

Lista 1

4) $\|x\| \geq -1 \quad \forall x \in \mathbb{R}^n.$

$$\|0\| = 0 \quad \|x\| \geq 0 \quad \forall x \in \mathbb{R}^n$$

\downarrow
vetor 0

$$\|v\| = 5 \quad \text{e} \quad \|w\| = 3$$

$$\begin{aligned} \|v - w\| &= \|v + (-w)\| \leq \|v\| + \|-w\| \\ &= \|v\| + \|w\| \\ &= 5 + 3 \\ &= 8 \end{aligned}$$

$$\Rightarrow \|v - w\| \leq 8 \quad \leftarrow$$

$$|v \cdot w| \leq \|v\| \|w\| \quad \|v + w\| \leq \|v\| + \|w\|$$

Segunda desigualdade triangular:

$$|\|v\| - \|w\|| \leq \|v - w\|,$$

\updownarrow

$$-\|v - w\| \leq \|v\| - \|w\| \leq \|v - w\|$$

$$\forall v, w$$

$$\|\cdot\| : \mathbb{R}^n \rightarrow \mathbb{R}_+ \quad |(1,2)| = \sqrt{5}$$

$$\|x\|, x \in \mathbb{R}^n$$

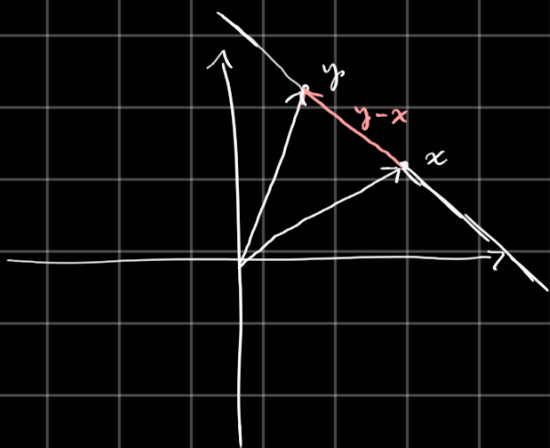
$$\|(1,2)\| = \sqrt{5}$$

$$|\cdot| : \mathbb{R} \rightarrow \mathbb{R}_+$$

$$|a|, a \in \mathbb{R}$$

$$\|(x_1, \dots, x_n)\| = \sqrt{\sum_{i=1}^n x_i^2} = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

$$\|(x_1, \dots, x_n)\|_1 = \sum_{i=1}^n |x_i| = |x_1| + |x_2| + \dots + |x_n|$$



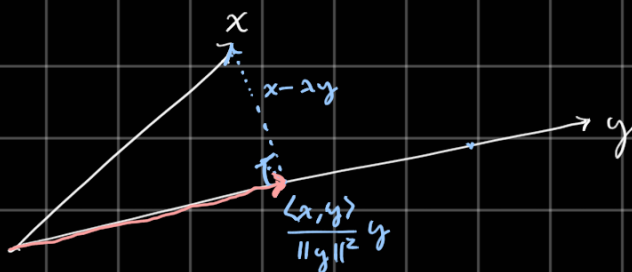
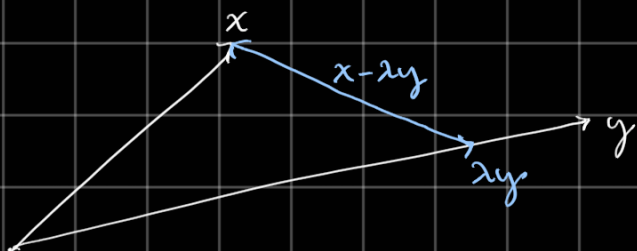
Retta que passa por x e y é

$$\{x + \lambda(y-x) : \lambda \in \mathbb{R}\}$$

10) Por ex: $i=1 : -x_2 + 2x_1 - x_0 = 1$

Cauchy Schwarz: $|\langle x, y \rangle| \leq \|x\| \|y\|$

$$\langle x - \lambda y, x - \lambda y \rangle \geq 0 \quad \dots$$



$$\begin{aligned} \left\| \frac{\langle x, y \rangle}{\|y\|^2} y \right\| &= \frac{|\langle x, y \rangle|}{\|y\|^2} \|y\| \\ &\stackrel{\substack{\in \mathbb{R}}}{=} \frac{|\langle x, y \rangle|}{\|y\|} \end{aligned}$$

Por pitágoras, $\left\| \frac{\langle x, y \rangle}{\|y\|^2} y \right\|^2 + \|x - \frac{\langle x, y \rangle}{\|y\|^2} y\|^2 = \|x\|^2$

$$\Rightarrow \left\| \frac{\langle x, y \rangle}{\|y\|^2} y \right\|^2 \leq \|x\|^2$$

$$\frac{|\langle x, y \rangle|^2}{\|y\|^2} \leq \|x\|^2$$

$$\Rightarrow |\langle x, y \rangle|^2 \leq \|y\|^2 \|x\|^2$$

$$\Rightarrow |\langle x, y \rangle| \leq \|y\| \|x\|$$

$$\left(\int_a^b f(x)g(x) dx \right) \leq \left(\int_a^b f(x)^2 dx \right) \left(\int_a^b g(x)^2 dx \right)$$

em \mathbb{R}^n :

$$\left(\sum_{i=1}^n x_i y_i \right) \leq \left(\sum_{i=1}^n x_i^2 \right) \left(\sum_{i=1}^n y_i^2 \right)$$

$f: [a, b] \rightarrow \mathbb{R}$ e $g: [a, b] \rightarrow \mathbb{R}$

$$\langle f, g \rangle = \int_a^b f(x)g(x) dx$$

$$\|f\| = \sqrt{\int_a^b f(x)^2 dx}$$

6) em \mathbb{R}^2 $u \cdot v = \|u\| \|v\| \cos \theta$
 θ é o ângulo entre u e v .
 $\theta \in [0, \pi]$