Place title of Group Project Here

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Abstract

Place Abstract Here.

1 Background/Movitation

Put your background motivation stuff here. You might reference something material here like [1].

2 Modeling PDE for Traffic Flow

To begin our first foray into modelling traffic flow, we will pull from Volume 4 and work with the given equation for traffic flow $u_t + V_\infty \left(1 - \frac{2u}{u_\infty}\right) u_x = 0$. We want to consider three cases:

1. What happens to the left of a light after a red light turns green?
2. What happens before a light when a light turns from green to red?
3. What happens both before and after a light when it turns from red to green?

2.1 Method of Characteristics

To begin, we will try to find an explicit solution using the method of characteristics.

Suppose the solution u is some curve parameterized by s as $u(x(s), t(s))$. Taking the derivative of u with respect to s gives

$$u'(x(s), t(s)) = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial s} = u_x \frac{\partial x}{\partial s} + u_t \frac{\partial t}{\partial s}$$

and matching terms with the original differential equation gives us that

$$\frac{\partial x}{\partial s} = V_\infty \left(1 - \frac{2}{u_\infty} u\right)$$
$$\frac{\partial t}{\partial s} = 1$$
$$\frac{\partial u}{\partial s} = 0.$$

Using the fact that $\frac{\partial t}{\partial s} = 1$, we have that

$$\begin{aligned}
\frac{\partial x}{\partial t} &= V_\infty(1 - \frac{2}{u_\infty}) \\
x(t) - x(0) &= \int_0^t V_\infty(1 - \frac{2}{u_\infty}u)dt \\
x &= V_\infty(1 - \frac{2}{u_\infty}u)t + x(0) \\
t &= \frac{1}{V_\infty(1 - \frac{2}{u_\infty}u)}x - x(0)
\end{aligned}$$

where to evaluate the integral, we used the fact that $\frac{\partial u}{\partial s}$ is 0, and so the characteristics are constant along the plane. This yields a few cases for the characterstic curves depending on what value of u the curve takes. Based on the physical properties of our model, we have that $u \in [0, u_\infty]$. The slope of the characteristics changed depending on the value of u, specifically the slope $m => 0$ when $u \in [0, \frac{u_\infty}{2})$, the slope is vertical or undefined when $u = \frac{u_\infty}{2}$, and $m < 0$ when $u \in (\frac{u_\infty}{2}, u_\infty]$.

Next, suppose that there are some values x_0, x_1 from which we draw characteristic curves, and suppose $u(x_0) < u(x_1)$. This will result in shocks, (assuming the shocks happen inside the domain of relevance). See Figures 1, 2, and 3 for examples.

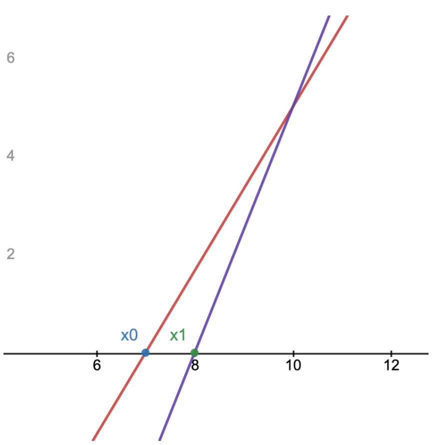

Figure 1: $u(x_0) < u(x_1), u(x_0), u(x_1) \in (0, \frac{u_\infty}{2})$

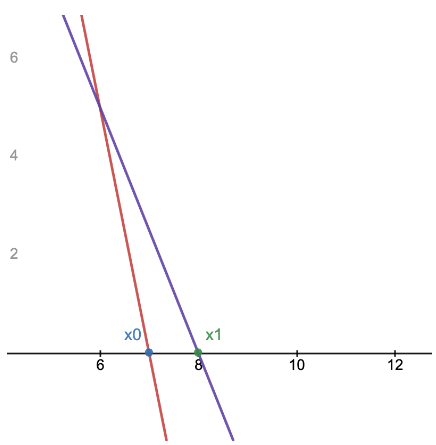

Figure 2: $u(x_0) < u(x_1), u(x_0), u(x_1) \in (\frac{u_\infty}{2}, u_\infty)$

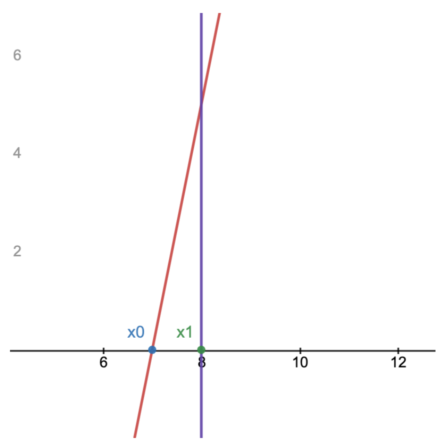

Figure 3: $u(x_0) < u(x_1), u(x_0) \in (\frac{u_\infty}{2}, u_\infty), u(x_1) = \frac{u_\infty}{2}$

Conversely, if the density of the initial condition is decreasing (i.e. when $x_0 < x_1 \implies u(x_0) < u(x_1)$), there are no shocks. See Figure

Write up numerical scheme with boundary conditions Try to figure out why sheme is unstable Apply to scenarios Note instability Try extending domain horizontally with an unrealistic rhs boundary condition OR have the cars come to red light (neumann) Figure out code for neumann b.c.

try case 2 (will need to knock out diffusion term)

combine simulations for case 3, TEEEHEHEEEEEEEEEEh

Get this on overleaf Get riley and Laren's stuff on here

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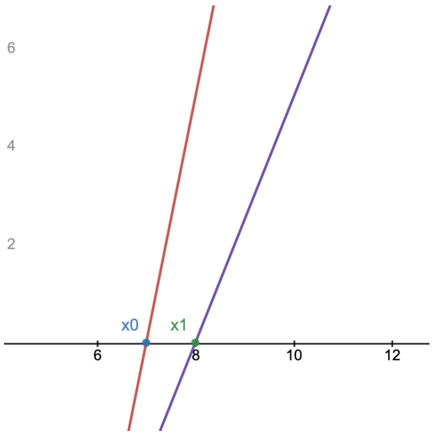

Figure 4: $u(x_0) > u(x_1), u(x_0), u(x_1) \in (0, \frac{u_\infty}{2})$

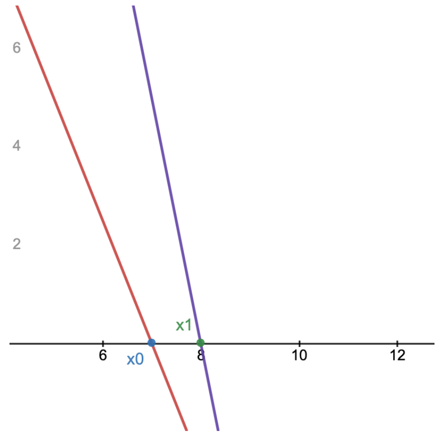

Figure 5: $u(x_0) > u(x_1), u(x_0), u(x_1) \in (\frac{u_\infty}{2}, u_\infty)$

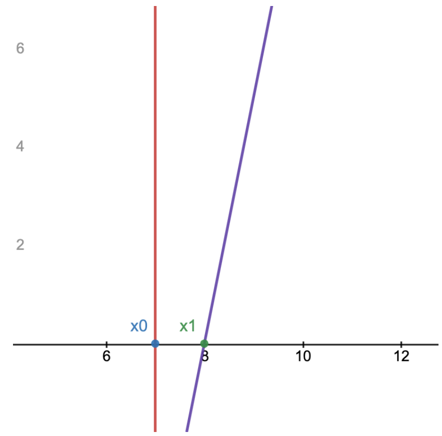

Figure 6: $u(x_0) < u(x_1), u(x_0) = \frac{u_\infty}{2}, u(x_0) \in (\frac{u_\infty}{2}, u_\infty)$

3 Results/Analysis

4 Modeling using SIR Model Type ODEs

5 Results/Analysis

6 Conclusions

References

[1] First reference goes here