# Class 12: Singular Value Decomposition

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# Agenda

- Singular Value Decomposition
- Working Groups
- Brook Guzder-Williams from the World Resources Institute

# Looking Ahead

Next week: Final Presentations!

- Sign up!
- Please be on time.

April 29: Final Project Due

# Singular Value Decomposition

Suppose you have a big $N \times K$ matrix. How can you understand it?

$$A = \begin{vmatrix} 3 & - & 5 & 3 & 2 \\ 4 & 3 & - & 2 & 1 \\ 3 & 5 & 9 & 2 & 4 \\ 1 & 4 & 0 & - & 9 \end{vmatrix}$$

What are some examples?

Suppose you have a big $N \times K$ matrix. How can you understand it?

$$A = \begin{vmatrix} 3 & - & 5 & 3 & 2 \\ 4 & 3 & - & 2 & 1 \\ 3 & 5 & 9 & 2 & 4 \\ 1 & 4 & 0 & - & 9 \end{vmatrix}$$

What are some examples?

- Netflix: Rows are users, columns are movies.
- Elections: Rows are Precinects, columns are Candidates.
- Demographics: Rows are Tracts, columns are income, race, age.

# Singular Value Decomposition

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What tasks can you accomplish?

# Singular Value Decomposition

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What tasks can you accomplish?

- Fill in missing values (assuming missing at random)
- Component analysis ("dimensionality reduction")
- Simplify patterns
- Predict new rows or columns

# Singular Value Decomposition

Singular Value Decomposition approximates matrix $A$ with

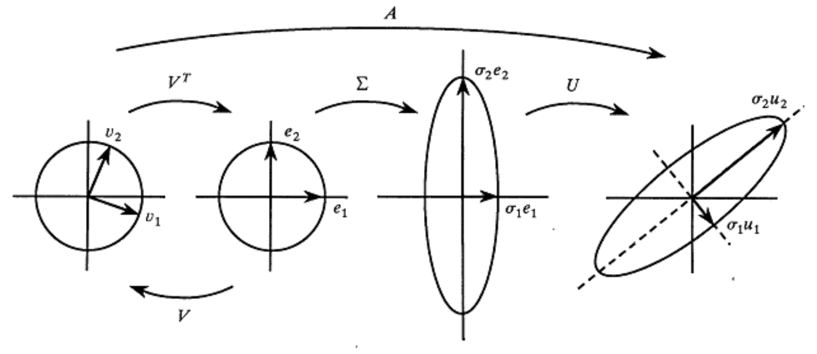
$A = UDV'$

- $A$ has dimensions $N \times K$.
- $U$ has dimensions $N \times M$.
- $D$ is a diagonal matrix $M \times M$.
- $V$ has dimensions $K \times M$.
- $U$ and $V$ are orthonormal.

# Singular Value Decomposition

Singular Value Decomposition approximates matrix $A$ with

$A = UDV'$




From https://blogs.sas.com/content/iml/2017/08/28/singular-value-decomposition-svd-sas.html

# Singular Value Decomposition

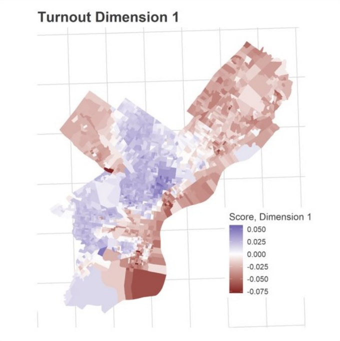
Singular Value Decomposition approximates matrix $A$ with

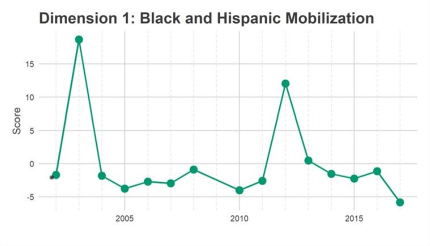
$A = UDV'$

$$A = \begin{vmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \\ U_{31} & U_{32} \\ U_{41} & U_{42} \end{vmatrix} \begin{vmatrix} D_{11} & 0 \\ 0 & D_{22} \end{vmatrix} \begin{vmatrix} V_{11} & V_{21} & V_{31} & V_{41} \\ V_{12} & V_{22} & V_{32} & V_{42} \end{vmatrix}$$

- U is components of the rows.
- V is components of the columns.
- D is in order of captured variance. Trimming D increases smoothing.
- Each row and each column gets a score in each dimension.
- Dimensions are orthogonal.
- Observation $A_{ij}$ is estimated as $U_{i1} V_{j1} D_{11} + U_{i2} V_{j2} D_{22} + \ldots$

# Turnout Dimensioons




$$U_{i1} V_{j1} D_{11} + U_{i2} V_{j2} D_{22} + \ldots$$

# Demo

# Working Groups

a. Python
b. R
c. Deployment (scraping, automating)
d. Methodology (Risk Scores, Regressions)
e. Paper writing (Question, Lit Review, etc.)