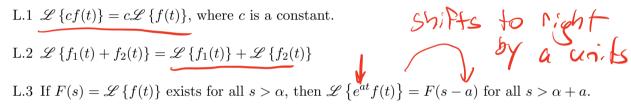
Common Laplace Transforms and Properties

f(t)	$F(s) = \mathcal{L}\left\{f(t)\right\}$
1	$\frac{1}{s}$, $s > 0$
e^{at}	$\frac{1}{s-a}, \ s > a$ $n!$
$t^n, n = 1, 2, \dots$	$\frac{n!}{s^{n+1}}, \ s > 0$
$\sin(bt)$	$\frac{b}{s^2 + b^2}$, $s > 0$
$\cos{(bt)}$	$\frac{s}{s^2 + b^2}, \ s > 0$
$e^{at}t^n, \ n=1,2,\dots$	$\frac{n!}{(s-a)^{n+1}}, \ s > a$
$e^{at}\sin\left(bt\right)$	$\frac{b}{(s-a)^2+b^2}, \ s>a$
$e^{at}\cos\left(bt\right)$	$\frac{s-a}{(s-a)^2+b^2}, \ s>a$

Properties:

L.1 $\mathscr{L}\left\{cf(t)\right\}=c\mathscr{L}\left\{f(t)\right\},$ where c is a constant.



L.4 If $F(s) = \mathcal{L}\{f(t)\}$ exists for all $s > \alpha$, then for all $s > \alpha$,

$$\mathscr{L}\left\{f^{(n)}(t)\right\} = s^n \mathscr{L}\left\{f(t)\right\} - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0).$$

L.5 If $F(s) = \mathcal{L}\{f(t)\}$ exists for all $s > \alpha$, then $\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n F}{ds^n}$ for all $s > \alpha$.

Explain in Words:

1. Describe properties 3, 4, and 5 in words. For example in property 3, multiplying f(t) by e^{at} and then taking the Laplace transform has what affect on $\mathcal{L}\{f(t)\}$?

Section 6.1: The Inverse Laplace Transform

Solving Diff. Eqs. with Inverse Laplace Transforms

$$t^n, \ n = 1, 2, \dots$$
 $\left| \frac{n!}{s^{n+1}}, \ s > 0 \right|$

2. Solve y'' - y = -t with y(0) = 0 and y'(0) = 1.

(a) Using the properties, apply the Laplace transform to both sides:

By L.1 and L.2
$$S_{5}y''-y_{3}=S_{5}y''_{3}-S_{5}y_{3}=-S_{5}t_{3}$$

By L.4 = $S_{5}S_{5}y't_{5}-S_{5}y_{5}-S_{5}y_{5}=-(\frac{1}{S^{\alpha}})$

= $S_{5}S_{5}y't_{5}-S_{5}y_{5}-S_{5}y_{5}=-(\frac{1}{S^{\alpha}})$

= $S_{5}S_{5}y_{5}-S_{5}y_{5}-S_{5}y_{5}=-\frac{1}{S^{\alpha}}$

We let (b) Using your answer in (b) answer in (b) solve for $\mathcal{L}\{y(t)\} = Y(s)$

$$S^{2}Y(s) - Y(s) = -\frac{1}{s^{2}} + \frac{1}{1}$$

$$Y(s)(s^{2}-1) = -\frac{1}{s^{2}} + \frac{1}{1}$$

$$Y(s) = -\frac{1}{s^{2}} + \frac{1}{1} + \frac{1}{s^{2}} + \frac{1}{s^{2$$

(c) Use the table of common Laplace transforms to identify what function y(t) has $\mathcal{L}\{y(t)\} = Y(s)$.

If
$$Y(s) = \frac{1}{s^2}$$
, then $Y(t) = t$ In. tig Value Poblem Solution

In 2c, we are apply the **Inverse Laplace Transform** to Y(s) in order to identify $y(t) = \mathcal{L}^{-1}\{Y(s)\}$.

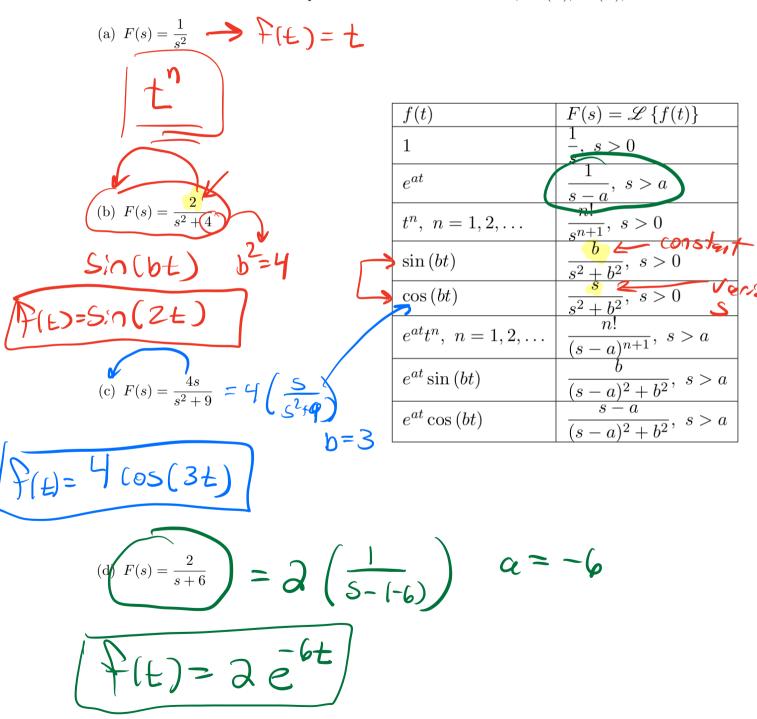
	2		
	f(t)	$F(s) = \mathcal{L}\left\{f(t)\right\}$	
	1	$\frac{1}{s}$, $s > 0$	A
	e^{at}	$\frac{1}{s-a}, \ s>a$	_ n!
n = 1	t^n , $n=1,2,\ldots$	$\frac{n!}{s^{n+1}}, \ s > 0$	2 (7+1
	$\sin\left(bt\right)$	$\frac{b}{s^2 + b^2}, \ s > 0 \chi$	5 5
	$\cos(bt)$	$\frac{s}{s^2 + b^2}, \ s > 0 \chi$	U = [
	$e^{at}t^n, \ n=1,2,\dots$	$\frac{1}{s-a}, s > a$ $\frac{n!}{s^{n+1}}, s > 0$ $\frac{b}{s^2 + b^2}, s > 0$ $\frac{s}{s^2 + b^2}, s > 0$ $\frac{n!}{(s-a)^{n+1}}, s > a$	•
	$e^{at}\sin\left(bt\right)$	$\left \frac{b}{(s-a)^2 + b^2}, \ s > a \right $	
	$e^{at}\cos\left(bt\right)$	$ \frac{c}{(s-a)^2 + b^2}, s > a $ $ \frac{s-a}{(s-a)^2 + b^2}, s > a $	
	P	7761)-1	
	Y(F) = 0	3 = t	
	V		

Section 6.1: Inverse Laplace Transforms

Given F(s), if there is a function f(t) that is continuous on $[0, \infty)$ and satisfies $\mathcal{L}\{f\} = F(s)$, then we say f(t) is the **inverse Laplace transform** of F(s) which is denoted by

$$\mathbf{f}(\mathbf{t}) = \mathscr{L}^{-1}\{\mathbf{F}(\mathbf{s})\}.$$

3. Determine whether the inverse Laplace transform is of the form t^n , $\cos(bt)$, $\sin(bt)$, or e^{at} .



- 4. Find the inverse Laplace transform of $F(s) = \frac{s+2}{s^2+4s+11}$ by answering the questions below.
 - (a) Complete the square for the expression in the denominator of F(s) to express $s^2+4s+11=(s-a)^2+b$.
 - (b) Use the table of common Laplace transforms to identify $\mathscr{L}^{-1}\{F(s)\}.$

5. Find the inverse Laplace transform of the function.

(a)
$$F(s) = \frac{5s - 10}{s^2 - 3s - 4}$$

(b)
$$F(s) = \frac{3s - 15}{2s^2 - 4s + 10}$$

(c)
$$F(s) = \frac{-5s - 36}{(s+2)(s^2+9)}$$