fractures

…looks like vertebroplasty was *way* worse for patients!

Vertebroplasty	Conservative care	Relative risk (95% confidence interval)
30/200 (15%)	15/200 (7.5%)	2.0 (1.1–3.6)

subsequent fractures

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3503514/

But wait...at time of initial fracture...

	Vertebroplasty N = 200	**Conservative care N = 200**
Age, y, mean ± SD	78.2 ± 4.1	79.0 ± 5.2
Weight, kg, mean ± SD	54.4 ± 2.3	53.9 ± 2.1
Smoking status, No. (%)	110 (55)	16 (8)

Age and weight are similar between groups. **Smoking Status** differs vastly.

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3503514/

So…let's stratify those results real quick

Smoke			**No smoke**		
Vertebroplasty	Conservative	RR (95% confidence interval)	Vertebroplasty	Conservative	RR (95% confidence interval)
23/110 (21%)	3/16 (19%)	1.1 (0.4, 3.3)	7/90 (8%)	12/184(7%)	1.2 (0.5, 2.9)

Risk of re-fracture is now similar within group

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3503514/

Nonparametric statistics

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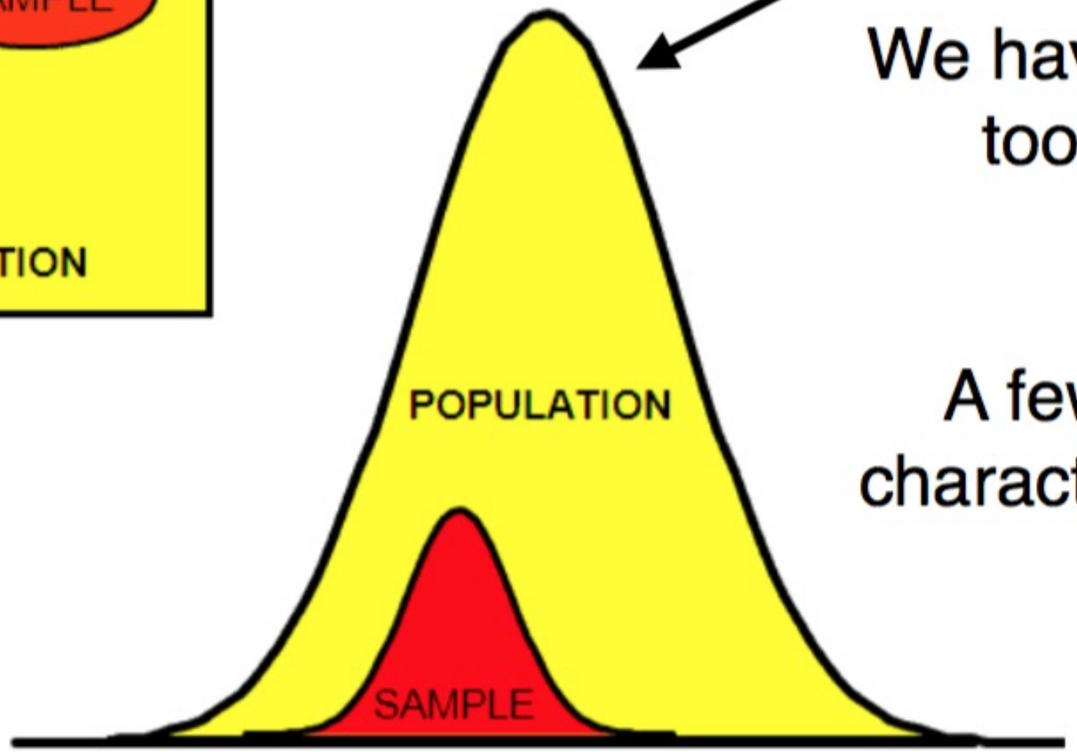
Non-parametric Statistics

Normal distribution
(nice and friendly)

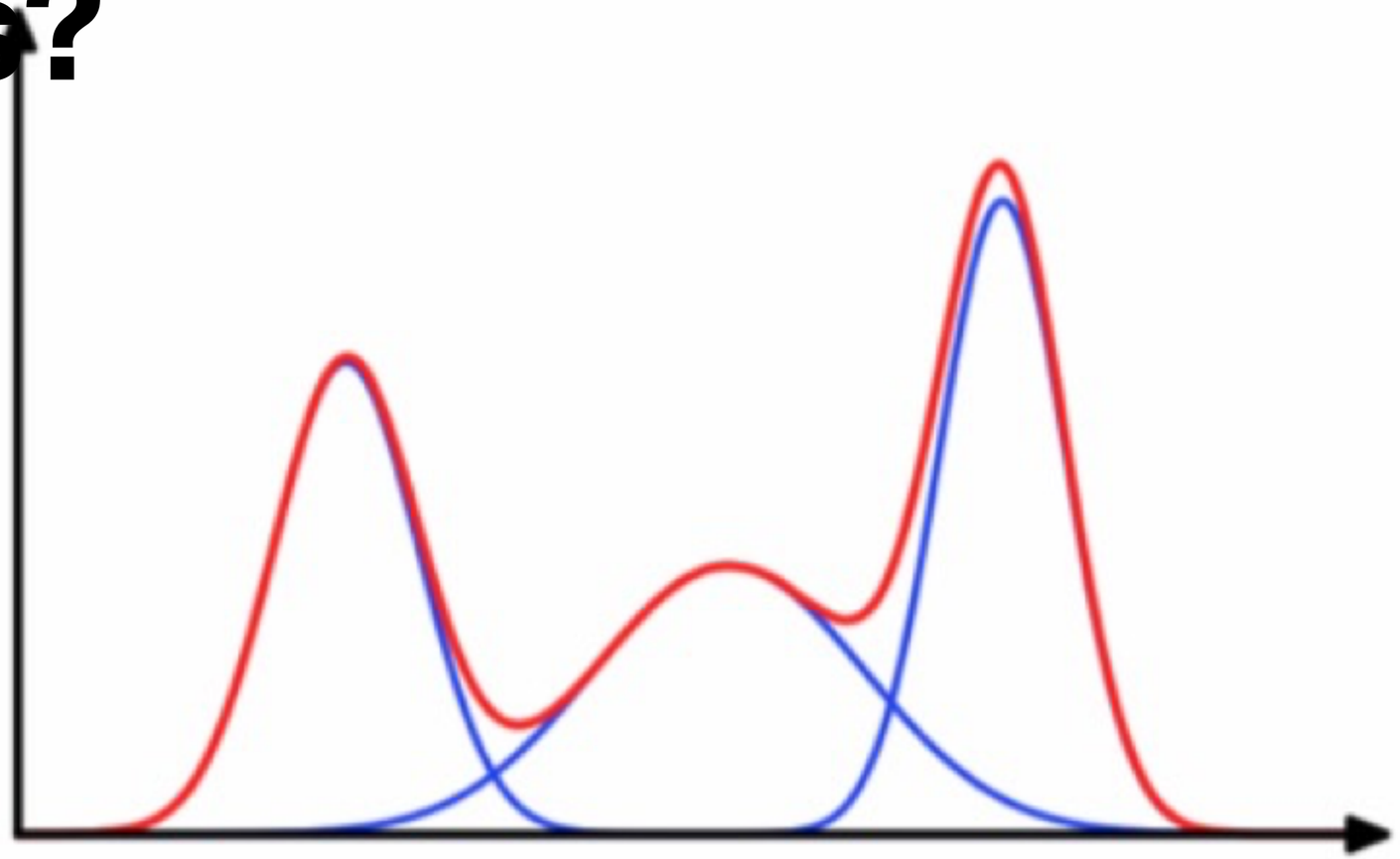
We have good math tools for this.



A few parameters **fully** characterize the distribution.

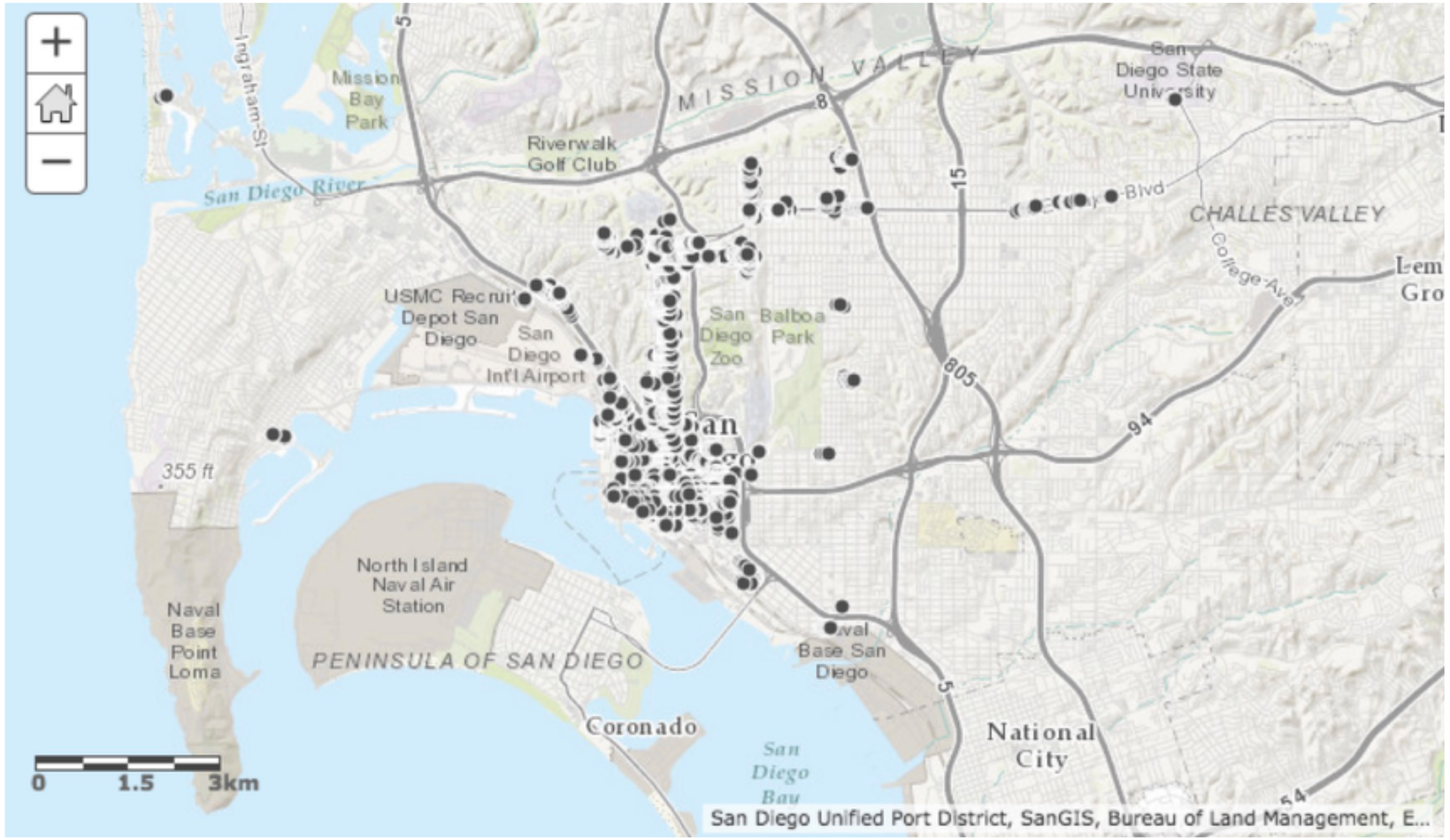
adapted from Brad Voytek

Non-parametric Statistics: What if your distribution looks like this?



adapted from Brad Voytek

Non-parametric Statistics: ...or like this?



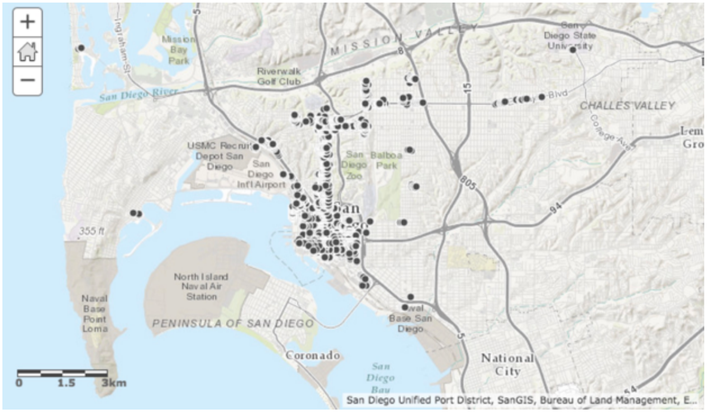
Parameters (like mean and variance) cannot fully and accurately capture this distribution!

Hence, we require **non-parametric statistics**.

adapted from Brad Voytek

When to turn to non-parametric statistics...

When underlying distributions are non-normal, skewed, or cannot be parameterized simply.



Like	Like Somewhat	Neutral	Dislike Somewhat	Dislike
1	2	3	4	5

Non-parametric Statistics: distribution-free

Myth: Non-parametric statistics does not use parameters.
Fact: Non-parametric statistics does not make *assumptions about* / parametrize the underlying distribution generating the data.

“Distribution-Free” statistics

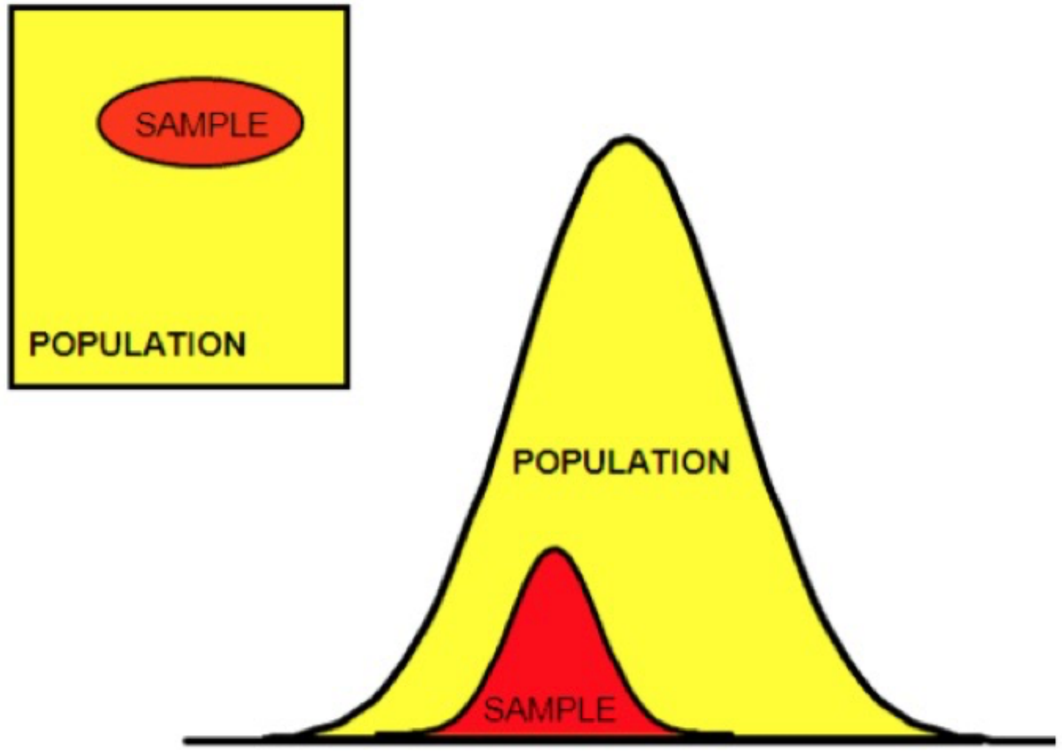
Meaning, it does not assume data-generating process (like heights) result in, *e.g.*, normally-distributed data

Resampling statistics: The What

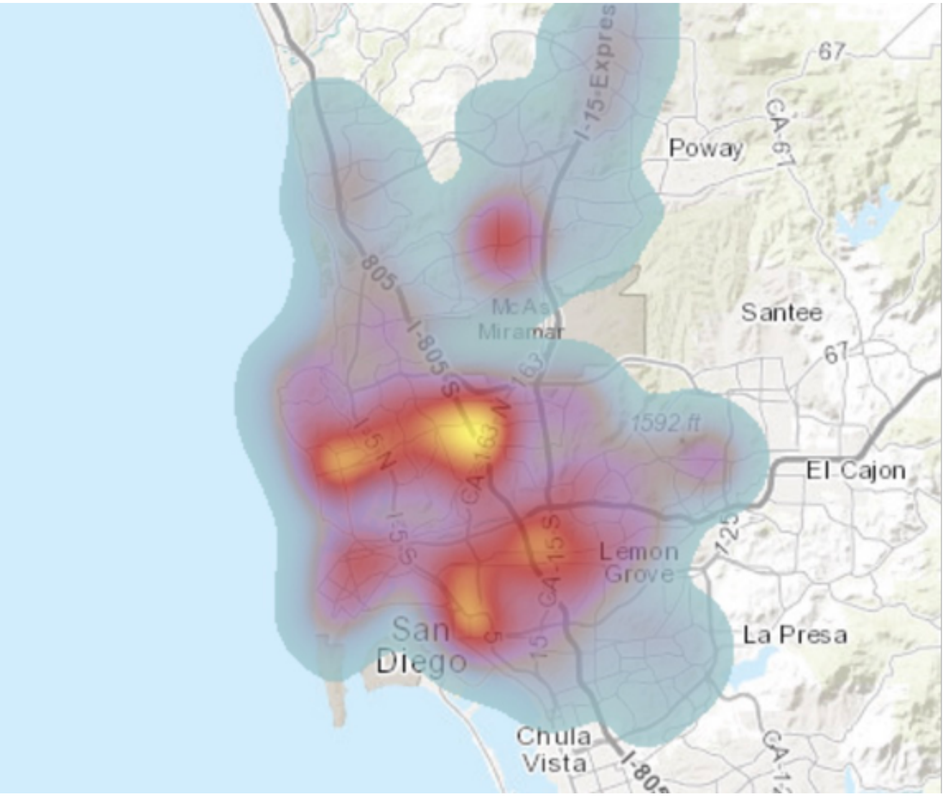
Empirical null distribution (Monte Carlo)
Rank Statistics (Mann Whitney U)
Kolmogorov-Smirnoff Test
Non-parametric prediction models

1) Bootstrapping (resampling)

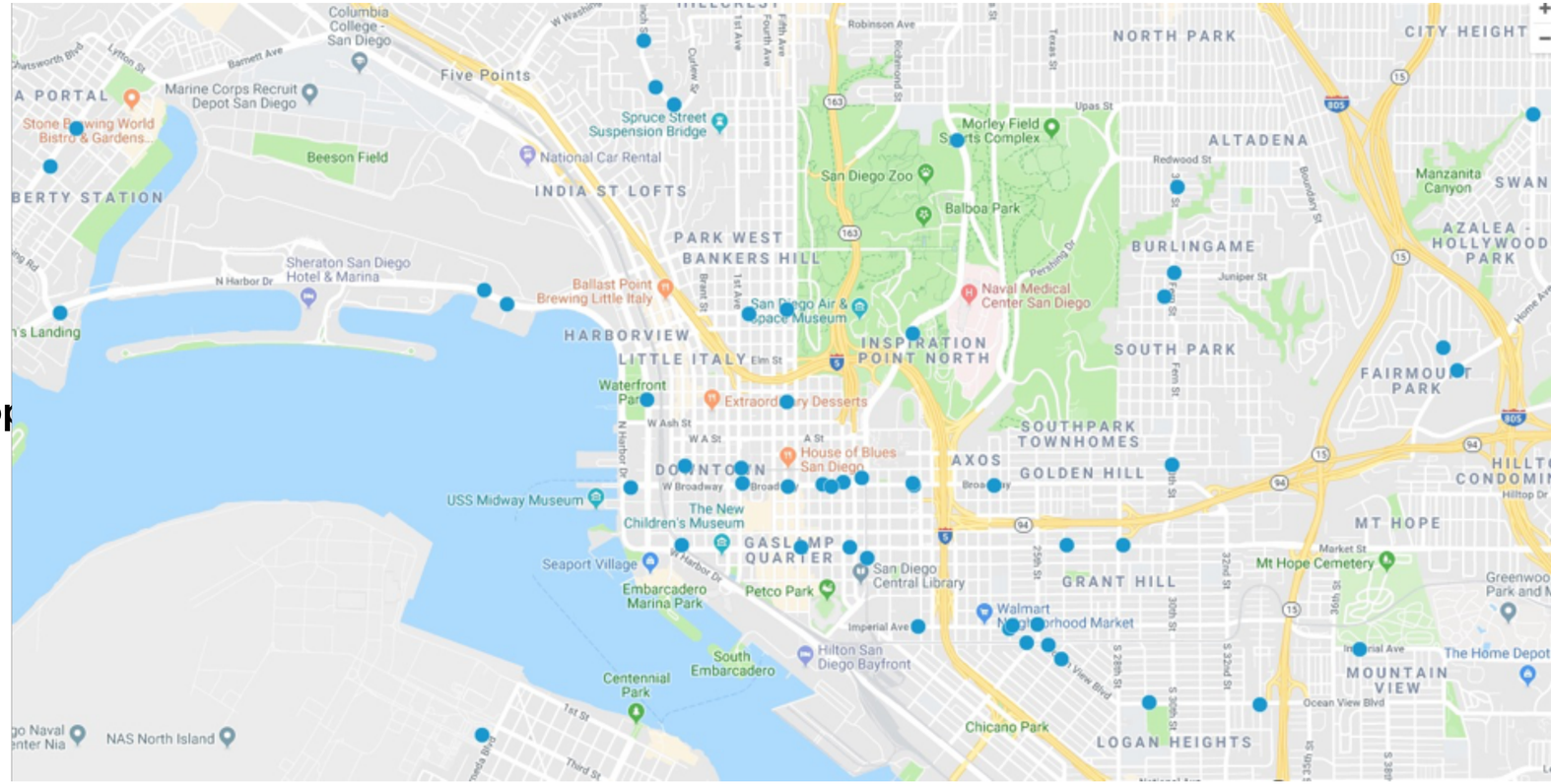
How can we build a more realistic “null distribution” for the sample estimate without knowing the population it’s drawn from?



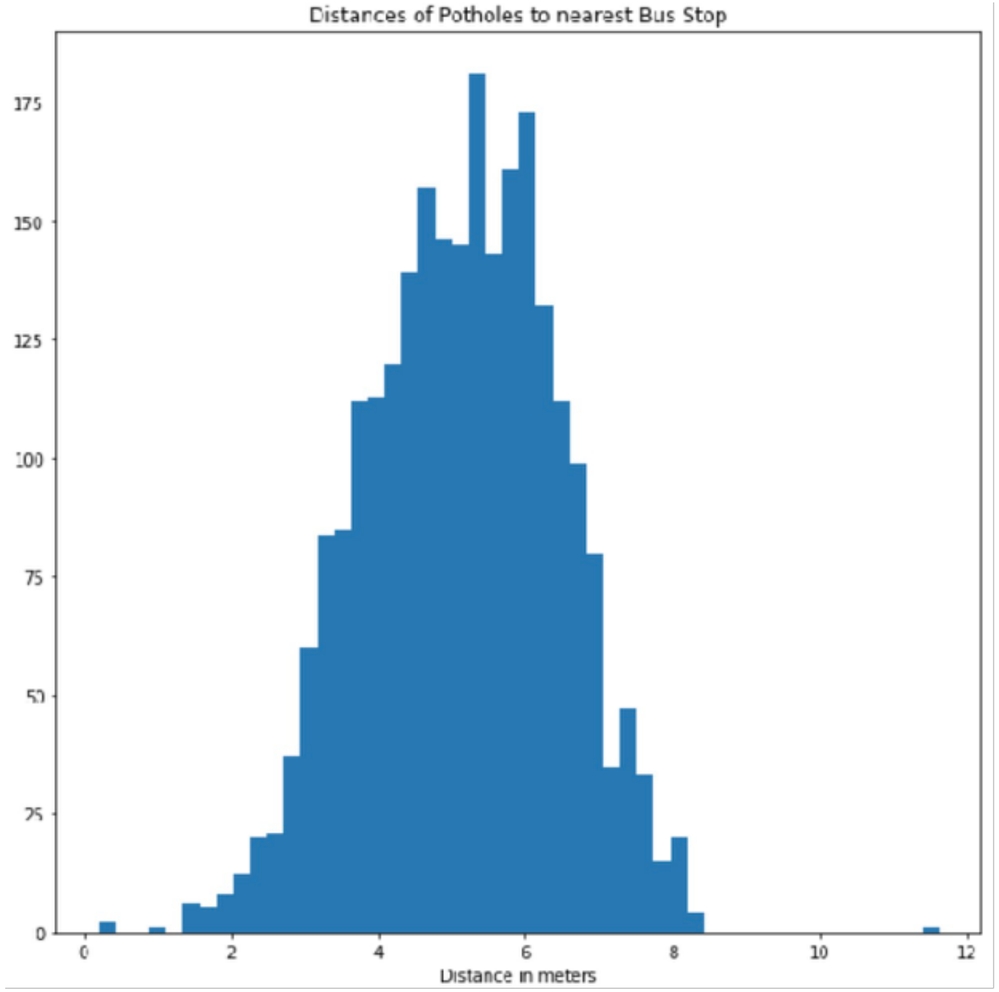
Bootstrapping (resampling)

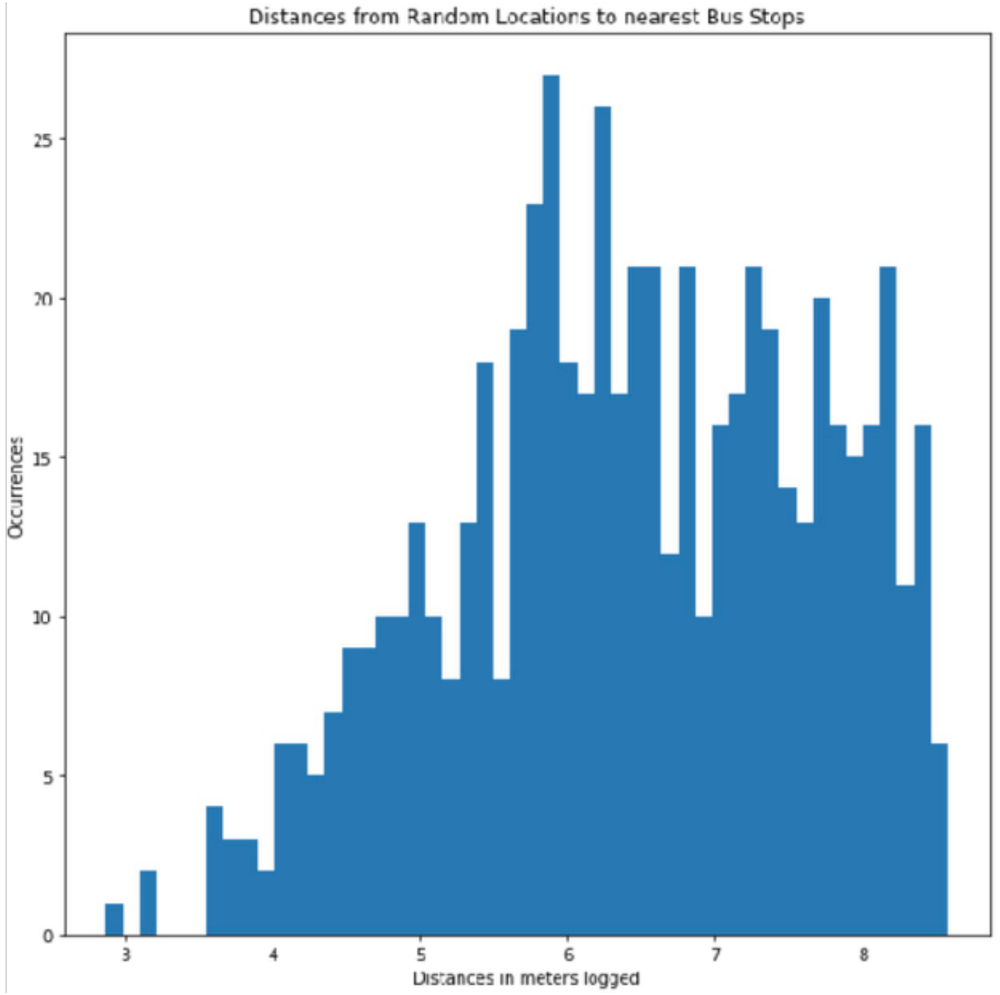


stop



Bootstrapping (resampling)





2) Rank Statistics

We rank things in the real world *all the time!*

International rankings (economics, happiness, government performance)
Sports (teams, players, leagues)
Search Engines
Academic Journals' prestige
Reviews online (1-4 stars)

Rank Statistics

quantitative data		ordinal data
1, 4.5, 6.6, 9.2	→	1, 2, 3, 4

Data are transformed from their quantitative value to their rank.

Ordinal data - categorical, where the variables have a natural order

Particularly helpful when data have a ranking but no clear numerical interpretation (i.e. movie reviews)

Ordinality

Which of the following variables contains ordinal data?

https://forms.gle/UUbbQd9nyz8tgzVh6

Wilcoxon rank-sum test (Mann Whitney U test)

Determine whether two independent samples were selected from the same populations, having the same distribution
Similar to t-test (but does not require normal distributions) & tests median

Assumptions:

Observations in each group are independent of one another
Responses are ordinal

H_o: distributions of both populations are equal
H_a: distributions are *not* equal

Mann-Whitney U: question example

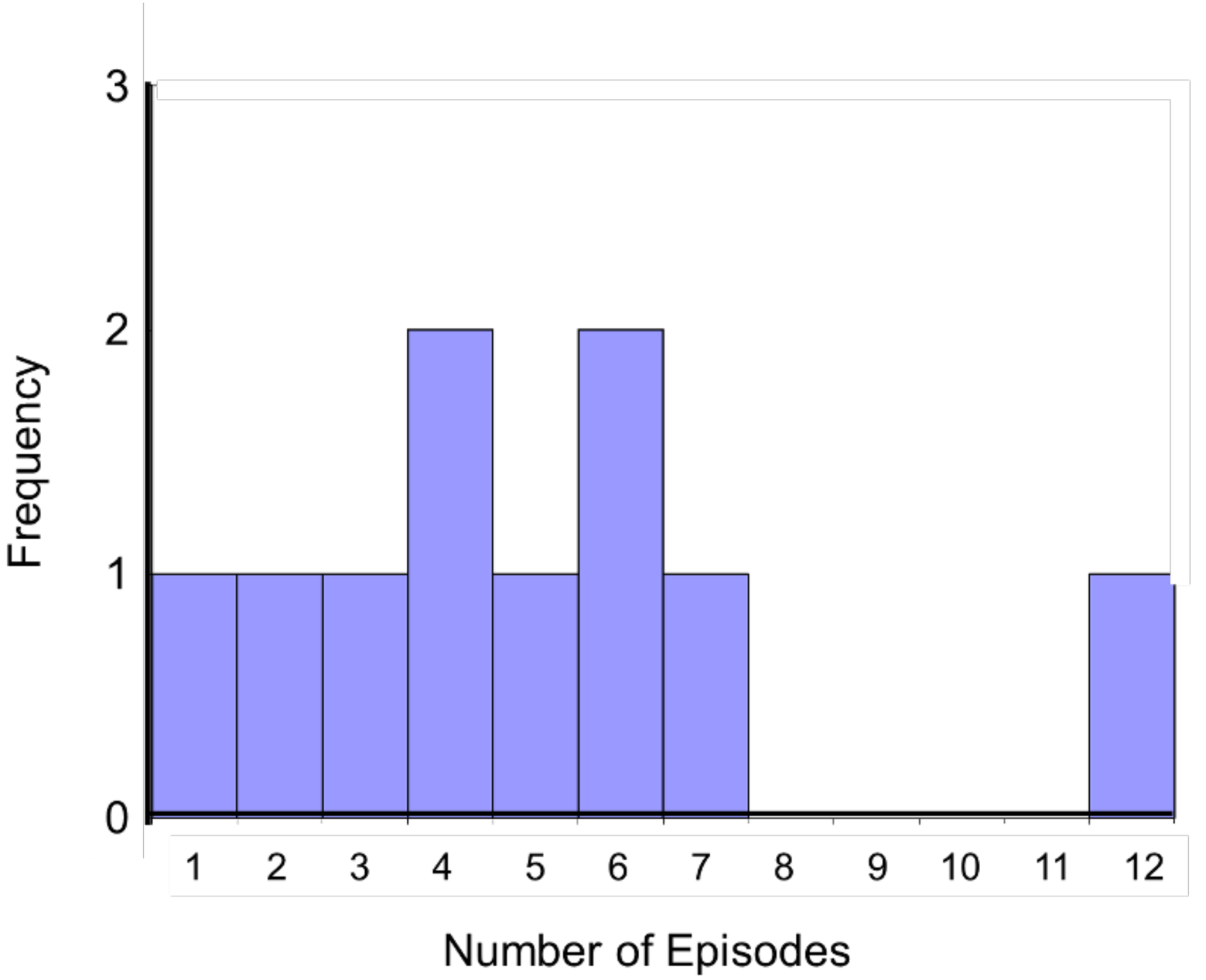
In a clinical trial, is there a difference in the number of episodes of shortness of breath between placebo and treatment?

Step 1: Participants record number of episodes they have.

Step 2: Episodes from both groups are combined, sorted, and ranked

Step 2: Resort the ranks into separate samples (placebo vs. treatment)

Step 3: Carry out statistical test



http://sphweb.bumc.bu.edu/otlt/mph-modules/bs/bs704_nonparametric/BS704_Nonparametric4.html

		Total Sample (Ordered Smallest to Largest)	Ranks
Placebo	New Drug		
7	3		
5	6		
6	4		
4	2		
12	1		

Sum of ranks:
Placebo = 37
New Drug = 18

http://sphweb.bumc.bu.edu/otlt/mph-modules/bs/bs704_

Mann-Whitney *U*

Ho: low and high scores are approximately evenly distributed in the two groups

Ha: low and high scores are NOT evenly distributed in the two groups (U <= 2)

$$U_A = n_a n_b + \frac{n_a(n_a+1)}{2} - T_A$$

The max possible value of TA

The observed sum of ranks for sample A

n_a = number of elements in group A

n_b = number of elements in group B

$U_{Placebo} = 3$

$U_{treatment} = 22$

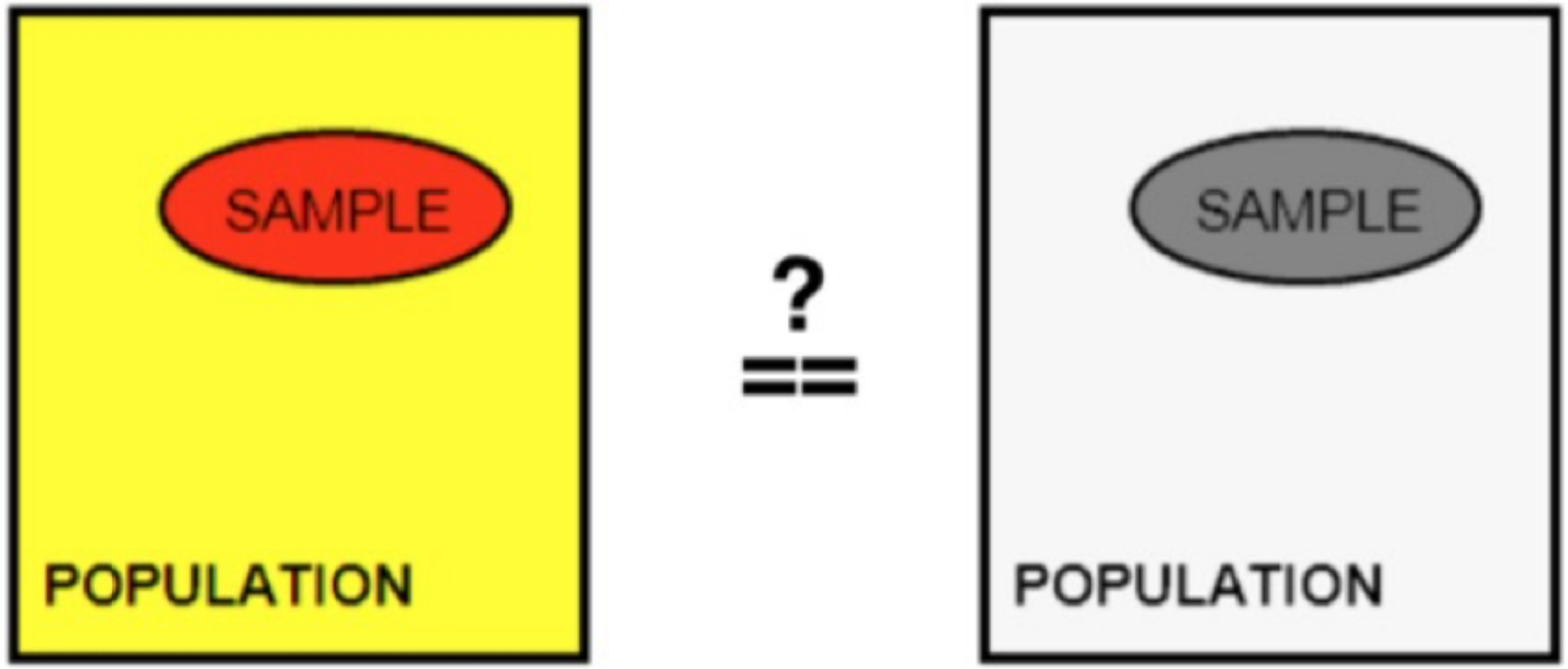
$0 < U < n_1 * n_2$

Complete separation → no separation

We reject the null if U is small.

3) Kolmogorov-Smirnov (KS) test

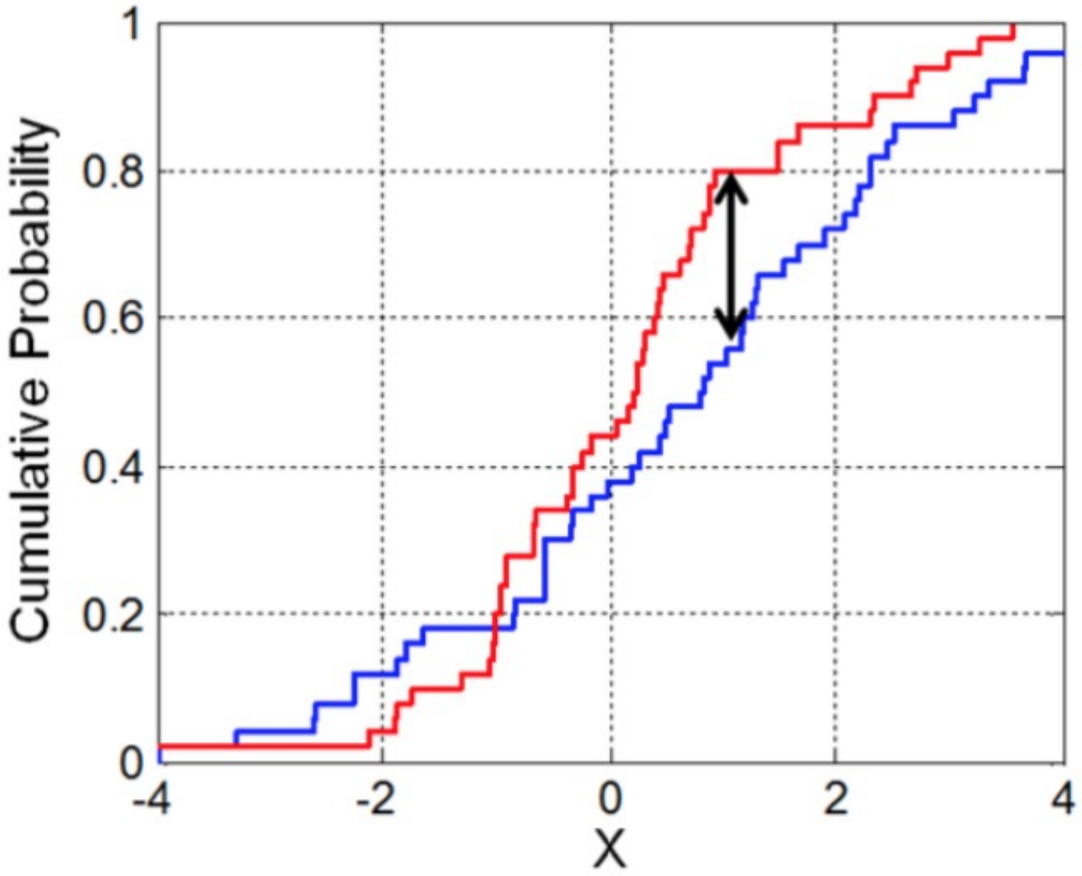
Given (limited) samples from two populations, how do we quantify whether they come from the same distribution?



Kolmogorov-Smirnov (KS) test

Comparing cumulative distributions empirically

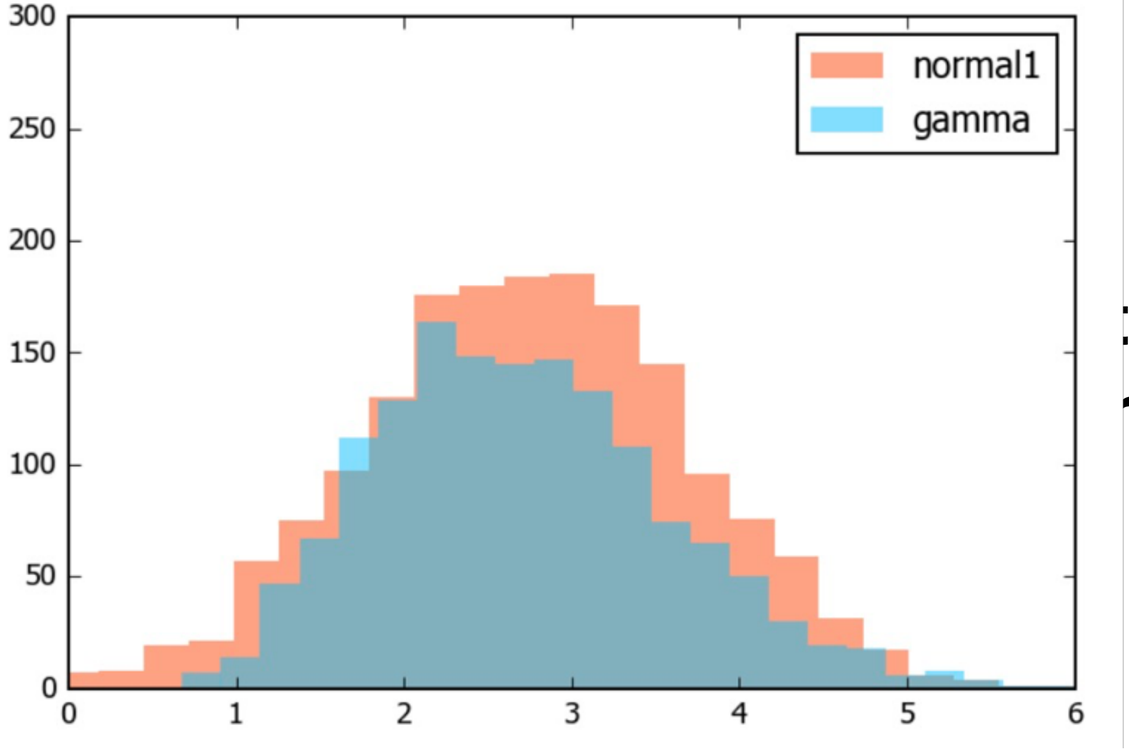
Find the maximum difference between the CDFs.

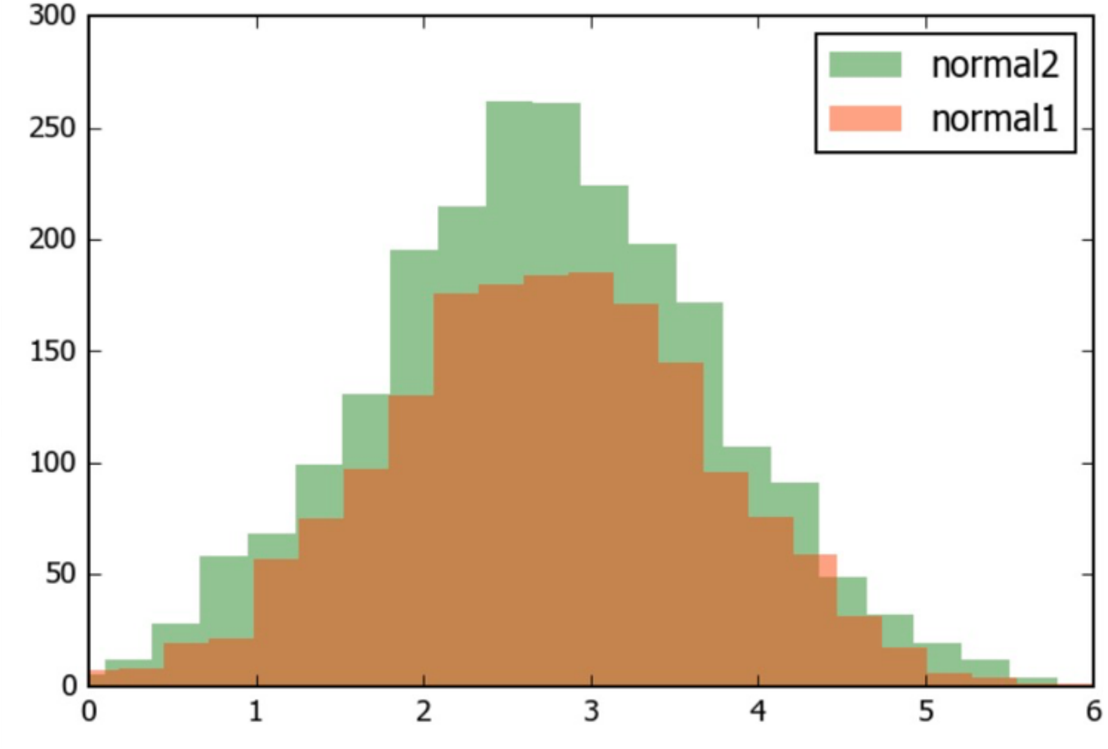


Tests:

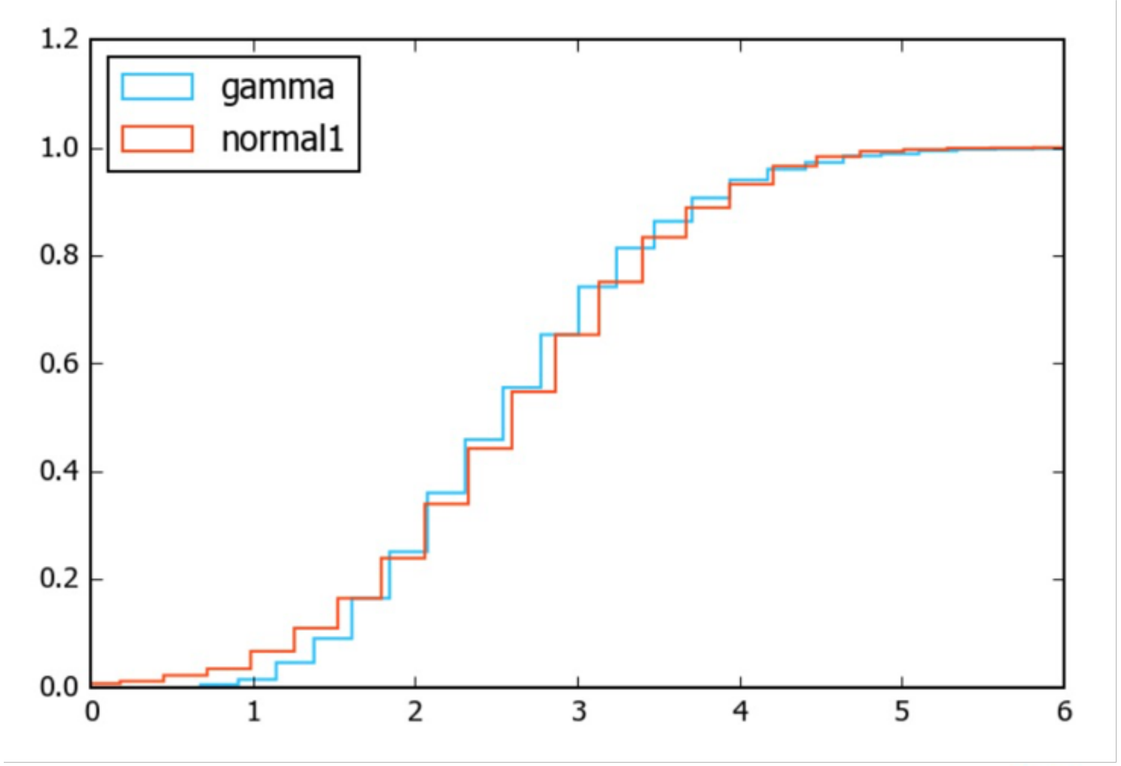
- whether a sample is drawn from a given distribution
- Whether two samples are drawn from the same distribution

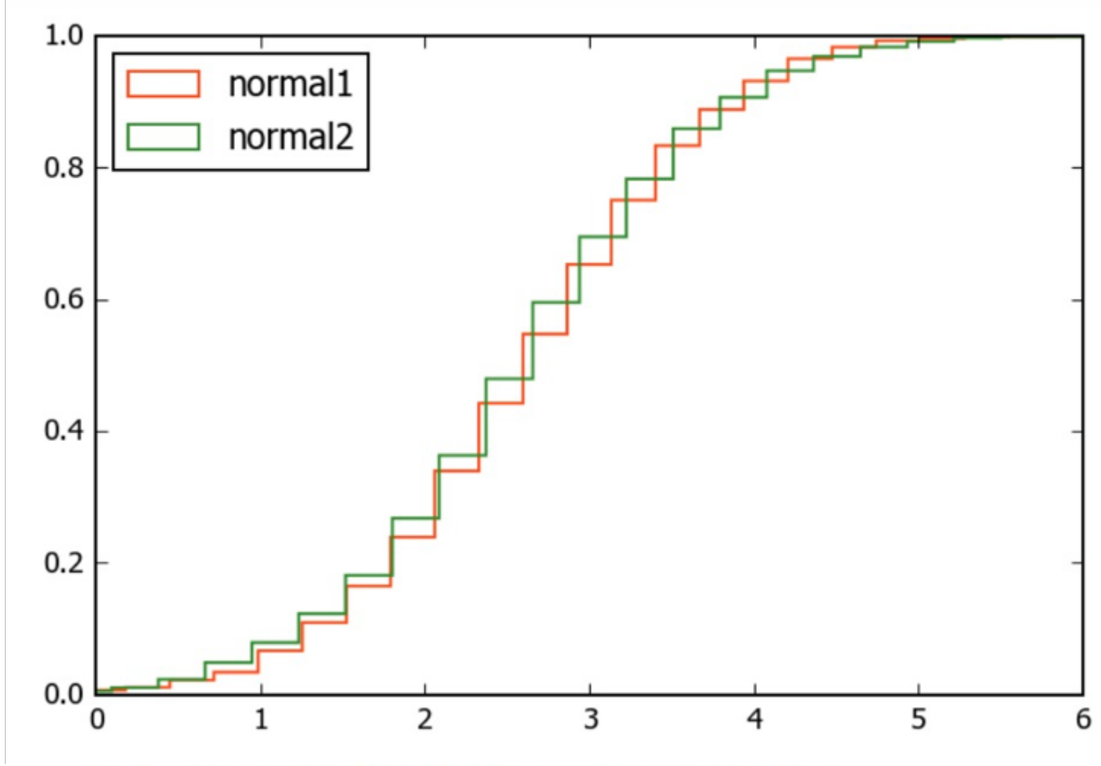
Kolmogorov-Smirnov (KS) test





Kolmogorov-Smirnov (KS) test






```
gamma vs. normal1: p = 0.010680362841 1
normal1 vs. normal2: p = 0.550735998243
```

4) Non-parametric prediction models

When you have lots of data and no prior knowledge
When you're not focused/worried about choosing the right features
Goal: fit training data while being able to generalize to unseen data

<u>Examples</u>:

- **KNN (K-Nearest Neighbors)**
- **Decision Trees and Decision Tree Regressions (CART)**
- **Quantile regression**

Why do we even teach/use parametric statistics anyway?

Parametric approaches:

- **Lots of data follow expected patterns**
- **Require less data**
- **More sensitive**
- **Quicker to run/train/predict**
- **More resistant to overfitting**