Linearly-constrained smooth optimization

DRSOM can be generalized to smooth linearly constrained optimization

$$\begin{array}{cc} \min\limits_{x} & f(x) \\ \text{subject to} & Ax = b \end{array}$$

by projecting directions to null-space

$$P_{A^{\perp}}[x] = (I_n - A^{\top}(AA^{\top})^{-1}A)x$$

and

$$\begin{array}{ccc} d^k & \leftarrow & \alpha^g P_{A^{\perp}}[g^k] + \alpha^m(\boldsymbol{x}^k - \boldsymbol{x}^{k-1}) \\ x^k & \leftarrow & \boldsymbol{x}^k + d^k \end{array}$$

Also possible to add inequality constraint $x \geq 0$.

Application: Linear programming

We can apply the above framework to solve LPs.

Problem	PInfeas	DInfeas.	Compl.	Problem	PInfeas	DInfeas.	Compl.
DLITTLE	1.347e-10	2.308e-10	2.960e-09	KB2	5.455e-11	6.417e-10	7.562e-11
AFIRO	7.641e-11	7.375e-11	3.130e-10	LOTFI	2.164e-09	4.155e-09	8.663e-08
AGG2	3.374e-08	4.859e-08	6.286e-07	MODSZK1	1.527e-06	5.415e-05	2.597e-04
AGG3	2.248e-05	1.151e-06	1.518e-05	RECIPELP	5.868e-08	6.300e-08	1.285e-07
BANDM	2.444e-09	4.886e-09	3.769e-08	SC105	7.315e-11	5.970e-11	2.435e-10
BEACONFD	5.765e-12	9.853e-12	1.022e-10	SC205	6.392e-11	5.710e-11	2.650e-10
BLEND	2.018e-10	3.729e-10	1.179e-09	SC50A	1.078e-05	6.098e-06	4.279e-05
BOEING2	1.144e-07	1.110e-08	2.307e-07	SC50B	4.647e-11	3.269e-11	1.747e-10
BORE3D	2.389e-08	5.013e-08	1.165e-07	SCAGR25	1.048e-07	5.298e-08	1.289e-06
BRANDY	2.702e-05	7.818e-06	1.849e-05	SCAGR7	1.087e-07	1.173e-08	2.601e-07
CAPRI	7.575e-05	4.488e-05	4.880e-05	SCFXM1	4.323e-06	5.244e-06	8.681e-06
E226	2.656e-06	4.742e-06	2.512e-05	SCORPION	1.674e-09	1.892e-09	1.737e-08
FINNIS	8.577e-07	8.367e-07	1.001e-05	SCTAP1	5.567e-07	8.430e-07	5.081e-06

- Solving LPs to 10^{-6} accuracy
- Matrix-free and scalable to dimension
- Applicable to other types of problems