basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

FEBRUARY/MARCH 2015

MARKS: 150

TIME: 3 hours

This question paper consists of 11 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - x - 20 = 0$ (2)

1.1.2 $2x^2 - 11x + 7 = 0$ (correct to TWO decimal places) (3)

1.1.3 $5x^2 + 4 > 21x$ (5)

1.1.4 $2^{2x} - 6.2^x = 16$ (4)

1.2 Solve for x and y simultaneously:

$y + 1 = 2x$
$x^2 - xy + y^2 = 7$ (6)

1.3 The roots of a quadratic equation are given by $x = \dfrac{-5 \pm \sqrt{20+8k}}{6}$, where $k \in \{-3;-2;-1;0;1;2;3\}$.

1.3.1 Write down TWO values of k for which the roots will be rational. (2)

1.3.2 Write down ONE value of k for which the roots will be non-real. (1)

1.4 Calculate a and b if $\sqrt{\dfrac{7^{2014} - 7^{2012}}{12}} = a\left(7^b\right)$ and a is not a multiple of 7. (4)

[27]

QUESTION 2

2.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d, the sum of the first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$. (4)

2.2 Calculate the value of $\sum_{k=1}^{50}(100-3k)$. (4)

2.3 A quadratic sequence is defined with the following properties:

$$T_2 - T_1 = 7$$
$$T_3 - T_2 = 13$$
$$T_4 - T_3 = 19$$

2.3.1 Write down the value of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

2.3.2 Calculate the value of T_{69} if $T_{89} = 23\,594$. (5)

[17]

QUESTION 3

Consider the infinite geometric series: 45 + 40,5 + 36,45 + …

3.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

3.2 Explain why this series converges. (1)

3.3 Calculate the sum to infinity of the series. (2)

3.4 What is the smallest value of n for which $S_\infty - S_n < 1$? (5)

[11]

QUESTION 4

Given: $g(x) = \dfrac{6}{x+2} - 1$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Calculate:

4.2.1 The y-intercept of g (1)

4.2.2 The x-intercept of g (2)

4.3 Draw the graph of g, showing clearly the asymptotes and the intercepts with the axes. (3)

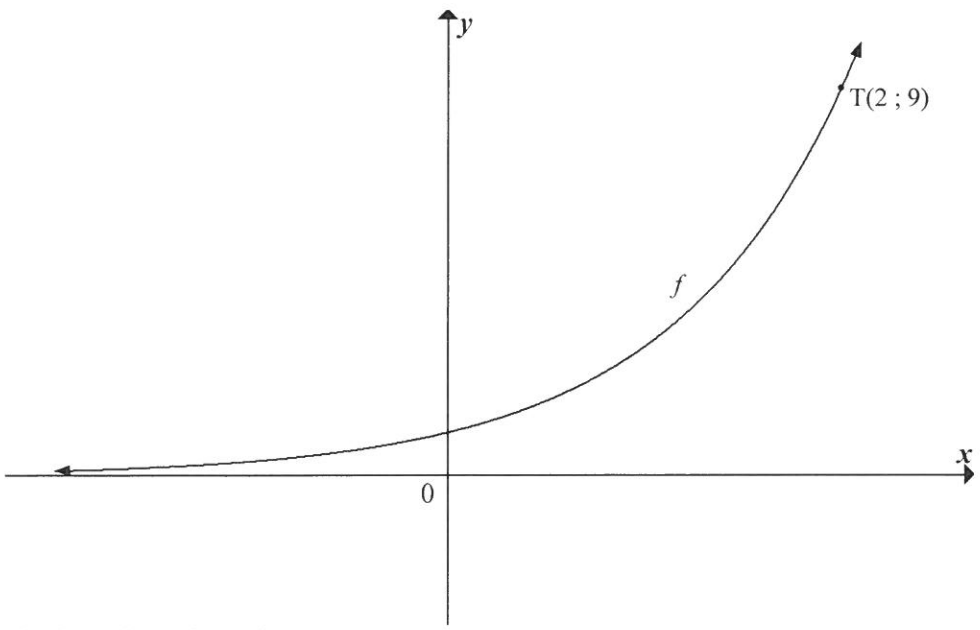
4.4 Determine the equation of the line of symmetry that has a negative gradient, in the form $y = ...$ (3)

4.5 Determine the value(s) of x for which $\dfrac{6}{x+2} - 1 \geq -x - 3$. (2)

[13]

QUESTION 5

The graph of $f(x) = a^x$, $a > 1$ is shown below. T(2 ; 9) lies on *f*.



5.1 Calculate the value of a. (2)

5.2 Determine the equation of $g(x)$ if $g(x) = f(-x)$. (1)

5.3 Determine the value(s) of x for which $f^{-1}(x) \geq 2$. (2)

5.4 Is the inverse of f a function? Explain your answer. (2)

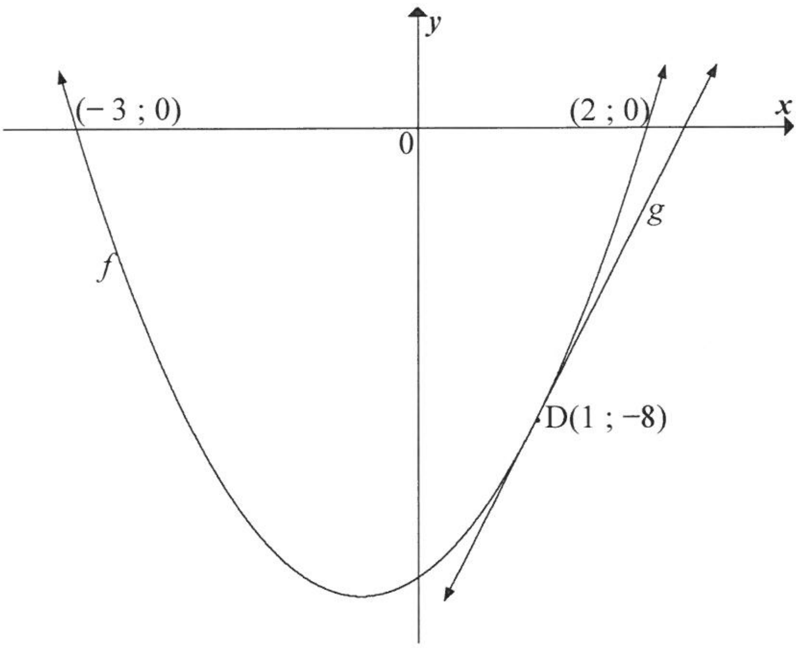
[7]

QUESTION 6

The graphs of $f(x) = ax^2 + bx + c$; $a \neq 0$ and $g(x) = mx + k$ are drawn below.

D(1 ; – 8) is a common point on f and g.

- f intersects the x-axis at $(-3 ; 0)$ and $(2 ; 0)$.
- g is the tangent to f at D.



6.1	For which value(s) of x is $f(x) \leq 0$?	(2)
6.2	Determine the values of a, b and c.	(5)
6.3	Determine the coordinates of the turning point of f.	(3)
6.4	Write down the equation of the axis of symmetry of h if $h(x) = f(x-7)+2$.	(2)
6.5	Calculate the gradient of g.	(3)
		[15]

QUESTION 7

7.1 Nomsa started working on 1 January 1970. At the end of January 1970 and at the end of each month thereafter, she deposited R400 into an annuity fund. She continued doing this until she retired on 31 December 2013.

7.1.1 Determine the total amount of money that she paid into the fund. (2)

7.1.2 The interest rate on this fund was 8% p.a., compounded monthly. Calculate the value of the fund at the time that she retired. (5)

7.1.3 On 1 January 2014 Nomsa invested R2 million in an account paying interest at 10% p.a. compounded monthly. Nomsa withdraws a fixed amount from this account at the end of each month, starting on 31 January 2014. If Nomsa wishes to make monthly withdrawals from this account for 25 years, calculate the maximum amount she could withdraw at the end of each month. (4)

7.2 For each of the three years from 2010 to 2012 the population of town X decreased by 8% per year and the population of town Y increased by 12% per year.

At the end of 2012 the populations of these two towns were equal.

Determine the ratio of the population of town X (call it P_X) to the population of town Y (call it P_Y) at the beginning of 2010. (4)

[15]

QUESTION 8

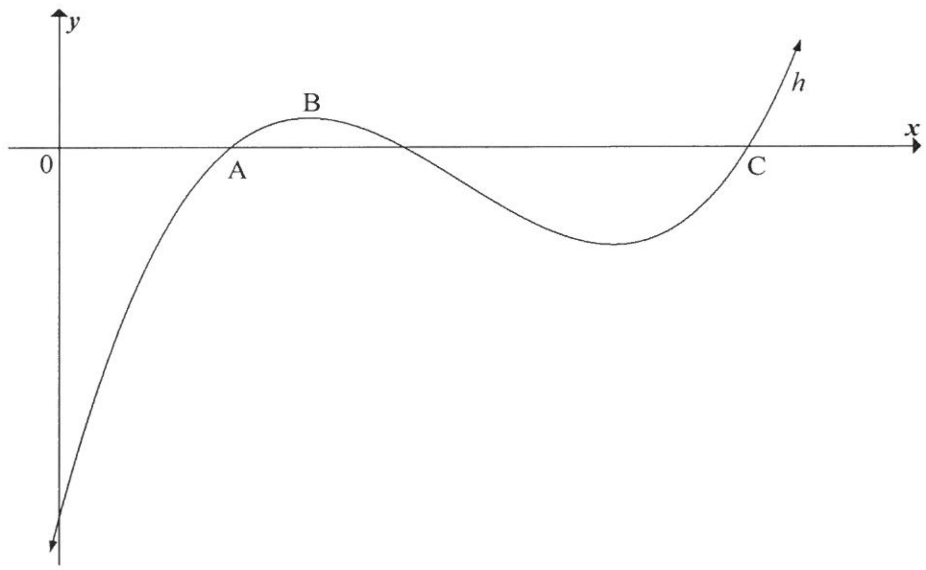
8.1 Determine the derivative of $f(x) = 2x^2 + 4$ from first principles. (4)

8.2 Differentiate:

8.2.1 $f(x) = -3x^2 + 5\sqrt{x}$ (3)

8.2.2 $p(x) = \left(\dfrac{1}{x^3} + 4x\right)^2$ (4)

8.3 The sketch below shows the graph of $h(x) = x^3 - 7x^2 + 14x - 8$. The x-coordinate of point A is 1. C is another x-intercept of h.



8.3.1 Determine $h'(x)$. (1)

8.3.2 Determine the x-coordinate of the turning point B. (3)

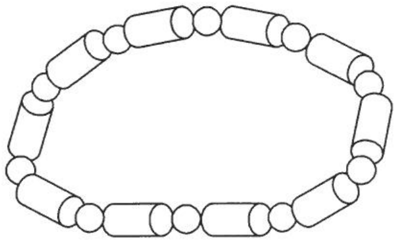
8.3.3 Calculate the coordinates of C. (4)

8.3.4 The graph of h is concave down for $x < k$. Calculate the value of k. (3)

[22]

QUESTION 9

A necklace is made by using 10 wooden spheres and 10 wooden cylinders. The radii, r, of the spheres and the cylinders are exactly the same. The height of each cylinder is h. The wooden spheres and cylinders are to be painted. (Ignore the holes in the spheres and cylinders.)



$$V = \pi r^2 h \qquad S = 2\pi r^2 + 2\pi r h$$

$$V = \frac{4}{3}\pi r^3 \qquad S = 4\pi r^2$$

9.1 If the volume of a cylinder is 6 cm^3, write h in terms of r. (1)

9.2 Show that the total surface area (S) of all the painted surfaces of the necklace is equal to $S = 60\pi r^2 + \dfrac{120}{r}$ (4)

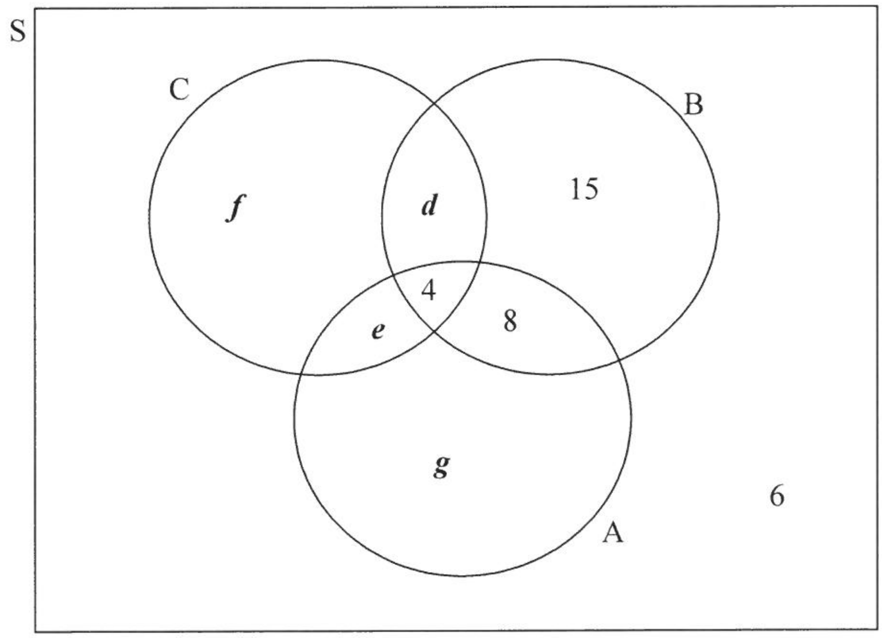
9.3 Determine the value of r so that the least amount of paint will be used. (4)
[9]

QUESTION 10

10.1 Research was conducted about driving under the influence of alcohol. Information obtained from traffic authorities in 54 countries on the methods that are used to measure alcohol levels in a person, are summarised below:

- 4 countries use all three methods (A, B and C).
- 12 countries use the alcohol content of breath (A) and blood-alcohol concentration (B).
- 9 countries use blood-alcohol concentration (B) and certificates issued by doctors (C).
- 8 countries use the alcohol content of breath (A) and certificates issued by doctors (C).
- 21 countries use the alcohol content of breath (A).
- 32 countries use blood-alcohol concentration (B).
- 20 countries use certificates issued by doctors (C).
- 6 countries use none of these methods.

Below is a partially completed Venn diagram representing the above information.



10.1.1 Use the given information and the Venn diagram to determine the values of d, e, f and g. (4)

10.1.2 For a randomly selected country, calculate:

(a) P(A and B and C) (1)

(b) P(A or B or C) (1)

(c) P(only C) (1)

(d) P(that a country uses exactly two methods) (1)

10.2 Nametso may choose DVDs from three categories as listed in the table below:

Drama	Romance	Comedy
• *Last Hero*	• *One Heart*	• *Laughing Dragon*
• *Midnight*	• *You and Me*	• *Falling Down*
• *Stranger Calls*	• *Love Song*	• *Sitting on the Stairs*
• *Missing in Action*	• *Bird's First Nest*	
• *Only 40 Seconds Left*		

10.2.1 Nametso must choose ONE DVD from the Drama category. What is the probability that she will choose *Midnight*? (2)

10.2.2 How many different selections are possible if her selection must include ONE drama, ONE romance and ONE comedy? (2)

10.2.3 Calculate the probability that she will have *Last Hero* and *Laughing Dragon* as part of her selection in QUESTION 10.2.2. (2)

[14]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$