Phase Plane Equations

Consider a system of two differential equations:

$$\frac{dx}{dt} = f(x, y)$$
$$\frac{dy}{dt} = g(x, y)$$

Recall from the chain rule we have

$$\frac{dy}{dx}\frac{dx}{dt} = \frac{dy}{dt},$$

which gives

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g(x, y)}{f(x, y)}.$$

1. Write and solve the corresponding phase plane equation for the system

$$\frac{dx}{dt} = 7y$$
$$\frac{dy}{dt} = -2x$$

2. Make a sketch of several solutions in the phase plane, include arrows to indicate how solutions behave with respect to time.

Equilibrium Solutions

A point (x_0, y_0) is called an **equilibrium** (or critical point) of the system

$$\frac{dx}{dt} = f(x, y)$$
$$\frac{dy}{dt} = g(x, y)$$

if both $f(x_0, y_0) = 0$ and $g(x_0, y_0) = 0$.

The corresponding solution $(x(t), y(t)) = (x_0, y_0)$ is called an **equilibrium solution**.

3. Find the equilibrium to the system.

(a) $\frac{dx}{dt} = 2x - y + 8$
$\frac{dy}{dt} = 3x + 6$

(b) $\frac{dx}{dt} = y^2 - xy$
$\frac{dy}{dt} = 2xy - 4$

4. Find the equilibrium. Then find and solve the phase plane equation.

(a) $\frac{dx}{dt} = 6x$
$\frac{dy}{dt} = 3y$

(b) $\frac{dx}{dt} = 4 - 4y$
$\frac{dy}{dt} = -4x$

(c) $\begin{array}{l} \frac{dx}{dt} = 2y^2 - y \\ \frac{dy}{dt} = x^2 y \end{array}$