Acoustic Control of Silica Beads

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Our objective is to create acoustic standing waves in one dimension in a fluid medium and use them to influence the position of free-floating and confined silica beads. Along the way, we propose the creation of a more direct means of controlling the fluid (as opposed to stage oscillation), broadening the space of possible experiments using Junior Lab's optical trap.

I. BEAD MECHANICS

We can model the motion of a silica bead in water using the framework of Newtonian mechanics. The position of a bead, $x(t)$, in an elastic medium whose state is specified by its displacement over all space and time by $w(\vec{x},t)$ is governed by

$$F = m\ddot{x} = D\left(\dot{w}(x,t) - \dot{x}\right). \tag{1}$$

The fluid is coupled to the bead via viscous drag, the magnitude of which is determined by the drag coefficient D and the magnitude of the relative velocity of fluid flowing past the bead. When $m \ll D$ (the low Reynolds number regime in which we are interested), Equation 1 reduces to

$$\dot{x} = \dot{w}. \tag{2}$$

We assume that the motion of the bead does not significantly alter the fluid. In so doing, the equation of motion for the fluid is simply the wave equation with dispersion relation $\omega = c\vec{k}$, where c is the speed of sound of the medium. These equations admit to standing wave solutions for the velocity of the fluid $w(\vec{x},t)$ and thereby the bead $x(t)$, confining the bead between velocity antinodes separated by $\lambda/2 = \pi/\vec{k}$.

II. SCALES & TRAP CHARACTERISTICS

In this section we compute the design specifications for the apparatus on the basis of the bead dynamics discussed above.

Our key design problem is to generate a standing wave in the fluid surrounding the bead that will trap it in place. We will generate the standing waves $w(x,t)$ by oscillating one side of the flow channel, so to design our oscillation mechanism we need to know $|w|$, how large the oscillations should be to trap the bead.

II.1. Calculation of Oscillation Amplitude

The amplitude of the velocity standing wave, $|\dot{w}|$, is related to the trapping force F by D, the drag coefficient, as in:

$$F \approx D|\dot{w}| \tag{3}$$

From optical trapping, we know that $D = 3\pi\eta d$ where η is the viscosity of the fluid and d is the diameter of the bead. Re-arranging 3, we get $|\dot{w}| \approx \frac{F}{3\pi\eta d}$.

For standing waves of wave-number $\vec{k}$, $|\dot{w}| = \omega|w|$ where $\omega = c|\vec{k}|$ is the angular frequency of the wave and $\lambda = \frac{2\pi}{|\vec{k}|}$ is the wavelength. Therefore, we have determined that:

$$|w| = \frac{\lambda F}{6\pi^2 c\eta d} \tag{4}$$

The diameter of the bead is $d = 1\mu m$. The force F needed to trap a $1\mu m$ silica bead is taken to be $20pN$, as determined in our previous experiments in optical trapping. In order to trap a $1\mu m$ silica bead, we need the wavelength λ of the standing wave to be at least $1\mu m$, giving $|\vec{k}| = \frac{2\pi}{10^{-6}m}$. We also have the speed of sound in water $c = 1.5 \times 10^3 m.s^{-1}$ and the viscosity of water $\eta = 10^{-3} Pa.s$. Substituting these values into the calculation, we obtain $|w| = 10^{-13}m$ or $0.1pm$ for the required size of the oscillations.

II.2. Impedance Matching

In order to eliminate reflections from the fluid back onto the transducer, we need to ensure that the impedance of the fluid is close to the impedance of the transducer.

The impedance of a coil of ?copper? with a magnet of mass m attached to it is ...

The impedance z of a fluid is known to be $z = \rho c$ where ρ is the density of the fluid and c is the speed of sound. The density of water is $\rho = 10^3 kg.m^{-3}$ and the speed of sound in water is $c = 1.5 \times 10^3 m.s^{-1}$. Therefore, the impedance of water is $z = 1.5 \times 10^6 kg.m^{-2}.s^{-1}$.

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III. APPARATUS

In this section we detail the construction of the flow channel and transducers.

The flow channel consists of a slide, two transparent spacers, and a slip forming the lower, middle, and upper layer of the flow channel respectively. The layers can be adhered together with water-tight epoxy. The stiffness of the membrane on either end of the flow channel as well as the mass of the attached magnet is critical because of impedance matching. With fixed cross section of 1 mm2, the longitudinal linear dimension of the magnet in the axis of the flow channel can be adjusted to adjust the mass of the oscillator. This dimension affects the magnetization of the magnet and therefore the current necessary to drive a wave of a particular amplitude given a number of turns of the coil N. Furthermore each coil must be impedance matched to the function generator with 50 impedance.

The coils are to be no bigger in cross section than a pencil. They should be easy both to remove and setup on the apparatus.

We'd like to construct both a channel with two transducers, and a channel with one transducer and an infinite impedance termination. We predict the latter will ease the creation of standing waves but not controllably in the aforementioned sense.

IV. MEASUREMENTS

V. SCHEDULE