Province of the
EASTERN CAPE
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2020

MATHEMATICS P1

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, including an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answer.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
5. Answers only will not necessarily be awarded full marks.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $2x^2 + x - 3 = 0$ (3)

1.1.2 $x(7x+2) = 1$ (correct to TWO decimal places) (4)

1.1.3 $-x^2 - x + 2 \leq 0$ (4)

1.1.4 $2^x + 2^{2-x} = \frac{17}{2}$ (5)

1.2 Given:

- $(x-2)^2 + y^2 = 25$ is an equation of a circle
- $x + 3 - 3y = 0$ is an equation of a straight line
- The graphs of the circle and line intersect at the points A and B

Determine, showing ALL necessary calculations, the coordinates of points A and B. (6)

1.3 Show that the roots of the equation $(x+m)(x+n) = 3p^2$ are real for all values of m, n and p. (4)

[26]

QUESTION 2

2.1 Given the quadratic pattern: 86 ; 119 ; 150 ; 179 ; …

2.1.1 Write down the next TWO terms of the pattern. (2)

2.1.2 Determine T_n, the general term of the pattern in the form $T_n = an^2 + bn + c$. (4)

2.1.3 Which term(s) of the pattern has a value of 326? (3)

2.1.4 Taine adds a constant, k to each of the terms in the pattern giving a new pattern P_n. Determine the general term of the new pattern. (2)

2.2 The first three terms of an arithmetic sequence are:

$2y-1$; $4y-1$; $6y-1$.

2.2.1 Determine T_{30} in terms of y. (3)

2.2.2 Determine the value of y, given that the sum of the first 30 terms of this sequence is -2820. (4)

[18]

QUESTION 3

3.1 Given the series: $1 + 2 + 3 + 4 + 5 + 6 + ... + 5000$

Write down the series in sigma notation if all the powers of 4 are removed from the series. (4)

3.2 Given that the following two geometric series are convergent:

$1 + x + x^2 + x^3 + ...$ and $1 - x + x^2 - x^3 + ...$

Determine the value(s) of x for which the sum of the two series is equal to 8. (6)

[10]

QUESTION 4

Given: $f(x) = \dfrac{a}{x-1} + 3$, where $a \in \mathbb{Z}$.

4.1 Write down the equations of the asymptotes of f. (2)

4.2 Determine the x and y intercepts of f in terms of a. (3)

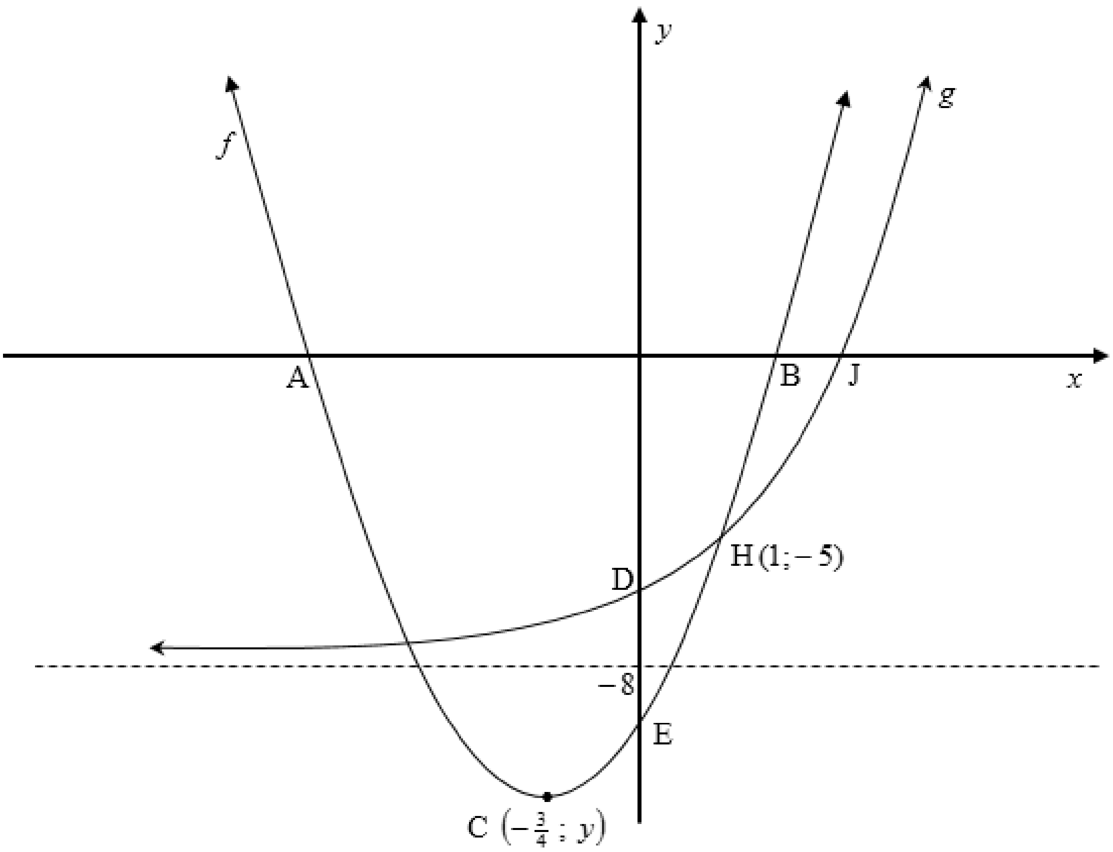
4.3 Given that $a = -1$, draw a neat sketch of f, clearly showing all asymptotes and intercepts with the axes. (4)

4.4 The graph of f is shifted 3 units to the left and 2 units downwards. Write down the new equation of f in terms of a. (2)

[11]

QUESTION 5

The diagram below shows the graphs of $f(x) = ax^2 + bx + c$ and $g(x) = b^x + q$.
A and B are the x-intercepts, E is the y-intercept and C$\left(-\frac{3}{4}\ ;\ y\right)$ is the turning point of f.
J is the x-intercept and D is the y-intercept of g. $y = -8$ is the equation of the asymptote of g. H$(1;-5)$ is one of the points of intersection of f and g.



5.1 Write down the coordinates of D. (1)

5.2 Write down the value of q. (1)

5.3 Show that $a = 2$, $b = 3$ and $c = -10$. (6)

5.4 Write down the range of g. (2)

5.5 The line with equation, $y + 9x = -28$, is the tangent of f at a point T. Determine the coordinates of T. (5)

5.6 Given that $h(x) = g(x) + 8$, write down $h^{-1}(x)$ in the form $y = ...$ (2)

5.7 Given that $p(x) = f(x) + 1$, determine the values of x for which $x.p(x) < 0$. (4)

[21]

QUESTION 6

6.1 Colby bought a laptop worth Rx for his university studies. The value of the laptop decreased at r % per annum using the reducing balance method.

After 4 years, the value of the laptop was worth $\frac{1}{3}$ of its original price.

Calculate r, the rate of depreciation. (3)

6.2 On 1 February 2014, Ncominkosi took a loan from a bank to buy a car. His first payment for the loan was due on 31 July 2014. Once he started paying the loan, it took him 6 years to fully pay the loan at an interest rate of 9,5% p.a. compounded monthly. In total, he paid the bank R596 458,10.

6.2.1 How much was his monthly instalment? (5)

6.2.2 How much money did he borrow from the bank? Write down your answer to the nearest rand. (6)

[14]

QUESTION 7

Determine:

7.1 $f'(x)$ from first principles if $f(x) = -2x^2$ (5)

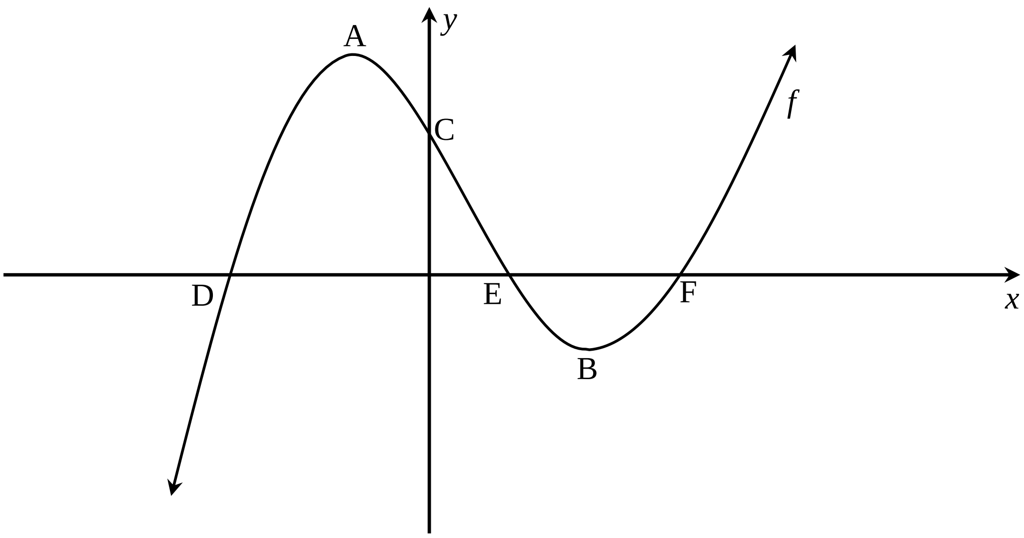
7.2 $\frac{dy}{dx}$ if $y = 7x^4 - \frac{2}{\sqrt{x^3}}$ (3)

7.3 $D_t\left[\frac{1}{2}gt^2 - \frac{5}{t} + 3g\right]$ (4)

[12]

QUESTION 8

In the diagram below, the graph of $f(x) = 2x^3 + x^2 - 12x + 9$ is drawn. A and B are the turning points of f and C is the y-intercept. D, E and F are the x-intercepts.



8.1 Write down the coordinates of C. (1)

8.2 Calculate the coordinates of D, E and F. (6)

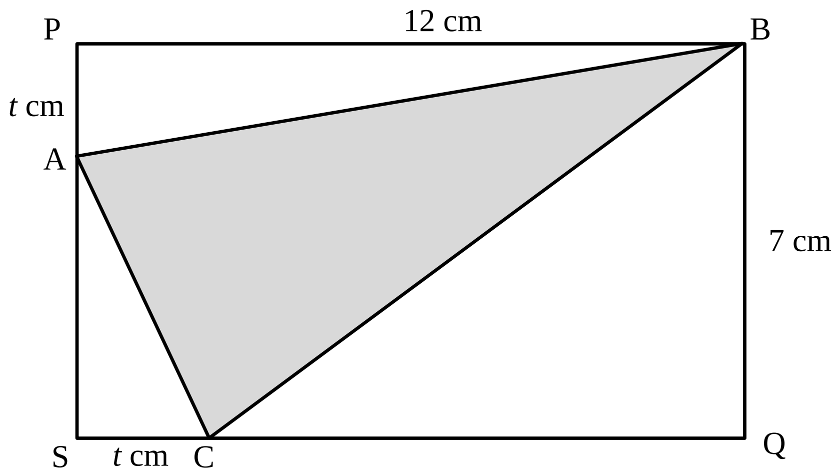
8.3 Determine the values of x for which f is concave down. (4)

8.4 Determine the values of x for which $f'(x) \leq 0$. (4)

[15]

QUESTION 9

The diagram below shows triangle ABC drawn such that its vertices lie on rectangle PBQS, as shown. PA = SC = t cm. PB = 12 cm and BQ = 7 cm.



Calculate the smallest possible area of ΔABC. **[6]**

QUESTION 10

At St Johns High School, a survey was carried out to determine the number of Grade 12 learners who take Mathematics (M), Physical Sciences (P) and Accounting (A). The following information was collected:

- 135 learners took part in the survey
- 5 learners take Mathematics and Accounting but not Physical Sciences
- 12 learners take Mathematics and Physical Sciences but not Accounting
- 24 learners take Physical Sciences and Accounting but not Mathematics
- y learners take Physical Sciences only
- x learners take all three subjects
- y learners take Accounting only
- $2y+3$ learners take Mathematics only
- 60 learners take Accounting
- The number of learners who take Mathematics is equal to the number of learners who take Physical Sciences

10.1 Represent the above information on a Venn-diagram. (4)

10.2 Determine the values of x and y. (4)

10.3 Calculate the probability that a learner chosen at random does Mathematics or both Physical Sciences and Accounting. (3)

[11]

QUESTION 11

Lwazi and Cwenga are the head boy and head girl of their school respectively. In addition, there are 3 boys and 2 girls who are prefects. They are all supposed to sit for two photos in a row.

11.1 In their first photo, order is not important. In how many ways can they sit? (2)

11.2 In their second photo, Lwazi and Cwenga can only sit on the third and fifth seats, either way.
What is the probability that the last seat is occupied by a boy and only a girl can sit on the fourth seat? (4)

[6]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$