Common Laplace Transforms and Properties

$f(t)$	$F(s) = \mathscr{L}\{f(t)\}$
$1 \cdot 0$	$\frac{1}{s},\ s > 0$
e^{at}	$\frac{1}{s-a},\ s > a$
$t^n,\ n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}},\ s > 0$
$\sin(bt)$	$\frac{b}{s^2+b^2},\ s > 0$
$\cos(bt)$	$\frac{s}{s^2+b^2},\ s > 0$
$e^{at}t^n,\ n = 1, 2, \ldots$	$\frac{n!}{(s-a)^{n+1}},\ s > a$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2},\ s > a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2},\ s > a$

$= \mathscr{L}\{1 \cdot 0\} = 0\mathscr{L}\{1\}$
$= 0$

Properties:

L.1 $\mathscr{L}\{cf(t)\} = c\mathscr{L}\{f(t)\}$, where c is a constant.

L.2 $\mathscr{L}\{f_1(t) + f_2(t)\} = \mathscr{L}\{f_1(t)\} + \mathscr{L}\{f_2(t)\}$

L.3 If $F(s) = \mathscr{L}\{f(t)\}$ exists for all $s > \alpha$, then $\mathscr{L}\{e^{at}f(t)\} = F(s-a)$ for all $s > \alpha + a$.

L.4 If $F(s) = \mathscr{L}\{f(t)\}$ exists for all $s > \alpha$, then for all $s > \alpha$,

$$\mathscr{L}\left\{f^{(n)}(t)\right\} = \boxed{s^n \underline{\mathscr{L}\{f(t)\}} - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0).}$$

n^{th} derivative

L.5 If $F(s) = \mathscr{L}\{f(t)\}$ exists for all $s > \alpha$, then $\mathscr{L}\{t^n f(t)\} = (-1)^n \frac{d^n F}{ds^n}$ for all $s > \alpha$.

$y(t) \quad \mathscr{L}\{y(t)\} = \underline{Y(s)}$

$n=2$ $\quad \mathscr{L}\{y''\} = s^2 Y(s) - s\,y(0) - y'(0)$

$n=1$ $\quad \mathscr{L}\{y'\} = s\,Y(s) - y(0)$

$n=3$ $\quad \mathscr{L}\{y'''\} = s^3 Y(s) - s^2 y(0) - s\,y'(0) - y''(0)$

Section 6.2: Solving ODE's

Step 1 Take the Laplace transform of both sides. Refer to properties. → Algebraic equation

Step 2 Rearrange and group like terms to solve for $\mathcal{L}\{y(t)\} = Y(s)$ ←

Step 3 Take the inverse Laplace transform and solve for $y(t) = \mathcal{L}^{-1}\{Y(x)\}$. ← Reverse step 1 more Algebra

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1. Solve the initial value problem using Laplace Transforms (not previous methods).

   L.1 and L.2

   (a) $y'' - 2y' + 5y = 0$ with $y(0) = 2$ and $y'(0) = 4$.

$$\mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} + 5\mathcal{L}\{y\} = \mathcal{L}\{0\}$$

By L.4 and table

$$\left(s^2 Y(s) - s\cdot 2 - 4\right) - 2\left(sY(s) - 2\right) + 5Y(s) = 0$$

$$Y(s)$$

$$\mathcal{L}^{-1}\{Y(s)\}$$

(b) $y'' - y' - 2y = 0$ with $y(0) = -2$ and $y'(0) = 5$.

(c) $y'' - 4y' - 5y = 4e^{3t}$ with $y(0) = 2$ and $y'(0) = 7$.

(d) $ty'' - ty' + y = 2$ with $y(0) = 2$ and $y'(0) = -1$.

(e) $y'' + ty' - y = 0$ with $y(0) = 0$ and $y'(0) = 3$.