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# NATIONAL SENIOR CERTIFICATE

**GRADE 12** 

## **SEPTEMBER 2020**

# **MATHEMATICS P2**

**MARKS: 150** 

TIME: 3 hours

This question paper consists of 15 pages, including a 1-page information sheet and an answer book of 25 pages.

#### INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

- 1. This question paper consists of 10 questions.
- 2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
- 3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
- 4. Answers only will NOT necessarily be awarded full marks.
- 5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
- 6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
- 7. Diagrams are NOT necessarily drawn to scale.
- 8. An information sheet with formulae is included at the end of the question paper.
- 9. Write neatly and legibly.

The following table shows a comparison of a school's Grade 12 final marks in 2019 and the learners' School Based Assessment (SBA) marks for the year.

LEARNERS	1	2	3	4	5	6	7	8	9	10
SBA MARK	99	93	77	74	63	62	63	63	47	37
FINAL MARK	94	81	73	65	59	58	55	49	43	31

- 1.1 Determine the equation of the least squares regression line for the data. (Round off your answer correct to 4 decimal places.) (3)
- 1.2 Determine the correlation coefficient between the SBA mark and the final mark. (1)
- 1.3 Comment on the correlation between the SBA mark and the final mark. (1)
- 1.4 Learner 11 scored 51% for SBA. Predict the final mark he should get, correct to the nearest unit. (2)
- 1.5 Given that the mean for the final mark is 60,8, calculate how many learners were within one deviation of the mean. (3)

  [10]

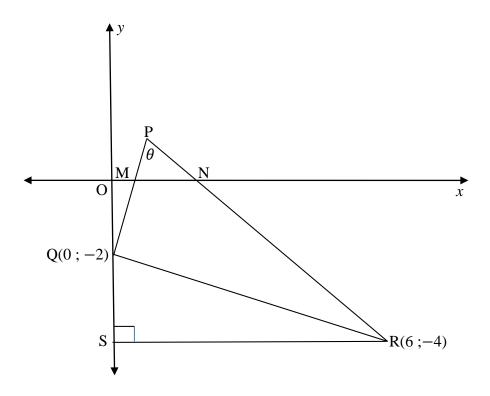
#### **QUESTION 2**

The speeds, in kilometres per hour, of cyclists that passed a point on the route of the Ironman Race were recorded and summarised in the table below:

Speed (km/h)	Frequency (f)	Cumulative Frequency
$0 < x \le 10$	10	10
$10 < x \le 20$		30
$20 < x \le 30$	45	
$30 < x \le 40$	72	
$40 < x \le 50$		170

- 2.1 Complete the above table in the ANSWER BOOK provided. (2)
- 2.2 Make use of the axes provided in the ANSWER BOOK to draw a cumulative frequency curve for the above data. (3)
- Indicate clearly on your graph where the estimates of the lower quartile  $(Q_1)$  and median (M) speeds can be read off. Write down these estimates. (2)
- 2.4 Draw a box and whisker diagram for the data. Use the number line in the ANSWER BOOK. (2)
- 2.5 Use your graph to estimate the number of cyclists that passed the point with speeds greater than 35 km/h. (1) [10]

In the diagram, P, Q (0; -2) and R (6; -4) are the vertices of triangle PQR. The equation of PQ is 3x - y - 2 = 0. The equation of PR is y = -x + 2. RS is the perpendicular from R to the y-axis. QPR =  $\theta$ .



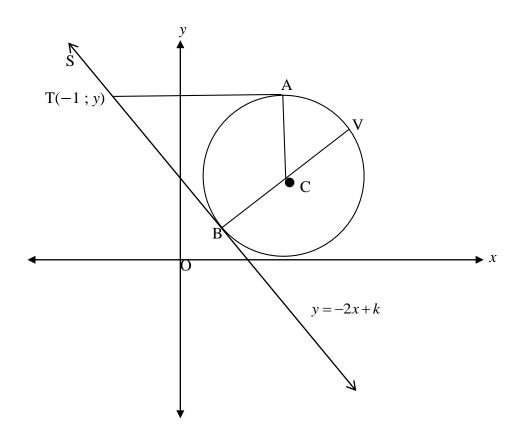
3.1 Calculate the gradient of QR. (2)

3.2 Prove that 
$$P\widehat{Q}R = 90^{\circ}$$
. (2)

- 3.3 Calculate the coordinates of P. (3)
- 3.4 Calculate the length of QR. Leave your answer in surd form. (2)
- 3.5 Determine the equation of the circle through Q, P and R. Give the answer in the form:  $(x a)^2 + (y b)^2 = r^2$ . (5)
- 3.6 Calculate the size of angle  $\theta$ . (5)
- 3.7 Calculate the area of  $\Delta PQR$ . (3) [22]

#### **QUESTION 4**

In the diagram below, C is the centre of the circle defined by  $x^2 - 6x + y^2 - 4y + 9 = 0$ . T (-1; y) is a point outside the circle. Two tangents are drawn to the circle from T. STB is tangent to the circle at B and has equation y = -2x + k. TA is tangent to the circle at A and is parallel to the x-axis. BV is a diameter of the circle.



- 4.1 Determine the coordinates of C. (4)
- 4.2 Determine the equation of BV. (3)
- 4.3 Determine the equation of line TA. (1)
- 4.4 Calculate the length of TB. Give reason(s). (4)
- 4.5 Determine the value of k. (2)
- 4.6 Calculate the size of AĈB. Give reason(s). (4) [18]

5.1 If  $\cos 22^{\circ} = p$ ; determine the following in terms of p:

$$5.1.1 \cos 158^{\circ}$$
 (2)

$$5.1.2 \sin 112^{\circ}$$
 (2)

$$5.1.3 \sin 38^{\circ}$$
 (4)

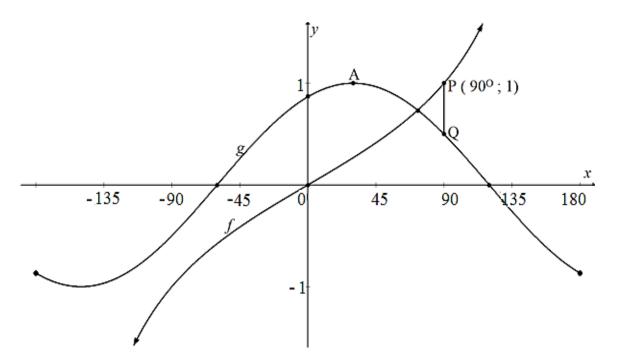
- 5.2 Determine all the values of P in the interval  $[0^\circ; 360^\circ]$  which satisfy the equation:  $\sin P = \sin 2P$ . (4)
- 5.3 If  $\triangle ABC$  is a scalene triangle, show that:  $\cos(A + B) = -\cos C$ . (2)
- 5.4 Prove the following identity:

$$\frac{\cos^2 x - \cos x - \sin^2 x}{2\sin x \cdot \cos x + \sin x} = \frac{1}{\tan x} - \frac{1}{\sin x}$$
 (5)

5.5 Determine the general solution of:  $4 + 7\cos\theta + \cos 2\theta = 0$ . (6) [25]

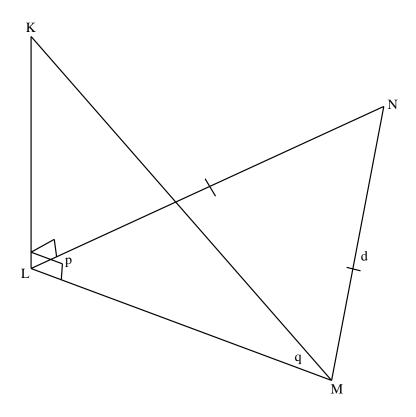
#### **QUESTION 6**

In the diagram below, the graphs of  $f(x) = \tan bx$  and  $g(x) = \cos(x - 30^\circ)$  are drawn on the same set of axes for  $-180^\circ \le x \le 180^\circ$ . The points P(90°;1) and Q lie on f and g respectively. Use the diagram to answer the following questions.



- 6.1 Determine the value of b. (1)
- 6.2 Write down the coordinates of A, the turning point of g. (2)
- 6.3 If PQ is parallel to the y-axis, determine the coordinates of Q. (2)
- 6.4 Write down the equation of the asymptote(s) of  $y = \tan b(x + 20^\circ)$  for  $x \in [-180^\circ; 180^\circ]$ . (1)
- 6.5 Determine the range of h if h(x) = 2g(x) + 1. (2)

Points L, M and N are in the same horizontal plane. KL is a vertical tower. The angle of elevation of K from M is  $q^{\circ}$ . N $\widehat{L}M = p^{\circ}$ ; NL = NM = d and KL = h.



7.1 Determine the size of  $L\widehat{N}M$  in terms of p. (2)

7.2 Prove that LM = 
$$\frac{d \sin 2p}{\sin p}$$
. (2)

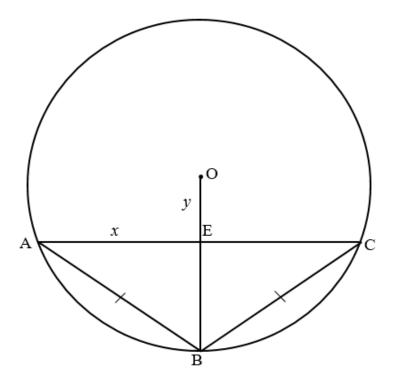
7.3 Hence, show that 
$$h = 2d \cos p \tan q$$
. (3)

#### **QUESTION 8**

8.1 Complete the following theorem statement:

The line drawn from the centre of a circle perpendicular to a chord ... (1)

8.2 In the diagram below, circle ABC with centre O is given. OB = 8 units and AB = BC = 10 units. E is the midpoint of AC. Let OE = y and AE = x.



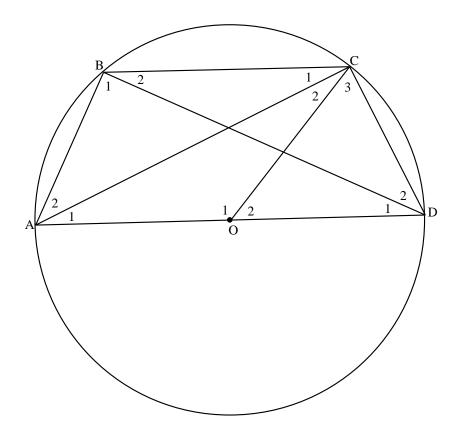
Calculate, with reasons, the length of OE.

(5)

8.3 Complete the following theorem statement:

The angle subtended by an arc at the centre of a circle is ... at the circle (on the same side of the chord as the centre). (1)

8.4 In the diagram, O is the centre of a circle ABCD. AOD is the diameter and OC is a radius. AB, BC, CD, AC and BD are straight lines.



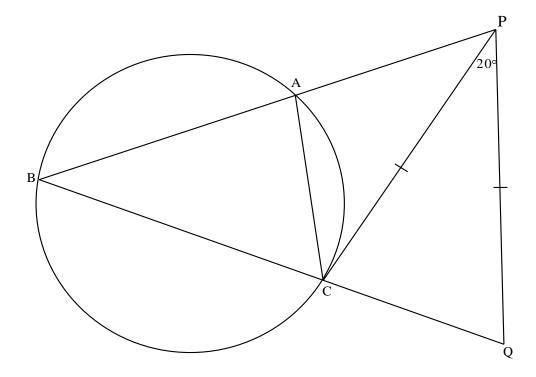
Write down, with reasons, an equation that expresses the relationship between each of the given groups of angles.

	ANGLES	EQUATION / RELATIONSHIP	REASON
e.g.	$\widehat{M}_3;\widehat{P}$	$\hat{\mathbf{M}}_3 = 2 \times \hat{\mathbf{P}}$	$\angle$ at centre = 2 $\times$ $\angle$ at circum.
8.4.1	$\hat{O}_2; \hat{B}_2$		
8.4.2	$\widehat{D}_1; \widehat{C}_3; \widehat{D}_2$		
8.4.3	$\widehat{B}_1; \widehat{B}_2; \widehat{D}_1; \widehat{D}_2$		
8.4.4	$\widehat{D}_1;\widehat{\mathcal{C}}_1$		

(8) [**15**]

## **QUESTION 9**

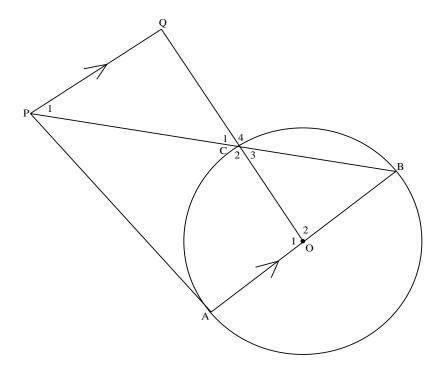
9.1 Given that PC is a tangent to the circle ACB; BAP and BCQ are straight lines. PC = PQ and  $C\hat{P}Q = 20^{\circ}$ .



Prove, stating reasons, that BC is NOT a diameter.

(5)

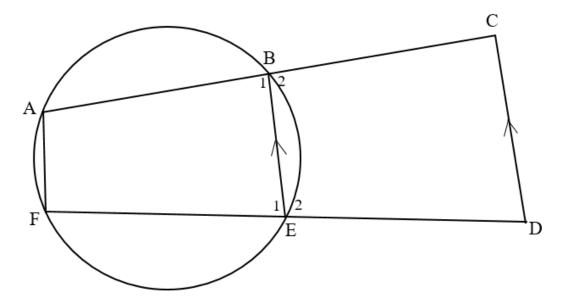
9.2 In the diagram below, O is the centre of circle ABC. The tangent PA to the circle and diameter AB meets at A. OCQ and BCP are straight lines. PQ||AB.



Prove, stating reasons, that PQ = QC.

(6)

9.3 In the diagram below, chords AB and FE of circles with centre O are produced to points C and D. BE||CD.

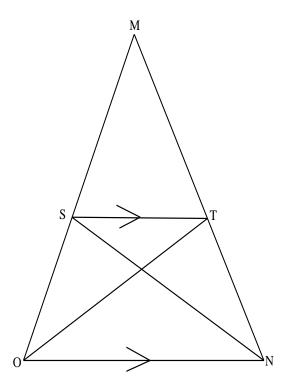


Prove that ACDF is a cyclic quadrilateral.

(5) [**16**]

## **QUESTION 10**

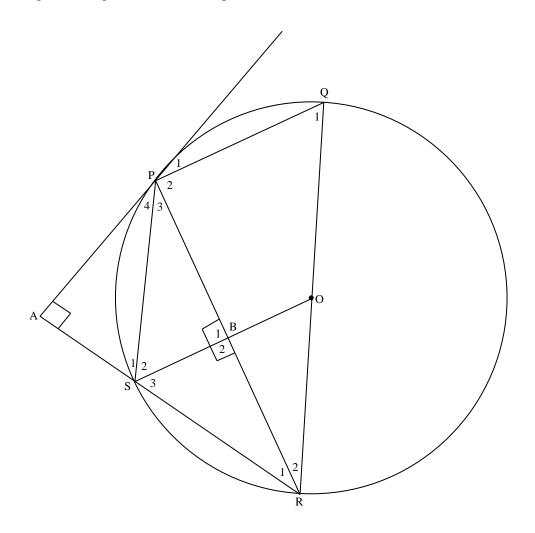
10.1 In the diagram,  $\Delta$ MON is drawn. S is a point on MO and T is a point on MN such that ST||ON. SN and OT are drawn.



Use the diagram to prove the theorem which states that a line parallel to one side of a triangle divides the other two sides proportionally. In other words, prove that:

$$\frac{MS}{SO} = \frac{MT}{TN}.$$
 (5)

10.2 In the diagram, O is the centre of the circle. PQRS is a cyclic quadrilateral. The tangent through P intersects RS produced at A. OB⊥PR and PA⊥AS.



Prove that:

10.2.1 
$$\triangle APS \parallel \triangle BRS$$
 (3)

10.2.2 AP . 
$$RS = BR$$
 .  $PS$  (1)

10.2.3 
$$\hat{P}_4 = \hat{R}_2$$
 (4)

10.2.4 BR . 
$$RQ = RS . RP$$
 (6) [19]

**TOTAL: 150** 

#### **INFORMATION SHEET: MATHEMATICS**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$\sum_{i=1}^n 1 = n \qquad \sum_{i=1}^n i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \quad ; \qquad r \neq 1 \qquad S_{\infty} = \frac{a}{1 - r} \quad ; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad p = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$In \ \Delta ABC: \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc \cdot \cos A \qquad area \ \Delta ABC = \frac{1}{2}ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta \qquad \sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta \qquad \cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$[\cos^2 \alpha - \sin^2 \alpha]$$

 $\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin \alpha . \cos \alpha$ 

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$