# Overview of DELPHI Model V2.0

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## 1 Introduction to the Model

The model underlying our predictions is DELPHI (Differential Equations Leads to Predictions of Hospitalizations and Infections). DELPHI is a compartamental model that is based on the widely successful SEIR model, but with many additions to account for realistic effects. In particular there are two important effects that we consider:

- **Underdetection**: In any pandemic, a lot of cases go undetected due to many factors (failure to record, unable to test, mistaken for other disease, etc). This is an important factor that if not appropriately accounted for, would underestimate the real reach and spread of the epidemic.
- **Governmental Response**: No epidemic exists in a world where it is allowed to spread completely freely. As the epidemic spreads, governments start to respond by enacting policies designed to limit the spread of the virus, and we explicitly design a framework to take such policies into account in the model.

The model separates people into 11 possible states of being in the epidemic:

- **Susceptible (S)**: The general populace who have not been infected.
- **Exposed (E)**: People who are currently infected, but are not contagious and within the incubation period.
- **Infected (I)**: People who are currently infected and contagious.
- **Undetected (AR) & (AD)**: People who are infected, and self-quarantined themselves at home due to the effects of the disease, but was not confirmed due to lack of testing. Here, we model it in a way that some of these people recover (AR) and some of these die (AD).
- **Detected, Hospitalized (DHR) & (DHD)**: People who are infected, confirmed, and hospitalized. Again, we model it in two separate states: some of these people recover (DHR) and some of these die (DHD).
- **Detected, Quarantine (DQR) & (DQD)**: People who are infected, confirmed, and home-quarantined rather than hospitalized. Similar as before, we have two states: (DQR) for those that recover, and (DQD) for those that die.
- **Recovered (R)**: People who have recovered from the disease (and assumed to be immune).
- **Death (D)**: People who have perished from the disease.

The separation of recovery and death states in the detection phase (including AR/AD, DQR/DQD, DHR/DHD) is required so that recovery and deaths can be tuned separately.

The interactions between the different states are summarized in the picture below, where the arrows indicate possible flow between the states:

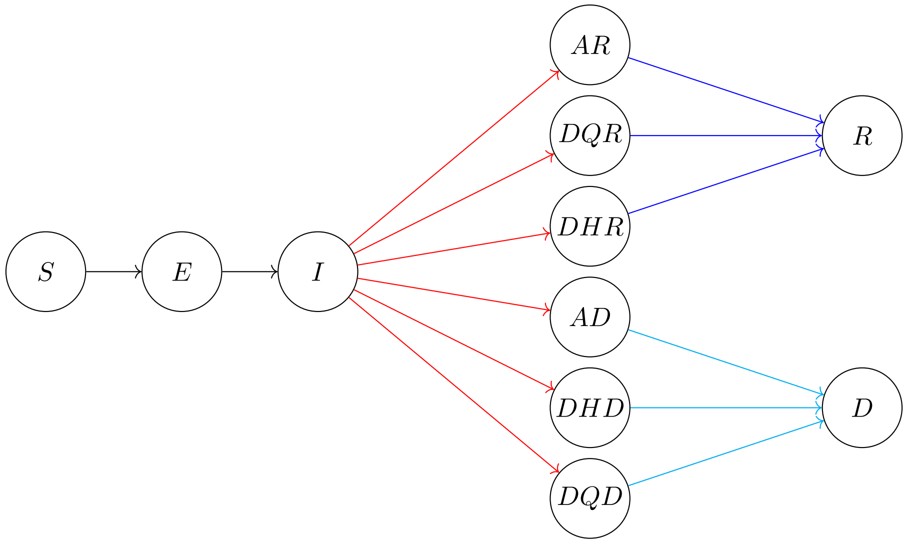


Figure 1: (Simplified) Flow Diagram of DELPHI (V2)

## 2 Detailed Model Formulation

In addition to main functional states, we also introduce helper states to help us calculate a few useful quantities. This includes Total Hospitalized (TH), Total Detected Deaths (DD) and Total Detected Cases (DT). The mathematical formulation of the model, along with these helper states, is as followed:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\alpha\gamma(t)S(t)I(t)$$
$$\frac{\mathrm{d}E}{\mathrm{d}t} = \alpha\gamma(t)S(t)I(t) - r_i E(t)$$
$$\frac{\mathrm{d}I}{\mathrm{d}t} = r_i E(t) - r_d I(t)$$
$$\frac{\mathrm{d}AR}{\mathrm{d}t} = r_d(1-p_{dth})(1-p_d)I(t) - r_{ri}AR(t)$$
$$\frac{\mathrm{d}DHR}{\mathrm{d}t} = r_d(1-p_{dth})p_d p_h I(t) - r_{rh}DHR(t)$$
$$\frac{\mathrm{d}DQR}{\mathrm{d}t} = r_d(1-p_{dth})p_d(1-p_h)I(t) - r_{ri}DQR(t)$$
$$\frac{\mathrm{d}AD}{\mathrm{d}t} = r_d p_{dth}(1-p_d)I(t) - r_{dth}AD(t)$$
$$\frac{\mathrm{d}DHD}{\mathrm{d}t} = r_d p_{dth} p_d p_h I(t) - r_{dth}DHD(t)$$
$$\frac{\mathrm{d}DQD}{\mathrm{d}t} = r_d p_{dth} p_d(1-p_h)I(t) - r_{dth}DQD(t)$$
$$\frac{\mathrm{d}TH}{\mathrm{d}t} = r_d p_d p_h I(t)$$
$$\frac{\mathrm{d}DD}{\mathrm{d}t} = r_{dth}(DHD(t) + DQD(t))$$
$$\frac{\mathrm{d}DT}{\mathrm{d}t} = r_d p_d I(t)$$
$$\frac{\mathrm{d}R}{\mathrm{d}t} = r_{ri}(AR(t) + DQR(t)) + r_{rh}DHR(t)$$
$$\frac{\mathrm{d}D}{\mathrm{d}t} = r_{dth}(AD(t) + DQD(t) + DHD(t))$$

We define each of the parameters below:

- $\alpha$ is the infection rate.
- $\gamma(t)$ measures the government response and is defined as:

$$\gamma(t) = \frac{2}{\pi} \arctan\left(\frac{-(t-a)}{b}\right) + 1$$

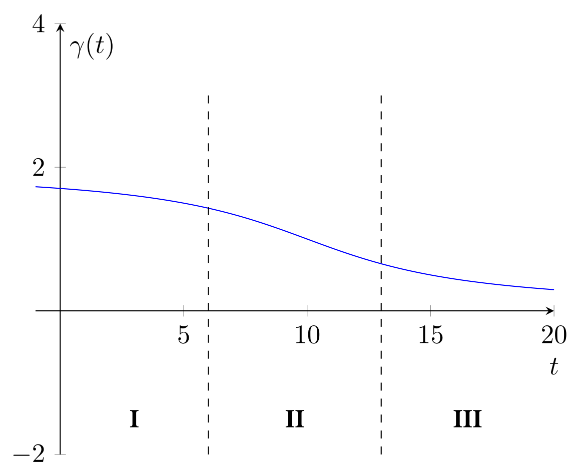
The details for choosing this function is explained in the sub-section below.

- $r_d$ is the rate of detection. This equals to $\frac{\log 2}{T_d}$ where $T_d$ is the median time to detection (assumed to be 2 days).
- $r_i$ is the rate of infection leaving incubation phase. This equals to $\frac{\log 2}{T_i}$ where $T_i$ is the median time to leave incubation (assumed to 5 days).
- $r_{ri}$ is the rate of recovery not under hospitalization. This equals to $\frac{\log 2}{T_{ri}}$ where $T_{ri}$ is the median time to recovery not under hospitalization (assumed to be 10 days).
- $r_{rh}$ is the rate of recovery under hospitalization. This equals to $\frac{\log 2}{T_{rh}}$ where $T_{rh}$ is the median time to recovery under hospitalization (assumed to be 15 days).
- $r_{dth}$ is the rate of death. This equals to $\frac{\log 2}{T_{dth}}$ where $T_{dth}$ is the time till death for dying patients. This parameter is fitted to the historical data.

- $p_{dth}$ is the mortality rate.
- $p_d$ is the percentage of infection cases detected.
- $p_h$ is the percentage of detected cases hospitalized.

## 2.1 Modeling Societal-Governmental Response

The rate of infection is never constant in an epidemic. As governments start responding to an epidemic, the rate of infection would start decreasing due to the measures being put in place. We hace decided to model the response by multiplying an initial infection rate with an arctan curve, as it captures the three phases of governmental response (the following graph is shown with $\gamma(t) = \frac{2}{\pi}\arctan(-\frac{t-10}{5})$, so $b = 5$ and $a = 10$):



- **Phase I**: This phase models the initial response when the government has just started to consider implementing policies for the epidemic. Some portion of the populace would have already changed their behavior responding to the reports of an epidemic, but a large portion of the population continues to experience life normally.
- **Phase II**: This phase is characterized by the sharp decline in infection rate as policies to control the spread go into full force (e.g. closing down part of the economy), and the society in whole experiences a shock event.
- **Phase III**: This phase models the inevitable flattening out of the response as the measures reach saturation. This is represented by the diminishing marginal returns (i.e. convexity) in the decline of infection rate.

Using parameters $a$ and $b$, we are able to control, respectively, when the measures start, and the strength of such measures. This formulation allows us to model a wide variety of policies that different governments impose under the same framework, including social distancing, stay-at-home policies, quarantines, among many others.