Dimension-reduced Interior Point Method

Discussion 8

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Potential reduction

- Working on C transformation
- The warm-start Lanczos improves by 20% in speed

The solver is designed for solving general problem

$$\begin{array}{rl} \min\limits_{\mathbf{x}} & f(\mathbf{x}) \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$

with smooth convex $f(\mathbf{x})$ via potential reduction

$$\phi(\mathbf{x}) \ := \ \rho \log(f(\mathbf{x}) - z) + \sum_{i=1}^{n} \log(x_i)$$

- A general framework exploiting curvature in potential reduction
- HSD embedding stands for $\mathbf{A} = \mathbf{e}^\top, \mathbf{b} = 1$ and $f(\mathbf{x}) = \frac{1}{2}\|\hat{\mathbf{A}}\mathbf{x}\|^2$

One useful observation

Recall that

$$\lambda_{\min}(\mathbf{X}\nabla^2\phi(\mathbf{x})\mathbf{X}) \leq \frac{-2\rho}{\|\mathbf{X}^{-1}(\mathbf{x}^* - \mathbf{x})\|^2} + 1.$$

- If $x_i \rightarrow 0$ while $x_i^* - x_i \neq 0$, then the curvature is harder to find

  In other words, centrality matters when exploiting the curvature

  $x_i + \alpha d_i \rightarrow 0^+$ makes next curvature hard to detect

- In practice, we now let line-search go less aggressively to ensure centrality

  ρ is also adjusted to balance centrality and optimality

HDSDP

Now integrating HDSDP into the next COPT release