# A Unified Scaling Law for Bootstrapped DQNs

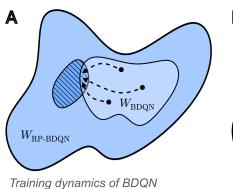
Roman Knyazhitskiy, Pascal van der Vaart, Neil Yorke-Smith

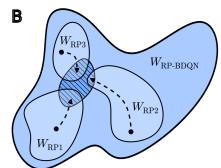


#### Introduction

Reinforcement Learning (RL) is concerned with finding a policy that maximizes cumulative reward over a sequence of actions. A successful algorithm has to balance (epistemic) exploration with exploitation [1].

Bootstrapped-based methods allow for such balancing by means of an approximate posterior over Q-networks.





Training dynamics of RP-B

We are analyzing and comparing the convergence behavior of Bootstrapped DQN (BDQN) [2] and BDQN with Randomized Priors (RP-BDQN) [3].

We show that the probability of discovering the only rewarding path on DeepSea [4] (PoD) for both methods is governed by a simple K-trials binomial law:

$$P(\text{discovery}) \approx 1 - (1 - \psi^n)^K$$

We also show that Randomized Priors help to prevent posterior collapse, which allows them to have higher ψ.

### Research Questions

- 1. How do BDQN and RP-BDQN scale with the ensemble size and DeepSea [4] environment size?
- 2. Can we describe the probability of these methods discovering a solution using a closed-form scaling law that is robust to changes in hyperparameters?
- 3. Where does the identified scaling law break down, and are there any properties of the ensemble that are related?

# Method & Results

We ran >40,000 experiments on DeepSea [4] environment; each one for 50k per-ensemble episodes.

We show that the probability of discovering the rewarding path is fairly well approximated by our simple model:

Algo	$\mathbf{prithm}$	Para	$\mathbf{G}$	Goodness-of-				-fit $(R^2)$			Dispersion			$\mathbf{MSE}$		
BDÇ	N	0.8					0.84 0.69			4.1 8.1			$0.024 \\ 0.049$			
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	BDQN vs $\psi=0.80$ law								RP-BDQN vs $\psi=0.87$ law							
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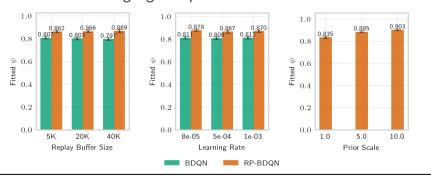
# Posterior Collapse

The introduced Q-Diversity metric – standard deviation of Q-values over different ensemble members on hold-out set of states – shows that randomized priors reduce posterior collapse.



# Hyperparameter Sensitivity

We find no effect on the law parameter from changing learning rate and replay buffer size, while a significant effect from changing the prior scale.



### Limitations

- 1. The law works well in medium-K regime, but fails in small- and large- K regimes. We attribute this to the compute budget limit and our member-independent model. The dispersion indicates an overall poor fit.
- 2. No use of more complex environments, because of no clearly defined "hardness" metric, which limits applicability.

#### Future work

- 1. Optimizing for  $\psi$ : Moving from a descriptive to a prescriptive use of our scaling law by designing algorithms that explicitly aim to maximize  $\psi$ .
- 2. Refining the Scaling Law: Developing a more nuanced model that accounts for the cooperative effects to better capture ensemble dynamics.

# GitHub



## References

- [1] "Reinforcement Learning: An Introduction" Sutton R. and Barto A.
- [2] "Deep exploration via bootstrapped dqn", Osband I et al
- [3] "Randomized prior functions for deep reinforcement learning". Osband I. et al.
- reinforcement learning", Osband I. et al [4] "Behaviour suite for reinforcement learning"
- [4] "Behaviour suite for reinforcement learnin Osband I. et al