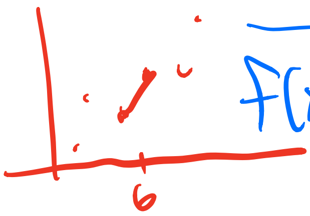
Welcome All!

Today we are working on Worksheet $\boxed{\text{08 Heating and Cooling}}$ → Reading Application 4.2

If



x	0	3	6	9	12
$f(x)$	3	12	15	16	20

$$f' = \frac{dy}{dx} \approx \boxed{\frac{\Delta y}{\Delta x}}$$

$$\frac{15-12}{3} = 1$$

Approximate

$$f'(6) \approx \frac{16-15}{9-6} \approx 1/3$$

$$f'(0) \approx \frac{12-3}{3-0} = 3$$

$$\boxed{\approx 2/3}$$

$$f'(6) \approx \frac{f(9)-f(3)}{9-3} = 2/3$$

Cooling Coffee

A group of students want to develop a rate of change equation to describe the cooling rate for hot coffee in order that they can make predictions about other cups of cooling coffee. Their idea is to use a temperature probe to collect data on the temperature of the coffee as it changes over time and then to use this data to develop a rate of change equation.

The data they collected is shown in the table below. The temperature C (in degrees Fahrenheit) was recorded every 2 minutes over a 14 minute period.

Time (min) t	Temp. (°F) C	$\frac{dC}{dt}$ (°F per min)
0	160.3	−19.95
2	120.4	−15.55
4	98.1	−8.9
6	84.8	−4.9
8	78.5	−2.6
10	74.4	−1.6
12	72.1	−0.725
14	71.5	−0.3

$$\left.\frac{dC}{dt}\right|_{t=2} \approx \frac{\overset{C(4)}{98.1} - \overset{C(0)}{160.3}}{4-0} = -15.55\ {}^\circ F/min$$

$$\left.\frac{dC}{dt}\right|_{t=0} \approx \frac{C(2)-C(0)}{2-0} = \frac{120.4 - 160.3}{2}$$

1. Figure out a way to use this data to fill in the third column whose values approximate $\frac{dC}{dt}$, where C is the temperature of the coffee.

2. Do you expect $\frac{dC}{dt}$ to depend on just the temperature C, on just the time t, or both the temperature C and the time t?

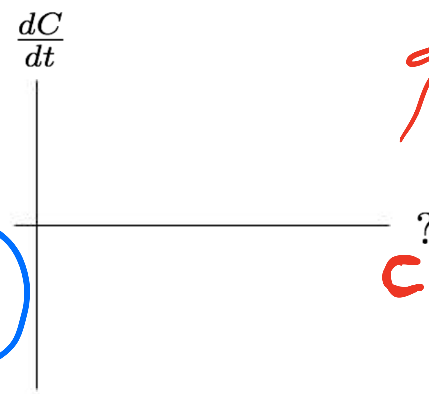
3. Sketch below your best guess for the graph of $\frac{dC}{dt}$.

Same t value – But different C values

If at t=0 two cups are placed in the room. Cup A has initial temp of 160°F. Cup B has initial temp of 90°F. Which cools at faster rate?

If cup 1 is 160°F at t=0. Then 6 min later cup 1 is 85°F at t=6 while a second cup is placed in the room and its initial temp is 85°F. Which cup cools at a faster rate?

Same C but different t



Newton's Law of Heating and Cooling

4.

(a) Input the data from your extended table in question 1 into the GeoGebra applet https://ggbm.at/uj2gbz3V to plot points for $\frac{dC}{dt}$ vs. C. Does this plot confirm or reject your sketch from question 3?

As $t \to \infty$

$C \to 70^\circ F$

(b) Toggle on the curve fitting tool and find an equation that fits your data.

$$\boxed{\frac{dC}{dt} = -0.3C + 21 = -0.3(C-70)}$$

$A = T_m$

Newton's Law of Heating and Cooling states that the temperature T of an object at time t changes at a rate which is proportional the difference of its temperature and the temperature of its surrounding:

$$\frac{dT}{dt} = -k(T - A)$$

constants

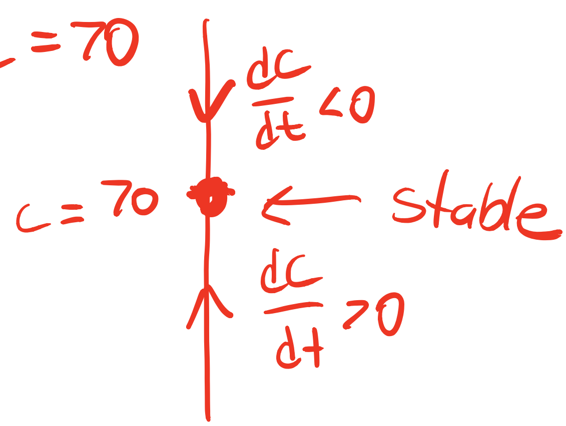
where A is a constant that denotes the ambient temperature and $k > 0$ is a constant that depends on the object.

5. What happens as $\mathbf{t} \to \infty$ if the initial temperature $\mathbf{T_0} > \mathbf{A}$? If $\mathbf{T_0} < \mathbf{A}$?

$$\frac{dC}{dt} = -\underset{k}{0.3}(C - \underset{A}{70}) = 0 \qquad C = 70$$

Equlibrium:

constant solutions: $\frac{dC}{dt} = 0$



Practice: Applications to Economics

6. Let $S(p)$ denote the number of units of a particular commodity supplied to the market at a price of p dollars per unit, and let $D(p)$ denote the corresponding number of units demanded by the market at the same price.

- In static circumstances, market equilibrium occurs at the price where demand equals supply.
- However, certain economic models consider a more dynamic economy in which price, supply, and demand are assumed to vary with time.
- One of these, the Evans price adjustment model, assumes that the rate of change of price with respect to time t is proportional to the shortage, which is the difference between the quantity demanded and the quantity supplied..

(a) Write a differential equation for the rate of the change of the price of the good with respect to time.

$$\frac{dp}{dt} = k(D-S) \quad \text{OR} \quad \frac{dp}{dt} = k(S-D)$$

(b) If we assume that supply and demand are linear functions given by

$$S(p) = 2 + p \qquad \text{and} \qquad D(p) = 8 - 2p,$$

Find a general solution to the differential equation in part (a).

$$\frac{dp}{dt} = k\big((8-2p)-(2+p)\big) = k(6-3p) = -3k(p-2)$$

$$\frac{1}{p-2}\,dp = -3k\,dt$$

$$\ln|p-2| = -3kt + C$$

$$|p-2| = Ce^{-3kt}$$

$$p-2 = Ce^{-3kt}$$

$$\boxed{p = Ce^{-3kt} + 2}$$

(c) If the price is \$5 at time $t = 0$ and \$3 at time $t = 2$, determine what happens to p in the long run.

(0,5) and $\boxed{(2,3)}$ are two given points

$\to 5 = Ce^{0} + 2$ so $C = 3$ $\quad p = 3e^{-3kt} + 2$

$3 = 3e^{-3k(2)} + 2$ gives $k \approx 0.183$ so

$$\boxed{P = 3e^{-3(0.183)t} + 2 = 3e^{-0.549t} + 2}$$

$T(0) = 90.3°F$

Practice: Applications to Forensic Science

7. A detective finds a murder victim at 9 am. The temperature of the body is measured at 90.3°F. One hour later, the temperature of the body is 89.0°F. The temperature of the room has been maintained at a constant 68°F.

   (a) Assuming the temperature, T, of the body obeys Newton's Law of Cooling, write a differential equation for T. Your equation will include the constant k (for now).

$$\frac{dT}{dt} = -k(T - A) = -k(T - 68)$$

ambient temp

   (b) Solve the differential equation to estimate the time the murder occurred.

Leave constant k in solution. Then use two given points to find constant of integration and k.

$$\frac{1}{T-68}\,dT = -k\,dt$$

$$\ln|T-68| = -kt + C$$

$$|T-68| = e^{C}e^{-kt} = Ce^{-kt}$$

$$|T-68| = \begin{cases} Ce^{-kt} & T \geq 68 \\ -Ce^{-kt} & T < 68 \end{cases}$$

$$\boxed{T - 68 = Ce^{-kt}}$$

A

$T = Ce^{-kt} + 68$

90.3 $t = 0$ 9 Am

$$T = 22.3e^{-kt} + 68$$

$t = 1$ 89° F

$$89 = 22.3e^{-k(1)} + 68$$

Solving for $k \approx 0.06$

$$\boxed{T = 22.3e^{-0.06t} + 68}$$

$$98.6 = 22.3e^{-0.06t} + 68$$

$t \approx -5.27$ hrs since 9 am

At about 3:44 Am