On the Practical Implementataion of a First-order Potential Reduction Algorithm for Linear Programming

Huikang Liu Wenzhi Gao Yinyu Ye

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Abstract

In this report, we present the detailed implementation details for a first-order potential reduction algorithm for linear programming (LP) problems. The algorithm applies dimension-reduced method to reduce the potential function defined over the well-known homogeneous self-dual model for LP and leverages the negative curvature of the potential function to accelerate convergence. A complete recipe on algorithm design and implementation is depicted in this report and some preliminary experiment results are given.

1 Introduction

1.1 First-order Potential Reduction Method for LP

In this report, we are interested in a first-order method for the standard LP problems.
Standard Primal-dual LP

$$\begin{array}{rc} \min\limits_{\mathbf{x}\in\mathbb{R}^n} & \mathbf{c}^\top\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x}=\mathbf{b} \\ & \mathbf{x}\geq\mathbf{0} \end{array}$$

$$\begin{array}{rc} \max\limits_{\mathbf{y}\in\mathbb{R}^m} & \mathbf{b}^\top\mathbf{y} \\ \text{subject to} & \mathbf{A}^\top\mathbf{y}+\mathbf{s}=\mathbf{c} \\ & \mathbf{s}\geq\mathbf{0} \end{array}$$

It is well-known that LPs admit a homogeneous self-dual (HSD) model

$$\begin{array}{rcl} \mathbf{A}\mathbf{x}-\mathbf{b}\tau & = & \mathbf{0} \\ -\mathbf{A}^\top\mathbf{y}-\mathbf{s}+\mathbf{c}\tau & = & \mathbf{0} \\ \mathbf{b}^\top\mathbf{y}-\mathbf{c}^\top\mathbf{x}-\kappa & = & 0 \\ (\mathbf{x},\mathbf{s},\kappa,\tau) & \geq & \mathbf{0}, \end{array}$$

where the two homogenizing variables κ, τ are introduced for infeasibility detection [2]. The first-order potential reduction method, initially proposed by [3], encodes the above HSD model into the following simplex-constrained QP

$$\begin{array}{cc} \min\limits_{(\mathbf{x},\mathbf{y},\mathbf{s},\kappa,\tau)} & \frac{1}{2}\left\|\mathbf{r}\left(\mathbf{x},\mathbf{y},\mathbf{s},\kappa,\tau\right)\right\|^2 \\ \text{subject to} & \mathbf{e}_n^\top\mathbf{x}+\mathbf{e}_n^\top\mathbf{s}+\kappa+\tau=1, \end{array}$$

where

$$\mathbf{r}\left(\mathbf{x},\mathbf{y},\mathbf{s},\kappa,\tau\right) \quad := \quad \left(\begin{array}{ccccc} \mathbf{0}_{m\times m} & \mathbf{A} & \mathbf{0}_{m\times n} & \mathbf{0}_{m\times 1} & -\mathbf{b} \\ -\mathbf{A}^\top & \mathbf{0}_{n\times n} & -\mathbf{I}_{n\times n} & \mathbf{0}_{n\times 1} & \mathbf{c} \\ \mathbf{b}^\top & -\mathbf{c}^\top & \mathbf{0}_{1\times n} & -1 & 0 \end{array}\right)\left(\begin{array}{c} \mathbf{y} \\ \mathbf{x} \\ \mathbf{s} \\ \kappa \\ \tau \end{array}\right)$$

and for brevity, we simplify the notation by re-defining $\mathbf{A}$ and $\mathbf{x}$ and consider the formulation below

$$\begin{array}{cc} \min\limits_{\mathbf{x}} & \frac{1}{2}\left\|\mathbf{A}\mathbf{x}\right\|^2 \quad =: f\left(\mathbf{x}\right) \\ \text{subject to} & \mathbf{e}^\top\mathbf{x}=1 \\ & \mathbf{x}\geq\mathbf{0}. \end{array}$$

Given the re-formulation, the first-order potential reduction method adopts the potential function

$$\phi\left(\mathbf{x}\right) := \rho\log\left(f\left(\mathbf{x}\right)\right)-\sum_{i=1}^{n}\log x_i$$

and applies a conditional gradient method to drive ϕ to $-\infty$. More detailedly, the gradient of ϕ is given by

$$\nabla\phi\left(\mathbf{x}\right)=\frac{\rho\nabla f\left(\mathbf{x}\right)}{f\left(\mathbf{x}\right)}-\mathbf{X}^{-1}\mathbf{e}.$$

At each iteration, we evaluate the gradient $\nabla\phi\left(\mathbf{x}^k\right)$, let $\Delta^k := \mathbf{x}^{k+1}-\mathbf{x}^k$ and solve following subproblem

$$\begin{array}{cc} \min\limits_{\Delta} & \left\langle\nabla\phi\left(\mathbf{x}^k\right),\Delta\right\rangle \\ \text{subject to} & \mathbf{e}^\top\Delta^k=0 \\ & \left\|\left(\mathbf{X}^k\right)^{-1}\Delta^k\right\|\leq\beta \end{array}$$

to update $\mathbf{x}^{k+1}\leftarrow\mathbf{x}^k+\Delta^k$. In the next section, we extend the basic potential reduction framwork by incorporating momentum term from the Dimension-reduced method proposed in [4].

1.2 Dimension-reduced Potential Reduction

In this section, we consider two direction extension of the potential reduction framework. In a word, by keeping track of one recent history iterate, we update

$$\begin{array}{rcl} \mathbf{d}^k & \leftarrow & \alpha^g\mathbf{P}_\Delta\left[\nabla\phi\left(\mathbf{x}^k\right)\right]+\alpha^m\left(\mathbf{x}^k-\mathbf{x}^{k-1}\right) \\ \mathbf{x}^k & \leftarrow & \mathbf{x}^k+\mathbf{d}^k \end{array}$$

where $\mathbf{P}_\Delta[\cdot]$ is the orthogonal projection onto null space of the simplex constraint $\mathbf{e}^\top \mathbf{x} = 0$. Since we leverage the dimension-reduced method, α^g, α^d are evaluated through the following model

$$\begin{array}{cc} \min\limits_{\mathbf{d},\alpha^g,\alpha^m} & \frac{1}{2}\mathbf{d}^\top \nabla^2 \phi\left(\mathbf{x}\right)\mathbf{d} + \nabla \phi\left(\mathbf{x}\right)^\top \mathbf{d} \\ \text{subject to} & \left\|\mathbf{X}^{-1}\mathbf{d}\right\| \leq \Delta \\ & \mathbf{d} = \alpha^g \mathbf{g}^k + \alpha^m \mathbf{m}^k \end{array}$$

where $\mathbf{g}^k := \mathbf{P}_\Delta\left[\nabla \phi\left(\mathbf{x}^k\right)\right]$, $\mathbf{m}^k := \mathbf{x}^k - \mathbf{x}^{k-1}$. If we define

$$\begin{aligned} \widetilde{\mathbf{H}} &:= \begin{pmatrix} \left\langle \mathbf{g}^k, \nabla^2\phi\left(\mathbf{x}^k\right)\mathbf{g}^k \right\rangle & \left\langle \mathbf{g}^k, \nabla^2\phi\left(\mathbf{x}^k\right)\mathbf{m}^k \right\rangle \\ \left\langle \mathbf{m}^k, \nabla^2\phi\left(\mathbf{x}^k\right)\mathbf{g}^k \right\rangle & \left\langle \mathbf{m}^k, \nabla^2\phi\left(\mathbf{x}^k\right)\mathbf{m}^k \right\rangle \end{pmatrix} \\ \widetilde{\mathbf{h}} &:= \begin{pmatrix} \left\|\mathbf{g}^k\right\|^2 \\ \left\langle \mathbf{g}^k, \mathbf{m}^k \right\rangle \end{pmatrix} \\ \mathbf{M} &:= \begin{pmatrix} \left\|\left(\mathbf{X}^k\right)^{-1}\mathbf{g}^k\right\|^2 & \left\langle \mathbf{g}^k, \left(\mathbf{X}^k\right)^{-2}\mathbf{m}^k \right\rangle \\ \left\langle \mathbf{m}^k, \left(\mathbf{X}^k\right)^{-2}\mathbf{g}^k \right\rangle & \left\|\left(\mathbf{X}^k\right)^{-1}\mathbf{m}^k\right\|^2 \end{pmatrix}, \end{aligned}$$

the above model simplies into a two-dimensional QCQP.

$$\begin{array}{cc} \min\limits_{\alpha} & \frac{1}{2}\alpha^\top \widetilde{\mathbf{H}}\alpha + \widetilde{\mathbf{h}}\alpha \quad =: m(\alpha) \\ \text{subject to} & \|\mathbf{M}\alpha\| \leq \Delta \end{array}$$

Note that $\nabla^2\phi\left(\mathbf{x}^k\right) = -\frac{\rho \nabla f\left(\mathbf{x}^k\right)\nabla f\left(\mathbf{x}^k\right)^\top}{f\left(\mathbf{x}^k\right)^2} + \rho\frac{\mathbf{A}^\top\mathbf{A}}{f\left(\mathbf{x}^k\right)} + \left(\mathbf{X}^k\right)^{-2}$ and we evaluate the above relations via

$$\begin{aligned} \left\langle \mathbf{a}, \nabla^2\phi\left(\mathbf{x}^k\right)\mathbf{a} \right\rangle &= \left\langle \mathbf{a}, -\frac{\rho\nabla f\left(\mathbf{x}^k\right)\nabla f\left(\mathbf{x}^k\right)^\top \mathbf{a}}{f\left(\mathbf{x}^k\right)^2} \right\rangle + \frac{\|\mathbf{A}\mathbf{a}\|^2}{f\left(\mathbf{x}^k\right)} + \left\|\left(\mathbf{X}^k\right)^{-1}\mathbf{a}\right\|^2 \\ &= -\rho\left(\frac{\nabla f\left(\mathbf{x}^k\right)^\top \mathbf{a}}{f\left(\mathbf{x}^k\right)}\right)^2 + \frac{\|\mathbf{A}\mathbf{a}\|^2}{f\left(\mathbf{x}^k\right)} + \left\|\left(\mathbf{X}^k\right)^{-1}\mathbf{a}\right\|^2 \\ \left\langle \mathbf{a}, \nabla^2\phi\left(\mathbf{x}^k\right)\mathbf{b} \right\rangle &= \left\langle \mathbf{a}, -\frac{\rho\nabla f\left(\mathbf{x}^k\right)\nabla f\left(\mathbf{x}^k\right)^\top \mathbf{b}}{f\left(\mathbf{x}^k\right)^2} \right\rangle + \frac{\langle \mathbf{A}\mathbf{a}, \mathbf{A}\mathbf{b}\rangle}{f\left(\mathbf{x}^k\right)} + \left\langle \mathbf{a}, \left(\mathbf{X}^k\right)^{-2}\mathbf{b} \right\rangle \\ &= -\rho\left(\frac{\nabla f\left(\mathbf{x}^k\right)^\top \mathbf{a}}{f\left(\mathbf{x}^k\right)}\right)\left(\frac{\nabla f\left(\mathbf{x}^k\right)^\top \mathbf{b}}{f\left(\mathbf{x}^k\right)}\right) + \frac{\langle \mathbf{A}\mathbf{a}, \mathbf{A}\mathbf{b}\rangle}{f\left(\mathbf{x}^k\right)} + \left\langle \mathbf{a}, \left(\mathbf{X}^k\right)^{-2}\mathbf{b} \right\rangle. \end{aligned}$$

To ensure feasibility, we always choose $\Delta \leq 1$ and adjust it based on the trust-region rule.

2 Accelerating the Dimension-reduced Potential Reduction

In this section, we summarize several techniques applied to improve the potential reduction method.

2.1 Scaling

As is often observed in the first-order type methods, proper scaling accelerates the performance of the algorithm. In practice, we scale

$$\mathbf{b} \leftarrow \frac{\mathbf{b}}{\|\mathbf{b}\|_1 + 1} \qquad \mathbf{c} \leftarrow \frac{\mathbf{c}}{\|\mathbf{c}\|_1 + 1}$$

and then apply Ruiz scaling [1] to $\begin{pmatrix} \mathbf{0}_{m\times m} & \mathbf{A} & \mathbf{0}_{m\times n} & \mathbf{0}_{m\times 1} & -\mathbf{b} \\ -\mathbf{A}^\top & \mathbf{0}_{n\times n} & -\mathbf{I}_{n\times n} & \mathbf{0}_{n\times 1} & \mathbf{c} \\ \mathbf{b}^\top & -\mathbf{c}^\top & \mathbf{0}_{1\times n} & -1 & 0 \end{pmatrix}$ to improve conditioning of the matrix.

2.2 Line-search

When a direction is assembled from the trust-region subproblem, instead of directly updating

$$\mathbf{x}^{k+1} \leftarrow \mathbf{x}^k + \mathbf{d}^k,$$

we allow a more aggressive exploitation of the direction by performing a line-search over

$$\phi\left(\mathbf{x} + \alpha\mathbf{d}\right), \alpha \in \left[0, 0.9\alpha_{\max}\right),$$

where $\alpha_{\max} = \max\left\{\alpha \geq 0, \mathbf{x} + \alpha\mathbf{d} \geq \mathbf{0}\right\}$. The line-search sometimes help accelerate convergence when the algorithm approaches optimality.

2.3 Escaping the Local Optimum

One most important accleration trick is to introduce the negative curvature as a search direction. Since potential function is nonconvex in nature, it's quite common that the algorithm stagates at a local solution. To help escape such local optimum, we make use of the negative curvature of $\nabla^2\phi\left(\mathbf{x}\right)$. In our case this can be done by finding the (minimal) negative eigenvalue and eigenvector

$$\lambda_{\min}\left\{\nabla^2\phi\left(\mathbf{x}\right) = \frac{2\rho\mathbf{A}^\top\mathbf{A}}{\|\mathbf{A}\mathbf{x}\|^2} - \frac{4\rho\mathbf{A}^\top\mathbf{A}\mathbf{x}\mathbf{x}^\top\mathbf{A}^\top\mathbf{A}}{\|\mathbf{A}\mathbf{x}\|^4} + \mathbf{X}^{-2}\right\}$$

and we wish to solve the eigen-problem

$$\begin{array}{ll} \min\limits_{\|\mathbf{v}\|=1} & \mathbf{v}^\top\left\{\frac{2\rho\mathbf{A}^\top\mathbf{A}}{\|\mathbf{A}\mathbf{x}\|^2} - \frac{4\rho\mathbf{A}^\top\mathbf{A}\mathbf{x}\mathbf{x}^\top\mathbf{A}^\top\mathbf{A}}{\|\mathbf{A}\mathbf{x}\|^4} + \mathbf{X}^{-2}\right\}\mathbf{v} \\ \text{subject to} & \mathbf{e}^\top\mathbf{v} = 0. \end{array}$$

In general there are two ways to compute a valid direction. The first method approaches the problem directly and uses Lanczos iteration to find the negative eigen-value of $\nabla^2\phi$. As for the second approach, we apply the scaling matrix $\mathbf{X}$ and solve

$$\begin{array}{ll} \min\limits_{\|\mathbf{X}\mathbf{v}\|=1} & \mathbf{v}^\top\left\{\frac{2\rho\mathbf{X}\mathbf{A}^\top\mathbf{A}\mathbf{X}}{\|\mathbf{A}\mathbf{x}\|^2} - \frac{4\rho\mathbf{X}\mathbf{A}^\top\mathbf{A}\mathbf{x}\mathbf{x}^\top\mathbf{A}^\top\mathbf{A}\mathbf{X}}{\|\mathbf{A}\mathbf{u}\|^4} + \mathbf{I}\right\}\mathbf{v} \\ \text{subject to} & \mathbf{x}^\top\mathbf{v} = 0. \end{array}$$

Since we are to find any negative curvature, it is safe to replace $\|\mathbf{X}\mathbf{v}\| = 1$ by $\|\mathbf{v}\| = 1$ and arrive at

$$\begin{array}{ll}\min\limits_{\|\mathbf{v}\|=1} & \mathbf{v}^\top \left\{\frac{2\rho\mathbf{X}\mathbf{A}^\top\mathbf{A}\mathbf{X}}{\|\mathbf{A}\mathbf{x}\|^2} - \frac{4\rho\mathbf{X}\mathbf{A}^\top\mathbf{A}\mathbf{x}\mathbf{x}^\top\mathbf{A}^\top\mathbf{A}\mathbf{X}}{\|\mathbf{A}\mathbf{u}\|^4} + \mathbf{I}\right\}\mathbf{v} \\ \text{subject to} & \mathbf{x}^\top\mathbf{v} = 0.\end{array}$$

Another useful technique when evaluating the curvature is to reduce the support of the curvature. Since it's likely that $v_j,\ j \in \{i : x_i \to 0\}$ will contribute a lot in the negative curvature, we can restrict the support of $\mathbf{v}$ to $\{i : x_i \geq \varepsilon\}$ for some $\varepsilon > 0$.

3 Algorithm Design

In this section, we discuss the design of the potential-reduction based solver.

3.1 Abstract Function Class

To allow further extension, we design the solver to solve general problem

$$\begin{array}{rc}\min\limits_{\mathbf{x}} & f(\mathbf{x}) \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}\end{array}$$

using potential reduction

$$\phi(\mathbf{x}) \quad := \quad \rho\log(f(\mathbf{x}) - z) + \log\sum_{i=1}^{n} x_i$$

over Null space of $\mathbf{A}$. To drive the method to work, the following methods are provided by f.

- Gradient evaluation $\nabla f(\mathbf{x})$
- Hessian vector product $\nabla^2 f(\mathbf{x})\mathbf{u}$
- Progress monitor (Optional)

The potential reduction framework requries $\mathbf{A}$ and maintains $\mathbf{x}, z, \rho$ to run

- Potential gradient evaluation

$$\nabla\phi(\mathbf{x}) = \frac{\rho}{f(\mathbf{x}) - z}\nabla f(\mathbf{x}) - \mathbf{X}^{-1}\mathbf{e}$$

- Potential (scaled) Hessian-vector product

$$\begin{aligned}\nabla^2\phi(\mathbf{x}) &= -\frac{\rho\nabla f(\mathbf{x})\nabla f(\mathbf{x})^\top}{(f(\mathbf{x}) - z)^2} + \frac{\nabla^2 f(\mathbf{x})}{f(\mathbf{x}) - z} + \mathbf{X}^{-2} \\ \mathbf{X}\nabla^2\phi(\mathbf{x})\mathbf{X}\mathbf{u} &= -\frac{\rho\mathbf{X}\nabla f(\mathbf{x})\nabla f(\mathbf{x})^\top\mathbf{X}\mathbf{u}}{(f(\mathbf{x}) - z)^2} + \frac{\mathbf{X}\nabla^2 f(\mathbf{x})\mathbf{X}\mathbf{u}}{f(\mathbf{x}) - z} + \mathbf{u}\end{aligned}$$

- (Scaled) projection onto Null space

$$\left(\mathbf{I} - \mathbf{A}^\top \left(\mathbf{A}\mathbf{A}^\top\right)^{-1} \mathbf{A}\right) \mathbf{u}$$

$$\left(\mathbf{I} - \mathbf{X}\mathbf{A}^\top \left(\mathbf{A}\mathbf{X}^2\mathbf{A}^\top\right)^{-1} \mathbf{A}\mathbf{X}\right) \mathbf{u}$$

- Scaled projected gradient and negative curvature
- Trust-region subproblem

$$\begin{array}{rl} \min\limits_{\alpha} & \frac{1}{2}\alpha^\top \mathbf{H}\alpha + \mathbf{g}^\top \alpha \\ \text{subject to} & \alpha^\top \mathbf{G}\alpha \leq \beta \end{array}$$

- Heuristic routines

  Line search, Curvature frequency, lower bound update

3.2 Numerical Operations

In this section, we introduce how to implement the numerical operations from the potential reduction method. Here we define $\widetilde{\mathbf{A}} := \begin{pmatrix} \mathbf{0}_{m\times m} & \mathbf{A} & \mathbf{0}_{m\times n} & \mathbf{0}_{m\times 1} & -\mathbf{b} \\ -\mathbf{A}^\top & \mathbf{0}_{n\times n} & -\mathbf{I}_{n\times n} & \mathbf{0}_{n\times 1} & \mathbf{c} \\ \mathbf{b}^\top & -\mathbf{c}^\top & \mathbf{0}_{1\times n} & -1 & 0 \end{pmatrix}$.

Residual setup

$$\begin{aligned} \mathbf{r}_1 &= \mathbf{A}\mathbf{x} - \mathbf{b}\tau \\ \mathbf{r}_2 &= -\mathbf{A}^\top \mathbf{y} - \mathbf{s} + \mathbf{c}\tau \\ r_3 &= \mathbf{b}^\top \mathbf{y} - \mathbf{c}^\top \mathbf{x} - \kappa. \end{aligned}$$

Objective value

$$f = \frac{1}{2}\left[\|\mathbf{r}_1\|^2 + \|\mathbf{r}_2\|^2 + r_3^2\right]$$

Gradient setup

$$\nabla f = \begin{pmatrix} -\mathbf{A}\mathbf{r}_2 + \mathbf{b}r_3 \\ \mathbf{A}^\top \mathbf{r}_1 - \mathbf{c}r_3 \\ -\mathbf{r}_2 \\ -r_3 \\ -\mathbf{b}^\top \mathbf{r}_1 + \mathbf{c}^\top \mathbf{r}_2 \end{pmatrix}$$

$$\nabla \varphi = \frac{\rho \nabla f}{f} - \begin{pmatrix} \mathbf{X}^{-1}\mathbf{e} \\ \mathbf{0}_m \\ \mathbf{S}^{-1}\mathbf{e} \\ \kappa^{-1} \\ \tau^{-1} \end{pmatrix}$$

Hessian-vector (with projection)

$$\begin{aligned}
\mathbf{u} &= \mathbf{x} - \frac{\mathbf{e}^\top \mathbf{x}}{n} \cdot \mathbf{e} \\
\nabla^2 \phi \mathbf{u} &= -\frac{\rho \left(\nabla f^\top \mathbf{u} \right)}{f^2} \nabla f + \frac{\rho}{f} \widetilde{\mathbf{A}}^\top \left(\widetilde{\mathbf{A}} \mathbf{u} \right) + \begin{pmatrix} \mathbf{X}^{-2} & & & & \\ & \mathbf{0}_{m\times m} & & & \\ & & \mathbf{S}^{-2} & & \\ & & & \kappa^{-2} & \\ & & & & \tau^{-2} \end{pmatrix} \mathbf{u}.
\end{aligned}$$

Lanczos Hessian-vector (with projection)

$$\mathbf{M} := \begin{pmatrix} \mathbf{I}_m & \\ & \mathbf{I}_n - \mathbf{x}\mathbf{x}^\top / \|\mathbf{x}\|^2 \end{pmatrix} \left[\frac{2\rho \mathbf{S} \widetilde{\mathbf{A}}^\top \widetilde{\mathbf{A}} \mathbf{S}}{\left\| \widetilde{\mathbf{A}} \mathbf{u} \right\|^2} - \frac{4\rho \mathbf{S} \widetilde{\mathbf{A}}^\top \widetilde{\mathbf{A}} \mathbf{u} \mathbf{u}^\top \widetilde{\mathbf{A}}^\top \widetilde{\mathbf{A}} \mathbf{S}}{\left\| \widetilde{\mathbf{A}} \mathbf{u} \right\|^4} + \begin{pmatrix} \mathbf{0}_m & \\ & \mathbf{I}_n \end{pmatrix} \right] \begin{pmatrix} \mathbf{I}_m & \\ & \mathbf{I}_n - \mathbf{x}\mathbf{x}^\top / \|\mathbf{x}\|^2 \end{pmatrix}$$

$$\begin{aligned}
\mathbf{x}' &\leftarrow \frac{\mathbf{x}}{\|\mathbf{x}\|} \\
\mathbf{v} &\leftarrow \begin{pmatrix} \mathbf{v}_\mathbf{y} \\ \mathbf{v}_\mathbf{x} - \left(\mathbf{x}'^\top \mathbf{v}_\mathbf{x} \right) \mathbf{x}' \end{pmatrix} \\
\mathbf{u}_1 &\leftarrow \begin{pmatrix} \mathbf{0} \\ \mathbf{v}_\mathbf{x} - \left(\mathbf{x}'^\top \mathbf{v}_\mathbf{x} \right) \mathbf{x}' \end{pmatrix} \\
\mathbf{u}_2 &\leftarrow \begin{pmatrix} \mathbf{I}_m & \\ & \mathbf{I}_n - \mathbf{x}'\mathbf{x}'^\top \end{pmatrix} \begin{pmatrix} \mathbf{I}_m & \\ & \mathbf{X} \end{pmatrix} \widetilde{\mathbf{A}}^\top \widetilde{\mathbf{A}} \begin{pmatrix} \mathbf{I}_m & \\ & \mathbf{X} \end{pmatrix} \mathbf{v} \\
\mathbf{u}_3 &\leftarrow \mathbf{g}^\top \begin{pmatrix} \mathbf{I}_m & \\ & \mathbf{X} \end{pmatrix} \mathbf{v} \begin{pmatrix} \mathbf{I}_m & \\ & \mathbf{I}_n - \mathbf{x}'\mathbf{x}'^\top \end{pmatrix} \begin{pmatrix} \mathbf{I}_m & \\ & \mathbf{X} \end{pmatrix} \mathbf{g} \\
\mathbf{M}\mathbf{v} &\leftarrow \frac{f^2 \mathbf{u}_1 + \rho f \mathbf{u}_2 - \rho \mathbf{u}_3}{\left\| \widetilde{\mathbf{A}} \mathbf{u} \right\|^4}
\end{aligned}$$

4 Numerical Experiments

We provide some preliminary computational results on the NETLIB LP problems. The results are obtained using `MATLAB` after 1000 iterations.

References

[1] Daniel Ruiz. A scaling algorithm to equilibrate both rows and columns norms in matrices. Technical report, CM-P00040415, 2001.

[2] Yinyu Ye. *Interior point algorithms: theory and analysis*. John Wiley & Sons, 2011.

[3] Yinyu Ye. On a first-order potential reduction algorithm for linear programming. 2015.

[4] Chuwen Zhang, Dongdong Ge, Bo Jiang, and Yinyu Ye. Drsom: A dimension reduced second-order method and preliminary analyses. *arXiv preprint arXiv:2208.00208*, 2022.

Problem	PInfeas	DInfeas.	Compl.	Problem	PInfeas	DInfeas.	Compl.
DLITTLE	1.347e-10	2.308e-10	2.960e-09	KB2	5.455e-11	6.417e-10	7.562e-11
AFIRO	7.641e-11	7.375e-11	3.130e-10	LOTFI	2.164e-09	4.155e-09	8.663e-08
AGG2	3.374e-08	4.859e-08	6.286e-07	MODSZK1	1.527e-06	5.415e-05	2.597e-04
AGG3	2.248e-05	1.151e-06	1.518e-05	RECIPELP	5.868e-08	6.300e-08	1.285e-07
BANDM	2.444e-09	4.886e-09	3.769e-08	SC105	7.315e-11	5.970e-11	2.435e-10
BEACONFD	5.765e-12	9.853e-12	1.022e-10	SC205	6.392e-11	5.710e-11	2.650e-10
BLEND	2.018e-10	3.729e-10	1.179e-09	SC50A	1.078e-05	6.098e-06	4.279e-05
BOEING2	1.144e-07	1.110e-08	2.307e-07	SC50B	4.647e-11	3.269e-11	1.747e-10
BORE3D	2.389e-08	5.013e-08	1.165e-07	SCAGR25	1.048e-07	5.298e-08	1.289e-06
BRANDY	2.702e-05	7.818e-06	1.849e-05	SCAGR7	1.087e-07	1.173e-08	2.601e-07
CAPRI	7.575e-05	4.488e-05	4.880e-05	SCFXM1	4.323e-06	5.244e-06	8.681e-06
E226	2.656e-06	4.742e-06	2.512e-05	SCORPION	1.674e-09	1.892e-09	1.737e-08
FINNIS	8.577e-07	8.367e-07	1.001e-05	SCTAP1	5.567e-07	8.430e-07	5.081e-06
FORPLAN	5.874e-07	2.084e-07	4.979e-06	SEBA	2.919e-11	5.729e-11	1.448e-10
GFRD-PNC	4.558e-05	1.052e-05	4.363e-05	SHARE1B	3.367e-07	1.339e-06	3.578e-06
GROW7	1.276e-04	4.906e-06	1.024e-04	SHARE2B	2.142e-04	2.014e-05	6.146e-05
ISRAEL	1.422e-06	1.336e-06	1.404e-05	STAIR	5.549e-04	8.566e-06	2.861e-05
STANDATA	5.645e-08	2.735e-07	5.130e-06	STANDGUB	2.934e-08	1.467e-07	2.753e-06
STOCFOR1	6.633e-09	9.701e-09	4.811e-08	VTP-BASE	1.349e-10	5.098e-11	2.342e-10

Table 1: Solving NETLIB LPs in 1000 iterations