Common Laplace Transforms and Properties

$f(t)$	$F(s) = \mathscr{L}\left\{f(t)\right\}$
1	$\frac{1}{s},\ s > 0$
e^{at}	$\frac{1}{s-a},\ s > a$
$t^n,\ n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}},\ s > 0$
$\sin(bt)$	$\frac{b}{s^2 + b^2},\ s > 0$
$\cos(bt)$	$\frac{s}{s^2 + b^2},\ s > 0$
$e^{at}t^n,\ n = 1, 2, \ldots$	$\frac{n!}{(s-a)^{n+1}},\ s > a$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2 + b^2},\ s > a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2},\ s > a$

Properties:

L.1 $\mathscr{L}\left\{cf(t)\right\} = c\mathscr{L}\left\{f(t)\right\}$, where c is a constant.

L.2 $\mathscr{L}\left\{f_1(t) + f_2(t)\right\} = \mathscr{L}\left\{f_1(t)\right\} + \mathscr{L}\left\{f_2(t)\right\}$

L.3 If $F(s) = \mathscr{L}\left\{f(t)\right\}$ exists for all $s > \alpha$, then $\mathscr{L}\left\{e^{at}f(t)\right\} = F(s-a)$ for all $s > \alpha + a$.

L.4 If $F(s) = \mathscr{L}\left\{f(t)\right\}$ exists for all $s > \alpha$, then for all $s > \alpha$,

$$\mathscr{L}\left\{f^{(n)}(t)\right\} = s^n\mathscr{L}\{f(t)\} - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0).$$

L.5 If $F(s) = \mathscr{L}\left\{f(t)\right\}$ exists for all $s > \alpha$, then $\mathscr{L}\left\{t^n f(t)\right\} = (-1)^n \frac{d^n F}{ds^n}$ for all $s > \alpha$.

Section 6.2: Solving ODE's

Step 1 Take the Laplace transform of both sides. Refer to properties.

Step 2 Rearrange and group like terms to solve for $\mathcal{L}\{y(t)\} = Y(s)$

Step 3 Take the inverse Laplace transform and solve for $y(t) = \mathcal{L}^{-1}\{Y(x)\}$.

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1. Solve the initial value problem using Laplace Transforms (not previous methods).

   (a) $y'' - 2y' + 5y = 0$ with $y(0) = 2$ and $y'(0) = 4$.

(b) $y'' - y' - 2y = 0$ with $y(0) = -2$ and $y'(0) = 5$.

(c) $y'' - 4y' - 5y = 4e^{3t}$ with $y(0) = 2$ and $y'(0) = 7$.

(d) $ty'' - ty' + y = 2$ with $y(0) = 2$ and $y'(0) = -1$.

(e) $y'' + ty' - y = 0$ with $y(0) = 0$ and $y'(0) = 3$.