Class 10: Spatial Regression

Jonathan Tannen

Agenda

- Spatial Regressions
- Mid-Point Presentations

Looking Ahead

Next Week: Code Review (or Implementation Review) for two projects. Phila Dept of Public Health Presenting.

April 8: World Resources Institute Presenting.

April 15/22: Final Presentations

April 29: Final Project Due

High-Level notes from Mid-Points

- Use the Literature Review to help the reader understand why your topic is important.
  - Include the *findings* of the papers.
- Do a lot explaining the *why* behind your methods.
  - What is the purpose of your controls?
  - What is the challenge in your risk score?
- Include a lot more summary stats and overall plots of your data.
  - Don't jump into regression too soon!

Spatial Regressions

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    - aka ϵ is not independent of X.
    - Introduces "Omitted Variable Bias".

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    - Introduces "Omitted Variable Bias".
  - $Cov(\epsilon_i, \epsilon_j) = 0$
    - When this is due to i and j being close, this is spatial autocorrelation.
    - You actually have fewer observations than you think.

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- Suppose we only used features of the house as X. Would the epsilons be correlated? Why?

Spatial Regressions

$$Y = X\beta + \epsilon$$

What are the solutions to spatial autocorrelation?

The answer hinges on whether you care about (a) estimating coefficients correctly, or (b) getting good predictions.

The three options:

- Introduce features that explain away the spatial correlation.
- Aggregate up to a granularity at which there is no spatial correlation.
- Use a fancy regression technique that corrects standard errors for autocorrelation.

What is your unit of analysis?

- Points
- Polygons

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This has implications for. . .

- What you control for (and how).
- How you specify spatial covariances.

Estimating Coefficients

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"What is the value of adding air conditioning?"

- What is the X we care about?
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All we care about is "controlling away" the spatial correlations.

Diagnosing Autocorrelation

$$Y = X\beta + \epsilon$$

Once we fit this model, is there evidence that $Cov(\epsilon_i, \epsilon_j)$ depends on distance?

- **Variogram**: Plot the covariance between values as a function of distance.
- **Moran's I**: $\hat{Cov}_W(\epsilon_i, \epsilon_j)/Cov(\hat{epsilon})$

Estimating Coefficients

Consider an example: Trying to predict the sales price of a house.

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Solutions:

- Add controls that explain the spatial correlation.
  - Proximity to schools? Neighborhood parks?
  - Question: Should we control for median income?

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- Add controls that explain the spatial correlation.
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- Use "Fixed Effects" (e.g. at Census Tracts) to blanket control for spatial patterns.
  - Add a dummy variable for each Census Tract.
  - $Y = X\beta + \gamma_0\delta_0 + \gamma_1\delta_1 + ... + \epsilon$
  - Pros: Blanket tool to control for unobserved variables that vary within tracts.
  - Cons:
    - Limits analysis to only within-tract comparisons.

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  - Cons:
    - Limits analysis to only within-tract comparisons.
- Random Effects, or Hierarchical Model
  - Allows you to add tract-level variables, estimates standard errors correctly.
  - $Y = X\beta + \gamma Z_{Tract} + \epsilon$
  - in R: `rmle` package.

Aggregating up to higher level of granularity

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All schools in a given catchment will receive the same school effect. Do analysis at the *school level* rather than the house level.

True autocorrelation: Smarter Regression Models

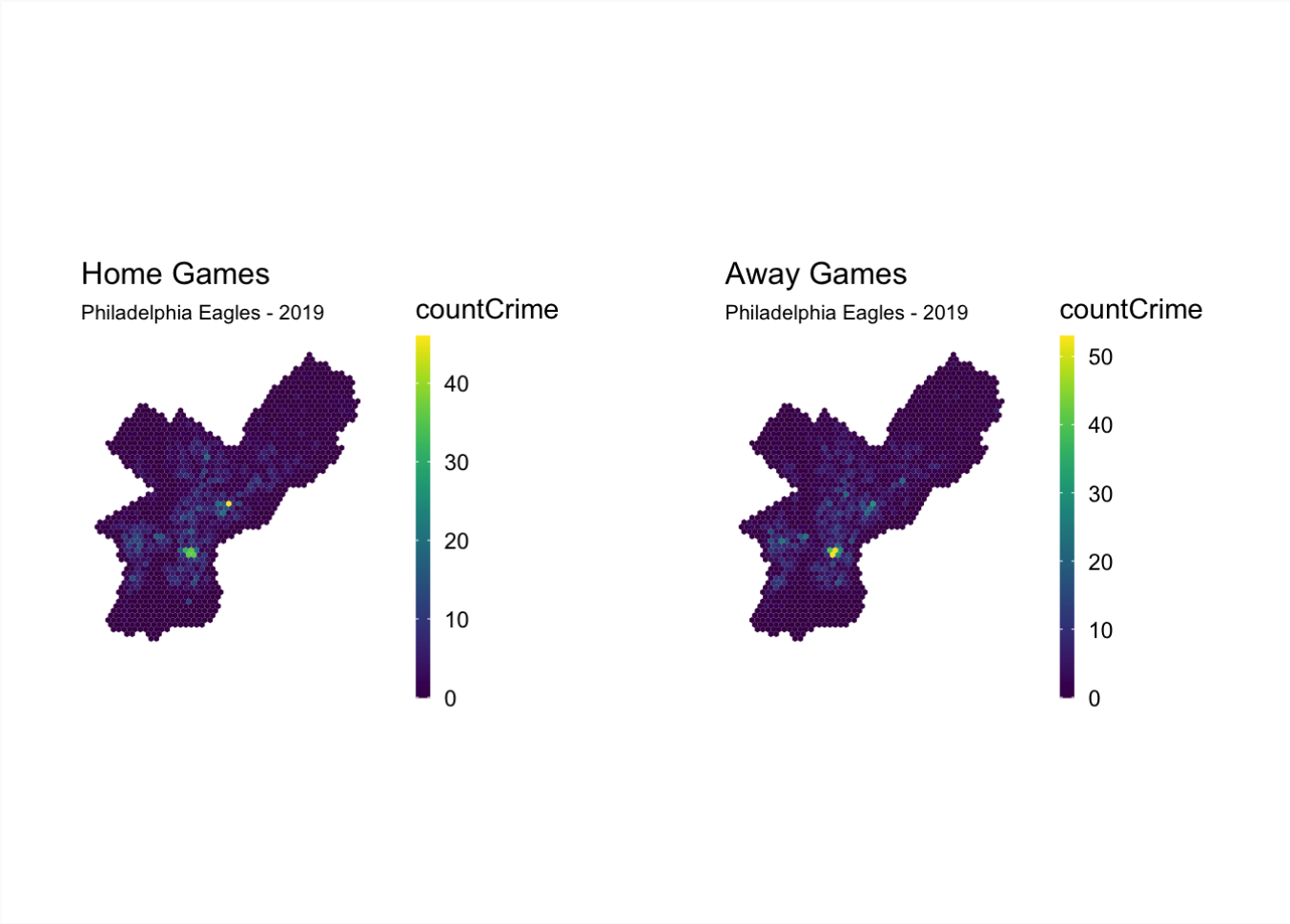
Consider an example: Trying to predict the sales price of a house.

$$Y = X\beta + \epsilon$$

Solution: Methods that simultaneously check and remove autocorrelation.

- Simultaneous Auto Regression (SAR) and Conditional Auto Regression methods.
- package `spatialreg` in R.

Crime in Philadelphia (McClellan 2022)



- Is there spatial autocorrelation?
- Is there omitted variable bias?
- How should we control spatial unobserved features?

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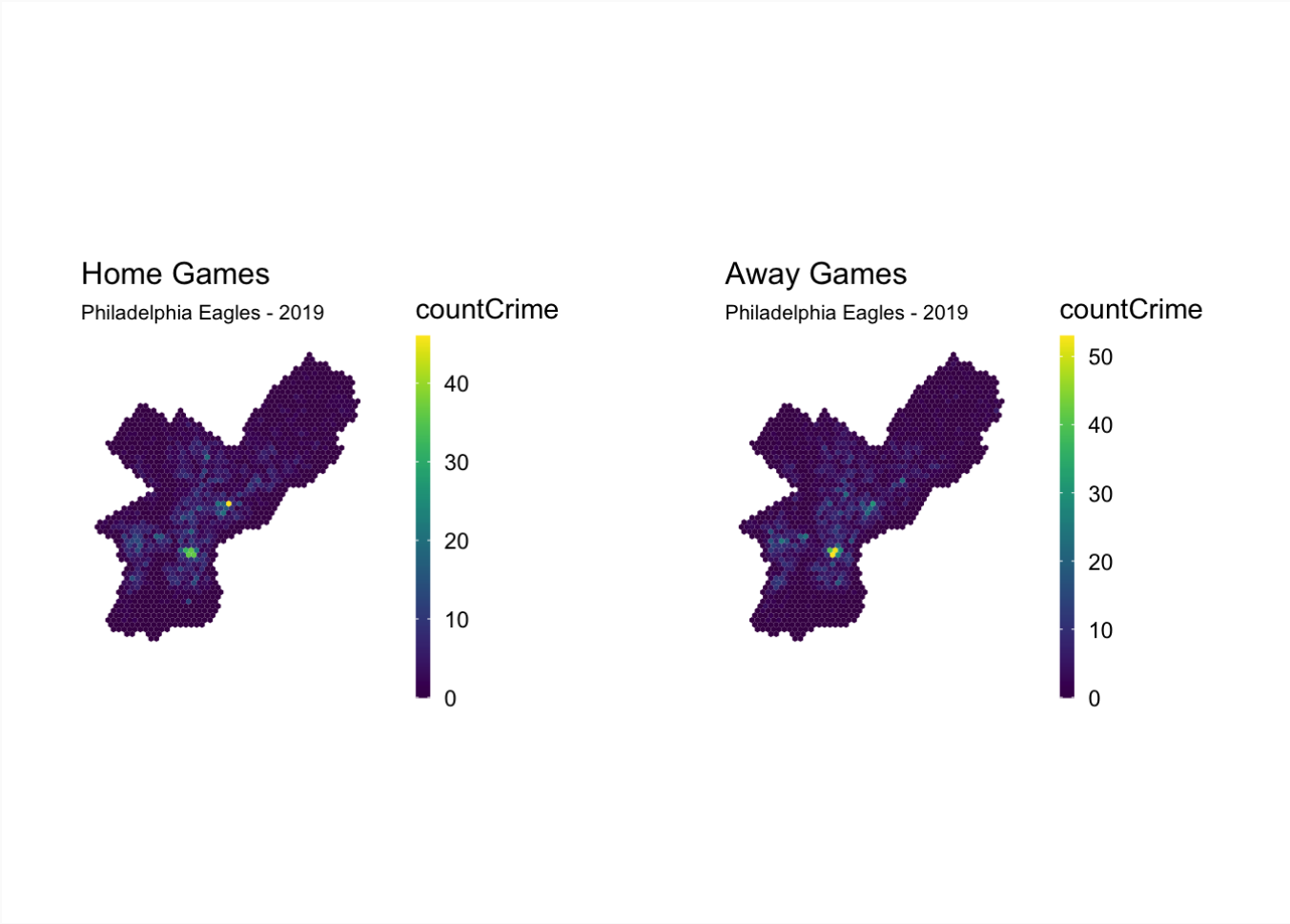

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$$crime = \alpha + \beta_0 anygame + \beta_1 homegame * anygame + \beta_2 Z + \epsilon$$

Demo

Mid-Point Presentations

Room A: Here Room B: 323

Add your name to a slot below.				
	Room A		Room B	
	Mar 18	Mar 25	Mar 18	Mar 25
Slot 1	Anran Zheng	Rui Jiang	Hanyu Gao	Lechuan Huang
Slot 2		Hasa	Ben Aiken	Ziyi Yang
Slot 3	Sean McClellan	Jonathon Sun	Yebei Yao	Aidan Cole
Slot 4	Chi Zhang	Will Friedrichs	Gianluca Mangiapane	Yuehui Gong
Slot 5	Jiamin Tan	Ziyuan Cai		Elisabeth Ericson
Slot 6		Tristan Grupp		Alex Nelms
Slot 7		xiaoyi		Hanpu Yao