

Mit 6.043 - Artificial Intelligence

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1 Reasoning: goal trees and rule-based expert systems

A **representation** consists of the following four fundamental parts:

- A lexical part that determines which symbols are allowed in the representation's vocabulary
- A structural part that describes constraints on how the symbols can be arranged
- A procedural part that specifies access procedures that enable you to create descriptions, to modify them, and to answer questions using them
- A semantic part that establishes a way associating a meaning with the description

A **semantic net** is a representation in which:

- Lexically, there are nodes, links, and application-specific link labels
- Structurally, each link connects a tail node to a head node
- Semantically, the nodes and links denote application-specific entities

With constructors that:

- Construct a node
- Construct a link, given a link label and two nodes to be connected

With readers that:

- Produce a list of all links departing from a given node
- Produce a list of all links arriving at a given node
- Produce a tail node, given a link
- Produce a head node, given a link
- Produce a link label, given a link

A **Semantic tree** is a representation, that is a semantic net in which:

- certain links are called **branches**. Each branch connects two nodes; the head node is called the **parent node** and the tail node is called the *child node*
- One node has no parent; it is called the root node. Other nodes have exactly one parent

- Some nodes have no children, they are called *leaf nodes*. When two nodes are connected to each other by a chain of two or more branches, one is said to be the *ancestor*; the other is said to be the descendant

With constructors that:

- Connect a parent node to a child node with a branch links

With readers that:

- Produce a list of a given node's children
- Produce a given node's parent

A **goal tree** is a semantic tree in which: nodes represent goals and branches indicate how you can achieve goals by solving one or more subgoals. Each node's children corresponds to **immediate subgoals**; each node's parent corresponds to the **immediate supergoal**. The top node, the one with no parent, is the **root** goal.

Some goals are satisfied directly, without reference to any other subgoals. These goals are called **leaf goals**, and the corresponding nodes are called **leaf nodes**.

Because goal trees always involve *And* nodes, or *Or* nodes, or both. they are often called **And-Or trees**.

To determine whether a goal has been achieved, you need a testing procedure. The key procedure, *REDUCE*, channels action into the *REDUCE-AND* and the *REDUCE-OR*.

Goal trees enable introspective question answering:

- how: the immediate subgoal (downstream)
- why the immediate supergoal (downstream)

1.1 Eliciting expert systems features

- Heuristic of specific situations: it is dangerous to limit inquiry to office interviews
- Heuristic of situation comparison: ask a domain expert for clarification whenever the domain expert's behavior varies in situations that look identical to the knowledge engineer.
- You should build a system and see when it cracks. Helps identifying missing rules.

2 Nets and Basic Search

A **search tree** is a representation, that is a semantic tree, in which:

- Nodes denote paths
- Branches connect paths to one-set path extensions

With writers that:

- Connect a path to a path description

With readers that:

- Produce a path's description

Each child denotes a path that is a one-city extension of the path denoted by its parent.

If a node has b children, it is said to have a **branching** factor of b . If the number of children is always b for every nonleaf node, the tree is said to have a branching factor of b .

Each path that does not reach the goal is called a **partial path**. Each path that does reach the goal is called a **complete path**, and the corresponding node is called a **goal node**.

Nodes are said to be **open** until they can be expanded, whereupon they become **closed**.

2.1 Blind methods

2.1.1 Depth-first Search

- Form a one-element queue consisting of a zero-length path that contains only the root node
- Until the first path in the queue terminates at the goal node or the queue is empty,
 - Remove the first path from the queue; create new paths by extending the first path to all the neighbors of the terminal node
 - Reject all new paths with loops
 - Add the new paths, if any to the **front** of the queue
- If the goal node is found, announce success; otherwise, announce failure

Good when you are confident that all partial paths either reach dead ends or become complete paths after a reasonable number of steps.

Bad with long or infinite paths, that neither reach dead ends nor become complete paths.

2.1.2 Breadth-first Search

- Form a one-element queue consisting of a zero-length path that contains only the root node
- Until the first path in the queue terminates at the goal node or the queue is empty,
 - Remove the first path from the queue; create new paths by extending the first path to all the neighbors of the terminal node
 - Reject all new paths with loops
 - Add the new paths, if any to the **back** of the queue
- If the goal node is found, announce success; otherwise, announce failure

Works with trees with infinitely deep paths.

Wasteful when all paths lead to the goal node at more or less the same depth.

2.1.3 Nondeterministic search

- Form a one-element queue consisting of a zero-length path that contains only the root node
- Until the first path in the queue terminates at the goal node or the queue is empty,
 - Remove the first path from the queue; create new paths by extending the first path to all the neighbors of the terminal node
 - Reject all new paths with loops
 - Add the new paths **at random places in the queue**
- If the goal node is found, announce success; otherwise, announce failure

2.2 Heuristically informed methods

2.2.1 Hill climbing

- Form a one-element queue consisting of a zero-length path that contains only the root node
- Until the first path in the queue terminates at the goal node or the queue is empty,
 - Remove the first path from the queue; create new paths by extending the first path to all the neighbors of the terminal node
 - Reject all new paths with loops
 - **Sort the new paths, if any, by the estimated distances between their terminal nodes and the goal**

- Add the new paths, if any to the **front** of the queue
- If the goal node is found, announce success; otherwise, announce failure

Possible issues:

- Foothills: an optimal point is found, but it is a local maximum
- Plateaus: for all but a small number of positions, all standard-step probes leave the quality measurement unchanged
- Ridges

2.2.2 Beam search

Like BFS in that it progresses level by level. Unlike BFS, beam search moves downward only through the best w nodes at each level; the other nodes are ignored.

2.2.3 Best-first search

While hill climbing demands forward motion from the most recently created open node. In the best-first search, **forward motion is from the best open node so far** no matter where that node is in the partially developed tree.

2.3 Which search type is good for me?

- Depth-first search is good when unproductive partial paths are never too long
- Breadth-first search is good when the branching factor is never too large
- Nondeterministic search is good when you are not sure whether depth-first search or breadth-first search would be better
- Hill climbing is good when there is a natural measurement of goal distance from each place to the goal and a good path is likely to be among the partial paths that appear to be good at all levels
- Beam search is good when there is a natural measure of goal distance and a good path is likely to be among the partial paths that appear to be good at all levels
- Best-first search is good when there is a natural measure of the goal distance and a good partial path may look like a bad option before more promising partial paths are played out

3 Nets and Optimal Search

3.1 British Museum procedure

Find all possible paths and select the best one from them.

3.2 Branch and Bound

- Form a one-element queue consisting of a zero-length path that contains only the root node
- Until the first path in the queue terminates at the goal node or the queue is empty:
 - Remove the first path from the queue; create new paths by extending the first path to all the neighbors of the terminal node
 - Reject all new paths with loops
 - add the remaining new paths, if any, to the queue
 - Sort the entire queue by path length with least-cost paths in front
- If the goal node is found, announce success; otherwise, announce failure

3.3 Branch and Bound with lower-bound estimate

- Form a one-element queue consisting of a zero-length path that contains only the root node
- Until the first path in the queue terminates at the goal node or the queue is empty:
 - Remove the first path from the queue; create new paths by extending the first path to all the neighbors of the terminal node
 - Reject all new paths with loops
 - Add the remaining new paths, if any, to the queue
 - Sort the entire queue by **the sum of the path length and a lower-bound estimate of the cost remaining, with least-cost paths in front**
- If the goal node is found, announce success; otherwise, announce failure

3.4 Branch and Bound with dynamic programming

- Form a one-element queue consisting of a zero-length path that contains only the root node
- Until the first path in the queue terminates at the goal node or the queue is empty:

- Remove the first path from the queue; create new paths by extending the first path to all the neighbors of the terminal node
- Reject all new paths with loops
- Add the remaining new paths, if any, to the queue
- **if two or more paths reach a common node, delete all those paths except the one that reaches the common node with the minimum cost**
- Sort the entire queue by **the sum of the path length with least-cost paths in front**
- If the goal node is found, announce success; otherwise, announce failure

3.5 A* procedure - Branch and bound with Underestimates and Dynamic Programming

- Form a one-element queue consisting of a zero-length path that contains only the root node
- Until the first path in the queue terminates at the goal node or the queue is empty:
 - Remove the first path from the queue; create new paths by extending the first path to all the neighbors of the terminal node
 - Reject all new paths with loops
 - Add the remaining new paths, if any, to the queue
 - **if two or more paths reach a common node, delete all those paths except the one that reaches the common node with the minimum cost**
 - Sort the entire queue by **the sum of the path length and a lower-bound estimate of the cost remaining, with least-cost paths in front**
- If the goal node is found, announce success; otherwise, announce failure

3.6 Which optimal search method is good for me?

- The British Museum procedure is good only when the search tree is small
- Branch-and-bound search is good when the tree is big and bad paths turn distinctly bad quickly
- Branch-and-bound search with a guess is good when there is a good lower-bound estimate of the distance remaining to the goal
- Dynamic programming is good when many paths converge on the same place

- The A* procedure is good when both branch-and-bound search with a guess and dynamic programming are good

4 Trees and Adversarial Search

A **game tree** is a representation:

- Nodes denote board configuration
- Branches denote moves

With writers that:

- Establish that a node is for the maximizer or for the minimizer
- Connect a board configuration with a board-configuration description

With readers that:

- Determine whether the node is for the minimizer or for the maximizer
- Produce a board configuration's description

4.1 Min-Max procedure

- If the limit of the search has been reached, compute the static value of the current position relative to the appropriate player. Report the result.
- Otherwise, if the level is a minimizing level, use MINMAX on the children of the current position. Report the minimum of the results
- Otherwise, the level is a maximizing level. Use MINMAX on the children of the current position. Report the maximum of the results

4.2 Alpha-Beta procedure

- If the level is the top level, let alpha be $-\infty$ and let beta be $+\infty$
- If the limit of search has been reached, compute the static value of the current position relative to the appropriate player. Report the result
- If the level is a minimizing level:
 - Until all children are examined with ALPHA-BETA or until alpha is equal to or greater than beta:
 - * Use ALPHA-BETA procedure, with the current alpha and beta values, on a child; note the value reported
 - * Compare the value reported with the beta value; if the reported value is smaller, reset beta to the new value
 - * Report beta
- Otherwise, the level is a maximizing level:

- Until all children are examined with ALPHA-BETA or until alpha is equal to or greater than beta:
 - * Use ALPHA-BETA procedure, with the current alpha and beta values, on a child; note the value reported
 - * Compare the value reported with the beta value; if the reported value is larger, reset alpha to the new value
 - * Report alpha

4.3 Heuristic Methods

4.3.1 Progressive deepening

For each depth level, compute the best move. By doing so, to compute up to level $d - 1$ it is necessary to evaluate:

$$b^0 + b^1 + \dots + b^{d-1} = \frac{b^d - 1}{b - 1} \approx b - 1$$

4.3.2 Forced move heuristic

The child move involving a *forced move* generally has a value that stands out from the rest.

4.3.3 Singular-extension heuristic

The search should continue as long as one move's static value stands out.

4.3.4 Tapered search

Arrange for the branching factor to vary with depth of penetration, possibly using tapered search to direct more effort into the more promising moves.

5 Symbolic constraints and propagation

Principle of convergent intelligence The world manifest constraints and regularities. If a computer is to exhibit intelligence, it must exploit those constraints and regularities, no matter of what the computer happens to be made.

Describe-to-explain principle The act of detailed description may turn probabilistic regularities into entirely deterministic constraints.

Marr's methodological principles

1. Identify the problem
2. Select or develop an appropriate representation
3. Expose constraints or regularities
4. Create particular procedures
5. Verify via experiments

Contraction net is a representation that is a *frame system*, in which:

- Lexically and structurally, certain frame classes identify a finite list of application-specific interpretations
- Procedurally, demon procedures enforce compatibility constraints among connected frames

5.1 Applications

5.1.1 3D object recognition in 2D images

Labeled drawing is a representation that is a *frame system*, in which:

- Lexically, there are line frames and junctions frames. Lines may be convex, concave, or boundary lines. Junctions may be *L*, *Fork*, *T*, or *Arrow* junctions
- Structurally, junctions frames are connected by line frames. Also, each junction frame contains a list of interpretation combinations for its connecting lines
- Semantically, line frames denote physical edges. Junction frames denote physical vertexes
- Procedurally, demon procedures enforce the constraint that each junction level must be compatible with at least one of the junction labels at each of the neighboring junctions

5.1.2 Time-interval relations and scheduling

Interval net is a representation that is a *contraction net*, in which:

- Lexically and semantically there are interval frames denoting time intervals and link frames denoting time relations: specifically: $\overrightarrow{\text{before}}$, $\overrightarrow{\text{during}}$, $\overrightarrow{\text{overlaps}}$, $\overrightarrow{\text{meets}}$, $\overrightarrow{\text{starts}}$, $\overrightarrow{\text{finishes}}$, $\overrightarrow{\text{isequalto}}$, and their mirrors
- Structurally, interval frames are connected by link frames
- Procedurally, demon procedures enforce the constraint that the interpretations allowed for a link frame between two intervals must be consistent with the interpretations allowed for the two link frames joining the two intervals to a third interval

5.2 Map coloring - Lecture notes

Definition 1 A *variable* v is something that can have an assignment

Definition 2 A *value* x is something that can be an assignment

Definition 3 A *domain* D is a bag of values

Definition 4 A *constraint* c is a limit on variable values

Procedure:

- For each *DFS* assignment
- For each variable v_i considered. A *considered variable* is some variable that may affect the assignment decision
- For each x_i in D_i
- For each constraint $c(x_i, x_j)$ where $x_j \in D_j$
- If $\nexists x_j$ such that $c(x_i, x_j)$ is satisfied, then remove x_i from D_i .
- If D_i is empty, backtrack

Examples of *considered variables*:

- Nothing: leads to wrong outputs
- Assignment: constraints are too slack
- Neighbors' only assignment: can succeed but depends on assignment ordering. If most constrained assignments are considered first, then it is fast; vice versa, might not end
- Propagate checking through variables with reduced domain reduced to a single value
- Propagate checking through variables with reduced domains
- Everything: too computationally intensive

6 K-D Trees and Nearest Neighbors

Consistency heuristic Whenever you want to guess a property of something, given nothing else to go on but a set of reference cases, find the most similar case, as measured by known properties, for which the property is known. Guess that the unknown property is the same as that known property.

Decision tree A decision tree is a representation, that is a semantic tree in which:

- Each node is connected to a set of possible answers
- Each nonleaf node is connected to a test that splits its set of possible answers into subsets corresponding to different test results
- Each branch carries a particular test result's subset to another node

k-d tree A k-d tree is a representation, that is a decision tree in which:

- The set of possible answers consists of points, one of which may be the nearest neighbor to a given point
- Each test specifies a coordinate, a threshold, and a neutral zone around the threshold containing no points
- Each test divides a set of points into two sets, according to on which side of the threshold each point lies

How to divide the cases into sets:

- If there is only one case, stop
- If this is the first division of cases, pick the vertical axis for comparison; otherwise, pick the axis that is different from the axis at the next higher level
- Considering only the axis of comparison, find the average position of the two middle objects. Call this average position the threshold, and construct a decision-tree test that compares unknowns in the axis of comparison againsts the threshold. Also note the position of the two middle objects in the axis of comparison. Call these positions the upper and lower boundaries
- Divide up all the objects into two subsets, according to on which side of the average position they lies
- Divide up the objects in each subset, forming a subtree for each, using this procedure

To find the nearest neighbor using the K-D procedure:

- Determine whether there is only one element in the set under consideration
 - If there is only one, report it
 - Otherwise, compare the unknown, in the axis of comparison, against the current node's threshold. The result determines the likely set
 - Find the nearest neighbor in the likely set using this procedure
 - Determine whether the distance to the nearest neighbor in the likely set is less than or equal to the distance to the other set's boundary in the axis of comparison:
 - * If it is, then report the nearest neighbor in the likely set
 - * If it is not, check the unlikely set using this procedure; return the nearer of the nearest neighbors in the likely set and in the unlikely set

7 Identification Trees

Identification Tree An identification tree is a representation, that is a decision tree in which:

- Each set of possible conclusion is established implicitly by a list of samples of known class

Average disorder $Average(disorder) = \sum_b \frac{n_b}{n_t} \sum_c -\frac{n_{bc}}{n_b} \log_2 \frac{n_{bc}}{n_b}$ where:

- n_b is the number of samples in branch b
- n_t is the total number of samples in all branches
- n_{bc} is the total of samples in branch b of class c

To generate an identification tree using SPROUTER:

- Until each leaf node is populated by as homogeneous a sample set as possible:
 - Select a leaf node with an inhomogeneous sample set
 - Replace that leaf node by a test node that divides the inhomogeneous sample set into minimally inhomogeneous subsets, according to some measure of disorder

To convert an identification tree into a rule set, execute the following procedure - PRUNER:

- Create one rule for each root-to-leaf path in the identification tree
- Simplify each rule by discarding antecedents that have no effect on the conclusion reached by the rule
- Replace those rules that share the most common consequent by a default rule that is triggered when no other rule is triggered. In the event of a tie, use some heuristic tie breaker to choose a default rule

8 Neural Nets

Neural Net A neural net is a representation, that is a arithmetic constraint net in which:

- Operation frames denote arithmetic constraints modeling synapses and neurons
- Demon procedures propagate stimuli through synapses and neurons

With demon procedures defines assignment:

- When a value is written into a synapse's input slot, write the product of the value and the synapse's weight into the synapse's output slot
- When a value is written into a synapse's output slot, check the following neuron to see whether all its input synapses' outputs have values:
 - If they do, add the outputs values of the input synapses together, pass the sum through the activation function, and write the appropriate value into the neuron's output slot
 - If they don't, do nothing

To do back propagation to train a neural net:

- Pick a rate parameter r
- Until performance is satisfactory,
 - For each input,
 - * Compute the resulting output
 - * Compute β for nodes in the output layers using $\beta_x = d_x - o_x$
 - * Compute β for all other nodes using: $\beta_j = \sum_k w_{jk} o_k (1 - o_k) \beta_k$
 - * Compute weight changes for all weight using: $\Delta w_{ij} = r o_i o_k (1 - o_k) \beta_j$
 - Add up the weight changes to all sample inputs and changes the weights

8.1 Back-propagation characteristics

- Training may require thousands of back propagations
- Back-propagation can be done in stages
- Back-propagation can train a net to learn to recognize multiple concepts simultaneously
- Trained neural nets can make predictions

- Excess weights lead to overfitting. Rule of thumb: be sure that the number of trainable weights influencing any particular output is smaller than the number of training samples
- Neural-net training is an article

8.2 Perceptrons

Perceptron A perceptron is a representation, that is a neural net in which:

- There is only one neuron
- The input sare binary
- Logic boxes may be interposed between the perceptron's inputs and the perceptron's weights. Each logic box can be viewed as a table that produces an output value of 0s or 1 for each combination os 0s and 1s that can appear at its inputs
- The output of the perceptron is 0s or 1 depending on whether the weighted sum of the logic-box outputs is greater than the threshold.

The perceptron convergence procedure guarantees success whenever success is possible.

To train a perceptron:

- Until the perceptron yields the correct result for each training sample, for each sample,
 - If the perceptron yields the wrong answer,
 - * If the perceptron says no when it should say yes, add the logic-box output vector to the weight vector
 - * Otherwise, subtract the logic-box output vector from the weight vector
 - Otherwise, do nothing

9 Genetic Algorithms

Chromosome A chromosome is a representation in which:

- There is a list of elements called genes
- The chromosome determines the overall fitness manifested by some mechanism that uses the chromosome's genes as a sort of blueprint

With constructors that:

- Create a chromosome, given a list of elements; this constructor might be called the *genesis constructor*
- Create a chromosome by crossing a pair of existing chromosomes

With writers that:

- Mutate an existing chromosome by changing one of the genes

With readers that:

- Produce a specified gene, given a chromosome

9.1 Fitness

The **standard method** for fitness computation is: $f_i = \frac{q_i}{\sum_j q_j}$

The **rank method** for fitness computation is:

- Sort the n individuals by quality
- Let the probability of selecting the i th candidate, given that the first $i - 1$ candidates have not been selected, be p , except for the final candidate, which is selected if no previous candidate has been selected
- Select a candidate using the computed probabilities

The **rank-space method** for fitness computation is:

- Sort the n individuals by quality
- Sort the n individuals by the sum of their inverse squared distances to already selected candidates (the lower the sum, the better the rank)
- Use the rank method, but sort on the sum of the quality rank and the diversity rank, rather than on quality rank only

9.2 Crossover

- Crossover enables to search high-dimensional spaces efficiently; it reduces the dimensionality of the optimum search space
- Crossover enables genetic algorithms to traverse obstructing moats

9.3 Natural selection

To mimic natural selection in general:

- Create an initial population of one chromosome
- Mutate one or more genes in one or more of the current chromosomes, producing one new offspring for each chromosome mutated
- Mate one or more pairs of chromosomes
- Add the mutated na doffspring chromosomes to the current population
- Create a new generation by keeping the best of the current population's chromosomes, along with other chromosomes selected randomly from the current population. Bias the random selection according to assessed fitness

10 Near Misses and Felicity Conditions

Near miss A *near miss* is a negative example that, for a small number of reasons, is not an instance of the class being taught.

10.1 Heuristics list

- The **require-link** heuristic is used when an evolving model has a link in a place where a near miss does not. The model link is converted to a *Must* form
- The **forbid-link** heuristic is used when a near miss has a link in a place where an evolving model does not. A *Must-not* form is installed in the evolving model
- The **climb-tree** heuristic is used when an object in an evolving model corresponds to a different object in an example. *Must-be-a* links are routed to the most specific common class in the classification tree above the model object and the example object
- The **enlarge-set** heuristic is used when an object in an evolving model corresponds to a different object in an example and the two objects are not related to each other through a classification tree. *Must-be-a* links are routed to a new class composed of the union of the objects' classes
- The **drop-link** heuristic is used when the objects that are different in an evolving model and in an example form an exhaustive set. The drop-link heuristic is also used when an evolving model has a link that is not in the example. The link is dropped from the model
- The **close-interval** heuristic is used when a number or interval in an evolving model corresponds to a number in an example. If the model uses a number, the number is replaced by an interval spanning the model's number and the example's number. If the model uses an interval, the interval is enlarged to read the example's number

10.2 Procedures

SPECIALIZE

- Match the evolving model to the example to establish correspondences among parts
- Determine whether there is a single, most important difference between the evolving model and the near miss
 - If there is a single, most important difference,
 - * If the evolving model has a link that is not in the near miss, use the require-link heuristic

- * If the near miss has a link that is not in the model, use the forbid-link heuristic
- Otherwise, ignore the example

GENERALIZE

- Match the evolving model to the example to establish correspondences among parts
- For each difference, determine the difference type:
 - If a link points to a class in the evolving model different from the class to which the link points in the example:
 - * If the classes are part of a class tree, use the climb-tree heuristic
 - * If the classes form an exhaustive set, use the drop-link heuristic
 - * Otherwise, use the enlarge-set heuristic
 - If a link is missing in the example, use the drop-link heuristic
 - If the difference is that different numbers, or an interval and a number out the interval, are involved, use the close-interval heuristic
 - Otherwise, ignore the difference

Notes:

- Specialize does nothing if it cannot identify a most important difference. You need a procedure that ranks all differences by difference type and by link type
- Both SPECIALIZE and GENERALIZE involve matching. You need a matching procedure

10.3 Learning

To learn using procedure W:

- Let the description of the first example, which must be an example, be the initial description
- For all subsection examples:
 - If the example is a near miss, use SPECIALIZE
 - If the example is an example, use GENERALIZE

Wait-and-see principle If there is doubt about what to do, do nothing.

Learning-facilitating teacher-student agreements are called **felicity conditions**.

No-altering principle When an object or situation known to be an example fails to match a general model, create a special-case exception model.

10.4 Identification

Similarity net A similarity net is a representation that is a semantic net in which:

- Nodes denotes models
- Links connect similar models
- Links are tied to difference descriptions

11 Support vector machines

Key idea:

- find the widest separating strip between the points belonging to the two classes.
- Find a vector w that is perpendicular to the median line to the strip.
- For any unknown u , classify as positive example if $f(u) = w \cdot u + b \geq 0$

11.1 Derivation of result

Assume the following constraints:

$$c_i = \begin{cases} f(x_+) = w \cdot x_+ + b \geq 1 & \text{for } x_+ \\ f(x_-) = w \cdot x_- + b \leq -1 & \text{for } x_- \end{cases}$$

The constraint can be generalized introducing:

$$y_i = \begin{cases} 1 & x_+ \\ -1 & x_- \end{cases}$$

So to obtain: $c_i : y_i(w \cdot x_i + b) - 1 \geq 0$

Also, assume that the constraint is satisfied with equality for observations at the edge of the strip.

Finding the width of the strip For any two points at the edge of the strip, one negative and one positive observation, it holds that:

$$\begin{aligned} \text{width} &= (x_+ - x_-) \cdot \frac{w}{\|w\|} \\ &= (1 - b) + (1 + b) \\ &= \frac{2}{\|w\|} \end{aligned} \tag{1}$$

Objective Maximize the width of the strip, reformulated as:

$$\text{objective} = \frac{1}{2} \|w\|^2 \tag{2}$$

11.2 Constrained optimization

The problem can be formulated as:

$$L(w) = \frac{1}{2} \|w\|^2 - \sum_i \alpha_i [y_i(w \cdot x_i + b) - 1] \tag{3}$$

$$\begin{cases} \frac{\Delta L}{\Delta w} : w - \sum \alpha_i y_i x_i = 0 \\ \frac{\Delta L}{\Delta b} : \sum_i \alpha_i y_i = 0 \end{cases} \tag{4}$$

Plugging the first f.o.c. into the Lagrangean, we obtain:

$$L = \sum_i a_i - \frac{1}{2} \sum_i \sum_j a_i a_j y_i y_j (x_i \cdot x_j) \quad (5)$$

Then check the sign of:

$$f(u) = \left(\sum a_i y_i (x_i u) \right) + b \quad (6)$$

Thus:

- w is a linear sum of the samples.
- Learning (w) depends only on the the dot product of observation pairs
- Classification depends only on the the dot product of observations with the unkown
- Exclusive reliance on dot products enables approach to problems in which samples cannot be separated by a straight line

When the classes are not linearly separable, handle non-linearities by applying a transformation Φ - the **kernel** to the dot products of the samples, so to solve the problem in a higher dimensional space.

Common kernels are:

- Polynomial: $(u \cdot v + 1)^n$
- Radial basis: $e^{-\frac{u-v}{\sigma}}$

12 Representations: classes, trajectories, transitions