Applications of Linear Systems

Linear Algebra

This work was initially funded by an Institutional OER Grant from the Colorado Department of Higher Education (CDHE). For similar OER materials in other courses funded by this project in the Department of Mathematical and Statistical Sciences at the University of Colorado Denver, visit
https://github.com/CU-Denver-MathStats-OER

Balancing Chemical Equations

In a chemical reaction, molecules recombine to produce other molecules.
The same number and type of atoms are present at the beginning and end of the reaction.

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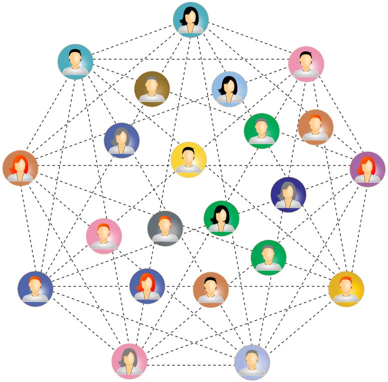
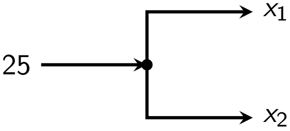
We thus have the balanced chemical equation $\mathrm{CH_4} + 2\mathrm{O_2} \to \mathrm{CO_2} + 2\mathrm{H_2O}$.

Network Flow

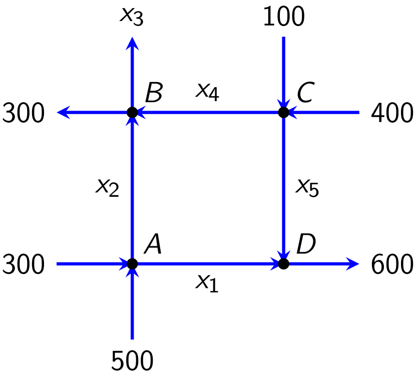
- A network consists of a set of points, called nodes with lines, called branches connecting some or all of the nodes.
- The direction of the flow is indicated by each branch (are things flowing in or out of the node?).
- The flow amount (or rate) is either given or denoted by a variable.
- We assume the total flow into a network equals the total flow out of the network.
- The goal is to determine the flow in each branch when partial information is known.
- Network flows have applications to current flow through a circuit, flow of goods through supply chains, social networks, and urban planning to name a few.





Traffic Flow in Baltimore

The network in the figure shows the flow of traffic (in vehicles per hour) over several one way streets in downtown Baltimore during a typical early afternoon. Determine the general flow pattern for the network.

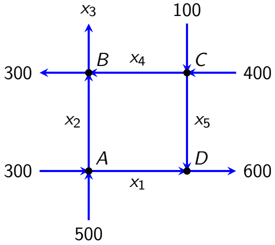


Intersection	Flow in		Flow out
A	$300+500$	$=$	x_1+x_2
B	x_2+x_4	$=$	$300+x_3$
C	$100+400$	$=$	x_4+x_5
D	x_1+x_5	$=$	600

$$
\begin{array}{rcrcrcrcrcr}
x_1 & + & x_2 & & & & & & & = & 800 \\
 & & x_2 & - & x_3 & + & x_4 & & & = & 300 \\
 & & & & & & x_4 & + & x_5 & = & 500 \\
x_1 & & & & & & & + & x_5 & = & 600 \\
 & & & & x_3 & & & & & = & 400
\end{array}
$$

Solving the System

We need to solve the following nonhomogeneous linear system of equations:



$$\begin{array}{rcrcrcrcrcl} x_1 & + & x_2 & & & & & & & = & 800 \\ & & x_2 & - & x_3 & + & x_4 & & & = & 300 \\ & & & & & & x_4 & + & x_5 & = & 500 \\ x_1 & & & & & & & + & x_5 & = & 600 \\ & & & & x_3 & & & & & = & 400 \end{array}$$

We have an augmented matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 800 \\ 0 & 1 & -1 & 1 & 0 & 300 \\ 0 & 0 & 0 & 1 & 1 & 500 \\ 1 & 0 & 0 & 0 & 1 & 600 \\ 0 & 0 & 1 & 0 & 0 & 400 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 600 \\ 0 & 1 & 0 & 0 & -1 & 200 \\ 0 & 0 & 1 & 0 & 0 & 400 \\ 0 & 0 & 0 & 1 & 1 & 500 \end{bmatrix} \rightarrow \begin{cases} x_1 = 600 - x_5 \\ x_2 = 200 + x_5 \\ x_3 = 400 \\ x_4 = 500 - x_5 \\ x_5 \text{ is free} \end{cases}$$