21-cm Radio Astrophysics

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We use a computer-controlled parabolic antenna, located on the roof of Building-26 at MIT, to collect radio waves at 21 cm from the sky. In the first part of the experiment, we measure the brightness temperature of the sun at 21cm. In the second part of the experiment, we observe the Doppler spectrum of the 21-cm hyperfine line of interstellar atomic hydrogen in various directions in the plane of the Milky Way galaxy. These spectra enable us to measure the rotation-curve and deduce certain features of the spiral-arms of the Milky Way.

I. PREPARATORY PROBLEMS

1. Explain the origin of the 21-cm line of atomic hydrogen.

2. Describe the size and shape of the Galaxy, giving our position from the center in light-years and parsecs (pc). What is the maximum Doppler shift you can expect to observe in the 21-cm line?

3. The parabolic dish antenna used in this experiment has a diameter of 7.5 feet. Suppose it is set so that a source of 21-cm radio emission drifts through its field of view. Plot the signal strength at the output of the diode detector as a function of angle from the center of the field of view.

4. Explain the 21-cm spectrum at $150°$ and $60°$ as shown in Figure 7.

5. Explain the signal chain in this experiment (e.g. how the hardware and software serve to convert the signal received by the telescope into a display of signal strength versus frequency.

II. PROGRESS CHECK FOR 2ND SESSION

What is the approximate temperature of the sun at 1420 MHz? Plot the power spectrum of galactic hydrogen at 40 degrees galactic longitude and identify all the salient characteristics of the spectrum. Does it agree with Van de Hulst's observation in Figure 7?

III. GENERAL STRUCTURE OF THE MILKY WAY

This description of the Galaxy is a summary of the material contained in Galactic Astronomy by Mihalas and Binney [1], a very useful reference for this experiment. It also contains in Chapter 8[1] a detailed presentation of the theoretical basis for interpreting the data on galactic kinematics obtained from observations of the Doppler profile of the 21-cm line. Another very nice introduction to the structure of the Milky Way is on-line at `http://cassfos02.ucsd.edu/public/tutorial/MW.html`. Other useful references for this lab are [2–8].

The Sun is one among the approximately 10^{11} stars that comprise our Galaxy. From a distant vantage point this vast array of stars would appear as a rotating spiral galaxy. Detailed examination of the Milky Way reveals a spheroidal component and a concentric disk component. The spheroidal component consists mostly of small ($M < 0.8M_\odot$), old (10-15 billion years) stars of which most are concentrated in a central "bulge" with a diameter of ~ 3 kpc and the rest are distributed in an extensive "halo" that extends to a radius of 30 kpc or more. The disk component is a thin, flat system of stars, gas and dust $\sim$200 pc thick and $\sim$30 kpc in radius. The disk stars are extremely heterogeneous, ranging in age from new-born to the age of the halo stars, and with masses from ~ 0.1 to $\sim 100M_\odot$.

The solar system is imbedded in the disk component at a distance of 8.0±0.5 kpc from the center at which radius the circular motion of the Galaxy has a period of approximately 2×10^8 years.

Between the stars of the disk is the interstellar medium (ISM) of gas and dust comprising approximately 10% of the total mass of the disk. The gas consists primarily of hydrogen and helium with a mass density ratio of 3 to 1 and an average total number density of about 1 atom cm^{-3}. The dust is composed mostly of graphite, silicates and other compounds of the light and common elements in microscopic grains containing a small fraction of higher-Z elements, primarily iron.

Comparison of visible spectra from the stars with the line spectra of elements in the laboratory yielded information on the composition and temperature of stars, and measurement of wavelength shifts due to the Doppler effect provided determinations of radial velocities which revealed the dynamical properties of systems. Most of the interstellar medium, however, is too cold to radiate in the visible part of the spectrum and its properties remained largely unknown until fifty years ago. Then, in the midst of World War II, a young Dutch astronomer, H. van de Hulst, examined the theoretical possibilities for detecting radio waves from the ISM, which would permit

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[1] This chapter can be found on 8.13 website.

measurements of physical conditions and radial motions like those available in the visible spectrum. He predicted that the spin-flip transition of atomic hydrogen would produce such a line at a wavelength near 21 cm, and that the unique conditions of low density and temperature in interstellar space are such as to allow time for hydrogen atoms, excited by collision to their hyperfine triplet state, to decay by radiation to the singlet ground state.

The line was observed in 1951 by Ewing and Purcell at Harvard, by Christiansen in Sydney, and by Muller and Oort in the Netherlands. Radio observations at 21 cm soon became a major tool of astronomy for exploring and measuring the structure of our Galaxy and many distant galaxies too.

Atomic hydrogen is the principal constituent of the interstellar medium, and one of the most interesting tracers of galactic structure. Whereas visible light is heavily obscured by dust in the ISM, radiation at 21 cm suffers little absorption.

Measurements of the Doppler shifts of the frequency of the 21-centimeter line determine the radial components (projections onto the line of sight) of motions of the interstellar medium. Analysis of the Doppler shifts in various directions around the galactic plane (i.e. around the Milky Way) reveal the kinematic structure of the galaxy, e.g., the tangential velocity of the matter in the Galaxy as a function of the distance from the center, and, by implication, the geometry of the spiral arms.

IV.   POSSIBLE THEORETICAL TOPICS

While doing this experiment, you might find it useful to have a basic know-how about the following:

1. Hyperfine splitting of the hydrogen ground state.

2. Radiative processes in the sun.

3. Antenna theory.

4. The structure and dynamics of our Galaxy.

5. Radiative transfer.

V.   SOME JARGON USED IN EXPERIMENT

Please look up the definitions of the following terms before you do the experiment.

1. Brightness Temperature

2. Antenna Temperature

3. Antenna Pattern or Antenna Function

4. Rotation Curve

5. Local Standard of Rest

6. Local Sidereal Time

7. Galactic Longitude

8. Azimuth and Elevation

VI.   EXPERIMENTAL APPARATUS

Many details regarding the antenna, calibration and experimental procedures, and documentation for the software can be found on the Junior Lab Student Wiki and the Junior Lab Website.

VI.1.   Hardware

VI.1.1.   Antenna & Motors

The small radio telescope, SRT, is a 7.5 foot diameter parabolic dish and receiver designed by Haystack Observatory and sold by Cassi Corporation. Complete specifications for the antenna, mount and receiver are available under the "Antenna and Mount Information" link at the SRT website (`http://www.haystack.mit.edu/edu/undergrad/srt/`). It has a focal length of 85.7cm. It has a beam width of approximately 7.0 degrees.[2]. The dish is mounted on a two-axis azimuth/elevation mount. It is supported by an aluminum frame constructed from C/Ku band mesh that reflects all incident microwave energy if the surface holes are less than 1/10th of the incident wavelength. The system is controlled using a computer running a java applet that communicates with the controller, which contains a Basic-Stamp microcontroller. The microcontroller in turn controls the motor functions. A very basic block diagram of the system is shown in Fig. 1.
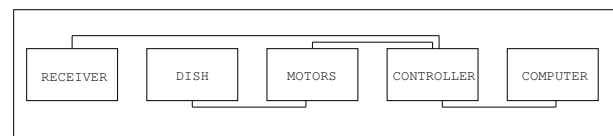

FIG. 1. A basic block diagram of the SRT

VI.1.2.   Receiver

The SRT uses a phasing-type single-sideband scanning receiver. Fig. 2 shows a block diagram of the radio receiver and subsequent signal processing stages. Radio power arriving from the sky is focused by reflection to

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[2] The first minimum of the antenna diffraction pattern is given by $1.22\lambda/d \approx 7°$.

the antenna feed horn. Signals then pass through a band pass filter, a low noise pre-amplifier and a mixer. The signal is digitized and sent back to the controlling computer over a serial RS-232 link. The central frequency of the synthesized local oscillator (LO) used in the mixer is user selectable from within the JAVA program. This permits the investigator to look at different spectral regions around 1420 MHz.
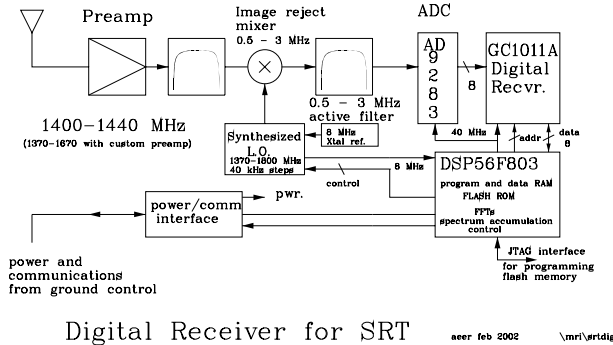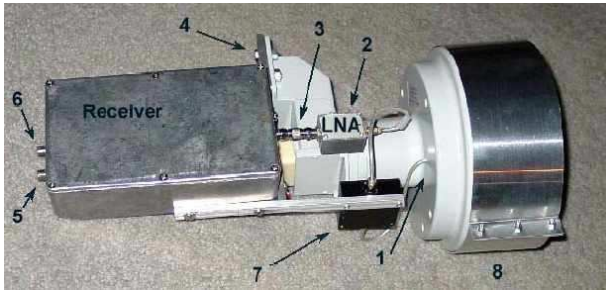

FIG. 2. A block diagram of the SRT Receiver

FIG. 3. A picture of the antenna feedhorn

  1 - L-Band probe
  2 - Low Noise Amplifier
  3 - Universal Male F to F Coupler
  4 - Receiver mounting bracket
  5 - Video Port - Analog signal
  6 - Power and Communication - Digital Signal
  7 - Band Pass Filter
  8 - Feed Horn Extension

Signals pass from the feed horn to the low noise amplifier. The LNA provides a 24dB gain whose output passes through a 40MHz bandpass filter that prevents band signals from producing intermodulation in the image rejection mixers. Mixers are circuit elements that form the product of two analog waveforms (in NMR these elements are also called "phase detectors"). It's two inputs and output are related by the trigonometric relationship:

$$\cos \omega_1 t \cos \omega_2 t = \frac{1}{2} \cos (\omega_1 + \omega_2)t + \frac{1}{2} \cos (\omega_1 - \omega_2)t. \quad (1)$$

Note that both sum and difference frequencies are generated in a mixer. For our application, we band-pass filter

the output and throw away the sum frequency contribution. See the "Receiver Information" link on the SRT website for more information.

VI.1.3. Electronic Noise Calibration

Electronic noise calibration is performed using a noise diode, whose intensity and spectral distribution is approximately equivalent to a 115 Kelvin blackbody.

Compare this to the known signal strengths of some radio sources in the sky, shown in Figure 4.

The electronic noise calibrator is a diode (Noise/Com NC302L) connected to a small dipole antenna attached to the center of the SRT dish. A small (6mA) current is sent through a controlled bulk avalanche mechanism, resulting in a wide band of frequencies. The dipole emits the signal which is detected by the receiver. To generate the correct wavelength signal, the dipole was fabricated to be 1/2 wavelength, or approximately 10.5 cm. To minimize signal from behind the dipole, the poles are set 1/4 wavelength, or approximately 5.25 cm, above the circular plate at the center of the SRT dish.

In practice, to calibrate the telescope you just have to point the antenna at an empty part of the sky, change the frequency to the hydrogen frequency, and hit the 'Calib' button on the software interface. The software outputs a single number called T_{sys} which represents the background noise level in the device.

The software measures the ratio of the received power when the noise diode is turned on and then off again. Complete details of the SRT calibration are available at `http://www.haystack.edu/edu/undergrad/srt/receiver/SRT_calibration.html` and `http://web.mit.edu/8.13/www/calibrator_report.pdf`.

VI.2. Software

The SRT control software interface is run from a computer which communicates with the ground controller in 26-630. The ground controller serves as the interface between the computer and the antenna stepper motors as well as the digital receiver located next to the antenna feed. RS-232 is used to send commands to the motors and to receive back the digitized spectral temperatures from the receiver[3]. The ground controller should be left ON so that students can remotely operate the dish from 4-361.

A printed copy of the **SRT Manual** is located next to the computer in 4-361 and is also available online on the Junior Lab Wiki. This manual is very helpful for understanding how to work with the software.

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[3] Full details and schematics of the ground controller are available at `http://www.haystack.edu/edu/undergrad/srt/receiver/schemgndctl2.pdf`
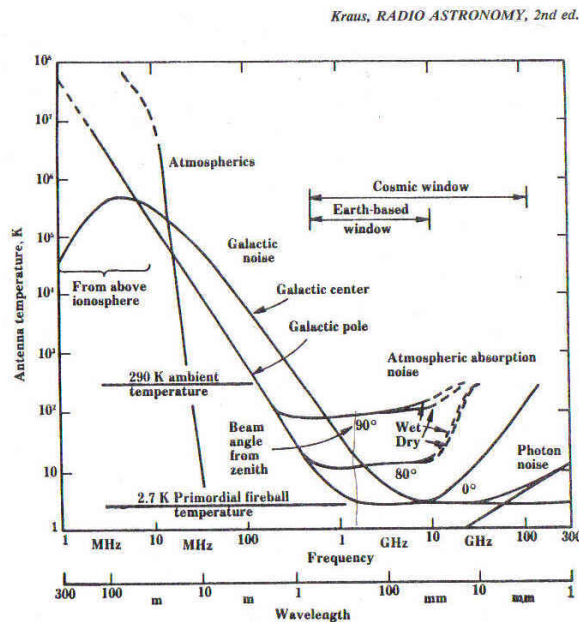
Kraus, RADIO ASTRONOMY, 2nd ed.

Fig. 8-6. Antenna sky noise temperature as a function of frequency and angle from zenith. A beam angle (HPBW) of less than a few degrees and 100 percent beam efficiency are assumed. (After *Kraus and Ko*, 1957, *cosmic noise between* 10 MHz *and* 1 GHz; *Penzias and Wilson*, 1965, *cosmic noise above* 1 GHz; *Croom*, 1964, *atmospheric noise*; *CCIR*, 1964, *atmospherics*; *and Radio Astronomy Explorer Satellite* RAE-2, *Novaco and Brown, cosmic noise below* 10 MHz.)

FIG. 4. Noise Temperature versus angle from zenith. Taken from Kraus, 2nd Edition.

Please take a while to look at the panel on the right-hand-side of the software window. Familiarize yourself with the information that is shown there.

The SRT is continually acquiring and processing RF signals incident upon the dish. The upper right plot in the software window shows individual power spectra updated every few seconds. The plot to the left in RED, shows an integrated power spectrum which may be cleared by pressing the "CLEAR" button.

It is convenient to check the system by generating a test signal at 1420.4 MHz. This is done with the Marconi RF synthesizer. Turn the device on, press the "RECL" button, then choose the first setting (enter '01'). This will set the generator to produce an RF signal at 710.2 MHz, which will also cause the antenna to emit the first harmonic, namely the desired 1420.4 MHz line. Press the "CARRIER ON/OFF" button to toggle the signal on and off. During this calibration, you can point the telescope at any clear part of the sky (avoid buildings – they emit and reflect all manner of infrared). Make sure that you don't leave this synthesizer on except for brief tests and system calibrations, because it will interfere with any other 21 cm experiment nearby.

You can execute commands via the toolbar located at the top of the console. Clicking on any of these buttons either initiates an automatic response or the software waits for text input by the user.

Azimuth and elevation locations are set using the Horizon Coordinate system. Azimuth and elevation are entered in degrees. The controller moves the telescope to the required position. If "tracking mode" is on (button text is YELLOW), when the telescope reaches the position, the controller continues to move the telescope to compensate for the Earth's rotation so as to keep the telescope pointing to the specified position in the sky. To turn tracking off, simply click on the button and it should turn RED.

Try the following to get started: Enter "180 40" in the text window and then press the Azel button. The dish should slowly move to point due south. Press STOW to return the dish to it's "parked" position at Azimuth=95°,Elevation=4°. Press the "RECORD" button to stop writing to the log file and open it up using a text editor.

It is useful to begin a log file ('.rad' file) which provides you with a continual record of your experiment. You can do this using the RECORD button. Hitting the record button again will cause the recording to stop.

Instead of using individual buttons from the toolbar, in some parts of the experiment it is useful to write a script (a '.cmd' file) and load the script to be run. Please see the SRT manual for sample scripts.

The program calculates the Local Sidereal and Universal Time based on the computer clock settings within the information sidebar on the right-hand side. **Since Universal Time = Eastern Time + 5 hours (under Standard Time) and = Eastern Time + 4 hours (under Eastern Daylight Savings Time), you must make sure the box "Automatically adjust clock for daylight savings" is checked when setting the computer's clock.** It also indicates the antenna direction in both Equatorial (RA and DEC) and Galactic ('l' and 'b').

When turning on the system, pay special attention to any error messages that may appear within the Message Board below the Total Power Chart Recorder. They may indicate that the system has failed to communicate through the coaxial cable (i.e. gnd 0 radio 1). If movement of the dish stops prematurely, "antenna drive status.." or "lost count" may appear in the Motion Status Display Area. If this occurs please turn off the control system (which in turn ends motor power) and have a technical staff member take a look at the motor drive gears. Be sure to return the dish to the stow position each time before powering down and quiting the java applet.

VII.   EXPERIMENTAL PROCEDURE

To prepare yourself for these measurements, start by reading the Haystack 'SRT Projects' link on the SRT wesite.

First of all, set the frequency to "1420.4 4" in order to set the bandwidth as wide as possible to reduce noise variations and calibrate the system temperature T_{sys}

VII.1.   The Sun's Brightness Temperature

Do an "npoint" scan of the Sun. By clicking on the "npoint" button, a series of measurements surrounding the sun in a 5×5 grid are made with the step size set by the antenna beamwidth defined in the "srt.cat" file. The default value for the beamwidth is $5°$. The measured power/temperature at each of these grid points is displayed just above the text input box and at the completion of the scan, a 2-D contour plot of the grid is displayed in the graph box at the top of the page. Information about the sweep is then displayed in the information sidebar. The telescope offsets are also adjusted so as to point the dish to the maximum observed signal within the sampled region.

The "npoint" scan is also useful for verifying the targeting accuracy of the system. If the computer clock is set incorrectly or the local latitude and longitude have been inadvertently changed, the sun will NOT appear well centered in this gridded scan. See for example, Figure 5.
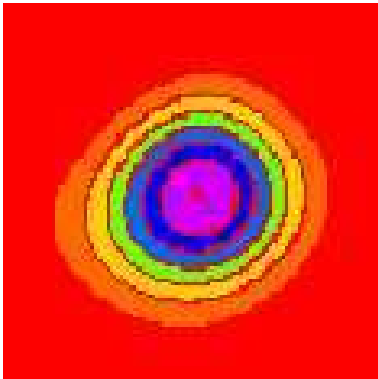

FIG. 5. Typical result from an "npoint" scan. The contour plot is generated from a 5×5 series of measurements spaced by 1/2 the antenna beamwidth as specified in the srt.cat text file within the CassiSRT folder.

The "npoint" scan will focus the dish at the maximum temperature location for the sun when completed. Pointing corrections should be set to the location of peak intensity before continuing the observation session.

To manually specify offsets, enter the offset in the command text area.

```
enter azimuth(degrees) elevation(degrees)
Press ``offset'' button
```

If you enter -5 -5, the effect is that the npoint plot will move right and up.

Do an azimuth scan and an elevation scan of the Sun. For this, you will need to write a .cmd file specifying the various offsets from the Sun that you want to take data. For example, these offsets may range from $-30°$ to $30°$ in steps of $2°$. Your antenna temperature vs. angular offset curve should resemble Figure 6.

You can also use the "drift" button to cause the dish to be pointed "ahead in time" and then stopped. This will permit the sun, or any other object, to drift through the antenna beam.

This data will enable you to derive an empirical value for the half-power beamwidth (HPBW) of the antenna, which can be compared to the theoretical value.

Measurement of the beam pattern can help the user discover problems with optical alignment or aid in the determination of antenna focus.

What is the convolution of a finite source with the radiation pattern of the antenna beam? Can you model this? An interesting comparison can be made by comparing your results with the solar flux measured by other antennas around the world, see `http://web.haystack.mit.edu/SRT/solar.html`.
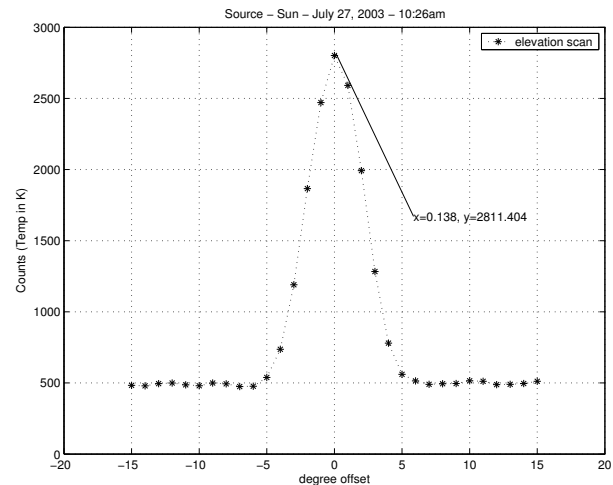

FIG. 6. A typical Azimuth or Elevation scan of the sun.

VII.2.   Rotation Curve of the Milky Way

To measure the rotation curve of the Milky Way, you will need to record hydrogen line profiles at a known set of galactic coordinates.

Your first job is to figure out at which coordinates to point the telescope. Use the software to determine when these regions of the galaxy will be visible to the telescope. Since the visible portion of the galaxy varies with time, you may find it necessary to share observation time with student pairs from other sections.

Derive the Galactic rotation curve, and estimate the mass of the Galaxy interior to the circle tangent to the lines of sight.

Observe the non-uniformities in the hydrogen distribution revealed by the doppler-induced structures of the line profiles. The multiple-component line profiles are evidence of the spiral-arm structure of the Galaxy.

VIII.   ANALYSIS

1. Compare your plot of the angular response function of the antenna obtained in the scans of the sun with the theoretical diffraction pattern of a circular aperture.

2. Derive an estimate of the brightness temperature of the sun at 21 cm from your measurements.

3. Using your Galactic data and with the help of the discussions presented in Mihalas and Binney[1] and by Shu[2], derive a plot of the velocity curve of the Galaxy as a function of radius. Why are you only able to do so for locations interior to the radial position of our solar system?

4. Optional. Using the Galactic data, construct some features of spiral-arm structure of the Milky Way.

[1] D. Mihalas and J. Binney, *Galactic Astronomy* (San Francisco, 1968).
[2] F. H. Shu, *The Physical Universe - Chapter 12* (University Science Books, Mill Valley, CA, 1982) this reference gives a clear description of the interpretation of 21cm spectra in terms of the rotation curve of the Galaxy.
[3] K. Rohlfs and T. L. Wilson, *Radio Astronomy, 2nd. ed.* (Springer, 1996) a modern handbook of radio astronomy techniques. Section 5.5 describes the relationship between antenna temperature and brightness temperature.[4] Chapter 6 provides a useful discussion of antenna theory.
[4] e. a. H. C. van de Hulst, *Bull. of the Astron.*, Vol. XII, 117 (Institutes of the Netherlands, 1954).
[5] P. Duffett-Smith, *Practical Astronomy With Your Calculator*, 3rd ed. (Cambridge, 1988).
[6] I. S. Shklovski, *Cosmic Radio Waves* (Cambridge; Harvard University Press, 1960).
[7] H. Horowitz and W. Hill, *The Art of Electronics, 2nd Edition* (Cambridge University Press, 1989).
[8] F. J. Kerr, Ann. Rev. Astron. Astroph. **Vol.7**, 39 (1969).

[4] Note, however, the following errors in Section 5.5: The equation between equation (5.59) and equation (5.60) should read $I_\nu = 2kT/\lambda^2$. Equation (5.62) should read $W = \frac{1}{2}A_e \int \int \frac{2kT_b(\theta,\phi)}{\lambda^2} P_n(\theta,\phi)d\Omega$.
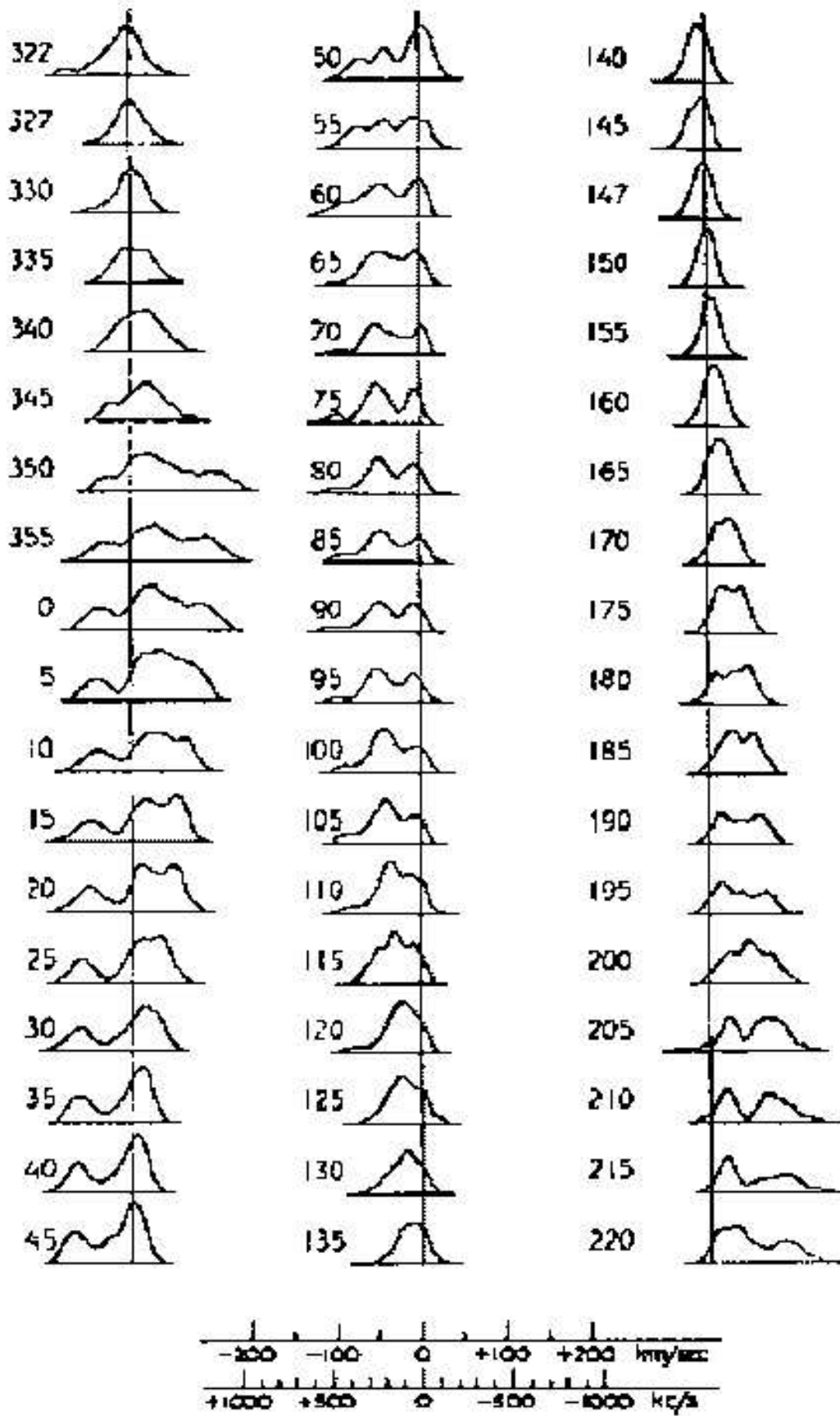
FIG. 7. Survey of hydrogen line profiles at various galactic longitudes. (from H.C. van de Hulst et. al., Bull. of the Astron. Inst. of the Netherlands, XII, 117, May 14, 1954) Note: Pictures are reversed from our images and longitude is given in the old system (l^I,b^I). $l^{II} = l^I + 32.31°$

Appendix A: The srt.cat file

The text file srt.cat, located in SRTCassi, is the primary configuration file for the SRT java interface program that is used to control the telescope.

The srt.cat file may be updated, but upon doing so, please stow the telescope, shut down the control program, and restart it so that it recognizes any updates made to the srt.cat control file.

The srt.cat file contains keywords which are case sensitive. Lines that are preceded by a "*" are ignored. Blank lines are ignored.

Keywords:
AXISTILT* - Use to set the azimuth and elevation axis tilt. Default is 0 0
AZEL* - Allows user to catalog a fixed catalog location by name.
AZLIMITS - This may be set to allow movement in a clockwise position from stow from 90° to 270°. At the midpoint the telescope will face due south. Degree limits should be set slightly below the physical limitations of the telescope. It is recommended to use 95° and 265°. The telescope movement is clockwise as seen from above through increasing degree.
BEAMWIDTH* - Antenna beamwidth in degrees, default is 7.0°.
CALCONS* - Gain correction constant. This is the ratio of temperature (K) per count. Change this during calibration (should be 1 to 1 ratio). Default is 1.0. This will be changed by software after calibration occurs and value will be available in the status bar.
COMM* - Communication port, default 1, use 0 for linux.
COUNTPERSTEP* - Counts per step for stepped antenna motion. The default is no stepped motion.
CURVATURE* - Optional correction for curvature in spectrum. The default is 0.
DIGITAL - Indicates that a digital receiver is being used. Comment out if using the analog receiver (set via jumper removal).
ELBACKLASH* - Optional correction for elevation backlash to improve pointing in flipped mode. The default is 0.
ELLIMITS - This may be set to allow movement in a clockwise position from stow from 0° to 180°. At the midpoint the telescope will face up. Degree limits should be set slightly below the physical limitations of the telescope. It is recommended to use 10° and 175°. The telescope movement is clockwise as seen from above through increasing degree.
GALACTIC - Set a galactic coordinate by location and name.
MANCAL* - Calibrates vane. 0 indicates auto-calibration, 1 indicates manual calibration. Is not a requirement of the digital setup.
Moon - Add the Moon to the catalog.
NOISECAL - Calibration temperature of the noise diode located at center of dish. Temperature is in K.
RECORDFORM* - Adds tabs to separate columns in output file. The default is space delimited. "VLSR" adds vlsr and "DAY" forces a file change at each new day.
SOU - Set a source in the catalog by location and name. For negative declination us "-" in from of dd.
SSAT* - Satellite ID and location (satellite name then longitude west).
STATION - Used to set latitude (in degrees), longitude (in degrees), and station name.
Sun - Add the Sun to the catalog.
TLOAD* - Load temperature. Default is 300K.
TOLERANCE* - Counts of error which can accumulate before command to drive stow occurs. Default value is 1.
TSPILL* - Antenna spill over temperature in K. Default is 20K.

File Format:
STATION deg(latitude) deg(longitude) name
AZLIMITS deg(lower limit) deg(upperlimit)
ELLIMITS deg(lower limit) deg(upper limit)
CALCONS value
TLOAD value(in temp K - default 300K)
MANCAL value(0 or 1 - default is 0)
TSPILL value(in temp K - default 20K)
BEAMWIDTH value(in degrees - default 7)
AXISTILT deg(azimuth axis tilt - default 0) deg(elevation axis tilt - default 0)
SSAT deg
AZEL deg(azimuth) deg(elevation) name
GALACTIC deg(longitude) deg(latitude) name
SOU ra(hh mm ss) dec(dd mm ss) name [epoc]
SUN
MOON
DIGITAL
NOISECAL value(in temp K)
TOLERANCE value(in counts)
COUNTPERSTEP value(in counts)
ELBACKLASH value(default 0)
CURVATURE value(default 0)
RECORDFORM value(i.e. "TAB", "—", default is " ")

* - represents required fields or fields that should be set by user

[ ] - represents optional field

Appendix B: Coordinate Systems

This material is taken in part from Duffett-Smith's book "Practical Astronomy With Your Calculator, 3rd Edition 1988, Cambridge Press". It contains many useful algorithms and is worth checking out of the library or purchasing. There are several coordinate systems which you may meet in astronomy and astrophysical work and here we shall be concerned with four of them: the horizon system (Figure 8), the equatorial system (Figure 10), the ecliptic system (Figure 13) and the galactic system (Figure 9).

Horizon Coordinates "Az" and "El" The horizon system, using Azimuth and Altitude (Elevation) is the system under which the SRT is operated.
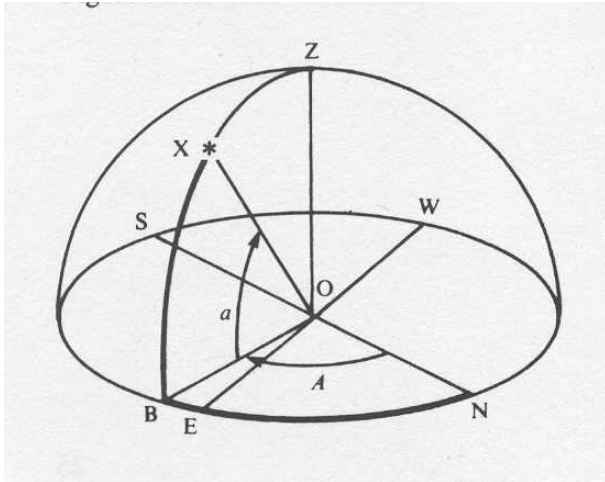
FIG. 8. Diagram of the Horizon coordinate system. Imagine an observer at point 'O'; then her horizon is the circle 'NESW' where North is the direction of the north pole on the Earth's rotation axis and not the magnetic north pole. **Azimuth** increases from 0° (north) through 360°. The **altitude** or **elevation** is 'how far up' in degrees (negative if below the horizon).

Galactic Coordinates, "l" and "b" When describing the relations between stars and other celestial objects within our own Galaxy, it is convenient to use the galactic coordinate system. The fundamental plane is the plane of the Galaxy and the fundamental directions the line joining our Sun to the center of the Galaxy as depicted in Figure 9.

Equatorial or Celestial Coordinates, "RA" and "DEC" are the spherical coordinates used to specify the location of a celestial object (Figures 10, 11 and 12). The north celestial pole (NCP) is the direction of the Earth's rotation axis, and the celestial equator is the projection onto the sky of the plane of the Earth's equator.

Right ascension (RA) is the celestial analog of geographic longitude. RA is measured eastward along the celestial equator from the vernal equinox ("V" in Figures 11 and 12) which is the ascending node of the plane defined by the sun's apparent motion (caused by the or-
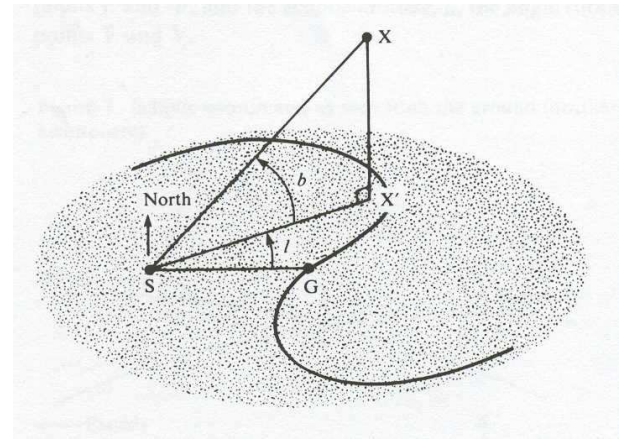

FIG. 9. Diagram of the Galactic coordinate system. The point marked "S" represents the Sun, "G" is the center of the Galaxy and "X" is a star which does not lie in the galactic plane.

bital motion of the Earth around the sun) and the celestial equator. In catalogs of celestial objects RA is generally specified in units of hours, minutes and seconds from 0 to 24 hours, but it is often more conveniently specified in degrees from 0° to 360° with a decimal fraction. Declination (DEC) is the celestial analog of geographic latitude. DEC is measured north from the celestial equator along a celestial meridian which is a great circle of constant RA. In catalogs DEC is generally specified in degrees, arc minutes (') and arc seconds ("), but it is also often more conveniently specified in degrees from -90° to +90° with a decimal fraction. (1 hour of RA at constant DEC corresponds to an angle of 15°*cos(DEC) degrees subtended at the origin).

The **ecliptic** is the intersection of the Earth's orbital plane with the celestial sphere. To an observer on Earth the sun appears to move relative to the background stars along the ecliptic with an angular velocity of about 1° per day. The angular velocity is not exactly constant due to the eccentricity of the Earth's orbit (e=0.016722). The period of the Earth's orbit is 365.256 days. The **inclination** (ie) of the Earth's equator to the ecliptic is 23° 27'. The **ascending node** of the ecliptic with respect to the celestial equator is the intersection of the ecliptic and the celestial equator (the vernal equinox) where the sun in its apparent motion crosses from south to north declinations on March 21. **Precession of the equinoxes** is the motion of the equinoxes along the ecliptic due to precession of the Earth's rotational angular momentum about the ecliptic pole. The precession is caused by the torque of the gravitational attractions between the sun and moon and the Earth's equatorial bulge. The period of the precession is approximately 25,000 years.

Ecliptic coordinates (Figures 14 and 13) are generally used to specify the positions and orientations of objects in the solar system. **Ecliptic longitude** (elon) is measured along the ecliptic eastward from the vernal
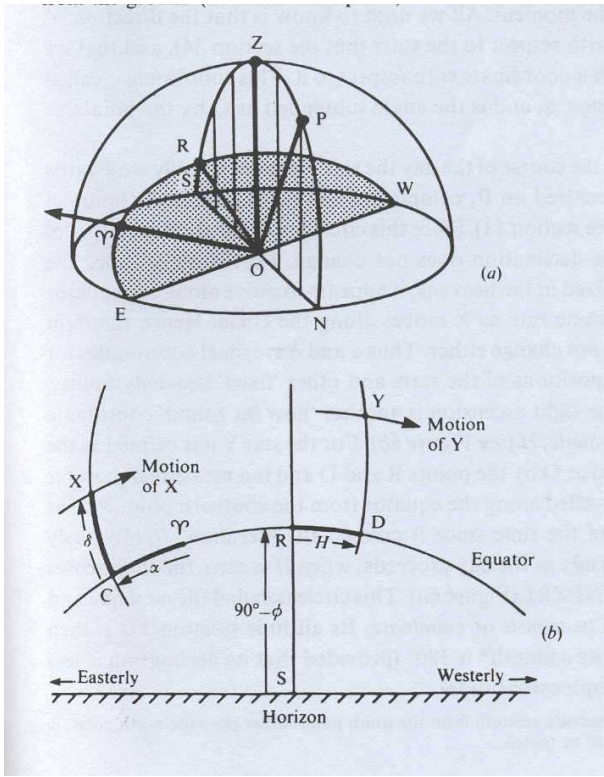
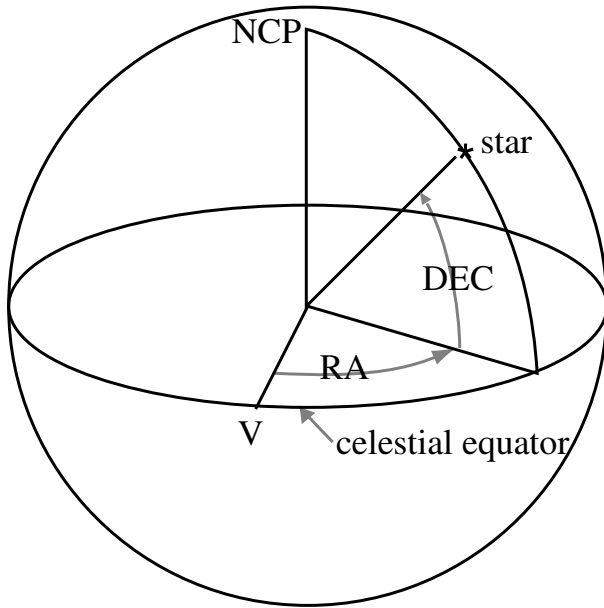FIG. 10. Diagram of the Equatorial coordinate system
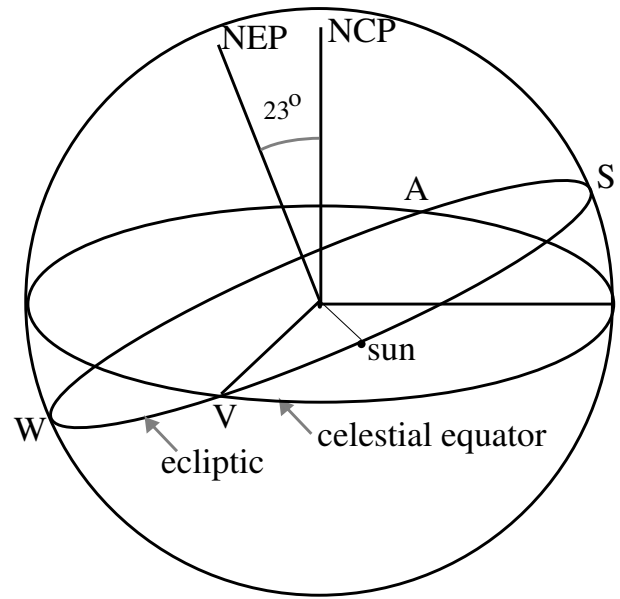
FIG. 11. Diagram of the celestial coordinate system

FIG. 12. Diagram of the celestial sphere, showing the celestial north pole (NCP), the celestial equator, the north ecliptic pole (NEP) and the ecliptic. The points labeled V, S, A, and W are, respectively, the vernal equinox, summer solstice, autumnal equinox, and winter solstice corresponding to the directions of the sun on March 21, June 21, September 21, and December 21. The point labeled "sun" is the direction of the sun on approximately April 21.
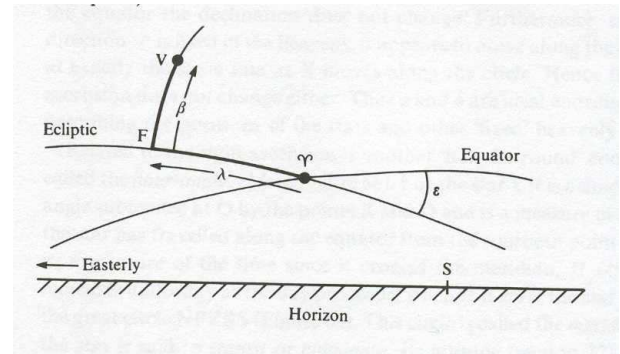
FIG. 13. Diagram of the Ecliptic coordinate system

rotation of the sun itself, as illustrated in Figure 14, is specified by the ecliptic longitude (ELON) of the ascending node of its equator and the inclination (INCL) of the equator to the ecliptic.

equinox. **Ecliptic latitude** (elat) is measured along a great circle northward from the ecliptic.

The orientation of the orbit of a planet is specified by 1) the ecliptic longitude of the ascending node of the orbital plane and 2) the inclination of the orbit to the ecliptic. Similarly, the orientation of a planet's rotation or the
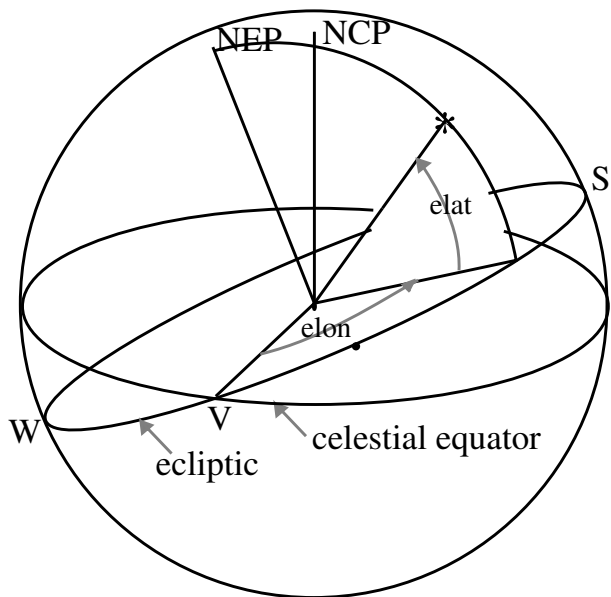
FIG. 14. Diagram showing the relation between the ecliptic and celestial coordinate systems
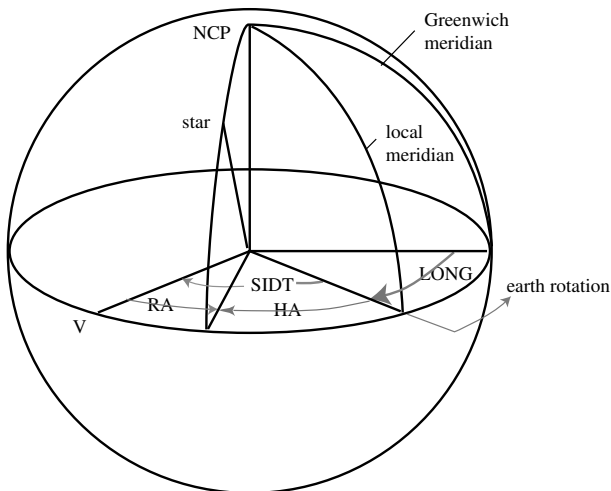

FIG. 15. Diagram of the relations between the quantities involved in setting the position of the telescope about the polar (HA or RA) axis

Appendix C: Signal Processing in 21-cm Radio Astrophysics

For a voltage $v(t)$ measured in the time domain we can compute the Fourier transform $V(\nu)$:

$$V(\nu) = \int v(t)e^{-2\pi i \nu t}dt \qquad (C1)$$

Note that $V(\nu)$ is *complex*, giving us an amplitude and a phase for each frequency component. In our experiment we are not interested in the phase information. Taking the square modulus of the Fourier transform gives us the voltage power spectrum:

$$S(\nu) = [V(\nu)]^2 \qquad (C2)$$

$S(\nu)$ is the quantity plotted by the data acquisition software. For the voltage power spectrum, the units are volts^2 (rms), and a sine wave with amplitude $V_o$rms will produce a signal of amplitude $V_o$rms/4, which becomes $V_o$rms/2 in the one-sided spectrum that is displayed (see below). The spectrum displayed on the Junior Lab computer screen has units dBm, which is power expressed in dB referred to one milliwatt. The spectrum values are converted to milliwatt units by taking into account the characteristic impedance of the transmission line, $Z_o = 50\,\Omega$. This is done according to $dBm = 10log\frac{Vrms^2}{.2236^2}$ where the $50\,\Omega$'s in the numerator and denominator have canceled. The power you measure $\frac{V_{rms}^2}{Z_0}$ is the power going down the cable to the A/D converter, related to, but not equal to, the power received at the antenna.

The above expression for the Fourier Transform, refer to a continuous signal measured for an infinite length of time. In practice, instead of $v(t)$, we in fact measure $v_i(t)$, discrete samples separated by a time interval δt, over a period of time Δt. Thus the number of points measured is $N = \Delta t / \delta t$. Our Fourier transform is now the sum

$$V_j(\nu) = \sum_{i=1}^{N} v_i(t)e^{-2\pi i \nu_j t}\delta t \qquad (C3)$$

This differs from the ideal $V(\nu)$ computed above in some important ways. We have to consider the effects of *sampling* and *windowing*. We also note that the power spectrum has the property of *symmetry about zero frequency.*

Sampling: We sample the signal at a rate $1/\delta t$, which means that only signals with frequency $2/\delta t$ or smaller can be reconstructed. In a properly designed spectrometer the sampling range and the *anti-aliasing filter* are chosen with this in mind. This is the reason for the 3 MHz bandpass filter (see Figure 2) and the sampling rate of just over 6 MHz. A strong signal with frequency larger than 6 MHz that "leaks" through the filter will appear in our power spectrum at an *aliased* frequency with the 0-3 MHz band. Thus, strong interference signals can corrupt your spectra, even though they are outside the bandpass.

Because we use the Fast Fourier Transform (FFT) algorithm to compute the Fourier transform, the spectra are also sampled with values known only at certain values of the frequency. Our frequency "channels" are centered at these values and are separated by

$$\delta\nu = \frac{1}{N\delta t} \qquad (C4)$$

Windowing: Because the time series is truncated (i.e., we only measure the voltage signal for a finite period of time), the features in the frequency domain are broadened. In other words, a pure harmonic signal, which in principle would appear as a delta function in the frequency domain, in fact appears as a feature with nonzero width. We can express this effect of the limited time span by multiplying by a "window function" $w(t)$ in the time domain:

$$v_m(t) = w(t)v(t) \qquad (C5)$$

For the simplest window function with no weighting, $w(t)$ is unity during the times of the measurements and zero otherwise:

$$w(t) = 1 \text{ for } -\frac{\Delta t}{2} \le t \le \frac{\Delta t}{2} \qquad (C6)$$

By the convolution theorem, the Fourier transform of $v_m(t)$ is the convolution of the Fourier transforms of $v(t)$ and $w(t)$:

$$V_m = W(\nu) * V(\nu) \qquad (C7)$$

The Fourier transform of the uniform $w(t)$ function above is a sinc function in the frequency domain (see Figure 1):

$$W(\nu) = \frac{\sin \pi\nu\Delta t}{\pi\nu} \qquad (C8)$$

The width of the sinc function is proportional to Δt. For example, the half width at the first null (HWFN) is $1/\Delta t$. Thus, each frequency "spike" in the frequency domain is broaden into a feature shaped like a sinc function with a width determined by the length of the data stream. Note that that the spacing between the nulls is equal to $1/\Delta t$ and that this is also the channel spacing. The spectral line is also characterized by "sidelobes," a sort of ringing that can extend far from the main spectral feature. For this choice of window and sampling, the width and sidelobes do not appear in the sampled values of the spectrum if the spectral line is at the center of a channel.

The uniform window is only one possible weighting scheme that can be applied to the data. By changing the weights, we can change the shape of the spectral line
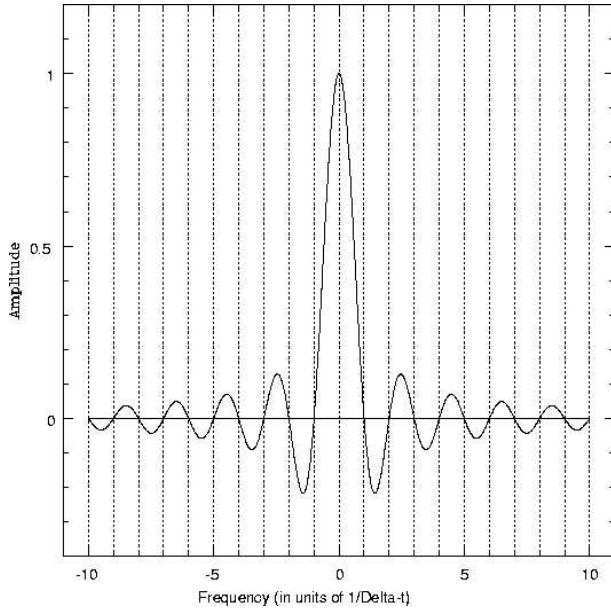
FIG. 16. The Fourier transform of a uniform window of width Δt. The vertical dotted lines show the centers of the frequency channels in the sampled power spectrum, assuming the spectral line is exactly at the center of one channel (which it may not be).

in the frequency domain. The choice should be based on considerations of the expected properties of the spectrum. For example, if we were interested in resolving two closely spaced spectral features, we would choose a weighting scheme that would produce a narrow spectral peak. If we were concerned about interfering signals, it would be prudent to choose a weighting function that produces small sidelobes, even at the cost of a broader spectral peak.

Symmetry About Zero Frequency   It can be shown that the Fourier transform of a real function is symmetric about zero frequency. For nonzero frequency components, the power is equally divided between $\pm\nu$. When we display the spectrum, we just sum the frequency components at $\pm\nu$ and present a "one-sided" spectrum. Thus, the first frequency bin is at $\nu = 0$, and the last frequency bin is at $\nu = \delta\nu(\frac{N}{2} - 1)$.

1.   Fourier Transform Spectroscopy

The radio astrophysics data acquisition program presents you with choices of:

- Blocks to average $= N_{\text{blocks}}$

- 2 Sided PS bins $= N_{\text{bins}}$ (corresponds to "n" above)

- Window

From these quantities the duty cycle and the frequency channel spacing are computed and displayed.

Blocks to average   Each "block" of data consists of $2^{19} = 524288$ samples collected in the time domain (taking approximately 79msec at a sampling rate of 6.67Msamples/sec). This is the minimum amount of data the program can take. After collecting a block of data, the computer is occupied for some time calculating the power spectrum from $n\delta t$ segments of the data, applying any user selected windowing function and converting to dBm. A power spectrum is calculated for each block of data, and these spectra are averaged. The fraction of time that the computer is collecting data (as opposed to processing it) is the "duty cycle" that is displayed on the screen. As computers get faster and faster, this duty cycle should approach unity.

2 Sided PS bins   The signal we measure is real, so the power spectrum we compute is symmetric about zero frequency. The spectrum presented on the PC screen has been "folded" so that the channels at plus and minus the same frequency have been summed. The channel spacing, $\delta\nu$, displayed on the screen is

$$\delta\nu = \frac{1}{N_{\text{bins}}\delta t} = \frac{6.67 \text{ MHz}}{N_{\text{bins}}}$$