Place title of Group Project Here

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Abstract

Place Abstract Here.

1 Background/Movitation

Put your background motivation stuff here. You might reference something material here like [1].

2 Modeling

Beginning with the equation for traffic flow $u_t + V_\infty \left(1 - \frac{2u}{u_\infty}\right) u_x = 0$, we will try to find an explicit solution using the method of characteristics.

Suppose the solution u is some curve parameterized by s as $u(x(s), t(s))$. Taking the derivative of u with respect to s gives

$$u'(x(s), t(s)) = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial s} = u_x \frac{\partial x}{\partial s} + u_t \frac{\partial t}{\partial s}$$

and matching terms with the original differential equation gives us that

$$\begin{aligned} \frac{\partial x}{\partial s} &= V_\infty \left(1 - \frac{2}{u_\infty} u\right) \\ \frac{\partial t}{\partial s} &= 1 \\ \frac{\partial u}{\partial s} &= 0. \end{aligned}$$

Using the fact that $\frac{\partial t}{\partial s} = 1$, we have that

$$\begin{aligned} \frac{\partial x}{\partial t} &= V_\infty (1 - \frac{2}{u_\infty}) \\ x(t) - x(0) &= \int_0^t V_\infty (1 - \frac{2}{u_\infty} u) dt \\ x &= V_\infty (1 - \frac{2}{u_\infty} u) t + x(0) \\ t &= \frac{1}{V_\infty (1 - \frac{2}{u_\infty} u)} x - x(0) \end{aligned}$$

where to evaluate the integral, we used the fact that $\frac{\partial u}{\partial s}$ is 0, and so the characteristics are constant along the plane. This yields a few cases for the characterstic curves depending on what value of u the curve takes. Based on the physical properties of our model, we have that $u \in [0, u_\infty]$. The slope of the characteristics changed depending on the value of u, specifically the slope $m => 0$ when $u \in [0, \frac{u_\infty}{2})$, the slope is vertical or undefined when $u = \frac{u_\infty}{2}$, and $m < 0$ when $u \in (\frac{u_\infty}{2}, u_\infty]$.

3 Results

4 Analysis/Conclusions

References

[1] First reference goes here