Vector Equations

Linear Algebra

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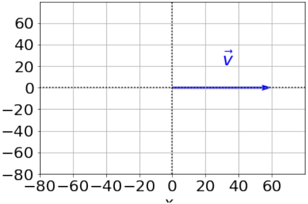
https://github.com/CU-Denver-MathStats-OER

Vectors

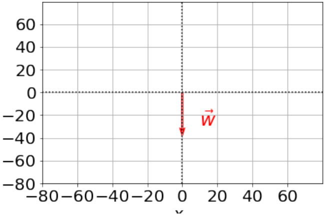
> A vector is a quantity that has both a length (also called magnitude) and a direction.

A car is traveling 60 miles per hour directly East.



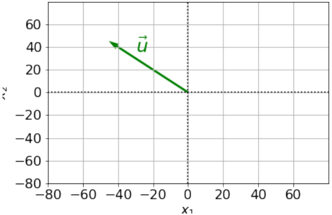
$$\mathbf{v} = \vec{v} = \begin{bmatrix} 60 \\ 0 \end{bmatrix}$$

A car is traveling 40 miles per hour directly South.



$$\mathbf{w} = \vec{w} = \begin{bmatrix} 0 \\ -40 \end{bmatrix}$$

A car is traveling 64 miles per hour directly Northwest.



$$\mathbf{u} = \vec{u} = \begin{bmatrix} -\frac{64}{\sqrt{2}} \\ \frac{64}{\sqrt{2}} \end{bmatrix}$$

Vectors in $\mathbb{R}^n$

A vector (or a column vector) is a matrix with only one column.

- We denote vectors in bold such as $\mathbf{v}$, and/or
- Using an arrow superscript such as $\vec{v}$.

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- The vector $\vec{v} = \begin{bmatrix} -12 \\ 8 \end{bmatrix}$ consists of an ordered pair of two real numbers. We say $\vec{v}$ is in $\mathbb{R}^2$ (read as "r two" or "r squared").
- If the vector $\vec{v}$ consists of n different real numbers, then we have

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \in \mathbb{R}^n.$$

Equality of Vectors

We say that two vectors $\vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$ are equal if:

- They have both the same direction and the same length, or equivalently,
- All corresponding entries of the two vectors are equal:

$$v_i = w_i \quad \text{for all } 1 \leq i \leq n.$$

Vector Addition

Given two vectors $\mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ in $\mathbb{R}^n$, we define their sum as the vector $\mathbf{u} + \mathbf{v} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ \vdots \\ u_n + v_n \end{bmatrix}$.

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Example

Add the vectors.

a) $\mathbf{u} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$.

b) $\mathbf{u} = \begin{bmatrix} 1 \\ -3 \\ 8 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 8 \\ 2 \\ -1 \\ 3 \end{bmatrix}$.

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b) $\mathbf{u} = \begin{bmatrix} 1 \\ -3 \\ 8 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 8 \\ 2 \\ -1 \\ 3 \end{bmatrix}$.

- We can only add two vectors if they have the same dimensions.
- When we add two vectors, the result is a vector with the same number of dimensions.

Scalar Multiplication

A quantity that has only a magnitude (and does not have a direction) is called a scalar. We usually refer to scalars as numbers. For example, -4, $\sqrt{5}$, 0, and e^2 are all scalars.

Given a vector $\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ and scalar c, we define the scalar product of c and $\mathbf{v}$ as $c\mathbf{v} = \begin{bmatrix} c \cdot v_1 \\ c \cdot v_2 \\ \vdots \\ c \cdot v_n \end{bmatrix}$.

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Compute the scalar product

a) $c = -3$ and $\mathbf{v} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$.

b) $c = 2$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$.

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- We can multiply any scalar and any vector.
- The result is a vector with the same dimension as the original vector.

Geometric Descriptions of Vectors in $\mathbb{R}^2$

We can think of the vector $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \in \mathbb{R}^2$ as a directed line segment (an arrow).

Consider two vectors $\mathbf{u}$ and $\mathbf{v} \in \mathbb{R}^2$. The sum $\mathbf{u} + \mathbf{v}$ is the result of first moving $\mathbf{u}$ and then moving $\mathbf{v}$.

Parallelogram Rule for Addition: If $\mathbf{u}$ and $\mathbf{v}$ are in $\mathbb{R}^2$, then the sum $\mathbf{u} + \mathbf{v}$ is the diagonal of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.

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Consider the vector $\mathbf{u} \in \mathbb{R}^2$ and nonzero scalar c. The scalar product $c\mathbf{u}$ gives a vector parallel to $\mathbf{u}$ whose magnitude has been scaled by a factor of c.

- If $c > 0$, then $c\mathbf{u}$ has the same direction as $\mathbf{u}$.
- If $c < 0$, then $c\mathbf{u}$ has the opposite direction as $\mathbf{u}$.
- If $|c| < 1$, then $c\mathbf{u}$ is a compression of $\mathbf{u}$.
- If $|c| > 1$, then $c\mathbf{u}$ is a stretching of $\mathbf{u}$.

Parallelogram Rule for Addition: If $\mathbf{u}$ and $\mathbf{v}$ are in $\mathbb{R}^2$, then the sum $\mathbf{u} + \mathbf{v}$ is the diagonal of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.

Properties of Vector Arithmetic

The zero vector is defined as the vector in $\mathbb{R}^n$ with no magnitude, and it is denoted by $\mathbf{0}$ or $\vec{0}$. As a result of this definition, it follows that $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$.

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For all $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in $\mathbb{R}^n$ and all scalars c and d:

(i) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(ii) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

(iii) $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$

(iv) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

(v) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$

(vi) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$

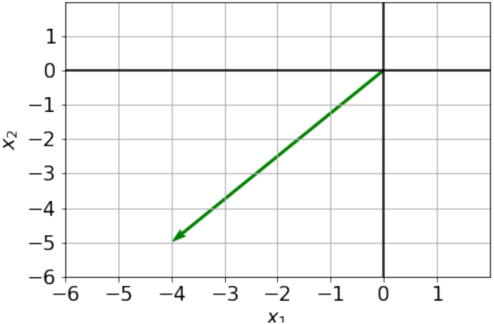
(vii) $c(d\mathbf{u}) = (cd)\mathbf{u}$

(viii) $1\mathbf{u} = \mathbf{u}$

Example

Find constants x_1 and x_2 such that

$$\begin{bmatrix} -4 \\ -5 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$



Giving Directions in Box-Grid City

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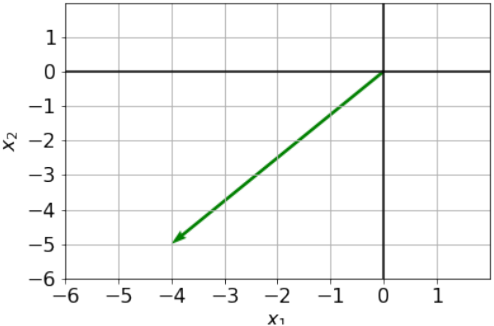


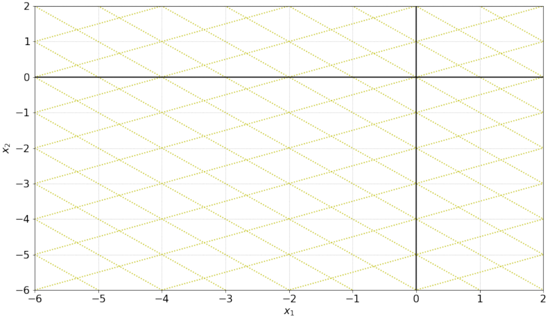
The vectors $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are called standard column vectors.

Giving Directions in Diagonaland

Example

Find constants x_1 and x_2 such that

$$\begin{bmatrix} -4 \\ -5 \end{bmatrix} = x_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$



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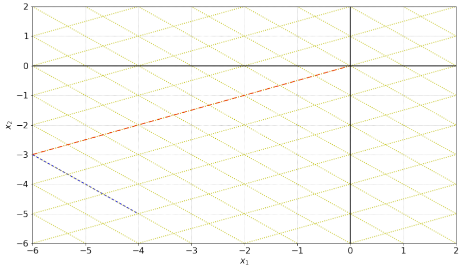


$$\begin{aligned} 2x_1 - x_2 &= -4 \\ x_1 + x_2 &= -5 \end{aligned}$$

$$\begin{bmatrix} 2 & -1 & -4 \\ 1 & 1 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -2 \end{bmatrix}$$

Thus, $\begin{cases} x_1 = -3 \\ x_2 = -2 \end{cases}$

Given any vector $\mathbf{w}$ in $\mathbb{R}^2$, we can find constants x_1 and x_2 such that

$$\mathbf{w} = x_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

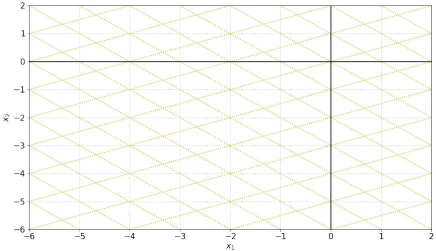
Linear Combinations

> Given vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ in $\mathbb{R}^n$ and scalars $c_1, c_2, \ldots, c_p$, the vector $\mathbf{y}$ defined by
>
> $$\mathbf{y} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \ldots + c_p\mathbf{v}_p = \sum_{i=1}^{p} c_i\mathbf{v}_i$$
>
> is called a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ with weights $c_1, c_2, \ldots, c_p$.

Can you think of two different vectors in $\mathbb{R}^2$ such that $\mathbf{w} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ **cannot** be written as a linear combination of the two vectors?

For example:

$$\begin{bmatrix} 2 \\ -3 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 2 \\ 1 \end{bmatrix} - \frac{8}{3}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$



The Span of a Set of Vectors

Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ be vectors in $\mathbb{R}^n$. The set of all linear combinations of the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ is called the subset of $\mathbb{R}^n$ spanned by $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$.
The set $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ is the collection of all vectors in $\mathbb{R}^n$ that can be written in the form

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \ldots + c_p\mathbf{v}_p = \sum_{i=1}^{p} c_i\mathbf{v}_i.$$

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$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \ldots + c_p\mathbf{v}_p = \sum_{i=1}^{p} c_i\mathbf{v}_i.$$

To determine whether a vector $\mathbf{y}$ is in $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$:

- We try to solve $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \ldots + c_p\mathbf{v}_p = \mathbf{y}$.
- In other words, we have the augmented matrix

$$\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \ldots & \mathbf{v}_p & \mathbf{y} \end{bmatrix}.$$

The Span of a Set of Vectors in $\mathbb{R}^2$

Describe the span of the set of vectors in $\mathbb{R}^2$.

a) $\text{Span}\left\{\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix}\right\}$

b) $\text{Span}\left\{\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\right\}$

c) $\text{Span}\left\{\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}-4\\-2\end{bmatrix}\right\}$

d) $\text{Span}\left\{\begin{bmatrix}0\\0\end{bmatrix}\right\}$

The Span of a Set of Vectors in $\mathbb{R}^3$

Describe the span of the set of vectors in $\mathbb{R}^3$.

a) $\text{Span}\left\{\begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$

b) $\text{Span}\left\{\begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$

c) $\text{Span}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$

d) $\text{Span}\left\{\begin{bmatrix}1\\0\\-2\end{bmatrix}, \begin{bmatrix}0\\1\\-1\end{bmatrix}\right\}$

Practice

Determine whether the given vector is in Span $\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$.

a) $\begin{bmatrix} 6 \\ -2 \\ -10 \end{bmatrix}$

b) $\begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$

Conceptual Practice

Let $\mathbf{v}_1, \mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{w}$ be vectors in $\mathbb{R}^n$. Prove that if $\mathbf{w}$ is in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ then $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}\} = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

Proof.

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Let $\mathbf{v}_1, \mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{w}$ be vectors in $\mathbb{R}^n$. Prove that if $\mathbf{w}$ is in $\operatorname{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ then $\operatorname{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}\} = \operatorname{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

Proof.

We need to show that the two sets $\operatorname{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}\}$ and $\operatorname{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are the same.
($\supseteq$) Suppose that $\mathbf{y} \in \operatorname{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Then there exist scalars c_1, c_2, c_3 such that $\mathbf{y} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3$. Then $\mathbf{y} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + 0\mathbf{w}$, and hence $\mathbf{y} \in \operatorname{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}\}$.

Conceptual Practice

Let $\mathbf{v}_1, \mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{w}$ be vectors in $\mathbb{R}^n$. Prove that if $\mathbf{w}$ is in Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ then Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}\} =$ Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

Proof.

We need to show that the two sets Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}\}$ and Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are the same.
($\supseteq$) Suppose that $\mathbf{y} \in$ Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Then there exist scalars c_1, c_2, c_3 such that $\mathbf{y} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3$. Then $\mathbf{y} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + 0\mathbf{w}$, and hence $\mathbf{y} \in$ Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}\}$.
($\subseteq$) Suppose that $\mathbf{y} \in$ Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}\}$. Then there exist scalars c_1, c_2, c_3, d such that $\mathbf{y} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + d\mathbf{w}$. Since $\mathbf{w}$ is in Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, there exist scalars b_1, b_2, b_3 such that $\mathbf{w} = b_1\mathbf{v}_1 + b_2\mathbf{v}_2 + b_3\mathbf{v}_3$. Substituting for $\mathbf{w}$ into the previous equation, we obtain

$$\begin{aligned}
\mathbf{y} &= c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + d\mathbf{w} \\
&= c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + d\left(b_1\mathbf{v}_1 + b_2\mathbf{v}_2 + b_3\mathbf{v}_3\right) \\
&= (c_1 + db_1)\mathbf{v}_1 + (c_2 + db_2)\mathbf{v}_2 + (c_3 + db_3)\mathbf{v}_3.
\end{aligned}$$

Since $c_1 + db_1$, $c_2 + db_2$, and $c_3 + db_3$ are scalars, $\mathbf{w}$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, and hence $\mathbf{y} \in$ Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Thus, Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}\} =$ Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. $\square$

Parametric Vector Form for Solutions of a Linear System

Find the solution set to the linear system

$$\begin{array}{rrrrrcl} x_1 & +2x_2 & & & -4x_5 & = & 0 \\ & & x_3 & & +7x_5 & = & 1 \\ & & & x_4 & +5x_5 & = & 2 \end{array}$$

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$$\begin{cases} x_1 = -2x_2 + 4x_5 \\ x_2 \text{ is free} \\ x_3 = 1 - 7x_5 \\ x_4 = 2 - 5x_5 \\ x_5 \text{ is free} \end{cases}$$

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$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2x_2 + 4x_5 \\ x_2 \\ 1 - 7x_5 \\ 2 - 5x_5 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 4 \\ 0 \\ -7 \\ -5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 4 \\ 0 \\ -7 \\ -5 \\ 1 \end{bmatrix}.$$

The weights s, t are any scalars. This is called parametric vector form.